Assignment-1

EE:1205 ($SignalsSystems$)
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QUESTION 11.9.3.9:

Find the sum to indicated number of terms in the geometric progression:

1, $-a$, a^2 , $-a^3$, ... n terms (if $a \neq -1$).
Solution:

PARAMETER	VALUE	DESCRIPTION
$x(0)$	1	First term
r	$(-a)$	common ratio
$x(n)$	$(-a)^n u(n)$	General term of the series

TABLE 0
PARAMETER TABLE1

$$y(n) = \sum_{k=0}^{n} (-a)^k = \sum_{k=-\infty}^{n} (-a)^k u(k) \tag{1}$$

$$y(n) = (-a)^n u(n) * u(n) \tag{2}$$

$$\implies Y(z) = f(z) \cdot \frac{1}{1 - z^{-1}} \tag{3}$$

$$f(z) = \sum_{n=-\infty}^{\infty} (-a)^n u(n) z^{-n} \tag{4}$$

$$\implies f(z) = \sum_{n=0}^{\infty} (-a)^n z^{-n} \tag{5}$$

$$\implies f(z) = \sum_{n=0}^{\infty} \left(\frac{-a}{z}\right)^n \tag{6}$$

$$\implies f(z) = \frac{1}{1 - \left(\frac{-a}{z}\right)}, \left|\frac{-a}{z}\right| < 1 \tag{7}$$

$$\implies f(z) = \frac{1}{1 + az^{-1}} \tag{8}$$

$$Y(z) = \frac{1}{1 + az^{-1}} \cdot \frac{1}{1 - z^{-1}} \tag{9}$$

$$\implies Y(z) = \frac{z^2}{(z + a)(z - 1)} \tag{10}$$

Using Z transform pairs to find the inverse Z-transform:

$$Y(z) = \frac{z^2}{a + 1} \left[\frac{1}{z - 1} - \frac{1}{z + a}\right] \tag{11}$$

$$= \frac{1}{a + 1} \left[\frac{z^2 - 1}{z - 1} + \frac{1}{z - 1} - \frac{z^2 - a^2}{z + a} - \frac{a^2}{z + a}\right] \tag{12}$$

$$= \frac{1}{a + 1} \left[(z - 1) + \frac{1}{z - 1} - (z - a) - \frac{a^2}{z + a}\right] \tag{13}$$

$$= 1 + \frac{1}{a + 1} \left[\frac{1}{z - 1} - \frac{a^2}{z + a}\right] \tag{14}$$

$$y(n) = \delta(n) + \frac{1}{a + 1} \left[1 - a^2 \cdot (-a)^n\right] \tag{15}$$

$$y(n) = \delta(n) + \frac{1 - (-a)^n}{1 - (-a)} \tag{16}$$

$$\tag{17}$$

Since $\delta(n)$ is zero for $n > 0$, thus:

$$y(n) = \frac{1 - (-a)^n}{1 - (-a)} \tag{18}$$

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