NCERT Question 11.9.3.9

EE23BTECH11019 - Faisal Imtiyaz *

Question: Find the sum to indicated number of terms in the geometric progression:
$1, -a, a^2, -a^3, ...n$ terms (if $a \neq -1$).

Solution:

Input Parameters	Values	Description
$x(0)$	1	First term
r	$(-a)$	Common ratio
$x(n)$	$(-a)^n u(n)$	General term

TABLE 1
GIVEN INPUTS

From Table 1,

$$X(z) = \frac{1}{1 + az^{-1}} \tag{1}$$

$$y(n) = (-a)^n u(n) * u(n) \tag{2}$$

$$\implies Y(z) = X(z) \cdot U(z) \tag{3}$$

$$= \frac{1}{1 + az^{-1}} \cdot \frac{1}{1 - z^{-1}} \tag{4}$$

$$\tag{5}$$

Using Z transform pairs to find the inverse Z-transform:

$$Y(z) = \frac{1}{a+1} \left[\frac{1}{1 - z^{-1}} - \frac{1}{1 + az^{-1}} \right] \tag{6}$$

$$= \frac{1}{a+1} \left[\frac{1 - z^{-2}}{1 - z^{-1}} + \frac{z^{-1}}{1 - z^{-1}} - \frac{1 - a^2 z^{-1}}{1 + az^{-1}} - \frac{a^2 z^{-1}}{1 + az^{-1}} \right] \tag{7}$$

$$= 1 + \frac{1}{a+1} \left[\frac{z^{-1}}{1 - z^{-1}} - \frac{a^2 z^{-1}}{1 + az^{-1}} \right] \tag{8}$$

$$y(n) = \delta(n) + \frac{1}{a+1} \left[1 - a^2 \cdot (-a)^n \right] \tag{9}$$

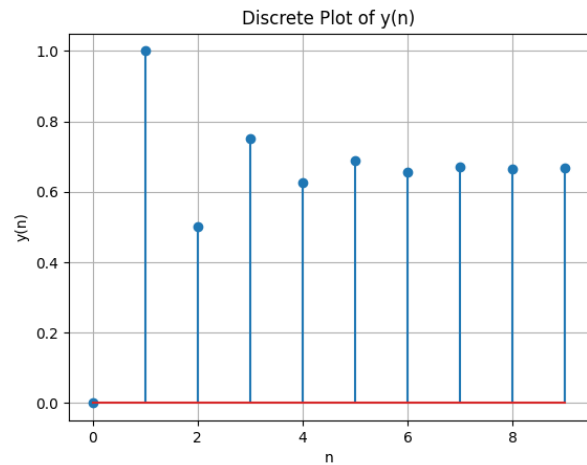$$y(n) = \delta(n) + \frac{1 - (-a)^n}{1 - (-a)} \tag{10}$$

Fig. 1. Plot of $y(n)$