Welcome to the Physics class

Principles of Physics II

Lecture 4

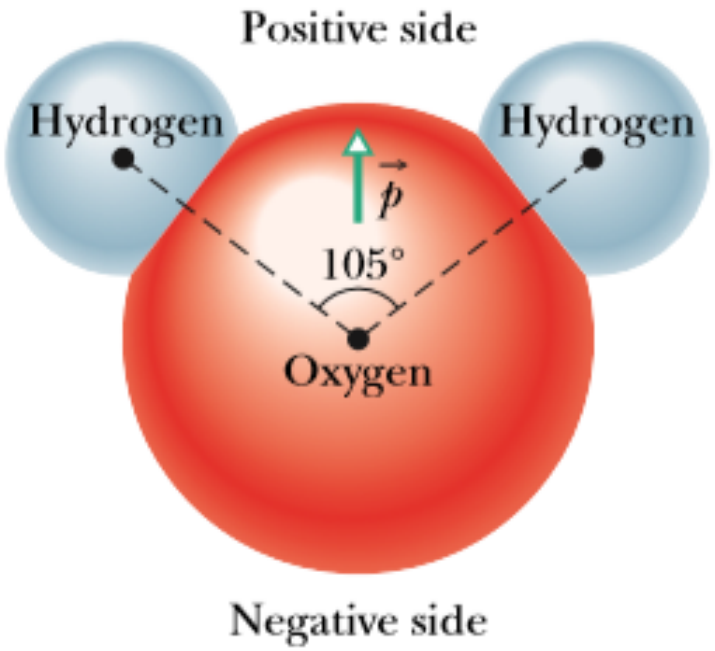


Fig. 22-18 A molecule of H_2O, showing the three nuclei (represented by dots) and the regions in which the electrons can be located. The electric dipole moment $\vec{p}$ points from the (negative) oxygen side to the (positive) hydrogen side of the molecule.

Where **p** = qd, the dipole moment of the dipole

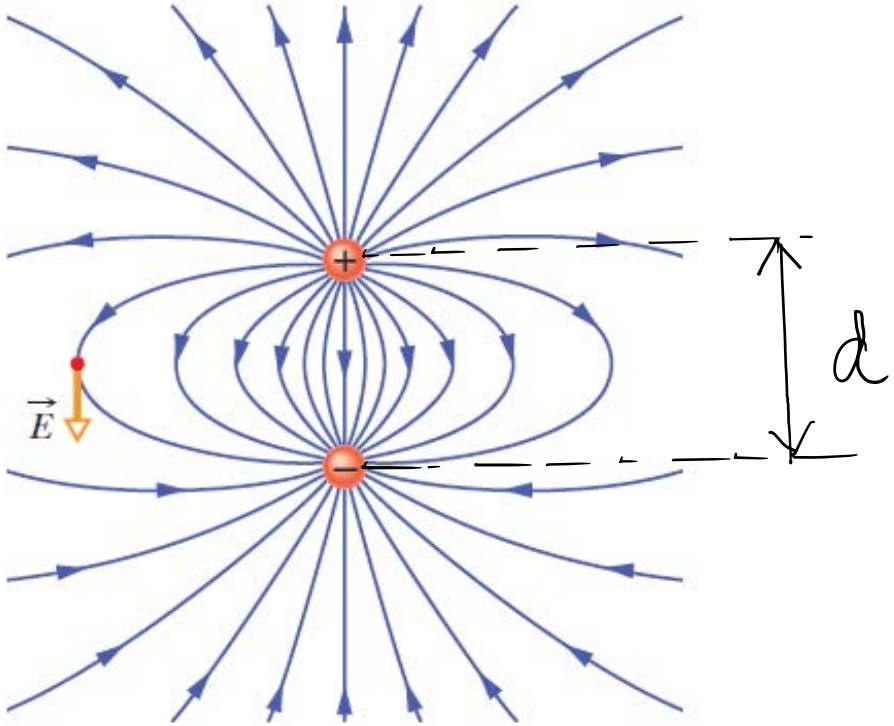


Figure 22-8 The pattern of electric field lines around an electric dipole, with an electric field vector $\vec{E}$ shown at one point (tangent to the field line through that point).

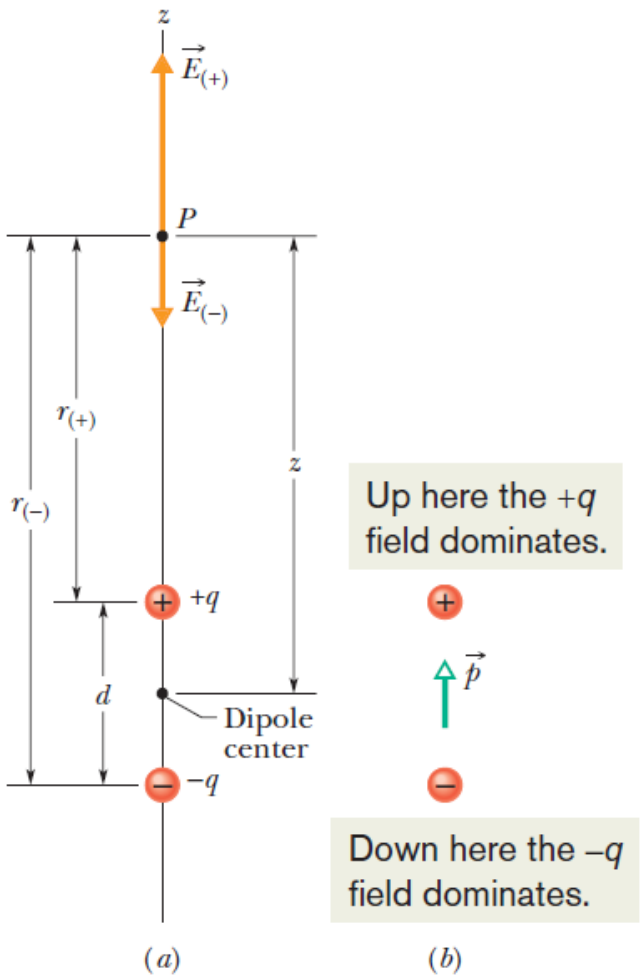


Figure 22-9 (*a*) An electric dipole. The electric field vectors $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at point P on the dipole axis result from the dipole's two charges. Point P is at distances $r_{(+)}$ and $r_{(-)}$ from the individual charges that make up the dipole. (*b*) The dipole moment $\vec{p}$ of the dipole points from the negative charge to the positive charge.

The electric field at point P

$$E = E_{(+)} + E_{(-)} = \frac{1}{4\pi\epsilon_0}\frac{q}{r_+^2} \quad \frac{1}{4\pi\epsilon_0}\frac{(-q)}{r_-^2}$$

or $$E = \frac{q}{4\pi\epsilon_0\left(z-\frac{d}{2}\right)^2} - \frac{q}{4\pi\epsilon_0\left(z+\frac{d}{2}\right)^2}$$

$$E = \frac{q}{4\pi\epsilon_0 z^2}\left[\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0 z^2}\,\frac{\left(1+d/2z\right)^2 - \left(1-d/2z\right)^2}{\left(1-\frac{d}{2z}\right)^2\left(1+\frac{d}{2z}\right)^2}$$

$$E = \frac{q}{\cancel{4}\,2\,\pi\,\epsilon_0\, z^2}\;\frac{\cancel{2}\cdot\cancel{2}\cdot 1\;\; d/2z}{\left(1^2 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\,\epsilon_0\, z^3}\;\frac{d}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}$$

$$E = \frac{p}{2\pi\,\epsilon_0\, z^3}\;\frac{1}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2} \qquad p = qd$$

At large distance

$$z \gg d \Rightarrow \frac{d}{2z} \ll 1$$

$$E = \frac{1}{2\pi\epsilon_0}\;\frac{p}{z^3}$$

E falls off as $\frac{1}{z^3}$ for dipole

E falls off as $\frac{1}{r^2}$ for pt charge (monopole)

THE ELECTRIC FIELD DUE TO A LINE OF CHARGE

discrete point charges to continuous distribution of charges.

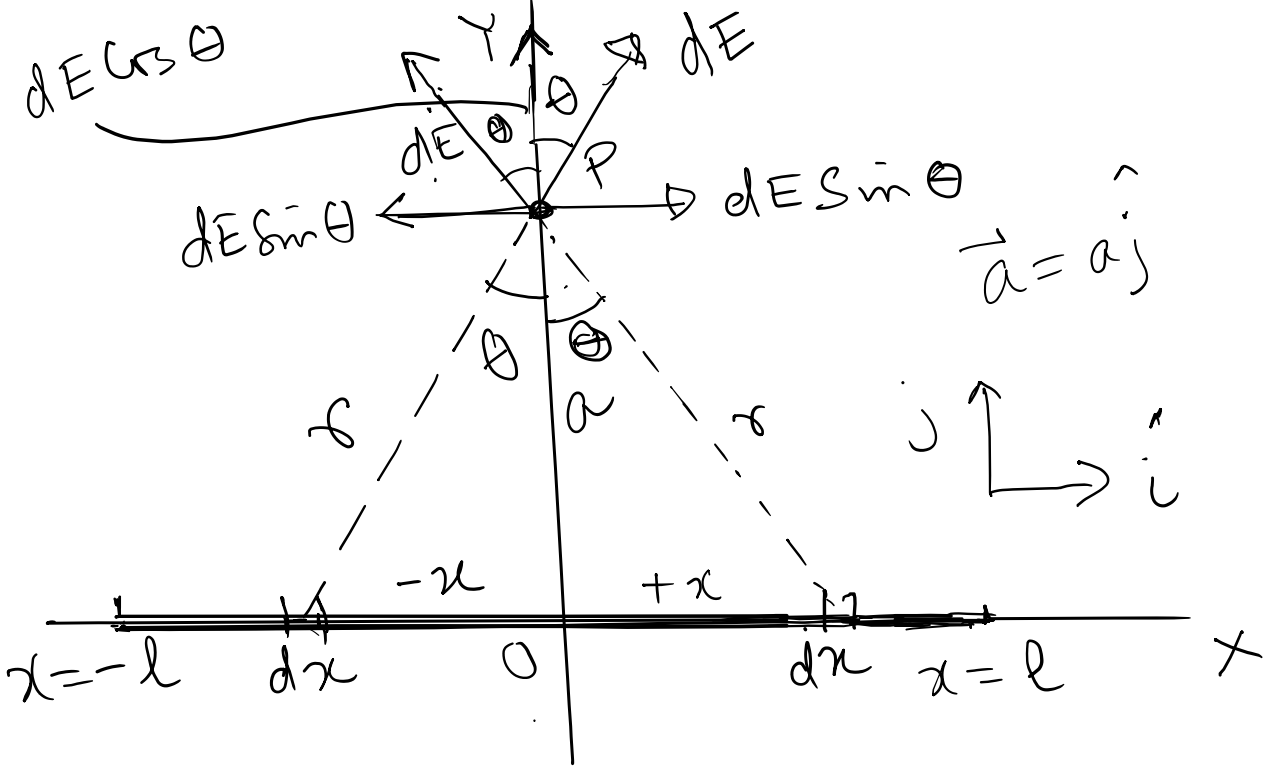
Superposition of fields.

$$\sum \Rightarrow \int$$

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

Charge elements infinitesimally small so that Coulombs law for point charges can be used.

Let us consider a straight line of charges Q uniformly distributed along a straight line of length "2l" along the x-axis. Let us calculate the electric field at a ⊥r distance "a" from the mid point of the uniform linear charge distribution.



$\vec{a} = a\hat{j}$

2-D coordinate system chosen as shown.

$$\lambda = \frac{Q}{2l}$$

dQ = charge element in line element dx at equal distance of x on either side of the origin.

$$= \lambda\, dx$$

dE at P due to dQ for each line element

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}$$

The x-components of the field cancel out. The y-component of dE has magnitude $dE\cos\theta$. Now $\cos\theta = \frac{a}{r}$

$$dE\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dx}{r^2}\frac{a}{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dx}{x^2+a^2}\frac{a}{(x^2+a^2)^{1/2}}$$

or $$dE\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dx\,a}{(x^2+a^2)^{3/2}}$$

$$\therefore \quad E = \int dE\cos\theta = \frac{\lambda a}{4\pi\epsilon_0}\int_{-l}^{l}\frac{dx}{(x^2+a^2)^{3/2}}$$

$$\left[\int\frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}\right]$$ Using the standard integral (see HRW appendix.)

$$E = \frac{\lambda a}{4\pi\epsilon_0}\left[\frac{x}{a^2\sqrt{a^2+x^2}}\right]_{-l}^{l} = \frac{\lambda a}{4\pi\epsilon_0}\left[\frac{2l}{a^2\sqrt{a^2+l^2}}\right]$$

$$E = \frac{\lambda(2l)\cancel{a}}{4\pi\epsilon_0} \frac{1}{a^{\cancel{2}}\sqrt{a^2+l^2}} = \frac{Q}{4\pi\epsilon_0} \frac{1}{a\sqrt{a^2+l^2}}$$

The electric field is in the y-direction, so in vector form $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{a\sqrt{a^2+l^2}} \hat{j}$

for a very short line segment of charge (or if P is at a very large distance) $a \gg l$

$$\therefore \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{a^2} \hat{j}$$

And for a long segment $l \gg a$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{a l \sqrt{\frac{a^2}{l^2}+1}} \quad \text{as } l >> a \quad \frac{a^2}{l^2} << 1$$

$$\therefore \vec{E} = \frac{1}{2\pi\epsilon_0} \frac{Q}{(2l)} \frac{1}{a} \hat{j} = \frac{\lambda}{2\pi\epsilon_0 a} \hat{j}$$

This is the electric field due to an infinitely long line of charge. At any point P at a $\perp r$ distance r from an infinite line of charge in any direction

E has the magnitude $E = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r}$

and vectorially $\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0} \frac{\hat{r}}{r}}$

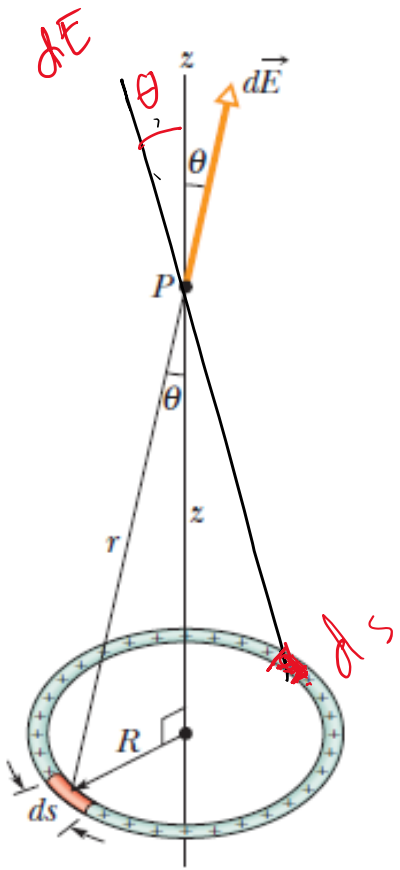


Figure 22-11 A ring of uniform positive charge. A differential element of charge occupies a length ds (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point P.

Ring of charge of radius R

E at a point P at a distance of z along the symmetry axis of the ring.

ds is the (arc) length of differential element

$$dq = \lambda ds$$

dE due to this element of charge dq

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{(z^2+R^2)}$$

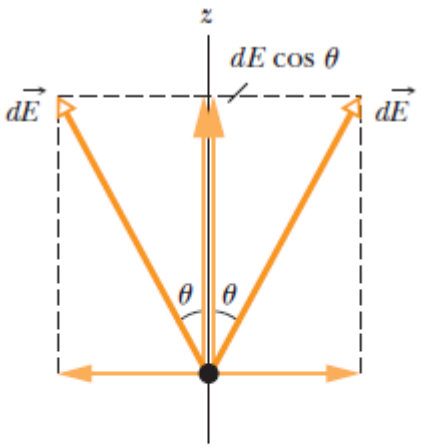


Figure 22-12 The electric fields set up at P by a charge element and its symmetric partner (on the opposite side of the ring). The components perpendicular to the z axis cancel; the parallel components add.

The horizontal component of dE (⊥r to the axis) cancel out with identical differential element diametrically opposite of ds.

The vertical component of dE has magnitude $dE\cos\theta$ $\qquad \cos\theta = \dfrac{z}{r}$

$$\cos\theta = \frac{z}{\sqrt{z^2+R^2}}$$

$$\therefore \quad dE\cos\theta = \frac{1}{4\pi\epsilon_0}\,\frac{\lambda\, ds}{z^2+R^2}\,\frac{z}{(z^2+R^2)^{1/2}} = \frac{z\lambda\, ds}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

$$\therefore \quad E = \int dE\cos\theta = \frac{z\lambda}{4\pi\epsilon_0 (z^2+R^2)^{3/2}} \int_0^{2\pi R} ds$$

$$\text{or} \quad E = \frac{z\lambda}{4\pi\epsilon_0 (z^2+R^2)^{3/2}} \Big[s \Big]_0^{2\pi R} = \frac{z\lambda (2\pi R)}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

Total charge q on the ring $= \lambda(2\pi R)$

$$\therefore \quad E = \frac{qz}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

away from the ring along the axis (z).

at the center of the ring $z = 0$ and $E = 0$ (why)

For negatively charged ring the electric field vector points towards the center of the ring.

At $z >> R$

$$E = \frac{qz}{4\pi\epsilon_0 z^3 \left(1 + \frac{R^2}{z^2}\right)^{3/2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2}$$

as if all the charges are concentrated at the center of the ring.

Figure 22-13*a* shows a plastic rod with a uniform charge $-Q$. It is bent in a 120° circular arc of radius r and symmetrically paced across an x axis with the origin at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?

KEY IDEA

Because the rod has a continuous charge distribution, we must find an expression for the electric fields due to differential elements of the rod and then sum those fields via calculus.

An element: Consider a differential element having arc length ds and located at an angle θ above the x axis (Figs. 22-11*b* and *c*). If we let λ represent the linear charge density of the rod, our element ds has a differential charge of magnitude

$$dq = \lambda\, ds. \tag{22-18}$$

The element's field: Our element produces a differential electric field $d\vec{E}$ at point P, which is a distance r from the element. Treating the element as a point charge, we can rewrite Eq. 22-3 to express the magnitude of $d\vec{E}$ as

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}. \tag{22-19}$$

The direction of $d\vec{E}$ is toward ds because charge dq is negative.

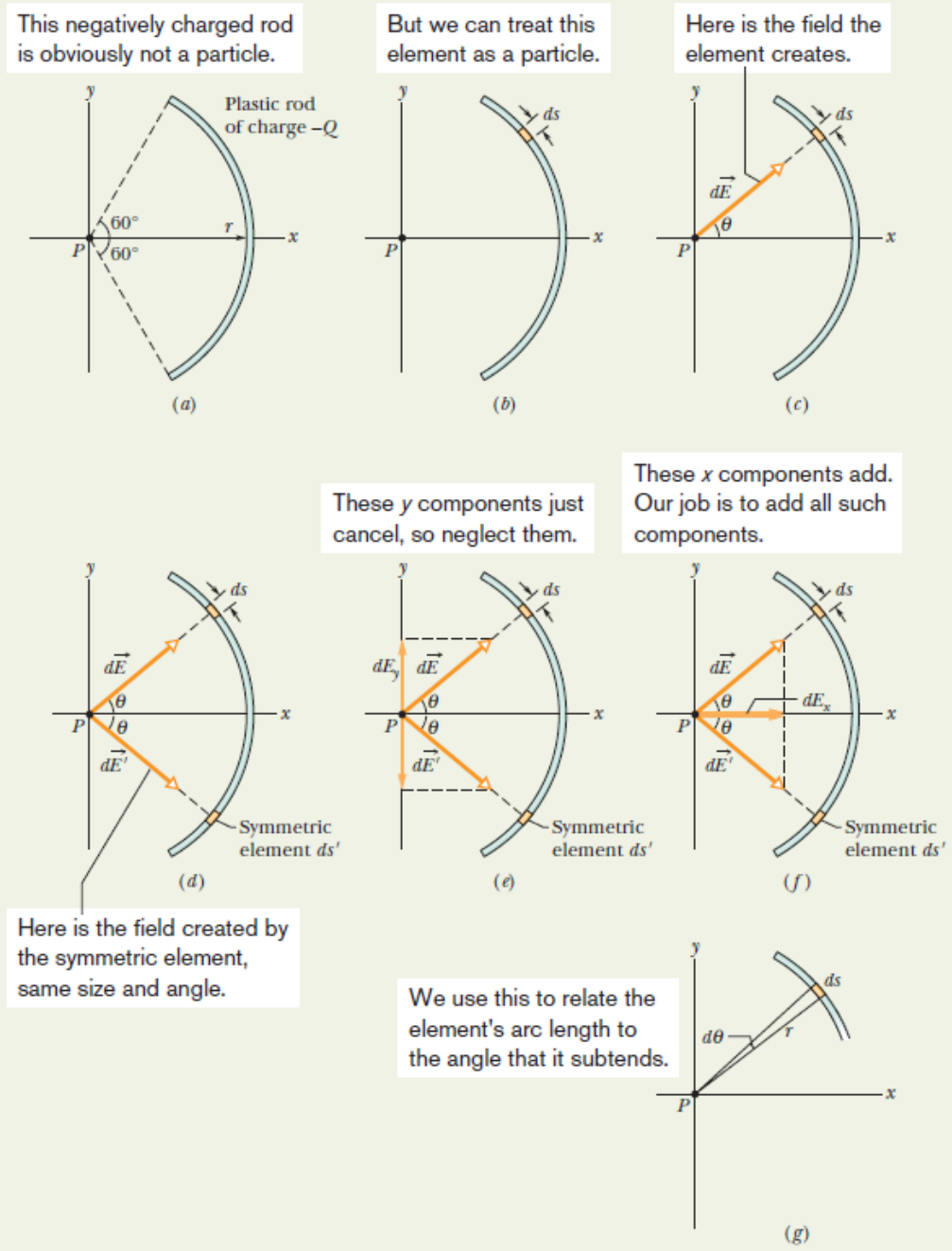


Figure 22-13 (*a*) A plastic rod of charge $-Q$ is a circular section of radius r and central angle 120°;

$$dE_x = dE\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, ds}{r^2}\cos\theta \qquad \text{Now } \underline{ds = r\,d\theta}$$

dE_y cancels out by the symmetric components of the line of charge (see figures d, e, f). The horizontal components dE_x adds up.

$$\therefore \; E \text{ at the center of the arc} = \int_{-60}^{60} \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\, d\theta$$

$$\text{or } E = \frac{\lambda}{4\pi\epsilon_0 r}\int_{-60}^{60}\cos\theta\, d\theta = \frac{\lambda}{4\pi\epsilon_0 r}\Big[\sin\theta\Big]_{-60}^{60}$$

$$= \frac{\lambda}{4\pi\epsilon_0 r}\left[\sin 60^\circ - \sin(-60^\circ)\right] = \frac{\lambda\sqrt{3}}{4\pi\epsilon_0 r} = \frac{1.73\lambda}{4\pi\epsilon_0 r}$$

The rod subtends an angle of 120°, so one third of a full circle. It's arc length is $\frac{2\pi r}{3}$

$$\lambda = \frac{\text{Charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477\,Q}{r}$$

$$\therefore \quad E = \frac{(1.73)(0.477\,Q)}{4\pi\epsilon_0 r^2} = \frac{0.83\,Q}{4\pi\epsilon_0 r^2}$$

$$\vec{E} = \frac{0.83\,Q}{4\pi\epsilon_0 r^2}\,\hat{i}$$

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \qquad \text{(charged ring).}$$

THE ELECTRIC FIELD DUE TO A CHARGED DISK

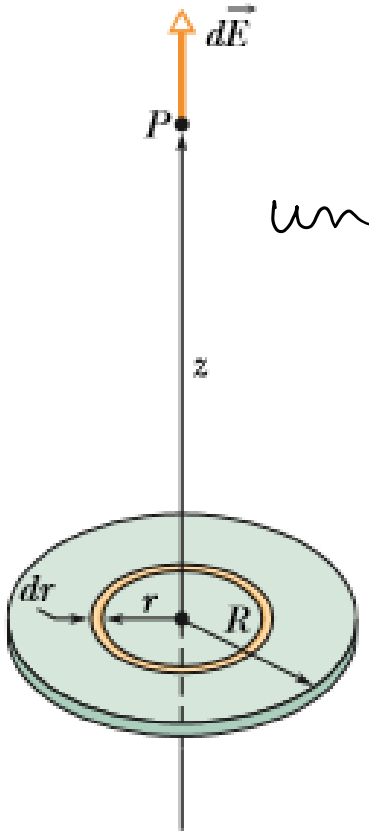


Fig. 22-13 A disk of radius R and uniform positive charge. The ring shown has radius r and radial width dr. It sets up a differential electric field $d\vec{E}$ at point P on its central axis.

Plastic disc of radius R with uniform surface charge density σ/m^2.

$\vec{E}$ at pt P along the symmetry axis of the disc: Consider an annular ring of radius r and width dr. The charge enclosed in this ring of infinitesimal width dr is

$$dq = \sigma\, dA = \sigma(2\pi r\, dr)$$

Using the formula for circular ring of charge

$$E = \frac{qz}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}} \quad \text{(charged ring).}$$

$$dE = \frac{z\, dq}{4\pi\epsilon_0 (z^2+r^2)^{3/2}}$$

$q \to dq$ & $R \to r$

$$= \frac{z(\sigma 2\pi r\, dr)}{4\pi\epsilon_0 (z^2+r^2)^{3/2}} = \frac{\sigma z}{4\epsilon_0} \frac{2r\, dr}{(z^2+r^2)^{3/2}}$$

$$E = \int dE = \frac{\sigma z}{4\epsilon_0} \int_0^R (z^2+r^2)^{-3/2} (2r)\, dr$$

$$I = \int (z^2+r^2)^{-3/2} 2r\, dr$$

Let $x = z^2 + r^2$

$dx = 2r\, dr$

$$= \int x^{-3/2}\, dx = \frac{x^{-3/2+1}}{-3/2+1}$$

$$\int x^m = \frac{x^{m+1}}{m+1}$$

$$I = \int x^{-3/2}\,dx = \frac{x^{-1/2}}{-1/2}$$

Substitute back x and dx

$$I = \int \left(z^2+r^2\right)^{-3/2} 2r\,dr = \frac{\left(z^2+r^2\right)^{-1/2}}{-1/2}$$

So, $$\int_0^R \left(z^2+r^2\right)^{-3/2}(2r)\,dr = \left[\frac{\left(z^2+r^2\right)^{-1/2}}{-1/2}\right]_0^R$$

$$= \left[\frac{\left(z^2+R^2\right)^{-1/2}}{-1/2} + \frac{z^{-1}}{1/2}\right]$$

$$\therefore\ E = \frac{\sigma z}{\cancel{4}_{2}\epsilon_0}\left[\frac{\left(z^2+r^2\right)^{-1/2}}{1}(-\cancel{2}) + \frac{z^{-1}}{1}\cancel{2}\right]$$

$$E = \frac{\sigma z}{2\epsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right] = \frac{\sigma}{2\epsilon_0}\left[\frac{z}{z} - \frac{z}{\sqrt{z^2+R^2}}\right]$$

or $$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$ charged disc of radius R

for $R \to \infty$

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{z\sqrt{1+R^2/z^2}}\right] = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{1+\left(\frac{R^2}{z^2}\right)}}\right]$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$ infinite sheet of charge of uniform surface density σ

Lecture 5 on PHY 112

Prof. DR. A.F.M. Yusuf Haider

A point charge in an electric field

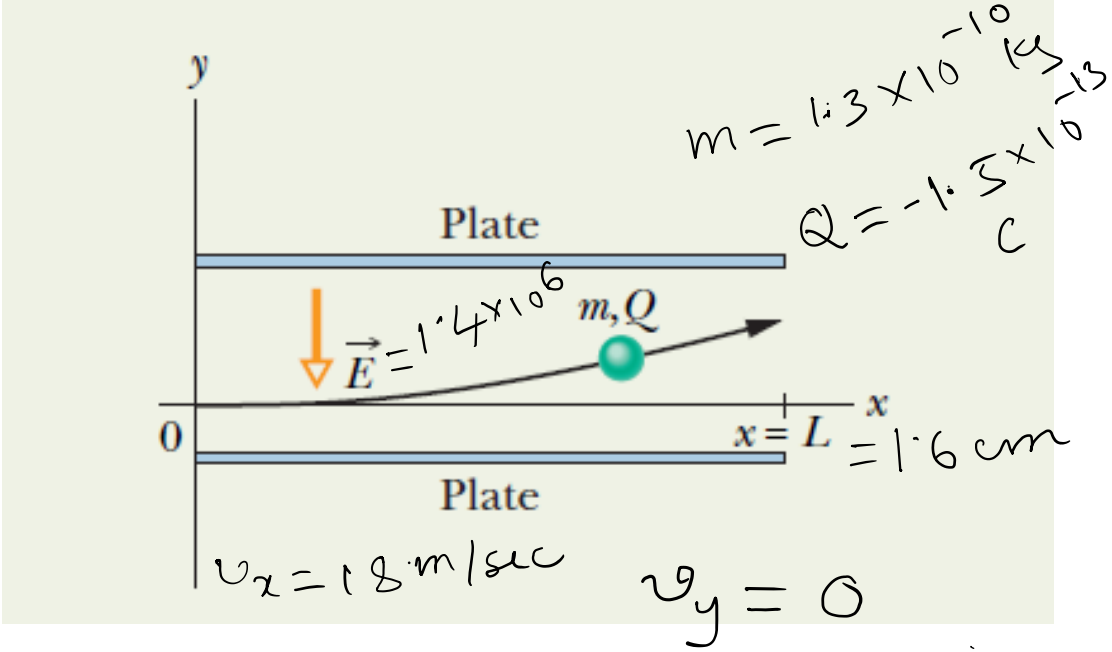
It experiences a force F given by

$$\vec{F} = q\vec{E}$$

q_s

$$\vec{F} = q\vec{E}$$

Figure shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x = 18$ m/s. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)



There is a force on the ink drop in the vertical direction $F_y = ma_y$

Now $F_y = QE$ $\therefore$ $a_y = \dfrac{F_y}{m} = \dfrac{QE}{m}$

The transit time of the ink drop through the plates is say "t" sec

During this time the vertical and horizontal displacements of the ink drop are

$$y = v_y t + \frac{1}{2} a_y t^2 \quad \text{and} \quad x(L) = v_x t + \frac{1}{2} a_x t^2$$

$$a_x = 0$$

$$y = \frac{1}{2} a_y t^2 \qquad \therefore \quad t = \frac{L}{v_x}$$

Substitute a_y and t in y

$$y = \frac{1}{2} \frac{QE}{m} \left(\frac{L}{v_x}\right)^2 = \frac{(1.5 \times 10^{-13}\,C)(1.4 \times 10^{6})(1.6 \times 10^{-2})^2}{2(1.3 \times 10^{-10})(18)^2}$$

$$= 6.4 \times 10^{-4}\,m$$

$$= 0.64\,mm$$

R

L

L

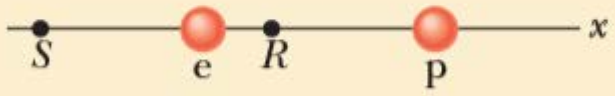
R

e

p

Checkpoint 1

The figure here shows a proton p and an electron e on an x axis. What is the direction of the electric field due to the electron at (a) point S and (b) point R? What is the direction of the net electric field at (c) point R and (d) point S?



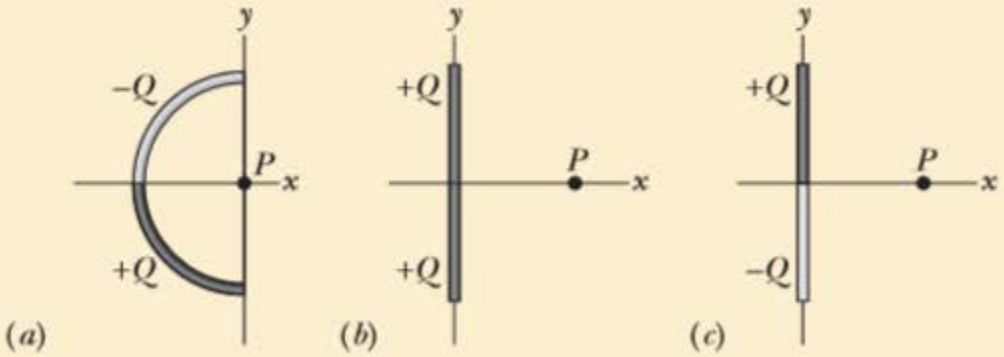
+y

+x

-y

Checkpoint 2

The figure here shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude Q along its top half and another along its bottom half. For each rod, what is the direction of the net electric field at point P?



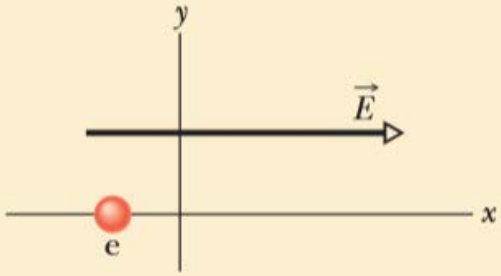
L

L

Decr

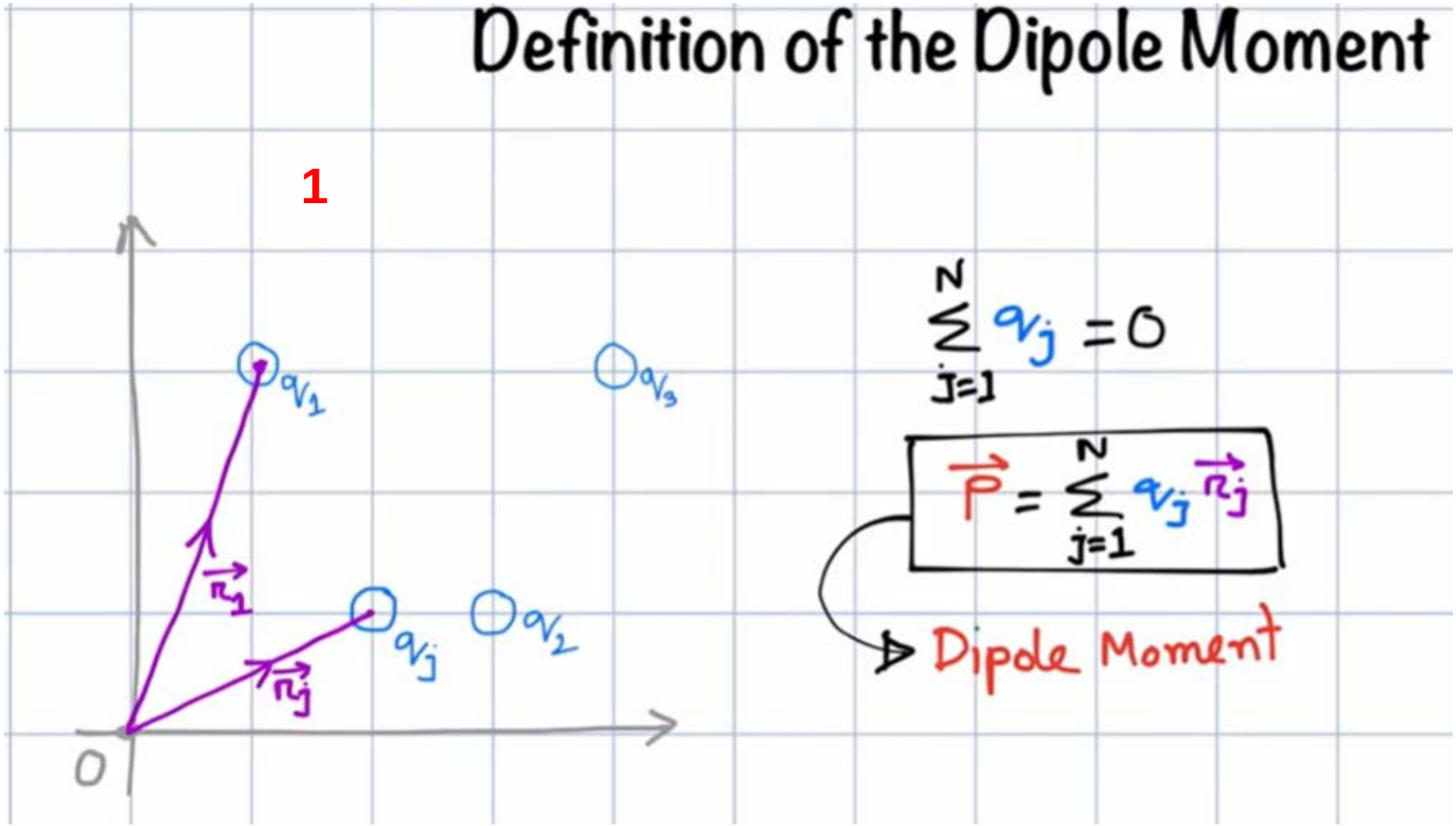
Checkpoint 3

(a) In the figure, what is the direction of the electrostatic force on the electron due to the external electric field shown? (b) In which direction will the electron accelerate if it is moving parallel to the y axis before it encounters the external field? (c) If, instead, the electron is initially moving rightward, will its speed increase, decrease, or remain constant?



Try yourself at home.

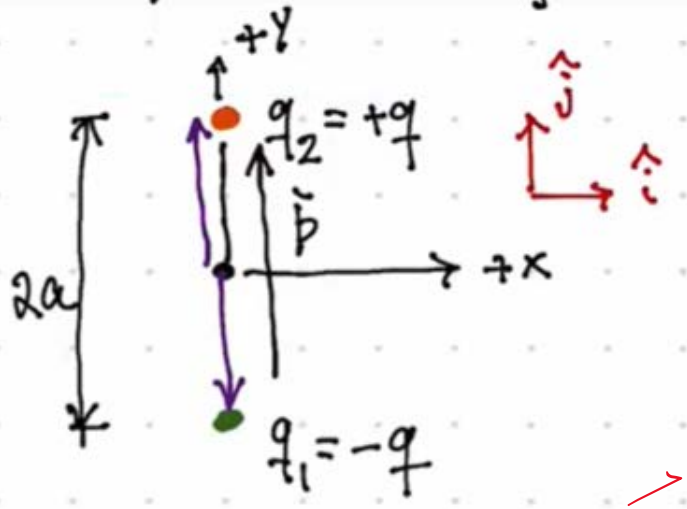
Definition of the Dipole Moment



Electric Dipole Moment of Two Charges

Electric Dipole Moment of 2 charges

1



$\vec{r}_1 = a(-\hat{j})$

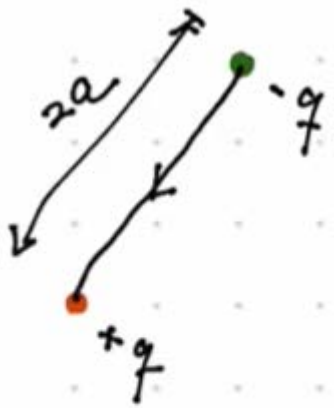
$\vec{r}_2 = a\hat{j}$

$$\vec{p} = q_1\vec{r}_1 + q_2\vec{r}_2$$

$$= (-q)\,a(-\hat{j}) + q\,a\hat{j}$$

$$= 2qa\hat{j}$$

2



$$|\vec{p}| = 2qa = qd \qquad d = 2a$$

In general

$$\sum_{i=1}^{n} q_i = 0$$

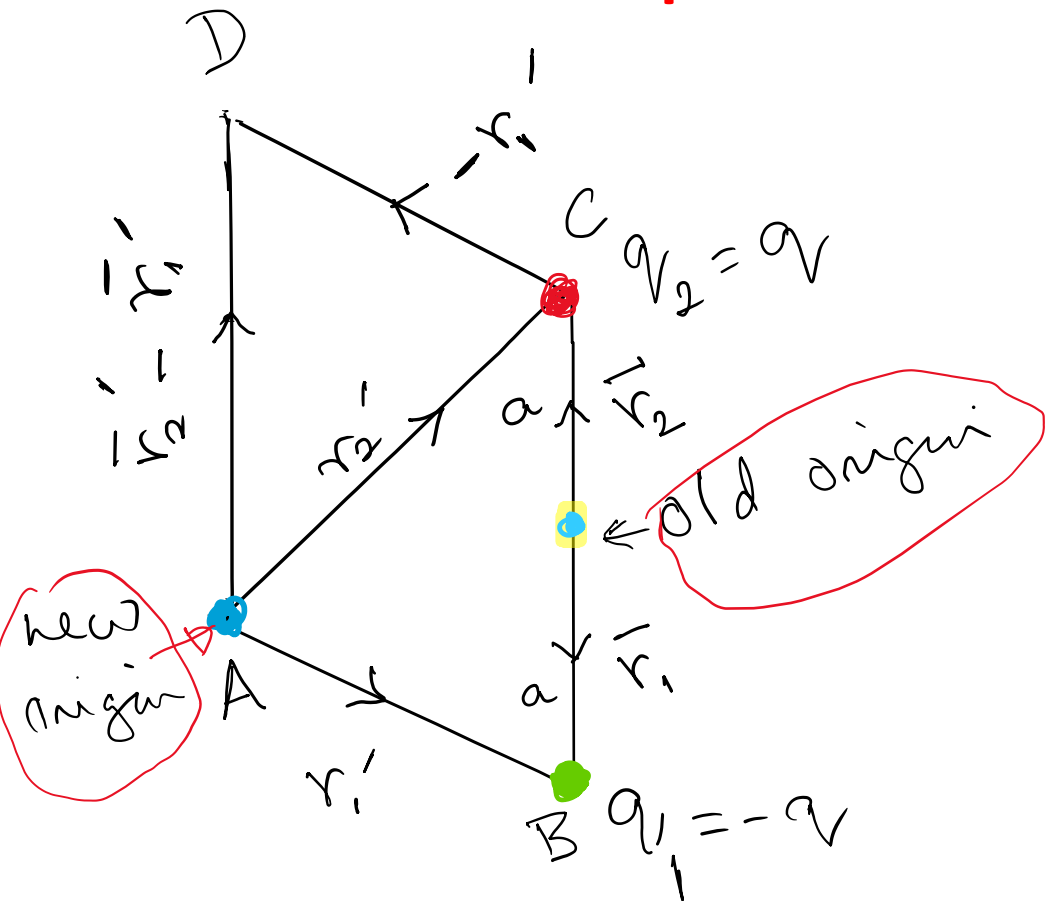
$q_1, q_2, q_3, \ldots$

3

$$\vec{p} = q_1\vec{r}_1 + q_2\vec{r}_2 + q_3\vec{r}_3 + \cdots$$

$$\vec{P} = \sum_{i=1}^{n} q_i\vec{r}_i$$

Dipole Moment is Independent of Origin



ABCD is a parallelogram

$AD = BC \qquad AD \parallel BC$

old origin

$$\vec{p} = q_1\vec{r}_1 + q_2\vec{r}_2$$
$$= (-q)a(-\hat{j}) + q\,a\hat{j}$$
$$= q\,a\hat{j} + q\,a\hat{j}$$
$$= q\,2a\,\hat{j}$$
$$\vec{p} = q\,d\,\hat{j}$$
$$d = 2a$$

New origin

$$\vec{p} = q_1\vec{r}_1' + q_2\vec{r}_2'$$
$$= (-q)(\vec{r}_1') + q\,\vec{r}_2'$$
$$= q(\vec{r}_2' - \vec{r}_1')$$
$$= q\,2a\,\hat{j}$$
$$\vec{p} = q\,d\,\hat{j} \qquad d = 2a$$

M RAL3 V6 | Force and Torque on a Dipole in a Uniform Field | P... What's it about? Watch later Share

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Force and Torque on a Dipole in a Uniform Field

SKIP

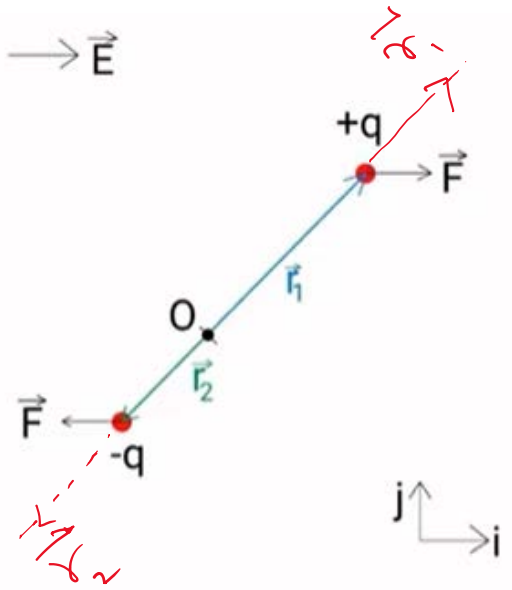
1. We can calculate the Coulomb force at point p if the electric field is given,

$$\vec{F}(p) = q\vec{E}(p) \tag{1}$$

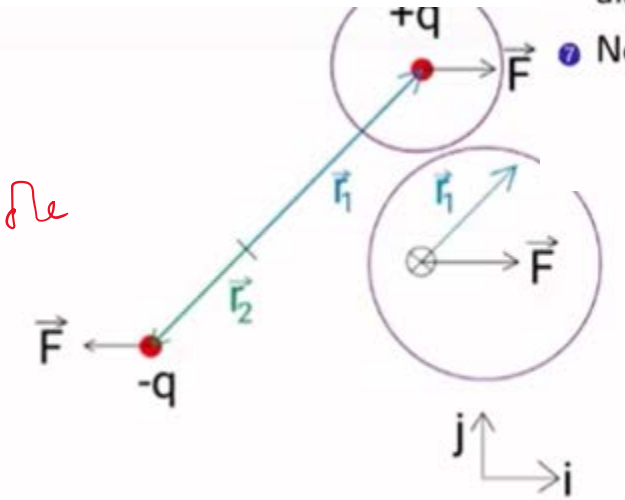
2. The torque $\vec{\tau}$ is defined by

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{2}$$

3. Torque on a object about some point is **independent** of the choice of origin if $\sum_j \vec{F}_j = 0$, that is the net force on the object is zero.
4. Electric dipole moment $\vec{p}$ is **independent** of the choice of origin if $\sum_j q_j = 0$, that is the net charge is zero.
5. We want to calculate the motion of the dipole in a constant electric field.



1. External constant electric field $\vec{E}_{ext}$ pointing from left to right.
2. Two point charge $+q$ and $-q$ connected by massless rigid rod and $\sum_j q_j = 0$. and constitute a dipole
3. Force on each charge $\vec{F} = q\vec{E}_{ext}$. and $\vec{F}_1 + \vec{F}_2 = 0$.
4. What about the torque $\vec{\tau}$? Definition $\vec{\tau} = \vec{r} \times q\vec{E}_{ext}$



5. For the $+q$ positive charge $\vec{\tau}_+ = \vec{r}_1 \times +q\vec{E}$ which points **into** the page.
6. For the $-q$ negative charge $\vec{\tau}_- = \vec{r}_2 \times -q\vec{E}$ which also points **into** the page.
7. Net torque

$$\vec{\tau} = \vec{\tau}_+ + \vec{\tau}_- \tag{3}$$

(SKIP)

Force and Torque on a Dipole in a Uniform Field contd.

SKIP

$$\vec{\tau} = \sum_j \vec{r}_j \times \vec{F}_j \tag{4}$$

$$= \sum_j \vec{r}_j \times q_j \vec{E}_{ext} \tag{5}$$

$$= \left(\sum_j \vec{r}_j q_j \right) \times \vec{E}_{ext} \tag{6}$$

$$\vec{\tau} = \vec{p} \times \vec{E}_{ext} \tag{7}$$

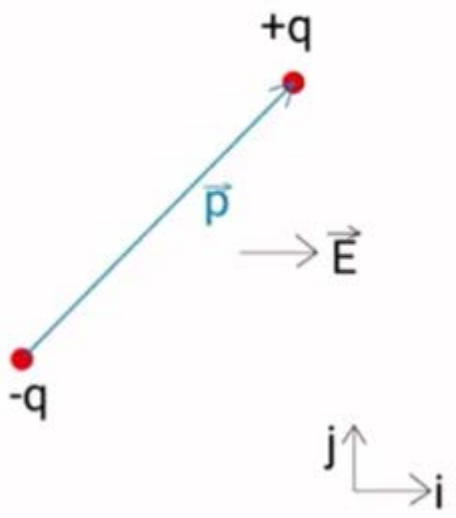
Where, $\vec{p}$ is the electric dipole moment.
caution : $\vec{F}_j$ is not the j-th component, rather the ~~j-th~~ force acting on j-th charge.

1. we assumed $\sum_i q_i = 0$. So $\vec{p}$ is independent of the choice of origin.
2. And we got

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}_{ext}} \tag{8}$$

SKIP

The Result



1. we assumed $\sum_i q_i = 0$. So $\vec{p}$ is independent of the choice of origin.
2. And we got

$$\vec{\tau} = \vec{p} \times \vec{E}_{ext} \tag{8}$$

$$\vec{\tau} = \vec{p} \times \vec{E}_{ext} = \vec{p} \times \vec{E}$$

put $\vec{E}_{ext} = \vec{E}$

$$\vec{\tau} = pE\sin\theta\,\hat{n}$$

$$|\tau| = pE\sin\theta$$

Important

1. If you have only one pointlike charged object in an external field

$$\vec{F} = q\vec{E}_{ext} = m\vec{a} \tag{9}$$

   It will feel an acceleration.
2. But for extended object, like the dipole we have here (water dipole is a real world example), it will not feel any net force but there will be a net torque.
3. For a dipole, the direction of the torque depends on the direction of the electric field.
4. A real world application of this is the microwave oven. It uses oscillating electric field and the food you put there has water in it. Dipole oscillates and food warms up.

Potential Energy of a Dipole in an External E field

$$dW = F\,dx$$

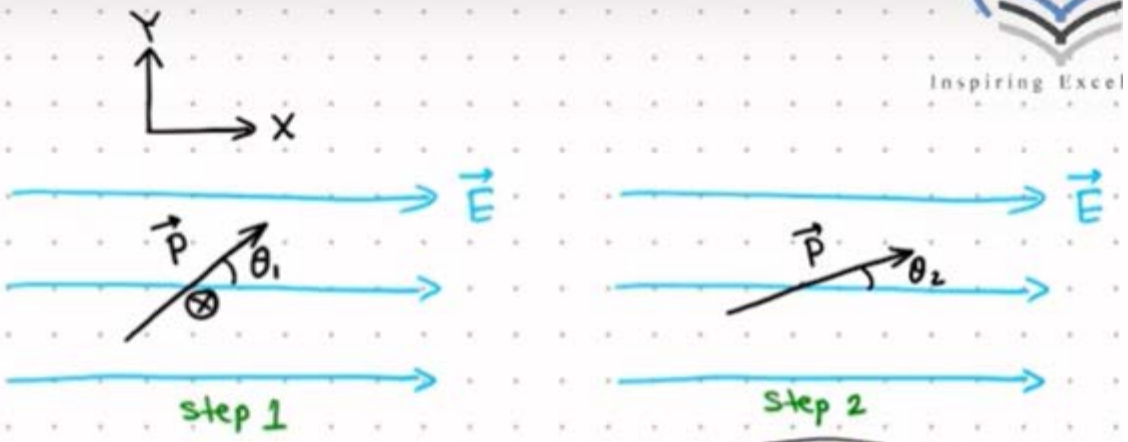
$$dW = -\tau\,d\theta \quad *$$

$$W = -\int_{\theta_1}^{\theta_2} \tau\,d\theta$$

$$= -PE\int_{\theta_1}^{\theta_2} \sin\theta\,d\theta$$

$$= -PE\,[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= PE(\cos\theta_2 - \cos\theta_1)$$



$$\vec{\tau} = \vec{P}\times\vec{E}$$

$$= PE\sin\theta\,\hat{n}$$

$\theta_1 \rightarrow \theta_2$

$\theta_1 > \theta_2$

⊗ ⊙

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* Clockwise torque is ⊖ve

Anticlockwise torque is ⊕ve

Potential Energy of a Dipole in an External E field

$$dW = \vec{F} \cdot d\vec{x}$$

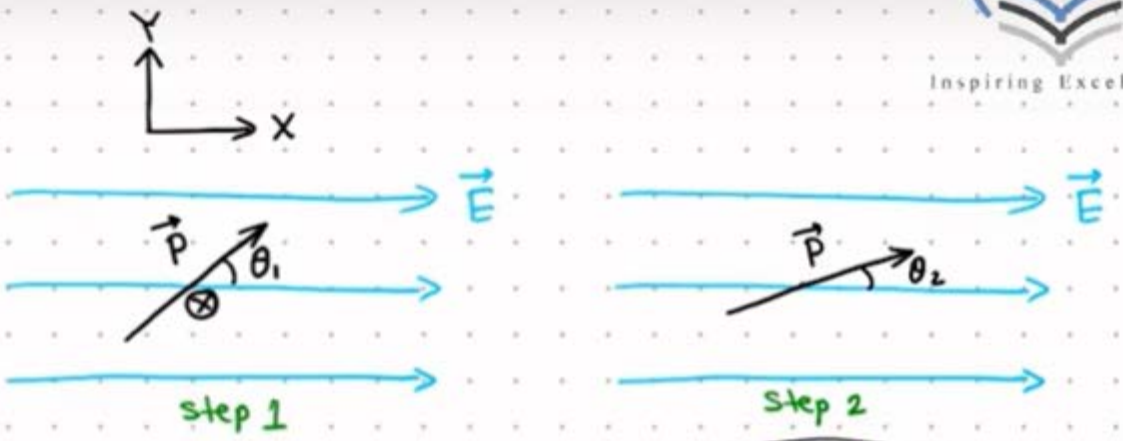
$$dW = -\vec{\tau} \cdot d\vec{\theta} \quad *$$

$$W = -\int_{\theta_1}^{\theta_2} \tau \, d\theta$$

$$= -PE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= -PE \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= PE\left(\cos\theta_2 - \cos\theta_1\right)$$



$$\vec{\tau} = \vec{P} \times \vec{E}$$

$$= PE \sin\theta \, \hat{n}$$

$\theta_1 \rightarrow \theta_2$

$\theta_1 > \theta_2$

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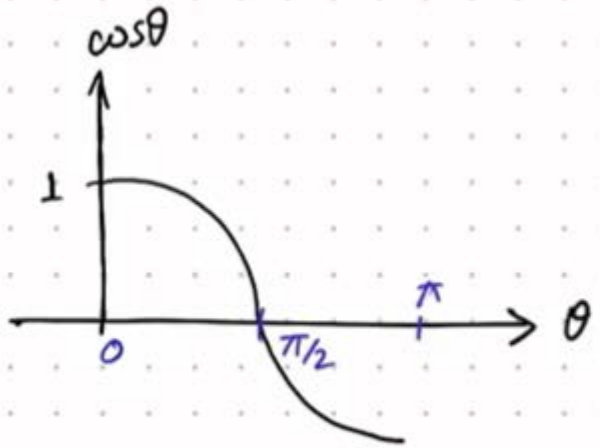
* Clockwise torque is ⊖ve

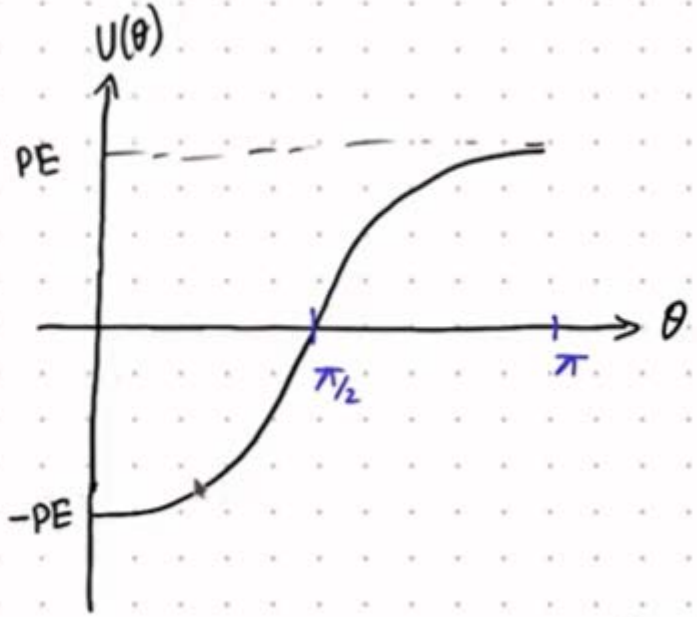
Anticlockwise torque is ⊕ve

$$W = -\Delta U \quad *$$
$$= -(U_2 - U_1)$$
$$= -(U(\theta_2) - U(\theta_1))$$

$$U(\theta) = -PE\cos\theta$$
$$= -\vec{P}\cdot\vec{E}$$

$$U(\theta) = -\vec{P}\cdot\vec{E}$$



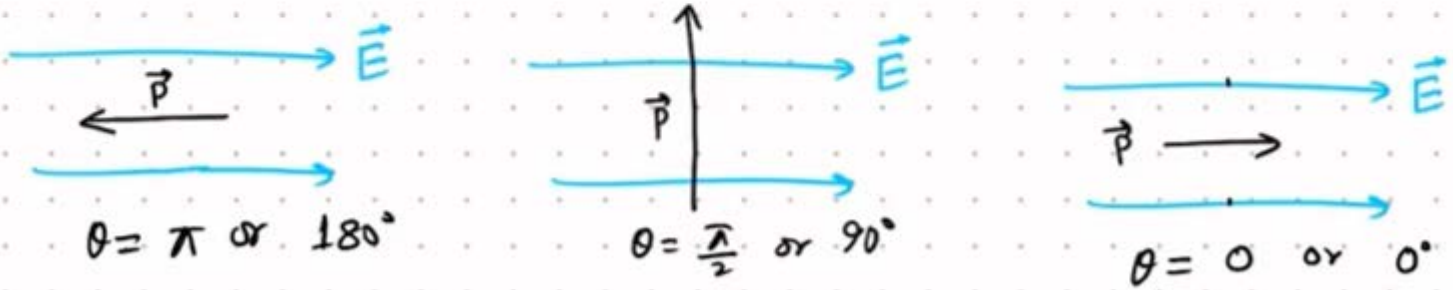


Work done by the system (W) = ⊖ve of the potential energy change (i.e. $-\Delta U$)

= ⊕ve of the work done by the external agent.

$$U(\theta) = -\vec{p}\cdot\vec{E}$$

$$U(\theta) = \begin{cases} -pE \text{ or min} & \text{if } \theta = 0 \\ 0 & \text{if } \theta = \pi/2 \\ pE \text{ or max} & \text{if } \theta = \pi \end{cases}$$



$$U(\theta) = -pE\cos\theta$$

$$\sum q_i = 0$$

$$\vec{p} = \sum_{j=1}^{n} q_j \vec{r}_j$$ dipole moment (electric)

$$\vec{F} = q\vec{E}$$

We know (PHYX 111)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$ torque on a dipole moment in a uniform electric field $\vec{E}$.

$$|\tau| = pE\sin\theta$$

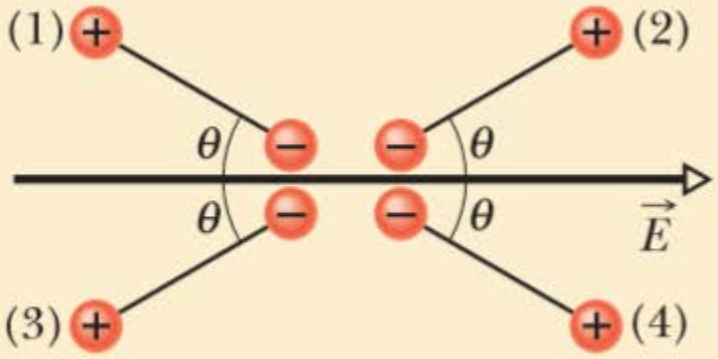
$$U(\theta) = -\vec{p}\cdot\vec{E}$$

$$= -pE\cos\theta$$

Potential Energy of an electric dipole moment $\vec{p}$ in a uniform electric field $\vec{E}$.

Checkpoint 4

The figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.



(a) all tie

(b) 1 and 3 tie

2 and 4 tie

$|\tau| = pE\sin\theta$

$U(\theta) = PE = -pE\cos\theta$

Try yourself at home.