Integral Calculus and Differential Equations

Assignment 2

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Section Number: 12

Ans no: 2

From the given integral we get the following equations

$$z = a^2 - x^2 - y^2, \quad z = 0, \quad x \geq 0, \quad y \geq 0$$

Thus, in cylindrical coordinates the limits are

$$0 \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq r \leq a, \quad 0 \leq z \leq a^2 - r^2$$

Thus,

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} \int_0^{a^2-x^2-y^2} x^2 \, dz \, dx \, dy$$

$$= \int_0^{\frac{\pi}{2}} \int_0^a \int_0^{a^2-r^2} (r^2 \cos^2\theta) r \, dz \, dr \, d\theta$$

Integrate with respect to z,

$$= \int_0^{\frac{\pi}{2}} \int_0^a r^3 \cos^2\theta \left[z\right]_0^{a^2-r^2} dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^a r^3 (a^2 - r^2) \cos^2\theta \, dr \, d\theta$$

Integrate with respect to r,

$$= \int_0^{\frac{\pi}{2}} \left[\frac{a^2 r^4}{4} - \frac{r^6}{6}\right]_0^a \cos^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{a^6}{4} - \frac{a^6}{6}\right) \cos^2\theta \, d\theta$$

Integrate with respect to θ

$$= \left(\frac{a^6}{4} - \frac{a^6}{6}\right)\left[\frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi a^6}{48}$$

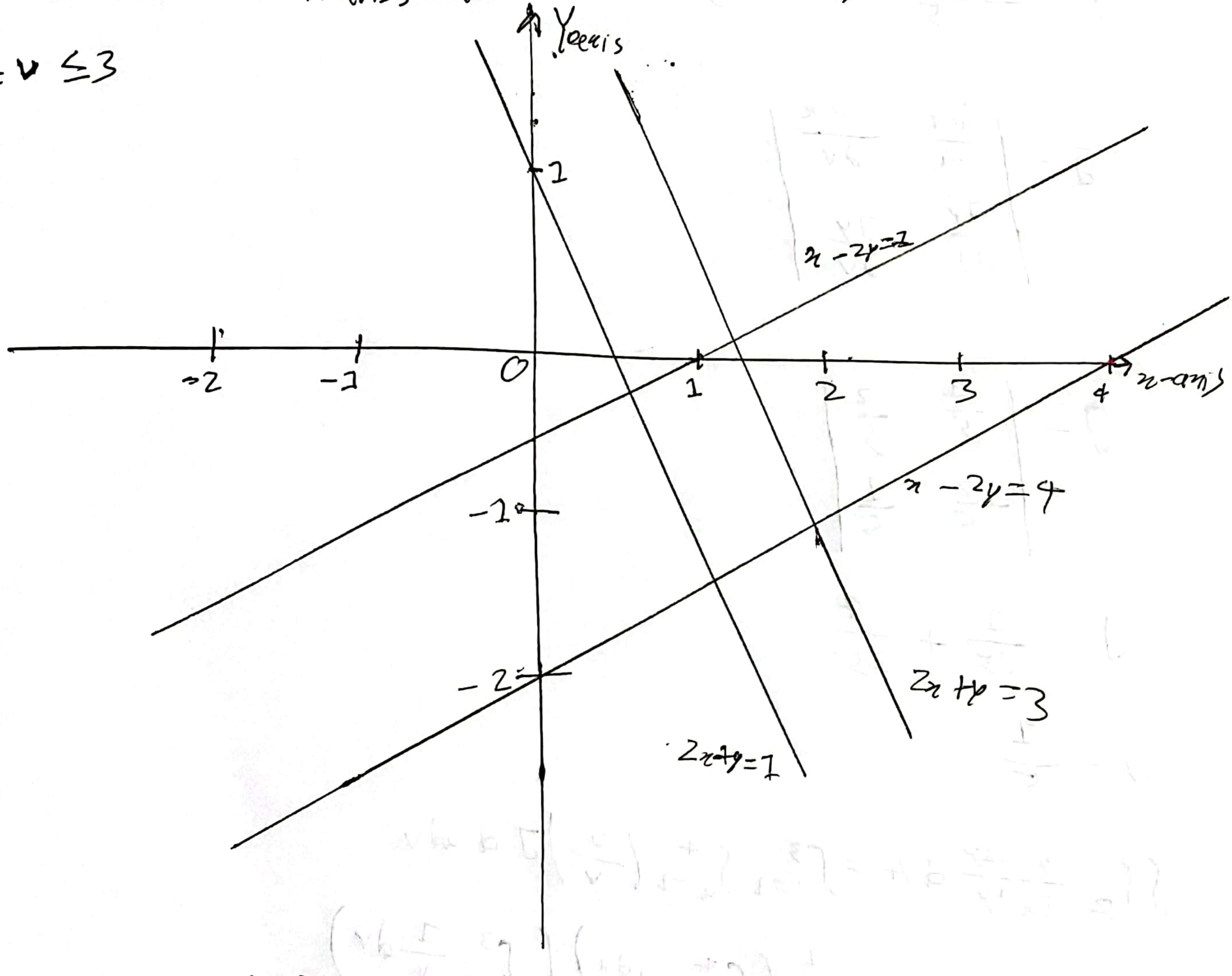
Ans no: 3

$\iint_R \frac{x-2y}{2x+y}\,dA$ where R is the rectangular region by the lines

$x-2y=1,\ x-2y=4,\ 2x+y=1,\ 2x+y=3$

If we use the transformation $u=x-2y,\ v=2x+y$ then $1\le u\le 4$,

$1\le v\le 3$



$u = x-2y$ ..... (1)

$v = 2x+y$ ..... (2)

Multiply 2 in equation (2) and add equation (2) and equation (1), we get

$u+2v = x-2y+4x+2y$

$u+2v = 5x$

$x = \frac{u+2v}{5}$ ...... (3)

and again Multiplying 2 in equation (1) and subtract equation (2) from equation (1), we get

$$2u - v = 2x - 4y - 2x - y$$

$$2u - v = -5y$$

$$y = \frac{v - 2u}{5} \quad \cdots\cdots (3)$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$J = \begin{vmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{vmatrix}$$

$$J = \frac{1}{25} + \frac{4}{25}$$

$$J = \frac{1}{5}$$

$$\iint_R \frac{x-2y}{2x+y}\, dA = \int_{v=1}^{3} \int_{u=1}^{4} \left(\frac{u}{v}\right) J \, du\, dv$$

$$\iint_R \frac{x-2y}{2x+y}\, dA = \frac{1}{5}\left(\int_{u=1}^{4} u\, du\right)\left(\int_{v=1}^{3} \frac{1}{v}\, dv\right)$$

$$= \frac{1}{5}\left[\frac{u^2}{2}\right]_{u=1}^{4} \left[\ln v\right]_{v=1}^{3}$$

$$= \frac{1}{5}\left[\frac{16}{2} - \frac{1}{2}\right]\left[\ln 3 - \ln 1\right]$$

$$= \frac{1}{5}\left[\frac{15}{2}\right]\left[\ln 3 - 0\right] = \frac{3\ln 3}{2}$$

Ans no: 1

Given,

$$3x+6y+4z=12$$

$$\Rightarrow 4z = 12-3x-6y$$

$$\therefore z = \frac{1}{4}(12-3x-6y)$$

And,

$$3x+6y=12$$

$$\therefore y=\frac{1}{6}(12-3x)$$

Now,

$$3x=12$$

$$\therefore x=4$$

$$\therefore 0 \le z \le \frac{1}{4}(12-3x-6y)$$

$$\therefore 0 \le y \le \frac{1}{6}(12-3x)$$

$$\therefore 0 \le x \le 4$$

$$\iiint_G dv$$

$$= \int_{x=0}^{4} \int_{y=0}^{\frac{1}{6}(12-3x)} \int_{z=0}^{\frac{1}{4}(12-3x-6y)} dz\,dy\,dx$$

$$= \int_0^4 \int_0^{\frac{1}{6}(12-3x)} [z]_0^{\frac{1}{4}(12-3x-6y)} dy\,dx$$

$$= \int_0^4 \int_0^{\frac{1}{6}(12-3x)} 12-3x-6y \, dy\, dx$$

$$= \int_0^4 \frac{1}{4}\left[12y - 3xy - 3y^2\right]_0^{\frac{1}{6}(12-3x)} dx$$

$$= \int_0^4 \frac{1}{4}\left[12\left\{\frac{1}{6}(12-3x)\right\} - 3x\left\{\frac{1}{6}(12-3x)\right\} - 3\left\{\left(\frac{1}{6}(12-3x)\right)^2\right\}\right] dx$$

$$= \frac{1}{4}\int_0^4 24-6x-6x+\frac{3}{2}x^2-12+6x-\frac{3}{4}x^2 \, dx$$

$$= \frac{1}{4}\int_0^4 12-6x+\frac{3}{4}x^2 \, dx$$

$$= \frac{1}{4}\left[12x-3x^2+\frac{1}{4}x^3\right]_0^4$$

$$= \frac{1}{4}\left[12\times4-3\times(4)^2+\frac{1}{4}\times(4)^3-0\right]$$

$$= \frac{1}{4}\times 16$$

$$= 4$$

The volume of the solid is 4.

(Ans)

Ans no: 4

Given,

$$(x+1)\frac{dy}{dx} + y = \ln x \; ; \quad y(1) = 10$$

Now,

$$\frac{dy}{dx} + \frac{y}{x+1} = \frac{\ln x}{x+1}$$

$$\therefore P(x) = e^{\int \frac{1}{x+1} dx}$$

$$= e^{\ln(x+1)}$$

$$= x+1$$

Integrating equation,

$$(x+1)y = \int \ln x \, dx$$

$$\Rightarrow (x+1)y = \int \ln x \cdot 1 \, dx$$

$$\Rightarrow (x+1)y = x\ln(x) - x + c$$

$$\therefore y = \frac{(x\ln|x| - x + c)}{x+1}$$

Putting $y(1) = 10$

$$\therefore y(1) = \frac{1\ln(1) - 1 + c}{1+1} = 10$$

$$\Rightarrow \frac{c-1}{2} = 10$$

$$\therefore c = 21$$

$$\therefore y = \frac{[x\ln(x) - x + 21]}{(x+1)}$$

(Ans)