Welcome to the Physics class

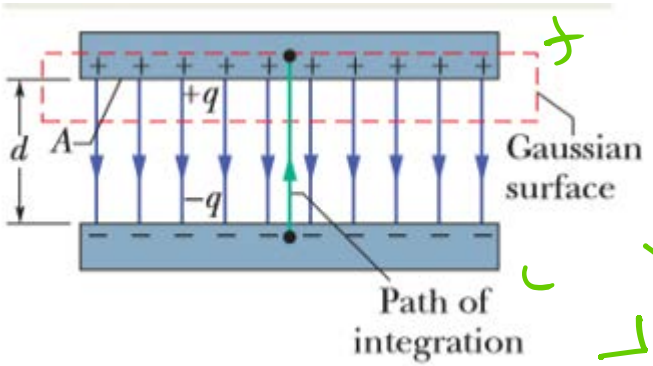
Principles of Physics II

Lecture 12

Parallel plate capacitor

Defining equation of capacitance: **C = Q/ V**

Q in Coulomb, V in Volts and C is in Farad (F)



$V_f = V$

$V_i = 0$

Gauss' law $\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$

$\therefore q = \epsilon_0 E A$

<u>Pot Dif</u>

$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$

$V - 0 = \int_-^+ E\,ds$ dir. of $\vec{E}$ and $d\vec{s}$ are opposite.

$\therefore V = \int_-^+ E\,ds$

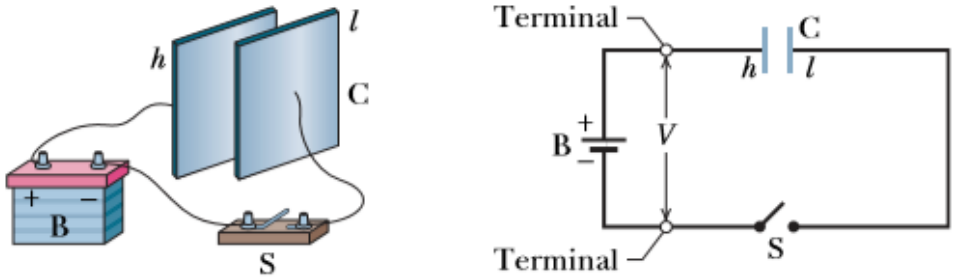


Fig. 25-4 (*a*) Battery B, switch S, and plates *h* and *l* of capacitor C, connected in a circuit. (*b*) A schematic diagram with the *circuit elements* represented by their symbols.

In terms of the geometry of the capacitor (e.g., the parallel plate capacitor)

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{(parallel-plate capacitor).}$$

We use Gauss' law to relate q and E. Then we integrate the E to get the potential difference.

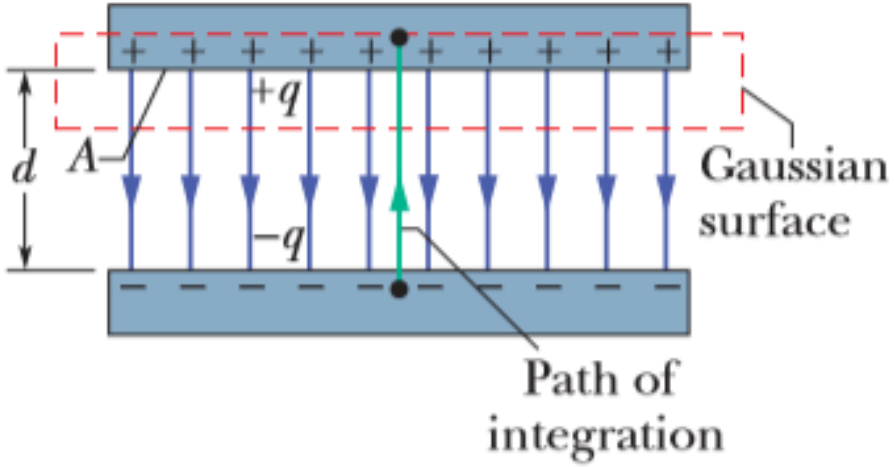


Fig. 25-5 A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration of Eq. 25-6 is taken along a path extending directly from the negative plate to the positive plate.

$q = \epsilon_0 E A$ (A is the plate area)

$$V = \int_-^+ E\,ds = E\int_-^+ ds = Ed$$

$$V = Ed$$

$$C = \frac{q}{V} = \frac{\epsilon_0 \cancel{E} A}{\cancel{E} d} = \frac{\epsilon_0 A}{d}$$

$C = \frac{\epsilon_0 A}{d}$ for ∥ plate capacitor with vacuum as the dielectric betn the plates.

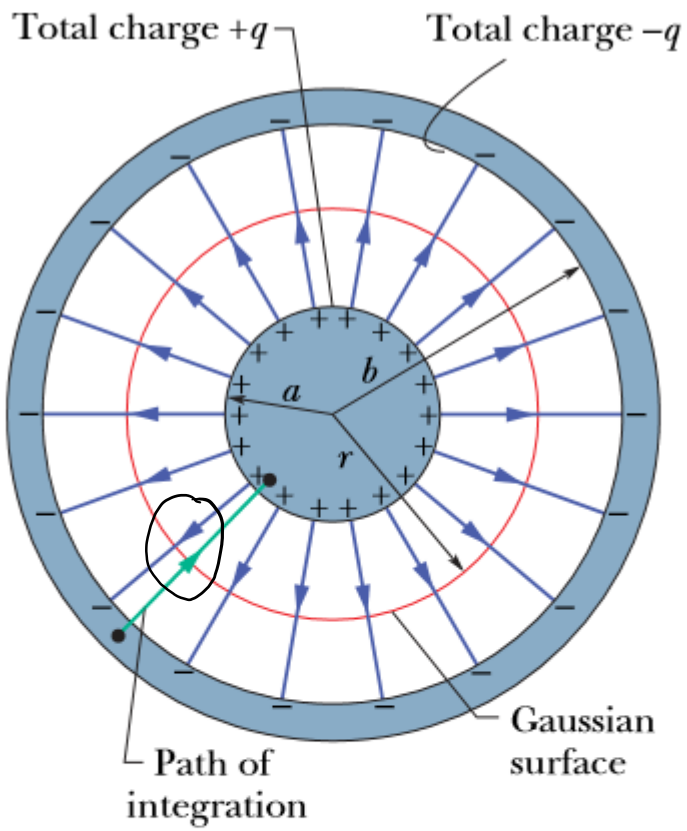
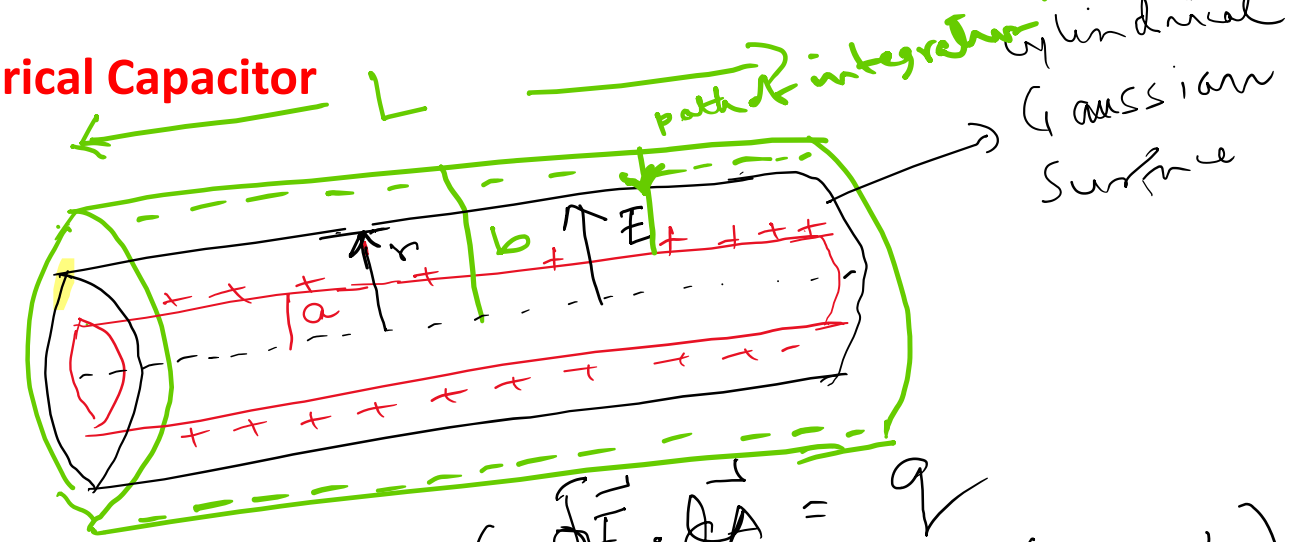


Fig. 25-6 A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate) and the radial path of integration along which Eq. 25-6 is to be applied. ~~This figure also serves to illustrate a spherical capacitor in a cross section through its center~~.

Cylindrical Capacitor



Gauss Law $\Rightarrow$ $\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$

$\epsilon_0 E A = q \Rightarrow q = \epsilon_0 E (2\pi r L)$

$\therefore E = \dfrac{q}{2\pi\epsilon_0 r L}$

$d\vec{r}$ and $d\vec{s}$ are oppositely directed.

$*$ $V = \int_{-}^{+} E\, ds = -\dfrac{q}{2\pi\epsilon_0 L} \int_{b(-)}^{a(+)} \dfrac{dr}{r}$

$V = \dfrac{-q}{2\pi\epsilon_0} \left[\ln r\right]_b^a = \dfrac{q}{2\pi\epsilon_0 L} \ln\left(\dfrac{b}{a}\right)$

$C = \dfrac{q}{V} = 2\pi\epsilon_0 \dfrac{L}{\ln(b/a)}$

$$V - 0 = -\int_{-}^{+} \vec{E} \cdot d\vec{s}$$

$$V = -\int^{+} E\, ds \cos 180^\circ$$

$$V = -\int_{-}^{+} E\, ds(-1)$$

$$= \int_{-}^{+} \frac{q\, ds}{2\pi\epsilon_0 r L}$$

We have to change the integration variable from ds to dr, since E is expressed in r. Now $ds = -dr$

$$\therefore V = \int_{b(-)}^{a(+)} \frac{q\,(-dr)}{2\pi\epsilon_0 r L}$$

$$\therefore V = -\frac{q}{2\pi\epsilon_0 L}\int_{b}^{a} \frac{dr}{r}$$

$$\text{or } V = -\frac{q}{2\pi\epsilon_0 L}\Big[\ln r\Big]_{b}^{a}$$

$$= \frac{q}{2\pi\epsilon_0 L}\Big[\ln r\Big]_{a}^{b}$$

$$\therefore V = \frac{q}{2\pi\epsilon_0 L}\Big[\ln b - \ln a\Big]$$

$$C = \frac{q}{V} = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}$$

Spherical Capacitor:

(A metallic sphere and a concentric metallic shell)

4L

$$\epsilon_0 \oint \vec{E}\cdot d\vec{A} = q$$

$$\epsilon_0 \oint E\, dA = q \Rightarrow \epsilon_0 E\, 4\pi r^2 = q$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$V = \int_{-}^{+} E\, ds = -\frac{q}{4\pi\epsilon_0}\int_a^b \frac{dr}{r^2}$$

$$d\vec{s} = -d\vec{r}$$

or $$V = -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_b^a = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\frac{b-a}{ab}$$

$$C = \frac{q}{V}$$

or $$C = 4\pi\epsilon_0 \frac{ab}{b-a}$$

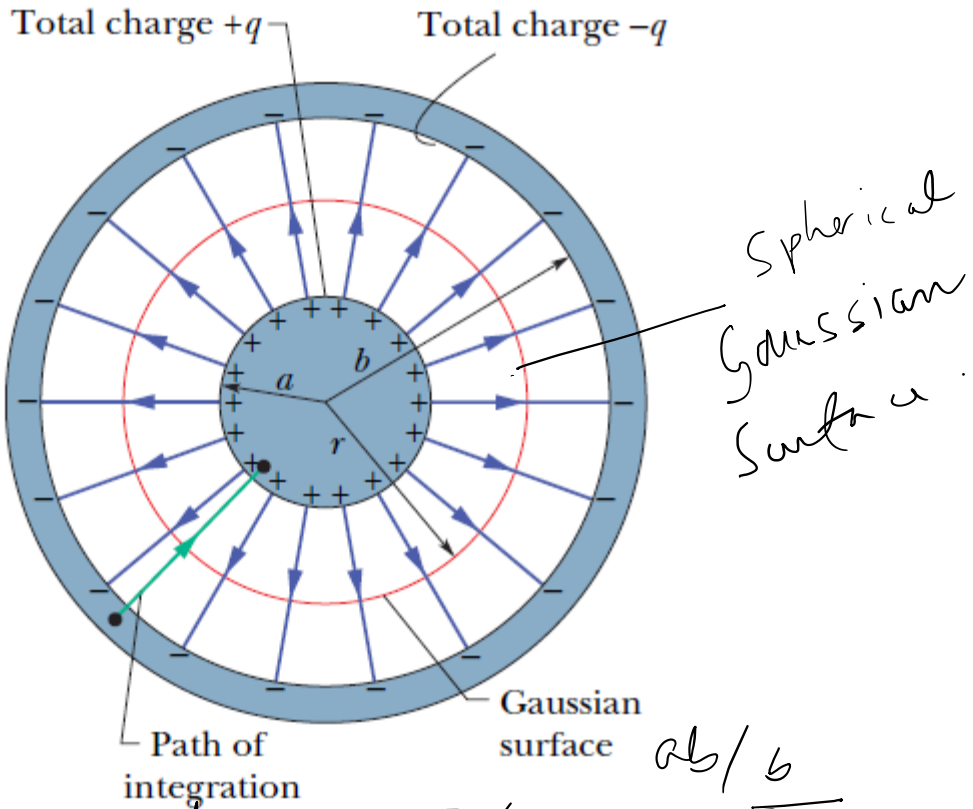
$b \gg a \; (b \to \infty)$ $$C = 4\pi\epsilon_0 \frac{ab}{b-a} = 4\pi\epsilon_0 \frac{ab/b}{\frac{b}{b}-\frac{a}{b}}$$

$$C = 4\pi\epsilon_0 \frac{a}{1-0}$$

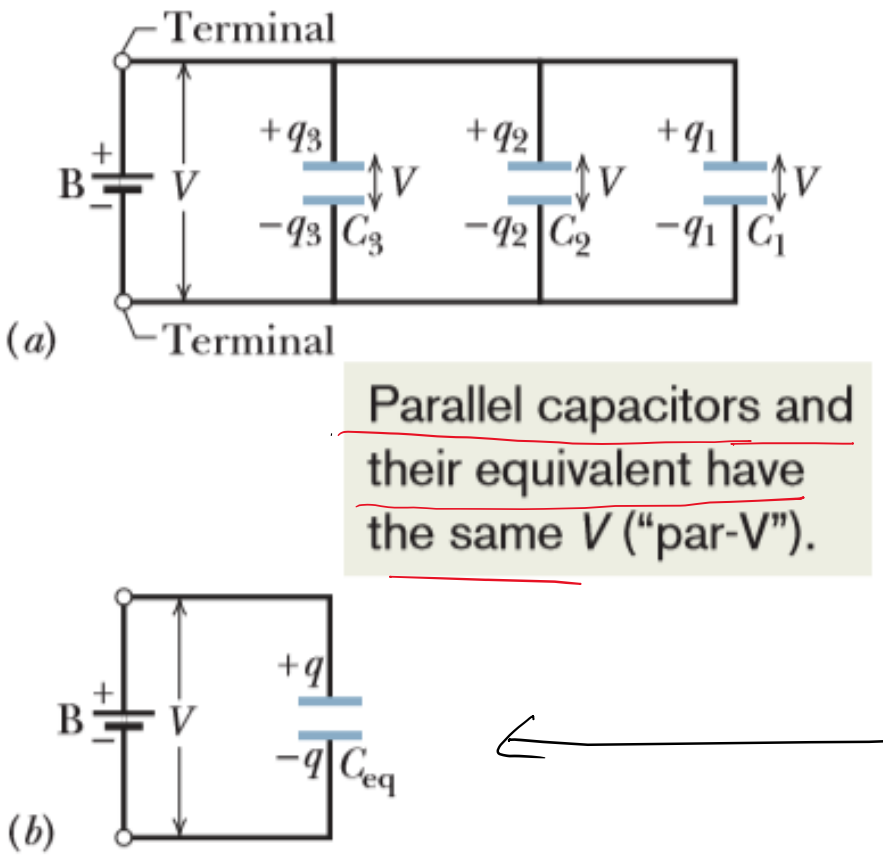
$$\Rightarrow \boxed{C = 4\pi\epsilon_0 R}$$

$a = R$

Capacitance of a metallic sphere of radius R.



Capacitors in parallel



Parallel capacitors and their equivalent have the same V ("par-V").

Fig. 25-8 (*a*) Three capacitors connected in parallel to battery B. The battery maintains potential difference V across its terminals and thus across *each* capacitor. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the parallel combination.

TA al charge stored in all capacitors

$$q = q_1 + q_2 + q_3$$
$$= C_1 V + C_2 V + C_3 V$$
$$= V(C_1 + C_2 + C_3)$$
$$\therefore \frac{q}{V} = C_1 + C_2 + C_3$$

$q_1 = C_1 V$

$q_2 = C_2 V$

$q_3 = C_3 V$

All capacitor have same pd across their plates.

Now

$$C_{eq} = \frac{q}{V}$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

$$C_{eq} = \sum_{j=1}^{n} C_j$$

Capacitors in Series

Same charge stored in all capacitor also in the equivalent capacitor.

$V = V_1 + V_2 + V_3$ splitted up across each capacitors depending on their capacitance.

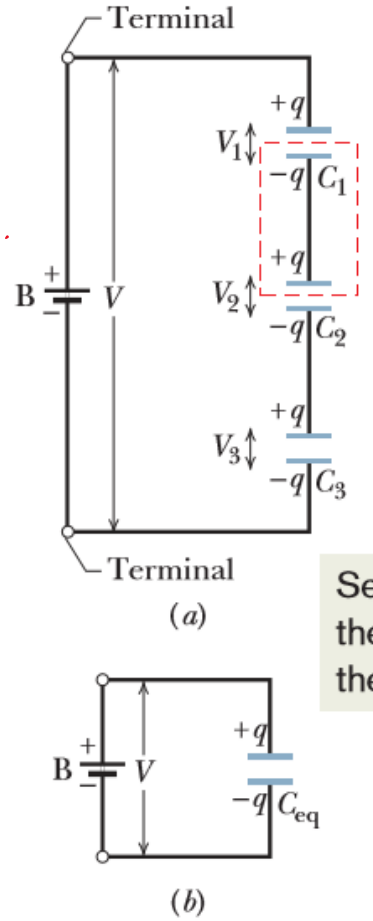
as $V_1 = \frac{q}{C_1}$; $V_2 = \frac{q}{C_2}$ & $V_3 = \frac{q}{C_3}$

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$C_{eq} = \frac{q}{V} \quad \text{or} \quad \frac{1}{C_{eq}} = \frac{V}{q} \quad \text{(Fig b)}$$

$$\frac{V}{q} = \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$\therefore \frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad \text{(n capacitors in Series)}$$



Series capacitors and their equivalent have the same q ("seri-q").

Fig. 25-9 (*a*) Three capacitors connected in series to battery B. The battery maintains potential difference V between the top and bottom plates of the series combination. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the series combination.

Capacitors

In Series

1. $V = V_1 + V_2 + V_2$

2. Charges in each capacitor are the same $q = q_1 = q_2 = q_3 = q_{eq}$

3. $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

In Parallel

1. P.D. across each capacitor are same

$V_1 = V_2 = V_2 = V =$ P.D. of the source.

2. $q_1 = C_1 V \quad q_2 = C_2 V \quad q_3 = C_3 V$

$q = q_1 + q_2 + q_3$

3. $C_{eq} = C_1 + C_2 + C_3$

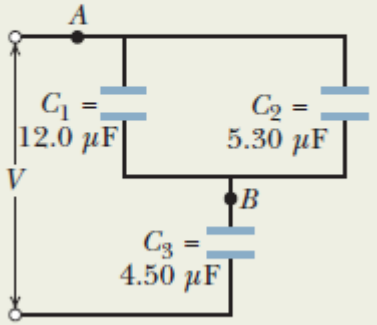
$$C_{eq} = \sum_{i=1}^{n} C_i$$

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10*a*, across which potential difference V is applied. Assume

$$C_1 = 12.0\ \mu F, \quad C_2 = 5.30\ \mu F, \quad \text{and} \quad C_3 = 4.50\ \mu F.$$

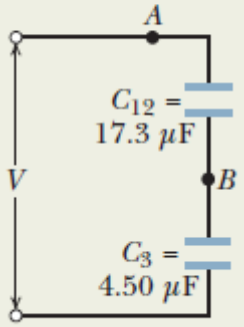
(b) The potential difference applied to the input terminals in Fig. 25-10*a* is $V = 12.5$ V. What is the charge on C_1?

We first reduce the circuit to a single capacitor.



(*a*)

The equivalent of parallel capacitors is larger.



(*b*)

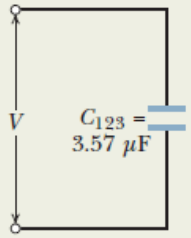
C_1 and C_2 are parallel $\therefore$ $C_{12} = C_1 + C_2 = 12 + 5.3 = 17.3\ \mu F$

C_{12} and C_3 are in series.

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{17.3\ \mu F} + \frac{1}{4.5\ \mu F} = 0.280\ (\mu F)^{-1}$$

$$\therefore C_{123} = \frac{1}{0.280} = 3.57\ \mu F$$

The equivalent of series capacitors is smaller.



(c)

$V = 12.5\ V$

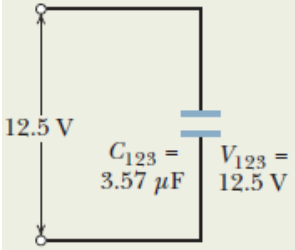
charge on equivalent capacitor

$q_{123} = C_{eq} V_{123}$ $\quad V_{123} = V = 12.5\ V$

$q_{123} = 3.57 \times 12.5 = 44.6\ \mu C$

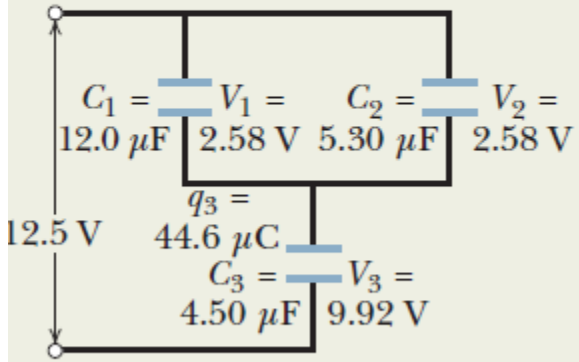
net charge stored

Next, we work backwards to the desired capacitor.



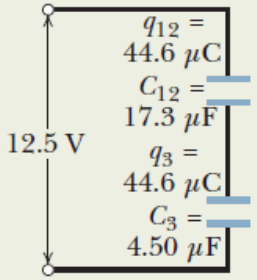
(d)

Parallel capacitors and their equivalent have the same V ("par-V").



(h)

Series capacitors and their equivalent have the same q ("seri-q").



(f)

$q_{12} = q_3 = q_{123}$ charge on equivalent capacitance.

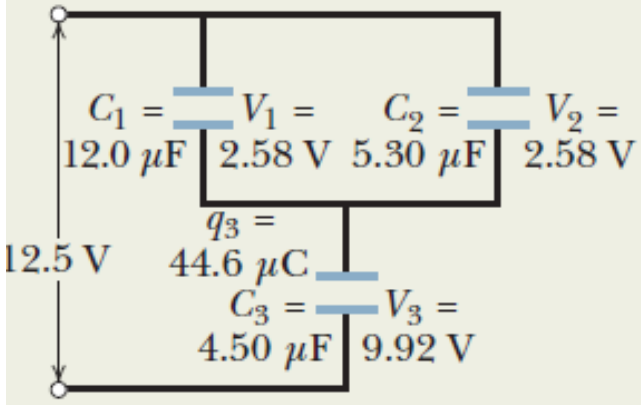
bcoz capacitors in series store same charge but will have diff. pd. dif. across them

$(V_{12} + V_3 = V_{123} = V = 12.5\ V.)$

NOW $V_{12} = \dfrac{q_{12}}{C_{12}} = \dfrac{44.6\ \mu C}{17.3\ \mu F} = 2.58$ volt

$= V_1 = V_2$

Parallel capacitors and their equivalent have the same V ("par-V").



(h)

$\therefore \; q_1 = C_1 V_{12} = 12 \times 2.58 = 30.96 \, \mu C$

$q_2 = C_2 V_{12} = 5.3 \times 2.58 = 13.67 \, \mu C$

$q_3 = 44.6 \, \mu C$

$q_{eq} = 44.6 \, \mu C$

$V_3 = \frac{q_3}{C_3} = \frac{44.6 \, \mu C}{4.5 \, \mu F}$

$= 9.92 \, V$