Welcome to the Physics class

Principles of Physics II

Lesson 18

Magnetic Field Due to a Current in a Circular Arc of Wire

angle betn $d\vec{s}$ and $\hat{r}$ is $\theta = 90^\circ$

BS law

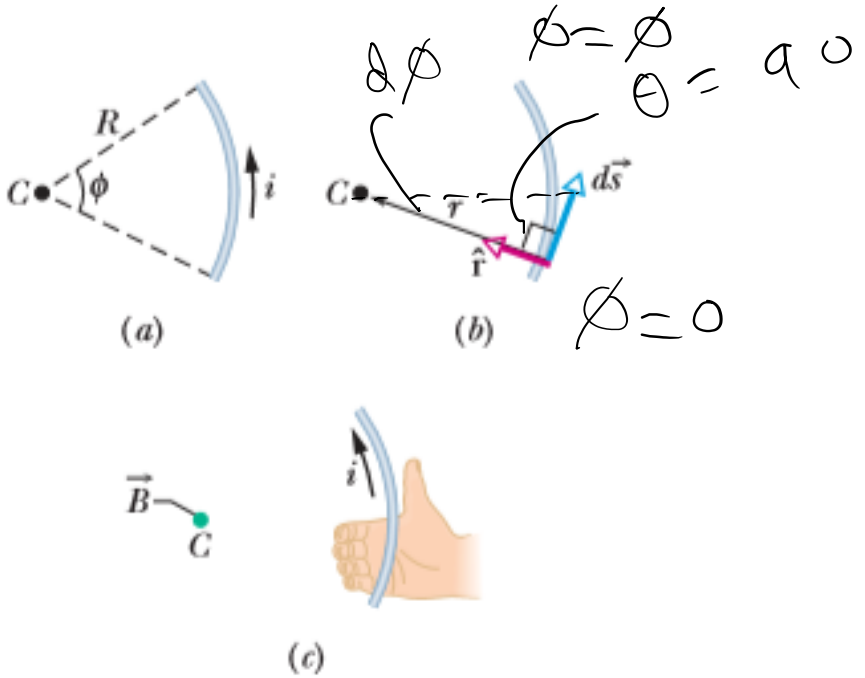
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{R^2}$$

$$= \frac{\mu_0}{4\pi} \frac{i\, ds}{R^2}$$

$$d\phi = \frac{ds}{R} \Rightarrow ds = R\, d\phi$$

$$\therefore B = \int_0^{\phi} dB = \int_0^{\phi} \frac{\mu_0}{4\pi} \frac{i R\, d\phi}{R^2}$$

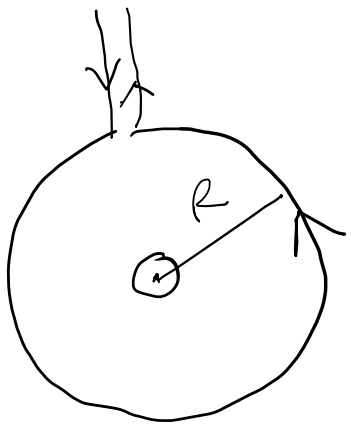
$$B = \frac{\mu_0 i}{4\pi R} \int_0^{\phi} d\phi = \frac{\mu_0 i}{4\pi R} \phi$$



The right-hand rule reveals the field's direction at the center.

Fig. 29-6 (*a*) A wire in the shape of a circular arc with center *C* carries current *i*. (*b*) For any element of wire along the arc, the angle between the directions of $d\vec{s}$ and $\hat{r}$ is 90°. (*c*) Determining the direction of the magnetic field at the center *C* due to the current in the wire; the field is out of the page, in the direction of the fingertips, as indicated by the colored dot at *C*.

$B = \frac{\mu_0 i}{4\pi R}\phi$ is the magnetic field at the center of an arc extending an angle ϕ at the center. The direction will be coming out of the plane of the paper (apply the R. H. Rule).



Magnetic field at the center of current carrying circular loop is coming out of the plane and the magnitude is

$$B = \frac{\mu_0 i}{4\pi R} 2\pi = \frac{\mu_0 i}{2R}$$

(for a complete circle $\phi = 2\pi$)

Force Between Two Parallel Currents (infinite conductors)

$$B = \frac{\mu_0 i}{2\pi R}$$

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

$$F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

The field due to a at the position of b creates a force on b.

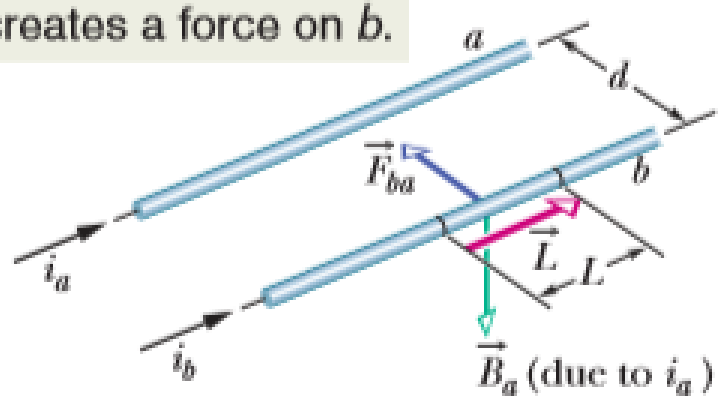


Fig. 29-9 Two parallel wires carrying currents in the same direction attract each other. $\vec{B}_a$ is the magnetic field at wire b produced by the current in wire a. $\vec{F}_{ba}$ is the resulting force acting on wire b because it carries current in $\vec{B}_a$.

To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

Remember

Parallel currents attract each other, and antiparallel currents repel each other.

Ampere's Law

Only the currents encircled by the loop are used in Ampere's law.

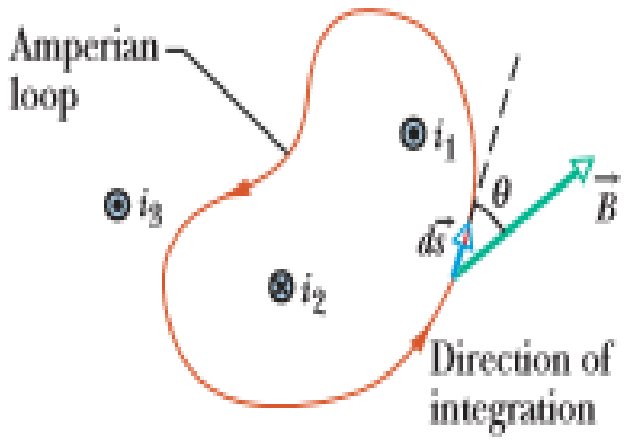


Fig. 29-11 Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{encl} \quad \text{(Ampere's Law)}$$

Meaning $\rightarrow$?

$$\oint \vec{B} \cdot d\vec{s} = \oint B\, ds \cos\theta = \mu_0 i_{encl}.$$

The closed loop of the line integral implies integration around the Amperian loop ?

Sign convention of the current ?

Refer to figure: $i_{encl} = i_1 - i_2$

Then $\oint B \cos\theta\, ds = \mu_0 (i_1 - i_2)$

Magnetic Field Outside a Long Straight Wire with Current

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

All of the current is encircled and thus all is used in Ampere's law.

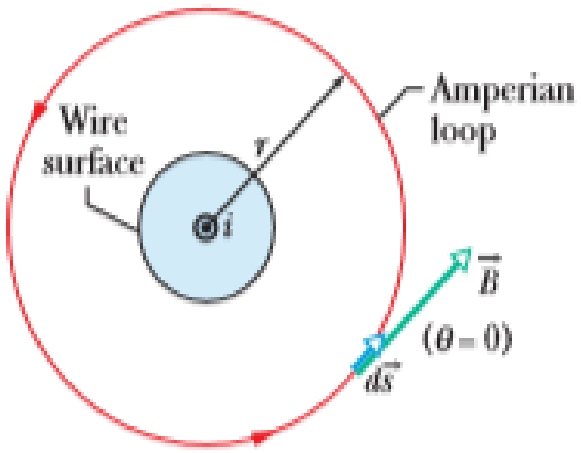


Fig. 29-13 Using Ampere's law to find the magnetic field that a current i produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

The field B has cylindrical symmetry about the wire.
Take concentric circular Amperian loop.

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = \underline{B(2\pi r). = \mu_0 i}$$

$$B = \frac{\mu_0 i}{2\pi r}$$

at a $\perp r$. distance r from a long (ideally) infinitely long current carrying wire

Magnetic Field Inside a Long Straight Wire with Current

Long straight (thick) conductor of radius R carrying a current i, coming out of the plane which is uniformly distributed across the x-section of the thick conductor.

B is cylindrically symmetric.

For B at a distance of r from the axis of the conductor, draw an Amperian loop of radius r. $r < R$

(1) $\oint \vec{B} \cdot d\vec{s} = B \oint ds = B\, 2\pi r = \mu_0 i_{encl}$

Now $i_{encl} = i \dfrac{\pi r^2}{\pi R^2} = i \dfrac{r^2}{R^2}$

Only the current encircled by the loop is used in Ampere's law.

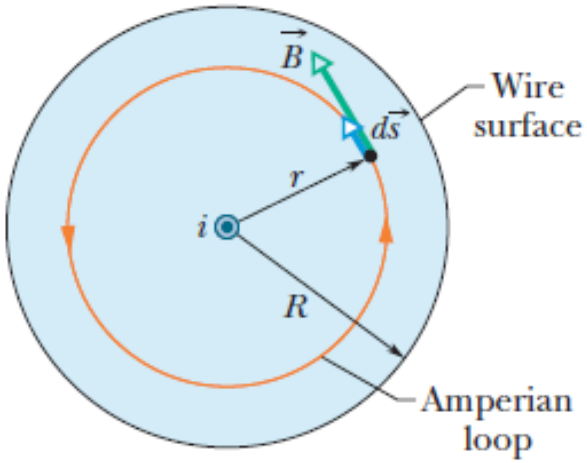


Fig. 29-14 Using Ampere's law to find the magnetic field that a current *i* produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

Using Ampere's law $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{encl}$.

$$B 2\pi r = \mu_0 i \frac{r^2}{R^2}$$

$$\therefore \quad B = \frac{\mu_0 i}{2\pi R^2} r$$ inside the thick conducting wire.

(2) at the surface of the conductor $r = R$

$$\therefore \quad B = \frac{\mu_0 i}{2\pi R^2} R = \frac{\mu_0 i}{2\pi R}$$

(3) For any point outside the conductor at a $\perp r$ distance r'

$$B = \frac{\mu_0 i}{2\pi r'}$$

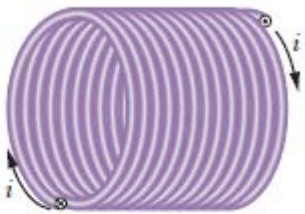


Fig. 29-16 A solenoid carrying current i.

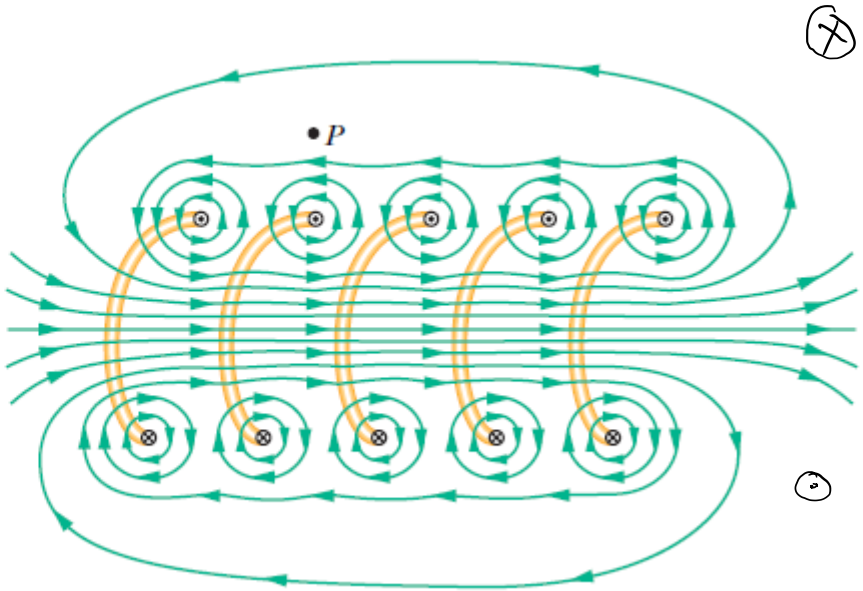


Fig. 29-17 A vertical cross section through the central axis of a "stretched-out" solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid. Each turn produces circular magnetic field lines near itself. Near the solenoid's axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.

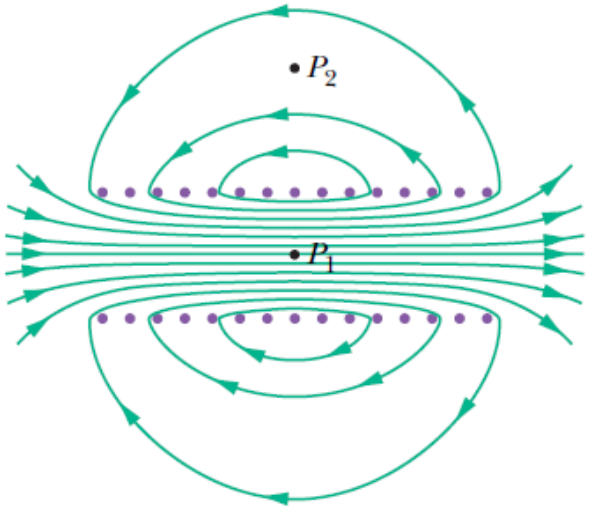


Fig. 29-18 Magnetic field lines for a real solenoid of finite length. The field is strong and uniform at interior points such as P_1 but relatively weak at external points such as P_2.

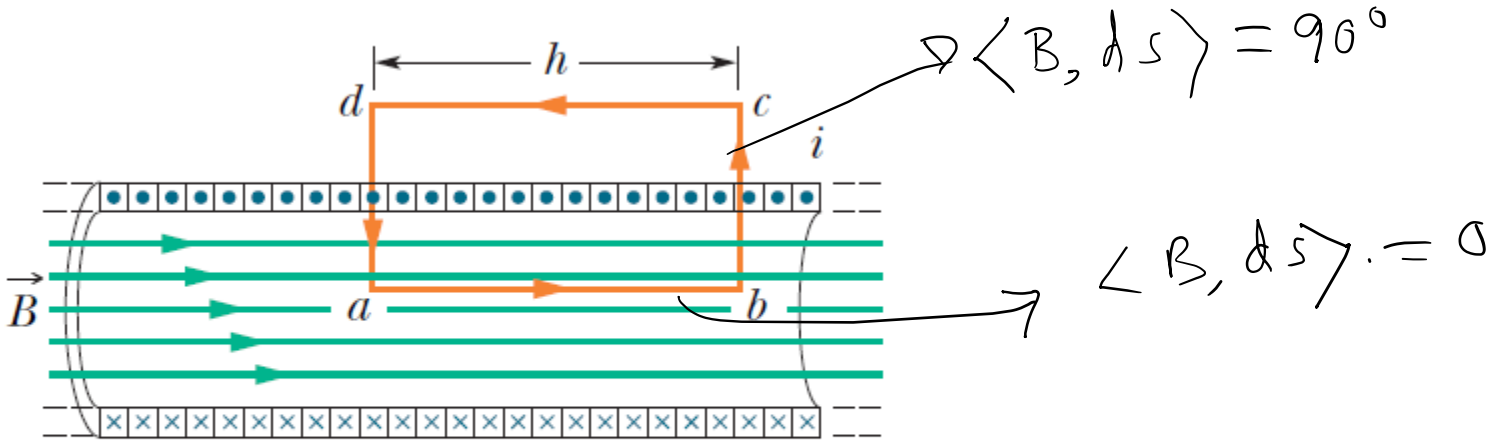
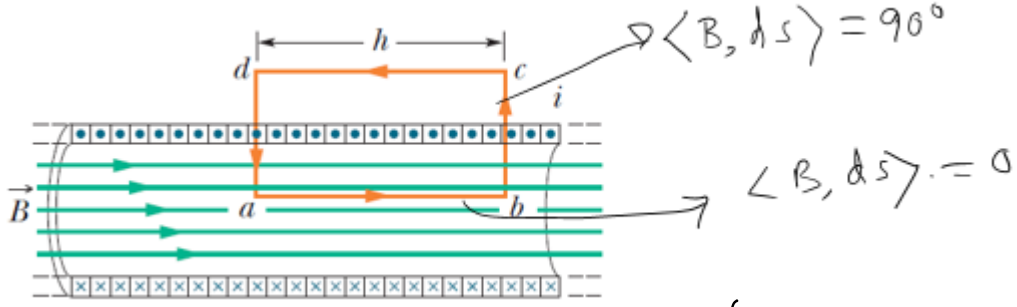


Fig. 29-19 Application of Ampere's law to a section of a long ideal solenoid carrying a current i. The Amperian loop is the rectangle $abcda$.



The golden rectangle is the Amperian loop

Ampere's law: $\oint \vec{B} \cdot d\vec{s} = \underbrace{\int_a^b \vec{B} \cdot d\vec{s}}_{Bh} + \underbrace{\int_b^c \vec{B} \cdot d\vec{s}}_{0} + \underbrace{\int_c^d \vec{B} \cdot d\vec{s}}_{0} + \underbrace{\int_d^a \vec{B} \cdot d\vec{s}}_{0}$

$B = 0$

or $\oint \vec{B} \cdot d\vec{s} = Bh + 0 + 0 + 0 = \mu_0 i_{encl}$

i_{encl} = current enclosed by the Amperian loop here

$i_{encl} = n \cdot h \cdot i$ $\quad$ n = number of turns per unit length.

$\therefore \quad Bh = \mu_0 n h i$

$B = \mu_0 n i$ $\quad$ where i is the current in the solenoid.

A solenoid has length L= 1.23 m and inner diameter d = 3.55 cm and carries a current i = 5.57 A

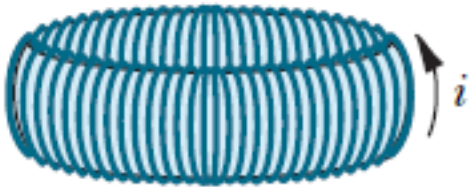
It consists of five closed packed layers each with 850 turns along L. What is B at the center

$$B = \mu_0 n i = 4\pi \times 10^{-7}\ \mathrm{T\,m\,A^{-1}} \cdot \frac{5 \times 850}{1.23} \times 5.57\ \mathrm{A}$$

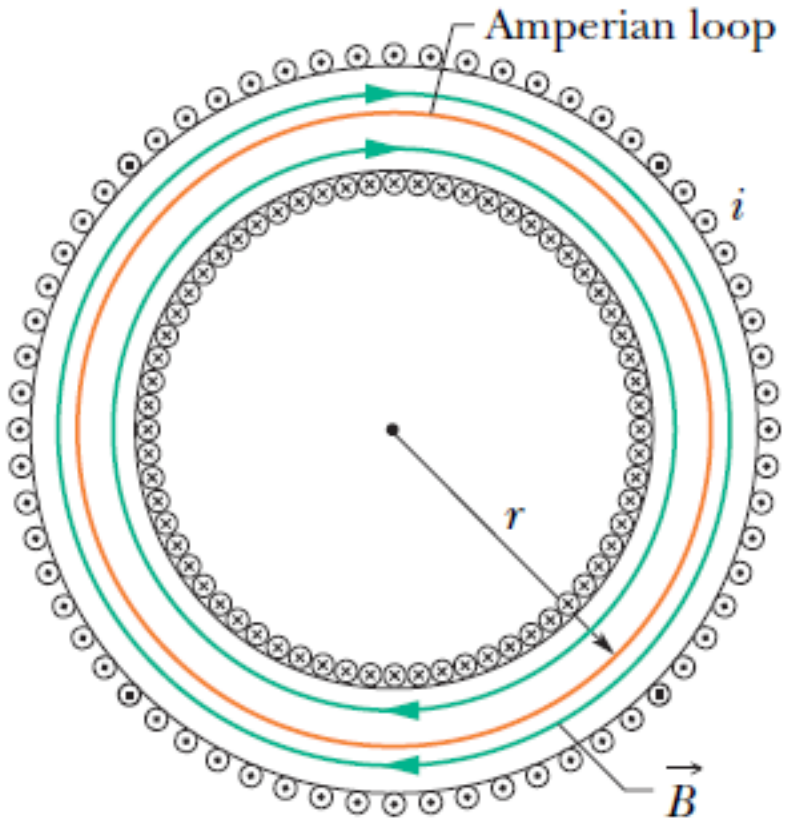
$$= 2.42 \times 10^{-2}\ \mathrm{T}$$

$$= 24.2\ \mathrm{mT}$$

Magnetic Field of a Toroid

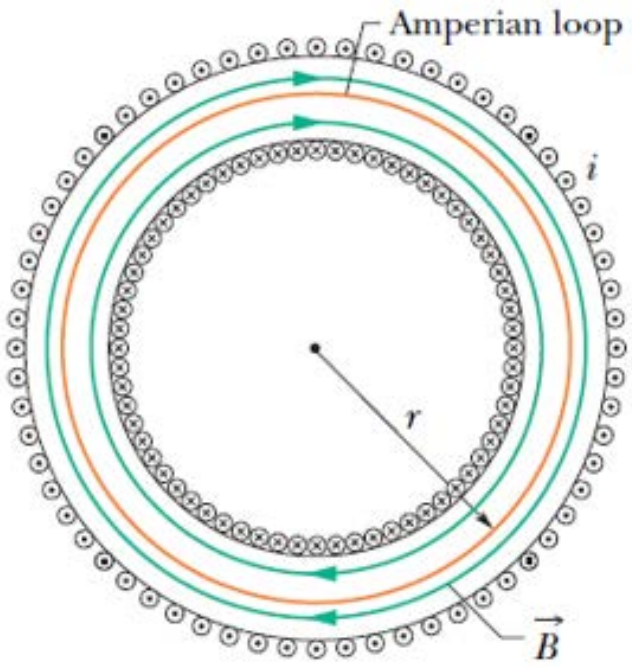


(*a*)



(*b*)

Fig. 29-20 (*a*) A toroid carrying a current *i*. (*b*) A horizontal cross section of the toroid. The interior magnetic field (inside the bracelet-shaped tube) can be found by applying Ampere's law with the Amperian loop shown.



(b)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (N i)$$

$$B\, 2\pi r = \mu_0 N i$$

$$B = \frac{\mu_0 N i}{2\pi} \frac{1}{r}$$

B is not uniform inside the toroid like the ideal solenoid..
But do not have edge effect (?)

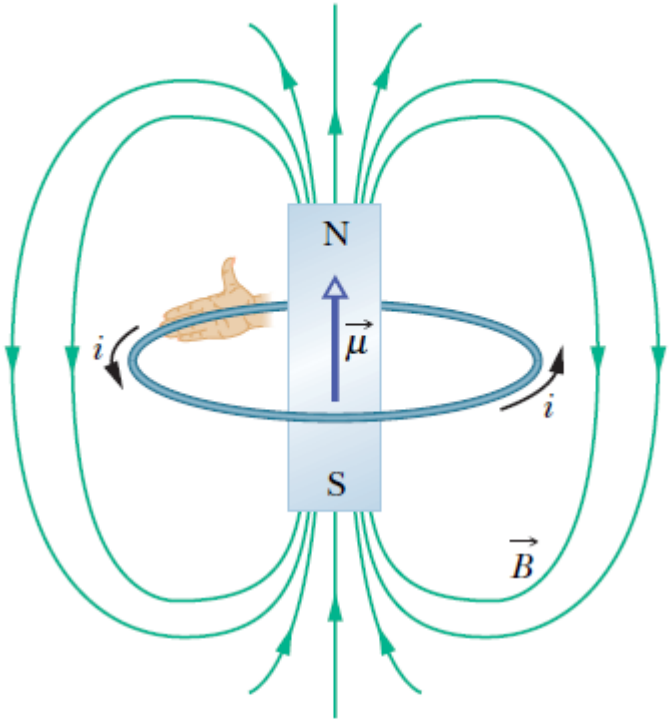


Fig. 29-21 A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment $\vec{\mu}$ of the loop, its direction given by a curled–straight right-hand rule, points from the south pole to the north pole, in the direction of the field $\vec{B}$ within the loop.

REVIEW & SUMMARY

The Biot–Savart Law The magnetic field set up by a current-carrying conductor can be found from the *Biot–Savart law*. This law asserts that the contribution $d\vec{B}$ to the field produced by a current-length element $i\,d\vec{s}$ at a point P located a distance r from the current element is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{r}}{r^2} \qquad \text{(Biot–Savart law).} \qquad (29\text{-}3)$$

Here $\hat{r}$ is a unit vector that points from the element toward P. The quantity μ_0, called the permeability constant, has the value

$$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T \cdot m/A}.$$

Magnetic Field of a Long Straight Wire For a long straight wire carrying a current i, the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance R from the wire,

$$B = \frac{\mu_0 i}{2\pi R} \qquad \text{(long straight wire).} \qquad (29\text{-}4)$$

Magnetic Field of a Circular Arc The magnitude of the magnetic field at the center of a circular arc, of radius R and central angle ϕ (in radians), carrying current i, is

$$B = \frac{\mu_0 i \phi}{4\pi R} \qquad \text{(at center of circular arc).} \qquad (29\text{-}9)$$

Force Between Parallel Currents Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length L of either wire is

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}, \qquad (29\text{-}13)$$

where d is the wire separation, and i_a and i_b are the currents in the wires.

Ampere's Law **Ampere's law** states that

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law).} \qquad (29\text{-}14)$$

The line integral in this equation is evaluated around a closed loop called an *Amperian loop*. The current i on the right side is the *net* current encircled by the loop. For some current distributions, Eq. 29-14 is easier to use than Eq. 29-3 to calculate the magnetic field due to the currents.

Fields of a Solenoid and a Toroid Inside a *long solenoid* carrying current i, at points not near its ends, the magnitude B of the magnetic field is

$$B = \mu_0 i n \qquad \text{(ideal solenoid),} \qquad (29\text{-}23)$$

where n is the number of turns per unit length. At a point inside a *toroid*, the magnitude B of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r} \qquad \text{(toroid),} \qquad (29\text{-}24)$$

where r is the distance from the center of the toroid to the point.

Field of a Magnetic Dipole The magnetic field produced by a current-carrying coil, which is a *magnetic dipole*, at a point P located a distance z along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}, \qquad (29\text{-}27)$$

where $\vec{\mu}$ is the dipole moment of the coil. This equation applies only when z is much greater than the dimensions of the coil.

INDUCTION AND INDUCTANCE

Faraday's law of induction

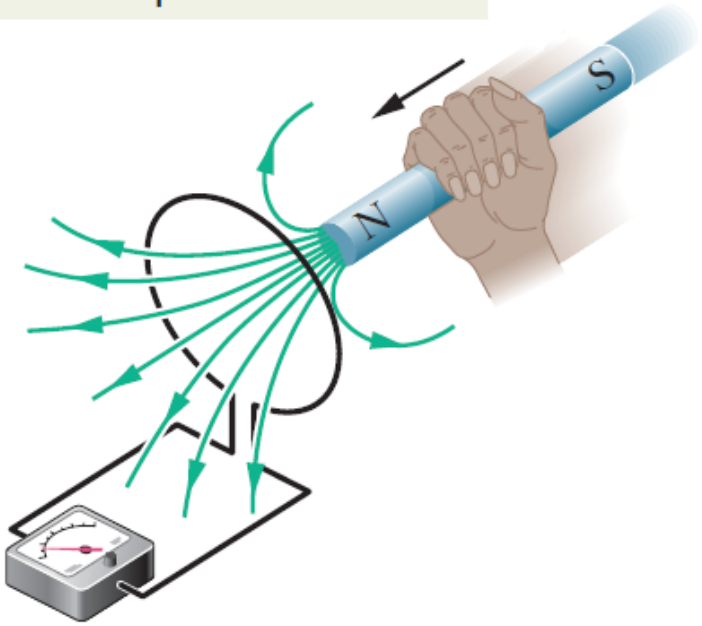


Fig. 30-1 An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

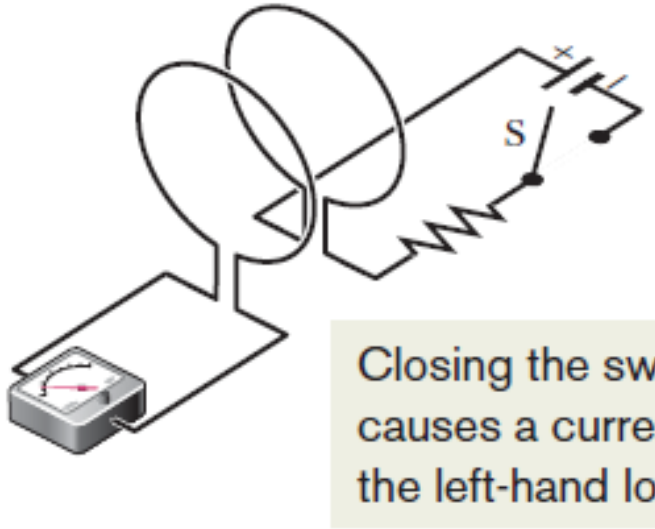


Figure 30-2 An ammeter registers a current in the left-hand wire loop just as switch S is closed (to turn on the current in the right-hand wire loop) or opened (to turn off the current in the right-hand loop). No motion of the coils is involved.

1) Current appears in the loop when there is a relative motion between the coil and the magnet.

(2) Faster motion produces stronger current.

(3) Increasing and decreasing lines of force through the coil produces current in opposite directions

(4) Switching on & off of the current in the 2nd circuit also produces current in the opposite direction.

(Next slide)

Magnetic flux

Suppose a conducting loop like the one in figure 30.1 enclosing an area A is placed in a magnetic field (like the one produced by the bar magnet in fig 30.1) $\vec{B}$, then the magnetic flux through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \qquad \text{(magnetic flux through area } A\text{)}$$

Similar to electric flux $\Phi_E = \int \vec{E} \cdot d\vec{A}$

$d\vec{A}$ is a vector of magnitude dA that is $\perp$r to the differential area dA. As a special case when $\vec{B}$ is $\perp$r to dA. Then

$$\Phi_B = BA \qquad (\vec{B} \perp \text{area } A,\ \vec{B} \text{ uniform}).$$

Unit; 1 weber = 1 Wb = 1 T · m^2.

(3) There was no cell / source of emf in the first coil (ckt). An emf is required to produce current in a circuit. Is there any emf produced in the coil when the magnetic flux through the coil changes (with time)? The answer is yes.

The magnitude of the emf $\mathcal{E}$ (xi) induced in a conducting loop is equal to the rate at which the magnetic flux ϕ_B through that loop changes with time.

Following is the mathematical statement of Faraday's law of electromagnetic induction:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)},$$

The minus sign indicating that the induced emf tends to oppose the flux change.

If there are N of turns in the coil (N number of loops) then:

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} \qquad \text{(coil of } N \text{ turns).}$$

30-4 Lenz's Law

Soon after Faraday proposed his law of induction, Heinrich Friedrich Lenz devised a rule for determining the direction of an induced current in a loop:

> An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.

The nature of the induced emf is to oppose the very cause to which it is due.

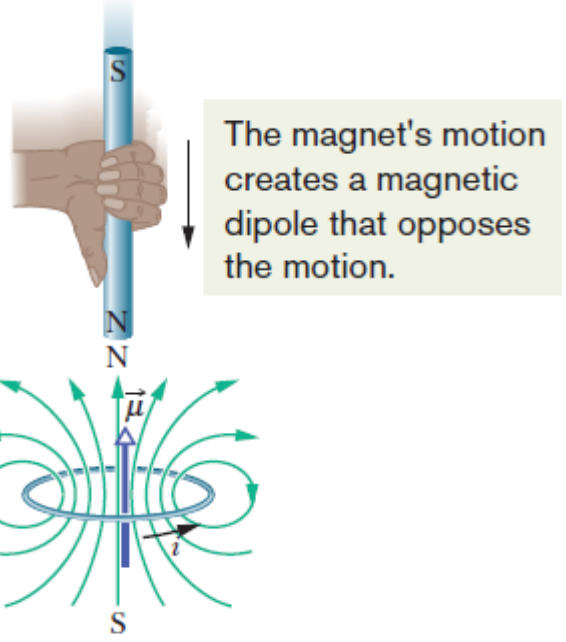


Fig. 30-4 Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

Explain Lenz's law! ① Opposition to pole movement.