## 4.4 Differential Equations

**Undetermined coefficients – Superposition Approach**

3. $y'' - 10y' + 25y = 30x + 3$

For complementary function:

$y'' - 10y' + 25y = 0$

Let $y = e^{mx} \Rightarrow y' = me^{mx} \Rightarrow y'' = m^2 e^{mx}$

$\therefore m^2 - 10m + 25 = 0, \quad e^{mx} \neq 0$

$(m - 5)^2 = 0$

$m_1 = m_2 = 5$

$y_c = c_1 e^{5x} + c_2 x e^{5x}$

For particular solution:

$y_p = Ax + B$

$y_p' = A, \quad y_p'' = 0$

Now $y_p'' - 10y_p' + 25y_p = 30x + 3$

$0 - 10A + 25Ax + 25B = 30x + 3$

Equating coefficients of like terms

$25Ax = 30x \qquad and \qquad -10A + 25B = 3$

$25A = 30 \qquad\qquad\qquad -10\left(\frac{6}{5}\right) + 25B = 3$

$A = \frac{30}{25} = \frac{6}{5} \qquad\qquad\qquad 25B = 3 + 12$

$$B = \frac{15}{25} = \frac{3}{5}$$

$y = y_c + y_p$

$y = c_1 e^{5x} + c_2 x e^{5x} + \frac{6}{5}x + \frac{3}{5}$

5. $\frac{1}{4}y'' + y' + y = x^2 - 2x$

For complementary function:

$\frac{1}{4}y'' + y' + y = 0$

Let $y = e^{mx} \Rightarrow y' = me^{mx} \Rightarrow y'' = m^2 e^{mx}$

$\therefore \frac{1}{4}m^2 + m + 1 = 0$

$m^2 + 4m + 4 = 0$

$(m + 2)^2 = 0$

$m_1 = m_2 = -2$

$y_c = c_1 e^{-2x} + c_2 x e^{-2x}$

For particular solution:

$y_p = Ax^2 + Bx + C \Rightarrow y_p' = 2Ax + B \Rightarrow y_p'' = 2A$

$\frac{1}{4}y'' + y' + y = x^2 - 2x$

$\frac{1}{4} 2A + 2Ax + B + Ax^2 + Bx + C = x^2 - 2x$

$\frac{A}{2} + 2Ax + B + Ax^2 + Bx + C = x^2 - 2x$

Equating coefficients of like terms

| $Ax^2 = x^2$ | $2Ax + Bx = -2x$ | $\frac{A}{2} + B + C = 0$ |
|---|---|---|
| $A = 1$ | $2A + B = -2$ | $\frac{1}{2} - 4 + C = 0$ |
| | $2 + B = -2$ | $C = 4 - \frac{1}{2}$ |
| | $B = -4$ | $C = \frac{7}{2}$ |

$y_p = Ax^2 + Bx + C$

$y_p = x^2 - 4x + \frac{7}{2}$   $\therefore y = y_c + y_p = c_1 e^{-2x} + c_2 x e^{-2x} + x^2 - 4x + \frac{7}{2}$