

Lecture 10

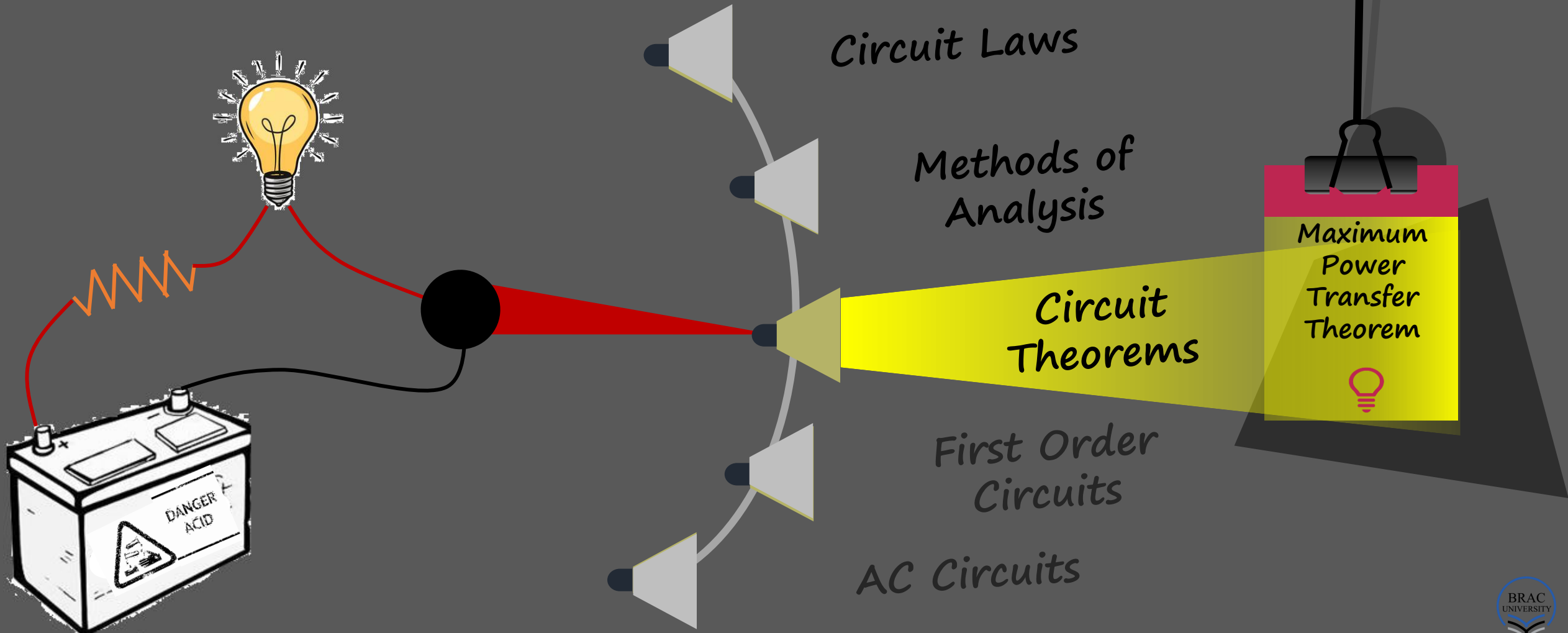
CSE250 - Circuits and Electronics

MAXIMUM POWER TRANSFER THEOREM



PURBAYAN DAS, LECTURER
Department of Computer Science and Engineering (CSE)
BRAC University

Course Outline: broad themes



Maximum Power Transfer

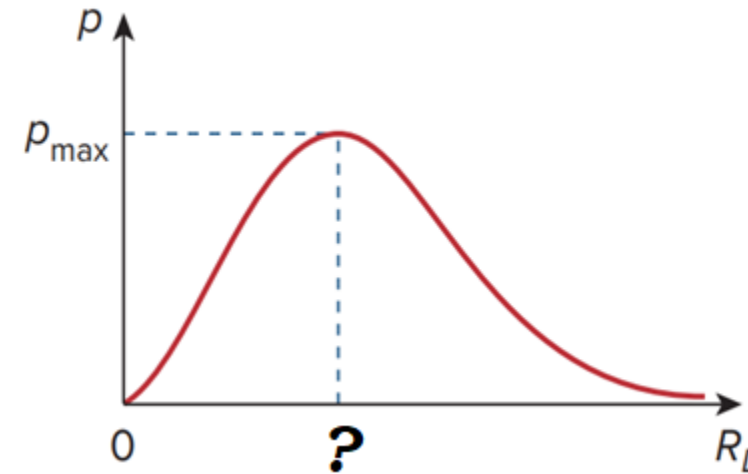
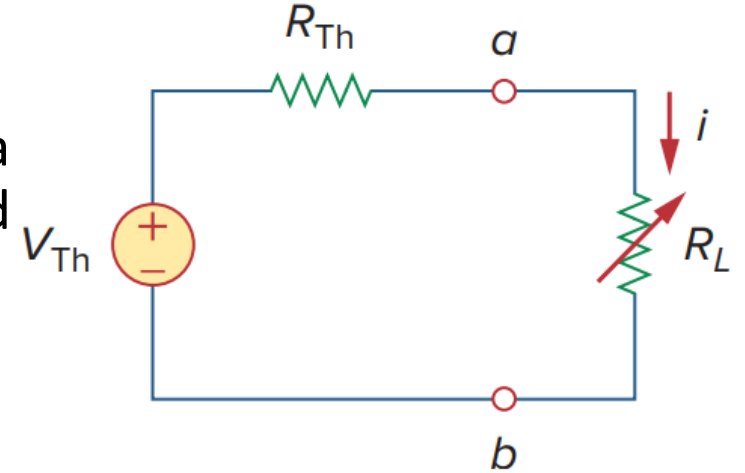
- In many practical situations, a circuit is designed to provide power to a load. There are applications in areas such as communications and amplification where it is desirable to maximize the power delivered to an antenna (load) and a speaker respectively.
- When designing a circuit, it is often important to be able to answer the question, *"What load should be applied to a system or what driving circuitry for a particular load should be designed to ensure that the load is receiving maximum power from the system or from the circuit respectively?"*
- Given a system with known internal losses the Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L .
- Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$). This is known as the *Maximum Power Transfer Theorem*.

Graphically

- Given any linear two terminal circuit, it can be reduced to a Thevenin equivalent as shown. Power delivered to the load by the Thevenin equivalent circuit is then,

$$p = i^2 R = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

- For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L , the power delivered to the load varies as sketched in the figure.
- Notice that the power is small for small or large values of R_L but maximum for some value of R_L between 0 and ∞ .
- Let's now see mathematically that this maximum power occurs when R_L is equal to R_{Th} .

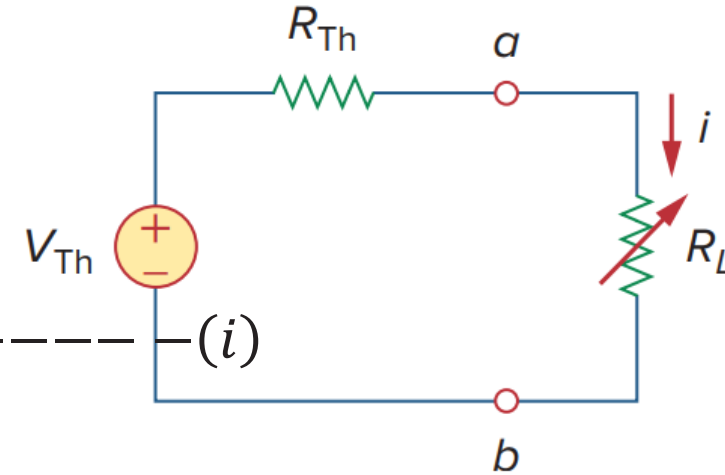


Mathematically

- The Thevenin equivalent circuit for a load R_L is shown below. The load current is, $i = \frac{V_{Th}}{R_{Th} + R_L}$.

- Power delivered to the load is,

$$p = i^2 R = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L = V_{Th}^2 \left[\frac{R_L}{(R_{Th} + R_L)^2} \right] \quad \text{--- (i)}$$



- Differentiating with respect to R_L ,

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 \frac{d}{dR_L}(R_L) - R_L \frac{d}{dR_L} \{(R_{Th} + R_L)^2\}}{(R_{Th} + R_L)^4} \right] = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L (R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right]$$

- Setting $\frac{dp}{dR_L}$ to zero will lead to the condition for maximum power transfer to the load.

Condition to P_{\max} transfer & P_{\max}

- For maxima/minima,

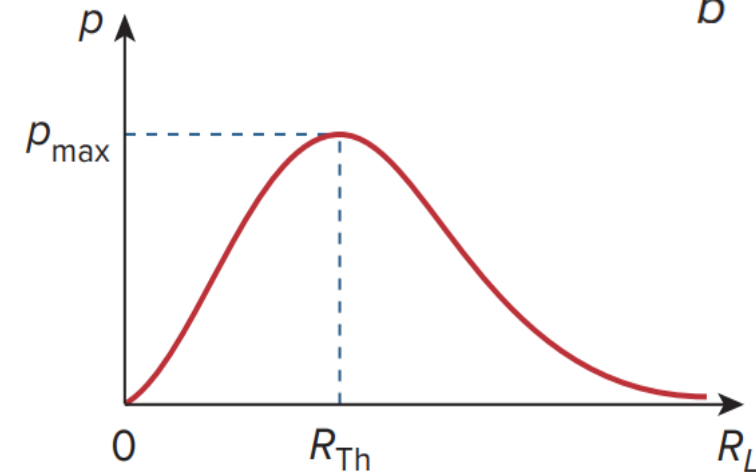
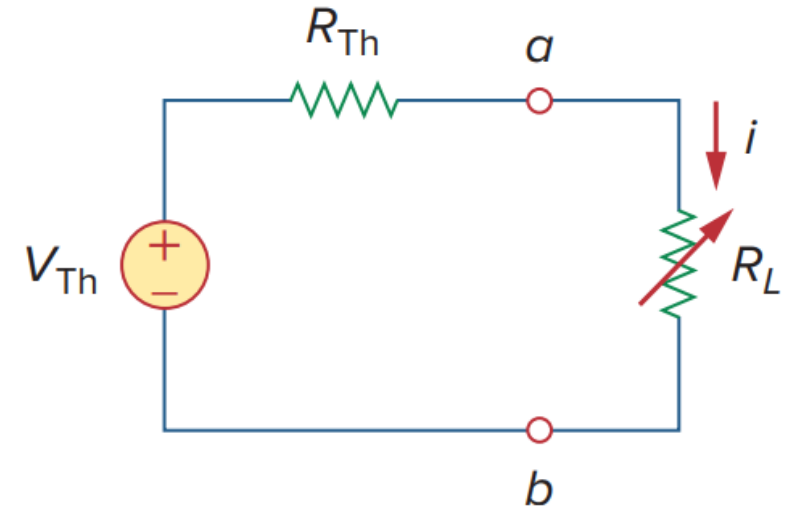
$$\frac{dp}{dR_L} = 0 = V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3} \right]$$

$$\Rightarrow R_{Th} + R_L - 2R_L = 0$$

$$\Rightarrow R_L = R_{Th}$$

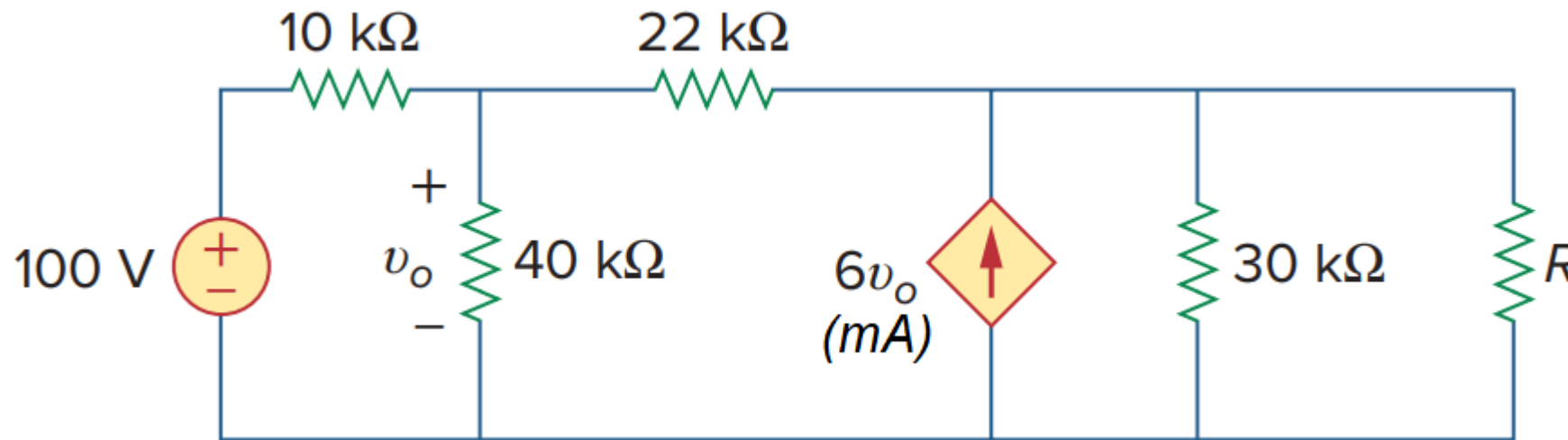
Substituting in (i) in the previous slide,

$$p_{\max} = \frac{V_{Th}^2}{4R_{Th}}$$



Example 1

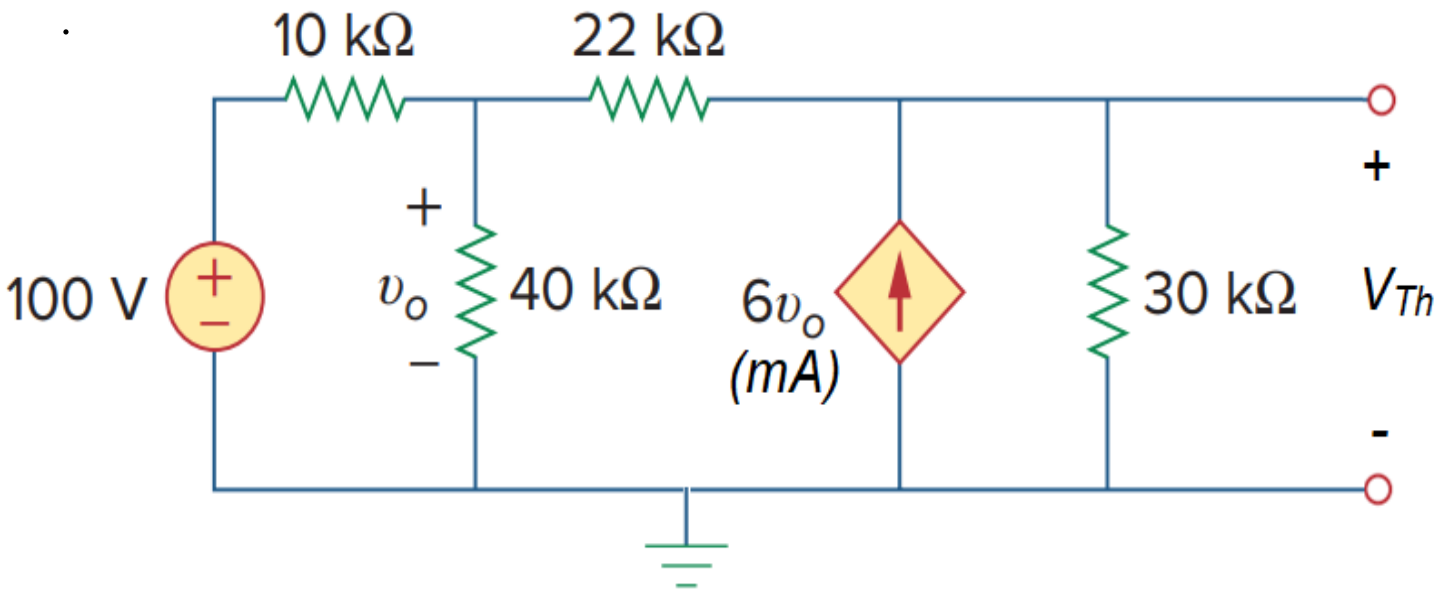
- Find the maximum power that can be delivered to the resistor R .



Ans: $V_{Th} = -231.304 \text{ V}$; $R_{Th} = -650 \Omega$; $p_{max} = \infty$ (Theoretically)

* See solution in the next slide if necessary

Example 1: finding V_{Th}



To find P_{max} , we have to first find V_{Th} and R_{Th} .

Let's use nodal analysis to find the V_{Th} .

KCL at node v_o ,

$$\frac{v_o - 100}{10} + \frac{v_o}{40} + \frac{v_o - V_{Th}}{22} = 0$$

$$\Rightarrow 75v_o - 20V_{Th} = 4400 \text{ --- (i)}$$

KCL at node V_{Th} ,

$$\frac{V_{Th} - v_o}{22} + \frac{V_{Th}}{30} = 6v_o$$

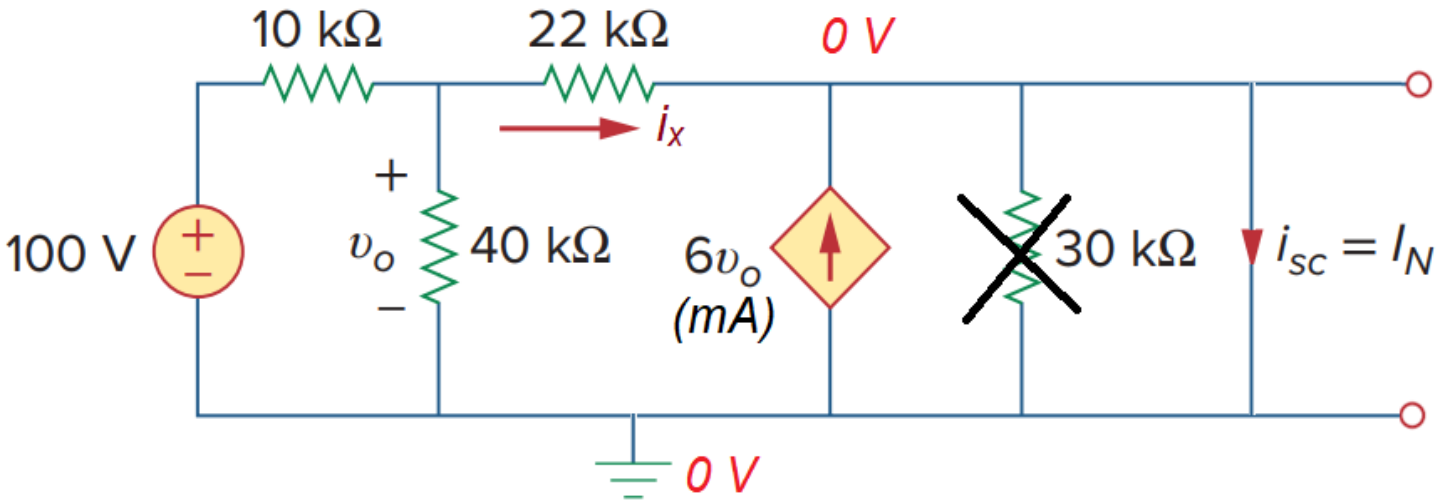
$$\Rightarrow 1995v_o - 26V_{Th} = 0 \text{ ---- (ii)}$$

Solving (i) and (ii),

$$V_{Th} = -231.304 \text{ V}$$



Example 1: finding R_{Th}



As $V_{Th} \neq 0$, let's use $R_{Th} = \frac{V_{Th}}{I_N}$ to determine the Thevenin equivalent resistance. The load terminals have been short circuited as shown in the figure.

Upon short circuiting the terminals $a - b$, the 10Ω is shorted out. The whole circuit to the left of the dependent source is shorted with respect to it. As a result, the $6v_o$ current supplied by the dependent source will only flow through the short circuit.

Let's use nodal analysis to solve for the current i_x going towards the short circuit through the $22 k\Omega$ resistor.

KCL at node v_o ,

$$\frac{v_o - 100}{10} + \frac{v_o}{40} + \frac{v_o - 0}{22} = 0$$

$$\Rightarrow 75v_o = 4400$$

$$\Rightarrow v_o = 58.667 V$$

$$\Rightarrow i_x = \frac{v_o - 0}{22} = 2.667 mA$$

So,

$$I_N = i_x + 6v_o = 354.669 mA$$

$$R_{Th} = \frac{V_{Th}}{I_N} = -650 \Omega$$



Example 1: finding P_{\max}

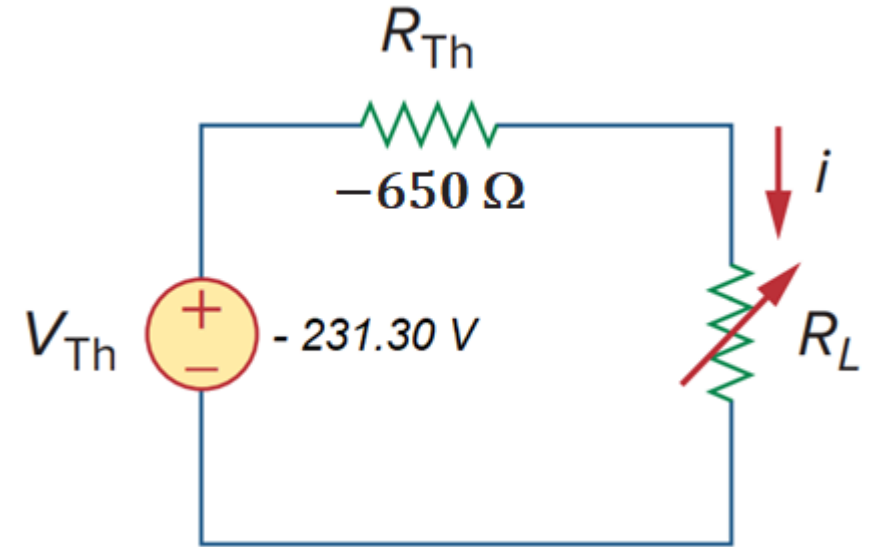
From the previous slides,

$$V_{Th} = -231.30 \text{ V}; \quad R_{Th} = -650 \Omega$$

What does a negative Thevenin resistance mean!

Negative Thevenin resistance is a part of the circuit model. The conversion of an actual circuit to a Thevenin equivalent is a mechanism for solving circuit problems and does not mean that the Thevenin equivalent circuit replaces the real circuit in all aspects.

Again, negative resistance means an active circuit. This means the circuit is trying to deliver infinite power to the load (assuming the load is practical, that is, $R_L > 0$).



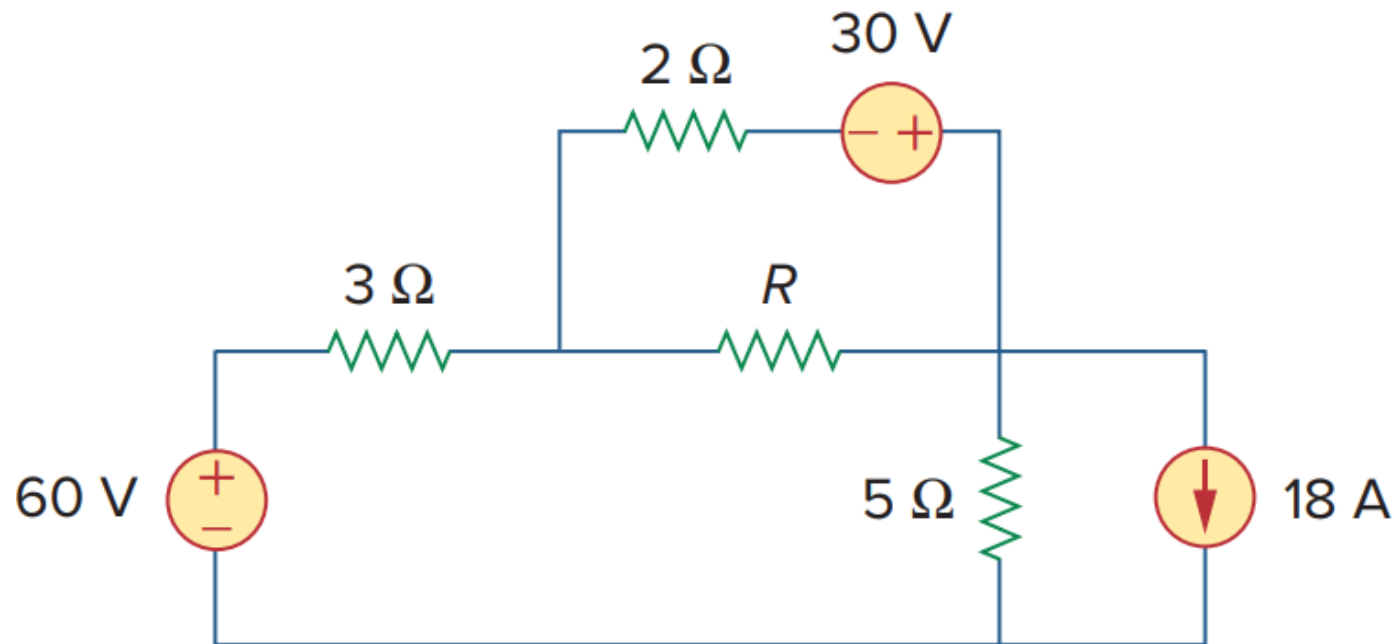
So, the correct answer is,

$$i = \frac{V_{Th}}{R_{Th} + R_L} = \frac{-231.30}{-650 + 650} = \infty$$

$$p_{\max} = i^2 R_L = \infty \text{ (theoretically)}$$

Problem 1

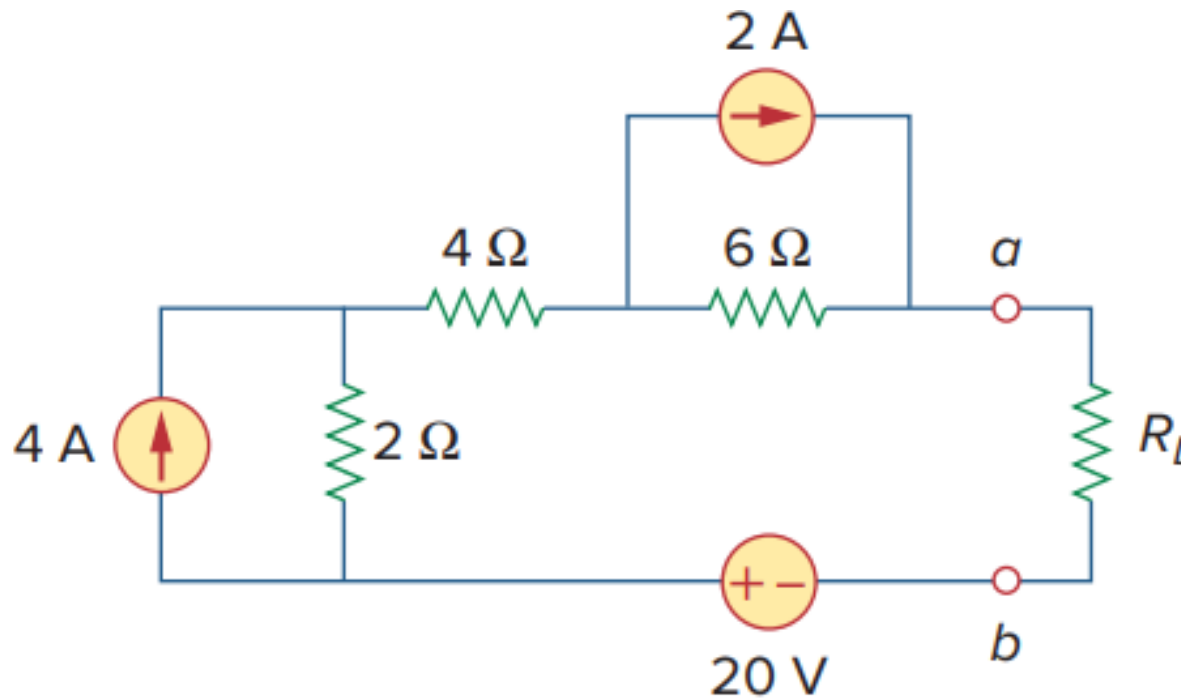
- Find the maximum power that can be delivered to the resistor R . Calculate the power efficiency at the maximum power point.



Ans: $V_{Th} = 6\text{ V}$; $R_{Th} = 1.6\ \Omega$; $p_{max} = 5.625\text{ W}$; $\eta = 50\%$

Problem 2

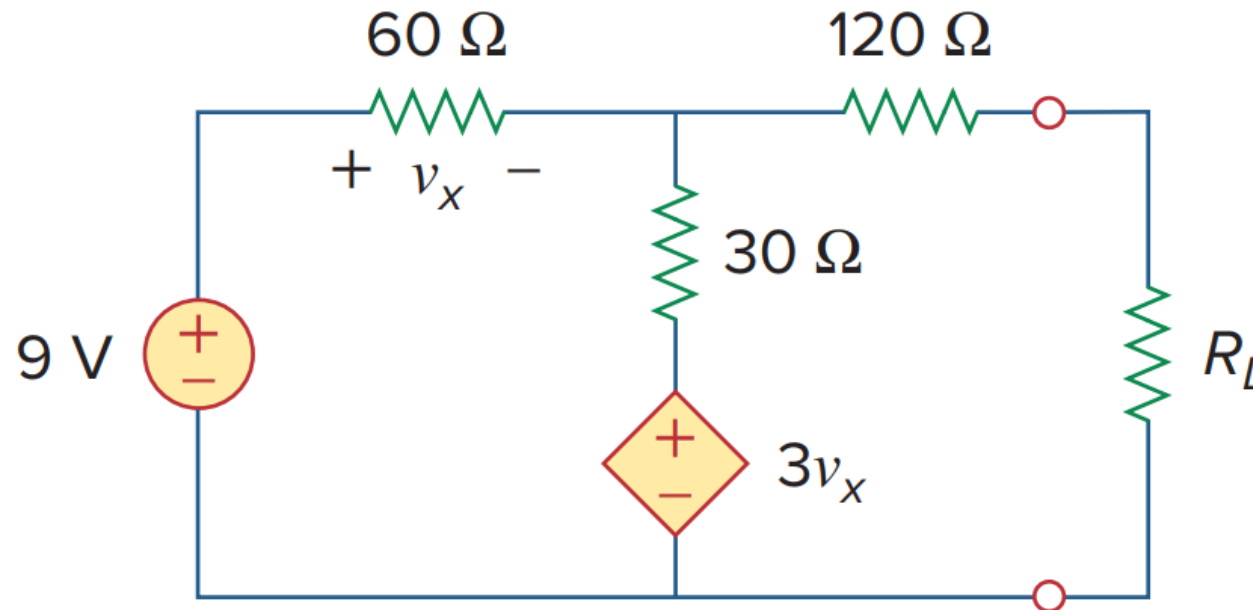
- (a) For the circuit in the figure below, obtain the Thevenin equivalent at terminals $a - b$.
- (b) Calculate the current if $R_L = 8 \Omega$.
- (c) Find R_L for maximum power deliverable to R_L .
- (d) Determine that maximum power.



Ans: $V_{Th} = 40 \text{ V}$; $R_{Th} = 12 \Omega$; $p_{max} = 33.33 \text{ W}$

Problem 3

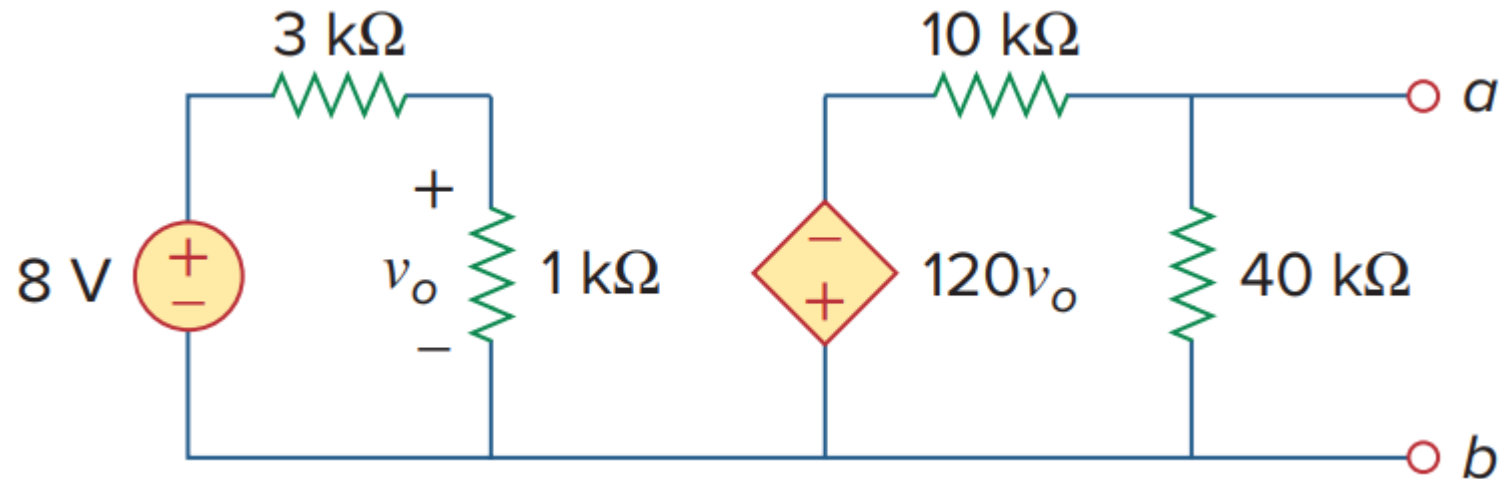
- Determine the value of R_L that will draw the maximum power from the rest of the circuit. Calculate the maximum power.



Ans: $R_L = 126.67 \Omega$; $p_{max} = 96.71 \text{ mW}$

Problem 4

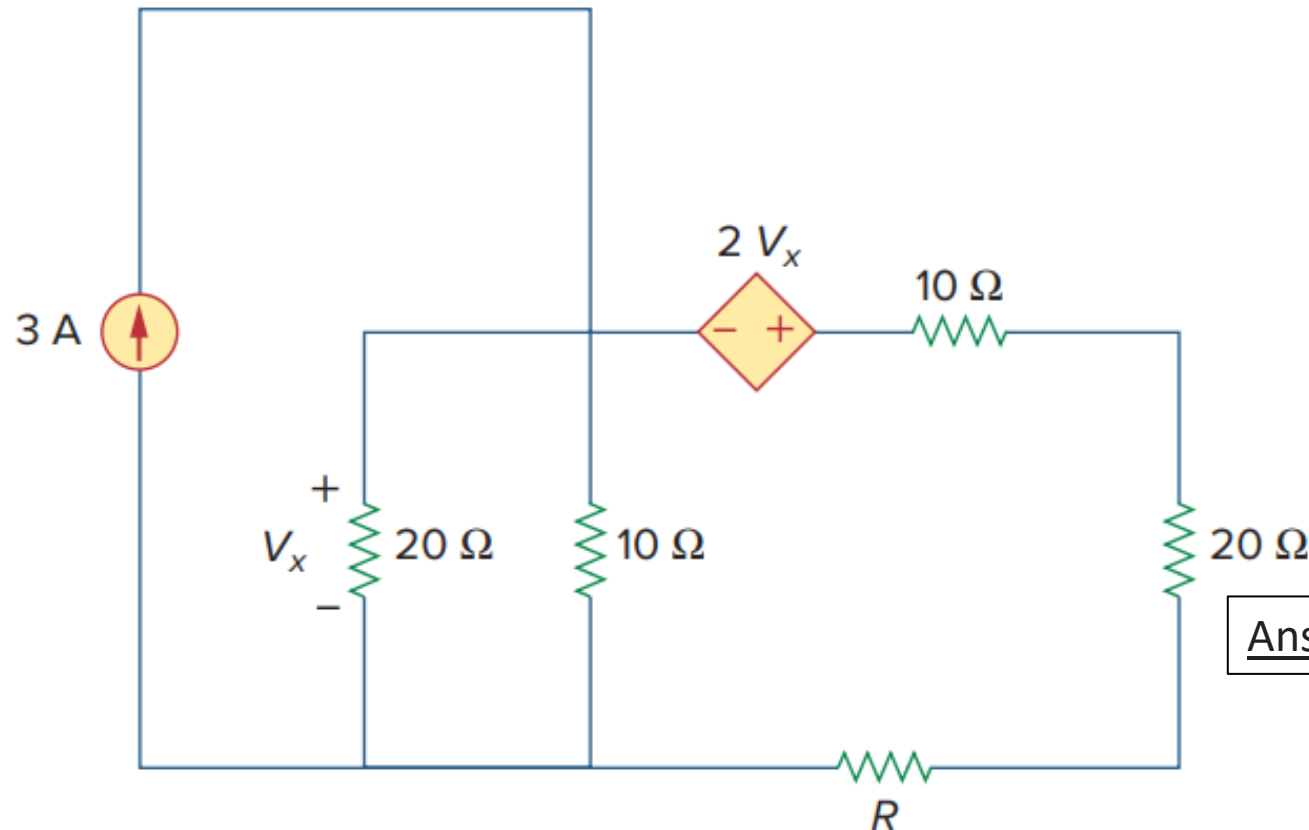
- What resistor connected across terminals will absorb maximum power from the circuit? What is that power?



Ans: $V_{Th} = -192\text{ V}$; $R_{Th} = 8\text{ k}\Omega$; $p_{max} = 1.152\text{ W}$

Problem 5

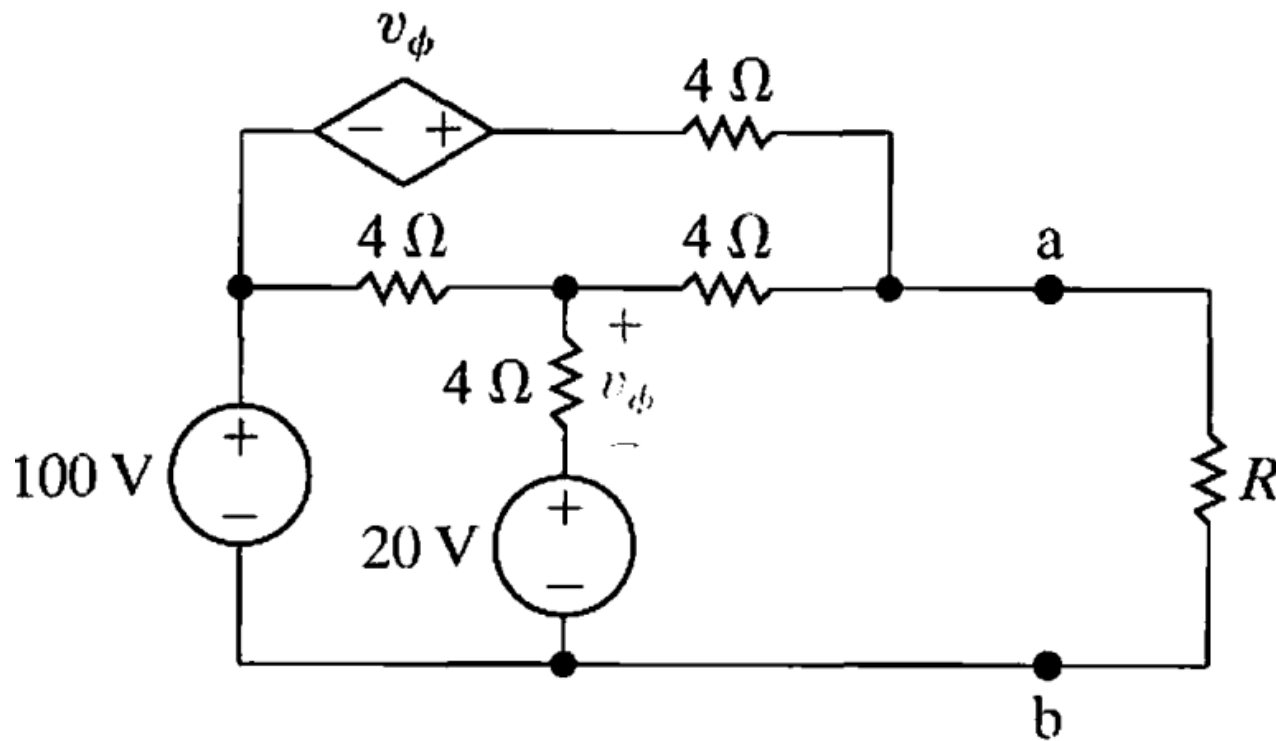
- Determine the maximum power delivered to the variable resistor R shown.



Ans: $V_{Th} = -60\text{ V}$; $R_{Th} = 50\ \Omega$; $p_{max} = 18\text{ W}$

Problem 6

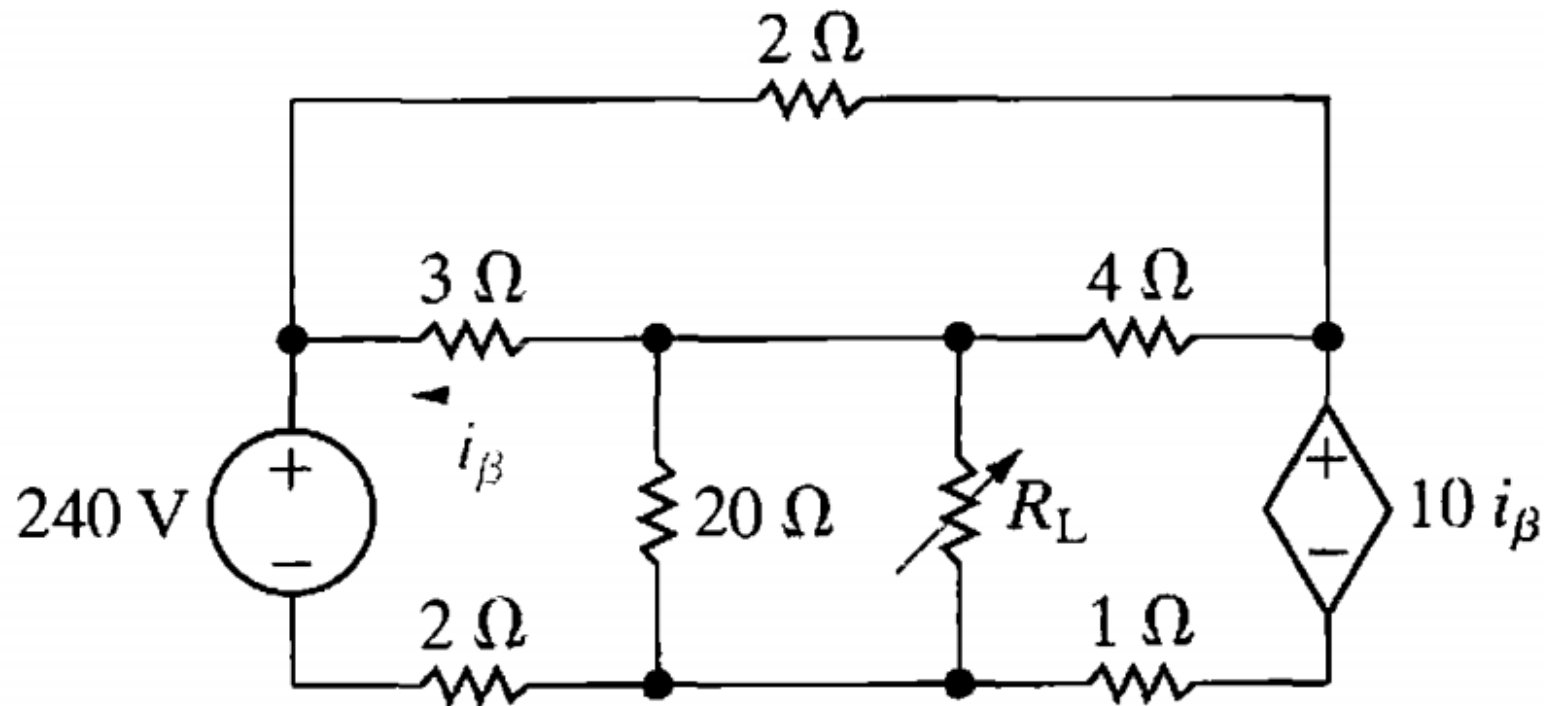
- Find the value of R that enables the circuit shown to deliver maximum power to the terminals $a - b$.
- Find the maximum power delivered to R .



Ans: (i) 3Ω ; (ii) $V_{Th} = 120 V$; $P_{max} = 1.2 kW$

Problem 7

- Find the value of R_L that enables the circuit shown to deliver maximum power to the load (R_L).
- Find the maximum power delivered to R_L .



Ans: (i) 6Ω (ii) $P_{max} = 24 W$

Practice Problems

- Additional recommended practice problems: [here](#)
- Other suggested problems from the textbook: [here](#)



Thank you for your attention

