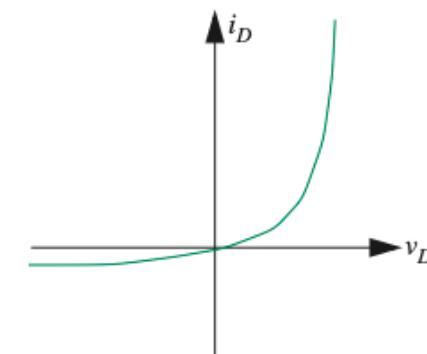
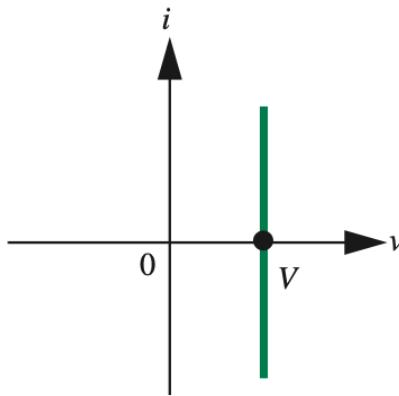
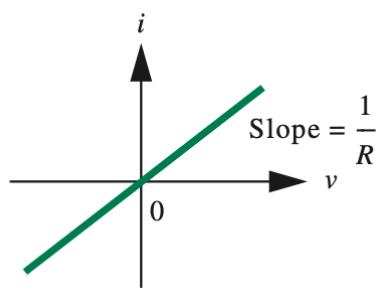
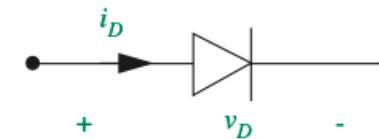
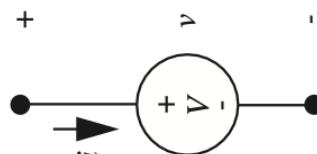
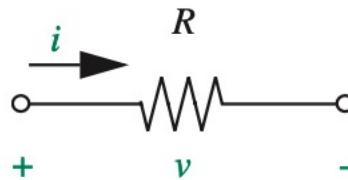


Three terminal devices

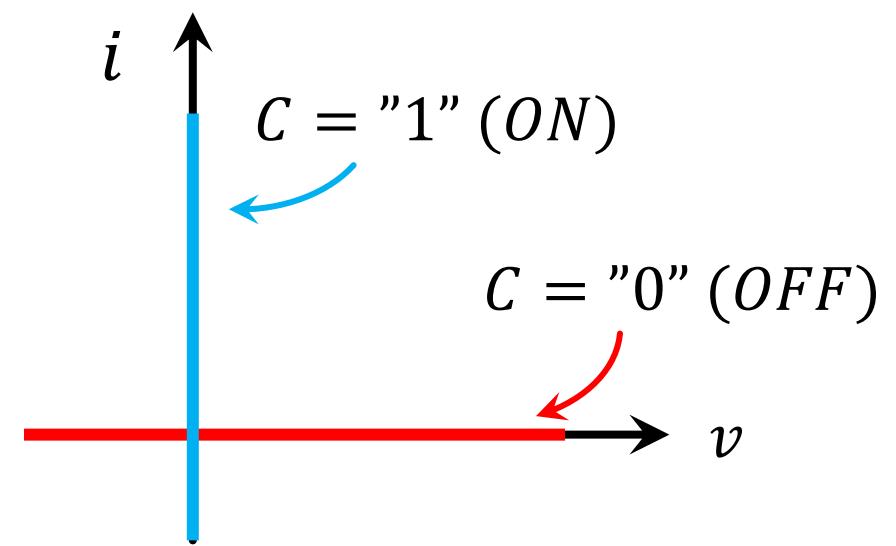
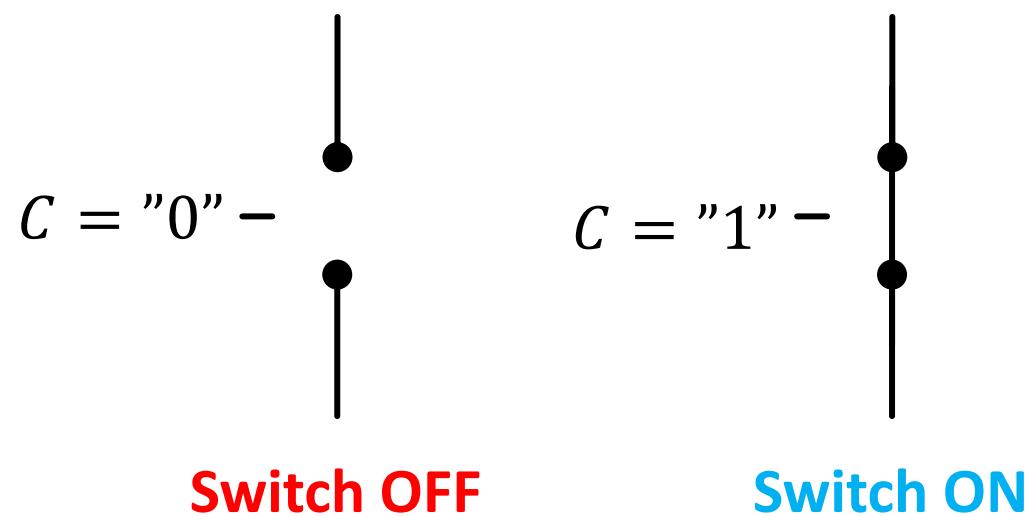
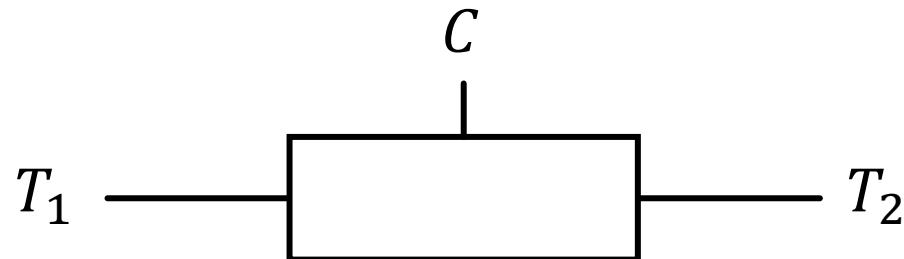
- Two terminal devices have fixed IV characteristics



- Three terminal devices – IV of two terminal can be controlled using the third terminal.
- Examples – Switch (linear), Transistors (non-linear)

Switch – IV Characteristics

- IV characteristics between terminal T_1 and T_2 is controlled by C



Switch – Types

- Depending on the control, the switch can be
 - **Analog:** Controlled using physical toggle/button
 - **Digital:** Controlled using voltage or current. Example – MOSFET (voltage controlled), BJT (current controlled)



Push button



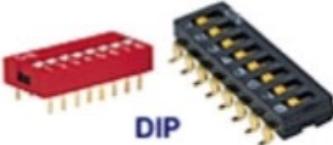
Rotary



Toggle



Slide

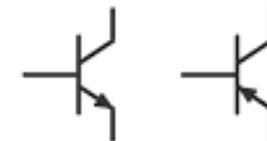


DIP

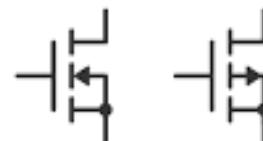


Dynamite
Plunger

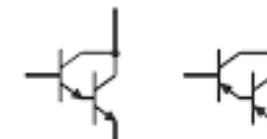
Analog switches



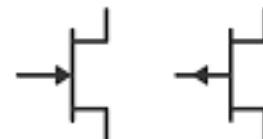
BJT



MOSFET



Darlington

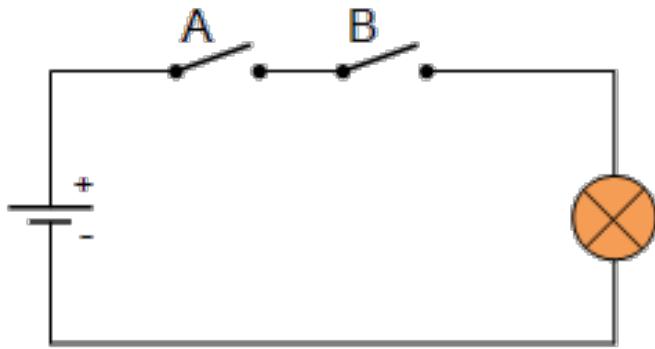


JFET

Digital switches (Transistors)

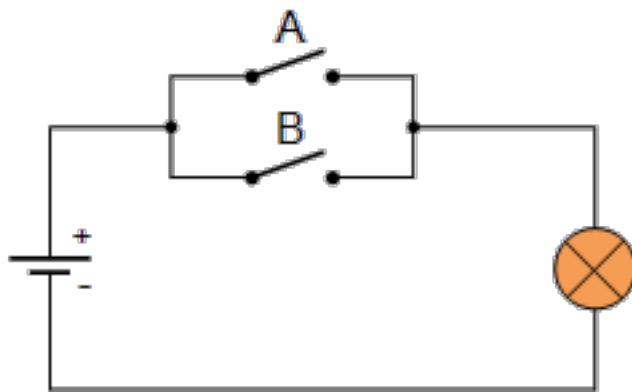
Switch Application – Logic Gates

- We can use switches to build logic gates



A	B	Bulb
0	0	OFF
0	1	OFF
1	0	OFF
1	1	ON

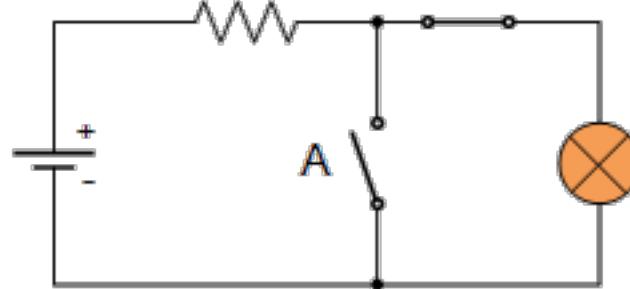
AND operation



A	B	Bulb
0	0	OFF
0	1	ON
1	0	ON
1	1	ON

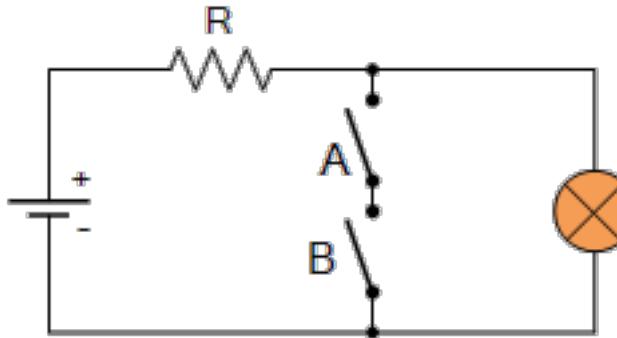
OR operation

Switch Application – Logic Gates



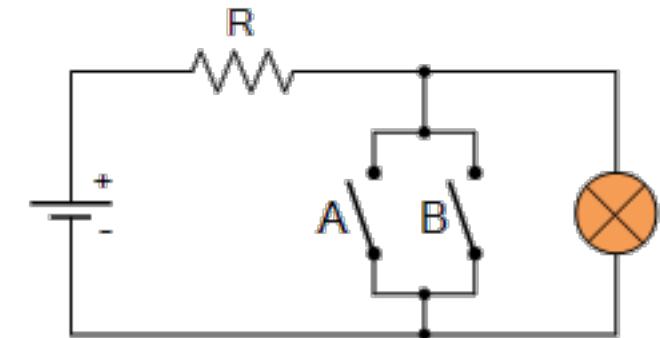
A	Bulb
0	ON
1	OFF

NOT operation



A	B	Bulb
0	0	ON
0	1	ON
1	0	ON
1	1	OFF

NAND operation



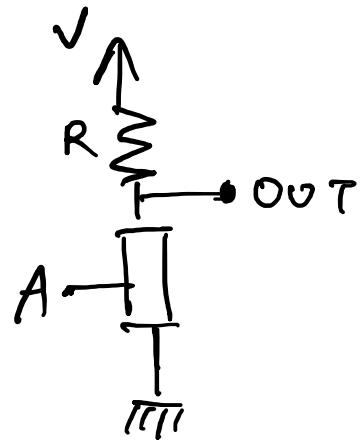
A	B	Bulb
0	0	ON
0	1	OFF
1	0	OFF
1	1	OFF

NOR operation

These circuits are “preferred” – because they can be cascaded to build combinational logic circuits
-> if we remove the bulb and use the voltage across instead to cascade and drive the next gate

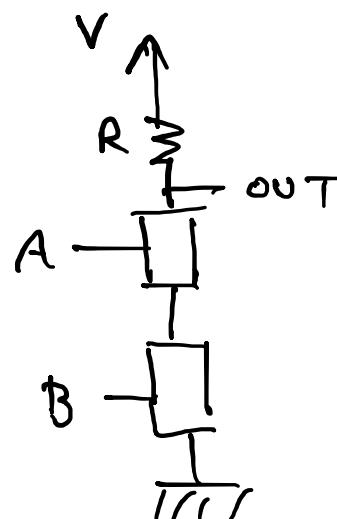
Switch Application – Logic Gates

Alternative representations:



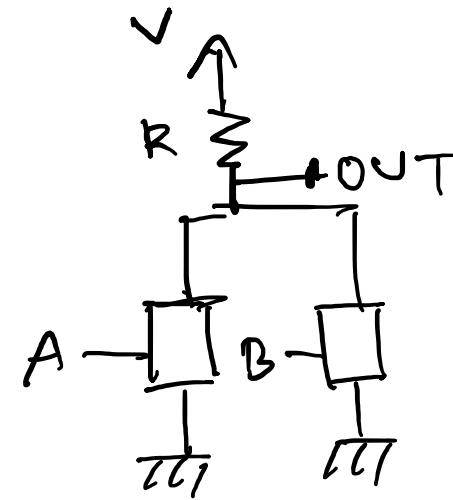
$$OUT = \overline{A}$$

(NOT)



$$OUT = \overline{AB}$$

(NAND)

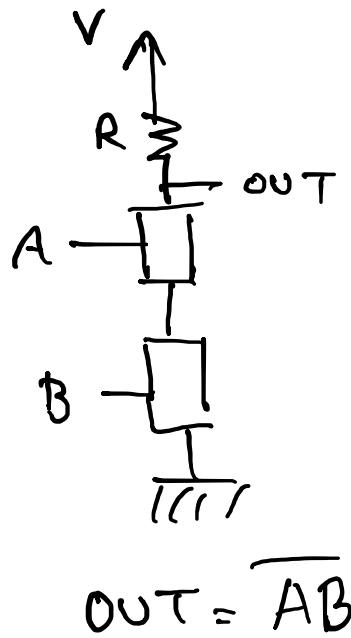


$$OUT = \overline{A+B}$$

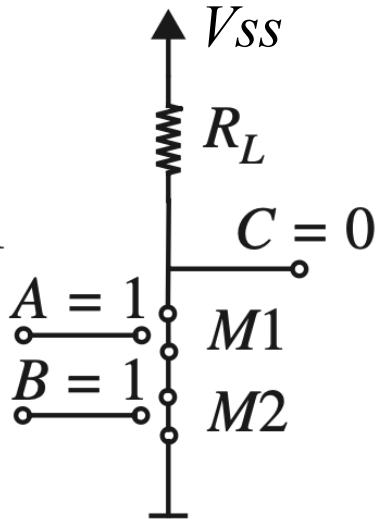
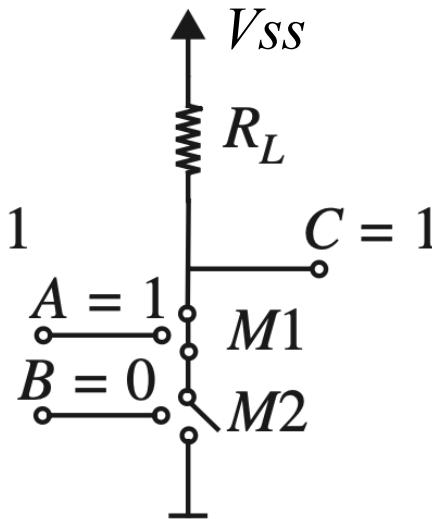
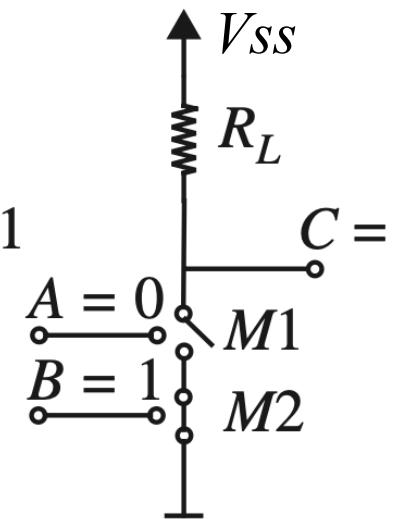
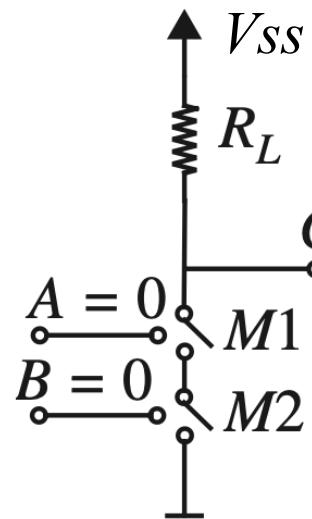
(NOR)

Switch Application – Logic Gates

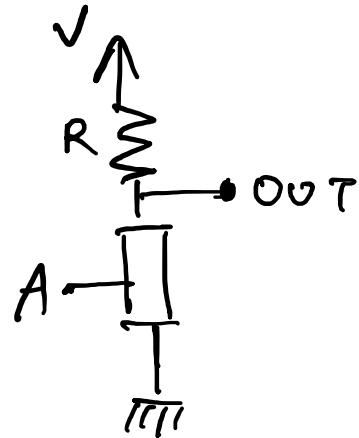
Alternative representations:



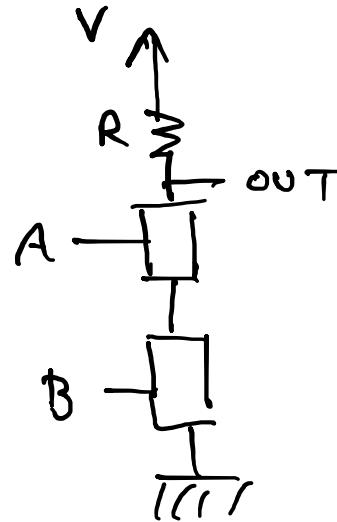
(NAND)



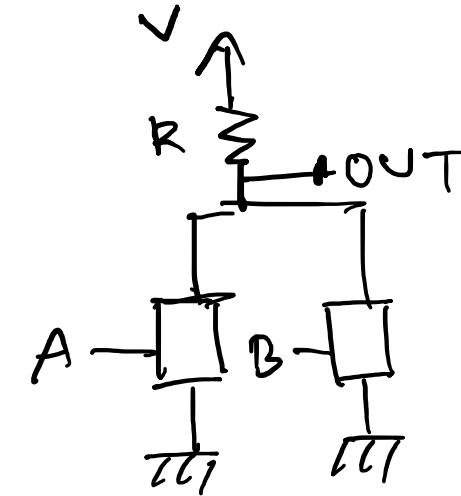
Switch Application – Logic Gates



$$OUT = \overline{A}$$



$$OUT = \overline{AB}$$



$$OUT = \overline{A+B}$$

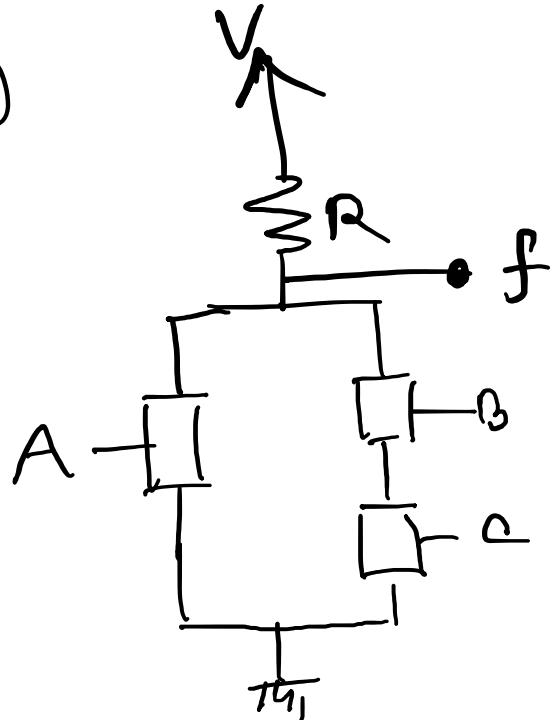
<i>A</i>	<i>V_{OUT}</i>
0	5V
1	0V

<i>A</i>	<i>B</i>	<i>V_{OUT}</i>
0	0	5V
0	1	5V
1	0	5V
1	1	0V

<i>A</i>	<i>B</i>	<i>V_{OUT}</i>
0	0	5V
0	1	0V
1	0	0V
1	1	0V

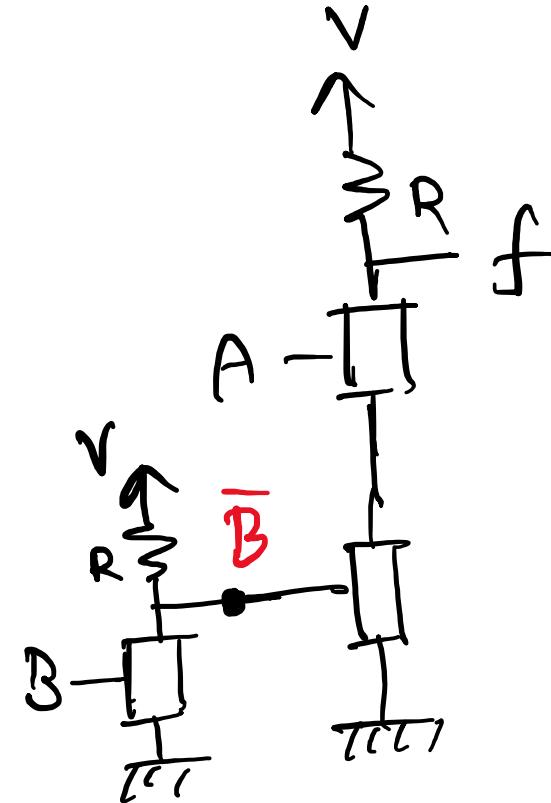
Examples

(1)



$$f = \overline{A + B.C}$$

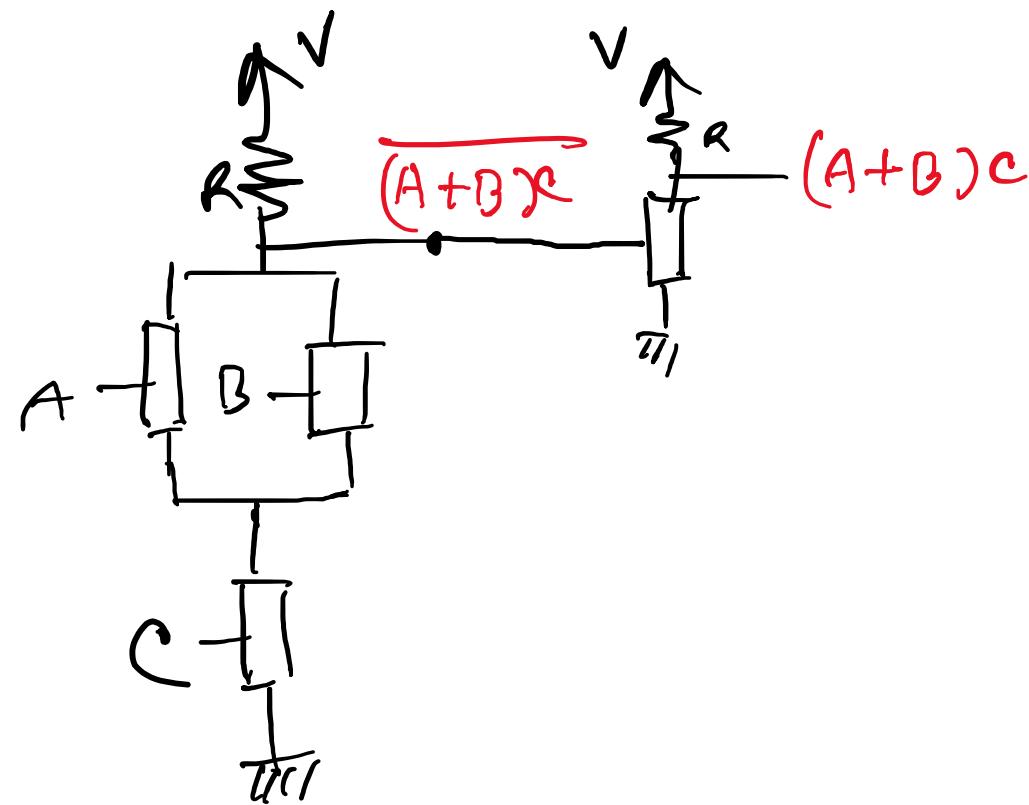
(2)



$$f = \overline{A\bar{B}}$$

Example

Implement using switches: $f = (A + B)C$



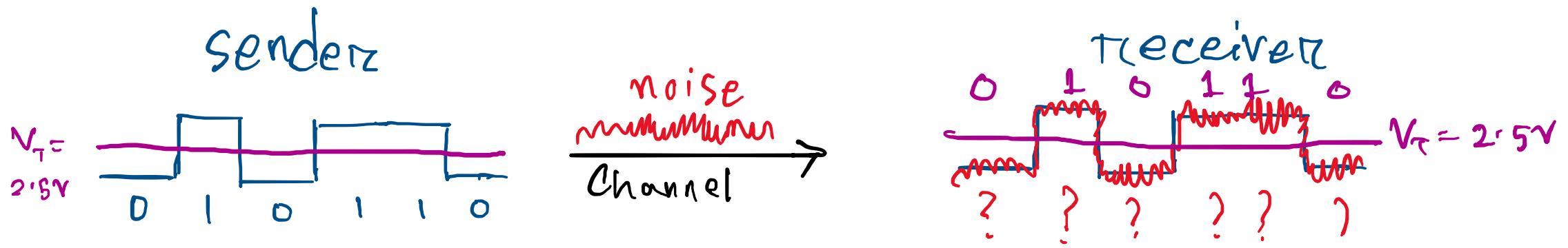
Digital Representation

- Binary → Two states (0/False, 1/True)
- Binary variables in circuit, need to use two states of device/parameters

Voltage	Current	State
5V → 1	2mA → 1	ON → 1
0V → 0	3mA → 0	OFF → 0
0V → 1		Low resistance → 1
3.3V → 0		High resistance → 0

Digital Representation

Suppose you want to send 010110



- Single value based representation fails in the presence of noise
- Better approach – threshold-based system
- Simplest: Logical 0 = $V < V_T$ Logical 1 = $V > V_T$

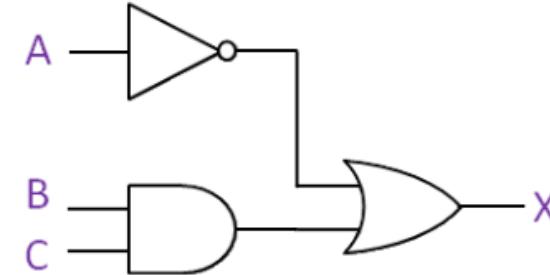
Static Discipline

- Specification for digital devices
- Requires devices to adhere to common representation to ensure that **valid input produces valid output**

- This means, if

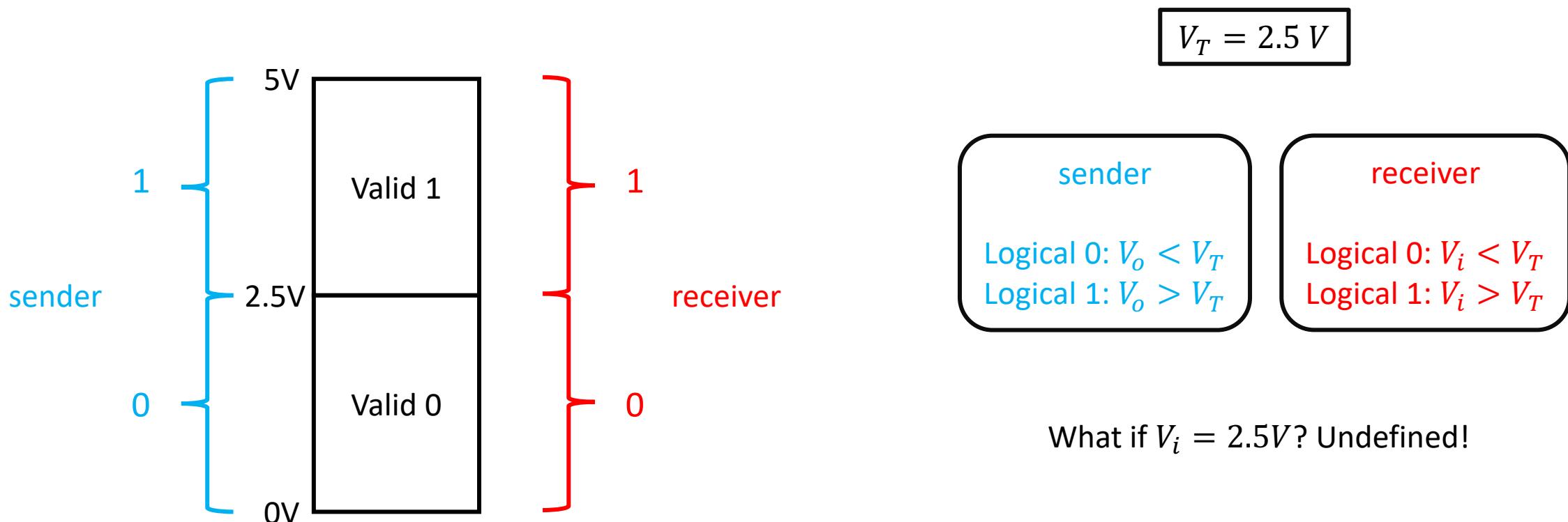
- Sender sends “0” $\xrightarrow[\text{Channel}]{\text{noise}} \text{Receiver interprets as “0”}$

- Sender sends “1” $\xrightarrow[\text{Channel}]{\text{noise}} \text{Receiver interprets as “1”}$



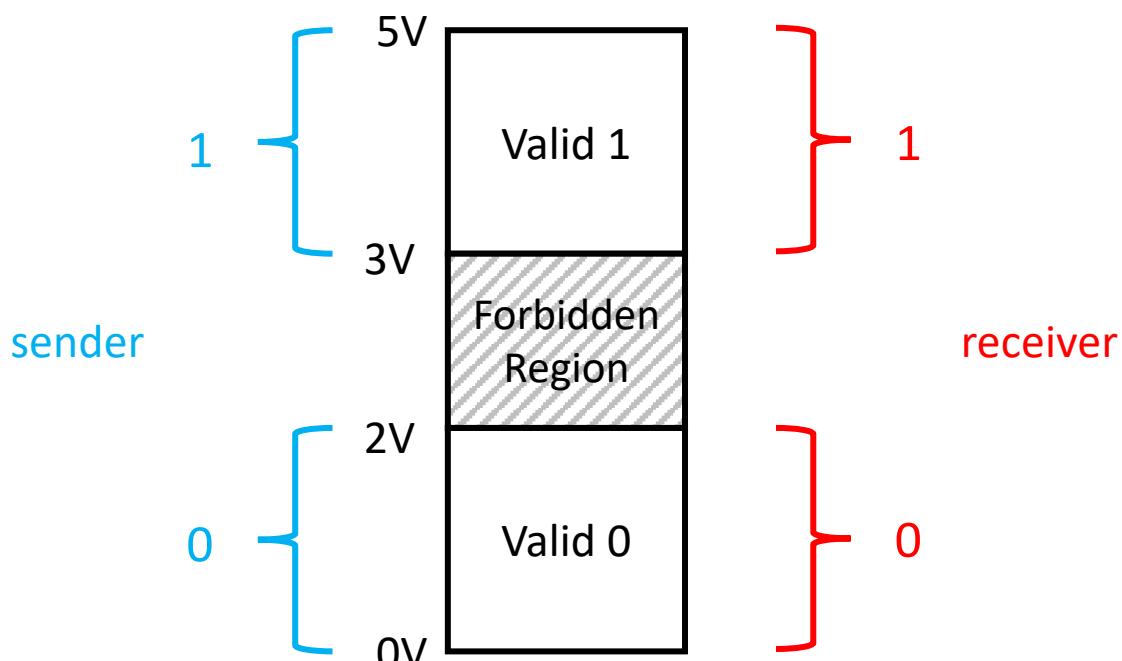
Static Discipline

Naïve approach: Single threshold based system



Static Discipline

Double threshold based system



V_H = High voltage threshold = 3V

V_L = Low voltage threshold = 2V

sender
Logical 0: $V_o < V_L$
Logical 1: $V_o > V_H$

receiver
Logical 0: $V_i < V_L$
Logical 1: $V_i > V_H$

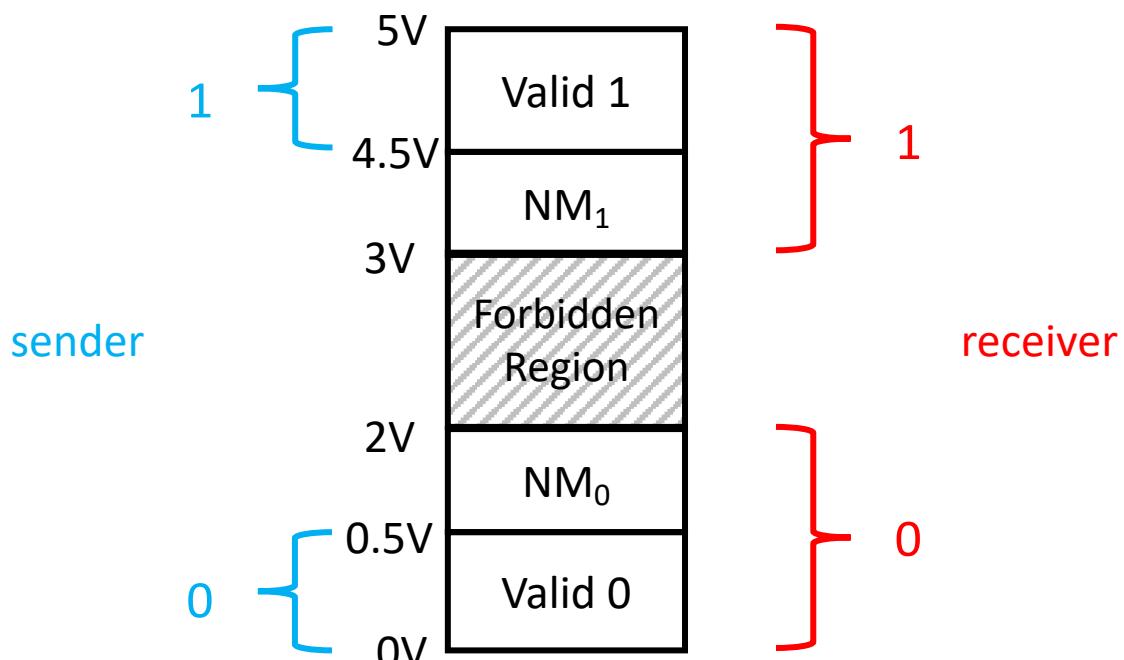
What if $V_o = 1.9V$ and channel noise is $0.5V$?

$V_i = 1.9V + 0.5V = 2.4V$ = invalid

→ valid output producing invalid input,
i.e., no margin for noise

Static Discipline

Four threshold based system → Tighter restriction on sender (**output**)



$$V_{OH} = \text{Output high voltage threshold} = 4.5V$$

$$V_{OL} = \text{Output low voltage threshold} = 0.5V$$

$$V_{IH} = \text{Input high voltage threshold} = 3V$$

$$V_{IL} = \text{Input low voltage threshold} = 2V$$

sender

receiver

$$\begin{aligned} \text{Logical 0: } & V_o < V_{OL} \\ \text{Logical 1: } & V_o > V_{OH} \end{aligned}$$

$$\begin{aligned} \text{Logical 0: } & V_i < V_{IL} \\ \text{Logical 1: } & V_i > V_{IH} \end{aligned}$$

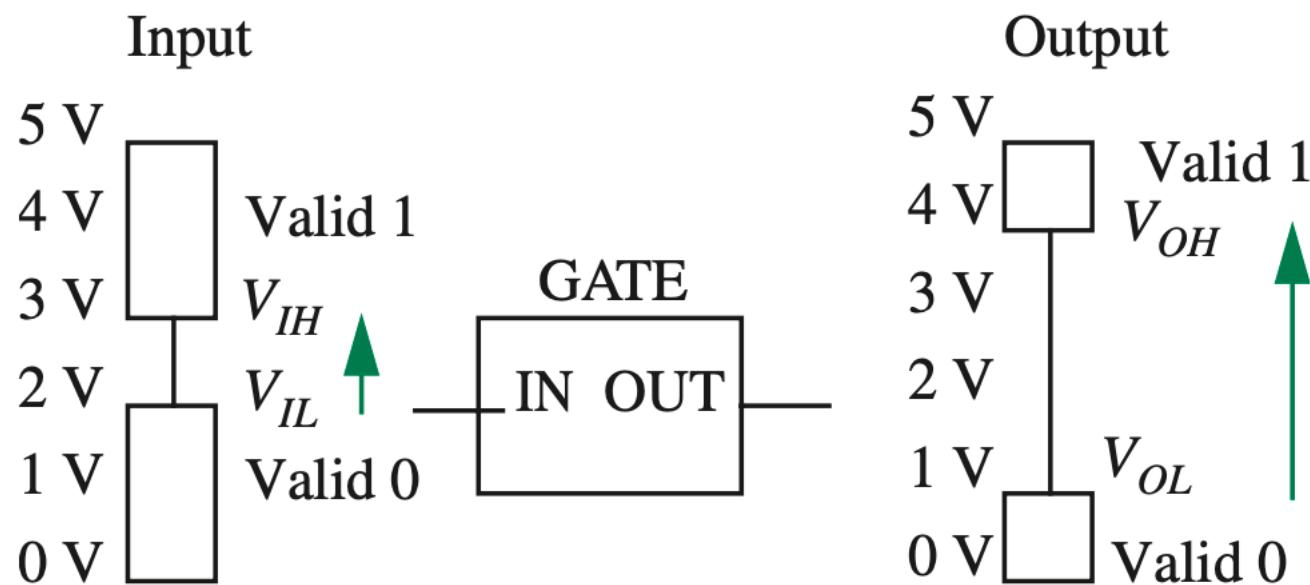
$$\text{For static discipline, } V_{OL} < V_{IL} < V_{IH} < V_{OH}$$

Noise margins (NM):

- $NM_1 = V_{OH} - V_{IH} = 4.5 - 3 = 1.5V$ (significance?)
- $NM_0 = V_{IL} - V_{OL} = 2 - 0.5 = 1.5V$ (significance?)

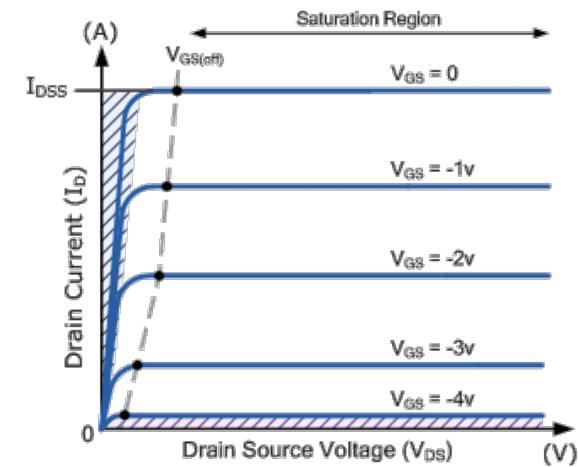
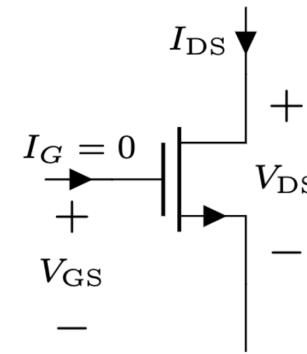
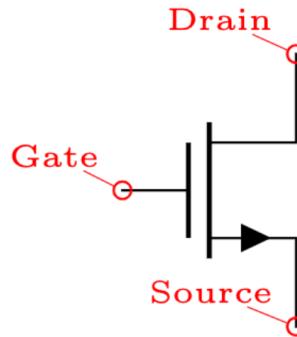
Static Discipline

Four threshold based system → Tighter restriction on sender (**output**)



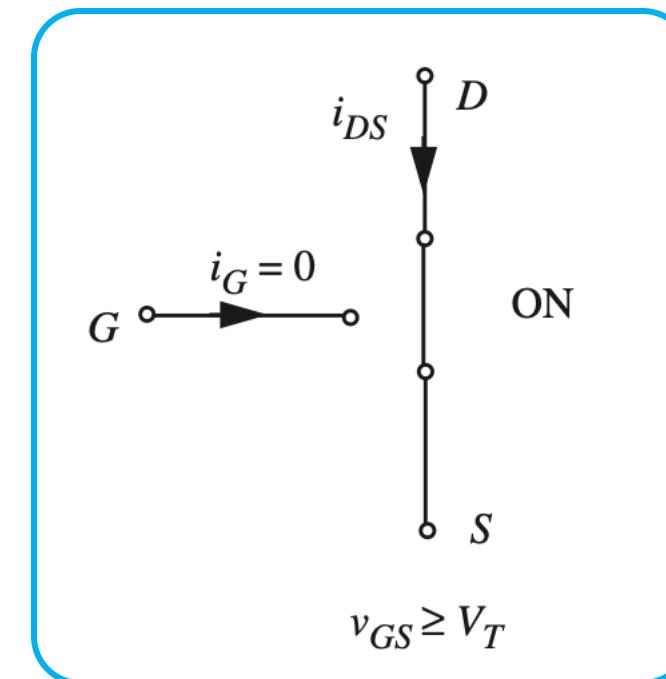
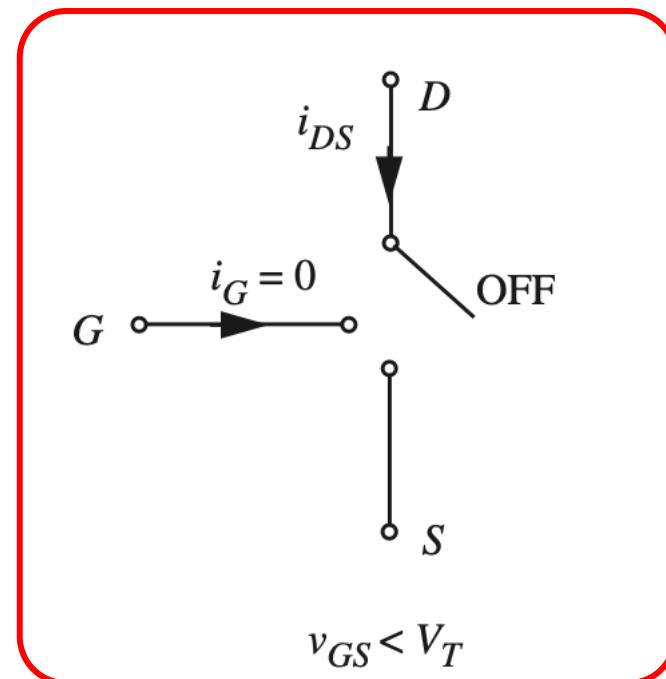
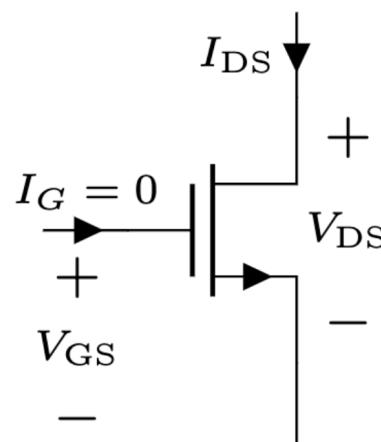
Transistors as Digital Switch

- Transistors are 3 terminal non-linear devices, can be used as switch
- 2 types – Voltage Controlled, Current Controlled
- **Metal Oxide Semiconductor Field Effect Transistor (MOSFET)** are voltage controlled
- Control, $C = V_{GS}$. The IV characteristics (I_{DS} vs V_{DS}) depends on V_{GS}
- Actual dependency is complex.
- Will start with a simple (but approximate) one – **S-Model (Switch Model)**

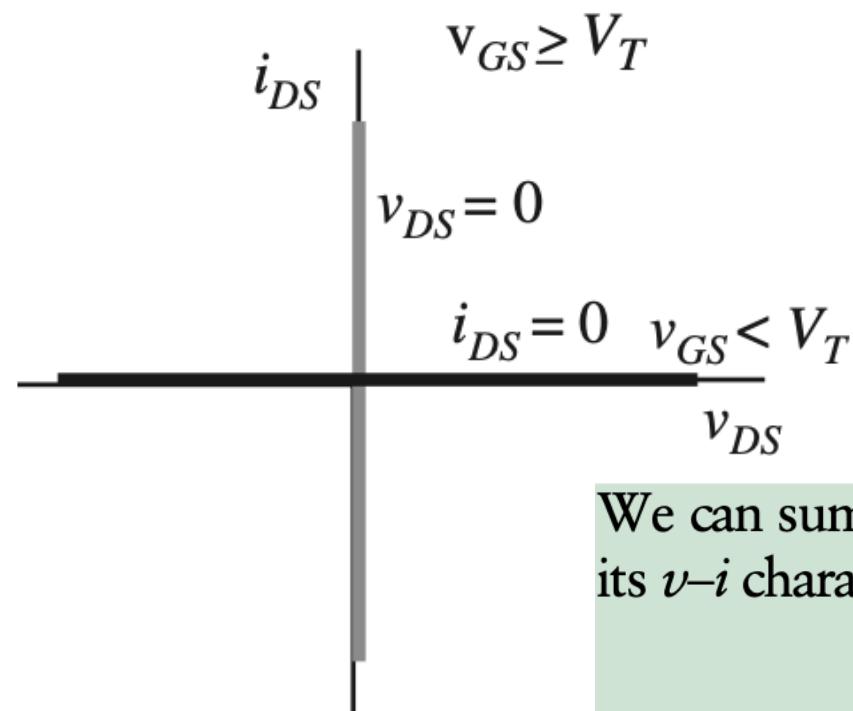


MOSFET S-Model

- The MOSFET (approximately) behaves like a switch
- $C = V_{GS}$. Here, $C = "0" \Rightarrow V_{GS} < V_T$, and $C = "1" \Rightarrow V_{GS} \geq V_T$



MOSFET S-Model



We can summarize the S model for the MOSFET in algebraic form by stating its $v-i$ characteristics as follows:

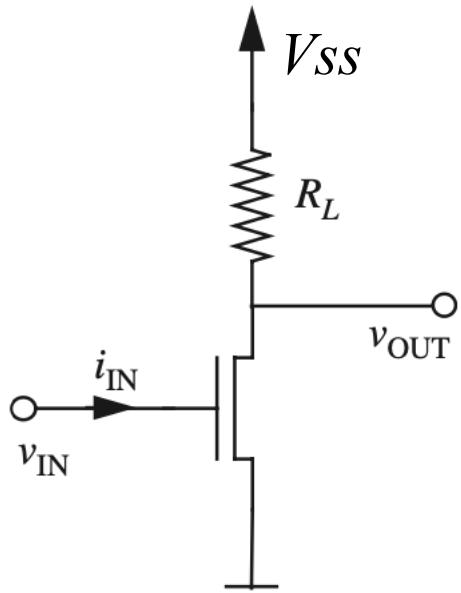
$$\text{for } v_{GS} < V_T, \quad i_{DS} = 0$$

and

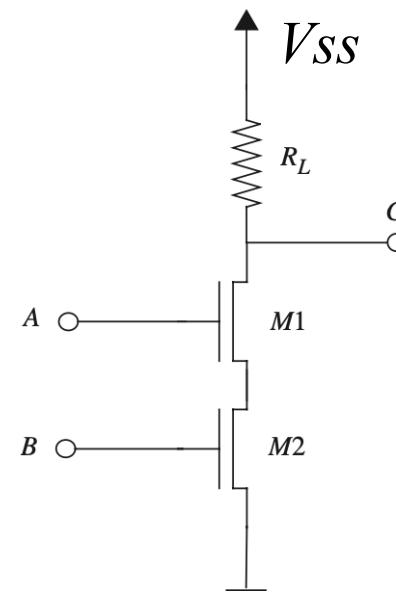
$$\text{for } v_{GS} \geq V_T, \quad v_{DS} = 0 \quad (6.2)$$

Logic Gates using MOSFET

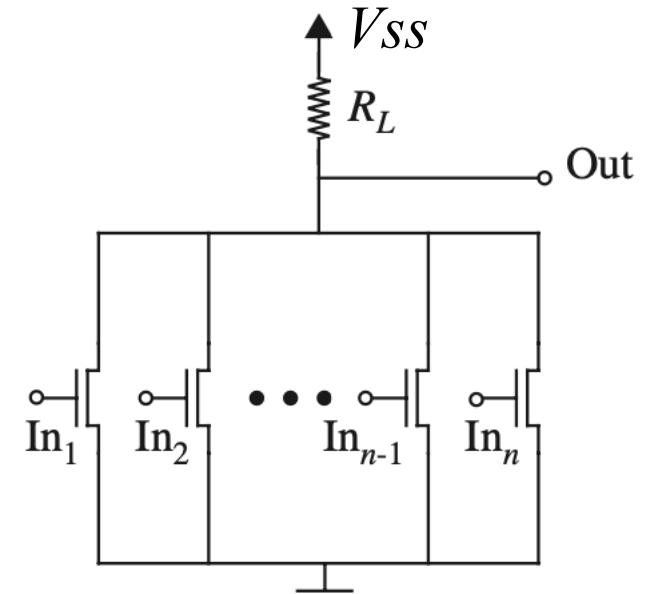
Just replace the switches with MOSFETs!



NOT Gate (Inverter)

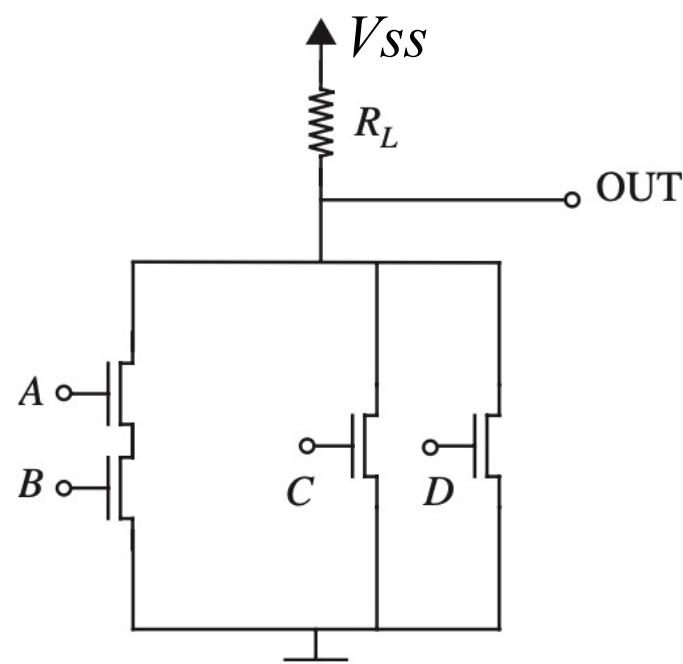


NAND Gate (Inverter)

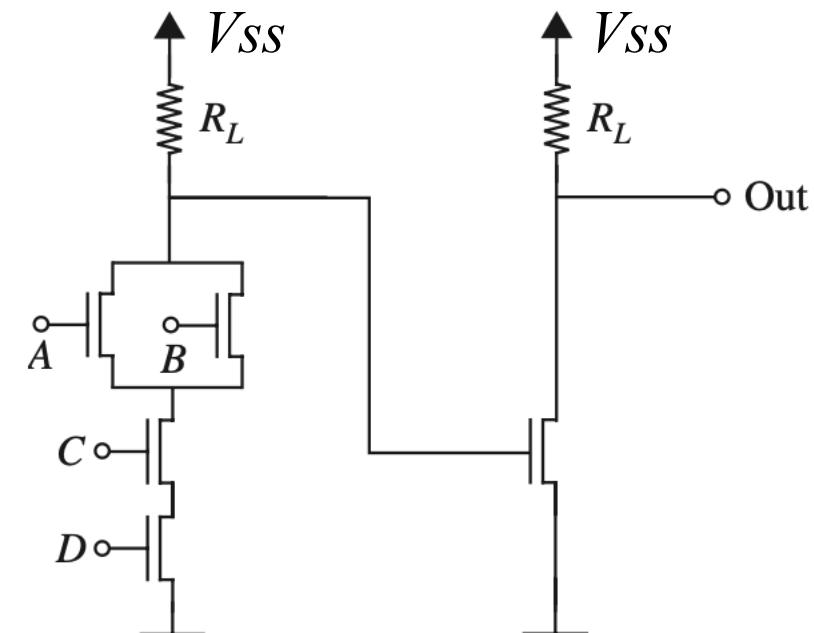


NOR Gate (Inverter)

MOSFET Logic Gates – More Examples



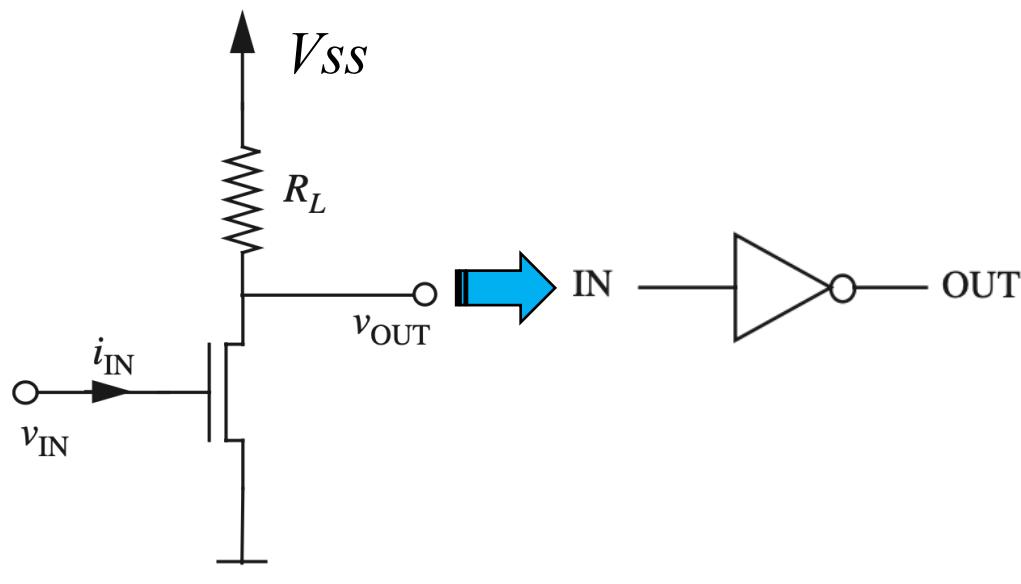
$$OUT = \overline{AB + C + D}$$



$$Out = \overline{\overline{(A + B)CD}} = (A + B)CD$$

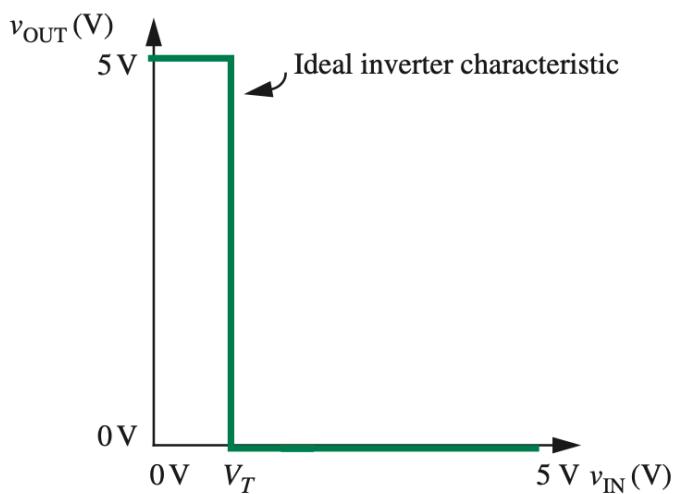
Voltage Transfer Characteristics (VTC)

- Reminder: VTC is a graph where x axis = input voltage, y axis = output voltage
- Why? Design logic gates to follow a given static discipline



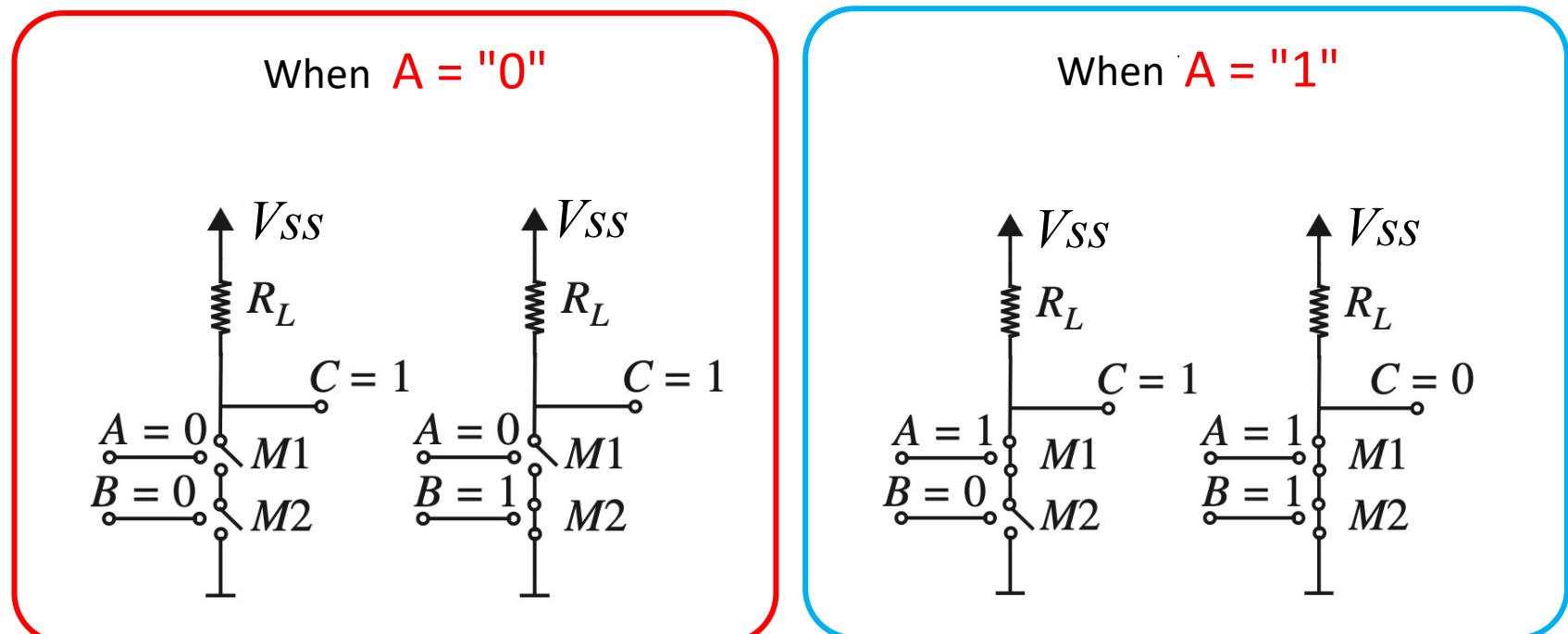
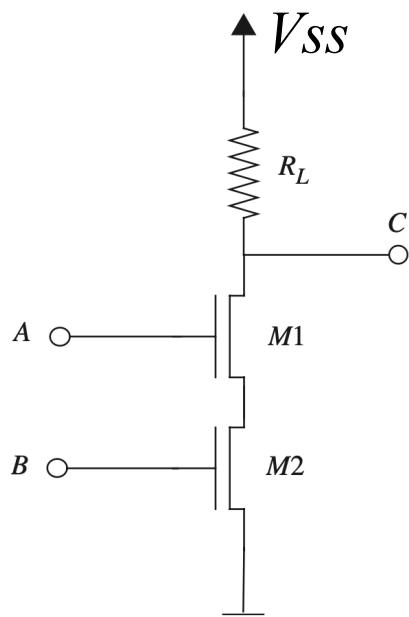
When $v_{IN} < V_T$ (Logical 0), $v_{OUT} = V_s = 5V$ (Logical 1)

When $v_{IN} \geq V_T$ (Logical 1), $v_{OUT} = 0$ (Logical 0)



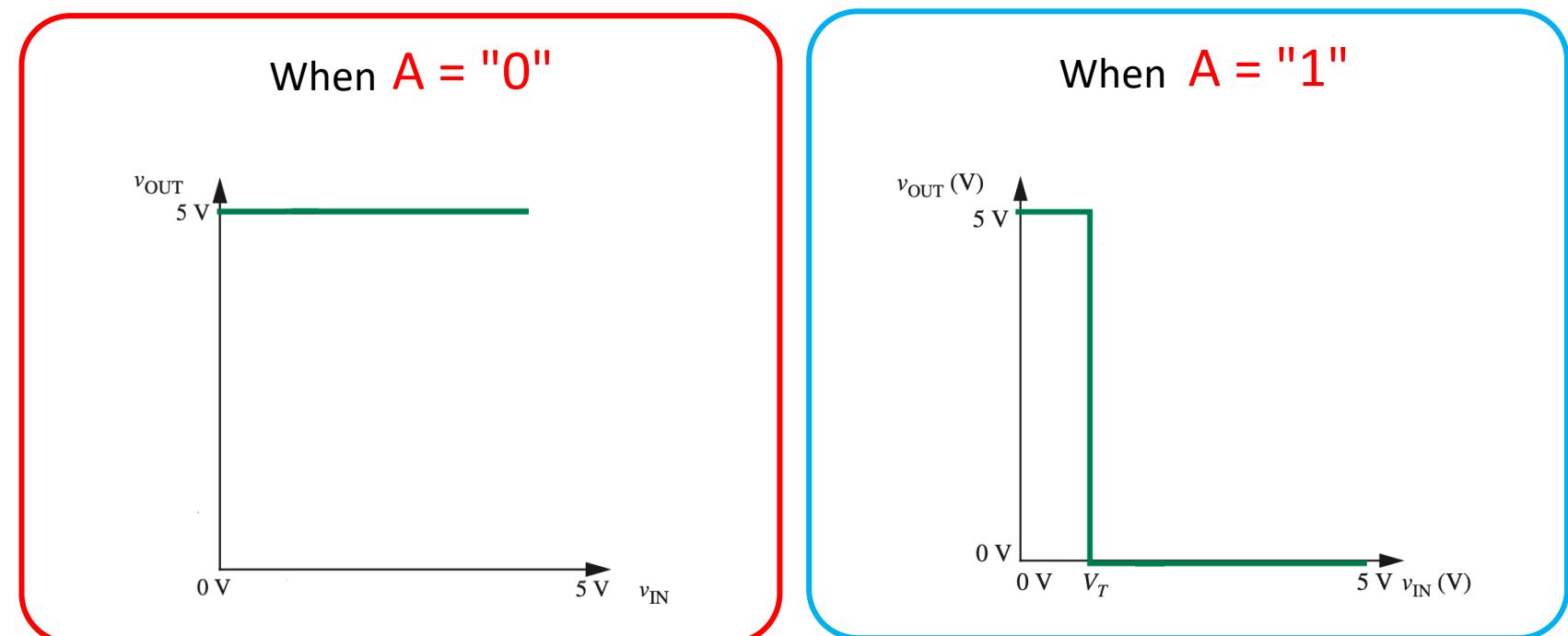
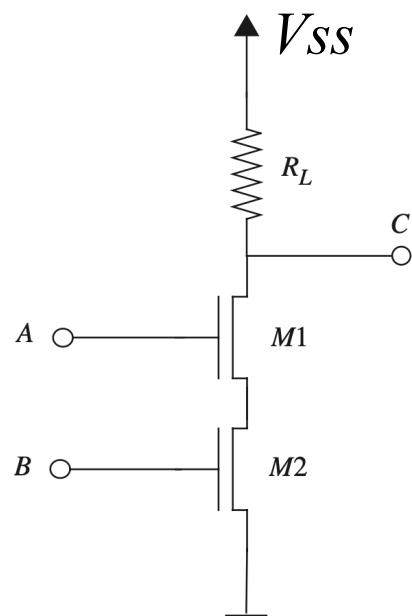
VTC of NAND gate

- We only have one x axis, but two inputs
- Solution: Draw two VTC, one considering $A = "0"$, one considering $A = "1"$



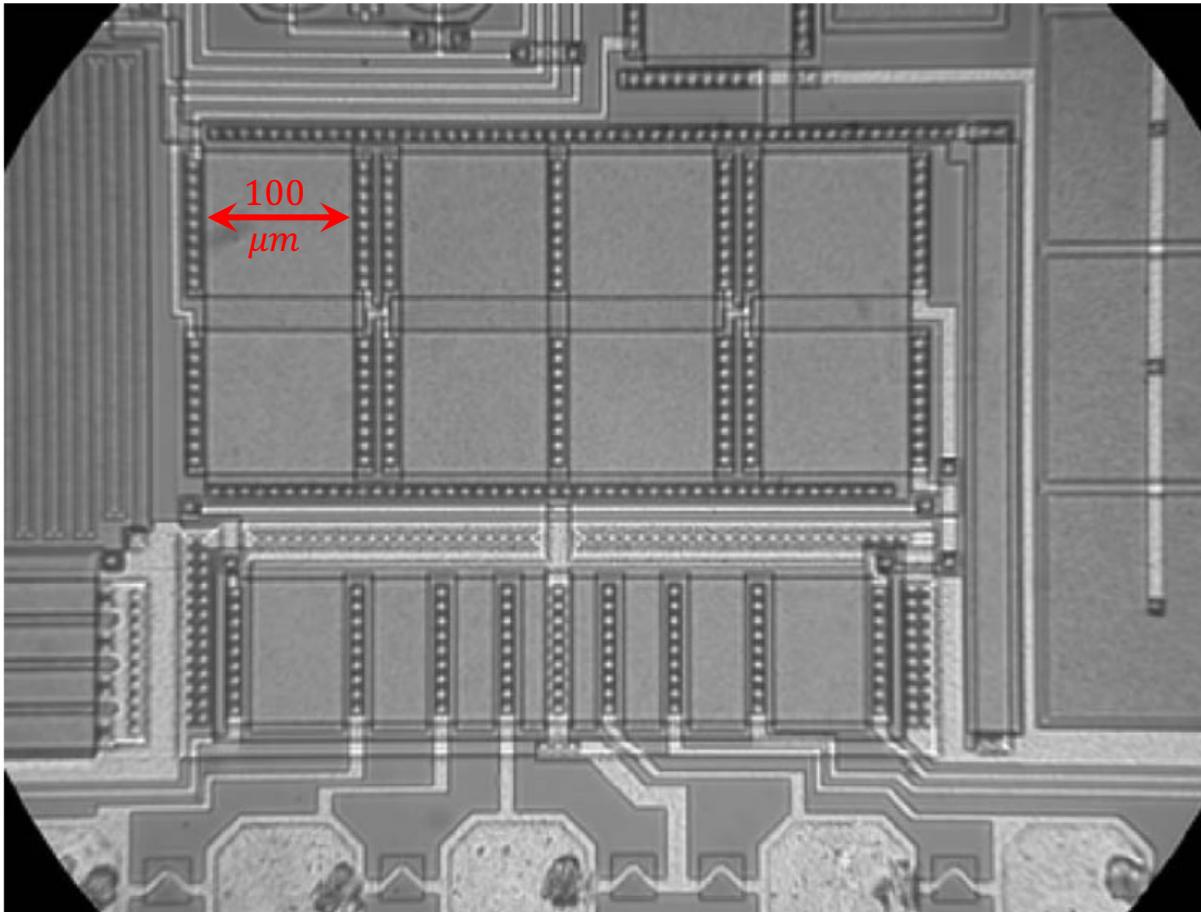
VTC of NAND gate

- We only have one x axis, but two inputs
- Solution: Draw two VTC, one considering $A = "0"$, one considering $A = "1"$



Homework: Find VTC for NOR gate

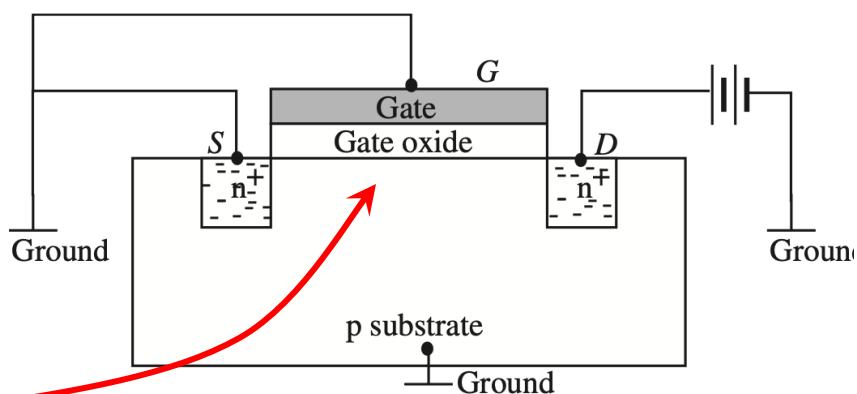
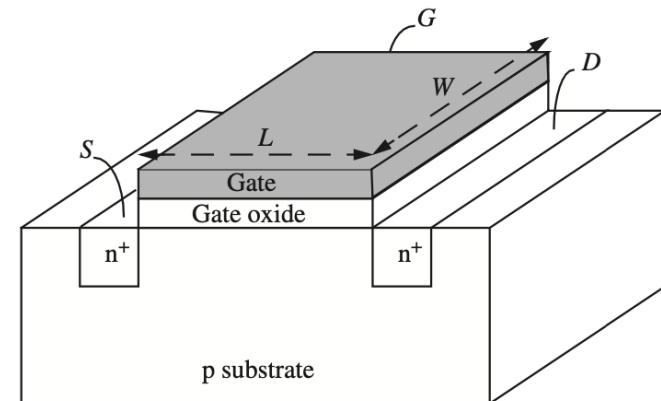
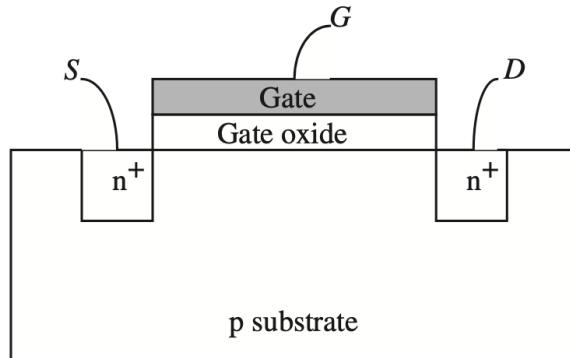
Construction of Real MOSFET



Top view of several n-channel MOSFETs fabricated on a chip. The square MOSFETs in the center of the photograph have a width and length of $100 \mu\text{m}$. (Photograph Courtesy of Maxim Integrated Products.)

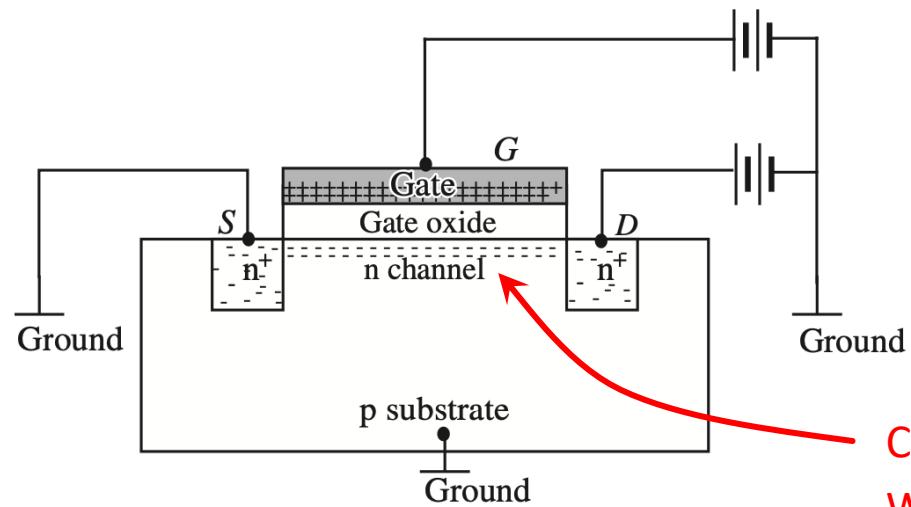
Construction of Real MOSFET

Simplified cross section
and 3D view



No channel, open ckt

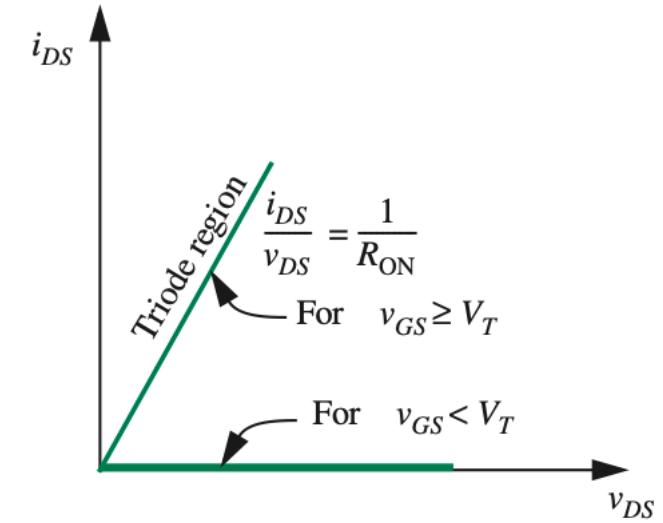
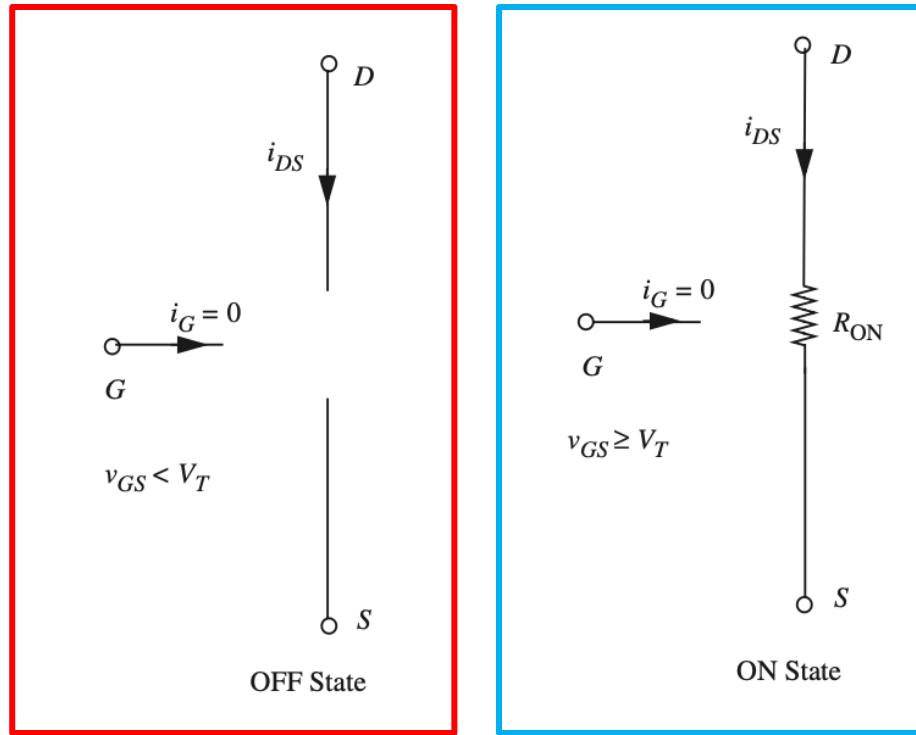
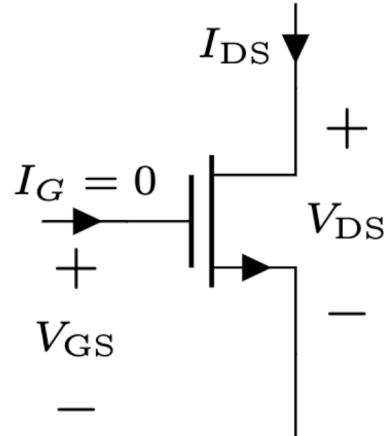
$$V_{GS} = 0V$$



$$V_{GS} > V_T$$

Channel created
Will have some R
-> SR model

SR Model

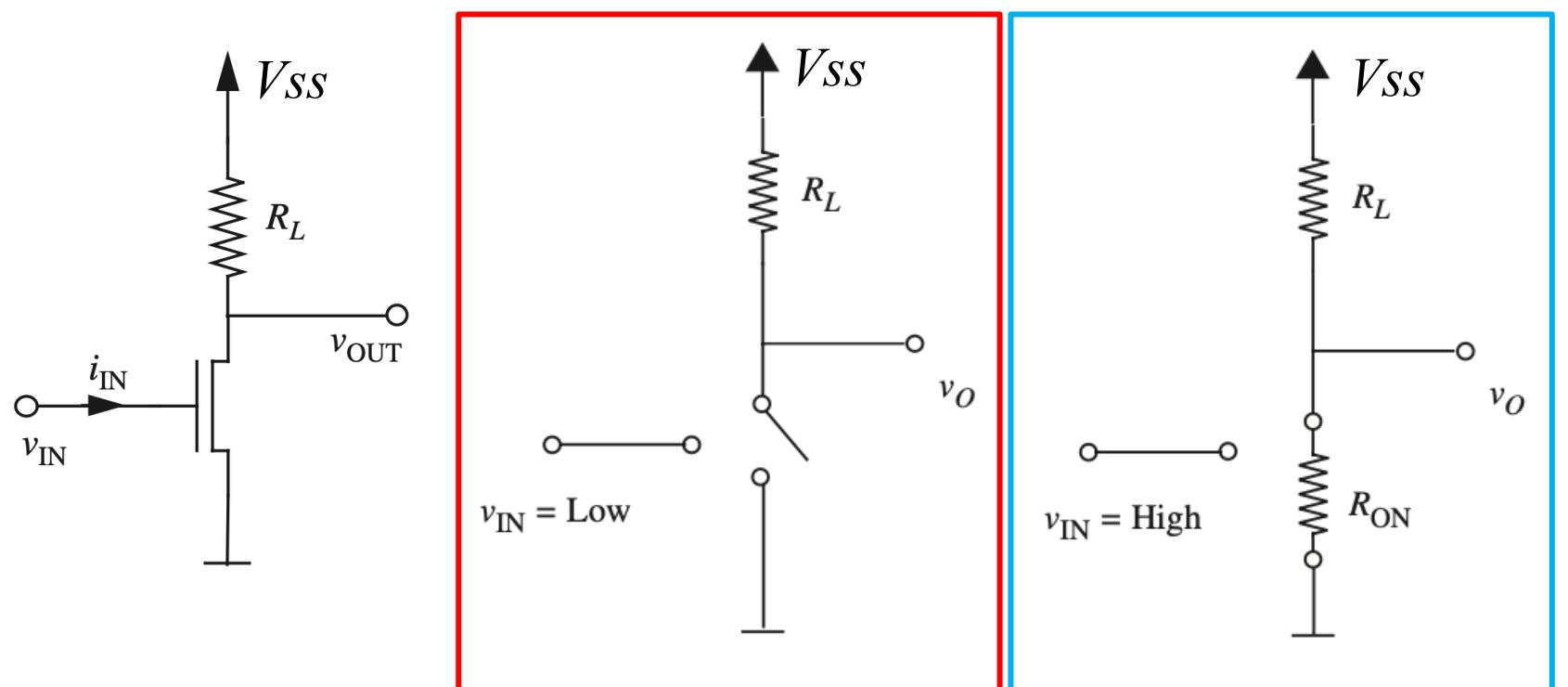


$$R_{ON} = \frac{1}{k'_n \frac{W}{L} (V_{GS} - V_T)} = \frac{1}{k V_{OV}}$$

Unit of $k = mA/V^2$

- SR model is a better approximation than S model
- However, still an approximation. This model fails when V_{DS} increases to around $V_{GS} - V_T$

SR Model - Inverter

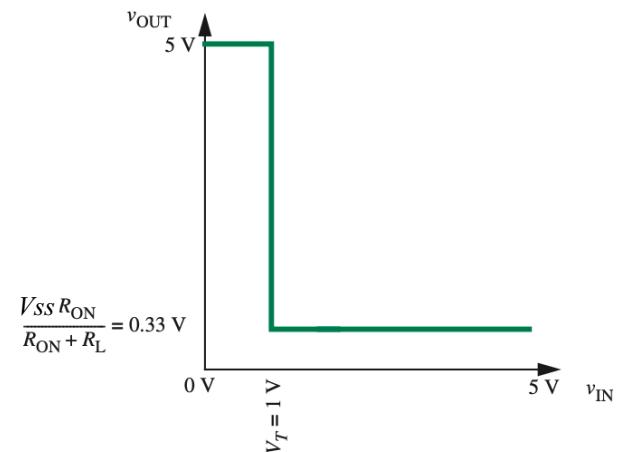


$$v_{OUT,High} = V_{SS}$$

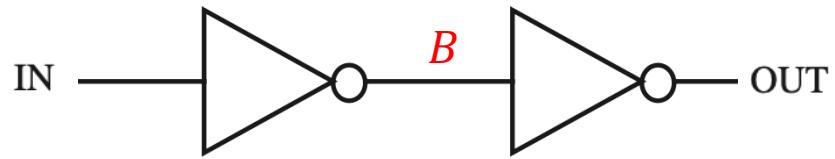
$$v_{OUT,Low} = V_{SS} \frac{R_{ON}}{R_{ON} + R_L} \neq 0$$

For example, if $V_{SS} = 5V$,
 $R_{ON} = 1 k\Omega$, $R_L = 14 k\Omega$

$$V_{SS} \frac{R_{ON}}{R_{ON} + R_L} = 0.33 \text{ V.}$$

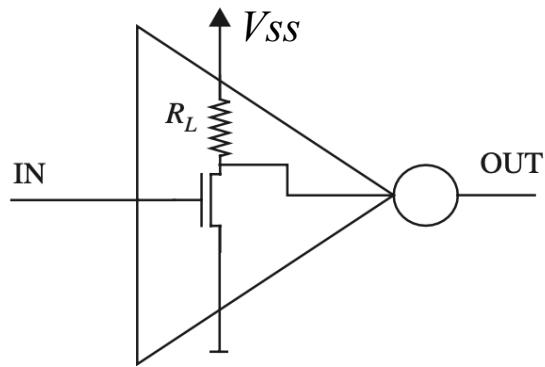


Design of logic gates



Expected

<i>IN</i>	<i>B</i>	<i>OUT</i>
Low ($V_{IN} < V_T$)	High (5V)	Low ($V_{OUT} = V_{OUT,Low}$)
High ($V_{IN} < V_T$)	Low ($V_{OUT,Low}$)	High (5V)



Actual

<i>IN</i>	<i>B</i>	<i>OUT</i>
Low ($V_{IN} < V_T$)	High (5V)	Low ($V_{OUT} = V_{OUT,Low}$)
High ($V_{IN} < V_T$)	Low ($V_{OUT,Low} = 0.5 V$)	Low ($V_{OUT} = V_{OUT,Low}$)

$$V_T = 0.4 \text{ V}, V_{SS} = 5 \text{ V}, R_{ON} = 1 \text{ k}\Omega, R_L = 9 \text{ k}\Omega$$

$$V_{OUT,High} = V_{SS} = 5 \text{ V}$$

$$V_{OUT,Low} = V_{SS} \frac{R_{ON}}{R_{ON} + R_L} = 5 \frac{1}{1 + 9} = 0.5 \text{ V}$$

Therefore, need to design logic gates properly such that

$$V_{SS} \frac{R_{ON}}{R_{ON} + R_L} < V_T.$$

Design of logic gates - Example

Assume the following values for the inverter circuit parameters: $V_S = 5\text{ V}$, $V_T = 1\text{ V}$, and $R_L = 10\text{ k}\Omega$. Assume, further, that $\frac{1}{k'_n V_{OV}} = 5$ for the MOSFET. Determine a $\frac{W}{L}$ sizing for the MOSFET so that the inverter gate output for a logical 0 is able to switch OFF the MOSFET of another inverter.

Solution:

$$\begin{aligned}V_S \frac{R_{ON}}{R_{ON} + R_L} &< V_T \\ \Rightarrow 5 \frac{R_{ON}}{R_{ON} + 10} &< 1 \\ \Rightarrow 5R_{ON} &< R_{ON} + 10 \\ \Rightarrow R_{ON} &< \frac{10}{4} = 2.5\end{aligned}$$

Now,

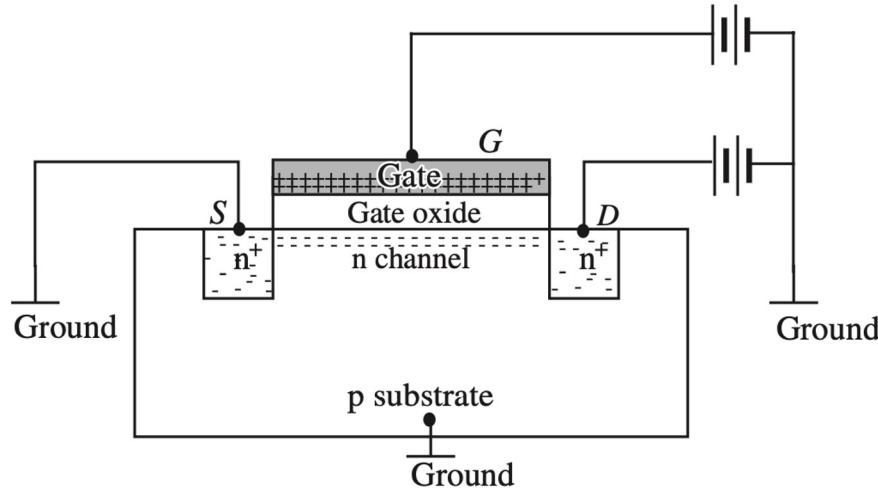
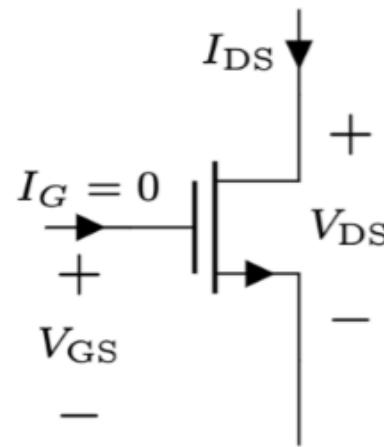
$$R_{ON} = \frac{1}{k'_n \frac{W}{L} V_{OV}} = 5 \times \frac{1}{W/L}$$

Hence

$$\frac{5}{W/L} < 2.5 \Rightarrow \frac{W}{L} > \frac{5}{2.5}$$

$$\Rightarrow \frac{W}{L} > 2$$

Review – MOSFET



Control = $V_{GS} = V_G - V_S$, controls the IV between drain-source (I_{DS} vs V_{DS})

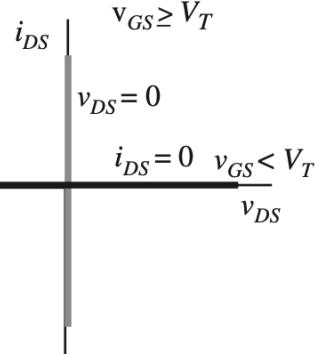
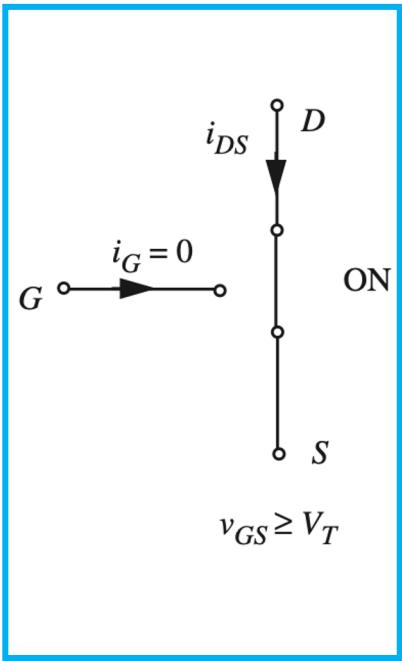
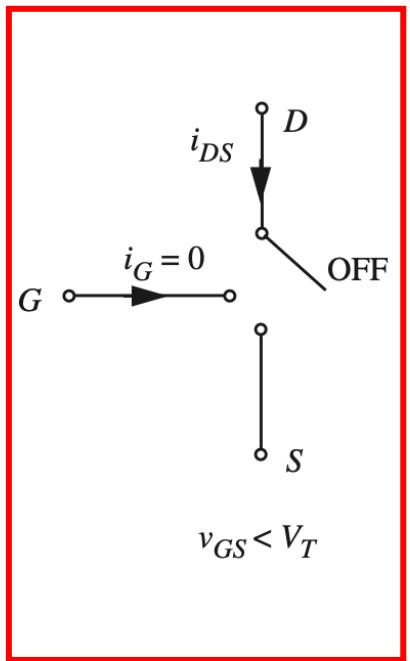
Threshold voltage = V_T , minimum voltage required to create the channel

Models

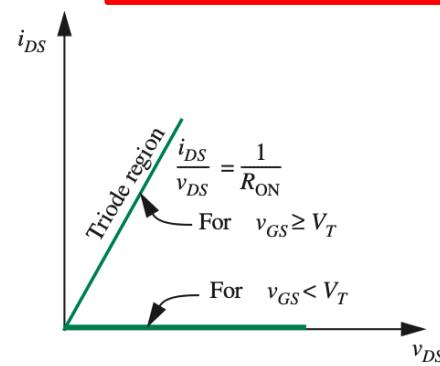
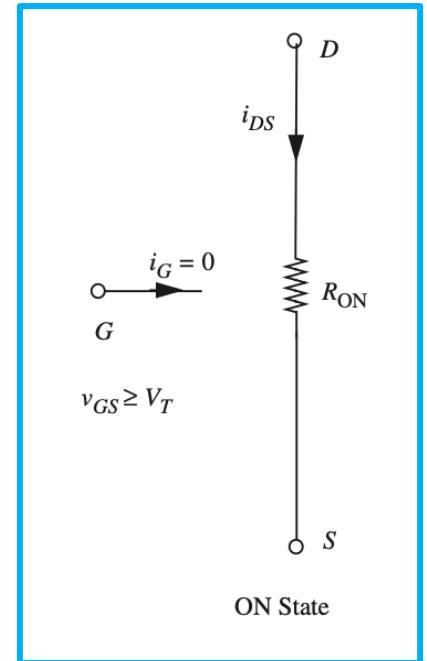
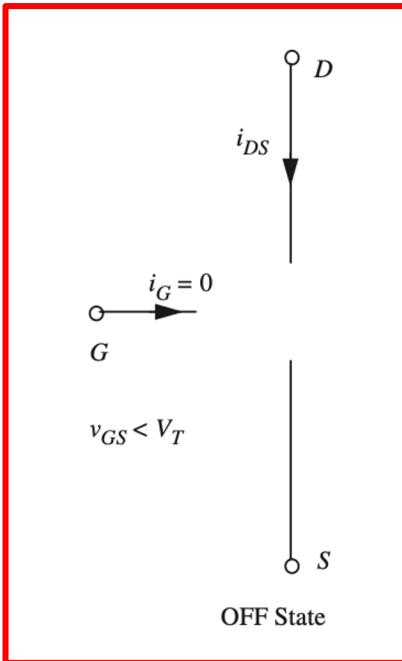
1. **S Mode:** Assumes an ideal channel with zero resistance
2. **SR Model:** Assumes finite channel resistance, R_{ON} , depends on $V_{GS} - V_T = V_{OV}$

MOSFET Linear Models

S Model



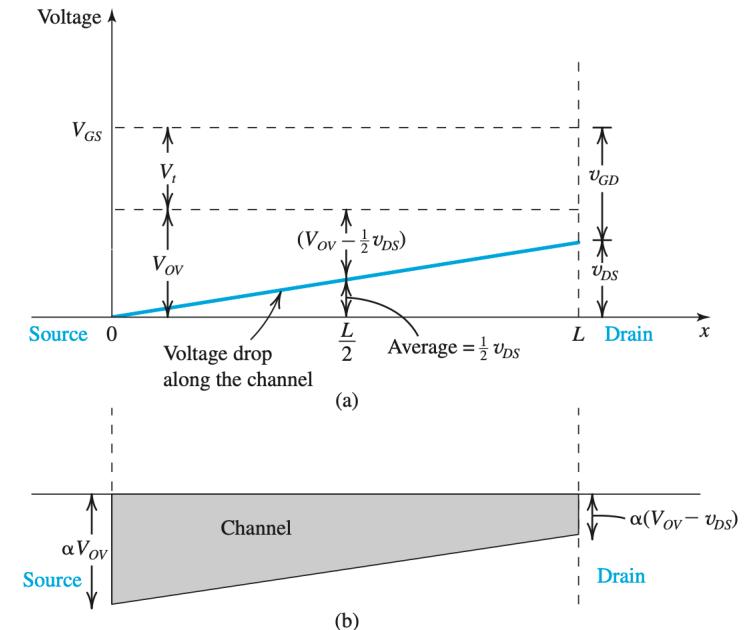
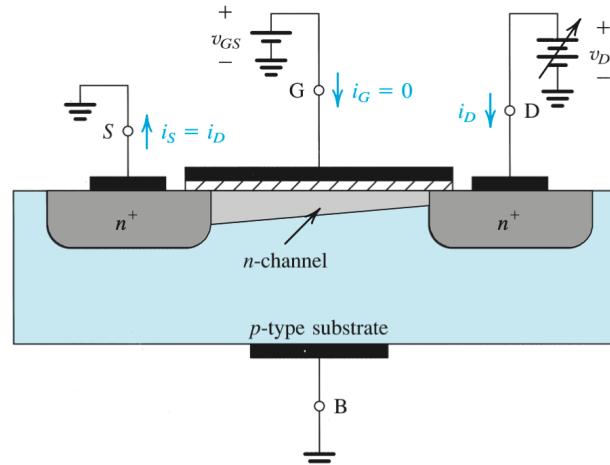
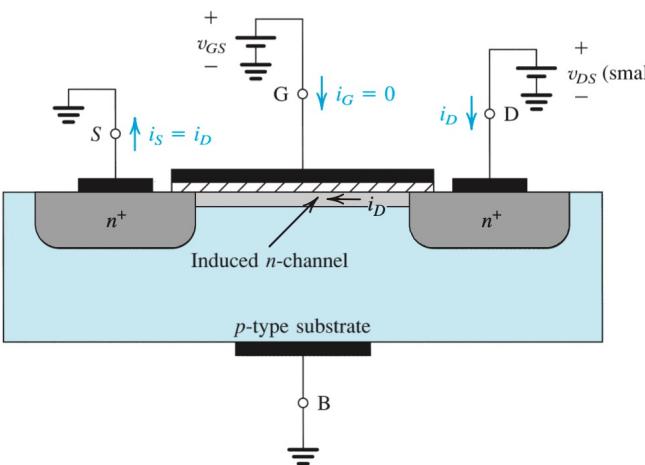
SR Model



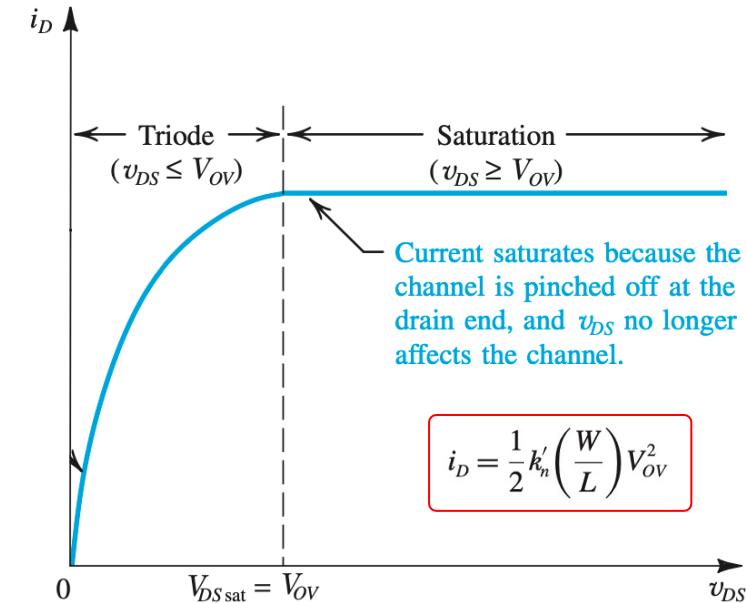
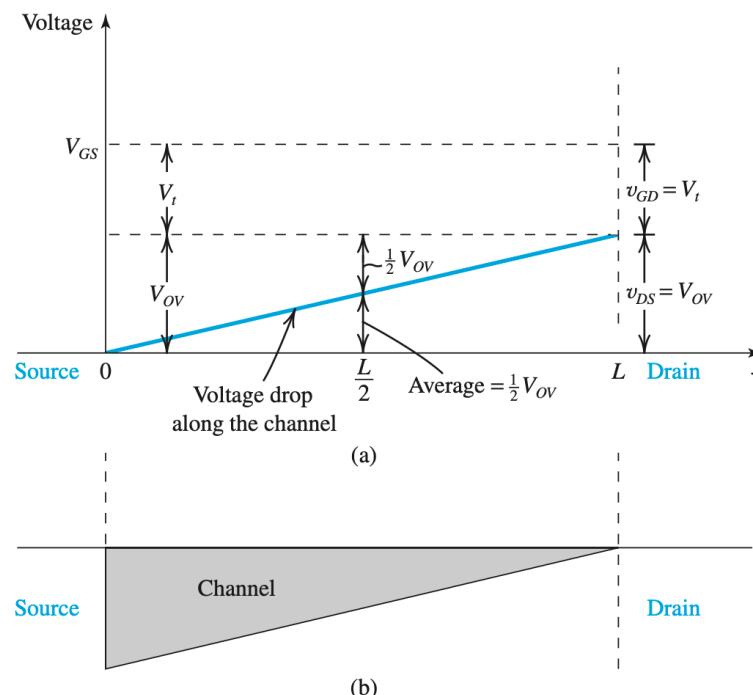
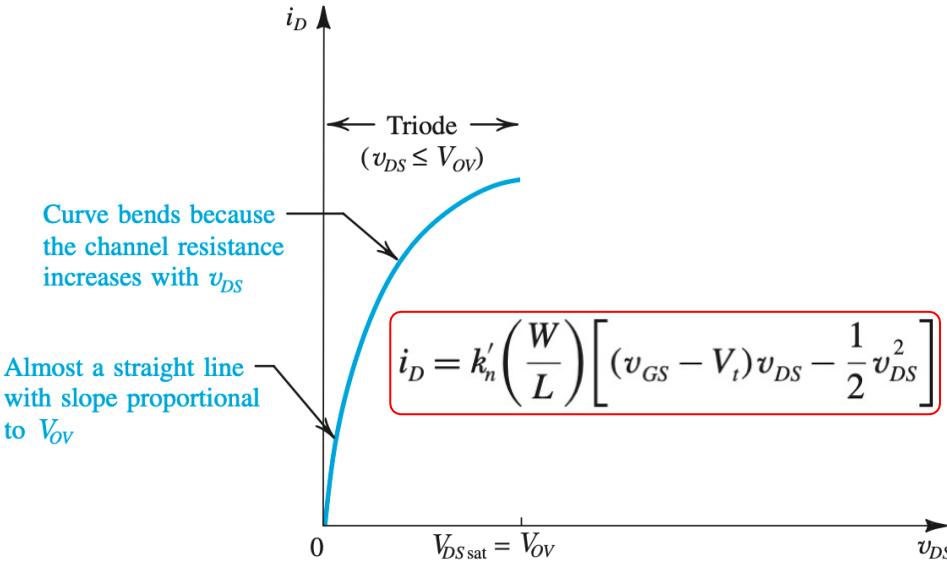
$$R_{ON} = \frac{1}{k'_n \frac{W}{L} (V_{GS} - V_T)} = \frac{1}{k V_{OV}}$$

Real MOSFET

- Why $R_{ON} = \frac{1}{k'_n \frac{W}{L} (V_{GS} - V_T)} = \frac{1}{kV_{OV}}$? Because channel width $\propto V_{OV}$, and $R \propto \frac{1}{\text{width}}$
- For small V_{DS} , uniform channel, hence fixed R_{ON} , therefore SR model valid.
- As V_{DS} is increased, channel becomes tapered cause $V_{GD} \downarrow$. Resistance \uparrow , slope \downarrow .
- This mode is called the **triode mode**. Condition: $V_{DS} < V_{OV}$

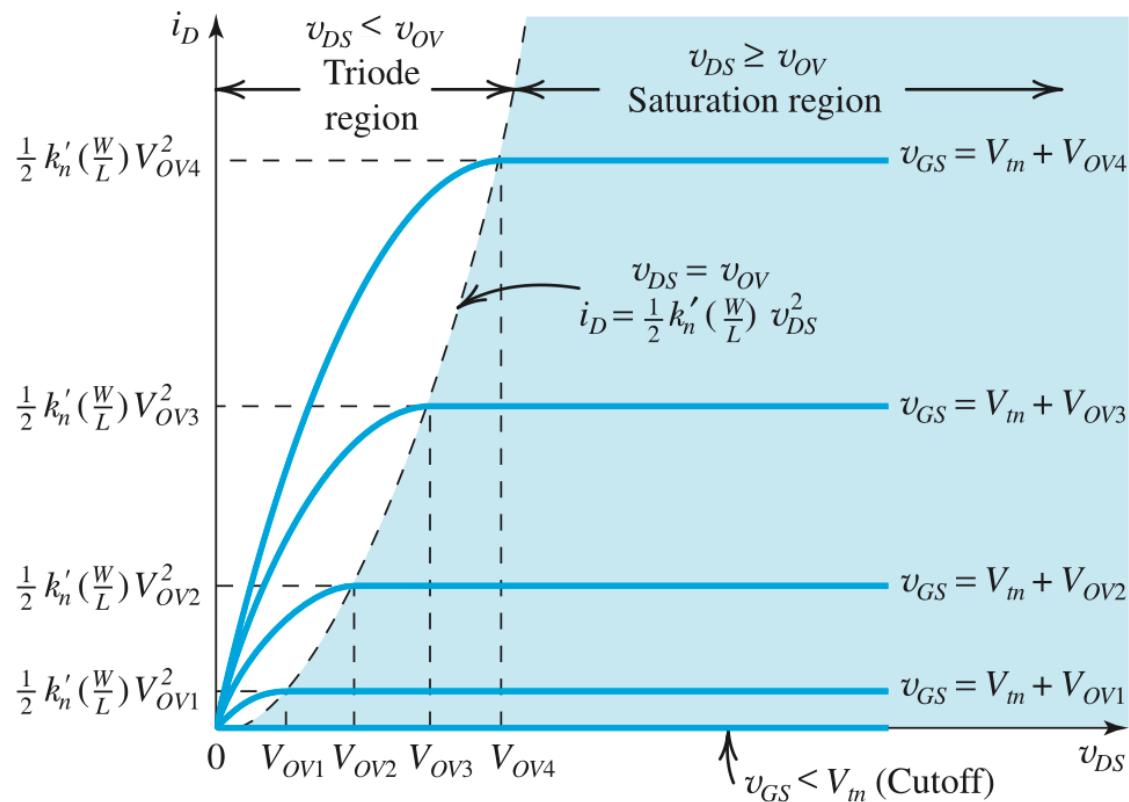


Real MOSFET



- When $V_{DS} = V_{OV}$, channel pinches off.
- Increasing V_{DS} further have no effect on channel shape. Hence, current saturates
- This mode is called the **saturation mode**. Condition: $V_{DS} \geq V_{OV}$
- Behaves like a current source (constant current) that depends on V_{OV}

IV Characteristics of Real MOSFET



Mode	Condition	Equation
Cutoff	$V_{GS} < V_T$	$I_D = 0$
Triode	$V_{GS} \geq V_T$ $V_{DS} < V_{OV}$	$I_D = k [V_{OV} V_{DS} - \frac{1}{2} V_{DS}^2]$
Saturation	$V_{GS} \geq V_T$ $V_{DS} \geq V_{OV}$	$I_D = \frac{k}{2} V_{OV}^2$

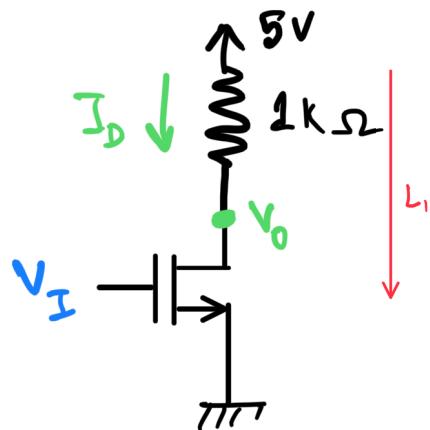
$$V_{OV} = V_{GS} - V_T$$

$$k = k'_n \left(\frac{W}{L}\right)$$

Solving Circuits with MOSFET

- Use **Method of Assumed State!**
- Three steps:
 - **Assume:** One of the modes (Cutoff, Triode, Saturation)
 - **Solve:** Use corresponding equation and KCL+KVL
 - **Verify:** Check if the conditions of V_{GS} and V_{DS} are satisfied. If not, repeat.
- Might need to solve quadratic equation ($ax^2 + bx + c = 0$).
- If we get two roots, choose the one that's *favorable* to your assumption

Example 1



The MOSFET is specified as $V_T = 1V$ and $k = 0.5 \text{ mA/V}^2$. Find I_D and V_o for $V_I = 2V$.

Solution:

Step 1: Assume the MOSFET in **saturation**

Step 2: $I_D = \frac{k}{2} V_{OV}^2$ Here, $V_{GS} = V_G - V_S = V_G - 0 = V_G = V_I = 2V$
Therefore, $V_{OV} = V_{GS} - V_T = 2 - 1 = 1V$

$$\therefore I_D = \frac{0.5}{2} (1)^2 = 0.25 \text{ mA}$$

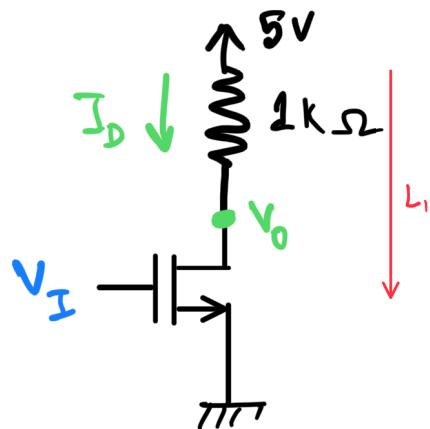
$$\text{Again, } V_{DS} = V_D - V_S = V_D - 0 = V_D = V_o$$

KVL along $\textcolor{red}{L_1}$: $I_D \times 1k\Omega + V_o = 5 - 0 \Rightarrow V_o = 5 - I_D \times 1k\Omega$
 $\Rightarrow V_o = 5 - 0.25 \times 1 = 4.75 \text{ V} = V_{DS}$

Step 3: $V_{GS} = 2V > V_T \checkmark$ Therefore, **assumption correct!**

$V_{DS} = 1V > V_{OV} \checkmark$ Correct ans: $I_D = 0.25 \text{ mA}, V_o = 4.75 \text{ V}$

Example 2



The MOSFET is specified as $V_T = 1V$ and $k = 0.5 \text{ mA/V}^2$. Find I_D and V_o for $V_I = 5V$.

Solution:

Step 1: Assume the MOSFET in **saturation**

Step 2: $I_D = \frac{k}{2} V_{OV}^2$ Here, $V_{GS} = V_G - V_S = V_G - 0 = V_G = V_I = 5V$
Therefore, $V_{OV} = V_{GS} - V_T = 5 - 1 = 4V$

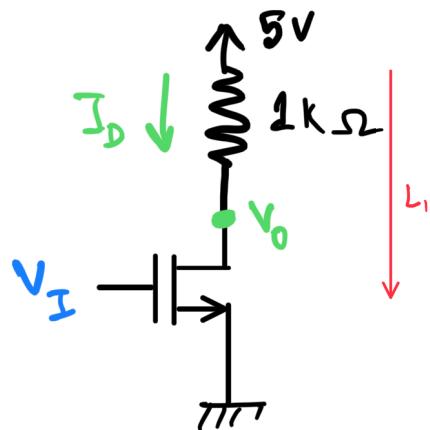
$$\therefore I_D = \frac{0.5}{2} (4)^2 = 4 \text{ mA}$$

$$\text{Again, } V_{DS} = V_D - V_S = V_D - 0 = V_D = V_o$$

$$\begin{aligned} \text{KVL along } L_1: I_D \times 1k\Omega + V_o &= 5 - 0 \Rightarrow V_o = 5 - I_D \times 1k\Omega \\ \Rightarrow V_o &= 5 - 4 \times 1 = 1V = V_{DS} \end{aligned}$$

Step 3: $V_{GS} = 5V > V_T \checkmark$ Therefore, **assumption wrong!**
 $V_{DS} = 1V \not> V_{OV} \times$

Example 2



The MOSFET is specified as $V_T = 1V$ and $k = 0.5 \text{ mA/V}^2$. Find I_D and V_o for $V_I = 5V$.

Repeat:

Step 1: Assume the MOSFET in **triode**

$$\text{Step 2: } I_D = k[V_{OV}V_{DS} - \frac{1}{2}V_{DS}^2]$$

$$\text{Here, } V_{GS} = V_G - V_S = V_G - 0 = V_G = V_I = 5V$$

$$\text{Therefore, } V_{OV} = V_{GS} - V_T = 5 - 1 = 4V$$

$$\text{Again, } V_{DS} = V_D - V_S = V_D - 0 = V_D = V_o. \text{ Assuming } V_{DS} = x$$

$$\text{KVL along } L_1: I_D \times 1k\Omega + V_o = 5 - 0 \Rightarrow I_D = \frac{5-V_{DS}}{1} = 5 - x$$

$$\therefore I_D = 0.5 \left[4 \times V_{DS} - \frac{1}{2} V_{DS}^2 \right] \Rightarrow (5 - x) = 0.5 \left[4x - \frac{1}{2} x^2 \right]$$

$$\Rightarrow 5 - x = 2x - 0.25x^2 \Rightarrow 0.25x^2 - 3x + 5 = 0$$

$$\text{Solving, } x = 2V, \cancel{x = 10V}$$

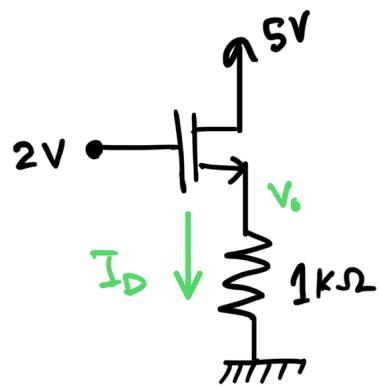
Since $V_{DS} = x$ is small in triode, smaller value of x is favorable

$$\text{Therefore, } V_o = V_{DS} = x = 2V, \text{ and } I_D = 5 - x = 3 \text{ mA}$$

$$\text{Step 3: } V_{GS} = 5V > V_T \quad \checkmark \quad \text{Therefore, assumption correct!}$$

$$V_{DS} = 2V < V_{OV} \quad \checkmark \quad \boxed{\text{Correct ans: } I_D = 3 \text{ mA}, V_o = 2V}$$

Example 3



The MOSFET is specified as $V_T = 1V$ and $k = 4 \text{ mA/V}^2$.

Find I_D and V_o

Solution:

Step 1: Assume the MOSFET in **saturation**

$$\text{Step 2: } I_D = \frac{k}{2} V_{ov}^2$$

Let's assume $V_0 = V_S = x$

$$\text{Here, } V_{GS} = V_G - V_S = V_G - V_0 = 2 - x$$

$$\text{Therefore, } V_{ov} = V_{GS} - V_T = (2 - x) - 1 = 1 - x$$

$$\text{Again, } V_{DS} = V_D - V_S = V_D - V_0 = 5 - x$$

$$\text{Ohm's law for the resistor: } I_D = \frac{V_0 - 0}{1k\Omega} = x$$

$$\therefore x = \frac{4}{2}(1 - x)^2 \Rightarrow x = 2(1 - 2x + x^2) \Rightarrow x = 2 - 4x + 2x^2 \\ \Rightarrow 2x^2 - 5x + 2 = 0$$

$$\text{Solving, } x = 0.5, \cancel{x = 2}$$

Since $V_{DS} = 5 - x$ is large in saturation
smaller value of x is favorable

$$\therefore V_0 = V_S = x = 0.5V, I_D = x = 0.5 \text{ mA},$$

$$V_{DS} = 5 - x = 4.5V, V_{GS} = 2 - x = 1.5V, \text{ and } V_{ov} = 1 - x = 0.5V$$

Step 3: $V_{GS} = 1.5V > V_T \checkmark$ Therefore, **assumption correct!**

$V_{DS} = 4.5V > V_{ov} \checkmark$ **Correct ans: $I_D = 0.5 \text{ mA}, V_o = 0.5 V$**

Practice

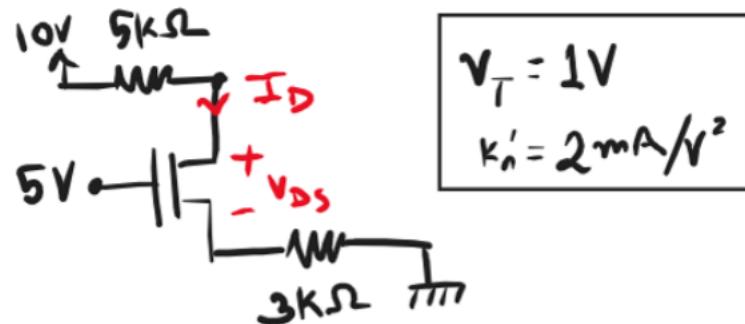
Question 4 [CO1, CO4]

10

Analyze the following circuit to find the values of I_D and V_{DS} using the Method of Assumed State. You must validate your assumptions.

[7 + 3]

Hint: Use I_D as unknown x . Use Ohm's law to represent V_D and V_S in terms of x .



Hint Explanation

Assume $I_D = x$. For $5k\Omega$: $I_D = \frac{10 - V_D}{5} \Rightarrow V_D = 10 - 5x$.

For $3k\Omega$: $I_D = \frac{V_S - 0}{3} \Rightarrow V_S = 3x$.

Therefore, $V_{GS} = V_G - V_S = 5 - 3x$, and $V_{OV} = V_{GS} - V_T = (5 - 3x) - 1$

Also, $V_{DS} = V_D - V_S = (10 - 5x) - 3x = 10 - 8x$

Now if you assume saturation:

$$I_D = \frac{k}{2} V_{OV}^2 \Rightarrow x = \frac{2}{2} (4 - 3x)^2$$

And if you assume triode:

$$I_D = k[V_{OV}V_{DS} - \frac{1}{2}V_{DS}^2]$$

$$\Rightarrow x = 2[(4 - 3x)(10 - 8x) - 0.5 \times (10 - 8x)^2]$$

Solve for x , take the _____ root