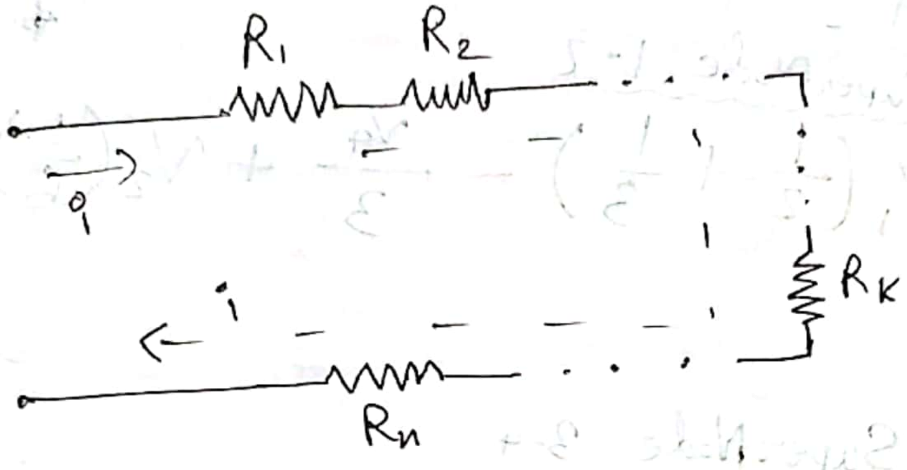
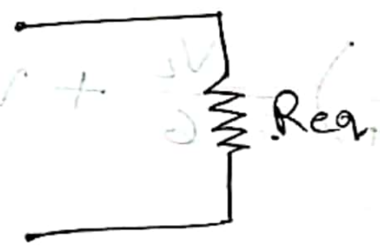
Lecture - 9: Circuit Simplification

Series - Parallel Review

Series

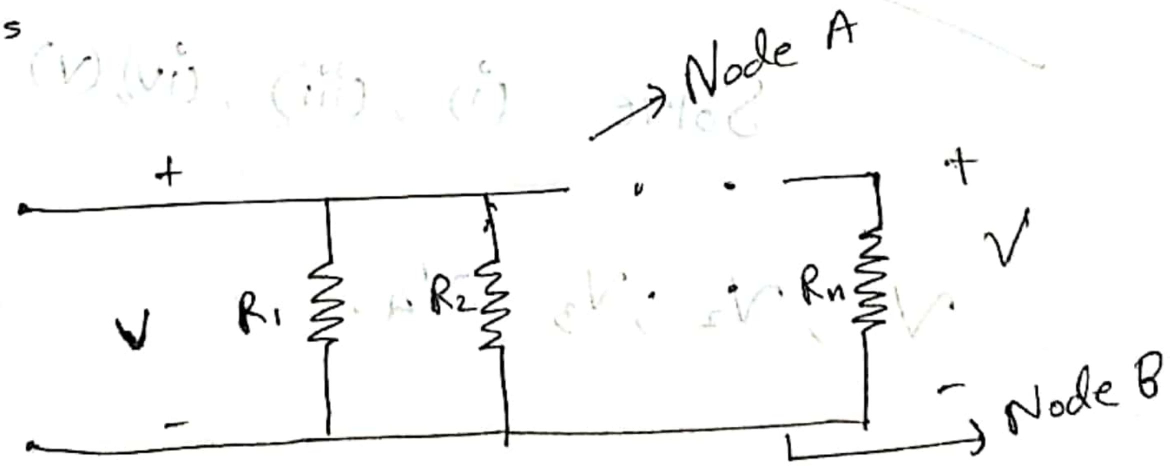


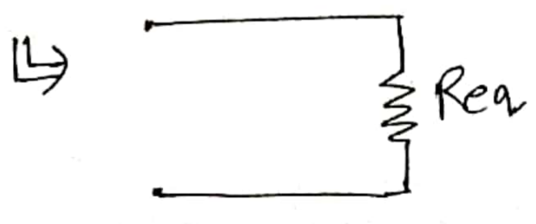


$$R_{eq} = R_1 + R_2 + \dots + R_K + \dots + R_n$$

R_s

Parallel



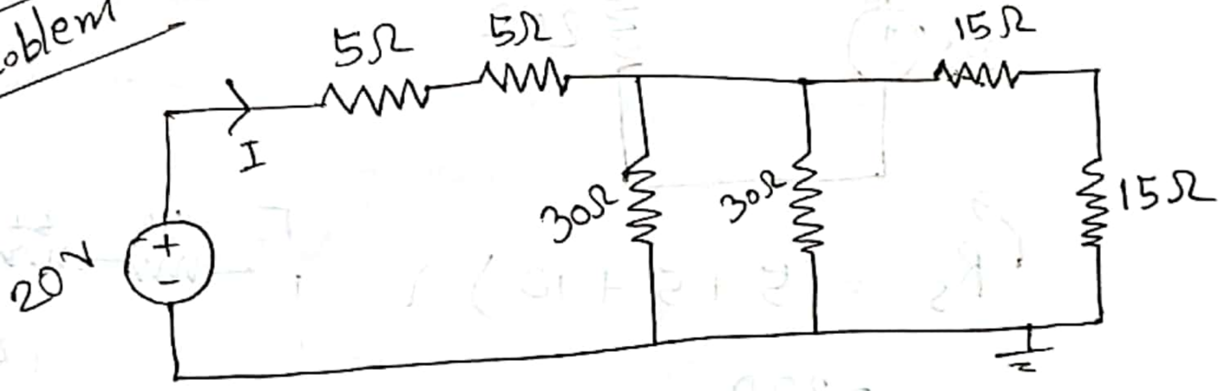


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$\Downarrow$

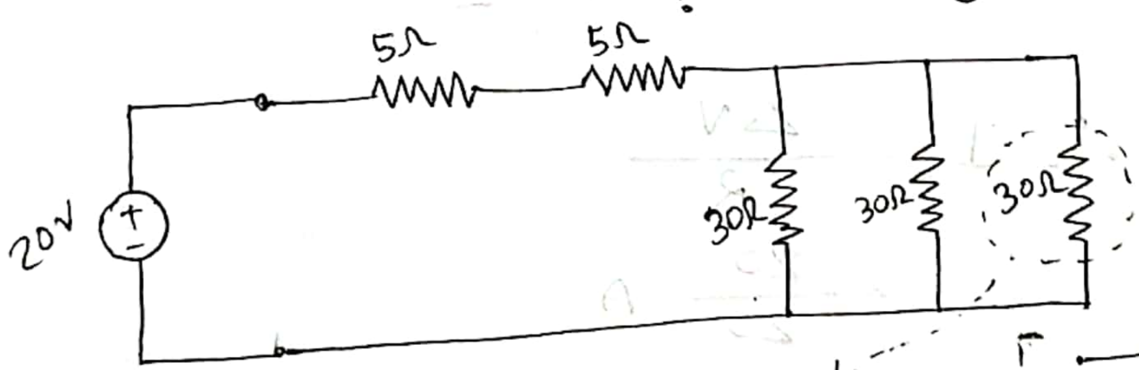
$R_{eq} \Leftrightarrow R_p$

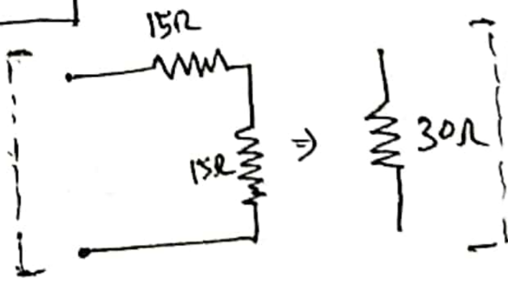
Example Problem



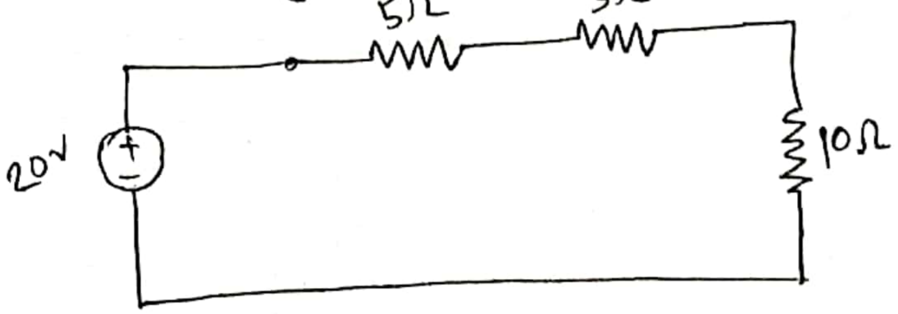
$I = ?$

$\Downarrow$





$R_3 = 15\Omega + 15\Omega = 30\Omega$



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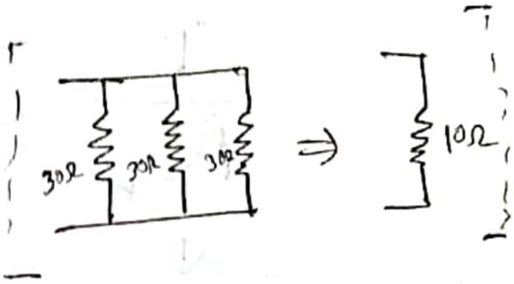
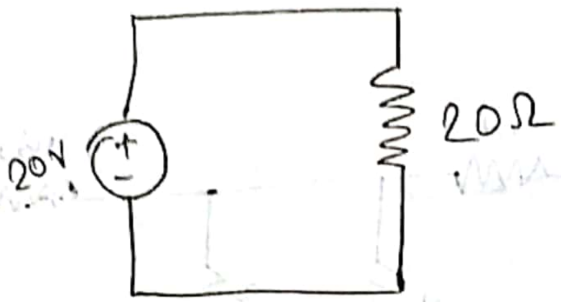
$$\frac{1}{R_p} = \frac{1}{30} + \frac{1}{30} + \frac{1}{30}$$

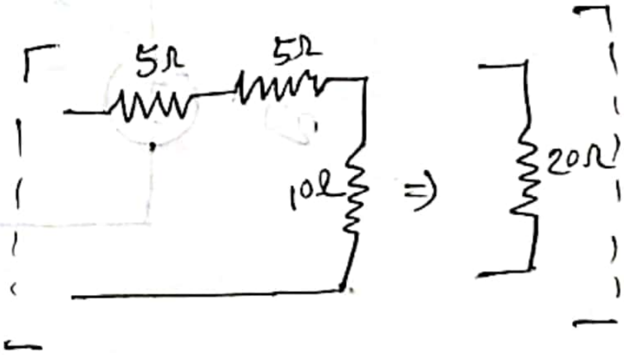


$\rightarrow R_p = 10\,\Omega$



$R_s = (5 + 5 + 10)\,\Omega$

$= 20\,\Omega$



$$\therefore I = \frac{\Delta V}{R}$$

$$= \frac{20}{20}\ A$$

$$= \boxed{1A}$$

(Ans.)

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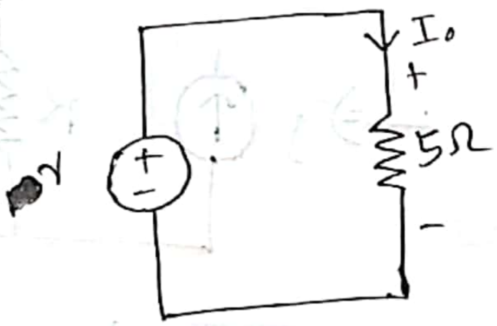
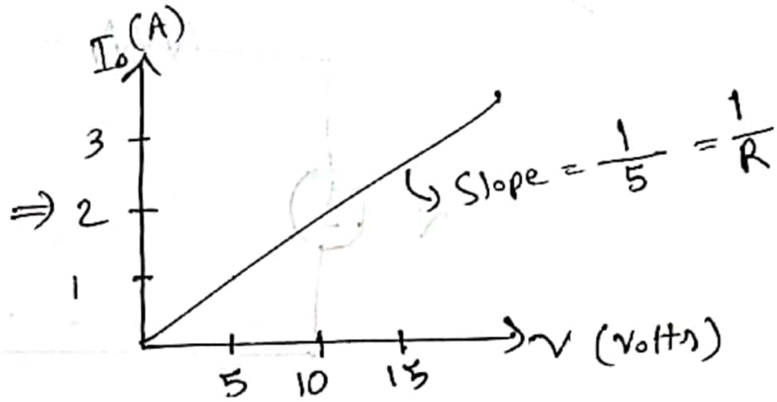
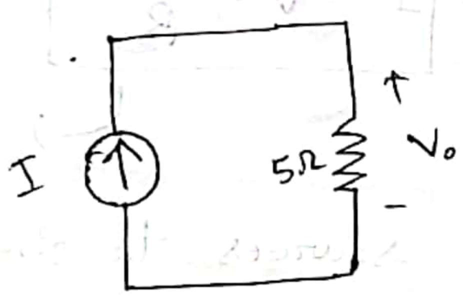
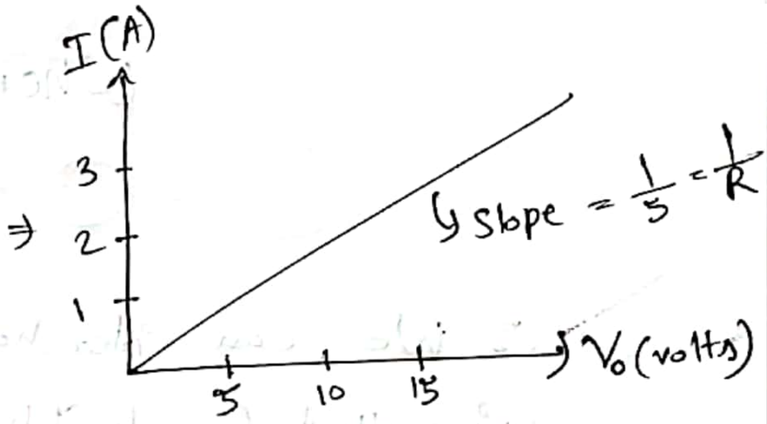
Source Transformation

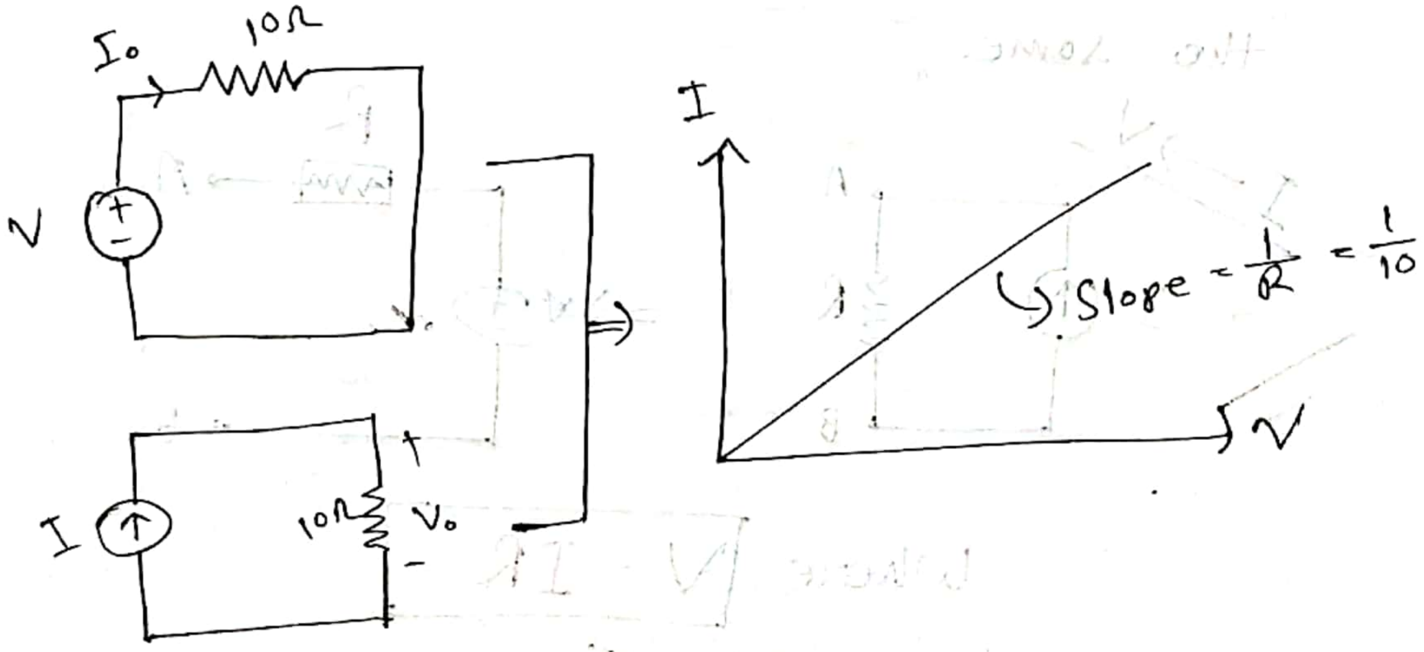
I-V Characteristics



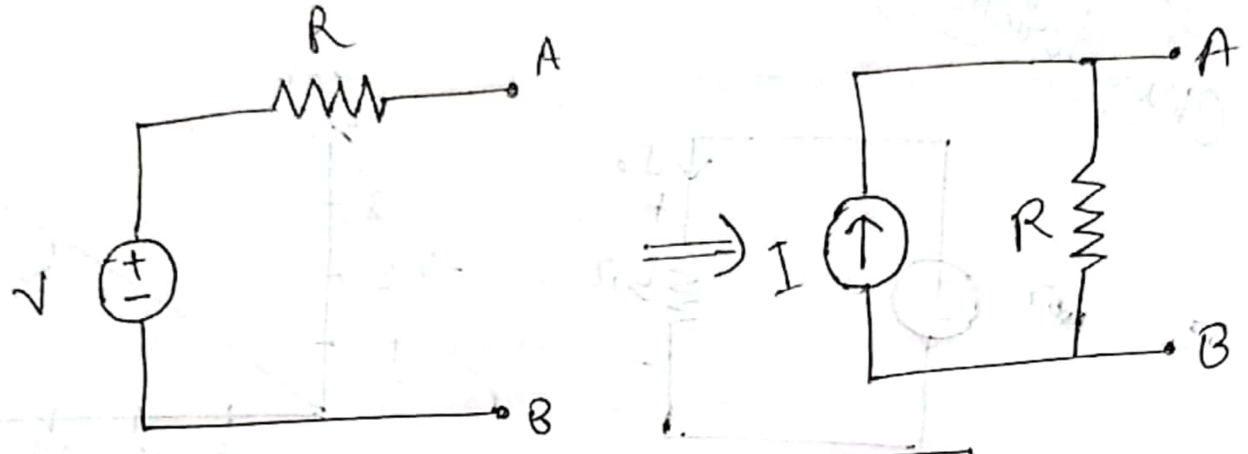








$V \rightarrow I$

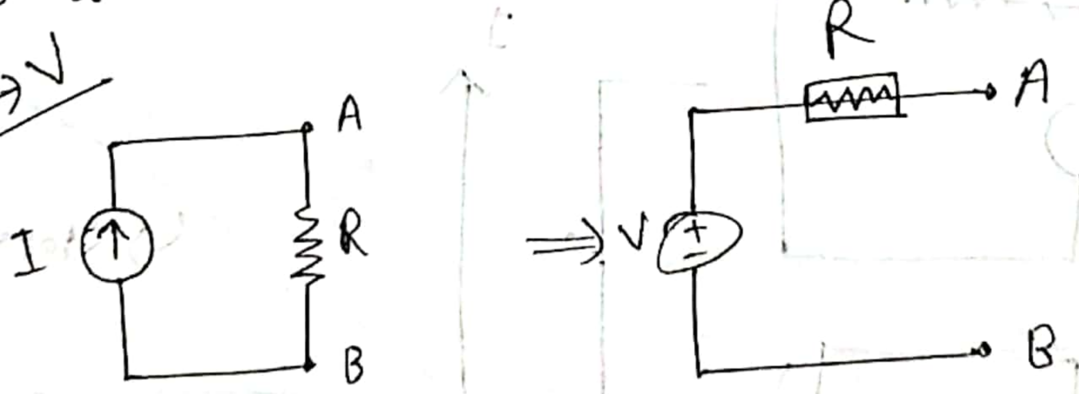


Where, $\boxed{I = V \cdot \frac{1}{R}} = \frac{V}{R}$

Slope

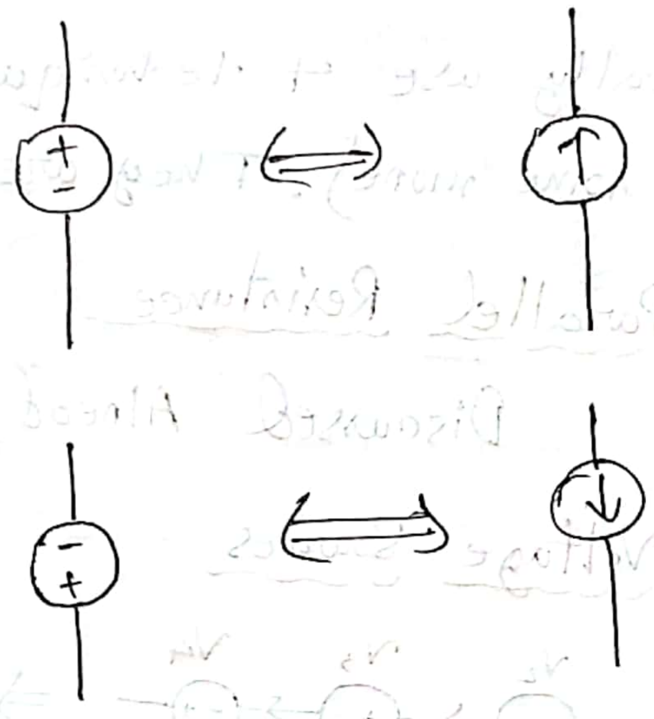
∴ We can interchange sources to simplifly circuit using that formula. The rest of the circuit will be the same.

$I \rightarrow V$

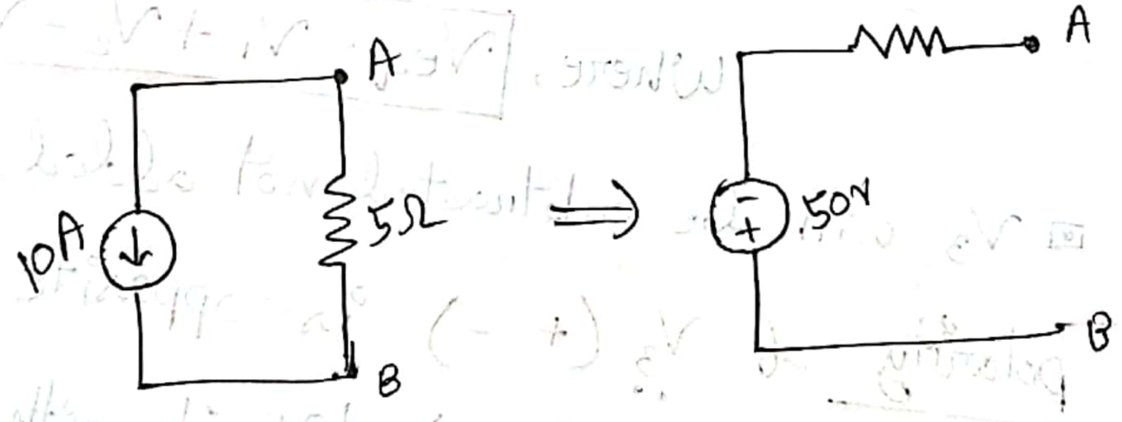


Where, $\boxed{V = IR}$

Signs



Ex.



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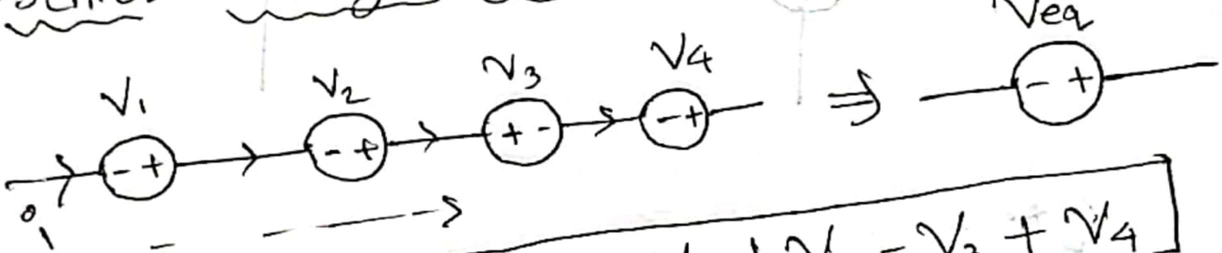
Circuit Simplification Techniques

We basically use 4 techniques (later on we may discuss some more). They are:

1. Series - Parallel Resistance

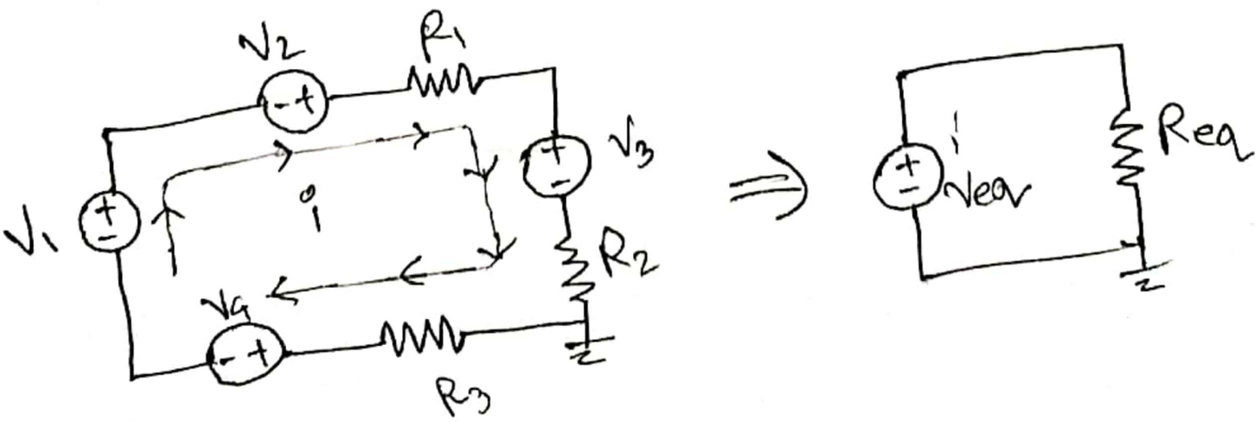
Discussed Already

2. Series Voltage Sources



Where, $\boxed{V_{eq} = V_1 + V_2 - V_3 + V_4}$

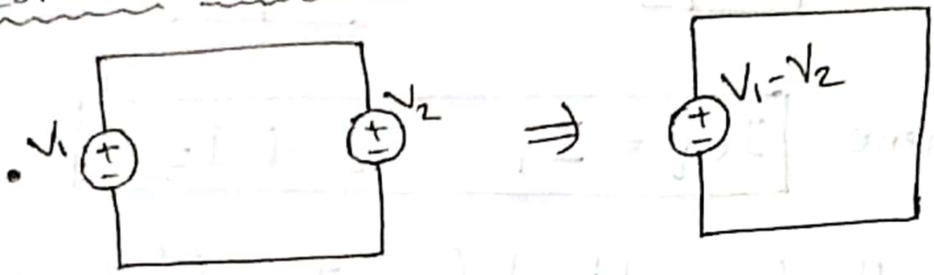
▣ V_3 will be subtracted, not added, because the polarity of V_3 (+ −) is opposite of the polarity of V_{eq}(− +). [Polarity with respect to the direction of current i (→)]



Where, $\boxed{V_{eq} = V_1 + V_2 - V_3 - V_4}$ | V_3, V_4 negative due to there polarity w.r.t current i.

$\boxed{R_{eq} = R_1 + R_2 + R_3}$ [Series Resistance]

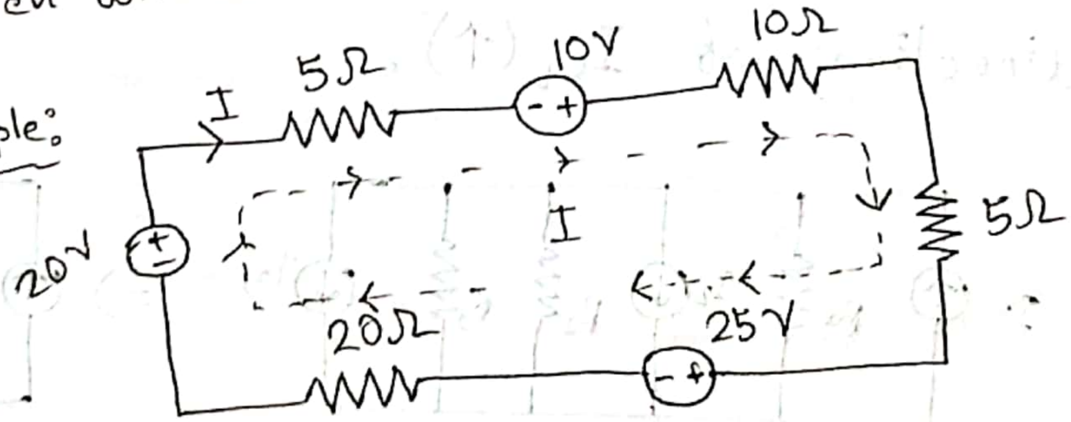
Common Mistakes



Here, $V_{eq} = V_1 - V_2$, not $V_1 + V_2$.

Watch where the current is entering from.

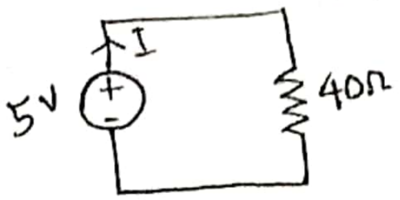
Example:



$I = ?$

Here, $V_{eq} = (20 + 10 - 25)V = 5V$

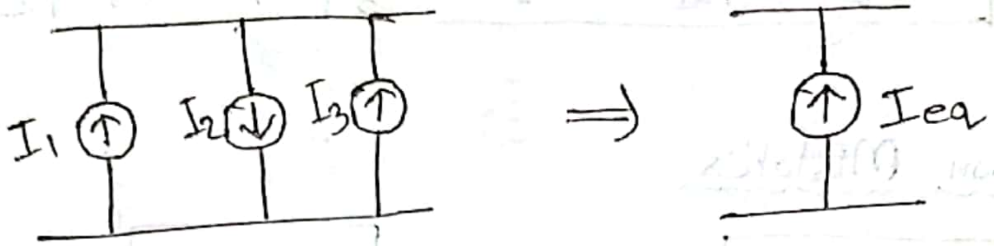
$R_{eq} = (5 + 10 + 5 + 20)\Omega = 40\Omega$



$\therefore I = \frac{V}{R} = \frac{5}{40} A = \boxed{0.125 A}$ (Ans.)

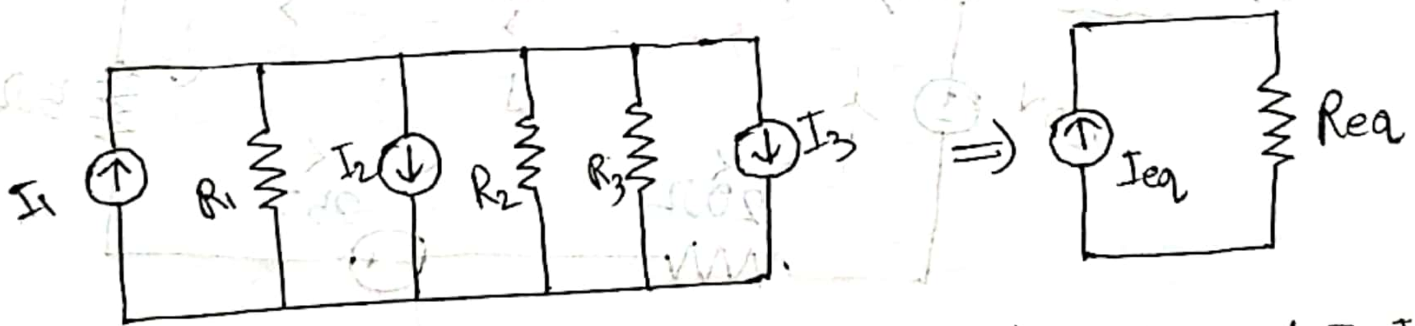
P.S: Watch video for verification

3. Parallel Current Sources



where, $\boxed{I_{eq} = I_1 - I_2 + I_3}$.

- I_2 will be subtracted, not added, because the direction of I_2 ($\downarrow$) is the opposite of the direction of I_{eq} ($\uparrow$).



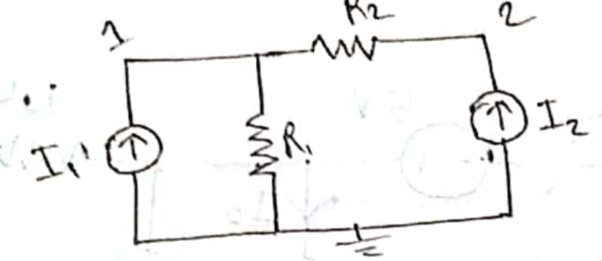
where, $\boxed{I_{eq} = I_1 - I_2 - I_3}$

Direction of I_2, I_3 downwards, I_{eq} upwards

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

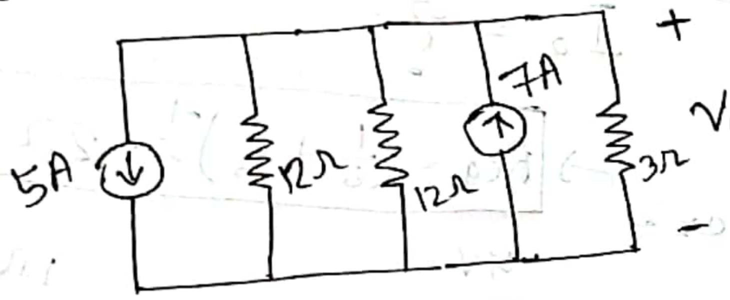
Parallel Resistance

Common Mistakes:

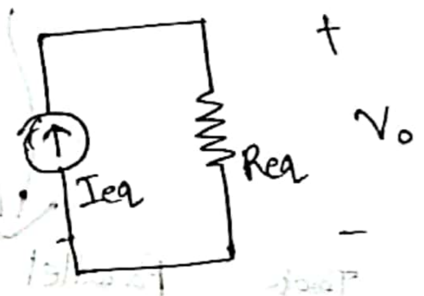


$\Rightarrow$ Here, you can't add I_1 & I_2 simply. Because they don't have the same nodes across them. Hence, they're not parallel.

Example:



$\Rightarrow$



Here, $I_{eq} = -5A + 7A = 2A$.

$I_1 \Rightarrow \downarrow$

$I_{eq} \Rightarrow \uparrow$

$$R_{eq} = \left(\frac{1}{12} + \frac{1}{12} + \frac{1}{3}\right)^{-1}$$

$$= \left(\frac{1}{2}\right)^{-1}$$

$$= 2\,\Omega$$

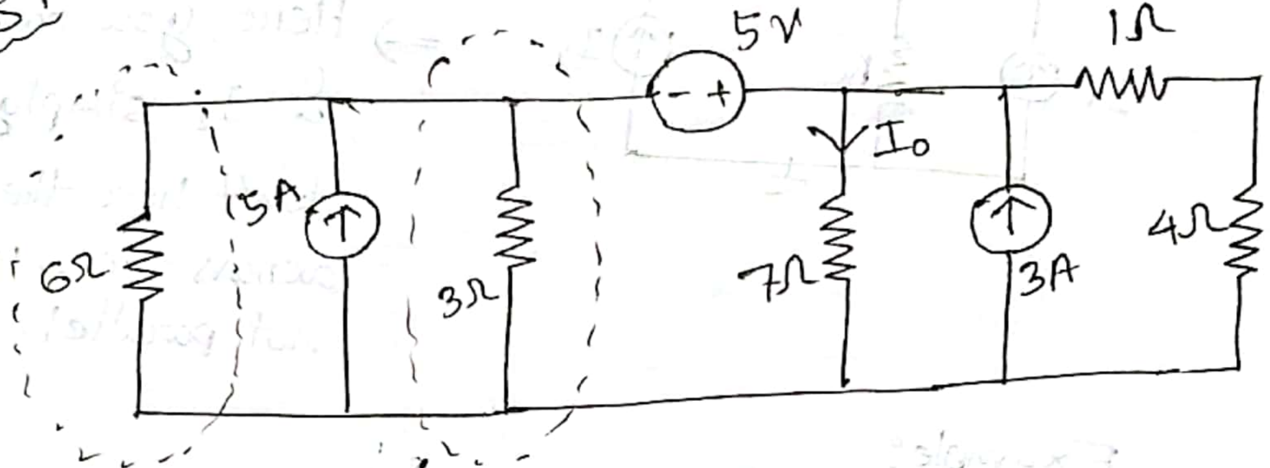
$$\therefore V_o = IR = I_{eq} \cdot R_{eq} = (2 \times 2)\,V$$

$$= \boxed{4V}$$

4. Source Transformation

Already discussed.

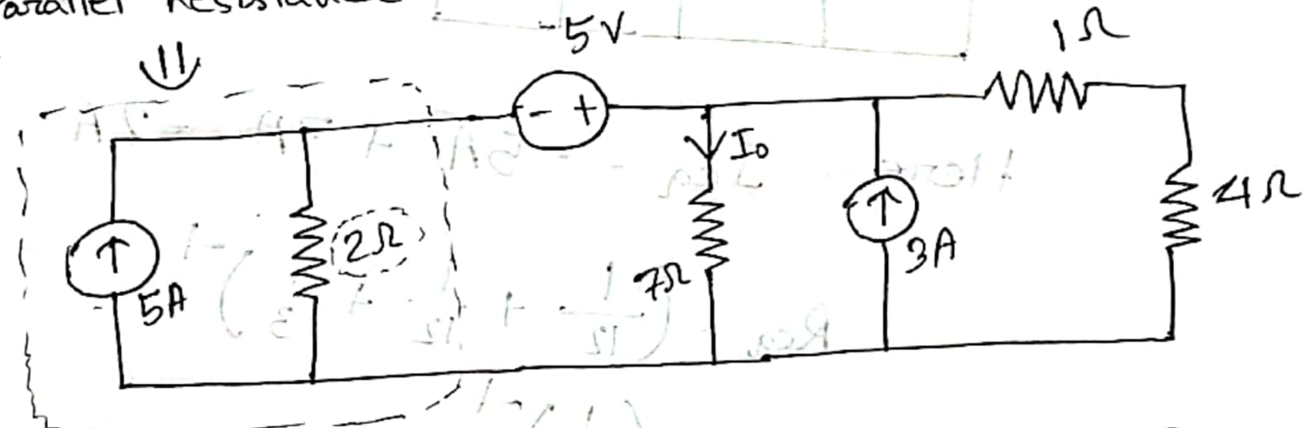
Example for ST full problem



$I_o = ?$

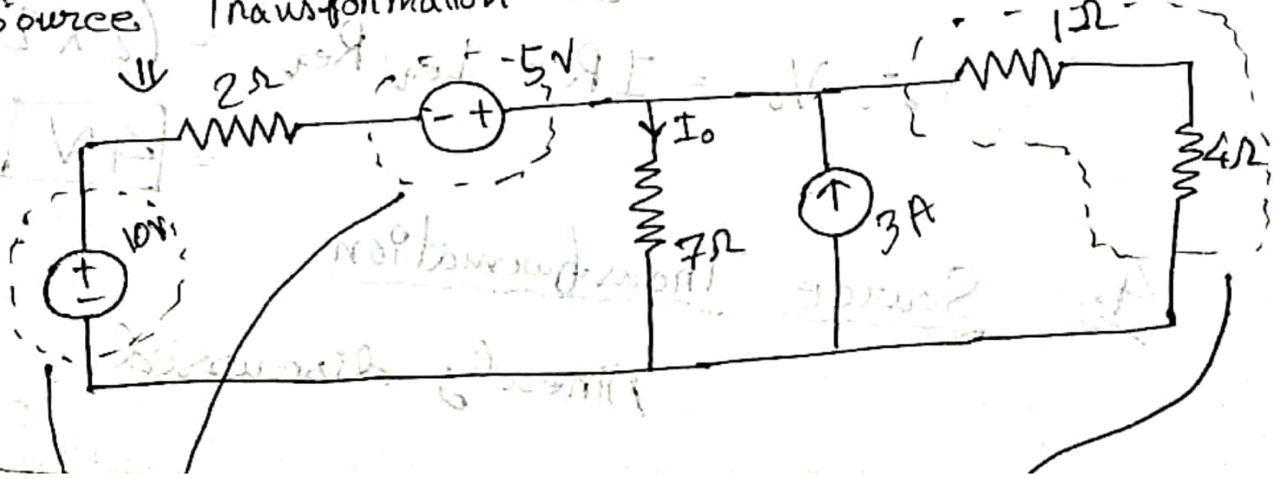
Parallel Resistance

$$\boxed{R_{eq} = \left(\frac{1}{6} + \frac{1}{3}\right)^{-1} \Omega = 2\Omega}$$



Source Transformation

$$\boxed{V = IR = 5 \times 2 \text{ V} = 10\text{V}}$$

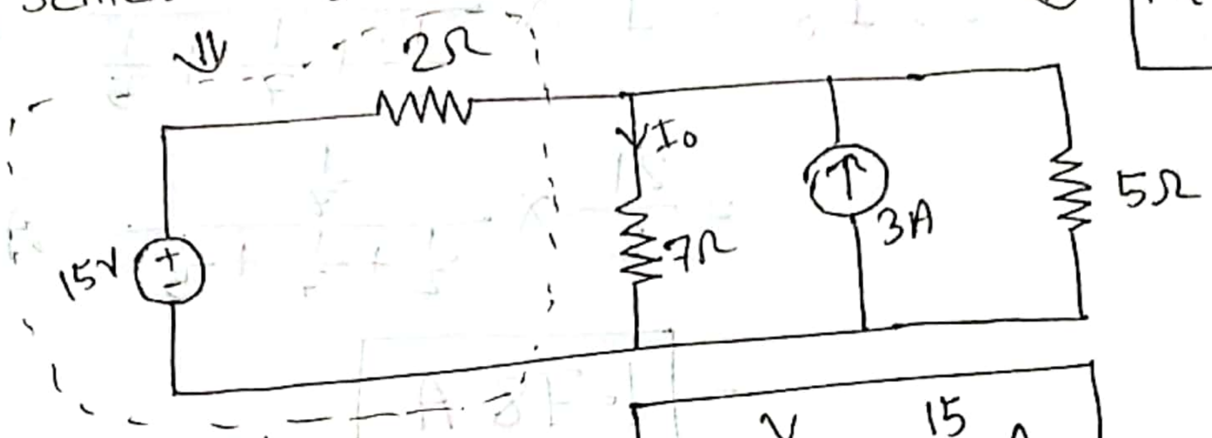


$$\boxed{V_{eq} = (10+5)\,V = 15V}$$

Series Voltage Source

Series Resistance

$$\boxed{R_{eq} = (1+4)\Omega = 5\Omega}$$

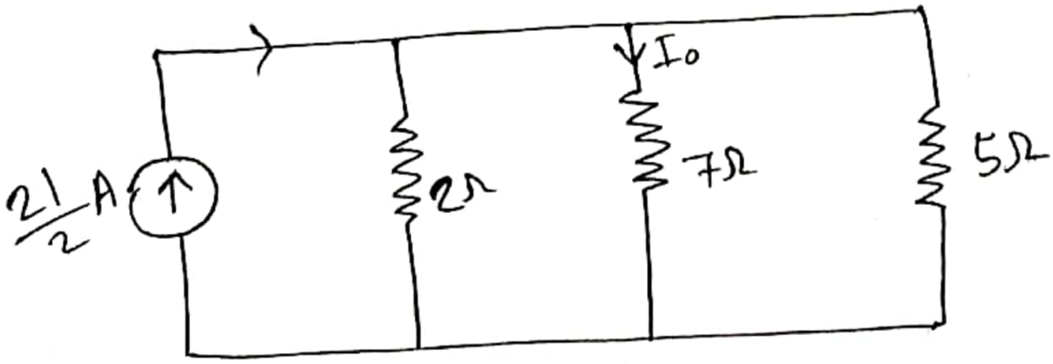


$$\boxed{I = \frac{V}{R} = \frac{15}{2}\,A}$$

Source Transformation

$$\boxed{I_{eq} = \left(\frac{15}{2} + 3\right)A = \frac{21}{2}\,A}$$

Parallel Current Source



∴ Now, we can directly apply Current Divider Rule [Lecture 3]

$$\therefore I_0 = 1 \times \frac{\frac{1}{7}}{\frac{1}{2}+\frac{1}{7}+\frac{1}{5}} \text{ A}$$

$$= \frac{21}{2} \times \frac{\frac{1}{7}}{\frac{1}{2}+\frac{1}{7}+\frac{1}{5}} \text{ A}$$

$$= \boxed{1.78 \text{ A}}$$

(Ans.)

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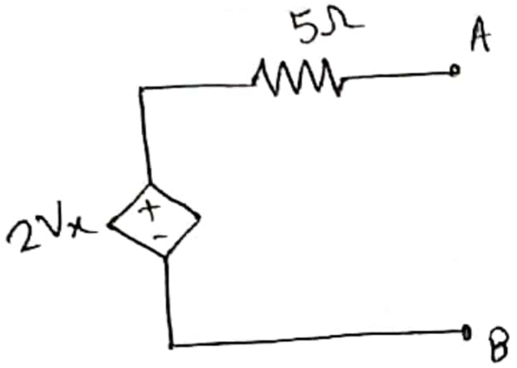
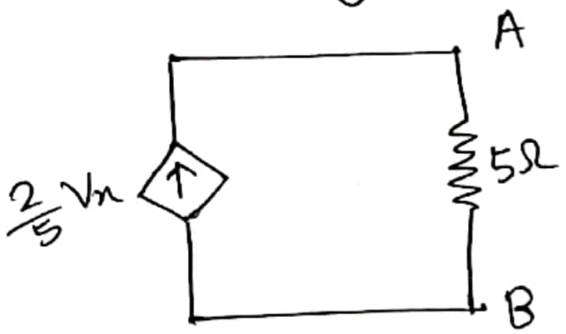
Source Transformation with Dependent Sources

We can do the ST for dependent sources using exactly the same procedure.

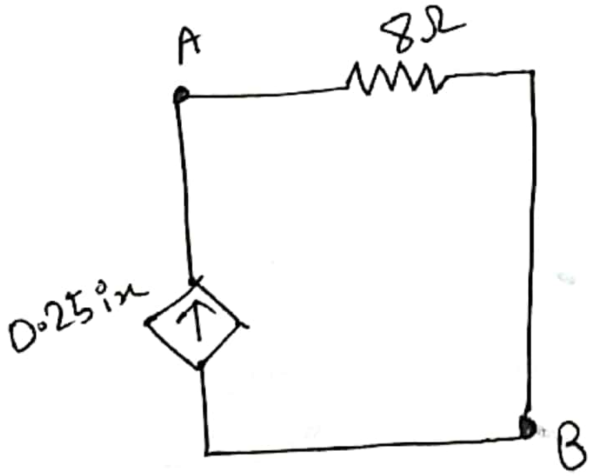


$$\Rightarrow I = \frac{V}{R} = \frac{2V_x}{5} = \frac{2}{5}V_x$$

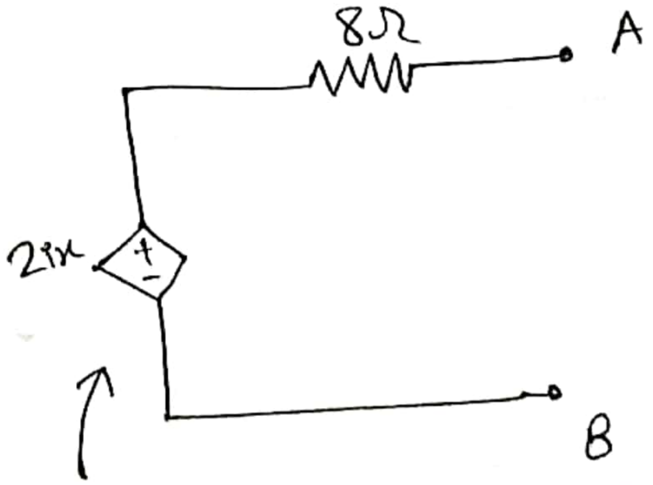
$$\Downarrow$$



Similarly,



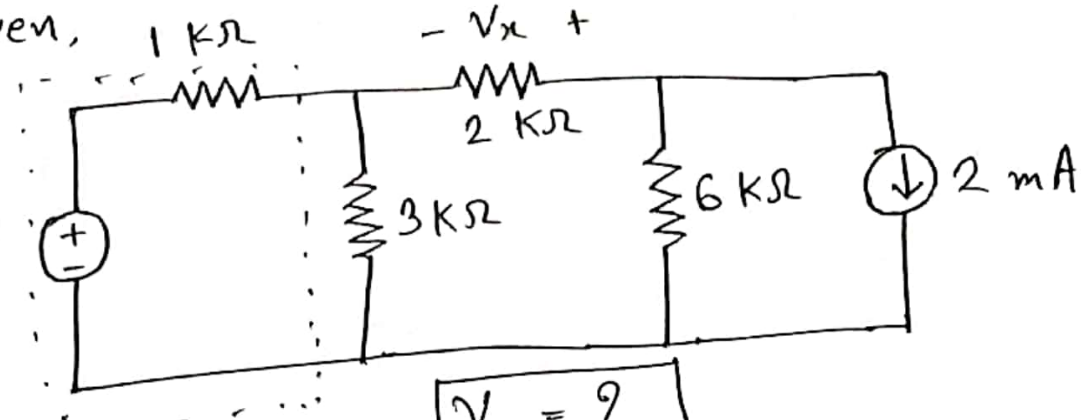
$\Rightarrow$



$$\boxed{V = IR = 0.25 i_x \times 8 = 2 i_x}$$

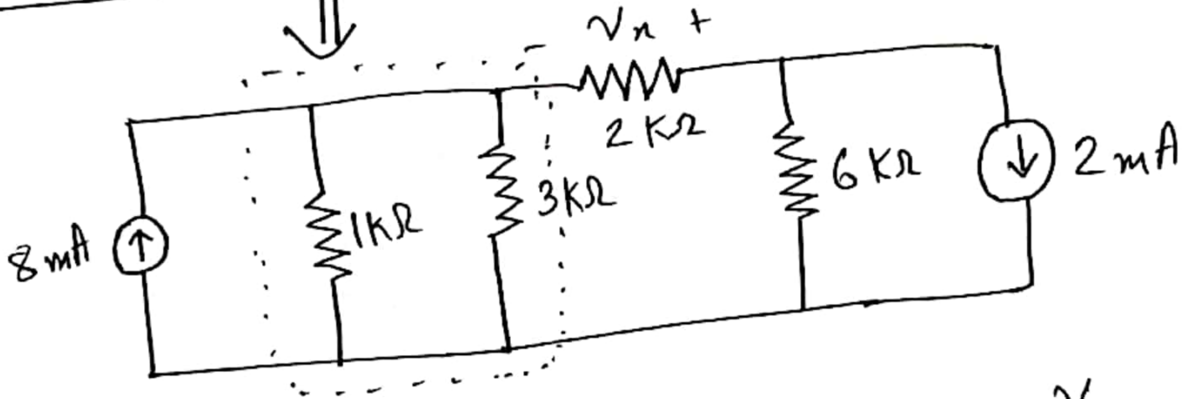
[2] Source Tx.

Given,



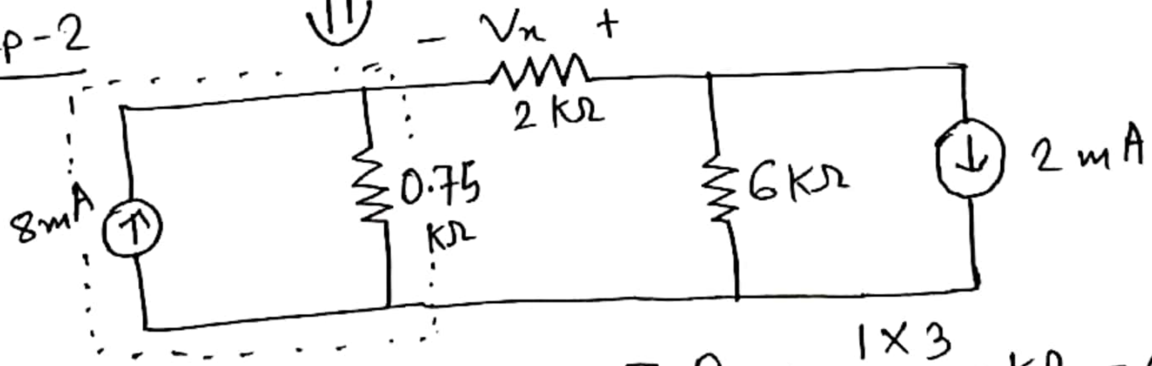
$\boxed{V_x = ?}$

Step-1



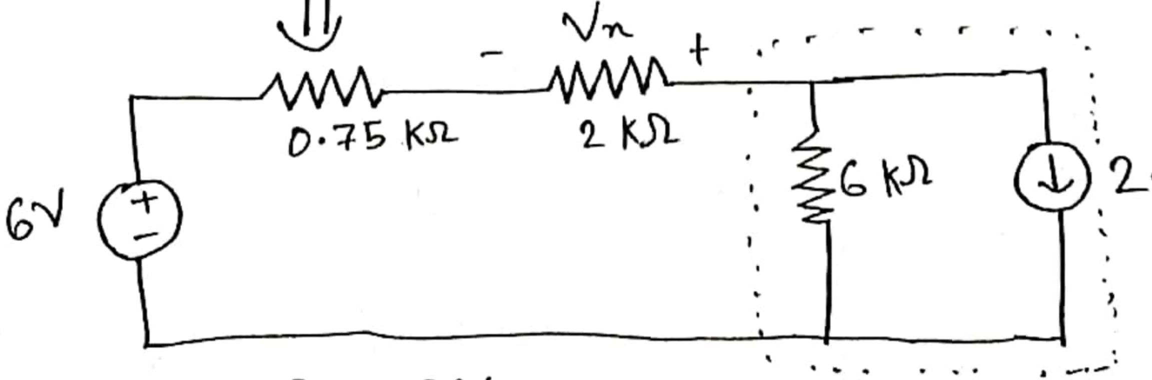
▫ $I = \frac{V}{R} = 8\ mA$

Step-2



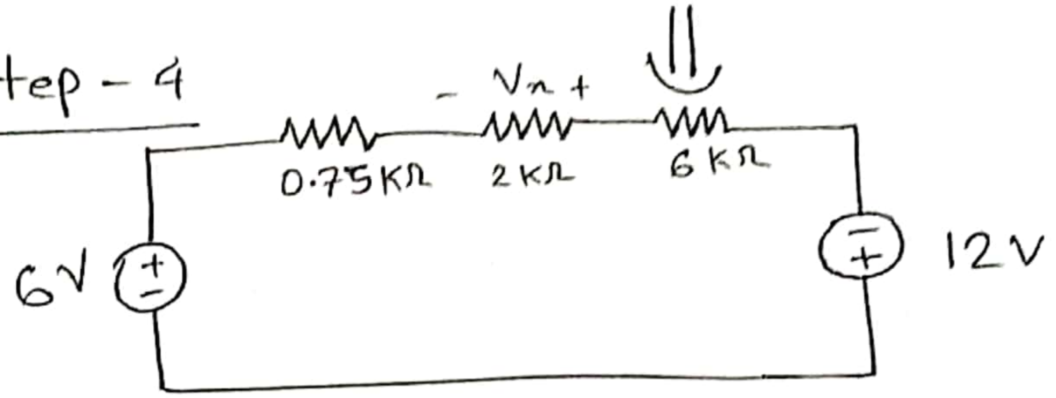
▫ $R_p = \frac{1 \times 3}{1+3}\ K\Omega = 0.75\ K\Omega$

Step-3



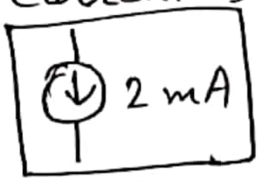
▫ $V = IR = 6V$

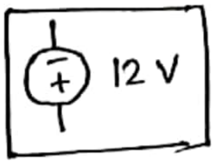
Step - 4

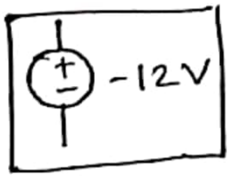


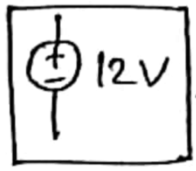
▣ $V = IR = 6 \times 2 = 12V$

Common Mistake → Wrong Polarity of Voltage Source.

Here, Current source was making current flow downwards. 

⇒ So the polarity of Voltage source will be 

or 

. Not 

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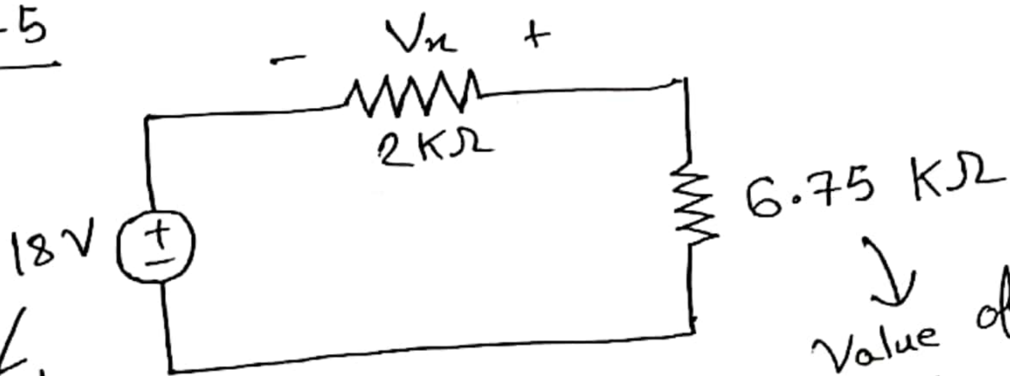
Step-5



Value of $V_1 = (6+12)V$
$= 18V$

Value of $R_1 = (6+0.75)$ KΩ
$= 6.75$ KΩ

Common Mistake → Mistake while adding V.

With respect to the complete loop, both Voltage sources are of same polarity. So the sum will be $(6+12) = 18V$; not 6V or -6V.

V_x Calculation

Using Voltage Divider Rule,

$$V_x = -\left(V_1 \times \frac{2}{6.75+2}\right)$$

$$= -\left(18 \times \frac{2}{8.75}\right)$$

$$= \boxed{-4.114\ V}$$

Common Mistake:

Wrong Polarity again. Here V_x is 

. The negative end is where the high voltage end of source meets.

So, V_x = (The Voltage drop at 2 KΩ resistance) X (−1)