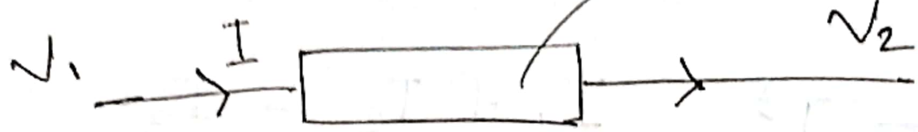
Week - 4

Nodal Analysis

Reviews

Resistance & Ohm's Law

A conductor



Ohm's Law $\longrightarrow$ $I \propto \Delta V$

$$\Rightarrow I \propto V_1 - V_2$$

Now, $I \propto \Delta V$ → A constant: Conductivity / Conductance

$$\Rightarrow I = G\Delta V$$

↓

How good a conductor is

↓

How easily current will flow.

$$G = \frac{1}{R} \quad \Bigg| \quad R = \frac{1}{G}$$

→ Unit: Ohm (Ω)

→ Resistance

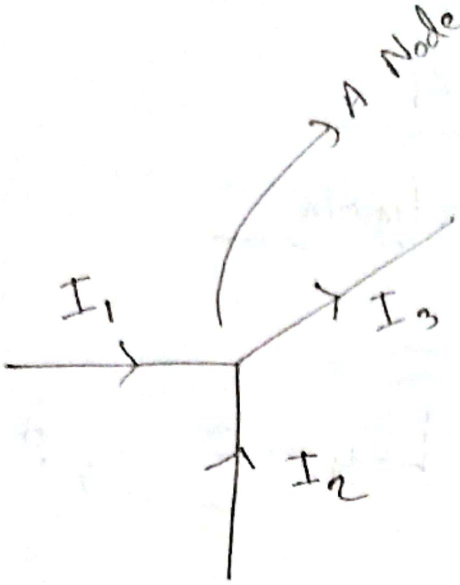
↓

How much it will resist current

⇓

$$\therefore \quad \boxed{I = \frac{\Delta V}{R}}$$

KCL



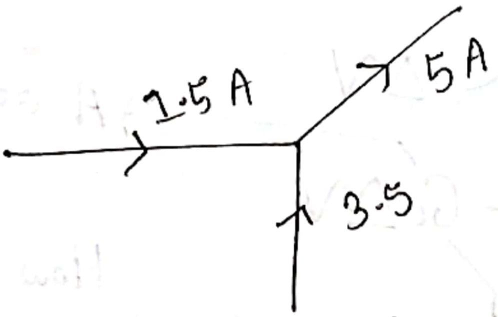
All currents entering

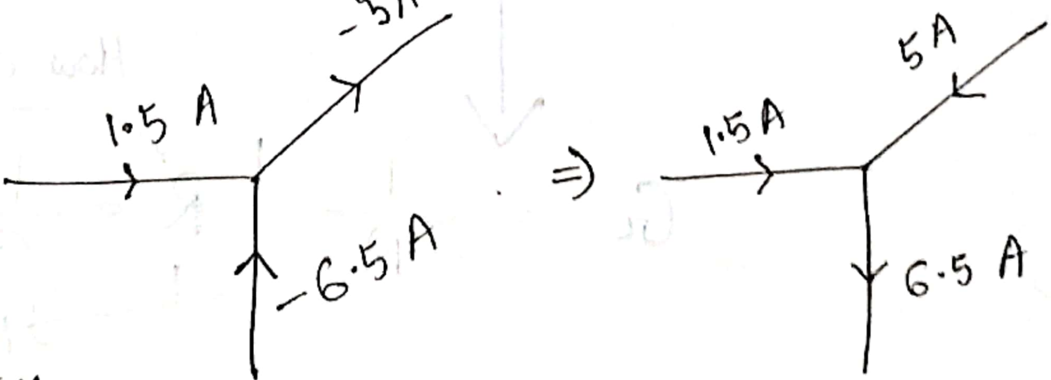
$$\sum I = 0$$

KCL → $I_1 + I_2 = I_3$

→ Total in = Total out

Example:





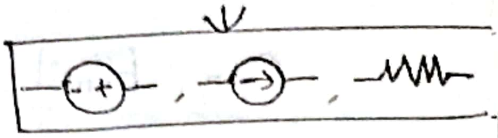
Passive Sign Convention

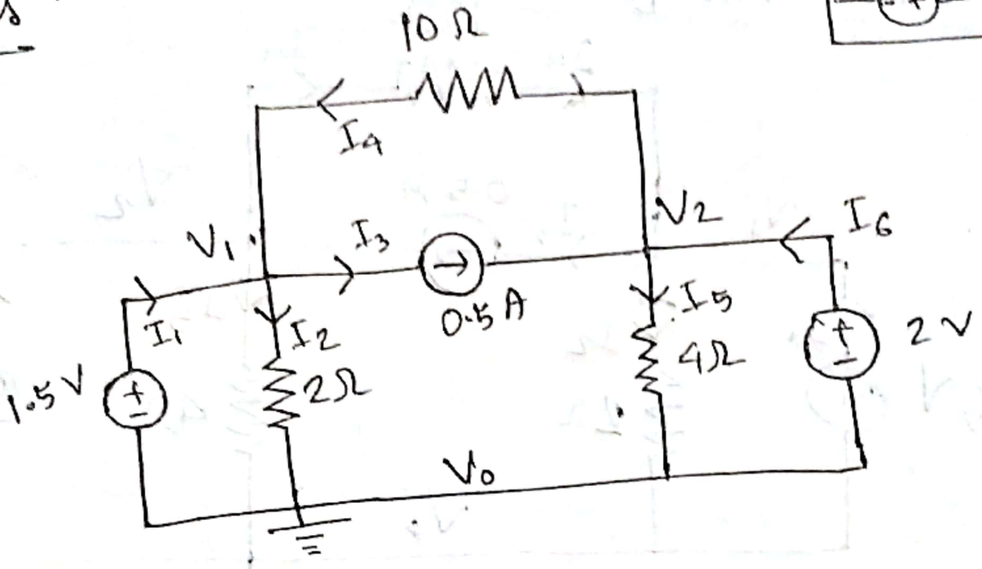
Current entering the Node → $\boxed{\text{Add}}$

Current exiting the Node → $\boxed{\text{Subtract.}}$

Nodes

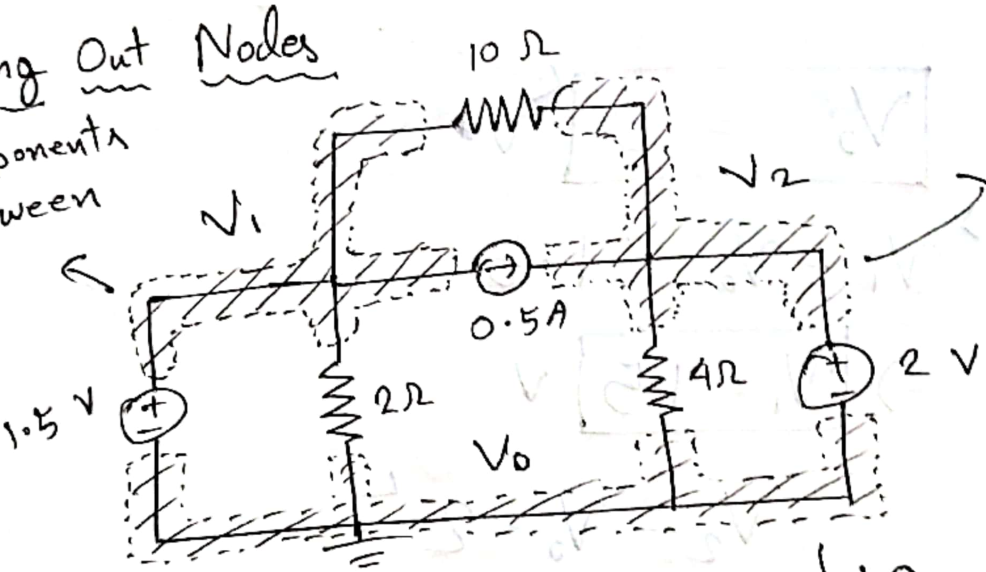
Connects two/more electric components





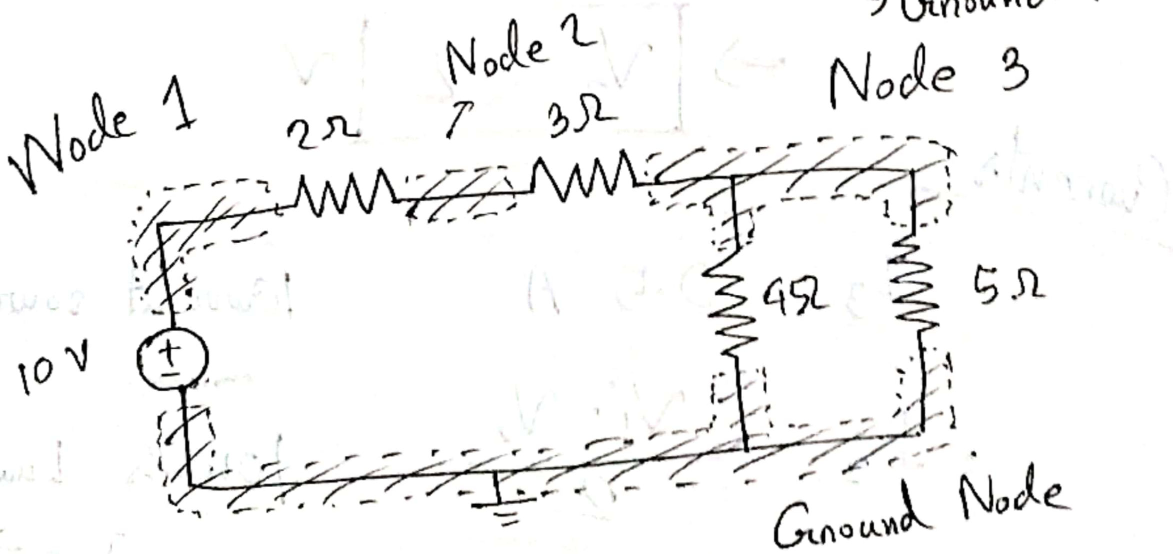
Finding Out Nodes

No Components in between

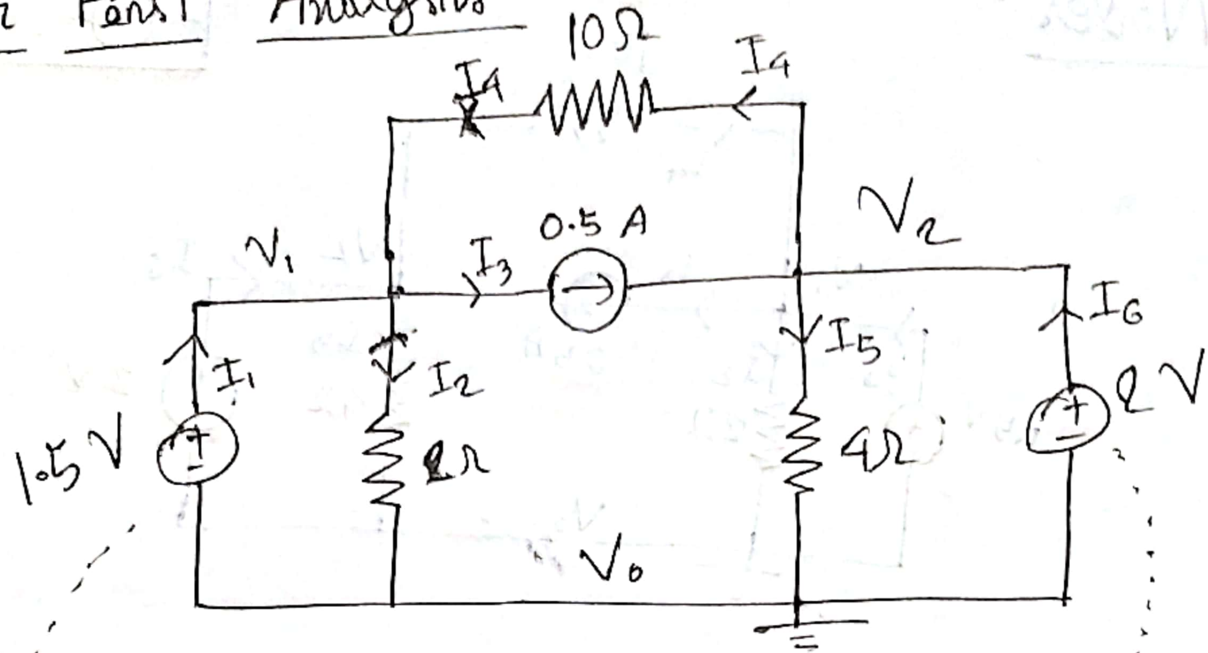


Same voltage in all points

Ground Node



Our First Analysis



Votages

$$\boxed{V_0 = 0}\ V$$

$$V_1 - V_0 = 1.5$$

$$\Rightarrow \boxed{V_1 = 1.5}\ V$$

$$V_2 - V_0 = 2$$

$$\Rightarrow \boxed{V_2 = 2}\ V$$

Currents

$I_3 = 0.5$ A | Current source |

$$I_2 = \frac{V_1 - V_0}{2} = 0.75 \text{ A}$$

| Ohm's Law, $I = \frac{\Delta V}{R}$ |

$$I_4 = \frac{V_2 - V_1}{10}$$

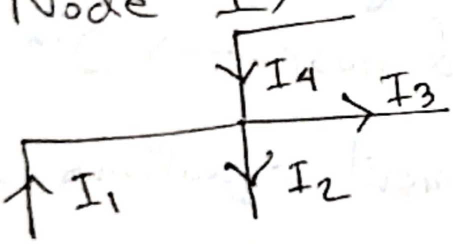
| Ohm's law, $I = \frac{\Delta V}{R}$ |

$$= \frac{2 - 1.5}{10}$$

$$= 0.05 \text{ A}$$

$I_1 = ?$ $\Longrightarrow$ Here comes KCL

For Node 1,



$\Rightarrow$ Entering Node 1
↳ I_4, I_1

↳ Exiting Node 1
↳ I_2, I_3

$$\therefore -I_1 + I_2 + I_3 - I_4 = 0$$

$$\Rightarrow I_1 + I_4 = I_2 + I_3$$

$$\Rightarrow I_1 = I_2 + I_3 - I_4$$

$$= \boxed{1.2 \text{ A}}$$

[Solved] → Try finding I_5, I_6.

Nodal Analysis

↳ Goal: Finding all Node voltages using KCL & co.

Steps

Step-1: Identifying all the nodes and marking them from 0 to N

↓

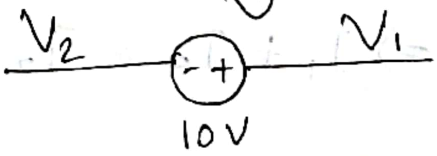
[Ground node → 0^{th} node]

Step-2: Write Component equations for all the voltage sources.

↓

[Voltage difference = labeled value]

⇓ example

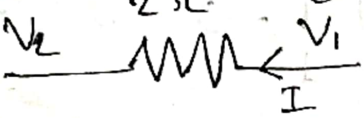


$\Rightarrow \boxed{V_1 - V_2 = 10}$

Step-3: Write the current through the resistors using Ohm's law, $I = \frac{\Delta V}{R}$. Passive

[Direction as per wish, but must follow sign convention]

⇓ example



$\Rightarrow \boxed{I = \frac{V_1 - V_2}{2}}$

Step-4: Write KCL equations for all the nodes and supernodes with unknown Voltage.

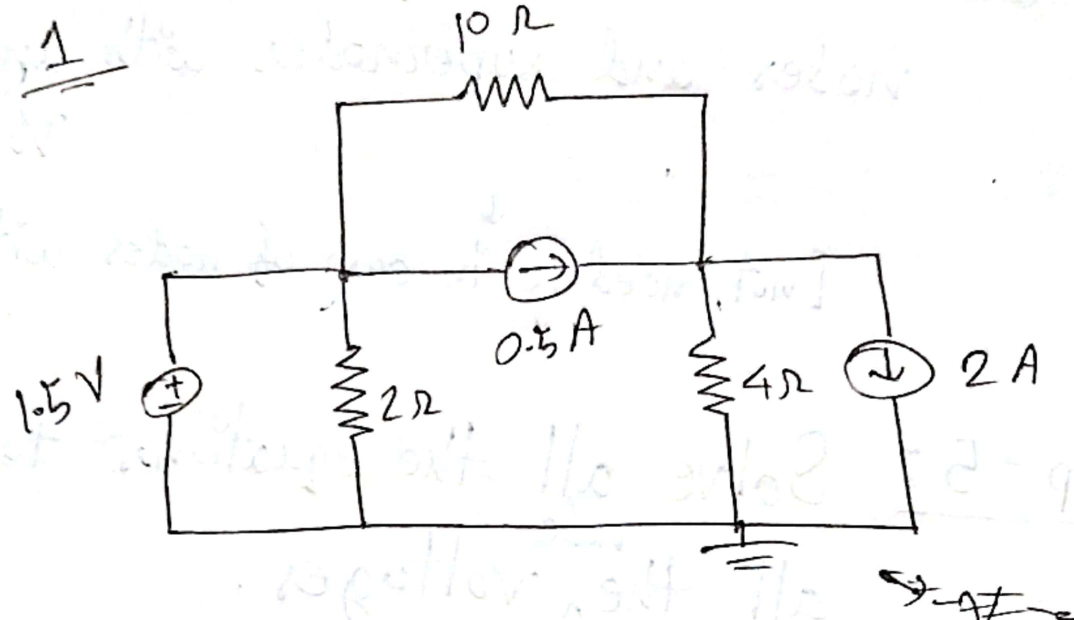
[not needed in case of nodes with known V value]

Step-5: Solve all the equations to find all the node voltages.

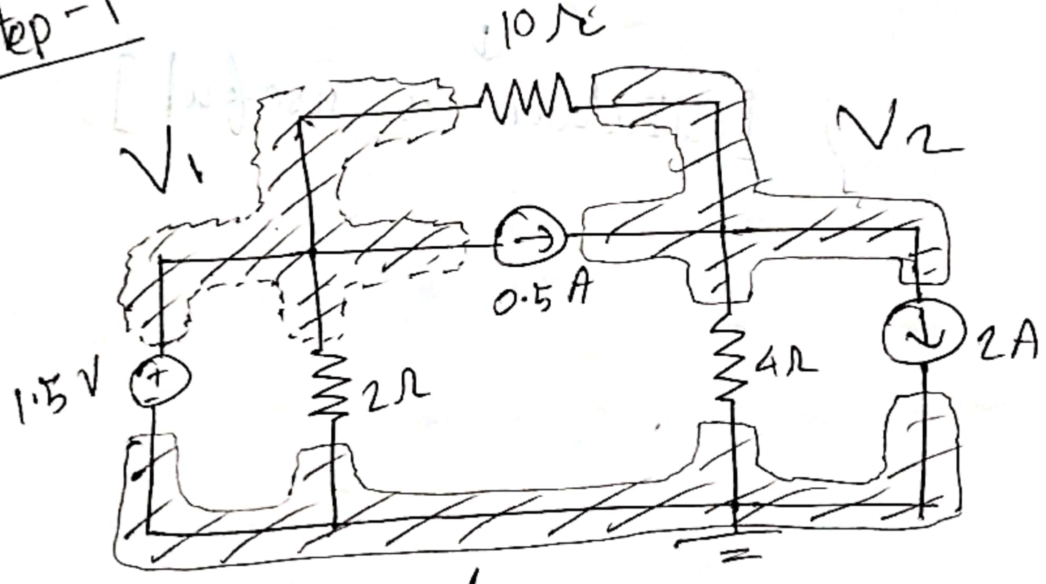
[Calculaton is useful]

Nodal Analysis

1



Step-1



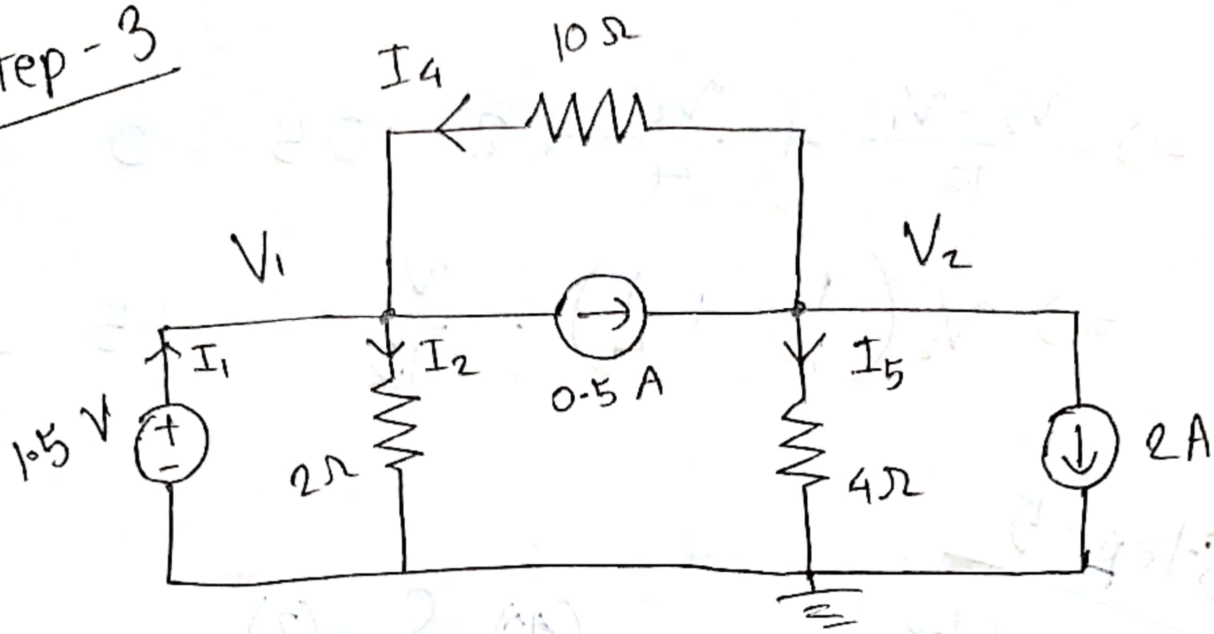
→ $V_0 = 0\,V$

Step-2

$$V_1 - V_0 = 1.5$$

$$\Rightarrow \boxed{V_1 = 1.5}\ V \quad \ldots\ldots (1)$$

Step-3



$$I_4 = \frac{V_2 - V_1}{10}$$

$$I_2 = \frac{V_1 - V_0}{2} = \frac{V_1}{2}$$

$$I_5 = \frac{V_2 - V_0}{4} = \frac{V_2}{4}$$

Step-4

Node 1 $\rightarrow$ $V_1 = 1.5\,V$

$\Downarrow$

No need to use KCL $\rightarrow$ Already Known voltage.

Node 2

$0.5 \longrightarrow$ Entering

$2, I_4, I_5 \longrightarrow$ Exiting

$$\therefore \quad I_4 + I_5 + 2 - 0.5 = 0 \quad \text{---} (E)$$

$$\Rightarrow \frac{V_2 - V_1}{10} + \frac{V_2}{4} + 2 - 0.5 = 0$$

$$\Rightarrow V_2\left(\frac{1}{10} + \frac{1}{4}\right) - \frac{V_1}{10} = -1.5 \quad \text{—— (2)}$$

Step :- 5

Using eqn. (1) & (2),

$$\boxed{\begin{aligned} V_1 &= 1.5\ V \\ V_2 &= -3.86\ V \end{aligned}}$$

| Using calculator

⇓

Can be used to answer any questions.

Current through 4Ω ?

$$I_5 = \frac{V_2 - 0}{4}$$

$$= \frac{(-3.86) - 0}{4}$$

$$= -0.965\ A$$

How much power consumed by $10\,\Omega$

$$P = \Delta V I = I^2 R = \frac{\Delta V^2}{R}$$

$$P = \frac{(V_2 - V_1)^2}{R}$$

$$= \frac{(-3.86 - 1.5)^2}{10}$$

$$= 2.87\ W$$

How much power connent source 2A supplying/consuming

$$P = \Delta V I$$

$$= (V_2 - V_0) \times 2$$

$$= (-3.86 - 0) \times 2$$

$$= -7.72\ W$$

$\hookrightarrow$ $\boxed{\text{Negative} \rightarrow \text{Not consuming} \rightarrow \text{Supplying}}$

How much power 1.5 V source consuming/supplying.

$$P = \Delta V I$$

$$= 1.5 \times I_1$$

Using KCL,

$$I_1 + I_4 = I_2 + 0.5$$

$$\Rightarrow I_1 = I_2 + 0.5 - I_4$$

$$= \frac{1.5}{2} + 0.5 - \frac{-3.86 - 1.5}{10}$$

$$= -1.486 \text{ A}$$

$$\therefore P = 1.5 \times (-1.486) \text{ W}$$

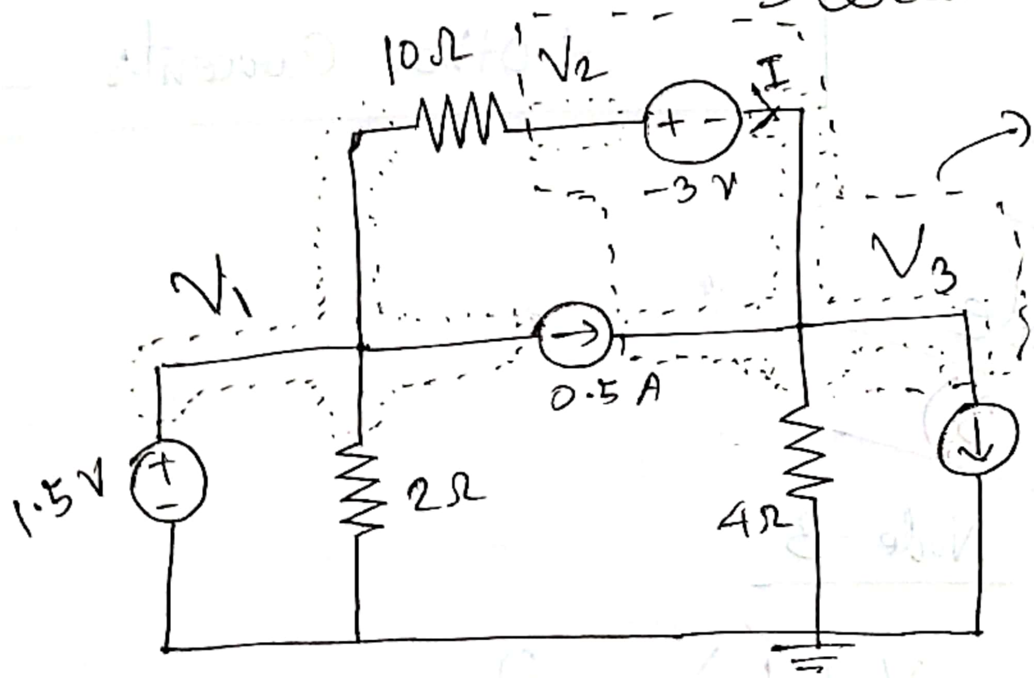
$$= -2.23 \text{ W}$$

↳ Supplying Power (-ve)

Nodal Analysis
2

Supernode → A voltage source between two non-ground nodes ⇒ $\boxed{\text{A floating Voltage Source}}$

↳ becomes $\boxed{\text{Supernode}}$



Step-1

Step-2

$$V_1 = 1.5 \quad \text{--- (1)}$$

$$\underbrace{V_2 - V_3 = -3}_{\text{Supernode}} \quad \text{(2)}$$

Shortcut to Step-4 → ~~Pre~~ See Previous Example too!

$$V_2\left(\begin{array}{c}\text{sum of} \\ \text{inverse of all resistances connected to node-2}\end{array}\right) - V_{\substack{\text{other side} \\ \text{of resistance}}}\left(\text{inverse of resistance}\right) + \text{Other Currents} = 0$$

In this Circuit

Supernode (2 & 3)

Node - 3

$$V_3\left(\frac{1}{4}\right) - \frac{0}{4} + 2 - 0.5 - I = 0 \quad \text{--- (a)}$$

Node - 2

$$V_2\left(\frac{1}{10}\right) - \frac{V_1}{10} + I = 0 \quad \text{--- (b)}$$

$$(a) + (b) \Rightarrow V_3\left(\frac{1}{4}\right) + V_2\left(\frac{1}{10}\right) - \frac{0}{4} - \frac{V_1}{10} + 2 - 0.5 = 0 \quad \text{--- (3)}$$

Step-5

$(1) \rightarrow V_1 = 1.5$

$(2) \rightarrow V_2 - V_3 = -3$

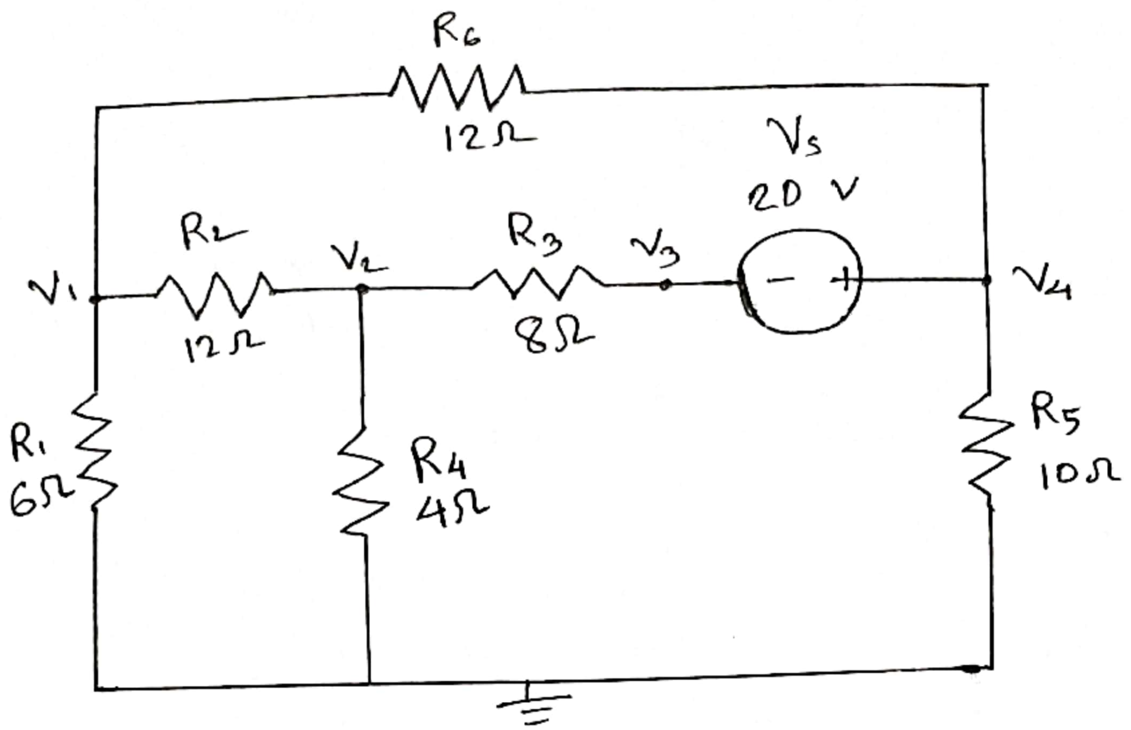
$(3) \rightarrow V_3\left(\frac{1}{4}\right) + V_2\left(\frac{1}{10}\right) - \frac{V_1}{10} - \frac{0}{4} + 2 - 0.5 = 0$

$\Downarrow$ Solution

$$\boxed{\begin{aligned} V_1 &= 1.5\ V \\ V_2 &= -6\ V \\ V_3 &= -3\ V \end{aligned}}$$

(Ans.)

[1] Nodal Analysis



Node 1

Using KCL,

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_4}{R_6} = 0$$

$$\Rightarrow V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_6}\right) - \frac{V_2}{R_2} - \frac{V_4}{R_6} = 0$$

In this circuit $\Rightarrow V_1\left(\frac{1}{6} + \frac{1}{12} + \frac{1}{12}\right) - \frac{V_2}{12} - \frac{V_4}{12} = 0$

Node 2

$$\frac{V_2}{R_4} + \frac{V_2 - V_1}{R_2} + \frac{V_2 - V_3}{R_3} = 0$$

$$\Rightarrow V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) - \frac{V_1}{R_2} - \frac{V_3}{R_3} = 0$$

$$\Rightarrow V_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) - \frac{V_1}{12} - \frac{V_3}{8} = 0$$

Common Mistake: Using $-\frac{V_1}{R_1}$ instead of $-\frac{V_1}{R_2}$.

Super Node

Node 3

$$\frac{V_3 - V_2}{R_3} - I = 0 \quad \text{------} \quad (1)$$

Node 4

$$\frac{V_4}{R_5} + \frac{V_4 - V_1}{R_6} + I = 0 \quad \text{------} \quad (2)$$

A) Component Equation $\Rightarrow V_4 - V_3 = V_s$

In this circuit $\Rightarrow V_4 - V_3 = 20$

|Answer of Ques. 3|

$\therefore$ Supernode Eqn. $\Rightarrow (1) + (2)$ |At Node 3 and 4|

$$\Rightarrow \frac{V_3}{R_3} + V_4\left(\frac{1}{R_5} + \frac{1}{R_6}\right) - \frac{V_1}{R_6} - \frac{V_2}{R_3} = 0$$

In this circuit, $\Rightarrow \frac{V_3}{8} + V_4\left(\frac{1}{10} + \frac{1}{12}\right) - \frac{V_1}{12} - \frac{V_2}{8} = 0$

|Answer of Ques. 4|

Common Mistakes:

1) Writing separate Node 3, 4 equations but not putting $-I, +I$ in them. Remember, the eqn. is incomplete without them.

2) Using $-\frac{V_1}{R_1}$ instead of $-\frac{V_1}{R_6}$.

Power

Solving for V_1, V_2, V_3, V_4 from the 4 equations,

$V_1 = 0.897\ V$ $\quad$ $V_2 = -3.34\ V$

$V_3 = -13.02\ V$ $\quad$ $V_4 = 6.98\ V$

Now, from Node-3 eqn., $\frac{V_3}{R_3} - \frac{V_2}{R_3} - I = 0$

$$\Rightarrow I = \frac{V_3}{R_3} - \frac{V_2}{R_3}$$

$$\therefore P = VI = V_s I$$

In this circuit,

$$I = \frac{-13.02}{8} + \frac{3.34}{8}$$

$$= -1.204A$$

$$\therefore P = 20 \times (-1.204)$$

$$= -24.08\ W$$

Common Mistake: Taking the sign of I wrong. Results in P becoming a positive value. Wrong according to passive sign convention which shows whether it's supplying or consuming power.