

ANALOG: 12.10.19

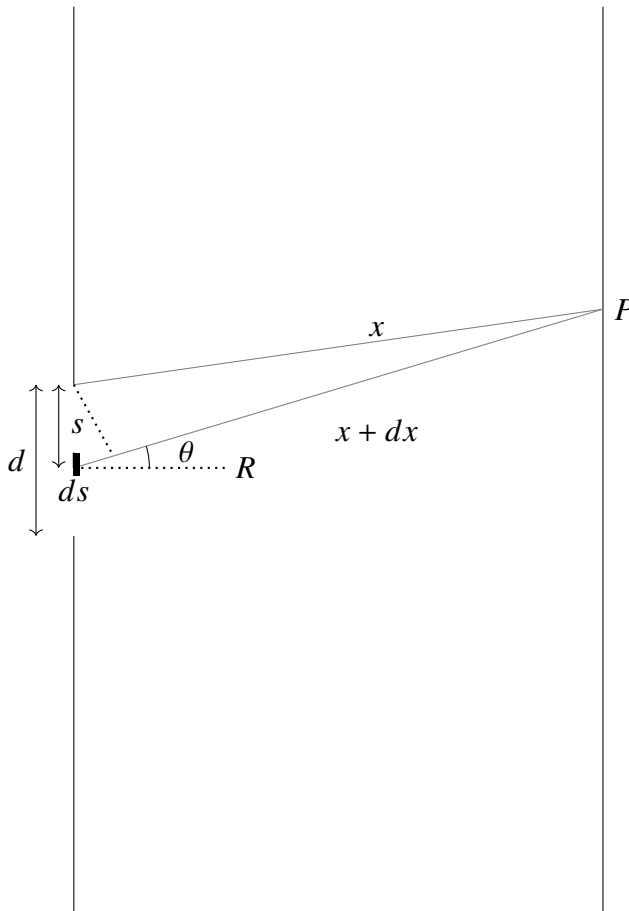
EE23BTECH11019 - Faisal Imtiyaz*

I. PROBLEM STATEMENT

A parallel beam of light with a wavelength of 500 nm falls on a narrow slit, and the resulting diffraction pattern is observed on a screen 1 m away. The distance to the first minimum from the center of the screen is 2.5 mm.

Find the width of the slit given that $y = 0.0025$ m, $L = 1$ m, and $\lambda = 5 \times 10^{-7}$ m.

II. DERIVATION



Let dE be the electric field at point P due to light from ds part of the slit. Since the Electric field dE is proportional to the small slit width ds , we can

say that:

$$dE = G \cos(k(x + dx) - \omega t) ds$$

$$\Rightarrow E = G \int_0^b \cos(k(x + dx) - \omega t) ds$$

Here, G is the constant of proportionality.

Now, from the figure it can be easily seen that $s \sin \theta = dx$, so,

$$\begin{aligned} E &= G \int_0^b \cos(k(x + s \sin \theta) - \omega t) ds \\ &= G \left[\frac{\sin((k \sin \theta)s + (kx - \omega t)s)}{k \sin \theta} \right]_{s=0}^{s=b} \\ &= G \left[\frac{\sin(kb \sin \theta + kx - \omega t) - \sin(kx - \omega t)}{k \sin \theta} \right] \\ &= \frac{2G}{k \sin \theta} \sin\left(\frac{kb \sin \theta}{2}\right) \cos\left(kx - \omega t + \frac{kb \sin \theta}{2}\right) \end{aligned}$$

Now, let $\frac{k\beta \sin \theta}{2} = \beta$. Also, the above equation represents a wave with amplitude A of $\frac{2G}{k \sin \theta} \sin\left(\frac{k\beta \sin \theta}{2}\right)$.

A can be written as $A = \frac{Gb \sin \beta}{\beta}$. The intensity of the wave will be proportional to A^2 . For minima, the intensity should be 0. Therefore, for minima we have:

$$\sin \beta = 0, \beta \neq 0$$

Therefore,

$$\sin \beta = n\pi$$

where n is a natural number.

So, the first minima should occur at:

$$\begin{aligned} \beta &= \pi, \\ \frac{kb \sin \theta}{2} &= \pi, \\ \frac{\pi \sin \theta}{\lambda} &= \pi, \\ \sin \theta &= \frac{\lambda}{b} \end{aligned}$$

III. SOLUTION

The first minimum is given by:

$$\sin \theta = \frac{\lambda}{b}$$

Where b is the width of the slit.

Now for a small angle θ , $\sin \theta$ can be assumed to be equal to θ as well as $\tan \theta$. So we can say $\sin \theta$ is approximately equal to $\tan \theta$.

Therefore:

$$\frac{\lambda}{b} = \frac{y}{L}$$

Solving the expression we get:

$$b = L \cdot \frac{\lambda}{y}$$

Plugging in the values, $b = 1 \times \frac{5 \times 10^{-7}}{0.0025} = 0.2 \text{ mm}$.