4.3 Homogeneous Linear Equation with Constant Coefficients

5. $\boldsymbol{y'' + 8y' + 16y = 0}$

$let, y = e^{mx}, then\ y' = me^{mx}\ \ and\ y'' = m^2e^{mx}$

Substitute into original equation

$m^2e^{mx} + 8me^{mx} + 16e^{mx} = 0$

$m^2 + 8m + 16 = 0, \quad \div by\ e^{mx}$

$(m + 4)^2 = 0$

$m + 4 = 0, m + 4 = 0$

$m_1 = -4, m_2 = -4,\ \ repeating\ roots$

$y = c_1e^{-4x} + c_2xe^{-4x}$

9. $\boldsymbol{y'' + 9y = 0}$

$m^2 + 9 = 0\ ,\ while\ y = e^{mx}$ and $y'' = m^2e^{mx}$

$m^2 = -9$

$m = \pm 3i$

$m_1 = 3i,\ m_2 = -3i$

$y = e^{(0)(x)}(c_1cos3x + c_2sin3x)$

$y = c_1cos3x + c_2sin3x$

23. $\boldsymbol{y^{iv} + y''' + y'' = 0}$

$m^4 + m^3 + m^2 = 0$

$m^2(m^2 + m + 1) = 0$

$m^2 = 0; \quad m^2 + m + 1 = 0$

$m_1 = m_2 = 0 \ \ (repeating\ roots); \ m = \frac{-1 \pm \sqrt{1-4}}{2} => m = \frac{-1 \pm i\sqrt{3}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

$y = c_1 + xc_2 + e^{-\frac{1x}{2}}(c_3 cos\frac{\sqrt{3}}{2}x + c_4 sin\frac{\sqrt{3}}{2}x)$

35. Solve the initial value problem:

$\boldsymbol{y''' + 12y'' + 36y' = 0;}$ $\boldsymbol{y(0) = 0, y'(0) = 1, y''(0) = -7}$

Let $y = e^{mx}$

Then $m^3 + 12m^2 + 36m = 0$

$m(m^2 + 12m + 36) = 0$

$m(m + 6)^2 = 0$

$m_1 = 0, m_2 = m_3 = -6$

$y = c_1 e^{(0)x} + c_2 e^{-6x} + c_3 x e^{-6x}$

$y = c_1 + c_2 e^{-6x} + c_3 x e^{-6x}$ ----------(a)

Substitute $y(0) = 0\ into\ (a)$

$c_1 + c_2 = 0$ ----------(1)

Differentiate EQUATION (a)

$y' = 0 - 6c_2 e^{-6x} + c_3[x(-6e^{-6x}) + e^{-6x}]$

$y\ ' = -6c_2 e^{-6x} - 6c_3 x e^{-6x} + c_3 e^{-6x}$ ----------(b)

Substitute $y'(0) = 1\ into\ (b)$

$-6c_2 + c_3 = 1$ ----------(2)

Differentiate EQUATION (b)

$y'' = 0 - 6c_2(-6e^{-6x}) - 6c_3[x(-6e^{-6x}) + e^{-6x}(1)] + c_3(-6e^{-6x})$

$y'' = 36c_2e^{-6x} + 36c_3xe^{-6x} - 6c_3e^{-6x} - 6c_3e^{-6x}$

$y'' = 36c_2e^{-6x} + 36c_3xe^{-6x} - 12c_3e^{-6x}$ ----------(c)

Substitute $y''(0) = -7\ into\ (c)$

$36c_2 - 12c_3 = -7$ ----------(3)

$(3) + (2) \times 6 => -6c_3 = -1 => c_3 = \frac{1}{6}$

Substitute $c_3 = \frac{1}{6}\ into\ (2)$

$-6c_2 + \frac{1}{6} = 1$

$c_2 = -\frac{5}{36}$

Substitute $c_2 = -\frac{5}{36}$ into (1)

$c_1 - \frac{5}{36} = 0 => c_1 = \frac{5}{36}$

Substitute $c_1, c_2, and\ c_3\ into\ (a)$

$y = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{1}{6}xe^{-6x}$