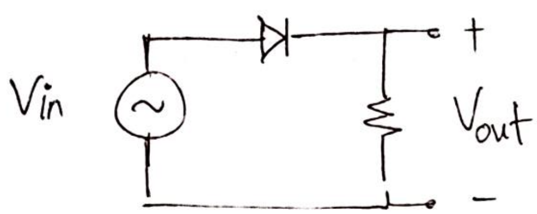
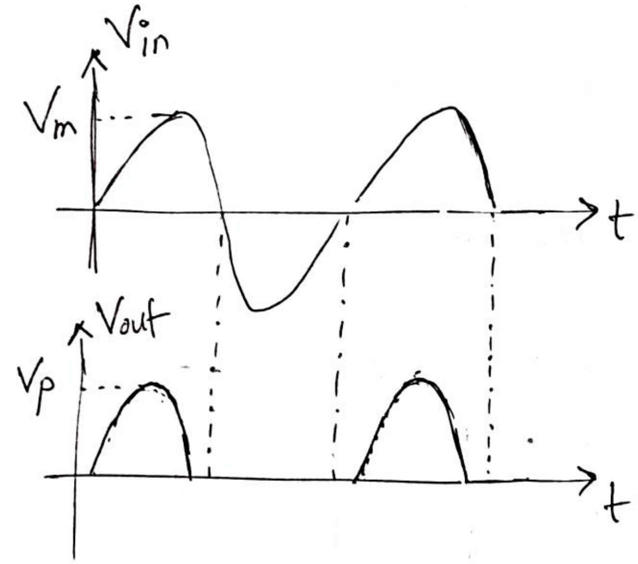
Rectifiers: Revisited

Half-Wave Rectifiers:





Suppose, input to the rectifier is $V_m \sin(\omega t)$.

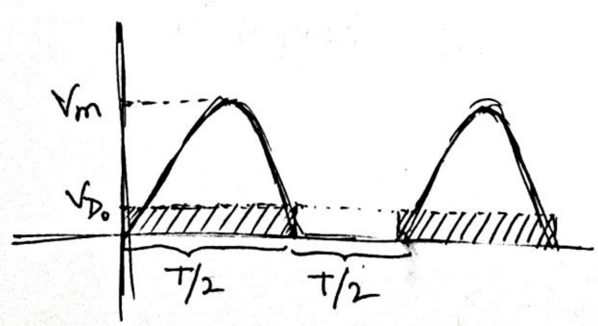
And, peak value of the output is V_p.

Then, V_p will be one diode-drop below V_m, i.e,

$$\boxed{V_p = V_m - V_{D_0}}$$

* Without Cap.

Now, what is the "average" value of the output? It is not actually straightforward, since, the output voltage does not start from $t=0$. We can make an approximation:



Here, if we subtract the area of the shaded portion, from the area of one of the lobes, we can approximately get the area of the output lobe. Then, averaging over 1 time period will give the average output voltage.

Now, Area of one lobe of the sine-wave $= \int_0^{T/2} V_m \sin(\omega t)\,dt$

$$= \frac{2}{\pi} V_m \cdot \left(\frac{T}{2}\right).$$

Area of the shaded region $= V_{D_0} \cdot (T/2)$.

$\therefore$ Approx. area of the output lobe $= \left(\frac{T}{2}\right) \cdot \left[\frac{2}{\pi} V_m - V_{D_0}\right]$

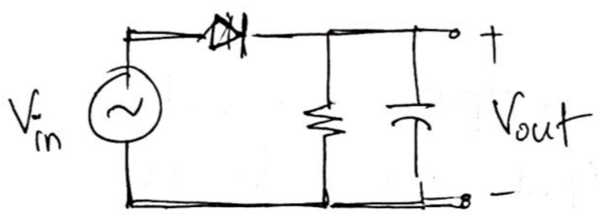
$\therefore$ Average value of voltage (over 1 Time-Period):

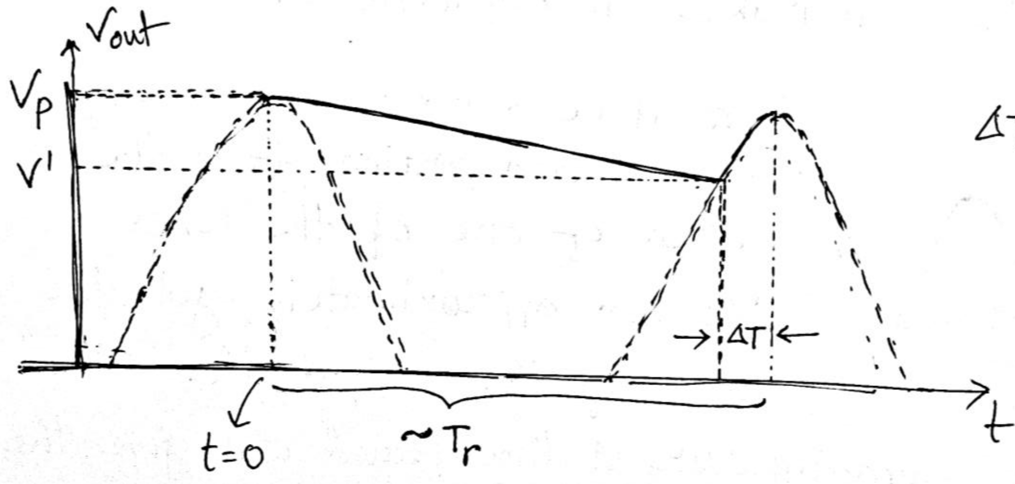
$$V_{avg} \text{ or, } V_{DC} = \frac{1}{T} \times \left(\frac{T}{2}\right)\left[\frac{2}{\pi} V_m - V_{D_0}\right]$$

$$\therefore \boxed{V_{Avg} \text{ or, } V_{DC} = \frac{1}{\pi} V_m - \frac{1}{2} V_{D_0}}$$

With Cap.

If a capacitor is added to the output,





ΔT = Charging Time.

Since the capacitor discharges through the resistor R, the output voltage t seconds after the peak V_p is,

$$V_{out}(t) = V_p e^{-t/RC}.$$

Assump^n (1):

If the discharge time $(T_r - \Delta T)$ is much less than RC, we can write,

$$V_{out}(T_r - \Delta T) = V_p \, e^{-(T_r - \Delta T)/RC}$$

$$= V_p \left(1 - \frac{T_r - \Delta T}{RC} + \frac{(T_r - \Delta T)^2}{2!\, R^2 C^2} - \cdots\cdots \right)$$

$$\approx V_p \left(1 - \frac{T_r - \Delta T}{RC}\right)$$

Then, the lowermost point of the voltage wave-form is,

$$V' \approx V_p \left(1 - \frac{T_r - \Delta T}{RC}\right) \qquad [T_r - \Delta T << RC]$$

Here, T_r = Time-period of Ripple

ΔT = Capacitor Charging Time.

Assump^n (2): If the capacitor charging time (ΔT) is much smaller than the ripple time-period (T_r), we may assume,

$$V' \approx V_p \left(1 - \frac{T_r}{RC}\right). \qquad [\Delta T << T_r]$$

~~Therefore~~

Then, the peak-to-peak ripple voltage is,

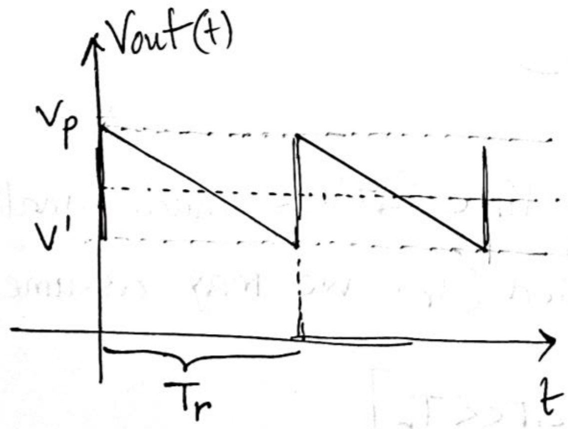
$$V_r(\text{peak-to-peak}) = V_p - V'$$
$$\approx V_p - V_p\left(1 - \frac{T_r}{RC}\right)$$
$$\approx V_p \cdot \frac{T_r}{RC}$$

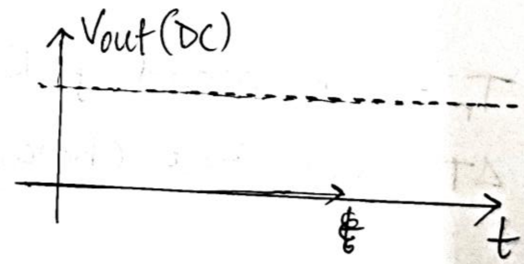
If the ripple-frequency, $f_r = \frac{1}{T_r}$, then,

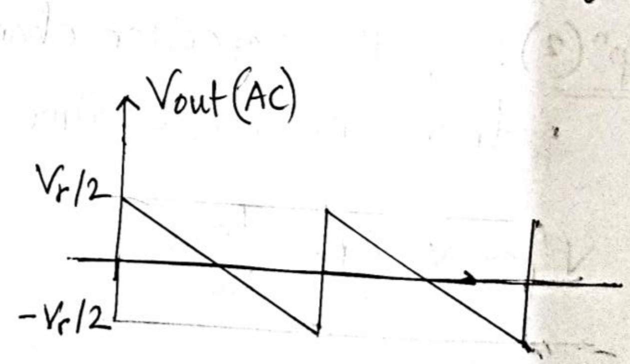
$$\boxed{V_r(p\text{-}p) = \frac{V_p}{f_r \cdot RC}}$$

for a Half-Wave Rectifier, time-period of ripple = time-period of input signal. $\therefore T_r = T_s \Rightarrow f_r = f_s$.

Also, if we neglect the charging time ΔT, the output wave can be approximated as below, which can be shown as the sum of 2 waves:







$$V_{out}(t) = V_{out[DC]} + V_{out[AC]}$$

* [$V_{out}(t)$ has been exaggerated in the figure for clearer understanding. Ripples will not be this big].

Since, the DC-value will go through the middle of the ripple, we have,

$$V_{DC} = \frac{V_p + V'}{2}$$

And, using $V_r(p\text{-}p) = V_p - V'$, we get,

$$\boxed{V_{DC} = V_p - \frac{V_r(p\text{-}p)}{2}}$$

Also, neglecting the charging time ΔT, the ripple waveform, or, the AC waveform will become almost like a triangular wave.

The r.m.s. value of the ripple is:

$$V_{AC} = V_r\,(\text{r.m.s.}) = \sqrt{\frac{1}{T}\int_0^T [V_{AC}(t)]^2\,dt}$$

Eq^n for triangular wave.

Assump^n:

Using the eq^n. $V_{AC}(t) = \frac{V_r}{2} - \frac{V_r}{T}\cdot t$ for $0 \leq t \leq T$,

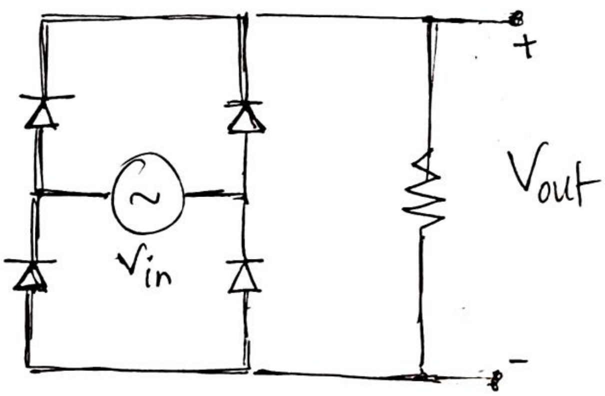
if we perform the integration, we get,

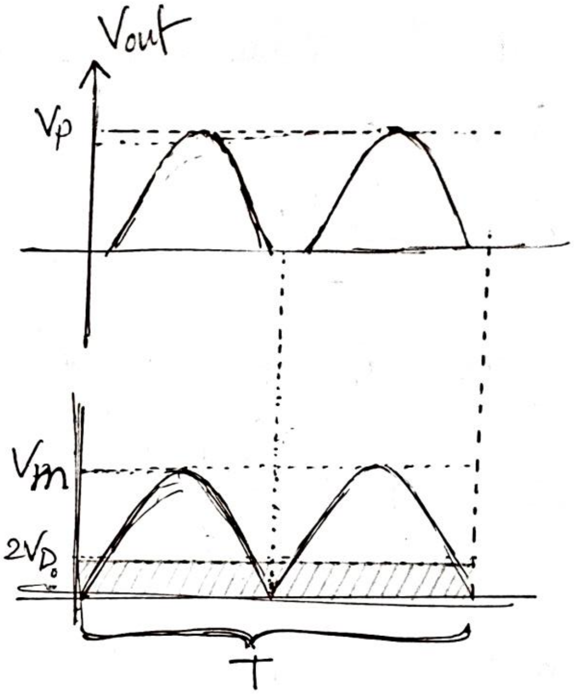
$$V_r\,(\text{r.m.s.}) = \frac{V_r(p\text{-}p)}{2\sqrt{3}}$$

[$V_r = V_r(p\text{-}p)$ in the above eq^n]

$$\therefore \boxed{V_r\,(\text{r.m.s.}) = \frac{V_p}{2\sqrt{3}\cdot f_r \cdot RC}}$$

Full-Wave Rectifier :





As before, input to the FW rectifier is,

$$V_{in} = V_m \sin(\omega t).$$

This time, the peak of the output will be 2 diode-drops below the input peak.

$$\therefore \boxed{V_p = V_m - 2V_{D_0}}$$

Without Cap

Like-wise, to find the average value of output, we may approximate by subtracting the area of the shaded portion from the area of the 2 lobes.

$$\text{Area of 2 lobes} = \frac{2}{\pi} V_m \cdot T$$

$$\text{Area of shaded portion} = 2V_{D_0} \cdot T.$$

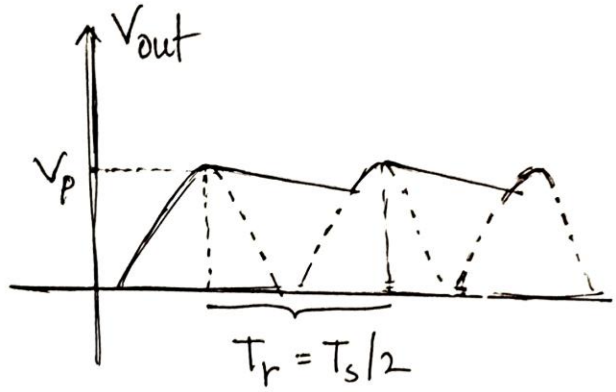
$\therefore$ Approx. Average value of voltage,

$$V_{Avg}, \text{ or, } V_{DC} = \frac{1}{T}\left[\frac{2}{\pi} V_m \cdot T - 2V_{D_0} \cdot T\right]$$

$$\therefore \boxed{V_{Avg}, \text{ or, } V_{DC} = \frac{2}{\pi} V_m - 2V_{D_0}}$$

With Capacitor

The ripple analysis of an FW rectifier will be ~~&~~ exactly the same as an HW rectifier. Except, one difference.



As can be seen, the ripple period is half the signal period. (Because, the capacitor will start charging if the input reaches its negative peak).

$$\therefore T_r = T_s/2 \Rightarrow f_r = 2f_s.$$

Except this change, all other formulas are same as before.

To sum up.

	HW	FW
V_p	$V_m - V_{D_0}$	$V_m - 2V_{D_0}$
f_r	f_s	$2f_s$
V_{DC} (without Cap)	$\frac{1}{\pi}V_m - \frac{1}{2}V_{D_0}$	$\frac{2}{\pi}V_m - 2V_{D_0}$

With Capacitor

$$V_r(p-p) = \frac{V_p}{f_r \cdot RC}$$

$$V_r(rms) = \frac{V_r(p-p)}{2\sqrt{3}}$$

$$V_r(rms) = \frac{V_p}{2\sqrt{3} f_r \cdot RC}$$

$$V_{DC} = V_p - \frac{V_r(p-p)}{2}$$