Lecture 16

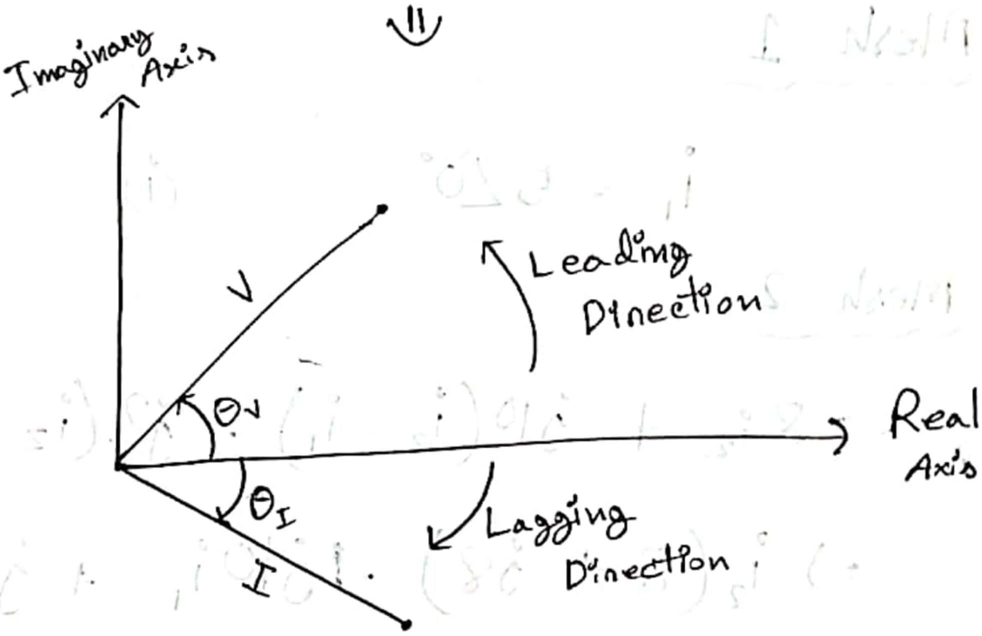
AC Power

Phasor Diagram

Phasors — $V\angle\theta_V$: $|V\cos(\omega t + \theta_V)|$

Phasors — $I\angle -\theta_I$: $|I\cos(\omega t - \theta_I)|$

⇓



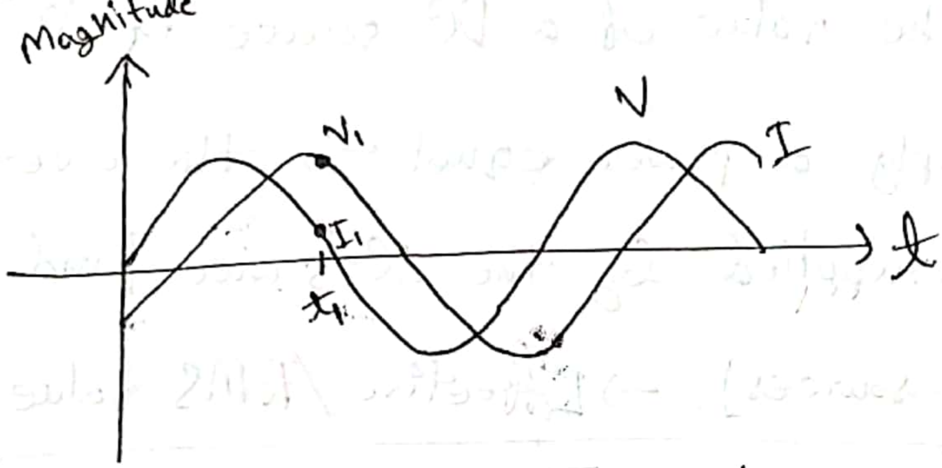
⇓

Representing a phasor in a Real-Imaginary → Complex plane. Such representations are called $\boxed{\text{Phasor Diagrams.}}$

▫ Here, Voltage is $\boxed{\text{leading}}$ current by $\boxed{\theta_V + \theta_I}$ degrees/radians.

AC Power

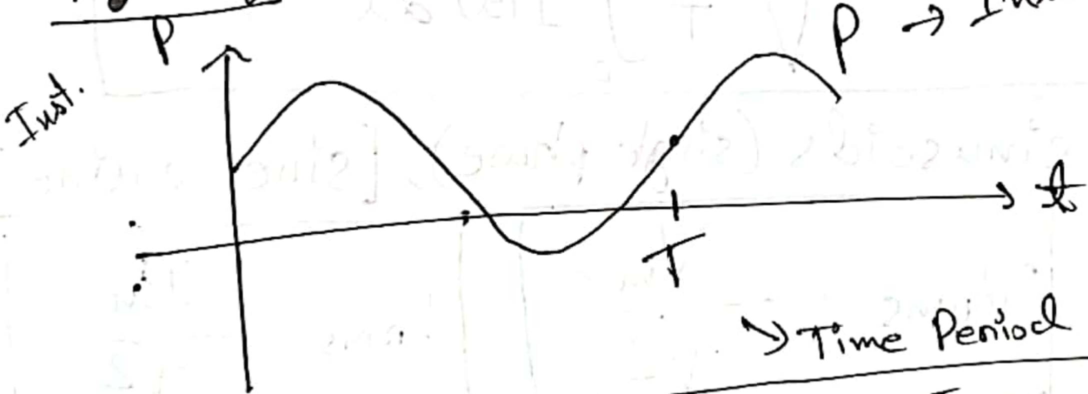
Instantaneous Power



At $\boxed{t_1}$ time instant → Intantaneous Power = $\boxed{V_1 I_1}$

- In DC, Instantaneous Power ⇒ Constant.

Average Power



Average Power, $\boxed{P_{avg} = \frac{1}{T}\int_0^T P \, dt}$

↓

The Power across a time period divived by the time period → Inst. Power per unit time.

RMS ~~Power~~ Value

The value of a DC source if it were to supply a power equal to the average power supplied by the AC source [and similar for non-sources]. → Effective / RMS value.

$$\boxed{V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V(t))^2 \, dt}}$$

$$\boxed{I_{rms} = \sqrt{\frac{1}{T}\int_0^T (I(t))^2 \, dt}}$$

For sinusoids (single phase), [sine, cosine]

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}, \quad \boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

Where, $V(t) = V_m \sin(\omega t + \theta_V)$ | or cos |

$I(t) = I_m \cos(\omega t + \theta_I)$ | or sin |

☐ Can be used in all DC formulas.

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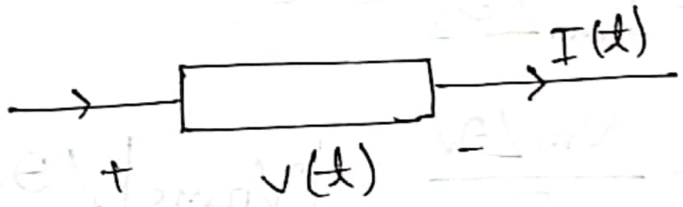
Apparent Power

Let,

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$I(t) = I_m \cos(\omega t + \theta_I)$$



$$|V_{rms}| = \frac{V_m}{\sqrt{2}} \quad , \quad |I_{rms}| = \frac{I_m}{\sqrt{2}}$$

$$\boxed{\text{Apparent Power} = |V_{rms}| \cdot |I_{rms}|}$$

Power Factor

↳ Cosine of the phase difference voltage and current / Load angle.

$$\therefore \boxed{pf = \cos(\theta_v - \theta_I)} = \cos(\theta_{load})$$

Complex Power $\boxed{S}$

Let,

$$V(t) = V_m \cos(\omega t + \theta_v) \qquad \cancel{V_{rms} \angle \theta_v}$$

$$= V_m \angle \theta_v$$

$$I(t) = I_m \cos(\omega t + \theta_I)$$

$$= I_m \angle \theta_I$$

$$\therefore V_{rms} = \frac{V_m \angle \theta_v}{\sqrt{2}} = |V_{rms}| \angle \theta_v$$

$$I_{rms} = \frac{I_m \angle \theta_I}{\sqrt{2}} = |I_{rms}| \angle \theta_I$$

~~Complex Power = I_{rms} V_{rms}~~

$$\boxed{\text{Complex Power, } S = |V_{rms}| \cdot |I_{rms}| \angle \theta_v - \theta_i}$$

$$= P + jQ$$

$$= V_{rms} \cdot I_{rms}^{*}$$

Here,

$P \longrightarrow$ Real Power

$Q \longrightarrow$ Reactive Power

Real Power

Complex Power

$$\boxed{P = Re(S) = S \cos(\theta_v - \theta_i)}$$

- Resistance absorbs Real power.

Reactive Power

$$\boxed{Q = Im(S) = S \sin(\theta_v - \theta_i)}$$

- Capacitance, Inductance

$\downarrow$

Reactive Power

$\downarrow$

Neither supply non dissipate power.
actually, but exchange power back and forth,

Examples
Power

Sadiku → 11.11

$$V(t) = 60\cos(\omega t - 10^\circ)\ V$$

$$I(t) = 1.5\cos(\omega t + 50^\circ)\ A$$

a) Complex Power $= |V_{rms}||I_{rms}| \angle \theta_v - \theta_i$

Here, $\theta_v = -10^\circ$

$\theta_i = 50^\circ$

$$|V_{rms}| = \frac{V_m}{\sqrt{2}} = \frac{60}{\sqrt{2}}$$

$$|I_{rms}| = \frac{I_m}{\sqrt{2}} = \frac{1.5}{\sqrt{2}}$$

$$\therefore V_{rms} = \frac{60}{\sqrt{2}} \angle -10^\circ$$

$$I_{rms} = \frac{1.5}{\sqrt{2}} \angle 50^\circ$$

$\therefore$ Complex Power, $S = V_{rms} \cdot I_{rms}^*$

$$= \frac{60}{\sqrt{2}} \cdot \frac{1.5}{\sqrt{2}} \angle -10^\circ - 50^\circ$$

$$= \boxed{45 \angle -60^\circ\ VA}$$ (Ans.)

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Apparent Power $= |V_{rms}||I_{rms}|$

$$= \frac{60}{\sqrt{2}} \times \frac{1.5}{\sqrt{2}} \text{ VA}$$

$$= \boxed{45 \text{ VA}}$$

b) $S = 45\angle -60^\circ = \underbrace{22.5}_{P} - j\underbrace{38.97}_{Q}$

$\therefore$ Real Power $= \boxed{22.5 \text{ W}}$

Reactive Power $= \boxed{-38.97 \text{ VAR}}$

c) $pf = \cos\angle(-10^\circ - 50^\circ) = \cos(-60^\circ)$

$= \boxed{0.5}$ (leading)

Current ahead

Load Impedance, $Z = \frac{V}{I}$

$$= \frac{60\angle -10^\circ}{1.5\angle 50^\circ} = \boxed{40\angle -60^\circ\,\Omega}$$

(Ans)

Capacitive

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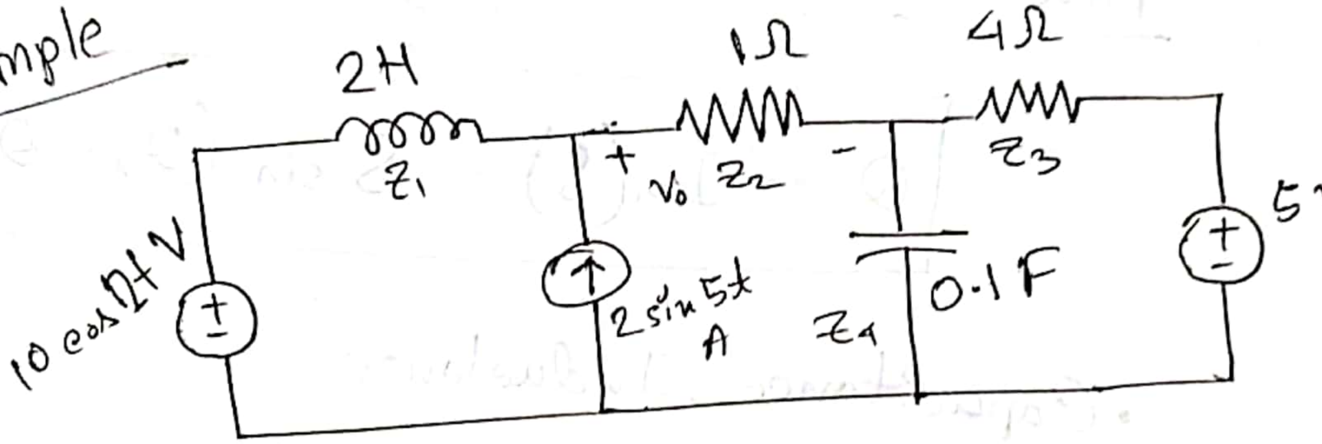
Superposition in AC

- Important when frequency diffenent in different sources
- Similan Concept as DC.

Example



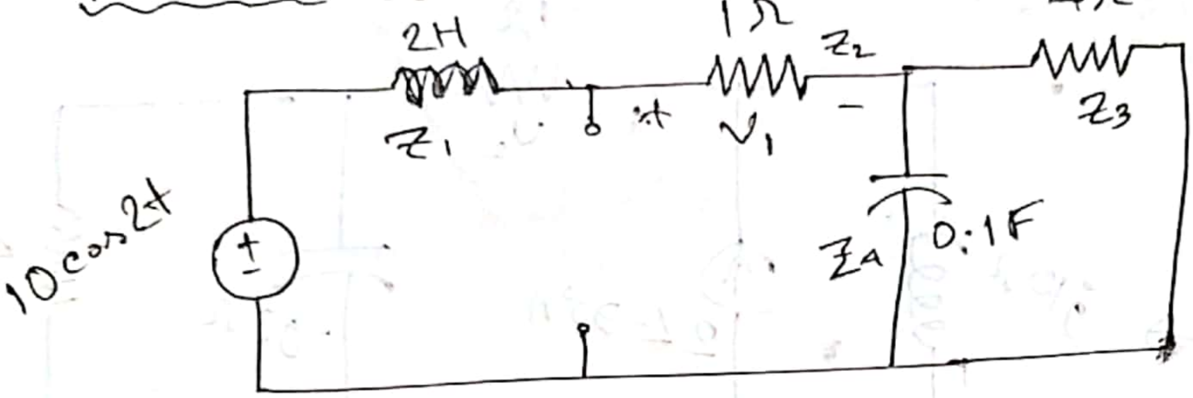
$$Z_1 = j\omega L = j\omega ?$$

↳ Different ω.

Superposition needed.

$\therefore V_o = V_1 + V_2 + V_3$ | One for each source |

$\underline{10 \cos 2t \ V \text{ source On}}$



$$Z_1 = j\omega L = j \times 2 \times 2 = 4j$$

$$Z_4 = \frac{1}{j\omega C} = \frac{1}{j \times 2 \times 0.1} = -5j$$

$$10 \cos 2t \ V \longrightarrow 10\angle 0^\circ \ V$$

$$\therefore V_1 = 10\angle 0^\circ \times \frac{Z_2}{Z_1 + Z_2 + Z_3 \| Z_4}$$ |Voltage Divider|

$$= 10\angle 0^\circ \times \frac{1}{4j + 1 + (-j5 \| 4)}$$

$$= 10\angle 0^\circ \times \frac{1}{4j + 1 + \frac{-j5 \times 4}{4 - j5}}$$

$$= 2.439 - j1.951$$

$$= 2.498 \angle -30.79^\circ$$

$$\therefore V_1(t) = 2.498 \cos(2t - 30.79^\circ)$$

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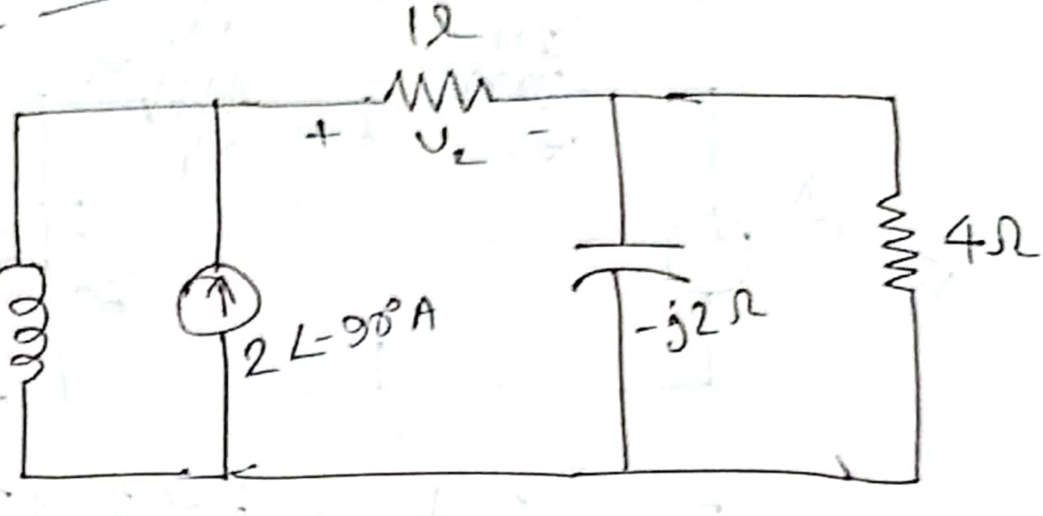
2 sin 5t A Source

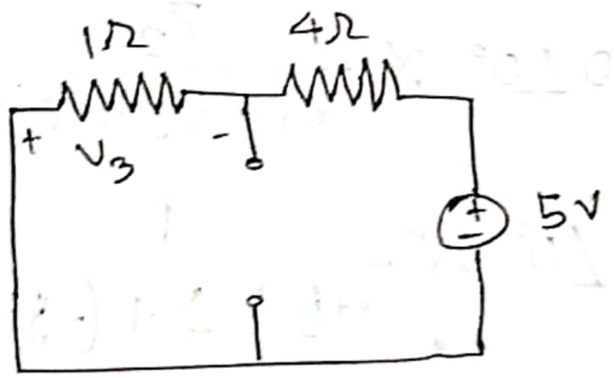
Using $\omega = 5$ → $j10\,\Omega$



Performing Similar Analysis,

$$V_2 = 2.33\cos(5t - 80^\circ)$$

5V source



DC circuit
↓
inductor → Short
capacitor → Open

$$\therefore V_3 = -\frac{1}{1+4} \times 5$$

$$= -1\,V$$

$$\therefore V_0 = V_1 + V_2 + V_3 = 2.498\cos(2t - 30.79^\circ) + 2.33\cos(5t - 80^\circ) - 1 \;V$$