01204371

Transform Techniques in Signal Processing

บทที่ 1 : วิทยาการสัญญาณ (The Science of Signals)

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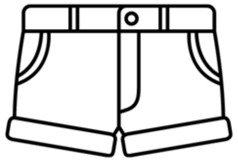
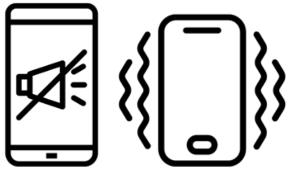
CLASSROOM RULES

NO Slipper

NO Shorts



SILENT Mode



NO Talks

RAISE Your Hand

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Today's Objective

1. รู้ความหมายของสัญญาณ
2. รู้ชนิดของสัญญาณ
3. อธิบายสัญญาณพื้นฐาน 9 สัญญาณได้
4. เขียนการแทนทางคณิตศาสตร์ของสัญญาณพื้นฐานได้

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Signal Meaning

Signals, Information, and Meaning

As an element of communication and control processes, a signal is strongly related to other concepts such as data, codes, protocols, messages, information, and meaning. However, our discussion of signals and signal processing will be, to a large degree, confined outside of the context of such facets attached to a signal.

Signals and Waveforms

In this book a signal is a time-varying waveform. It may be an information-carrying element of a communication process that transmits a message. It may be the unwanted disturbance that interferes with communication and control processes, distorts the message, or introduces errors. It may represent observations of a physical system and our characterizations of it regardless of its influence (or lack thereof) on other systems.

ที่มา: Nahvi, M. 2014. Signals and Systems. McGraw-Hill, New York, NY.

"Signals are a continual quantity of something to be measured, either by sensor or non-sensor."

Your Teacher

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Signal Meaning

Ex Natural Signals

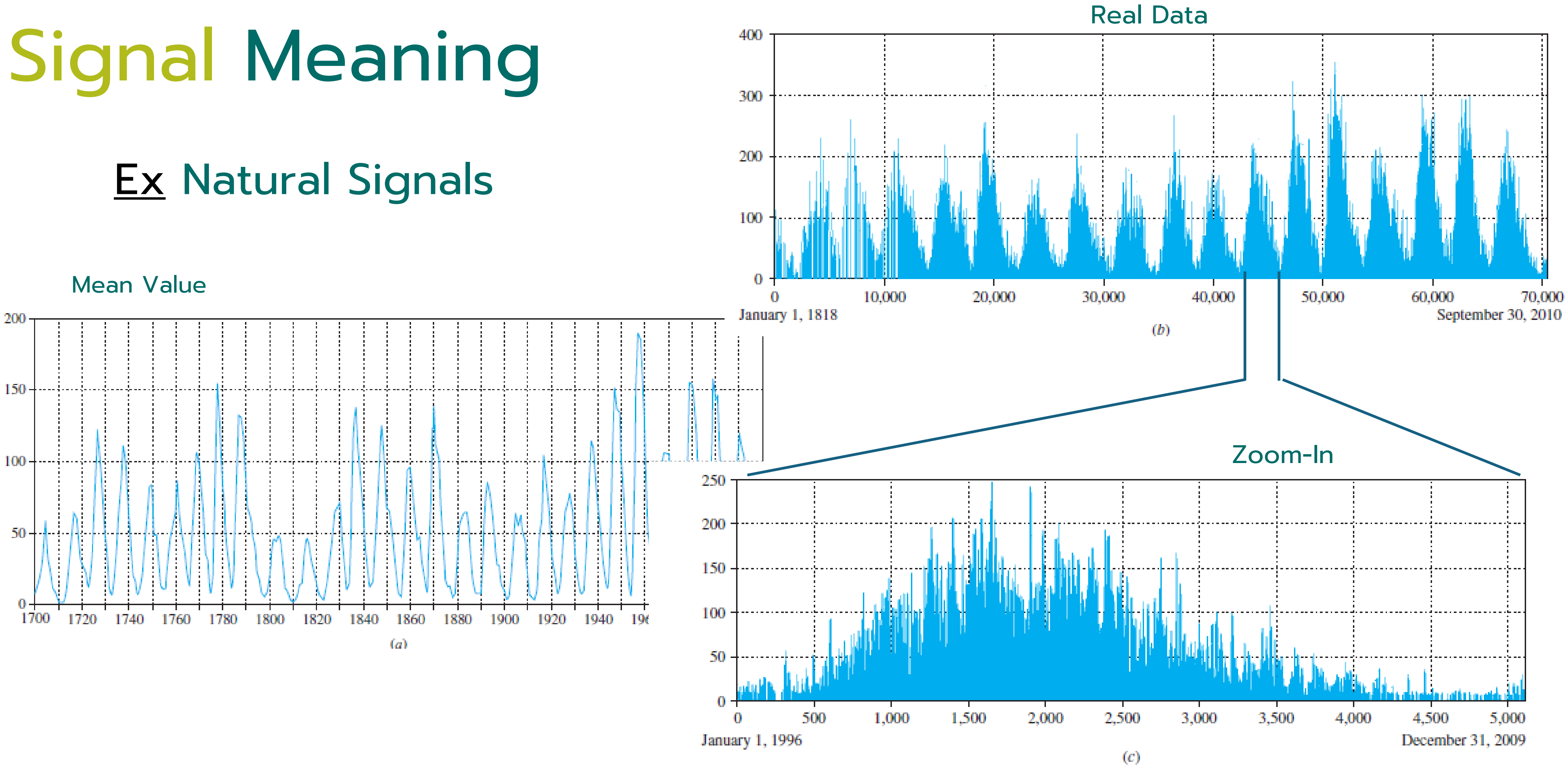

FIGURE 1.1 Sunspot numbers. *(a)* Mean annual values (AD 1700–2010); *(b)*, *(c)*, *(d)*, and *(e)* daily values for selected time intervals during 1818 to 2010.

ที่มา: Nahvi, M. 2014. Signals and Systems. McGraw-Hill, New York, NY.

Signal Meaning

Ex Seismic Signals

FIGURE 1.3 Sharp twisting of Wallace Creek in Carrizo Plane, central California, due to earth plate movements. The stream channel crosses the San Andreas fault at the twisted segment. The sharp twist (made of two bends of 90°) indicates the narrowness of the fault at that location. The gradual movements of the earth at the fault (at an average rate of 2 inches per year) along with the sudden movements due to earthquakes have twisted the channel and shifted its downstream path by about 100 meters toward the northwest (approximate direct measurement by the author).

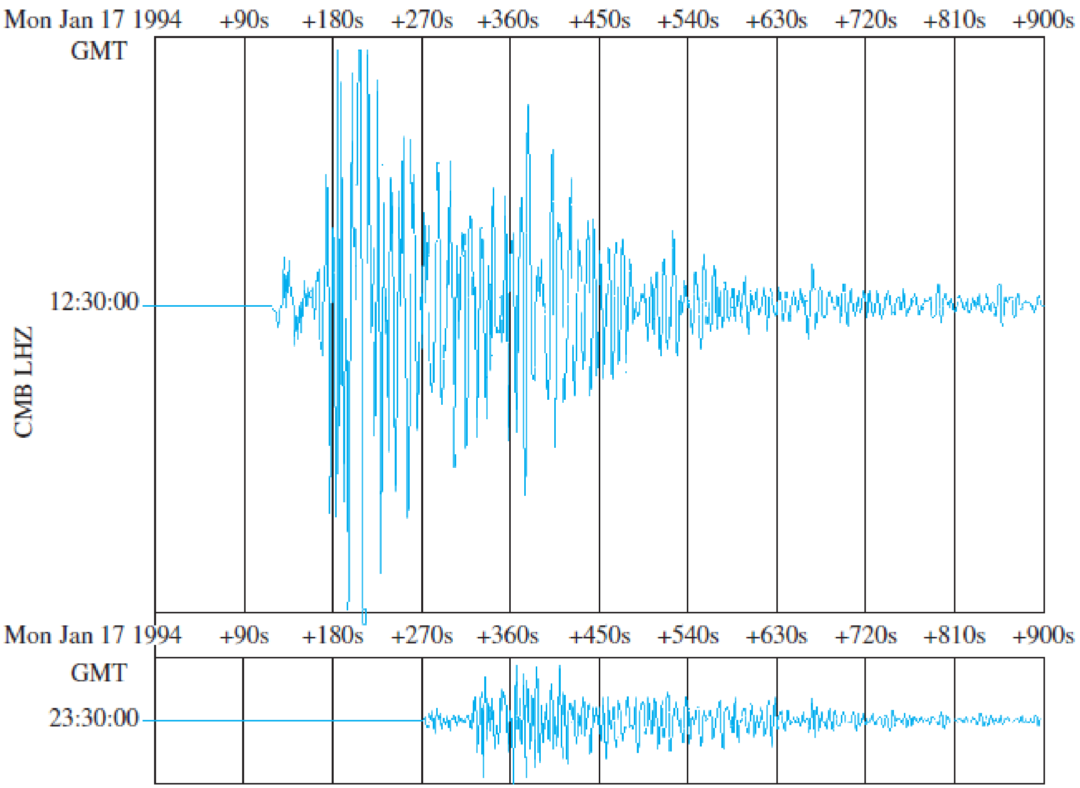

FIGURE 1.4 The seismogram of the Northridge earthquake of January 17, 1994, with a magnitude of 6.7. The main shock of the earthquake occurred at 4:30:55 a.m. local time (PST), 12:30:55 Coordinated Universal Time (UTC). The center of the earthquake was located at 1 mile south-southwest of Northridge and 20 miles west-northwest of Los Angeles. The seismogram in this figure shows vertical motion of the ground. It is plotted from the digital seismic data (sampled at the rate of 1 sample per second) which was recorded by Berkeley Digital Seismic Network (BDSN) at an earthquake station in Berkeley, California. The station is at a distance of 525 km from the earthquake location. The seismogram shows arrival of the main shock 63 seconds after the earthquake occurred in Northridge, indicating a travel speed of 8,333 m/s through the earth. An

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Signal Meaning

Ex Societal Signals

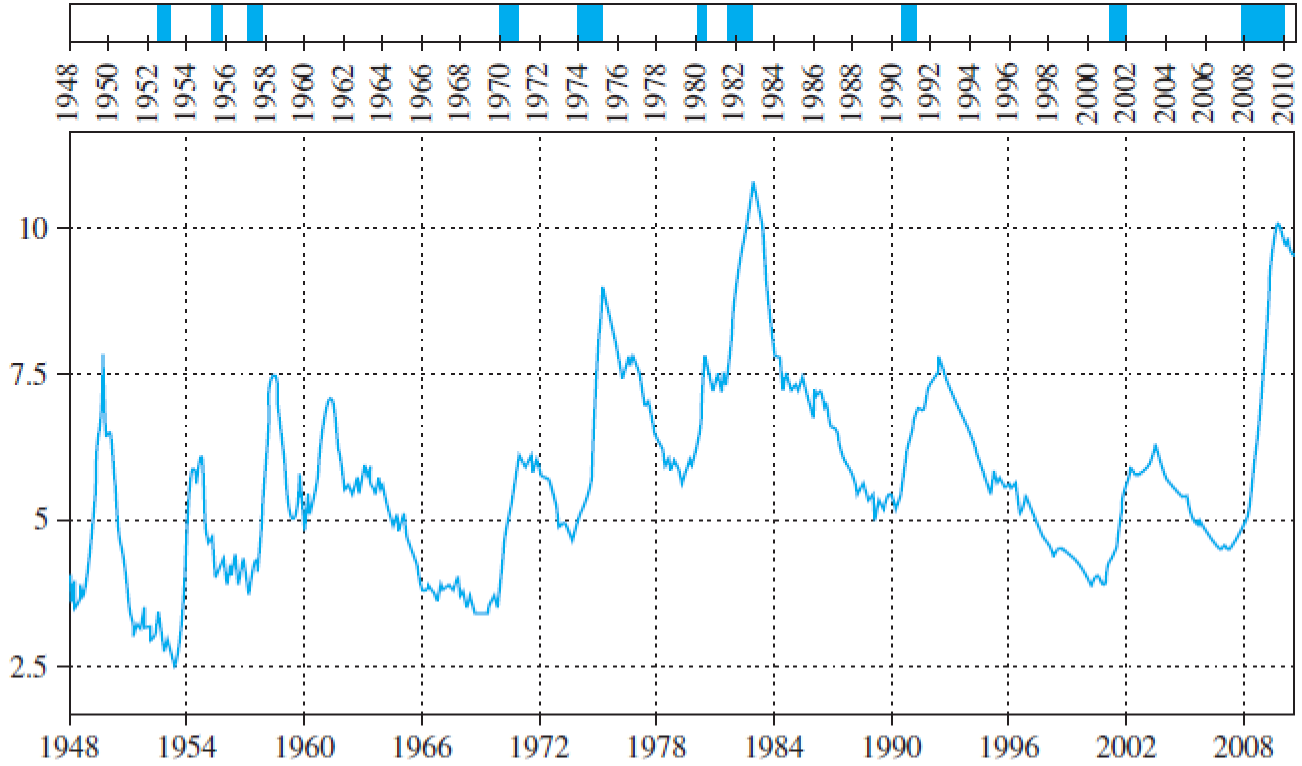

FIGURE 1.5 Plot of monthly unemployment rate in the United States as a percentage of the labor force from January 1948 to October 2010. The vertical bands on the top display periods of recession. Note the association between recessions and the rise in umemployment.

Source: Bureau of Labor Statistics, http://data.bls.gov.

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Signal Meaning

Ex Voice and Speech Signals

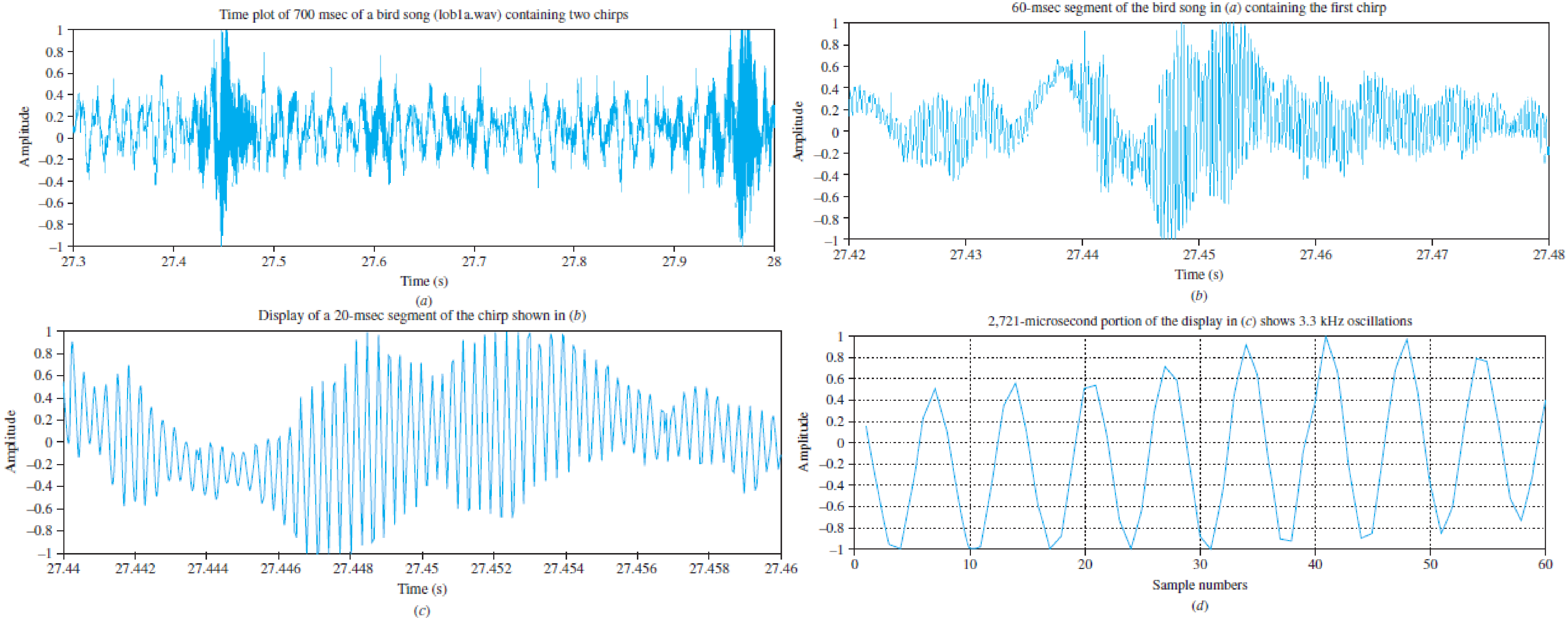

FIGURE 1.6 The waveform of a bird song (lob1a.wav, 22,050 samples per sec., 8 bits per sample, mono). The plot in (a) is 700 msec long and contains two *chirps*. The plots in (b), (c), and (d) reveal the fine structure of the *chirp*.

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Signal Meaning

Ex Communication Signals

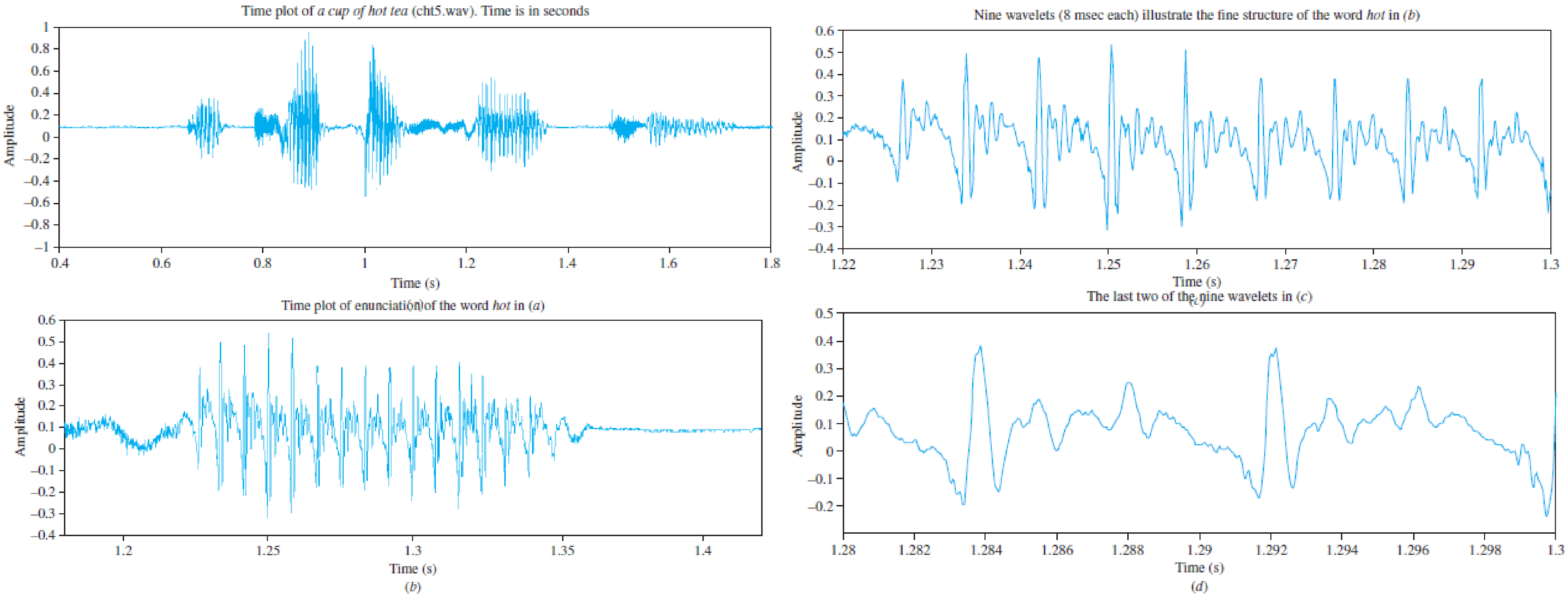

FIGURE 1.7 *(a)* The waveform of the utterance *a cup of hot tea* (cht5.wav, 22,050 samples per sec, 8 bits per sample, mono). The plot in *(b)* shows the waveform for *hot*. The fine structure of *(b)* is revealed in *(c)* and *(d)*.

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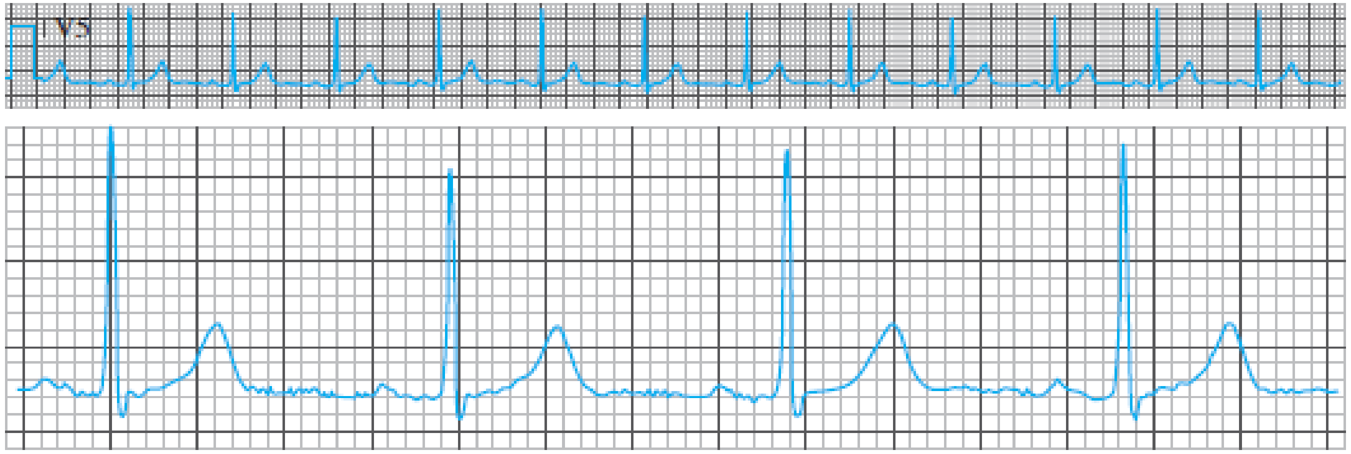
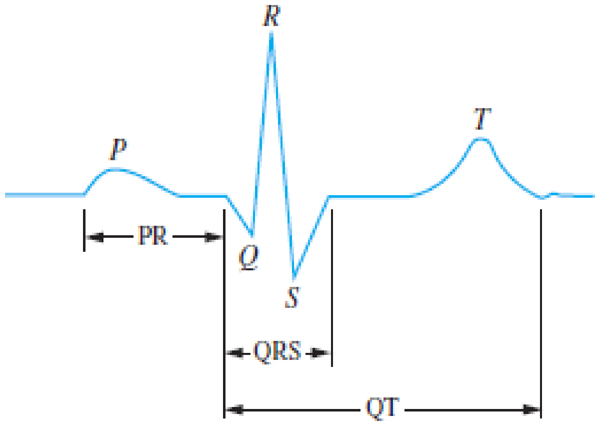
Signal Meaning

Ex Biological Signals



(a)

FIGURE 1.11 *(a)* Twelve consecutive cycles of continuous EKG recording (lead V_5 recorded by the electrode on the left side of the chest). Each division on the chart paper is 1 mm. One division on the time axis is 40 msec (a total of about 3 seconds and 4 beats on the lower trace), resulting in a rate of about 70 beats per minute (BPM). In practice, the BPM number is read directly off the chart (see text). One division on the vertical axis represents 100 μV, resulting in a base-to-peak value of 1.5 mV. Note the remarkable reproduction of the waveform in each cycle. The schematic of one cycle of EKG and its noted components are shown in Figure 1.11(*b*).



P-wave (depolarization of the atria)
QRS complex (depolarization of the ventricular muscle)
ST-T wave (repolarization of the ventricular muscle)

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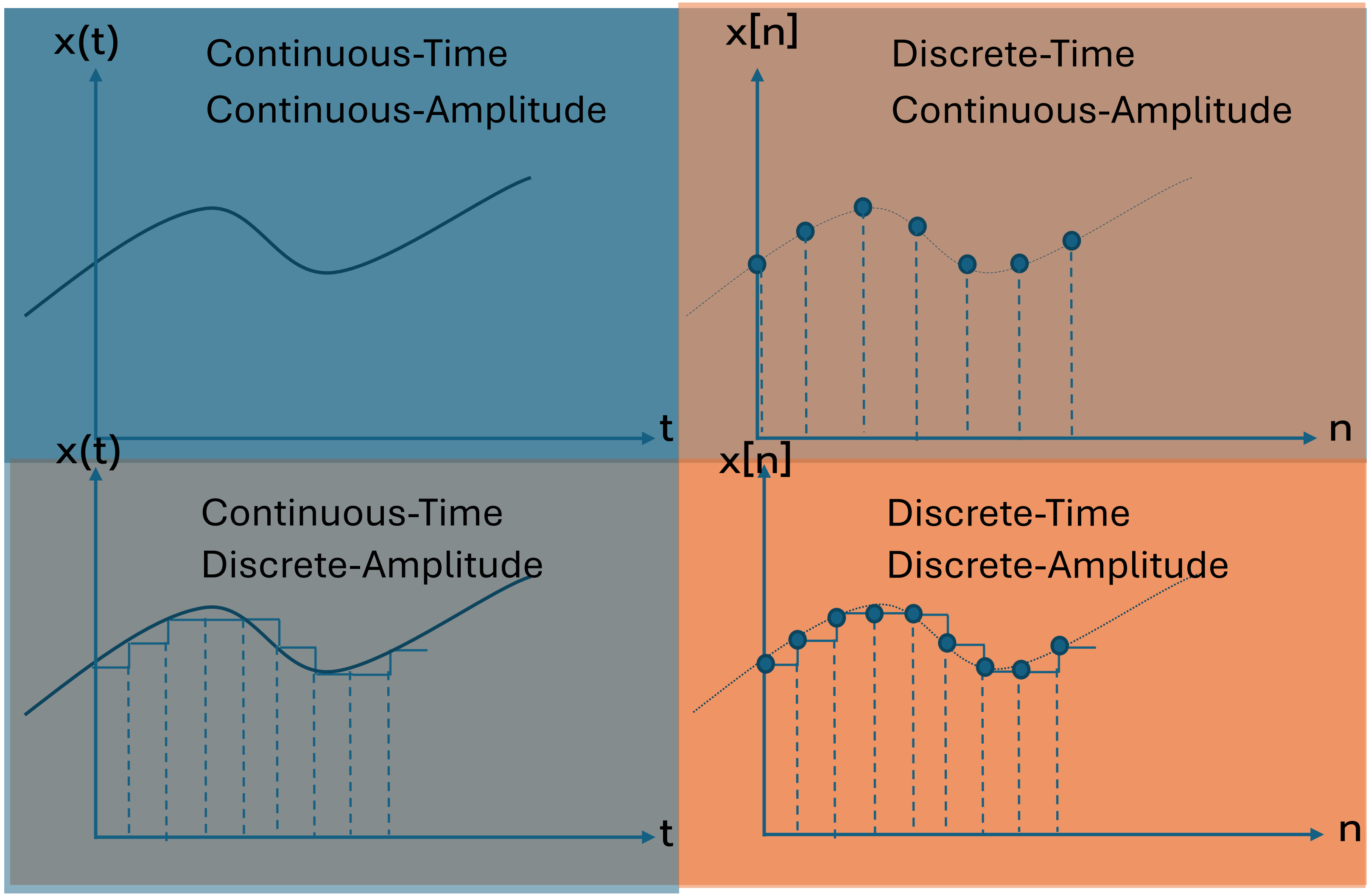
Signal Meaning

Ex BitCoin Price

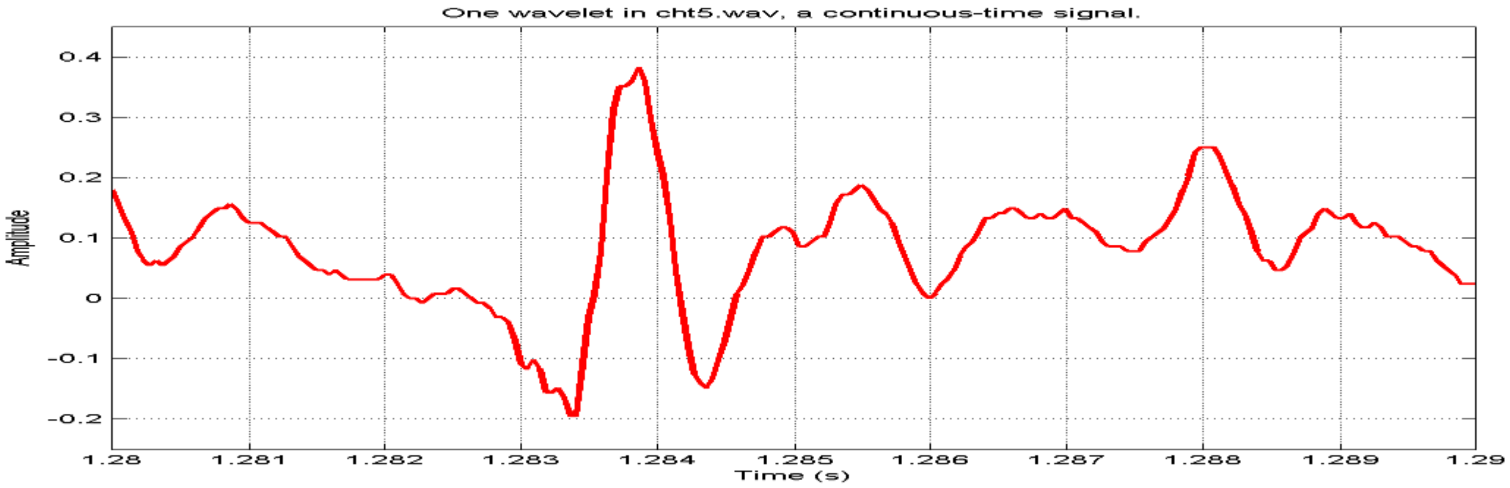
ที่มา: TradingView on 26 Nov 2024 8:44 AM

Signal Type


x(t)
Continuous-Time
Continuous-Amplitude
t
x[n]
Discrete-Time
Continuous-Amplitude
n
x(t)
Continuous-Time
Discrete-Amplitude
t
x[n]
Discrete-Time
Discrete-Amplitude
n

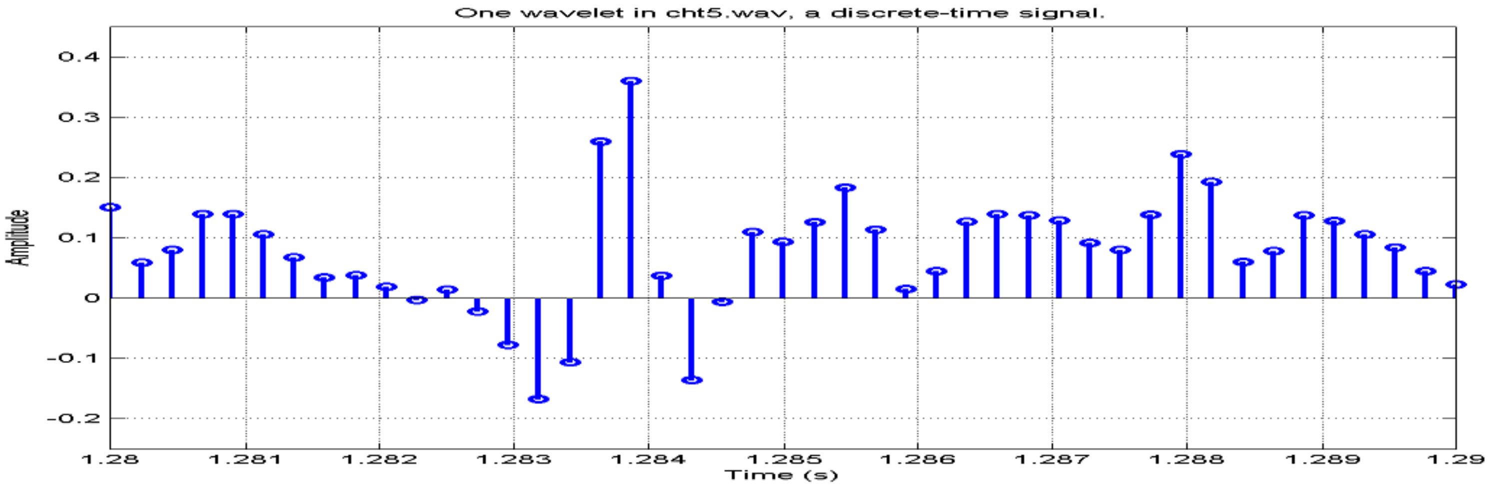
Signal Type

Continuous-Time Signal



vs.

Discrete-Time Signal



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Signal Type

Ex จงตรวจสอบสัญลักษณ์ทางคณิตศาสตร์ของสัญญาณและระบบข้อใดต่อไปนี้ "เป็นจริง"/"ไม่เป็นจริง" พร้อมอธิบายเหตุผลหากสัญลักษณ์ไม่เป็นจริง

1. x(0)
2. x(1.1)
3. x(2)
4. x(2.0)
5. x[1.1
6. x[2]
7. x[2.0]

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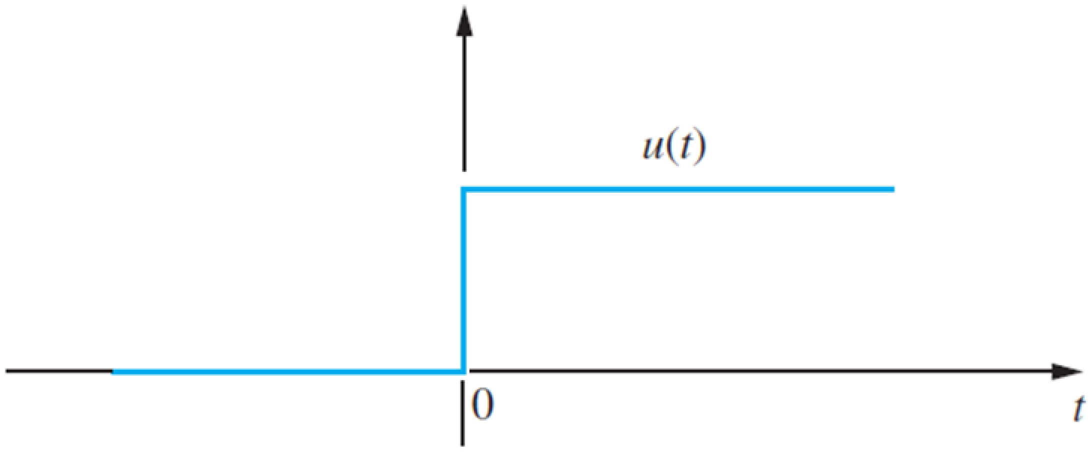
Mathematical Representation – CT Signals

1. Unit Step

The unit-step function $u(t)$ is defined by

$$u(t) = \begin{cases} 1, & \text{for } t \geq 0 \\ 0, & \text{for } t < 0 \end{cases}$$



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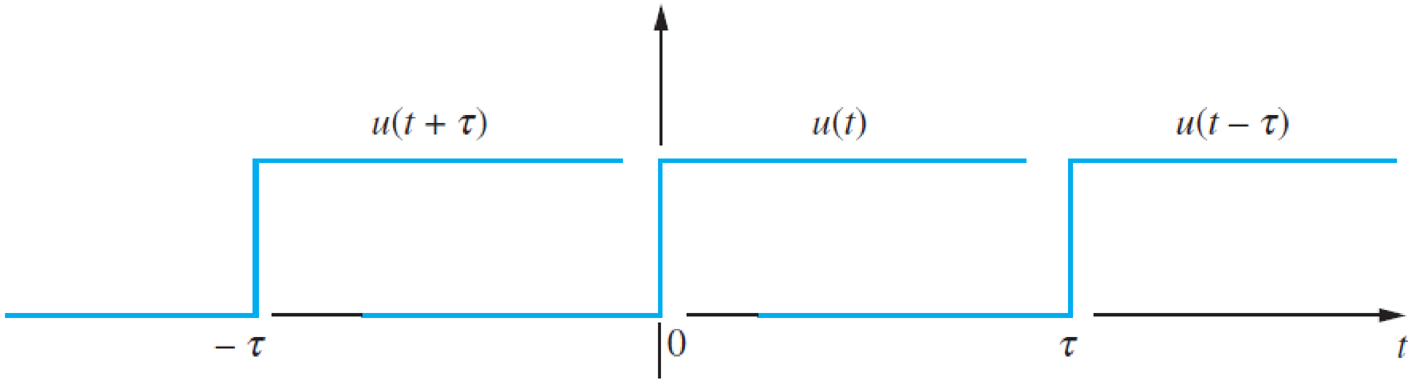
Mathematical Representation – CT Signals

1. Unit Step

A delay of τ produces $u(t-\tau)$, while an advance of τ produces $u(t+\tau)$.



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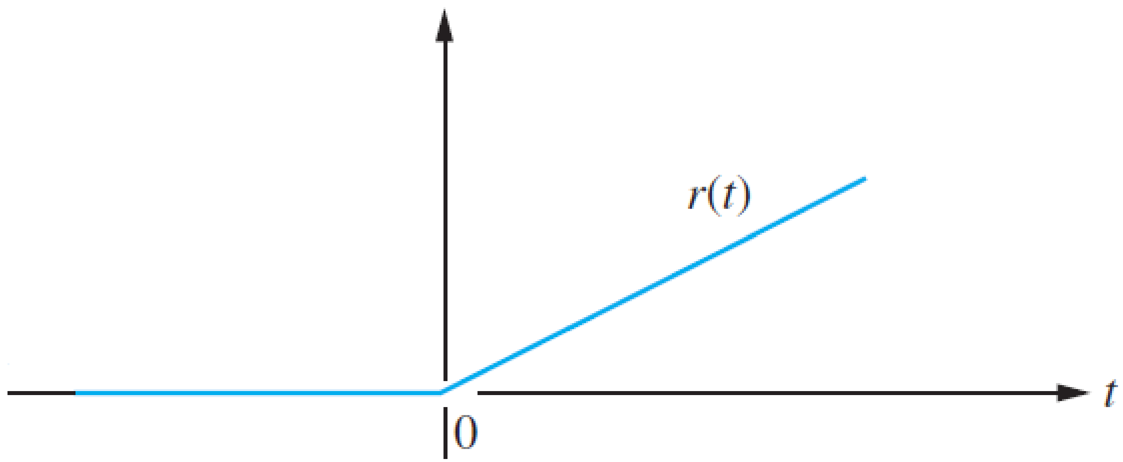
Mathematical Representation – CT Signals

2. Unit Ramp

The unit-ramp function $r(t)$ is defined by

$$r(t) = \begin{cases} t, & \text{for } t \geq 0 \\ 0, & \text{for } t < 0 \end{cases}$$



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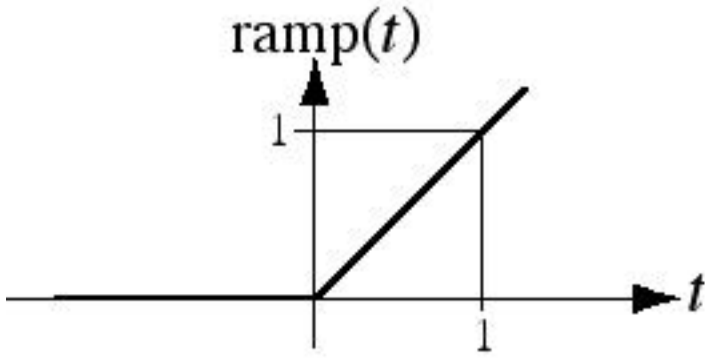
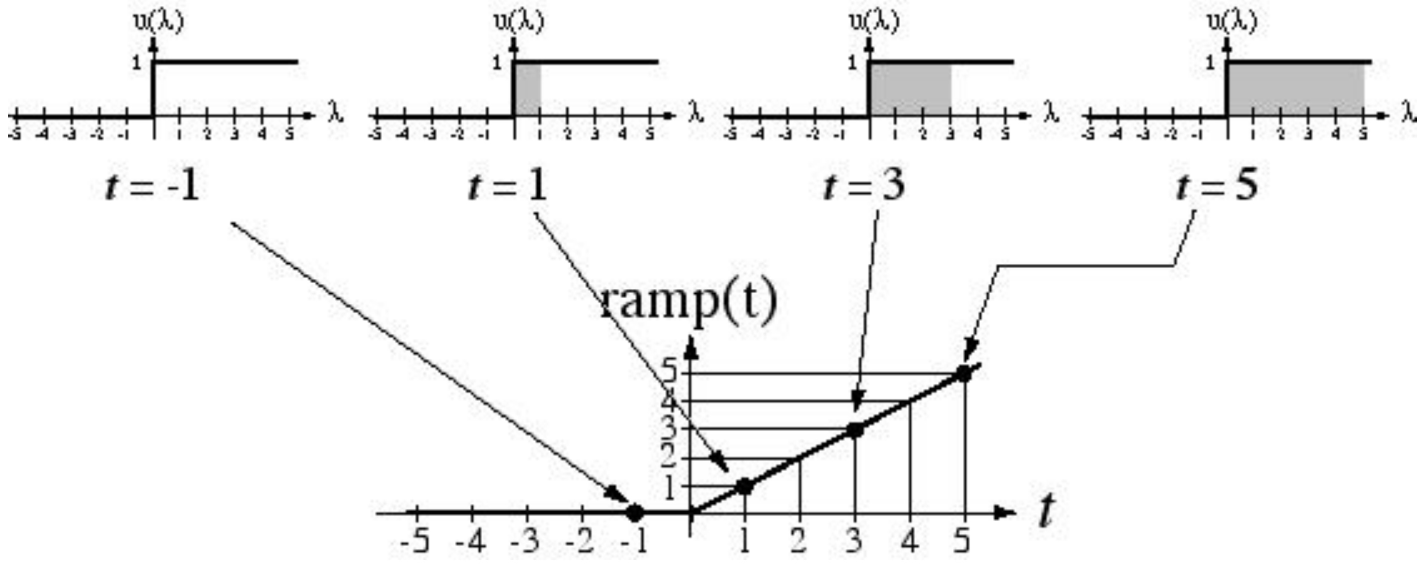
Mathematical Representation – CT Signals

2. Unit Ramp



$$\mathrm{ramp}(t) = \begin{cases} t & , \ t > 0 \\ 0 & , \ t \le 0 \end{cases} = \int_{-\infty}^{t} \mathrm{u}(\lambda)\, d\lambda = t\,\mathrm{u}(t)$$



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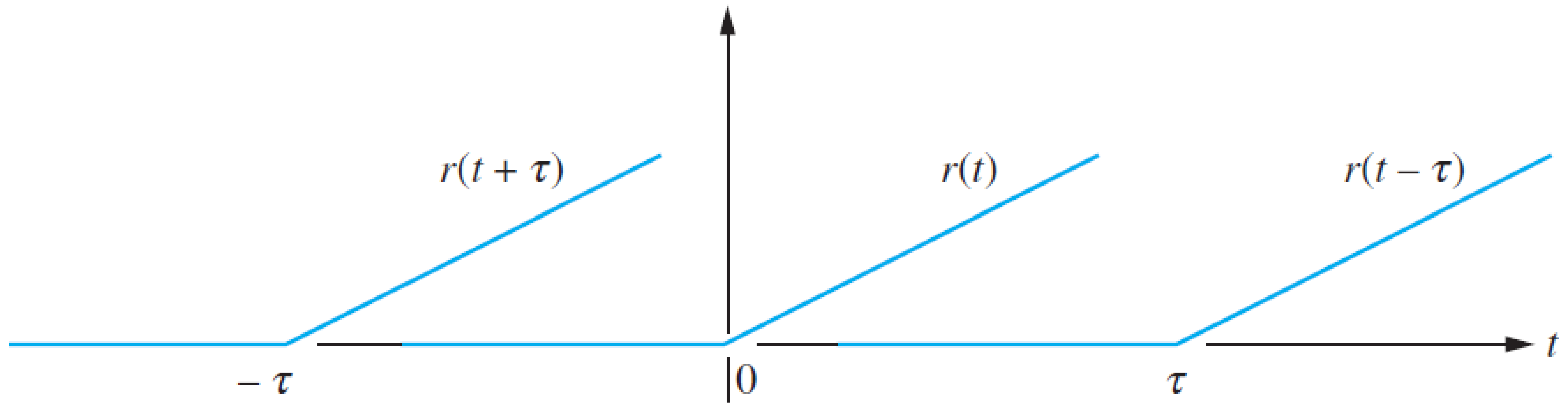
Mathematical Representation – CT Signals

2. Unit Ramp

A delay of τ produces $r(t-\tau)$, while an advance of τ produces $r(t+\tau)$.



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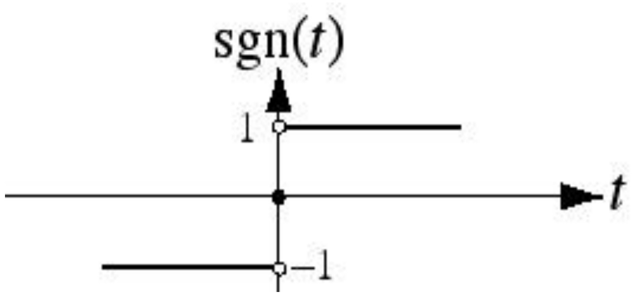
Mathematical Representation – CT Signals

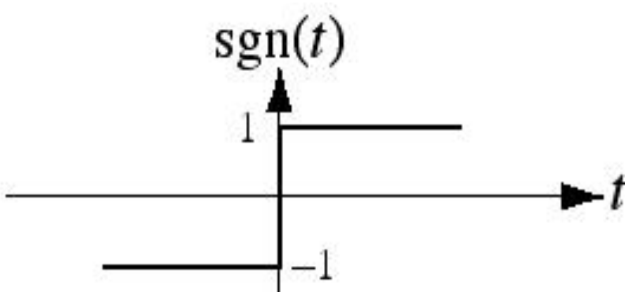
3. Signum

$$\operatorname{sgn}(t) = \begin{cases} 1 & , \ t > 0 \\ 0 & , \ t = 0 \\ -1 & , \ t < 0 \end{cases}$$

Precise Graph



Commonly-Used Graph



The signum function, in a sense, returns an indication of the sign of its argument.

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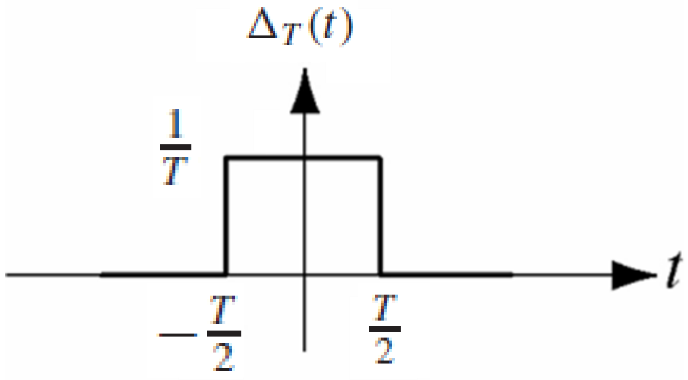
Mathematical Representation – CT Signals

Ex Find the signum function in the form of u(t).

Mathematical Representation – CT Signals

4. Rectangular Pulse

$$\Delta_T(t) = \begin{cases} \frac{1}{T}, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & \text{elsewhere} \end{cases}$$



Note !!!

The wider the pulse width, The lower the pulse amplitude.

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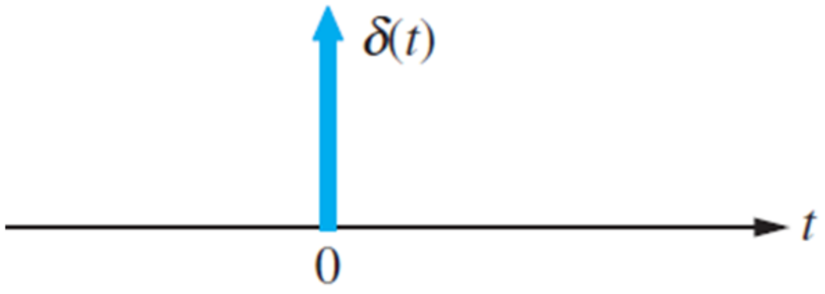
Mathematical Representation – CT Signals

5. (Unit) Impulse

The unit-impulse function $\delta(t)$[9] (also called *Dirac's delta* function) is represented by a double-stem up arrow



One such definition is

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t)dt = 1$$

Unbounded Property

Unit Area Property

<u>Note !</u> It is not a function in the ordinary sense.

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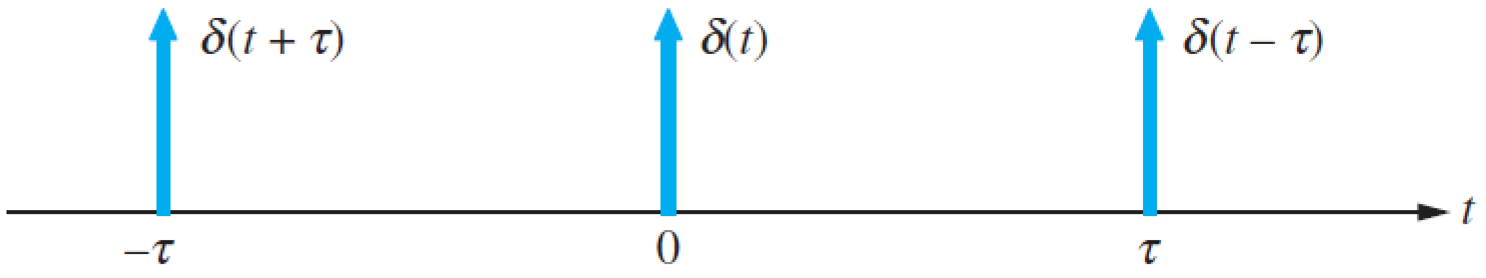
Mathematical Representation – CT Signals

5. (Unit) Impulse

A delay of τ produces $\delta(t-\tau)$, while an advance of τ produces $\delta(t+\tau)$.



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Mathematical Representation – CT Signals

5. (Unit) Impulse

Another definition involves the limit of a narrow pulse $\Delta_T(t)$ with unit area, as the pulse-width approaches zero, but the area remains equal to 1. For example, let

$$\Delta_T(t) = \begin{cases} \frac{1}{T}, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & \text{elsewhere} \end{cases}$$

Then,

$$\delta(t) = \lim_{T \to 0} \Delta_T(t)$$

Note that $\int_{-\infty}^{\infty} \Delta_T(t)dt = 1$.

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Mathematical Representation – CT Signals

5. (Unit) Impulse

Rectangular pulses, Figure 1.20(*a*):

$$x_1(t) = \begin{cases} \frac{1}{T}, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & \text{elsewhere} \end{cases} \qquad -\infty < t < \infty \qquad \lim_{T\to 0} x_1(t) = \delta(t)$$

Exponential pulses, Figure 1.20(*b*):

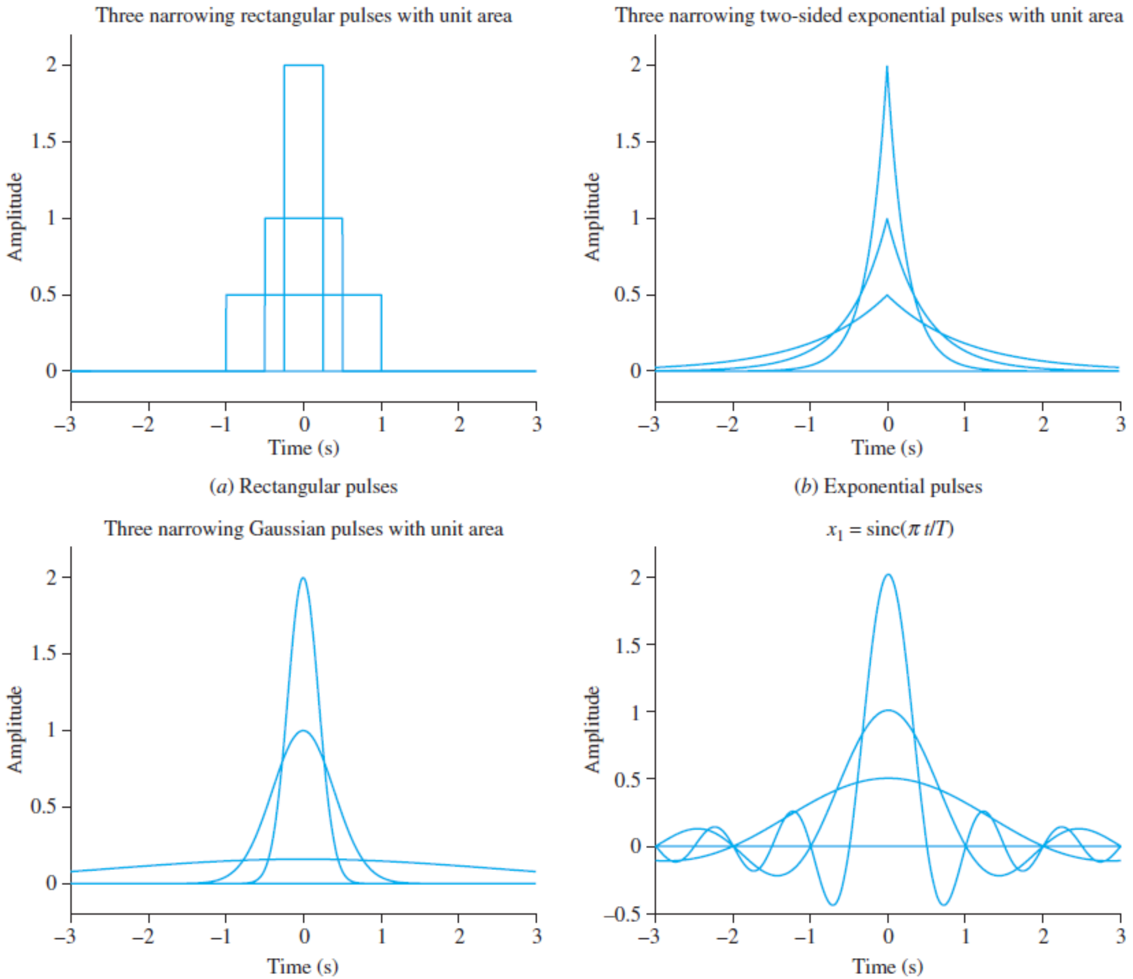
$$x_2(t) = \frac{1}{2\tau} e^{-\frac{|t|}{\tau}} \qquad -\infty < t < \infty \qquad \lim_{\tau\to 0} x_2(t) = \delta(t)$$

Gaussian pulses, Figure 1.20(*c*):

$$x_3(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \qquad -\infty < t < \infty \qquad \lim_{\sigma\to 0} x_3(t) = \delta(t)$$

Sinc pulses, Figure 1.20(*d*):

$$x_4(t) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} \qquad -\infty < t < \infty \qquad \lim_{T\to 0} x_4(t) = \delta(t)$$



(*a*) Rectangular pulses
(*b*) Exponential pulses
(*c*) Gaussian pulses
(*d*) Sinc pulses

Generation of an impulse function.

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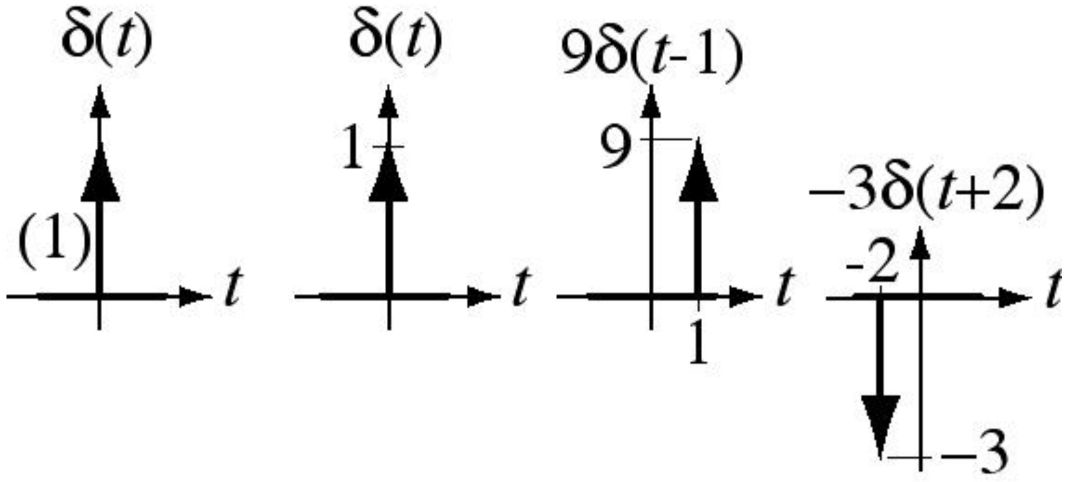
Mathematical Representation – CT Signals

5. (Unit) Impulse

The impulse is not a function in the ordinary sense because its value at the time of its occurrence is not defined. It is represented graphically by a vertical arrow. Its strength is either written beside it or is represented by its length.



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Mathematical Representation – CT Signals

5. (Unit) Impulse

Sieving Property of Impulse

A more rigorous definition of $\delta(t)$, based on generalized functions and distribution, is given by its "sieving" property

$$\int_{-\infty}^{\infty} f(t)\delta(t - t_0)dt = f(t_0)$$

where $f(t)$ is a well-behaved function and continuous at t_0.

**This equation can "extract" value of function at a point.

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Mathematical Representation – CT Signals

Ex Evaluate the following integration.

(1) $\int_{-5}^{5} \delta(t)dt =$

(2) $\int_{-\infty}^{10} \delta(t)dt =$

(3) $\int_{-3}^{-1} \delta(t)dt =$

(4) $\int_{5}^{10} \delta(t-8)dt =$

(5) $\int_{-\infty}^{\infty} \delta(t-5)dt =$

(6) $\int_{0}^{\infty} \delta(t-5)dt =$

(7) $\int_{10}^{\infty} \delta(t-5)dt =$

(8) $\int_{2}^{10} \delta(t-5)dt =$

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Mathematical Representation – CT Signals

5. (Unit) Impulse

The Scaling Property

$$\delta(a(t-t_0)) = \frac{1}{|a|}\delta(t-t_0)$$

This property illustrates that the impulse is different from ordinary mathematical functions.

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Mathematical Representation – CT Signals

Ex Plot the impulse function.

(1) $\delta(2t)$

(2) $\delta\left(\frac{t-1}{3}\right)$

(3) $\delta\left(\frac{t}{4}-1\right)$

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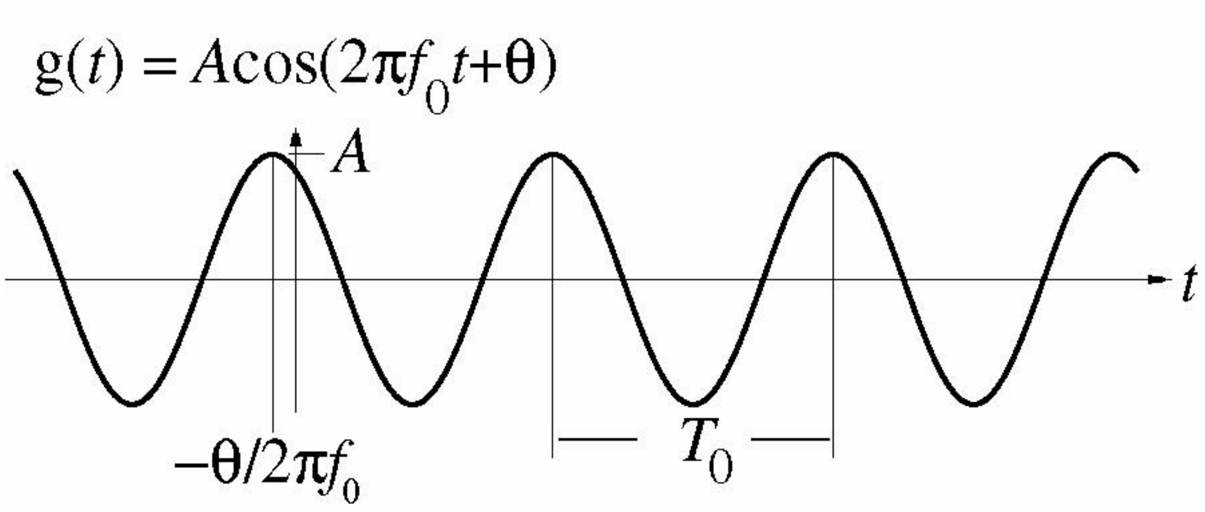
Mathematical Representation – CT Signals

6. Sinusoid

$$g(t) = A\cos(2\pi t / T_0 + \theta) = A\cos(2\pi f_0 t + \theta) = A\cos(\omega_0 t + \theta)$$

- A: Amplitude
- T_0: Period (s)
- θ: Phase Shift (radians)
- f_0: Cyclic Frequency (Hz)
- ω_0: Radian Frequency (radians/s)



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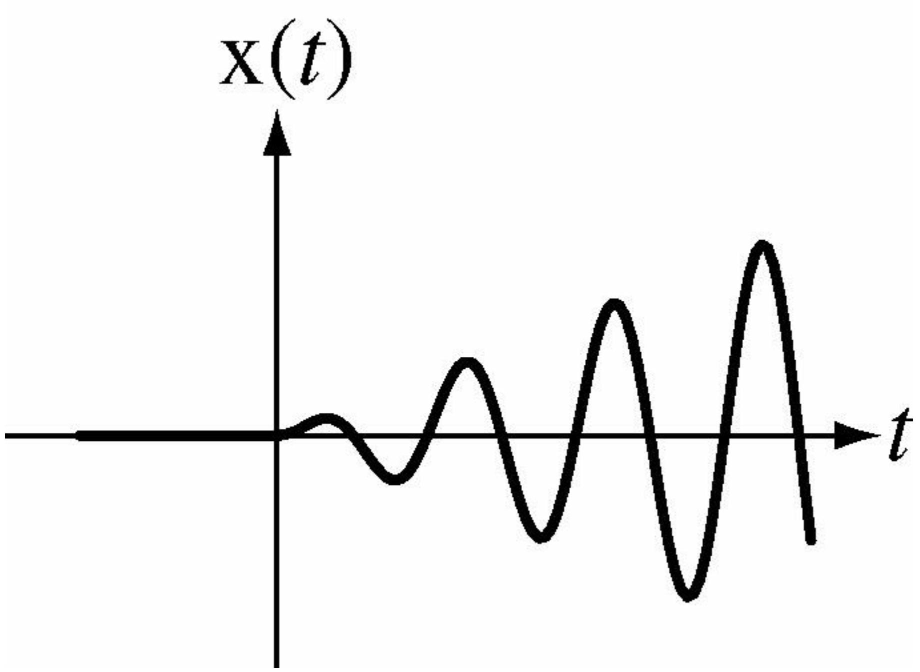
Mathematical Representation – CT Signals

6. Sinusoid

Product of a sine wave and a ramp function.



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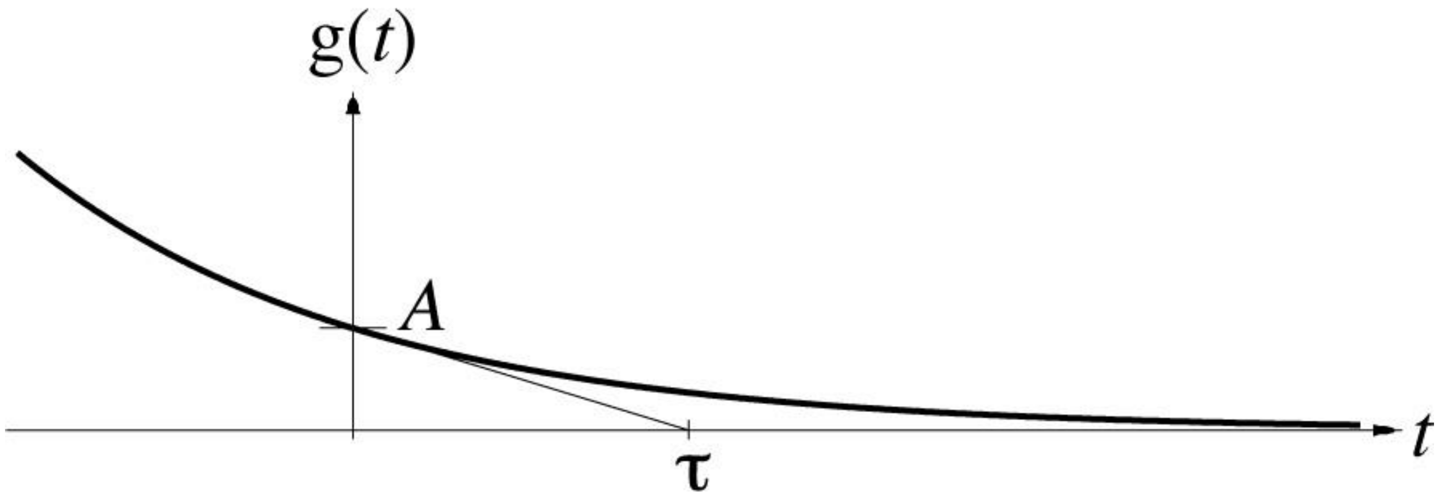
Mathematical Representation – CT Signals

7. Exponential

$$g(t) = Ae^{-t/\tau}$$

↑ Amplitude ↑ Time Constant (s)



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Mathematical Representation – CT Signals

7. Exponential

Determining a time constant

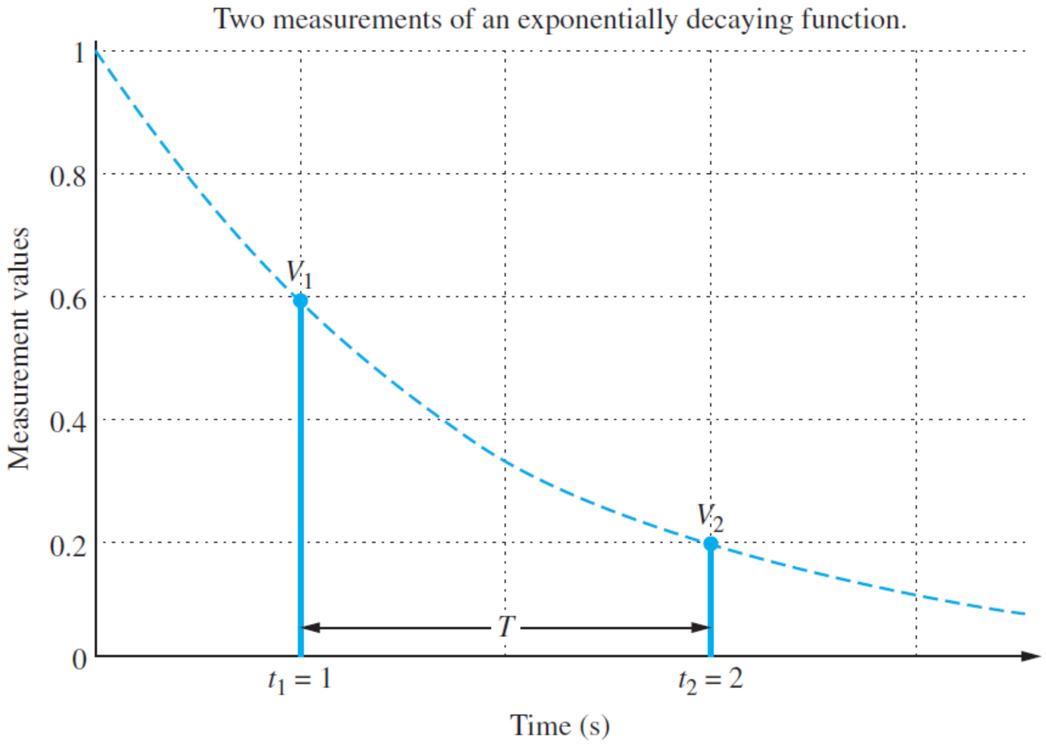
A signal is known to be exponentially decaying toward its zero final value. Derive its time constant from the ratio of two measurements taken T seconds apart.

Solution

An exponentially decaying signal with zero final value can be represented by $v(t) = Ae^{-t/\tau}$, where A is the value of the signal at $t = 0$ and τ is its time constant. Let $V_1 = Ae^{-t_1/\tau}$ and $V_2 = Ae^{-t_2/\tau}$ be the measured values at t_1 and t_2, respectively. The time constant can be derived from a knowledge of V_1/V_2 and $T = t_2 - t_1$ by the following:

$$\frac{V_1}{V_2} = \frac{Ae^{-t_1/\tau}}{Ae^{-t_2/\tau}} = e^{(t_2-t_1)/\tau} = e^{T/\tau}, \quad \text{and} \quad \ln\left(\frac{V_1}{V_2}\right) = T/\tau, \quad \tau = \frac{T}{\ln\left(\frac{V_1}{V_2}\right)}$$

For example, let $V_1 = 0.6$, $V_2 = 0.2$, and $T = 1$ s, as shown in Figure 1.25. Then $\tau = T/\ln(\frac{V_1}{V_2}) = \frac{1}{\ln 3} = 0.91$ s. Having found the time constant, the function can now be expressed by $v(t) = V_1 e^{-t/\tau}$ (taking the time of the first measurement as the time origin). If the time origin is already set, then the parameter A (the value of the exponential at $t = 0$) can be found from $V_1 = Ae^{-t_1/\tau}$ or $A = V_1 e^{-t_1/\tau}$.



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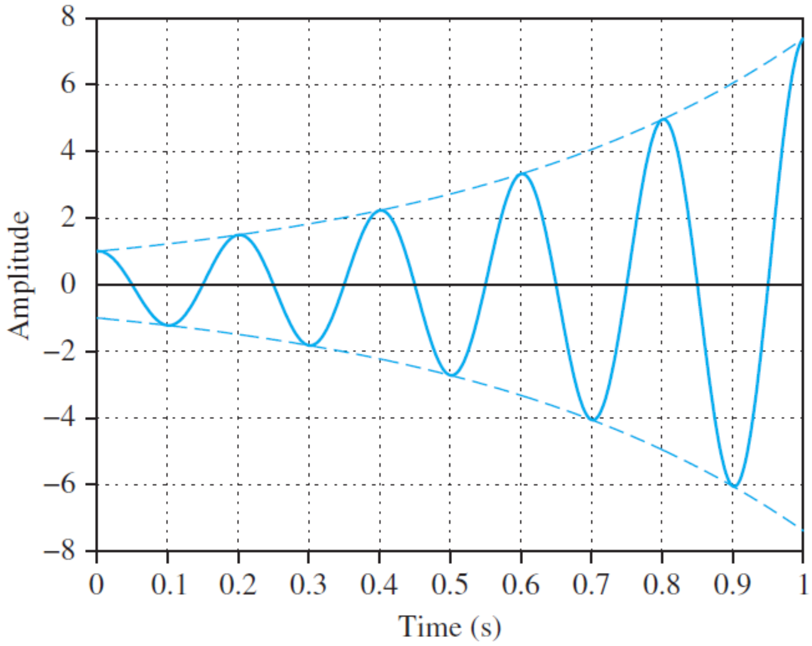
Mathematical Representation – CT Signals

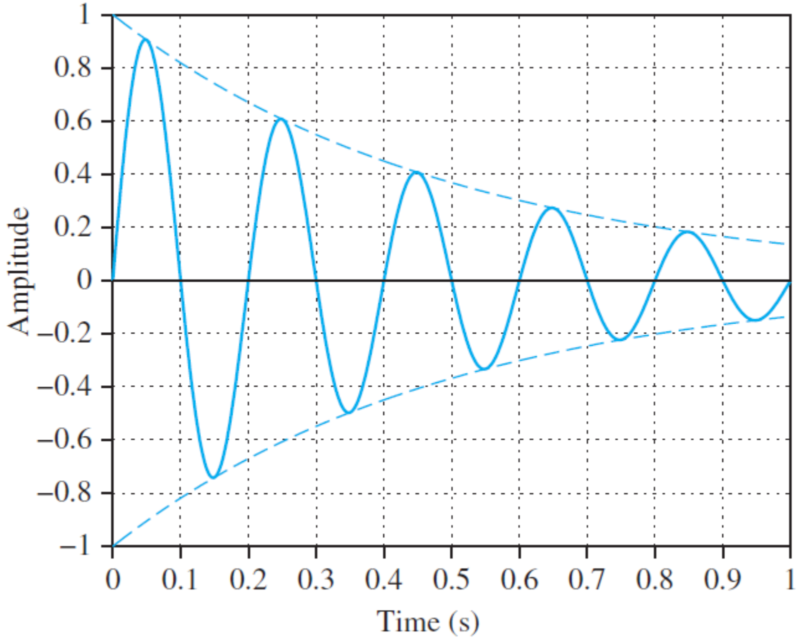
Combination between Sinusoid and Exponential

Growing Sinusoid



$e^{2t}\cos(10\pi t)$

Decaying Sinusoid



$V_0 e^{\sigma t}\sin(2\pi f t)$ and its envelope.

$V_0 = 1,\ f = 5\text{ Hz},\ \sigma = -2\ s^{-1}$

ที่มา: Nahvi, M. 2014. Signals and Systems. McGraw-Hill, New York, NY.

Mathematical Representation – CT Signals

8. sinc function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

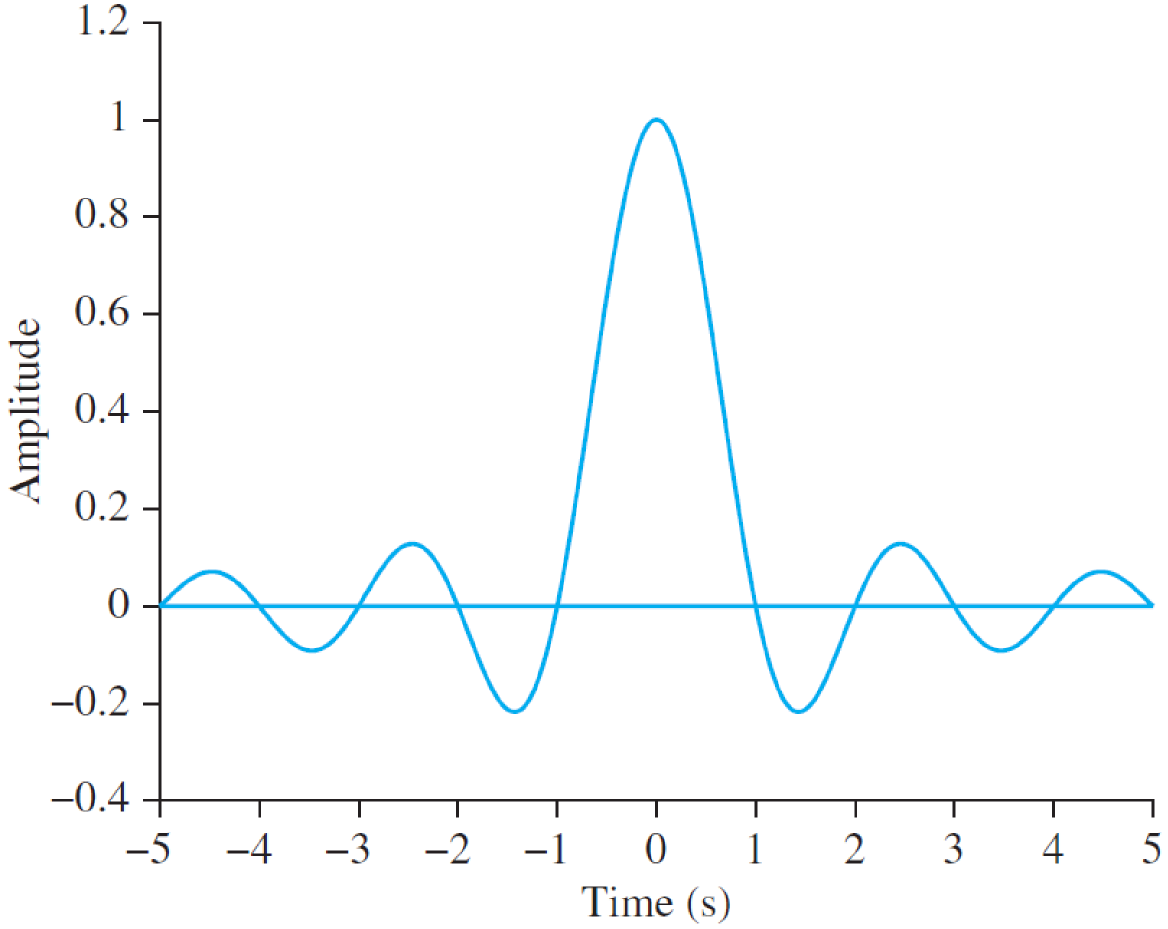
The definition is not standard.

However, this definition is also used in engineering and MATLAB.

For mathematician,

they normally uses $\dfrac{\sin(t)}{t}$

ที่มา: Phillips, C.L., et al.. 2014. Signals, Systems., and Transforms. Pearson, Essex, England.



ที่มา: Nahvi, M. 2014. Signals and Systems. McGraw-Hill, New York, NY.

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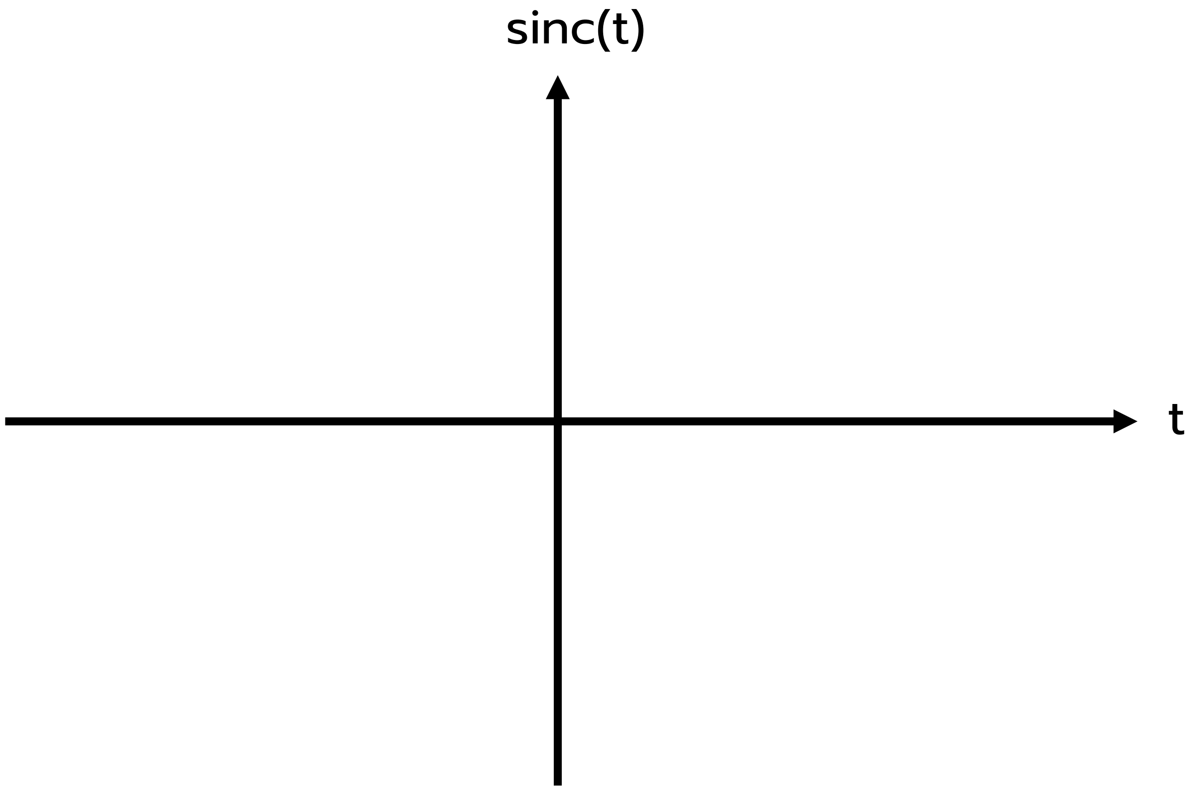
Ex Plot sinc function.



t	sinc(t)
-1.5	
-1	
-3/4	
-1/2	
-1/4	
0	
1/4	
1/2	
3/4	
1	
1.5	
2	
2.5	
3	

Note!

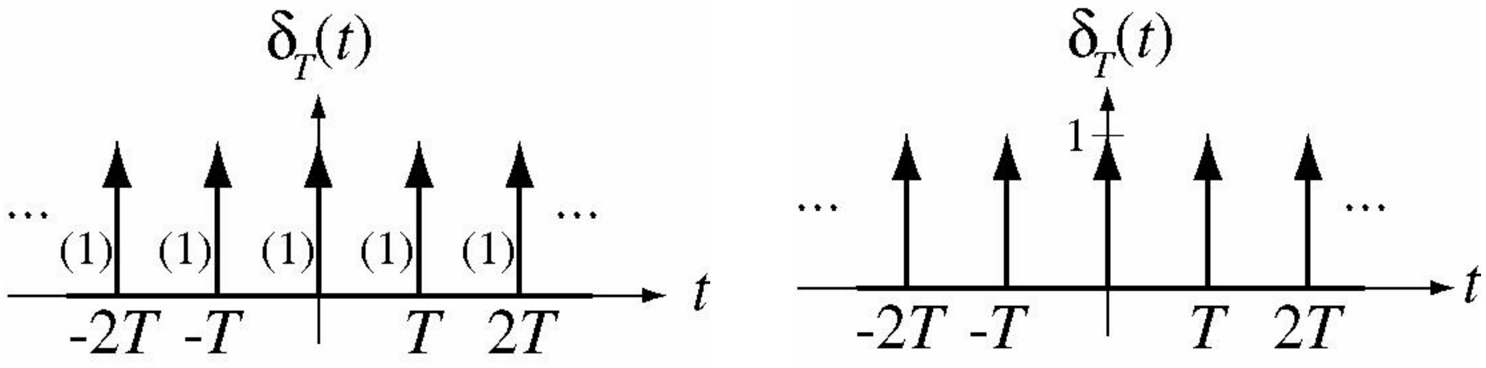
sinc(t) has a value of unity at t=0.
and has zeros at t=+/-1, +/-2, ...

Mathematical Representation – CT Signals

9. Unit Periodic Impulse

The unit periodic impulse is defined by

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \ , \quad n \text{ an integer}$$

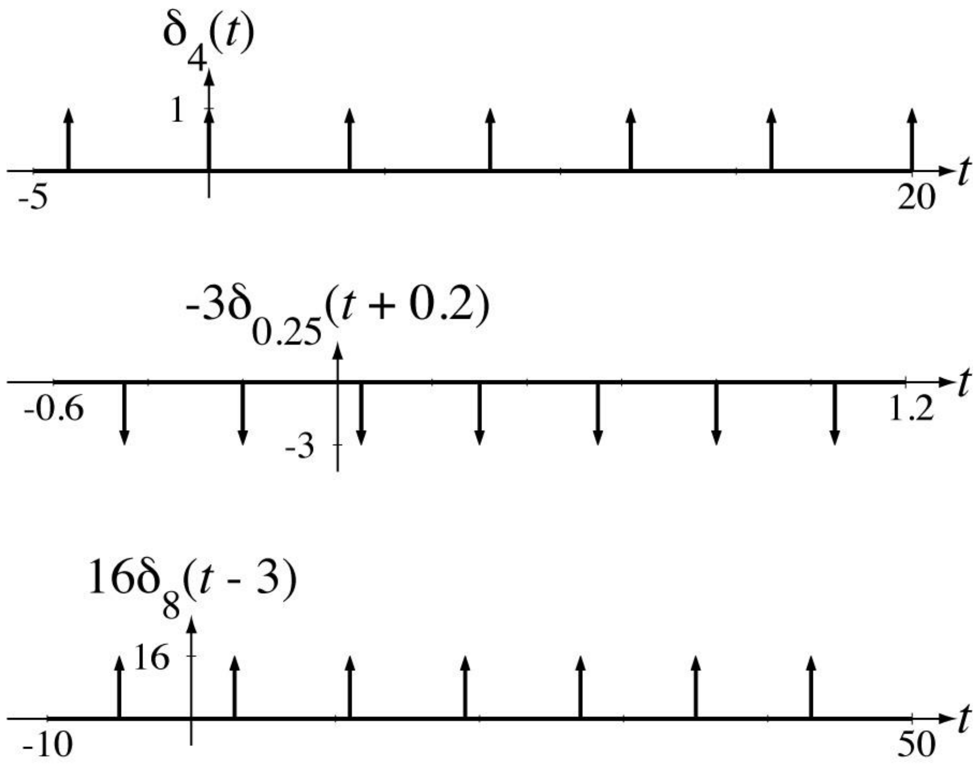


The periodic impulse is a sum of infinitely many uniformly-spaced impulses.

ที่มา: Robert, M.J. 2018. Signals and Systems. 3rd ed. McGraw-Hill, New York, NY.

Mathematical Representation – CT Signals

9. Unit Periodic Impulse



ที่มา: Robert, M.J. 2018. Signals and Systems. 3rd ed. McGraw-Hill, New York, NY.

Chapter 1

The Science of Signals