# Welcome to the Physics class

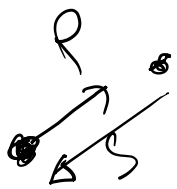
# Principles of Physics II

# Lecture 10

# Electric Potential Energy and Electric Potential Contd.

$W = \vec{F} \cdot \vec{s}$

$= q_t \vec{E} \cdot \vec{s}$




$U_f$

$\vec{E}$

$W_a$

$q_t$

$q_t$

$\vec{F} = q_t \vec{E}$

$q_s$

Potential Energy

Potential at a point

$V_f$

$\Delta U = -W$

$\Delta U = W_a$

$\Delta U = -W = W_a$

$q_t$

$U_i$

$\Delta U = U_f - U_i$

$= -W$

$V_f = U_f / q_t$

$V_i = U_i / q_t$

$V_i$

$(qx)$

$f$ $i$

Potential at a point in an electric field

$$= \frac{\text{Potential Energy}}{\text{test charge}}$$

$$= \text{Potential Energy per unit charge}$$

$$= V$$

Q5

$\Delta V = V_f - V_i$

Potential difference between points $i$ and $f$

$V_f$

$$\Delta V = V_f - V_i = \frac{U_f}{q_t} - \frac{U_i}{q_t} = \frac{U_f - U_i}{q_t} = \frac{-W}{q_t}.$$

$$V_f - V_i = \frac{-W}{q_t} = \frac{Wa}{q_t}$$

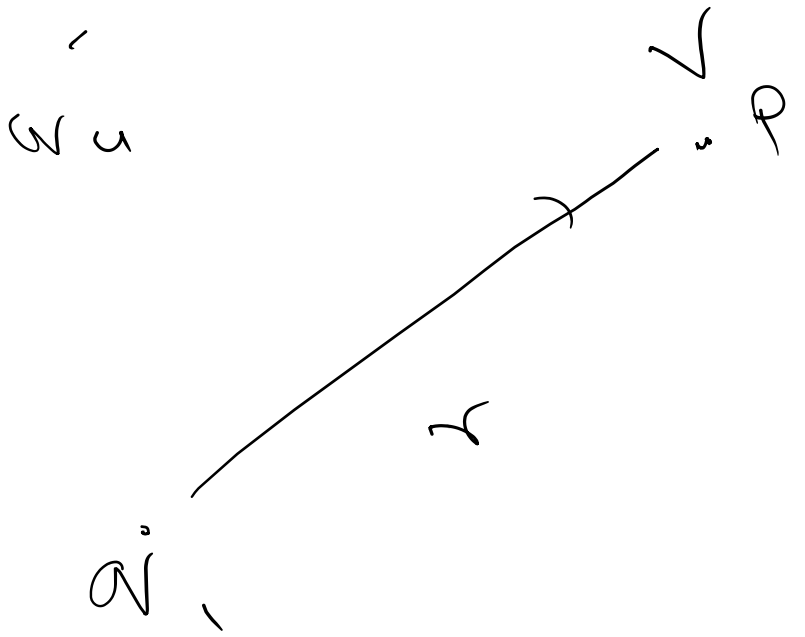
Let us say that the test charge is at infinity and the potential at ~~the~~ infinity $= 0$.

The potential at $f$ is $V$ the potential at point $f$

$$V - 0 = \frac{-W\alpha}{q_t} = \frac{W\alpha}{q_t}$$

potential at ~~point~~ any point

$$V = \frac{W\alpha}{q_t}$$




$$V = K \frac{q}{r}$$ is a scalar ~~Q~~ quantity.

$$K = \frac{1}{4\pi\epsilon_0}$$

$$V(P) = K \sum_{i=1}^{n} \frac{q_i}{r_i}$$

$$\boxed{U = qV}$$

Potential Energy $U$

Potential $V$

Potential due to a continuous charge distribution.

$\sum \rightarrow \int$ over the charge distribution.

$dq \rightarrow$ infinitesimal charge. Small enough to invoke the formula for potential due to a point charge as the following

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$ for potential

for the entire charge distribution,

$$V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$ etc.

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \quad (n \text{ charged particles}).$$

# Potential due to a continuous charge distribution

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \quad \text{(positive or negative } dq\text{)}.$$

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}.$$

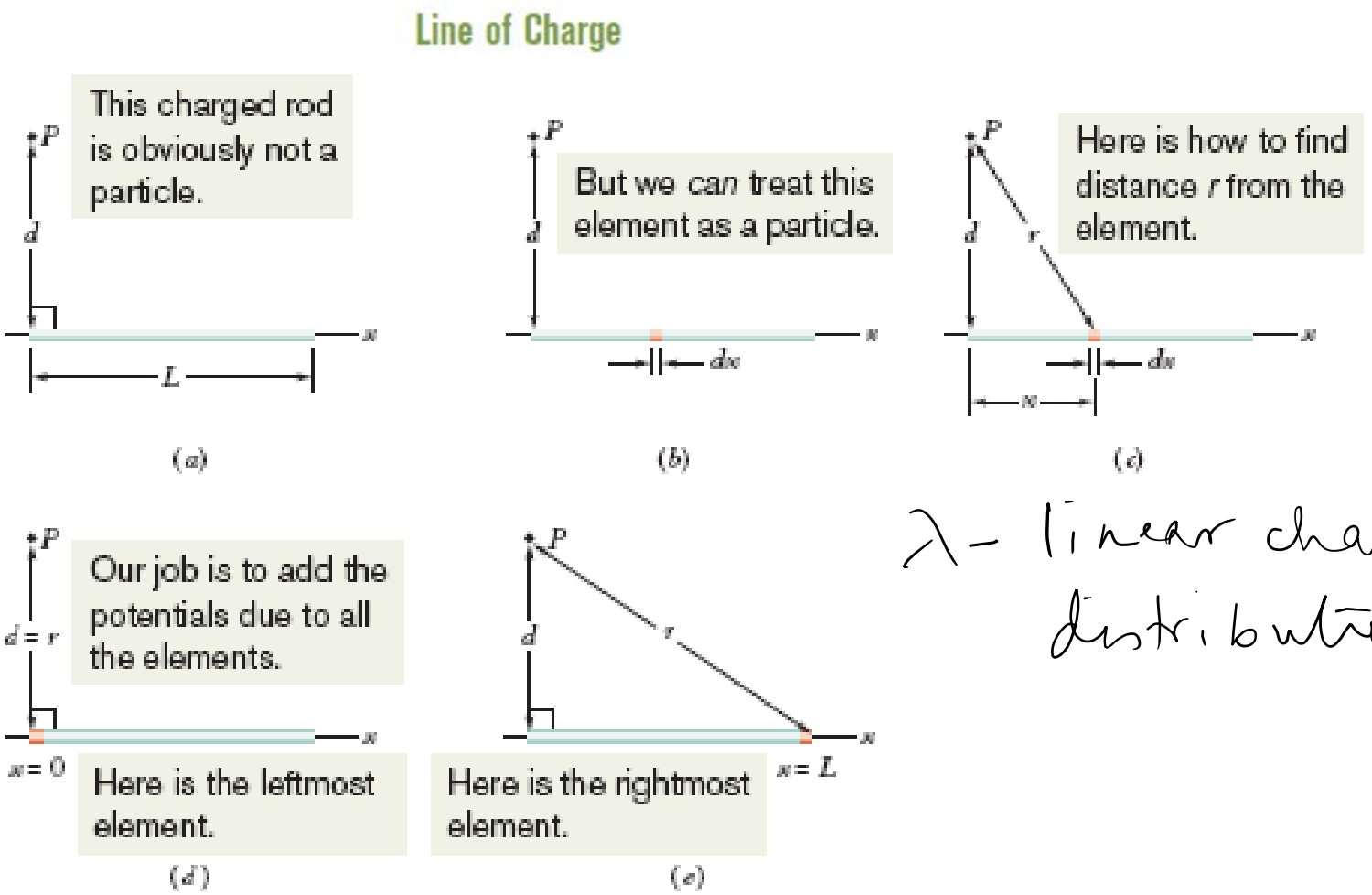
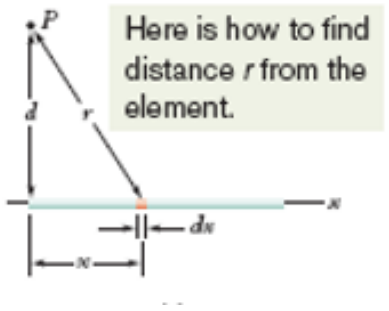



**Fig. 24-12** (*a*) A thin, uniformly charged rod produces an electric potential $V$ at point $P$. (*b*) An element can be treated as a particle. (*c*) The potential at $P$ due to the element depends on the distance $r$. We need to sum the potentials due to all the elements, from the left side (*d*) to the right side (*e*).

$\lambda$ - linear charge distribution.

$dq = \lambda dx$ The potential at point P due to the the element of charge $dq$



Here is how to find distance $r$ from the element.

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(x^2+d^2)^{1/2}}$$

Total V produced at P is obtained by integrating the above w.r.t. $x$ and within the limit $x=0$ to $x=L$

$$V = \int dV = \int_0^L \frac{1}{4\pi\epsilon_0}\frac{\lambda}{(x^2+d^2)^{1/2}}\, dx$$

$$= \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{(x^2+d^2)^{1/2}}$$

$$V = \frac{\lambda}{4\pi\epsilon_0}\left[\ln\left(x + (x^2+d^2)^{1/2}\right)\right]_0^L$$

Using the formula 17 in Appendix E of RHW.

$$\int \frac{dx}{(x^2+a^2)^{1/2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$$

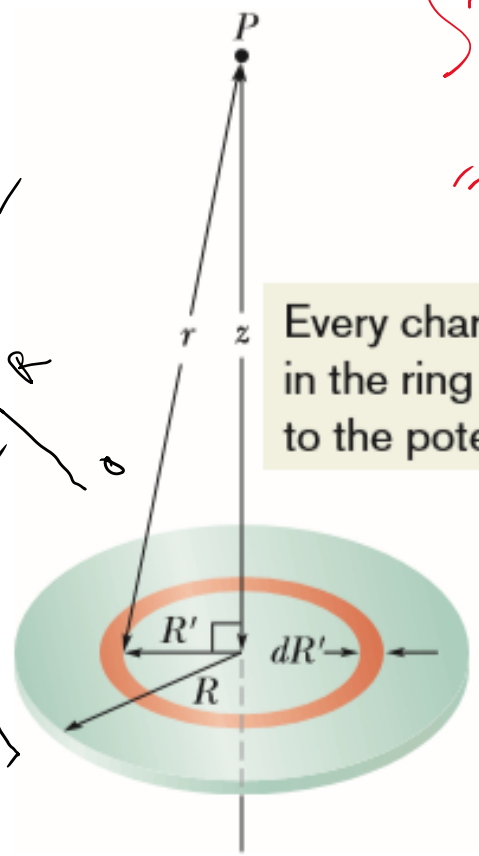
in our case $a$ is $d$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[\ln\left(L+(L^2+d^2)^{1/2}\right) - \ln d\right]$$

$$= \frac{\lambda}{4\pi\epsilon_0}\ln\left[\frac{L+(L^2+d^2)^{1/2}}{d}\right]$$

$V$ is $\oplus$ve why ?

## Charged Disk




Every charge element in the ring contributes to the potential at $P$.

**Fig. 24-13** A plastic disk of radius $R$, charged on its top surface to a uniform surface charge density $\sigma$. We wish to find the potential $V$ at point $P$ on the central axis of the disk.

$$dq = \sigma(2\pi R')dR'$$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{\sigma(2\pi R')dR'}{(z^2+R'^2)^{1/2}}$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{R'\,dR'}{(z^2+R'^2)^{1/2}}$$

$$= \frac{\sigma}{2\epsilon_0}\left[(z^2+R'^2)^{1/2}\right]_0^R$$

$$= \frac{\sigma}{2\epsilon_0}\left[(z^2+R^2)^{1/2} - z\right]$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int \frac{x\,dx}{(x^2+a^2)^{1/2}}$$

Let $x^2+a^2 = y$

$\therefore 2x\,dx = dy \Rightarrow x\,dx = \frac{dy}{2}$

The integral becomes

$$\int \frac{dy/2}{y^{1/2}} = \frac{1}{2}\int y^{-1/2}$$

$$\therefore \int \frac{x\,dx}{(x^2+a^2)^{1/2}} = (x^2+a^2)^{1/2}$$

Replace $x$ by $R'$ and $a$ by $z$

# Calculating the Field from the Potential

$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$

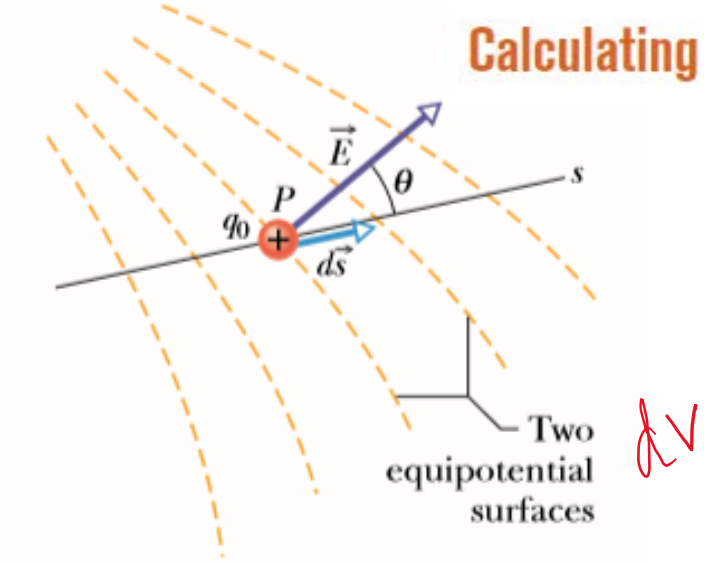



**Fig. 24-14** A test charge $q_0$ moves a distance $d\vec{s}$ from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $d\vec{s}$ makes an angle $\theta$ with the direction of the electric field $\vec{E}$.

The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

$dW = \vec{F}\cdot d\vec{s}$

$dW = q_0\vec{E}\cdot d\vec{s}$ also $dU = q_0 dV$

We know

$dU = -dW$

$q_0 dV = -q_0\vec{E}\cdot d\vec{s} = -q_0 E\, ds \cos\theta$

$= -q_0 E \cos\theta\, ds$

$\cancel{q_0} dV = -\cancel{q_0} E\cos\theta\, ds$

$E\cos\theta = -\dfrac{dV}{ds}$

Component of E along ds, Let's say $E_s$

$\therefore$

$$E_s = -\frac{\partial V}{\partial s}.$$

s is any direction. If $s \to \begin{cases} x \\ y \\ z \end{cases}$

$$E_x = -\frac{\partial V}{\partial x}; \qquad E_y = -\frac{\partial V}{\partial y}; \qquad E_z = -\frac{\partial V}{\partial z}.$$

$\vec{E} = -\vec{\nabla} V$

$= -\text{grad}\, V$

**For Uniform Electric field**

$$E = -\frac{\Delta V}{\Delta s}, \quad = -\frac{V}{d} \qquad \Delta s \to d$$

$$\vec{E}_s = -\frac{dV}{ds}$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$\left.\begin{aligned} \vec{E}_x &= -\hat{i}\,\frac{\partial V}{\partial x} \\ \vec{E}_y &= -\hat{j}\,\frac{\partial V}{\partial y} \\ \vec{E}_z &= -\hat{k}\,\frac{\partial V}{\partial z} \end{aligned}\right\} \longrightarrow \left\{\begin{aligned} \vec{E} &= -\vec{\nabla} V \\ &= -\left[\hat{i}\,\frac{\partial V}{\partial x} + \hat{j}\,\frac{\partial V}{\partial y} + \hat{k}\,\frac{\partial V}{\partial z}\right] \\ \vec{E}_x &= -\hat{i}\,\frac{\partial V}{\partial x} \\ &\text{etc.} \end{aligned}\right.$$

$\vec{\nabla}$ = is a vector operator

$= \hat{i}\,\partial/\partial x + \hat{j}\,\partial/\partial y + \hat{k}\,\partial/\partial z$

# Electric Potential Energy of a System of Point Charges

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}.$$

**Figure 24-18** Two charges held a fixed distance $r$ apart.

## Potential Energy of two charges

$$U = (W_a =)\, q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}.$$

The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.

Figure 24-19 shows three charged particles held in fixed positions by forces that are not shown. What is the electric potential energy $U$ of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

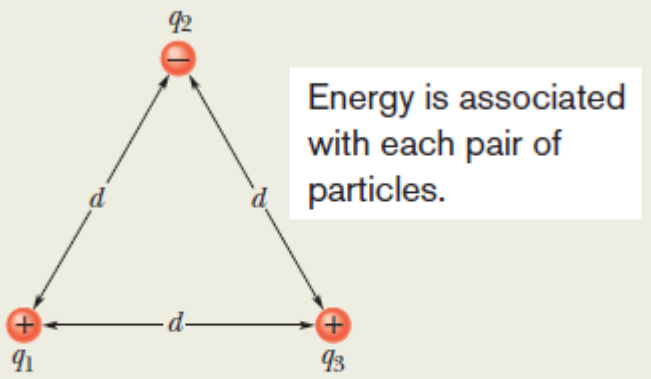



**Figure 24-19** Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

$U_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{d} = W_{12}$ is the amount of work done by an "external agent" in bringing $q_2$ from infinity while $q_1$ is in place. Then bring $q_3$ from infinity when $q_1$ and $q_2$ are in place.

$$W_{13} + W_{23} = U_{13} + U_{23} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_3}{d} + \frac{q_2 q_3}{d}\right)$$

$$U = U_{12} + U_{13} + U_{23}$$

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{d} + \frac{q_1 q_3}{d} + \frac{q_2 q_3}{d}\right]$$

$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{+q(-4q)}{d} + \frac{(q)(2q)}{d} + \frac{(-4q)(2q)}{d}\right)$$

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{-4q^2}{d} + \frac{2q^2}{d} - \frac{8q^2}{d}\right]$$

$$= \frac{-10q^2}{4\pi\epsilon_0}$$

$$U = -\frac{10 q^2}{4\pi\epsilon_0 d}$$

$$= -\frac{(8.99 \times 10^9 \, N.m^2/C^2)\, 10\, (150 \times 10^{-9} C)^2}{0.12 \, m}$$

$$= -1.7 \times 10^{-2} \, J$$

$$= -17 \, mJ$$

# Potential of a Charged Isolated Conductor

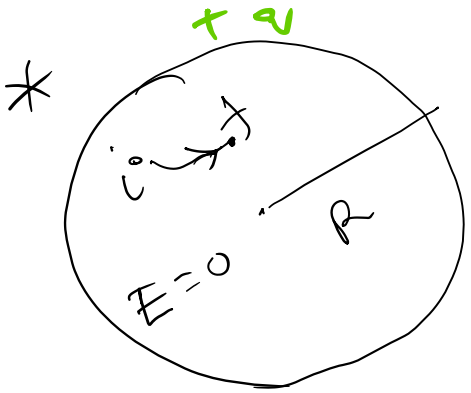
# Potential of a Charged Isolated Conductor

Contd.

An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

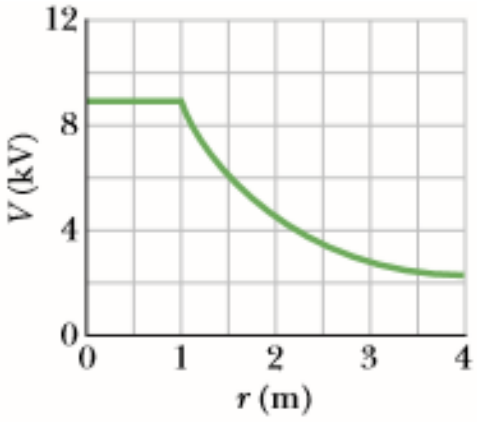
Since $\vec{E} = 0$ for all points within a conductor, it follows directly that $V_f = V_i$ for all possible pairs of points $i$ and $f$ in the conductor.




$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

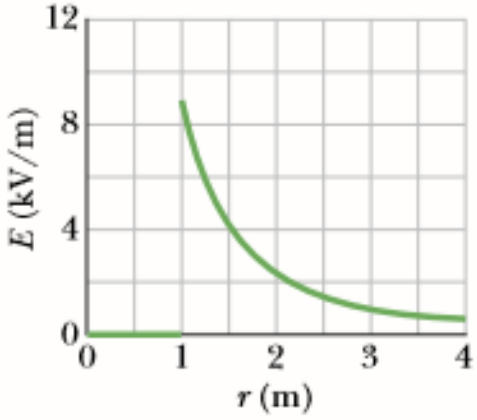
$E = 0$

$\therefore V_f = V_i$




(a)

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$




(b)

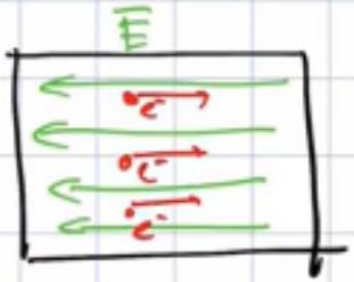
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{E} = -\frac{dV}{dr}$$

**Fig. 24-18** (*a*) A plot of $V(r)$ both inside and outside a charged spherical shell of radius 1.0 m. (*b*) A plot of $E(r)$ for the same shell.

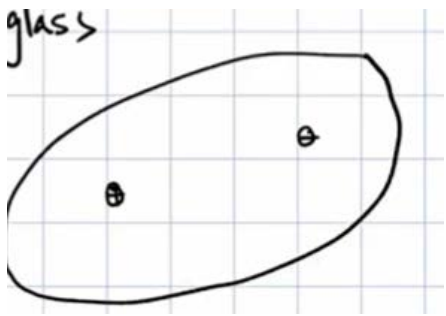
# Conductors, Insulators and the Electrostatic Equilibrium

## Conductors and Insulators

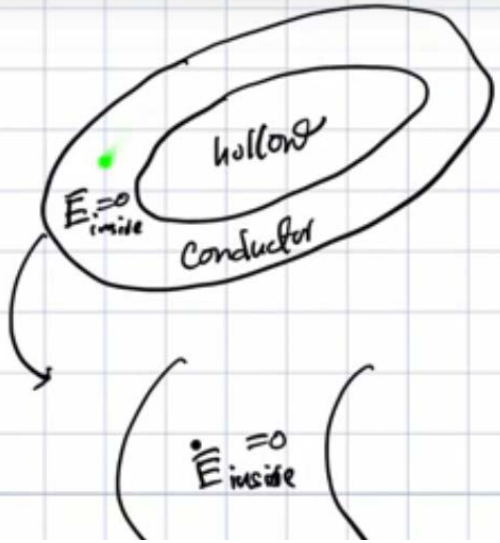



plastic
wood
paper
glass

1. Copper
2. Aluminium
3. Stainless steel
4. Gold.

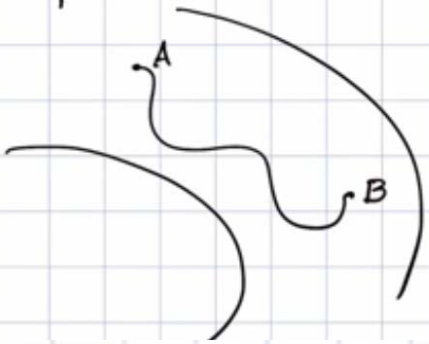



1.



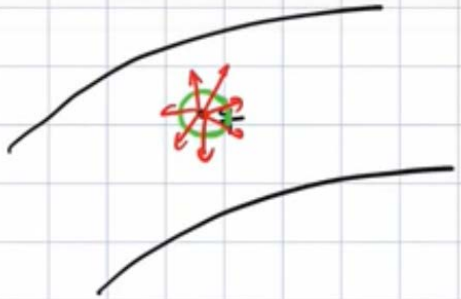

2. The whole conductor is at the same electrostatic potential.

$$V(B) - V(A) = -\int_A^B \underbrace{\bar{E}_{inside}}_{=0} \cdot d\bar{s}$$




$$V(B) = V(A)$$

3.



$$\lim_{\ell \to 0} \iint_{g.s.} \underbrace{\bar{E}_{inside}}_{=0} \cdot d\bar{a} = \frac{q_{encl}}{\varepsilon_0}$$

$$0 = q_{encl}$$

## REVIEW & SUMMARY

**Electric Potential Energy** The change $\Delta U$ in the electric potential energy $U$ of a point charge as the charge moves from an initial point $i$ to a final point $f$ in an electric field is

$$\Delta U = U_f - U_i = -W, \tag{24-1}$$

where $W$ is the work done by the electrostatic force (due to the external electric field) on the point charge during the move from $i$ to $f$. If the potential energy is defined to be zero at infinity, the **electric potential energy** $U$ of the point charge at a particular point is

$$U = -W_\infty. \tag{24-2}$$

Here $W_\infty$ is the work done by the electrostatic force on the point charge as the charge moves from infinity to the particular point.

**Electric Potential Difference and Electric Potential** We define the **potential difference** $\Delta V$ between two points $i$ and $f$ in an electric field as

$$\Delta V = V_f - V_i = -\frac{W}{q}, \tag{24-7}$$

where $q$ is the charge of a particle on which work $W$ is done by the electric field as the particle moves from point $i$ to point $f$. The **potential** at a point is defined as

$$V = -\frac{W_\infty}{q}. \tag{24-8}$$

Here $W_\infty$ is the work done on the particle by the electric field as the particle moves in from infinity to the point. The SI unit of potential is the *volt*: 1 volt = 1 joule per coulomb.

Potential and potential difference can also be written in terms of the electric potential energy $U$ of a particle of charge $q$ in an electric field:

$$V = \frac{U}{q}, \tag{24-5}$$

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}. \tag{24-6}$$

**Equipotential Surfaces** The points on an **equipotential surface** all have the same electric potential. The work done on a test charge in moving it from one such surface to another is independent of the locations of the initial and final points on these surfaces and of the path that joins the points. The electric field $\vec{E}$ is always directed perpendicularly to corresponding equipotential surfaces.

**Finding $V$ from $\vec{E}$** The electric potential difference between two points $i$ and $f$ is

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}, \tag{24-18}$$

where the integral is taken over any path connecting the points. If the integration is difficult along any particular path, we can choose a different path along which the integration might be easier. If we choose $V_i = 0$, we have, for the potential at a particular point,

$$V = -\int_i^f \vec{E} \cdot d\vec{s}. \tag{24-19}$$

**Potential Due to Point Charges** The electric potential due to a single point charge at a distance $r$ from that point charge is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}, \tag{24-26}$$

where $V$ has the same sign as $q$. The potential due to a collection of point charges is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}. \tag{24-27}$$

**Potential Due to an Electric Dipole** At a distance $r$ from an electric dipole with dipole moment magnitude $p = qd$, the electric potential of the dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \tag{24-30}$$

for $r \gg d$; the angle $\theta$ is defined in Fig. 24-10.

**Potential Due to a Continuous Charge Distribution** For a continuous distribution of charge, Eq. 24-27 becomes

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}, \tag{24-32}$$

in which the integral is taken over the entire distribution.

**Calculating $\vec{E}$ from $V$** The component of $\vec{E}$ in any direction is the negative of the rate at which the potential changes with distance in that direction:

$$E_s = -\frac{\partial V}{\partial s}. \tag{24-40}$$

The $x$, $y$, and $z$ components of $\vec{E}$ may be found from

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z}. \tag{24-41}$$

When $\vec{E}$ is uniform, Eq. 24-40 reduces to

$$E = -\frac{\Delta V}{\Delta s}, \tag{24-42}$$

where $s$ is perpendicular to the equipotential surfaces. The electric field is zero parallel to an equipotential surface.

**Electric Potential Energy of a System of Point Charges** The electric potential energy of a system of point charges is equal to the work needed to assemble the system with the charges initially at rest and infinitely distant from each other. For two charges at separation $r$,

$$U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}. \tag{24-43}$$

**Potential of a Charged Conductor** An excess charge placed on a conductor will, in the equilibrium state, be located entirely on the outer surface of the conductor. The charge will distribute itself so that the following occur: (1) The entire conductor, including interior points, is at a uniform potential. (2) At every internal point, the electric field due to the charge cancels the external electric field that otherwise would have been there. (3) The net electric field at every point on the surface is perpendicular to the surface.

$U = qV$

2nd Quiz:

On 4th July during the regular class time.

Syllabus: Gauss' Law & Electric Potential Energy and Electric potential