ANALOG: 12.10.19

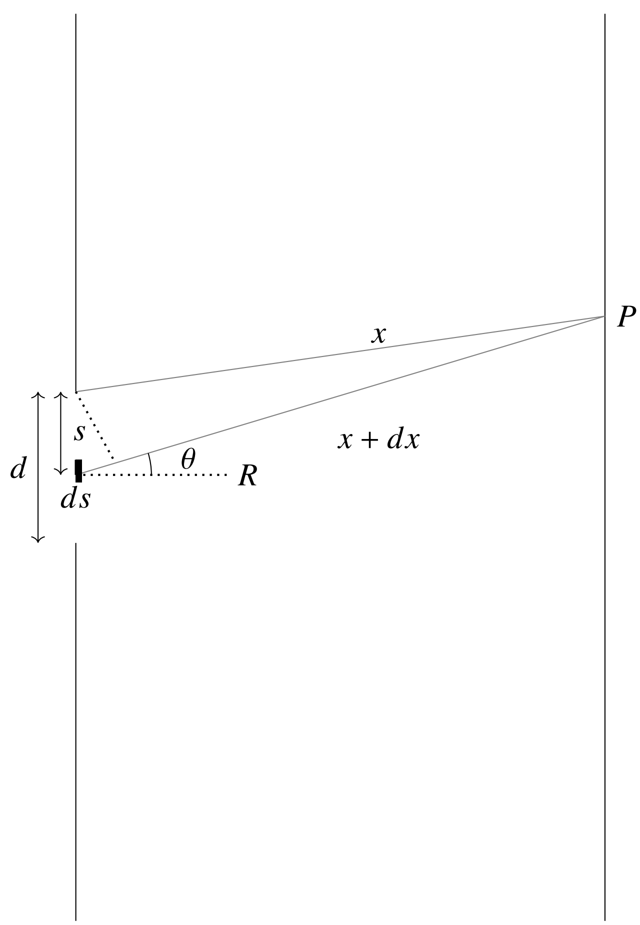
EE23BTECH11019 - Faisal Imtiyaz*

I. Problem Statement

A parallel beam of light with a wavelength of 500 nm falls on a narrow slit, and the resulting diffraction pattern is observed on a screen 1 m away. The distance to the first minimum from the center of the screen is 2.5 mm.

Find the width of the slit given that $y = 0.0025$ m, $L = 1$ m, and $\lambda = 5 \times 10^{-7}$ m.

II. Derivation



Let dE be the electric field at point P due to light from ds part of the slit. Since the Electric field dE is proportional to the small slit width ds, we can say that:

$$dE = G\cos(k(x+dx)-\omega t)\,ds$$
$$\implies E = G\int_0^b \cos(k(x+dx)-\omega t)\,ds$$

Here, G is the constant of proportionality.

Now, from the figure it can be easily seen that $s\sin\theta = dx$, so,

$$\begin{aligned}
E &= G\int_0^b \cos(k(x+s\sin\theta)-\omega t)\,ds\\
&= G\left[\frac{\sin((k\sin\theta)s+(kx-\omega t)s)}{k\sin\theta}\right]_{s=0}^{s=b}\\
&= G\left[\frac{\sin(kb\sin\theta+kx-wt)-\sin(kx-wt)}{k\sin\theta}\right]\\
&= \frac{2G}{k\sin\theta}\sin\left(\frac{kb\sin\theta}{2}\right)\cos\left(kx-wt+\frac{kb\sin\theta}{2}\right)
\end{aligned}$$

Now, let $\frac{kb\sin\theta}{2} = \beta$. Also, the above equation represents a wave with amplitude A of $\frac{2G}{k\sin\theta}\sin\left(\frac{kb\sin\theta}{2}\right)$.

A can be written as $A = \frac{Gb\cdot\sin\beta}{\beta}$. The intensity of the wave will be proportional to A^2. For minima, the intensity should be 0. Therefore, for minima we have:

$$\sin\beta = 0, \beta \neq 0$$

Therefore,

$$\sin\beta = n\pi$$

where n is a natural number.

So, the first minima should occur at:

$$\begin{aligned}
\beta &= \pi,\\
\frac{kb\sin\theta}{2} &= \pi,\\
\frac{\pi\sin\theta}{\lambda} &= \pi,\\
\sin\theta &= \frac{\lambda}{b}
\end{aligned}$$

III. Solution

The first minimum is given by:

$$\sin\theta = \frac{\lambda}{b}$$

Where b is the width of the slit.

Now for a small angle θ, $\sin\theta$ can be assumed to be equal to θ as well as $\tan\theta$. So we can say $\sin\theta$ is approximately equal to $\tan\theta$.

Therefore:

$$\frac{\lambda}{b} = \frac{y}{L}$$

Solving the expression we get:

$$b = L \cdot \frac{\lambda}{y}$$

Plugging in the values, $b = 1 \times \frac{5\times10^{-7}}{0.0025} = 0.2$ mm.