

Efficient Texture Image Retrieval of Improved Completed Robust Local Binary Pattern

Arrie Kurniawardhani
Department of Informatics
Universitas Islam Indonesia
Yogyakarta, Indonesia
e-mail : arrie.kurniawardhani@uii.ac.id

Agus Eko Minarno
Department of Informatics
Universitas Muhammadiyah Malang
Malang, Indonesia
e-mail : agoes.minarno@gmail.com

Fitri Bimantoro
Department of Informatics
Universitas Mataram
Mataram, Indonesia

Abstract— Improved Completed Robust Local Binary Pattern is one of the robust texture extraction for image retrieval that rotation invariant (ICRLBP). ICRLBP has proven that can increase the precision, recall, and computation time from its previous work by 21.14%, 20.03%, and 56 times, respectively, on four different texture image dataset. ICRLBP, however, has a lot of feature, thus require more time during recognition process. Moreover, it leads to high time consuming and curse of dimensionality. To overcome those issues, in this paper, we try to reduce insignificant or unnecessary ICRLBP attributes and examine the effect of reducing number of attributes on precision and recall of the retrieving images. The methods we used to reduce the ICRLBP attributes are Correlation-based Feature Selection (CFS) and Pearson's-based Correlation. The experiment results show that the optimal number of attributes that can be reduced is 95% for S_M_C feature and 4.2% for M_C feature. It indicated that Correlation-based Feature Selection (CFS) and Pearson's-based Correlation can reduce the attributes effectively.

Keywords—Attribute Selection, Batik, Content Based Image Retrieval, Correlation-based Feature Selection, Improved Completed Robust Local Binar.

I. INTRODUCTION

Since 1990s, the amounts of researches have been conducted for content-based image retrieval (CBIR) [1]. It attracts much attention because images became the most important media format for communication which they contain a rich amount of information. Image retrieval to retrieve texture image still have big challenge to achieve high retrieval accuracy and less computational complexity.

A number of studies have been conducted to retrieve texture image precisely. Kokare, Biswas, and Chatterji did texture image retrieval using rotated wavelet filters based approach to improves the retrieval rate of traditional discrete wavelet transform [2][3]. Reference [4][5] did texture image retrieval using Gabor feature based approach. Reference [6][7][8][9][10] did image retrieval using texton or texture of element based

approach that compiled of texture, color, and shape feature extraction simultaneously. Reference [11] did image retrieval using Improved Completed Robust Local Binary Pattern (ICRLBP) based approach to improves Local Binary Pattern (LBP) in order to rotation invariant and insensitive to noise.

The performance of ICRLBP was evaluated by retrieving 18 images that have different rotation angle based on a query image. ICRLBP has proven that it can improve the previous work, Completed Robust Local Binary Pattern (CRLBP). ICRLBP increased precision, recall, and computation time by 21.14%, 20.03%, and 56 times, respectively, on four different texture image dataset. The optimal precision and recall of ICRLBP can achieve are 84.86% and 80.39%, respectively, using S_M_C feature. S_M_C feature of ICRLBP, however, has a lot of feature, i.e. 2592 attributes, thus require more time during recognition process. Moreover, it leads to high time consuming and curse of dimensionality.

To overcome those issues, in this paper, we try to reduce insignificant or unnecessary ICRLBP attributes and examine the effect of reducing number of attributes on precision and recall of the retrieving images. The methods we used to reduce the ICRLBP attributes are Correlation-based Feature Selection (CFS) [12][13]. and Pearson's-based Correlation [14].

II. PREVIOUS WORK

A. Improved Completed Robust Local Binary Pattern

This section will explain a brief review previous work, Improved Completed Robust Local Binary Pattern [11][15]. Improved Completed Robust Local Binary Pattern (ICRLBP) is proposed to improve the performance of Completed Robust Local Binary Pattern (CRLBP) [16] in order to be rotation invariant. ICRLBP improves the performance of CRLBP by inserting LBPROT algorithm [17] to CRLBP algorithm.

ICRLBP represents the texture locally based on local difference sign intensity of neighbor pixels to the threshold value

(CRLBP_S), magnitude intensity of neighbor pixels to the threshold value (CRLBP_M), and center pixel intensity (CRLBP_C), as defined in (1), (4), and (5), respectively.

$$\begin{aligned} CRLBP_S &= \sum_{p=0}^{P-1} s(I_{p,R} - WLG_c) 2^p \\ &= \sum_{p=0}^{P-1} s\left(I_{p,R} - \frac{\sum_{i=0}^{P-1} I_{ci,R} + \alpha I_c}{P + \alpha}\right) \end{aligned} \quad (1)$$

where, *Weighted Local Gray Level* (WLG) is a threshold value as defined in (2) and $s(x)$ is defined in (3)

$$WLG = \frac{\sum_{p=0}^{P-1} I_{p,R} + \alpha I_c}{P + \alpha} \quad (2)$$

$$s(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (3)$$

P is the total number of involved neighbor pixels, R is radius between center pixel and neighbor pixels, I_c is value of center pixel intensity, $I_{p,R}$ is value of p^{th} involved neighbor pixel intensity ($p = 0, 1, \dots, P-1$) with radius R , α is a parameter set by user, and $I_{ci,R}$ is value of i^{th} involved neighbor pixel intensity ($i = 0, 1, \dots, P-1$) with radius R from center pixel I_c .

$$CRLBP_M = \sum_{p=0}^{P-1} s(m_p - c) 2^p \quad (4)$$

where,

$$m_p = |WLG_p - WLG_c| = \left| \frac{\sum_{i=0}^{P-1} I_{pi,R} + \alpha I_{p,R}}{P + \alpha} - \frac{\sum_{i=0}^{P-1} I_{ci,R} + \alpha I_c}{P + \alpha} \right|$$

$I_{pi,R}$ is value of i^{th} involved neighbor pixel intensity ($i = 0, 1, \dots, P-1$) with radius R from center pixel $I_{p,R}$. Where as c is threshold value calculated of mean value of m_p of a whole image. CRLBP_M is calculated the local variance of WLG.

CRLBP_C operator represents value of center pixel intensity as defined in (5).

$$CRLBP_C = s(WLG_c - c_l) \quad (5)$$

Where c_l is threshold calculated of mean value of *Average Local Gray Level* (ALG) of a whole image. ALG is defined in Eq. (6).

$$ALG = \frac{\sum_{p=0}^{P-1} I_{p,R} + I_c}{P} \quad (6)$$

After the value of CRLBP_S, CRLBP_M, and CRLBP_C are obtained, LBPROT algorithm is inserted after binary value of sign vector and magnitude vector is obtained. LBPROT looks for the smallest combination of binary value of sign vector and magnitude vector in each pixel, as defined in (7). Where $CRLBP_{p,R}$ is the binary value obtained from texture feature extraction of CRLBP, and P is the total number of shifted

combination that has the same total number of involved neighbor pixels.

$$LBPROT_{p,R} = \min\{ROT(CRLBP_{p,R}, p)\} \quad (7)$$

The smallest combination of binary value is converted to decimal value. From the decimal value, feature histogram of ICRLBP_Sign (ICRLBP_S), ICRLBP_Magnitude (ICRLBP_M), and ICRLBP_Center (ICRLBP_C) can be plotted.

In grayscale image, if the value of P is 8, then histogram of $CRLBP_S$, $CRLBP_M$, and $CRLBP_C$ consist of 36, 36, and 2 bin features, respectively. Those histograms can be combined together to get the final histogram of ICRLBP. Based on Guo [18], there are two ways to combined those histograms, namely concatenate and jointly.

B. Dataset

Four image datasets are used for evaluating the performance of ICRLBP, namely Batik, textile, Brodatz, and Corel. The size of Batik, textile, and Brodatz image is 128x128 pixels, whereas the size of Corel image is 80x80 pixels. All images are in JPEG format. Batik and Brodatz dataset consist of 112 types of image or class, whereas textile and Corel dataset consist of 50 types of image or class. Each types of those images is rotated by 0°, 5°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°, 50°, 55°, 60°, 65°, 70°, 75°, 80°, 85°, 90°. Therefore, There are 19 images in each class. The example of images in one class as shown in Fig.1.

Batik is the traditional pattern on a fabric drawn with traditional method [19]. Batik has been recognized by UNESCO as one of the indigenous cultural heritage of Indonesia, on October 2, 2009. Batik and textile datasets are obtained from the Laboratory of Vision, Image Processing and Graphics (VIP-G) Institut Teknologi Sepuluh Nopember. Brodatz dataset is obtained from (<http://sipi.usc.edu/database/database.php?volume=textures>).

Corel dataset used in this study is Corel 5,000 which is obtained from (<http://www.ci.gxnu.edu.cn/cbir/Dataset.aspx>). Corel 5,000 consists of 50 categories, and each category consists of 100 images. But in this study, just 1 image of each category is used that is chosen randomly. Batik, textile, Brodatz, and Corel datasets are showed in Fig. 2-5 respectively.

III. METHODOLOGY

In this study, we use the same dataset and algorithm as in [11]. According to the [11], the feature combinations that can achieve the optimal precision and recall are M_C feature when using Modified Canberra distance and S_M_C feature when using L1 distance, as defined in Table 1. Because of that, the ICRLBP features that we observe in this study are only M_C feature and S_M_C feature.

As previously mentioned in [11], M_C feature of ICRLBP has 72 attributes that we need to reduce, whereas S_M_C feature has 2592 attributes. We reduce those attributes after we generate them and before we fed them to distance measure. Attribute selection methods we used are Correlation-based Feature Selection (CFS) [12][13]. and Pearson's-based Correlation [14].

A. Correlation-based Feature Selection

Define.

Correlation-based Feature Selection (CFS) produces a subset of attributes or reduced attributes without losing the essential signal in the data [13]. CFS identifies attributes that are strongly associated to the target class and weakly associated with each other attribute. CFS evaluates the worth of a subset of attributes by considering the individual predictive ability of each feature along with the degree of redundancy between them. Subsets of features that are highly correlated with the class while having low intercorrelation are preferred.

B. Pearson's-based Correlation

Pearson's-based Correlation evaluates the worth of an attribute by measuring the correlation based on Pearson's between it and the class.

C. Distance Measure

The similarity between query images and images in the database is measured using L1 and modified Canberra distance [9], that are defined in equation (8) and (9), respectively.

$$LD(T, Q) = \sum_{i=1}^F |I_i - Q_i| \quad (8)$$

$$CD(T, Q) = \sum_{i=1}^F \frac{|I_i - Q_i|}{|I_i + \mu I| + |Q_i + \mu Q|} \quad (9)$$

Where I is image in the dataset, Q is query image, F is the number of feature vector of each image, $\mu I = \sum_{i=1}^F \frac{I_i}{F}$ and $\mu Q = \sum_{i=1}^F \frac{Q_i}{F}$.

D. Performance Measure

The performance of ICRLBP is measured using precision and recall which are defined in equation (10) and (11), respectively.

$$\text{Precision} = Ir/n \cdot 100\% \quad (10)$$

$$\text{Recall} = Ir/m \cdot 100\% \quad (11)$$

Where I_r is the number of retrieved images, n is the number of relevant images that must be retrieved, and m is the number of all relevant data in the dataset.

Fig. 1. Example of images in one class of Batik dataset.



Fig. 2. Example of Batik dataset.

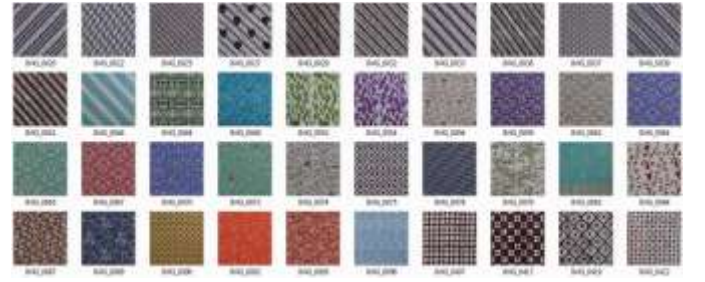


Fig. 3. Example of Textile dataset.

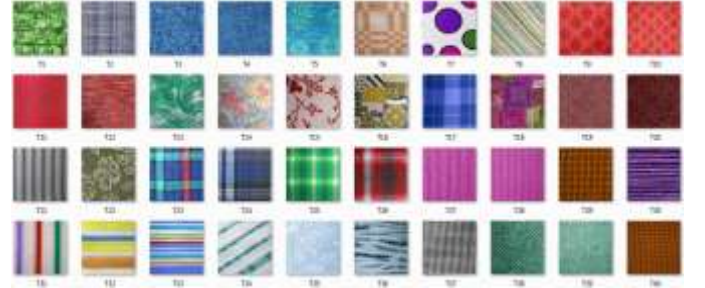


Fig. 4. Example of Brodatz dataset.



Fig. 5. Example of Corel dataset.



IV. RESULT AND DISCUSSION

The performance of ICRLBP is evaluated with six scenarios on the four databases. The first scenario looks for the ICRLBP precision, recall, and computation time of unreduced M_C. The second scenario looks for the ICRLBP precision, recall, and computation time of unreduced S_M_C feature. The third scenario looks for the ICRLBP precision, recall, and computation time of reduced M_C feature using CFS. The fourth scenario looks for the ICRLBP precision, recall, and

computation time of unreduced S_M_C feature using CFS. The fifth scenario looks for the ICRLBP precision, recall, and computation time of reduced M_C feature using Pearson's-based Correlation. The sixth scenario looks for the ICRLBP precision, recall, and computation time of unreduced S_M_C feature using Pearson's-based Correlation.

In each scenario, each image of each database becomes a query image alternately. The total number of image that are retrieved in each retrieval process of one query image is 18 images, because there are 18 images in one class that are rotated in different angles.

Table 1 show the result of first and second scenario. Table 1 indicate that the optimal precisions of unreduced M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset are 80.75%, 65.73%, 64.06%, and 81.92%, respectively. Whereas, the optimal recall of unreduced M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset are 76.5%, 62.27%, 60.69%, and 77.61%, respectively. Those all optimal precision and recall are achieved using Modified Canberra distance. The unreduced M_C feature has 72 attributes.

Table 1 also indicate about the optimal precisions of unreduced S_M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset, that is 80.48%, 66.24%, 59.92%, and 84.86%, respectively. Whereas, the optimal recall of unreduced S_M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset are 76.25%, 62.75%, 56.76%, and 80.39%, respectively. The unreduced S_M_C feature has 2592 attributes.

Table 2 show the result of third and fourth scenario. Table 2 indicate that the optimal precisions of reduced M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset are 84.02%, 69.84%, 68.77%, and 86.8%, respectively. Whereas, the optimal recall of reduced M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset are 79.6%, 66.16%, 65.15%, and 82.34%, respectively. Those all optimal precision and recall are achieved using Modified Canberra distance. The reduced M_C feature using CFS has approximately 38 attributes. It is indicated that CFS can reduce the attributes of M_C feature around 47%, without reducing the value of precision and recall instead increasing them around 4.2% and 4%, respectively.

Table 2 also indicate about the optimal precisions of unreduced S_M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset, that is 87.21%, 74.18%, 71.78%, and 88.2%, respectively. Whereas, the optimal recall of unreduced S_M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset are 82.62%, 70.27%, 68.01%, and 83.56%, respectively. Those all optimal precision and recall are achieved using Modified Canberra distance. The reduced S_M_C feature using CFS has approximately 124.5 attributes. It is indicated that CFS can reduce the attributes of S_M_C feature around 95%, without reducing the value of precision and recall instead increasing them around 7.5% and 7.1%, respectively.

Table 3 show the result of fifth and sixth scenario. Table 3 indicate that the optimal precisions of reduced M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset are 83.74%, 69.17%, 67.22%, and 85.8%, respectively. Whereas,

the optimal recall of reduced M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset are 79.33%, 65.53%, 63.68%, and 81.29%, respectively. Those all optimal precision and recall are achieved using Modified Canberra distance. The reduced M_C feature using Pearson's-based Correlation has approximately 40.5 attributes. It is indicated that Pearson's-based Correlation can reduce the attributes of M_C feature around 44%, without reducing the value of precision and recall instead increasing them around 3.4% and 3.2%, respectively.

Table 3 also indicate about the optimal precisions of unreduced S_M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset, that is 86.24%, 71.86%, 70.05%, and 87.99%, respectively. Whereas, the optimal recall of unreduced S_M_C feature that can achieve of Batik, Brodatz, Corel, and Textile dataset are 81.70%, 68.08%, 66.36%, and 83.36%, respectively. Those all optimal precision and recall are achieved using Modified Canberra distance. The reduced S_M_C feature using Pearson's-based Correlation has approximately 148 attributes. It is indicated that Pearson's-based Correlation can reduce the attributes of S_M_C feature around 94%, without reducing the value of precision and recall instead increasing them around 6.2% and 5.8%, respectively.

From Table 1 until Table 3, we can show that the optimal distance measure to do image retrieval in this study is Modified Canberra distance. Beside, it show that CFS is more optimal for attribute reducing than Pearson's-based Correlation in this study. Moreover, CFS is faster when searching the significant attributes than Pearson's-based Correlation's. Tabel 4 shows the improvement performance using CFS that compare to unreduced ICRLBP using Modified Canberra distance.

According to the results mentioned above, CFS and Pearson's-based Correlation can reduce the attributes effectively. It is indicate that there are some attributes that are not significant or necessary. Some attributes become insignificant because they are irrelevant attribute, i.e. the attribute values of all data are constant e.g. zero value for all data, or they have low variance. Furthermore, some attributes become unnecessary because they are redundant attribute. Those insignificant and unnecessary attribute lead to curse of dimensionality, extend the image retrieval runtime, and sometimes deteriorate the precision and recall rate because of overfitting.

V. CONCLUSION

This study try to reduce insignificant or unnecessary ICRLBP attributes and examine the effect of reducing number of attributes on precision and recall of the retrieving images. The methods we used to reduce the ICRLBP attributes are Correlation-based Feature Selection (CFS) and Pearson's-based Correlation. The experiment results show that Correlation-based Feature Selection (CFS) and Pearson's-based Correlation can reduce the attributes effectively. The optimal number of attributes that can be reduced is 95% for S_M_C feature and 4.2% for M_C feature. It is indicate that there are some attributes that are insignificant or unnecessary for image retrieval. Thus, those can extend the image retrieval runtime, and sometimes degraded the retrieval performance.

TABLE I. PRECISION AND RECALL OF UNREDUCED ICRLBP IN EACH DATASET

Dataset		Batik		Brodatz		Corel		Textile	
Features		M_C	S_M_C	M_C	S_M_C	M_C	S_M_C	M_C	S_M_C
L1	Precision	61,86	79,22	53,19	66,24	46,31	59,92	79,77	84,86
	Recall	58,61	75,05	50,39	62,75	43,87	56,76	75,57	80,39
Canberra	Precision	80,75	80,48	65,73	61,87	64,06	44,58	81,92	69,44
	Recall	76,5	76,25	62,27	58,61	60,69	42,23	77,61	65,78
computation time (s)		143,2	2012	139,2	1950	29	343,8	28,87	345,2
number of attributes		72	2592	72	2592	72	2592	72	2592

TABLE II. PRECISION AND RECALL OF REDUCED ICRLBP USING CFS IN EACH DATASET

Dataset		Batik		Brodatz		Corel		Textile	
Features		M_C	S_M_C	M_C	S_M_C	M_C	S_M_C	M_C	S_M_C
L1	Precision	61,45	78,11	52,69	64,59	45,19	55,88	79,76	85,01
	Recall	58,22	74	49,91	61,19	42,81	52,94	75,56	80,54
Canberra	Precision	84,02	87,21	69,84	74,18	68,77	71,78	86,8	88,2
	Recall	79,6	82,62	66,16	70,27	65,15	68,01	82,23	83,56
computation time (s)		134,5	176,7	121,2	160	25,28	28,23	26,4	34,81
number of attributes		42	148	34	112	28	69	48	169

TABLE III. PRECISION AND RECALL OF REDUCED ICRLBP USING PEARSON'S-BASED CORRELATION IN EACH DATASET

Dataset		Batik		Brodatz		Corel		Textile	
Features		M_C	S_M_C	M_C	S_M_C	M_C	S_M_C	M_C	S_M_C
L1	Precision	61,88	76,36	53,27	63,86	46,31	58,18	79,78	83,09
	Recall	58,63	72,34	50,47	60,5	43,87	55,12	75,58	78,72
	computation time (s)	135,6	388,4	132,4	163,8	29,27	33,62	28,89	33,29
	number of attributes	68	149	61	150	68	146	67	147
Canberra	Precision	83,74	86,24	69,18	71,86	67,22	70,05	85,81	87,99
	Recall	79,33	81,7	65,53	68,08	63,68	66,36	81,29	83,36
	computation time (s)	128,8	388,4	129,1	162,8	28,62	34,87	25,76	33,29
	number of attributes	48	149	48	149	53	147	13	147

TABLE IV. THE AVERAGE IMPROVEMENT PERFORMANCE OF ICRLBP

	The average improvement of							
	M_C feature				S_M_C feature			
	precision (%)	recall (%)	computation time (times)	number of attributes (%)	precision (%)	recall (%)	computation time (times)	number of attributes (%)
CFS	4,2425	4,018	1,114	47	7,468	7,078	11,42	95
Pearson's-based Correlation	3,371075	3,193	1,081	44	6,16	5,838	9,346	94

References

- [1] Liu, Y., Zhang, D., Lu, G., & Ma, W. Y. "A survey of content-based image retrieval with high-level semantics." *Pattern recognition* 40.1 (2007): 262-282.
- [2] Kokare, M., Biswas, P.K. and Chatterji, B.N., 2005. Texture image retrieval using new rotated complex wavelet filters. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6), pp.1168-1178.
- [3] Kokare M, Biswas PK, Chatterji BN. Texture image retrieval using rotated wavelet filters. *Pattern recognition letters*. 2007 Jul 15;28(10):1240-9.
- [4] Zhang D, Wong A, Indrawan M, Lu G. Content-based image retrieval using Gabor texture features. In *IEEE Pacific-Rim Conference on Multimedia*, University of Sydney, Australia 2000 Dec 13 (pp. 392-395).
- [5] Han, J. and Ma, K.K., 2007. Rotation-invariant and scale-invariant Gabor features for texture image retrieval. *Image and vision computing*, 25(9), pp.1474-1481.
- [6] Liu, G.H. and Yang, J.Y., 2008. Image retrieval based on the texton co-occurrence matrix. *Pattern Recognition*, 41(12), pp.3521-3527.
- [7] Liu GH, Zhang L, Hou YK, Li ZY, Yang JY. Image retrieval based on multi-texton histogram. *Pattern Recognition*. 2010 Jul 31;43(7):2380-9.
- [8] Liu GH, Li ZY, Zhang L, Xu Y. Image retrieval based on micro-structure descriptor. *Pattern Recognition*. 2011 Sep 30;44(9):2123-33.
- [9] Minarno, A.E., Munarko, Y., Bimantoro, F., Kurniawardhani, A. and Suciati, N., 2014, February. Batik image retrieval based on enhanced micro-structure descriptor. In *Computer Aided System Engineering (APCASE)*, 2014 Asia-Pacific Conference on (pp. 65-70). IEEE.
- [10] Minarno, A.E., Kurniawardhani, A. and Bimantoro, F., 2016. Image Retrieval Based on Multi Structure Co-occurrence Descriptor. *TELKOMNIKA (Telecommunication Computing Electronics and Control)*, 14(3).
- [11] Kurniawardhani, A., Suciati, N. and Ariesianti, I., 2015. Texture Feature Extraction Using Improved Completed Robust Local Binary Pattern for Batik Image Retrieval. *International Journal of Advancements in Computing Technology*, 7(6), p.69.
- [12] M. A. Hall (1998). *Correlation-based Feature Subset Selection for Machine Learning*. Hamilton, New Zealand.
- [13] Hall M, Holmes G: Benchmarking attribute selection techniques for discrete class data mining. *IEEE Trans Knowl Data Eng*. 2003, 15 (6): 1437-1447. 10.1109/TKDE.2003.1245283
- [14] [Patil, M.D. and Sane, D.S.S., 2014. Effective Classification after Dimension Reduction: A Comparative Study. *International Journal of Scientific and Research Publications*, 4(7), p.1.
- [15] Kurniawardhani, A., Suciati, N. and Ariesianti, I., 2014. Klasifikasi Citra Batik Menggunakan Metode Ekstraksi Ciri Yang Invariant Terhadap Rotasi. *JUTI: Jurnal Ilmiah Teknologi Informasi*, 12(2), pp.48-60.
- [16] Y Zhao, W Jia, RX Hu, H Min, "Completed robust local binary pattern for texture classification", *Journal of Neurocomputing (ScienceDirect)*, vol. 106, pp.68-76, 2013
- [17] M Pietikäinen, T Ojala, Z Xu, "Rotation-invariant texture classification using feature distributions", *Journal of Pattern Recognition (ScienceDirect)*, vol. 33, no. 1, pp.43-52, 2000.
- [18] ZH Guo, L Zhang, D Zhang, "A completed modeling of local binary pattern operator for texture classification", *Journal of Image Processing (IEEE)*, vol. 19, no. 6, pp.1657-1663, 2010.
- [10] A Haake, "The role of symmetry in Javanese Batik patterns", *Journal of Computers & Mathematics with Applications (ScienceDirect)*, vol. 17, no.4-6, pp.815-826, 1989.