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Q 1.1) DH table of the manipulator →

Joint	α	a	d	θ
1	$\frac{\pi}{2}$	0	l_1	θ_1
2	0	l_3	$-l_2$	θ_2
3	$-\frac{\pi}{2}$	l_4	0	θ_3

Q 2.1) For the given system,

$$x = -l_2 \sin\theta + l_3 \cos\theta_2 \cos\theta_1 + l_4 \cos(\theta_1+\theta_3) \cos\theta_1$$

$$y = l_2 \cos\theta + l_3 \cos\theta_2 \sin\theta_1 + l_4 \cos(\theta_2+\theta_3) \sin\theta_1$$

$$z = l_1 + l_3 \sin\theta_2 + l_4 \sin(\theta_2+\theta_3)$$

Let,

$$p = l_3 \sin\theta_2 + l_4 \sin(\theta_2+\theta_3)$$

$$q = l_3 \cos\theta_2 + l_4 \cos(\theta_2+\theta_3)$$

$$\therefore p^2+q^2 = l_3^2 + l_4^2 + 2l_3 l_4 \cos\theta_3 \quad \text{--- (i)}$$

Next,

$$z - l_1 = p$$

$$\Rightarrow (z-l_1)^2 = p^2 \quad \text{--- (ii)}$$

$$x^2+y^2 = l_2^2 + q^2 \quad \text{--- (iii)}$$

$\therefore$ Using (ii) & (iii),

$$x^2+y^2 - l_2^2 + (l_1-z)^2 = l_3^2 + l_4^2 + 2l_3 l_4 \cos\theta_3$$

$$\Rightarrow \theta_3 = \cos^{-1}\left[\frac{x^2+y^2+z^2+l_1^2-l_2^2-l_3^2-l_4^2-2l_1 z}{2l_3 l_4}\right] \quad \text{--- (iv)}$$

Substituting (iv) in z,

$$\Rightarrow z = l_1 + l_3 \sin\theta_2 + l_4 \sin(\theta_2+\theta_3) \quad \text{--- (v)}$$

we consider,

$$\frac{x + l_2 \sin\theta_1}{\cos\theta_1} = \frac{y - l_2 \cos\theta_1}{\sin\theta_1}$$

$$\Rightarrow x \sin\theta_1 + l_2 \sin^2\theta_1 = y \cos\theta_1 - l_2 \cos^2\theta_1$$

$\Rightarrow x\sin\theta_1 - y\cos\theta_1 + l_2 = 0$

$\Rightarrow (x\sin\theta_1 - y\cos\theta_1 + l_2)^2 = 0$

Let $a = -y$, $x = b$, $c = l_2$

$a = r\cos\alpha$, $b = r\sin\alpha$, $c = \sqrt{a^2+b^2}$

$\tan\alpha = \frac{b}{a} \Rightarrow \alpha = \tan^{-1}\left(\frac{b}{a}\right)$

$r\cos(\theta_1 \alpha) = \frac{-c}{\sqrt{a^2+b^2}}$

$\therefore \theta_1 = \pm\cos^{-1}\left[\frac{-c}{\sqrt{a^2+b^2}}\right] + \arctan[b, a]$

$\Rightarrow \theta_1 = \pm\cos^{-1}\left[\frac{-l_2}{\sqrt{x^2+y^2}}\right] + \arctan\left(\overset{x,-y}{\cancel{b,a}}\right)$ —— (vi)

Again, substituting (vi) in (v), we calculate θ_2