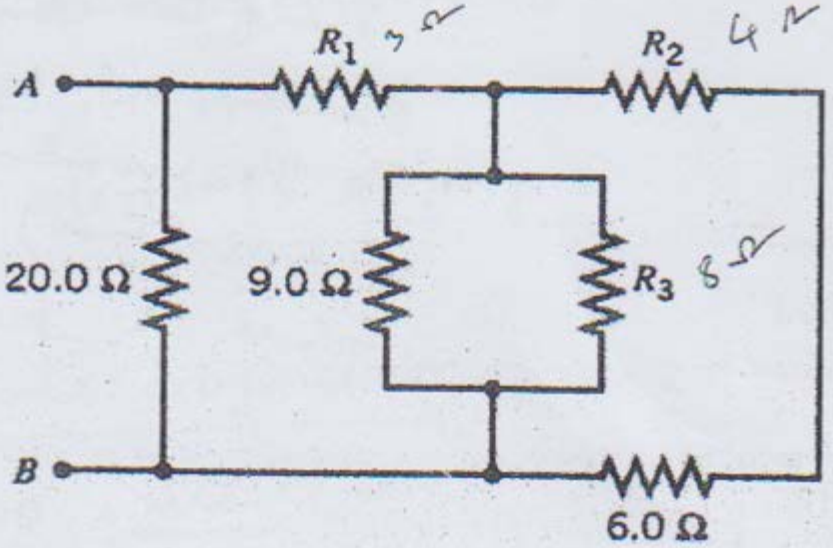
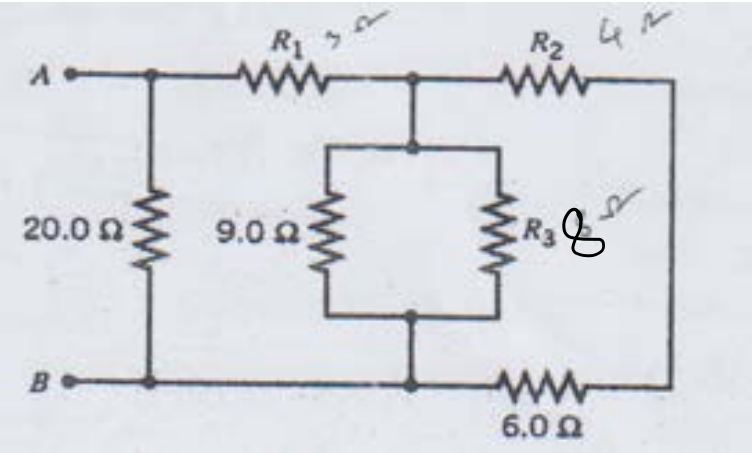
Welcome to the Physics class

Principles of Physics II

Lecture 16

Determine the equivalent resistance between the points A and B for the group of resistors in the drawing. Use $R_1 = 3.0\ \Omega$, $R_2 = 4.0\ \Omega$, and $R_3 = 8.0\ \Omega$.





$R_{46} = 4 + 6 = 10\,\Omega$ ($R_2 = 4\,\Omega$ and $6\,\Omega$ are in series)

$9\,\Omega$ and R_3 ($8\,\Omega$) and R_{46} ($10\,\Omega$) are all in ||

$$\frac{1}{R_{9,8,46}} = \frac{1}{9} + \frac{1}{8} + \frac{1}{10} = \frac{80+90+72}{720} = \frac{242}{720}$$

$$\therefore R_{9,8,4,6} = \frac{720}{242} = 2.98\,\Omega$$

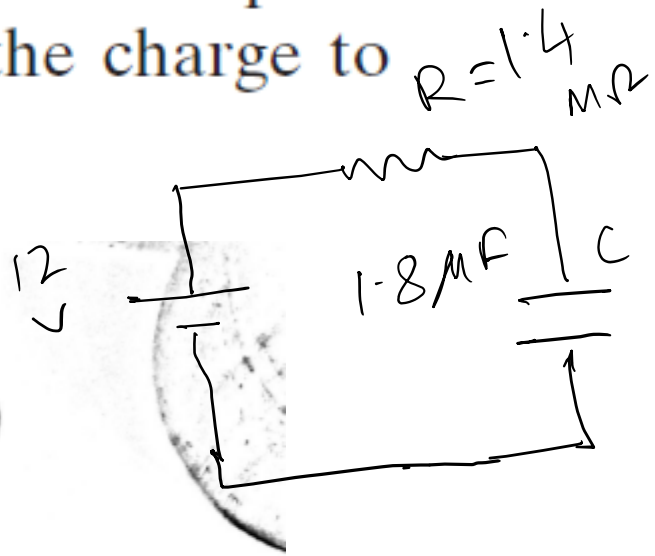
Now $R_{9,8,46}$ and R_1 ($3\,\Omega$) are in series $= 2.98 + 3 = 5.98\,\Omega$

Finally

$$\frac{1}{R_{eq}} = \frac{1}{R_{9846}3} + \frac{1}{20} = \frac{1}{5.98} + \frac{1}{20} = \frac{20+5.98}{5.98 \times 20} = \frac{25.98}{119.6}$$

$$\therefore R_{eq} = \frac{119.6}{25.98} = 4.603\,\Omega$$

•58 In an RC series circuit, emf $\mathscr{E} = 12.0$ V, resistance $R = 1.40\ \text{M}\Omega$, and capacitance $C = 1.80\ \mu\text{F}$. (a) Calculate the time constant. (b) Find the maximum charge that will appear on the capacitor during charging. (c) How long does it take for the charge to build up to $16.0\ \mu\text{C}$?



58. (a) $\tau = RC = (1.4 \times 10^{6})(1.80 \times 10^{-6})$

$= 2.52$ sec.

(b) $q^{max} = \mathcal{E}C = 12 \times 1.8\ \mu F = 21.6\ \mu C$

(c) The charging of a capacitor takes place according to the following equation.

$$q(t) = q_0\left(1 - e^{-t/RC}\right)$$

q_0 = final charge $= CE$

q_0 = final charge $= q^{max} = 21.6\ \mu C$

$RC = \tau = 2.52$ sec ; $q(t) = 16\ \mu C$. $t = ?$

$$\frac{q(t)}{q_0} = 1 - e^{-t/RC}$$

$$\therefore\ 1 - \frac{q(t)}{q_0} = e^{-t/RC}$$

$$\therefore\ \frac{q_0 - q}{q_0} = e^{-t/RC}$$

or $$-\ln\left(\frac{q_0 - q}{q_0}\right) = +t/RC$$

$$\therefore\ \ln\frac{q_0}{q_0 - q} = t/RC$$

$$\therefore\ RC\ln\frac{q_0}{q_0 - q} = \boxed{t} = \cancel{2.52\ln\left(\frac{21.6\ \mu C}{(21.6 - 16.0)\ \mu C}\right)}$$

$$\cancel{\boxed{t = 3.40\ \text{sec}}}$$

$$\therefore\ t = RC\ln\frac{q_0}{q_0 - q} = 2.52\ln\frac{21.6\ \mu C}{21.6 - 16\ \mu C}$$

$$= 3.40\ \text{sec}$$

REVIEW & SUMMARY

Emf An **emf device** does work on charges to maintain a potential difference between its output terminals. If dW is the work the device does to force positive charge dq from the negative to the positive terminal, then the **emf** (work per unit charge) of the device is

$$\mathscr{E} = \frac{dW}{dq} \quad \text{(definition of } \mathscr{E}\text{)}. \tag{27-1}$$

The volt is the SI unit of emf as well as of potential difference. An **ideal emf device** is one that lacks any internal resistance. The potential difference between its terminals is equal to the emf. A **real emf device** has internal resistance. The potential difference between its terminals is equal to the emf only if there is no current through the device.

Analyzing Circuits The change in potential in traversing a resistance R in the direction of the current is $-iR$; in the opposite direction it is $+iR$ (resistance rule). The change in potential in traversing an ideal emf device in the direction of the emf arrow is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$ (emf rule). Conservation of energy leads to the loop rule:

Loop Rule. *The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.*

Conservation of charge gives us the junction rule:

Junction Rule. *The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.*

Single-Loop Circuits The current in a single-loop circuit containing a single resistance R and an emf device with emf $\mathscr{E}$ and internal resistance r is

$$i = \frac{\mathscr{E}}{R + r}, \tag{27-4}$$

which reduces to $i = \mathscr{E}/R$ for an ideal emf device with $r = 0$.

Power When a real battery of emf $\mathscr{E}$ and internal resistance r does work on the charge carriers in a current i through the battery, the rate P of energy transfer to the charge carriers is

$$P = iV, \tag{27-14}$$

where V is the potential across the terminals of the battery. The rate P_r at which energy is dissipated as thermal energy in the battery is

$$P_r = i^2 r. \tag{27-16}$$

The rate P_{emf} at which the chemical energy in the battery changes is

$$P_{\text{emf}} = i\mathscr{E}. \tag{27-17}$$

Series Resistances When resistances are in **series,** they have the same current. The equivalent resistance that can replace a series combination of resistances is

$$R_{\text{eq}} = \sum_{j=1}^{n} R_j \quad (n \text{ resistances in series}). \tag{27-7}$$

Parallel Resistances When resistances are in **parallel,** they have the same potential difference. The equivalent resistance that can replace a parallel combination of resistances is given by

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad (n \text{ resistances in parallel}). \tag{27-24}$$

***RC* Circuits** When an emf $\mathscr{E}$ is applied to a resistance R and capacitance C in series, as in Fig. 27-15 with the switch at a, the charge on the capacitor increases according to

$$q = C\mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor)}, \tag{27-33}$$

in which $C\mathscr{E} = q_0$ is the equilibrium (final) charge and $RC = \tau$ is the **capacitive time constant** of the circuit. During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC} \quad \text{(charging a capacitor)}. \tag{27-34}$$

When a capacitor discharges through a resistance R, the charge on the capacitor decays according to

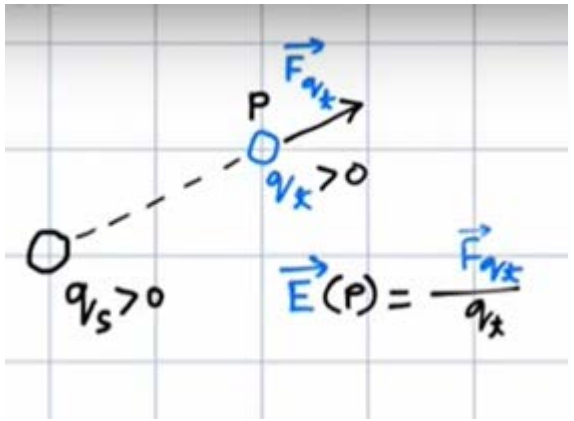
$$q = q_0 e^{-t/RC} \quad \text{(discharging a capacitor)}. \tag{27-39}$$

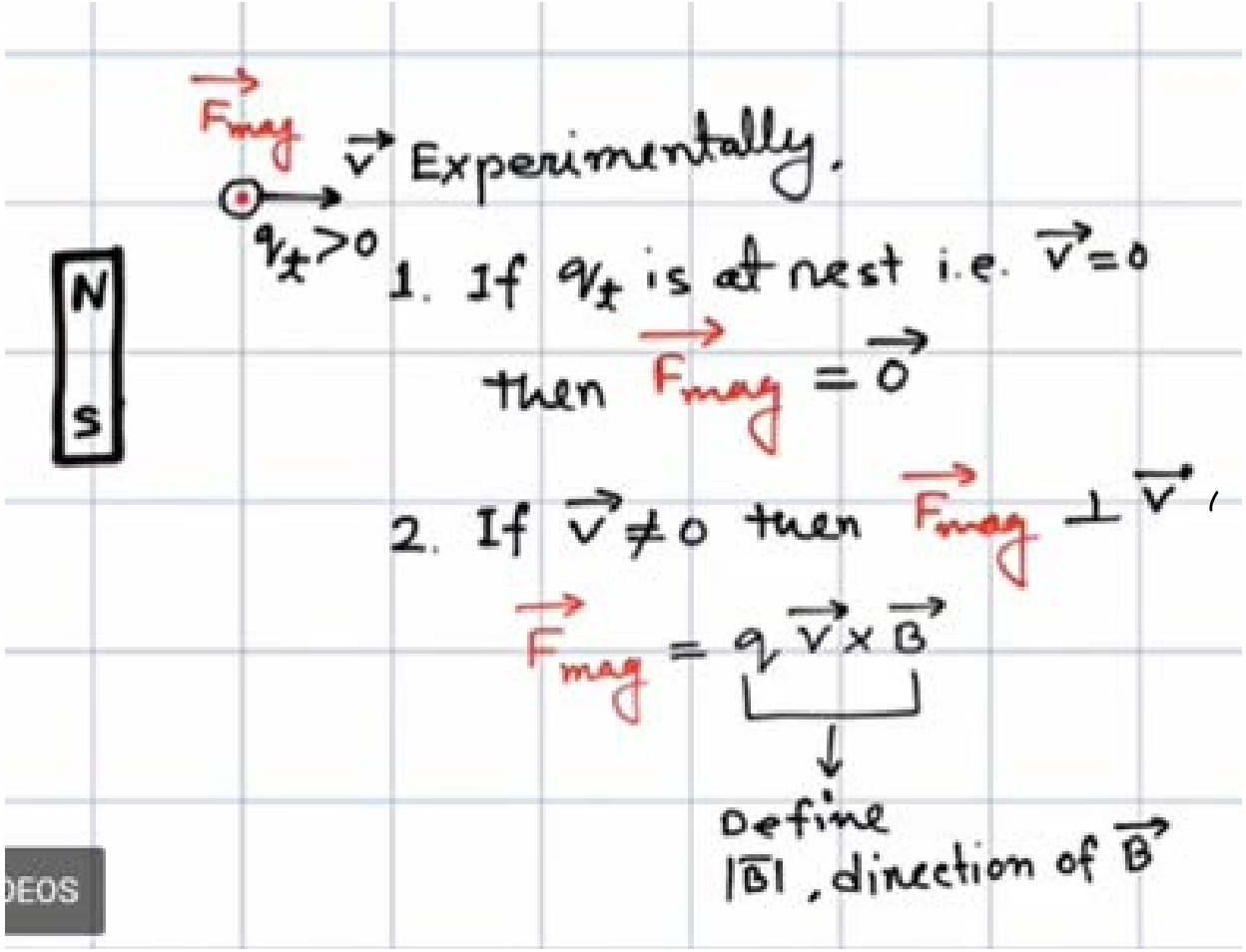
During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \quad \text{(discharging a capacitor)}. \tag{27-40}$$

MAGNETIC FIELDS

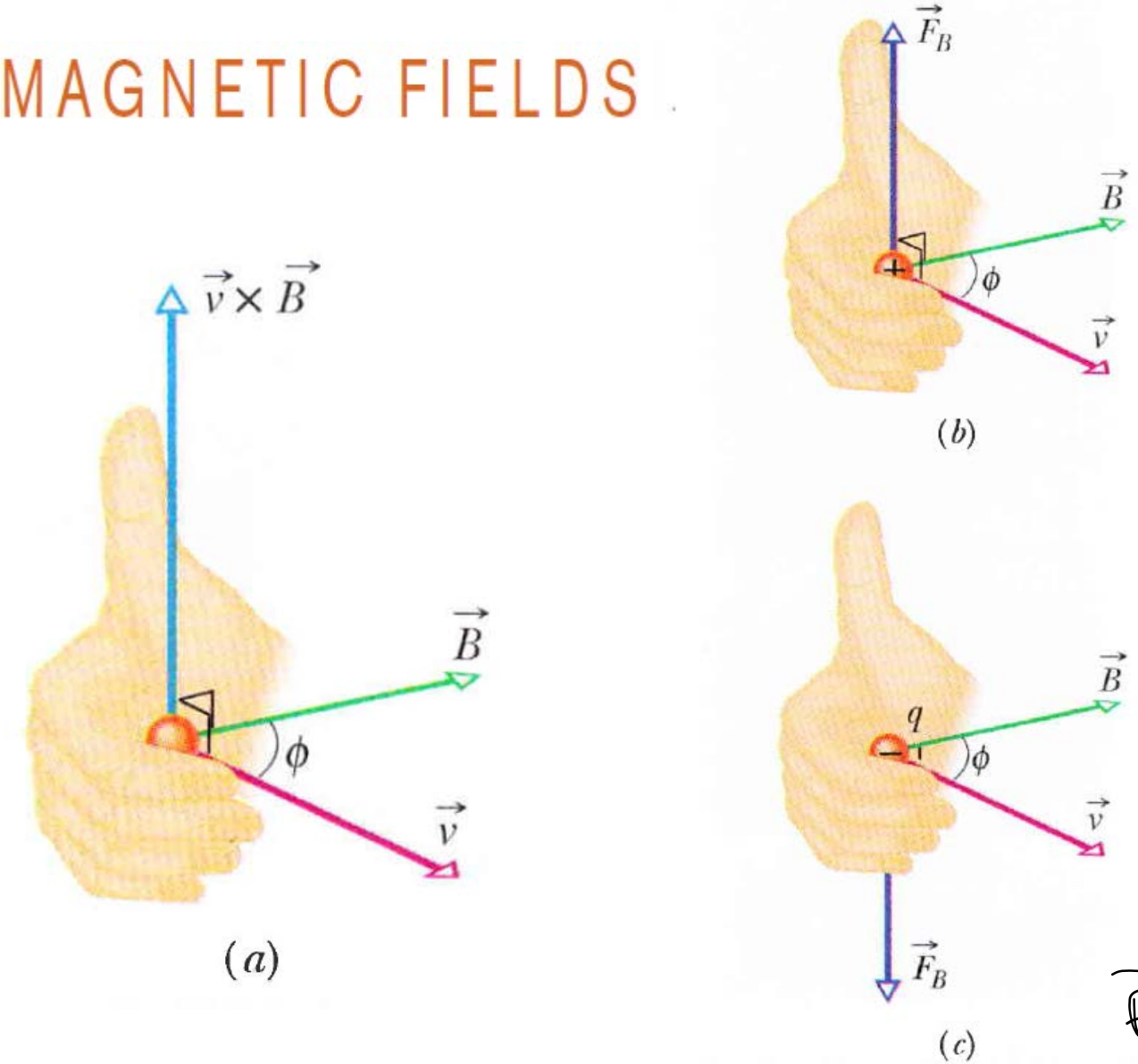
Force on a charged particle moving in a magnetic field.





Lorentz Force Law

MAGNETIC FIELDS



$\vec{F}_B = q\vec{v} \times \vec{B}$ for a ⊕ve charge

FIG. 28-2 (*a*) The right-hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle ϕ between them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb. (*b*) If q is positive, then the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$. (*c*) If q is negative, then the direction of $\vec{F}_B$ is opposite that of $\vec{v} \times \vec{B}$.

$\vec{F}_B = -q\vec{v} \times \vec{B}$ for a negative charge

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

Definition of B.

Magnitude of $|F_B| = |q| v B \sin\phi$ --- (1)

where ϕ is the angle betn $\vec{v}$ and $\vec{B}$

or $|F_B| = q v B$ if v and B are $\perp$r.

We can now define a magnetic field B which has a magnitude B given by equation (1) and the direction given by the vector $\vec{v} \times \vec{B}$ if the charged particle is (+)ve or $-\vec{v} \times \vec{B}$ if the particle is (−)vely charged.

$$\therefore |B| = \frac{F_B}{|q| v}$$

Unit is Tesla or T.

$$B = \frac{F_B}{|q|v} = \frac{\text{Newton}}{(\text{Coul})\ \text{meter/sec}} = \frac{N}{\frac{C}{s}\ m} = \frac{N}{A.m} = T$$

Newton / Amp. m

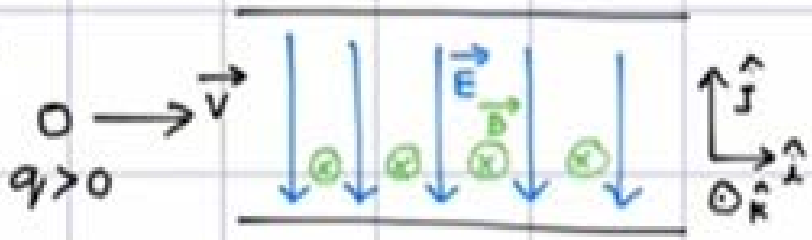
$$1\ T = 1\ \text{tesla} = 10^4\ \text{Gauss}$$

Velocity selector

If $\vec{E} \neq 0$, $\vec{B} \neq 0$ & $\vec{v} \neq 0$

Lorentz Force $\leftarrow$ $\vec{F}_q = q\vec{E} + q\vec{v} \times \vec{B}$

Example:



Slit

When does the charged particle undergoes straight line/ linear motion?

$$\vec{F}_q = \vec{0}$$

$$\Rightarrow q\vec{E} + q\vec{v} \times \vec{B} = \vec{0}$$

$$\Rightarrow -qE\hat{j} + \underbrace{q(v\hat{i} \times -B\hat{k})}_{qvB\hat{j}} = \vec{0}$$

$$\Rightarrow -qE + qvB = 0$$

$\vec{E} = E(-\hat{j})$
$\vec{B} = B(-\hat{k})$
$\vec{v} = v(\hat{i})$

Velocity Selector

$$\boxed{v = \frac{E}{B}}$$

Velocity Selector

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{k} = -\hat{j}$

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)

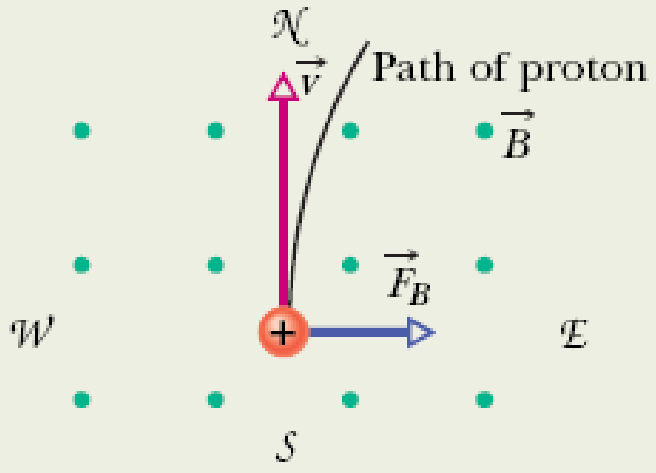
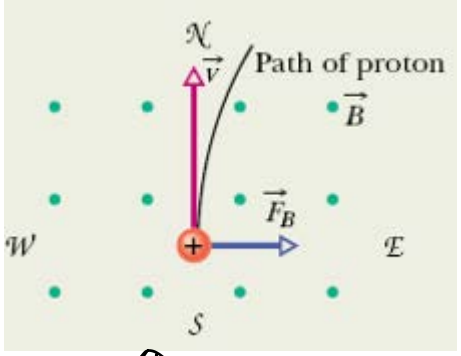


Fig. 28-6 An overhead view of a proton moving from south to north with velocity $\vec{v}$ in a chamber. A magnetic field is directed vertically upward in the chamber, as represented by the array of dots (which resemble the tips of arrows). The proton is deflected toward the east.



$\vec{B} = 1.2 \times 10^{-3}\ T\ \hat{k}$

$\vec{v} = v\,\hat{j} = 3.2 \times 10^{7}\ m/s\,\hat{j}$

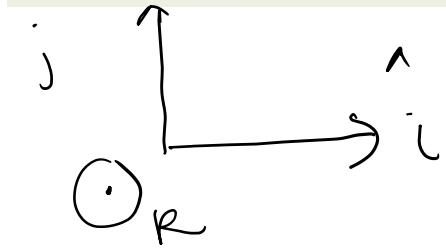
KE given $= 5.3$ MeV

$m_p = 1.67 \times 10^{-27}$ kg

$K = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$

$v = \left[\frac{2 \times 5.3\ MeV \times 1.60 \times 10^{-13}\ J}{1.67 \times 10^{-27}}\right]^{1/2}$

$= 3.2 \times 10^{7}\ m/sec.$

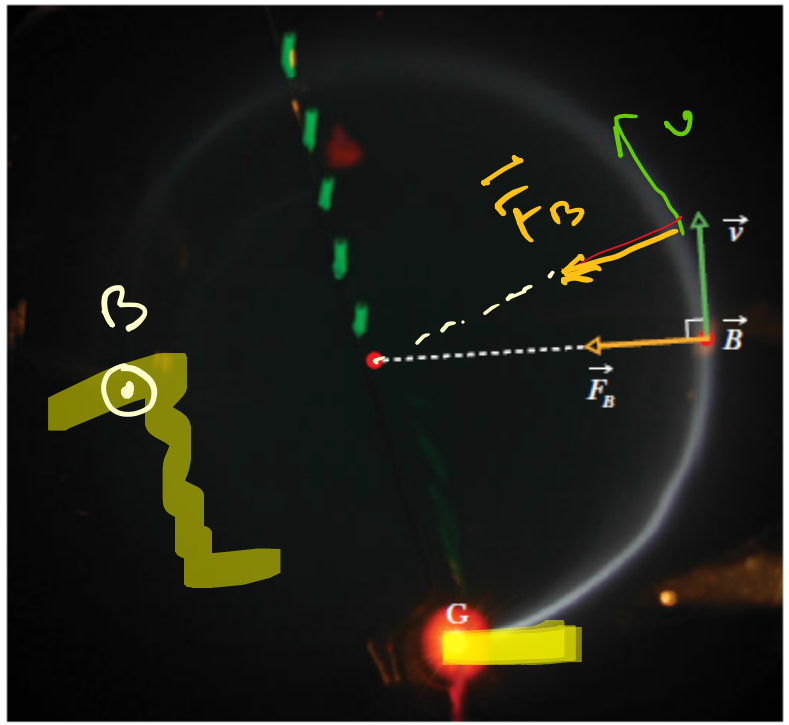


$\vec{F}_B = q\,\vec{v} \times \vec{B} = (1.6 \times 10^{-19}\ C)(3.2 \times 10^{7}\ m/s\,\hat{j}) \times (1.2 \times 10^{-3}\ T)\,\hat{k}$

$\vec{F}_B = 6.1 \times 10^{-15}\ N\ \hat{i}$

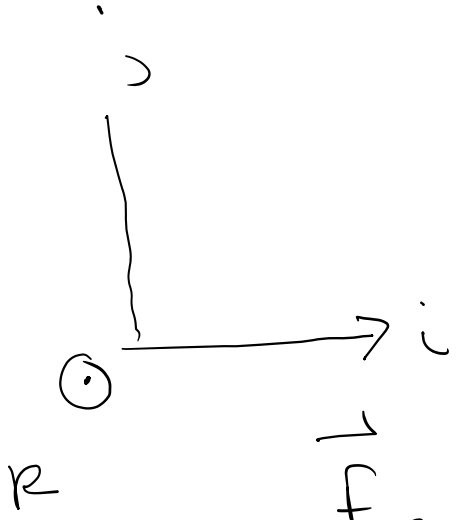
$i \times j = k$

$j \times k = i$



Courtesy Jearl Walker

Figure 28-10 Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field $\vec{B}$, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force $\vec{F}_B$; for circular motion to occur, $\vec{F}_B$ *must* point toward the center of the circle. Use the right-hand rule for cross products to confirm that $\vec{F}_B = q\vec{v} \times \vec{B}$ gives $\vec{F}_B$ the proper direction. (Don't forget the sign of q.)



$$\vec{B} = B\hat{R}$$

$$\vec{v} = v\hat{j}$$

$$\vec{F}_B = -q\vec{v} \times \vec{B}$$

$$= -qvB\,\hat{j} \times \hat{R}$$

$$= -qvB\,\hat{i}$$ at the position shown. The electron will be deflected towards $-x$ ($-i$) direction. At a later point it will deflect towards a central point, because v and B will be always $\perp$r to each other. So F_B will deflect the electron in $\perp$r direction of v. This deflection

will cause the electron to follow a circular path.

A particle moving in a circular path will be acted upon by a centripetal force $F_c = \frac{mv^2}{r}$. This centripetal force is provided by the force $F_B = -qvB$

$$\therefore \quad |F_B| = F_c \qquad |q|vB = \frac{mv^2}{r}$$

$$\therefore \quad r = \frac{mv}{|q|B}$$ the radius of the circular path of the charged particle (Here it is the electron)

The time period of the rotational motion

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{|q|B} = \frac{2\pi m}{|q|B}$$ (Time period T)

frequency of revolution (rotation/sec) = f

and $$f = \frac{1}{T} = \frac{|q|B}{2\pi m}$$ (Cyclotron frequency)

The angular frequency of rotation is

$$\omega = 2\pi f = 2\pi \frac{|q|B}{2\pi m} = \frac{|q|B}{m}$$

MAGNETIC FIELDS Contd.

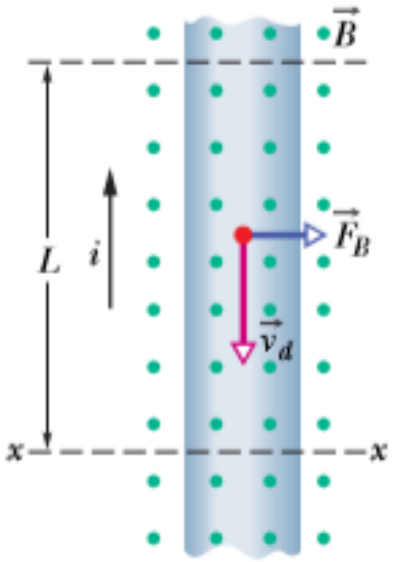
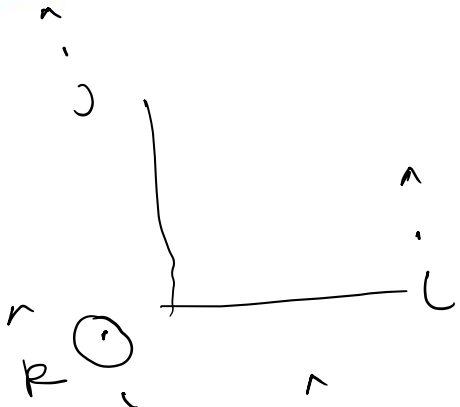


Fig. 28-15 A close-up view of a section of the wire of Fig. 28-14*b*. The current direction is upward, which means that electrons drift downward. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

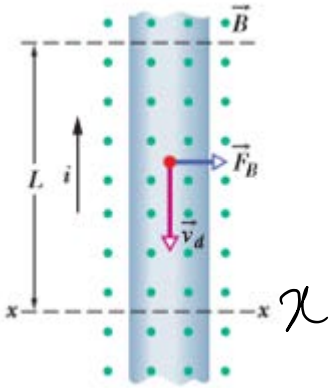


$\vec{B} = B\hat{k}$

$\vec{v}_d = v_d(-\hat{j})$

$$\vec{F}_B = -e\,\vec{v}_d \times \vec{B}$$
$$= -e\,v_d B(-\hat{j}) \times (\hat{k})$$
$$= e\,v_d B(\hat{j} \times \hat{k})$$
$$= e\,v_d B\,\hat{i}$$

So the electrons in the conductor will experience a force in $+\hat{i}$ or $+x$ direction (or in the right direction in this case). Hence the conductor as a whole move in the right direction.



Consider a length L of the conductor. All the conduction electrons within this length will pass the plane at "x" in time $t = \frac{L}{v_d}$

Now in time "t" the amount of charge (no. of electrons) passing the plane at x is $q = it = i\frac{L}{v_d}$

$$\therefore \quad v_d = \frac{iL}{q}$$

The magnetic force that acts on a straight wire of length L is then $|F_B| = q v_d B$

$$\therefore \quad |F_B| = q\frac{iL}{q}B = iLB$$

where L and B are $\perp r$.

The force is perpendicular to both the field and the length.

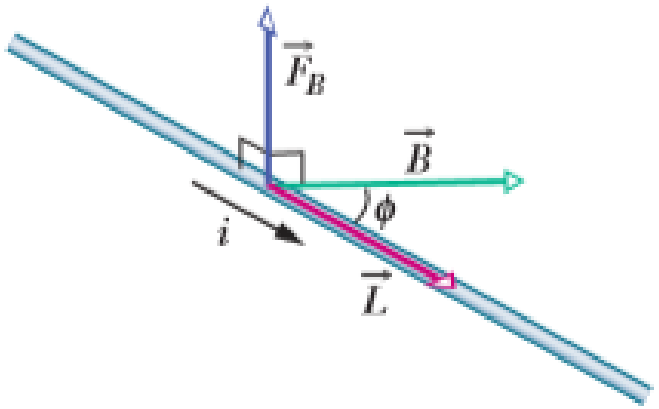


Fig. 28-16 A wire carrying current i makes an angle ϕ with magnetic field $\vec{B}$. The wire has length L in the field and length vector $\vec{L}$ (in the direction of the current). A magnetic force $\vec{F}_B = i\vec{L} \times \vec{B}$ acts on the wire.

If B is not $\perp$r to the wire then

$$|F_B| = iLB\sin\phi$$

where ϕ is the angle bet^n L and B. Vectorially we can write

$$\vec{F}_B = i\vec{L} \times \vec{B}$$ considering

L as a vector in the direction of i (the conventional direction of current).