Assignment 01

Name : Faisal Ahmed

ID : 21301186

Section : 07

Course : CSE422 (IBU)

3) (a)

	Library		Cafe	
	Morning	Evening	Morning	Evening
Quiet	40/200 =0.2	30/200 =0.15	25/200 =0.125	45/200 =0.225
Moderate	20/200 =0.1	10/200 =0.05	15/200 =0.075	15/200 =0.075

(b) From the above table we can see that the Probability of students who preferred the cafe in the Evening with a Quiet environment i.e. $P(L=\text{Cafe}, T=\text{Evening}, N=\text{Quiet}) = 0.225$

(c) The marginal probability of $P(L=\text{Library})$

$=(0.2+0.15+0.1+0.05) = 0.5$

Again, the marginal probability of $P(N=\text{Quiet})$

$=(0.2+0.15+0.125+0.225) = 0.7$

(d) From the joint probability table we can see that $P(L=\text{Library}, T=\text{Morning}, N=\text{Quiet}) = 0.2$

~~Now~~ Again, $P(L = Library) = 0.5$ [From 'c']

$P(T = Morning) = 0.2 + 0.1 + 0.125 + 0.075$

$= 0.5$

$P(N = Quiet) = 0.7$ [From 'c']

So, $P(Library) \cdot P(T = Morning) \cdot P(N = Quiet)$

$= 0.5 \times 0.5 \times 0.7 = 0.175$ which is not equal to $P(L = Library, T = Morning, N = Quiet)$

So, the factors Location, Time of Day and Noise level are not independent.

(e) $P(L = Library / T = Morning, N = Quiet)$

$$= \frac{P(L = Library, T = Morning, N = Quiet)}{P(T = Morning, N = Quiet)}$$

$$= \frac{0.2}{(0.2 + 0.125)} = 0.615 \text{ (Ans)}$$

7) Given, E and F are conditionally independent given both G and $\neg G$.

So, $P(E \cap F | G) = P(E|G) \times P(F/G)$

$= 0.5 \times 0.6 = 0.3$

Again, $P(E \cap F / \neg G) = P(E/\neg G) \times P(F/\neg G)$

$= 0.4 \times 0.3$

$= 0.12$

(Ans)

11) $P(Y/A \cap R) = \dfrac{P(A \cap R/Y)\, P(Y)}{P(A \cap R)}$

Considering the presence of action scenes and scary scenes are independent given the genre. So,

$P(A \cap R/Y) = P(A/Y) \cap P(R/Y)$

$= 0.3 \times 0.8 = 0.24$

Now, assuming the same,

$$P(A \cap R / X) = P(A / X) \cap P(R / X)$$

$$= 0.7 \times 0.2 = 0.14$$

Now, $P(A \cap R) = P(A \cap R / X)\, P(X) + P(A \cap R / Y)\, P(Y)$

$$= (0.14 \times 0.6) + (0.24 \times 0.4)$$

$$= 0.084 + 0.096$$

$$= 0.18$$

So, $P(Y / A \cap R) = \dfrac{0.24 \times 0.4}{0.18} = 0.533$

(Ans)

16) We have to calculate $P(\text{Party}A / M, H, C)$

We know,

$$P(\text{Party}A / M, H, C) = \frac{P(M, H, C / \text{Party}A)\, P(\text{Party}A)}{P(M, H, C)}$$

Now, for, $P(M, H, C \mid \text{Party } A)$, considering age, income and education are independent given the party.

So, $P(M, H, C \mid \text{Party } A) = P(M \mid \text{Party } A) \times P(H \mid \text{Party } A)$
$\times P(C \mid \text{Party } A)$

$= 0.4 \times 0.3 \times 0.5$

$= 0.06$

~~Now, $P(M/H, C/\text{Party } B) = P(M/\text{Party } B) \times$~~
~~$P(H/\text{Party } B) \times$~~
~~$P(C/\text{Party } B)$~~

So, $P(\text{Party } A \mid M, H, C) = \dfrac{0.06 \times 0.45}{P(M, H, C)}$

$= 0.027$ of $P(M \cap H \cap C)$

So, 0.027 among the Middle-aged, high income and college educated voter will choose Party A.

9) We have to calculate $P(S/R \cap V)$

We know, $P(S/R \cap V) = \dfrac{P(R \cap V/S)\, P(S)}{P(R \cap V)}$

To calculate $P(R \cap V/S)$, assuming voter registration and past voting frequency are independent variables given that estimate support is provided.

So, $P(R \cap V/S) = P(R/S) \cap P(V/S)$

$= 0.9 \times 0.1 = 0.09$

Now calculating

$P(R \cap V/O) = P(R/O) \cap P(V/O)$

$= 0.6 \times 0.3 = 0.18$

So, $P(R \cap V) = P(R \cap V/S) \times P(S) + P(R \cap V/O) \times P(O)$

$= (0.09 \times 0.7) + (0.18 \times 0.3)$

$= 0.063 + 0.054 = 0.117$

So, $P(S/R\cap V) = \dfrac{P(R\cap V/S)\, P(S)}{P(R\cap V)}$

$= \dfrac{0.09 \times 0.7}{0.117}$

$= 0.5384$ (Ans)

20) We have to calculate and compare

~~P(M)~~ $P(\text{Hire}/ C, U, M)$ and $P(\text{Not hire}/C,U,M)$

Calculating $P(\text{Hire}/C, U, M)$,

$P(\text{Hire}/C, U, M) = \dfrac{P(C/\text{Hire})\, P(U/\text{Hire})\, P(M/\text{Hire})\, P(\text{Hire})}{P(C,U,M)}$

So, $P(\text{Hire}/C, U, M) = \dfrac{0.9 \times 0.85 \times 0.2 \times 0.7}{P(C, U, M)}$

$= \dfrac{0.1071}{P(C,U,M)}$

Since, we do not have the value of $P(C, U, M)$

We have to ~~calat~~ calculate, $P(\text{Not Hire} \mid C, U, M)$:

$$P(\text{NotHire}/C,U,M) = \frac{P(\text{NotHire}) \cdot P(C/\text{NotHire}) P(U/\text{NotHire}) \cdot P(M/\text{NotHire})}{P(C,U,M)}$$

$$= \frac{0.3 \times 0.4 \times 0.3 \times 0.8}{P(C,U,M)}$$

$$= \frac{0.0288}{P(C,U,M)}$$

So, $$\frac{P(\text{Hire}/C,U,M)}{P(\text{Not Hire}/C,U,M)} = \frac{0.1071}{0.0288}$$

$$= 3.718\ \%$$

So, The chance of ~~hig~~ hiring a minority is 3.718% higher than not hiring a ~~mor~~ minority.

Again, for, $P(\text{Hire}/C,U,N)$.

$$P(\text{Hire}/C,U,N) = \frac{P(C/\text{Hire}) P(U/\text{Hire}) P(N/\text{Hire}) P(\text{Hire})}{P(C,U,N)}$$

$$\therefore P(Hire/C,U,N) = \frac{0.9 \times 0.85 \times 0.8 \times 0.7}{P(C,U,N)}$$

$$= \frac{0.4284}{P(C,U,N)}$$

Again, as $P(C,U,N)$ is not given, we have to calculate $P(NotHire/C,U,N)$.

$$P(NotHire/C,U,N) = \frac{P(C/NotHire) \times P(U/NotHire) \times P(N/NotHire) \times P(NotHire)}{P(C,U,N)}$$

$$= \frac{0.4 \times 0.3 \times 0.2 \times 0.3 = 0.0072}{P(C,U,N)}$$

So, $\frac{P(Hire/C,U,N)}{P(NotHire/C,U,N)} = \frac{0.4284}{0.0072} = 59.5\%$

So, the chance of hiring a non-minority is 59.5% more than non-hiring a non-minority

So, we can see that, the chance of hiring a minority than not hiring is 3.718 % whereas the chance of hiring a non-minority is 59.5% ~~per~~ than not hiring, which shows a clear racial biasness towards the non-minority candidates and it shows how the minority candidates are suffering of deprivation although having same coding experiences and from a top-ranking university."

Bonus

1) (a) From the data table, we can see that

$$P(X=1, Y=2) = 0.1$$

(b) $P(X=1) = 0.05 + 0.25 + 0.1 = 0.4$

(c) $P(Y=2) = 0.03 + 0.1 + 0.15$

$$= 0.28$$

(d) $P(X=1, Y=2) = 0.1.$

Again, $P(X=1) = 0.4$ and $P(Y=2) = 0.28$

So, $P(X=1) \cdot P(Y=2) = 0.4 \times 0.28$

$$= 0.112$$

$$\neq 0.1$$

So, X and Y are not independent

2) (a)

Size \ Color	Red	Blue
Small	20	10
Large	30	40

(b) From the data table,

$$P(C = Red, S = Large) = \frac{30}{100} = 0.3$$

(c) Joint probability of $P(C = Red, S = Large)$.

From the table, we can see that,

$$P(C = Red \mid S = Large) = \frac{30}{70} = 0.4286$$

and $P(S = Large) = \frac{30+40}{100} = 0.7$

We know,

$$P(C = Red, S = Large) = P(C = Red \mid S = Large) * P(Large)$$

[Joint Probability formula]

$$= 0.4286 \times 0.7$$

$$= 0.3 \text{ (Ans)}$$

(d) $P(C = Red) = \frac{20+30}{100} = 0.5$

$P(S = Large) = \frac{70}{100} = 0.7$

$P(C = Red) * P(S = Large) = 0.5 \times 0.7 = 0.35$

Again, we got $P(C = Red, S = Large) = 0.3$ from the data table. So, color and size are not independent.

(e) $$P(S = Small \mid C = Red) = \frac{P(S = Small) \cap P(C = Red)}{P(C = Red)}$$

$$= \frac{20}{20+30}$$

$$= \frac{2}{5} = 0.4 \text{ (Ans)}$$

4) We know,

$$P(A \cap B / C) = P(A/C) \times P(B/C)$$

$$= 0.4 \times 0.5 = 0.2$$

Again,

$$P(A \cap B / C) = \frac{P(A \cap B \cap C)}{P(C)}$$

So, $P(A \cap B \cap C) = P(A \cap B / C) * P(C)$

$$= 0.2 \times 0.2 = 0.04 \text{ (Ans)}$$

5) Given, $P(B|\neg C) = 0.6$

~~So, $P(\neg C/B) =$~~ P

So, $P(B/C) = 0.4$

Therefore, $P(A \cap B|C) = P(A/C) \times P(B/C)$

$= 0.3 \times 0.4 = 0.12$

$P(A \cap B/\neg C) = P(A/\neg C) \times P(B|\neg C)$

$= 0.2 \times 0.6$

$= 0.12$ (Ans)

6) $P(A \cap B/D) = P(A/D) * P(B/D)$

$= 0.4 * 0.5 = 0.2$

~~P(A∩B~~ $P(A \cap B|\neg D) = P(A/\neg D) * P(B/\neg D)$

$= 0.2 \times 0.3$

$= 0.06$

8) $P(A \cap B / C) = P(A/C) * P(B/C)$

$= 0.4 \times 0.5$

$= \cancel{0.02}\ 0.2$

$P(A \cap B \cap C) = P(A \cap B / C)\, P(C) = \cancel{0.02 \times 0} = 0.2 \times 0.3$

$\cancel{P(A \cap B \cap C) = P(A \cap B / C) \cdot P(C) = 0.2 \times 0.2}$

$= 0.06$

$\cancel{= 0.06}$

So, not conditionally independent

9) $P(D \cap E / F) = P(D/F) * P(E/F)$

$= 0.6 * 0.7 = 0.42$

$P(D \cap E \cap F) = P(D \cap E / F)\, P(F) = 0.42 \times 0.2 = 0.084$ (Not independent)

10) $P(G \cap H / I) = P(G/I) * P(H/I)$

$= 0.3 \times 0.4$

$= 0.12$

$P(G \cap H \cap I) = P(G \cap H / I) * P(I)$

$= 0.12 \times 0.5 = 0.06$

So, not conditionally independent.

12) we have to find $P(D/M \cap B)$

we know, from naive bayes theorem,

$$P(D/M \cap B) = \frac{P(M \cap B/D)\, P(D)}{P(M \cap B)}$$

$$P(M \cap B/D) = P(M/D) * P(B/D)$$

$$= 0.3 \times 0.7 = 0.21$$

For, $P(M \cap B)$ we need to find

$$P(M \cap B/E) = P(M/E) * P(B/E)$$

$$= 0.8 * 0.2 = 0.16$$

So, $P(M \cap B) = P(M \cap B/D)\, P(D) + P(M \cap B/E)\, P(E)$

$$= (0.21 \times 0.3) + (0.16 \times 0.7)$$

$$= 0.175$$

So, $$P(D/M\cap B) = \frac{P(M\cap B/D)\cdot P(D)}{P(M\cap B)}$$

$$= \frac{0.21 \times 0.3}{0.175} = 0.36 \text{ (Ans)}$$

13) $$P(P/EP\cap EC) = \frac{P(EP\cap EC/P)\cdot P(P)}{P(EP\cap EC)}$$

So, $P(EP\cap EC/P) = P(EP/P) * P(EC/P)$

$= 0.8 \times 0.4 = 0.32$

$P(EP\cap EC/C) = P(EP/C) * P(EC/C)$

$= 0.3 * 0.7 = 0.21$

$\therefore P(EP\cap EC) = P(EP\cap EC/P) * P(P) + P(EP\cap EC/C) * P(C)$

$= (0.32 \times 0.5) + (0.21 \times 0.5)$

$= 0.265$

So, $P(P/EP \cap EC) = \dfrac{P(EP \cap EC/P) * P(P)}{P(EP \cap EC)}$

$= \dfrac{0.32 \times 0.5}{0.265}$

$= 0.6038$

<u>14></u> $P(\text{Click}/Y \cap F) = \dfrac{P(Y \cap F/\text{Click}) * P(\text{Click})}{P(Y \cap F)}$

$P(Y \cap F/\text{click}) = P(Y/\text{Click}) * P(F/\text{Click})$

$= 0.4 * 0.7$

$= 0.28$

$P(Y \cap F/\text{No click}) = P(Y/\text{No click}) * P(F/\text{No click})$

$= 0.2 * 0.3 = 0.06$

So, $P(Y \cap F) = P(Y \cap F/\text{click}) * P(\text{click}) + P(Y \cap F/\text{No click}) * P(\text{No click})$

$= (0.28 \times 0.3) + (0.06 * 0.7)$

$= 0.126$

So, $P(\text{click}/Y \cap F) = \dfrac{P(Y \cap F/\text{click}) * P(\text{click})}{P(Y \cap F)}$

$= \dfrac{0.28 \times 0.3}{0.126} = 0.67$

(Ans)

15> $P(\text{High Risk} | S \cap I) = \dfrac{P(S \cap I / \text{High Risk}) * P(\text{High Risk})}{P(S \cap I)}$

$P(S \cap I/\text{High Risk}) = P(S/\text{High Risk}) * P(I/\text{High Risk})$

$= 0.6 * (1 - 0.3)$

$= 0.42$

$P(S \cap I/\text{Low Risk}) = P(S/\text{Low Risk}) * P(I/\text{Low Risk})$

$= 0.3 * (1-0.7)$

$= 0.3 * 0.3 = 0.09$

$\therefore P(S \cap I) = P(S \cap I / \text{High Risk}) * P(\text{High Risk})$
$+ P(S \cap I / \text{Low Risk}) * P(\text{Low Risk})$

$= (0.42 \times 0.25) + (0.09 * 0.75)$

$= 0.1725$

So, $P(\text{High Risk}/S \cap I) = \dfrac{P(S \cap I / \text{High Risk}) \, P(\text{High Risk})}{P(S \cap I)}$

$= \dfrac{0.42 \times 0.25}{0.1725}$

$= 0.6087$ (Ans)

17) $P(\text{Sci-Fi} / SF \cap PK \cap P) = ?$

we know,

$P(SF \cap PK \cap P / \text{Sci-Fi}) = P(SF/\text{Sci-Fi}) * P(PK/\text{Sci-Fi}) * P(P/\text{SciFi})$

$= 0.7 \times 0.8 \times (1 - 0.5) = 0.28$

$P(SF \cap PK \cap P / Rom\text{-}com) = P(SF / Rom\text{-}com) * P(PK / Rom\text{-}com) * P(P / Rom\text{-}com)$

$= (1 - 0.8) * (1 - 0.6) * 0.5$

$= 0.04$

So, $P(SF \cap PK \cap P) = P(SF \cap PK \cap P / Sci\ Fi) * P(Sci\ Fi) + P(SF \cap PK \cap P / Rom\text{-}com) * P(Rom\text{-}com)$

$= (0.28 \times 0.6) + (0.04 \times 0.4)$

$= 0.184$

So, $P(SciFi / SF \cap PK \cap P) = \dfrac{0.28 \times 0.6}{0.184}$

≈ 0.913

18) $P(\text{High Risk} / S \cap HS \cap Y)$

$$= \frac{P(S \cap HS \cap Y / \text{High Risk}) \times P(\text{High Risk})}{P(S \cap HS \cap Y)}$$

So, $P(S \cap HS \cap Y / \text{High Risk}) = P(S / \text{High Risk})$

$* P(HS / \text{High Risk})$

$* P(Y / \text{High Risk})$

$= 0.7 \times 0.6 \times 0.4$

$= 0.168$

$P(S \cap HS \cap Y | \text{Low Risk}) = P(S / \text{Low Risk})$

$* P(HS / \text{Low Risk}) * P(Y / \text{Low Risk})$

$= (1-0.8) * (1-0.9) * (1-0.85)$

$= 0.003$

So, $P(S \cap HS \cap Y) = P(S \cap HS \cap Y / \text{High Risk}) * P(\text{High Risk})$

$+ P(S \cap HS \cap Y / \text{Low Risk}) * P(\text{Low Risk})$

$$= (0.168 \times 0.3) + (0.003 \times 0.7)$$

$$= 0.0525$$

So, $P(\text{High Risk} / SA\,HS \cap Y) = \dfrac{0.168 \times 0.3}{0.0525}$

$$= 0.96 \quad \underline{\underline{(\text{Ans})}}$$