Welcome to the Physics class

Principles of Physics II

Lesson 17

The force is perpendicular to both the field and the length.

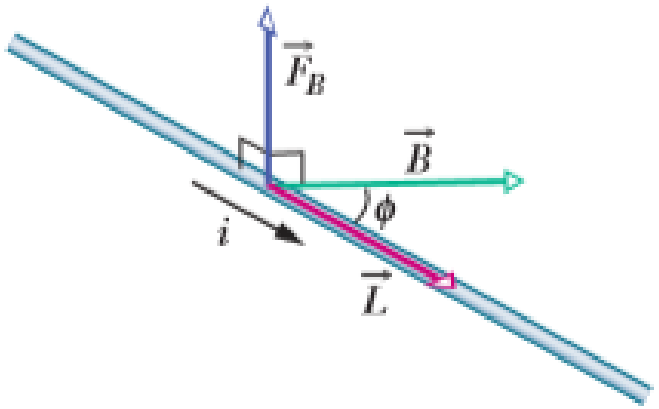


Fig. 28-16 A wire carrying current i makes an angle ϕ with magnetic field $\vec{B}$. The wire has length L in the field and length vector $\vec{L}$ (in the direction of the current). A magnetic force $\vec{F}_B = i\vec{L} \times \vec{B}$ acts on the wire.

If B is not $\perp$r to the wire then

$$|F_B| = iLB\sin\phi$$

where ϕ is the angle bet^n L and B. Vectorially we can write

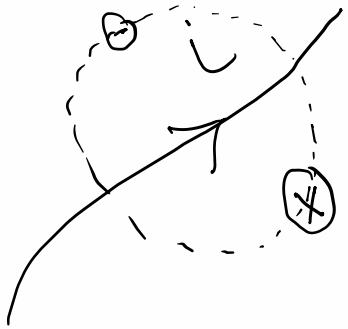
$$\vec{F}_B = i\vec{L} \times \vec{B}$$ considering

L as a vector in the direction of i (the conventional direction of current).

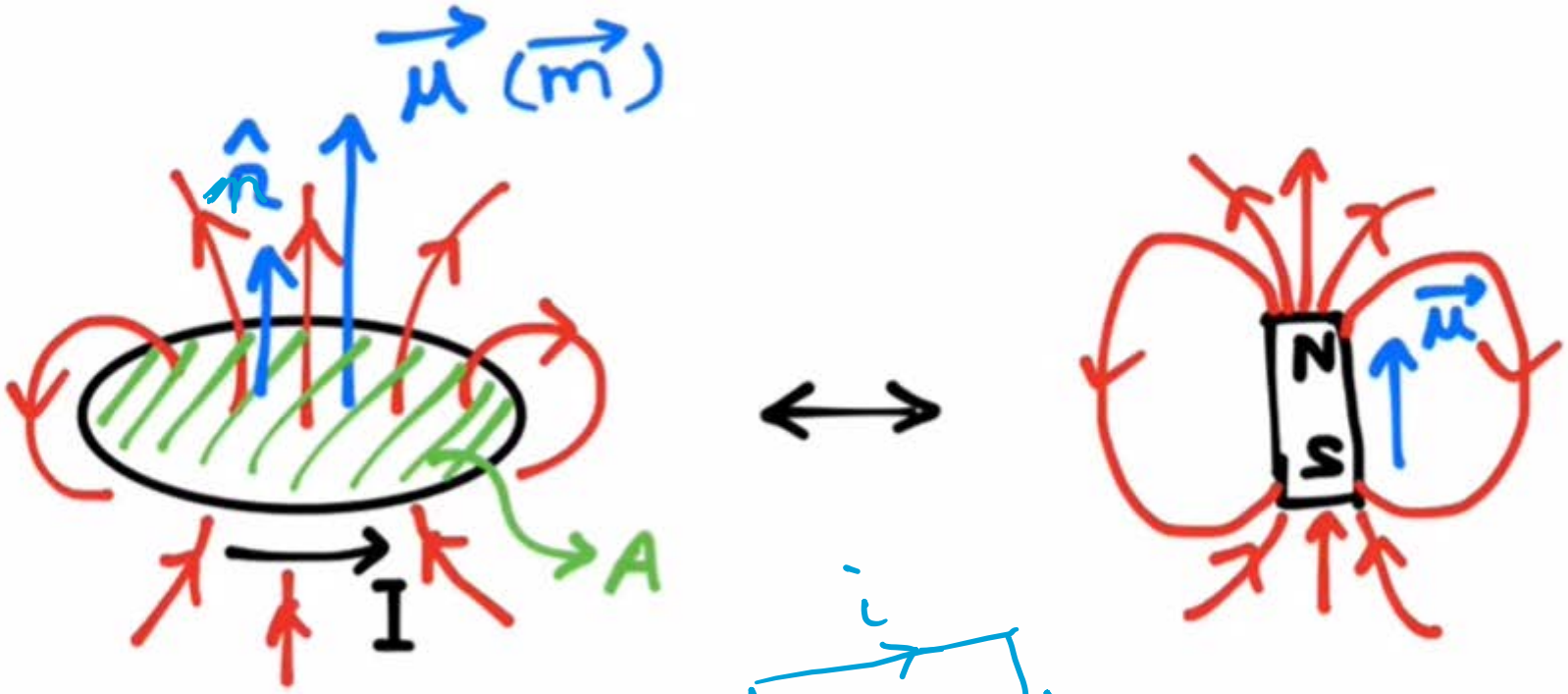
L16V3

Magnetic Dipole Moment

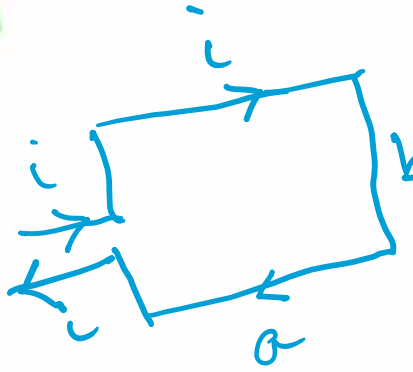
Magnetic effect of a current



THE MAGNETIC DIPOLE MOMENT



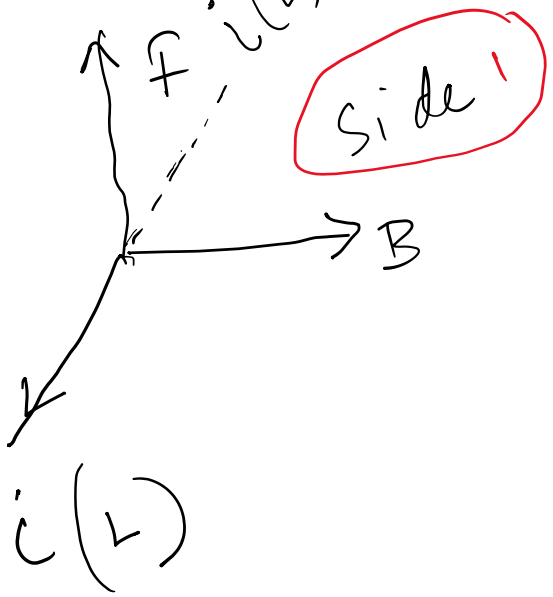
$$\vec{\mu} \equiv I A \hat{n}$$



$$\vec{\mu} = I A \hat{n}$$
$$= I a b \hat{n}$$

TORQUE ON A CURRENT LOOP

$i\vec{L} \times \vec{B} = F_B$ *



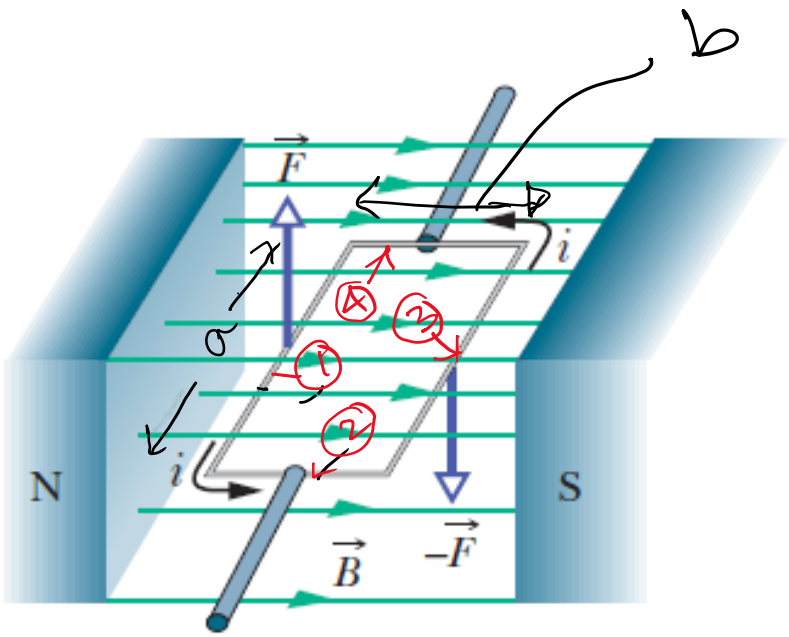
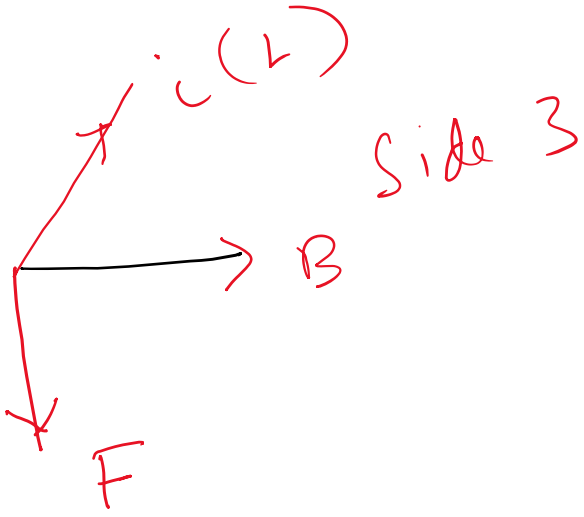


Figure 28-18 The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.



* For Side 1 & 3

$i\vec{L} \times \vec{B} = F_1 \; (F_3)$

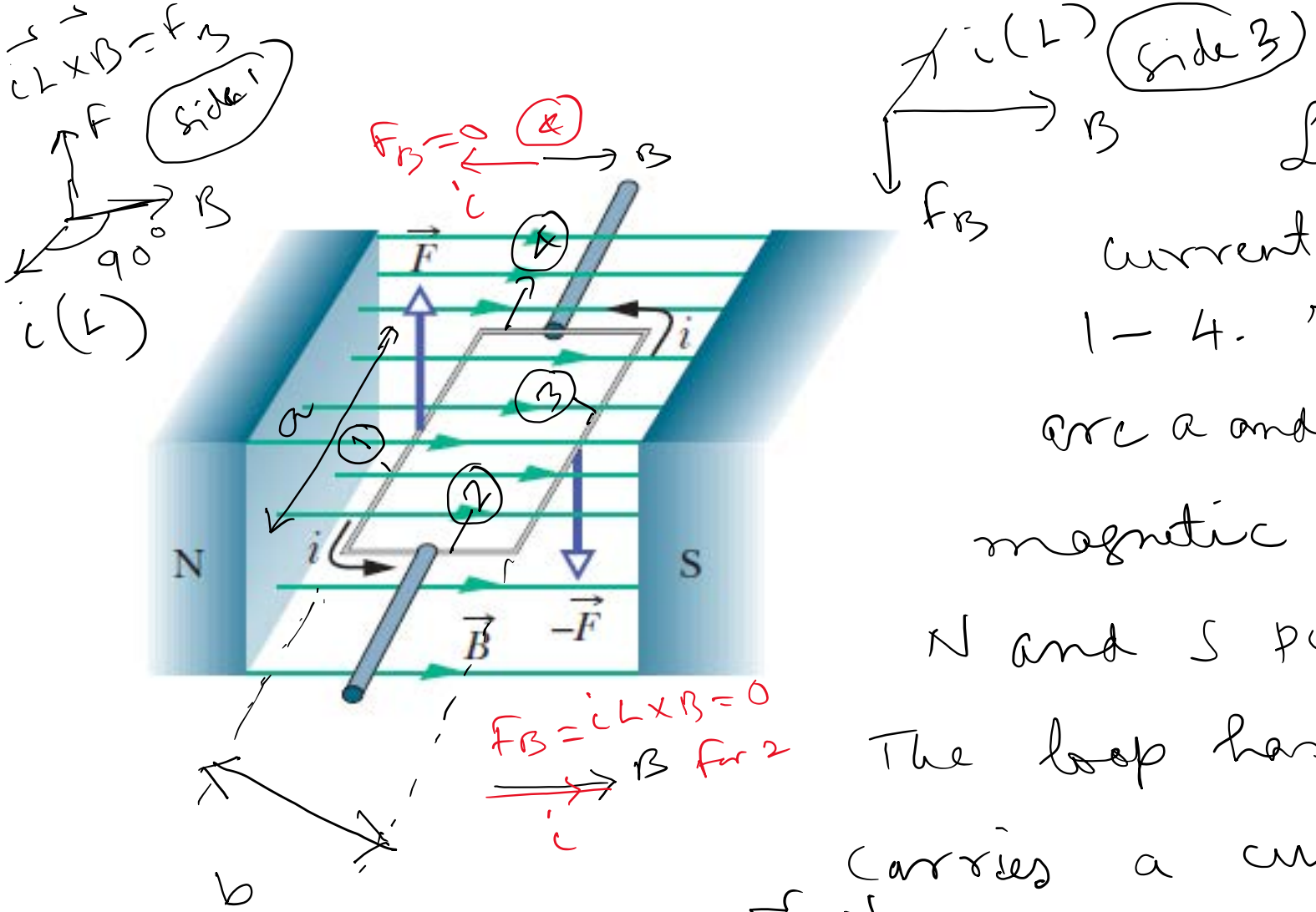
$iaB = F_1 \; (F_3)$

L & B are ⊥r

For Side 2 and 4

F = 0 because the angle betn L (i) and B are either zero or 180°

TORQUE ON A CURRENT LOOP



Let us consider a current loop with sides labelled 1 – 4. The loop length and breadth are a and b and it is placed in the magnetic field B produced by the N and S pole pieces of a magnet. The loop has a single turn and carries a current i. Applying

$$F_B = i\vec{L}\times\vec{B}$$

on four sides of the coil (loop)

for side 1 $\quad F_1 = iaB$ in the upward direction

For side (3) $F_3 = iaB$ in the downward direction.

For (2) and (4) the forces are zero because i and B are in the same or in the opposite direction ($\phi = 0^\circ$ or 180°)

F_1 and F_3 are equal and opposite and are separated by a distance. Hence they will constitute a torque to rotate the loop around the axle. Let the torque be τ'.

$\vec{\tau} = \vec{r} \times \vec{F}$

$$\tau' = \frac{b}{2} \times F_1 + \frac{b}{2} \times F_3 = \frac{aiBb}{2} + \frac{aiBb}{2} = iabB$$

$$\tau' = iAB \qquad \boxed{\vec{\tau}' = i\vec{A} \times \vec{B}}$$

A = Area of the coil.

Now if the normal to the plane of the coil ($\vec{A} = A\hat{n}$) makes an angle θ with B then

$$\tau' = iAB\sin\theta.$$

If there are N turns in the coil (loop) the

total torque $\tau = N\tau' = NAiB\sin\theta$

$$\vec{\tau} = Ni\,\vec{A}\times\vec{B}$$

$\tau = Ni\vec{A}\times\vec{B}$

$\vec{\tau} = \vec{p}\times\vec{E}$

for a single turn

$$\vec{\tau}' = i\,\vec{A}\times\vec{B}$$

$i\vec{A}$ or $Ni\vec{A}$ are magnetic dipole moments of a (single turn or N turn) coil in the direction of the normal to the plane of the coil.

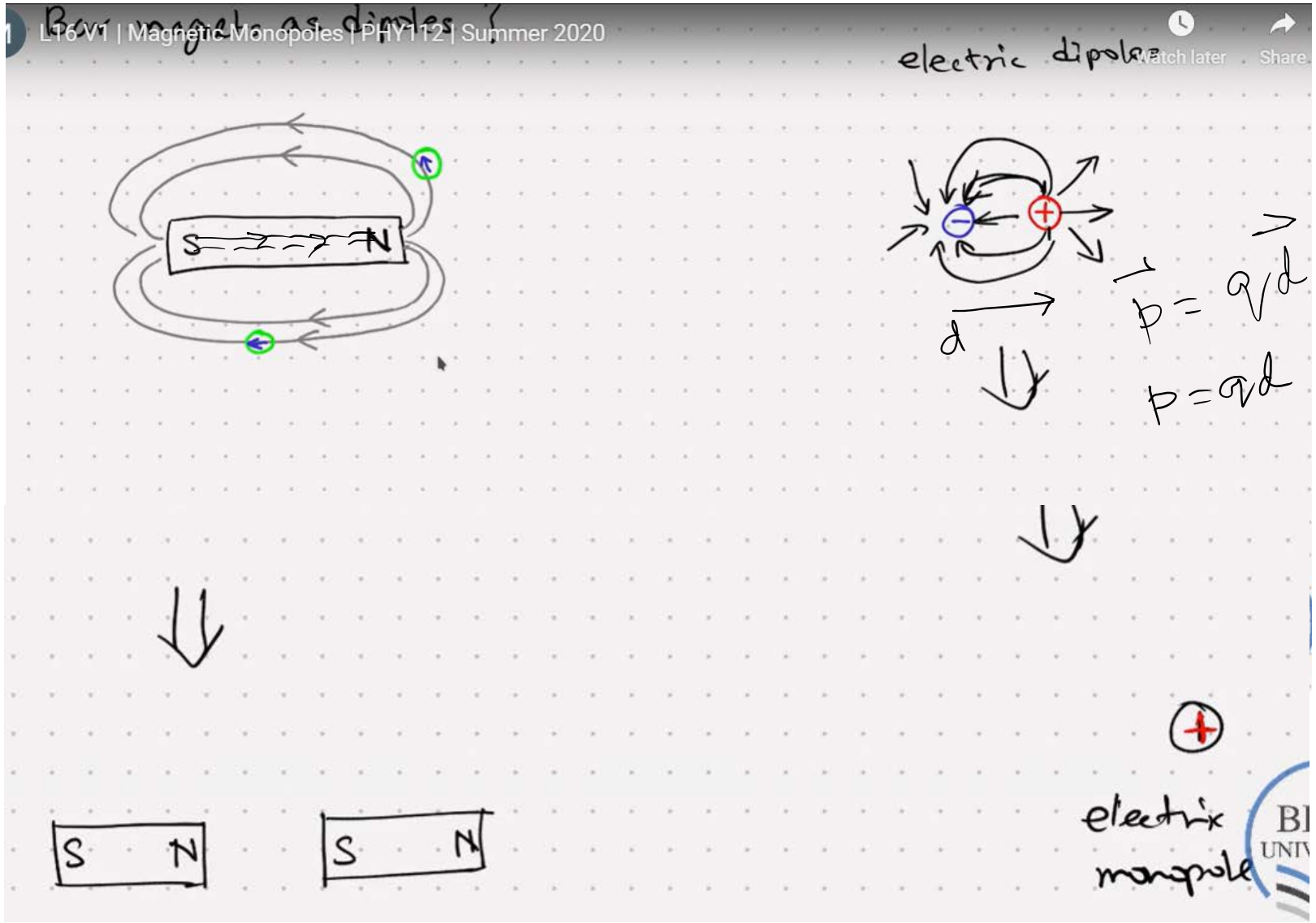
$$\vec{\mu} = i\,\vec{A}\ (Ni\vec{A})$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

Compare with the expression of torque of an electric dipole $\vec{p}$ in an Electric field E

Magnetic Monopoles do not exist

electric Monopoles exist

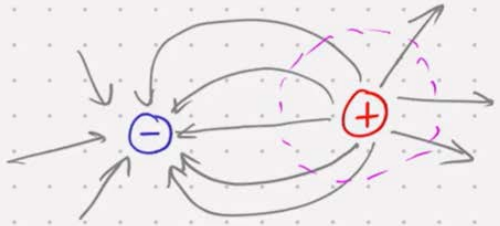


L16 V2 | First two Maxwell's Equations | PHY112 | Summer 2020

First Two Maxwell's Equations

1. Gauss's Law : electric monopoles are allowed

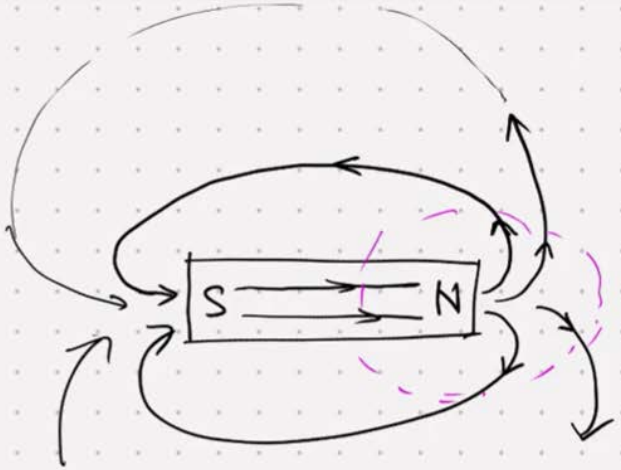
$$\oiint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}$$



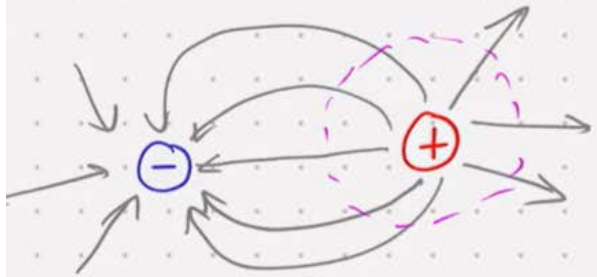
$q_{enc} = +q$

2. Magnetic Gauss's Law : No magnetic monopoles

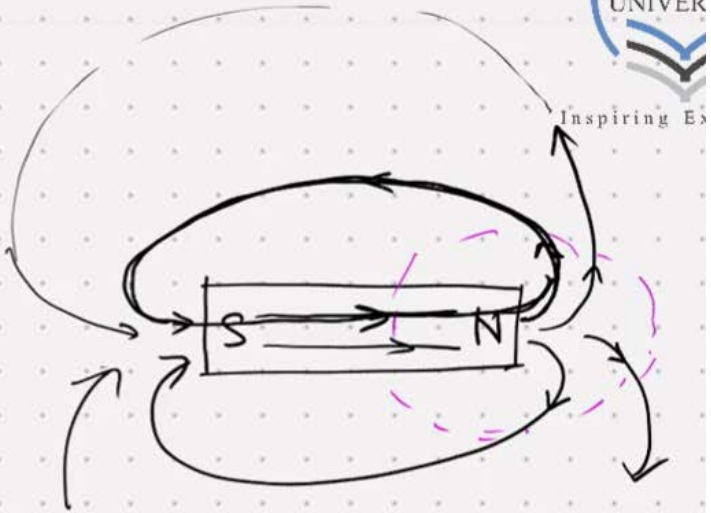
$$\oiint \vec{B}\cdot d\vec{a} = 0$$



magnetic flux



electric dipoles



magnetic dipoles

Lorentz Force law

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Magnetic Monopoles does not exist and we know this as experimental fact.

Magnetic field lines always form closed loop. **Where electric field lines does not**

Magnetic Gauss's law tells us that magnetic monopoles does not exist.

REVIEW & SUMMARY

Magnetic Field $\vec{B}$ A **magnetic field** $\vec{B}$ is defined in terms of the force $\vec{F}_B$ acting on a test particle with charge q moving through the field with velocity $\vec{v}$:

$$\vec{F}_B = q\vec{v} \times \vec{B}. \tag{28-2}$$

The SI unit for $\vec{B}$ is the **tesla** (T): $1\ \text{T} = 1\ \text{N/(A} \cdot \text{m)} = 10^4$ gauss.

The Hall Effect When a conducting strip carrying a current i is placed in a uniform magnetic field $\vec{B}$, some charge carriers (with charge e) build up on one side of the conductor, creating a potential difference V across the strip. The polarities of the sides indicate the sign of the charge carriers.

A Charged Particle Circulating in a Magnetic Field A charged particle with mass m and charge magnitude $|q|$ moving with velocity $\vec{v}$ perpendicular to a uniform magnetic field $\vec{B}$ will travel in a circle. Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r}, \tag{28-15}$$

from which we find the radius r of the circle to be

$$r = \frac{mv}{|q|B}. \tag{28-16}$$

The frequency of revolution f, the angular frequency ω, and the period of the motion T are given by

$$f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{|q|B}{2\pi m}. \tag{28-19, 28-18, 28-17}$$

Magnetic Force on a Current-Carrying Wire A straight wire carrying a current i in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}. \tag{28-26}$$

The force acting on a current element $i\,d\vec{L}$ in a magnetic field is

$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}. \tag{28-28}$$

The direction of the length vector $\vec{L}$ or $d\vec{L}$ is that of the current i.

Torque on a Current-Carrying Coil A coil (of area A and N turns, carrying current i) in a uniform magnetic field $\vec{B}$ will experience a torque $\vec{\tau}$ given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}. \tag{28-37}$$

Here $\vec{\mu}$ is the **magnetic dipole moment** of the coil, with magnitude $\mu = NiA$ and direction given by the right-hand rule.

Orientation Energy of a Magnetic Dipole The orientaion energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}. \tag{28-38}$$

If an external agent rotates a magnetic dipole from an initial orientation θ_i to some other orientation θ_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i. \tag{28-39}$$

In the last class we have calculated the torque $\vec{\tau}$ on a current carrying coil in a magnetic field B as

$$\vec{\tau} = N i \vec{A} \times \vec{B}$$

(vectorially)

OR $\vec{\tau} = \vec{\mu} \times \vec{B}$ → True for any magnetic moment in a magnetic field

where $\vec{\mu} = N i \vec{A}$ is the magnetic dipole moment of a current carrying coil of N turns and area A. This expression $\vec{\tau} = \vec{\mu} \times \vec{B}$ is very similar to the expression of the torque $\vec{\tau}$ on an electric dipole moment $\vec{p}$ in an electric field E

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$\vec{p} \rightarrow \vec{\mu}$$

$$\vec{E} \rightarrow \vec{B}$$

In case of an Electric dipole moment p in an electric field E, the torque it experiences is $\vec{\tau} = \vec{p} \times \vec{E}$

and its P.E is

$$P.E = -\vec{p} \cdot \vec{E}$$

We learned these in our earlier lectures. We are going to do the same for μ and B. Remember

$p \rightarrow \mu$
$E \rightarrow B$

The corresp. magnetic counterparts.

Similarly, in analogy with electric dipole and electric field we can write for the magnetic potential as

$$U(\theta) = -\vec{\mu} \cdot \vec{B}$$

$$= -\mu B \cos\theta$$

(1) For $\theta = 0$ i.e when the magnetic dipole is aligned with the magnetic field $U(\theta) = -\mu B \cos\theta = -\mu B \cos 0 = -\mu B$ is the minimum p.E situation.

(2) when the dipole is anti-aligned with the magnetic field $U(\theta) = -\mu B \cos\theta = -\mu B \cos(180^\circ) = \mu B$ maximum potential energy configuration.

(3) For $\theta = 90$ $U(\theta) = ?$

MAGNETIC FIELDS DUE TO CURRENTS

$i \rightarrow$ current

$ds \Rightarrow$ element of length of the conductor

$\vec{r}$ is the radius vector of the point P from the line element ds,

with $\hat{r}$, the unit vector in the direction of $\vec{r}$.

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

This element of current creates a magnetic field at P, into the page.

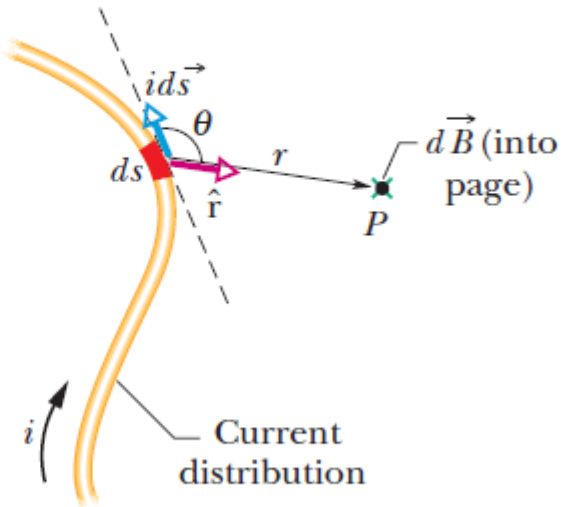


Fig. 29-1 A current-length element $i\,d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point P. The green × (the tail of an arrow) at the dot for point P indicates that $d\vec{B}$ is directed *into* the page there.

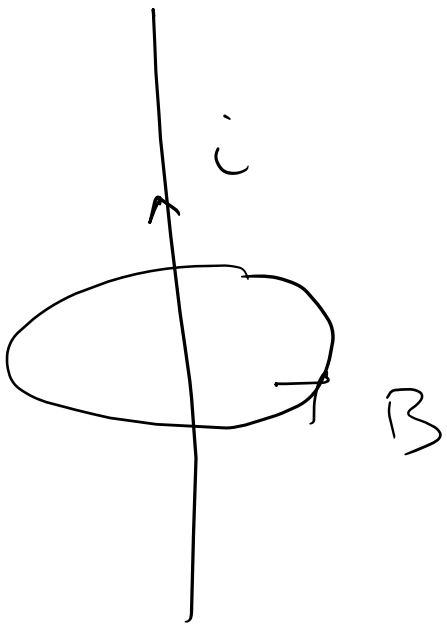
$\mu_0 = 4\pi \times 10^{-7}$ T.m/A permeability constant of free space.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2} \qquad \text{(Biot–Savart law).}$$

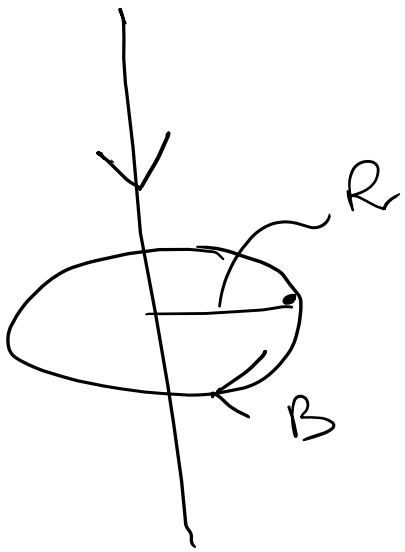
at P, B is into the plane of the board.

i.e in the direction of $i d\vec{s} \times \hat{r}$

Magnetic field due to a current in a long (ideally infinitely long) straight wire.



CCW.



CW

$$B = \frac{\mu_0 i}{2\pi R}$$

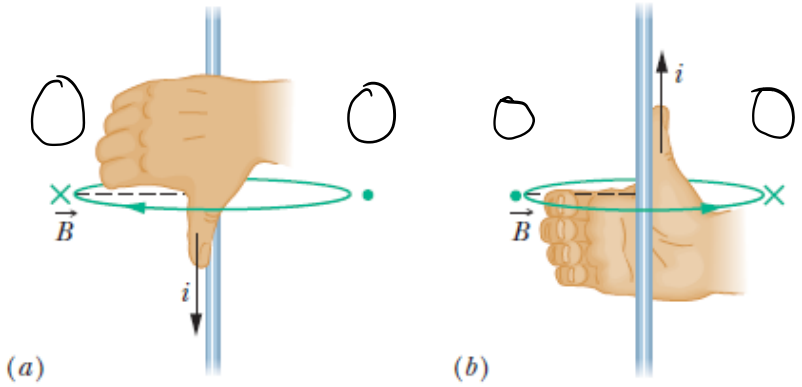


Fig. 29-4 A right-hand rule gives the direction of the magnetic field due to a current in a wire. (*a*) The situation of Fig. 29-2, seen from the side. The magnetic field $\vec{B}$ at any point to the left of the wire is perpendicular to the dashed radial line and directed into the page, in the direction of the fingertips, as indicated by the ×. (*b*) If the current is reversed, $\vec{B}$ at any point to the left is still perpendicular to the dashed radial line but now is directed out of the page, by the dot.

The thumb is in the current's direction. The fingers reveal the field vector's direction, which is tangent to a circle.

The magnetic field vector at any point is tangent to a circle.

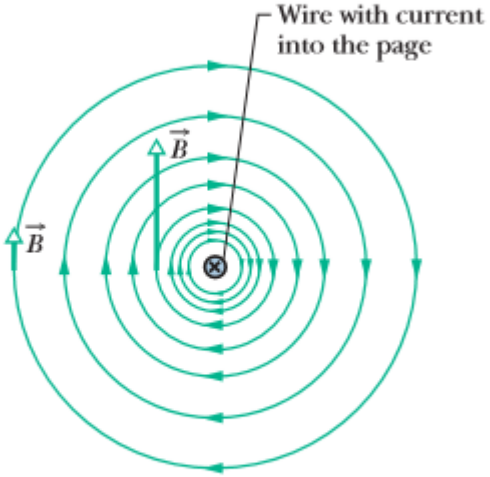


Fig. 29-2 The magnetic field lines produced by a current in a long straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the ×.

Here is a simple right-hand rule for finding the direction of the magnetic field set up by a current-length element, such as a section of a long wire:

Right-hand rule: Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

Field at P for a conducting wire carrying a current i at a ⊥r distance R due to the current-element $i\,d\vec{s}$ located at a distance of $\vec{r}$ from P is given by B-S law as

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

The direction of dB is that of the vector $d\vec{s}\times\vec{r}$ directed into the page

$$B = \int_{-\alpha}^{\alpha} dB = 2\int_0^{\alpha} dB = \cancel{2}\cdot\frac{\mu_0 i}{\cancel{4}_2\pi}\int_0^{\alpha}\frac{\sin\theta\,ds}{r^2}$$

$r = \sqrt{s^2+R^2}$ and

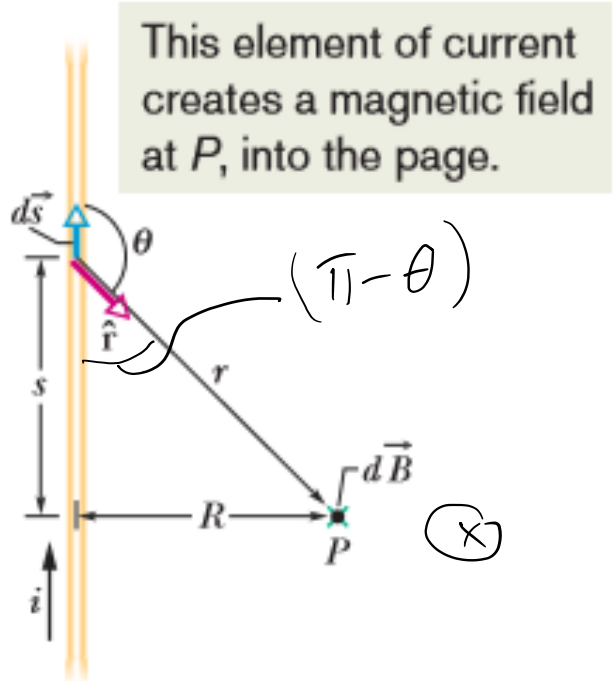


Fig. 29-5 Calculating the magnetic field produced by a current i in a long straight wire. The field $d\vec{B}$ at P associated with the current-length element $i\,d\vec{s}$ is directed into the page, as shown.

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{r} = \frac{R}{\sqrt{s^2+R^2}}$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^{\alpha} \frac{R}{\sqrt{s^2+R^2}}\,\frac{ds}{s^2+R^2} = \frac{\mu_0 i}{2\pi}\int_0^{\alpha}\frac{R\,ds}{(s^2+R^2)^{3/2}}$$

Standard integral $\displaystyle\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$

$x \to s$

$a \to R$

$$\int_0^{\alpha}\frac{R\,ds}{(s^2+R^2)^{3/2}} = \left[\frac{R\,s}{R^2(s^2+R^2)^{1/2}}\right]_{s=0}^{s=\alpha} =$$

$$= \frac{1}{R}\left[\frac{\cancel{s}\,1}{\cancel{s}\left(\frac{s^2}{s^2}+\frac{R^2}{s^2}\right)^{1/2}}\right]_0^{\alpha}$$

$$= \frac{1}{R}\left[\frac{1}{(1+0)} - \frac{1}{\left(1+\frac{R^2}{0}\right)}\right] = \frac{1}{R}$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi} \frac{1}{R}$$

$$\therefore \quad \boxed{B = \frac{\mu_0 I}{2\pi R}}$$

The magnetic field due an infinitely long straight current carrying conductor at a $\perp r$ distance R from the wire is given by the above equation.

and the direction is given by $d\vec{s} \times \vec{r}$

(or the right hand rule)