Association Rule Mining - II

Dr. Faisal Kamiran

Factors Affecting Complexity

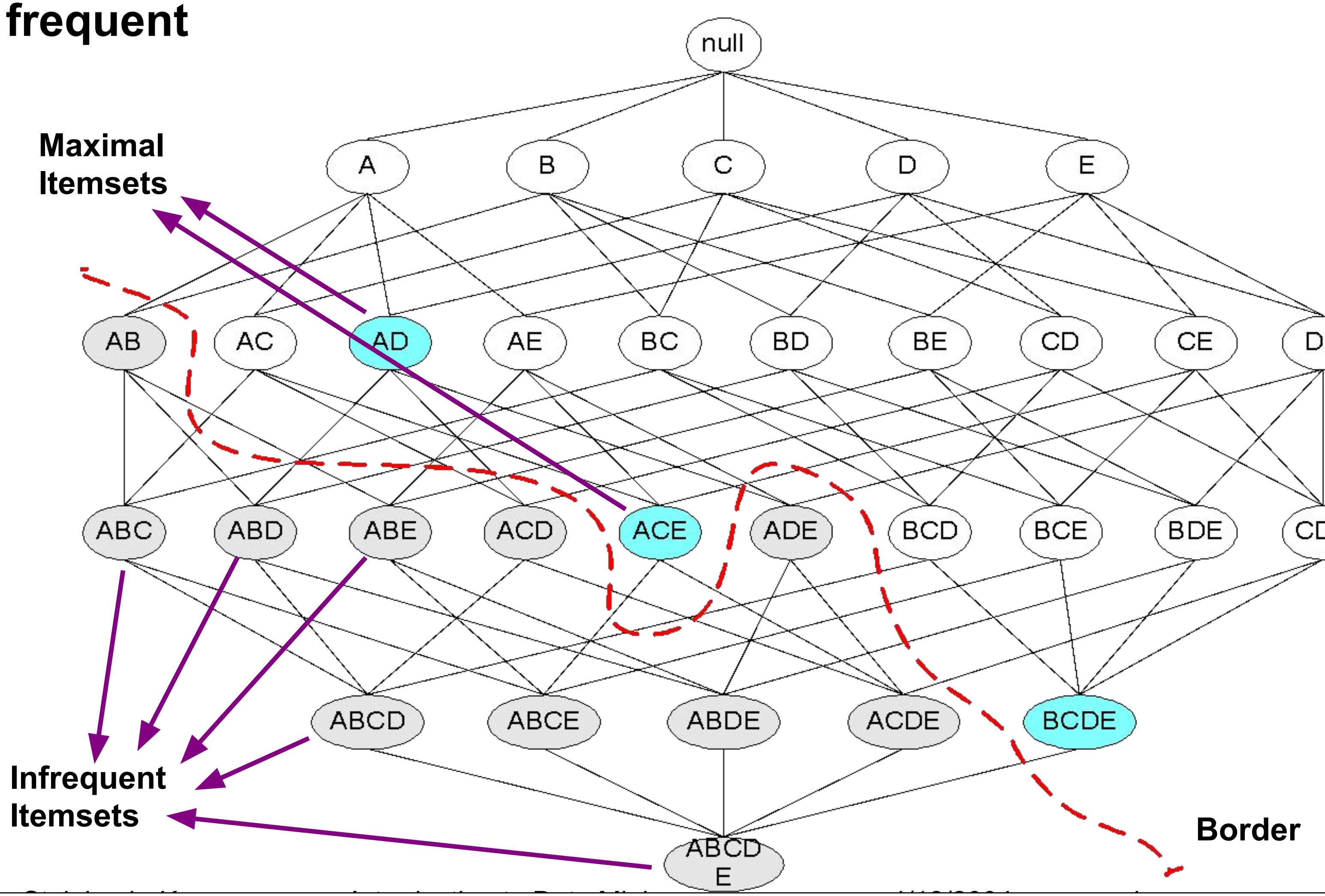
- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - transaction width increases with denser data sets
  - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

Compact Representation

- Practically, too many frequent itemsets
- Need to identify a small representative set of itemsets to derive all itemsets
- Need to have compact representation of frequent itemsets:
  - **Maximal frequent itemsets**
  - **Closed frequent itemsets**

Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets is frequent



Maximal Frequent Itemset

- Provide a valuable representation of dataset that can generate too many frequent itemsets
- We need an efficient algorithms to find maximal frequent itemsets
- Maximal frequent itemsets do not contain support information about their subsets

Closed Itemset

- Minimal representation without losing the support information
- An itemset is closed if none of its immediate supersets has the same support as the itemset

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

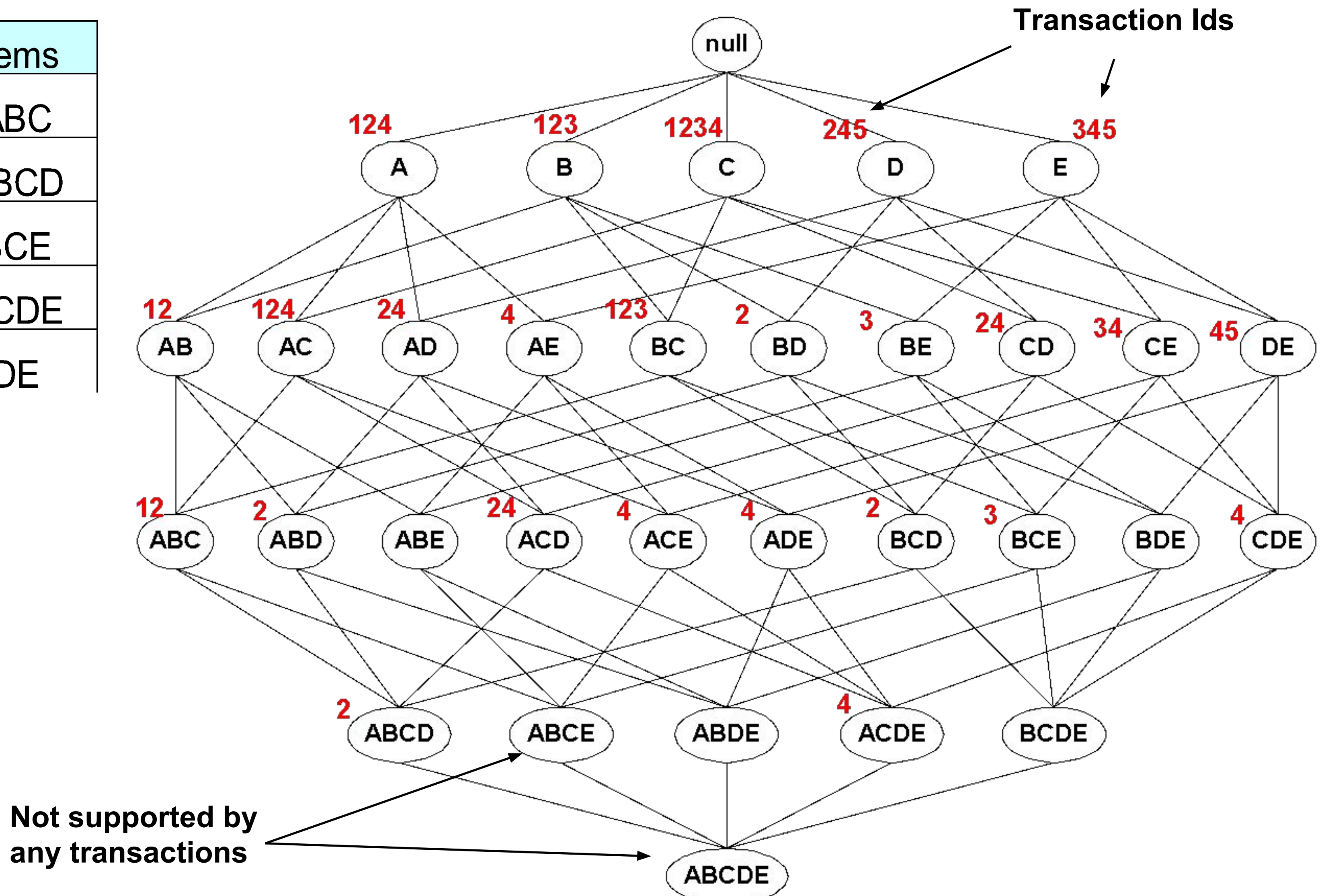
Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

Closed Itemset

- An closed itemset is closed frequent itemset if its support is greater than the min support threshold
- Useful for removing some of the redundant association rules
- The rule X → Y is redundant if another rule X\` → Y\` exists with where $X \in X\`\ \ and\, Y \in Y\`$ and the support and confidence for both rules are identical.

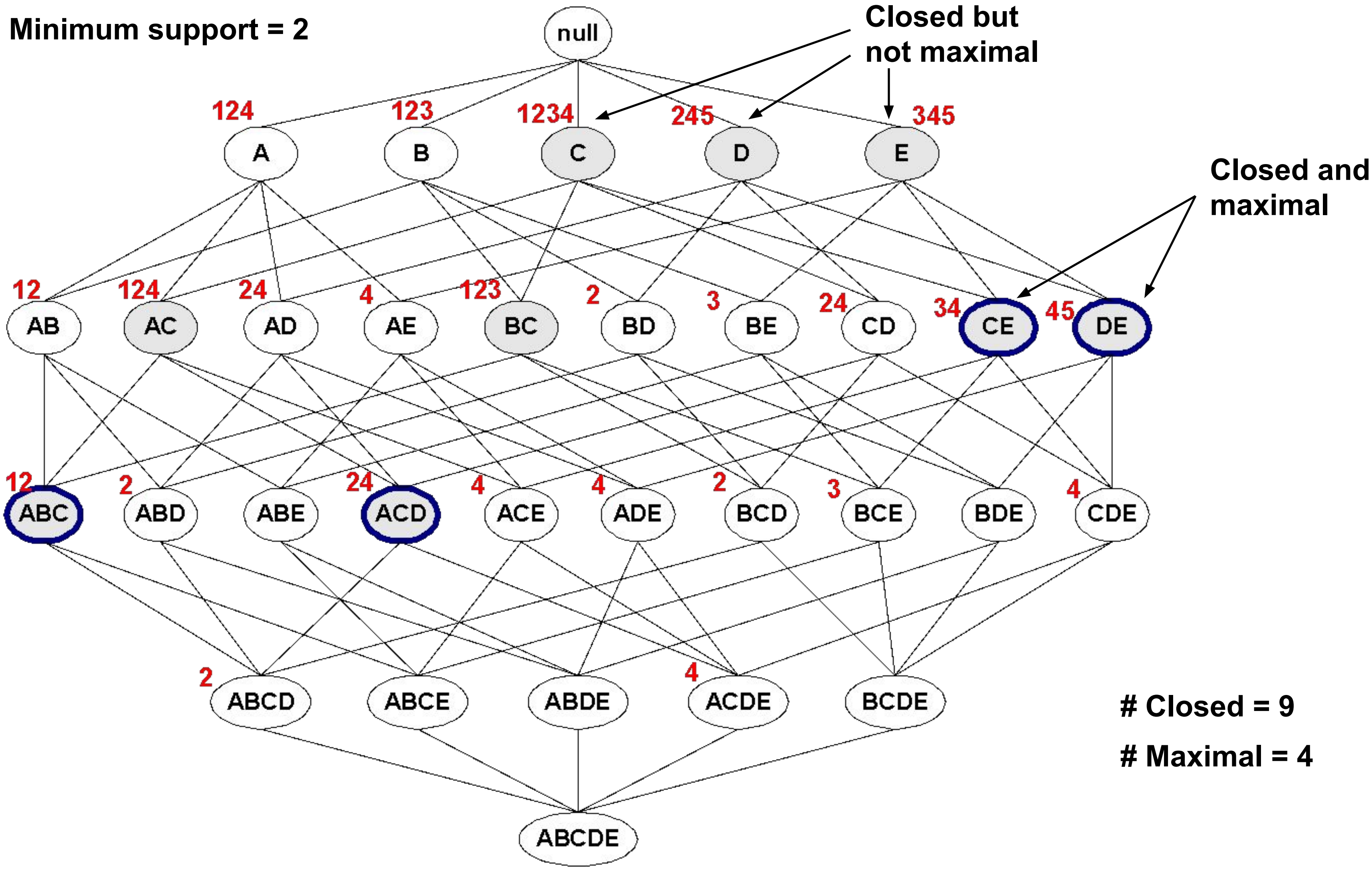
Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal vs Closed Frequent Itemsets

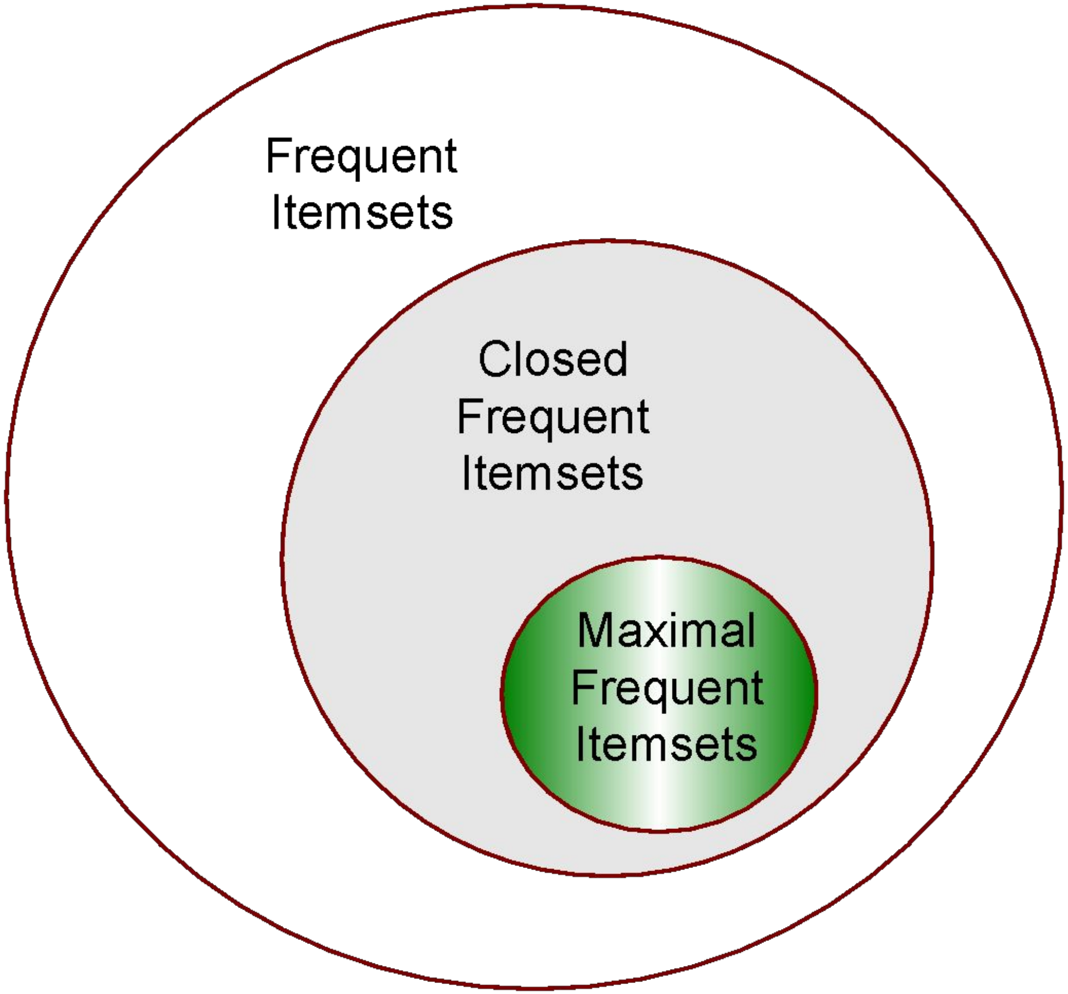
Minimum support = 2



Closed = 9

Maximal = 4

Maximal vs Closed Itemsets



Alternative Methods for Frequent Itemset Generation

- Performance of Apriori algorithm degrades over dense data because of increased width of transactions.
- Need to overcome this limitation and to improve the efficiency
- Conceptually the search for frequent itemset can be viewed as a traversal on the itemset lattice
- Need better search and traversal strategies

FP-growth Algorithm

- Use a compressed representation of the database using an FP-tree

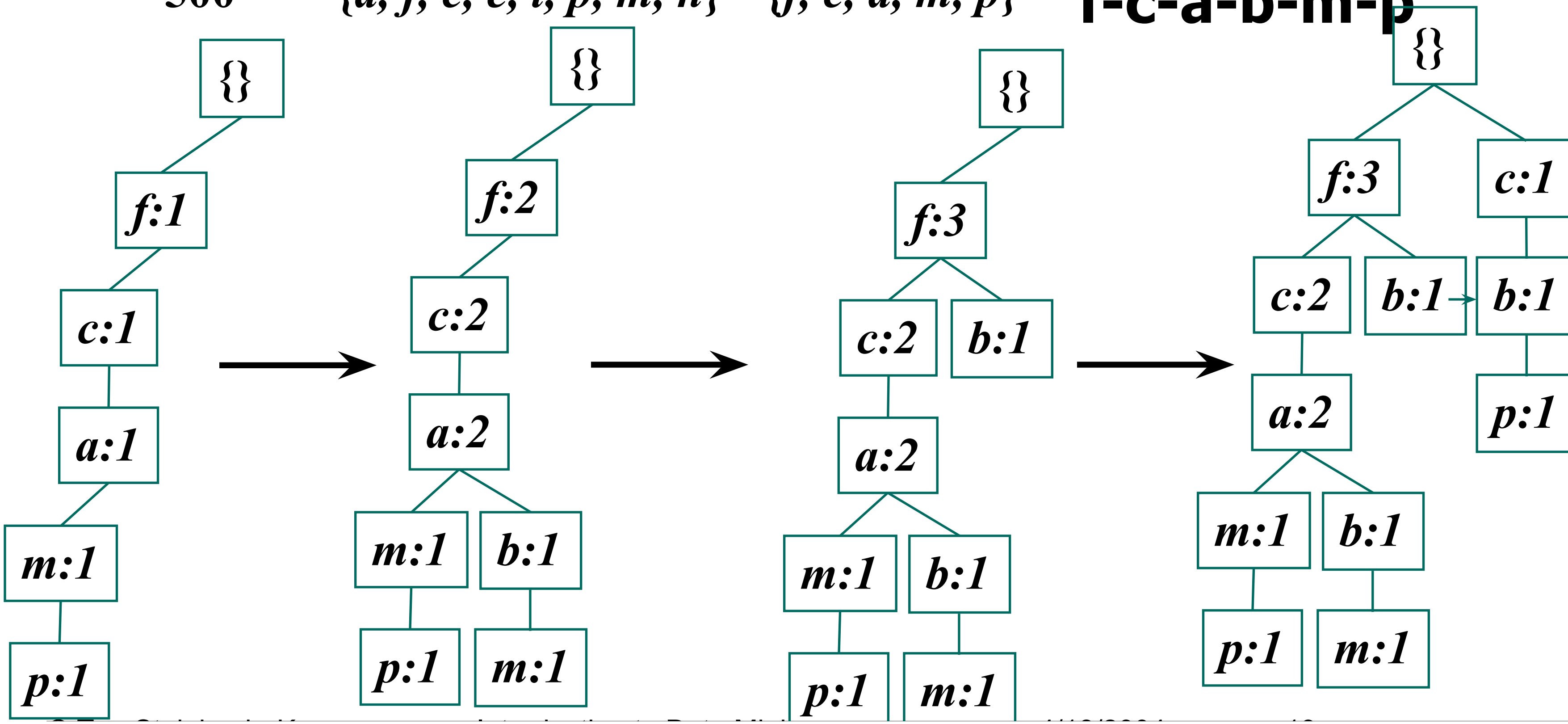
- Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets
- Steps:
  - FP tree construction
  - Conditional pattern base construction
  - Conditional FP-trees
  - Frequent pattern generation

FP-Tree Construction

TID	*Items bought*	*(ordered) frequent items*
100	*{f, a, c, d, g, i, m, p}*	*{f, c, a, m, p}*
200	*{a, b, c, f, l, m, o}*	*{f, c, a, b, m}*
300	*{b, f, h, j, o, w}*	*{f, b}*
400	*{b, c, k, s, p}*	*{c, b, p}*
500	*{a, f, c, e, l, p, m, n}*	*{f, c, a, m, p}*

min_support = 3

F-list = f-c-a-b-m-p



FP-Tree Construction

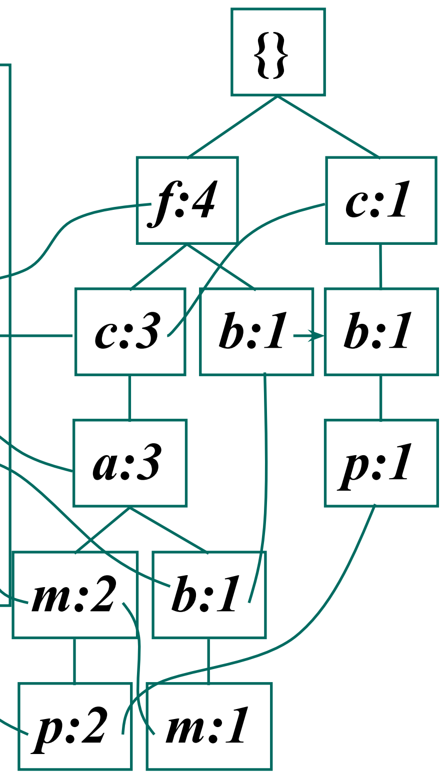
TID	*Items bought*	*(ordered) frequent items*
100	*{f, a, c, d, g, i, m, p}*	*{f, c, a, m, p}*
200	*{a, b, c, f, l, m, o}*	*{f, c, a, b, m}*
300	*{b, f, h, j, o, w}*	*{f, b}*
400	*{b, c, k, s, p}*	*{c, b, p}*
500	*{a, f, c, e, l, p, m, n}*	*{f, c, a, m, p}*

min_support = 3

1. **Scan DB once, find frequent 1-itemset (single item pattern)**
2. **Sort frequent items in frequency descending order, f-list**
3. **Scan DB again, construct FP-tree**

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



F-list = f-c-a-b-m-p

Partition Patterns and Databases

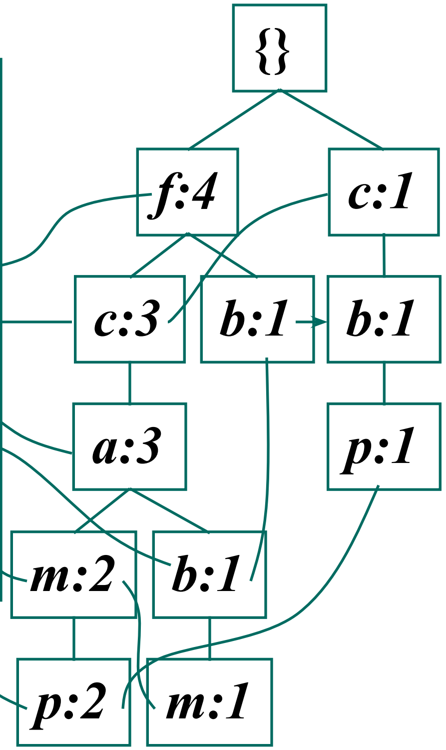
- Frequent patterns can be partitioned into subsets according to f-list
  - F-list = f-c-a-b-m-p
  - Patterns containing p
  - Patterns having m but no p
  - …
  - Patterns having c but no a nor b, m, p
  - Pattern f
- Completeness and non-redundency

Conditional Pattern Base Construction

- **Conditional Pattern Base** is a sub-database which consists of the set of prefix paths in the FP tree co-occurring with the suffix pattern or item. For example *prefix paths (fcam, cb)* of item *p* to form *p*'s conditional pattern base.

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	

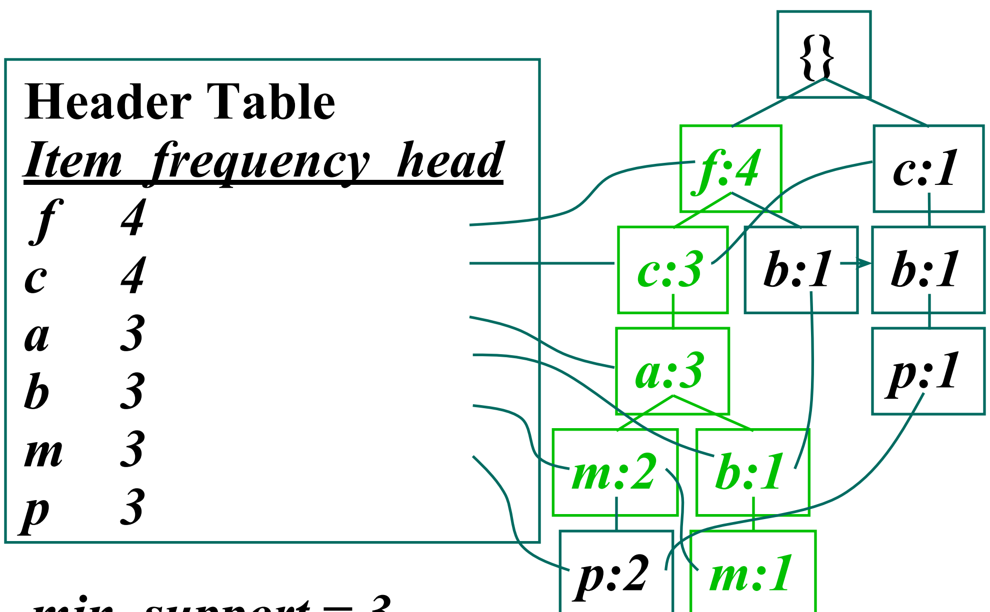


Conditional **pattern bases**

item	*cond. pattern base*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

From Conditional Pattern-bases to Conditional FP-trees

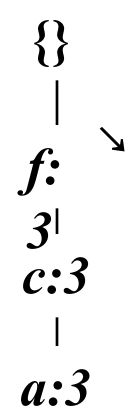
- For each pattern-base
  - Accumulate the count for each item in the base
  - Construct the FP-tree for the frequent items of the pattern base



min_support = 3

m-conditional **pattern base:**

fca:2, fcab:1



m-conditional **FP-tree**

All frequent patterns relate to *m*

m,

fm, cm, am,

fcm, fam, cam,

fcam

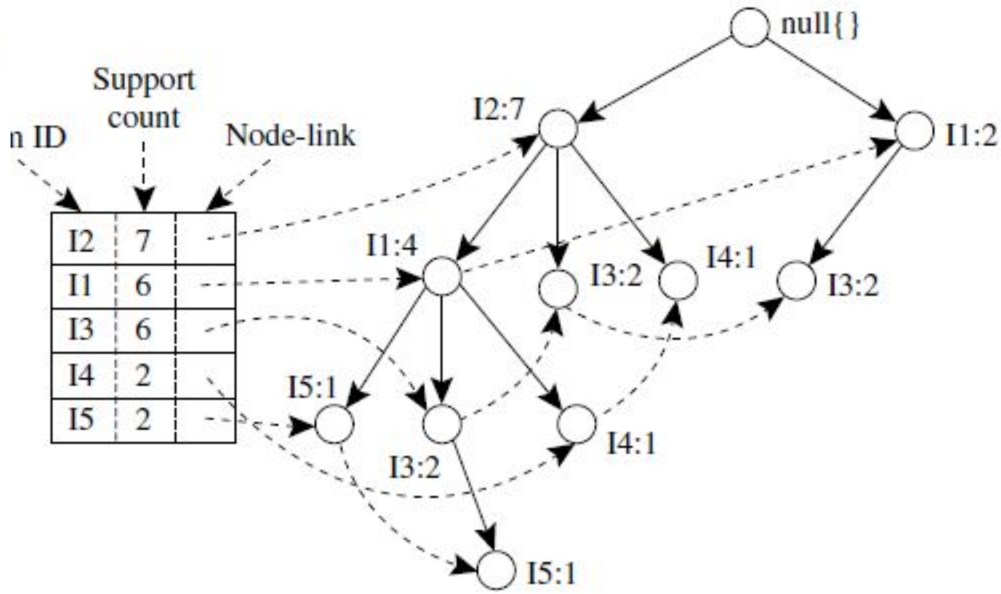
FP Growth: Example

Min supp=2

Data:

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

FP Tree:



Mining the FP-Tre

Item	*Conditional Pattern Base*	*Conditional FP-tree*	*Frequent Patterns Generated*
I5	{{I2, I1: 1}, {I2, I1, I3: 1}}	⟨I2: 2, I1: 2⟩	{I2, I5: 2}, {I1, I5: 2}, {I2, I1, I5: 2}
I4			
I3			
I1			

Benefits of the FP-tree Structure

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are gone
  - Items in frequency descending order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database (not count node-links and the *count* field)

ECLAT

- For each item, store a list of transaction ids (tids)

Horizontal Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

TID-list

ECLAT

- Determine support of any k-itemset by intersecting tid-lists of two of its (k-1) subsets.

A
1
4
5
6
7
8
9

$\wedge$

B
1
2
5
7
8
10

$\rightarrow$

AB
1
5
7
8

- 3 traversal approaches:
  – top-down, bottom-up and hybrid
- Advantage: very fast support counting
- Disadvantage: intermediate tid-lists may become too large for memory

Rule Generation

- Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

    $ABC \rightarrow D$, $ABD \rightarrow C$, $ACD \rightarrow B$, $BCD \rightarrow A$,
    $A \rightarrow BCD$, $B \rightarrow ACD$, $C \rightarrow ABD$, $D \rightarrow ABC$
    $AB \rightarrow CD$, $AC \rightarrow BD$, $AD \rightarrow BC$, $BC \rightarrow AD$,
    $BD \rightarrow AC$, $CD \rightarrow AB$,

- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \varnothing$ and $\varnothing \rightarrow L$)

Rule Generation

- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property

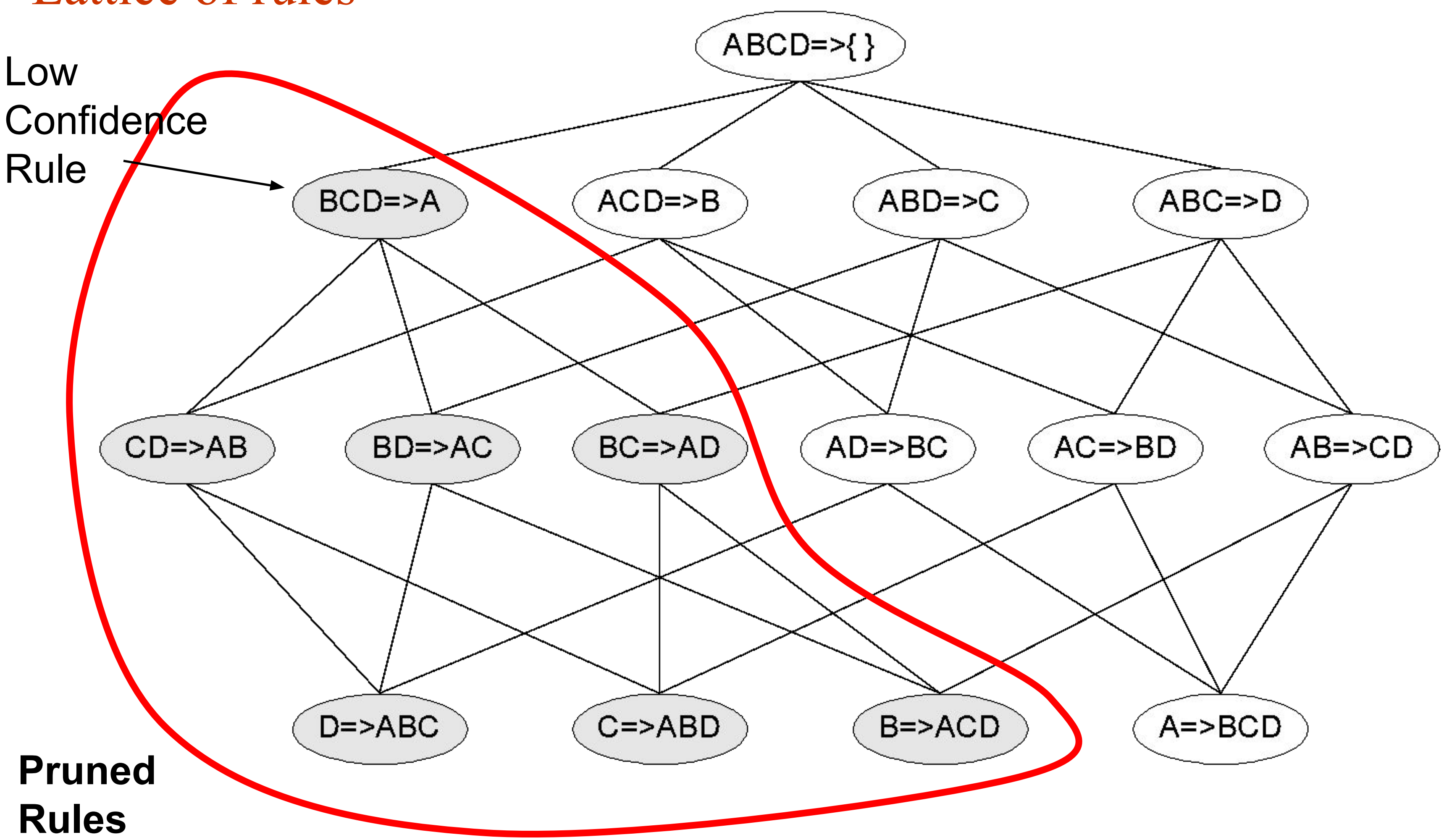
    $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

  - But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., $L = \{A,B,C,D\}$:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

Rule Generation for Apriori Algorithm

Lattice of rules

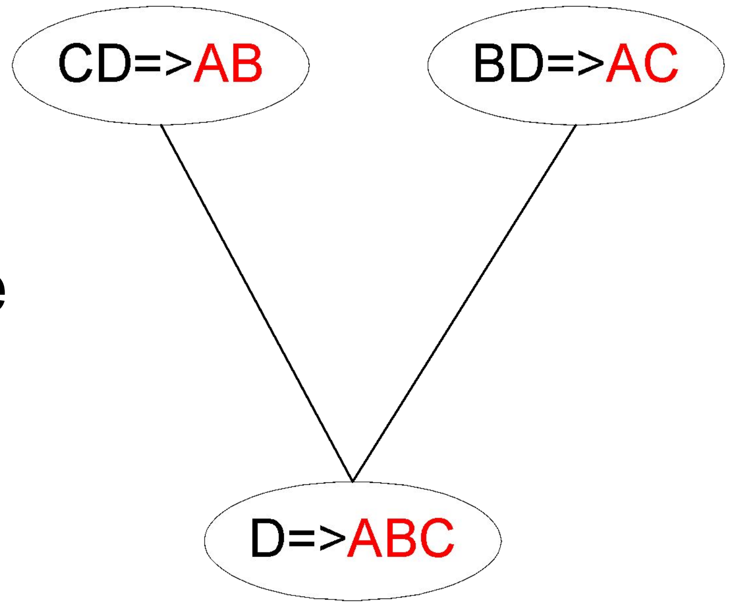


Rule Generation for Apriori Algorithm

- Candidate rule is generated by merging two rules that share the same prefix in the rule consequent

- join(CD=>AB,BD=>AC) would produce the candidate rule D => ABC

- Prune rule D=>ABC if its subset CD=>AB does not have high confidence



Pattern Evaluation

- Association rule algorithms tend to produce too many rules
  - many of them are uninteresting or redundant
  - Redundant if $\{A,B,C\} \rightarrow \{D\}$ and $\{A,B\} \rightarrow \{D\}$ have same support & confidence

- Interestingness measures can be used to prune/rank the derived patterns

- In the original formulation of association rules, support & confidence are the only measures used

Computing Interestingness Measure

- Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	$\|T\|$

f_{11}: support of X and Y
f_{10}: support of X and $\overline{Y}$
f_{01}: support of $\overline{X}$ and Y
f_{00}: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea → Coffee

- Confidence= P(Coffee|Tea) = 0.75
- but P(Coffee) = 0.9
- Although confidence is high, rule is misleading
- P(Coffee|$\overline{\text{Tea}}$) = 0.9375

Statistical Independence

- Population of 1000 students
  - 600 students know how to swim (S)
  - 700 students know how to bike (B)
  - 420 students know how to swim and bike (S,B)

  - $P(S \wedge B) = 420/1000 = 0.42$
  - $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

  - $P(S \wedge B) = P(S) \times P(B)$ => Statistical independence
  - $P(S \wedge B) > P(S) \times P(B)$ => Positively correlated
  - $P(S \wedge B) < P(S) \times P(B)$ => Negatively correlated

Statistical-based Measures

- Measures that take into account statistical dependence

$$Lift = \frac{P(Y \mid X)}{P(Y)}$$

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

⇒ Lift = 0.75/0.9= 0.8333 (< 1, therefore is negatively associated)