NCERT Question 11.9.3.9

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Question: Find the sum to indicated number of terms in the geometric progression:
$1, -a, a^2, -a^3, ...n$ terms (if $a \neq -1$).

Solution:

TABLE 1

Input Parameters	Values	Description
$x(0)$	1	First term
r	$(-a)$	Common ratio
$x(n)$	$(-a)^n u(n)$	General term

TABLE 1

GIVEN INPUTS

Signal	Transform
$\frac{1}{1-z^{-1}}$	$u(n)$
$\frac{1}{1-az^{-1}}$	$(a)^n u(n)$

TABLE 2

GIVEN INPUTS

Poles of the respecttive signals are as:

$$1 - z^{-1} = 0 \tag{1}$$
$$\implies |z| = 1 \tag{2}$$
$$1 - az^{-1} = 0 \tag{3}$$
$$\implies |z| = \frac{1}{a} \tag{4}$$
$$\tag{5}$$

Both the Signals are right sided so ROC is :

$$|z| > 1 \text{ and } |z| > \frac{1}{a} \tag{6}$$
$$\tag{7}$$

So for $0 < a < 1$ ROC is $|z| > \frac{1}{a}$ and otherwise ROC is $|z| > 1$

From Table 1,

$$X(z) = \frac{1}{1 + az^{-1}} \tag{8}$$
$$y(n) = (-a)^n u(n) * u(n) \tag{9}$$
$$\implies Y(z) = X(z) \cdot U(z) \tag{10}$$

$$= \frac{1}{1 + az^{-1}} \cdot \frac{1}{1 - z^{-1}} \tag{11}$$
$$\tag{12}$$

Using Z transform pairs to find the inverse Z-transform:

$$Y(z) = \frac{1}{a+1} \left[\frac{1}{1-z^{-1}} - \frac{1}{1+az^{-1}} \right] \tag{13}$$

$$= \frac{1}{a+1} \left[\frac{1-z^{-2}}{1-z^{-1}} + \frac{z^{-1}}{1-z^{-1}} - \frac{1-a^2z^{-1}}{1+az^{-1}} - \frac{a^2z^{-1}}{1+az^{-1}} \right] \tag{14}$$

$$= 1 + \frac{1}{a+1} \left[\frac{z^{-1}}{1-z^{-1}} - \frac{a^2z^{-1}}{1+az^{-1}} \right] \tag{15}$$

$$y(n) = \delta(n) + \left[\frac{1}{a+1} \left[1 - a^2 \cdot (-a)^n \right] \right] u(n) \tag{16}$$

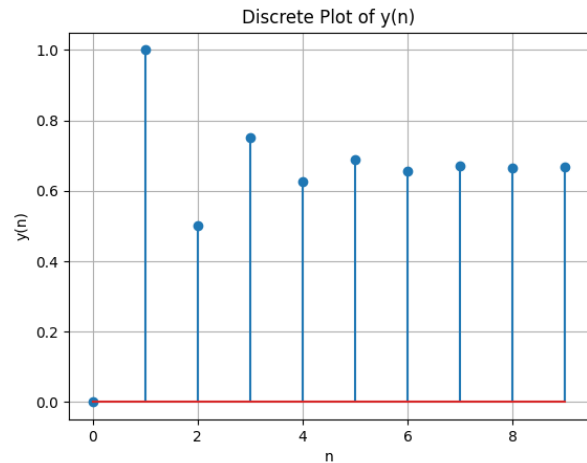$$y(n) = \delta(n) + \left[\frac{1-(-a)^n}{1-(-a)} \right] u(n) \tag{17}$$

Fig. 1. Plot of $y(n)$