Understanding Graphs & Extracting Data from them

First Order RC & RL Circuits (Transient Analysis)

3 parameters: Final Voltage, Initial voltage, Time constant

Common Formula

$$\text{Voltage } v(t) = V(\infty) + \left(V(0) - V(\infty)\right)e^{-t/\tau}$$

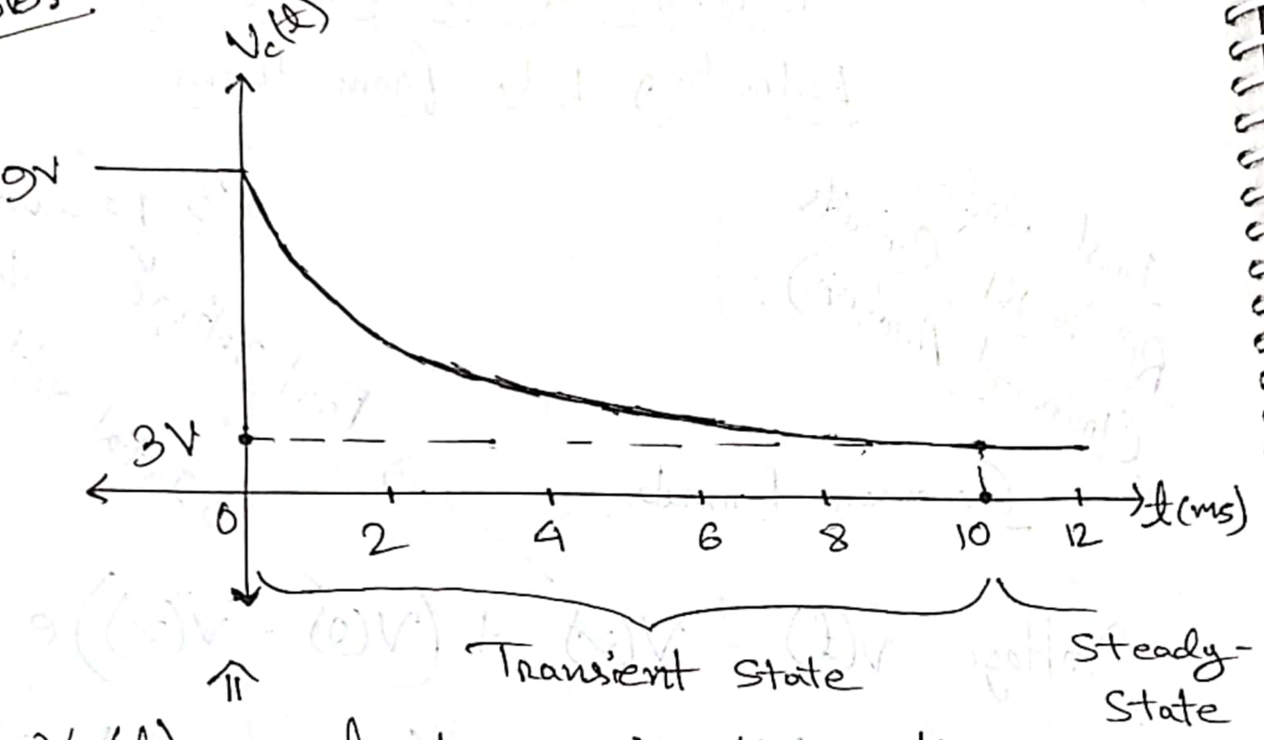
$$\text{Current } i(t) = i(\infty) + \left(i(0) - i(\infty)\right)e^{-t/\tau}$$

Final Current, Initial Current, Time Constant

Trick: If you are told to generate the equation from a given graph, find the three parameters from the graph (initial value, final value, Time Constant) and put them in the equation. Time constant may come from charging/discharging time.

Examples

1.



$V_c(t)$ vs. t here is the voltage response of a capacitor in a series RC circuit with $R_{eq} = 2\,K\Omega$.

i) Here Time Constant (τ) = ? C = ?

Ans.) From the graph, the full discharging time, 5τ is 10 ms.

$$\therefore 5\tau = 10 \text{ ms}$$

$$\Rightarrow \tau = \frac{10}{5} \text{ ms}$$

$$\therefore \tau = \boxed{2 \text{ ms.}}$$

(Ans.)

Again, We Know, $\tau = R_{eq} C$

$$\Rightarrow C = \left(\frac{R_{eq}}{\tau}\right)^{-1} = \frac{\tau}{R_{eq}}$$

$$= \left(\frac{2\,K\Omega}{2\,ms}\right)^{-1} = \frac{2\,ms}{2\,K\Omega}$$

$$= \boxed{1\,\mu F}$$

(Ans.)

ii) $v_c(t) = ?$ for $t > 0$.

Ans./ From the graph, Initial voltage $V_c(0) = 9V$

Final voltage $V_c(\infty) = 3V$

From i, Time Constant $\tau = 2\,ms$

$= 0.002\,s$

∴ Using the basic formula,

$$v_c(t) = V_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau}\ V$$

$$= 3 + (9-3)e^{-t/0.002}\ V$$

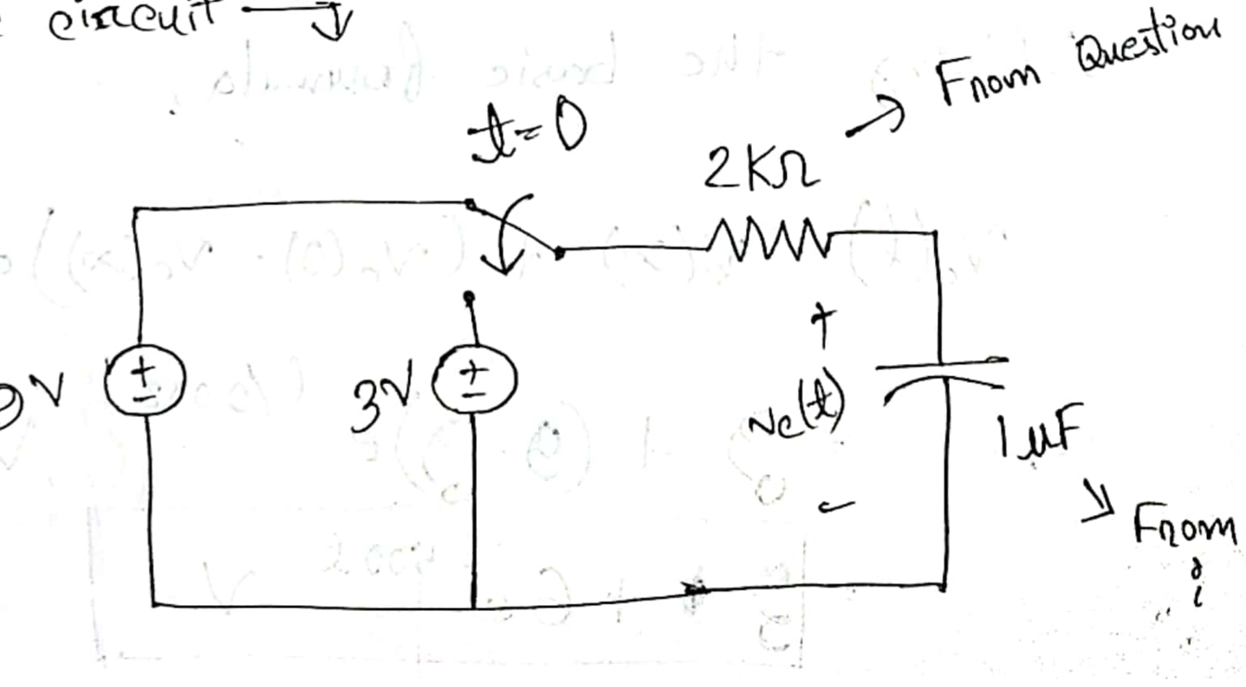
$$= \boxed{3 + 6e^{-500t}\ V}$$

(Ans.)

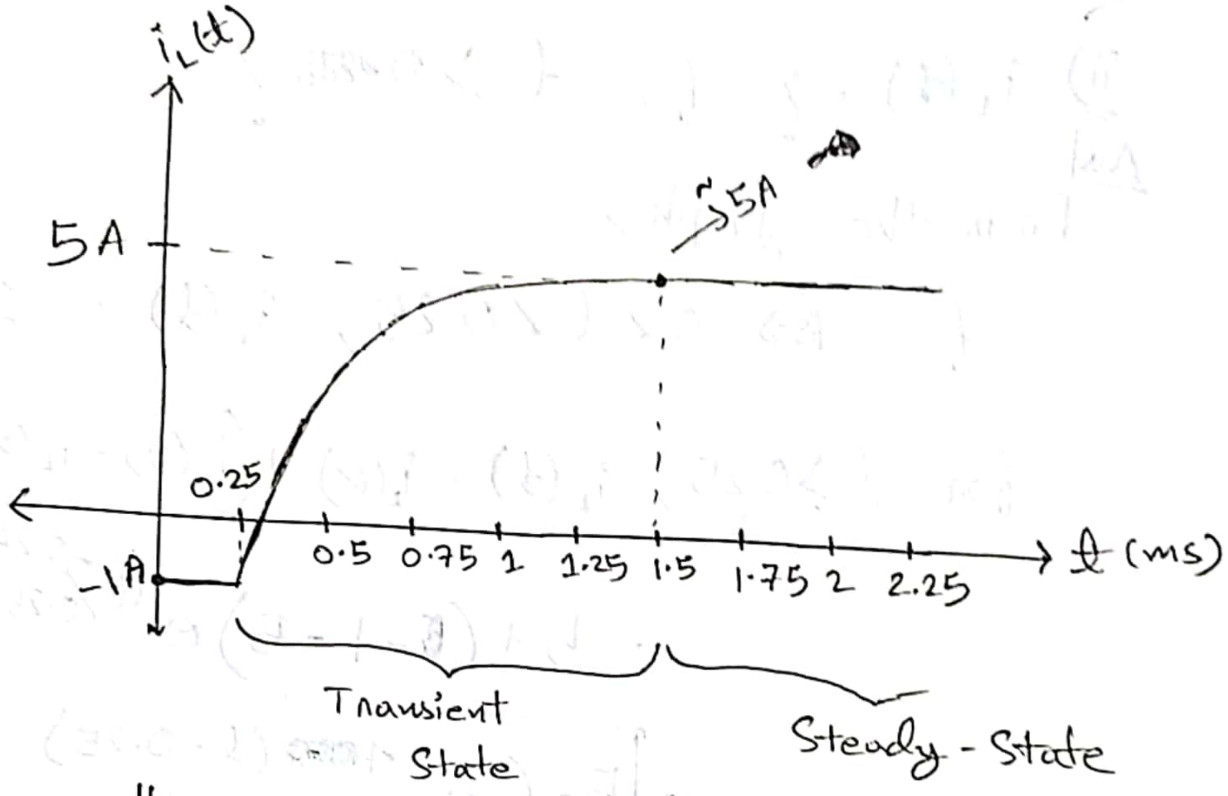
(iii) Predict & draw the circuit that generates $v_c(t)$.

Ans.) There can be multiple answers. But the simple answer here would be the simplest RC circuit with two voltage sources generating the initial and final voltages. For currents in RL circuits, it would be two current sources generating the initial and final current.

So, for $v_c(t) = 3 + (9-3)e^{-t/0.002}$ V, the circuit →



2)



⇓

Current response of an inductor in a series RL Circuit with $R_{eq} = 4\,K\Omega$.

i) $\tau = ?$ $L = ?$

Here, Charging time, $5\tau = (1.5 - 0.25)$ ms | From the graph

$= 1.25$ ms

$\therefore \tau = \frac{L}{R_{eq}} = \frac{1.25}{5}$ ms. $= \boxed{0.25 \text{ ms.}}$

$\therefore L = \tau \cdot R_{eq} = 0.25$ ms $\times 4\,K\Omega$

$= \boxed{1\,H}$

ii) $i_L(t) = ?$ for $t > 0$?

Ans

From the graph,

for $0 < t < 0.25$, $i_L(t) = -1\,A$

for $t \geq 0.25$, $i_L(t) = i_L(\infty) + \left(i_L(0) - i_L(\infty)\right)e^{-(t-0.25)/\tau}\ A$

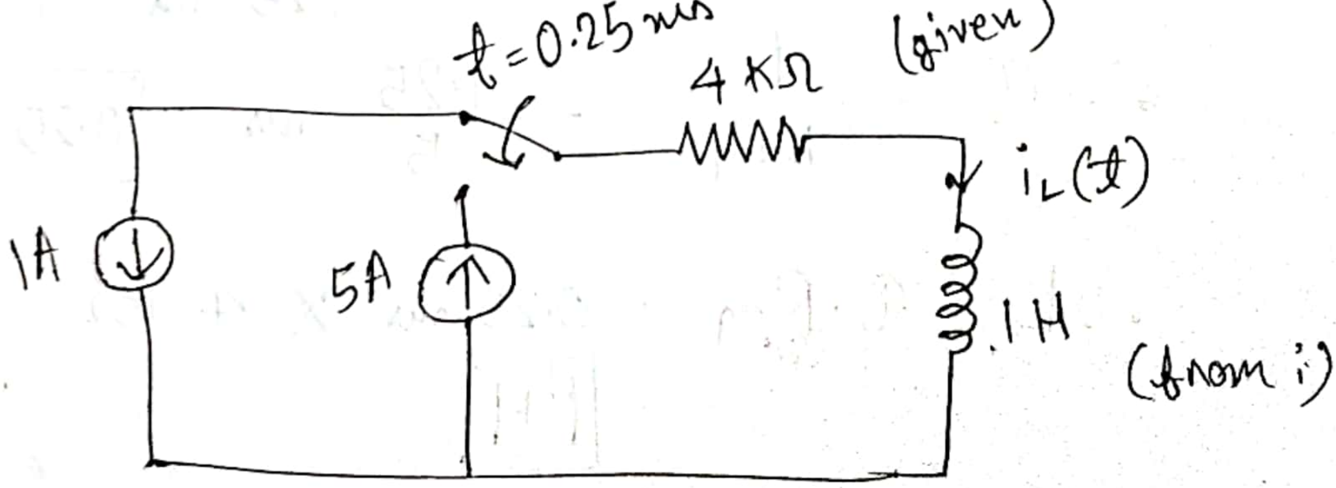
$= 5 + (-1-5)e^{-(t-0.25)/0.25}\ A$

$= \boxed{5 - 6e^{-4(t-0.25)}\ A}$

This -0.25 comes due to time shift.

The switching happens at $t = 0.25\,ms$

(iii) Circuit $\rightarrow$?



AC Circuits

Common Formula

$$\omega = \frac{2\pi}{T}$$

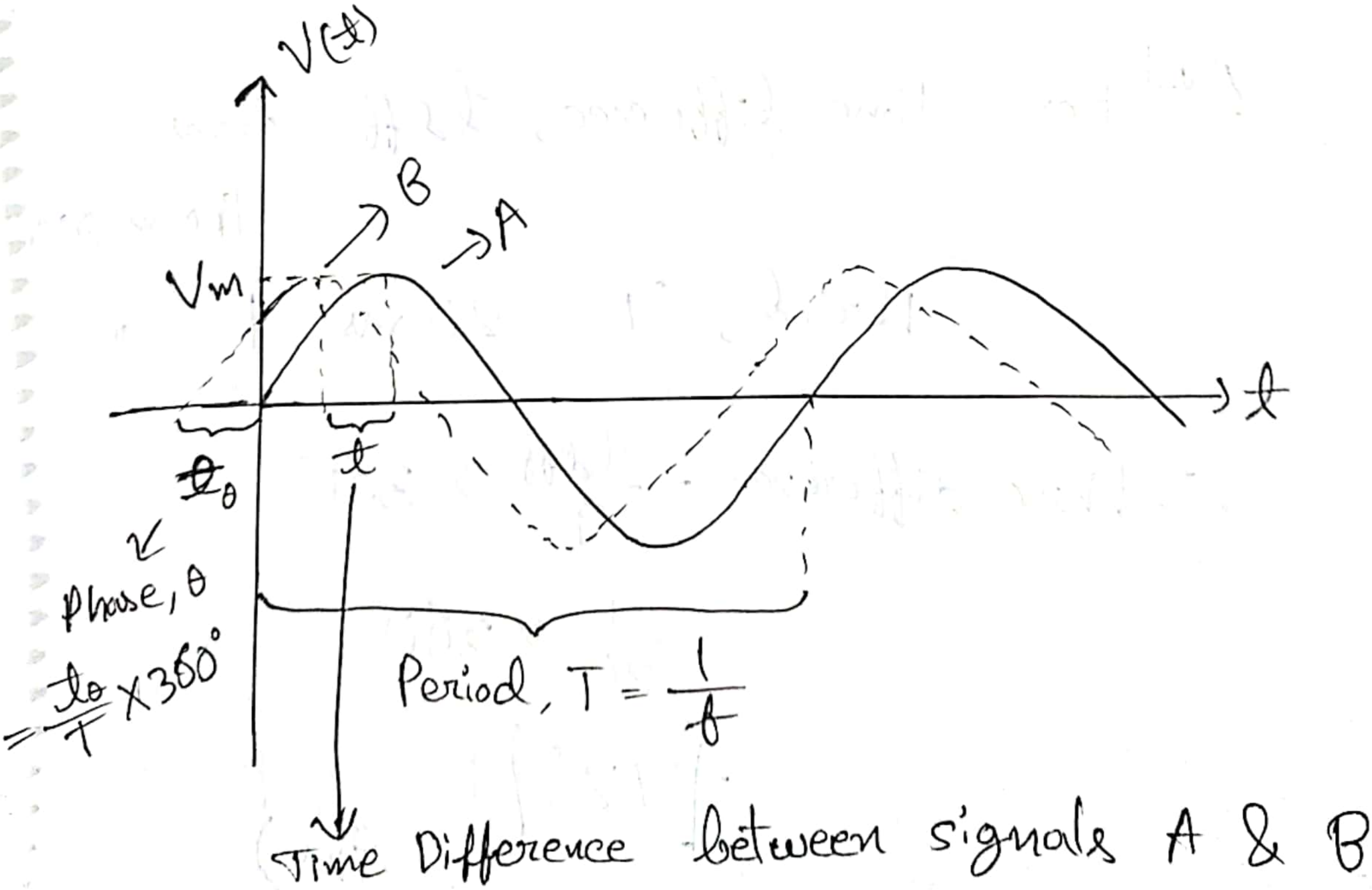
$$V(t) = V_m \cos(\omega t + \theta) \text{ or, } V_m \sin(\omega t + \theta)$$

[Similar for $I(t)$]

$$\text{Phase shift} = \frac{t}{T} \times 360^\circ$$

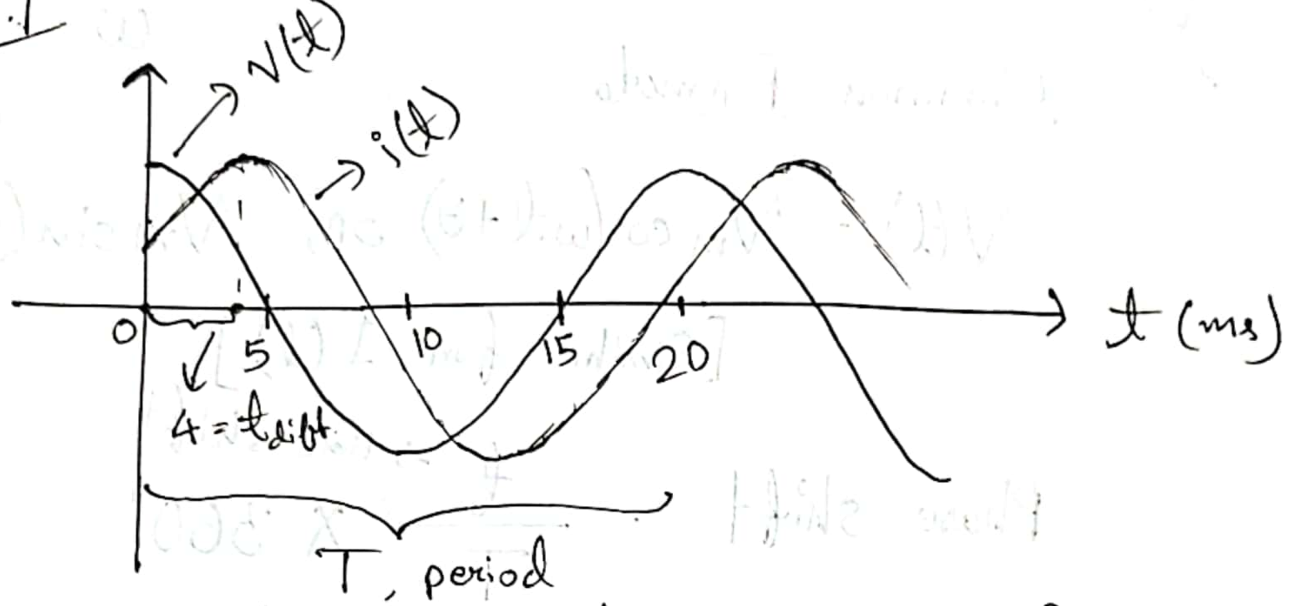
t → time shift

T → Period



Examples

1.1



i) Phase difference between v(t) & i(t) = ?

Ans. Here, time difference, t_{diff} = 4 ms

[From graph]

Period, T = 20 ms ["]

$\therefore$ Phase difference $= \frac{t_{diff}}{T} \times 360^\circ$

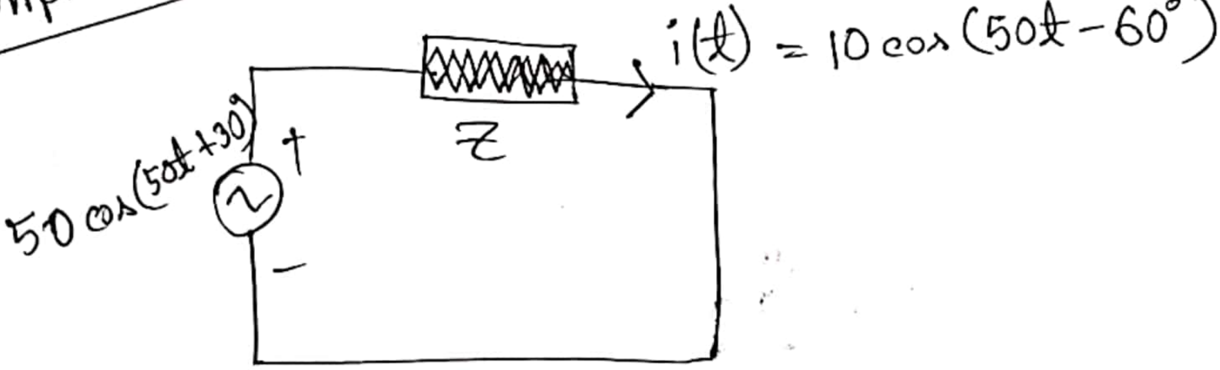
$= \frac{4}{20} \times 360^\circ$

$= \boxed{72^\circ}$ (Ans.)

ii) Is $i(t)$ leading $v(t)$, or lagging?

Ans.) $i(t)$ reaches peak later than $v(t)$. From the graph, so $\boxed{i(t) \text{ is lagging } v(t)}$.

Example 2



$Z = ?$ What is it? What's it's value?

Ans.) $Z = \dfrac{V}{I} = \dfrac{50\angle 30^\circ}{10\angle -60^\circ} = 5\angle 90^\circ = \boxed{5j}$

$5j \rightarrow$ Imaginary and positive $\rightarrow$ $\boxed{\text{So, Inductor}}$

$\therefore Z = j\omega L = 5j \Rightarrow \omega L = 5$

$\Rightarrow 50L = 5 \Rightarrow L = \dfrac{5}{50}$

$\therefore L = \boxed{0.1\ H}$

Note: Inductor $\rightarrow$ making voltage lead current.