NCERT Question 11.9.3.9

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Question: Find the sum to indicated number of terms in the geometric progression:
$1, -a, a^2, -a^3, ...n$ terms (if $a \neq -1$).
Solution:

Input Parameters	Values	Description
$x(0)$	1	First term
r	$(-a)$	Common ratio
$x(n)$	$(-a)^n u(n)$	General term

TABLE 1
GIVEN INPUTS

From Table 1,

$$X(z) = \frac{1}{1 + az^{-1}} \tag{1}$$

$$y(n) = (-a)^n u(n) * u(n) \tag{2}$$

$$\implies Y(z) = X(z) \cdot \; U(z) \tag{3}$$

$$Y(z) = \frac{1}{1 + az^{-1}} \cdot \frac{1}{1 - z^{-1}} \tag{4}$$

$$\implies Y(z) = \frac{z^2}{(z + a)(z - 1)} \tag{5}$$

Using Z transform pairs to find the inverse Z-transform:

$$Y(z) = \frac{z^2}{a + 1}\left[\frac{1}{z - 1} - \frac{1}{z + a}\right] \tag{6}$$

$$= \frac{1}{a + 1}\left[\frac{z^2 - 1}{z - 1} + \frac{1}{z - 1} - \frac{z^2 - a^2}{z + a} - \frac{a^2}{z + a}\right] \tag{7}$$

$$= \frac{1}{a + 1}\left[(z - 1) + \frac{1}{z - 1} - (z - a) - \frac{a^2}{z + a}\right] \tag{8}$$

$$= 1 + \frac{1}{a + 1}\left[\frac{1}{z - 1} - \frac{a^2}{z + a}\right] \tag{9}$$

$$y(n) = \delta(n) + \frac{1}{a + 1}\left[1 - a^2 \cdot (-a)^n\right] \tag{10}$$

$$y(n) = \delta(n) + \frac{1 - (-a)^n}{1 - (-a)} \tag{11}$$

$$\tag{12}$$

Since $\delta(n)$ is zero for $n > 0$, thus:

$$y(n) = \frac{1 - (-a)^n}{1 - (-a)} \tag{13}$$

$$\tag{14}$$