Normal Subgroups and Factor Groups

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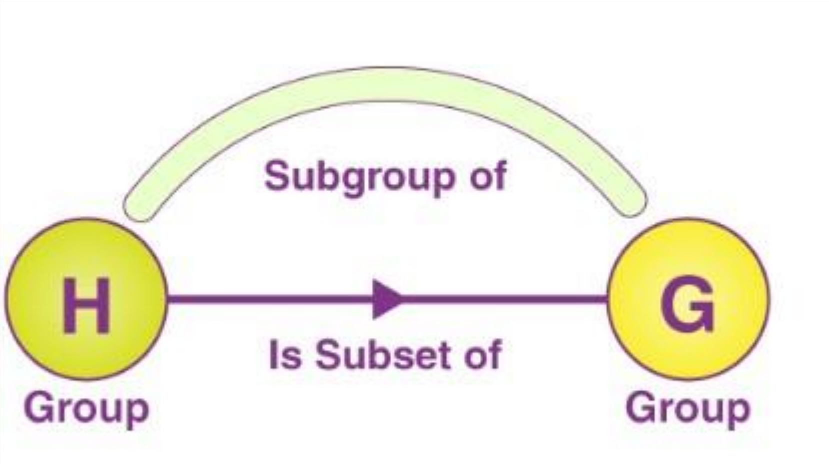
Contents

Introduction to Group

Groups are sets with a binary operation that satisfies closure, associativity, identity, and inverses. They are essential for studying symmetry and algebraic structures.



Subgroup

Contains the Identity: The identity element of G must be in H.

Closure: If a,b∈H, then $a*b$ (the group operation) must also be in H.

Inverses: For every element a∈H, its inverse a^−1 must also be in H.

Non-empty: A subgroup cannot be empty, and it must satisfy the above conditions under the same operation as G.

Normal Subgroup

- A normal subgroup N of G is a subgroup that satisfies the condition:

$$gN = Ng \text{ for all } g \in G.$$

- Notation: $N \triangleleft G$.
- This property allows the formation of factor groups.
- Example: In (Z,+), 2Z is a normal subgroup.

Cosets and Their Role in Normal Subgroups

01

Left Coset: $gN = \{gn \mid n \in N\}$

02

Right Coset: $Ng = \{ng \mid n \in N\}$

03

If N is normal, $gN=Ng$, meaning the left cosets are equal to the right cosets.

04

Cosets form a partition of the group G.

Key Differences Between Subgroups and Normal Subgroups

Subgroups

- Cosets Not always equal.
- Factor Groups Not applicable.
- Conjugacy Not required.

Normal Subgroups

- Cosets Always equal
- Factor Groups Can form factor groups
- Conjugacy Required (closed under conjugacy)

Factor Group

- A factor group (or quotient group) G/N is the set of cosets of a normal subgroup N in G.

- The operation on cosets is defined as:

$$(g1N)(g2N)=(g1g2)N.$$

Example of Factor Groups

- Let $G=Z$ (integers) and $N=3Z$(multiples of 3).

- The cosets of N in G are:

$[0]=\{\ldots,-6,-3,0,3,6,\ldots\},[1]=\{\ldots,-5,-2,1,4,7,\ldots\},[2]=\{\ldots,-4,-1,2,5,8,\ldots\}$

- The factor group is $Z/3Z=\{[0],[1],[2]\}$.

Applications of Normal Subgroups and Factor Groups

01

Symmetry groups in geometry and physics:

Understanding symmetries often involves identifying normal subgroups.

02

Cryptography:

In RSA encryption, normal subgroups and factor groups are used in constructing secure cryptographic systems.

03

Finite groups:

Simplifying the structure of finite groups by examining their normal subgroups.

Conclusion

- Normal subgroups are essential for understanding the structure of groups and for simplifying groups into manageable factor groups.
- Factor groups provide a powerful tool for studying group properties through coset formation.
- These concepts are foundational in algebra, cryptography, and the study of symmetries in mathematics.

References

Books:

Dummit, D. S., & Foote, R. M. (2004). Abstract Algebra. Wiley.

Herstein, I. N. (1996). Topics in Algebra. Wiley.

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Online Resources:

Wikipedia: Normal Subgroup

MathWorld: Factor Group

Thank You