

CSE - 331 Assignment -2:

sec. 02

submitted by,

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(a) $L_1 = \{w \in \{0,1,2\}^* : 0^n 1^n 2^n \text{ where } n \geq 0\}$ is not regular

Assume, L_1 is a regular language then it should have a pumping length p and also can be divided into three parts $s = xyz$ satisfying the following conditions,

$$|xy| \leq p$$

$$|y| > 0$$

$$xy^i z \in L_1, i \geq 0$$

now,

$$\text{Let, } s = 0^p 1^p 2^p$$

According to assume, $p = 4$

therefore,

$$w = 0^4 1^4 2^4$$

now,

$$w = \underbrace{0000}_{x \quad y} \underbrace{1111}_{z} 2222$$

$$xy^i z, i = 2$$

$\Rightarrow xy^2 z = 00000011112222$ this string violates

pumping lemma ~~there~~ fore thus L_1 is not a regular language.

Therefore,

$$(b) L_2 = \{ w \in \{0,1\}^* : 0^x 1^y 0^z \text{ where } z > x+y, x, y \geq 0 \}$$

Let's assume L_2 is a regular language. then it must have a pumping length p and can be divided in 3 slices $S = xyz$. And satisfy the following.

- * $|xy| < p$
- * $|y| > 0$
- * $xy^i z \in L_2, i \geq 0$

Let, $w = 0^p 1^{2p} 0^{p+2p+p+1}$

$$x = p$$

$$y = 2p$$

$$z = p+3p+1$$

Assuming, $p=2$

$$w = 0^2 1^4 0^7$$

$$w = \underbrace{00}_x \underbrace{1111}_y \underbrace{0000000}_z$$

$$xy^i z \quad i=2$$

$$\Rightarrow xy^2 z = 0011111110000000. \text{ here } z \neq x+y$$

therefore, L_2 is not a regular language.

(c) $L_3 = \{w \in \{01\}^* : w \text{ is a palindrome}\}$

Assuming L_3 is a regular language. It is bound to have a pumping length of p such that any string S in L_3 can be split in three different parts. $S = xyz$, fulfilling conditions.

* $|xy| \leq p$.

* $|y| > 0$

* $xy^iz \in L \quad i \geq 0$.

therefore,

choosing $w = 0^p 1^p 0^p$, which is palindrome which belongs to L_3 . Here, $|S| = 2p + p = 3p$ so S is longer than p .

According to the pumping lemma, S can also be split in xyz where $|xy| \leq p$ and $|y| > 0$. Both x and y must consist of 0's because the first p characters of S are all 0's. Considering pumping y , the string xy^iz would look like $0^{p+|y|} 1^p 0^p$ which makes the new string not a palindrome. Thus, making it an irregular language.

Therefore, L_3 is not a regular language.

(d) $L_4 = \{ w \in \{a,b\}^* \mid \text{number of } a \text{ in } w \text{ is a prime number} \}$

Assume L_4 is a regular language then it has a pumping length of p also it can be divided in three parts $S = xyz$ also it should satisfy the following

$$* |xy| < p$$

$$* |y| > 0$$

$$* xy^i z \in L, i \geq 0$$

Let, $w = a^{p+1}$ [here, $p+1 = \text{prime number}$]

$$\therefore w \in L_4$$

now,

Let,

$$p = 4$$

$$w = \underbrace{a a a a}_x \underbrace{a}_y \underbrace{a}_z$$

now,

$$(1) |xy| = 4 = p$$

$$(2) |y| = 1 > 0$$

$$(3) xy^2z = xy y z = a a a a a a$$

here length of w is 6 which is not a prime number.

$\therefore L_4$ is not a regular language.

$$(c) L_5 = \{w \in \{0,1\}^* \mid 0^{3^m} \text{ where } m \geq 0\}$$

Assume L_5 is a regular language so it must have a pumping length p and can be divided in 3 parts.

now,

$$\text{let, } w = 0^{3^p}$$

$$\text{hence } p \geq 0$$

~~now~~

$$\text{let, } p = 2$$

$$w = 0^{3^2} = 0^9 \\ = \underbrace{000}_x \underbrace{000}_y \underbrace{000}_z$$

$$(i) |xy| = 6 \leq p$$

$$(ii) |y| = 3 > 0$$

$$(iii) xy^2z = xy yz = 000 000 000 000$$

where the no. number of 0's don't follow 3^m pattern

thus, L_5 is not a regular language.