# Assignment-1

EE:1205 ($SignalsSystems$)
Indian Institute of Technology, Hyderabad

Faisal Imtiyaz
EE23BTECH11019

### ***Question 11.9.3.9:***

Find the sum to indicated number of terms in the geometric progression:

1, $-a$, $a^2$ , $-a^3$ , ... n terms (if $a \neq -1$).

### ***Solution:***

To be found:

$$S(n) = \sum_{k=0}^{n}(-a)^k = \sum_{k=-\infty}^{n}(-a)^k u(k) \tag{1}$$
$$S\,(n) = (-a)^n u(n) * u\,(n) \tag{2}$$
$$\implies S\,(z) = f\,(z) \cdot \frac{1}{1-z^{-1}} \tag{3}$$

$$f\,(z) = \sum_{n=-\infty}^{\infty}(-a)^n u(n) z^{-n} \tag{4}$$
$$\implies f\,(z) = \sum_{n=0}^{\infty}(-a)^n z^{-n} \tag{5}$$
$$\implies f\,(z) = \sum_{n=0}^{\infty}\left(\frac{-a}{z}\right)^n \tag{6}$$
$$\implies f\,(z) = \frac{1}{1-\left(\frac{-a}{z}\right)}, \left|\frac{-a}{z}\right| < 1 \tag{7}$$
$$\implies f\,(z) = \frac{1}{1+az^{-1}} \tag{8}$$

$$S\,(z) = \frac{1}{1+az^{-1}} \cdot \frac{1}{1-z^{-1}} \tag{9}$$
$$\implies S\,(z) = \frac{z^2}{(z+a)(z-1)} \tag{10}$$

Using contour integration to find the inverse Z-transform:

$$\implies S\,(n) = \frac{1}{2\pi j}\oint_C S\,(z)\; z^{n-1}\,dz \tag{11}$$
$$= \frac{1}{2\pi j}\oint_C \frac{z^{n+1}}{(z+a)(z-1)}\,dz \tag{12}$$
$$= \frac{1}{2\pi j}(2\pi j)(R_1+R_2) \tag{13}$$
$$= (R_1+R_2) \tag{14}$$
$$\tag{15}$$

To find R1 and R2, we need to first find the poles which are given by:

$$(z+a)(z-1) = 0 \tag{16}$$
$$\implies z = -a \quad \text{and} \quad 1 \tag{17}$$

Therefore we find R1 and R2 as follows:

$$R_1 = \lim_{z\to -a}(z+a)\cdot\frac{z^{n+1}}{(z+a)(z-1)} \tag{18}$$
$$= \frac{(-a)^{n+1}}{-a-1} \tag{19}$$
$$R_2 = \lim_{z\to 1}(z-1)\cdot\frac{z^{n+1}}{(z+a)(z-1)} \tag{20}$$
$$= \frac{1}{1+a} \tag{21}$$
$$\tag{22}$$

Therefore, we conclude that:

$$S\,(n) = \frac{(-a)^{n+1}}{-a-1} + \frac{1}{1+a} \tag{23}$$
$$= \frac{1-(-a)^{n+1}}{1-(-a)} \tag{24}$$
$$\tag{25}$$

This gives the sum of n terms of the G.P