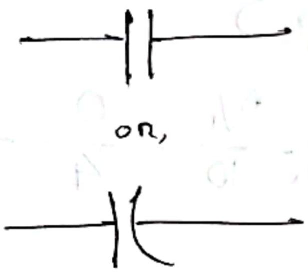
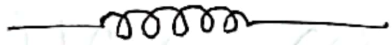
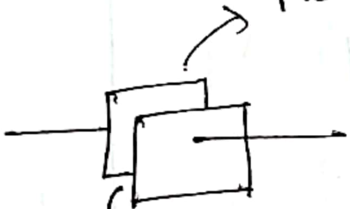
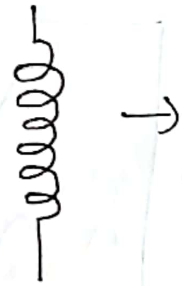
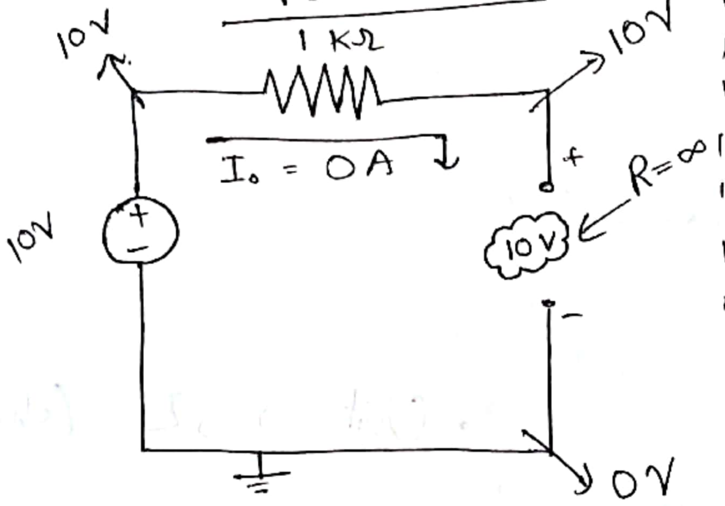
Week - 8

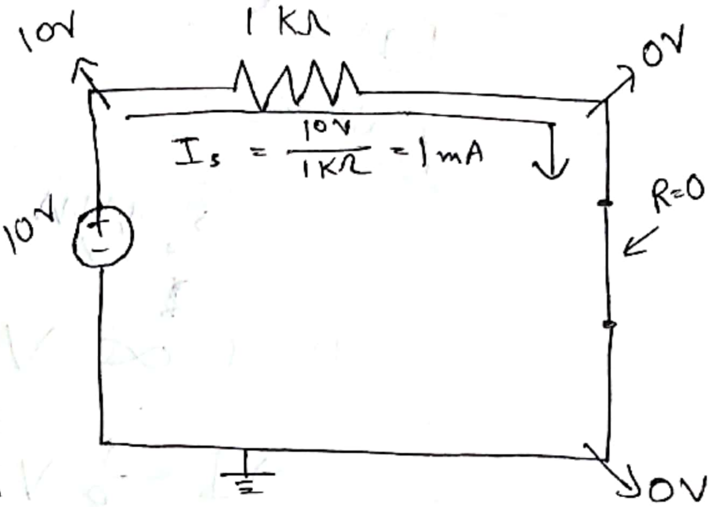
Transient Circuit Analysis

	Capacitors	Inductors
Symbol →		
Physical Structure →	Metal Sheets  Separated by air or some other non-conducting material	 → Simply a coil of wire
Basic Property →	Stores Charges when voltage is applied across. ⇓ Electrical Energy	Can store magnetic energy according to Faraday's law.

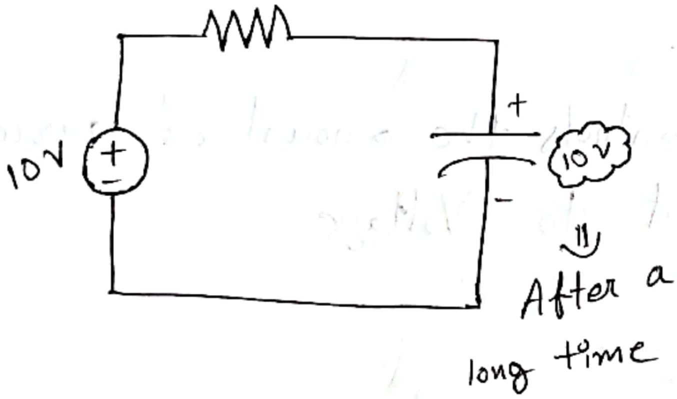
Open Circuit



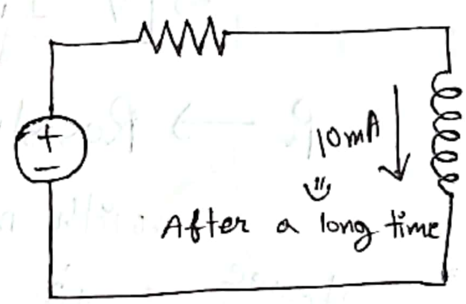
Short Circuit



⇓



⇓

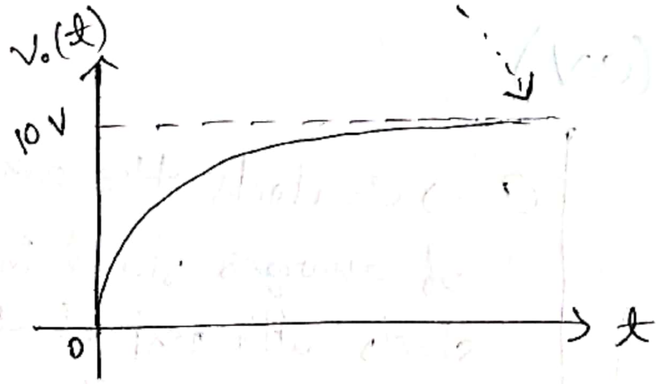


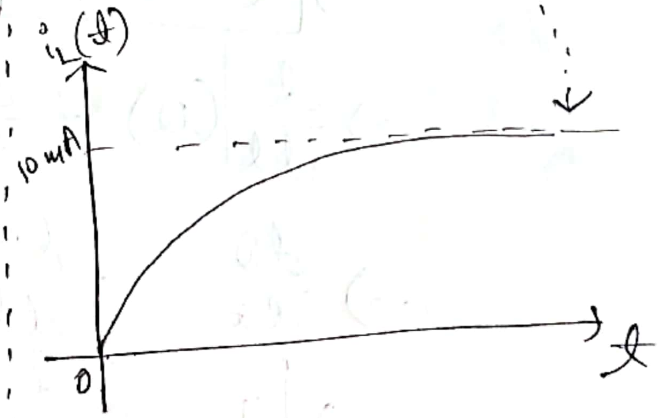
⇓

Eventually acts like a Open Circuit

⇓

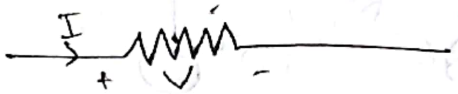
Eventually acts like a Short circuit.





Component Equations

Resistance



$$I \propto V$$

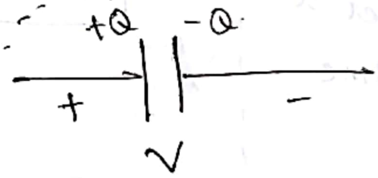
$$\Rightarrow I = \frac{1}{R} V$$

$$\Rightarrow \boxed{V = IR}$$

• Unit → Ω (ohm).

R → Resists / Controls the amount of current with respect to Voltage.

Capacitance



$$Q \propto V$$

$$\Rightarrow \boxed{Q = CV}$$

• Unit → F (Faraday)

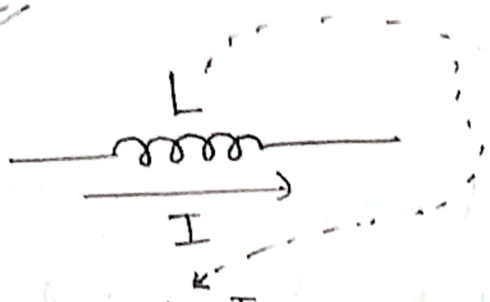
$$\Rightarrow \frac{d}{dt}(Q) = \frac{d}{dt}(CV)$$

$$\Rightarrow \frac{dQ}{dt} = C\frac{dV}{dt}$$

$$\Rightarrow \boxed{I = C\frac{dV}{dt}}$$

C → Controls the amounts of charges stored in metal sheets with respect to 'Change of Voltage'.

Inductance



Magnetic Flux ← $\varphi = LI$

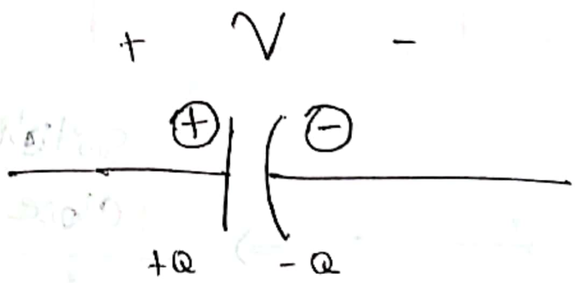
• Unit → H (Henry)

$$\Rightarrow \frac{d\varphi}{dt} = L\frac{dI}{dt}$$

$$\Rightarrow \boxed{V = L\,dI/dt}$$

L → Controls the amount of flux induced with respect to 'Change in Current'.

Why Change for a small time initially?



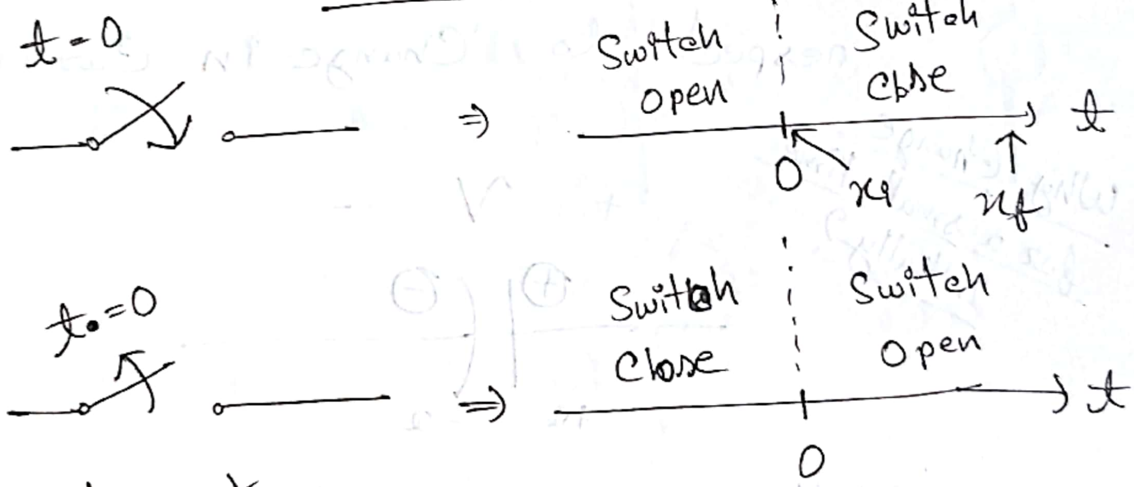
⇒ Valtage applied → Changes start to flow towands the sheets (Creating Current) → A certain amount of change stoned in the sheet → Voltage Diffenence created, Current flow stopped (almost) → Steady State.

⊡ Similan in Inductons.

Transient Analysis

- A function of time.
- There is an initial value and a final (steady state) value.
- After the initial changes, the value doesn't change with time too much anymore.

Switch



Basic Formula for Exponential Transient Analysis

$$\boxed{x(t) = x_f + (x_i - x_f)\,e^{-t/\tau}}$$

- $x(t) \rightarrow$ Voltage / Current etc.
- $x_i \longrightarrow$ Initial value
- $x_f \rightarrow$ final value
- $\tau \longrightarrow$ Time constant (sec).

Time Constant

τ → Dictates the rise time etc of the graph.

Capaciton

$$\boxed{\tau = RC}$$

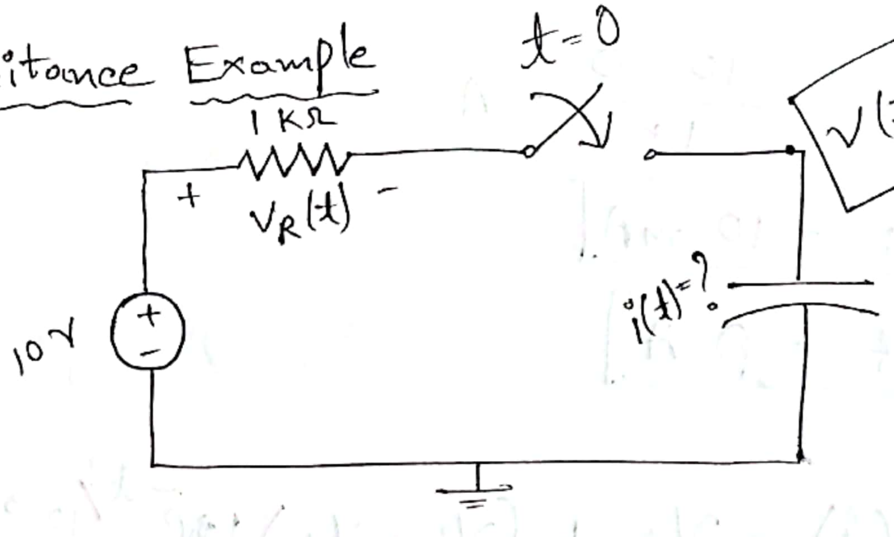
• Unit → sec

Inducton

$$\boxed{\tau = \frac{L}{R}}$$

• Unit → sec.

Capacitance Example



$\boxed{v(t) = ?}$

From Basic formula,

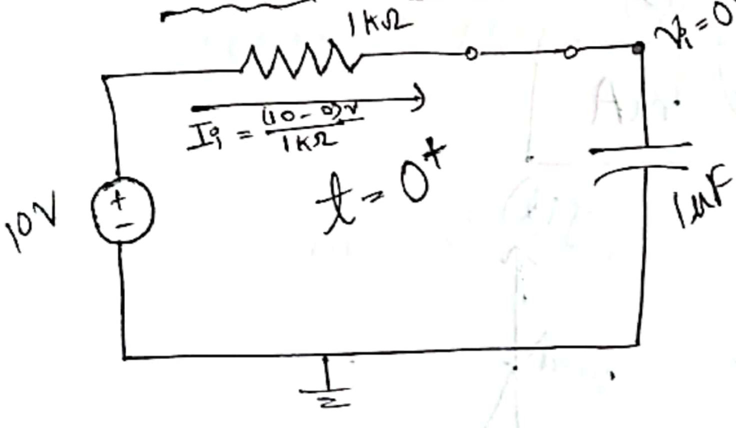
$$\boxed{v(t) = V_f + (V_i - V_f)\, e^{-t/\tau}}$$

$\boxed{\tau = RC}$ | for Capacitor |

$= 1\,K\Omega \times 1\,\mu F$

$= 10^{-3}$ s

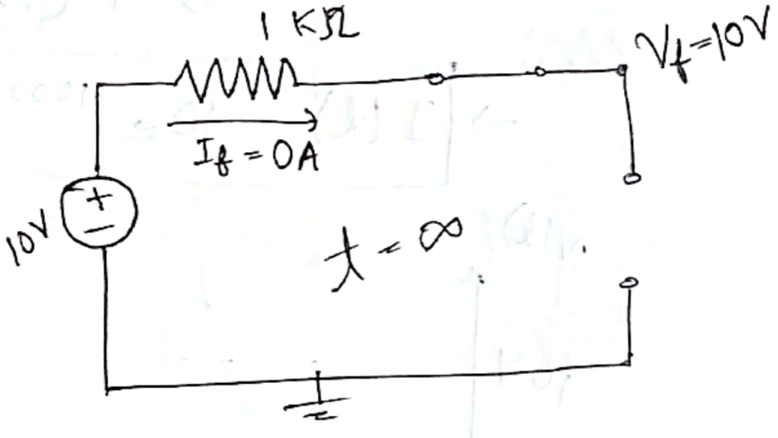
Initial Circuit (V_i)



$\boxed{V_i = 0V}$

Tip: Think about the graph & V before t=0

Final Circuit (V_f)



$\boxed{V_f = 10\ V}$

$$I_i = \frac{10-0}{1K} A$$

$$\therefore \boxed{I_i = 10 \text{ mA}}$$

$$\boxed{I_f = 0 \text{ A}}$$

$$\therefore V(t) = V_f + (V_i - V_f) e^{-t/10^{-3}} \quad |\tau = 10^{-3}|$$

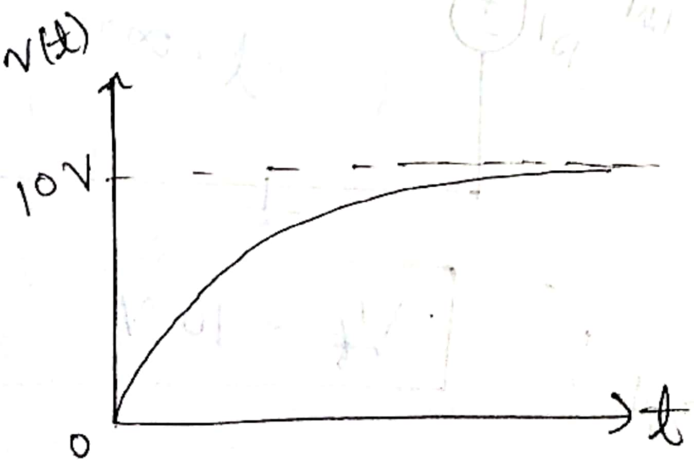
$$= 10 + (0 - 10) e^{-1000t} \text{ V}$$

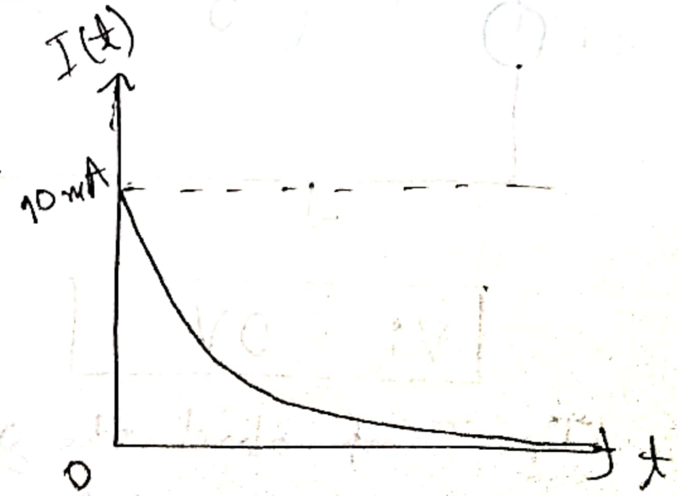
$$\Rightarrow \boxed{V(t) = 10\left(1 - e^{-1000t}\right) \text{V}}$$

$$I(t) = I_f + (I_i - I_f) e^{-t/10^{-3}} \quad |\tau = 10^{-3}|$$

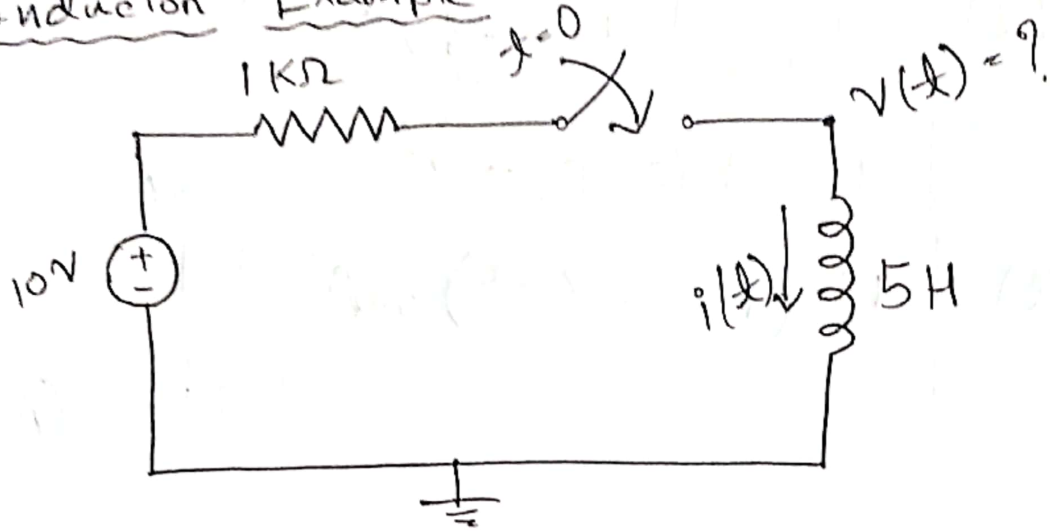
$$= 0 + (10 - 0) e^{-t/10^{-3}}$$

$$\Rightarrow \boxed{I(t) = 10 e^{-1000t} \text{ mA}}$$





Inductor Example

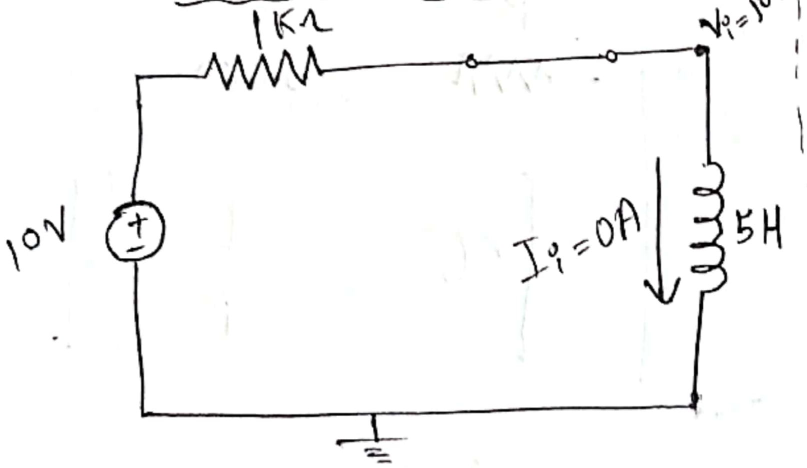


$$\boxed{\tau = \frac{L}{R}}$$ |for inductor|

$$= \frac{5}{1K}$$

$$= 0.005 \text{ s}$$

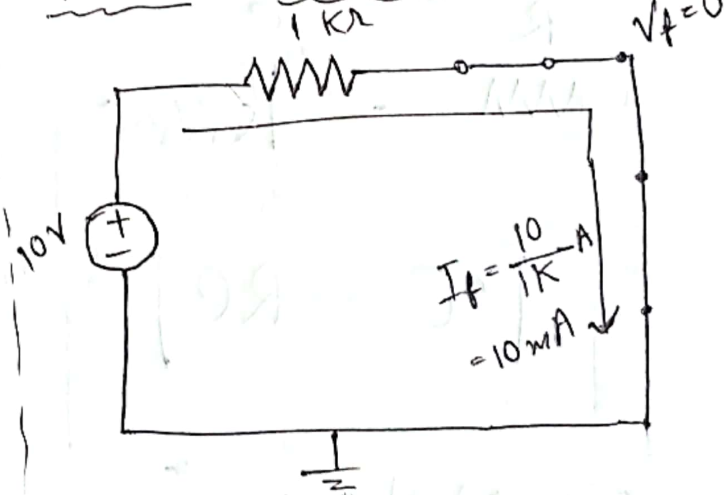
Initial Circuit (I_i)



$$\boxed{I_i = 0 A}$$

Tip: Same as before.

Final Circuit (I_f)



$$\boxed{I_f = 10 \text{ mA}}$$

$$I(t) = I_f + (I_i - I_f)e^{-t/\tau}$$

$$= 10 + (0 - 10)e^{-t/0.005} \quad mA$$

$$\Rightarrow \boxed{I(t) = 10\left(1 - e^{-200t}\right) \, mA}$$

(Ans.)