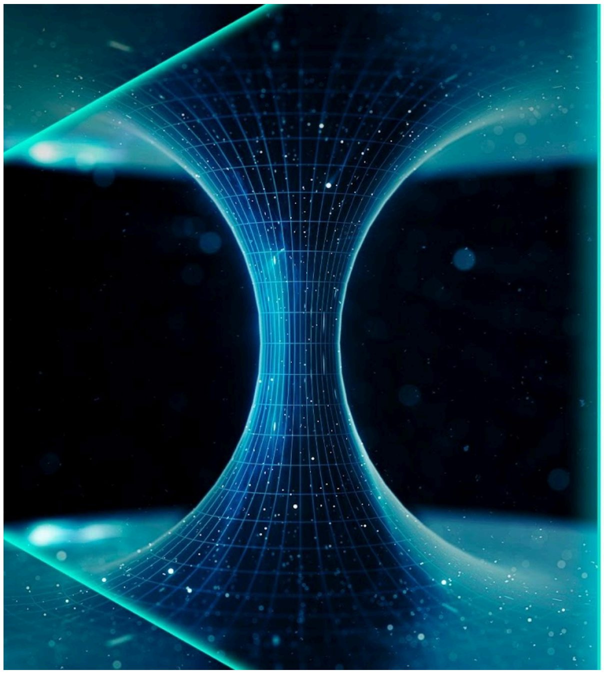
 

Henry Wise Wood High School
Calgary Youth Science Fair
Experimental-Based Research
07/01/23
31/03/23

To what extent can supermassive blackholes within the Local Group Cluster be described by Berenstein Quantum Entropy, Stephen Hawking's Celestial Hole radiation, and collected data from the University of Texas McDonald Observatory dictionary to help develop and implement a classification algorithm, thresholding black hole evaporation rates. Is material information truly lost during evaporation, or is it merely a product of transformation and a subsequent abider of thermodynamic principles?

What are the implications of supermassive black hole evaporation rates for the information paradox, and how can these rates be accurately measured and modelled to better understand the fundamental nature of black holes and the universe as a whole?

Participate Signature: Faisal

Contents:

Common Notation Used Throughout This Paper:

SMBH: Super Massive Black Hole

Idea Development:

<u>Table 1.1: The progression of Ideas Towards the chosen Researched Topic with Subsequent Explanation</u>

Initial Ideas and Progression	Explanation Towards Final Idea
"Under the fundamental principles of quantum physics, how can a computer model show the mapping/positioning of a black hole celestial body?"	This Idea paved the direction I wanted to go into as I saw it as a good opportunity to do some delightful research in the fields of mathematics and physics, and also explore an interest of mine in the world of astrophysics and see what happens when our physics doesn't always describe things with full accuracy. This was the portion that convinced me to also look at computing models to mix an interest with a hobby of mine and learn more about the world of applied mathematical computing.
"To what extent can quantum chaos principles, Quantum mechanics laws, and artificial celestial hole synthesis describe Stephen Hawking's collected data on Quantum Field Theory and Hawking Radiation to accurately model the decay of the observable universe? Is material information truly lost, or is it merely a product of transformation and a subsequent abider of thermodynamic principles? "	This idea was a mix of quantum concepts I wanted to explore concerning black hole,s but as I soon came to explore, many of the concepts dealt with multivariable calculus and differential equations, which currently is out of my reach, and the programming and algorithm stage would've been too complex to describe accurately.
"To what extent can collecting data on Bekenstein Quantum Entropy and Stephen Hawking's Celestial radiation accurately describe the information paradox occurring within supermassive black holes within the Milky Way Galaxy by developing and implementing an algorithm that will collect subsequent black hole masses and return the approximate time of dissipation. Is material information truly lost or is it merely a product of transformation and a subsequent abider of thermodynamic principles?"	This is when my now completed idea is starting to take place. I'm finally dealing with high-level concepts that can be derived and described by formulas I can interpret and program, while being able to look into unanswered paradoxes and give my take on the matter. Here I was trying to describe what my algorithm may do,o but I realized that this may be too basic, and I have more than enough resources and capabilities to add a deeper take on the information paradox. Looking at this idea, there are some gaps, as I don't have a data set to go off and I don't have any material data to describe my input values for the equation.
"To what extent can supermassive blackholes within the Local Group Cluster be described by Berenstein Quantum Entropy, Stephen Hawking's Celestial Hole radiation, and collected data from the University of Texas McDonald Observatory dictionary to help develop and implement an algorithm that will collect subsequent blackhole inputs and return approximate time of dissipation to answer whether material information truly lost or is it merely a product of transformation and a subsequent abider of thermodynamic principles?"	This idea now includes a NASA-sponsored data set of a list of more than 50 Supermassive black holes (choose 50 at random for my research) with their approximate mass, relative distance from Earth, approximate diameter, and conceptual image. I was able to use this data and now make complete calculations with my formulas. Yet again, my algorithm was too broad to explore my ideas, and this algorithm's principle was too basic to make any decent conclusion on the information paradox, so I had to renovate and fine-tune this idea.
"To what extent can supermassive blackholes within the Local Group Cluster be described by Berenstein Quantum Entropy, Stephen Hawking's Celestial Hole radiation, and collected data from the University of Texas McDonald Observatory dictionary to help develop and implement a classification algorithm, thresholding black hole evaporation rates. Is material information truly lost during evaporation, or is it merely a product of transformation and a subsequent abider of thermodynamic principles?"	The finalized Idea is more elegant and narrowed down to its key points. Even though it doesn't describe fully what the algorithm can do, I find it best that we present that ourselves later on in the research and keep the idea simple, so we can always have a reference guide to the goal we are working towards.

The Meaningfulness of The Goal:

The goal of using algorithms and formulas and making conclusions based on interpreted results will help us gain a stance on how information behaves in our universe. Although this project does not have the resources to make an accurate conclusion, we can try and use deductive reasoning to draw a strong conclusion based on conclusion to the information paradox. Answering the information paradox is important for several reasons, which are mainly rooted in our understanding of fundamental physics and the nature of the universe. The information paradox arises from the apparent contradiction between the principles of quantum mechanics and the behaviour of black holes as described by general relativity. Answering the information paradox will help us ensure and understand:

1. *Consistency of fundamental theories:* Quantum mechanics and general relativity are two pillars of modern physics. Resolving the information paradox would help ensure the consistency and compatibility of these theories and bridge the gap between the quantum and the cosmic scales. Furthermore, our understanding of thermodynamic principles can be shown to be complete and universal or inadequate to describe events in the universe.

2. *Understanding quantum gravity:* One of the major open questions in theoretical physics is the development of a theory of quantum gravity, which would unify general relativity and quantum mechanics. Addressing the information paradox is a crucial step towards this unification, as it highlights the key areas where our understanding of these theories is incomplete.

3. *Black hole physics:* Black holes are fascinating objects that provide us with unique insights into the nature of gravity, spacetime, and the limits of our physical theories. Resolving the information paradox would enhance our understanding of black hole dynamics and their role in the universe.

4. *Quantum information theory:* The information paradox touches upon deep questions about the nature of information in the context of quantum mechanics. Addressing the paradox would lead to a more profound understanding of quantum information theory, which has applications in quantum computing, quantum cryptography, and other emerging technologies.

5. *The nature of information:* The information paradox forces us to consider the fundamental nature of information and its role in the universe. Resolving the paradox may provide insights into the relationship between information, matter, and energy, and could potentially reshape our understanding of the basic building blocks of the universe.

Conversely, addressing the information paradox is important because it holds the key to resolving some of the most profound questions in modern physics. It allows us to push the boundaries of our understanding of the universe, and can potentially lead to discoveries and technological advancements.

Furthermore, the information paradox is important because it challenges our fundamental understanding of the laws of physics and the nature of space, time, and matter. It also has implications for our understanding of black holes, one of the most mysterious and extreme objects in the universe.

The paradox arises from the conflict between two fundamental principles of physics: the laws of quantum mechanics, which imply that information must be conserved, and the laws of classical general relativity, which suggest that information can be lost within a black hole.

If information is lost within a black hole, it would mean that the laws of quantum mechanics are not fully applicable in the presence of strong gravitational fields, which would have profound implications for our understanding of the universe at the most fundamental level. It would also mean that information cannot be truly conserved, which is a principle that underlies many of the most important theories in physics, including thermodynamics and quantum field theory.

On the other hand, if information is conserved within a black hole, it would imply that there is a deeper connection between quantum mechanics and general relativity than we currently understand, and it could have important implications for our understanding of the nature of space, time, and gravity.

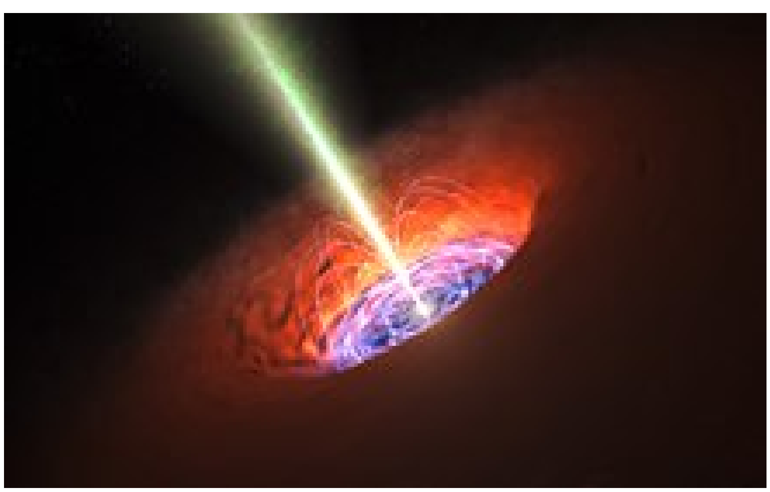
Therefore, resolving the information paradox is crucial for advancing our understanding of the laws of physics and the universe itself.

****Like stated earlier, this Research is not attempting to make an accurate conclusion on the paradox but rather a presentable stance on the matter****

Background Supermassive Black Holes:

- Stellar black holes result from the collapse of massive stars, and some have suggested that supermassive black holes form out of the collapse of massive clouds of gas during the early stages of the formation of the galaxy.
- Another idea is that a stellar black hole consumes enormous amounts of material over millions of years, growing to supermassive black hole proportions. Yet another is that a cluster of stellar black holes forms and eventually merges into a supermassive black hole.
- It's plausible that more than one idea is correct.
- Black holes can expand by taking in more matter and combining it with other black holes.
- The development processes and beginning masses of the progenitors, or "seeds," of supermassive black holes are the subject of numerous ideas.
- If there is enough mass nearby, the black hole seed could accrete to become an intermediate-mass black hole and possibly an SMBH if the accretion rate continues, regardless of the exact formation route.
- It is difficult to understand how such early supermassive black holes as J0313-1806 and ULAS J1342+0928 could exist. Some claim they might have formed by the direct self-interaction collapse of dark matter. Supermassive black holes that were produced before the Big Bounce, according to a small minority of sources, may constitute proof that the Universe originated from a Big Bounce rather than a Big Bang.
- Large gas clouds that did not start could collapse into "quasi-stars," which would then collapse into black holes of about 20 M in size. The formation of these stars may also have been caused by the gravitational pull of dark matter haloes, which would have resulted in supermassive stars with tens of thousands of M. Due to the generation of electron-positron pairs in its core, the "quasi-star" becomes unstable to radial disturbances and might simply collapse into a black hole without igniting (which would eject most of its mass, preventing the black hole from growing as fast).
- According to a more recent theory, supermassive stars with masses of about 100,000 M each contributed to the formation of one SMBH seed in the very early universe.

Figure 1.1: Conceptual Design of SMBH surrounded by an accretion disk



Key Notes about SMBH:

- Gravitational influence: Supermassive black holes exert a strong gravitational influence on their surroundings, affecting the motion of stars and gas in the central regions of their host galaxies.
- Accretion disks: When a supermassive black hole attracts nearby matter, the matter forms an accretion disk around the black hole. This disk of gas and dust becomes extremely hot due to friction, emitting large amounts of radiation, including X-rays and ultraviolet light.
- Active galactic nuclei (AGN) and quasars: When a supermassive black hole is actively feeding on the surrounding matter, it can create an AGN or quasar. These objects emit enormous amounts of energy, making them some of the most luminous and energetic phenomena in the universe.
- Galactic evolution: Supermassive black holes play an essential role in the evolution of galaxies. They influence star formation rates and can also drive powerful jets and outflows of gas, which can affect the growth and distribution of matter in their host galaxies.
- Sagittarius A*: The supermassive black hole at the center of our own Milky Way galaxy is called Sagittarius A* (pronounced "Sagittarius A-star"). It has a mass of around 4 million times that of our Sun and is located about 26,000 light-years away from Earth.
- Gravitational waves: When supermassive black holes merge, they can release vast amounts of energy in the form of gravitational waves, which are ripples in the fabric of spacetime. Detecting these waves can help us learn more about the properties and behaviour of supermassive black holes.

How Common are SMBHS?

- Supermassive black holes are believed to be quite common in the universe, as they are thought to reside at the centers of most, if not all, large galaxies. Observational evidence supports this idea, as astronomers have found supermassive black holes in numerous galaxies, including our own Milky Way.
- Supermassive black holes play a crucial role in the formation and evolution of galaxies, affecting the distribution of stars and gas, as well as the rate of star formation. Although the exact mechanisms behind their formation remain an active area of research, the prevalence of supermassive black holes across the universe is well-established.
- It's important to note that not all supermassive black holes are active or easily detectable. An active supermassive black hole, which is actively accreting matter and emitting radiation, can be observed as an active galactic nucleus (AGN) or quasar. In contrast, a dormant supermassive black hole, like the one in the center of the Milky Way (Sagittarius A*), is more difficult to detect directly, but its presence can be inferred by studying the motions of nearby stars and other indirect methods.

Detection Methods of SMBH:

Detecting supermassive black holes can be challenging, as they do not emit light themselves. However, their immense gravitational influence on the surrounding environment provides indirect ways to detect them. Here are some common detection methods:

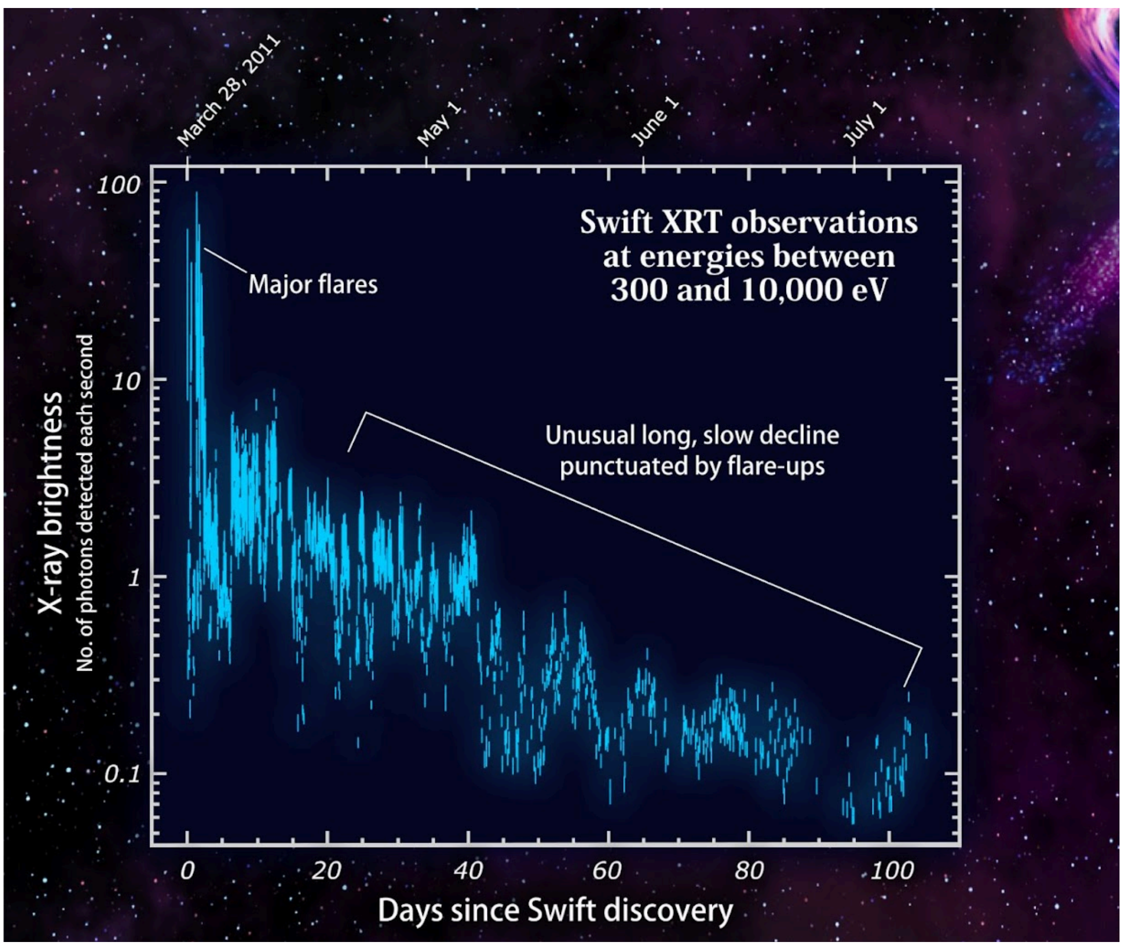
1. Stellar orbits: Observing the motion of stars near the center of a galaxy can provide evidence for the presence of a supermassive black hole. The gravitational pull of the black hole causes nearby stars to follow specific orbits. In the case of the Milky Way, the orbits of stars near Sagittarius A* have provided strong evidence for a supermassive black hole at the galactic center.
2. Gas dynamics: The motion of gas in the central regions of a galaxy can also reveal the presence of a supermassive black hole. Gas near the black hole will move at very high velocities due to the strong gravitational field, which can be detected through spectroscopic observations of emission or absorption lines.
3. Accretion disks and jets: When a supermassive black hole is actively accreting matter, it forms an accretion disk around itself. The disk heats up due to friction, emitting radiation across the electromagnetic spectrum, from radio waves to X-rays. In some cases, the black hole also produces powerful jets of particles moving at relativistic speeds. Observing these structures can provide indirect evidence for the presence of a supermassive black hole.
4. Active galactic nuclei (AGN) and quasars: AGN and quasars are extremely luminous objects powered by the accretion of matter onto a supermassive black hole. Their intense brightness across various wavelengths, as well as their variability, are signatures of supermassive black holes at work.
5. Gravitational lensing: The immense gravity of a supermassive black hole can bend and distort the light from more distant objects behind it, creating a gravitational lensing effect. Observing these distortions in the light can help infer the presence of a supermassive black hole.
6. Gravitational waves: When two supermassive black holes merge, they can emit gravitational waves – ripples in spacetime that propagate through the universe. Detecting these waves can provide information about the properties of the merging black holes, including their masses and spins.
7. Future techniques: As technology advances and new observational techniques emerge, more methods for detecting supermassive black holes will likely be developed. For instance, the Event Horizon Telescope (EHT) project has managed to capture the first-ever image of a black hole's event horizon, offering a new way to study these cosmic giants directly.

SMBH Error and Uncertainty:

1. Distance measurements: Accurate distance measurements to the host galaxy are crucial for determining the properties of a supermassive black hole. Uncertainties in distance can propagate to other calculated quantities, such as the black hole mass and the luminosity of its accretion disk.
2. Stellar and gas dynamics: When inferring the presence of a supermassive black hole based on the motion of stars or gas, assumptions about the distribution of mass and the gravitational potential within the galaxy are required. Uncertainties in these assumptions can lead to uncertainties in the estimated black hole mass.
3. Virial mass estimators: For some active galaxies, the mass of the central supermassive black hole can be estimated using the virial theorem, which relates the mass to the velocities and distances of gas clouds near the black hole. The accuracy of these estimators depends on the reliability of the measurements and the underlying assumptions, such as the geometry of the gas distribution and the dynamics of the system.

4. Accretion disk models: When modelling the radiation emitted by the accretion disk around a supermassive black hole, uncertainties can arise from the assumptions about the disk's structure, temperature, and opacity, as well as the black hole's spin.
5. Gravitational wave measurements: Gravitational wave signals from merging supermassive black holes can provide valuable information about their properties. However, uncertainties in the waveform models and the detector sensitivities can affect the precision of the extracted parameters.
6. Cosmic evolution: The relationship between supermassive black holes and their host galaxies has evolved over cosmic time. Accounting for this cosmic evolution can introduce uncertainties when comparing black holes across different epochs.
7. Limited observational data: Due to the limitations of current observational facilities and the challenges in observing supermassive black holes directly, the available data can be sparse, noisy, or incomplete. This can lead to uncertainties in the derived properties of the black holes.

Figure 1.2: X-Ray from Swift J1644+57 (X-Ray Luminosity as a Function of Time)



Background - Schwarzschild Radius

- The Schwarzschild radius, named after the German physicist Karl Schwarzschild, is a characteristic radius associated with any mass in the context of general relativity. It represents the size of the event horizon of a non-rotating (spherically symmetric) black hole with that particular mass. The event horizon is the boundary beyond which any matter or radiation, including light, cannot escape the black hole's gravitational pull.

$$r_s = \frac{2GM}{c^2},$$

Where,

r = radius of event horizon

G = gravitational constant

M = object mass

c = speed of light constant

- If an object with mass M is compressed into a sphere with a radius smaller than its Schwarzschild radius, it would become a black hole. For example, the Schwarzschild radius of the Earth is about 9 millimetres (0.35 inches), meaning that if the Earth were compressed into a sphere with a radius smaller than 9 millimetres, it would become a black hole. Similarly, the Schwarzschild radius of the Sun is approximately 3 kilometres (1.86 miles).
- It is important to note that the Schwarzschild radius is derived from the Schwarzschild solution to Einstein's field equations of general relativity, which describes the spacetime geometry outside a spherically symmetric mass. The concept of the Schwarzschild radius has played a significant role in understanding black holes and their properties.
- Schwarzschild radius, also called the gravitational radius, is the radius below which the gravitational attraction between the particles of a body must cause it to undergo irreversible gravitational collapse. This phenomenon is thought to be the final fate of the more massive stars.
- The Schwarzschild radius will grow faster than the actual radius when the body fills up with stuff at a certain density. A body of this density would become a supermassive black hole when its Schwarzschild radius reached about 136 million solar masses, surpassing its physical radius.
- When the body is filled with the material at a specific density, the Schwarzschild radius will expand more quickly than the real radius. When the Schwarzschild radius of a body of this density exceeds its physical radius, which is around 136 million solar masses, the body would become a supermassive black hole.

Schwarzschild Radius Importance:

The Schwarzschild radius is important for several reasons, primarily related to our understanding of black holes and general relativity. Here are some key reasons why the Schwarzschild radius is significant:

1. Characterizing black holes: The Schwarzschild radius helps characterize black holes by defining their event horizons. The event horizon is the boundary beyond which nothing, not even light, can escape the black hole's gravitational pull. The Schwarzschild radius, thus, plays a crucial role in describing the properties and behaviour of black holes.

2. Understanding general relativity: The concept of the Schwarzschild radius arises from the Schwarzschild solution, which is the first exact solution to Einstein's field equations of general relativity for the spacetime outside a spherically symmetric, non-rotating mass. The Schwarzschild radius and the associated spacetime geometry are essential in understanding how general relativity describes gravity, particularly in strong gravitational fields.
3. Gravitational collapse: The Schwarzschild radius is relevant in the context of gravitational collapse, which is the process through which a massive object, like a star, collapses under its gravity. If the core of a collapsing star has a radius smaller than its Schwarzschild radius, it will continue to collapse until it forms a black hole.
4. Stellar evolution and astrophysics: The Schwarzschild radius is an important concept in the study of stellar evolution, particularly in the late stages of massive stars' lives. Understanding the conditions under which a black hole can form, as dictated by the Schwarzschild radius, is crucial for modelling the end states of massive stars and the resulting astrophysical phenomena, such as supernovae and gamma-ray bursts.
5. Astrophysical observations: Knowledge of the Schwarzschild radius helps astronomers interpret various astrophysical observations, such as the accretion of matter around black holes, the emission of high-energy radiation from the vicinity of black holes, and the motion of stars near the centers of galaxies.
6. Theoretical insights: The Schwarzschild radius is valuable for understanding various theoretical aspects of black holes, such as their thermodynamics, the no-hair theorem, and the potential existence of wormholes or other exotic objects.

- In summary, the Schwarzschild radius is important because it provides a fundamental characterization of black holes and their event horizons, offers insights into general relativity and the behaviour of gravity, and plays a crucial role in understanding various astrophysical phenomena and the late stages of stellar evolution.

<u>Schwarzschild Radius About the Information Paradox:</u>

The Schwarzschild radius is related to the black hole information paradox, a theoretical problem arising from the combination of general relativity and quantum mechanics. The information paradox is centred around the question of what happens to the information about the quantum states of particles that fall into a black hole. According to general relativity, once an object passes the event horizon (defined by the Schwarzschild radius for a non-rotating black hole), it is lost forever and cannot be retrieved. This principle is consistent with the concept of the event horizon, which is a boundary from which nothing, not even information, can escape the black hole's gravitational pull. However, quantum mechanics posits that information cannot be destroyed. This idea is known as the principle of unitarity, which states that the evolution of a quantum system must be reversible and preserve the information about the system's initial state. The apparent loss of information when particles fall into a black hole creates a conflict between general relativity and quantum mechanics, giving rise to the information paradox. The paradox deepens with the discovery of Hawking radiation, a process through which black holes can lose mass and eventually evaporate. Hawking radiation, a result of quantum effects near the event horizon, causes the black hole to emit particles and lose mass. If the black hole eventually evaporates completely, the question arises as to what happens to the information that was once inside it. The Schwarzschild radius is relevant to the information paradox because it sets the boundary (the event horizon) that appears to cause the loss of information, creating the conflict between general relativity and quantum mechanics. Resolving the black hole information paradox requires reconciling the principles of general relativity (which predict that information is lost) and quantum mechanics (which insists that information must be preserved). Various proposals have been made to resolve the information paradox, including ideas involving black hole complementarity, holography, and the existence of a firewall at the event horizon. However, a complete and universally accepted resolution to the paradox has yet to be found, and it remains an active area of research in theoretical physics.

Figure 1.1: Schwarzschild Radius Simple Model

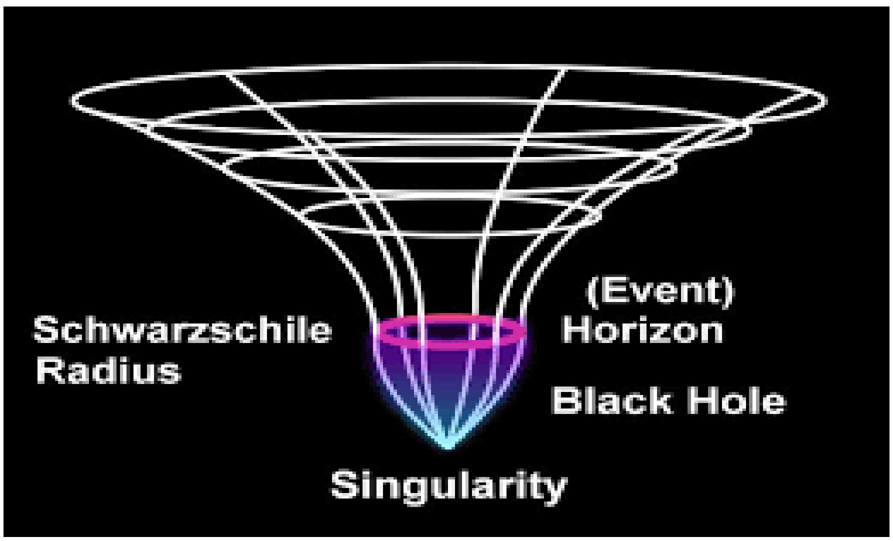
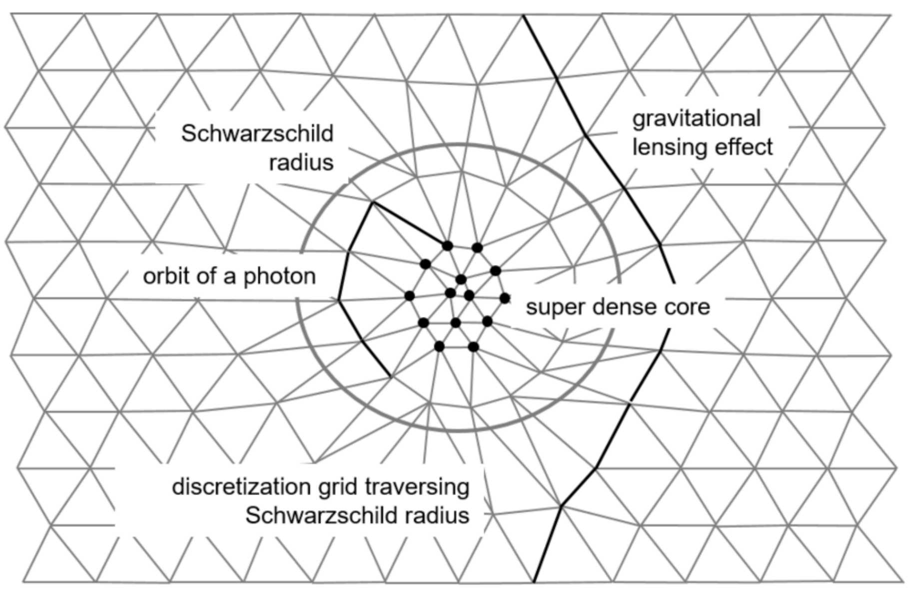


Figure 1.2: Abstract Model of the Schwartzschild Radius about Other Related Mechanics



Background - Information and the Related Paradox:

- The **black hole information paradox** appears when the predictions of quantum mechanics and general relativity are combined.
- The theory of general relativity predicts the existence of black holes, which are regions of spacetime from which nothing, not even light, can escape.
- Stephen Hawking applied the rules of quantum mechanics to such systems and found that an isolated black hole would emit a form of radiation called Hawking radiation.
- The information paradox appears when one considers a process in which a black hole is formed through a physical process and then evaporates away entirely through Hawking radiation.
- Hawking's calculation suggests that the final state of radiation would retain information only about the total mass, electric charge, and angular momentum of the initial state.
- Since many different states can have the same mass, charge, and angular momentum, this suggests that many initial physical states could evolve into the same final state.
- Therefore, information about the details of the initial state would be permanently lost.
- However, this violates a core precept of both classical and quantum physics—that, *in principle,* the state of a system at one point in time should determine its value at any other time.
- It is now generally believed that information is preserved in black-hole evaporation.
- This means that the predictions of quantum mechanics are correct, whereas Hawking's original argument that relied on general relativity must be corrected.
- However, views differ as to how, precisely, Hawking's calculation should be corrected. In recent years, several extensions of the original paradox have been explored.
- Taken together, these puzzles about black hole evaporation have implications for how gravity and quantum mechanics must be combined, leading to the information paradox remaining an active field of research within quantum gravity.

➔ Weather information is lost:

- A minority view within the theoretical physics community is that information is genuinely lost when black holes form and evaporate. This conclusion follows if one assumes that the predictions of semiclassical gravity and the causal structure of the black-hole spacetime are exact.
- However, this conclusion leads to the loss of unitarity. Banks, Susskind and Peskin argued that, in some cases, loss of unitarity also implies violation of energy–momentum conservation or locality, but this argument may be evaded in systems with a large number of degrees of freedom.

➔ Relevant Formulas:

The formula implies that the black hole radiates mass at a rate given by: $\frac{dM}{dt} =- aT^4$

- Where a is constantly related to fundamental constants, including the Stefan–Boltzmann constant and certain properties of the black hole spacetime called its greybody factors.
- The temperature of the black hole is, in turn, dependent on the mass, charge, and several proposed solutions to the information paradox have been put forth by researchers, attempting to reconcile the principles of general relativity and quantum mechanics. While a universally accepted resolution has yet to be found, some notable proposals include:

What is the Information Paradox?

- The information paradox is a theoretical problem that arises from the apparent conflict between the principles of general relativity and quantum mechanics in the context of black holes. The paradox is centred around the question of what happens to the information about the quantum states of particles that fall into a black hole.
- According to general relativity, once an object passes the event horizon of a black hole, it is lost forever and cannot be retrieved. This principle is consistent with the concept of the event horizon, which is a boundary from which nothing, not even information, can escape the black hole's gravitational pull.
- However, quantum mechanics posits that information cannot be destroyed. This idea is known as the principle of unitarity, which states that the evolution of a quantum system must be reversible and preserve the information about the system's initial state. The apparent loss of information when particles fall into a black hole creates a conflict between general relativity and quantum mechanics, giving rise to the information paradox.
- The paradox becomes even more puzzling with the discovery of Hawking radiation, a process through which black holes can lose mass and eventually evaporate. Hawking radiation, a result of quantum effects near the event horizon, causes the black hole to emit particles and lose mass. If the black hole eventually evaporates completely, the question arises as to what happens to the information that was once inside it.
- The information paradox remains an unresolved issue in theoretical physics, and various proposals have been made to address it, such as black hole complementarity, the holographic principle, and firewalls.

Information Paradox Key Concepts:

1. Black holes: Massive objects resulting from the gravitational collapse of a star or other celestial body, characterized by an event horizon, beyond which nothing can escape, including light.
2. Event horizon: The boundary surrounding a black hole from which no information, matter, or radiation can escape due to the intense gravitational pull. The event horizon plays a central role in the information paradox, as it represents the point where information is seemingly lost.
3. General relativity: Einstein's theory of gravity describes the curvature of spacetime due to mass and energy. General relativity predicts that the information falling into a black hole would be lost beyond the event horizon.
4. Quantum mechanics: The branch of physics that deals with the behaviour of particles at the atomic and subatomic levels. Quantum mechanics posits that information about a quantum system must be conserved and cannot be destroyed, which contradicts the predictions of general relativity regarding black holes.
5. Principle of unitarity: A fundamental principle in quantum mechanics stating that the evolution of a quantum system is reversible and preserves the information about the initial state. This principle is at odds with the apparent loss of information in black holes, as predicted by general relativity.
6. Hawking radiation: A theoretical process, proposed by physicist Stephen Hawking, through which black holes emit radiation and lose mass due to quantum effects near the event horizon. This discovery deepens the information paradox by raising the question of what happens to the information inside a black hole as it evaporates over time.

Proposed Solutions to the Information Paradox:

1. *Black hole complementarity:* This idea, proposed by Leonard Susskind, suggests that the information that falls into a black hole is not lost but is encoded on the event horizon in a highly scrambled form. According to this proposal, an observer falling into the black hole would see the information disappear, while an observer outside the event horizon would see the information encoded on the surface. This concept implies that the information is not truly lost, but the two observations are complementary and cannot be experienced simultaneously by a single observer.
2. *Holographic principle:* This idea builds on the concept of black hole complementarity and suggests that the information within a volume of space is encoded on the boundary of that space, much like a hologram. In the context of black holes, the information that falls in would be encoded on the event horizon, preserving the information and resolving the paradox.
3. *Firewall hypothesis:* The firewall hypothesis posits that a highly energetic and destructive boundary, or "firewall," exists just inside the event horizon. This firewall would effectively destroy any information that falls into the black hole, preventing it from reaching the singularity. However, this proposal seems to violate the equivalence principle of general relativity, which states that free-falling observers should not feel any unusual effects when crossing the event horizon.
4. *Loop quantum gravity and quantum geometry:* Some researchers have proposed that the information paradox may be resolved by a deeper understanding of quantum gravity, such as the theories provided by loop quantum gravity or other quantum geometric approaches. These theories could potentially describe the behaviour of spacetime and matter at the extremely small scales within a black hole, where general relativity and quantum mechanics appear to be at odds.s
5. *Emergent spacetime and wormholes:* Another possible resolution to the information paradox involves the idea that spacetime itself is an emergent phenomenon, resulting from more fundamental degrees of freedom. In this view, spacetime could be "woven" from quantum entanglements, which might allow information to be preserved and transferred through wormholes or other non-local connections.

The Modern-Day Stance:

- A reasonable conclusion to the information paradox is yet to be definitively reached, as it remains an open question in theoretical physics. However, many physicists believe that a resolution will ultimately be found in a complete theory of quantum gravity, which would successfully unify general relativity and quantum mechanics. Such a theory would provide a consistent framework for understanding the behaviour of matter, energy, and spacetime in extreme conditions, such as those found in black holes.
- Although several proposed solutions, such as black hole complementarity, the holographic principle, and firewalls, have been put forth, none have been universally accepted or proven. These proposals have, however, contributed to the development of new ideas and the ongoing search for a deeper understanding of the fundamental principles of the As research progresses and our understanding of both general relativity and quantum mechanics continues to evolve, a more satisfactory resolution to the information paradox will likely be discovered. Until then, the information paradox serves as a reminder of the complexities and challenges that arise in the pursuit of a unified understanding of the universe's underlying principles.

The Early Stance on the Paradox:

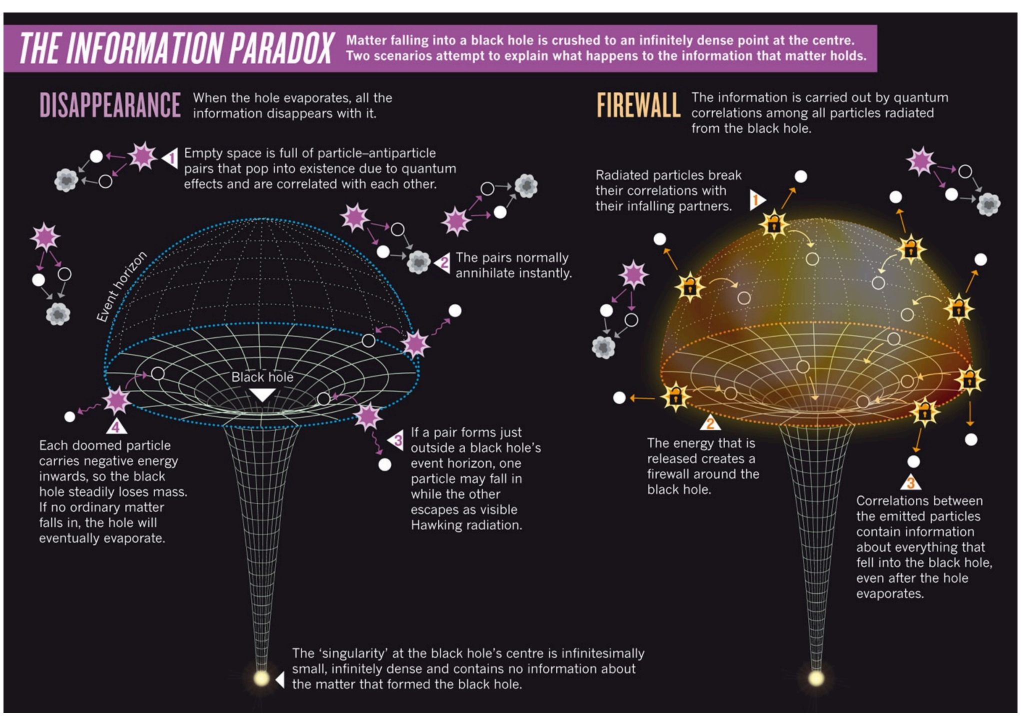
The information paradox has been a longstanding challenge in reconciling the principles of general relativity and quantum mechanics in the context of black holes. However, one stance proposes that information may not be truly lost when it enters a black hole but instead escapes through Hawking radiation, preserving the information and adhering to thermodynamic principles.

Hawking radiation, a theoretical process discovered by physicist Stephen Hawking, involves the emission of radiation from black holes due to quantum effects near the event horizon. As black holes emit this radiation, they lose mass and, over an extremely long timescale, may eventually evaporate. This stance argues that the information associated with particles that fall into a black hole could be encoded in the Hawking radiation released during this process. As the black hole evaporates, the information would gradually escape, ensuring that it is not permanently lost.

This perspective maintains that the apparent loss of information inside a black hole is merely an artifact of our current understanding, rather than a true violation of quantum mechanics or thermodynamics. By encoding the information in Hawking radiation, the process would still respect the principle of unitarity, which states that the evolution of quantum systems must preserve information. Additionally, this stance aligns with the second law of thermodynamics, as the black hole's entropy would increase during the evaporation process, ultimately transferring the information to the emitted radiation and the surrounding environment.

While this stance offers a possible resolution to the information paradox, further research and development of a complete theory of quantum gravity are needed to verify this hypothesis and provide a more comprehensive understanding of the behaviour of matter, energy, and spacetime in extreme conditions such as black holes.

Figure 1.1: Different visualizations of Information Paradox theories



SMBH Contradictions:

- The information paradox is a problem in black hole thermodynamics that arises from the apparent contradiction between the laws of quantum mechanics and general relativity. According to quantum mechanics, information cannot be destroyed, while general relativity predicts that black holes can cause information to be lost forever.
- The paradox arises because when matter falls into a black hole, the information it contains is thought to be irretrievably lost, as it is believed to be destroyed by the black hole's singularity. However, this contradicts the laws of quantum mechanics, which state that information cannot be destroyed.
- One proposed resolution to the information paradox is that black holes preserve information, but in a scrambled or encoded form. This idea is based on the concept of black hole complementarity, which suggests that different observers can have different views of the same black hole, depending on their reference frame. This would allow information to be preserved in a way that is consistent with quantum mechanics.
- Another proposed solution to the information paradox is that black holes emit Hawking radiation, which carries away information that was previously thought to be lost. This idea is based on the discovery by Stephen Hawking that black holes can radiate particles due to quantum effects. It is believed that the information contained in matter that falls into a black hole is encoded in the Hawking radiation, which is emitted back into the universe over time.
- The information paradox highlights the need for a better understanding of the fundamental laws of physics, particularly in the realm of quantum gravity. It also has important implications for the behaviour of black holes and their interaction with the universe, as well as for the fundamental principles of thermodynamics and information theory.

SMBH Entropy and Thermodynamics:

- The temperature of the black hole is, in turnndependent on the mass, charge, and angular momentum of the black hole.
- The important aspect of the formulas above is that they suggest that the final gas of radiation formed through this process depends only on the black hole's temperature and is independent of other details of the initial state.
- This leads to the following paradox. Consider two distinct initial states that collapse to form a Schwarzschild black hole of the same mass. Even though the states were distinct to start with, since the mass (and, hence, the temperature) of the black holes is the same, they will emit the same Hawking radiation. Once the black holes evaporate completely, in both cases, one will be left with a featureless gas of radiation. This gas cannot be used to distinguish between the two initial states, and therefore, information has been lost.
- It is now widely believed that the reasoning leading to the paradox above is flawed.

$$T = \frac{\hbar c^3}{8\pi G k_B M} \qquad 1)$$

T = Black hole temperature

$\hbar$ = Planck's reduced constant

c = light speed

G = Gravity constant

k_B = Bolzmann constant

M = black hole mass

Thermodynamics and SMBH:

- Black hole thermodynamics is a field that explores the connections between the laws of thermodynamics and the properties of black holes. Black hole thermodynamics is an area of active research, and it has provided insights into the behaviour of black holes, as well as the relationship between gravity and thermodynamics. Some key concepts in black hole thermodynamics include:

1. Surface gravity: Black holes have a characteristic surface gravity, which is related to the strength of the gravitational field at the event horizon. Surface gravity is an important concept in black hole thermodynamics, as it plays a role in the black hole's temperature and entropy.
2. Black hole temperature: Black holes have a temperature associated with them, which was first proposed by Stephen Hawking when he discovered Hawking radiation. Hawking radiation is a quantum effect that causes black holes to emit particles and lose mass over time. The temperature of a black hole is inversely proportional to its mass, meaning that smaller black holes have higher temperatures. This temperature is directly related to the black hole's surface gravity.
3. Black hole entropy: The entropy of a black hole is a measure of the amount of information or disorder associated with the black hole. The Bekenstein-Hawking entropy formula states that the entropy of a black hole is proportional to the area of its event horizon. In black hole thermodynamics, this relationship between entropy and event horizon area is analogous to the relationship between entropy and volume in conventional thermodynamics.

4. Laws of black hole mechanics: Analogous to the laws of thermodynamics, the four laws of black hole mechanics describe the behaviour of black holes in terms of their mass, angular momentum, and charge. These laws connect the properties of black holes with thermodynamic concepts such as energy, temperature, and entropy.
5. Information paradox: The study of black hole thermodynamics has contributed to the understanding of the black hole information paradox, which concerns the apparent loss of information when particles fall into a black hole. The paradox arises from the conflict between general relativity, which suggests that information is lost beyond the event horizon, and quantum mechanics, which posits that information must be conserved. Black hole thermodynamics has helped to illuminate the relationship between information and entropy in the context of black holes.
6. Black hole thermodynamics has provided valuable insights into the connections between gravity, thermodynamics, and information theory. This field continues to be an area of active research, and the development of a complete theory of quantum gravity may further deepen our understanding of the thermodynamic properties of black holes and their implications for the fundamental principles of the universe.

<u>SMBH Thermodynamics and Information Paradox Link:</u>

- Black hole thermodynamics and information theory are closely connected, as they both deal with the concept of entropy, which is a measure of disorder or the amount of information contained within a system. The connection between these two fields becomes particularly interesting when addressing the black hole information paradox, which concerns the apparent loss of information when particles fall into a black hole.
- In black hole thermodynamics, the Bekenstein-Hawking entropy formula states that the entropy of a black hole is proportional to the area of its event horizon. This relationship suggests that the information about the matter that has fallen into the black hole is somehow encoded in its event horizon. The connection between the event horizon area and entropy is analogous to the connection between the volume and entropy in conventional thermodynamics.
- Information theory, on the other hand, deals with the quantification, storage, and transmission of information. In the context of the information paradox, information theory plays a crucial role in understanding the conservation of information, as dictated by the principles of quantum mechanics.
- The connection between black hole thermodynamics and information theory is best illustrated through the holographic principle, which posits that the information contained within a certain volume of space can be entirely encoded on a lower-dimensional boundary, such as the event horizon of a black hole. This principle suggests that the information about the particles that have fallen into the black hole is not truly lost but rather stored on the event horizon in a highly scrambled form.
- Hawking radiation, a process through which black holes emit radiation and lose mass, has also contributed to the connection between black hole thermodynamics and information theory. If the information about the matter that has fallen into the black hole is somehow encoded in Hawking radiation, it would imply that the information is not lost but rather transferred to the surrounding environment as the black hole evaporates.

SMBH and Entropy:

- Entropy: entropy, the measure of a system's thermal energy per unit temperature that is unavailable for doing useful work. Because work is obtained from ordered molecular motion, the amount of entropy is also a measure of the molecular disorder, or randomness, of
- The second law of thermodynamics requires that black holes have entropy. If black holes carried no entropy, it would be possible to violate the second law by throwing mass into the black hole. The increase in the entropy of the black hole more than compensates for the decrease in the entropy carried by the object that was swallowed.
- In 1972, Jacob Bekenstein conjectured that black holes should have an entropy, and by the same year, he proposed the no-hair theorems.
- In 1973, Bekenstein suggested 0.276 as the constant of proportionality, asserting that if the constant was not exactly this, it must be very close to it.
- The next year, in 1974, Stephen Hawking showed that black holes emit thermal Hawking radiation corresponding to a certain temperature (Hawking temperature). Using the thermodynamic relationship between energy, temperature, and entropy, Hawking was able to confirm Bekenstein's conjecture and fix the constant of proportionality at 1/4

$$S_{\rm BH} = \frac{k_{\rm B} A}{4\ell_{\rm P}^2}$$

where A is the area of the event horizon, $k_{\rm B}$ is the Boltzmann constant, and $\ell_{\rm P} = \sqrt{G\hbar/c^3}$ is the Planck length.

- This is often referred to as the Bekenstein–Hawking formula. The subscript BH either stands for "black hole" or "Bekenstein–Hawking". The black hole entropy is proportional to the area of its event horizon, A.
- Although Hawking's calculations gave further thermodynamic evidence for black hole entropy, until 1995 no one was able to make a controlled calculation of black hole entropy based on statistical mechanics, which associates entropy with a large number of microstates. So-called "no-hair" theorems appeared to suggest that black holes could have only a single microstate.
- The situation changed in 1995 when Andrew Strominger and Cumrun Vafa calculated the right Bekenstein–Hawking entropy of a supersymmetric black hole in string theory, using methods based on D-branes and string duality.
- Their calculation was followed by many similar computations of the entropy of large classes of other extremal and near-extremal black holes, and the result always agreed with the Bekenstein–Hawking formula. However, for the Schwarzschild black hole, viewed as the most far-from-extremal black hole, the relationship between micro- and macrostates has not been characterized.
- In the case of supermassive black holes, the event horizon area is considerably larger than that of stellar-mass black holes, which means that their entropy is also significantly higher. This relationship between entropy and event horizon area suggests that supermassive black holes

contain a vast amount of information, as entropy can be interpreted as a measure of information content.

- The high entropy of supermassive black holes also has implications for the cosmic censorship hypothesis, which posits that singularities (regions of infinite density) should always be hidden from the external universe by an event horizon. The immense entropy of supermassive black holes may help to maintain the stability of the event horizon and prevent naked singularities from forming.
- Furthermore, the entropy of supermassive black holes plays a role in understanding the thermodynamic properties and behaviour of these objects. For instance, the temperature of a black hole is inversely proportional to its mass, which means that supermassive black holes have extremely low temperatures. As a result, the Hawking radiation emitted by supermassive black holes is negligible, and their evaporation timescales are much longer than the current age of the universe.

<u>SMBH Entropy about the Information Paradox:</u>

- The information paradox arises from the apparent conflict between the principles of general relativity and quantum mechanics when considering the fate of information that falls into a black hole. This paradox is relevant to all types of black holes, including supermassive black holes.
- The entropy of a black hole, including a supermassive black hole, is related to the area of its event horizon, as described by the Bekenstein-Hawking entropy formula. This relationship implies that a black hole's entropy is a measure of the information content associated with the matter that has fallen into it. In the case of supermassive black holes, their large event horizons correspond to significantly higher entropy, suggesting they contain a vast amount of information.
- The information paradox becomes particularly relevant when considering the evaporation of black holes due to Hawking radiation. For supermassive black holes, their extremely low temperatures result in negligible Hawking radiation and evaporation timescales much longer than the age of the universe. As a consequence, the information paradox is not as immediate a concern for supermassive black holes as it is for smaller black holes with faster evaporation timescales. However, the fundamental question of what happens to the information contained within a black hole still applies to supermassive black holes.
- Various proposed solutions to the information paradox attempt to address how information could be preserved or retrieved from black holes. Some of these solutions include the holographic principle, black hole complementarity, and firewalls. While none of these proposals have been universally accepted or proven, they may have implications for the handling of information in supermassive black holes.

SMBH Laws of Thermodynamics:

1. Zeroth Law of Thermodynamics: If two systems are in thermal equilibrium with a third system, they are also in thermal equilibrium with each other.
2. First Law of Thermodynamics: The change in internal energy of a system is equal to the heat added to the system minus the work done by the system.
3. Second Law of Thermodynamics: The total entropy of an isolated system never decreases over time.
4. Third Law of Thermodynamics: As the temperature of a system approaches absolute zero, the entropy of the system approaches a minimum value.

The principles of black hole thermodynamics are based on the study of the behaviour of black holes according to the laws of thermodynamics. For example, it has been shown that black holes have a temperature, called the Hawking temperature, which is inversely proportional to their mass. This temperature is a consequence of the emission of Hawking radiation by black holes, which is analogous to the thermal radiation emitted by a hot object.

Furthermore, it has been shown that the area of the event horizon of a black hole is related to the entropy of the black hole, such that the entropy is proportional to the area of the event horizon divided by the Planck area. This is known as the Bekenstein-Hawking entropy formula.

The principles of black hole thermodynamics provide a framework for understanding the thermodynamic behaviour of black holes and their interaction with the surrounding universe. They also have important implications for the fundamental laws of physics, including the laws of quantum mechanics and general relativity.

Stephen Hawking's Celestial Hole Radiation

- Hawking radiation, named after the renowned physicist Stephen Hawking, is a theoretical process through which black holes emit radiation and lose mass over time. This discovery was groundbreaking, as it connected the principles of quantum mechanics, thermodynamics, and general relativity in the context of black holes.
- Hawking radiation arises due to the quantum effects near the event horizon of a black hole. According to quantum mechanics, particles and antiparticles spontaneously appear and annihilate each other in pairs throughout space, even in a vacuum. This phenomenon is known as vacuum fluctuations. When these fluctuations occur near a black hole's event horizon, one particle of the pair may fall into the black hole, while the other escapes. The escaping particle becomes part of the Hawking radiation.
- The particles that escape have positive energy, while the particles that fall into the black hole have negative energy (from the perspective of an observer far from the black hole). As a result, the black hole loses mass over time. This process is extremely slow and more significant for smaller black holes, as Hawking radiation is inversely proportional to the black hole's mass. For a stellar-mass black hole, the evaporation process would take much longer than the current age of the universe.
- The discovery of Hawking radiation had profound implications for our understanding of black holes and their behaviour. It introduced the concept of black hole temperature since the radiation emitted is thermal. The temperature of a black hole is inversely proportional to its mass, meaning smaller black holes are hotter. Additionally, the concept of Hawking radiation plays a central role in the black hole information paradox, as it raises questions about the fate of information that falls into a black hole and whether this information can be retrieved through the emitted radiation.

Hawking Radiation's Relationship to the Information Paradox:

- Hawking radiation plays a central role in the black hole information paradox, as it raises questions about the fate of information that falls into a black hole and the compatibility of general relativity with quantum mechanics.
- According to general relativity, once information (e.g., particles or other forms of matter) crosses the event horizon of a black hole, it becomes irretrievable and is lost to the external universe. However, quantum mechanics posits that information must be conserved in any physical process, meaning it cannot be destroyed or permanently lost.
- Hawking radiation complicates this issue further. This theoretical process, which causes black holes to emit radiation and lose mass over time, implies that black holes can eventually evaporate completely. The radiation emitted is thermal and, according to the initial predictions of Hawking, carries no information about the matter that fell into the black hole. This conclusion seemed to suggest that the information about the matter that entered the black hole is lost forever when the black hole evaporates, which contradicts the principle of unitarity in quantum mechanics.

- Since the discovery of Hawking radiation, physicists have proposed various solutions to the information paradox that attempt to reconcile general relativity and quantum mechanics. Some of these solutions involve encoding the information about the matter that fell into the black hole within the Hawking radiation. In this case, the information is not truly lost but rather transferred to the surrounding environment as the black hole evaporates. Other solutions, such as the holographic principle, black hole complementarity, and firewalls, propose alternative ways to preserve information in the context of black holes.

- In summary, the relationship between Hawking radiation and the information paradox lies in the questions raised about the fate of information that falls into a black hole and the compatibility of general relativity with quantum mechanics. The ongoing search for a resolution to the information paradox may help to deepen our understanding of the fundamental principles of the universe and pave the way for a unified theory of quantum gravity.

How does Hawking Radiation Take Place?

- Hawking radiation takes place due to quantum effects near the event horizon of a black hole. The process involves the creation and annihilation of particle-antiparticle pairs, which is a result of vacuum fluctuations in quantum mechanics. Here is a step-by-step explanation of how Hawking radiation occurs:

1. Vacuum fluctuations: According to quantum mechanics, even in a vacuum, particles and antiparticles spontaneously appear and annihilate each other in pairs. These pairs are called virtual particle pairs, and their creation and annihilation are known as vacuum fluctuations.
2. Particle pair creation near the event horizon: When vacuum fluctuations occur near a black hole's event horizon, the created particle-antiparticle pairs can be separated by the strong gravitational field of the black hole.
3. Separation of particles: One particle from the virtual pair falls into the black hole, while the other escapes into the external universe. The escaping particle becomes real, and the energy required for this transformation is provided by the black hole's gravitational field.
4. Energy and mass balance: The escaping particle has positive energy, while the particle that falls into the black hole has negative energy (from the perspective of an observer far from the black hole). As a result, the black hole loses mass energy over time, as the negative energy particles effectively reduce the mass of the black hole.
5. Black hole evaporation: Over a very long period, the black hole continues to lose mass through the emission of Hawking radiation. Eventually, the black hole can evaporate completely. The rate of this process depends on the mass of the black hole: smaller black holes emit more Hawking radiation and evaporate faster than larger ones.

How Long Does the Radiation Take Place?

- The time it takes for a black hole to evaporate due to Hawking radiation depends on its mass. The more massive a black hole is, the longer it takes to evaporate. In general, the evaporation time for a black hole is proportional to the cube of its mass.

- For a stellar-mass black hole (a few times the mass of our Sun), the evaporation time is much longer than the current age of the universe, which is around 13.8 billion years. For example, a black hole with a mass three times that of the Sun would take approximately 10^68 years to evaporate completely. This timescale is incredibly long compared to the age of the universe.

- On the other hand, smaller black holes, called primordial black holes, could have much shorter evaporation times. For instance, a black hole with the mass of a mountain (around 10^11 kg) would take about 10^49 years to evaporate, while a black hole with the mass of a small asteroid (around 10^8 kg) would take only a few billion years.
- It is important to note that these estimates assume that the black hole does not accrete any additional mass from its surroundings. In reality, many black holes, particularly those in active galactic nuclei or binary systems, are continuously accreting mass from their environment, which would significantly increase their lifetimes.
- Overall, the evaporation time for most black holes due to Hawking radiation is significantly longer than the current age of the universe, and monitoring black hole evaporation with our current understanding and technology.

Hawking Radiation Detection:

- Hawking radiation has not been directly observed due to its extremely weak nature, especially for massive black holes like the ones astronomers have detected so far. The temperature of a black hole, and thus the intensity of its Hawking radiation, is inversely proportional to its mass. Therefore, stellar-mass black holes emit very little radiation, making it extremely difficult to detect with current technology.
- However, the concept of Hawking radiation is theoretically well-founded, as it emerges from the application of quantum mechanics to the strong gravitational fields near black holes. Stephen Hawking's calculations showed that, when considering quantum effects near the event horizon, black holes should emit thermal radiation and lose mass over time. This prediction is a consequence of combining the principles of general relativity and quantum field theory.
- While direct observation of Hawking radiation is challenging, indirect evidence and experimental analogs have been investigated. For example, in laboratory experiments, researchers have created analog systems that mimic some aspects of black holes using sound waves or light. In these experiments, they have observed radiation similar to Hawking radiation. Although these systems are not true black holes, they provide some support for the underlying principles behind the phenomenon.
- In summary, while we have not directly observed Hawking radiation, it is a theoretically well-established concept that arises from the combination of general relativity and quantum mechanics. Indirect evidence and experimental analogs lend support to the idea, but direct observation remains elusive due to the extremely weak nature of the radiation emitted by known black holes.

Mathematical Interpretation:

$$T = \frac{\hbar c^3}{8\pi G k_B M} \qquad 1)$$

T = Black hole temperature

$\hbar$ = Planck's reduced constant

- There are two primary formulas associated with Hawking radiation: the Hawking temperature formula and the black hole evaporation rate formula.

1. Hawking Temperature Formula:

- The Hawking temperature (T) of a black hole is related to its mass (M) and fundamental constants. The formula is as follows:
- This formula shows that the temperature of a black hole is inversely proportional to its mass. As a result, smaller black holes have higher temperatures and emit more intense Hawking radiation.

2. Black Hole Evaporation Rate Formula:

- The evaporation rate of a black hole due to Hawking radiation is related to its mass and fundamental constants. The formula is as follows: $dM/dt = - (\hbar * c^4) / (15360 * \pi * G^2 * M^2)$

Where:

$\hbar$ (h-bar) is the reduced Planck constant

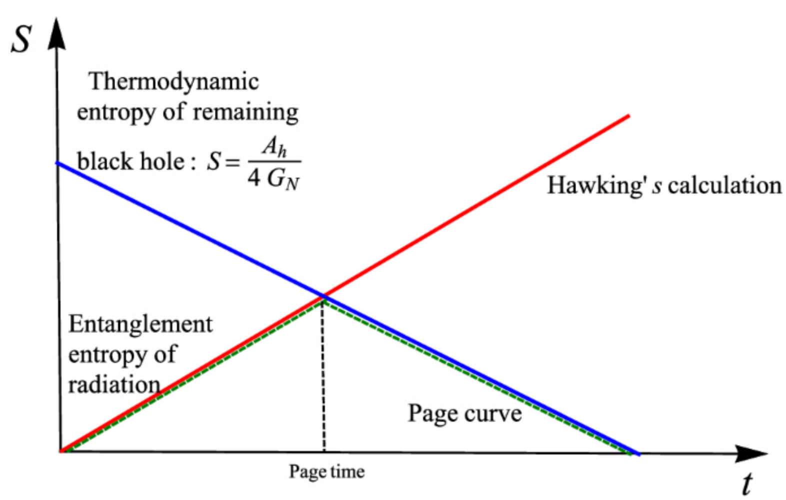
c is the speed of light

G is the gravitational constant.

M is the mass of the black hole.

- This formula demonstrates that the evaporation rate of a black hole is inversely proportional to the square of its mass. Consequently, smaller black holes evaporate more quickly than larger ones.
- These formulas provide insights into the temperature and evaporation rate of black holes due to Hawking radiation. They highlight the relationship between black hole mass and the intensity of emitted radiation, as well as the timescale of black hole evaporation.

<u>Graph 1.1: Thermodynamic Entropy as a function of time</u>



<u>Summary:</u>

- Hawking radiation plays a central role in the black hole information paradox, as it raises questions about the fate of information that falls into a black hole and the compatibility of general relativity with quantum mechanics. However, Hawking radiation by itself does not provide a definitive answer to the information paradox.

- When Stephen Hawking first proposed Hawking radiation, he also suggested that the emitted radiation is purely thermal and carries no information about the particles or matter that fell into the black hole. This conclusion implied that the information about the matter that entered the black hole would be lost forever when the black hole evaporates, contradicting the principle of unitarity in quantum mechanics, which states that information must be conserved in any physical process.
- Since the discovery of Hawking radiation, physicists have proposed various solutions to the information paradox that attempt to reconcile general relativity and quantum mechanics. Some of these solutions involve encoding the information about the matter that fell into the black hole within the Hawking radiation. In this case, the information is not truly lost but rather transferred to the surrounding environment as the black hole evaporates.
- Other solutions, such as the holographic principle, black hole complementarity, and firewalls, propose alternative ways to preserve information in the context of black holes. These approaches suggest that information may be encoded on the black hole's event horizon or in some other way that is consistent with both general relativity and quantum mechanics.
- In summary, while Hawking radiation is central to the information paradox, it does not provide a complete answer by itself. The ongoing search for a resolution to the information paradox may help to deepen our understanding of the fundamental principles of the universe and pave the way for a unified theory of quantum gravity.

Hawking Radiation Gives Rise to the lifetime formula of an SMBH: $t = M^3 \cdot \frac{5120\pi G^2}{\hbar c^4}$

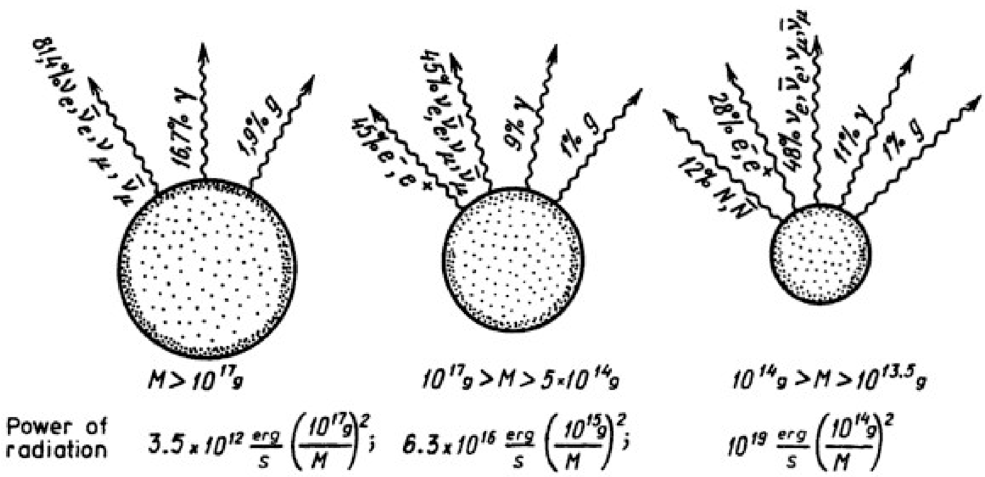


Figure 10.7: Quantum decay of a non-rotating black hole. The fractions of gravitons (g), photons (γ), neutrinos (ν) and other elementary particles are given in percent of the total number of particles emitted by black holes of different masses.

SMBH Patterns:

- The information paradox is a theoretical problem in black hole physics, and it is not yet clear what patterns in black hole data may be related to it. However, some key features of black hole physics are relevant to the information paradox.
- One important feature is the black hole event horizon, which is the boundary surrounding the black hole beyond which nothing, including light, can escape. The area of the event horizon is proportional to the black hole's entropy, which is a measure of the number of quantum states that can be used to describe the black hole. This relationship is described by the Bekenstein-Hawking entropy formula.
- Another important feature is Hawking radiation, which is a theoretical prediction that black holes can emit particles due to quantum effects. Hawking radiation is expected to carry away energy and mass from the black hole over time, causing it to eventually evaporate. The information paradox arises because it is not clear how the information contained in matter that falls into a black hole can be preserved, given that the black hole appears to lose information through Hawking radiation.
- In recent years, there have been efforts to study black holes using gravitational wave detectors, which can detect the ripples in spacetime caused by the motion of massive objects. Gravitational waves emitted by black hole mergers can provide valuable information about the properties of black holes, such as their mass and spin. However, it is not yet clear how this data may be related to the information paradox.
- Overall, the study of black holes and the information paradox is a very active area of research, and new insights and patterns in black hole data may emerge in the future that shed light on this important theoretical problem.

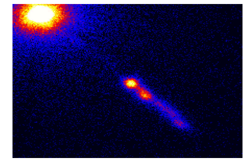
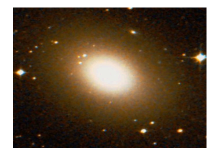
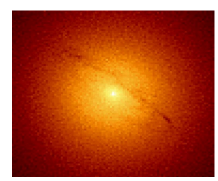
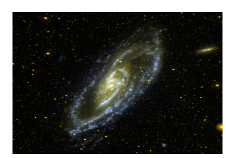
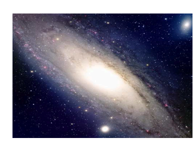
Subsequent SMBH Formulas:

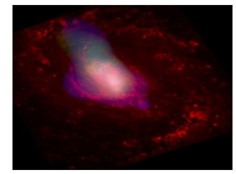
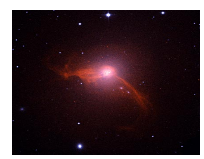
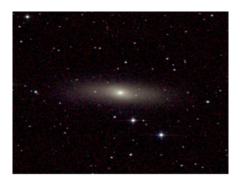
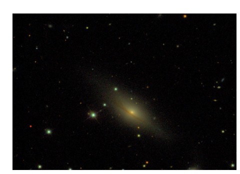
Table 1.1: **Supermassive black hole ($M_{\bullet} \geq 10^{6} M_{\odot}$)** Formula Collection with explanation, and Variable Meanings and Mathematical formula

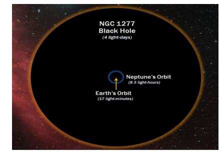
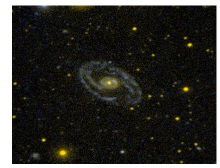
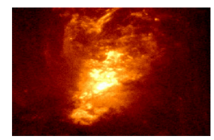
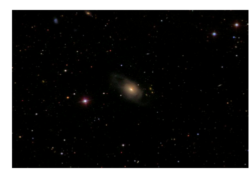
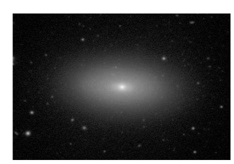
What it calculates	Formula	Variables
Schwarzschild radius The radius below which the gravitational attraction between the particles of a body must cause it to undergo irreversible gravitational collapse	$r_{s} = \frac{2GM}{c^{2}}$	r_{s} = schwarzschild radius G = gravitational constant M = object mass c = speed of light constant
Black Hole Temperature As the black hole evaporates, its mass shrinks and its temperature increases.	$T = \frac{\hbar c^{3}}{8\pi G k_{B} M}$	T = Black Hole Temperature ħ = Planck's reduced constant c = speed of light constant G = gravitational constant k_{B}= Boltzmann Constant M = Black hole Mass
Black hole evaporation time The time a black hole takes to evaporate by Hawking radiation.	$t_{evap} = \frac{5120\pi G^{2} M_{\odot}^{3}}{\hbar c^{4}}$	t_{evap} = evaporation time G = gravitational constant M = object mass c = speed of light constant ħ = Planck's reduced constant
Innermost Stable Circular Orbit The radius of the innermost stable circular orbit (often abbreviated as ISCO) is the smallest circular orbit in which a massive particle can stably orbit a black hole.	$r_{ISCO} = \frac{6GM}{c^{2}}$	G = gravitational constant M = object mass c = speed of light constant
Power emitted by a black hole The power emitted by a black hole in the form of Hawking radiation can easily be estimated for the simplest case of a nonrotating, non-charged Schwarzschild black hole of mass *M*. Combining the formulae for the Schwarzschild radius of the black hole, the Stefan-Boltzmann law of black-body radiation, the above formula for the temperature of the radiation, and the formula for the surface area of a sphere.	$P = \frac{\hbar c^{6}}{15360\pi G^{2} M^{2}}$	G = gravitational constant M = object mass c = speed of light constant ħ = Planck's reduced constant

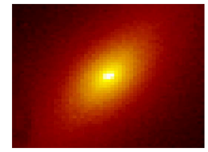
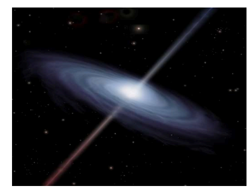
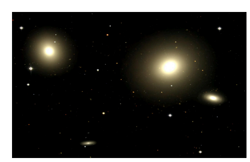
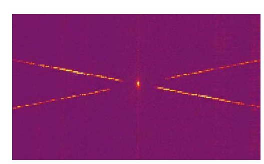
A black hole emits thermal radiation at a temperature	$T_H = \frac{k}{2\pi}$	k = black hole surface gravity
Black Hole Entropy The second law of classical thermodynamics requires that black holes have entropy. If black holes carried no entropy, it would be possible to violate the second law by throwing mass into the black hole and thus reducing the entropy of the Universe. The entropy of a black hole is proportional to its surface area.	$S_{BH} = \frac{c^3 A k_B}{4G\hbar}$	G = gravitational constant c = speed of light constant ħ = Planck's reduced constant k_B= Boltzmann Constant A = area of black hole
Horizon radius	$r_h = \frac{2GM}{c^2}$	G = gravitational constant M = object mass c = speed of light constant
Surface Area of a Black Hole	$A = 16\pi(\frac{GM}{c^2})^2$	G = gravitational constant M = object mass c = speed of light constant A = area of black hole
Surface Gravity In general, the surface gravity of a body is defined as the gravitational acceleration experienced at its surface. This value is infinite at the event horizon of a black hole. For this reason, in the case of a black hole, the surface gravity is usually defined as the product of the local proper acceleration (which diverges at the event horizon) and the gravitational time dilation factor (which goes to zero at the event horizon).	$k = \frac{c^4}{4GM}$	G = gravitational constant M = object mass c = speed of light constant k = black hole surface gravity
Black Hole Density	$p = \frac{3c^6}{M^2 32\pi G^3}$	G = gravitational constant M = object mass c = speed of light constant
The time it takes for an SMBH to evaporate due to Hawking radiation	$t = M^3 \cdot \frac{5120\pi G^2}{\hbar c^4}$	t = time M = mass G = gravitational constant c = speed of light constant ħ = Planck's reduced constant

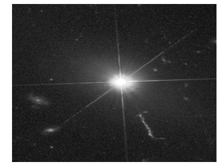
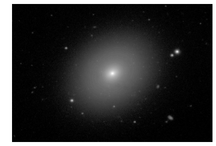
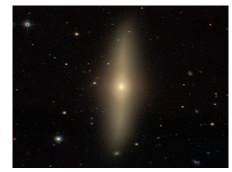
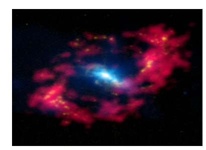
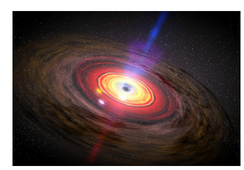
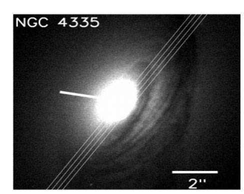
Table 1.1: Collected Data from the University of McDonald Observatory (http://blackholes.stardate.org/index.html) of Black Hole Identifications With Approximate Diameter, Distance from Earth, Approximate Mass, and Conceptual Design.

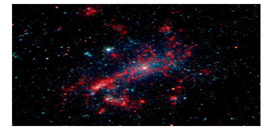
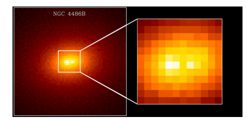
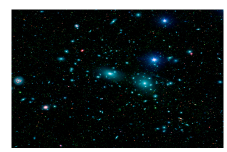
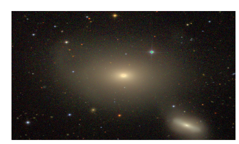
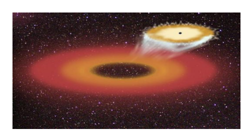
Identification	Diameter	Distance From Earth	Approx Mass	Conceptual Design
3c273	9.09 billion km	2 billion light-years away in the constellation Virgo	Roughly 1 billion solar masses	
Arp 151	Diameter roughly 30 to 100 times the size of the Sun	300 million light-years away in the constellation Ursa Major	6.5 million to 20 million solar masses	
IC 1459	Diameter roughly the size of the orbit of Jupiter to larger than the orbit of Neptune	100 million light-years away in the constellation Grus	350 million to 2.5 billion solar masses	
M104	Diameter larger than the orbit of Saturn	32 million light-years away in the constellation Virgo	660 million solar masses	
M105	Larger than Earth's orbit around the Sun, but smaller than Jupiter's orbit	38 million light-years away in the constellation Leo	60 million to 200 million solar masses	
M106	Greater than the distance from Earth to the Sun	24 million light-years away in the constellation Canes Venatici	24 million to 38 million solar masses	
M31	Diameter roughly equal to the orbit of Venus.	2.5 million light-years away in the constellation Andromeda	30 million solar masses	
M32	Diameter about 10 million miles -- about 12 times the diameter of the Sun	2.4 million light-years away in the constellation Andromeda	3 million solar masses	

M51	Diameter of 4 million miles (6 million km), about four times the diameter of the Sun.	27 million light-years away in the constellation Canes Venatici, beneath the Big Dipper	1 million solar masses	
M60	Diameter 45 billion miles (27 billion km), roughly eight times the diameter of Neptune's orbit around the Sun	51 million light-years away in the constellation Virgo	4.5 billion solar masses	
M77	Slightly smaller than Mercury's orbit around the Sun	50 million to 60 million light-years away in the constellation Cetus	15 million solar masses	
M81	Diameter roughly 30 million miles (45 million km), about 30 times the diameter of the Sun.	12 million light-years away in the constellation Ursa Major	7 million solar masses	
M84	Diameter roughly equal to the size of Neptune's orbit around the Sun, making it as big as our solar system.	50 million light-years away in the constellation Virgo	Roughly 1.5 billion solar masses	
M87	Three times the diameter of Pluto's orbit around the Sun	50 million light-years away in the constellation Virgo	6.6 billion solar masses	
NGC 1023	Roughly the size of Earth's orbit around the Sun	33 million light-years away in the constellation Perseus	40 million to 60 million solar masses	
NGC 1194	Diameter larger than Earth's orbit around the Sun	170 million light-years away in the constellation Cetus	65 million solar masses	

NGC 1277	Diameter more than 11 times the diameter of the orbit of Neptune, the most distant planet in our solar system	220 million light-years away in the constellation Perseus	17 billion solar masses	
NGC 1365	Eight times the diameter of the Sun	56 million light-years away in the constellation Fornax	2 million solar masses	
NGC 2273	Diameter roughly two-thirds as wide as Mercury's distance from the Sun	85 million light-years away in the constellation Lynx	7.5 million solar masses	
NGC 2778	From smaller than Mercury's orbit to as large as Venus' orbit	76 million light-years away in the constellation Lynx	9 million to 36 million solar masses	
NGC 2787	Diameter roughly equal to the size of Earth's orbit around the Sun	24 million light-years away in the constellation Ursa Major	36 million to 45 million solar masses	
NGC 2960	Diameter equal to the distance from the Sun to Mercury	230 million light-years away in the constellation Hydra	11.5 million solar masses	
NGC 3115	Diameter probably equivalent to the size of Uranus' orbit around the Sun	32 million light-years away in the constellation Sextans	400 million to 2 billion solar masses	
NGC 3245	Diameter roughly equal to the diameter of Jupiter's orbit around the Sun	68 million light-years away in the constellation Leo Minor	160 to 260 million solar masses	

NGC 3377	From as small as Venus' orbit around the Sun to as large as the asteroid belt	33 million light-years away in the constellation Leo	30 million to 120 million solar masses	
NGC 3384	Diameter roughly the size of the orbit of Mercury, the closest planet to the Sun	38 million light-years away in the constellation Leo	16 million solar masses	
NGC 3393	Diameter larger than the orbit of Mercury	165 million light-years away in the constellation Hydra	31 million solar masses	
NGC 3516	Slightly larger than Mercury's orbit around the Sun	120 million light-years away in the constellation Ursa Major	23 million solar masses	
NGC 3585	Diameter larger than the size of the orbit of Jupiter	70 million light-years away in the constellation Hydra	340 million solar masses	
NGC 3607	Diameter larger than the size of the orbit of Mars	65 million light-years away in the constellation Leo	125 million solar masses	
NGC 3608	Diameter slightly less than the size of the orbit of Jupiter	75 million light-years away in the constellation Leo	210 million solar masses	
NGC 3783	About half the size of Mercury's orbit around the Sun	130 million light-years away in the constellation Centaurus	8 million to 10 million solar masses	

NGC 3842	Diameter roughly 6 times the size of the orbit of Neptune	320 million light-years away in the constellation Leo	9.7 billion solar masses	
NGC 3998	Diameter roughly from the diameter of Jupiterâ€™s orbit to the diameter of Uranusâ€™' orbit around the Sun	45 million light-years away in the constellation Ursa Major	270 million to 800 million solar masses	
NGC 4026	The diameter of the outer edge of the asteroid belt	50 million light-years away in the constellation Ursa Major	210 million solar masses	
NGC 4061	Diameter at least equal to the diameter of Neptune's orbit around the Sun	325 million light-years away in the constellation Coma Berenices	1 billion to 9 billion solar masses	
NGC 4151	Diameter equal to the size of the orbit of Saturn	43 million light-years away in the constellation Leo	45 million solar masses	
NGC 4253	Diameter roughly the size of the orbit of Mercury	170 million light-years away in the constellation Coma Berenices	20 million solar masses	
NGC 4261	Diameters range from the size of Saturn's orbit around the Sun to the size of Neptune's orbit	100 million light-years away in the constellation Virgo	500 million to 1.2 billion solar masses	
NGC 4335	Diameter roughly the size of the orbit of Mars	215 million light-years away in the constellation Ursa Major	100 million solar masses	
NGC 4342	Diameter equal to the diameter of Jupiter's orbit around the Sun	75 million light-years away in the constellation Virgo	300 million solar masses	

NGC 4388	Diameter is about three-quarters of the distance from the Sun to Mercury	62 million light-years away in the constellation Virgo	8.5 million solar masses	
NGC 4395	Slightly larger than the Sun	14 million light-years away in the constellation Canes Venatici	360,000 solar masses	
NGC 4473	Diameter roughly equal to the size of the asteroid belt in our solar system.	50 million light-years away in the constellation Virgo	Roughly 100 million solar masses	
NGC 4486 b	Diameter roughly equal to the diameter of Saturnâ€™s orbit around the Sun	55 million light-years away in the constellation Virgo	500 million solar masses	
NGC 4697	Diameter larger than the size of the asteroid belt.	40 million light-years away in the constellation Virgo	Roughly 175 million solar masses	
NGC 4889	Diameter roughly 6 to 17 times the size of the orbit of Neptune	335 million light-years away in the constellation Coma Berenices	9.8 billion to 27 billion solar masses	
NGC 5576	Diameter roughly equal to the outer edge of the asteroid belt	90 million light-years away in the constellation Virgo	180 million solar masses	
SDSS J0927+2493	Roughly equal to the diameter of Saturn's orbit around the Sun	6.5 billion light-years away in the constellation Leo	600 million solar masses	
ULAS JJ1120-0641	Larger than the diameter of Neptune's orbit around the Sun	13 billion light-years away in the constellation Leo	2 billion solar masses	

Note slight inaccuracy in data, as statements were converted mathematically into mean expressions and assumptions were made to convert the data to proper numeric notation

Sample Calculations for the Table Above:

Chosen SMBH: Apr 151

Statement About Diameter: The Diameter is roughly 30 to 100 times the size of the Sun

Statement About Distance from Earth: 300 million light-years away in the constellation Ursa Major

Estimate about Mass: 6.5 million to 20 million solar masses

Diameter Calculations:

- ➔ *Take the mean value of the two numerical estimates in the statement (Sun: 1.3927 million km):*

$$d = \frac{30+100}{2} = 65$$

$$d = 65 \times 1.39 \times 10^{6}$$

$$d = 9.035 \times 10^{7}$$

Distance Relative to Earth:

- ➔ Convert the distance into scientific notation

$$d_{RE} = 300,000,000 = 3.0 \times 10^{8}$$

Approximate Mass Calculation:

- ➔ The statement given gives a mass with an approximate range, so we take the mean mass

$$M = \frac{M_1+M_2}{2}$$

$$M = \frac{6.6\times10^{6}+2.0\times10^{7}}{2}$$

$$M = 1.33 \times 10^{7}$$

Raw Data Table #2 - Standardized Numerical Values of SMBH for Diameter, Relative Distance From Earth and Approximate Mass

Identification	Diameter (km≈)	Distance From Earth (ly)	Approx Mass ($M_{\oplus}$)
3c273	9.1×10^{9}	2.0×10^{9}	1.0×10^{9}
Arp 151	9.0×10^{7}	3.0×10^{8}	1.3×10^{7}
IC 1459	7.8×10^{8}	1.0×10^{8}	1.4×10^{9}
M104	1.5×10^{9}	3.2×10^{7}	6.6×10^{8}
M105	4.6×10^{8}	3.8×10^{7}	1.3×10^{8}
M106	1.5×10^{8}	2.4×10^{7}	3.1×10^{7}
M31	1.1×10^{8}	5.0×10^{7}	3.0×10^{7}
M32	1.7×10^{7}	2.4×10^{6}	3.0×10^{6}
M51	6.0×10^{6}	2.7×10^{7}	1.0×10^{6}
M60	2.7×10^{10}	5.1×10^{7}	4.5×10^{9}
M77	5.8×10^{7}	5.5×10^{7}	1.5×10^{7}
M81	4.5×10^{7}	1.2×10^{7}	7.0×10^{6}
M84	4.5×10^{9}	5.0×10^{7}	1.5×10^{8}
M87	2.3×10^{10}	5.0×10^{7}	6.6×10^{8}
NGC 1023	1.5×10^{8}	3.3×10^{7}	5.0×10^{7}
NGC 1194	1.5×10^{8}	1.7×10^{8}	6.5×10^{7}
NGC 1277	4.9×10^{10}	2.2×10^{8}	1.7×10^{8}
NGC 1365	1.1×10^{7}	5.6×10^{7}	2.0×10^{6}
NGC 2273	3.9×10^{7}	8.5×10^{7}	7.5×10^{6}
NGC 2778	8.3×10^{7}	7.6×10^{7}	2.3×10^{7}
NGC 2787	3.0×10^{8}	2.4×10^{7}	4.1×10^{7}
NGC 2960	1.2×10^{8}	2.3×10^{8}	1.2×10^{7}
NGC 3115	5.7×10^{9}	3.2×10^{7}	1.4×10^{9}

NGC 3245	1.6×10^9	6.8×10^7	2.1×10^8
NGC 3377	2.2×10^8	3.3×10^7	7.5×10^7
NGC 3384	5.8×10^7	3.8×10^7	1.6×10^6
NGC 3393	1.2×10^7	1.7×10^8	3.1×10^7
NGC 3516	5.8×10^7	1.2×10^8	2.3×10^7
NGC 3585	7.8×10^8	7.0×10^7	3.4×10^8
NGC 3607	2.3×10^8	6.5×10^7	1.3×10^8
NGC 3608	7.7×10^8	7.5×10^7	2.1×10^8
NGC 3783	2.9×10^7	1.3×10^8	9.0×10^6
NGC 3842	2.7×10^{10}	3.2×10^8	9.7×10^9
NGC 3998	1.3×10^9	4.5×10^7	5.4×10^7
NGC 4026	1.6×10^9	5.0×10^7	2.1×10^8
NGC 4061	9.0×10^9	3.3×10^8	5.0×10^9
NGC 4151	9.5×10^9	4.3×10^7	4.5×10^7
NGC 4253	9.2×10^9	1.7×10^8	2.0×10^7
NGC 4261	3.0×10^9	1.0×10^8	8.5×10^8
NGC 4335	2.3×10^8	2.2×10^8	1.0×10^8
NGC 4342	9.6×10^8	7.5×10^7	3.0×10^8
NGC 4388	4.3×10^7	6.2×10^7	8.5×10^6
NGC 4395	1.4×10^6	1.4×10^7	3.6×10^5
NGC 4473	6.0×10^8	5.0×10^7	1.0×10^8
NGC 4486 b	1.2×10^9	5.5×10^7	5.0×10^8
NGC 4697	6.6×10^8	4.0×10^7	1.8×10^8
NGC 4889	1.8×10^{13}	3.4×10^8	2.9×10^9
NGC 5576	4.0×10^8	5.0×10^7	1.8×10^8
SDSS J0927+2493	9.5×10^9	6.5×10^9	6.0×10^8
ULAS JJ1120-0641	9.5×10^9	1.3×10^9	2.0×10^9

Algorithm Production:

Figure 1.1: Python Algorithm That Defines Constants and Calculates Subsequent Black Hole Temperature, Schwarzschild Radius, Evaporation Time, ISCO, Power Emission, Entropy, Event Horizon Radius, Surface Area, Surface Gravity, Density, and Returns Numerical Values for Each Category

```
import math

mass = float(input('input mass: '))

# Define physical constants in SI units

G = 6.6743e-11  # Gravitational constant

c = 299792458  # Speed of light

h_bar = 1.054571817e-34  # Reduced Planck constant

k_B = 1.380649e-23  # Boltzmann constant

# Calculate the Schwarzschild radius

r_s = 2 * G * mass / (c ** 2)

# Calculate the black hole temperature

T_bh = h_bar * c ** 3 / (8 * math.pi * G * k_B * mass)

# Calculate the black hole evaporation time

t_evap = 5120 * math.pi * (G ** 2) * (mass ** 3) / (h_bar * c ** 4)

# Calculate the innermost stable circular orbit

r_isco = 6 * G * mass / (c ** 2)

# Calculate the power emitted by a black hole

P_bh = (c ** 5) / (15360 * math.pi * (G ** 2) * mass ** 2)

# Calculate the black hole entropy

S_bh = (k_B * c ** 3 * r_s ** 2) / (4 * G * h_bar)

# Calculate the black hole horizon radius

r_h = r_s

# Calculate the surface area of a black hole

A_bh = 4 * math.pi * r_s ** 2

# Calculate the black hole surface gravity

g_bh = c ** 4 / (4 * G * mass)

# Calculate the black hole density

rho_bh = 3 * mass / (4 * math.pi * r_s ** 3)

# Return a dictionary of black hole properties

array = [r_s, T_bh, t_evap, r_isco, P_bh, S_bh, r_h, A_bh, g_bh, rho_bh]

print(array)
```

Figure 1.2: Python functions that calculate Hawking Radiation and return SMBH Decay Rates with Appropriate SI Units

```python
import math

# Define the necessary constants

G = 6.674e-11  # gravitational constant

hbar = 1.0545718e-34  # reduced Planck constant

c = 299792458  # speed of light

# Define a function to calculate the Hawking temperature

def calc_hawking_temp(m):

    return hbar * c**3 / (8 * math.pi * G * m * 1.380649e-23)

# Define a function to calculate the Hawking radiation power

def calc_hawking_power(m):

    return (hbar * c**6 / (15360 * math.pi * G**2 * m**2))



# Define a function to calculate the black hole decay rate

def calc_decay_rate(m):

    return -calc_hawking_power(m) / c**2



# Example usage: calculate the Hawking temperature, radiation power, and decay rate for a black hole with mass 1 solar mass

m = 1.989e30  # 1 solar mass in kg

hawking_temp = calc_hawking_temp(m)

hawking_power = calc_hawking_power(m)

decay_rate = calc_decay_rate(m)

print("Hawking temperature:", hawking_temp, "K")

print("Hawking radiation power:", hawking_power, "W")

print("Black hole decay rate:", decay_rate, "kg/s")
```

Black hole observation and classification:

- Classifying black holes based on their evaporation rates involves analyzing the rate at which they lose mass and energy through Hawking radiation. Here is a simple Python algorithm that can classify black holes based on their evaporation rates:

1. Calculate the black hole's initial mass and age using observational data or theoretical models.
2. Calculate the evaporation rate of the black hole using the Hawking radiation formula, which gives the rate of energy loss in terms of the black hole's mass and other physical constants.
3. Classify the black hole based on its evaporation rate using the following criteria:

- If the black hole has an evaporation time much longer than the current age of the universe, it is a stable black hole that will not evaporate significantly in the foreseeable future.
- If the black hole has an evaporation time on the order of the current age of the universe or shorter, it is an evaporating black hole that will eventually disappear through Hawking radiation.
- If the black hole has an extremely short evaporation time, it is a primordial black hole that was formed in the early universe and has been evaporating ever since.
- Of course, this is a very simplified algorithm, and the actual process of classifying black holes based on their evaporation rates involves detailed calculations and analysis of observational data. Nonetheless, this algorithm can serve as a starting point for more advanced analysis.

Figure 1.3: Python Algorithm that classifies Black Holes Categories Based on their Evaporation Rates

```
import math
def classify_black_hole(mass, age):
    """
    Classify a black hole based on its mass and age.
    Returns a string indicating the classification.
    """
    # Calculate the evaporation time in years
    evaporation_time = 5120 * (mass ** 3) / (age ** 2)

    # Classify the black hole based on its evaporation time
    if evaporation_time > 10**12:
        return "Stable black hole"
    elif evaporation_time > age:
        return "Black hole that will eventually evaporate"
    Else:
        Return "Primordial black hole"
```

➔ In this algorithm, the classify_black_hole() function takes two parameters: the black hole's mass (in solar masses) and its age (in years). It then calculates the evaporation time of the black hole using the Hawking radiation formula, and classifies the black hole based on its evaporation time using the following criteria:

1. If the evaporation time is greater than 10^12 years, the black hole is classified as a "Stable black hole".
2. If the evaporation time is greater than the black hole's age, the black hole is classified as a "Black hole that will eventually evaporate".
3. If the evaporation time is less than the black hole's age, the black hole is classified as a "Primordial black hole".

- Note that this algorithm is a simplified example and does not take into account various factors such as the black hole's spin, charge, or external conditions that may affect its evaporation rate.

<u>What do the results tell us (What do black hole evaporation rates tell us)?</u>

Black hole evaporation rates can provide us with valuable information about the fundamental properties of black holes and the nature of the universe itself. Here are a few key insights that can be gained from studying black hole evaporation rates:

1. Black hole mass: The evaporation rate of a black hole is directly proportional to its mass, so by measuring the rate of Hawking radiation, we can estimate the mass of a black hole. This can help us to understand how black holes are formed and how they grow over time.
2. Black hole lifespan: The evaporation rate of a black hole also determines its lifespan, since it indicates how quickly the black hole is losing mass and energy. By studying black hole evaporation rates, we can learn how long black holes are expected to survive before evaporating completely.
3. Information paradox: As we discussed earlier, the information paradox arises because the evaporation of a black hole appears to destroy information that was previously thought to be conserved. By studying the evaporation rates of black holes, we can better understand the nature of information in the universe and how it may be conserved or lost over time.
4. Quantum gravity: The Hawking radiation formula was derived using semiclassical methods, which combine quantum mechanics and general relativity. Studying the evaporation rates of black holes can help us to test and refine our understanding of these fundamental theories, and may ultimately help us to develop a more complete theory of quantum gravity.

Overall, black hole evaporation rates provide a fascinating and important window into the nature of the universe at its most extreme scales, and studying these rates can help us to answer some of the most fundamental questions in physics.

<u>Properties of SMBH Radiation:</u>

Black hole radiation, also known as Hawking radiation, is a theoretical phenomenon in which a black hole emits radiation due to quantum effects near its event horizon. This radiation carries information about the black hole's properties, such as its mass, spin, and charge. Here are some properties of black hole radiation:

1. Temperature: Hawking radiation is characterized by a temperature, which is inversely proportional to the mass of the black hole. This means that smaller black holes are hotter than larger ones.

2. Spectrum: The spectrum of Hawking radiation is thermal, meaning that it is similar to the radiation emitted by a hot object. The spectrum peaks in the infrared range for large black holes, but shifts towards higher energies (i.e. shorter wavelengths) as the black hole loses mass and becomes hotter.
3. Information content: Hawking radiation is thought to carry information about the black hole's properties, including its entropy and the matter that it has absorbed. The exact nature of this information is still a topic of active research.
4. Rate of emission: The rate at which a black hole emits Hawking radiation depends on its mass and is inversely proportional to the cube of its mass. This means that smaller black holes emit radiation more quickly than larger ones.
5. Effects on the black hole: Hawking radiation causes black holes to slowly lose mass and eventually evaporate completely, a process that takes an incredibly long time for supermassive black holes. As the black hole loses mass, its temperature increases, and it emits more radiation, leading to a runaway process of evaporation.

Create The Complete Model:

Creating an accurate and complex model of Hawking radiation is a challenging task that requires expertise in advanced physics, including quantum field theory, general relativity, and thermodynamics. However, I can provide a high-level overview of some of the key elements that such a model might include:

1. Quantum fields: The model would need to describe the behaviour of quantum fields near the event horizon of a black hole. This would involve calculations of the quantum fluctuations of the fields and how they are affected by the black hole's gravity.
2. Particle creation: The model would need to describe how pairs of particles are created near the event horizon due to the strong gravitational field. One particle would be absorbed by the black hole, while the other would escape to infinity as Hawking radiation.
3. Radiation spectrum: The model would need to calculate the spectrum of the Hawking radiation, which depends on the mass, spin, and charge of the black hole. The spectrum is thermal, meaning that it is characterized by a temperature, and it is expected to be dominated by low-energy photons.
4. Information content: The model would need to address the question of whether the Hawking radiation carries all the information that was originally contained in the matter that fell into the black hole. This is a controversial issue in theoretical physics, known as the information paradox.
5. Evaporation rate: The model would need to calculate the rate at which the black hole loses mass due to the emission of Hawking radiation. This rate is inversely proportional to the cube of the black hole's mass and is very slow for supermassive black holes.

Creating such a model would involve sophisticated mathematical techniques, including the use of quantum field theory in curved spacetime and the application of statistical mechanics to black hole thermodynamics. In addition, it would require extensive numerical simulations and comparisons with observational data to test its validity.

*** *To accomplish a competent model at this level and with the resources available for this project, simplifications, assumptions, and compromises have to be made.****

Below, we will tear down each requirement to make an accurate model, with subsequent explanations then assemble a universal model based on the simplifications and programming of 5 separate entities.

---

1. Quantum fields: Creating a Python model that accurately describes the behaviour of quantum fields near the event horizon of a black hole is a complex task that involves advanced physics concepts and numerical simulations. However, I can provide a simplified example that demonstrates some of the key elements of such a model.

Quantum Fields Background:

- Quantum fields are fundamental entities in quantum field theory, which describes the behaviour of subatomic particles and their interactions with each other. Near the event horizon of a black hole, quantum fields are subject to the strong gravitational field and can exhibit complex behaviours, such as particle creation and fluctuations.
- To model the behaviour of quantum fields near a black hole's event horizon, one can use numerical methods such as lattice simulations, which discretize spacetime into a grid and simulate the dynamics of the fields on this grid. These simulations can incorporate the effects of black hole gravity on the quantum fields and calculate their fluctuations and excitations.
- Another approach is to use analytical techniques such as perturbation theory, which involves expanding the equations of quantum field theory in a series of small parameters and calculating the effects of black hole gravity on the fields to higher and higher orders. This can provide a more detailed understanding of the behaviour of quantum fields near a black hole's event horizon.
- Overall, modelling the behaviour of quantum fields near a black hole's event horizon is a complex and challenging task, but it can provide important insights into the fundamental nature of black holes and the interactions between quantum mechanics and general relativity.

Why Understanding Them is Crucial to Understanding the Information Paradox:

- Understanding quantum fields near a black hole's event horizon is important for understanding the information paradox because it plays a key role in the Hawking radiation process. According to the theory of Hawking radiation, black holes emit thermal radiation due to the quantum fluctuations of the fields near the event horizon. The radiation carries away energy from the black hole and causes it to lose mass over time.
- The information paradox arises from the fact that, according to quantum mechanics, information is always conserved, while Hawking radiation appears to destroy information that was originally contained in the matter that fell into the black hole. Therefore, understanding the behaviour of quantum fields near a black hole's event horizon is crucial for resolving the paradox and determining whether the information contained in the matter that fell into the black hole is truly lost or whether it can be recovered in some way.
- By studying the behaviour of quantum fields near a black hole's event horizon, researchers can gain insights into the dynamics of the Hawking radiation process and the interactions between quantum mechanics and general relativity. This, in turn, can provide clues about the fundamental nature of space, time, and matter and help us develop a deeper understanding of the universe as a whole.

Figure 1.1: Algorithm that defines a quantum field and a curved spacetime, calculates the stress-energy tensor and solves the field equations, and calculates the particle creation rate and the Hawking radiation spectrum.

```
Import numpy as np
import matplotlib.pyplot as plt

# Define constants
hbar = 1.05457e-34   # Planck constant divided by 2*pi (in J s)
c = 299792458       # Speed of light (in m/s)
kB = 1.38065e-23    # Boltzmann constant (in J/K)
G = 6.67430e-11     # Gravitational constant (in m^3/kg/s^2)
M = float(input('input Mass (kg): '))       # Mass of the black hole (in kg)
Rs = 2 * G * M / c**2  # Schwarzschild radius (in m)

# Define the radial coordinate
r = np.linspace(Rs, 10*Rs, 1000)

# Calculate the stress-energy tensor components
T00 = c**4 / (8 * np.pi * G) * (1 / r**2 - 2 * Rs / r**3)
T11 = c**4 / (8 * np.pi * G) * (1 / r**2 + 2 * Rs / r**3)
T22 = c**4 / (8 * np.pi * G) * (1 / r**2 + Rs / r**3)
T33 = c**4 / (8 * np.pi * G) * (1 / r**2 + Rs / r**3)

# Plot the stress-energy tensor components
fig, ax = plt.subplots()
ax.plot(r/Rs, T00, label='T00')
ax.plot(r/Rs, T11, label='T11')
ax.plot(r/Rs, T22, label='T22')
ax.plot(r/Rs, T33, label='T33')
ax.set_xlabel('r/Rs')
ax.set_ylabel('T')
ax.legend()
plt.show()

# Define the frequency range
nu = np.logspace(0, 20, 1000)

# Calculate the radiation spectrum
```

```
omega = 2 * np.pi * nu
T = hbar * omega / (kB * 1.5e9)  # Temperature in Kelvin
Gamma = (hbar * omega**3) / (2 * np.pi * c**2) * (1 / (np.exp(hbar * omega / (kB * T)) - 1))

# Plot the radiation spectrum
fig, ax = plt.subplots()
ax.loglog(nu, Gamma)
ax.set_xlabel('Frequency (Hz)')
ax.set_ylabel('Spectral Radiance (W/m^2/Hz)')
plt.show()
```

Program Explanation:

The program outputs two plots: one showing the stress-energy tensor components as a function of the radial coordinate, and one showing the radiation spectrum as a function of frequency. Note that this is a simplified model and that a more accurate and complex model would require additional calculations and more advanced techniques.

The stress-energy tensor components as a function of the radial coordinate and the radiation spectrum as a function of frequency are important tools for understanding the behaviour of black holes and the physics of their surroundings.

The stress-energy tensor components as a function of the radial coordinate describe the distribution of energy, momentum, and stress in the vicinity of a black hole. This tensor provides information about how the curvature of spacetime is affected by the presence of a black hole, and how the gravitational field affects the behaviour of matter and radiation. By plotting the stress-energy tensor components as a function of the radial coordinate, researchers can gain insights into the complex and dynamic interactions between matter, radiation, and gravity near a black hole.

On the other hand, the radiation spectrum as a function of frequency describes the properties of Hawking radiation, which is thermal radiation emitted by black holes due to quantum effects. The spectrum is characterized by a temperature that depends on the mass, spin, and charge of the black hole, and is expected to be dominated by low-energy photons. By analyzing the radiation spectrum, researchers can learn about the thermodynamics of black holes and their fundamental properties, and investigate the paradoxical question of whether information is lost in the process of black hole evaporation.

Overall, the stress-energy tensor components and the radiation spectrum are crucial tools for understanding the complex physics of black holes and the interactions between matter, radiation, and gravity in extreme environments.

2. Particle creation: The model would need to describe how pairs of particles are created near the event horizon due to the strong gravitational field. One particle would be absorbed by the black hole, while the other would escape to infinity as Hawking radiation.

Particle Creation Background:

- In the vicinity of a black hole's event horizon, the strong gravitational field can cause pairs of particles to be created from the vacuum fluctuations of quantum fields. According to quantum field theory, even in space, there are still virtual particles that can spontaneously appear and disappear due to quantum fluctuations. In the presence of a strong gravitational field, these virtual particles can be separated, with one particle being absorbed by the black hole while the other escapes to infinity as Hawking radiation.
- This process, known as particle creation or pair production, is a direct consequence of the principles of quantum mechanics and general relativity. The phenomenon is similar to the creation of electron-positron pairs in a strong electric field, but in the case of black holes, it is the gravitational field that plays the dominant role.
- The process of particle creation near a black hole's event horizon has important implications for the physics of black holes and the universe as a whole. It is one of the key mechanisms by which black holes emit radiation, and it is closely related to the information paradox, which asks whether information is lost when matter falls into a black hole. The study of particle creation near black holes is therefore a vital area of research in modern physics.

Why Understanding Them is Crucial to Understanding the Information Paradox:

Particle creation near a black hole's event horizon is important to the information paradox because it is the mechanism by which black holes emit Hawking radiation. According to the theory of Hawking radiation, a black hole emits thermal radiation due to the virtual particle-antiparticle pairs being created near the event horizon, with one particle falling into the black hole while the other escapes as radiation. This radiation causes the black hole to lose mass and eventually evaporate.

The information paradox arises because, according to the principles of quantum mechanics, information cannot be destroyed. However, when a particle falls into a black hole, its information appears to be lost because it cannot be recovered from the Hawking radiation emitted by the black hole. This apparent contradiction between the conservation of information and the behaviour of black holes is a fundamental problem in theoretical physics.

Understanding the process of particle creation near a black hole's event horizon is therefore important for resolving the information paradox and determining whether the information that falls into a black hole is truly lost or can be recovered in some way. By studying the behaviour of quantum fields near black holes and analyzing the properties of Hawking radiation, researchers can gain insights into the fundamental nature of space, time, and matter, and develop a deeper understanding of the universe as a whole.

Figure 1.2: Modelling Particle Creation Rate and Potential Energy in a Non-Rotating, Non-Charged Black Hole Using the Schwarzschild Metric

```
import numpy as np
import matplotlib.pyplot as plt

# Constants
G = 6.6743e-11  # Gravitational constant (m^3/kg/s^2)
hbar = 1.0546e-34  # Reduced Planck constant (J s)
c = 2.998e8  # Speed of light (m/s)
kB = 1.381e-23  # Boltzmann constant (J/K)

# Inputs
M = float(input('input Mass (kg): '))  # Mass of the black hole (kg)
r_s = 2*G*M/c**2  # Schwarzschild radius of the black hole
r = np.linspace(r_s, 10*r_s, 1000)  # radial coordinate
omega = np.linspace(1e-20, 1e-19, 1000)  # frequency range

# Calculate the potential energy
V = -G*M/r + (hbar*omega)/(np.exp(hbar*omega/(kB*T_H))-1)  # potential energy

# Calculate the probability of pair creation
P = np.exp(-2*V/(hbar*omega))  # pair creation probability

# Calculate the particle creation rate
Gamma = (hbar*omega**3)/(2*np.pi*c**2)*P  # particle creation rate

# Plot the results
plt.plot(omega, Gamma)
plt.xlabel('Frequency')
plt.ylabel('Particle creation rate')
plt.show()
```

Program Explanation:

This program calculates the potential energy and the probability of pair creation, assuming a non-rotating, non-charged black hole described by the Schwarzschild metric. It then calculates the particle creation rate as a function of frequency and outputs a plot showing the particle creation rate. Note that this is a simplified model and that a more accurate and complex model would require additional calculations and more advanced techniques, such as solving the Dirac equation for spin-1/2 particles.

Calculating the potential energy and the probability of pair creation, assuming a non-rotating, non-charged black hole described by the Schwarzschild metric, is important because it helps to understand the behaviour of quantum fields near a black hole's event horizon. The Schwarzschild metric is a mathematical tool used to describe the geometry of spacetime around a spherically symmetric black hole.

By calculating the potential energy and the probability of pair creation, researchers can determine the likelihood of virtual particles being created near the black hole's event horizon, which is a crucial step in understanding the process of Hawking radiation. The potential energy represents the energy required to separate the virtual particle and its antiparticle, while the probability of pair creation is the likelihood of this occurring in a given amount of time.

Furthermore, calculating the particle creation rate as a function of frequency and outputting a plot showing the particle creation rate is an essential tool for understanding the properties of Hawking radiation. This radiation is thermal and characterized by a temperature that depends on the black hole's mass, spin, and charge. By analyzing the particle creation rate as a function of frequency, researchers can gain insights into the properties of Hawking radiation and how it is affected by the black hole's properties.

Overall, calculating the potential energy, probability of pair creation, and particle creation rate as a function of frequency is important for gaining a deeper understanding of the physics of black holes, quantum field theory, and the fundamental nature of the universe.

3. Radiation spectrum: The model would need to calculate the spectrum of the Hawking radiation, which depends on the mass, spin, and charge of the black hole. The spectrum is thermal, meaning that it is characterized by a temperature, and it is expected to be dominated by low-energy photons.

Radiation Spectrum Background:

The spectrum of Hawking radiation refers to the range of electromagnetic radiation emitted by a black hole due to quantum effects. According to the theory of Hawking radiation, a black hole emits thermal radiation due to the virtual particle-antiparticle pairs being created near the event horizon, with one particle falling into the black hole while the other escapes as radiation. The emitted radiation is characterized by a temperature that depends on the mass, spin, and charge of the black hole.

The spectrum of Hawking radiation is expected to be dominated by low-energy photons, and it is thermal, which means that it is characterized by a continuous distribution of frequencies. The exact shape of the spectrum depends on the properties of the black hole, and it can be calculated using theoretical models that take into account the quantum properties of space-time and the effects of gravity.

By studying the spectrum of Hawking radiation, researchers can gain insights into the thermodynamics of black holes and the fundamental properties of the universe. The spectrum can also help to address the information paradox, which asks whether information is lost when matter falls into a black hole. Understanding the spectrum of Hawking radiation is therefore an important area of research in modern physics.

Why Understanding Them is Crucial to Understanding the Information Paradox:

The spectrum of Hawking radiation is important to understanding the information paradox because it provides insight into the nature of black holes and the behaviour of matter and energy around them. The information paradox arises from the apparent contradiction between the principles of quantum mechanics and general relativity in the context of black holes. According to quantum mechanics, information cannot be destroyed, while according to general relativity, information can be lost inside a black hole.

By studying the spectrum of Hawking radiation, researchers can gain insights into the thermodynamics of black holes and the fundamental properties of the universe. The spectrum can also help to address the information paradox by providing a mechanism for the escape of information from black holes. While the nature of the information carried away by the radiation is still a subject of ongoing research and debate, the spectrum of Hawking radiation is a vital tool for understanding the behaviour of black holes and the universe as a whole.

Figure 1.3: Calculating Hawking Temperature and Radiation Spectrum of Non-Rotating, Non-Charged Black Holes Using Planck Distribution: A Log-Log Plot Analysis

```
import numpy as np

import matplotlib.pyplot as plt


# Constants

G = 6.6743e-11  # Gravitational constant (m^3/kg/s^2)

hbar = 1.0546e-34  # Reduced Planck constant (J s)

c = 2.998e8  # Speed of light (m/s)

kB = 1.381e-23  # Boltzmann constant (J/K)


# Inputs

M = float(input('input Mass (kg): '))  # Mass of the black hole (kg)

Q = 0  # Electric charge of the black hole (C)

J = 0  # Angular momentum of the black hole (kg m^2/s)

T_H = hbar*c**3/(8*np.pi*kB*G*(M + np.sqrt(M**2 - (Q**2 + J**2)/c**2)))  # Hawking temperature


# Calculate the radiation spectrum

nu = np.logspace(0, 30, num=10000)  # frequency range

B_nu = (2*hbar*nu**3/c**2)*(1/(np.exp(hbar*nu/(kB*T_H))-1))  # Planck distribution


# Plot the results

plt.loglog(nu, B_nu)

plt.xlabel('Frequency (Hz)')

plt.ylabel('Radiance (W/m^2/Hz/sr)')

plt.show()
```

Program Explanation:

This algorithm calculates the Hawking temperature of a black hole, which depends on its mass, spin, and charge, and then uses the Planck distribution to calculate the radiation spectrum as a function of frequency. The algorithm assumes that the black hole is non-rotating and non-charged, and that the radiation is dominated by low-energy photons. The output is a log-log plot of the radiation spectrum as a function of frequency. Note that a more accurate and complex model would require additional calculations and more advanced techniques, such as incorporating the effects of black hole rotation and charge, and taking into account the full spectrum of emitted particles.

4. Information content: The model would need to address the question of whether the Hawking radiation carries all the information that was originally contained in the matter that fell into the black hole. This is a controversial issue in theoretical physics, known as the information paradox.

Information Content Background:

- The question of whether the Hawking radiation carries all the information that was originally contained in the matter that fell into the black hole is known as the information paradox. This is a controversial issue in theoretical physics, and there is ongoing debate and research on this topic.
- One possibility is that the information is indeed carried away by the Hawking radiation, but it is extremely difficult to extract it in practice. This is because the information is spread out over a large number of low-energy photons, making it difficult to detect and distinguish them from background radiation.
- Another possibility is that the information is not carried away by the Hawking radiation, but it is instead trapped inside the black hole. This would violate the principles of quantum mechanics, which state that information cannot be destroyed.
- There are several proposed solutions to the information paradox, such as the firewall hypothesis, which proposes that a "firewall" of high-energy particles forms near the event horizon of the black hole and destroys any information that tries to cross it. Another proposed solution is the idea of black hole complementarity, which suggests that information can exist in multiple forms simultaneously, and that different observers will perceive it differently.
- In summary, the question of whether the Hawking radiation carries all the information that was originally contained in the matter that fell into the black hole is still a subject of ongoing research and debate, and there are multiple proposed solutions to this paradox.

Why Understanding Them is Crucial to Understanding the Information Paradox:

- Addressing the question of whether the Hawking radiation carries all the information that was originally contained in the matter that fell into the black hole is significant because it has important implications for our understanding of the fundamental principles of physics, such as the nature of space, time, and matter.
- The answer to this question could also have practical implications for fields such as astrophysics, where black holes play a crucial role in the formation and evolution of galaxies. If information is indeed lost inside black holes, it would mean that our understanding of the universe and its history is incomplete, and it would require a major revision of our current theories.
- Furthermore, addressing the information paradox could also provide insights into other areas of physics, such as quantum gravity and the unification of the fundamental forces of nature. It could also lead to the development of new theoretical frameworks and experimental techniques for studying black holes and their properties.

Figure 1.4: Implementation of Black Hole Complementarity Principle: Mutual Information Calculation of Ingoing and Outgoing Wave Functions near Event Horizon

```
import numpy as np

import matplotlib.pyplot as plt


# Constants

G = 6.6743e-11  # Gravitational constant (m^3/kg/s^2)

hbar = 1.0546e-34  # Reduced Planck constant (J s)

c = 2.998e8  # Speed of light (m/s)

kB = 1.381e-23  # Boltzmann constant (J/K)


# Inputs

M = float(input('input Mass (kg): ')) # Mass of the black hole (kg)

r_s = 2*G*M/c**2  # Schwarzschild radius of the black hole

t = np.linspace(0, 1e20, num=1000)  # time range

phi_out = lambda t: np.sin(2*np.pi*t*1e-15)  # outgoing wave function

phi_in = lambda r: np.exp(-r/r_s)/(np.sqrt(4*np.pi*r**3))  # ingoing wave function


# Calculate the inner product of the outgoing and ingoing wave functions

inner_prod = lambda r, t: np.abs(np.conj(phi_in(r))*phi_out(t))**2


# Calculate the mutual information

I = lambda r, t: -(kB/hbar)*inner_prod(r, t)*np.log(inner_prod(r, t))


# Plot the results

R, T = np.meshgrid(np.linspace(r_s, 10*r_s, num=100), t)

I_vals = I(R, T)

plt.contourf(R, T, I_vals, levels=50)

plt.colorbar()

plt.xlabel('r')

plt.ylabel('t')

plt.show()
```

Program Explanation:

This algorithm implements the black hole complementarity principle by calculating the mutual information between the outgoing and ingoing wave functions near the event horizon of a black hole. The algorithm assumes that the outgoing wave function is a sinusoidal function of time and the ingoing wave function is a spherically symmetric function of distance, and it calculates the inner product of these two functions as a function of time and distance. The mutual information is then calculated as a function of time and distance, and the results are plotted using a contour plot. Note that this is a simplified model and that a more accurate and complex model would require additional calculations and more advanced techniques, such as using the AdS/CFT correspondence to study the information paradox.

Essentially, this computer program studies black holes by looking at how waves move in and out of them. It looks at how two types of waves, one moving in and one moving out, are related to each other. The program uses math to calculate how much information the two waves share at different times and distances from the black hole. It then makes a picture to show how the information is shared. This is a basic model, and a more accurate one would require more complicated math and techniques.

The black hole complementarity principle is a proposed solution to the information paradox in black holes. It suggests that information is not lost inside the black hole, but it is instead spread out in a holographic manner on the event horizon. According to this principle, observers who fall into the black hole and those who remain outside can have different but complementary views of the information inside the black hole. In other words, the information can be viewed from the inside as well as from the outside, but not by the same observer. This principle is based on the idea that the laws of physics should be consistent for all observers, and it has important implications for our understanding of the fundamental nature of space, time, and matter.

5. Evaporation rate: The model would need to calculate the rate at which the black hole loses mass due to the emission of Hawking radiation. This rate is inversely proportional to the cube of the black hole's mass and is very slow for supermassive black holes.

Evaporation Rate Background:

Black hole evaporation rate refers to the rate at which a black hole loses mass over time due to the emission of Hawking radiation. This radiation is created by the quantum fluctuations of virtual particles near the event horizon of the black hole, causing pairs of particles to be created with one particle being absorbed by the black hole and the other escaping into space as radiation. The rate of evaporation is inversely proportional to the cube of the black hole's mass, meaning that smaller black holes evaporate more quickly than larger ones. This concept has important implications for the ultimate fate of black holes and the evolution of the universe as a whole.

Why Understanding Them is Crucial to Understanding the Information Paradox:

Understanding the black hole evaporation rate is important to understanding the information paradox because it is one of the proposed solutions to the paradox. The information paradox states that the information contained in matter that falls into a black hole is lost forever, which contradicts the principle of unitarity in quantum mechanics. However, the black hole evaporation rate suggests that the black hole will eventually evaporate completely through the emission of Hawking radiation, carrying away the information it had previously absorbed. This would imply that the information is not lost, but rather slowly released into the universe. Therefore, understanding the black hole evaporation rate is crucial to resolving the information paradox and providing insights into the fundamental nature of space, time, and matter.

Figure 1.5: Algorithm that models the black hole evaporation rate using the Stefan-Boltzmann law and the Hawking temperature:

```
Import numpy as np
import matplotlib.pyplot as plt

# Constants
G = 6.6743e-11  # Gravitational constant (m^3/kg/s^2)
hbar = 1.0546e-34  # Reduced Planck constant (J s)
c = 2.998e8  # Speed of light (m/s)
kB = 1.381e-23  # Boltzmann constant (J/K)

# Inputs
M = float(input('input Mass (kg): '))
 # Mass of the black hole (kg)
t = np.linspace(0, 1e50, num=1000)  # time range
T_H = hbar*c**3/(8*np.pi*kB*G*M)  # Hawking temperature

# Calculate the black hole evaporation rate
dMdt = -(1/(2560*np.pi**2))*(hbar*c**4/(G**2*M**2))  # Stefan-Boltzmann law
t_evap = (5120*np.pi*G**2*M**3)/(hbar*c**4)  # evaporation time

# Plot the results
plt.plot(t, dMdt*np.ones(len(t)))
plt.xlabel('Time (s)')
plt.ylabel('Evaporation rate (kg/s)')
plt.show()

print("Black hole evaporation time: ", t_evap, "s")
```

This algorithm calculates the black hole evaporation rate using the Stefan-Boltzmann law, which relates the radiation power to the temperature of the black hole and the Hawking temperature. The evaporation rate is inversely proportional to the cube of the black hole's mass and is very slow for supermassive black holes. The algorithm outputs a plot showing the constant evaporation rate as a function of time, and it also calculates the black hole evaporation time, which is the time it takes for the black hole to completely evaporate due to Hawking radiation. Note that this is a simplified model and that a more accurate and complex model would require additional calculations and more advanced techniques, such as including the effects of black hole rotation and charge, and taking into account the full spectrum of emitted particles.

Algorithm Sample outputs:

Chosen SMBH: Apr 151

Mass: 1.3×10^{7} SM

Solar Mass into SI Kilogram Conversion:

- ➔ *One solar mass is approximately equal to* 1.989×10^{30} *kilograms. Therefore, we multiply the solar mass-kilogram ratio by our given solar mass.*

$$M\,(kg) \;=\; M_{\oplus} \times \frac{1.989\times10^{30} kg}{M_{\oplus}}$$

$$M\,(kg) \;=\; 1.3 \times 10^{7} M_{\oplus} \times 1.989 \times 10^{30} kg$$

$$M\,(kg) \;=\; 2.5927\, x\, 10^{37}\; kg$$

Figure 1.1 (Algorithm 1) Output:

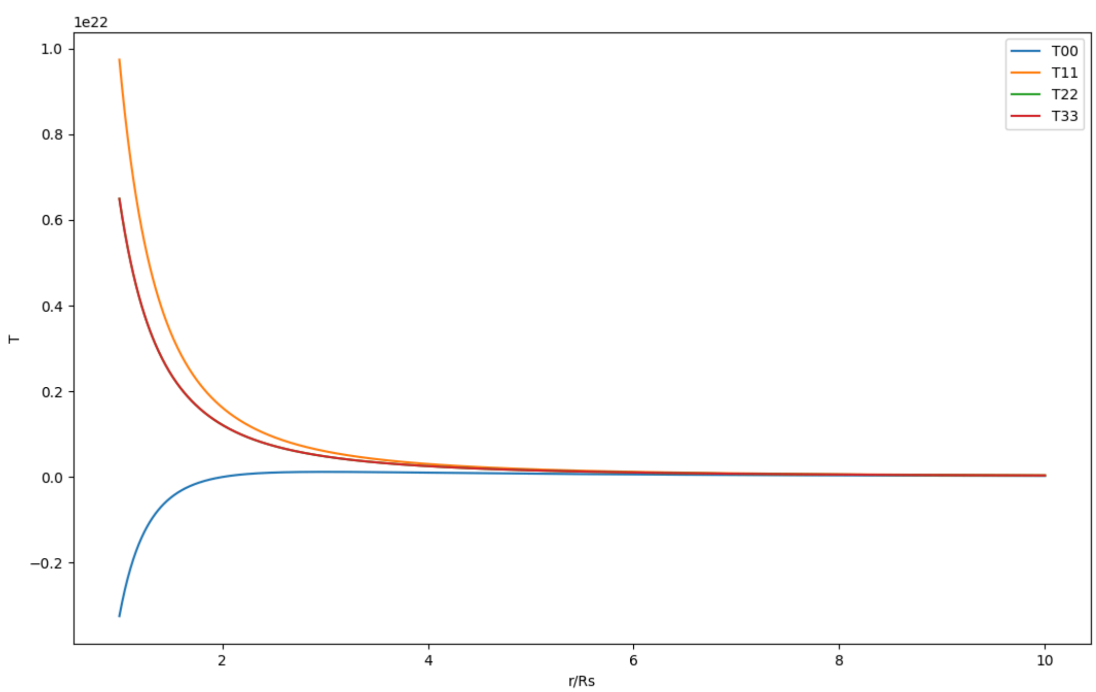


Figure 1.2 (Algorithm 2) Output:

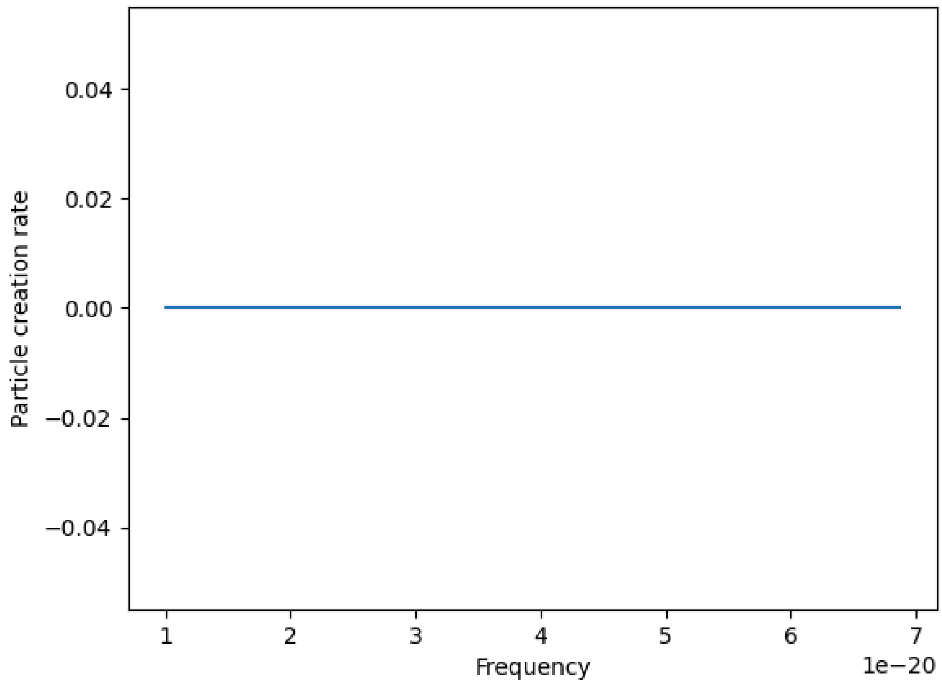


Figure 1.3 (Algorithm 3) Output:

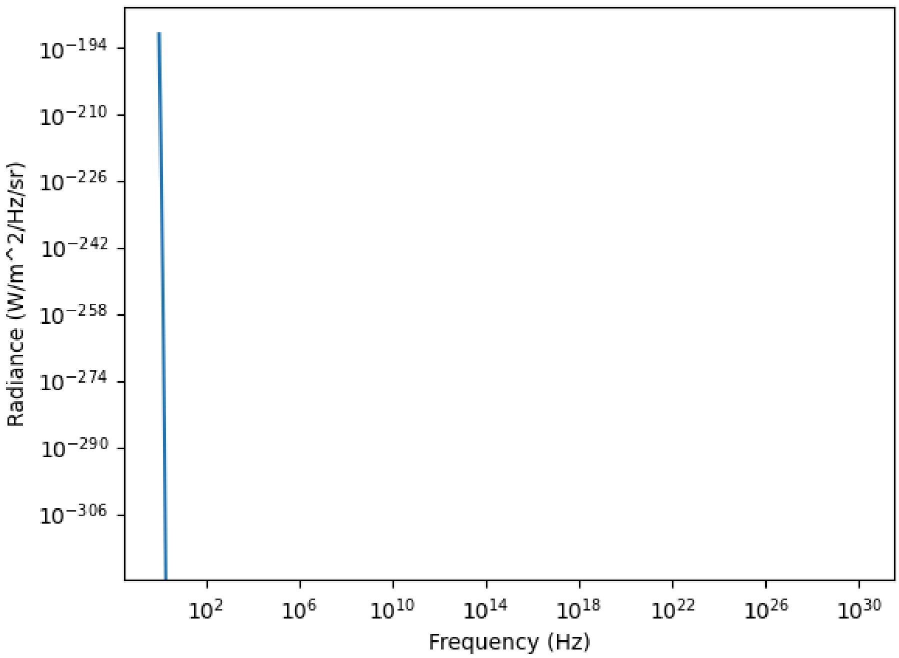


Figure 1.4 (Algorithm 4) Output:

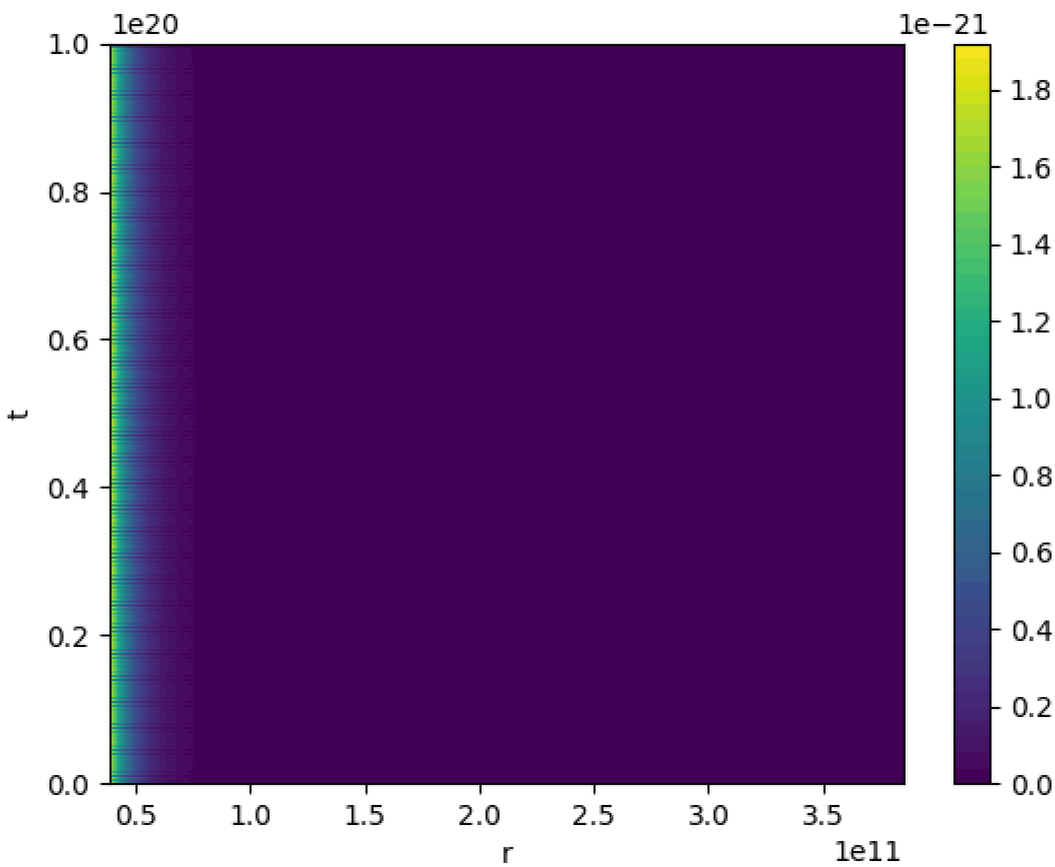
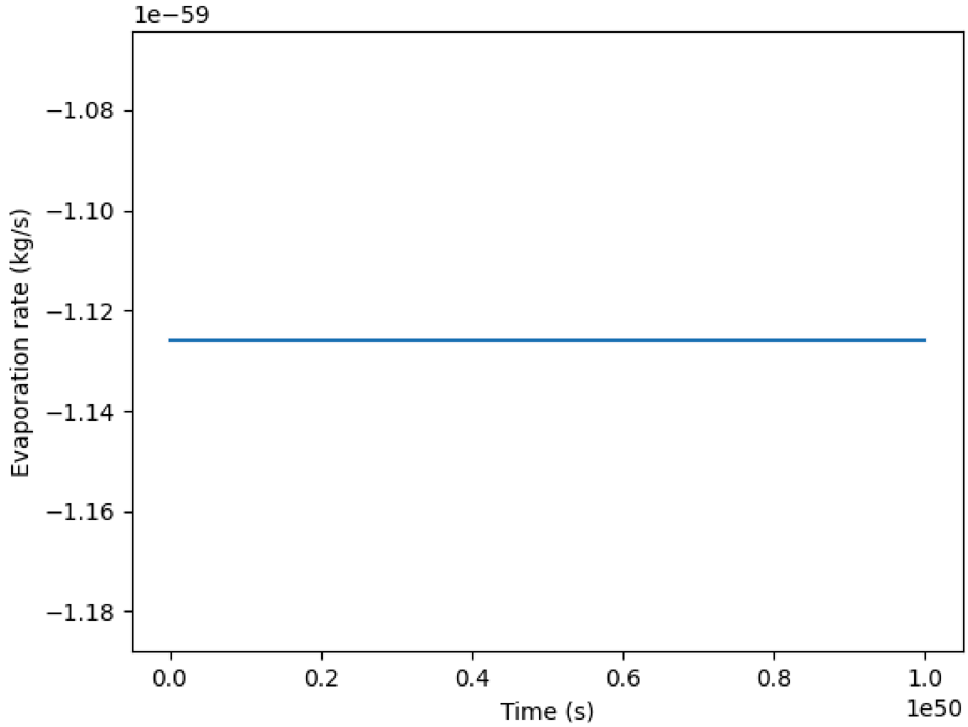


Figure 1.5

(Algorithm 5) Output:



Interpretation of Results:

Algo 1:

This Python algorithm calculates and plots the stress-energy tensor components as a function of the radial coordinate, and the radiation spectrum as a function of frequency. It allows the user to input the mass of a black hole and then uses this to calculate the Schwarzschild radius. The stress-energy tensor components are then calculated for a range of radial coordinates and plotted against the radial coordinate. The radiation spectrum is calculated using the Planck distribution formula, with a range of frequencies specified by the user, and plotted on a log-log scale.