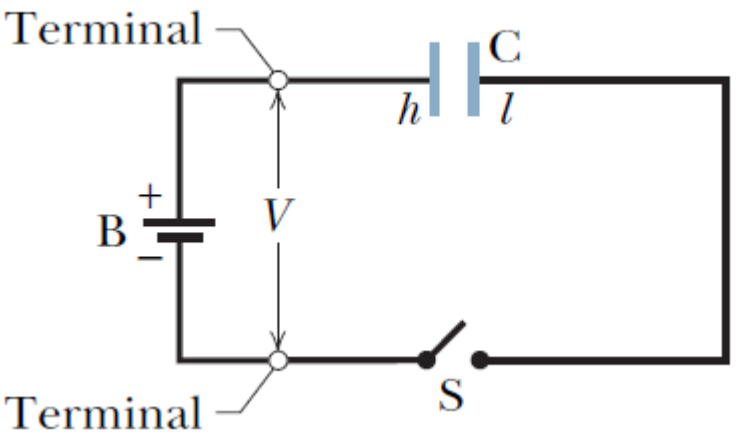
Welcome to the Physics class

Principles of Physics II

Lecture 13

Energy Stored in an Electric Field



$(C = \varepsilon_0 A/d)$

$$dW = V'\,dq' = \frac{q'}{C}\,dq'.$$

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

$$U = \frac{q^2}{2C} \qquad \text{(potential energy).}$$

$q = CV$

$$U = \tfrac{1}{2}CV^2 \qquad \text{(potential energy).}$$

Very Important and frequently needed

Energy Density $= u$

$$u = \frac{\text{Potential energy Stored}}{\text{Volume}}$$

$$= \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\epsilon_0 \frac{\cancel{A}}{d} V^2}{\cancel{A}d}$$

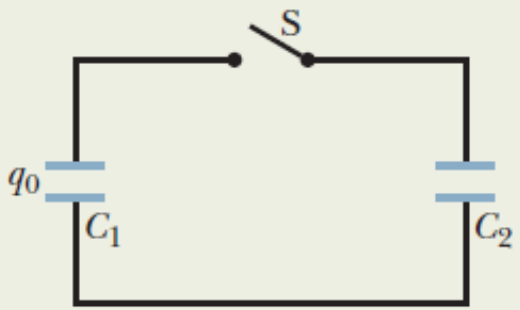
$$= \frac{1}{2}\epsilon_0 \left(\frac{V}{d}\right)^2$$

$$= \frac{1}{2}\epsilon_0 E^2$$

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density).}$$

Capacitor 1, with $C_1 = 3.55\ \mu F$, is charged to a potential difference $V_0 = 6.30$ V, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95\ \mu F$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

Fig. 25-11 A potential difference V_0 is applied to capacitor 1 and the charging battery is removed. Switch S is then closed so that the charge on capacitor 1 is shared with capacitor 2.



Calculations: Initially, when capacitor 1 is connected to the battery, the charge it acquires is, ~~from Eq. 25-1~~,

$$q_0 = C_1 V_0 = 3.55 \times 10^{-6} \times 6.30$$
$$= 22.365 \times 10^{-6}\ C$$

When S is closed the charge from C_1 will flow to C_2 until $V_1 = V_2$.

Then $V_1 = \frac{q_1}{C_1}$ and $V_2 = \frac{q_2}{C_2}$

where q_1 and q_2 will be the final charges on the two capacitors. (Since $V_1 = V_2$)

$$\therefore \frac{q_1}{C_1} = \frac{q_2}{C_2} \text{ (finally)}$$

But the total charge is $q_0 = q_1 + q_2 \Rightarrow q_2 = q_0 - q_1$

$$\therefore \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_0 - q_1}{C_2} \quad \therefore \quad q_1 C_2 = q_0 C_1 - C_1 q_1$$

$$q_1 (C_2 + C_1) = q_0 C_1$$

$$\text{or } q_1 = \frac{C_1}{C_1 + C_2} q_0$$

$$= \frac{3.55}{8.95 + 3.55} \times 22.365 \times 10^{-6} \text{ C}$$

$$= 0.284 \times 22.365 \times 10^{-6}$$

$$= 6.35 \ \mu\text{C}$$

$$q_2 = q_0 - q_1$$

$$= (22.365 - 6.35) \times 10^{-6}$$

$$= 16.015 \times 10^{-6} \text{ C.}$$

$$= 16.02 \times 10^{-6} \text{ C}$$

a. Calculate the P.E stored in C_1 before connecting to C_2

b. Calculate the PE stored in C_1 and C_2 after C_1 and are connected and equilibrium is reached

(a) $C_1 = 3.55 \times 10^{-6}\ \mu F$

$q_0 = 22.36 \times 10^{-6}\ \mu C$

$$\therefore (PE)_0 = \frac{q_0^2}{2C_1}$$

$$= \frac{(22.36 \times 10^{-6})^2}{2 \cdot 355 \times 10^{-6}}$$

$$= 70.44\ \mu J$$

(b) After connecting the two capacitor.

We have $C_1 = 3.55 \times 10^{-6}\ \mu F$ and $C_2 = 8.95 \times 10^{-6}\ \mu F$

$q_1 = 6.35 \times 10^{-6}\ C$ $\qquad$ $q_2 = 16.2 \times 10^{-6}\ C$

$$\therefore (PE)_1 = \frac{q_1^2}{2C_1} = \frac{(6.35 \times 10^{-6})^2}{2 \times 3.55 \times 10^{-6}}$$

$$= 5.67 \times 10^{-6}\ J$$

$$(PE)_2 = \frac{q_2^2}{2C_2}$$

$$= \frac{(16.2 \times 10^{-6})^2}{2 \times 8.95 \times 10^{-6}}$$

$$= 14.33 \times 10^{-6}\ J$$

Capacitor with a Dielectric

Dielectric Constant

$$K = \frac{\epsilon}{\epsilon_0}$$

$$C_0^* = \frac{\epsilon_0 A}{d}$$

Capacitance with the space filled with vacuum

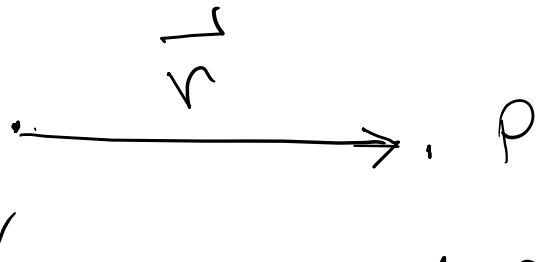
Capacitor filled with a dielectric of permittivity ϵ will have a new capacitance

$$C = \frac{\epsilon A}{d} = \frac{\frac{\epsilon}{\epsilon_0}\,\epsilon_0 A}{d} = \frac{K \epsilon_0 A}{d} = K C_0^*$$

$$\therefore \; C = K C_0$$

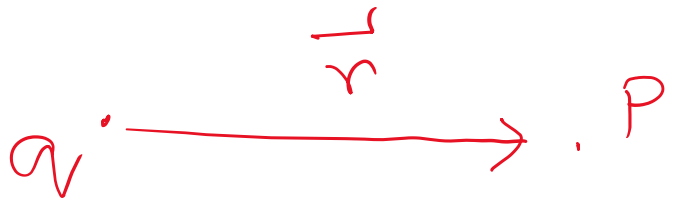
If the region is completely filled with a material of dielectric constant K, all electrostatic equations involving ϵ_0 are to be modified by replacing ϵ_0 with $K \epsilon_0$

For example (field in vacuum)



electric field at P in vacuum

$$\vec{E}_0 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$



(Field in a medium)

The electric field at the same point P and for the same charge at the same distance r but the region is filled with a material of dielectric constant K is

$$\vec{E} = \frac{1}{4\pi K\epsilon_0}\frac{q}{r^2}\hat{r} = \frac{E_0}{K}$$

Similarly

In vacuum

$$E_0 = \frac{\sigma}{\epsilon_0}$$

$$E = \frac{\sigma}{K\epsilon_0} = \frac{E_0}{K}$$ (In medium)

When NaCl is dissolved in water it dissociates into Na^+ & Cl^-

why?

Table 25-1 Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

[a]Measured at room temperature, except for the water.

Capacitor with a Dielectric

$C = \kappa C_0$. κ = dielectric Constant

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

dielectric Strength ?

V = a constant

(*a*)

q = a constant

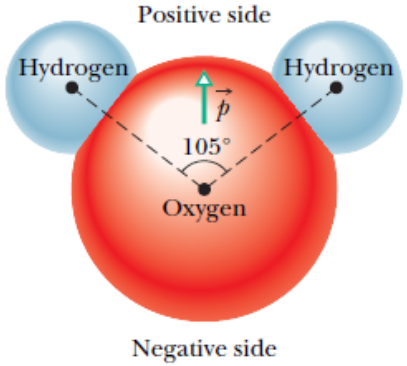
(*b*)

more charge same pd

Same charge less pd.

Figure 25-13 (*a*) If the potential difference between the plates of a capacitor is maintained, as by battery B, the effect of a dielectric is to increase the charge on the plates. (*b*) If the charge on the capacitor plates is maintained, as in this case, the effect of a dielectric is to reduce the potential difference between the plates. The scale shown is that of a *potentiometer,* a device used to measure potential difference (here, between the plates). A capacitor cannot discharge through a potentiometer.

$C = Q/V$



$\uparrow p$

Fig. 22-18 A molecule of H_2O, showing the three nuclei (represented by dots) and the regions in which the electrons can be located. The electric dipole moment $\vec{p}$ points from the (negative) oxygen side to the (positive) hydrogen side of the molecule.

Polar dielectrics.

Nonpolar dielectrics.

Polar dielectrics.

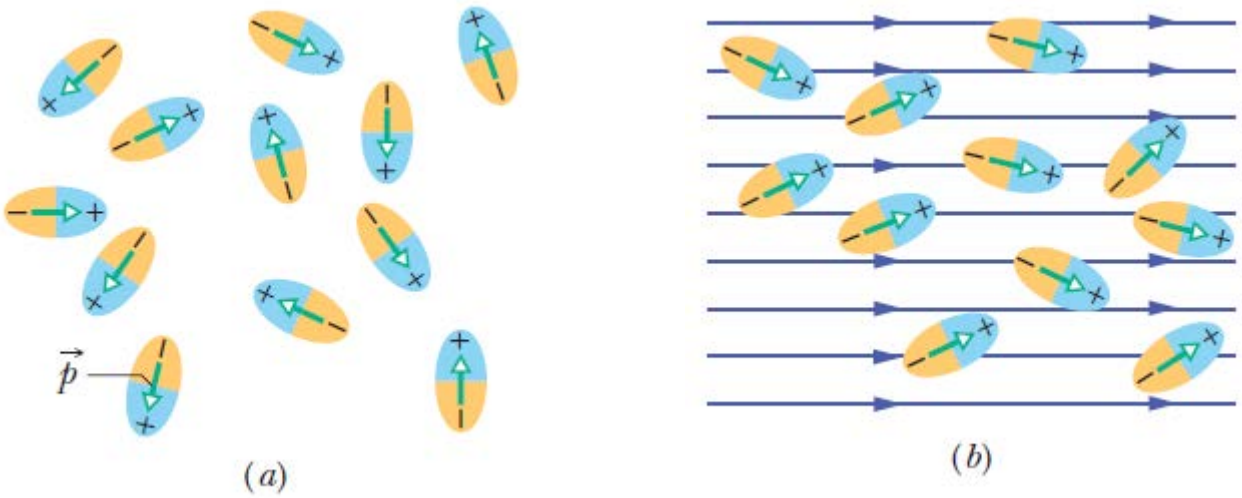


Figure 25-14 (*a*) Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field. (*b*) An electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.

Explain E_0, E and E'

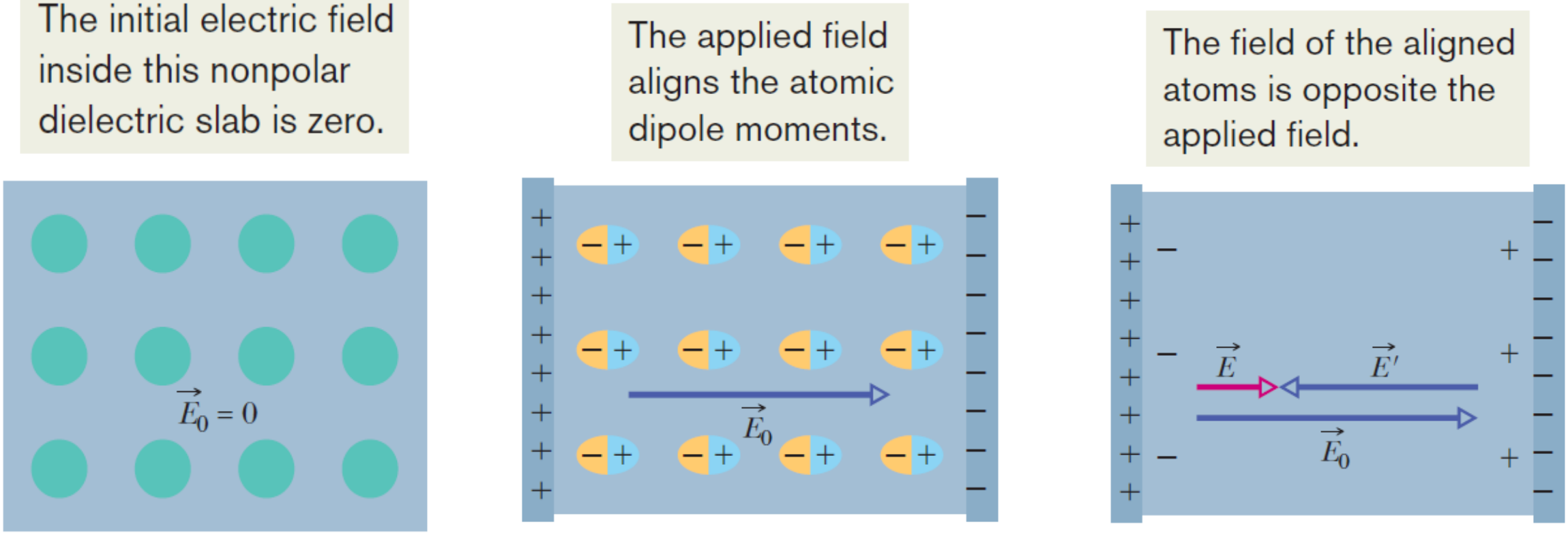


Fig. 25-15 (*a*) A nonpolar dielectric slab. The circles represent the electrically neutral atoms within the slab. (*b*) An electric field is applied via charged capacitor plates; the field slightly stretches the atoms, separating the centers of positive and negative charge. (*c*) The separation produces surface charges on the slab faces. These charges set up a field $\vec{E}'$, which opposes the applied field $\vec{E}_0$. The resultant field $\vec{E}$ inside the dielectric (the vector sum of $\vec{E}_0$ and $\vec{E}'$) has the same direction as $\vec{E}_0$ but a smaller magnitude.

$E_0 = 0$

$E = 0$

$E_0 \neq 0$

E_0 $E' \neq 0$

$E = E_0 - E'$

Dielectrics and Gauss' Law

without dielectric

$\epsilon_0 \oint \vec{E}_0 \cdot d\vec{A} = q$

$\epsilon_0 E_0 A = q$

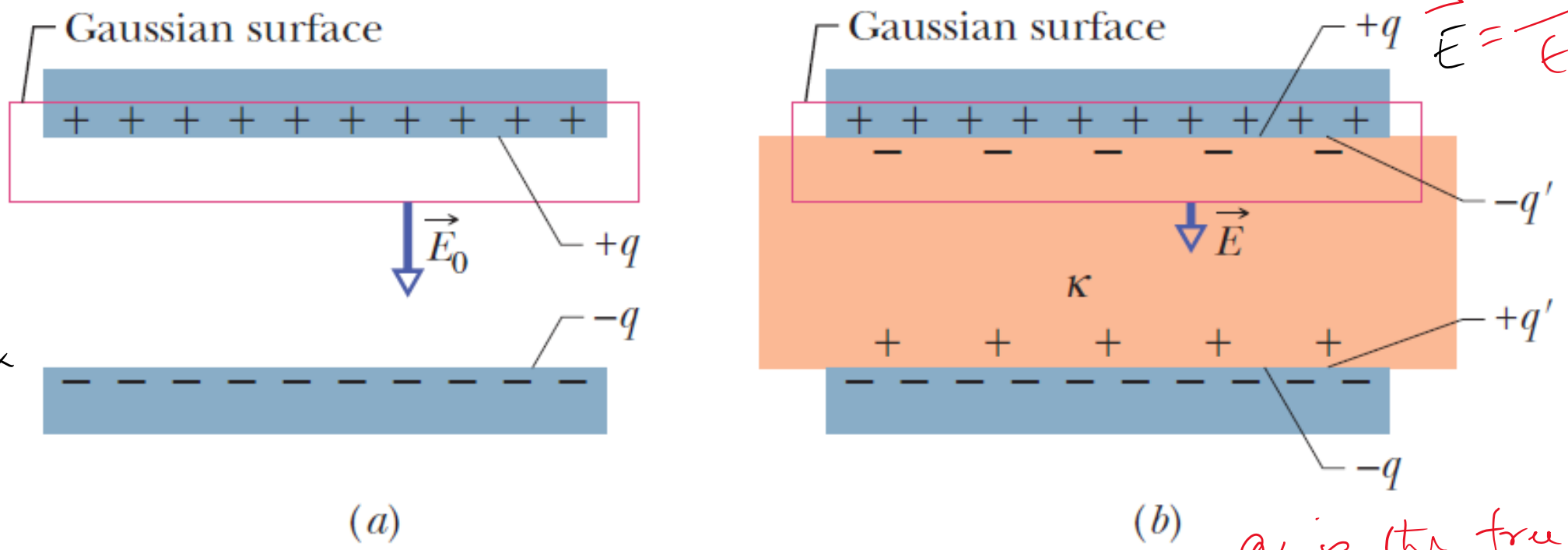
$\therefore E_0 = \dfrac{q}{\epsilon_0 A}$

With dielectric

$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q - q'$

$\epsilon_0 E A = q - q'$

$\vec{E} = \dfrac{q - q'}{\epsilon_0 A}$



q is the free charge

q' is the induced bound charge

Figure 25-16 A parallel-plate capacitor (a) without and (b) with a dielectric slab inserted. The charge q on the plates is assumed to be the same in both cases.

We have $E_0 = \dfrac{q}{\epsilon_0 A}$ $\quad E = \dfrac{q - q'}{\epsilon_0 A}$ $\quad q' = ?$

Also $E = \dfrac{E_0}{K} = \boxed{\dfrac{q}{K\,\epsilon_0 A} = E = \dfrac{q - q'}{\epsilon_0 A}}$

$\therefore \quad q - q' = \dfrac{q}{K} \quad \therefore \quad q' = q - \dfrac{q}{K}$

$E = \dfrac{E_0}{K}$

$E_0 = KE$

Gauss' Law with dielectric

The Gauss' law $\quad \epsilon_0 \oint \vec{E}_0 \cdot d\vec{A} = q$

Now $E_0 = KE$ $\quad \boxed{\therefore \epsilon_0 \oint K\vec{E} \cdot d\vec{A} = q}$

(1) The flux integral in the Gauss law now involves KE not just $\vec{E}$.

(2) The charge q enclosed by the Gaussian surface now is only the free charge. The induced, surface bound charges are deliberately ignored on the right hand side of the equation having been taken into consideration by introducing the dielectric constant k on the left hand side.

The electric displacement vector D

$$\epsilon_0 \oint K \vec{E} \cdot d\vec{A} = q$$ Gauss' law with dielectric

$$\oint \epsilon_0 K \vec{E} \cdot d\vec{A} = q$$

$$\oint \vec{D} \cdot d\vec{A} = q$$

$\vec{D} = \epsilon_0 K \vec{E}$ is called the electric displacement vector.

$$= \epsilon_0 \frac{\epsilon}{\epsilon_0} \vec{E}$$

$$= \epsilon \vec{E}$$

$$\boxed{\begin{aligned} \vec{D} &= \epsilon_0 K \vec{E} \\ \vec{D} &= \epsilon \vec{E} \end{aligned}}$$

Work and energy when a dielectric is inserted into a capacitor

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference $V = 12.5$ V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

KEY IDEA

We can relate the potential energy U_i of the capacitor to the capacitance C and either the potential V (with Eq. 25-22) or the charge q (with Eq. 25-21):

$$U_i = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}.$$

Calculation: Because we are given the initial potential V (= 12.5 V), we use Eq. 25-22 to find the initial stored energy:

$$U_i = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(13.5 \times 10^{-12}\ \text{F})(12.5\ \text{V})^2$$
$$= 1.055 \times 10^{-9}\ \text{J} = 1055\ \text{pJ} \approx 1100\ \text{pJ}. \quad \text{(Answer)}$$

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

KEY IDEA

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change.

Calculations: Thus, we must now use Eq. 25-21 to write the final potential energy U_f, but now that the slab is within the capacitor, the capacitance is κC. We then have

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055\ \text{pJ}}{6.50}$$
$$= 162\ \text{pJ} \approx 160\ \text{pJ}. \quad \text{(Answer)}$$

When the slab is introduced, the potential energy decreases by a factor of κ.

The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162)\ \text{pJ} = 893\ \text{pJ}.$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.

REVIEW & SUMMARY

Capacitor; Capacitance A **capacitor** consists of two isolated conductors (the *plates*) with charges $+q$ and $-q$. Its **capacitance** C is defined from

$$q = CV, \tag{25-1}$$

where V is the potential difference between the plates.

Determining Capacitance We generally determine the capacitance of a particular capacitor configuration by (1) assuming a charge q to have been placed on the plates, (2) finding the electric field $\vec{E}$ due to this charge, (3) evaluating the potential difference V, and (4) calculating C from Eq. 25-1. Some specific results are the following:

A *parallel-plate capacitor* with flat parallel plates of area A and spacing d has capacitance

$$C = \frac{\varepsilon_0 A}{d}. \tag{25-9}$$

A *cylindrical capacitor* (two long coaxial cylinders) of length L and radii a and b has capacitance

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}. \tag{25-14}$$

A *spherical capacitor* with concentric spherical plates of radii a and b has capacitance

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}. \tag{25-17}$$

An *isolated sphere* of radius R has capacitance

$$C = 4\pi\varepsilon_0 R. \tag{25-18}$$

Capacitors in Parallel and in Series The **equivalent capacitances** C_{eq} of combinations of individual capacitors connected in **parallel** and in **series** can be found from

$$C_{eq} = \sum_{j=1}^{n} C_j \qquad (n \text{ capacitors in parallel}) \tag{25-19}$$

and

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \qquad (n \text{ capacitors in series}). \tag{25-20}$$

Equivalent capacitances can be used to calculate the capacitances of more complicated series–parallel combinations.

Potential Energy and Energy Density The **electric potential energy** U of a charged capacitor,

$$U = \frac{q^2}{2C} = \tfrac{1}{2}CV^2, \tag{25-21, 25-22}$$

is equal to the work required to charge the capacitor. This energy can be associated with the capacitor's electric field $\vec{E}$. By extension we can associate stored energy with any electric field. In vacuum, the **energy density** u, or potential energy per unit volume, within an electric field of magnitude E is given by

$$u = \tfrac{1}{2}\varepsilon_0 E^2. \tag{25-25}$$

Capacitance with a Dielectric If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ, called the **dielectric constant,** which is characteristic of the material. In a region that is completely filled by a dielectric, all electrostatic equations containing ε_0 must be modified by replacing ε_0 with $\kappa\varepsilon_0$.

The effects of adding a dielectric can be understood physically in terms of the action of an electric field on the permanent or induced electric dipoles in the dielectric slab. The result is the formation of induced charges on the surfaces of the dielectric, which results in a weakening of the field within the dielectric for a given amount of free charge on the plates.

Gauss' Law with a Dielectric When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q. \tag{25-36}$$

Here q is the free charge; any induced surface charge is accounted for by including the dielectric constant κ inside the integral.

Midterm Exam Summer 2023

PHY 112 Section 4

Date : 20-Jul-23 (Thursday)

Time : 11:00 AM to 12:00 PM

Place : UB 20204

Syllabus : Chapters 21 to 24 of HRW, Edition 10 (extended)

Topics : Coulomb's law, Electric fields, Gauss' law, Electric Potential

CURRENT AND RESISTANCE

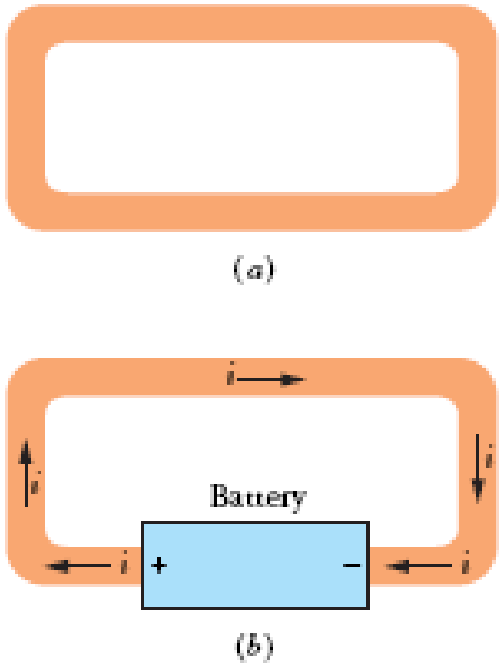


Fig. 26-1 (*a*) A loop of copper in electrostatic equilibrium. The entire loop is at a single potential, and the electric field is zero at all points inside the copper. (*b*) Adding a battery imposes an electric potential difference between the ends of the loop that are connected to the terminals of the battery. The battery thus produces an electric field within the loop, from terminal to terminal, and the field causes charges to move around the loop. This movement of charges is a current *i*.

The current is the same in any cross section.

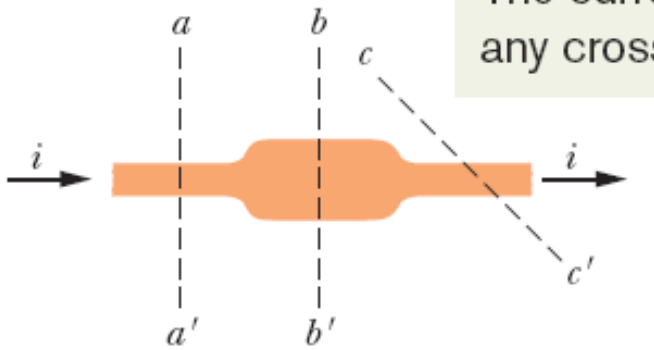


Fig. 26-2 The current *i* through the conductor has the same value at planes *aa′*, *bb′*, and *cc′*.

$i = \frac{dq}{dt}$ definition of current

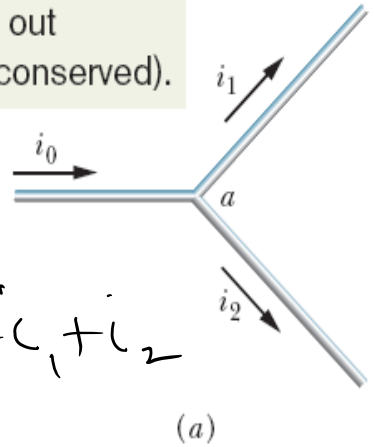
$i_{av} = \frac{q}{t}$ Also $dq = i\,dt$

$\therefore q = \int dq = \int_0^t i\,dt$ unit = Amp

A = C/S

direction of current ?

The current into the junction must equal the current out (charge is conserved).



$i_0 = i_1 + i_2$