Summer 2023

CSE250 - Circuits and Electronics

SERIES AND PARALLEL NETWORKS

BRAC UNIVERSITY
Inspiring Excellence

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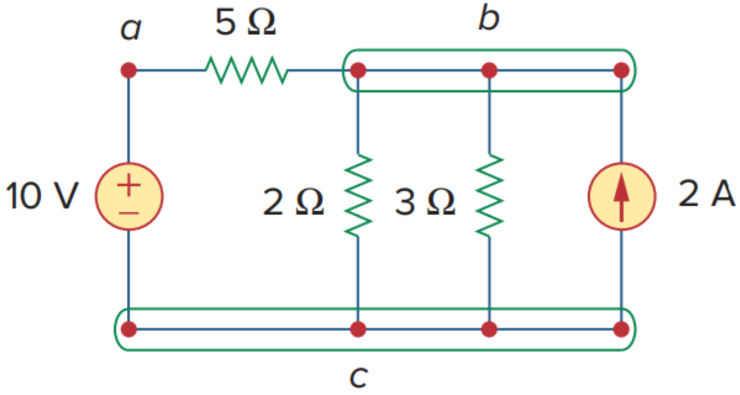
Nodes, Branches, Loops, & Mesh

- A *branch* represents a single element such as a voltage source or a resistor. In other words, a branch represents a two-terminal element.
- A *node* is the point of connection between two or more branches.
- A *loop* is a path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- A loop is said to be *independent* if contains at least one branch which is not part of any other independent loops.
- A *mesh* is a loop which does not contain any other loops within it.



☞ 5 branches: $10\,V$ source, $2\,\Omega$, $3\,\Omega$, and $5\,\Omega$ resistors, $2\,A$ current source

☞ 3 nodes (n): a, b, c

☞ 3 independent loops (l)

☞ 3 dependent loops (l_T)

☞ 3 meshes

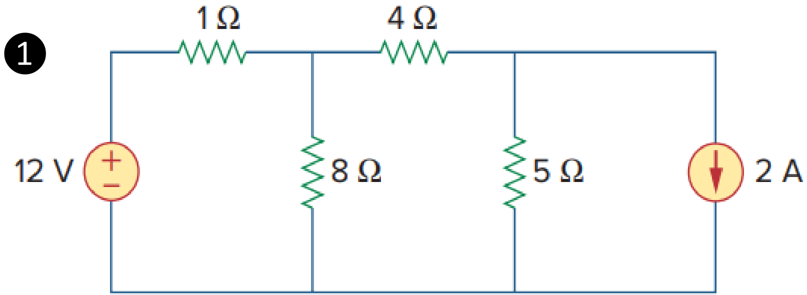
Problem 1

Ans:

1. $b = 6;\ n = 4;\ l_T = 3 + 3;\ m = 3$
2. $b = 5;\ n = 3;\ l_T = 3 + 3;\ m = 3$
3. $b = 6;\ n = 2;\ l_T = 5 + 10;\ m = 5$

- Determine the number of **branches**, **nodes, loops,** and **meshes** in the following circuits.

❶



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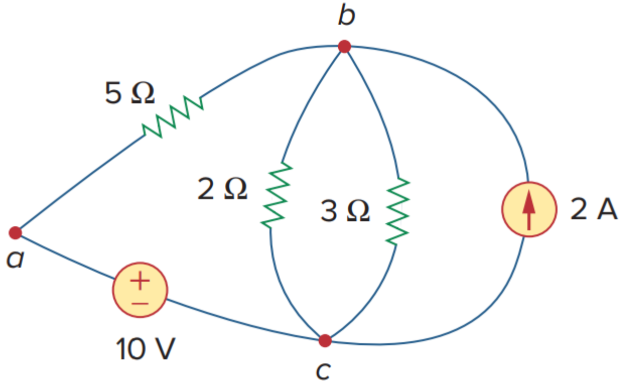

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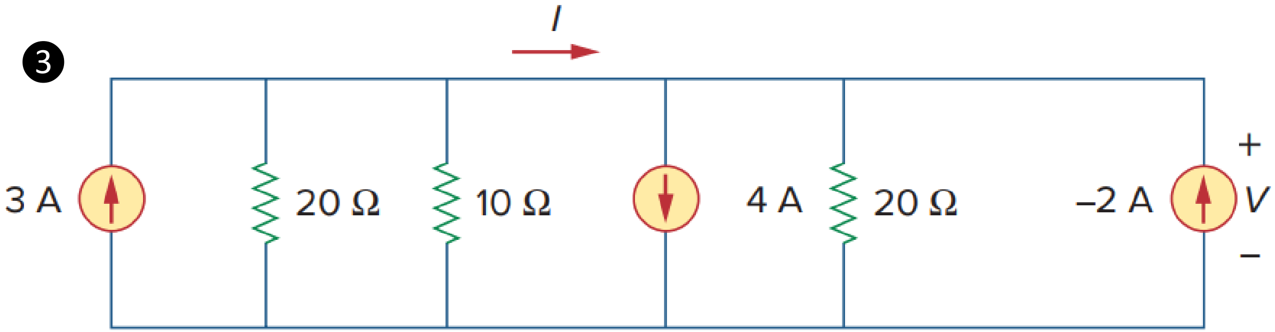
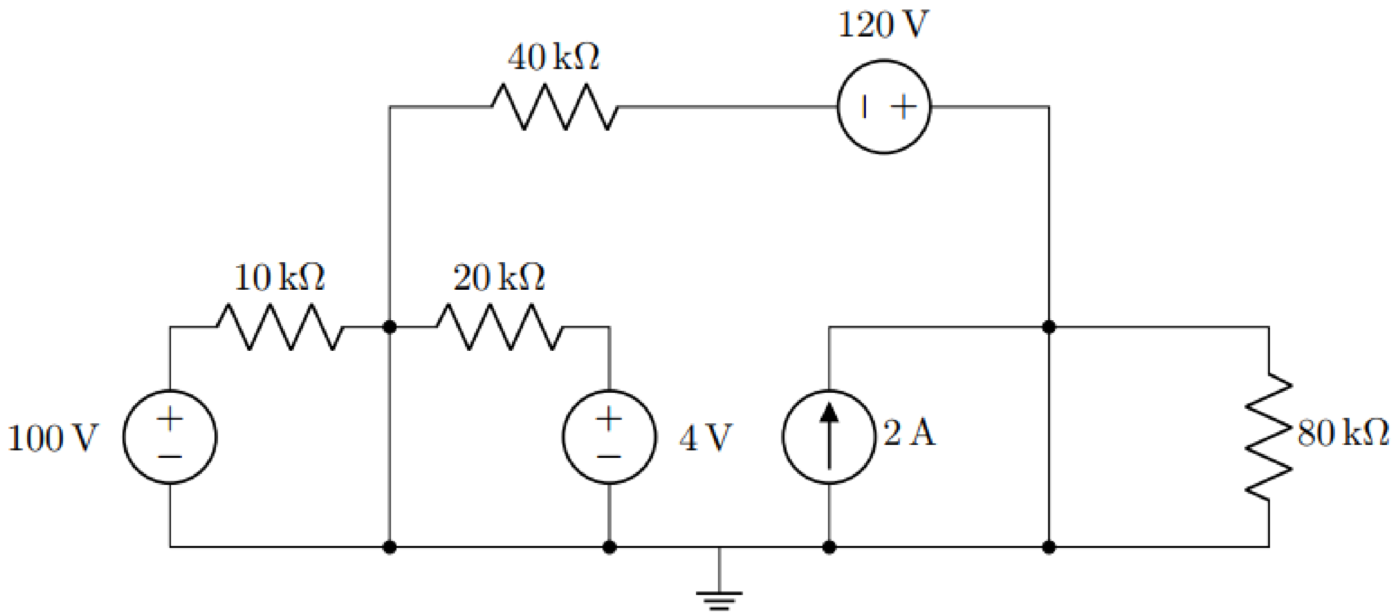


The three can be related as, $l = b - n + 1$, where l is the number of independent loops

Problem 2

- Determine the number of **nodes** and **meshes** in the following circuit.

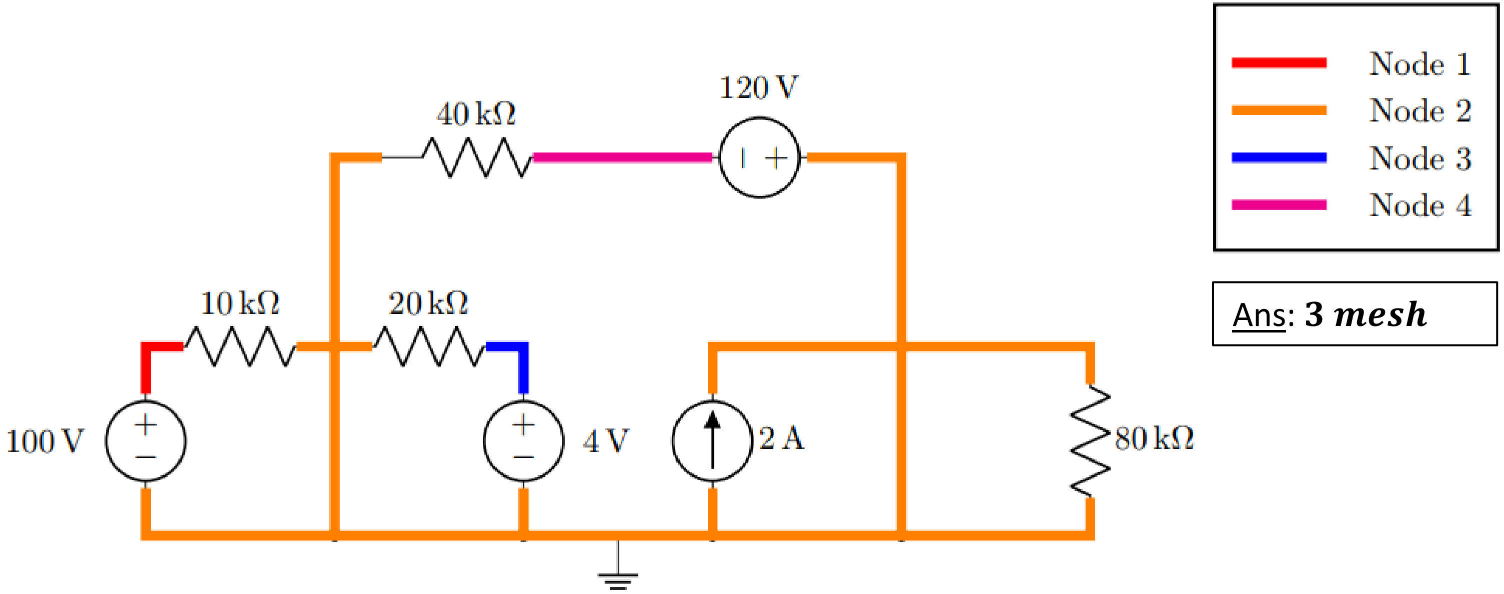


Ans: **Try yourself**

Problem 2: Solution

- Question: Determine the number of **nodes** in the following circuit.

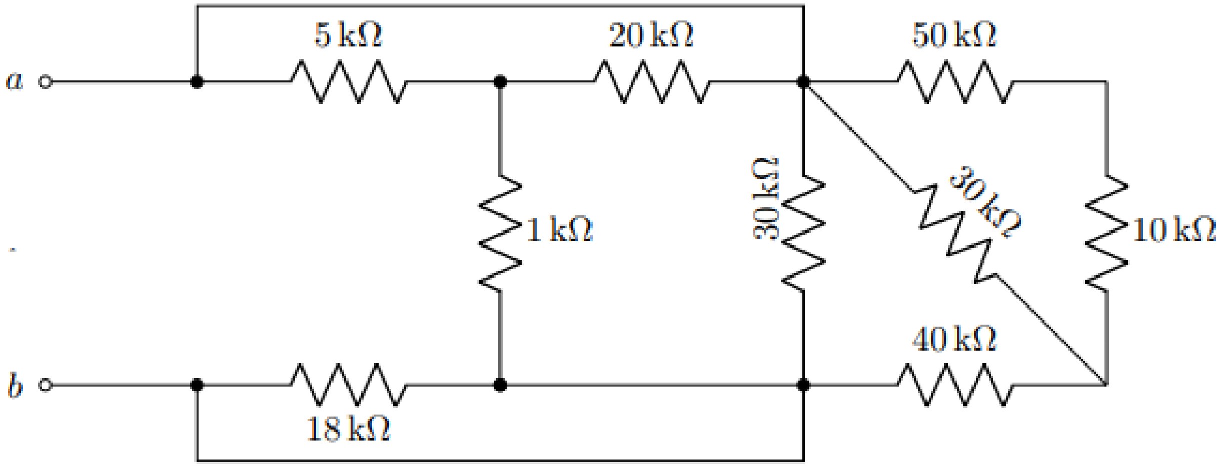
Solution:



Ans: **3 *mesh***

Problem 3

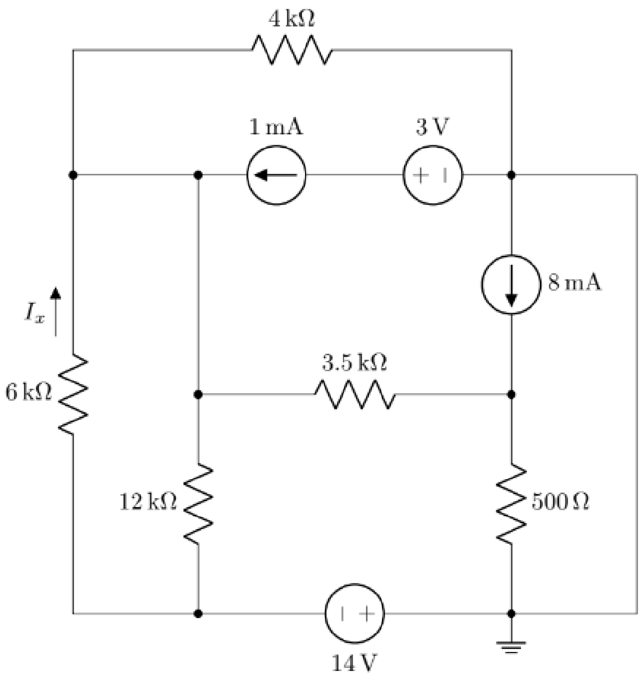
- Determine the number of **nodes** in the following circuit.



Ans: **Try yourself**

Problem 4

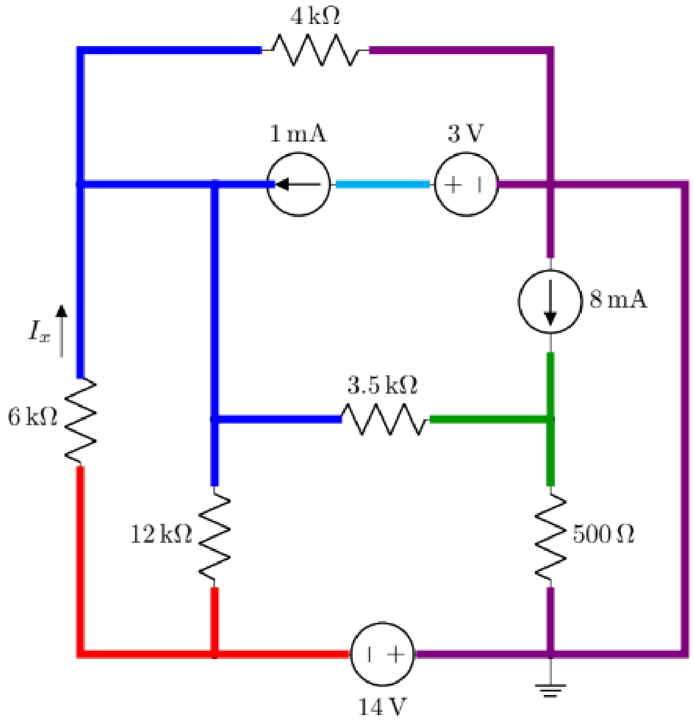
- Determine the number of **nodes** in the following circuit.



Ans: **Try yourself**

Problem 4: solution

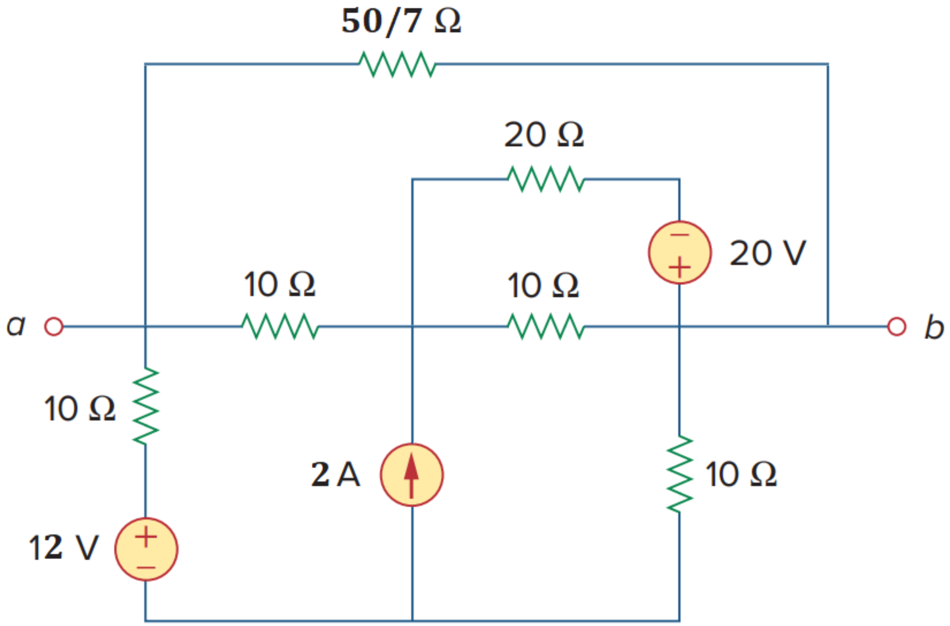
- Question: Determine the number of **nodes** and **meshes** in the following circuit.



Ans: $\mathbf{5}\ \boldsymbol{nodes};\ \mathbf{5}\ \boldsymbol{meshes}$

Problem 5

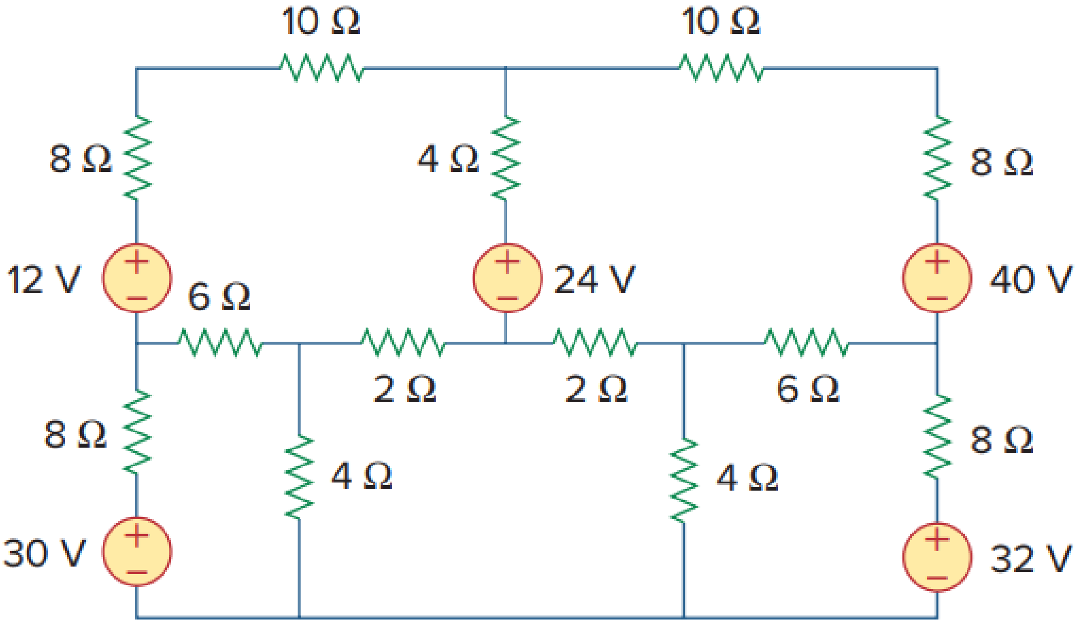
- Determine the number of **nodes** in the following circuit.



Ans: **Try yourself**

Problem 6

- How many **nodes** and **meshes** are there in the following circuit.



Ans: **14 nodes**; **5 meshes**

Circuit Configurations

- Circuit elements can be connected to each other in two main ways: series connection and parallel connection.

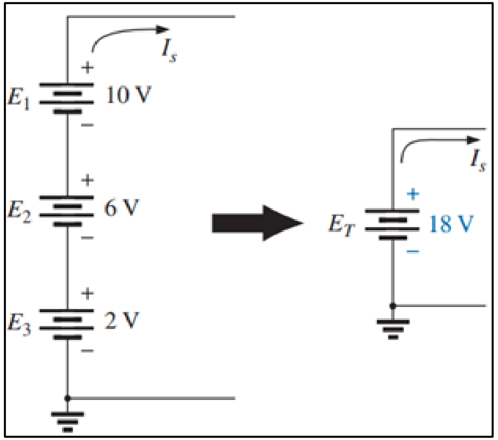
- In *series configuration*, components are connected end-to-end in a line. The same current flows through all the components. If one component breaks down, the whole circuit will burn out. *So, if same current flows through two circuit elements having a common node, they are said to be in series.*

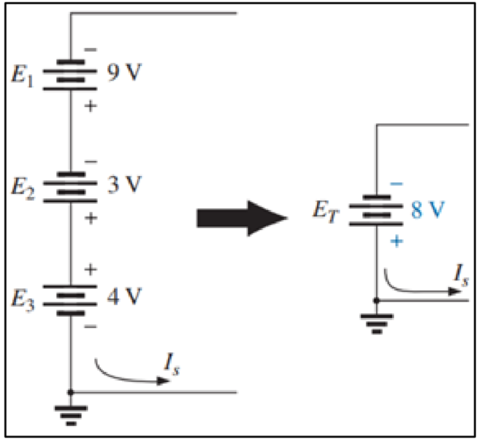
- In *parallel configuration*, all components are connected across each other leads forming exactly two sets of electrically common points. There are many paths for current flow, but only one voltage across all components. *So, if multiple circuit elements are connected between the same two nodes, they are said to be in parallel.*

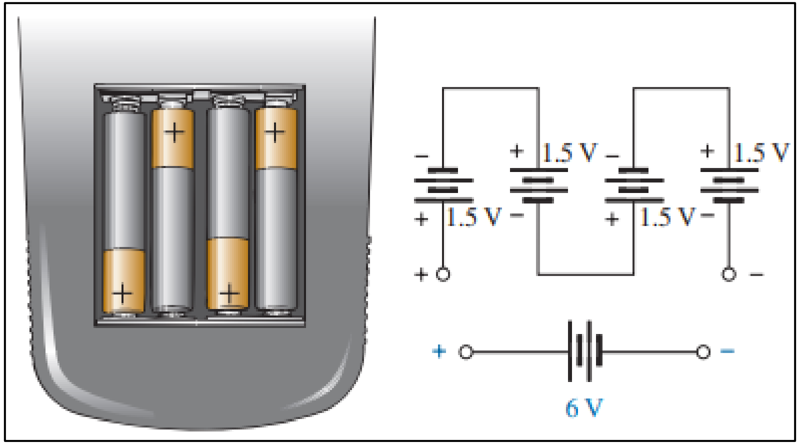
- Another configuration occurs when the circuit components are not connected in series or parallel but rather in a 'Y' or 'Δ' configuration. *Wye-Delta transformation* is required to simplify such configuration.

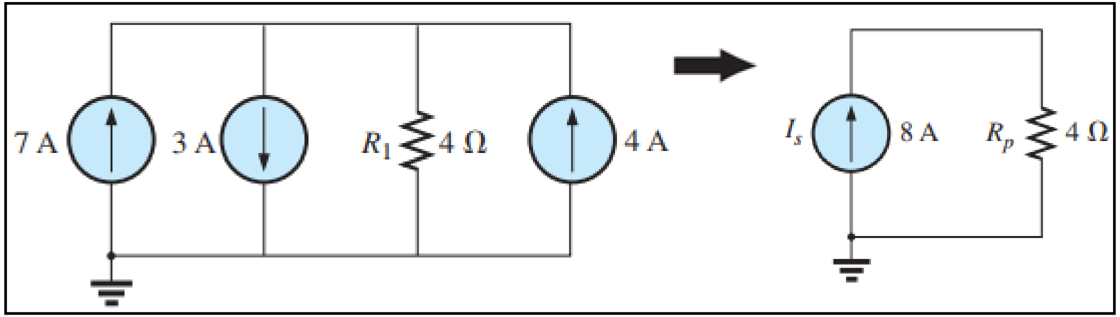
- The majority of electric circuits use all configurations simultaneously.

Different arrangements of sources





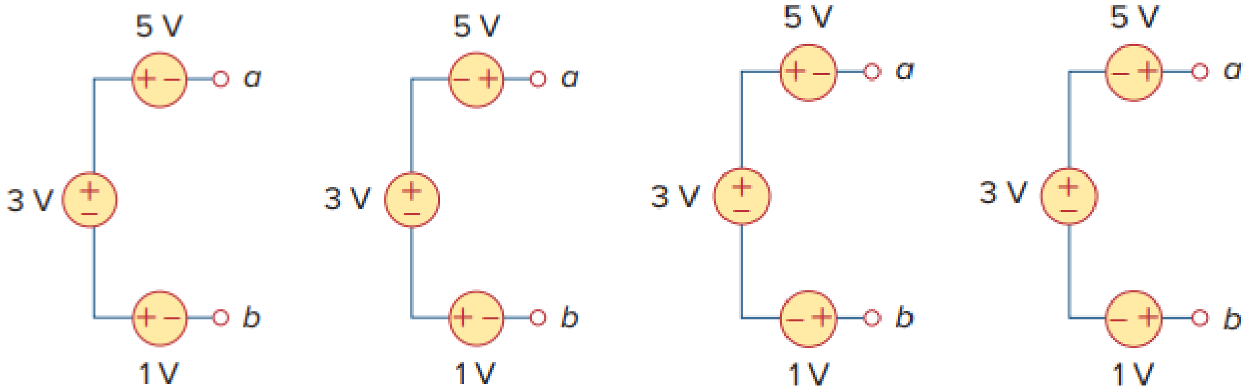




It is not practical to connect voltage sources of unequal ratings in parallel and current sources of unequal currents in series due to the direct violation of KVL and KCL respectively.

Problem 7

- For each of the circuits shown below, calculate V_{ab}

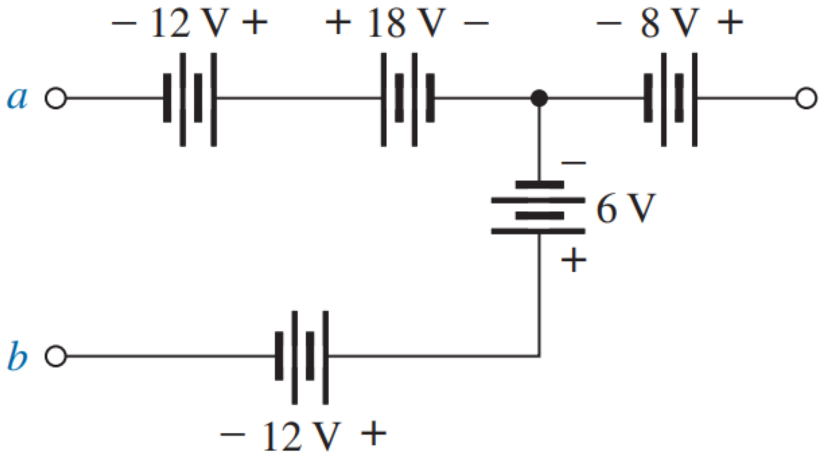


(a) (b) (c) (d)

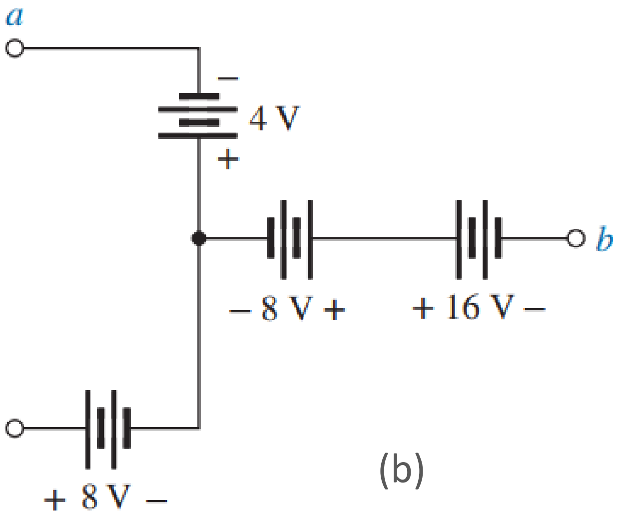
Ans: $(a)\ V_{ab} = -1\ V$; $(b)\ V_{ab} = 10\ V$; $(c)\ V_{ab} = -3\ V$; $(d)\ V_{ab} = 7\ V$;

Problem 8

- For each of the circuits shown below, calculate V_{ab}



(a)

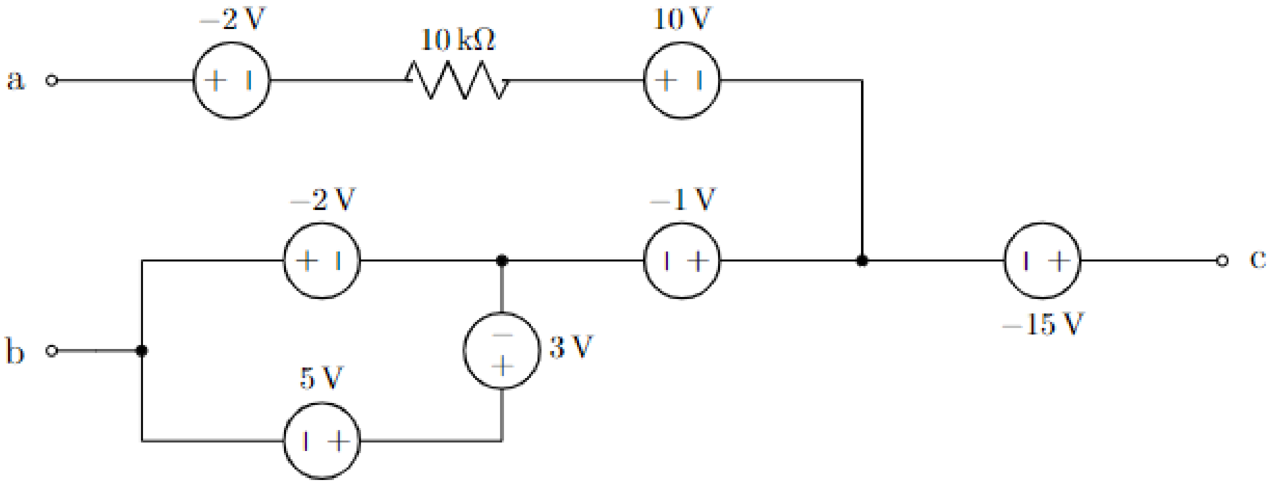


(b)

<u>Ans</u>: $(a)\ \boldsymbol{V_{ab} = 12\ V};\ (b)\ \boldsymbol{V_{ab} = 4\ V}$

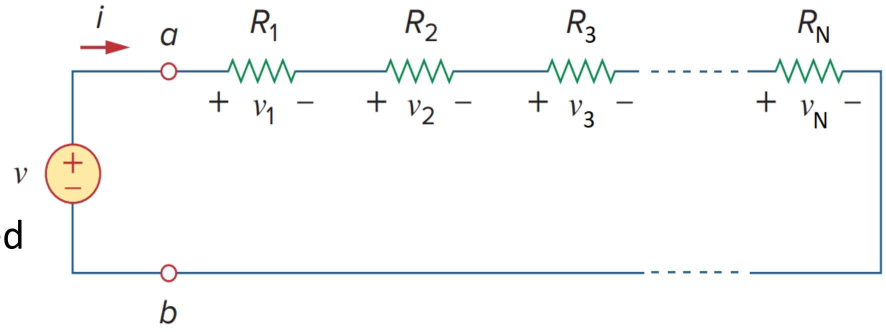
Problem 9

- For the circuit shown below, calculate V_{ac} and V_{bc}



Ans: $\boldsymbol{V_{ac} = -23\ V;\ V_{bc} = -14\ V}$

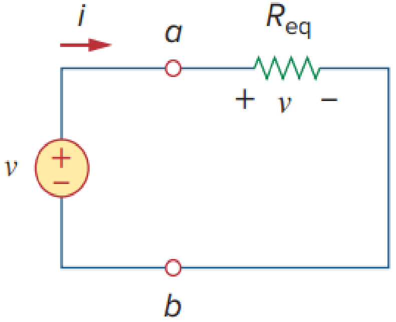
Series resistors



- Consider N number of resistors connected in series with a power supply of v volts.
- If the current flowing through the series circuit is i, then the voltage drops across the resistors can be written as,
- $v_1 = iR_1, v_2 = iR_2, v_3 = iR_3, \dots \dots, v_N = iR_N$
- According to the KVL, $v = v_1 + v_2 + v_3 + \dots \dots + v_N$

$$\Rightarrow \quad v = i(R_1 + R_2 + R_3 + \dots \dots + R_N)$$

$$\Rightarrow \quad i = \frac{v}{R_1 + R_2 + R_3 + \dots \dots + R_N}$$

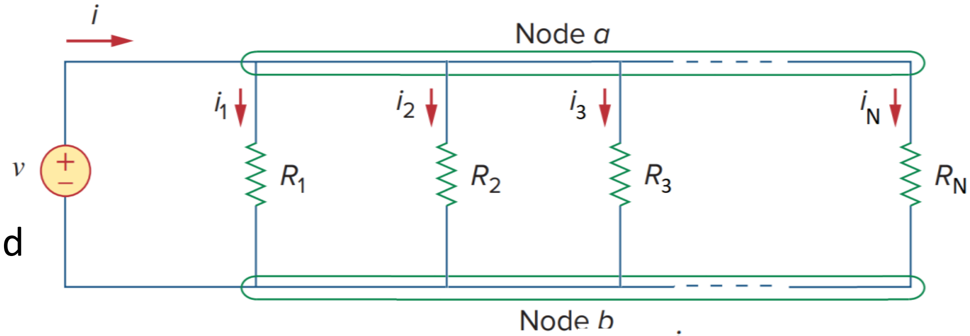


- It can be written as, $v = iR_{eq}$, implying that the series resistors can be replaced by an equivalent resistor R_{eq}; that is,

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \dots \dots + R_N}$$

- The two circuits have the same voltage-current relationships at terminal $a - b$, hence, they are equivalent to each other.

Parallel resistors



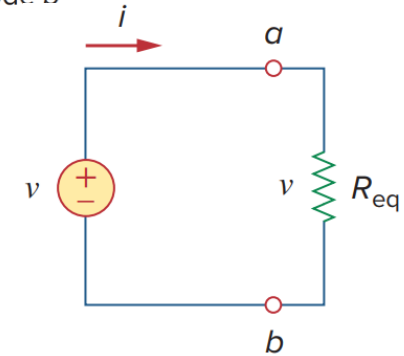
- Consider N number of resistors connected in parallel with a power supply of v volts.
- Therefore, they have the same voltage v across them. So,

$$v = i_1R_1 = i_2R_2 = i_3R_3 = \ \dots\dots = i_NR_N$$

- According to the KCL, $\quad i = i_1 + i_2 + i_3 + \ \dots\dots + \ i_N$

$$\Rightarrow \quad i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} + \ \dots\dots + \frac{v}{R_N}$$

$$\Rightarrow \quad i = v\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ \dots\dots + \frac{1}{R_N}\right)$$



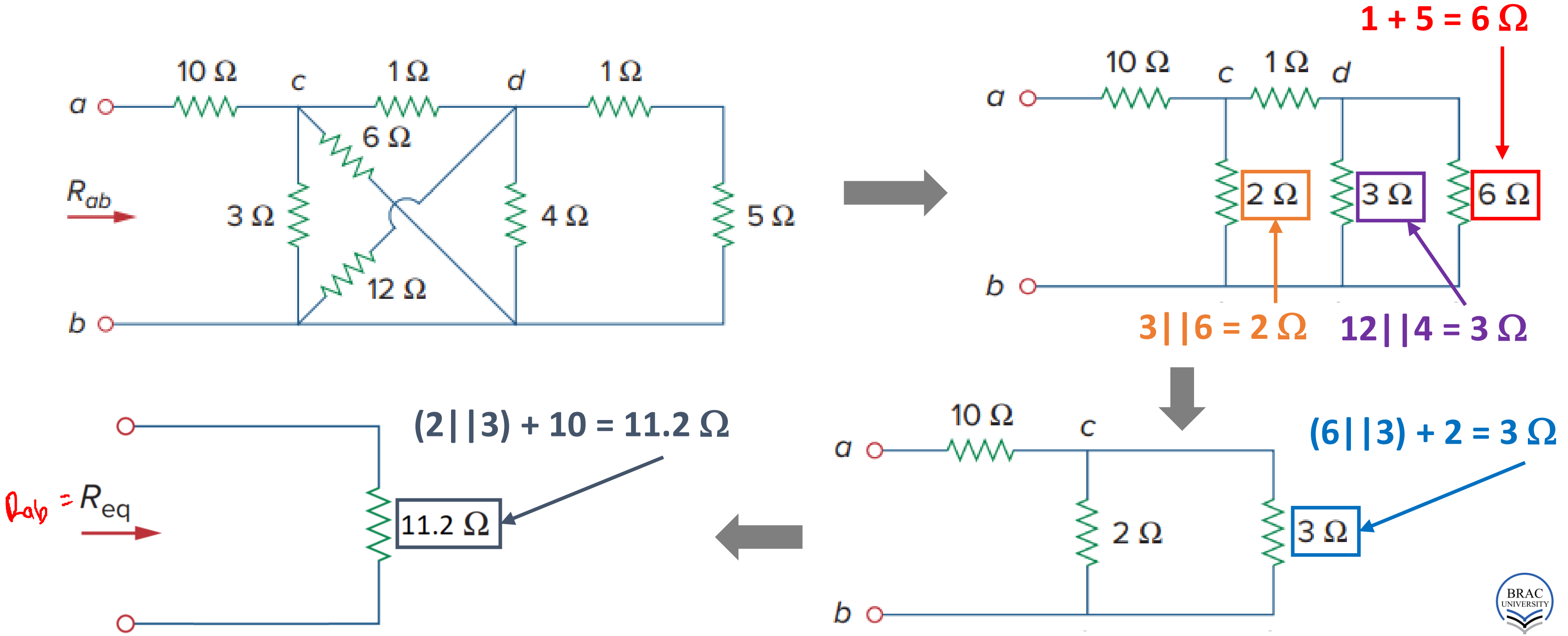
- It can be written as, $i = \frac{v}{R_{eq}}$, implying that the parallel resistors can be replaced by an equivalent resistor R_{eq}; that is,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ \dots\dots + \frac{1}{R_N}}$$

- The two circuits have the same voltage-current relationships at terminal $a - b$, hence, are equivalent to each other.

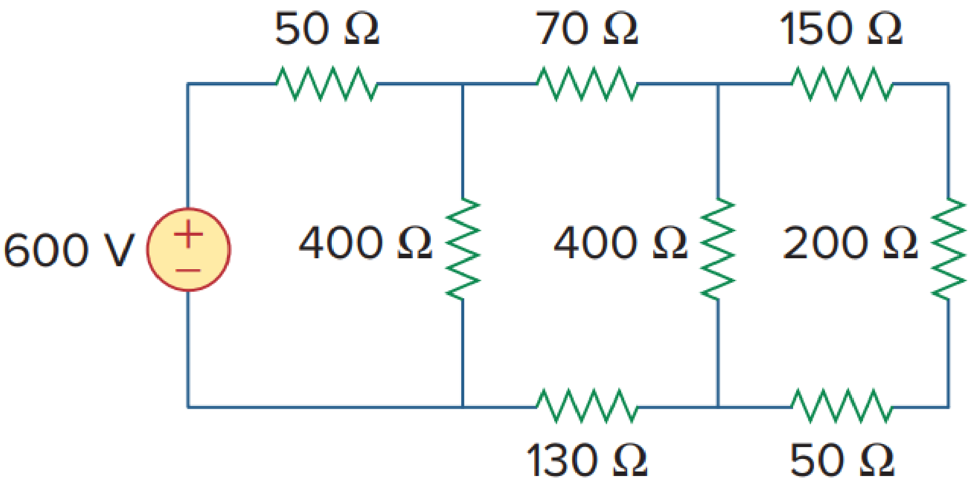
Example 1

- Using series/parallel resistance combination, find R_{ab} for the circuit shown below.



Problem 10

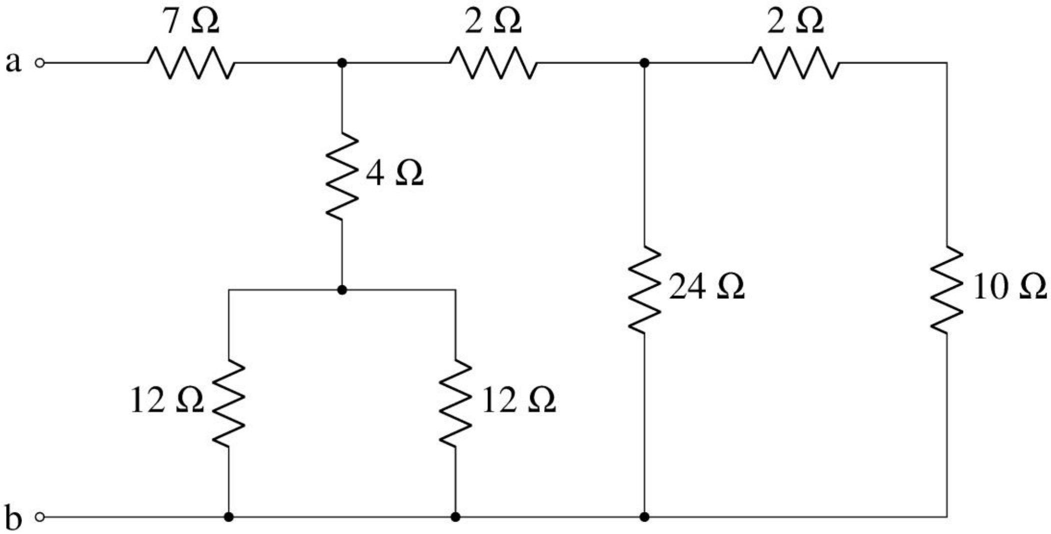
- Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit below.



Ans: $R_{eq} = 250\ \Omega$

Problem 11

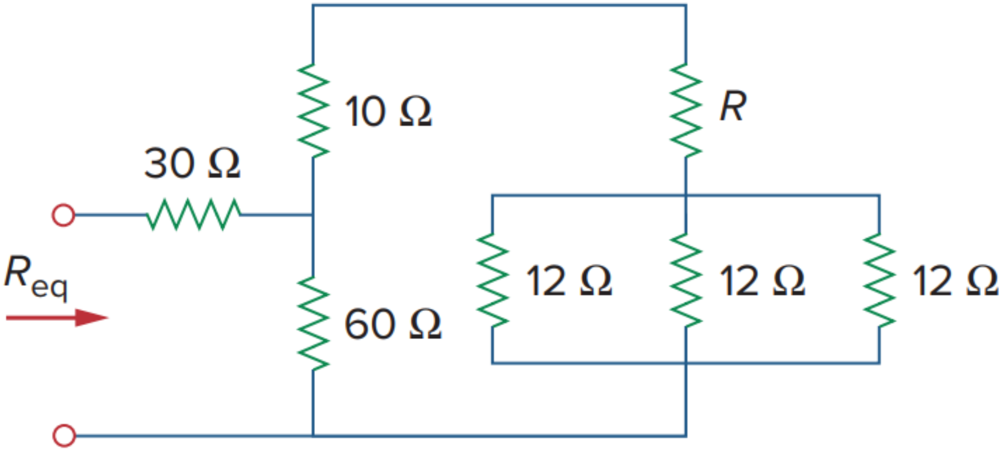
- Find the equivalent resistance between terminals a and b.



Ans: $\boldsymbol{R_{ab}} = \mathbf{12}\ \Omega$

Problem 12

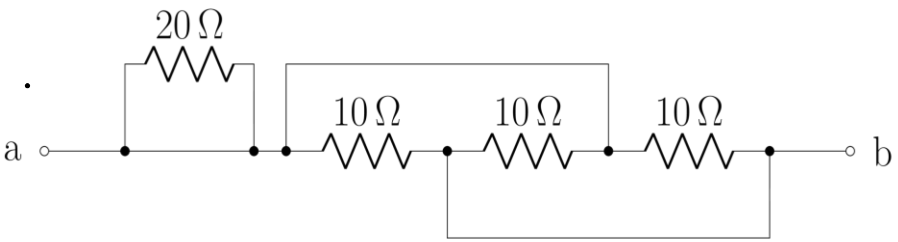
- If $R_{eq} = 50\ \Omega$ in the circuit, find R.

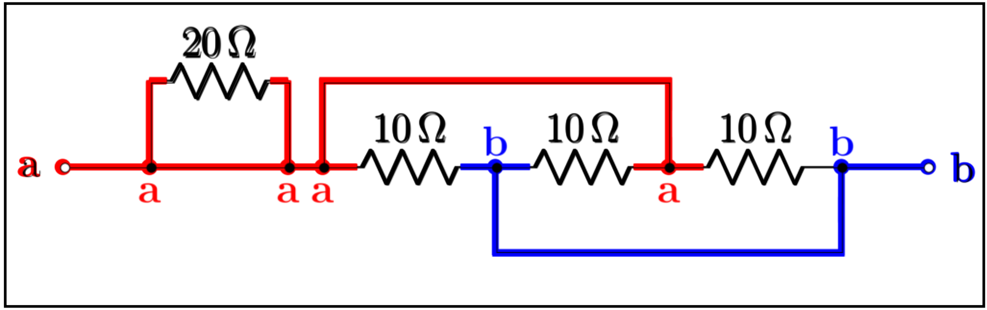


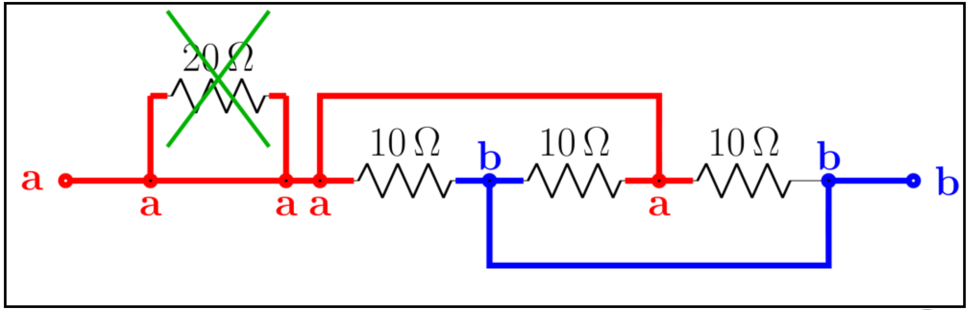
Ans: $\boldsymbol{R} = \mathbf{16}\ \Omega$

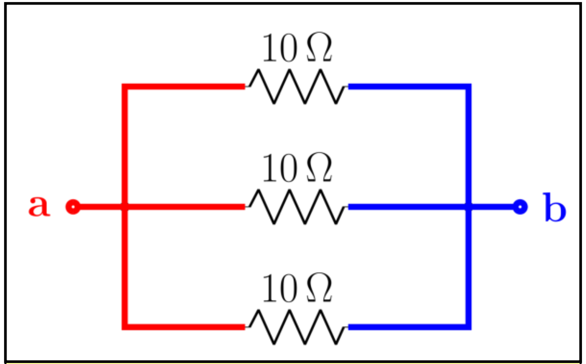
Example 2

- Find R_{ab}





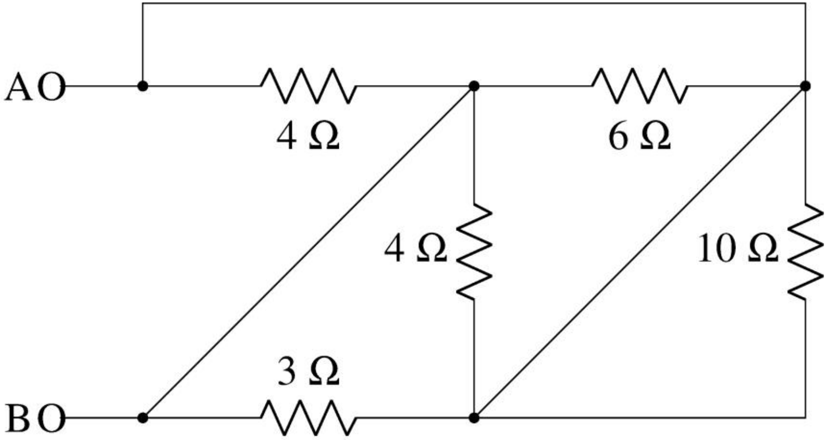




$R_{ab} = 10 \,||\, 10 \,||\, 10 = \frac{10}{3}\,\Omega$

Problem 13

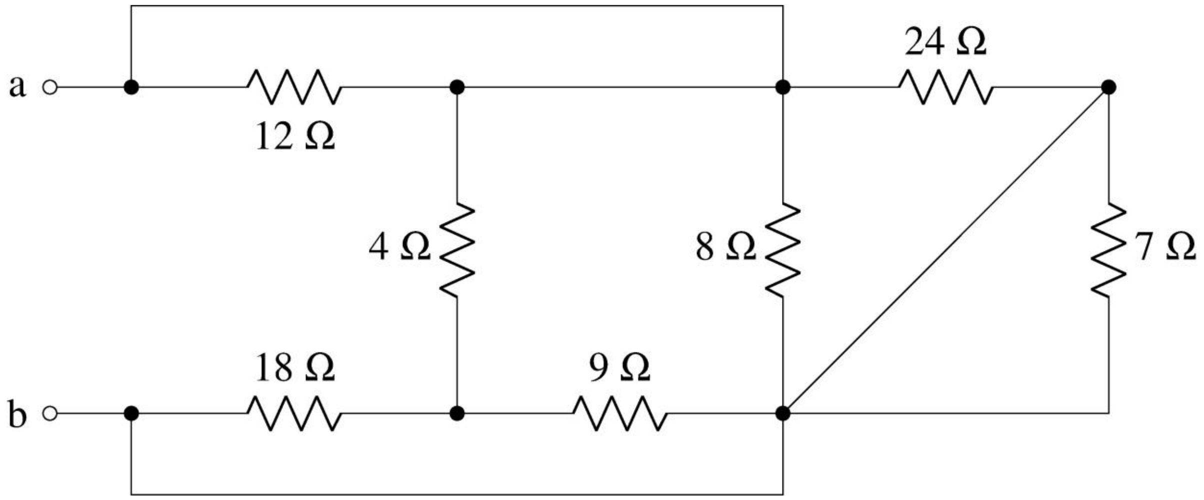
- Find the equivalent resistance between terminals A and B.



Ans: $\boldsymbol{R_{AB}} = \mathbf{1}\,\Omega$

Problem 14

- Find the equivalent resistance between terminals a and b.



Ans: $\boldsymbol{R_{ab}} = \mathbf{3.75}\ \Omega$

Voltage Division Rule

- The voltage division rule permits the determination of the voltage across a series resistor without first having to determine the current of the circuit.

- The current through the series circuit can be found using Ohm's law as,

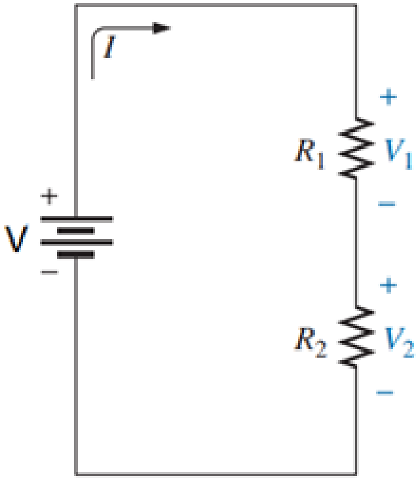
$$I = \frac{V}{R_1 + R_2}$$

- Applying Ohm's law to each of the resistors,

$$V_1 = IR_1 \quad \text{and} \quad V_2 = IR_2$$

$$\Rightarrow \quad V_1 = \frac{V}{R_1+R_2} R_1 \quad \text{and} \quad V_2 = \frac{V}{R_1+R_2} R_2$$

$$\Rightarrow \quad V_1 = \frac{R_1}{R_1+R_2} \times V \quad \text{and} \quad V_2 = \frac{R_2}{R_1+R_2} \times V$$

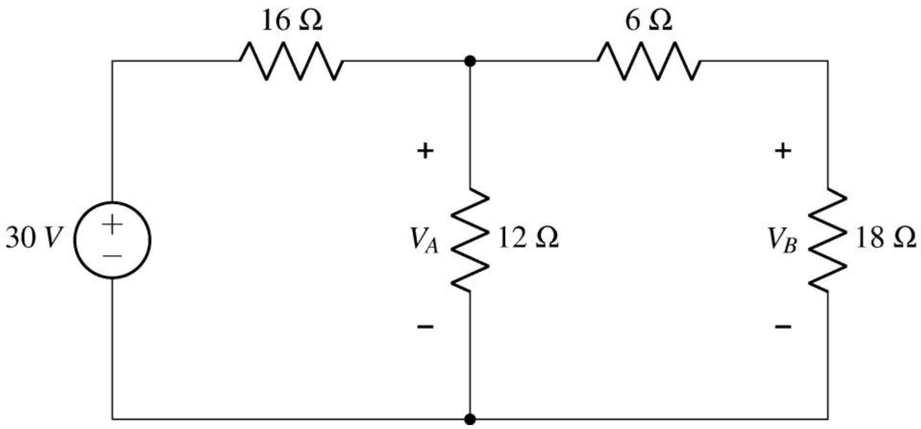


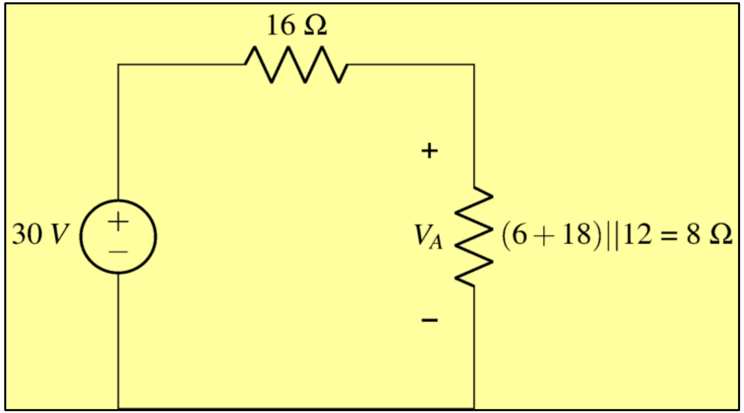
- In general, for any number of resistors connected in series to a supply voltage, the voltage across any particular resistor R_x is,

$$\boxed{V_x = \frac{R_x}{R_1 + R_2 + R_3 + \ldots\ldots + R_N} \times V}$$

Example 3

- Using the voltage divider rule, find the voltages V_A and V_B. Don't calculate currents.



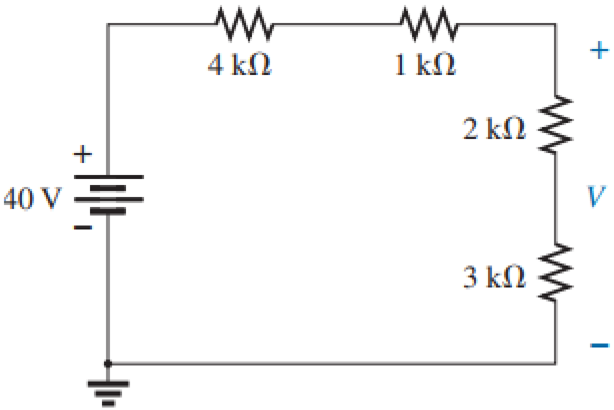


$$V_A = \frac{8}{8+16} \times 30 = 10\ V$$

$$V_B = \frac{18}{18+6} \times V_A = 7.5\ V$$

Problem 15

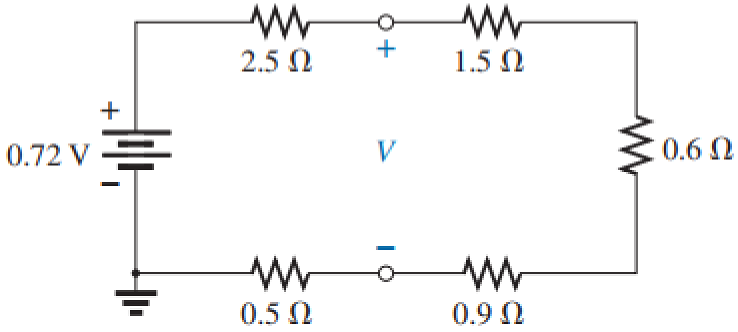
- Using the voltage divider rule, find the indicated voltage. Don't calculate current.



Ans: **V = 20 V**

Problem 16

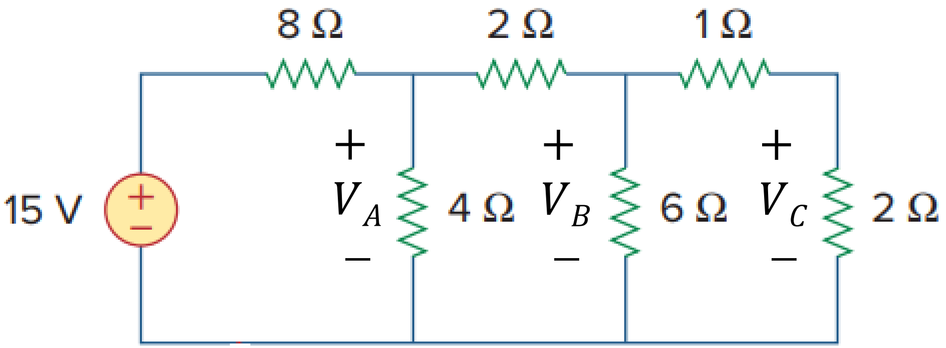
- Using the voltage divider rule, find the indicated voltage. Don't calculate current.



Ans: $V = 0.36\ V$

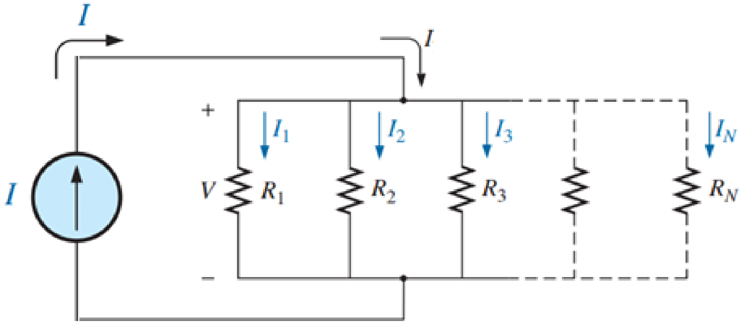
Problem 17

- Using the voltage divider rule, find the voltages V_A, V_B, and V_C. Don't calculate currents.



Ans: $\boldsymbol{V_A = 3\,V}$; $\boldsymbol{V_B = 1.5\,V}$; $\boldsymbol{V_C = 1\,V}$

Current Division Rule



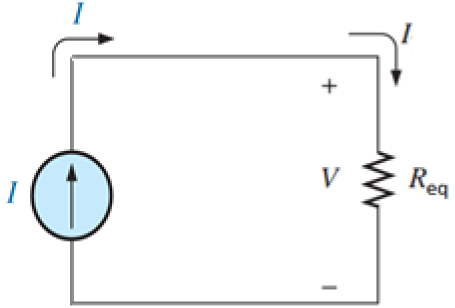
- The current division rule permits the determination of the currents through resistors connected in parallel without first having to determine the voltage across them.
- Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \ \dots\dots = I_NR_N$$

- Substituting V with $V = IR_{eq}$,

$$IR_{eq} = I_1R_1 = I_2R_2 = I_3R_3 = \ \dots\dots = I_NR_N$$

$$\Rightarrow \quad I_1 = \frac{R_{eq}}{R_1} \times I, \qquad I_2 = \frac{R_{eq}}{R_2} \times I, \qquad I_3 = \frac{R_{eq}}{R_3} \times I$$

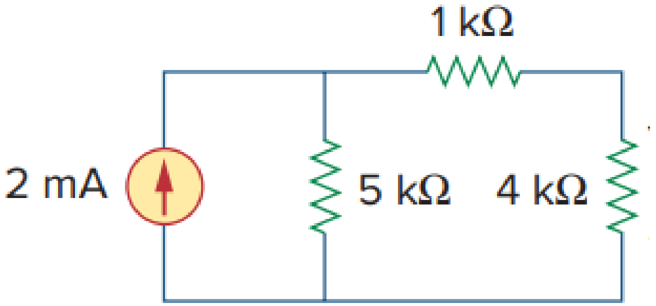


- In general, for any number of resistors connected in parallel to a supply current, the current through any particular resistor R_x is,

$$\boxed{I_x = \frac{R_{eq}}{R_x} \times I, \text{or}, I_x = \frac{(R_x)^{-1}}{(R_1)^{-1} + (R_2)^{-1} + \dots + (R_N)^{-1}} \times I}$$

Example 4

- Calculate the current through the $5\ k\Omega$ resistor using current division rule. Do not use Ohm's Law.



<u>Solution</u>

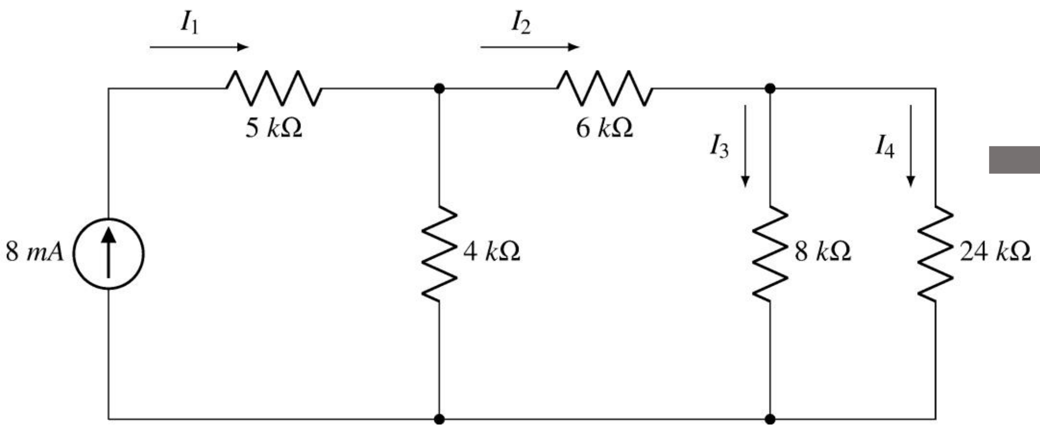
Current through the $5\ k\Omega$ resistor is,

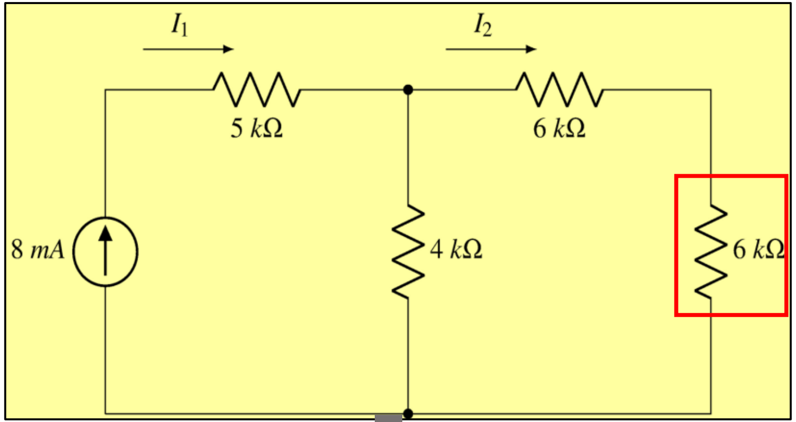
$$\frac{5^{-1}}{(1+4)^{-1}+5^{-1}} \times 2\ mA$$

$$= 1\ mA$$

Example 5

- Calculate the currents I_1 to I_4 using current division rule. Don't calculate voltage.





$$I_1 = 8\ mA$$

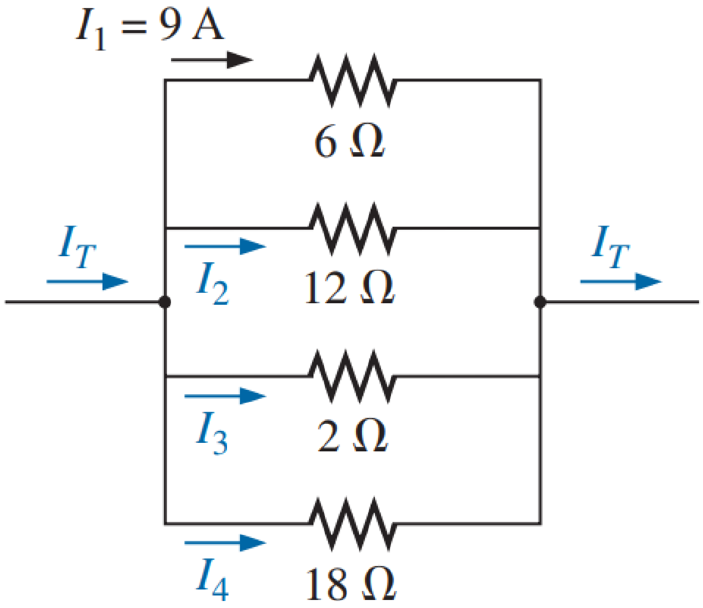
$$I_2 = \frac{12^{-1}}{4^{-1} + 12^{-1}} \times 8\ mA = 2\ mA$$

$$I_3 = \frac{8^{-1}}{24^{-1} + 8^{-1}} \times I_2 = 1.5\ mA$$

$$I_4 = I_2 - I_3 = 0.5\ mA$$

Problem 18

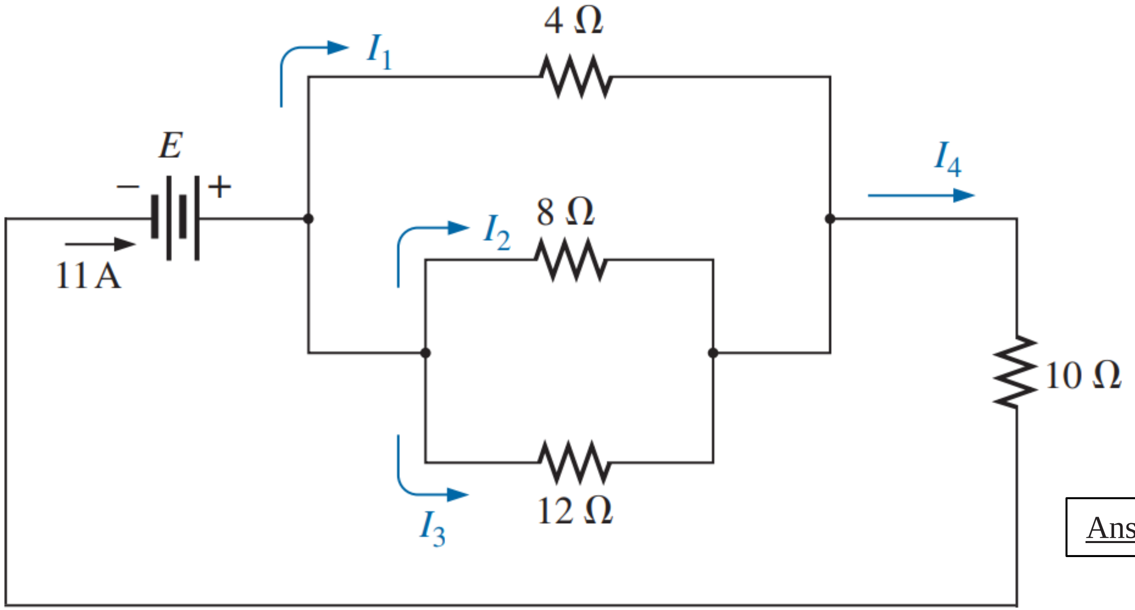
- Based solely on the resistor values, determine all the currents. Do not use Ohm's law.



Ans: $I_T = 43.5\ A$; $I_2 = 4.5\ A$; $I_3 = 27\ A$; $I_4 = 3\ A$

Problem 19

- Determine the unknown currents. Do not use Ohm's law.



Ans: $I_1 = 5\ A$; $I_2 = 3.6\ A$; $I_3 = 2.4\ A$; $I_4 = 11\ A$

Problem 20

- Determine the currents i_1 to i_5 using current division rule.

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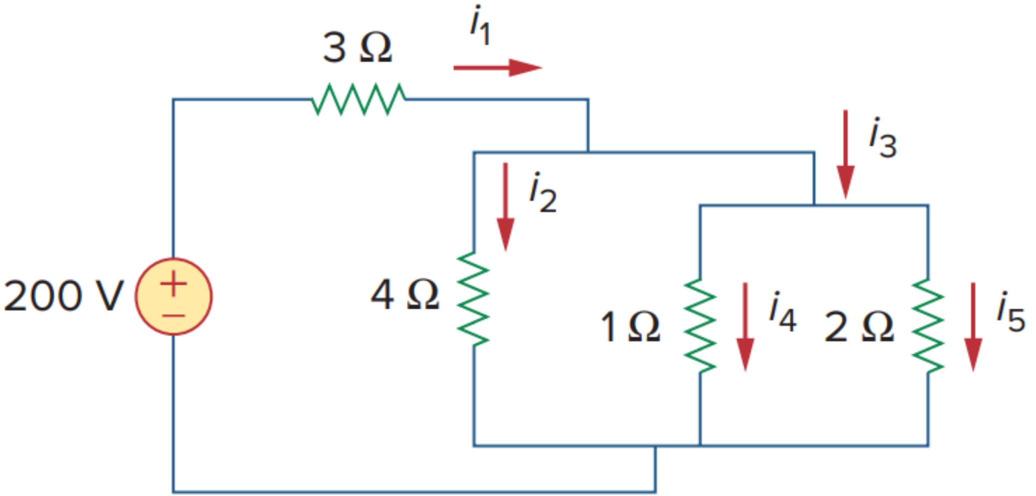

Ans: $\boldsymbol{i_1 = 56\ A;\ i_2 = 8\ A;\ i_3 = 48\ A;\ i_4 = 32\ A;\ i_5 = 16\ A.}$

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here

Thank you for your attention