MAT 120
Integral Calculus and Differential Equations
Assignment 2
Spring 2023

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Section Number: 12

1. $3x+6y+4z=12$

at $z=0$:

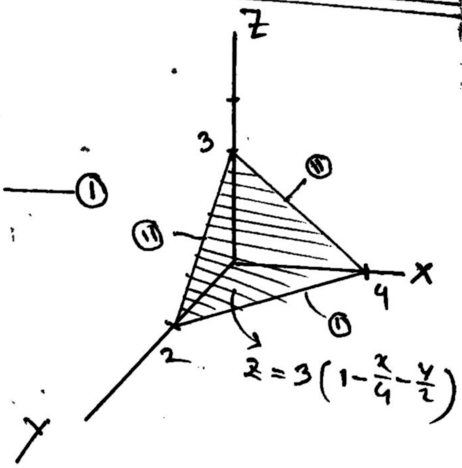
$$3x+6y=12 \Rightarrow x+2y=4 \quad \text{— (i)}$$

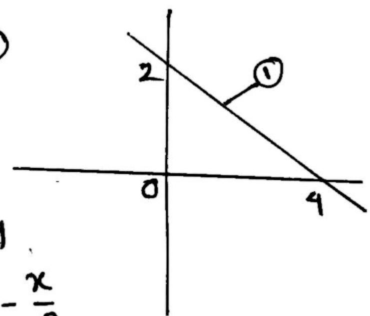
at $y=0$

$$3x+4z=12 \quad \text{— (ii)}$$

at $x=0$

$$6y+4z=12 \Rightarrow 3y+2z=6 \quad \text{— (iii)}$$





(i) $\Rightarrow x+2y=4$

$\therefore y = 2-\frac{x}{2}$

here.

$x=0,\ x=4$

$y=0,\ y=2-\frac{x}{2}$

$$\text{volume} = \int_{x=0}^{4}\int_{y=0}^{2-\frac{x}{2}}\int_{z=0}^{3(1-\frac{x}{4}-\frac{y}{2})} dz\,dy\,dx$$

$$= \int_0^4\int_0^{2-\frac{x}{2}} \left[z\right]_0^{3(1-\frac{x}{4}-\frac{y}{2})} dy\,dx$$

$$= 3\int_0^4\int_0^{2-\frac{x}{2}} \left(1-\frac{x}{4}-\frac{y}{2}\right) dy\,dx$$

$$= 3\int_0^4 \left[y-\frac{xy}{4}-\frac{y^2}{4}\right]_0^{2-\frac{x}{2}} dx$$

$$= 3\int_0^4 \left(2-\frac{x}{2}-\frac{x}{2}+\frac{x^2}{8}-\frac{1}{4}\left(4+\frac{x^2}{4}-2x\right)\right) dx$$

$$= 3\int_0^4 \left(2 - x + \frac{x^2}{8} - 1 - \frac{x^2}{16} + \frac{x}{2}\right) dx$$

$$= 3\int_0^4 \left(1 - \frac{x}{2} + \frac{x^2}{16}\right) dx$$

$$= 3\left[x - \frac{x^2}{4} + \frac{x^3}{48}\right]_0^4 dx$$

$$= 3\left(4 - \frac{16}{4} + \frac{64}{48}\right)$$

$$= 3\left(4 - 4 + \frac{64}{48}\right)$$

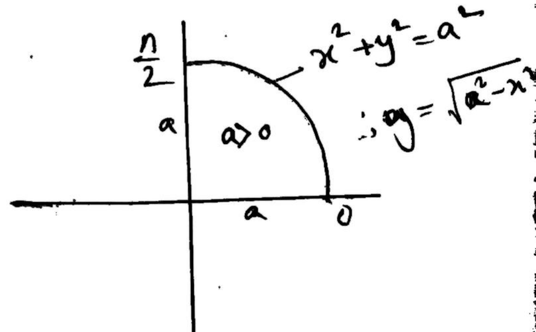
$$= \boxed{4}$$

2. $$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{a^2-x^2-y^2} x^2 \, dz\, dy\, dx \quad \text{——} (1)$$

as per cylindrical cordinates,

$x = r\sin\theta \qquad x^2 + y^2 = r^2$

$\therefore x^2 = r^2 \sin^2\theta$



$$\int_0^{\frac{\pi}{2}}\int_0^{a}\int_0^{a^2-r^2} r^2\sin^2\theta \cdot r \cdot dz\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{a} r^3\sin^2\theta\,[z]_0^{a^2-r^2}\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{a} \sin^2\theta\,(a^2r^3 - r^5)\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^2\theta \left[\frac{a^2r^4}{4} - \frac{r^6}{6}\right]_0^a d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^2\theta \left(\frac{a^6}{4} - \frac{a^6}{6}\right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^2\theta \cdot \frac{a^6}{12}\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{a^6}{12}\sin^2\theta\, d\theta = \int_0^{\frac{\pi}{2}} \frac{a^6}{12}\cdot\frac{1}{2}(1-\cos 2\theta)\, d\theta$$

$$= \frac{a^6}{24}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{a^6}{24}\left(\frac{\pi}{2} - \frac{1}{2}\cdot\sin\pi\right)$$

$$= \frac{a^6}{24}\times\frac{\pi}{2}$$

$$= \frac{a^6}{48}\pi$$

3. $x - 2y = 1$

$x - 2y = 4$

$2x + y = 1$

$2x + y = 3$

Let's consider

$x - 2y = u \qquad 1 \leqslant u \leqslant 4$

$2x + y = v \qquad 1 \leqslant v \leqslant 3$

$$J(u,v) = \begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} \frac{du}{dx} & \frac{du}{dy} \\ \frac{dv}{dx} & \frac{dv}{dy} \end{vmatrix}^{-1} = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}^{-1}$$

$$= \frac{1}{5}$$

$$\iint_R \frac{x-2y}{2x+y}\, dA = \int_1^4 \int_1^3 \frac{u}{v}\left(\frac{1}{5}\right) dv\, du$$

$$= \frac{1}{5}\int_1^4 u \left[\ln v\right]_1^3 du$$

$$= \frac{1}{5}\int_1^4 u \ln 3\, du$$

$$= \frac{1}{5}\left[\frac{u^2}{2}\right]_1^4 \ln 3$$

$$= \frac{1}{5} \cdot \frac{15}{2} \ln 3$$

$$= \frac{3}{2}\ln 3$$

$$= \ln 3^{\frac{3}{2}} = \ln 3\sqrt{3}$$

1

4. $(x+1)\frac{dy}{dx} + y = \ln x$

$$\frac{dy}{dx} + \frac{1}{x+1}y = \frac{\ln x}{x+1}$$

Integrating factor $\mu(x) = e^{\int \frac{1}{x+1} dx}$

$= e^{\ln(x+1)}$

$= x+1$

$\therefore (x+1)\frac{dy}{dx} + y = \ln x$

$\Rightarrow \int \left[(x+1)\frac{dy}{dx} + y\right] dx = \int \ln x \, dx$

$\Rightarrow (x+1)y = \int \ln x \, dx$

$\Rightarrow (x+1)y = x\ln x - \int x \cdot \frac{1}{x} dx$

$\Rightarrow (x+1)y = x\ln x - x + c$

$\therefore y = \frac{x\ln x - x + c}{x+1}$

$y(1) = 10$

$\Rightarrow \frac{\ln(1) - 1 + c}{2} = 10$

$\therefore c = 21$

$\therefore \boxed{y = \frac{x(\ln x - 1) + 21}{x+1}}$