CSE 250

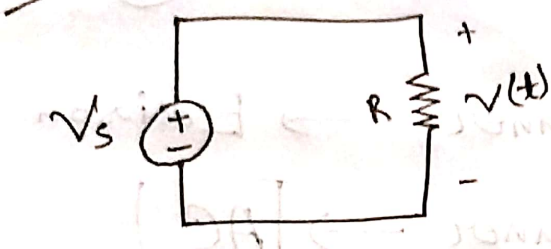
Circuits & Electronics

Week - 9

AC Circuit Fundamentals + Complex Number Review

DC vs Transient vs AC

DC



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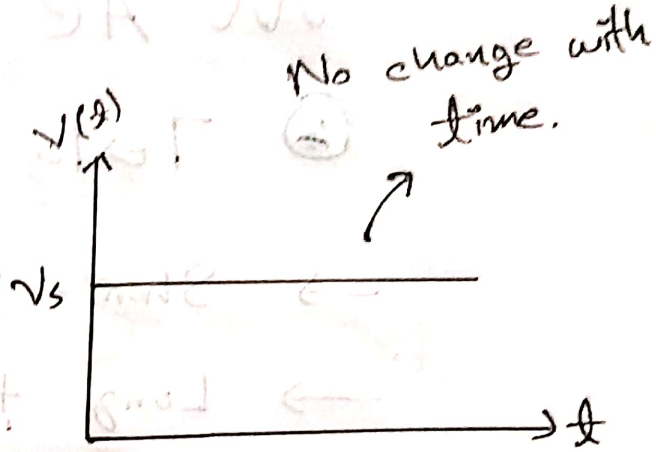
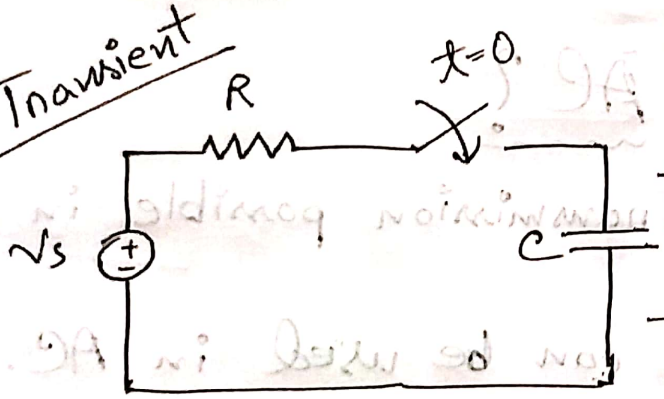


Transient



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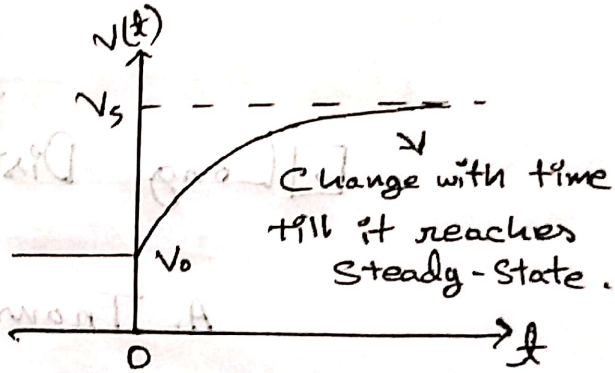
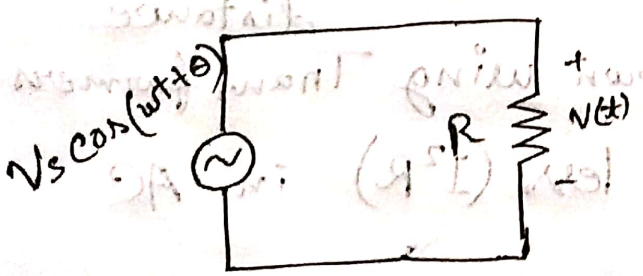


AC



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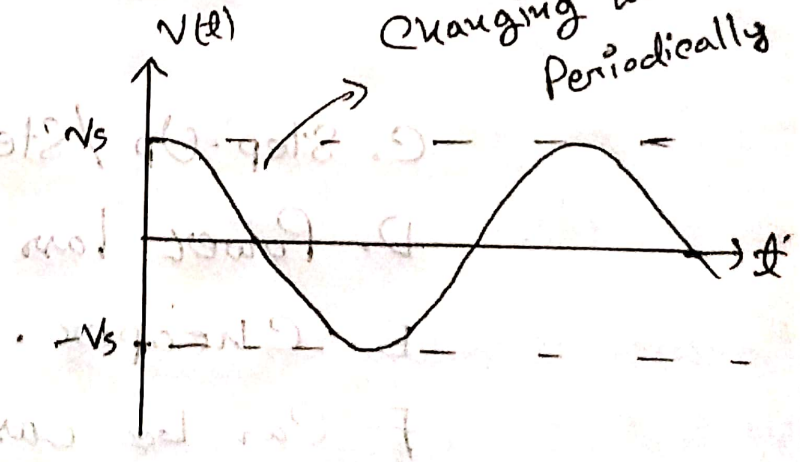


The WAR of Currents

AC vs DC —

Tesla vs Edison

→ Short term Winner → Edison

→ Long term Winner → **AC**

Why AC?

- Long Distance Transmission possible in AC.

⇓

A. Transformers can be used in AC.

B. Also, AC can generate High voltages.

↓

Easier to transmit in long distance.

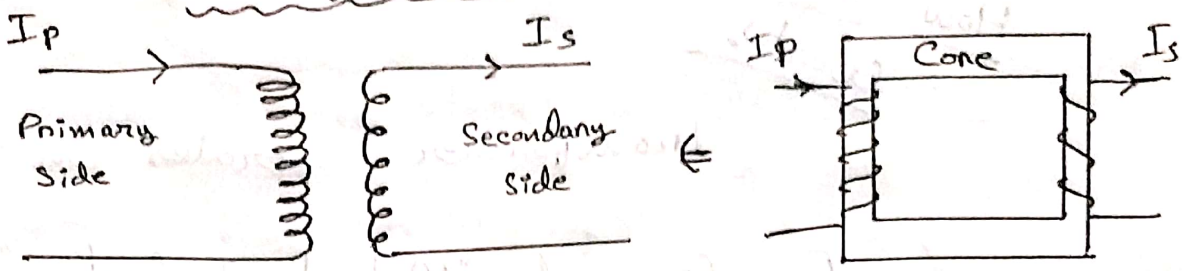
C. Step-Up / Step-Down using Transformers.

D. Power loss is less (I^2R) in AC

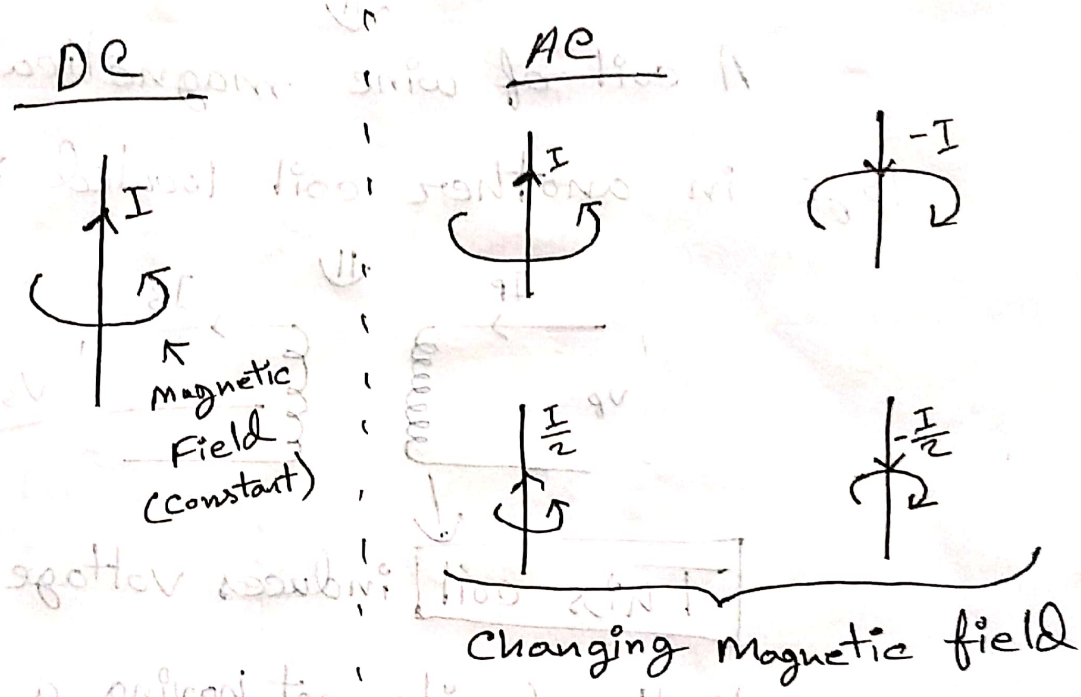
E. Cheaper.

F. Can be easily converted to DC.

Transformers



Magnetic Effect



So, Current through a wire → Magnetic Field

Constant for DC

Varying for AC

Again,

Varying Magnetic field → Induces current in a wire. (Constant MF doesn't)

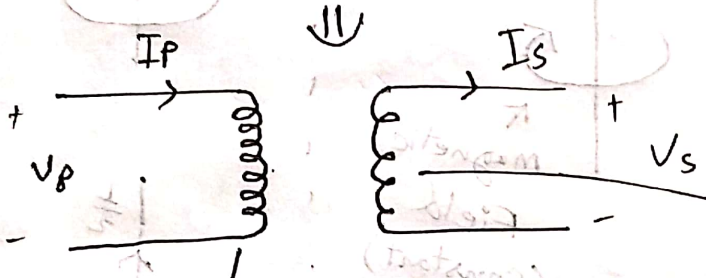
How Transformer Works

Transformer operates on the principle of Faraday of "Mutual Induction."

⇓

A coil of wire magnetically induces voltage in another coil located in close proximity.

⇓



This coil induces voltage in this coil magnetically despite not having a direct connection.

⇓

Power (VI) conserved

⇓

$$\therefore V_p I_p = V_s I_s$$

⇓

Can transform low voltage, high current to High voltage, low current and vice versa.

How this Step Up/Down

⇓

Depends on the ratio of number of turns in the primary and secondary coils, because the magnetic flux (ϕ) depends on it.

⇓

$$\text{Turns Ratio}, n = \frac{N_P}{N_S} = \frac{\text{Number of turns in primary}}{\text{Number of turns in secondary}}$$

⇓

$$n = \frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{I_S}{I_P}$$

P → primary

S → secondary

⇓

To turn 200 V into 20000 V for transmission,

$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{200}{20000} = \frac{1}{100}$$

⇓

Using a transformer of 100 times more turns in secondary coil compared to the primary coil.

Effect in Power loss

$$\frac{V_P}{V_S} = \frac{1}{100} = \frac{I_S}{I_P}$$

$$\therefore I_S = \frac{I_P}{100}$$

This means the current in the transmission line (secondary side in this case), will be 100 times less.

Now, we know,

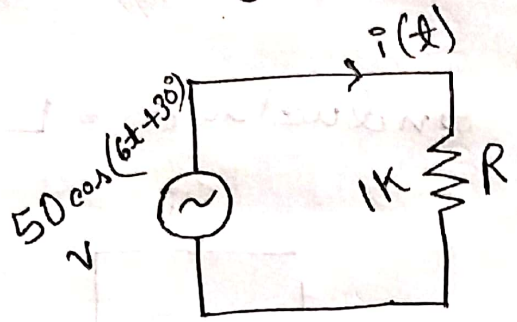
$$\text{Power loss} = I^2R \propto I^2$$

$\therefore$ Power loss will be $100^2 = 10000$ times less.

$\Downarrow$

Possible to transmit a long distance

AC Circuit Analysis



$$v_s(t) = 50\cos(6t + 30^\circ)\ V$$

$$R = 1K\ \Omega$$

$$I = i(t) = ?$$

$$v_s(t) = i(t) \times R$$

$$\Rightarrow i(t) = \frac{v_s(t)}{R}$$

$$= \frac{50\cos(6t + 30^\circ)}{1000}$$

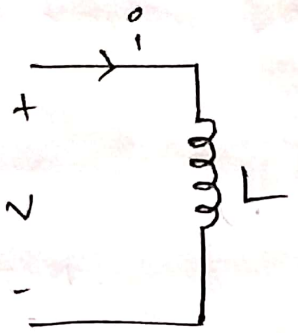
$$= \frac{1}{20}\cos(6t + 30^\circ)$$

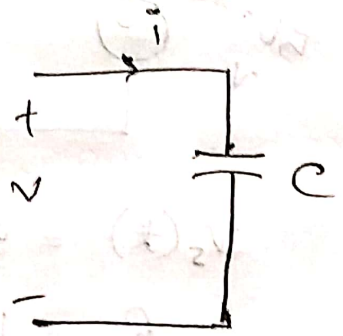
(Ans.)

⇒ With just Resistance, almost identical to DC analysis. Things are different when we add Capacitance and Inductance.

We Know,

if Capacitance = C, Inductance = L,





$$V = L\frac{di}{dt}$$

In case of DC,

$$\frac{di}{dt} = 0$$ [No change with time]

$\therefore V = 0$

$\Rightarrow$ Short Circuit.

$$i = C\frac{dv}{dt}$$

In case of DC,

$$\frac{dv}{dt} = 0$$ [No change with time]

$\therefore i = 0$

$\Rightarrow$ Open Circuit

But in case of AC,

$$\frac{di}{dt} \neq 0, \quad \frac{dv}{dt} \neq 0$$

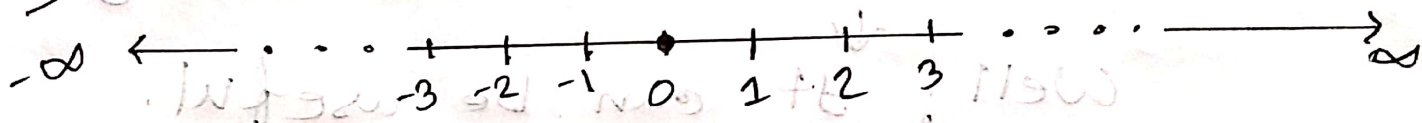
$\therefore$ Analysis will be a bit more "Complex".

Complex Number
(Review)

Real Number $\xrightarrow{+}$ $\boxed{\Sigma}$ $\Rightarrow$ Complex Number

Imaginary Number $\xrightarrow{+}$

Real Number $\longrightarrow$ Unit is '1'.



Remember সংখ্যা কোন?

- 1, 0, 19, 236012, 222 million (:3) . . .
- $\frac{1}{2}$, $\frac{3}{7}$, $\frac{1369}{5251}$, $\frac{9}{29}$, 0.25813 . . .
- -85, -41, $-\frac{81}{2}$, -5.8125 . . . .
- π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{19}$.

$\Downarrow$

All of them Real Numbers

$\Downarrow$

All / Each of them is somewhere in Number line.

Imaginary Number

→ Unit is 'i'.

Timeline

- René Descartes → 17th century

↓

coined in as a derogatory, useless (:3) term.

- Euler → 18th century

↓

Well! It can be useful.

- Cauchy, Gauss → 19th century

↓

It's super useful!!

Concept

$$i = \sqrt{-1}$$

[Doesn't make sense right?]

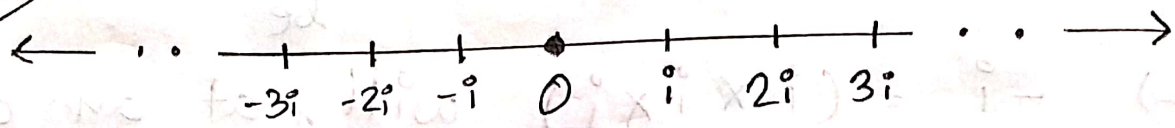
Examples

$$\sqrt{-3} = \sqrt{-1} \times \sqrt{3}$$

$$= \sqrt{3}\, i$$

$$\sqrt{-9} = \sqrt{-1} \times \sqrt{9} = 3i$$

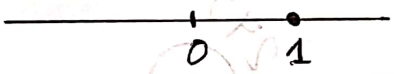
Complex Numbers: Real & Imaginary Axis



$$\sqrt{0} = \sqrt{-0} = 0$$

Same origin as Real axis (সাংখ্যা রেখা)

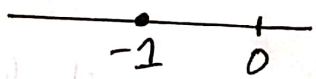
Now,

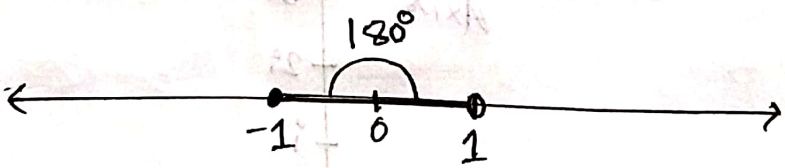


$\Rightarrow 1$

$1 \times i = i$

$i \times i = i^2 = -1 \Leftarrow$





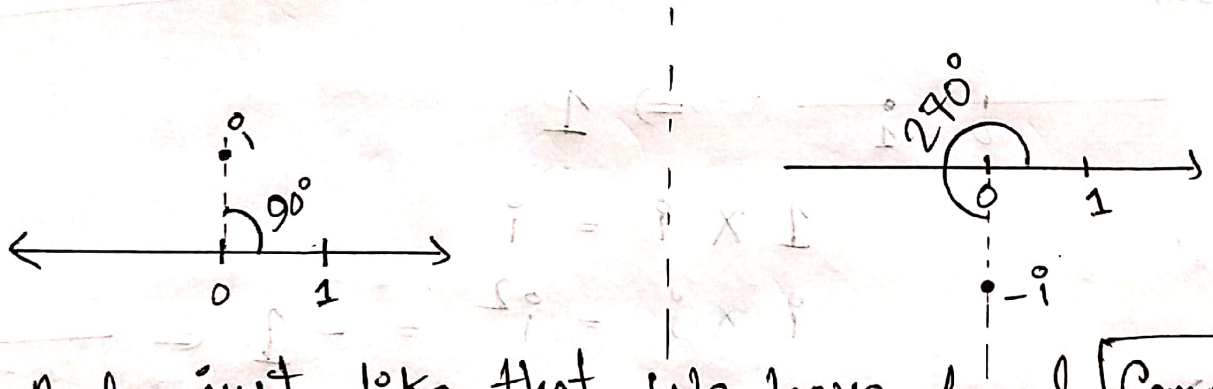
∴ Multiplying 1 twice by i has introduced a 180° notation.

∴ Multiplying by i once will i will introduce a notation of $\frac{180^\circ}{2} = \boxed{90^\circ}$.

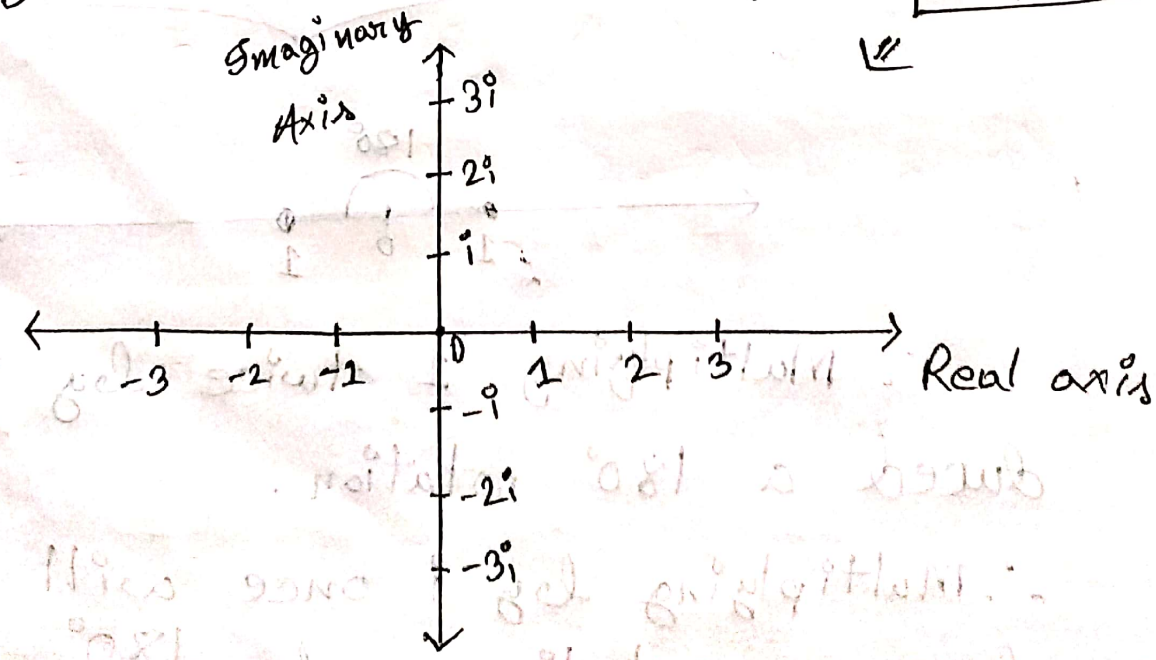
$\therefore$ i will be situated at angle of $90°$ with 1.

$\Rightarrow$ $-i = (i \times i \times i)$ will be at an angle of $90° \times 3 = 270°$ with 1.

$\Rightarrow$ Let's plot them with respect to real axis.



And just like that, we have found $\boxed{\text{Complex plane}}$



A Complex Number

$$a + bi$$

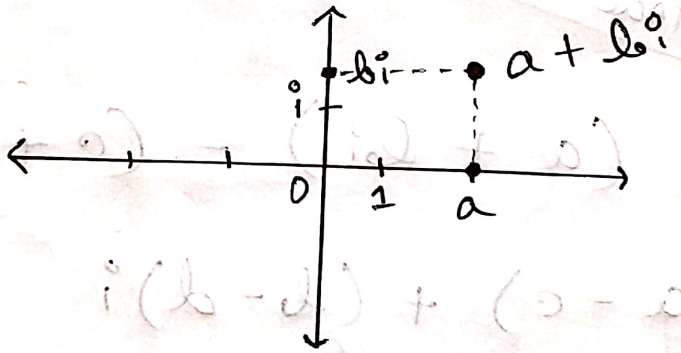
Real → a ; Imaginary → bi

Example: $3 + 5i$, $-3 + 7i$, $2.51 + 3.47i$,

$\sqrt{2} + \sqrt{3}i$, $\sqrt{2} - \sqrt{3}i$.

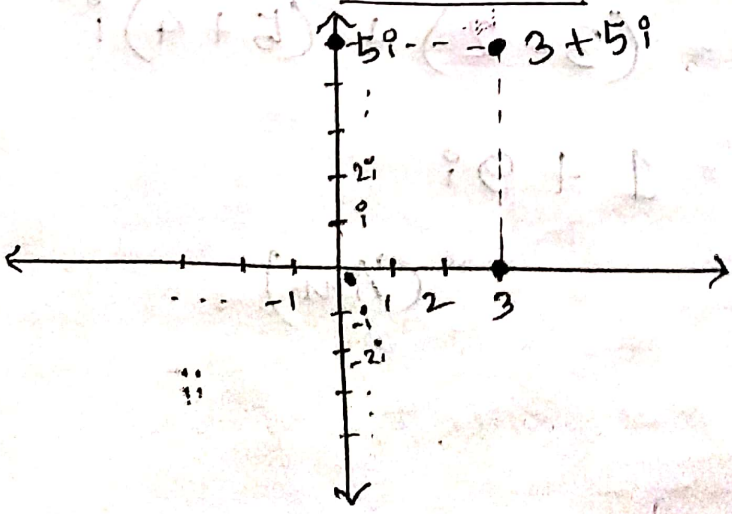
Real part → $\sqrt{2}$; Imaginary part → $\sqrt{3}$

In Complex Plane

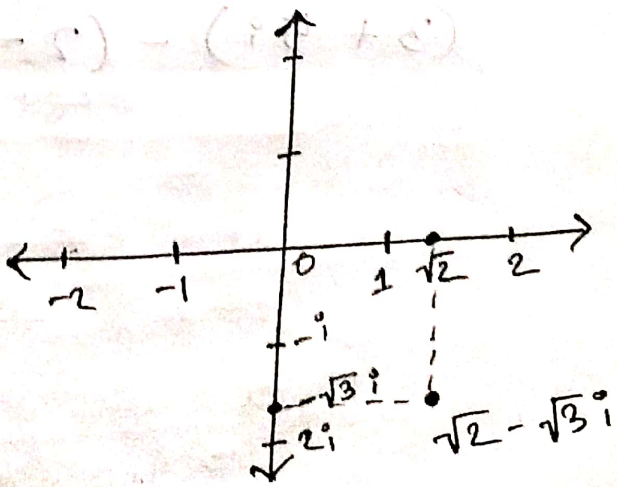


⇓

$3 + 5i$



$\sqrt{2} - \sqrt{3}i$



Complex operations

Addition

$$(a + bi) + (c + di)$$

$$= \underbrace{(a + c)}_{\text{Real part}} + \underbrace{(b + d)}_{\text{Imaginary}}i$$

Ex: $(2 + 3i) + (5 + 2i) = ?$

$$(2 + 3i) + (5 + 2i) = (2 + 5) + (3 + 2)i$$

$$= 7 + 5i$$

(Ans.)

Subtraction

$$(a + bi) - (c + di)$$

$$= (a - c) + (b - d)i$$

Ex: $(3 + 5i) - (2 - 4i) = ?$

$$(3 + 5i) - (2 - 4i) = (3 - 2) + (5 + 4)i$$

$$= 1 + 9i$$

(Ans.)

Multiplication

$$(a + bi) \times (c + di)$$

$$= ac + c \cdot bi + a \cdot di + bi \cdot di$$

$$= ac + bci + dai + bd \cdot i^2$$

$$= (ac - bd) + (bc + da)i \quad [\because i^2 = -1]$$

Ex: $(3 + 5i) \times (2 + 7i)$

$$= (3 \times 2) + (2 \times 5)i + (3 \times 7)i + (5 \times 7) \cdot i^2$$

$$= (6 - 35) + (10 + 21)i$$

$$= -29 + 31i$$

(Ans.)

Complex Conjugate

$\longrightarrow$ Same Real part, Complex part same value with opposite sign.

$$z \xrightarrow{\text{conjugate}} \bar{z}$$

$$a + bi \xrightarrow[\text{Conjugate}]{\text{Complex}} a - bi$$

Ex:

	Complex Conjugate
$3 + 5i$	$3 - 5i$
$6 - 8i$	$6 + 8i$
$\sqrt{3} + \sqrt{2}i$	$\sqrt{3} - \sqrt{2}i$

Division

$$\frac{a+bi}{c+di}$$

Trick → Multiply both the numenaton and denominaton by the complex conjugate of denominaton.

$$\Rightarrow \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{ac+b\cdot ci-a\cdot di-bi\cdot di}{c^2-c\cdot di+c\cdot di-(di)^2}$$

$$= \frac{ac+bci-dai-bd\cdot i^2}{c^2-d^2\cdot i^2}$$

$$= \frac{(ac+bd)+(bc-da)i}{c^2+d^2}$$

This way the real and complex part of the answer is separated

Ex: $\frac{3+4i}{5-2i}$

$$= \frac{(3+4i)(5+2i)}{(5-2i)(5+2i)}$$

$$= \frac{15+20i+6i+8i^2}{25-10i+10i-4i^2}$$

$$= \frac{(15-8)+26i}{25+4}$$

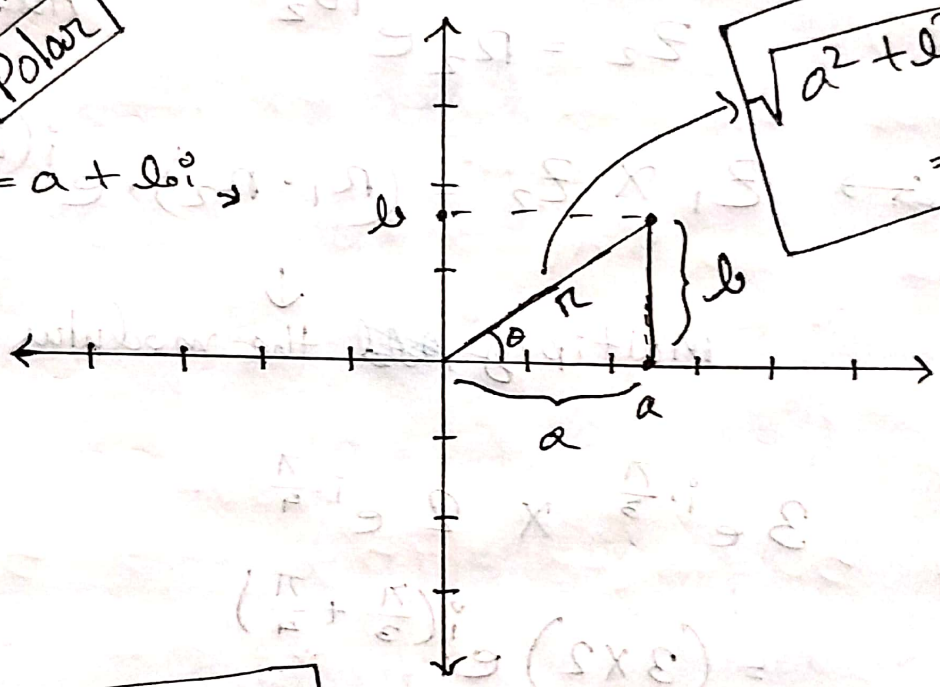
$$= \frac{7}{29} + \frac{26}{29} i$$

(Ans.)

Another Representation

$\boxed{\text{Polar}}$

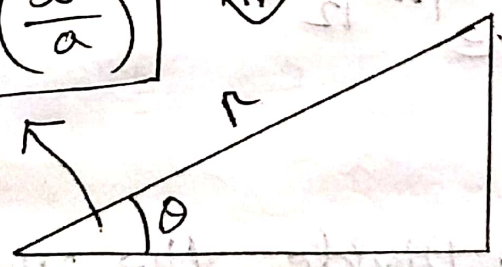
- $Z = a + bi$



$$\boxed{\sqrt{a^2+b^2} = \sqrt{z \cdot \bar{z}} = |z| = r}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{b}{a}\right)}$$

$\Downarrow$



$b = r\sin\theta$

$a = r\cos\theta$

$$\therefore Z = a + bi = r\underbrace{(\cos\theta + i\sin\theta)}_{e^{i\theta} = \cos\theta + i\sin\theta}$$

$\Downarrow$

Euler's Formula

Cartesian Polar

$$\therefore \boxed{z = a + bi = re^{i\theta}} \quad ☆$$

Polar Multiplication

Let, $z_1 = r_1 e^{i\theta_1}$

$z_2 = r_2 e^{i\theta_2}$

Add the angle

$$\Rightarrow z_1 \times z_2 = (r_1 \cdot r_2) \cdot e^{i(\theta_1 + \theta_2)}$$

Multiply the modulus

Ex:

$$3e^{i \cdot \frac{\pi}{3}} \times 2e^{i \cdot \frac{\pi}{4}}$$

$$= (3 \times 2) e^{i\left(\frac{\pi}{3} + \frac{\pi}{4}\right)}$$

$$= 6e^{i \cdot \frac{7\pi}{12}}$$

Division

- Divide the modulus
- Subtract the angle.

$z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$

$$\Rightarrow \frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

Complex number in AC calculation

- $\sqrt{-1} = j$ | Not i because it's used for current |

- $re^{i\theta} \longrightarrow \boxed{R\angle\theta^\circ} \longrightarrow$ Phasors.

Ex:

$$z_1 = 3e^{i\cdot\frac{\pi}{3}} = 3\angle\frac{\pi}{3} = 3\angle 60^\circ$$

$$z_2 = 2e^{i\cdot\frac{\pi}{4}} = 2\angle\frac{\pi}{4} = 2\angle 45^\circ$$

$$z_1 \cdot z_2 = 3\angle 60^\circ \times 2\angle 45^\circ$$

$$= (3\times 2)\angle(60^\circ + 45^\circ)$$

$$= 6\angle 105^\circ$$

- Addition, Subtraction $\longrightarrow$ Harder in Polar.

Advantage of using Phasor

An Example $\longrightarrow$ $\sqrt[3]{1} = 1, -\frac{1}{2} + i\frac{\sqrt{3}}{2}, -\frac{1}{2} - i\frac{\sqrt{3}}{2}$.

$\Downarrow$

In Phasor format,

$$\sqrt[3]{1} = 1\angle 0^\circ, 1\angle 120^\circ, 1\angle -120^\circ.$$

$\Downarrow$

Try plotting it in complex plane.

Voltage & Current in Phasor Domain

That's how AC voltage/ current sources are usually represented.

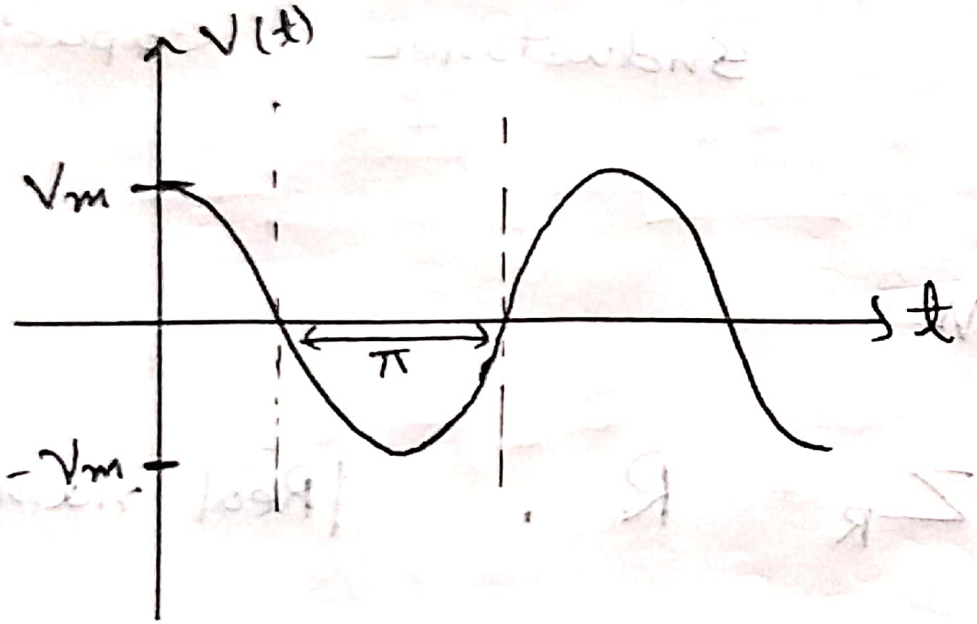
Time Domain		Phasor Domain
$V_m \cos(\omega t + \theta)$	$\longrightarrow$	$V_m \angle \theta$
$V_m \sin(\omega t + \theta)$	$\longrightarrow$	$V_m \angle \theta - 90^\circ$
$I_m \cos(\omega t + \theta)$	$\longrightarrow$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$\longrightarrow$	$I_m \angle \theta - 90^\circ$

Ex.

- $50 \cos(2t + 75^\circ) \longrightarrow 50 \angle 75^\circ$
- $10 \cos 4t \longrightarrow 10 \angle 0^\circ$
- $5 \cos(100t - 30^\circ) \longrightarrow 5 \angle -30^\circ$
- $10 \sin(5t + 45^\circ) \longrightarrow 10 \angle (45^\circ - 90^\circ) = 10 \angle -45^\circ$

Sinusoids

↳ A signal that has the form of the sine or cosine function.



$$V(t) = V_m \cos(\omega t + \theta)$$

$$= V_m \angle \theta$$

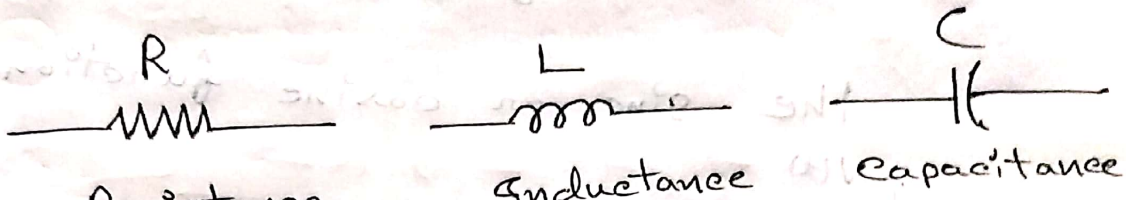
- V_m = Amplitude
- ω = Angular frequency

Period, $T = \frac{2\pi}{\omega}$

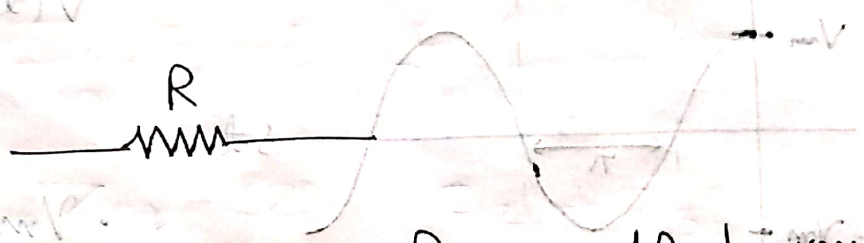
- θ = Phase, argument.

Impedence

$$\boxed{V = IZ}$$



Resistors



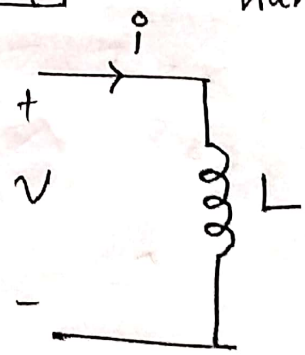
Impedence, $Z_R = R$. |Real number|

Inductors

Impedence, $\boxed{Z_L = j\omega L}$ |Imaginary number|

How?

We Know,



$$v = L\frac{di}{dt}$$

here, $i = I_m \cos(\omega t + \theta)$

$$\therefore v = L\frac{d}{dt}\{I_m \cos(\omega t + \theta)\}$$

$$= -\omega L I_m \sin(\omega t + \theta)$$

$$= +\omega L I_m \angle \theta + 90^\circ$$

$$= \omega L I_m \angle \theta \cdot 1\angle 90^\circ$$

$$= \omega L I \cdot e^{j90^\circ}$$

$$= j\omega L \cdot I$$

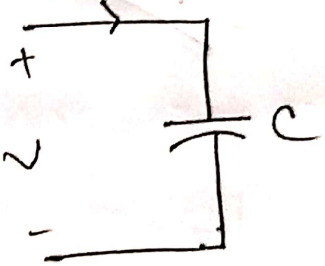
$$\therefore V = j\omega L \cdot I$$

Now, $V = IZ \Rightarrow Z = \frac{V}{I}$

$$\therefore \boxed{Z_L = \frac{V}{I} = j\omega L}$$

Capacitors

$$i = C\frac{dv}{dt}$$

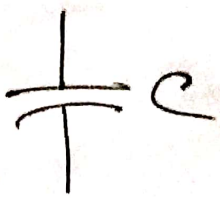


Similar analysis like

Inductors

$$I = j\omega C V \Rightarrow \frac{V}{I} = \boxed{Z_c = \frac{1}{j\omega C}}$$

Ex: $C = 10\ mF$, $\omega = 4$, $Z_c = ?$



$$\therefore Z_c = \frac{1}{j\omega C} = \frac{1}{j \times 4 \times 10 \times 10^{-3}}$$

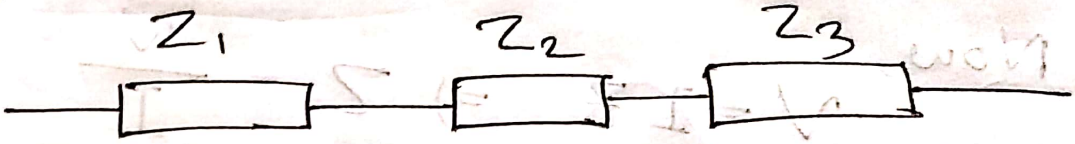
$$= \frac{1}{j} 25\ \Omega = -j25\ \Omega$$

(Ans.)

AC Circuit Combinations

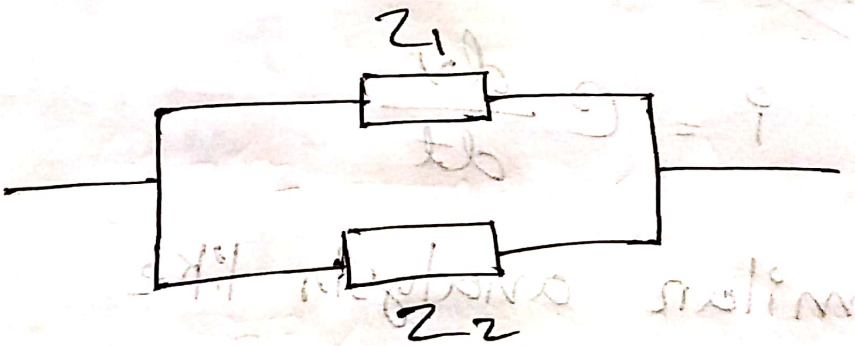
Impedence Combinations

Series



$$Z_{eq} = Z_1 + Z_2 + Z_3$$

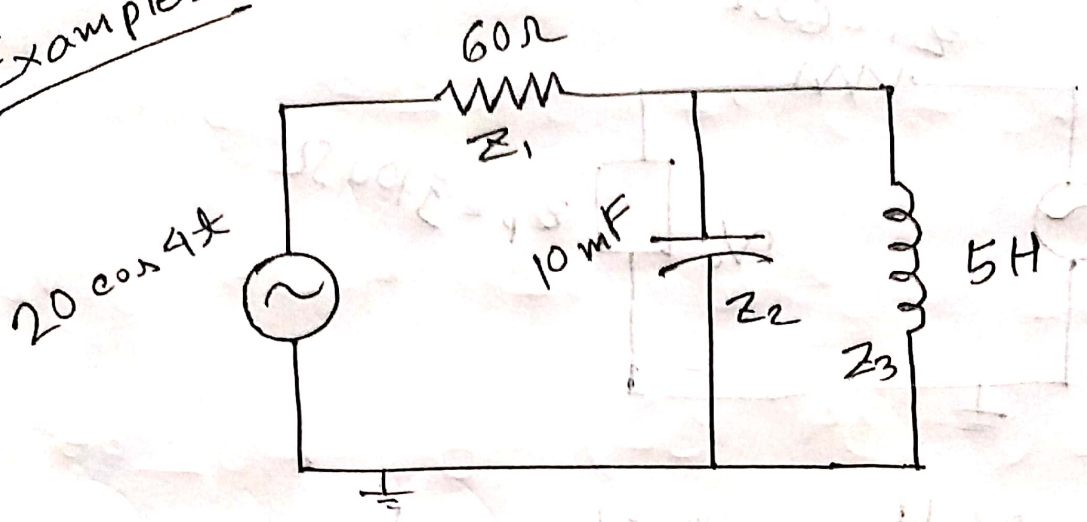
Parallel



$$Z_{eq} = Z_1 \parallel Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1}$$

Examples



$$V(t) = 20\cos 4t = 20\angle 0^\circ$$

$$\omega = 4$$

$$Z_1 = 60\,\Omega$$

$$Z_2 = \frac{1}{j\omega C} = \frac{1}{j \times 4 \times 10 \times 10^{-3}}\,\Omega$$

$$= -j25\,\Omega$$

$$Z_3 = j\omega L = j \times 4 \times 5\,\Omega$$

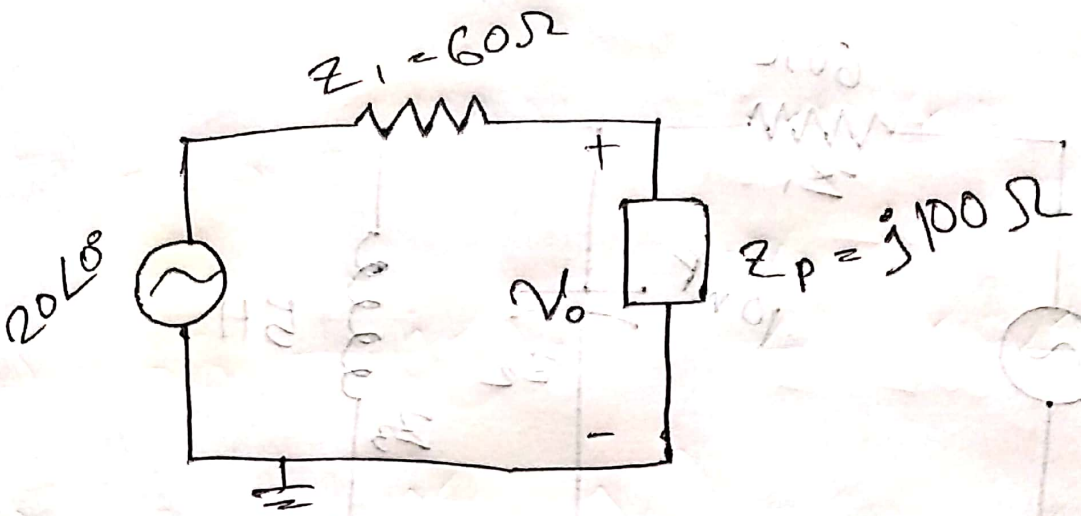
$$= j20\,\Omega$$

Let,

$$Z_p = Z_2 \parallel Z_3 = \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= \frac{-j25 \times j20}{-j25 + j20} = \frac{-500j^2}{-j5}$$

$$= j100\,\Omega$$



Using Voltage divider rule,

$$V_o = \frac{Z_p}{Z_1 + Z_p} \times V(t)$$

$$= \frac{j100}{60 + j100} \times 20\angle 0^\circ$$

$$= (0.735 + j0.44) \times 20\angle 0^\circ$$

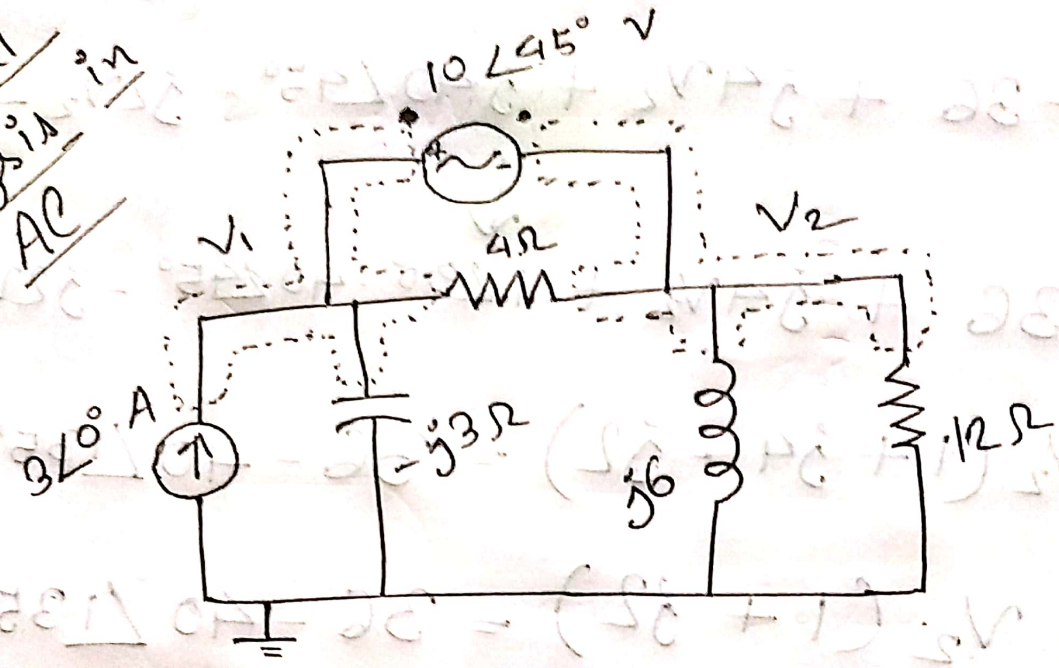
$$= 0.857\angle 31^\circ \times 20\angle 0^\circ$$

$$= (0.857 \times 20)\angle(31^\circ + 0^\circ)$$

$$= 17.14\angle 31^\circ$$

(Ans.)

Nodal Analysis in AC



Supernode

$$V_1 - V_2 = 10\angle 45^\circ \quad \ldots (i)$$

$$-3\angle 0^\circ + \frac{V_1 - 0}{-j3} + \frac{V_2 - 0}{j6} + \frac{V_2 - 0}{12} = 0 \quad \ldots (ii)$$

From (i),

$$V_1 = V_2 + 10\angle 45^\circ$$

$\therefore$ Substituting it in (ii)

$$-3\angle 0^\circ + \frac{V_2 + 10\angle 45^\circ}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12} = 0$$

$$\Rightarrow -36 - \frac{1}{j}\left(4V_2 + 40\angle 45^\circ\right) + \frac{1}{j}2V_2 + V_2 = 0$$

$$\Rightarrow -36 + j\left(4V_2 + 40\angle 45^\circ\right) - j2V_2 + V_2 = 0$$

$$\Rightarrow -36 + j4V_2 + j\,40\angle 45^\circ - j2V_2 + V_2 = 0$$

$$\Rightarrow -36 + j4V_2 + 1\angle 90^\circ \cdot 40\angle 45^\circ - j2V_2 + V_2 = 0$$

$$\Rightarrow V_2(1 + j4 - j2) = 36 - 40\angle 135^\circ$$

$$\Rightarrow V_2(1 + j2) = 36 - 40\angle 135^\circ$$

$$\therefore V_2 = \frac{36 - 40\angle 135^\circ}{1 + 2j}$$

$$= 31.41\angle -87.18^\circ$$

(Ans.)

$$V_1 = V_2 + 10\angle 45^\circ$$

$$= 31.41\angle -87.18^\circ + 10\angle 45^\circ$$

$$= 25.78\angle -70.48^\circ$$

(Ans.)