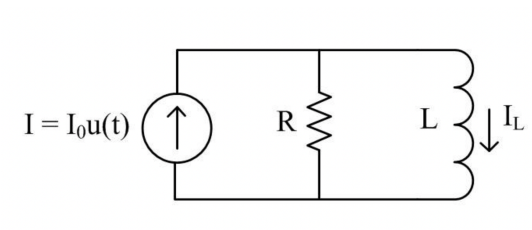
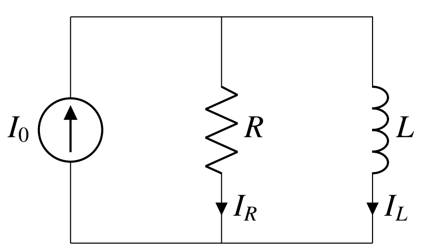
NCERT Question 11.9.3.9

EE23BTECH11019 - Faisal Imtiyaz *

Question: The R-L circuit with $R = 10k\Omega$ and $L = 1mH$ is excited by a step current $I_0u(t)$. At $t = 0^-$, there is a current $I_L = I_0/5$ flowing through the inductor. The minimum time taken for the current through the inductor to reach 99% of its final value is $\ldots \mu s$ (rounded off to two decimal places).

Solution:





Transform	Signal
$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$
$\frac{1}{s+a}$	e^{-at}

TABLE 1
Inverse Laplace transform pairs

$$I_0u(t) = I_R + I_L \tag{1}$$

From KVL, we have:

$$I_R(R) - L\frac{dI_L}{dt} = 0 \tag{2}$$

$$(I_0u(t) - I_L)(R) - L\frac{dI_L}{dt} = 0 \tag{3}$$

$$(\frac{I_0}{s} - I(s))R - L(sI(s) - I_L(0^-)) = 0 \tag{4}$$

After Simplyfying we have:

$$I(s) = \frac{I_0R + LsI(0^-)}{s(R + Ls)} \tag{5}$$

$$I(s) = \frac{I_0R}{L}\frac{1}{s(s+\frac{R}{L})} + \frac{I_0}{5}\frac{1}{\frac{R}{L}+s} \tag{6}$$

From Table 1, we have:

$$I(t) = \frac{I_0R}{L}\left[\frac{1}{\frac{R}{L}}(1 - e^{-\frac{R}{L}t})\right] + \frac{I_0}{5}e^{-\frac{R}{L}} \tag{7}$$

$$I(t) = I_0 - \frac{4}{5}I_0e^{-\frac{R}{L}t} \tag{8}$$

$$I(t) = I_0 - \frac{4}{5}I_0e^{-10^7t} \tag{9}$$

$$\lim_{t\to\infty} I(t) = I_0 \tag{10}$$

Now time when current in inductor is 99% of its final value is given by:

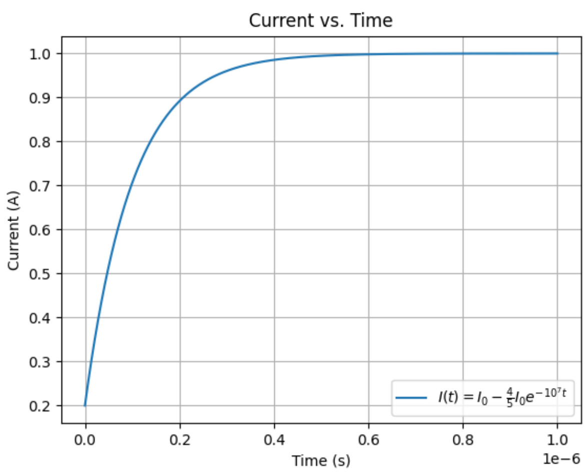
$$0.99I_0 = I_0 - \frac{4}{5}I_0e^{-\frac{R}{L}t} \tag{11}$$

$$0.01I_0 = \frac{4}{5}I_0e^{-\frac{R}{L}t} \tag{12}$$

$$t = \frac{L}{R}\ln(80) \tag{13}$$

$$t = 10^{-7}\ln(80)\mu s \tag{14}$$

$$t = 0.43\mu s \tag{15}$$


Current vs. Time
Current (A)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
1e−6
$I(t) = I_0 - \frac{4}{5} I_0 e^{-10^7 t}$