Proof by Induction

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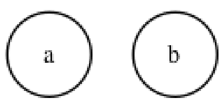
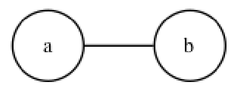
Theorem 1 *Show that every graph with two of more nodes contains two nodes that have equal degrees.*

Basis: A graph G with two nodes must be one of the two followings: In both cases, we



case A: graph G



case B: graph G

see that a and b have the same degree. Therefore, the statement holds.

Inductive Hypothesis: Let's assume that the statement holds for any graph $G = (V, E)$ with vertices $|V| = k \geq 2$.

Induction Step: Let's assume, we have a graph $G' = (V', E')$ with $|V'| = k + 1$ many vertices. If we assume that no two vertices share the same degree then the degree of any vertex $w \in V'$ will range from 0 to k.

However not all the vertices with different degrees can occur in the same graph because a vertex with degree 0 cannot co-exist with a vertex with degree k. So, there are vertices with degree 1 to k. So, G' can exhibit at most k values among it's $k+1$ vertices. According to pigeon hole principle, there is at least two vertices that have equal degrees.

Hence, every graph with two of more nodes contains two nodes that have equal degrees.