Welcome to the Physics class

Principles of Physics II

Lecture 14

CURRENT AND RESISTANCE

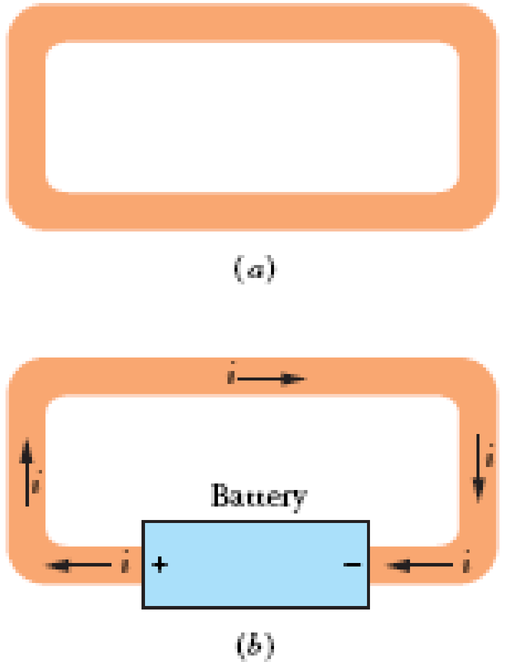


Fig. 26-1 (*a*) A loop of copper in electrostatic equilibrium. The entire loop is at a single potential, and the electric field is zero at all points inside the copper. (*b*) Adding a battery imposes an electric potential difference between the ends of the loop that are connected to the terminals of the battery. The battery thus produces an electric field within the loop, from terminal to terminal, and the field causes charges to move around the loop. This movement of charges is a current *i*.

The current is the same in any cross section.

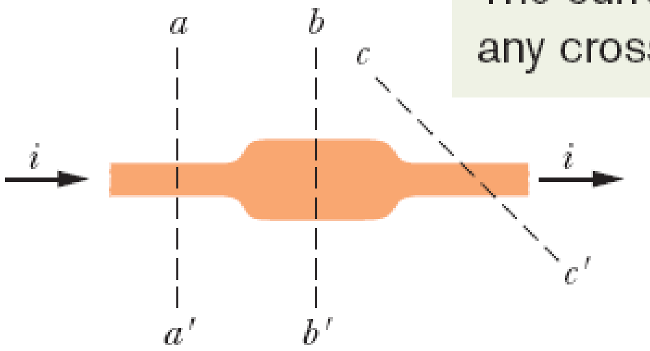


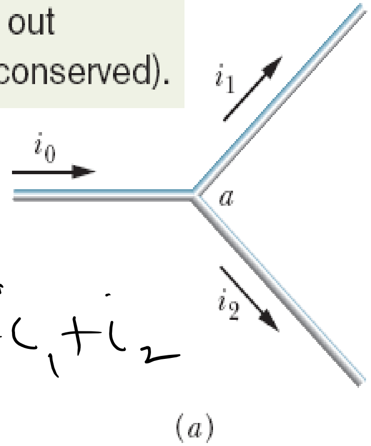
Fig. 26-2 The current *i* through the conductor has the same value at planes aa', bb', and cc'.

$$i = \frac{dq}{dt}$$ definition of current

$$i_{av} = \frac{q}{t}$$ Also $dq = i\,dt$

$$\therefore q = \int dq = \int_0^t i\,dt$$ unit = Amp, $A = C/s$

The current into the junction must equal the current out (charge is conserved).



$$i_0 = i_1 + i_2$$

direction of current

Current density $\vec{J}$

$\vec{J}$ is a vector "||" to the drift velocity of ⊕ve charge.

Current through an element of area $\vec{dA}$

$$di = \vec{J} \cdot \vec{dA}$$

$$i = \int di = \iint \vec{J} \cdot \vec{dA}$$

for uniform J

$$i = \iint J\, dA = J \iint dA = J A$$

$$J = \frac{i}{A}$$ magnitude of uniform J.

unit of $J = \frac{A}{m^2}$

Drift Velocity

$v_{drift} \sim 10^{-5}$ to 10^{-4} m/s

Current is said to be due to positive charges that are propelled by the electric field.

$v_{random} \rightarrow \sim 10^{6}$ m/sec

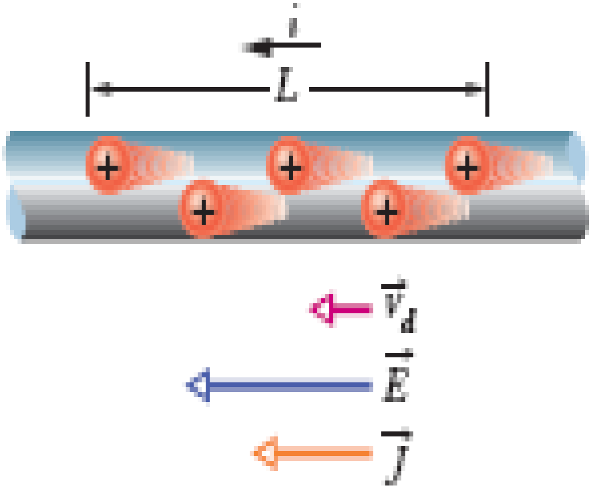


Fig. 26-5 Positive charge carriers drift at speed v_d in the direction of the applied electric field $\vec{E}$. By convention, the direction of the current density $\vec{J}$ and the sense of the current arrow are drawn in that same direction.

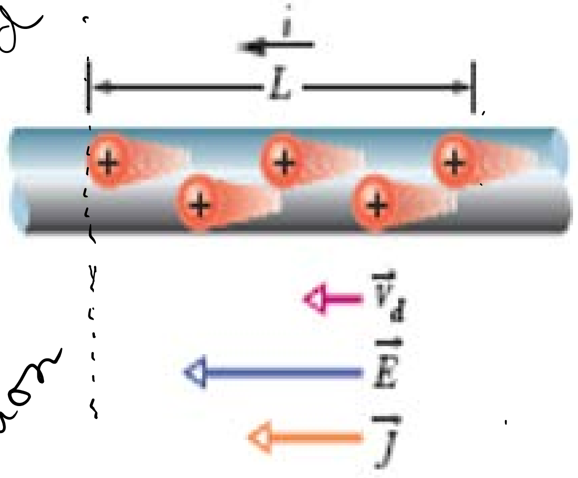
Random motion of electrons in a conductor when there is no current flowing

$i = nAev_d$

Drift Velocity Contd.

Current is said to be due to positive charges that are propelled by the electric field.

$J = nev_d$



x-section

$t = \frac{L}{v_d}$

Assume J in uniform across A

No. of charge carriers in length L of the conductor

$n \times (AL)$

charge carrier density / volume

Total charge in that volume

$q = e \times (nAL)$ passes through any x-section of the conductor in t sec.

$t = \frac{L}{v_d}$

$$i = \frac{q}{t} = \frac{enAL}{\frac{L}{v_d}} = nAev_d$$

$$\therefore v_d = \frac{i}{nAe} = \frac{J}{ne}$$

Resistance and Resistivity

Current is driven by a potential difference.

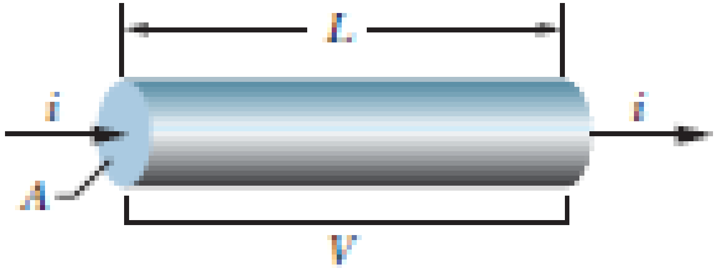


Figure 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

$$R = \frac{V}{i}$$

definition of R

$$\Omega = \frac{\text{Volt}}{\text{Amp}} = \frac{V}{A}$$

ohm

$$E = \frac{V}{L}$$

$$V \Rightarrow E$$

$$J = \frac{i}{A}$$

$$i \Rightarrow J$$

$$R = ?$$

$$R \Rightarrow \rho$$

$R =$ Resistance of a resistor

$\rho =$ Resistivity is a property of the material of which the object resistor is made of.

We define $\rho = \dfrac{E}{J} \Rightarrow \vec{E} = \rho \vec{J}$

Now, we define conductivity $\sigma = \dfrac{1}{\rho}$ (def of conductivity)

$\therefore \quad J = \dfrac{E}{\rho} = \sigma E$

$\vec{J} = \sigma \vec{E}$ microscopic ohm's law

Calculating Resistance from Resistivity

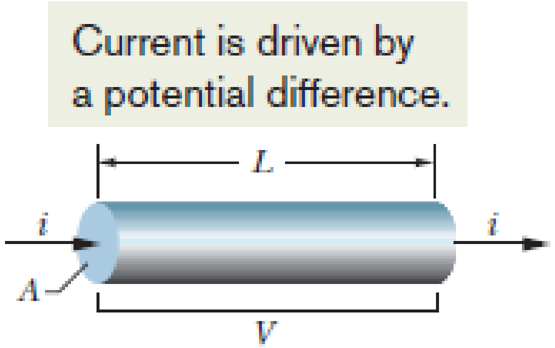


Figure 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

Uniform J

E and J will be constant

$E = \frac{V}{L}$ & $J = \frac{i}{A}$

defined $\Rightarrow$ $\rho = \frac{E}{J} = \frac{V/L}{i/A} = \frac{V}{L} \times \frac{A}{i}$

$\rho = \frac{V}{i} \frac{A}{L} = R \frac{A}{L}$

$\therefore R = \rho \frac{L}{A}$ for homogeneous / isotropic / uniform X-section

Table 26-1 Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot m$)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

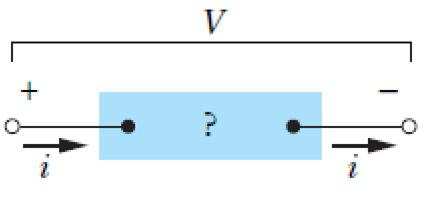
Resistance is a property of an object. Resistivity is a property of a material.

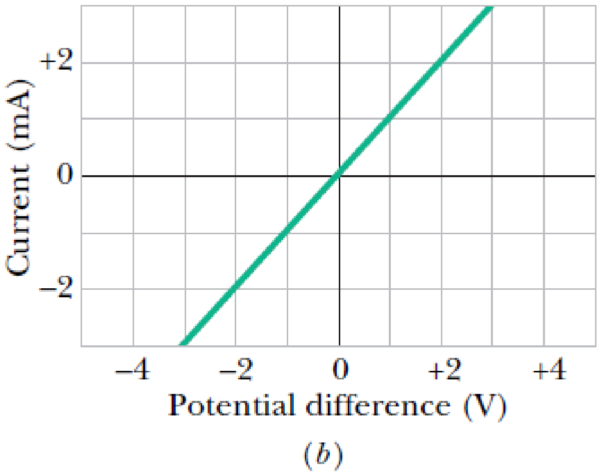
Ohm's Law

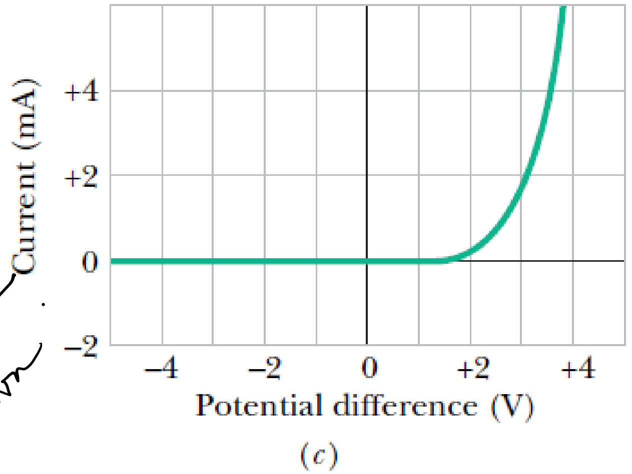
$i \propto V$

$i = \frac{V}{R}$

$\frac{1}{R}$ is the prop. constant.







non ohmic

non-linear relation between I and V

Fig. 26-11 (*a*) A potential difference V is applied to the terminals of a device, establishing a current i. (*b*) A plot of current i versus applied potential difference V when the device is a 1000 Ω resistor. (*c*) A plot when the device is a semiconducting *pn* junction diode.

ohmic

linear relation between current and voltage

The battery ~~at the left~~ supplies energy to the conduction electrons that form the current.

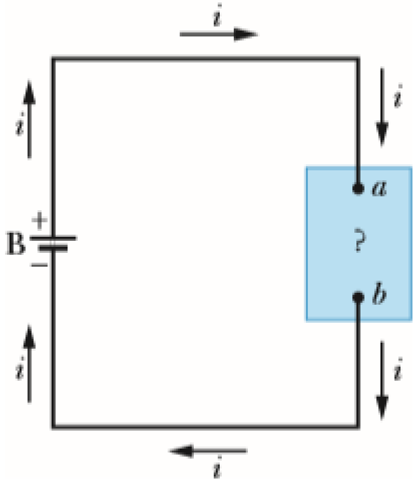


Fig. 26-13 A battery B sets up a current i in a circuit containing an unspecified conducting device.

Power in Electric Circuits

The battery ~~at the left~~ supplies energy to the conduction electrons that form the current.

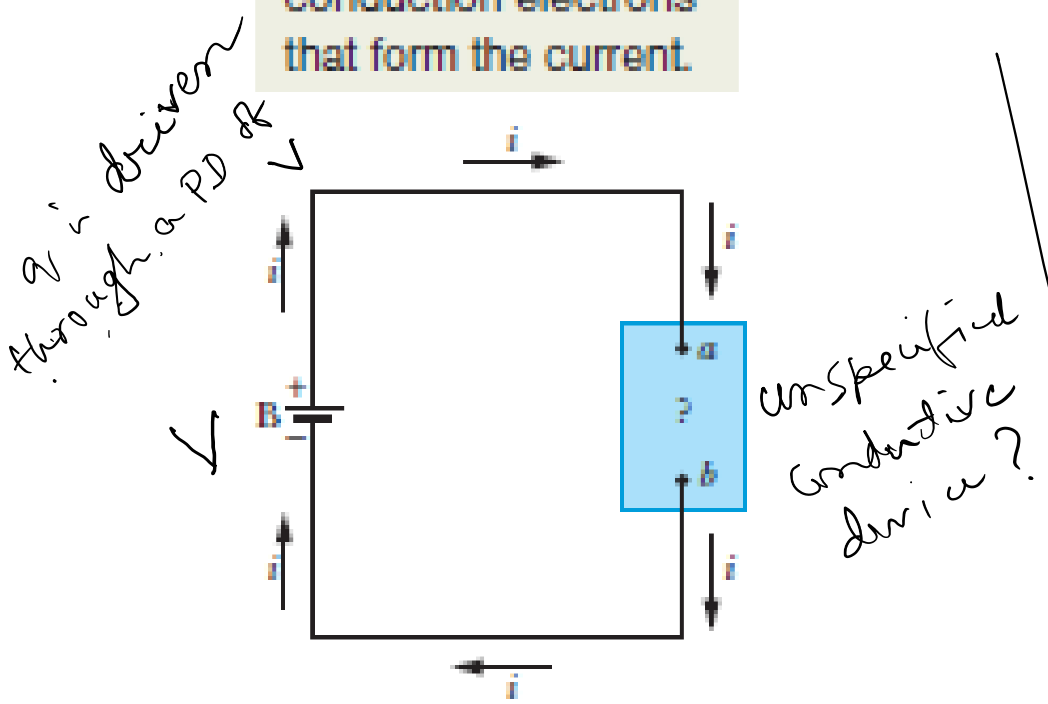


Figure 26-13 A battery B sets up a current i in a circuit containing an unspecified conducting device.

$$P = \frac{W}{t} = \frac{qV}{t} = iV$$

$$P = IV$$ → general.

$= \text{Volt} \cdot \text{Amp}$

$$\frac{J}{\cancel{C}} \cdot \frac{\cancel{C}}{s} = \frac{J}{s} = \text{Watt} = W$$

For a Resistive device (passive element)

$$R = \frac{V}{i} \Rightarrow V = Ri$$

$$\therefore P = i \cdot Ri = i^2 R$$

$$= \frac{V^2}{R}$$

Current An **electric current** i in a conductor is defined by

$$i = \frac{dq}{dt}. \tag{26-1}$$

Here dq is the amount of (positive) charge that passes in time dt through a hypothetical surface that cuts across the conductor. By convention, the direction of electric current is taken as the direction in which positive charge carriers would move. The SI unit of electric current is the **ampere** (A): 1 A = 1 C/s.

Current Density Current (a scalar) is related to **current density** $\vec{J}$ (a vector) by

$$i = \int \vec{J} \cdot d\vec{A}, \tag{26-4}$$

where $d\vec{A}$ is a vector perpendicular to a surface element of area dA and the integral is taken over any surface cutting across the conductor. $\vec{J}$ has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

Drift Speed of the Charge Carriers When an electric field $\vec{E}$ is established in a conductor, the charge carriers (assumed positive) acquire a **drift speed** v_d in the direction of $\vec{E}$; the velocity $\vec{v}_d$ is related to the current density by

$$\vec{J} = (ne)\vec{v}_d, \tag{26-7}$$

where ne is the *carrier charge density.*

Resistance of a Conductor The **resistance** R of a conductor is defined as

$$R = \frac{V}{i} \quad \text{(definition of } R\text{)}, \tag{26-8}$$

where V is the potential difference across the conductor and i is the current. The SI unit of resistance is the **ohm** (Ω): 1 Ω = 1 V/A. Similar equations define the **resistivity** ρ and **conductivity** σ of a material:

$$\rho = \frac{1}{\sigma} = \frac{E}{J} \quad \text{(definitions of } \rho \text{ and } \sigma\text{)}, \tag{26-12, 26-10}$$

where E is the magnitude of the applied electric field. The SI unit of resistivity is the ohm-meter ($\Omega \cdot$m). Equation 26-10 corresponds to the vector equation

$$\vec{E} = \rho \vec{J}. \tag{26-11}$$

The resistance R of a conducting wire of length L and uniform cross section is

$$R = \rho \frac{L}{A}, \tag{26-16}$$

where A is the cross-sectional area.

Change of ρ with Temperature The resistivity ρ for most materials changes with temperature. For many materials, including metals, the relation between ρ and temperature T is approximated by the equation

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0). \tag{26-17}$$

Here T_0 is a reference temperature, ρ_0 is the resistivity at T_0, and α is the temperature coefficient of resistivity for the material.

Ohm's Law A given device (conductor, resistor, or any other electrical device) obeys *Ohm's law* if its resistance R, defined by Eq. 26-8 as V/i, is independent of the applied potential difference V. A given *material* obeys Ohm's law if its resistivity, defined by Eq. 26-10, is independent of the magnitude and direction of the applied electric field $\vec{E}$.

Resistivity of a Metal By assuming that the conduction electrons in a metal are free to move like the molecules of a gas, it is possible to derive an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}. \tag{26-22}$$

Here n is the number of free electrons per unit volume and τ is the mean time between the collisions of an electron with the atoms of the metal. We can explain why metals obey Ohm's law by pointing out that τ is essentially independent of the magnitude E of any electric field applied to a metal.

Power The power P, or rate of energy transfer, in an electrical device across which a potential difference V is maintained is

$$P = iV \quad \text{(rate of electrical energy transfer)}. \tag{26-26}$$

Resistive Dissipation If the device is a resistor, we can write Eq. 26-26 as

$$P = i^2 R = \frac{V^2}{R} \quad \text{(resistive dissipation)}. \tag{26-27, 26-28}$$

In a resistor, electric potential energy is converted to internal thermal energy via collisions between charge carriers and atoms.

Semiconductors *Semiconductors* are materials that have few conduction electrons but can become conductors when they are *doped* with other atoms that contribute free electrons.

Superconductors *Superconductors* are materials that lose all electrical resistance at low temperatures. Recent research has discovered materials that are superconducting at surprisingly high temperatures.