Welcome to the Physics class

Principles of Physics II

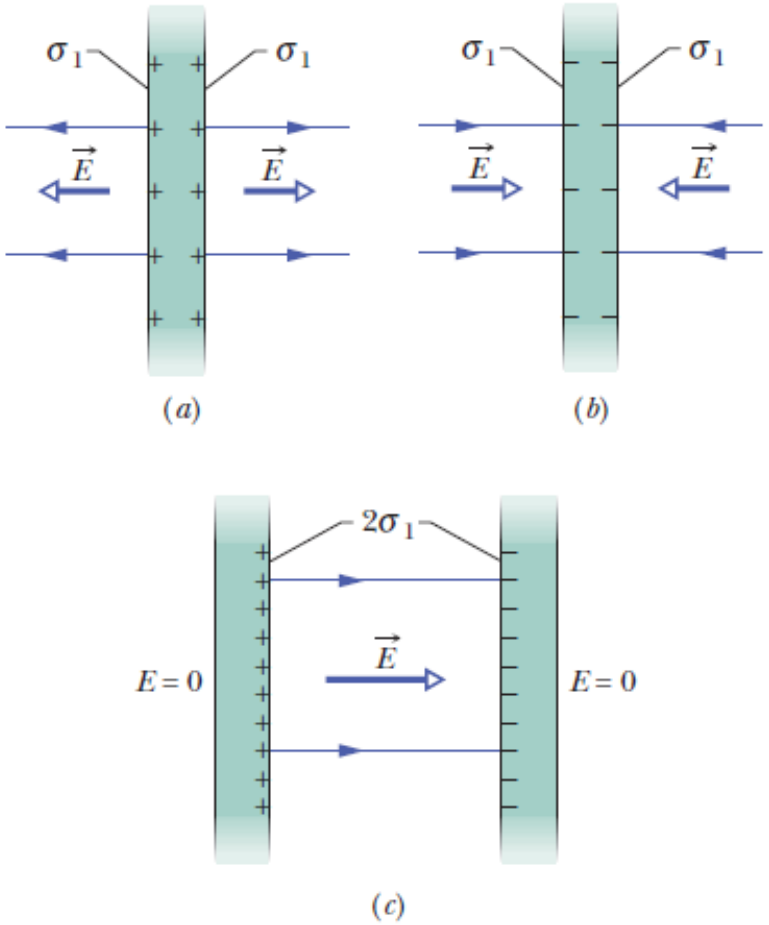
Lecture 8

Gauss' Law Contd

Two Conducting Plates

$E = \frac{\sigma_1}{\varepsilon_0}$ outward

$E = \frac{\sigma_1}{\varepsilon_0}$ inward



$E = \frac{2\sigma_1}{\varepsilon_0}$ inside

$E = 0$ outside

Figure 23-18 (*a*) A thin, very large conducting plate with excess positive charge. (*b*) An identical plate with excess negative charge. (*c*) The two plates arranged so they are parallel and close.

Inf. Line of charge

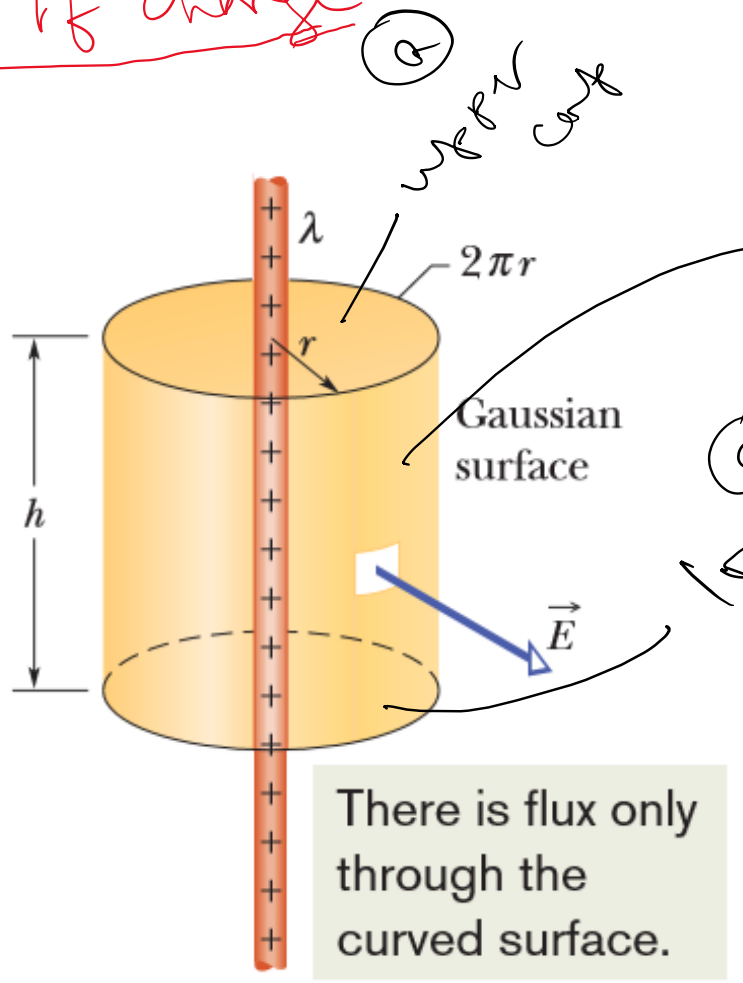
Flux through the Gaussian closed cylindrical surface

$\phi = \oint \vec{E} \cdot d\vec{A}$

$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}$

$= 0 + E \cdot 2\pi r h + 0$

$\phi = E 2\pi r h$



(a) curved surface

(b) cylindrical surface

(c) flat surface

Figure 23-14 A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

Gauss' Law

$\phi = \left\{ \begin{array}{l} q_{encl} = \lambda h \\ \epsilon_0 \end{array} \right.$

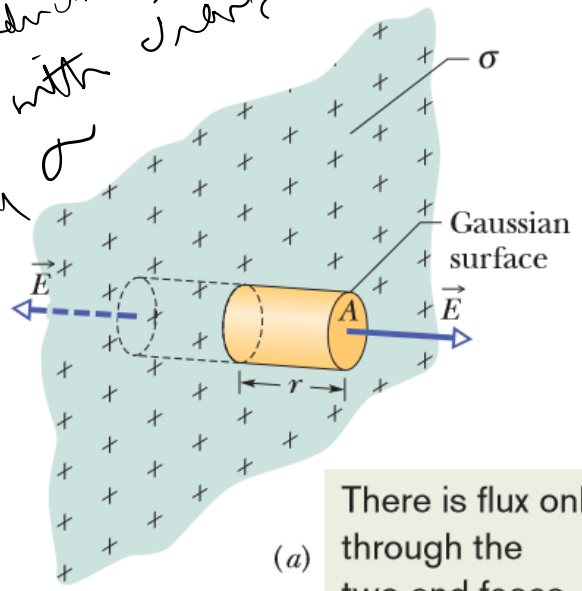
$\lambda h / \epsilon_0$

$E 2\pi r h = \frac{\lambda}{\epsilon_0}$

$E = \frac{\lambda}{2\pi \epsilon_0 r}$

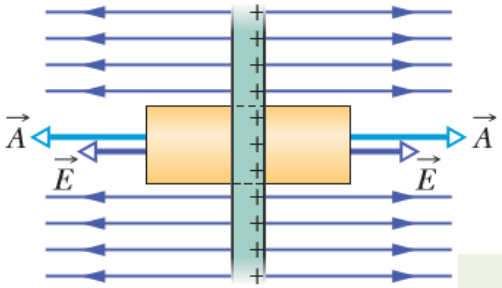
infinite line of charge.

non conducting surface with charge density σ



(a)

There is flux only through the two end faces.



(b)

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(sheet of charge).}$$

$\varepsilon_0 \Phi = q_{encl} = \sigma A$

$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{encl} = \sigma A$

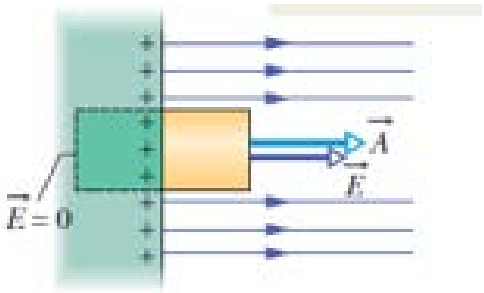
$\varepsilon_0 (EA + 0 + EA) = \sigma A$

$2EA\varepsilon_0 = \sigma A$

σA is the charge enclosed by the Gaussian pill box.

$$E = \frac{\sigma}{2\varepsilon_0}$$

Compare this result with that of an infinitely extended nonconducting plate with surface charge density σ.



$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(conducting surface).}$$

Figure 23-17 (*a*) Perspective view and (*b*) side view of a portion of a very large, thin plastic sheet, uniformly charged on one side to surface charge density σ. A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

E inside the shell of charge is zero $\oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$

$q_{encl} = 0 \Rightarrow E = 0$

E outside

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$

$E\oint dA = \frac{q}{\epsilon_0}$

$E\,4\pi r^2 = \frac{q}{\epsilon_0}$

$E = \frac{1}{4\pi\epsilon_0}\, q/r^2$

as if all the charge are concentrated at the centre

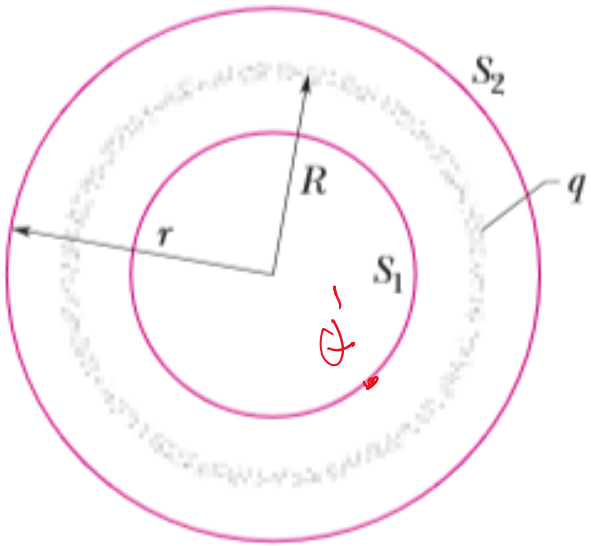


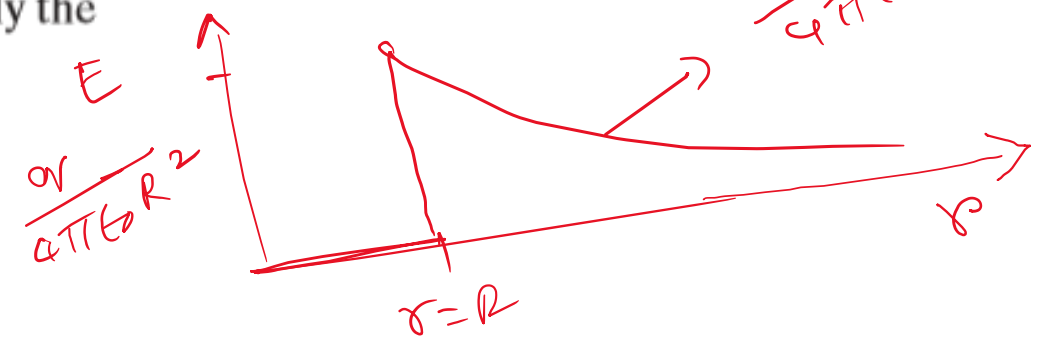
Figure 23-20 A thin, uniformly charged, spherical shell with total charge q, in cross section. Two Gaussian surfaces S_1 and S_2 are also shown in cross section. Surface S_2 encloses the shell, and S_1 encloses only the empty interior of the shell.

$\vec{F} = Q\vec{E}$

Q

① Force on Q outside the charged shell?

② Force on Q' inside the charged shell?



E outside (a)

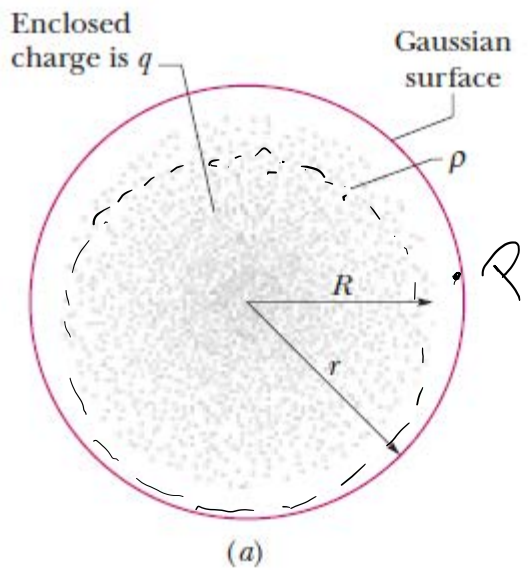
$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

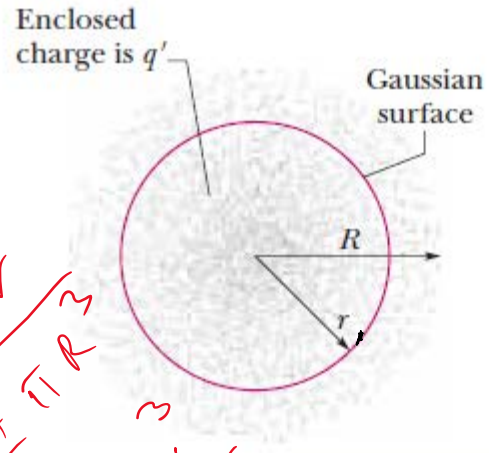
$r > R$

E inside (b)

$r' < R$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q'}{r'^2}$$





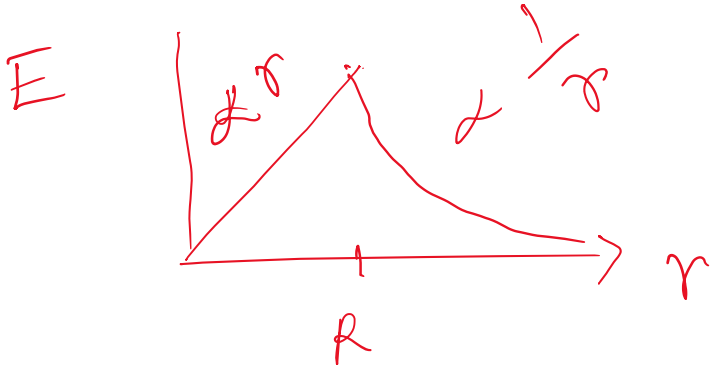
The flux through the surface depends on only the *enclosed* charge.

$$\frac{q'}{\frac{4}{3}\pi r'^3} = \frac{q}{\frac{4}{3}\pi R^3}$$

$$q' = q\frac{r'^3}{R^3}$$

$$\therefore E = \frac{q r'}{4\pi\epsilon_0 R^3}$$

Figure 23-21 The dots represent a spherically symmetric distribution of charge of radius R, whose volume charge density ρ is a function only of distance from the center. The charged object is not a conductor, and therefore the charge is assumed to be fixed in position. A concentric spherical Gaussian surface with $r > R$ is shown in (a). A similar Gaussian surface with $r < R$ is shown in (b).



For a dipole there are some parts of the Gaussian surface where the electric field is coming out of the surface hence the flux through there is positive and there are some regions of the Gaussian surface where the situation is exactly opposite rendering the net flux through the closed surface as zero, but the electric field E is not zero.

1. Zero flux does not necessarily mean zero electric field.
2. Evaluating double integral on an arbitrary surface can be very complicated but if we know $q_{enc} = 0$ then form Gauss's Law the surface integral is zero. That's a clever way of finding Electric flux through a closed surface.

Gauss's law can't be used to calculate electric field near an electric dipole.

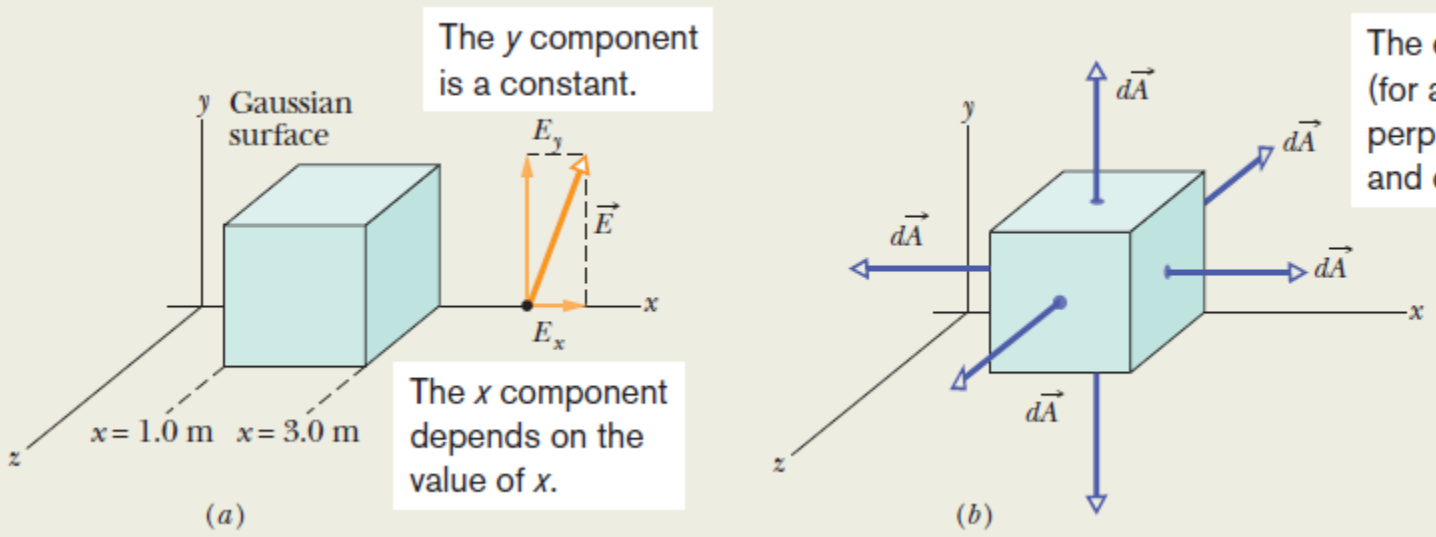
To apply gauss law, **there must exist a symmetric distribution of charges**, such that the Gaussian surface could have a single value of electric field for the given surface in the equation

$q/\epsilon_0 = \int E.dA.$

Electric dipoles have no surface that have a symmetric distribution of electric field around, thus Gauss' Law cannot be used for the task.

Sample Problem 23.02 Flux through a closed c

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-7*a*. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)



$$d\vec{A} = dA\hat{i}.$$

$$\Phi_r = \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i})$$

$$= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i}]$$

$$= \int (3.0x\, dA + 0) = 3.0 \int x\, dA.$$

$$\Phi_r = 3.0 \int (3.0)\, dA = 9.0 \int dA.$$

The integral $\int dA$ merely gives us the area $A = 4.0\ \text{m}^2$ of the right face, so

$$\Phi_r = (9.0\ \text{N/C})(4.0\ \text{m}^2) = 36\ \text{N} \cdot \text{m}^2/\text{C}. \qquad \text{(Answer)}$$

Left face: We repeat this procedure for the left face. However, two factors change. (1) The element area vector $d\vec{A}$ points in the negative direction of the x axis, and thus $d\vec{A} = -dA\hat{i}$ (Fig. 23-7*d*). (2) On the left face, $x = 1.0$ m. With these changes, we find that the flux Φ_l through the left face is

$$\Phi_l = -12\ \text{N} \cdot \text{m}^2/\text{C}. \qquad \text{(Answer)}$$

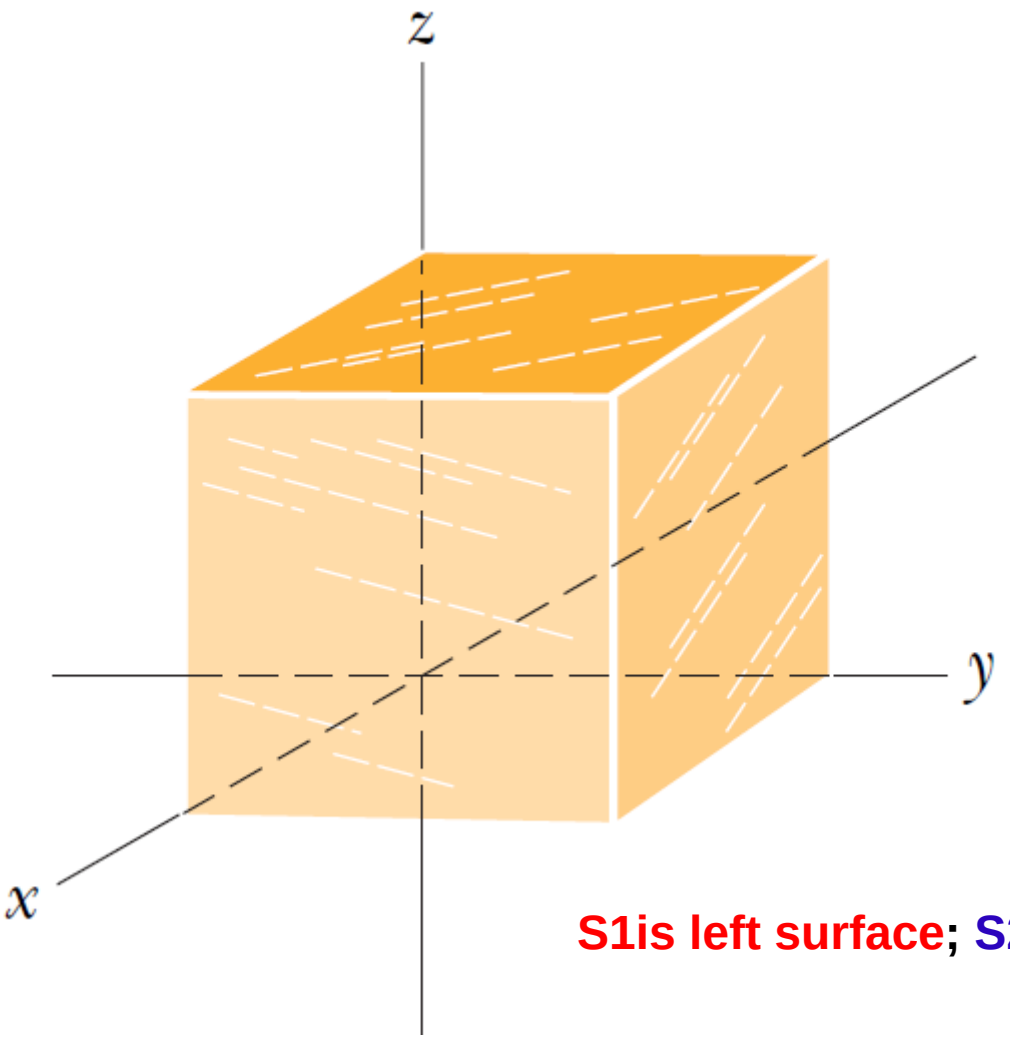
Top face: Now $d\vec{A}$ points in the positive direction of the y axis, and thus $d\vec{A} = dA\hat{j}$ (Fig. 23-7*e*). The flux Φ_t is

$$\Phi_t = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{j})$$

$$= \int [(3.0x)(dA)\hat{i} \cdot \hat{j} + (4.0)(dA)\hat{j} \cdot \hat{j}]$$

$$= \int (0 + 4.0\, dA) = 4.0 \int dA$$

$$= 16\ \text{N} \cdot \text{m}^2/\text{C}. \qquad \text{(Answer)}$$



A cube has sides of length L = 0.300 m and is placed with one corner at the origin as shown in figure. The electric field is not uniform but is given by E = (-5.00 N/C.m. x i + 3.00 N/C.m. z k.) (a) Find the electric flux through each of the cubic faces S1, S2, S3, S4, S5 and S6. (b) Find the total electric charge inside the cube.

S1is left surface; **S2 is top**; **S3 is right**; **S4 is bottom**; **S5 is front**; **S6 is back**

For S1: $y=0 \quad d\vec{A} = -\hat{j}\,dA.$

$$\phi_{S1} = \iint_{S1} \left(-5.\,x\,\hat{i} + 3z\,\hat{k}\right)_{y=0} \cdot (-\hat{j})\,dA) = 0$$

$\hat{i}\cdot\hat{j} = 0 \quad \hat{k}\cdot\hat{j} = 0$

For S2; $z = 0.3\,m \quad d\vec{A} = \hat{k}\,dA.$

$$\phi_{S2} = \iint_{S2} \left(-5.\,x.\,\hat{i} + 3z.\,\hat{k}\right)_{z=0.3m} \cdot \hat{k}\,dA$$

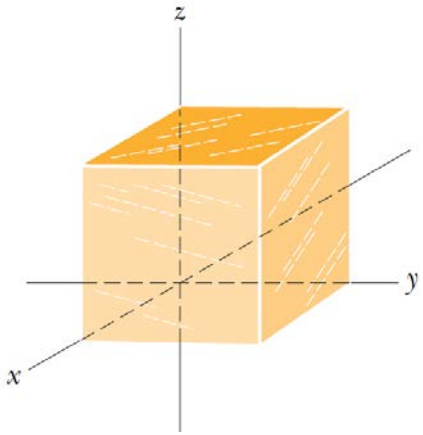
$$= \iint_{S2} 3\times 0.3\,dA = .9\iint_{S2} dA = .9\times(.3)^2 = 0.081\ \frac{Nm^2}{C}$$

For S3 $\Rightarrow$ $y = 0.3\,m \quad d\vec{A} = \hat{j}\,dA.$

$$\phi_{S3} = \iint \left(-5x\,\hat{i} + 3z\,\hat{k}\right)\cdot(\hat{j}\,dA) = 0$$

$$\phi_{S4} = \iint_{S4} \left(-5x\,\hat{i} + 3z\,\hat{k}\right)_{z=0} \cdot (-\hat{k})\,dA.$$

$$= \iint_{S} -3\times 0\times dA = 0$$



For S5: $x = 0.3 \quad d\vec{A} = \hat{i}\,dA.$

$$\phi_{S5} = \iint_{S5} \left(-5x\,\hat{i} + 3z\,\hat{k}\right)_{x=0.3} \cdot (\hat{i}\,dA)$$

$$= \iint -5\times 3\,dA = -1.5\iint dA = -1.5\times(.3)^2$$

$$= -.135\ \frac{Nm^2}{C}$$

For S6 $\quad x = 0 \quad d\vec{A} = -\hat{i}\,dA$

$$\phi_{S6} = \iint_{S6} \left(-5x\,\hat{i} + 3z\,\hat{k}\right)_{x=0} \cdot (-\hat{i}\,dA) = 0.$$

$$\phi_{total} = \phi_{S1} + \phi_{S2} + \phi_{S3} + \phi_{S4} + \phi_{S5} + \phi_{S6}.$$

$$= 0 + 0.081\ \frac{Nm^2}{C} + 0 + 0 - 0.135\ \frac{Nm^2}{C} + 0$$

$$= -0.054\ \frac{N}{C}\ m^2$$

Applying Gauss law

$$\phi = \frac{q_{encl}}{\varepsilon_0} \qquad q_{encl} = \phi\,\varepsilon_0 = (-)0.054\ \frac{N}{C}\,m^2 \times 8.85\times10^{-12}\ \frac{C^2}{Nm^2}$$

$$q_{encl} = -0.4779\times10^{-12}\ C$$

S1is left surface; S2 is top; S3 is right; S4 is bottom; S5 is front; S6 is back

REVIEW & SUMMARY

Gauss' Law *Gauss' law* and Coulomb's law are different ways of describing the relation between charge and electric field in static situations. Gauss' law is

$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law)}, \tag{23-6}$$

in which q_{enc} is the net charge inside an imaginary closed surface (a *Gaussian surface*) and Φ is the net *flux* of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface)}. \tag{23-4}$$

Coulomb's law can be derived from Gauss' law.

Applications of Gauss' Law Using Gauss' law and, in some cases, symmetry arguments, we can derive several important results in electrostatic situations. Among these are:

1. An excess charge on an isolated *conductor* is located entirely on the outer surface of the conductor.
2. The external electric field near the *surface of a charged conductor* is perpendicular to the surface and has magnitude

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface)}. \tag{23-11}$$

   Within the conductor, $E = 0$.
3. The electric field at any point due to an infinite *line of charge* with uniform linear charge density λ is perpendicular to the line of charge and has magnitude

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \text{(line of charge)}, \tag{23-12}$$

   where r is the perpendicular distance from the line of charge to the point.
4. The electric field due to an *infinite nonconducting sheet* with uniform surface charge density σ is perpendicular to the plane of the sheet and has magnitude

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(sheet of charge)}. \tag{23-13}$$

5. The electric field *outside a spherical shell of charge* with radius R and total charge q is directed radially and has magnitude

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{(spherical shell, for } r \geq R\text{)}. \tag{23-15}$$

   Here r is the distance from the center of the shell to the point at which E is measured. (The charge behaves, for external points, as if it were all located at the center of the sphere.) The field *inside* a uniform spherical shell of charge is exactly zero:

$$E = 0 \qquad \text{(spherical shell, for } r < R\text{)}. \tag{23-16}$$

6. The electric field *inside a uniform sphere of charge* is directed radially and has magnitude

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r. \tag{23-20}$$

Welcome to the Physics class

Principles of Physics II

Lecture 9

Electric Potential Energy (U) and Electric potential (V) at a point in an electric field

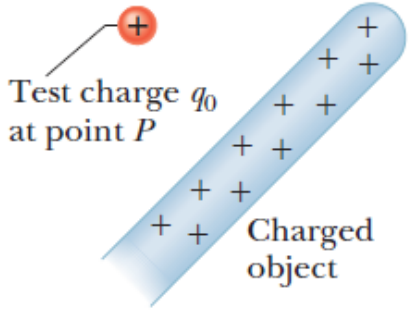
Electric Potential Energy

Pot Energy — The System — External Agent

U — The field + charged particle

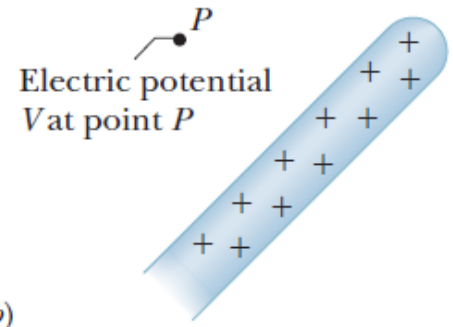
$\Delta U = U_f - U_i = -W$

Figure 24-1 Particle 1 is located at point P in the electric field of particle 2.



(a)

The rod sets up an electric potential, which determines the potential energy.



(b)

Figure 24-2 (a) A test charge has been brought in from infinity to point P in the electric field of the rod. (b) We define an electric potential V at P based on the potential energy of the configuration in (a).

Work done by the field $W = -\Delta U$ change in the potential energy

or

$\Delta U = -W$ negative of the work done

$\Delta U = W_a$ work done by an external agent.

$\boxed{\Delta U = -W = W_a}$

U = potential energy

V = potential at a point.

$U = -W_\alpha$

U_i

$E \rightarrow q$

U_f

$F = Fq$

work done by the system $= W$

$U_i = -W_f$

q_s

$\Delta U = U_f - U_i$

$\Delta U = U_f - 0 = -W_\alpha$

$\Delta U = U$

$U_f = U$

q_s

initial position

α

$U_i(\alpha) = 0$

f $\quad$ i $\quad$ U_i $\quad$ q

U_f

qV_f

V_f is the potential at point f

V_i is the potential at pt i

$$\Delta U = U_f - U_i$$

per unit charge

$$\frac{\Delta U}{q} = \frac{U_f}{q} - \frac{U_i}{q} = \frac{-W}{q}$$

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

Ui (all particles at infinity) = 0 and Uf = U (particles at final configuration).
Work done by the field to bring charges to the point in the configuration = W_∞.

$$U = -W_\infty.$$

$$\Delta U = U_f - U_i = -W.$$

Usually Ui is the ref. configuration at infinity and taken as zero. and

$U_f = U$

$\therefore U = -W_\infty$

CHECKPOINT 1

In the figure, a proton moves from point i to point f in a uniform electric field directed as shown. (a) Does the electric field do positive or negative work on the proton? (b) Does the electric potential energy of the proton increase or decrease?

(a) -ve
(b) increase

Electric Potential

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}.$$

substitute $-W$ for ΔU

$$\Delta V = V_f - V_i = -\frac{W}{q} \quad \text{(potential difference defined).}$$

$V_f = V$ the potential at a point

$V_i = V(\infty) = 0$

$$V = -\frac{W_\infty}{q} \quad \text{(potential defined),}$$

$E = N/C = \frac{Nm}{mC} = \frac{J}{mC}$

$= \frac{J}{Cm} = V/m$

1 volt = 1 joule per coulomb. V = J/C J = V.C E = N/C

$1\,eV = e(1V) = 1.602 \times 10^{-19}\ J$

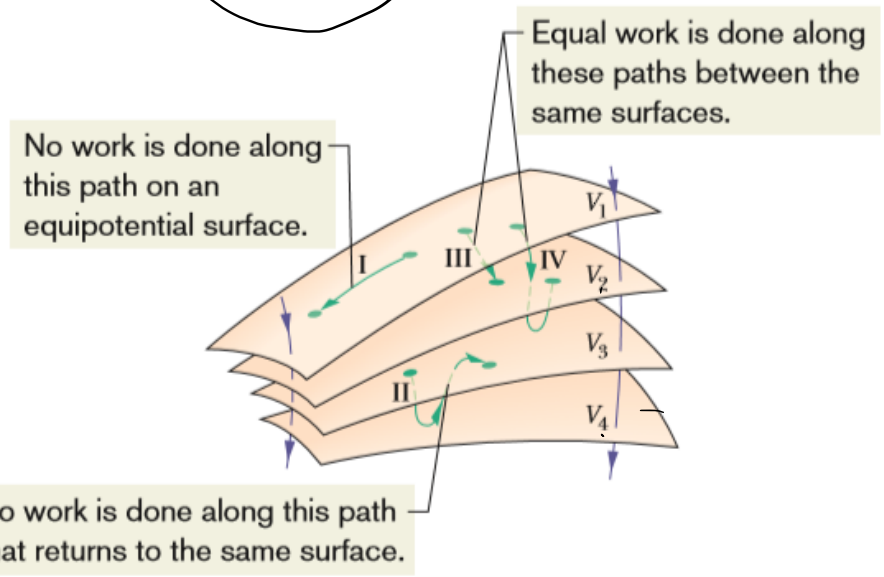


Fig. 24-2 Portions of four equipotential surfaces at electric potentials $V_1 = 100$ V, $V_2 = 80$ V, $V_3 = 60$ V, and $V_4 = 40$ V. Four paths along which a test charge may move are shown. Two electric field lines are also indicated.

electric field lines are $\perp$ to the equipotential surface

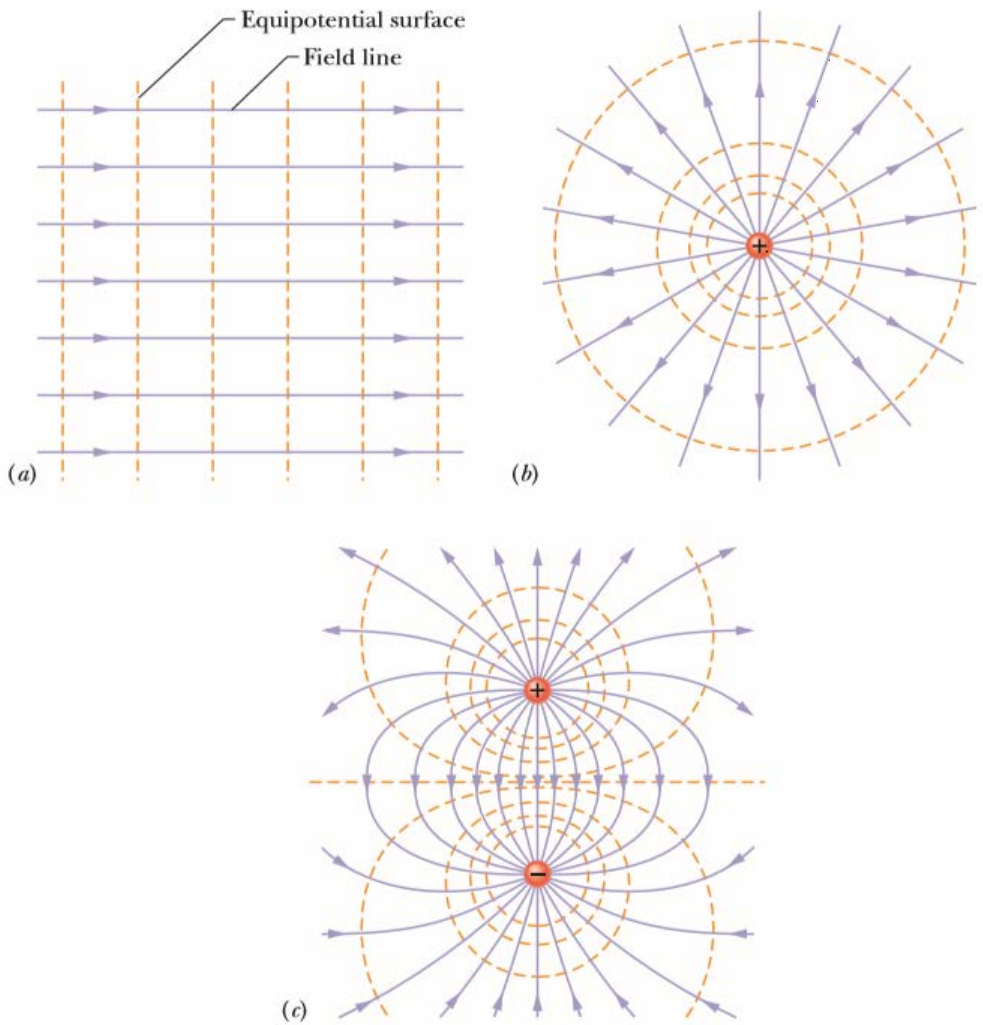


Fig. 24-3 Electric field lines (purple) and cross sections of equipotential surfaces (gold) for (*a*) a uniform electric field, (*b*) the field due to a point charge, and (*c*) the field due to an electric dipole.

$$\Delta U = -W = W_a$$

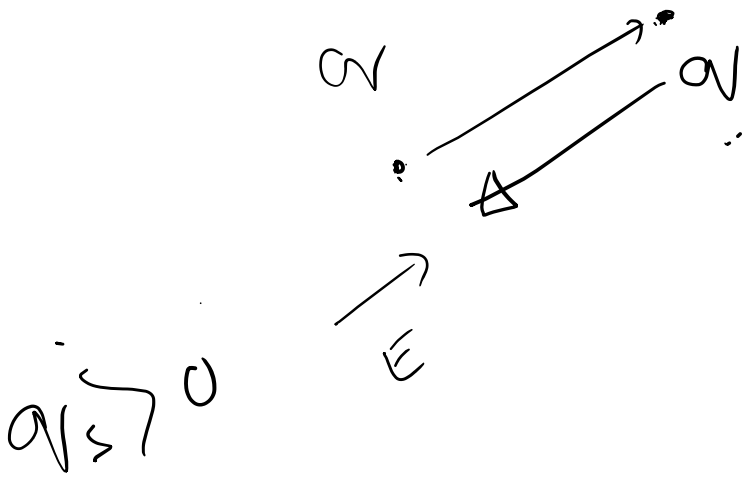
ΔU → change in PE / Energy

W → work done by the field (system)

W_a → work done by an external Agent.

U = Potential Energy

V = Is the potential at a point in an electric field.



Potential difference between two points "i" and "f" in an electric field

$$\Delta V = V_f - V_i = \frac{U_f}{q_0} - \frac{U_i}{q_0} = \frac{\Delta U}{q_0}$$

$$\therefore \underline{\underline{\Delta U}} = q_0 \underline{\underline{\Delta V}}$$

Also,

$$\Delta V = V_f - V_i = \frac{U_f - U_i}{q_0} = \frac{\Delta U}{q_0} = \frac{-W}{q_0}$$

Potential difference can be ⊕ve, ⊖ve or "0"

If we set $V_i = V_\alpha = 0$ that is potential at infinity is zero and $V_f = V$ the potential at a point in an electric field then

$$\Delta V = V_f - V_i = V - 0 = -\frac{W}{q_0}$$

$$\therefore \quad V = -\frac{W_\alpha}{q_0}$$ (defines potential at a point)

$$\text{or} \quad V = \frac{W_a^\alpha}{q_0}$$

$$E = \frac{N}{C} = \frac{Nm}{C\,m} = \frac{J}{C \cdot m}$$

$$= \frac{V}{m}$$

$$1 \text{ Volt } (1V) = \frac{1\,J}{1\,C} \qquad \text{Volt} = J/C$$

$$\boxed{E = \frac{V}{m}}$$

Electric Potential from the electric field intensity E

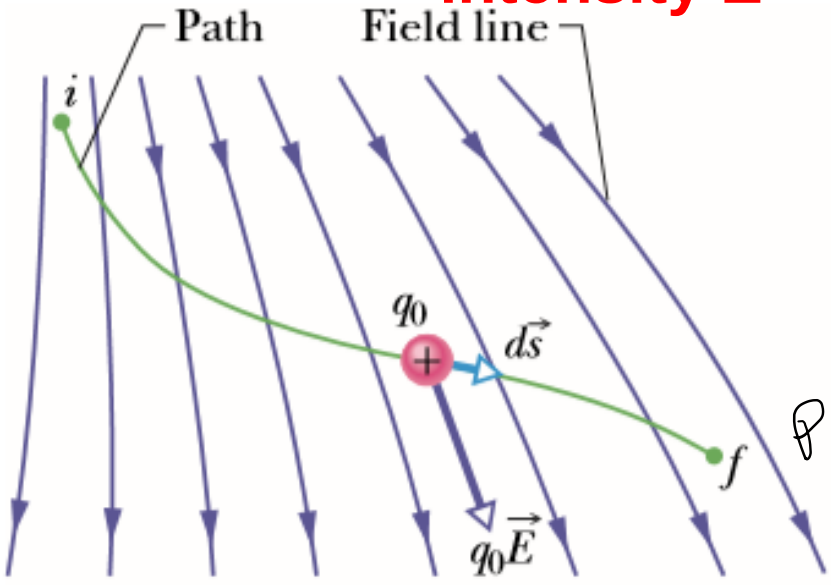


Fig. 24-4 A test charge q_0 moves from point i to point f along the path shown in a nonuniform electric field. During a displacement $d\vec{s}$, an electrostatic force $q_0\vec{E}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.

Differential amount of work done by the field $dW = \vec{F}\cdot d\vec{s} = q_0\vec{E}\cdot d\vec{s}$

$\therefore \quad W = \int_i^f q_0\vec{E}\cdot d\vec{s}$ Now $\Delta U = -W$

$$U_f - U_i = -q_0\int_i^f \vec{E}\cdot d\vec{s}$$

or $$\frac{U_f}{q_0} - \frac{U_i}{q_0} = -\int_i^f \vec{E}\cdot d\vec{s}$$

or $$\boxed{V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}} \quad \text{--- (1)}$$

if $i \to \infty$ and we then set $V_i(\infty) = 0$

$f \to P$ $\quad V_f = V$ the potential at P

$$V = -\int_\infty^f \vec{E}\cdot d\vec{s}$$

Potential due to point charge (24-6)

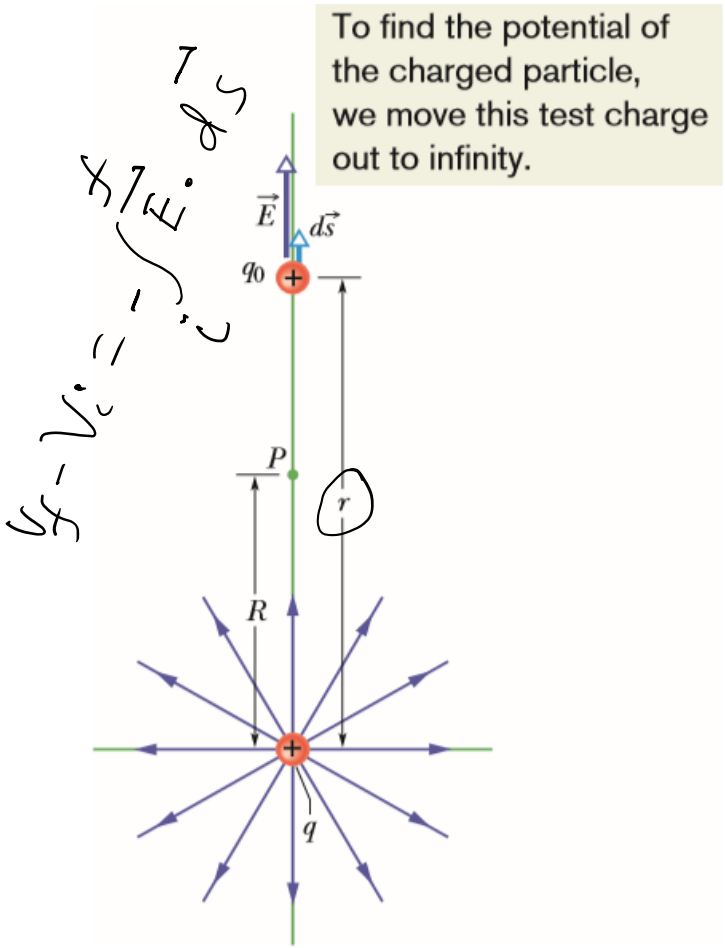


Fig. 24-6 The positive point charge q produces an electric field $\vec{E}$ and an electric potential V at point P. We find the potential by moving a test charge q_0 from P to infinity. The test charge is shown at distance r from the point charge, during differential displacement $d\vec{s}$.

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$$

Imagine a +ve test charge q_0 moves from P to infinity

$i \to R \quad f \to \infty$

$$V_f - V_i = -\int_R^\infty \vec{E}\cdot d\vec{s} = -\int_R^\infty E\,ds \qquad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$V_f = 0$ (at ∞) and $V_i = V$ (at R) $\quad ds = dr$

$$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$

$$-V = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Potential at a point due to a number of charges q_i

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \quad (n \text{ charged particles}).$$

Electric field E at a point P

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\vec{F} = \vec{E} q_t = \frac{1}{4\pi\epsilon_0} \frac{q q_t}{r^2} \hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_t}$$

Potential V at a point P

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$U = q_t V = \frac{1}{4\pi\epsilon_0} \frac{q q_t}{r}$$

$\vec{E}$ and $\vec{F}$ are vectors

V and U are scalars

$$\Delta U = W_a = -W$$

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electric force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150$ N/C and is directed downward. What is the change ΔU in the electric potential energy of a released electron when the electric force causes it to move vertically upward through a distance $d = 520$ m (Fig. 24-3)? Through what potential change does the electron move?

$\Delta V = ?$

See note

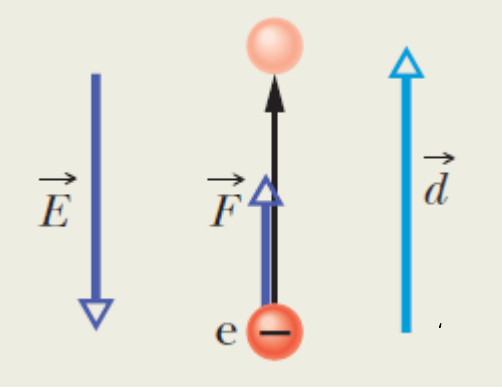


Figure 24-3 An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electric force $\vec{F}$ due to an electric field $\vec{E}$.

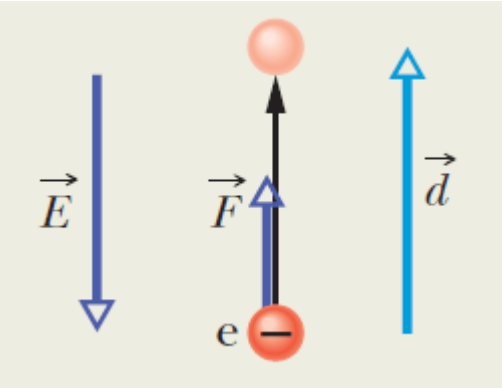


Figure 24-3

$\Delta U = -W$ $\quad W = \vec{F}\cdot\vec{d}$ $\quad \vec{F} = q\vec{E}$ $\quad q = -e = -1.6\times10^{-19}$ Coul, $d = 520$ m

$W = \vec{F}\cdot\vec{d} = q\vec{E}\cdot\vec{d} = qEd\cos\theta$

$= -1.602\times10^{-19} \times 150 \times 520 \cos 180^\circ$ $\quad$ E is downward, d moves upward

$= 1.2\times10^{-14}$ J = work done by the system.

During the ascent of the electron, the potential energy of the electron decreases by 1.2×10^{-14}. $\quad \Delta U = -W = -1.2\times10^{-14}$ J

$\Delta U = q\Delta V$

$$\Delta V = \frac{\Delta U}{q} = \frac{-1.2\times10^{-14}\ \text{J}}{-1.6\times10^{-19}\ \text{C}} = 75\ \text{KV}$$

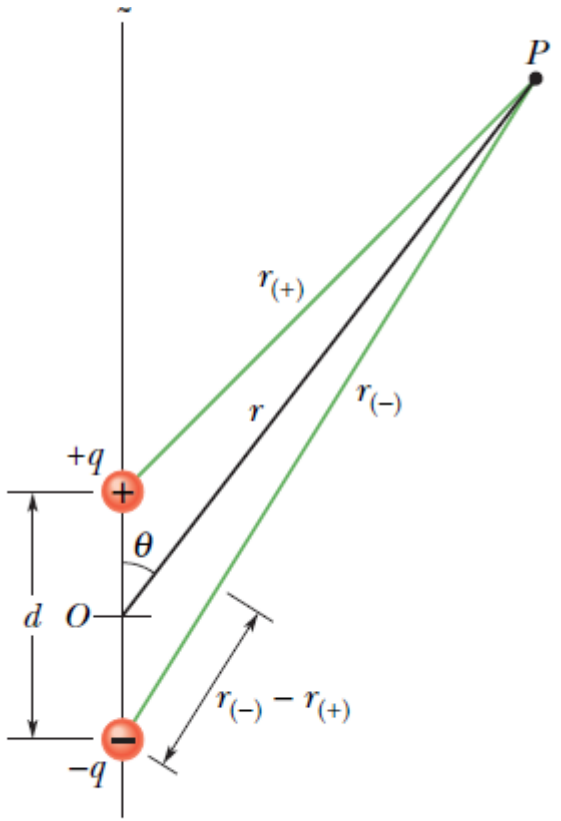
The electron moved through a potential difference of 75 KV

Potential Due to an Electric Dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \text{(electric dipole)},$$

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \quad (n \text{ charged particles}).$$

The net potential at point P for the dipole



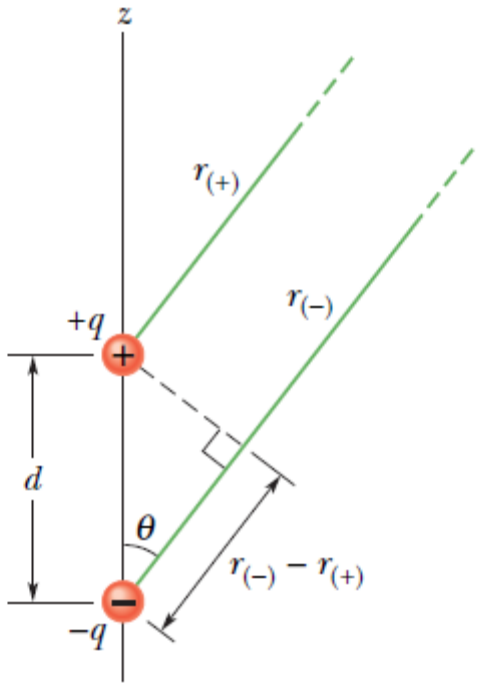
(a)

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{r_{(-)} - r_{(+)}}{r_{(+)}\, r_{(-)}}$$

r > > d



(b)

$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

or $$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$p = qd$ is the magnitude of the dipole moment.

Potential due to a continuous charge distribution.

$\sum \rightarrow \int$ over the charge distribution.

$dq \rightarrow$ infinitesimal charge. Small enough to invoke the formula for potential due to a point charge as the following

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$ for potential

for the entire charge distribution,

$$V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$ etc.

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \quad (n \text{ charged particles}).$$

Potential due to a continuous charge distribution

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \quad \text{(positive or negative } dq\text{)}.$$

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}.$$

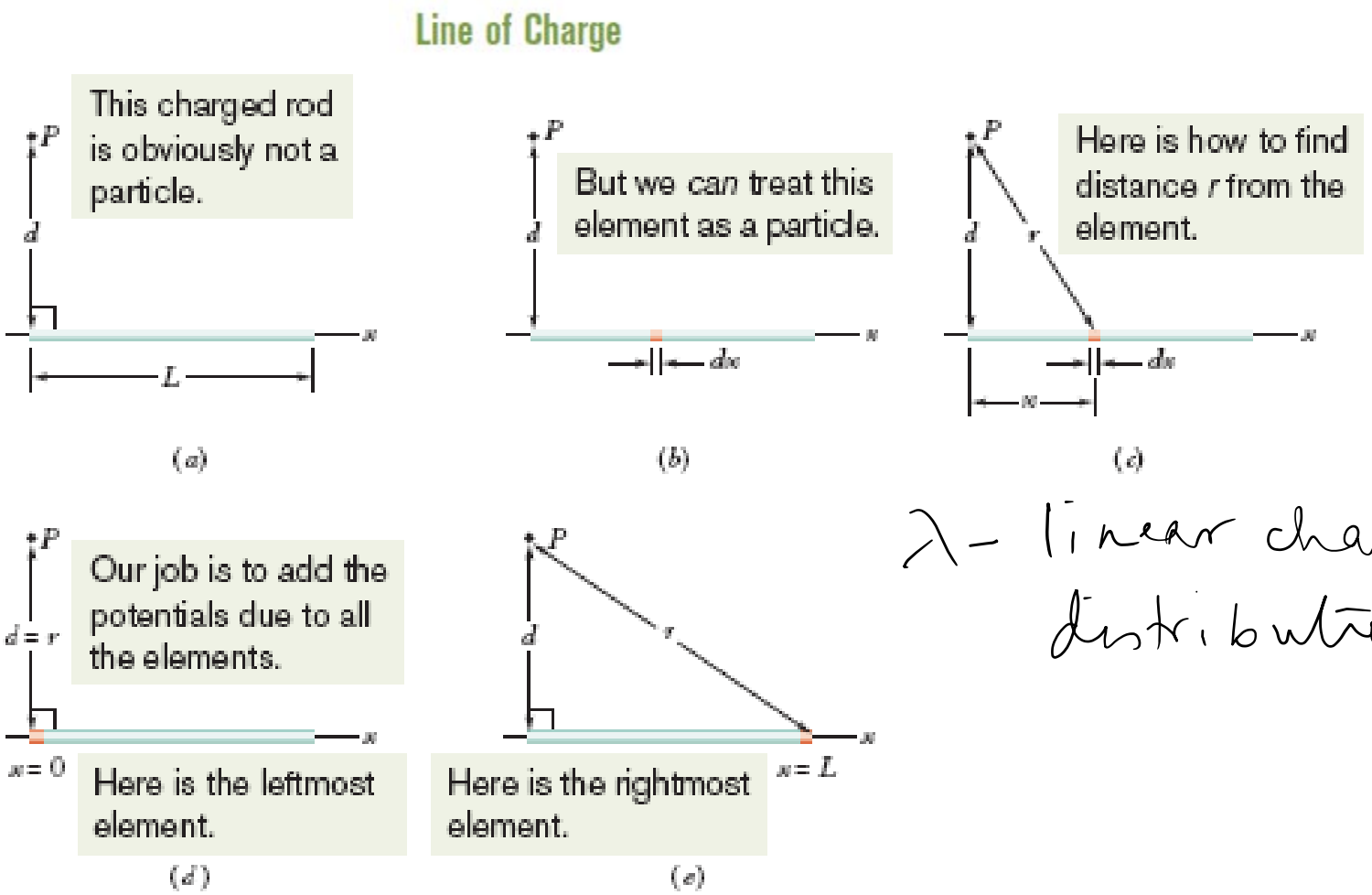


Fig. 24-12 (*a*) A thin, uniformly charged rod produces an electric potential V at point P. (*b*) An element can be treated as a particle. (*c*) The potential at P due to the element depends on the distance r. We need to sum the potentials due to all the elements, from the left side (*d*) to the right side (*e*).

λ - linear charge distribution.