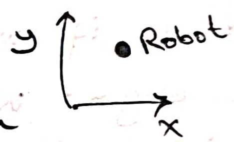
Name : MD Faizal Karim
Roll No : 202212100 4

1(a) Rigid body transformation : It is a operation that combines rotation and translation, to move an object such that the shape and size of the object remains the same. For eg, given an object at origin with no rotation, we can rotate it ~~by~~ to the desired orientation by multiplying the points using a 3x3 rotational matrix. This rotation matrix preserves the shape and size of the object.

(b) Configuration space : It is the space of all posible positions and orientation that a robot can achieve. For eg, the configuration space of a circular robot that can move freely in 2D space is R^2.



(c) Workspace : It refers to the physical region of space within which the robot can operate. For eg, a robotic arm in industry is used for pick and place, works in the 3D space. ~~Hence, its workspace is~~ Hence, the workspace is a sphere of radius the maximum length of the robot.

(d) Task space : Task space refers to the space in which tasks are defined and executed, usually expressed in terms of the end effector's position and orientation. For eg, a robotic arm working at a conveyer belt will have its task space as the region near the conveyer belt. This space is a subset of the workspace.

(e) Degree of freedom : It is the minimum number of coroodinate needed to represent the configuration space. For eg, a prismatic joint that can only move in one direction has a dof of 1. A cylindrical joint has 2 DOF (one for translation and one for rotation).

2 (a) Implicit ~~representation~~ parameterization represents the n dimensional space in terms of more than n dimensions which having constraints on these extra dimensions which reduces the no. degrees of freedom.

Explicit parametrization simply uses n ~~dimensions~~ parameters to represent n dimensional space.

Pros and cons ⟶

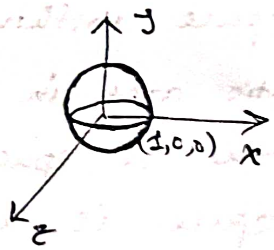
(i) Implicit uses more parameters but avoids singularities.

(ii) Explicit is straight forward and uses the same no. of parameters as needed but suffers from singularities.

(b) Coordinate singularity is when the choice of coordinates fails to uniquely describe a point in the space, leading to infinite ways to represent that point.

For eg., if we have a sphere of unit radius, centered at the origin. We represent each coordinate using the latitude and longitude.

$$x(\theta,\phi) = [\cos\theta\sin\phi,\ \sin\theta\cos\phi,\ \sin\phi]^T$$



where $\theta \in [-\pi, \pi]$ is the longitude and

$\phi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ is the latitude.

However, $x(\theta, \frac{\pi}{2}) = [0\ 0\ 1]^T$ and

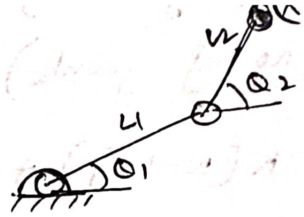
$x(\theta, -\frac{\pi}{2}) = [0\ 0\ -1]^T$

Hence for any θ and $\phi = \frac{\pi}{2}$ we can represent the point $[0\ 0\ 1]^T$ and for any θ and $\phi = -\frac{\pi}{2}$ we can represent $[0\ 0\ -1]^T$.

Therefore singularity occurs at the points $[0\ 0\ 1]^T$ and $[0\ 0\ -1]^T$.

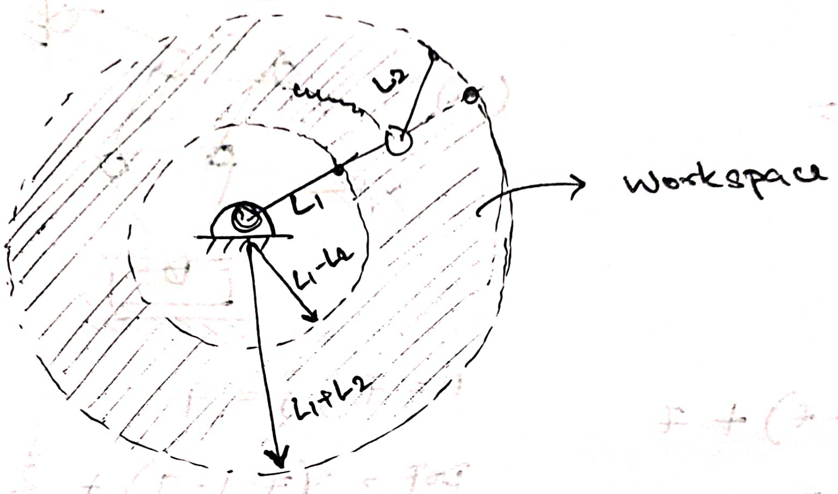
6. $x = 2\cos\theta_1 + \cos(\theta_1+\theta_2)$

$y = 2\sin\theta_1 + \sin(\theta_1+\theta_2)$



(a) Configuration space : $S^1 \times S^1$

(b) Workspace of robot : Annulus.



3(a) Joint space variables : $[\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]$

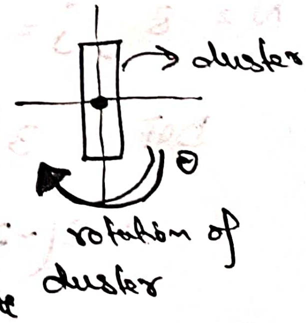
θ_i is the i^{th} joint and $\theta_i \in S^1$

(b) Task space variables = pose of the end effector

$= [x, y, z, \theta]$

where x, y, z represents the eef position and ~~α, β, γ are~~ θ is ~~the roll, pitch, yaw~~ angle of the duster.

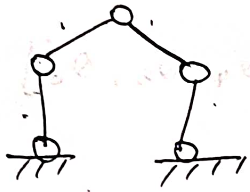
Topological space = $\mathbb{R}^3 \times S^1$



(c) Workspace = $\mathbb{R}^3 \times S^1 \times S^1 \times S^1$

This is assuming the endeffector can ~~orient~~ rotate in all the 3 directions. For the duster task, we are constraining the end effector to rotate in only one axis.

4 (a)



$N = 5$ (no. of links)

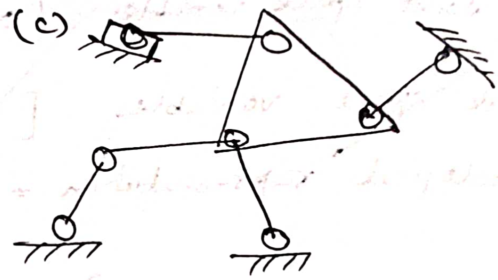
$j = 5$ (no. of joints)

$\therefore DOF = K(N-1-j) + \sum_{i=1}^{j} f_i$

$= 3(5-1-5) + 5$

$= -3+5 = 2$

(c)



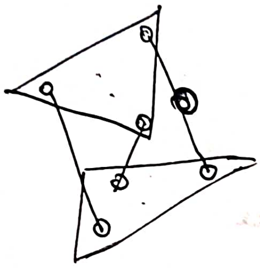
$N = 8,\ j = 10$

$\therefore DOF = 3(8-1-10) + \sum_{i=1}^{j} f_i$

$= 3(-3) + 10$

$= 1.$

(b)



$N = 6$

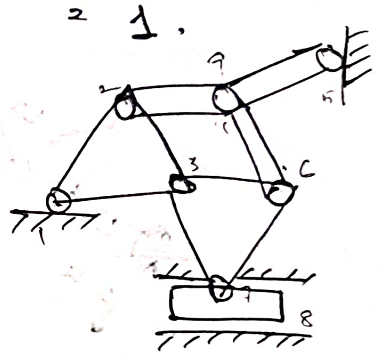
$j = 7$

$\therefore DOF = 3(6-1-7) + 7$

$= -6+7$

$= 1.$

(d)

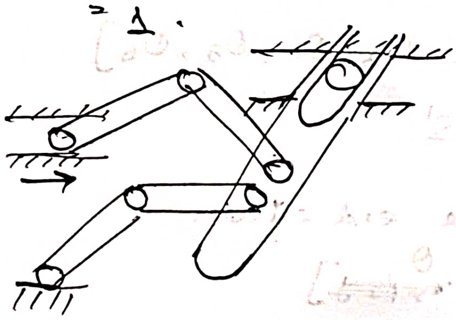


$N = 7,\ j = 9$

$DOF = 3(7-1-9) + \sum_{i=1}^{j} f_i$

$= 3(-3) + 9 = 0$

(e)



$N = 8$, $j =$ ~~7+2 (2 for the fork joint)~~ 9

$\therefore DOF = 3(8-9-1) + \sum_{i=1}^{j} f_i$

$= 3(-2) + \underbrace{6\times1}_{\text{(revolute)}} + \underbrace{2\times1}_{\text{(prismatic)}} + 2\text{ (fork)}$

$= -6 + 10 = 4$

5(a) The mechanism consists of 4 similar arms.

Each arm has a 4 bar linkage with revolute joints.

We consider the serial subchain as a ground along with a 6 dof joint.

DOF = 4 bar linkage dof + 4 revolute + 1 prismatic

= 1 + 4 + 1 = 6

So, dof of the subchain = 6

Applying grubler's formula

$N = 1$ plate + 1 ground $= 2$

$K = 6$ (3D space)

$J = 4$

$\therefore$ dof $= 6(2-1-4) + \sum_{i=1}^{4} f_i$

$= -6 + 24 = 18$

$\therefore$ DOF $= 18$

(b) given we have n legs.

Applying grubler's equation,

$$DOF = K(N-1-j) + \sum_{i=1}^{j} f_i$$

$$= 6(2-1-n) + 6n$$

$$= 6 - 6n + 6n = 6$$

$\therefore$ the generalized dof doesn't change with n legs.