Welcome to the Physics class

Principles of Physics II

Lecture 6

Gauss’ Law

Electric Flux

Flux ?

Flat Surface, Uniform Field.

$$\Delta\Phi = (E\cos\theta)\,\Delta A. \qquad \Delta\phi = \vec{E}\cdot\Delta\vec{A}$$

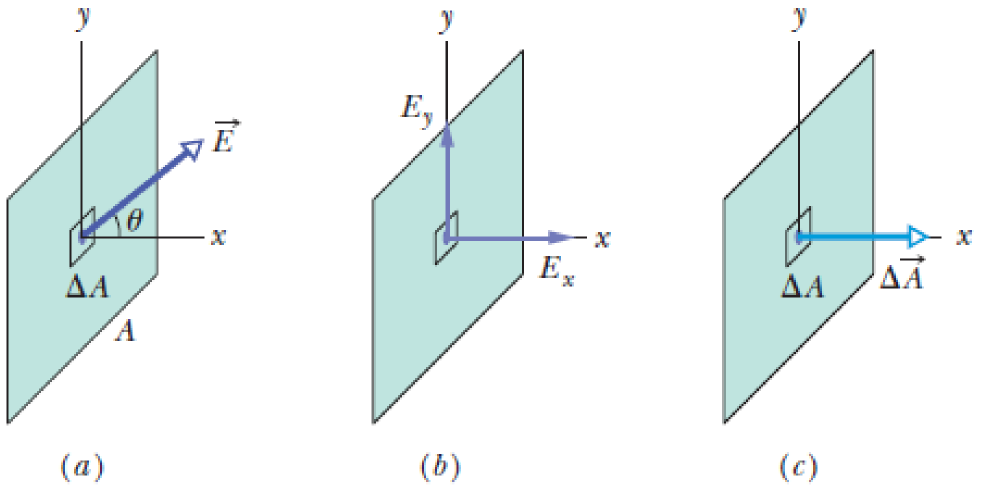


Figure 23-4 (*a*) An electric field vector pierces a small square patch on a flat surface. (*b*) Only the *x* component actually pierces the patch; the *y* component skims across it. (*c*) The area vector of the patch is perpendicular to the patch, with a magnitude equal to the patch's area.

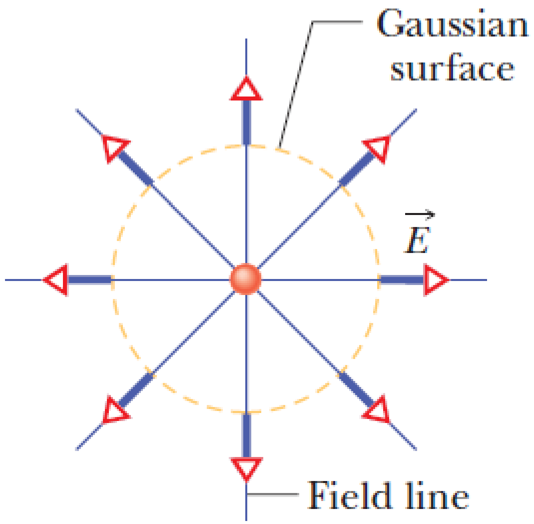


Figure 23-1 Electric field vectors and field lines pierce an imaginary, spherical Gaussian surface that encloses a particle with charge $+Q$.

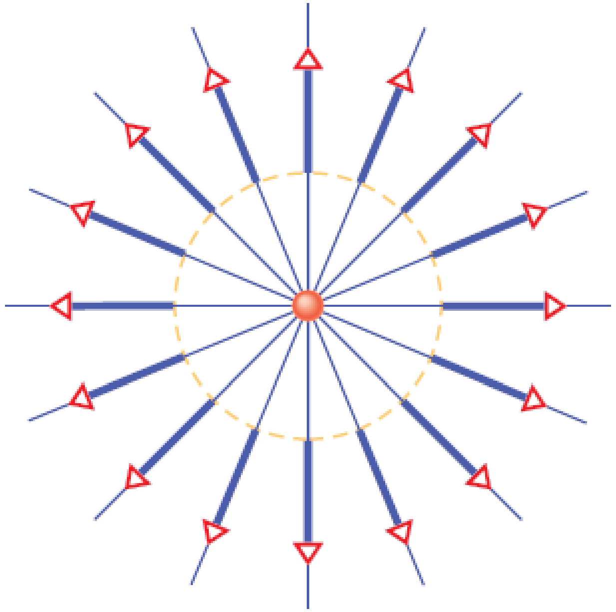


Figure 23-2 Now the enclosed particle has charge $+2Q$.

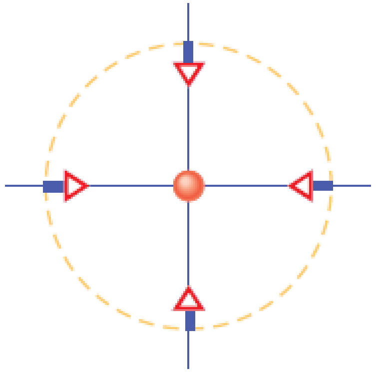


Figure 23-3 Can you tell what the enclosed charge is now?

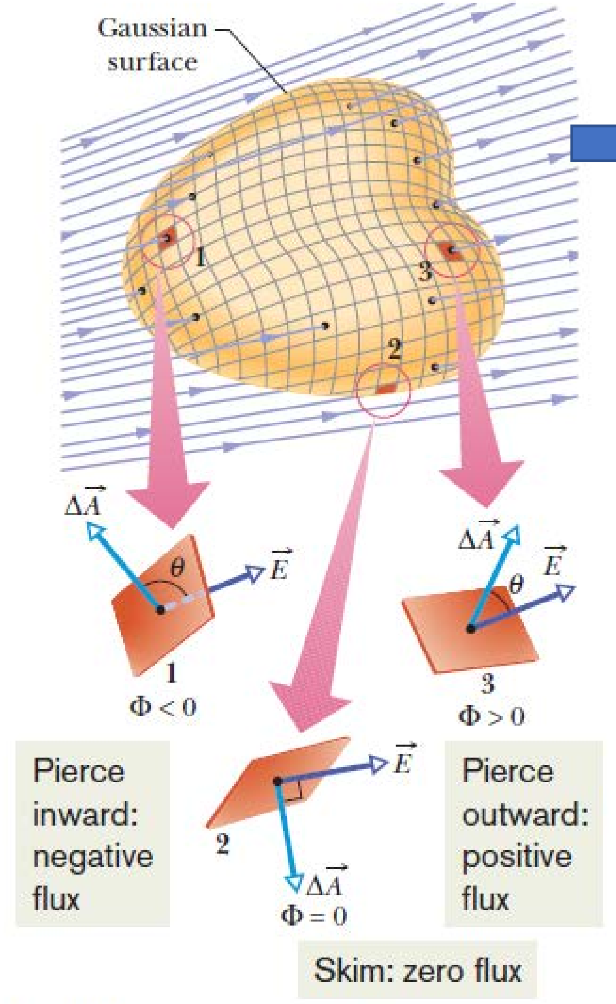


Figure 23-5 A Gaussian surface of arbitrary shape immersed in an electric field. The surface is divided into small squares of area ΔA. The electric field vectors $\vec{E}$ and the area vectors $\Delta\vec{A}$ for three representative squares, marked 1, 2, and 3, are shown.

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface).}$$

✓ unit of flux is $\frac{N}{C} m^2$

An inward piercing field is negative flux. An outward piercing field is positive flux. A skimming field is zero flux.

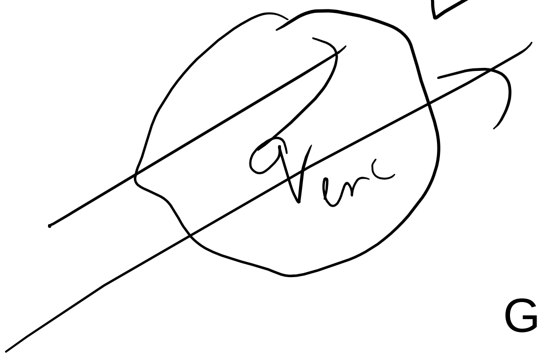
Flux through a Gaussian surface (closed surface).

$$\phi = \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i$$

$$\Sigma \to \int$$

$$\phi = \oint \vec{E} \cdot d\vec{A} \quad \text{etc}$$

ϵ_0 $\vec{E}$



Surface

Gauss' Law

Gauss' law relates the net flux of an electric field through a closed surface (called a Gaussian surface) to the net charge q enclosed by the Gaussian according to the following equation

In some books

$$\phi = \frac{q_{encl}}{\epsilon_0}$$

$$\phi = \oint \vec{E} \cdot d\vec{A}$$

$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law).}$$

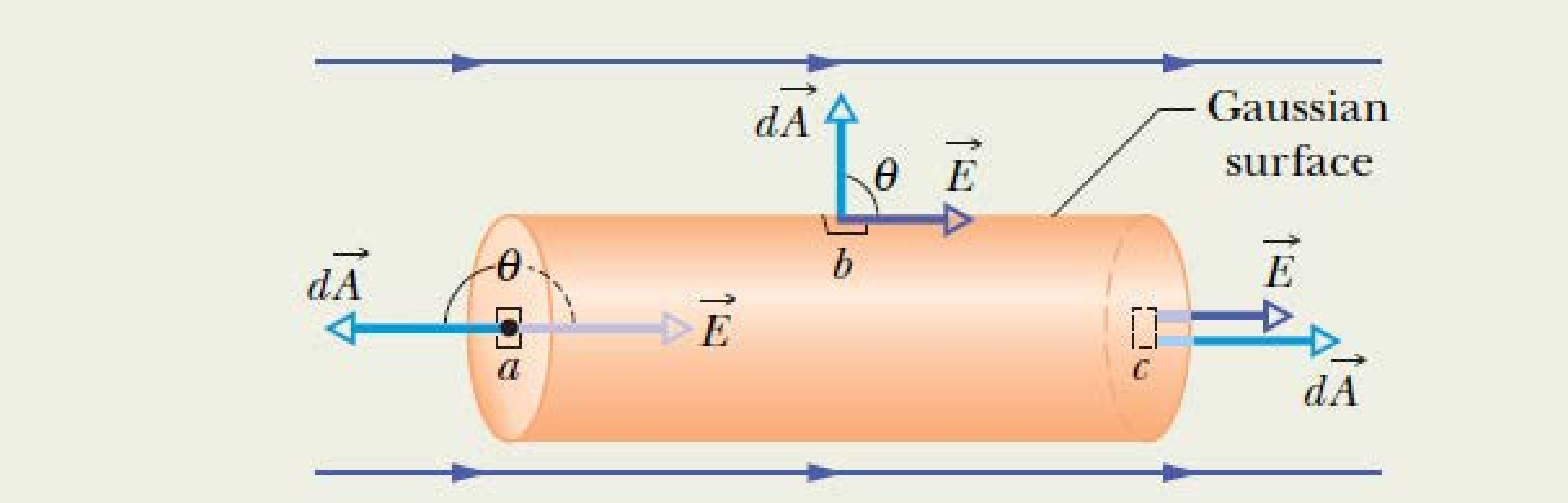
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \qquad \text{(Gauss' law).}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$$

Figure 23-6 . shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field $\vec{E}$, with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}.$$



$$\int_a \vec{E} \cdot d\vec{A} = \int_a E\,dA\cos 180^\circ$$

$$= -E\int dA = -EA$$

$$\int_c \vec{E} \cdot d\vec{A} = \int_c E\,dA\cos 0 = EA$$

$$\int_b \vec{E} \cdot d\vec{A} = \int_b E\,dA\cos 90 = 0$$

$$\therefore\ \phi = -EA + 0 + EA = 0$$

Electric field intensity $\vec{E}$

Electric flux through a surface

$$\phi = \int \vec{E} \cdot d\vec{A}$$

Gaussian surface : closed surface :

Flux through Gaussian surface

$$\phi = \oint \vec{E} \cdot dA \qquad \oplus ve \text{ and } \ominus ve$$

Gauss Law :

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

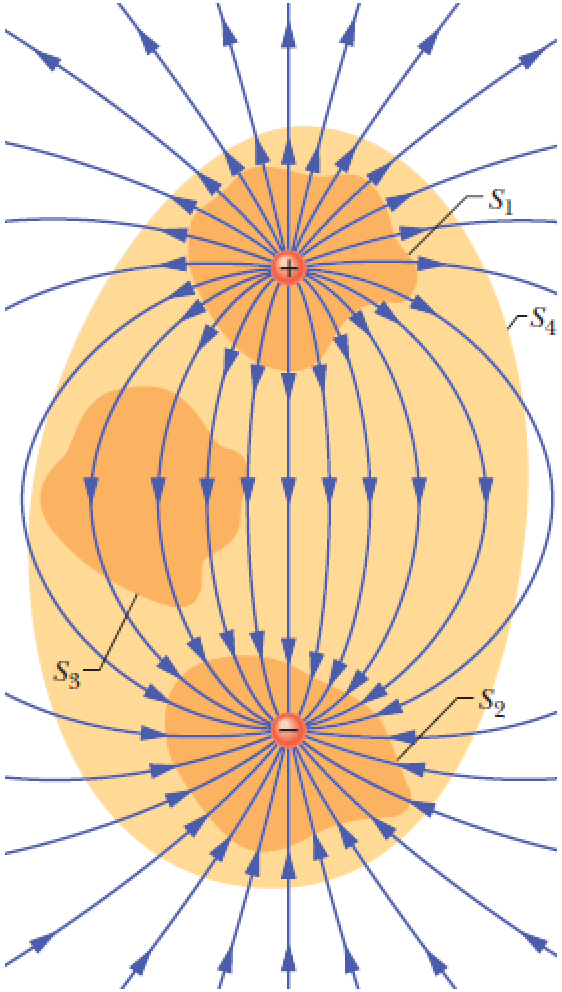
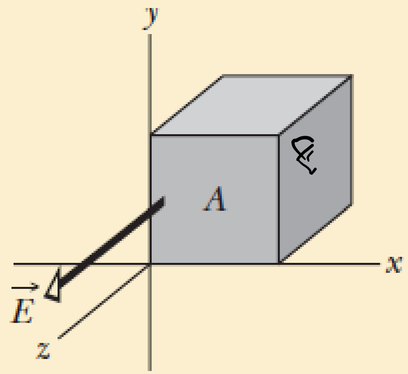


Figure 23-8 Two charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section. Surface S_1 encloses the positive charge. Surface S_2 encloses the negative charge. Surface S_3 encloses no charge. Surface S_4 encloses both charges and thus no net charge.

EA
-EA
Ø
0

Checkpoint 1

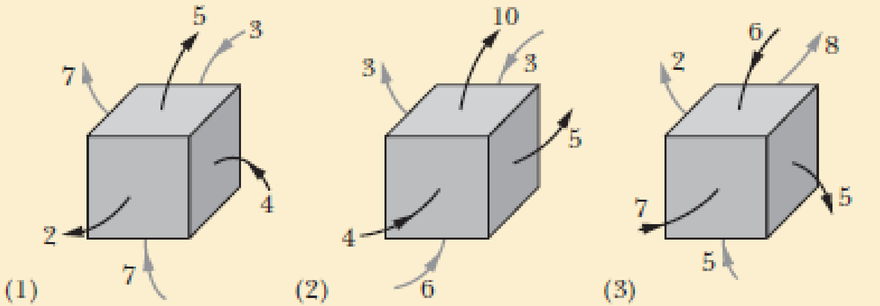
The figure here shows a Gaussian cube of face area A immersed in a uniform electric field $\vec{E}$ that has the positive direction of the z axis. In terms of E and A, what is the flux through (a) the front face (which is in the xy plane), (b) the rear face, (c) the top face, and (d) the whole cube?



2
3
1

Checkpoint 2

The figure shows three situations in which a Gaussian cube sits in an electric field. The arrows and the values indicate the directions of the field lines and the magnitudes (in $N \cdot m^2/C$) of the flux through the six sides of each cube. (The lighter arrows are for the hidden faces.) In which situation does the cube enclose (a) a positive net charge, (b) a negative net charge, and (c) zero net charge?



Welcome to the Physics class

Principles of Physics II

Lecture 7

Gauss' Law and Coulomb's Law

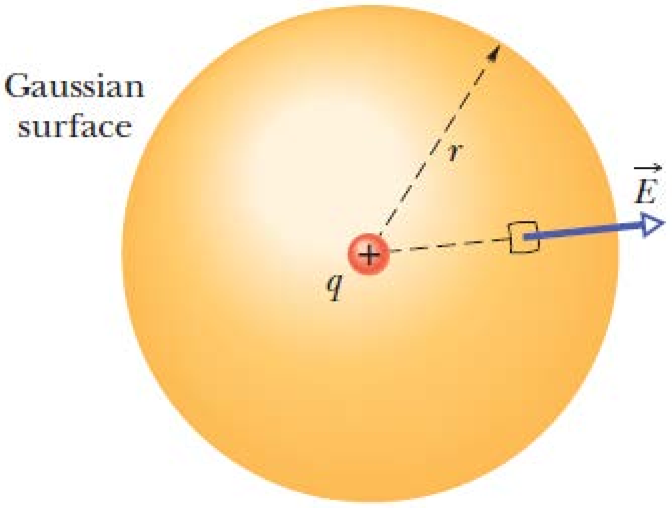


Figure 23-9 A spherical Gaussian surface centered on a particle with charge q.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}.$$

$$\varepsilon_0 E \oint dA = q.$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

CHECKPOINT 3

There is a certain net flux Φ_i through a Gaussian sphere of radius r enclosing an isolated charged particle. Suppose the enclosing Gaussian surface is changed to (a) a larger Gaussian sphere, (b) a Gaussian cube with edge length equal to r, and (c) a Gaussian cube with edge length equal to $2r$. In each case, is the net flux through the new Gaussian surface greater than, less than, or equal to Φ_i?

equal

Gauss' Law

Easier to calculate the electric field due to a continuous charge distribution.

This is best achieved if the charge distribution has a regular symmetric distribution.

Because symmetric charge distribution produce a symmetric electric field. That is the electric field produced by a charge distribution reflects the symmetry of the charge distribution.

planar → spherical — cylindrical charge distribution etc.

$$\int \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$$

$$\vec{E} = 0 \Rightarrow q_{encl} = 0$$

A Charged isolated conductor:

An important theorem about conductors.

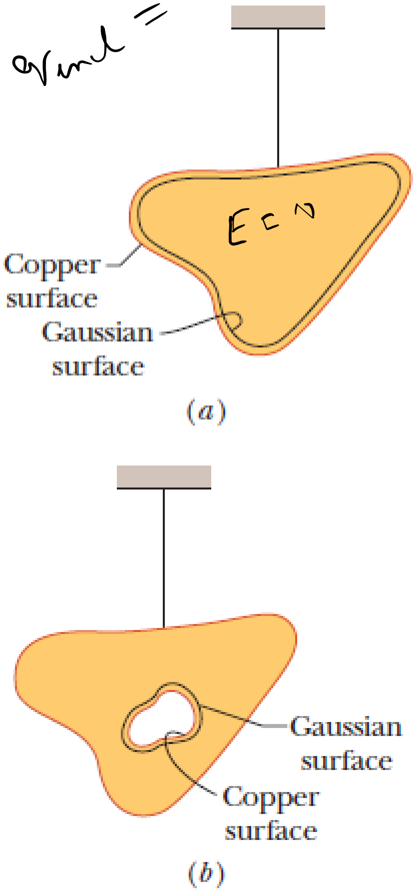
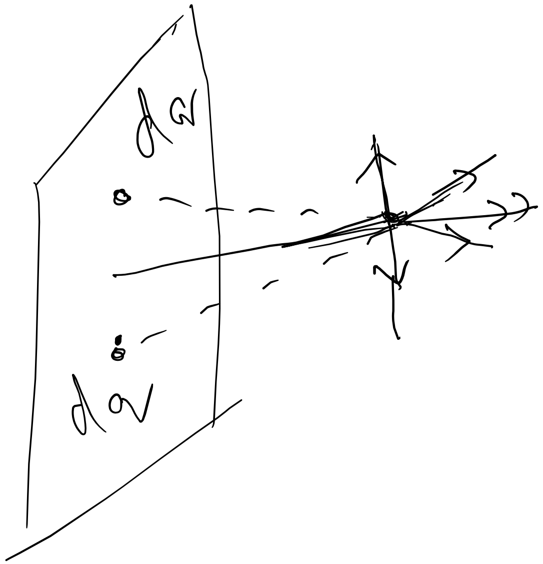


Figure 23-11 (*a*) A lump of copper with a charge *q* hangs from an insulating thread. A Gaussian surface is placed within the metal, just inside the actual surface. (*b*) The lump of copper now has a cavity within it. A Gaussian surface lies within the metal, close to the cavity surface.

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

The external Electric field due to a uniformly charged conductor (constant σ) of infinite extent



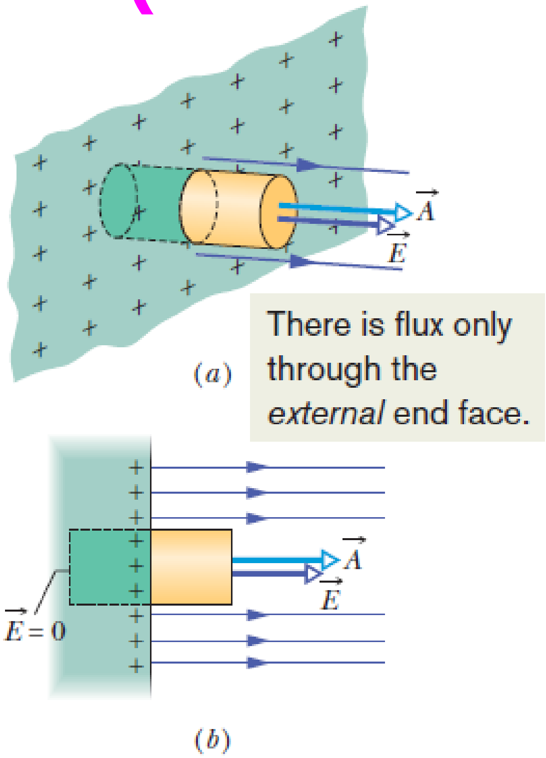


Figure 23-12 (*a*) Perspective view and (*b*) side view of a tiny portion of a large, isolated conductor with excess positive charge on its surface. A (closed) cylindrical Gaussian surface, embedded perpendicularly in the conductor, encloses some of the charge. Electric field lines pierce the external end cap of the cylinder, but not the internal end cap. The external end cap has area A and area vector $\vec{A}$.

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface).}$$

$\phi = EA$

$q_{encl} = \sigma A$

$\varepsilon_0 \phi = q_{encl} = \sigma A$

$\varepsilon_0 E A = \sigma A$

$E = \sigma / \varepsilon_0$

The +q charge distribution on the inner surface is skewed (?)

$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\epsilon_0$

$0 = \dfrac{-q + q}{\epsilon_0} = 0$

Figure 23-13*a* shows a cross section of a spherical metal shell of inner radius R. A particle with a charge of $-5.0\,\mu C$ is located at a distance $R/2$ from the center of the shell. If the shell is electrically neutral, what are the (induced) charges on its inner and outer surfaces? Are those charges uniformly distributed? What is the field pattern inside and outside the shell?

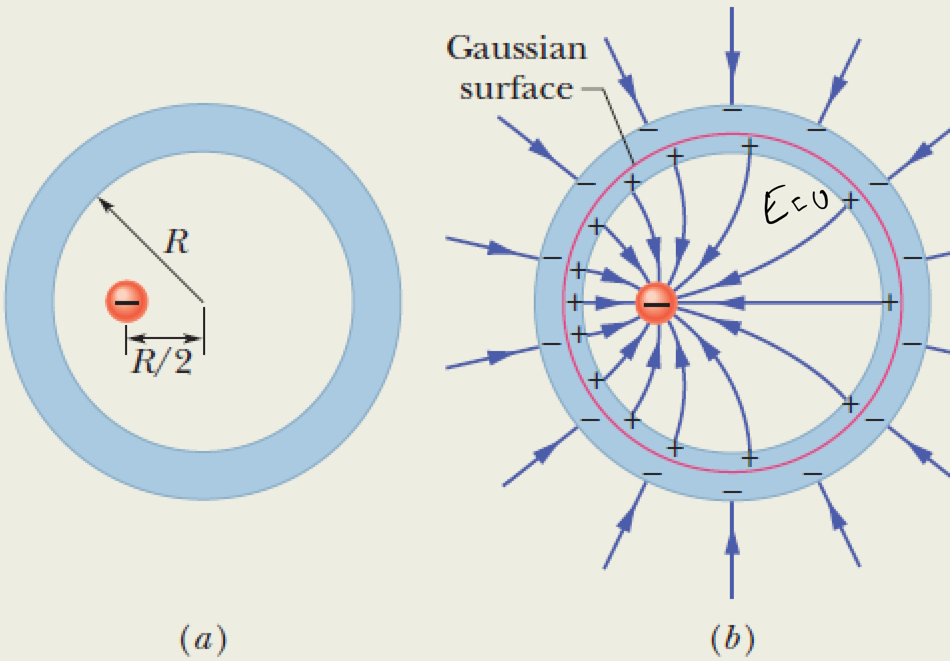


Figure 23-13 (*a*) A negatively charged particle is located within a spherical metal shell that is electrically neutral. (*b*) As a result, positive charge is nonuniformly distributed on the inner wall of the shell, and an equal amount of negative charge is uniformly distributed on the outer wall.

+q on inner surface or the outer surface charge −q. uniformly distributed on the outer surface (?)