Lecture 10: Circuit Theorems

Thevenin's Theorem

Load

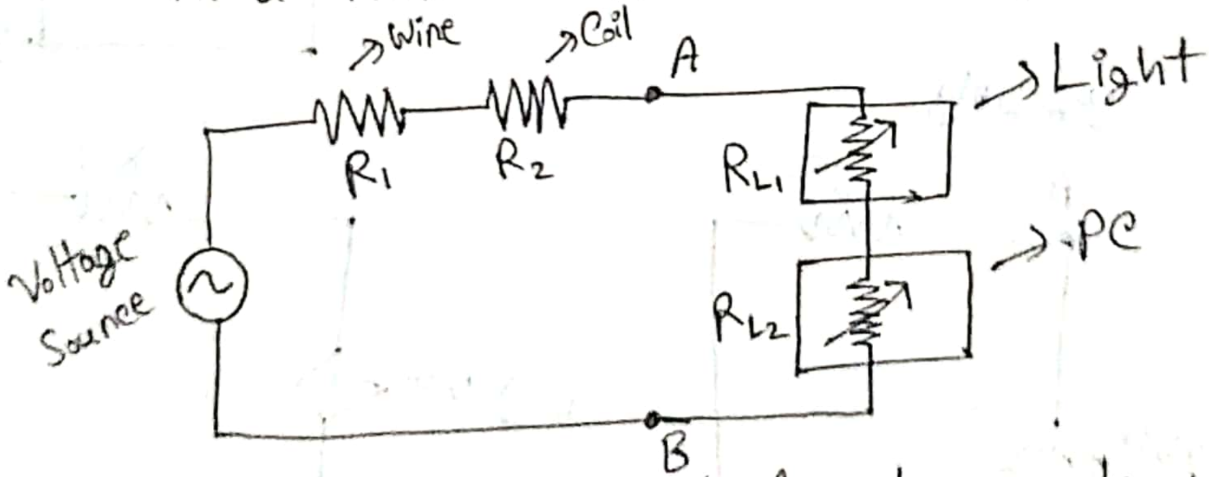
Often within a circuit

- → Some elements → Fixed (Value)
- → Some other element → Variable

That variable element (often consists of resistors/inductors/capacitors etc) is called $\boxed{\text{Load}}$.

example:

In a Household outlet circuit (simplified)

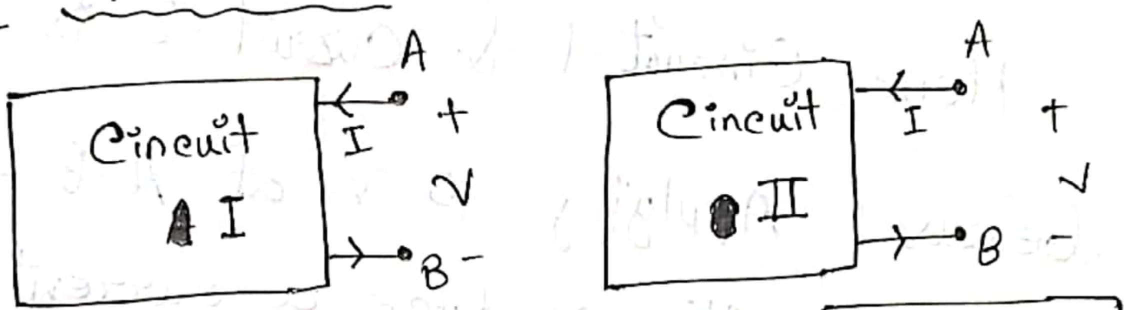


Here, the A-B terminal denotes a household outlet terminal. The wire resistance R_1, coil resistance R_2 are $\boxed{\text{fixed}}$. But household related resistances (Light R_{L_1}, PC R_{L_2}) are $\boxed{\text{variable}}$.

→ They are variable because they can be changed due to switching on/off, unlike wire resistances. ⟶ They are $\boxed{\text{Loads.}}$

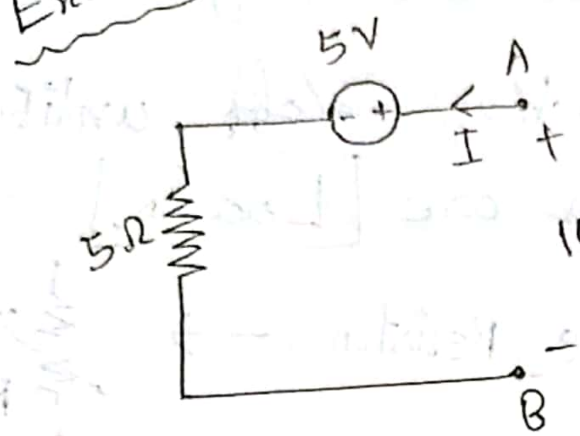
- Symbol of variable Resistor ⟶ 

Circuit Equivalence

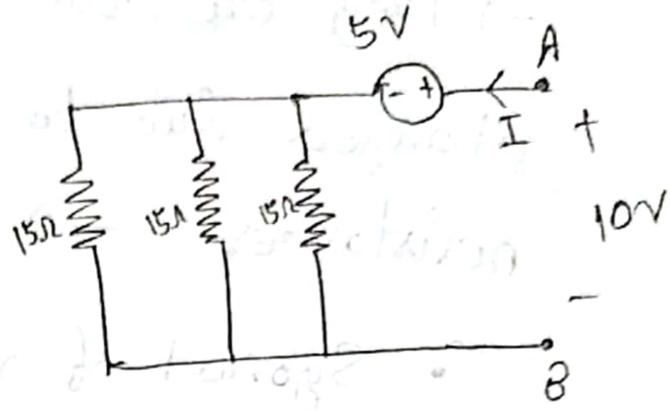


Circuit - I & Circuit - II are $\boxed{\text{Equivalent}}$ if they have the same Current voltage relationship at their terminals (A - B). That means, If we apply V voltage difference across the A-B terminals, the same current I will flow from A and come out through B (and Vice-Versa).

Example



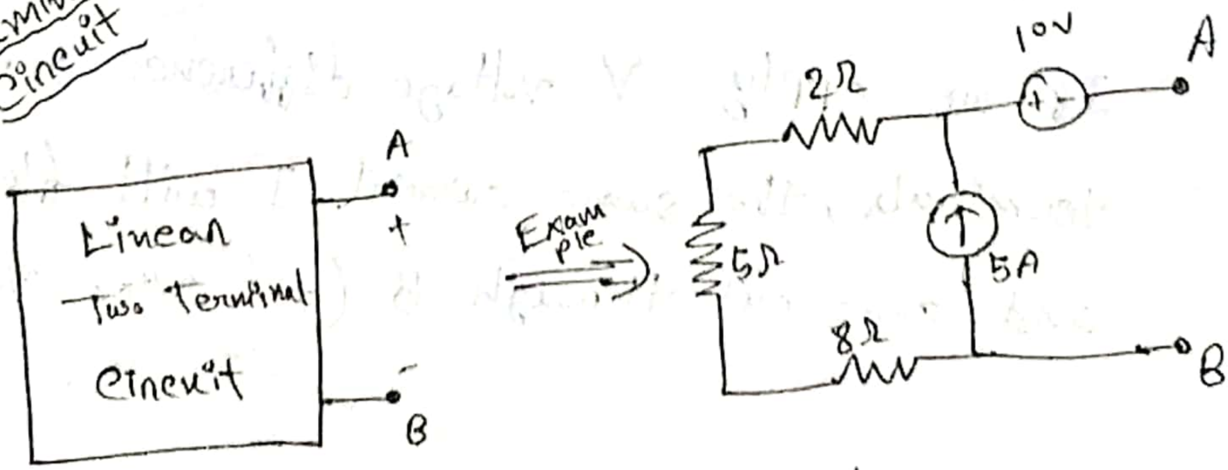
Circuit -1



Circuit -2

Here, Circuit-1 & Circuit-2 is equivalent because: Applying 10 V at A-B terminal, both circuits produce a current I = 1A (same for both) flowing from A.

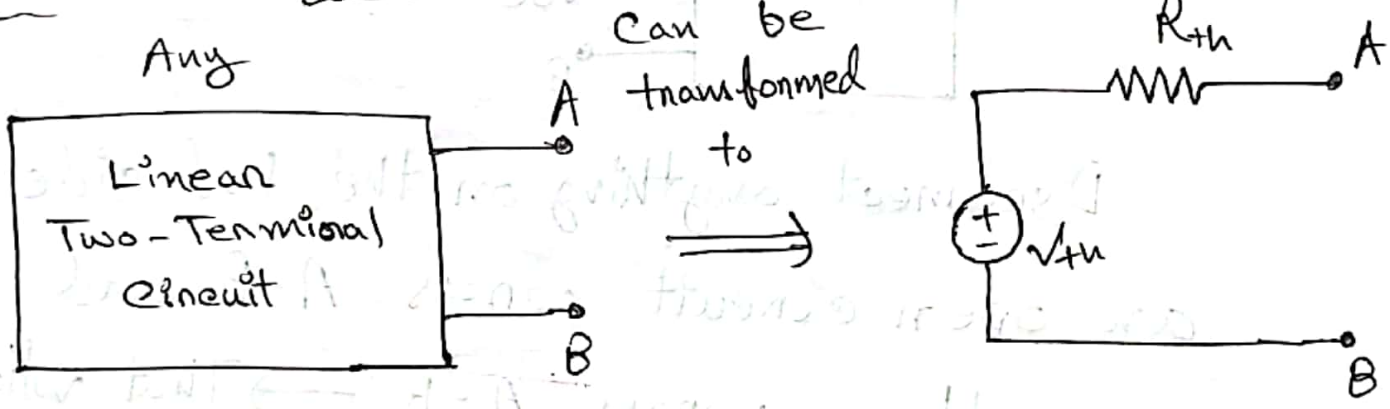
Linear Two-Terminal Circuit



This can be very complex.

Thevenin's Theorem

"A Linear Two-terminal circuit can be replaced by an Equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}, ... [Sadiku 5th Edition page 139]"



⇓

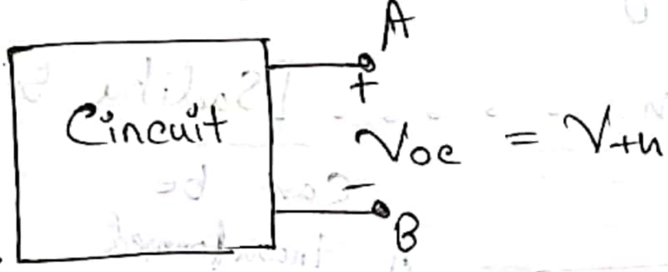
Can be used to replace the fixed part (often very complex) of a circuit by an equivalent simple circuit.

How to Calculate V_{th}, R_{th}

Step-1

Find V_{oc} $\xrightarrow{}$ $V_{open\text{-}circuit}$ across the terminals.

⇓



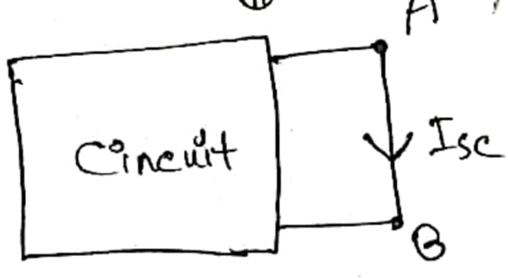
Disconnect anything on the load side to make an open circuit across A-B and calculate the voltage across A-B ⟶ That voltage = V_{oc}

$$V_{oc} = V_{th}.$$

Step-2

Find I_{sc} $\xrightarrow{}$ $I_{short\text{-}circuit}$ through the terminals.

⇓



Ignore all other elements on the load side to ~~ma~~ short A & B terminal and calculate the current flowing from A to B. → That current = I_{sc}.

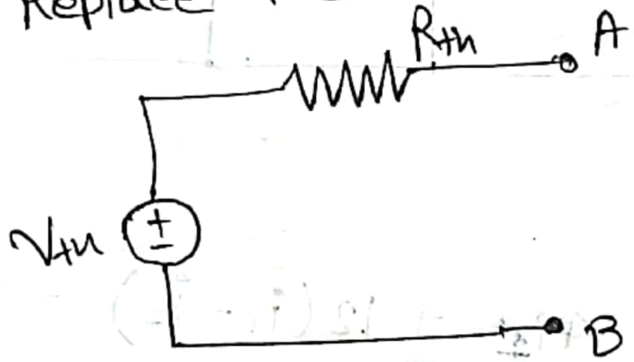
Step-3

Find R_{th}.

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

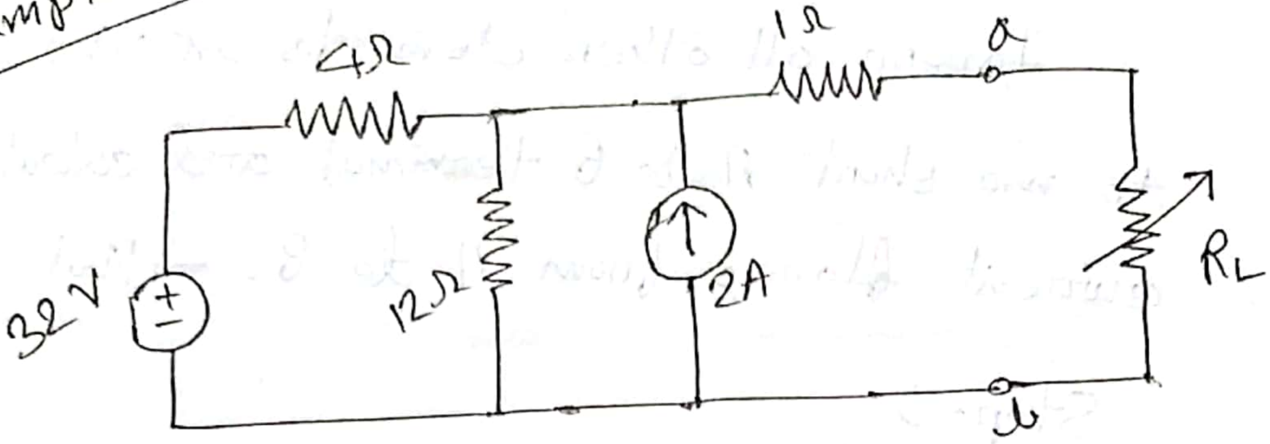
Step-4

Replace the circuit with the Thevenin Equivalent circuit.



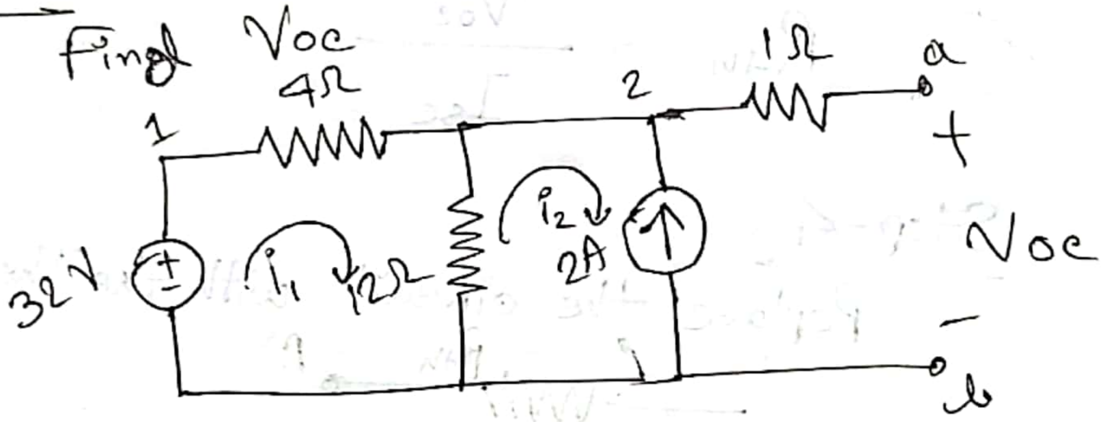
Scanned with CamScanner

Example-14.8



Step-1

Find Voc



Loop-1

$$-32 + 4i_1 + 12(i_1 - i_2) = 0$$

$$\Rightarrow 16i_1 - 12i_2 = 32 \quad \text{--- (i)}$$

Loop-2

$$i_2 = -2 \quad \text{--- (ii)}$$

$\therefore$ Solving (i), (ii) $\longrightarrow$ $i_1 = 0.5\,A$

$i_2 = -2\,A$

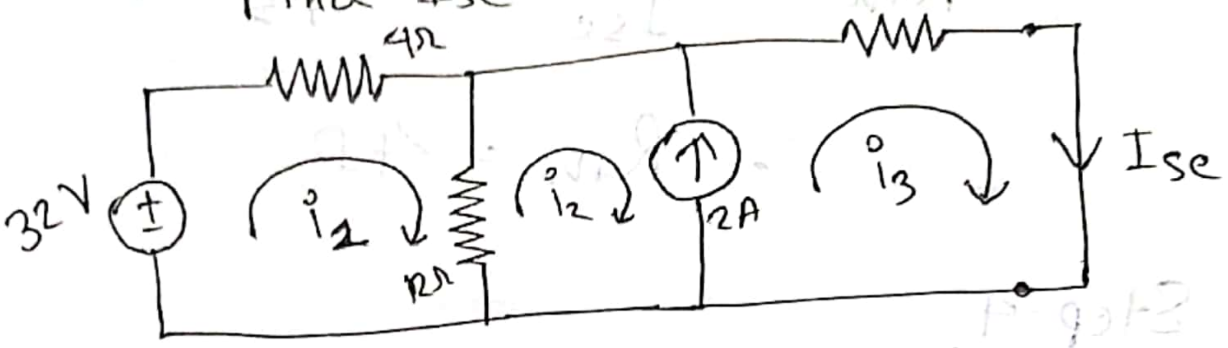
$$\therefore V_{th} = V_2 = 12(i_1 - i_2) = 12(0.5 + 2)\ V$$

$= 30\,V$

$\therefore V_{th} = 30\,V$

Step-2

Find I_{sc}



Loop-1

$$16 i_1 - 12 i_2 = 32 \quad \text{--- (i)}$$

Loop-2 & 3 (Supermesh)

$$i_3 - i_2 = 2 \quad \text{--- (ii)}$$

$$12(i_2 - i_1) + i_3 = 0$$

$$\Rightarrow -12 i_1 + 12 i_2 + i_3 = 0 \quad \text{--- (iii)}$$

$\therefore$ Solving (i), (ii), (iii),

$$i_1 = 6.125\,A$$

$$i_2 = 5.5\,A$$

$$i_3 = 7.5\,A$$

$$\therefore I_{sc} = i_3 = 7.5\,A$$

Step-3

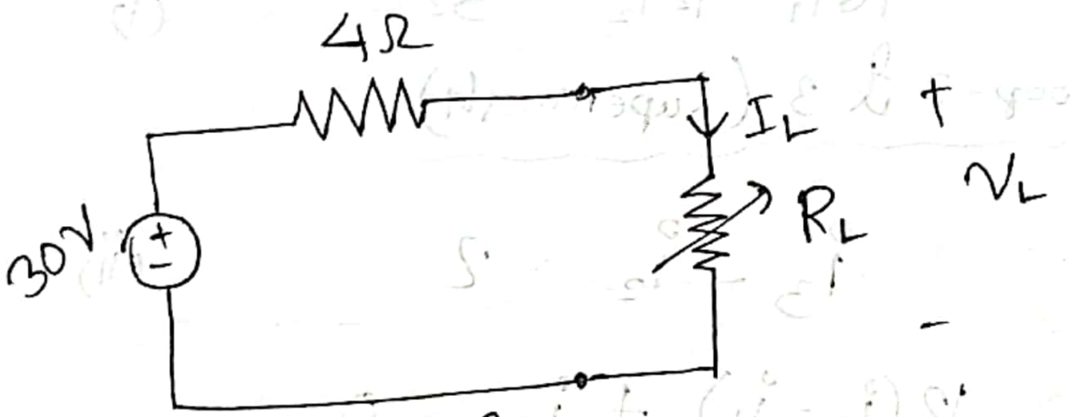
Find R_{th},

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{30}{7.5}\ \Omega$$

$$\therefore R_{th} = 4\,\Omega$$

Step-4

Construct the simplified circuit.



$V_L = ?$

$I_L = ?$

$R_L = 6\,\Omega \longrightarrow$ Using Voltage Divider Rule,

$$V_L = 30 \times \frac{6}{4+6} = \boxed{18\,V}$$

$$I_L = \frac{\Delta V}{R_{eq}} = \frac{30}{4+6} = \boxed{3\,A}$$

$R_L = 16\,\Omega \longrightarrow I_L = \frac{30}{4+16} = \boxed{1.5\,A}$

$R_L = 36\,\Omega \longrightarrow I_L = \frac{30}{4+36} = \boxed{0.75\,A}$ (Ans.)

An Alternative way to calculate R_{th}

This method is only applicable when there is no dependent source in the circuit.

V_{th} & R_{th} Calculation

Step-1

Same as before.

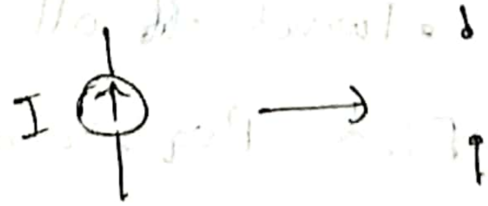
Step-2

Skip

Step-3

a) Replace all voltage sources with short circuit →

b) Replace all current sources with open circuit →



c) Find the Equivalent Resistance Req seeing from a-b terminal.

Step-4 → Same as before.

The previous problem

Step-1

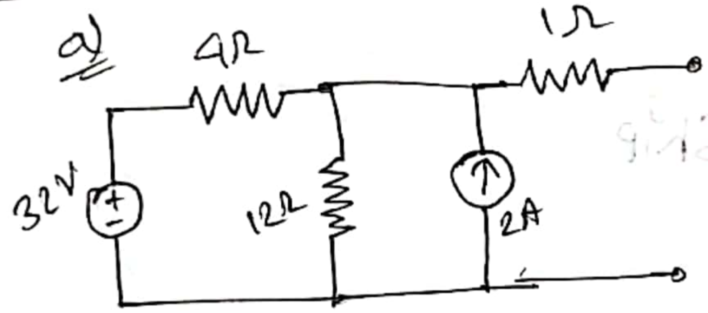
Same as before

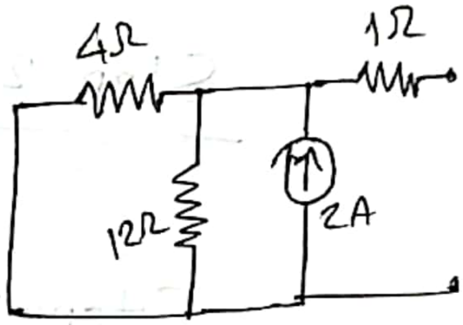
Step-2

Skip

Step-3

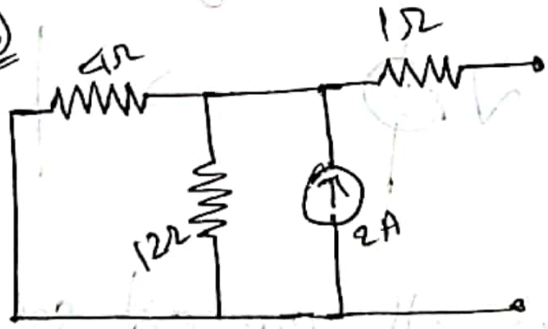
a)

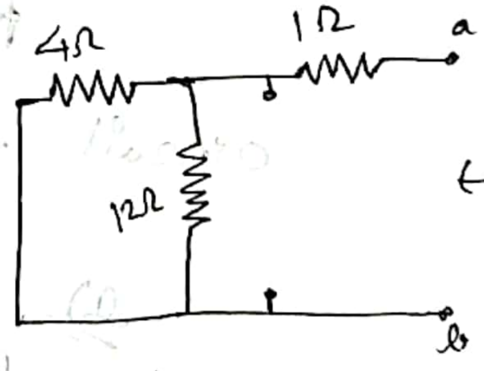




• Turned off all the voltage sources

b)

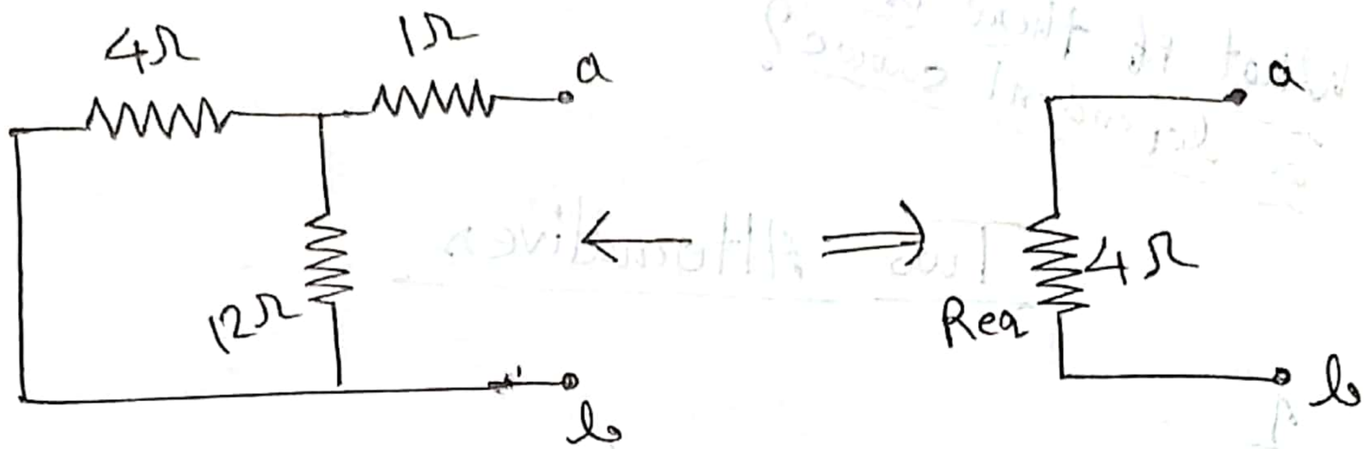




• Turned off all the current sources

c) Find Req, seen from a-b.

$$Req = 1 + 4 \| 12$$
$$= 1 + \left(\frac{1}{4} + \frac{1}{12}\right)^{-1} \Omega = \boxed{4\Omega}$$



$$R_{th} = R_{eq} = 4\Omega$$

Step-4

Same as before.

What if there is a dependent source?

Two Alternatives.

1/ Use the first approach

$V_{th} \rightarrow I_{sc} \longrightarrow R_{th} = \frac{V_{th}}{I_{sc}}$

simply using Mesh Analysis everytime.
/Nodal
/ST

2/ Use the second approach using a dummy voltage source

⇓

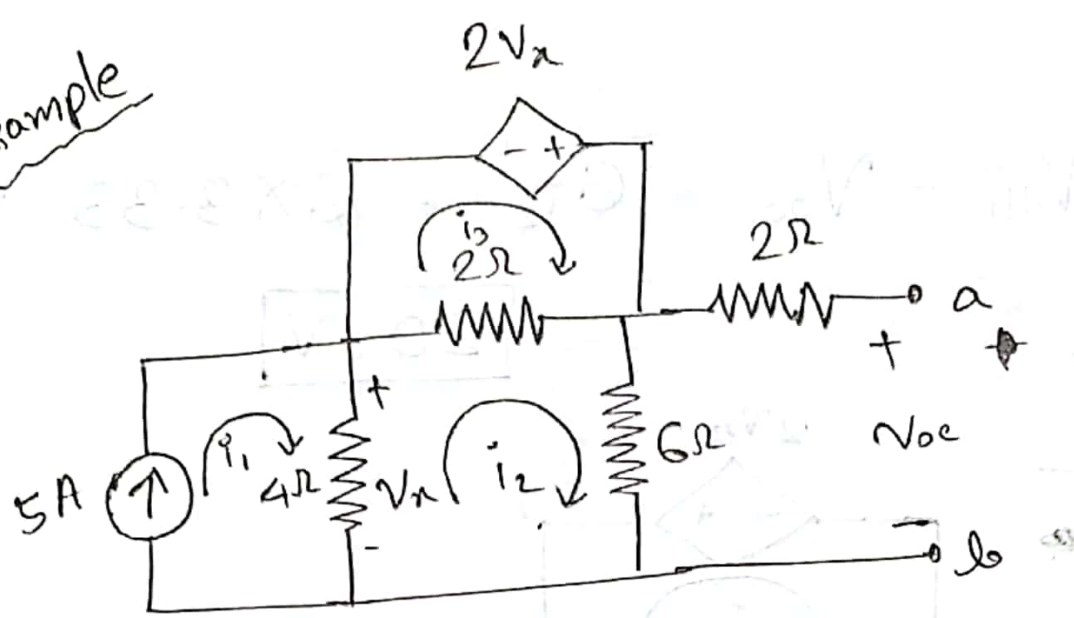
a) Between a-b → place a 1V voltage source

b) Turn all other independent sources off while keeping the dependent sources intact.

c) Analyse the circuit to find out the current i_o flowing through a.

d) $R_{th} = 1/i_o$

Example



$$V_x = 4(i_1 - i_2)$$

Step-1 → V_{th}

Loop-1

$$i_1 = 5 \quad \text{———— (i)}$$

Loop-2

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

$$\Rightarrow -4i_1 + 12i_2 - 2i_3 = 0 \quad \text{———— (ii)}$$

Loop-3

$$2(i_3 - i_2) - 2V_x = 0$$

$$\Rightarrow 2i_3 - 2i_2 - 2\cdot 4(i_1 - i_2) = 0$$

$$\Rightarrow -8i_1 + 6i_2 + 2i_3 = 0 \quad \text{———— (iii)}$$

Solving i, ii, iii,

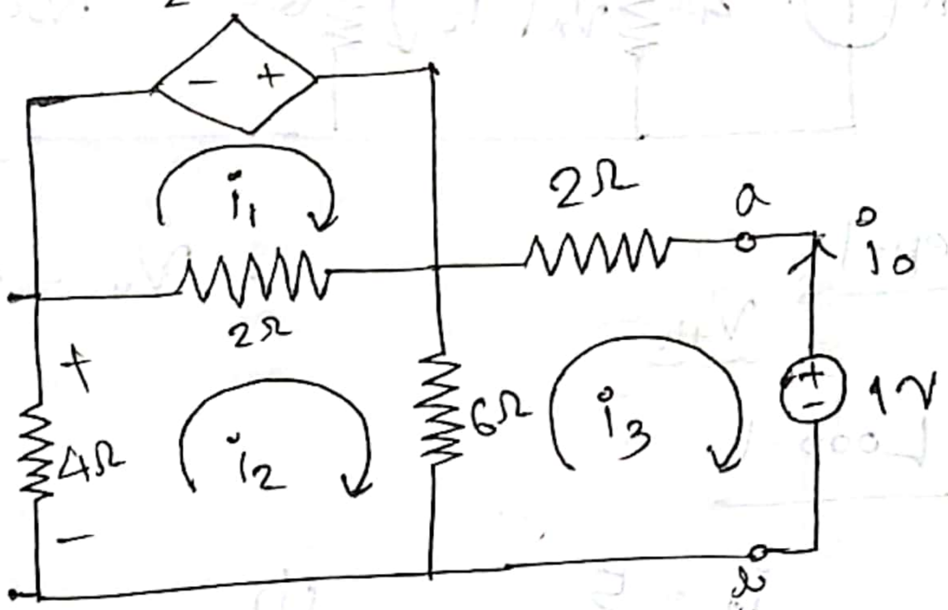
$$i_1 = 5\,A, \quad i_2 = 3.33\,A, \quad i_3 = 10\,A.$$

$$V_{th} = V_{oc} = 6 i_2 = 6 \times 3.33 \text{ V}$$

$$= \boxed{20 \text{ V}}$$

Step-2

R_{th}



$$V_x = -4 i_2$$

Loop-1

$$2(i_1 - i_2) - 2V_x = 0$$

$$\Rightarrow 2i_1 - 2i_2 - 2\cdot(-4i_2) = 0$$

$$\Rightarrow 2i_1 + 6i_2 = 0 \quad \text{———} (i)$$

Loop-2

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

$$\Rightarrow -2i_1 + 12i_2 - 6i_3 = 0 \quad \text{———} (ii)$$

Loop-3

$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

$$\Rightarrow -6i_2 + 8i_3 = -1 \quad \text{———} (iii)$$

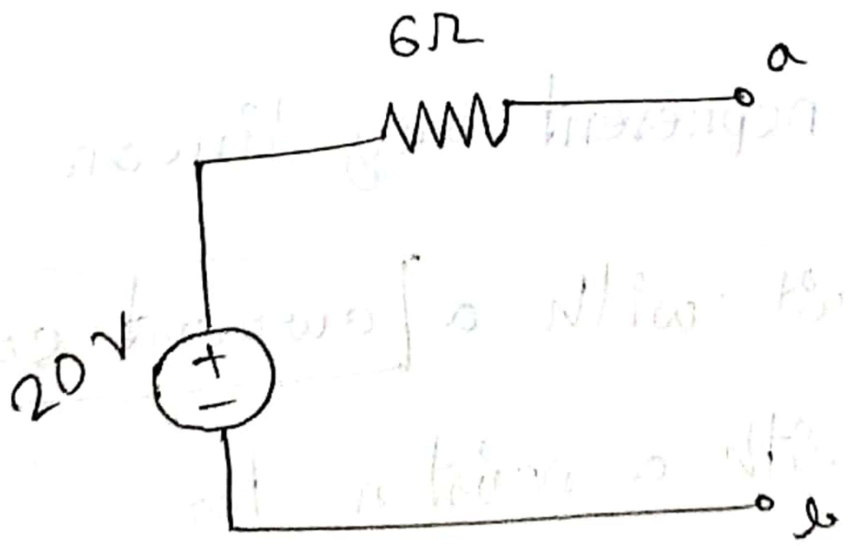
Solving for i, ii, iii,

$$i_3 = -0.1667 \text{ A}$$

Here, $i_o = -i_3$

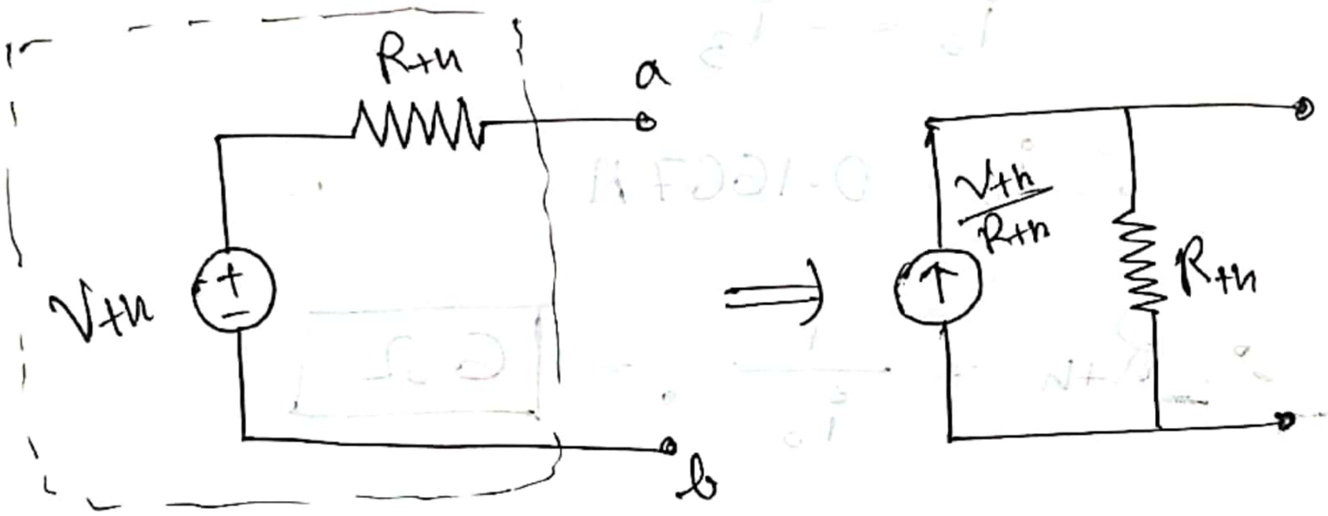
$$\therefore i_o = 0.1667 \text{ A}$$

$$\therefore R_{th} = \frac{1}{i_o} = \boxed{6 \Omega}$$



Thevenin Equivalent Circuit.

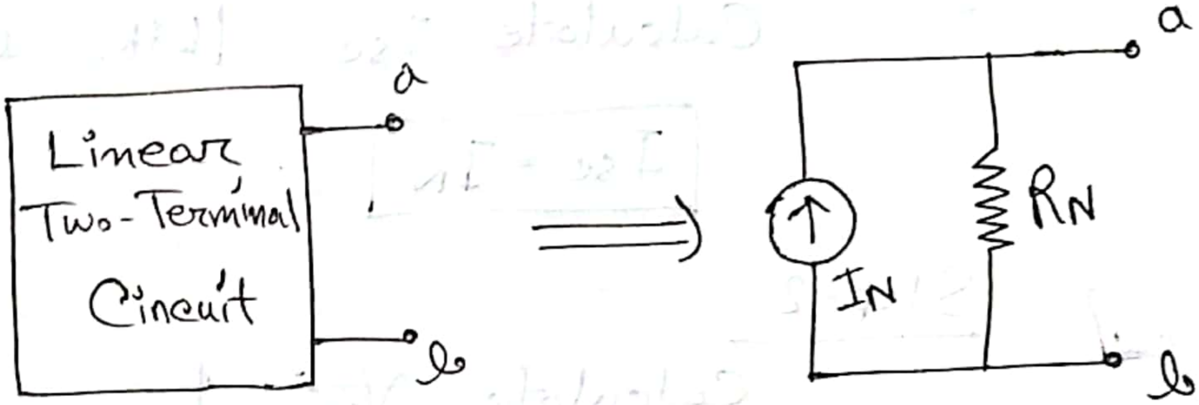
Applying Source Transformation to a Thevenin Eq. Circuit



∴ We can represent any linear two terminal circuit with a current source in parallel with a resistor too.

Norton's Theorem

Sadiku 5th Edi. → Page 145.



Relationships with Thevenin Eq. circuit

- $R_N = R_{th}$

- $I_N = \dfrac{V_{th}}{R_{th}}$

$I_N = I_{sc}$, | The I_{sc} we calculated in step-2 of Thevenin |

How to Calculate I_N, R_N

Step-1

Calculate I_{sc} |Like before|

$$\boxed{I_{sc} = I_N}$$

Step-2

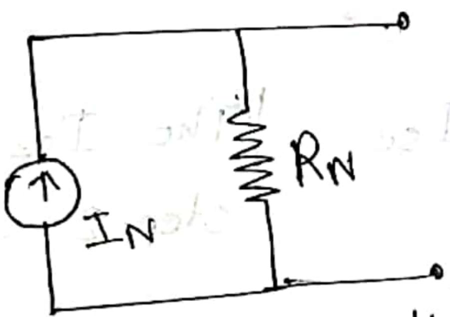
Calculate V_{oc} | " |

Step-3

Find R_N

$$\boxed{R_N = \frac{V_{oc}}{I_{sc}}}$$ | " |

Step-4

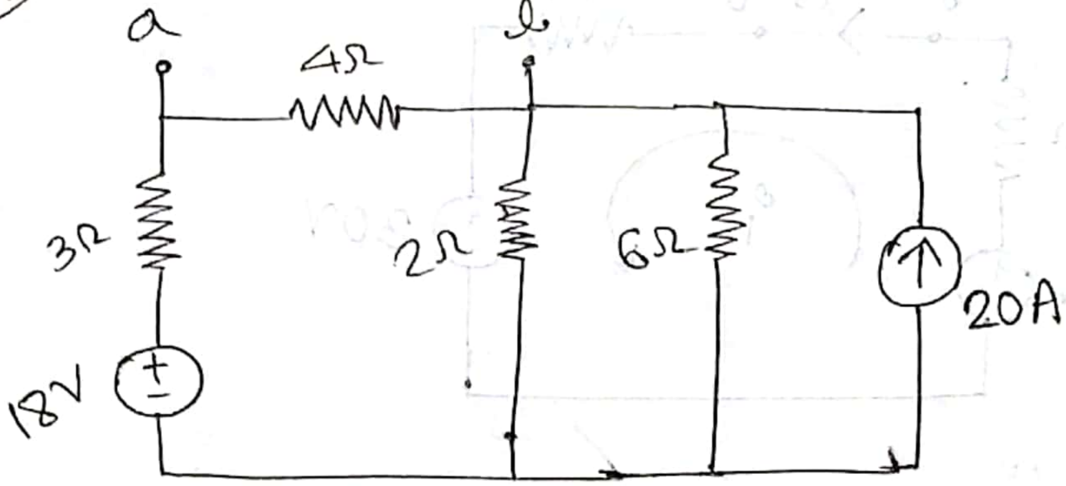


Norton Equivalent Circuit

Alternative ways

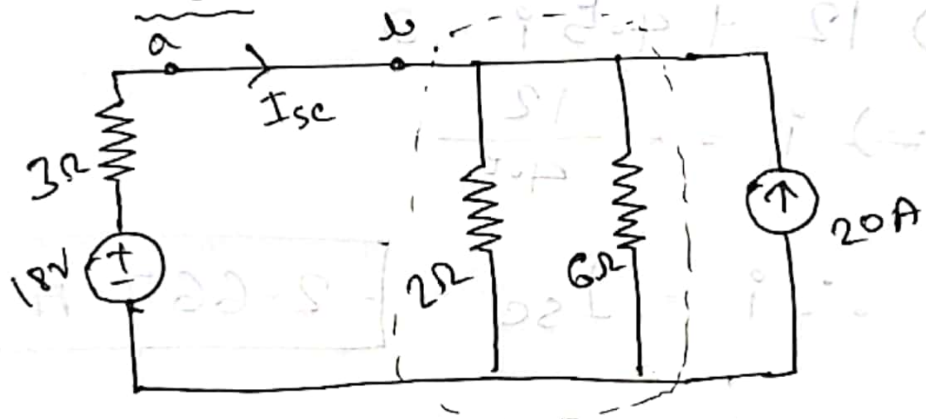
$R_N = R_{th}$. ∴ Everything else exactly the same as Thevenin.

Example

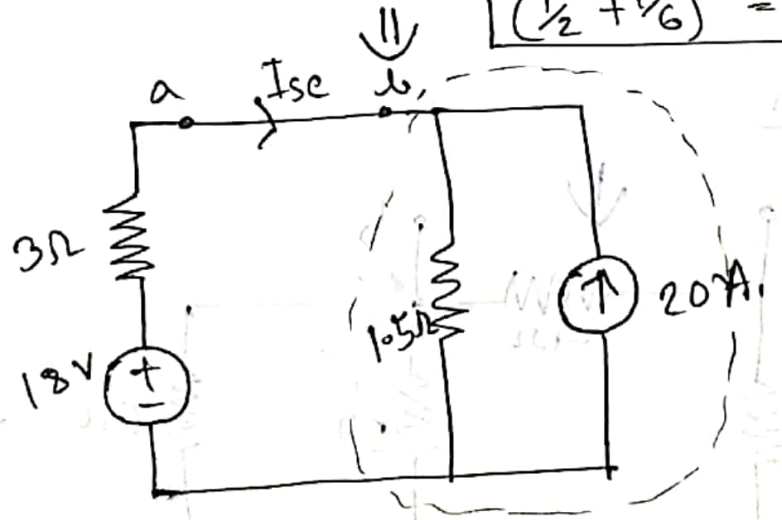


Step-1

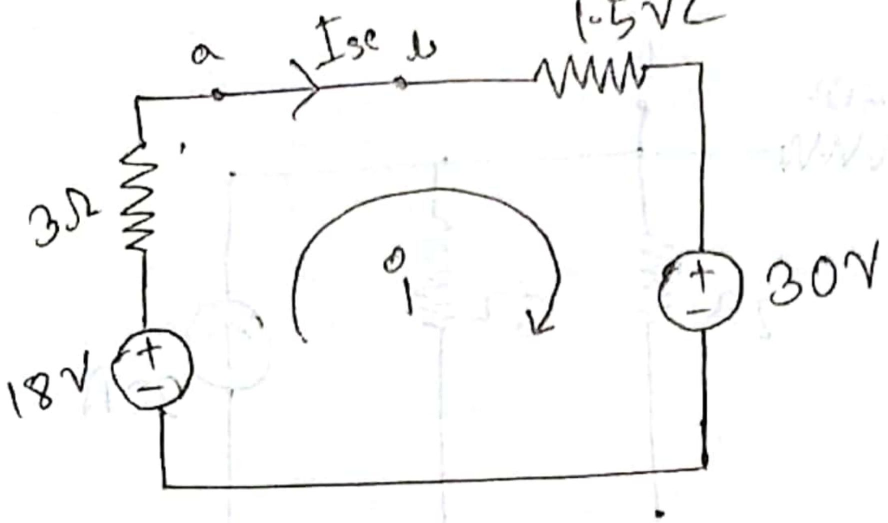
Isc



$\boxed{\left(\frac{1}{2}+\frac{1}{6}\right)^{-1} = 1.5}$ ← Req



$\boxed{V = 20 \times 1.5 = 30V}$



$$\therefore -18 + 3i + 1.5i + 30 = 0$$

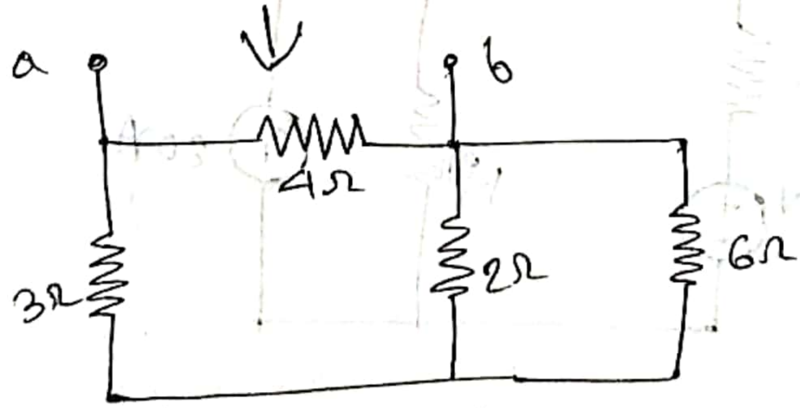
$$\Rightarrow 12 + 4.5i = 0$$

$$\Rightarrow i = -\frac{12}{4.5}$$

$$\therefore i = I_{sc} = \boxed{-2.667\ A}$$

Step-2

Rth



{ 18V voltage source shorted, 20A current source open circuit

$$R_{eq} = (3 + 2 \| 6) \| 4$$

{ 2Ω & 6Ω in parallel, they are in series with 3Ω. All others in parallel with 4Ω

$$= \left(3 \cancel{\|} + \left(\frac{1}{2} + \frac{1}{6}\right)^{-1}\right) \| 4$$

$$= (3 + 1.5) \| 4$$

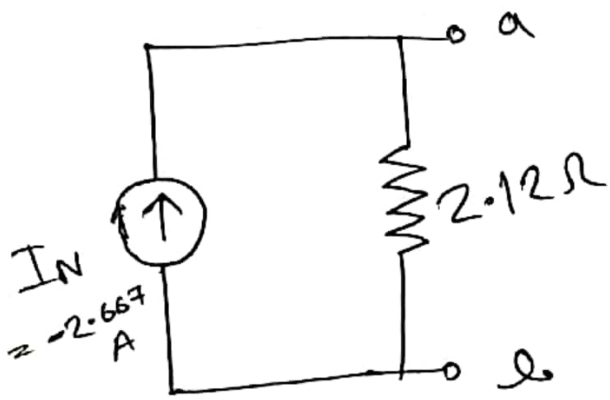
$$= 4.5 \| 4$$

$$= \left(\frac{1}{4.5} + \frac{1}{4}\right)^{-1}$$

$$= 2.12\,\Omega$$

$$\therefore R_{eq} = R_N = 2.12\,\Omega$$

Norton Equivalent Circuit



$\Downarrow$