Department of Computer Science and Engineering (CSE)
BRAC University

Lecture 3

CSE250 - Circuits and Electronics

KCL AND KVL

BRAC UNIVERSITY
Inspiring Excellence

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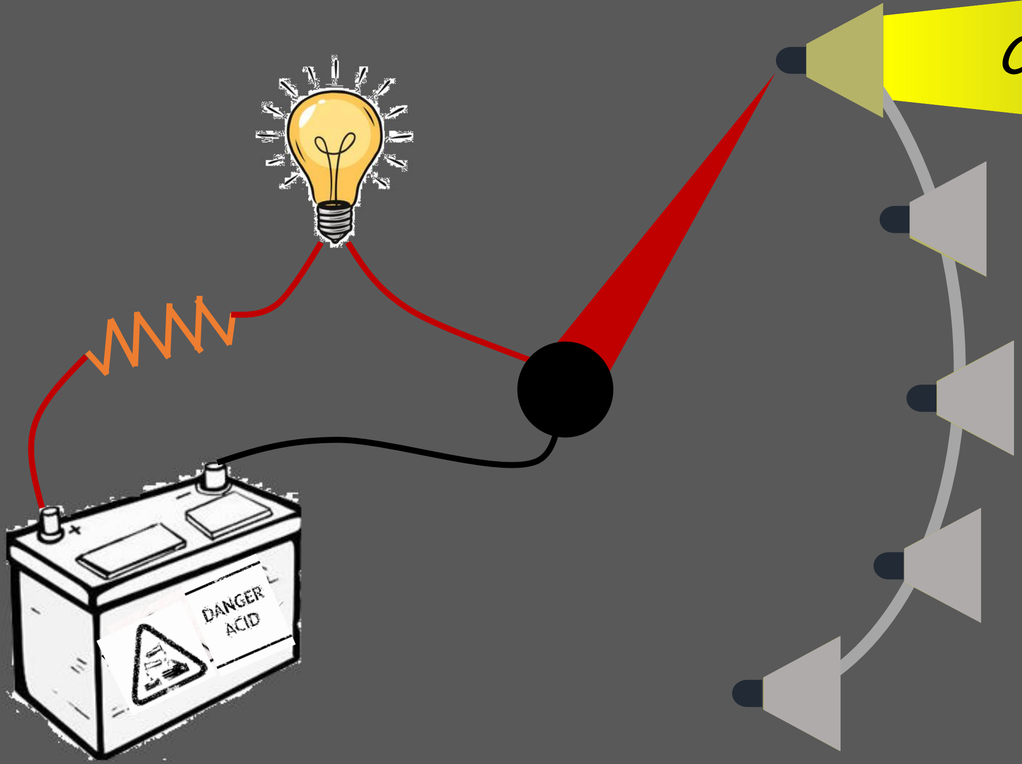
Course Outline: broad themes



- Circuit Laws
  - Kirchhoff's Current Law
- Methods of Analysis
- Circuit Theorems
- First Order Circuits
- AC Circuits

Kirchhoff's Current Law (KCL)

- Kirchhoff's current law (KCL) the algebraic sum of the currents entering a node is equal to the algebraic sum of the currents leaving the node.

- Mathematically, $\sum_{n=1}^{N} i_n = 0$, where N is the number of branches connected to the node and I_n is the nth current entering (or leaving) the node.

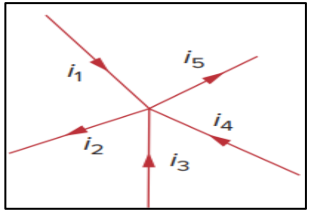
- Assume a set of currents $i_k(t), k = 1, 2, \ldots,$ flow into a node. The algebraic sum of currents at the node is, $i_{total}(t) = i_1(t) + i_2(t) + i_3(t) + \ldots\ldots$

- Integrating both sides, $q_{total}(t) = q_1(t) + q_2(t) + q_3(t) + \ldots\ldots,\ [q_k(t) = \int i_k(t)\,dt]$

- The *law of conservation of electric charge* requires that the algebraic sum of electric charges at the node must not change; that is, the node stores no net charge. Thus, $q_{Total}(t) = 0 \rightarrow i_T(t) = 0$, confirming the validity of KCL.

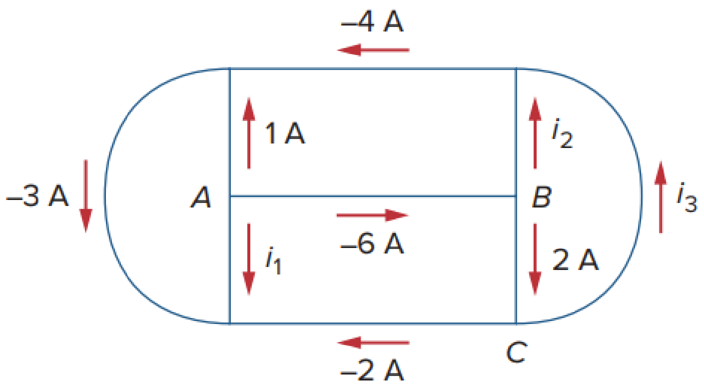
- For the node shown beside, $i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$

$$or, i_1 + i_3 + i_4 = i_2 + i_5$$



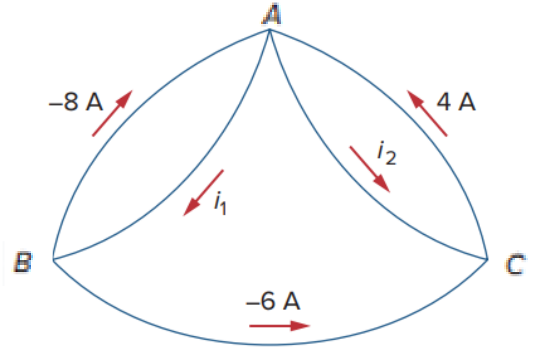
Example 1

- (i) Find i_1, i_2, and i_3



Note that, in both the circuits A, B, and C are the same nodes. It is more appropriate to call them junctions in this case.

(ii) Find $i_1, and\ i_2$



KCL at junction A,	KCL at junction C,
$i_1 + 1 + (-6) = 0$	$2 = (-2) + i_3$
$\Rightarrow \quad i_1 = 5\ A$	$\Rightarrow \quad i_3 = 4\ A$
KCL at junction B,	
$i_2 + 2 = -6$	
$\Rightarrow \quad i_2 = -8\ A$	

KCL at junction B,

$$i_1 = (-8) + (-6)$$

$$\Rightarrow \quad i_1 = -14\ A$$

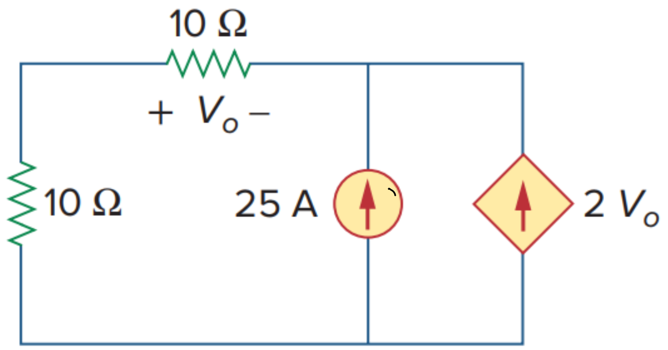
KCL at junction C,

$$i_2 + (-6) = 4$$

$$\Rightarrow \quad i_2 = 10\ A$$

Example 2

- Find V_0 and power absorbed/supplied by the dependent source with appropriate $\pm$ sign.



Current through the series resistances = $25 + 2V_0$

According to the Ohm's law,

$$V_0 = -10 \times (25 + 2V_0)$$
$$V_0 = -11.9\ V$$

The voltage across the dependent source is,

$$V_x = (10 + 10) \times (25 + 2V_0) = 24\ V$$

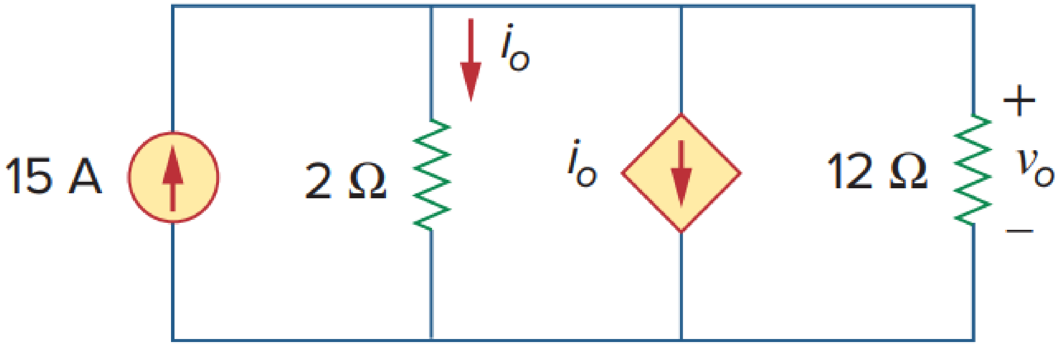
With the polarity of V_x and the direction of the current $(2V_0)$ given, according to the passive sign convention, the dependent source is supplying power. So,

$$p = -\ 24 \times 2V_0 = 571.2\ W$$

The power is positive, hence, the dependent source is actually absorbing power. This is true as V_0 is negative, the current $2V_0$ is actually flowing in the opposite direction.

Problem 1

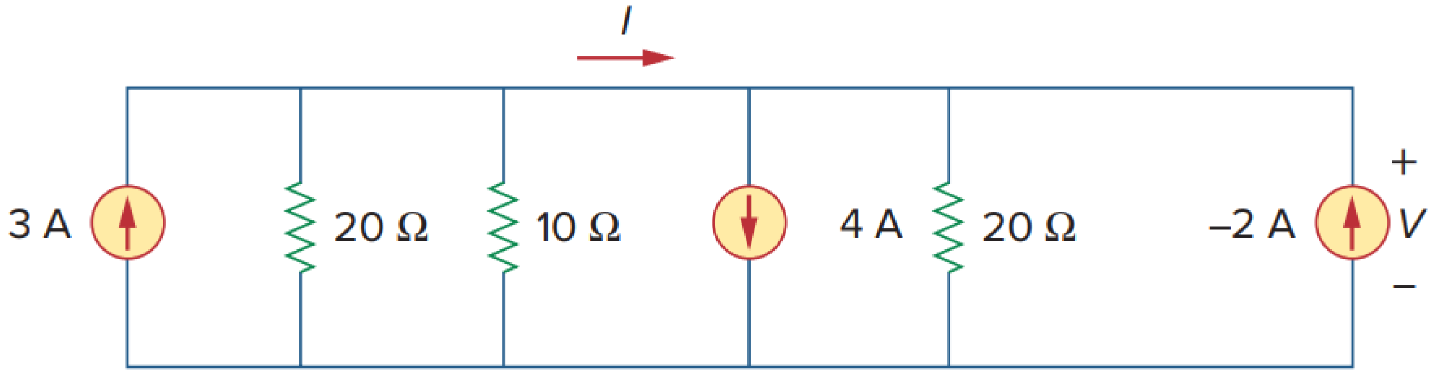
- Determine v_0 and i_0.



Ans: $\boldsymbol{v_0 = 13.85\,V;\ i_0 = 6.92\,A.}$

Problem 2

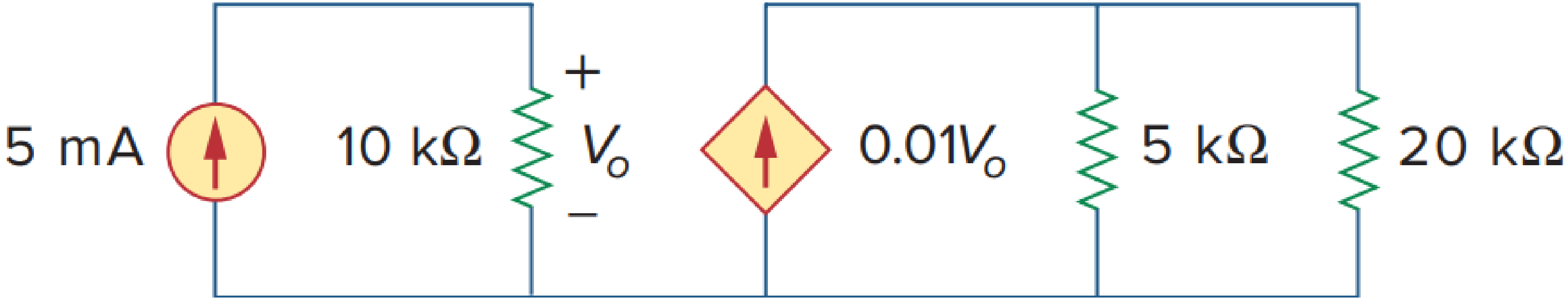
- Find the I and V shown in the following circuit.



Ans: $\boldsymbol{V = -15\,V;\ I = 5.25\,A.}$

Problem 3

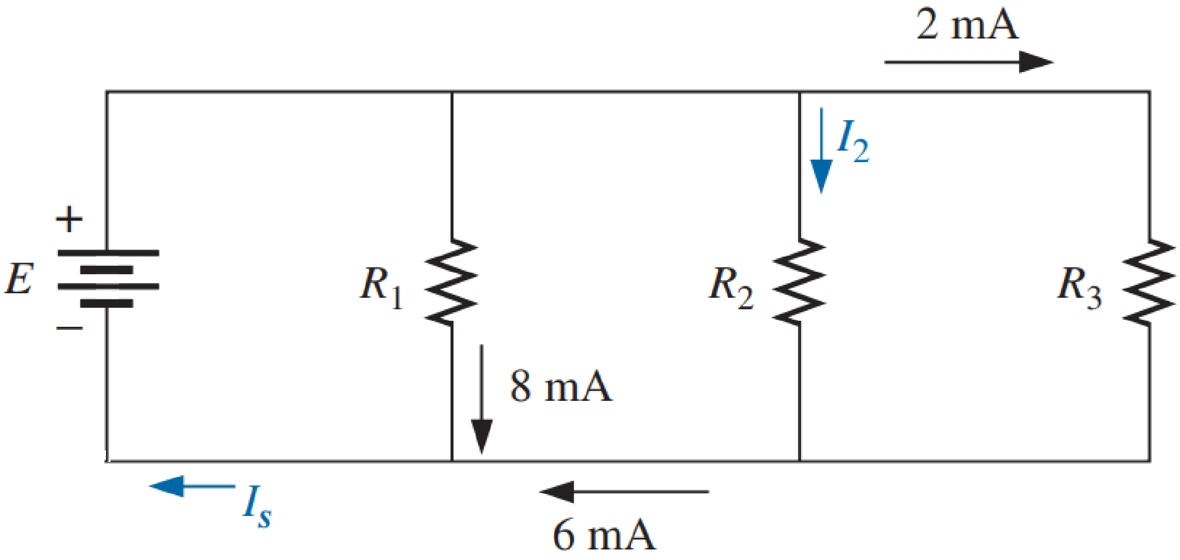
- For the network shown below, find the current, voltage, and power associated with the 20 $k\Omega$ resistor.



Ans: $\mathbf{0.1\ mA, 2\ V, 0.2\ mW}$

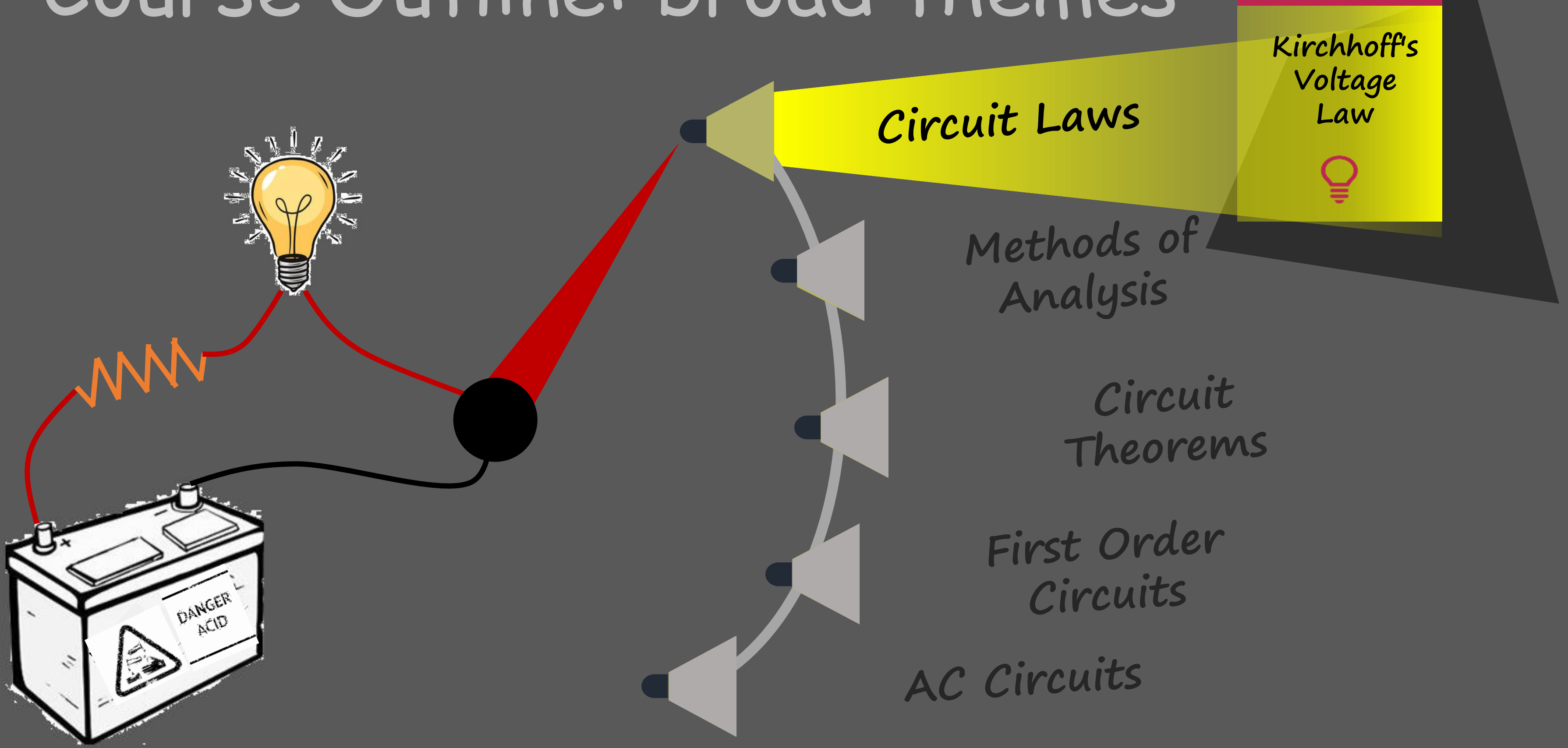
Problem 4

- Using KCL, determine the unknown currents.



Ans: $I_2 = 4\ mA, I_s = 14\ mA$

Course Outline: broad themes



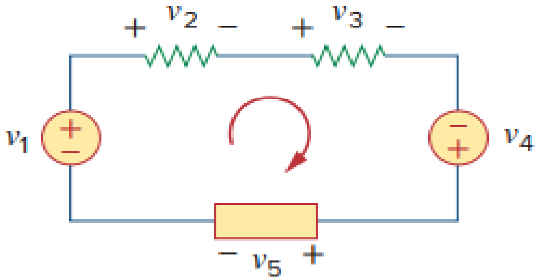
Kirchhoff's Voltage Law (KVL)

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a path (or loop) is zero. KVL can be applied both in loop or path consisting of open circuits.
- Mathematically, $\sum_{m=1}^{M} v_m = 0$, where M is the number of voltages (or branches) in the loop and v_m is the m^{th} voltage.
- To illustrate KVL, consider the circuit shown. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop.
- If we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1$, $+v_2$, $+v_3$, $-v_4$, and $+v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence, we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $+v_4$. Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

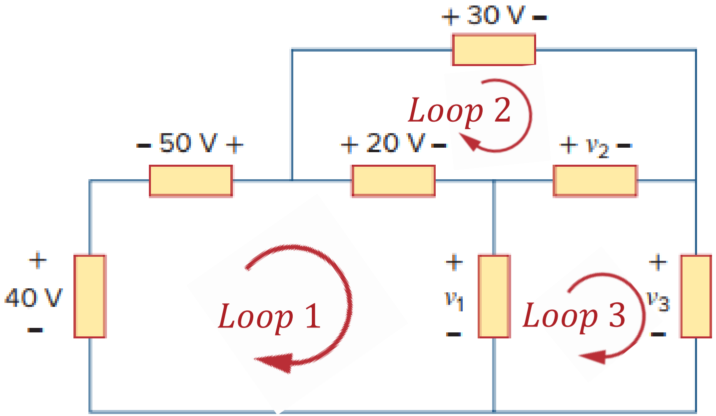
$$or, \quad v_2 + v_3 + v_5 = v_1 + v_4$$

Sum of voltage drops = Sum of voltage rises



Example 3

- Determine v_1, v_2, v_3 using KVL



KVL at loop 1,

$$-40 - 50 + 20 + v_1 = 0$$
$$v_1 = 70\ V$$

KVL at loop 2,

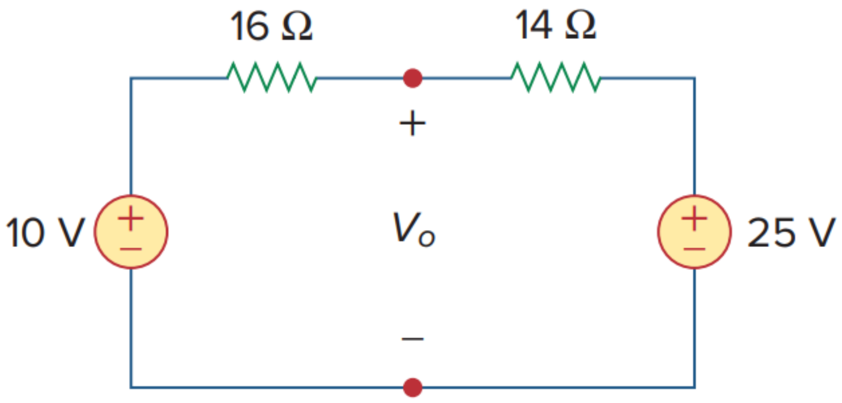
$$-20 + 30 - v_2 = 0$$
$$v_2 = 10\ V$$

KVL at loop 3,

$$-v_1 + v_2 + v_3 = 0$$
$$-70 + 10 + v_3 = 0$$
$$v_3 = 60\ V$$

Example 4

- Determine V_0 using KVL.



Let's assume that the current through the series circuit is i.

Applying KVL around the loop,

$$-10 + 16i + 14i + 25 = 0$$

$$i = -0.5\ A$$

V_0 can be found either by applying KVL through the loop consisting of V_0, 14 Ω, and 25 V or applying KVL through the loop consisting of V_0, 16 Ω, and 10 V. That is,

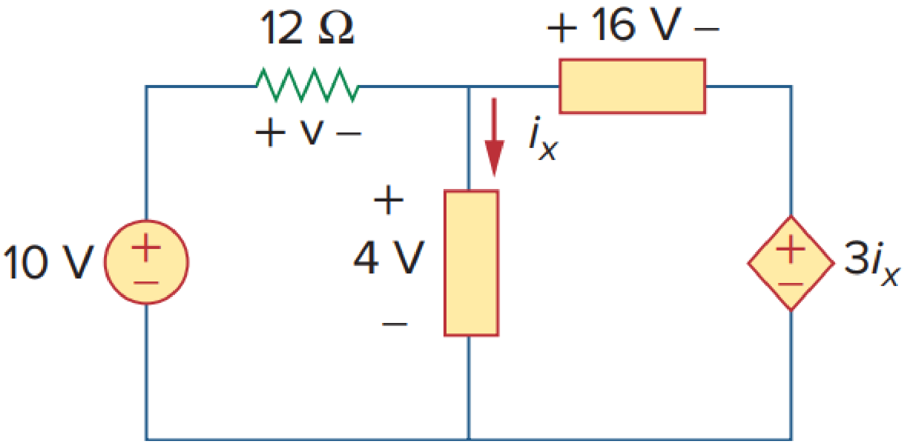
$-V_0 + 14i + 25 = 0$, or $V_0 = 18\ V$

Or,

$-10 + 16i + V_0 = 0$, or $V_0 = 18\ V$

Problem 5

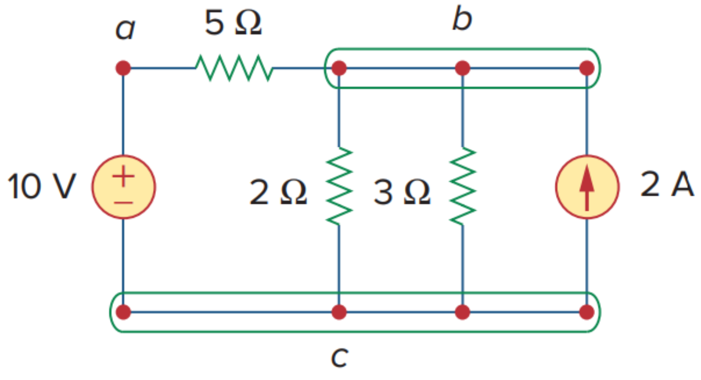
- Find v and i_x in the following circuit.



Ans: $\boldsymbol{v = 6\,V;\ i_x = -4\,A.}$

of Solvable KVL Equations

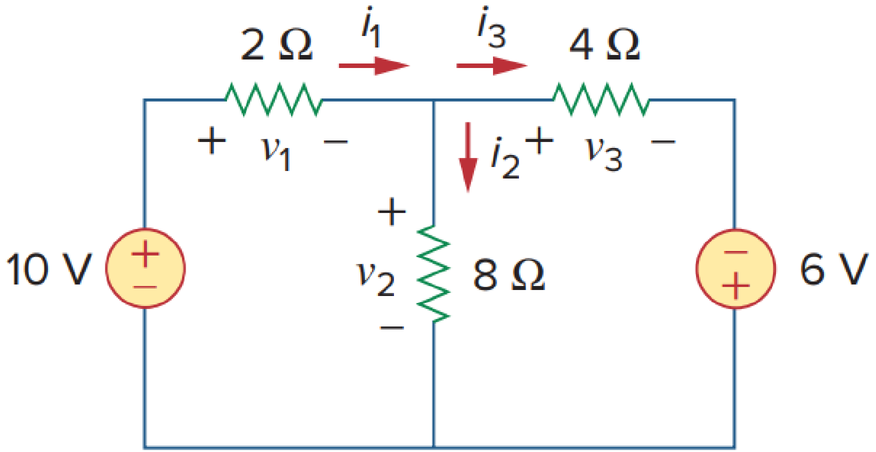
- In general, a circuit with n nodes and b branches will have $b - n + 1$ *independent KVL equations*.
- It is necessary to apply KVL only to these loops, which will in total, traverse each branch at least once in the process. For example, in the adjacent circuit, the number of independent KVL equations will be $5 - 3 + 1\ or\ 3$. However, not any 3 equations will be solvable.
- For example, if 3 KVL equations are formed from the 3 loops: (i) $10\,V$, $5\ \Omega$, $2\ \Omega$, (ii) $10\,V$, $5\ \Omega$, $3\,\Omega$, and (ii) $2\ \Omega$, $3\ \Omega$, there will be no finite set of solution as the $2\,A$ branch is not traversed in any of the three equations.
- They are not independent as any of the 3 equations will be a linear combination of the other two.



- Try applying KVL to every mesh first.
- Be careful not to allow the number of variables in the equations exceed the number of meshes.

Problem 6

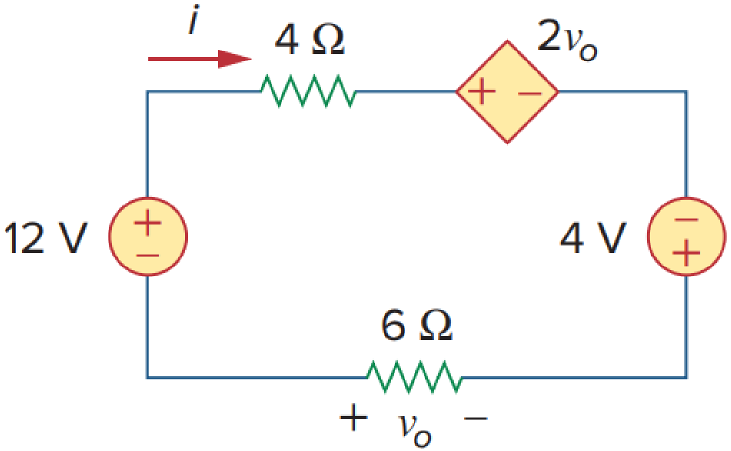
- How many KVL equations can be formed from the circuit shown below? Form all the KVL equations in terms of v_1, v_2, and v_3. Are the simultaneous equations solvable? Why? Determine all the voltages and currents labelled.



Ans: $v_1 = 6\ V$; $v_2 = 4\ V$; $v_3 = 10\ V$.
$i_1 = 3\ A$; $i_2 = 0.5\ A$; $i_3 = 2.5\ A$

Problem 7

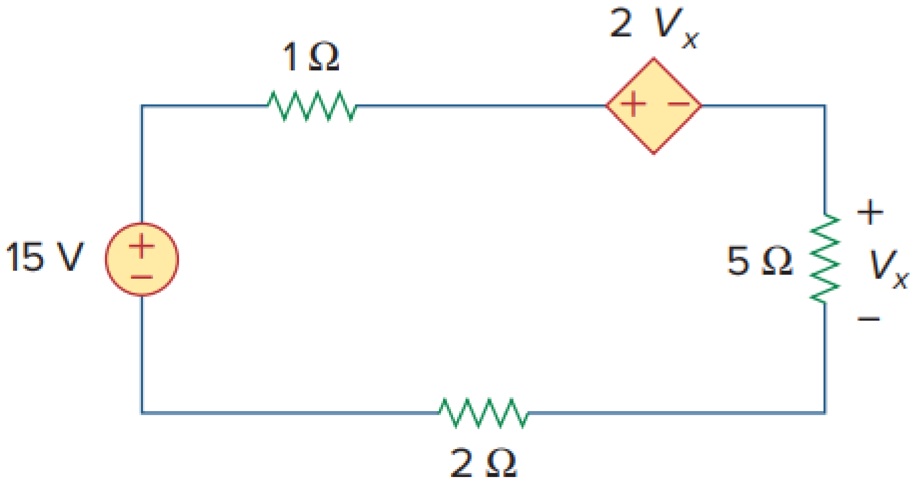
- Find v_0 and i in the circuit



Ans: $\boldsymbol{v_0 = 48\,V;\ I = -8\,A.}$

Problem 8

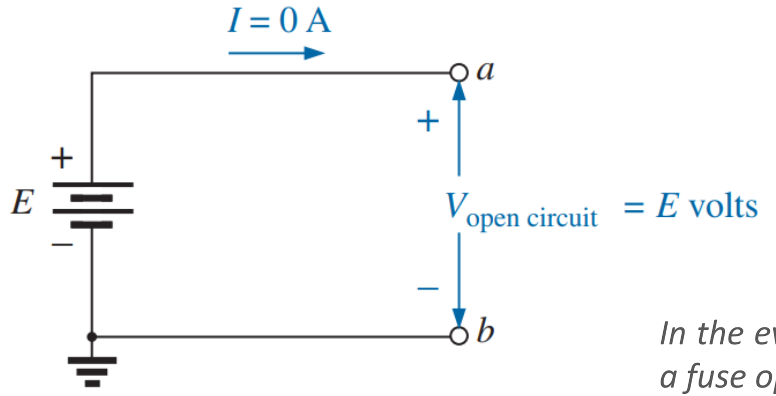
- Find V_x

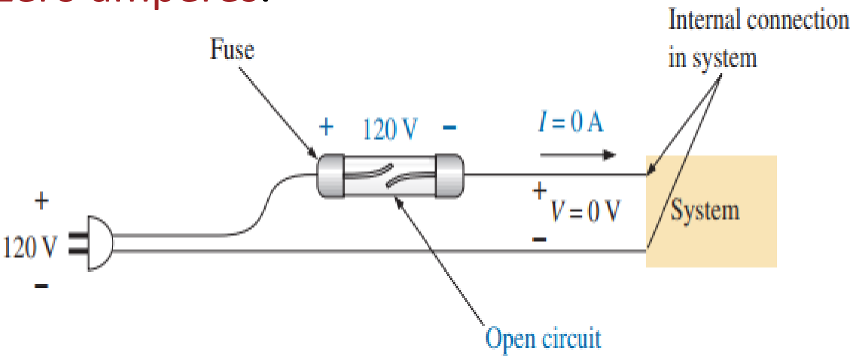


Ans: $\boldsymbol{V_x = 4.167\ V.}$

Open circuit

- An *open circuit* is two isolated terminals not connected by an element of any kind. It is the limiting case of a resistor where the resistance approaches infinite.
- Any element with $R \to \infty$ is an open circuit. $i = 0 = \lim\limits_{R \to \infty} \frac{v}{R}$
- Indicating that, an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

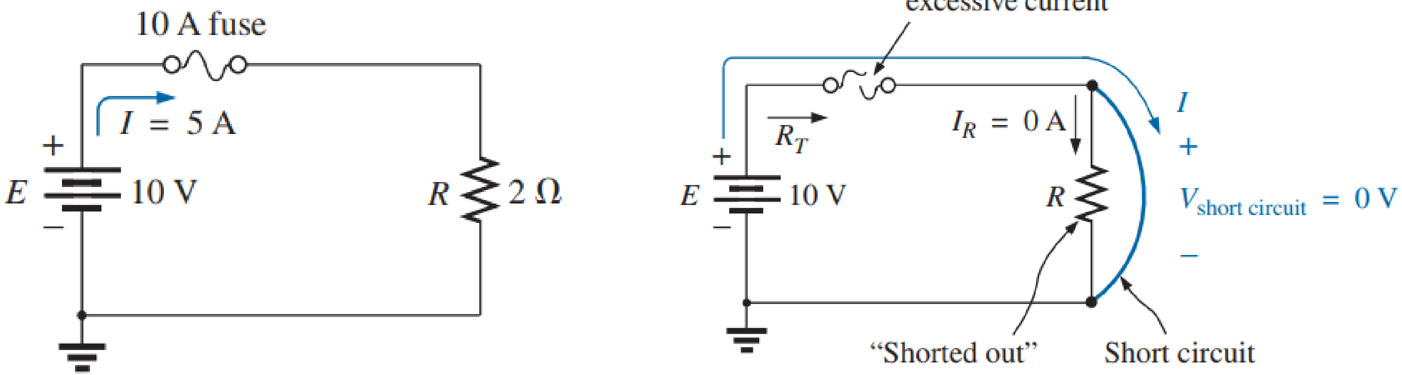




In the event of an excessive current flow, a fuse opens to protect appliances.

Short circuit

- A *short circuit* is a very low resistance, direct connection between two terminals of a network. It is the limiting case of a resistor where the resistance approaches zero.
- Any element with $R = 0$ is a short circuit. $v = 0 = \lim_{R \to 0} iR$
- Indicating that, a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

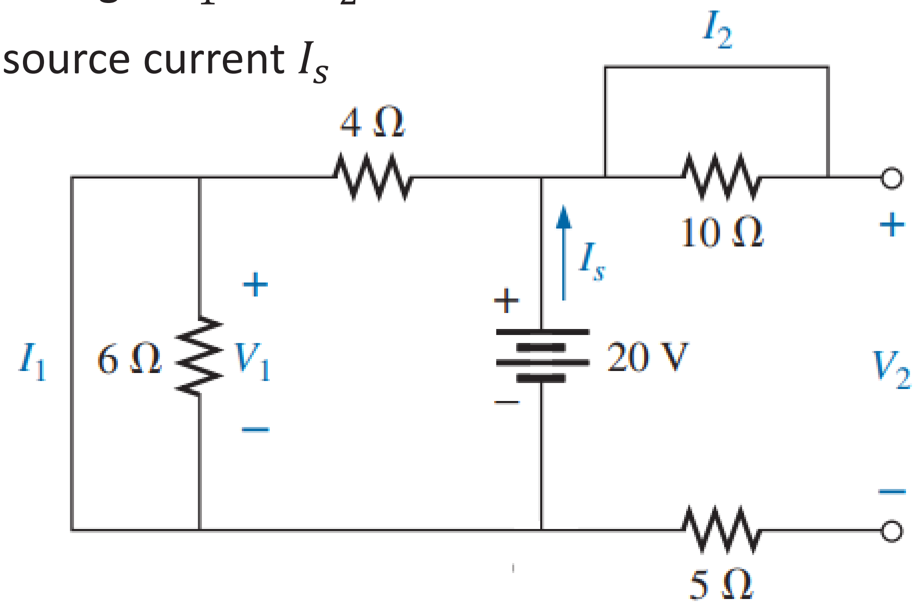


Problem 9

a. Determine the short circuit currents I_1 and I_2.

b. The voltages V_1 and V_2.

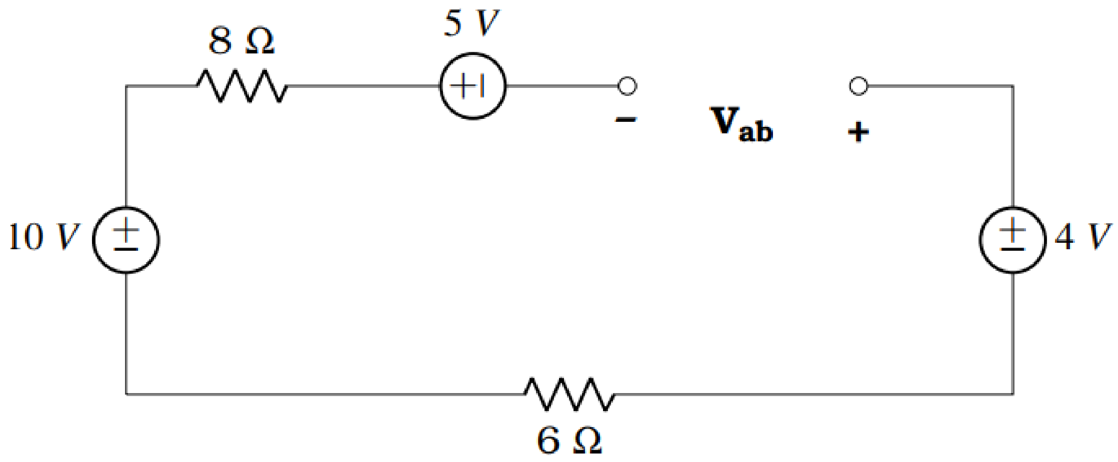
c. The source current I_s



Ans:
a. 5 A, 0 A
b. 0 V, 20 V
c. 5 A

Problem 10

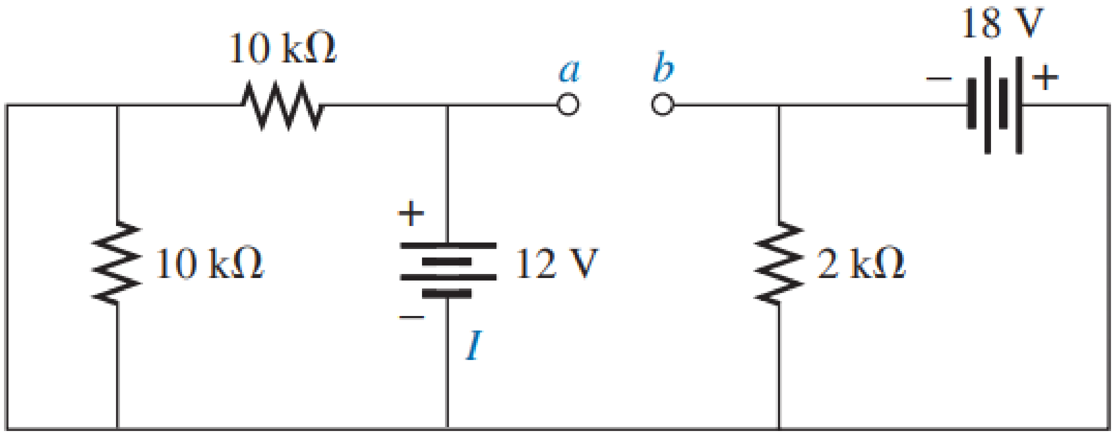
- Determine the voltage V_{ab} as indicated.



$\underline{\text{Ans}}$: $\boldsymbol{V_{ab} = -1\, V}$.

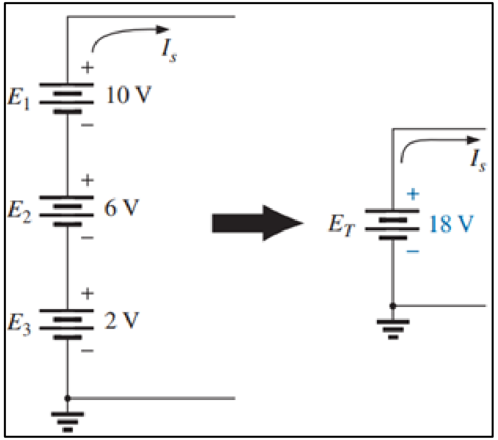
Problem 11

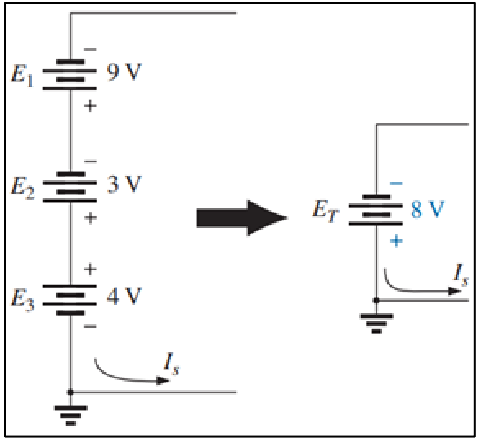
- Determine the voltage between terminals a and b and the current I for the network shown below.

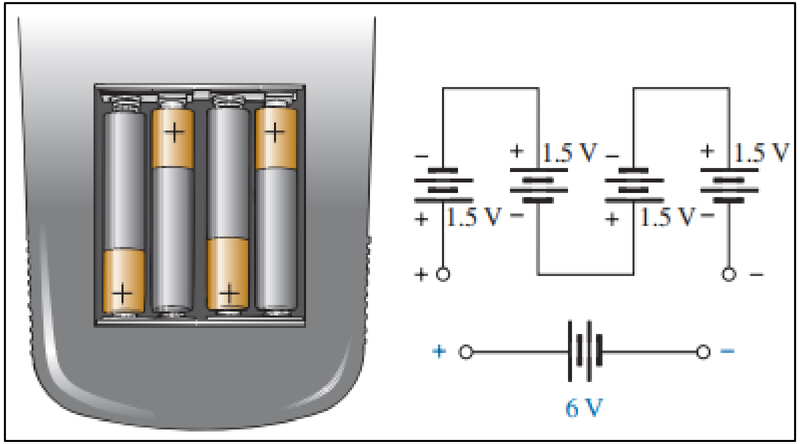


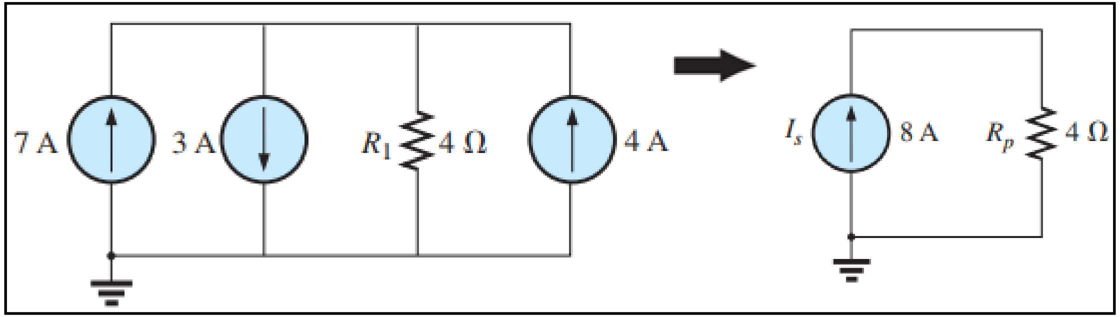
Ans: $V_{ab} = 30\ V; I = 1.2\ mA.$

Series and Parallel sources





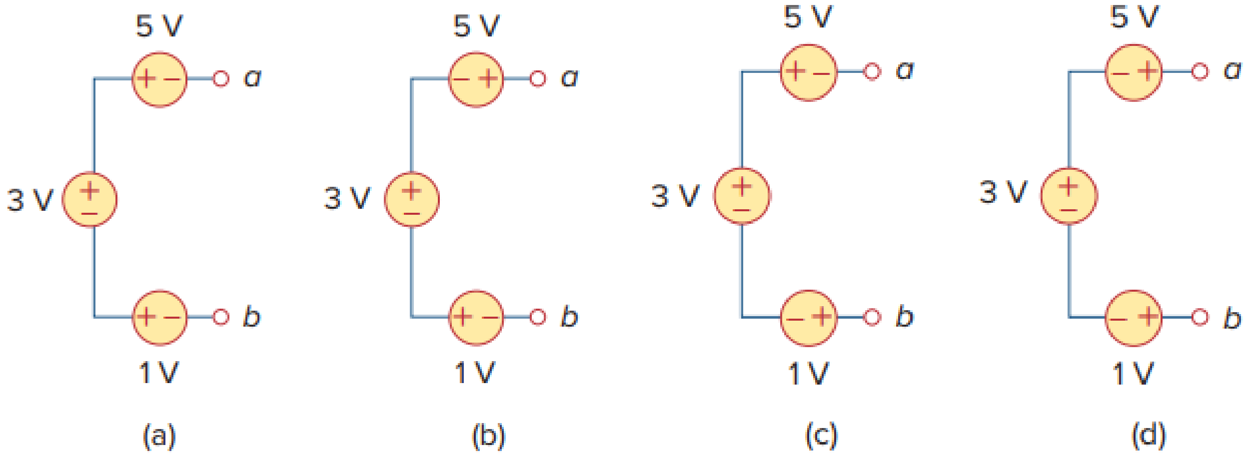




It is not practical to connect voltage sources of unequal ratings in parallel and current sources of unequal currents in series due to the direct violation of KVL and KCL respectively.

Problem 12

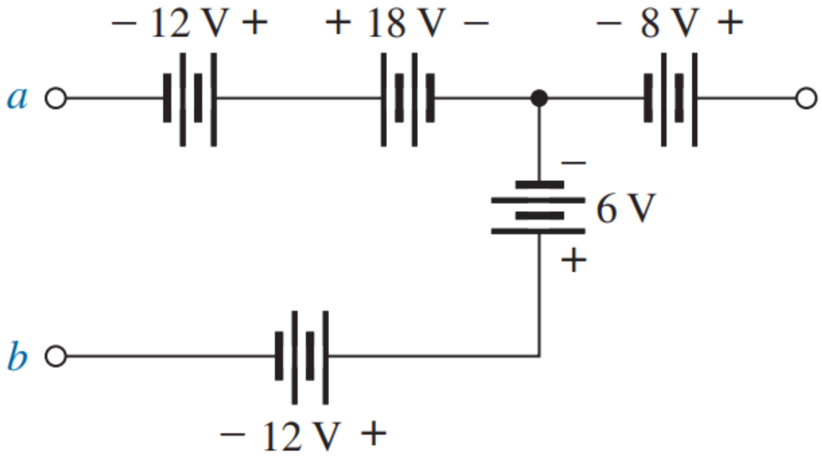
- For each of the circuits shown below, calculate V_{ab}

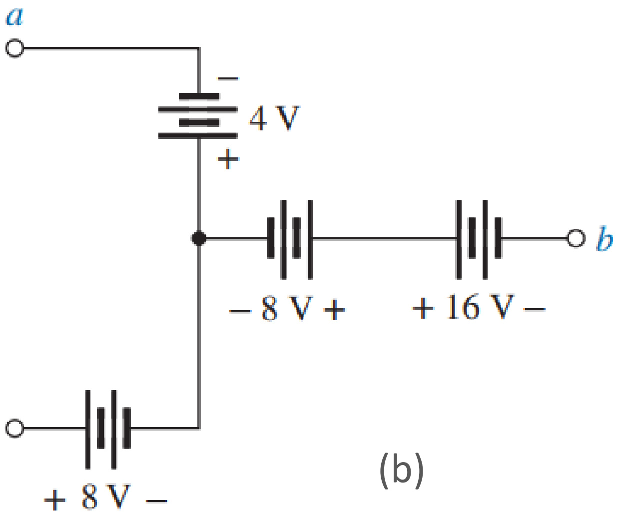


Ans: $(a)\ \boldsymbol{V_{ab} = -1\ V};\ (b)\ \boldsymbol{V_{ab} = 10\ V};\ (c)\ \boldsymbol{V_{ab} = -3\ V};\ (d)\ \boldsymbol{V_{ab} = 7\ V};$

Problem 13

- For each of the circuits shown below, calculate V_{ab}



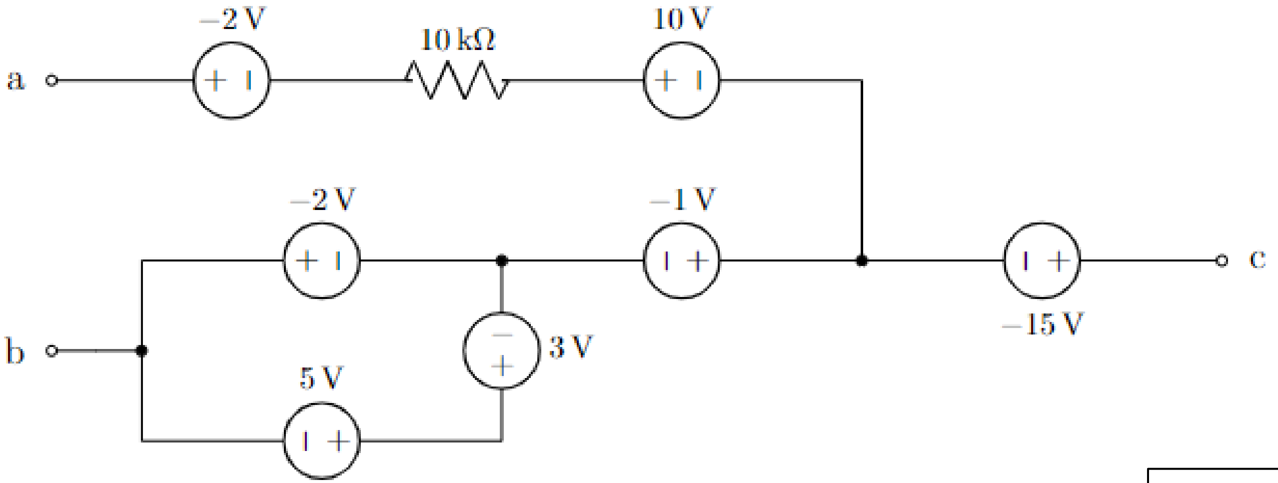


(b)

Ans: $(a)\ \boldsymbol{V_{ab}} = \mathbf{12\ V};\ (b)\ \boldsymbol{V_{ab}} = \mathbf{4\ V}$

Problem 14

- For the circuit shown below, calculate V_{ac} and V_{bc}



Ans: $\boldsymbol{V_{ac} = 23\ V;\ V_{bc} = 14\ V}$

Voltage Division Rule

- The voltage division rule permits the determination of the voltage across a series resistor without first having to determine the current of the circuit.

- The current through the series circuit can be found using Ohm's law as,

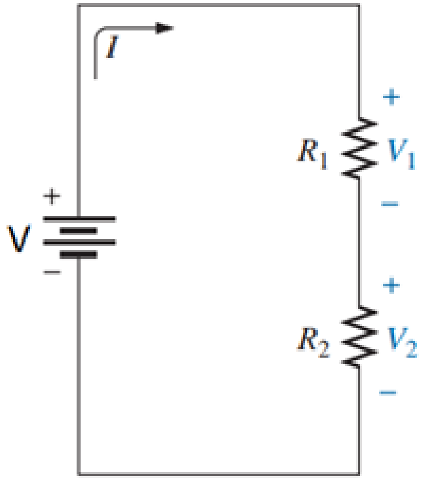
$$I = \frac{V}{R_1 + R_2}$$

- Applying Ohm's law to each of the resistors,

$$V_1 = IR_1 \quad \text{and} \quad V_2 = IR_2$$

$$\Rightarrow \quad V_1 = \frac{V}{R_1+R_2} R_1 \quad \text{and} \quad V_2 = \frac{V}{R_1+R_2} R_2$$

$$\Rightarrow \quad V_1 = \frac{R_1}{R_1+R_2} \times V \quad \text{and} \quad V_2 = \frac{R_2}{R_1+R_2} \times V$$

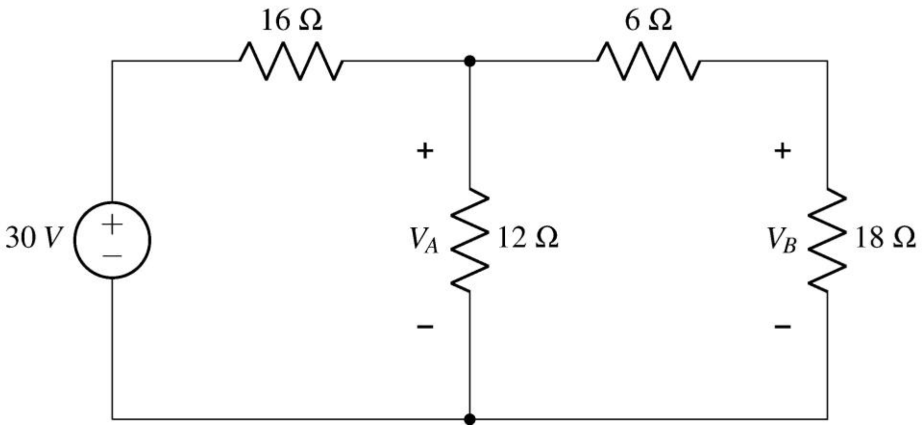


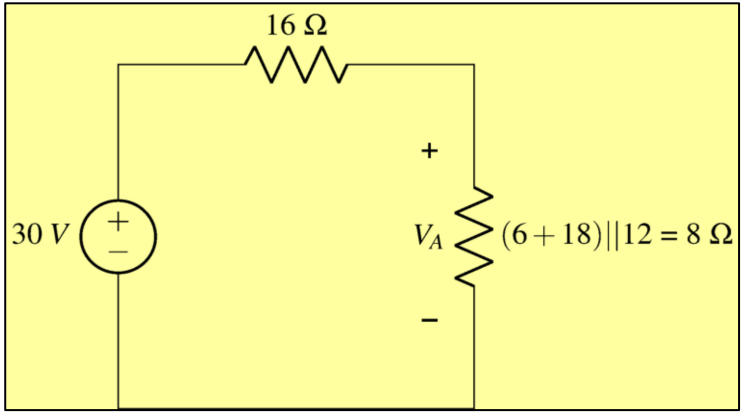
- In general, for any number of resistors connected in series to a supply voltage, the voltage across any particular resistor R_x is,

$$\boxed{V_x = \frac{R_x}{R_1 + R_2 + R_3 + \ \ldots \ldots + R_N} \times V}$$

Example 5

- Using the voltage divider rule, find the voltages V_A and V_B. Don't calculate currents.



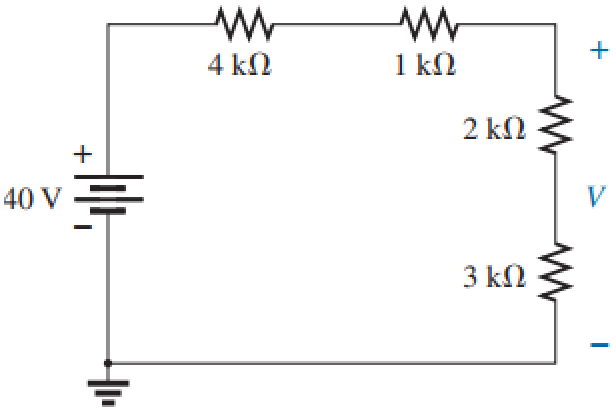


$$V_A = \frac{8}{8+16} \times 30 = 10\ V$$

$$V_B = \frac{18}{18+6} \times V_A = 7.5\ V$$

Problem 15

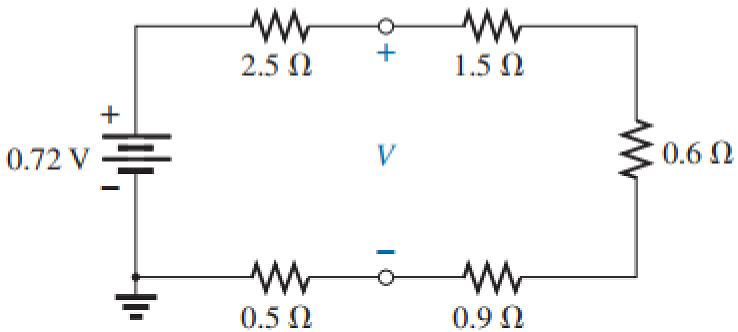
- Using the voltage divider rule, find the indicated voltage. Don't calculate current.



Ans: $V = 20\ \mathrm{V}$

Problem 16

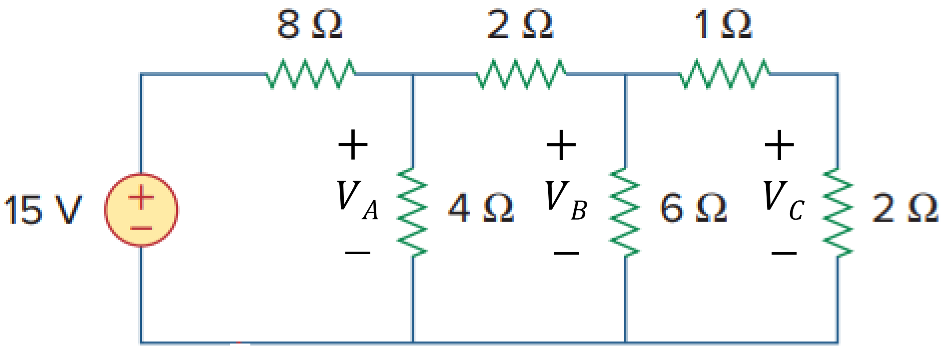
- Using the voltage divider rule, find the indicated voltage. Don't calculate current.



Ans: $V = 0.36\,V$

Problem 17

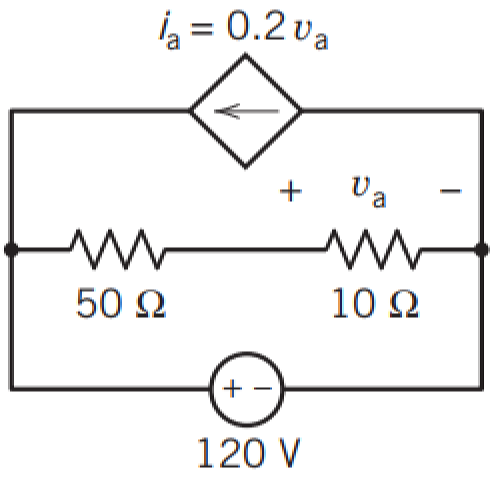
- Using the voltage divider rule, find the voltages V_A, V_B, and V_C. Don't calculate currents.



Ans: $\boldsymbol{V_A = 3\ V;\ V_B = 1.5\ V; V_C = 1\ V}$

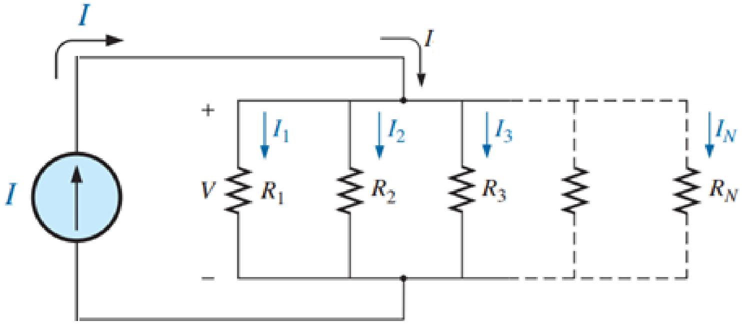
Problem 18

- Determine the power of the dependent source. Don't use Ohm's Law.



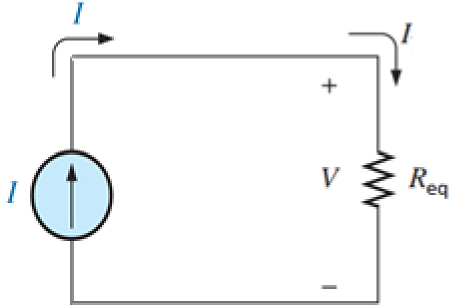
Ans: $-\mathbf{480}\ \boldsymbol{W}$

Current Division Rule



- The current division rule permits the determination of the currents through resistors connected in parallel without first having to determine the voltage across them.
- Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \ \ldots\ldots = I_NR_N$$



- Substituting V with $V = IR_{eq}$,

$$IR_{eq} = I_1R_1 = I_2R_2 = I_3R_3 = \ \ldots\ldots = I_NR_N$$

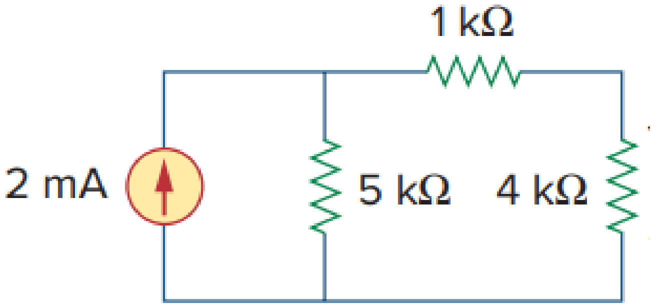
$$\Rightarrow \quad I_1 = \frac{R_{eq}}{R_1} \times I, \qquad I_2 = \frac{R_{eq}}{R_2} \times I, \qquad I_3 = \frac{R_{eq}}{R_3} \times I$$

- In general, for any number of resistors connected in parallel to a supply current, the current through any particular resistor R_x is,

$$\boxed{I_x = \frac{R_{eq}}{R_x} \times I, \mathrm{or}, I_x = \frac{(R_x)^{-1}}{(R_1)^{-1} + (R_2)^{-1} + \cdots + (R_N)^{-1}} \times I}$$

Example 6

- Calculate the current through the $5\ k\Omega$ resistor using current division rule. Do not use Ohm's Law.



Solution

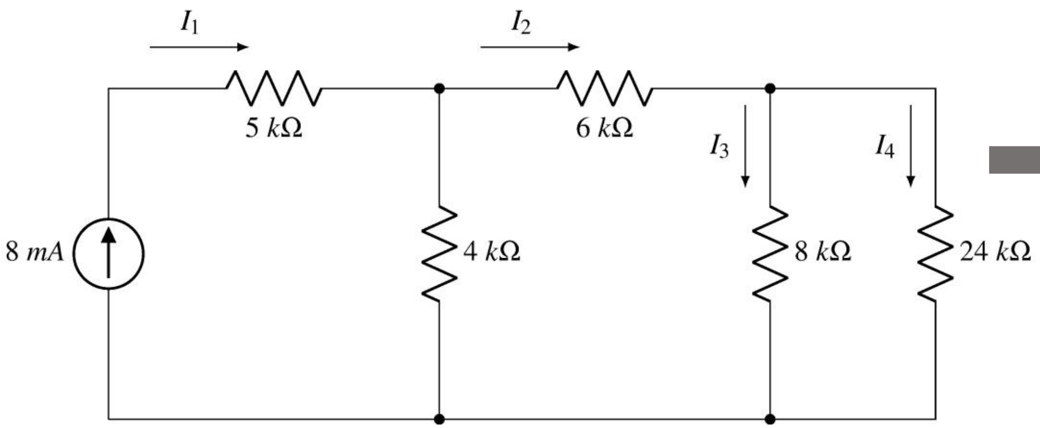
Current through the $5\ k\Omega$ resistor is,

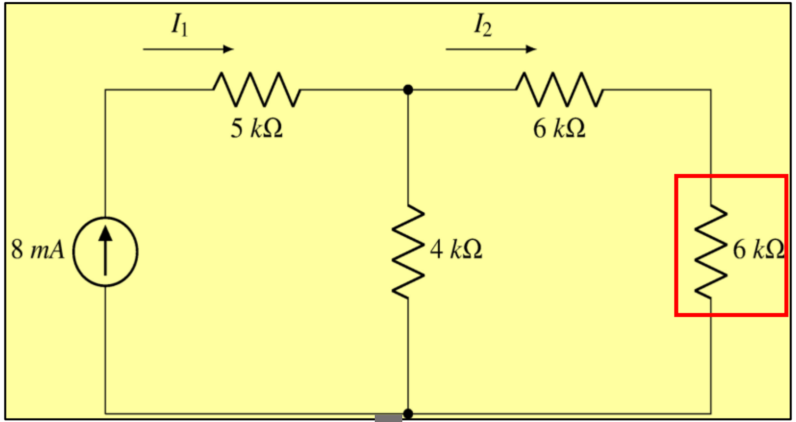
$$\frac{5^{-1}}{(1+4)^{-1}+5^{-1}} \times 2\ mA$$

$$= 1\ mA$$

Example 7

- Calculate the currents I_1 to I_4 using current division rule. Don't calculate voltage.





$$I_1 = 8\,mA$$

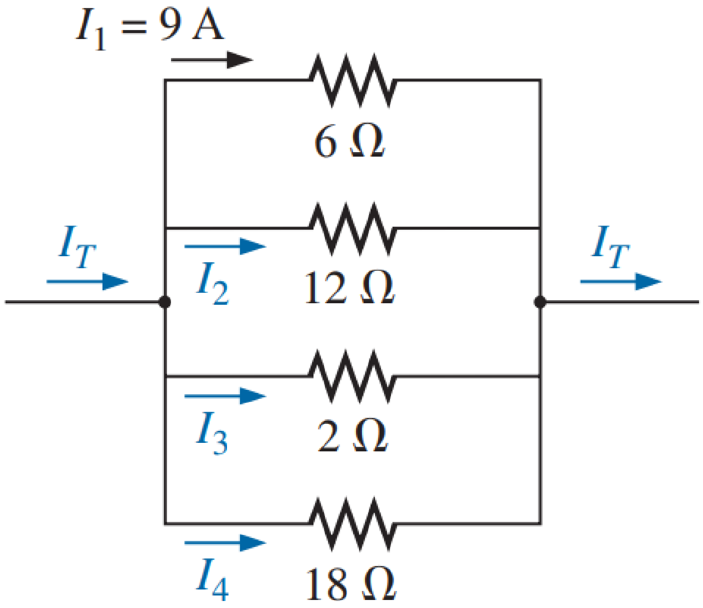
$$I_2 = \frac{12^{-1}}{4^{-1} + 12^{-1}} \times 8\,mA = 2\,mA$$

$$I_3 = \frac{8^{-1}}{24^{-1} + 8^{-1}} \times I_2 = 1.5\,mA$$

$$I_4 = I_2 - I_3 = 0.5\,mA$$

Problem 19

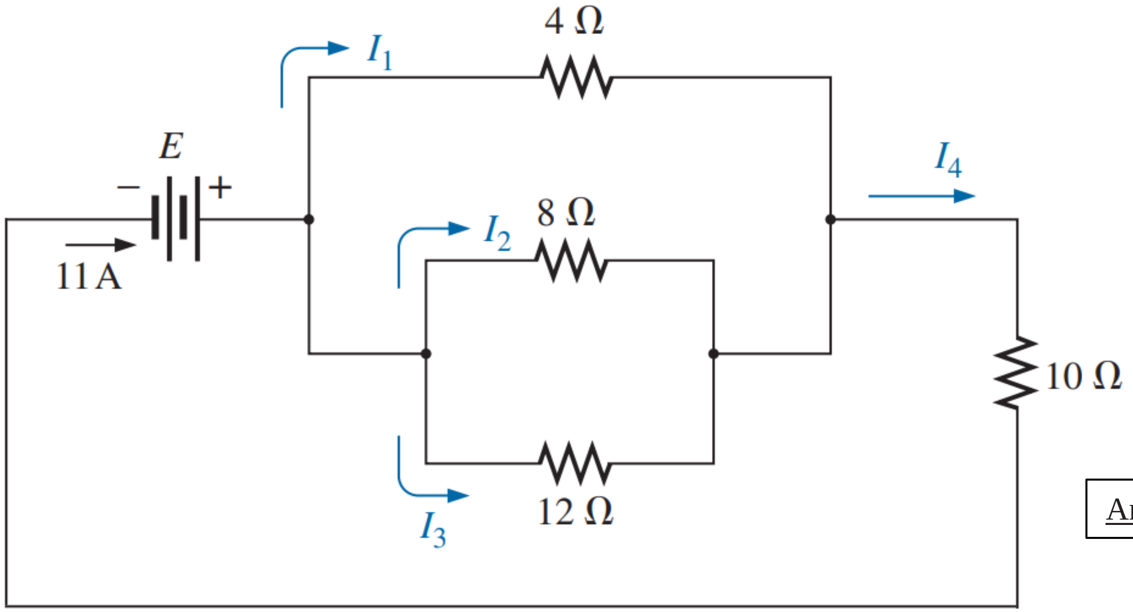
- Based solely on the resistor values, determine all the currents. Do not use Ohm's law.



Ans: $I_T = 43.5\ A;\ I_2 = 4.5\ A;\ I_3 = 27\ A;\ I_4 = 3\ A$

Problem 20

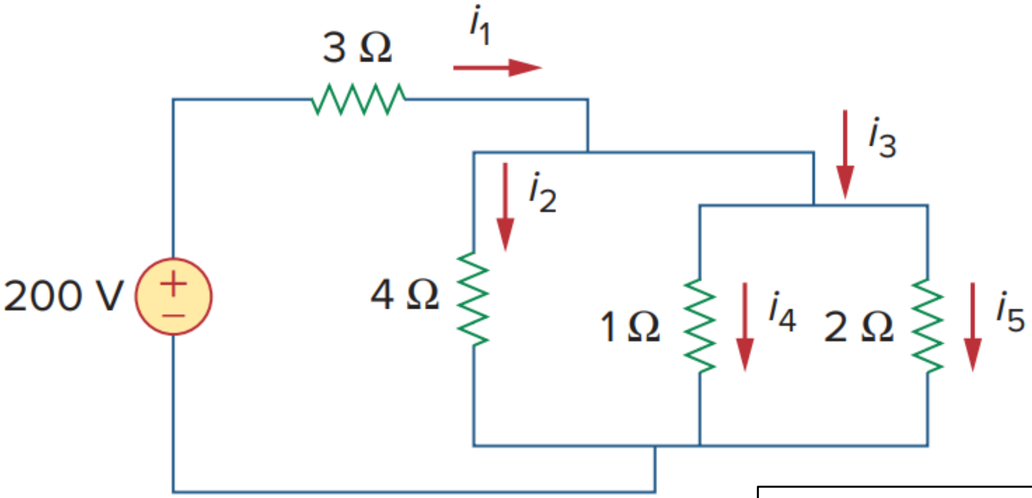
- Determine the unknown currents. Do not use Ohm's law.



Ans: $I_1 = 6\ A$; $I_2 = 3\ A$; $I_3 = 2\ A$; $I_4 = 11\ A$

Problem 21

- Determine the currents i_1 to i_5 using current division rule.



<u>Ans</u>: $\boldsymbol{i_1 = 56\,A;\ i_2 = 8\,A;\ i_3 = 48\,A;\ i_4 = 32\,A;\ i_5 = 16\,A.}$

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here

Thank you for your attention