NCERT Question 11.9.3.9

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Question: Find the sum to indicated number of terms in the geometric progression:
$1, -a, a^2, -a^3, ...n$ terms (if $a \neq -1$).

Solution:

Input Parameters	Values	Description
$x(0)$	1	First term
r	$(-a)$	Common ratio
$x(n)$	$(-a)^n u(n)$	General term

TABLE 1
GIVEN INPUTS

From Table 1,

$$X(z) = \frac{1}{1+az^{-1}} \tag{1}$$

$$y(n) = \sum_{k=0}^{n}(-a)^k = \sum_{k=-\infty}^{n}(-a)^k u(k) \tag{2}$$

$$y(n) = (-a)^n u(n) * u(n) \tag{3}$$

$$\implies Y(z) = X(z) \cdot U(z) \tag{4}$$

$$Y(z) = \frac{1}{1+az^{-1}} \cdot \frac{1}{1-z^{-1}} \tag{5}$$

$$\implies Y(z) = \frac{z^2}{(z+a)(z-1)} \tag{6}$$

Using Z transform pairs to find the inverse Z-transform:

$$Y(z) = \frac{z^2}{a+1}\left[\frac{1}{z-1} - \frac{1}{z+a}\right] \tag{7}$$

$$= \frac{1}{a+1}\left[\frac{z^2-1}{z-1} + \frac{1}{z-1} - \frac{z^2-a^2}{z+a} - \frac{a^2}{z+a}\right] \tag{8}$$

$$= \frac{1}{a+1}\left[(z-1) + \frac{1}{z-1} - (z-a) - \frac{a^2}{z+a}\right] \tag{9}$$

$$= 1 + \frac{1}{a+1}\left[\frac{1}{z-1} - \frac{a^2}{z+a}\right] \tag{10}$$

$$y(n) = \delta(n) + \frac{1}{a+1}\left[1 - a^2 \cdot (-a)^n\right] \tag{11}$$

$$y(n) = \delta(n) + \frac{1-(-a)^n}{1-(-a)} \tag{12}$$

(13)

Since $\delta(n)$ is zero for $n > 0$, thus:

$$y(n) = \frac{1-(-a)^n}{1-(-a)} \tag{14}$$

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