2 | Rootfinding

Newton's Method

Newton's Method

The Newton's method is to find a root x close to your initial guess x_0 a function f, i.e. $f(x) = 0$. You start with the function f, the intial guess x_0, a tolerance ε, and a max number of iteration $ITMAX$.

1. Calculate the next iteration $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, this is equivalent of finding the zero of the linear approximation of f and x_n, which is $f'(x_n)(x - x_n) + f(x_n) = 0$.
2. If convergence is satisfactory, i.e. $|f(x_{n+1})| < \varepsilon$, or $|x_{n+1} - x_n| < \varepsilon$, or iteration is $ITMAX$, return x_{n+1} and stop iterating.

Secant's Method

Secant's Method

It is the same as Newton's method, except you replace $f'(x_n)$ by its approximation $f'(x_n) \simeq \frac{f(x_n)-f(x_{n-1})}{x_n-x_{n-1}}$. You start with the function f, the intial guess x_0, a tolerance ε, and a max number of iteration $ITMAX$.

1. Calculate the next iteration $x_{n+1} = x_n - \frac{x_n-x_{n-1}}{f(x_n)-f(x_{n-1})} f(x_n)$, this is equivalent of finding the zero of the linear approximation of f and x_n, which is $f'(x_n)(x - x_n) + f(x_n) = 0$.
2. If convergence is satisfactory, i.e. $|f(x_{n+1})| < \varepsilon$, or $|x_{n+1} - x_n| < \varepsilon$, or iteration is $ITMAX$, return x_{n+1} and stop iterating.

Fixed point

Fixed Point

Fixed point iteration is a method to find the fixed point of a function, $g(x) = x$. It can also be used to find the root of a function f by setting up $g(x) = f(x) - x$. You start with the function g, the intial guess x_0, a tolerance ε, and a max number of iteration $ITMAX$.

1. Calculate the next iteration $x_{n+1} = g(x_n)$.
2. If convergence is satisfactory, i.e. $|g(x_{n+1}) - x_{n+1}| < \varepsilon$, or $|x_{n+1} - x_n| < \varepsilon$, or iteration is $ITMAX$, return x_{n+1} and stop iterating.

The fixed point iteration algorithm does not always converges, it can be shown that it does converges when $|g'(x)" < 1$.