NCERT Question 11.9.3.9

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Question: Find the sum to indicated number of terms in the geometric progression:
$1, -a, a^2, -a^3, ...n$ terms (if $a \neq -1$).

Solution:

Input Parameters	Values	Description
$x(0)$	1	First term
r	$(-a)$	Common ratio
$x(n)$	$(-a)^n u(n)$	General term

TABLE 1
GIVEN INPUTS

Signal	Transform
$\frac{1}{1-z^{-1}}$	$u(n)$
$\frac{1}{1-az^{-1}}$	$(a)^n u(n)$

TABLE 2
GIVEN INPUTS

$$x(n) = (-a)^n u(n) \tag{1}$$

$$X(z) = \frac{1}{1+az^{-1}} \tag{2}$$

$$\text{The ROC is :} \quad |z| > |a| \tag{3}$$

$$\tag{4}$$

From Table **??**,

$$y(n) = (-a)^n u(n) * u(n) \tag{5}$$

$$\implies Y(z) = X(z) \cdot U(z) \tag{6}$$

$$= \frac{1}{1+az^{-1}} \cdot \frac{1}{1-z^{-1}} \tag{7}$$

$$\tag{8}$$

Using Z transform pairs to find the inverse Z-transform:

$$Y(z) = \frac{1}{a+1}\left[\frac{1}{1-z^{-1}} - \frac{1}{1+az^{-1}}\right] \tag{9}$$

$$= \frac{1}{a+1}\left[\frac{1-z^{-2}}{1-z^{-1}} + \frac{z^{-1}}{1-z^{-1}} - \frac{1-a^2z^{-1}}{1+az^{-1}} - \frac{a^2z^{-1}}{1+az^{-1}}\right] \tag{10}$$

$$= 1 + \frac{1}{a+1}\left[\frac{z^{-1}}{1-z^{-1}} - \frac{a^2z^{-1}}{1+az^{-1}}\right] \tag{11}$$

$$y(n) = \delta(n) + \left[\frac{1}{a+1}\left[1 - a^2 \cdot (-a)^n\right]\right] u(n) \tag{12}$$

$$y(n) = \delta(n) + \left[\frac{1-(-a)^n}{1-(-a)}\right] u(n) \tag{13}$$

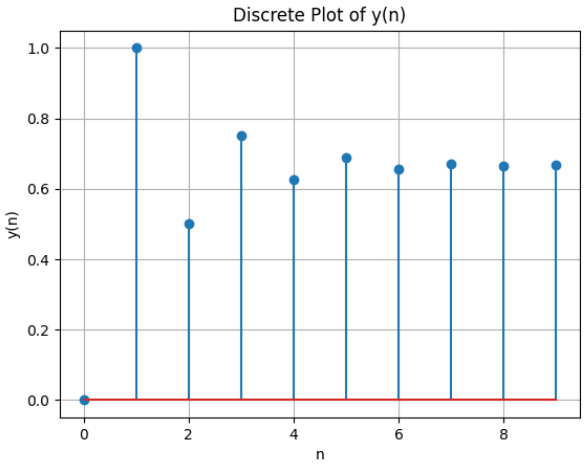

Fig. 1. Plot of $y(n)$