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Lab 04 & 05: Projectile Motion

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Projectile Motion – Mini Launcher

Equipment Required

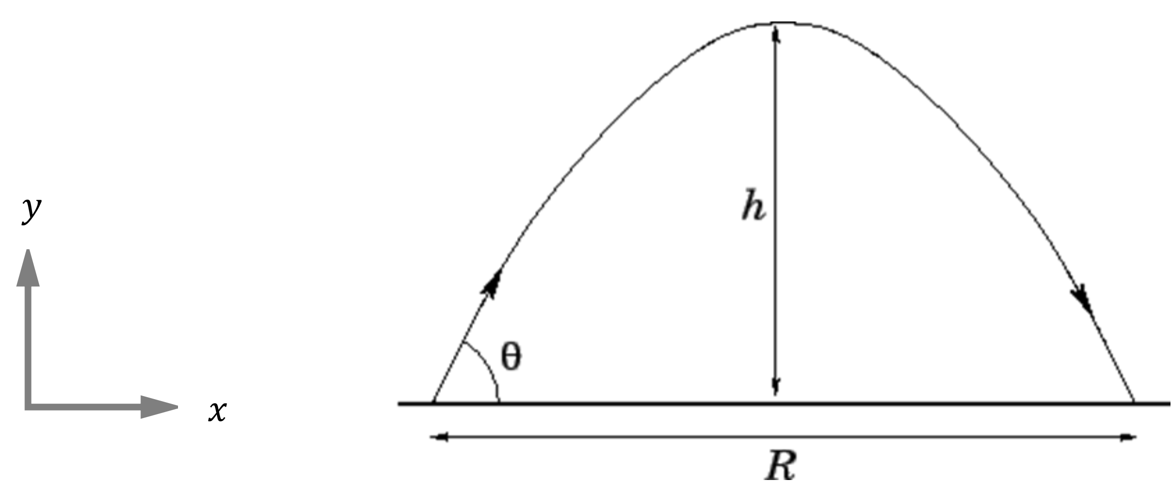
- Mini Launcher and two steel ball
- Collision Attachment
- Movable vertical target board, plank
- Photogates
- Meter rule
- White and carbon paper

Abstract

In first two experiments the range of a ball launched at an angle was predicted and verified using Mini Launcher. In first, the initial velocity of the ball was determined by shooting it horizontally and measuring the range and the height of the Launcher. In the second, Photogates were used to determine the initial velocity of the ball. In third, relation between launch angle, horizontal position, and vertical height of launcher examined. In the last experiment, relation between vertical height the ball drops and horizontal range was used to determine initial velocity. It was compared with initial velocity from the first experiment.In last experiment, conservation of momentum was observed but systematic error caused values to deviate.

Theory

A projectile motion is ideally a 2D motion of an object under the action of gravity. The projectile is the object undergoing this motion.



In projectile motion, the horizontal velocity of object remains constant.

$$V_{ix} = V_{fx} = V_i \cos\Theta$$

While vertical velocity changes due to acceleration due to gravity and initial velocity is given by:

$$V_{iy} = V_i \sin\Theta$$

Time of trajectory:

The total time required to complete the parabolic pathway is called the time of trajectory.

According to the first equation of motion;

$$V_{fy} = V_{iy} + at$$

$$V_{fy} = V_{iy} - gt$$

At max height Vfy = 0, so:

$$V_{iy} - gt = 0$$

$$t = \frac{V_i \sin\theta}{g}$$

Total time of trajectory is hence:

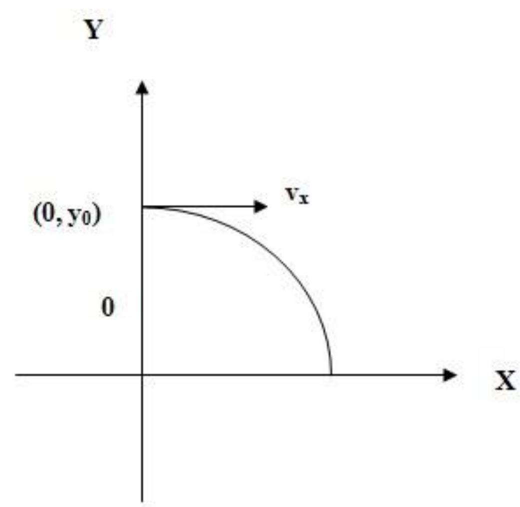
$$t = \frac{2V_i \sin\theta}{g}$$

HEIGHT OF PROJECTILE:According to the second equation of motion:

$$y_f = v_i \sin\theta\, t - \frac{1}{2}gt^2$$

RANGE OF PROJECTILE: The maximum horizontal distance covered by a projectile, where ‘t’ is total time of flight.

$x_f = V_i cos\theta t$



If the projectile is thrown from a certain height, y_f horizontally, then the vertical component of initial vertical velocity is taken as zero:

$$y = y_f + V_i \sin\theta\, t - \frac{1}{2}gt^2$$

since, $V_i sin\theta = 0$ as $\theta = 0$.

$$y = y_f - \frac{1}{2}gt^2$$

Time of trajectory then becomes:
When y = 0.

$$t = \sqrt{\frac{2y_f}{g}}$$

Horizontal distance travelled remains as before since horizontal velocity remains unchanged:

$$x_f = V_i cos\theta t$$

Initial velocity can then be calculated using:

$$V_i = \frac{x_f}{t}$$

Experiment 1: Projectile Motion and Horizontal Range

Part A: Determining the Initial Velocity of the Ball

Procedure

1. Mount the Mini Launcher such that the square nut that the Launcher pivots around should be slid to the front of the launcher. So, that changing the angle of the launcher, does not change tthe launch position.
2. Adjust the angle of the mini launcher to zero degrees.
3. Load the ball in the mini launcher and shoot the ball to locate the point where the ball hits the table.
4. Tape white paper and carbon paper (carbon side down) to the point where the ball hits the table so that when the ball hits the table, it leaves a mark there.
5. Mark a point that is directly below the release point of the ball and measure the vertical distance from the release point to that marked point. This distance is labelled as, $\boldsymbol{y_m}$.
6. Load the steel ball again with mini launcher at 0° and fire the ball. Measure the horizontal distance beneath the launching point of the ball to the point it strikes the table. This distance will be horizontal range, $\boldsymbol{x_o}$.

Calculations:

$$Range = x_o = 0.918\,m$$

$$vertical\ height = y_m = 0.413m$$

$$time\ of\ flight = t = \sqrt{\frac{2y_m}{g}}$$

$$t = \sqrt{\frac{2(0.413)}{9.8}}$$

$$\boldsymbol{t = 0.29\ s}$$

Now,

$$v_o = \frac{x_o}{t} = \frac{(0.918)}{(0.29)}$$

$$\boldsymbol{v_o = 3.16\ m/s}$$

Part B: Predicting the Range of the Ball Shot at 20° degree

Procedure

1. Adjust the angle of the mini launcher to twenty degrees.
2. Load the ball in the mini launcher and shoot the ball to locate the point where the ball hits the table.
3. Tape white paper and carbon paper (carbon side down) to the point where the ball hits the table so that when the ball hits the table, it leaves a mark there.
4. Load the steel ball again with mini launcher at 20° and fire the ball. Measure the horizontal distance beneath the launching point of the ball to the point it strikes the table. This distance will be horizontal range at twenty degree, $\boldsymbol{x_o}$.

Calculations:

Trial	Measured range $\boldsymbol{x_o}$ (cm)
1	124.0
2	124.2
3	124.7

θ = 20° $vertical\ height = y_m = 0.413m$

$$\boldsymbol{x_o} = \frac{124+124.2+124.7}{3} = 124.3\text{ cm}$$

x_o = 1.243 m

Calculating Predicted Range:

$$y = y_f + V_i \sin\theta\, t - \frac{1}{2} g t^2$$

$$y = y_m + v_o \sin\theta\, t - \frac{1}{2} g t^2$$

$$0 = 0.413 + 3.16 \sin(20°)\, t - \frac{1}{2}(9.8) t^2$$

$$0 = 2(0.413) + 2(3.16 \sin(20°)\, t) - (9.8) t^2$$

$$9.8t^2 - 2.162t - 0.826 = 0$$

By quadratic formula,
t = 0.4209 , t = - 0.2003

As time is always positive, so we neglect the negative value, so
t = 0.4209 s

Now for theoretical horizontal range,

$$x_f = V_i cos\theta t$$

$$x_o = (3.16) cos(20°)(0.4209)$$

x_o= 1.250 m

Percentage error = $\frac{\text{Theoretical}-\text{experimental}}{\text{Theoretical}}$ x 100%

= $\frac{1.250-1.243}{1.250}$ x 100%

Percentage error = **0.56%**

Experiment 2: Projectile Motion using Photogates

Procedure:

1. Attach the photogates to the launcher at the distance of 3cm from the end of the muzzle.
2. Plug the photogates into a timer.
3. Load the steel ball in the mini launcher and set the timer to measure time.
4. Fire the ball. The sensors of the photogates detect the motion of the ball and time taken by the ball to travel through the gates is displayed on the timer.
5. Take three readings for the time to reduce error.

Calculations:

$$mean\ time = t = \frac{(0.0314 + 0.0312 + 0.0314)}{3}$$

$$\boldsymbol{t = 0.0313\ s}$$

Trial	Time t (s)
1	124.0
2	124.2
3	124.7

where the distance between photogates, x= 10 cm= 0.1 m

$$v_o = \frac{x}{t} = \frac{0.1}{0.0313}$$

$$v_o = \boldsymbol{3.19\ m/s}$$

Calculating Predicted Range:

$$y = y_f + V_i \sin\theta\, t - \frac{1}{2} g t^2$$

$$y = y_m + v_o \sin\theta\, t - \frac{1}{2} g t^2$$

$$0 = 0.413 + 3.19\sin(20°)\,t - \frac{1}{2}(9.8)t^2$$

$$0 = 2(0.413) + 2(3.19\sin(20°)\,t) - (9.8)t^2$$

$$9.8t^2 - 2.182t - 0.826 = 0$$

By quadratic formula,
t = 0.4223 , t = - 0.1996

As time is always positive, so we neglect the negative value, so
t = 0.4223 s

Now for theoretical horizontal range,

$$x_f = V_i cos\theta t$$

$$x_o = (3.19)cos(20°)(0.4223)$$

$\boldsymbol{x_o}$**= 1.266 m**

Where measured range from experiment 1 is $\boldsymbol{x_o}$ **= 1.243 m**

Percentage error = $\frac{\text{Theoretical}-\text{experimental}}{\text{Theoretical}}$ x 100%

= $\frac{1.266-1.243}{1.266}$ x 100%

Percentage error **= 1.8%**

Dominant Source of Error:

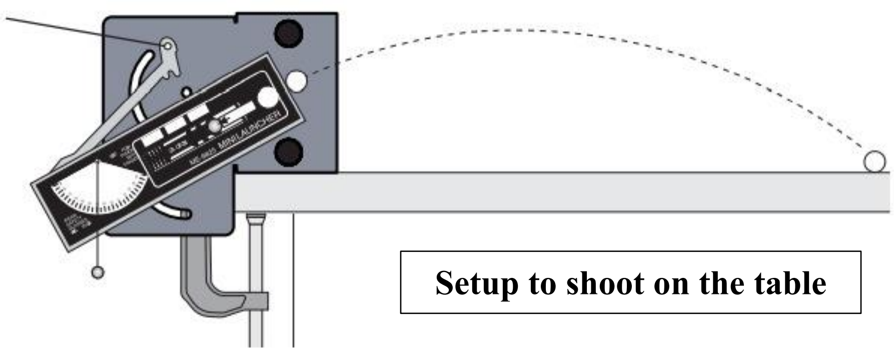
1. There is uncertainty in the value of velocity calculated in experiment 1 because the reading of the horizontal range was taken only once.
2. Change of position of stand while loading steel ball in launcher.

Experiment 3: Projectile Range Versus Angle

Procedure:

- **Shooting on the Table:**

1. Clamp the mini launcher to one end of the table and load the steel ball in the muzzle.
2. Adjust the angle of the mini launcher to 25°.
3. First, shoot the ball to find at what point the ball hits the table. At this point, tape the white paper and carbon paper to the table (carbon side down) so that when the ball hits the table, it leaves a mark. This mark will help in measuring the horizontal distance i.e., Range.
4. Take three readings of the range to minimize the error at 25°.
5. Measure the horizontal distance from the launching position of the ball to the points it strikes the table. Record these distances in a table. Take the average of your readings.
6. Repeat the same process by increasing the angle by 10° and repeat for angles up to 65°. Record the readings in the table.

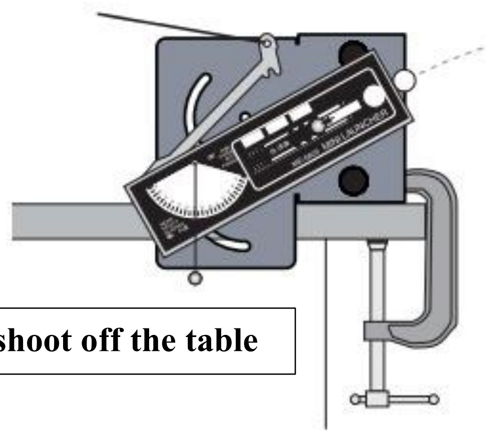


Setup to shoot on the table

Trials\Angles	25°	35°	45°	55°	65°
1.	75.5	95.9	101.2	95.9	78.2
2.	75.4	94.6	100.5	95.4	78.7
3.	75.4	93.9	101.3	95.2	78.6
Mean $\overline{x}$ (cm)	*75.46*	*94.8*	*101*	*95.5*	*78.5*

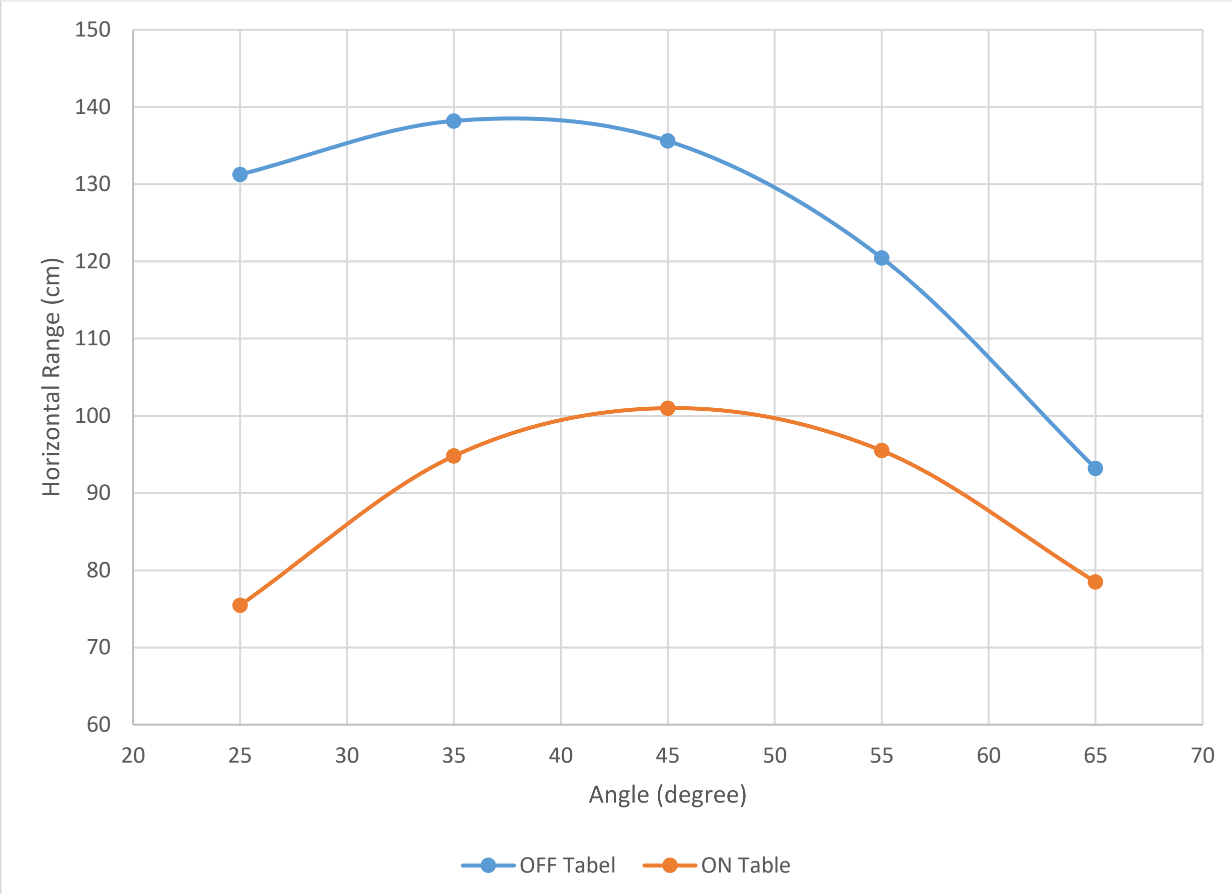
- **Shooting off the Table:**

1. Clamp the mini launcher as shown in the figure such that the steel ball hits the floor. Then repeat the above steps from 1 to 6 and record the data in the form of a table.



Setup to shoot off the table

Trials\Angles	25°	35°	45°	55°	65°
1.	131.3	138.2	135.2	120.6	92.8
2.	131.2	138	135.8	120.2	93
3.	131.3	138.4	135.8	120.6	93.8
Mean $\bar{x}$ (cm)	**131.26**	**138.2**	**135.6**	**120.46**	**93.2**



Experiment 4: Projectile Path

Theory:

The vertical distance the ball drops is related to the horizontal distance the ball travels when launched horizontally. Where V_i is initial velocity, x is horizontal range,and Time of flight, t:

$$t = \frac{V_i}{x}$$

Vertical distance the ball drops in time, t:

$$y = \frac{1}{2}gt^2$$

Substituting t gives:

$$y = \left(\frac{g}{2V_i^2}\right)x^2$$

Plotting y against x^2 will give a straight line with gradient, m

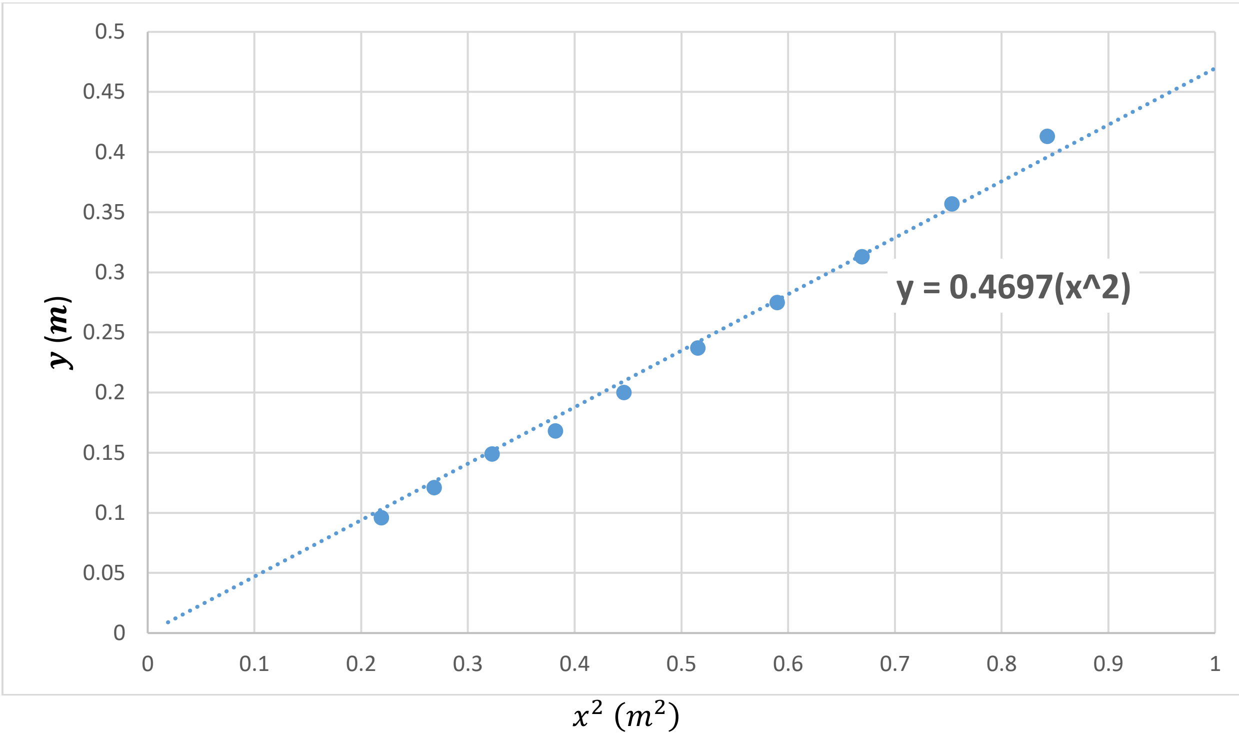
$$m = \frac{g}{2V_i^2}$$

Procedure:

- Clamp the base to a vertical rod of clamp stand placed on table
- Set the angle of Mini Launcher at zero degrees, 0°.
- Measure vertical height of launch position from table, $\boldsymbol{y_m}$
- Shoot on the short-range setting to determine the landing position of ball
- Place the plank vertically, covered on side with white paper, at that place and tape a carbon paper on it.
- Measure the horizontal distance from the launch position of the launcher to the plank, $\boldsymbol{x}$
- Shoot the ball.
- Move the plank 5 cm closer to the Launcher. Now range will be $\boldsymbol{x - 5.}$
- Repeat last two steps until change in height, $\boldsymbol{y}$ is about 10 cm.
- Measure height of each dot from base of plank, $\boldsymbol{h}$ and record it in table.

Data Analysis:

Horizontal Range, $\boldsymbol{x}$ $(\boldsymbol{m})$	$\boldsymbol{x^2}$ $(\boldsymbol{m^2})$	**Height of dot from table,** **h** $(\boldsymbol{m})$	**Vertical Distance ball falls,** $\boldsymbol{y = y_m - h}$ $(\boldsymbol{m})$
0.918	0.8427	0.000	0.413
0.868	0.7534	0.056	0.357
0.818	0.6691	0.100	0.313
0.768	0.5898	0.138	0.275
0.718	0.5155	0.176	0.237
0.668	0.4462	0.213	0.200
0.618	0.3819	0.245	0.168
0.568	0.3226	0.264	0.149
0.518	0.2683	0.292	0.121
0.468	0.2190	0.317	0.096



Note: Line of best fit is drawn and its equation is calculated using Microsoft Word.

Gradient = $m = 0.4697$ using the equation $m = \frac{g}{2V_i^2}$

$$V_i = \sqrt{\frac{g}{2m}} = \sqrt{\frac{9.8}{2(0.4679)}} = 3.23\ ms^{-1}$$

Comparing V_i , calculated in this experiment with V_o ,calculated in experiment 1:

$$mean = \frac{V_o + V_i}{2} = \frac{3.17 + 3.23}{2} = 3.20$$

$$\text{Percentage Difference} = (100)\frac{|V_o - V_i\,|}{mean} = (100)\frac{|3.17 - 3.23|}{3.2} = (100)\frac{0.06}{3.2}$$
$$= 1.857 \sim 1.9\%$$

$$\mathbf{Percentage\ Difference} = \mathbf{1.9}\%$$

Experiment 5: Conservation of Momentum in Two Dimensions

Theory

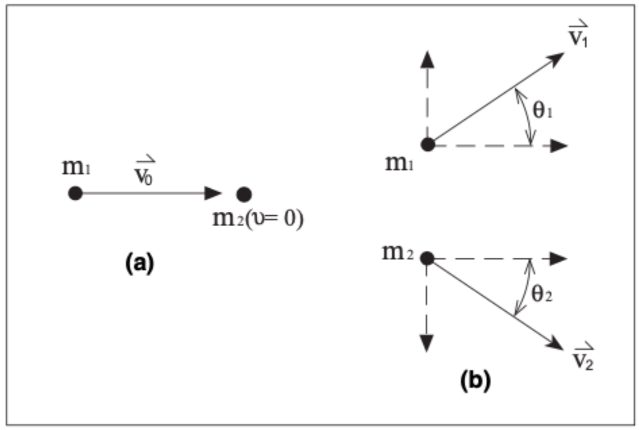
There is no force in horizontal plane, x-y plane so momentum is conserved before and after collision:

x-axis:

$$m_1 v_0 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

y-axis: as there is no motion in y-axis before collision.

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2$$



Identical steel ball used, so $m_1 = m_2$

For elastic collision Kinetic energy is conserved:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$v_0^2 = v_1^2 + v_2^2$$

x-axis: $v_0 = v_1 \cos\theta_1 + v_2 \cos\theta_2$

y-axis: $0 = v_1 \sin\theta_1 - v_2 \sin\theta_2$

Squaring and adding both equations:

$$v_0^2 = v_1^2 + v_2^2 + 2v_1 v_2 \cos(\theta_1 + \theta_2)$$

As $v_0^2 = v_1^2 + v_2^2$:

$$0 = 2v_1 v_2 \cos(\theta_1 + \theta_2)$$

$$\theta_1 + \theta_2 = \cos^{-1}(0)$$

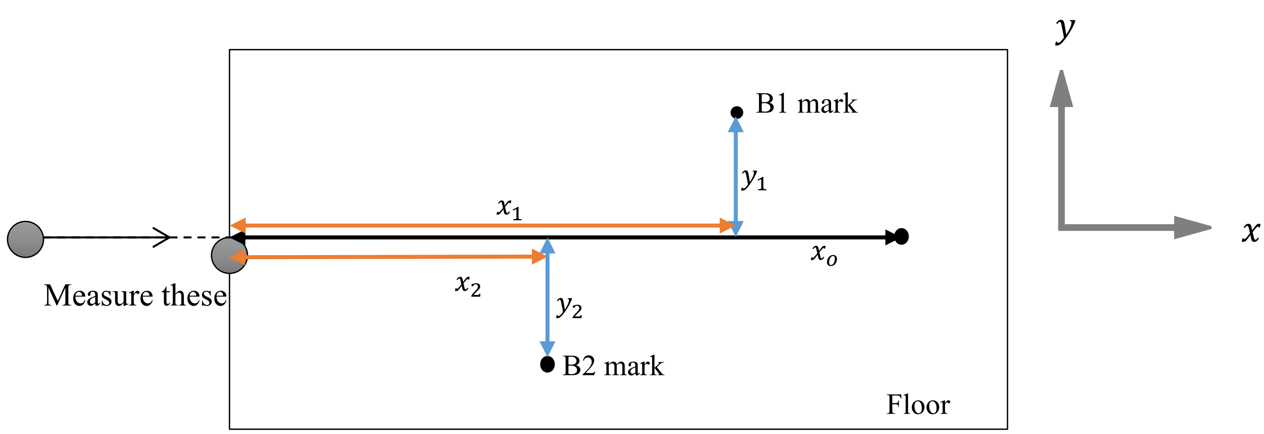
$$\theta_1 + \theta_2 = 90°$$

Thus for elastic collision, $\boldsymbol{\theta_1 + \theta_2 = 90°}$

Thus for non-elastic collision, $\boldsymbol{\theta_1 + \theta_2 > 90°}$,as $v_0^2 > v_1^2 + v_2^2$ because kinetic energy is not conserved.

Procedure

1. Adjust the angle of Mini Launcher to zero degrees.
2. Load the steel ball into the Mini Launcher and fire a test shot to approximate the range.
3. Tape the white paper and carbon paper (carbon side down) to the point where the ball hit. Fire the ball again and this time it will leave a mark at the point it hits the table.
4. Draw a line from point beneath the launching point of the ball to the mark. Length of this line will be horizontal range, $\boldsymbol{x_o}$.
5. Now attach the collision attachment to the mini launcher. Rotate the tee from side to side so that neither ball rebounds into the launcher after the collision and both balls land on the table.
6. Load one ball into the mini launcher and place the other on the tee launcher. The two balls must at the same height. Fire the test shot to determine where both the balls land.
7. Place a white and carbon paper (carbon side down) at each site so that a point is marked when the balls strike.
8. Elastic Collision: Using two balls, load one ball in the mini launcher and put the other ball on the tee.
9. Inelastic Collision: Load one ball, B1 in the mini launcher and stick some tape to the tee ball, B2 before placing it on the tee launcher. This will result in an inelastic collision.
10. x_1 and x_2 will be the distances covered by both balls in the x-direction and y_1 and y_2 will be the distances covered in the y-direction by ball 1, B1 and ball 2, B2 respectively.



- **For Elastic Collision :**

Along x-axis:	Along y-axis:	Angle $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
x_0 =0.850 m	y_0=0.0	$\theta_o = 0°$
x_1=0.390 m	y_1=0.419 m	$\theta_1 = 47.05°$
x_2= 0.531 m	y_2=0.370 m	$\theta_2 = 34.87°$

$$\theta_1 + \theta_2 = 81.92°$$

Percentage error = $\frac{\text{Theoretical}-\text{experimental}}{\text{Theoretical}}$ x 100%

$$= \left(\frac{90 - 81.92}{90}\right) 100$$

Percentage error = $\mathbf{9.0}$%

- **For Inelastic Collision :**

Along x-axis:	Along y-axis:	Angle $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
x_0 =0.850 m	y_0=0.0	$\theta_o = 0°$
x_1= 0.288 m	y_1=0.315 m	$\theta_1 = 47.56°$
x_2= 0.63 m	$y_2 = 0.282$ m	$\theta_2 = 24.11°$

$$\boldsymbol{\theta_1 + \theta_2 = 71.67°}$$

As it is round about 10^0 less than total angle in elastic collision, so it proves that this one is an inelastic collision.

Systamatic error: x_1 *and* x_2 were measured from point beneath launching point of Launcher rather than from point of collision, due to which angles are less than actual angles by certain degrees.

Conclusion:

From first two experiments, presence of errors and/or uncertainties while calculating the predicted(theoretical) value may increase or decrease percentage error between experimental and predicted value. In third experiment, When launching position is at surface level range is maximum when launch angle is 45°, while as distance between launch position and surface increases, the angle at which range is maximum decreases. When launch angle is zero degree vertical distance ball drops is directly proportional to square of horizontal range with proportionality constant $\frac{g}{2V_i^2}$,where V_i is initial velocity. Momentum is also conserved in two dimensions for elastic and inelastic collisions, and Kinetic energy is conserved only in Elastic collision.