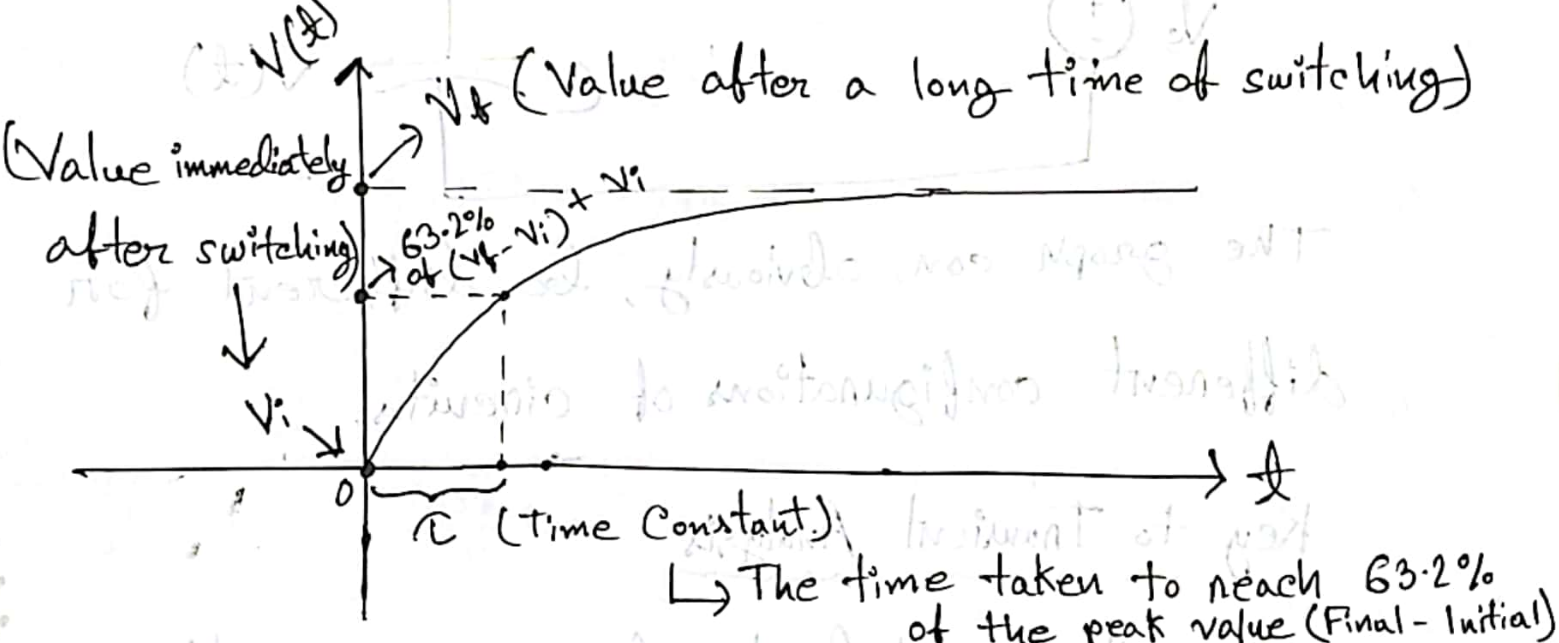
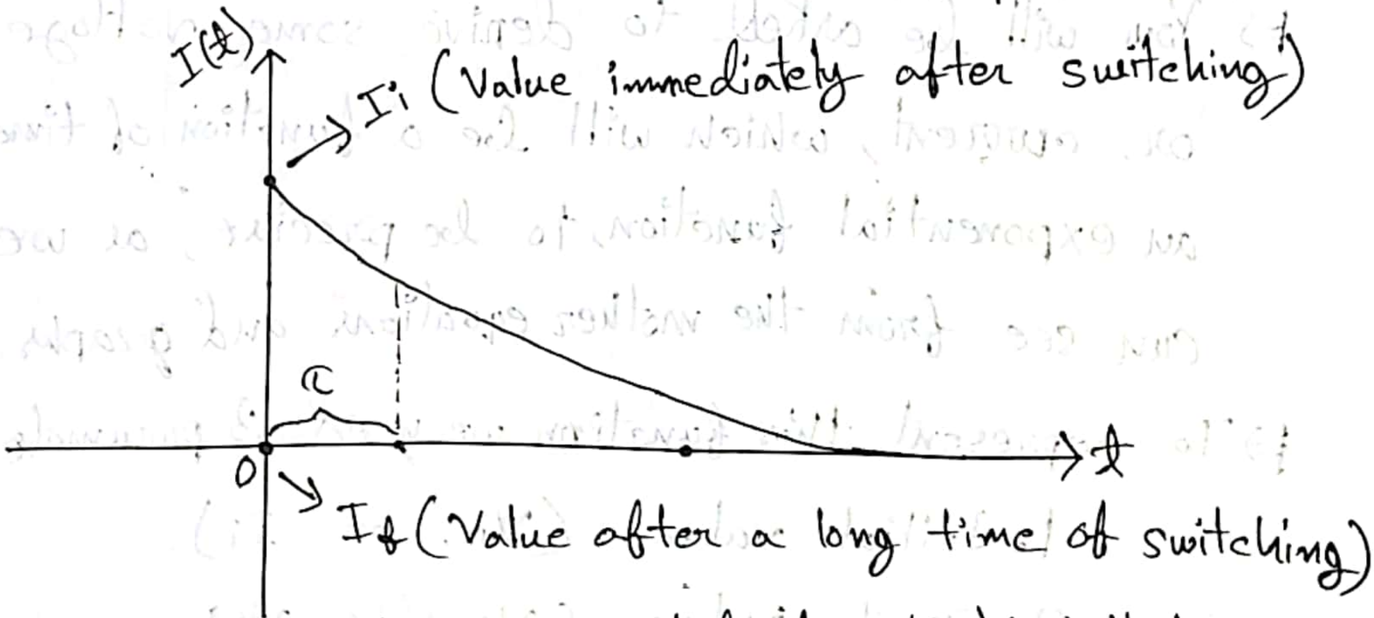
Transient Analysis (Step by Step)

Mother Equations

- $V(t) = V_f + (V_i - V_f)e^{-t/\tau}$

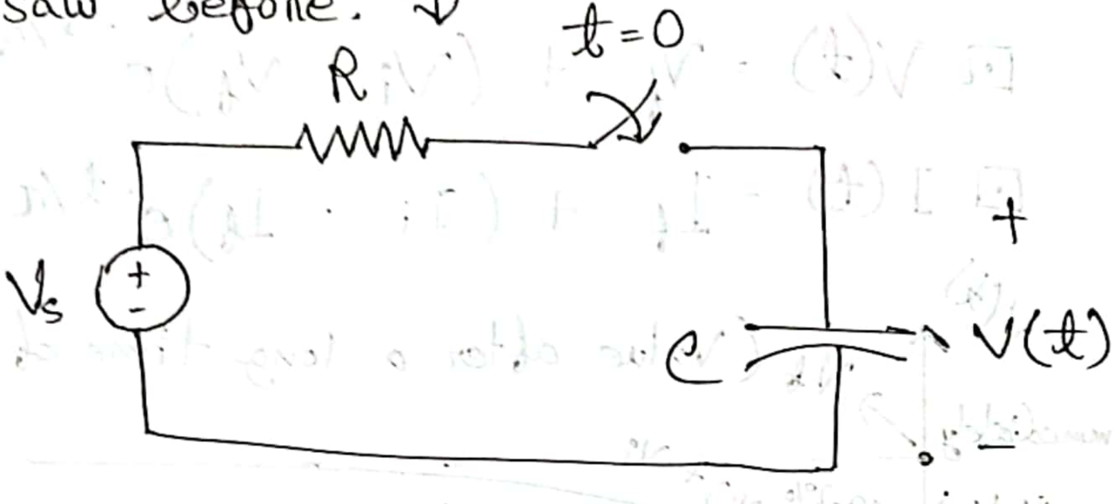
- $I(t) = I_f + (I_i - I_f)e^{-t/\tau}$





N.B:
- $V_f \rightarrow$ Also represented by $V(\infty)$; similarly, $I_f \rightarrow I(\infty)$
- $V_i \rightarrow$ Also represented by $V(0)$; similarly, $I_i \rightarrow I(0)$

The graphs correspond to the capacitive circuit we saw before. ↓



The graph can, obviously, be different for different configurations of circuits.

<u>Key to Transient Analysis</u>

⇛ You will be asked to derive some voltage or, current, which will be a function of time, an exponential function, to be precise, as we can see from the mother equations and graphs.

⇛ To represent this function, we need 3 parameters.

1. Initial value (V_i or I_i)
2. Final Value (V_f or V_i)
3. Time Constant (τ)

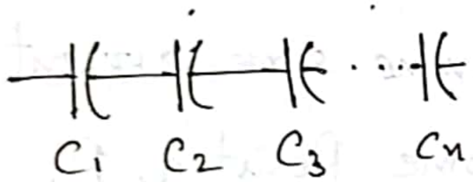
Steps of Transient Analysis

Case 1: Capacitive (RC) Circuit

Step -1

Determine τ; $\boxed{\tau = R_{eq} C_{eq}}$

- R_{eq} can be determined from the circuit after switching, taking the terminals across the capacitor.
- C_{eq} can be determined using series/parallel formulas if there are more than one capacitors.



$$\Rightarrow C_s^{-1} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

$$\Rightarrow C_p = C_1 + C_2 + \dots + C_m$$

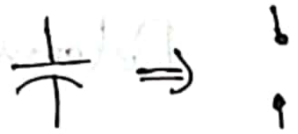
Step - 2

Determine V_i (not current), from initial circuit.

- As voltage across the capacitor doesn't change instantaneously, V_i can be determined from the circuit before switching.

Step - 3

Determine V_f from the final circuit.

- After a long time, capacitor acts as an open circuit. So, in the final circuit →  .

Step - 4

Determine V(t) from the mother equation.

- $$V(t) = V_f + (V_i - V_f)e^{-t/\tau}$$

☆ **Step - 5** (If asked to determine some current).

Case 1: Current through some Resistor, R.

↦ Use Ohm's law → $\boxed{I(t) = \frac{V(t)}{R}}$ → Voltage across that Resistor.

Case 2: Current through some Capacitor, C.

↦ Use Component Equation of Capacitor.

$$\boxed{I(t) = C\frac{dv(t)}{dt}}$$

Here, Initial Circuit = Circuit immediately after switching.

Final Circuit = Circuit a long time after switching.

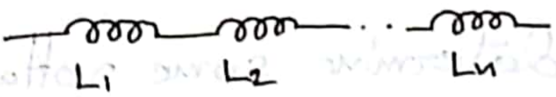
Case 2: Inductive Circuit (RL)

Step - 1

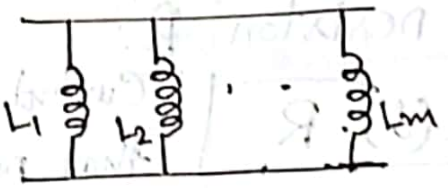
Determine Time Constant, τ ; $\boxed{\tau = \frac{L_{eq}}{R_{eq}}}$

- R_{eq} can be determined from the circuit after switching (Think: why after switching?) taking the terminals across the inductor.

- L_{eq} can be determined using series/parallel formulas if there are more than one inductors.



$\Rightarrow L_s = L_1 + L_2 + \dots + L_n$



$\Rightarrow L_p^{-1} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_m}$

Step - 2

Determine I_i (not Voltage), from initial circuit.

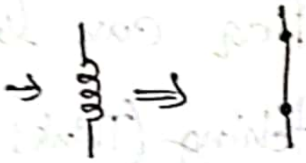
- As current through inductor doesn't change instantaneously, I_i can be determined from the circuit before switching. (Inductor can be replaced by short circuit there).

Step-3

Determine I_f from the final circuit.

- After a long time (in steady state), inductor acts as a short circuit.

So, in the final circuit →



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Step-4

Determine $I(t)$ using mother ~~forms~~ equation.

- $I(t) = I_f + (I_i - I_f)e^{-t/\tau}$

★ Step-5 (If asked to determine some voltage)

Case-1: Voltage across some resistor, R.

⇒ Use Ohm's law → $\boxed{V(t) = I(t) \cdot R}$ → Current through that resistor.

Case-2: Voltage across some inductor, L.

⇒ Use component equation of inductor.

$$\boxed{V(t) = L\frac{dI(t)}{dt}}$$

To Summarize,

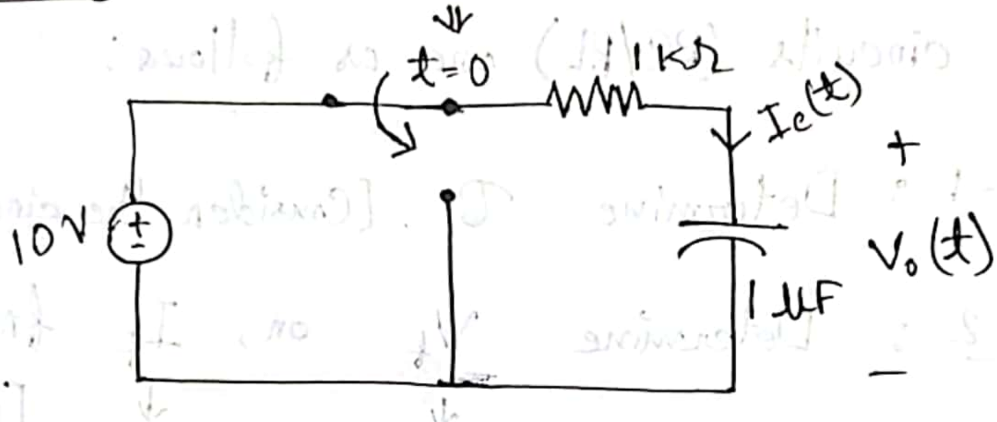
The general steps for transient analysis of first order circuits (RC/RL) are as follows:

- **Step - 1** : Determine τ. [Consider the circuit after switching]
- **Step - 2** : Determine V_f or, I_f from final circuit
  $V_f \downarrow$ (RC), $I_f \downarrow$ (RL)
  [Capacitor → Open, Inductor → Short]
- **Step - 3** : Determine V_i or, I_i from initial circuit.
  $V_i \downarrow$ (RC), $I_i \downarrow$ (RL)
  [Tip: Consider the circuit before switching]
- **Step - 4** : Determine $V(t)$ or, $I(t)$ from mother equation.
  $V(t) \downarrow$ (RC), $I(t) \downarrow$ (RL)
- ☆ **Step - 5** : Determine $I(t)$ or, $V(t)$ from ohm's law or, differential component equations.
  $I(t) \downarrow$ (RC), $V(t) \downarrow$ (RL)

Tip: It's better to draw these 3 circuits at the beginning → 1. Circuit before switching.
2. Initial Circuit (immediately after switching).
3. Final Circuit (Long time after switching).

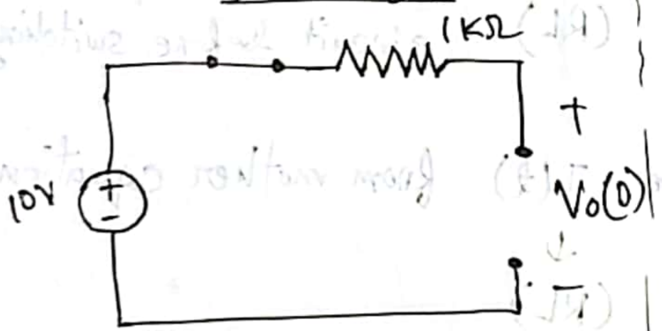
Some Examples of Step-by-Step Transient Analysis

Example 1: Source Free RC Circuit



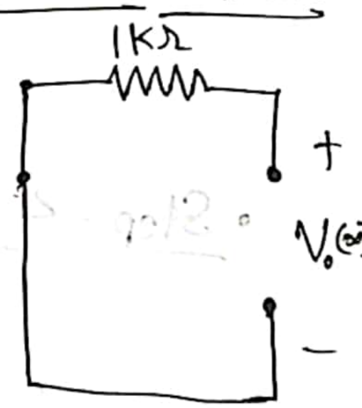
Circuits at different times

Circuit before switching



Initial Circuit

Final Circuit



Here, this kinda circuit is called the source free RC circuit since there are no active sources and the capacitor is discharged through a reverse flow of current.

Step-1: Determining τ.

From initial circuit, $R_{eq} = 1K\Omega$, $C_{eq} = 1\mu F$

$$\therefore \tau = R_{eq} C_{eq} = 1K\Omega \times 1\mu F$$
$$= 10^3 \times 10^{-6} \text{ s} = \boxed{10^{-3} \text{ s}}.$$

Step-2: Determining $V_o(0)$ from initial circuit.

Since, voltage across C can't change instantaneously, we can consider the circuit before switching to determine the initial voltage. From that circuit, we can see,

$$\boxed{V_o(0) = 10 \text{ V}}$$

Step-3: Determining $V_o(\infty)$ from final circuit.

Since no active sources or current sources ofcourse, $\boxed{V_o(\infty) = 0 \text{ V}}$.

Step-4: Determining $V_o(t)$ from mother equation.

$$V_o(t) = V_{o,final} + (V_{o,initial} - V_{o,final}) e^{-t/\tau}$$
$$= V_o(\infty) + (V_o(0) - V_o(\infty)) e^{-t/\tau}$$
$$= 0 + (10 - 0) e^{-t/10^{-3}} \text{ Volts.}$$
$$= \boxed{10 e^{-1000t} \text{ Volts}}$$

Step - 5 : Determining $I_c(t)$ from differential component equation.

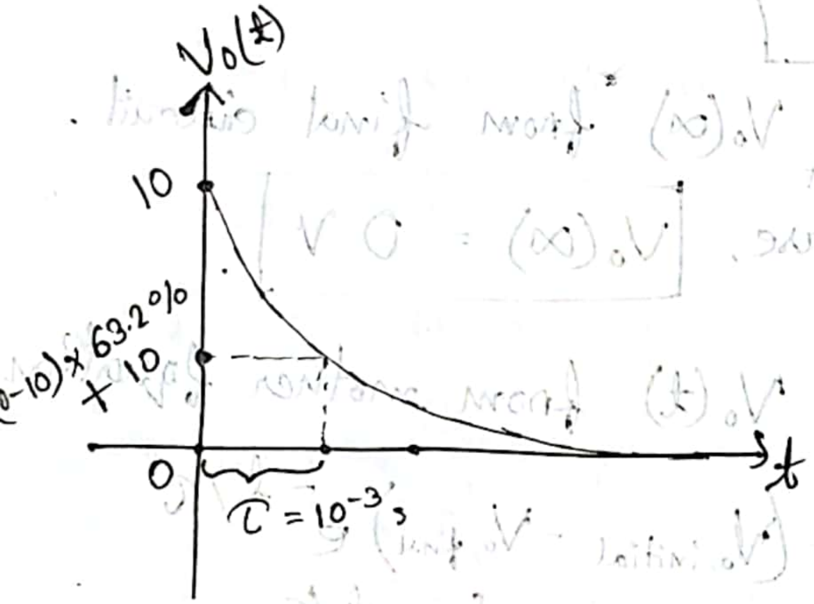
$$I_c(t) = C\frac{dV_o(t)}{dt}$$

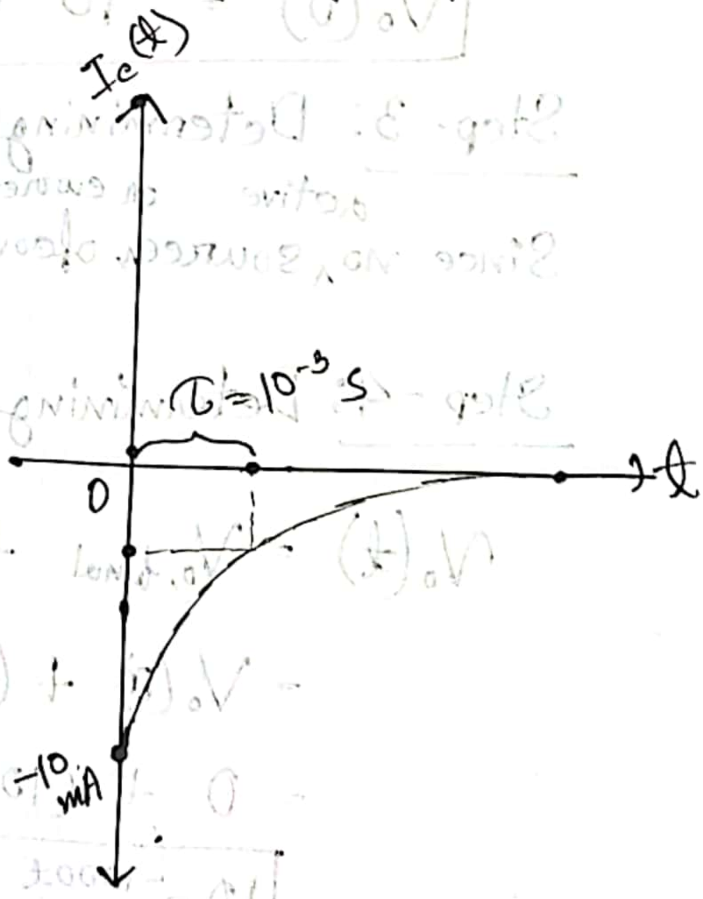
$$= 10^{-6} \times \frac{d}{dt}\left(10e^{-1000t}\right) A.$$

$$= 10^{-6} \times 10 \times (-1000) e^{-1000t} A$$

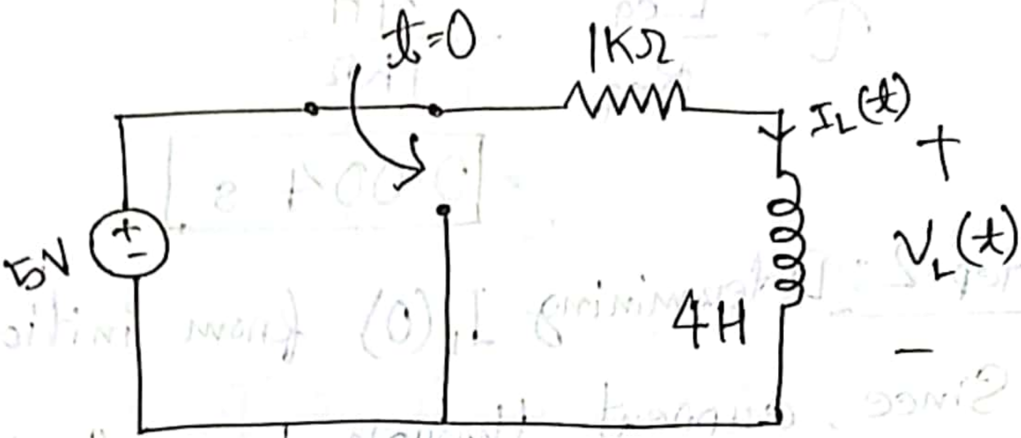
$$= -10^{-2} e^{-1000t} A$$

$$= \boxed{-10e^{-1000t} \ mA}$$

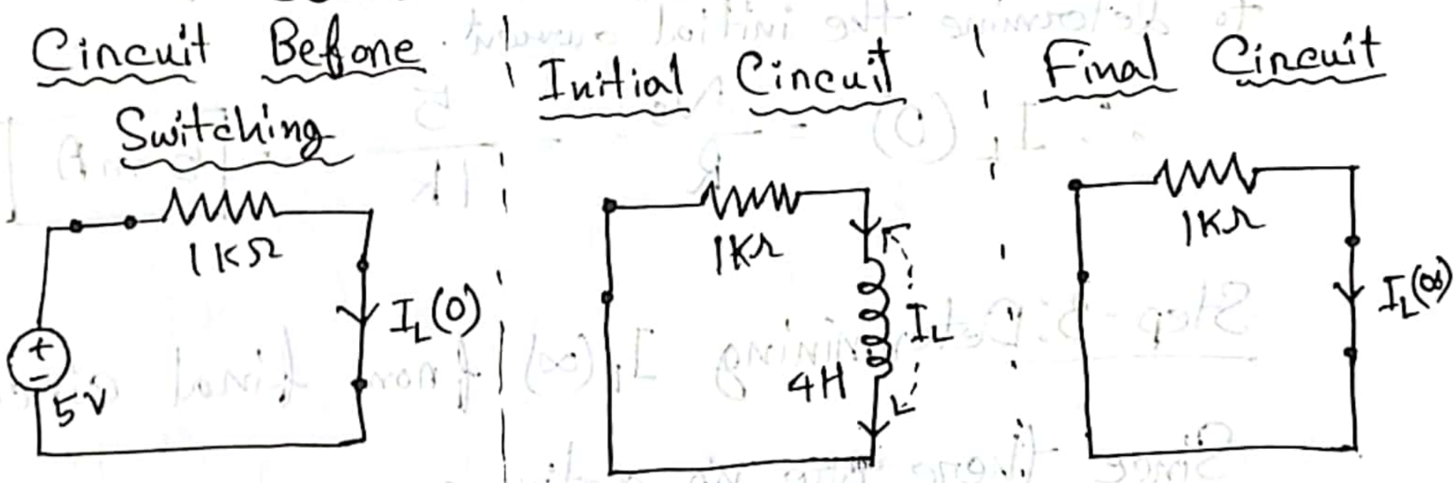




Source Free RL Circuit (Example-2)



Circuits at Different times



Here, this kinda circuit is called Source free RL circuit since there are no active sources and the inductor is discharged (using up the stored magnetic flux) through forward flow of current.

Step-1: Determining τ.

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{4H}{1K\Omega}$$

$$= \boxed{0.004 \text{ s}}$$

Step-2: Determining $I_L(0)$ from initial circuit.

Since, current through L can't change instantaneously, we can consider the circuit before switching to determine the initial current.

$$\therefore I_L(0) = \frac{V_s}{R} = \frac{5}{1K} = \boxed{5 \text{ mA}}$$

Step-3: Determining $I_L(\infty)$ from final circuit.

Since there are no active sources,

$$I_L(\infty) = \boxed{0 \text{ mA}} = 0 \text{ A}$$

Step-4: Determining $I_L(t)$ from mother equation.

$$I_L(t) = I_L(\infty) + (I_L(0) - I_L(\infty))e^{-t/\tau}$$

$$= 0 + (5-0)e^{-t/0.004} \text{ mA}$$

$$= \boxed{5e^{-250t} \text{ mA}}$$

Step-5: Determining $V_L(t)$ from differential component equation

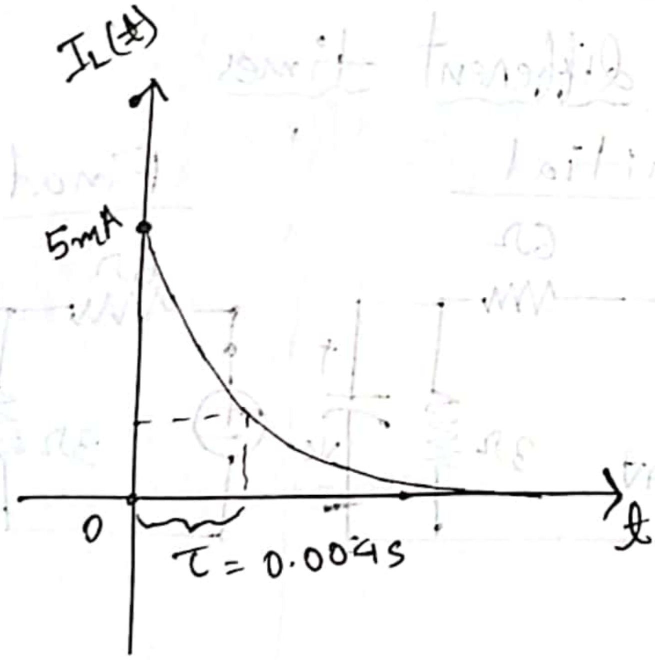
equation, $$V_L(t) = L \frac{dI_L(t)}{dt}$$

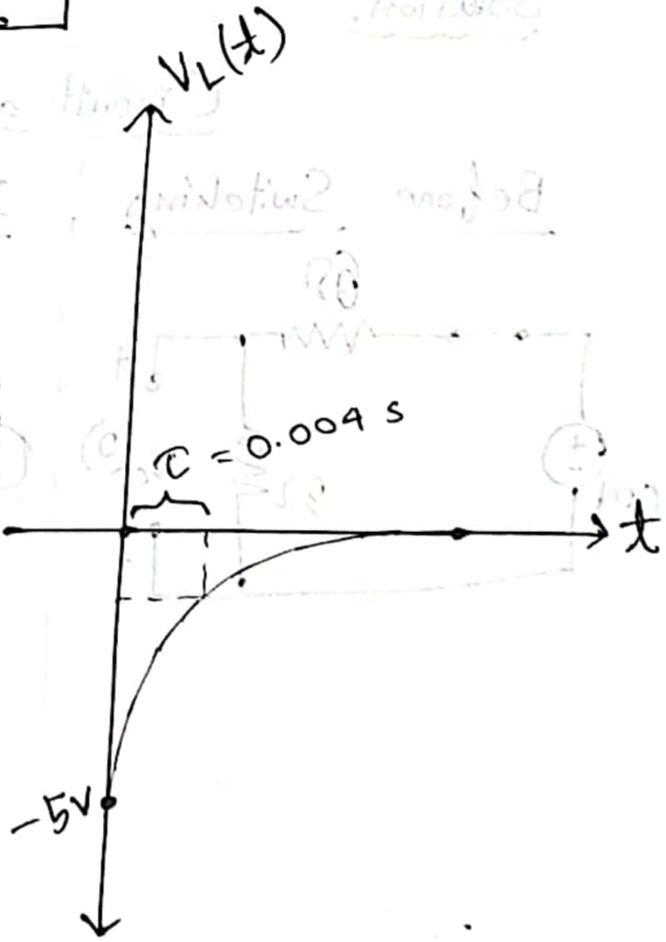
$$= 4H \times \frac{d}{dt}\left(5e^{-250t}\right) \text{ mA}$$

$$= 4 \times 5 \times 10^{-3} \times \frac{d}{dt}\left(e^{-250t}\right) \text{ A}$$

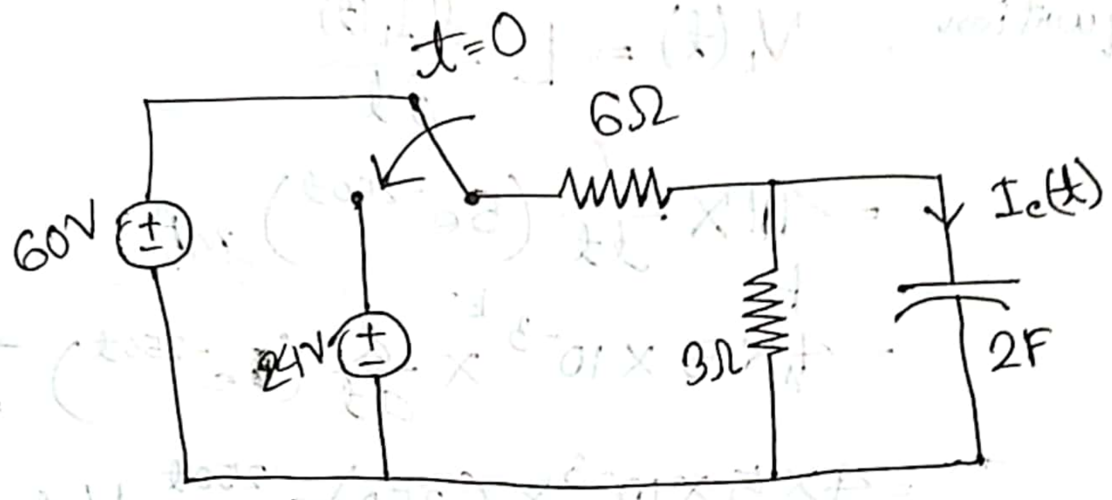
$$= 4 \times 5 \times 10^{-3} \times (-250)e^{-250t} \text{ V}$$

$$= \boxed{-5e^{-250t} \text{ V}}$$





Example 3

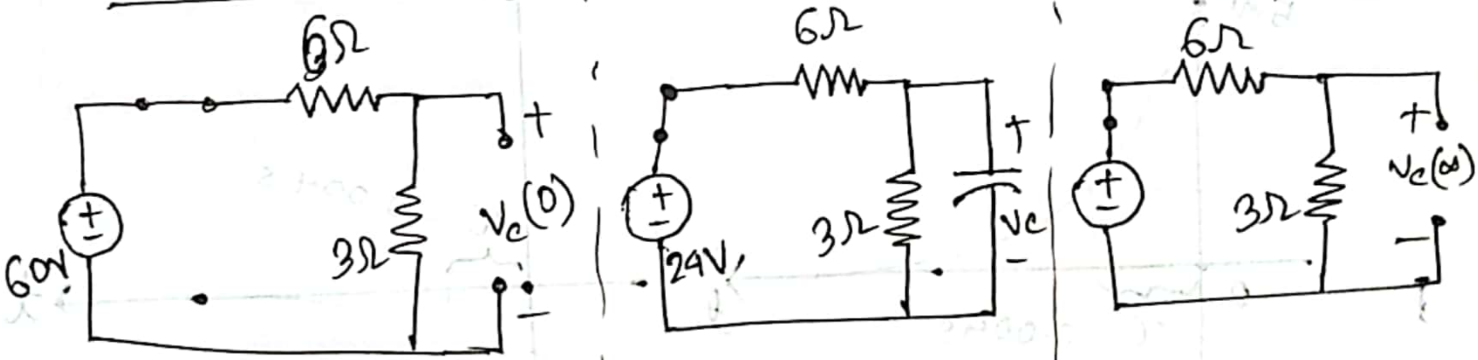


$I_c(t) = ?$ for $t > 0$

Solution:

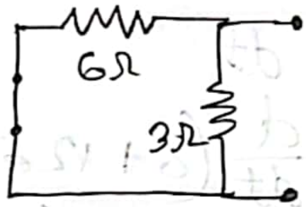
Circuit at different times

Before Switching | Initial | Final



Step-1

$$\tau = R_{eq} C \quad , \quad C = 2F$$

$R_{eq} \rightarrow$ 

$\Leftarrow R_{eq} = 6 \parallel 3 \ \Omega$

$= 2\Omega$

$$\therefore \tau = 2 \times 2 = \boxed{4s}$$

Step-2

From the circuit before switching,

$$V_c(0) = 60 \times \frac{3}{6+3} \ V$$

$$= \boxed{20 \ V}$$

Step-3

From the final circuit,

$$V_c(\infty) = 24 \times \frac{3}{6+3} \ V$$

$$= \boxed{8 \ V}$$

Step-4

$$V_c(t) = V_c(\infty) + (V_c(0) - V_c(\infty)) e^{-t/\tau}$$

$$= 8 + (20 - 8) e^{-t/4} \ V$$

$$= 8 + 12e^{-0.25t} \ V$$

Step - 5

$$I_c(t) = C \frac{dV_c(t)}{dt}$$

$$= 2 \times \frac{d}{dt}\left(8 + 12e^{-0.25t}\right) A$$

$$= 2 \times 12 \times (-0.25)e^{-0.25t} A$$

$$= \boxed{-6e^{-0.25t} \; A}$$

(Ans.)

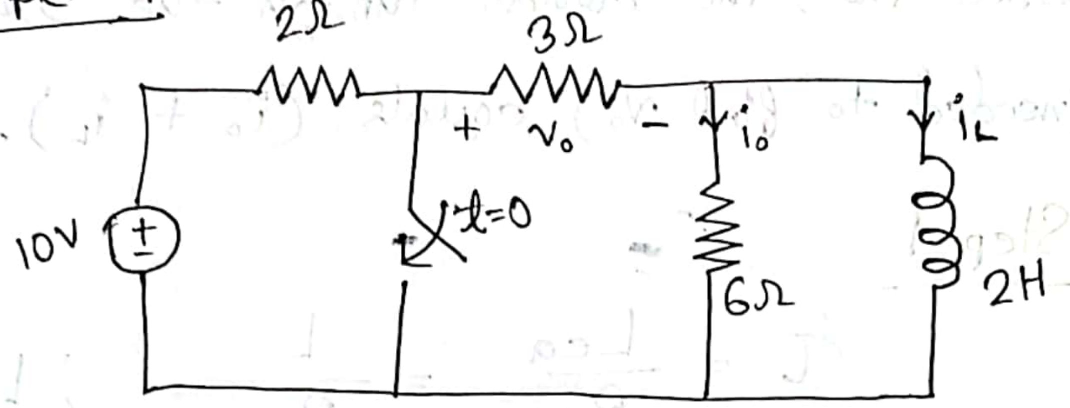
• What's the current I_c at $t = 0.5$ s?

$\Rightarrow I_c(t)$, at $t = 0.5 \Rightarrow I_c(0.5) = \boxed{-6e^{-0.25 \times 0.5} A}$

$$= \boxed{-5.29 \; A}$$

(Ans.)

Example - 4



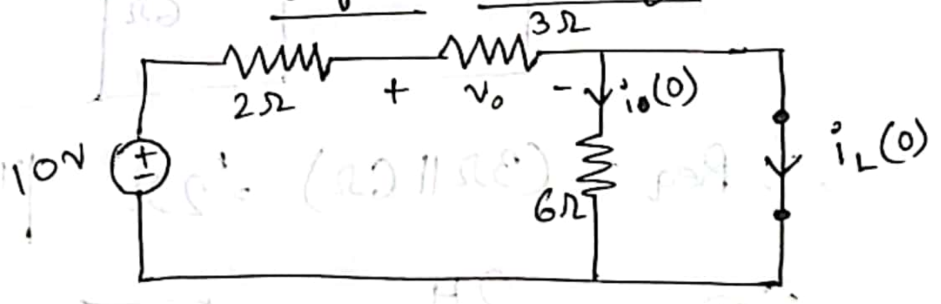
$i_0 = ?$, $v_0 = ?$, $i_L = ?$

For all time.

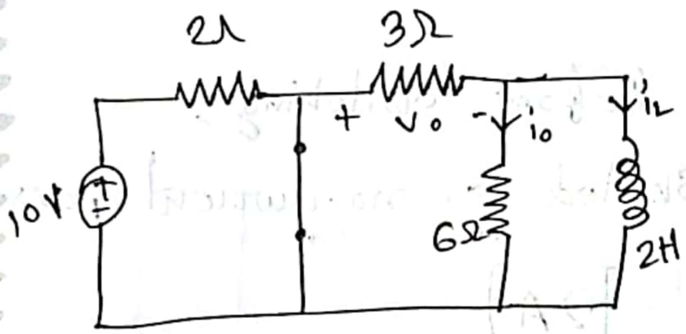
Solution:

Since Inductive (RL) circuit, Let's find currents.

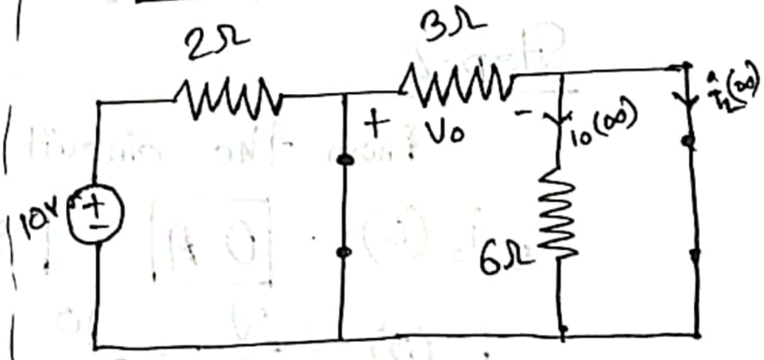
Before switching



Initial Circuit



Final Circuit



Notice that, the current through the 3Ω resistor (needed to find v_o) equals $(i_o + i_L)$.

Step-1

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{L}{R_{eq}} \quad ; L = 2H.$$

R_{eq}, from the circuit after switching, terminals across the inductor $\Rightarrow$

$\therefore R_{eq} = (3\Omega \parallel 6\Omega) = 2\Omega$ | The 2Ω got shorted |

$$\therefore \tau = \frac{2H}{2\Omega} = \boxed{1\,s}$$

Step-2

From the circuit before switching,

$i_o(0) = \boxed{0\,A}$ | Shorted, so no current pass |

$$i_L(0) = \frac{V}{R} = \frac{10}{2+3} = \boxed{2A}$$

Step - 3

From the final circuit,

$i_o(\infty) = 0A$ | Shorted, so no current pass |

$\Downarrow$

Wait, both $i_o(0)$ & $i_o(\infty)$ are 0. Doesn't sound correct, right?

Actually, current can't change instantaneously through inductor, but it can, through resistor. So, we can't find $i_o(0)$ from the circuit before switching.

But we can find $i_o(t)$ directly from the voltage across the 6Ω resistor using Ohm's law, and that voltage is the same as the voltage across the inductor (Parallel).

So, let's focus on $i_L(t)$ for now.

$i_L(\infty) = 0A$ | Shorted, so no current flow |

Step - 4

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-t/\tau}$$

$$= 0 + (2-0).e^{-t/1}$$

$$= \boxed{2e^{-t} A}$$

Step-5

Now, we need the voltage across the inductor to find $i_o(t)$.

$$\therefore v_L(t) = L \frac{di_L(t)}{dt}$$

$$= 2 \times \frac{d}{dt}(2e^{-t}) V$$

$$= -4e^{-t} V$$ = Voltage across the 6Ω resistor.

$$\therefore i_o(t) = \frac{v_L(t)}{R} = \frac{-4e^{-t}}{6}$$

$$= \boxed{-\frac{2}{3}e^{-t} A}$$

Also, the current through the 3Ω resistor,

$$= i_o(t) + i_L(t) \quad |KCL|$$

$$= -\frac{2}{3}e^{-t} + 2e^{-t} \text{ A}$$

$$= \frac{4}{3}e^{-t} \text{ A}$$

$$\therefore V_o(t) = IR$$

$$= \frac{4}{3}e^{-t} \times 3 \text{ V}$$

$$= 4e^{-t} \text{ V}$$

$\therefore$ For $t > 0$,

$$i_o(t) = -\frac{2}{3}e^{-t} \text{ A}$$

$$i_L(t) = 2e^{-t} \text{ A}$$

$$V_o(t) = 4e^{-t} \text{ V}$$

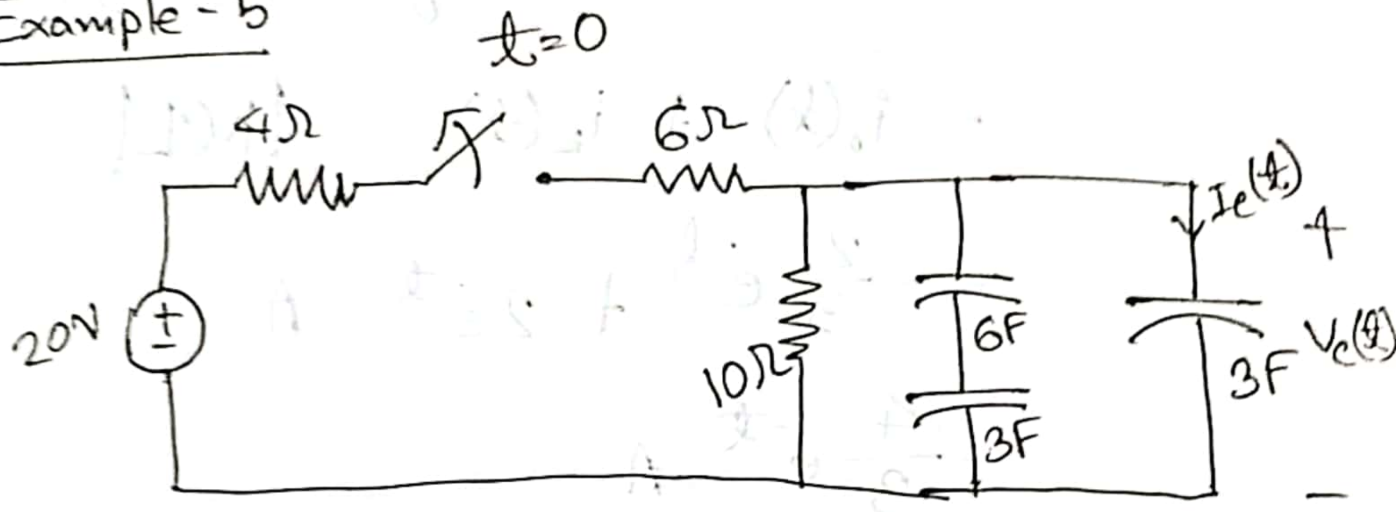
Also, examining the circuit before switching, for $t < 0$,

$$i_o(t) = 0 \text{A}$$

$$i_L(t) = 2 \text{A}$$

$$V_o(t) = 10 \times \frac{3}{2+3} \text{ V} = 6 \text{ V}$$

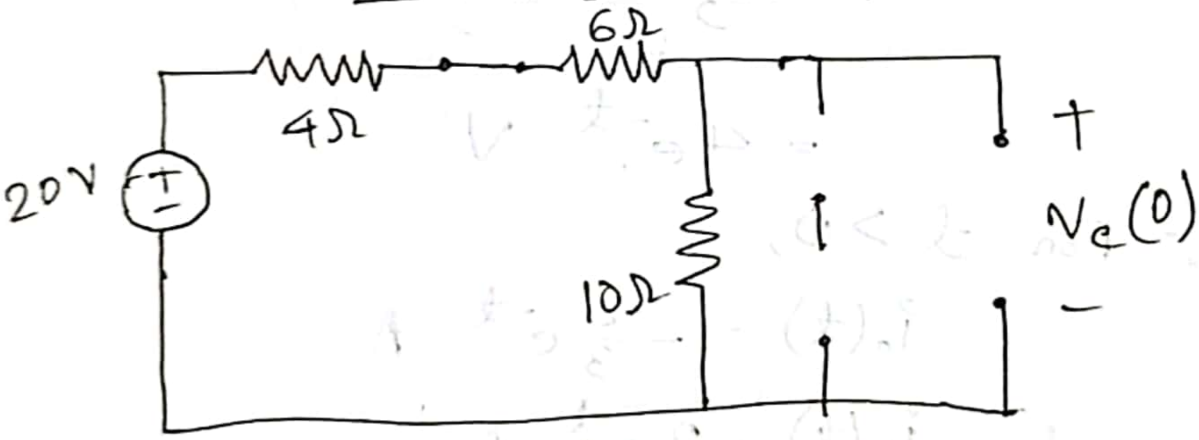
Example - 5



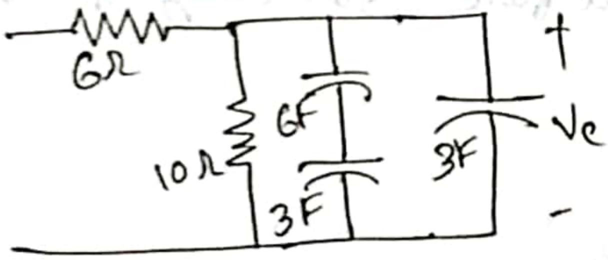
$$V_c(t),\ i_c(t) = ?$$

Solution:

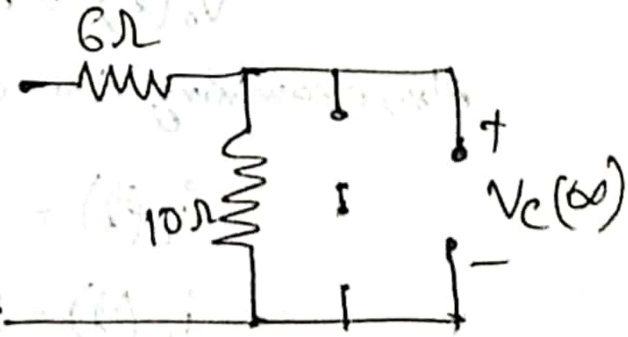
Before Switching



Initial



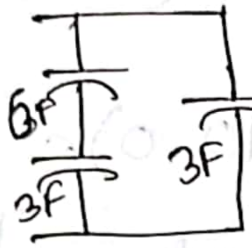
Final



Step-1

$$\tau = R_{eq} C_{eq}$$

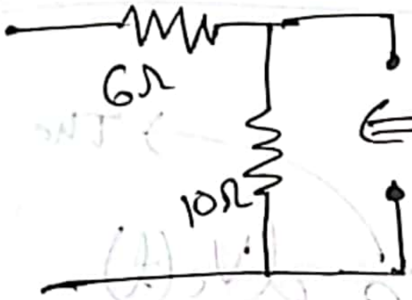
$C_{eq} \rightarrow$



$$\Rightarrow C_{eq} = \left[\left(\frac{1}{6} + \frac{1}{3}\right)^{-1} + 3\right] F$$

$$= 5F$$

$R_{eq} \rightarrow$



$\Leftarrow R_{eq}$ (From C_{eq} seen)

$$\therefore R_{eq} = 10\,\Omega$$

$$\therefore \tau = 5 \times 10 \text{ s} = 50 \text{ s}$$

Step-2

From the circuit before switching,

$$V_c(0) = 20 \times \frac{10}{4+6+10} \text{ V} = 10 \text{ V}$$

Step-3

From the final circuit,

$V_c(\infty) = 0$ V | no active source |

Step - 4

$$\therefore V_c(t) = V_c(\infty) + (V_c(0) - V_c(\infty))e^{-t/RC}$$

$$= 0 + (10 - 0)e^{-t/50} \text{ V}$$

$$= \boxed{10e^{-0.02t} \text{ V}}$$

Step - 5

The capacitor through which the current passing.

$$I_c(t) = C\frac{dV_c(t)}{dt}$$

$$= 3 \times \frac{d}{dt}\left(10e^{-0.02t}\right)$$

$$= 3 \times 10 \times (-0.02)e^{-0.02t} \text{ A}$$

$$= \boxed{-0.6e^{-0.02t} \text{ A}}$$