

NCERT Question 11.9.3.9

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Question: Find the sum to indicated number of terms in the geometric progression:
 $1, -a, a^2, -a^3, \dots, n$ terms (if $a \neq -1$).

Solution:

| Input Parameters | Values | Description |
|------------------|---------------|--------------|
| $x(0)$ | 1 | First term |
| r | $(-a)$ | Common ratio |
| $x(n)$ | $(-a)^n u(n)$ | General term |

TABLE 1
GIVEN INPUTS

From Table 1,

$$X(z) = \frac{1}{1 + az^{-1}} \quad (1)$$

$$y(n) = (-a)^n u(n) * u(n) \quad (2)$$

$$\Rightarrow Y(z) = X(z) \cdot U(z) \quad (3)$$

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TABLE 2
GIVEN INPUTS

From Table 1,

$$X(z) = \frac{1}{1 + az^{-1}} \quad (4)$$

$$y(n) = (-a)^n u(n) * u(n) \quad (5)$$

$$\Rightarrow Y(z) = X(z) \cdot U(z) \quad (6)$$

$$Y(z) = \frac{1}{1 + az^{-1}} \cdot \frac{1}{1 - z^{-1}} \quad (7)$$

$$\Rightarrow Y(z) = \frac{z^2}{(z + a)(z - 1)} \quad (8)$$

Using Z transform pairs to find the inverse Z-transform:

$$Y(z) = \frac{z^2}{a + 1} \left[\frac{1}{z - 1} - \frac{1}{z + a} \right] \quad (9)$$

$$= \frac{1}{a + 1} \left[\frac{z^2 - 1}{z - 1} + \frac{1}{z - 1} - \frac{z^2 - a^2}{z + a} - \frac{a^2}{z + a} \right] \quad (10)$$

$$= \frac{1}{a + 1} \left[(z - 1) + \frac{1}{z - 1} - (z - a) - \frac{a^2}{z + a} \right] \quad (11)$$

$$= 1 + \frac{1}{a + 1} \left[\frac{1}{z - 1} - \frac{a^2}{z + a} \right] \quad (12)$$

$$y(n) = \delta(n) + \frac{1}{a + 1} \left[1 - a^2 \cdot (-a)^n \right] \quad (13)$$

$$y(n) = \delta(n) + \frac{1 - (-a)^n}{1 - (-a)} \quad (14)$$

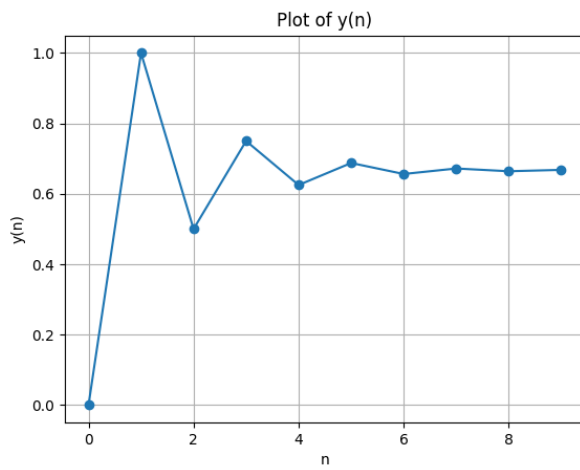


Fig. 1. Plot of $y(n)$