# Data Structures
Instructor: Hafiz Tayyeb Javed

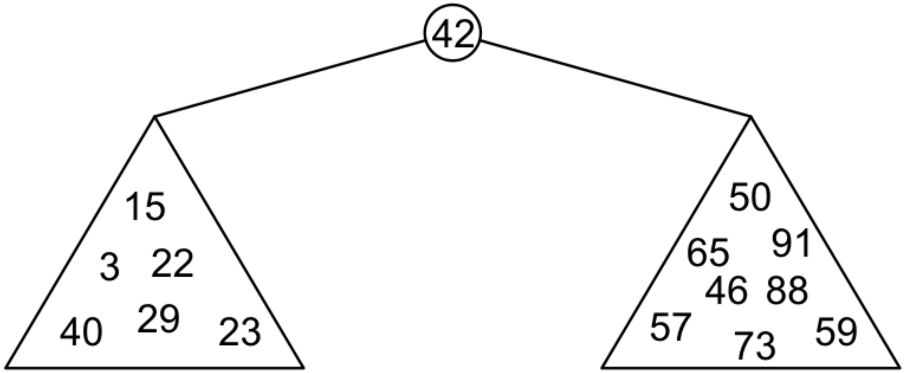
**17. Binary Search Tree**

**Weeek-07-Lecture-02**

# Binary Search Tree (BST)

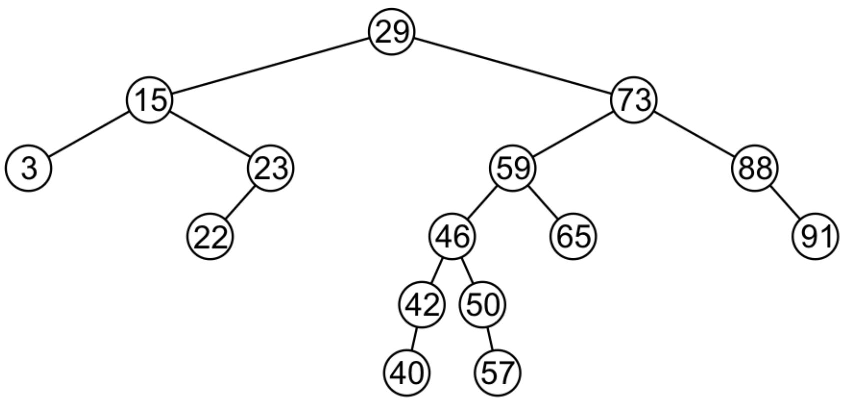
- With a binary tree, we can dictate an order on the two children

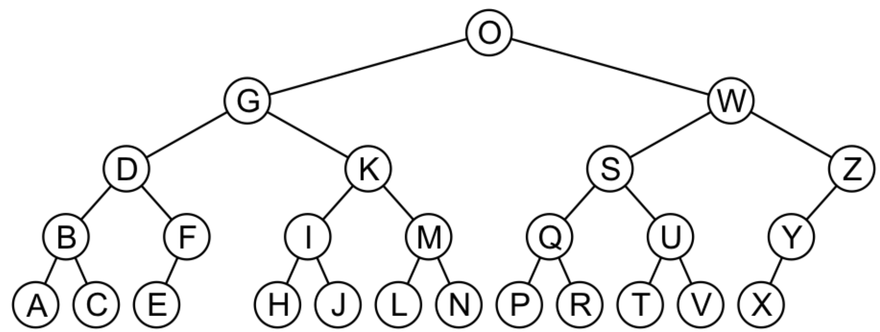
- Binary Search Tree (BST) defines the following order:
  - All elements in the left sub-tree to be less than the element stored in the root node, and
  - All elements in the right sub-tree to be greater than the element in the root object

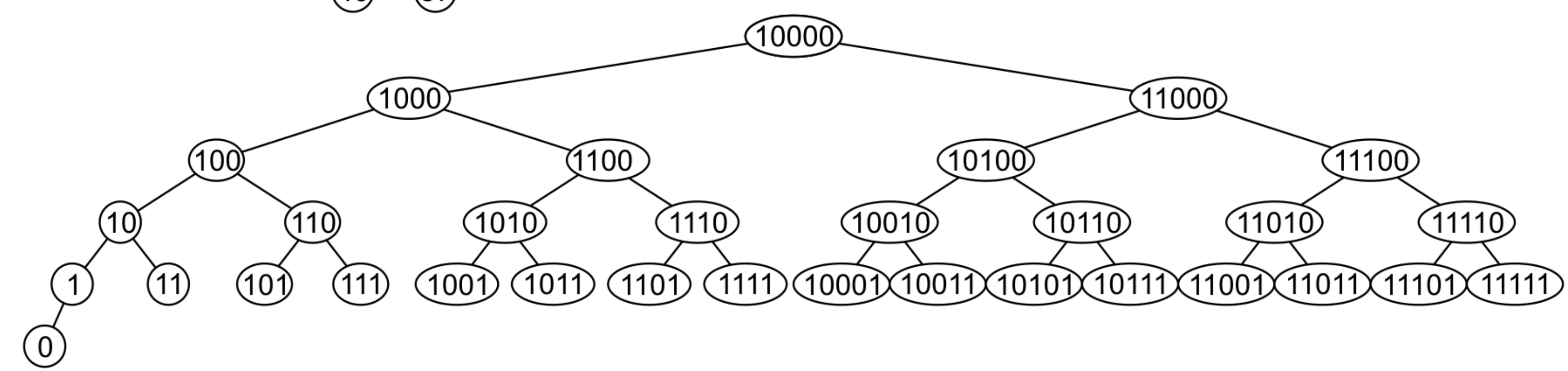



subtrees will themselves be binary search trees

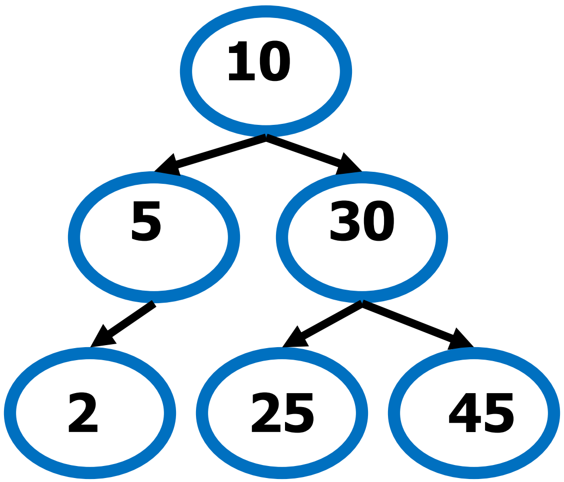
# Binary Search Tree (BST) – Example (1)



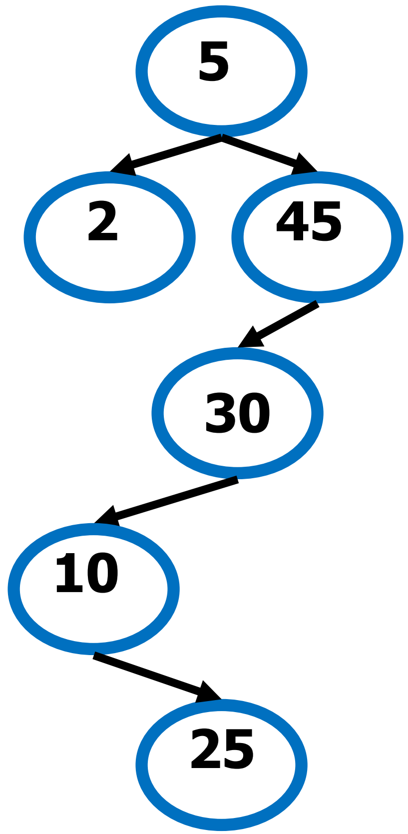



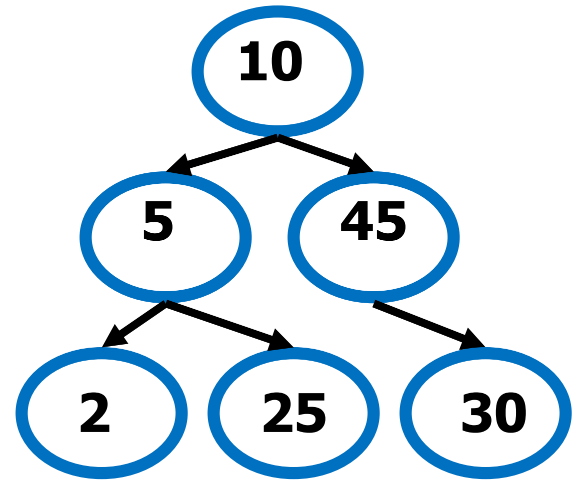
# Binary Search Tree (BST) – Example (2)




BST




BST




Not BST

# BST Operations

- Many operations one can perform on a binary search tree
  - Creating a binary search tree
  - Finding a node in a binary search tree
  - Inserting a node into a binary search tree
  - Deleting a node in a binary search tree
  - Traversing a binary search tree
- In the following, we will examine the algorithms and examples for all of the above operations

# Creating BST

- A simple class that implements a binary tree to store integer values
  - A class called `IntBinaryTree`

- Node of binary search tree

```
struct TreeNode
{
     int value;
     TreeNode *left;
     TreeNode *right;
};
```

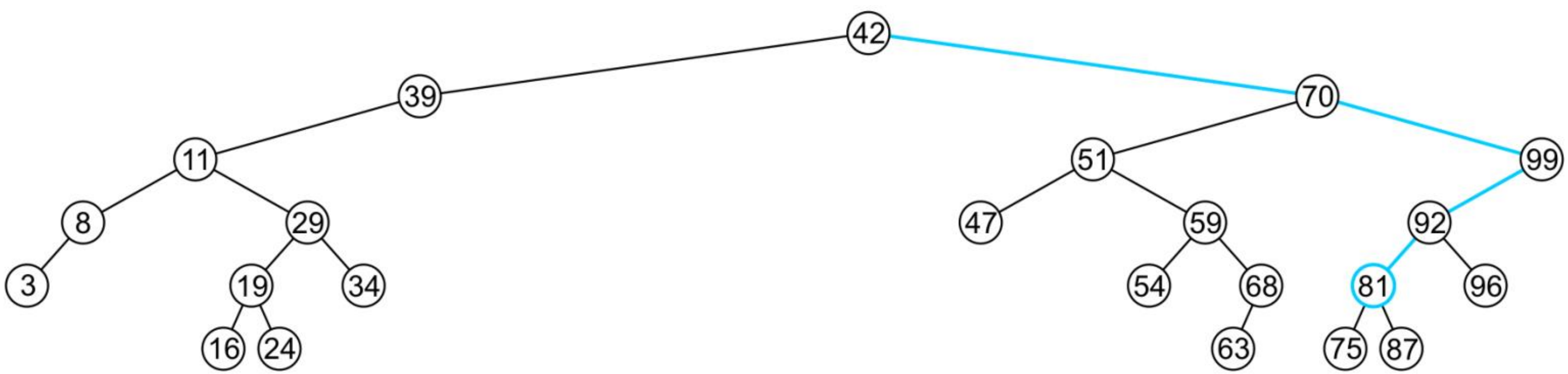
# Creating BST – Class Definition

```
class IntBinaryTree {
   private:
      TreeNode *root;  // Pointer to the root of BST

      void destroySubTree(TreeNode *); //Recursively delete all tree nodes
      void deleteNode(int, TreeNode *&);
      void makeDeletion(TreeNode *&);
      void displayInOrder(TreeNode *);
      void displayPreOrder(TreeNode *);
      void displayPostOrder(TreeNode *);

   public:
      IntBinaryTree()              { root = NULL; }
      ~IntBinaryTree()             { destroySubTree(root); }
      void insertNode(int);
      bool find(int);
      void remove(int);
      void showNodesInOrder()      { displayInOrder(root);   }
      void showNodesPreOrder()     { displayPreOrder(root);  }
      void showNodesPostOrder()    { displayPostOrder(root); }
};
```

# Finding a Node in BST

- Recall that a BST has the following key property (invariant):
  - Smaller values in left sub-tree
  - Larger values in right sub-tree

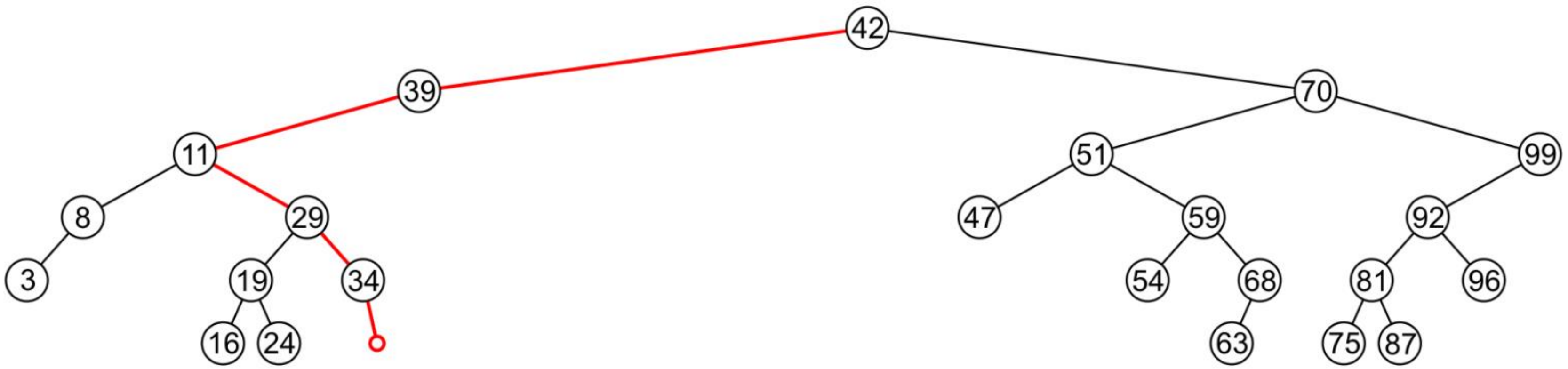
- For example: `find (81)`
  - Returns true if found



# Finding a Node in BST

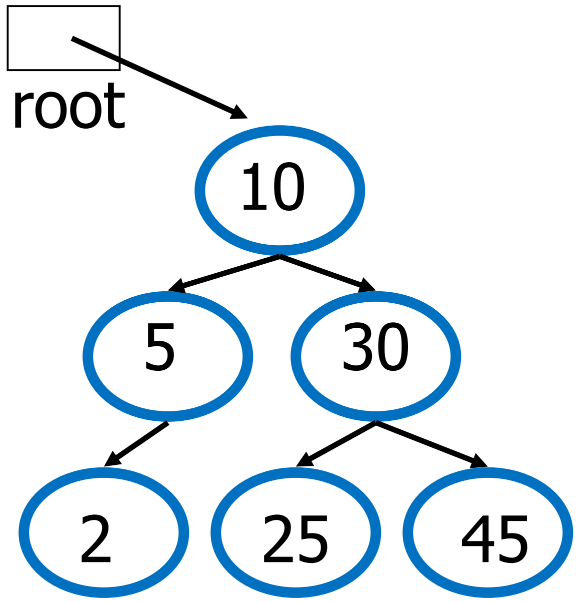
- Recall that a BST has the following key property (invariant):
  - Smaller values in left sub-tree
  - Larger values in right sub-tree

- For example: `find (36)`
  - Returns false if not found



# Finding a Node in BST

- Example: `find(2)`



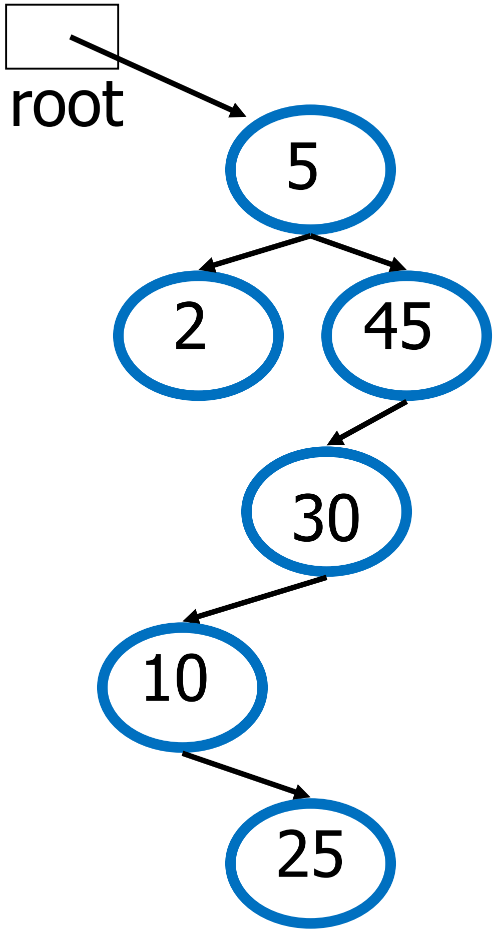

10 > 2, left

5 > 2, left

2 = 2, found

# Finding a Node in BST

- Example: `find(25)`




5 < 25, right

45 > 25, left

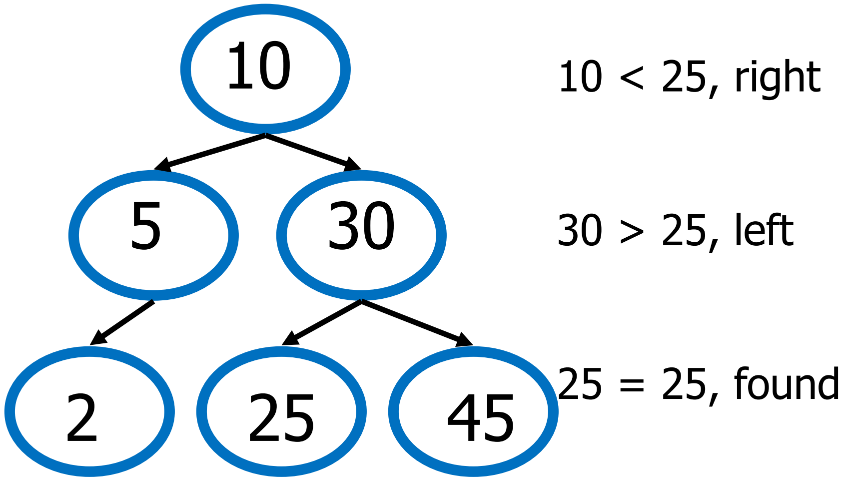
30 > 25, left

10 < 25, right

25 = 25, found

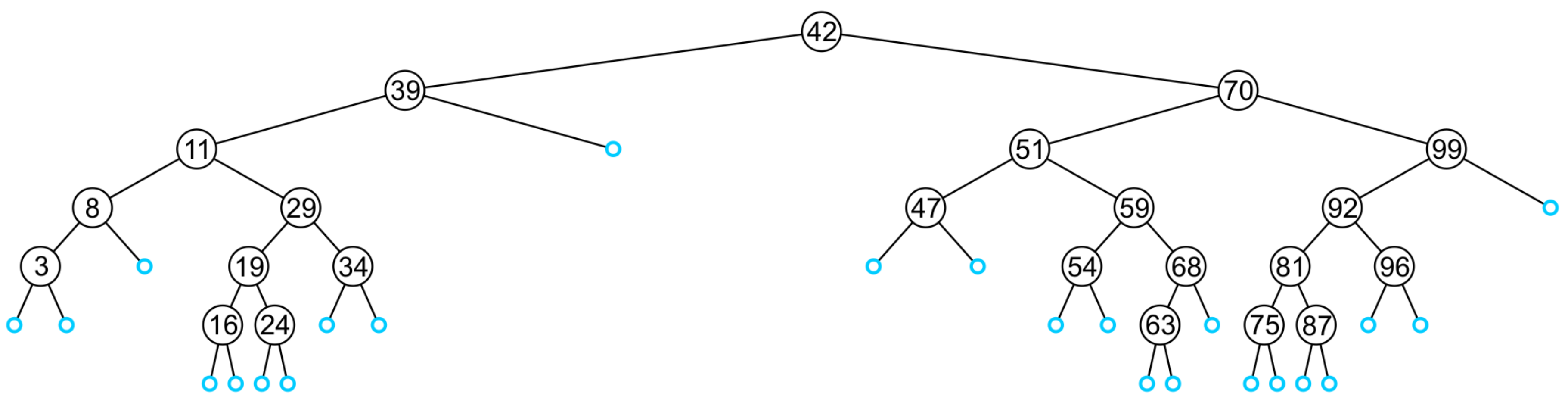
# Finding a Node in BST – Implementation

```
bool IntBinaryTree::Find(int num){
   // The function starts from the root
   TreeNode *nodePtr = root;

   while (nodePtr) {
      if (nodePtr->value == num)
         return true;  // value is found
      else if (num < nodePtr->value)
         nodePtr = nodePtr->left;
      else
         nodePtr = nodePtr->right;
   }
   return false; // value not found
}
```



# Inserting a Node in BST

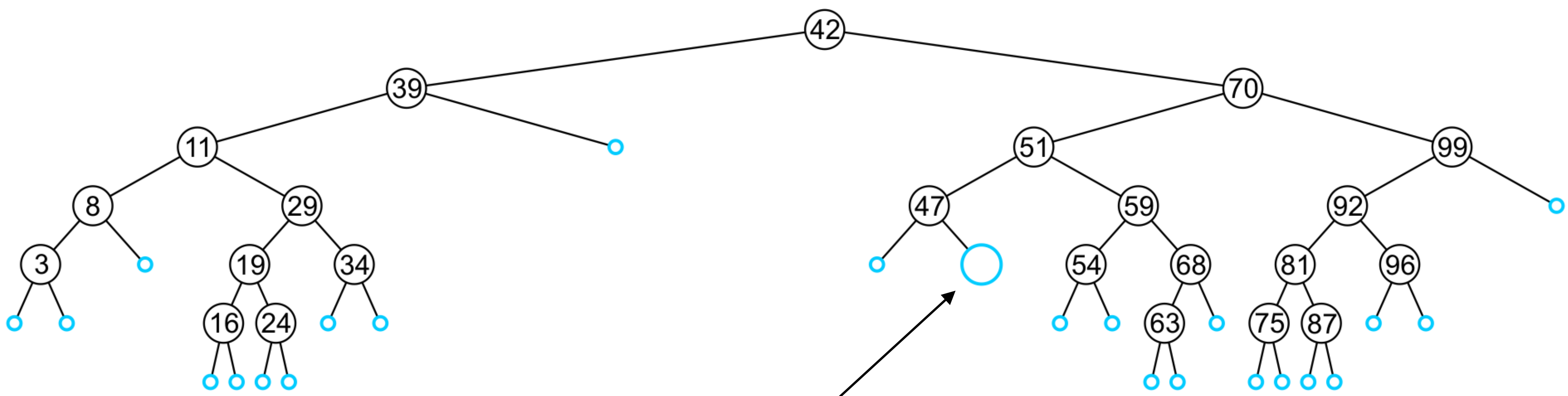
- An insertion will be performed at a leaf node
  - Any empty node is a possible location for an insertion




- Values which may be inserted at any empty node depend on the surrounding nodes

# Inserting a Node in BST

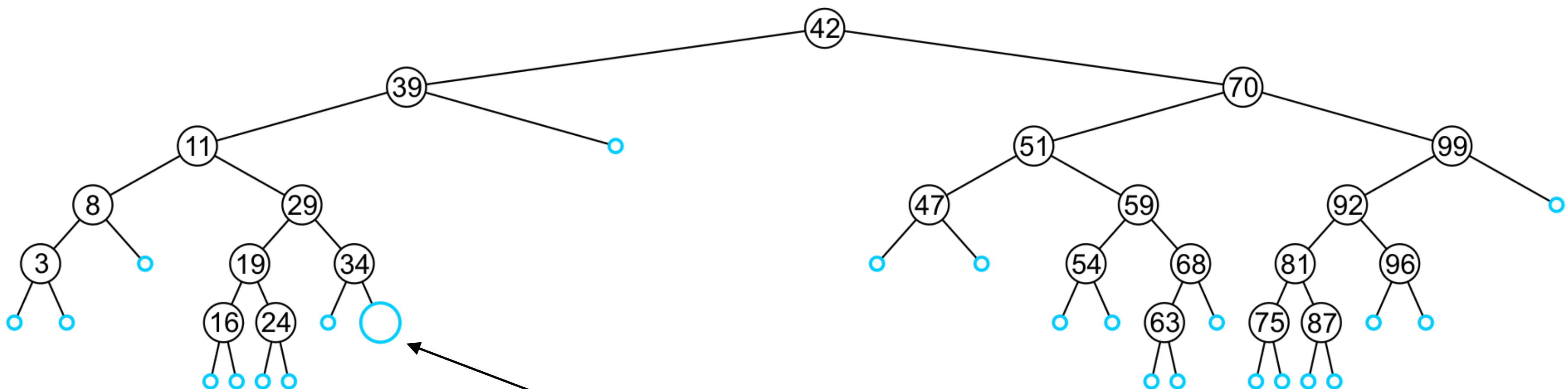
- Which values can be held by empty node?




This node may hold
48, 49, or 50

# Inserting a Node in BST

- Which values can be held by empty node?




This node may hold
35, 36, 37 or 38

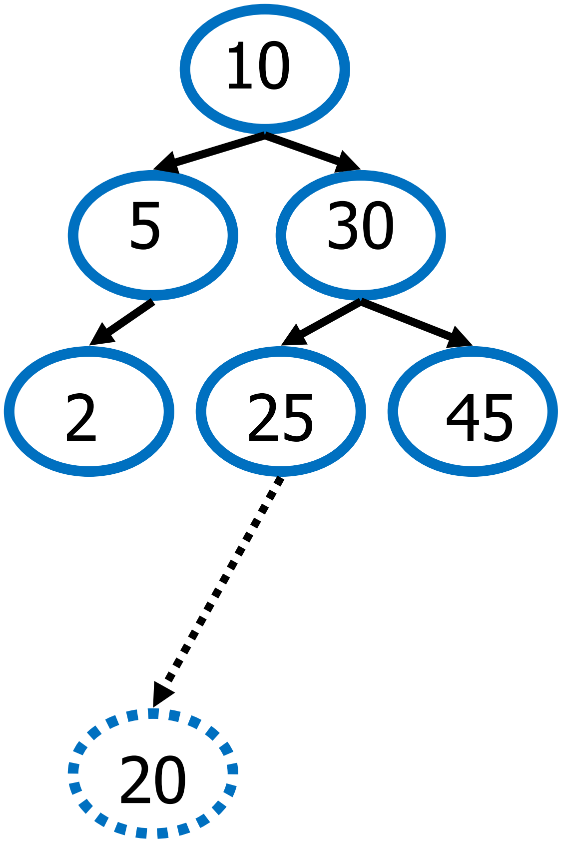
# Inserting a Node in BST – Algorithm

- Like find, algorithm will step through the tree
  - If algorithm find the object already in the tree, it will return
    - The object is already in the binary search tree (no duplicates)
  - Otherwise, algorithm will arrive at an empty node
  - The object will be inserted into that location

- Why no duplicates?
  - In reality, it is seldom the case where duplicate elements in a BST must be stored as separate entities

# Inserting a Node in BST – Example

- insertNode(20)




10 < 20, right

30 > 20, left

25 > 20, left

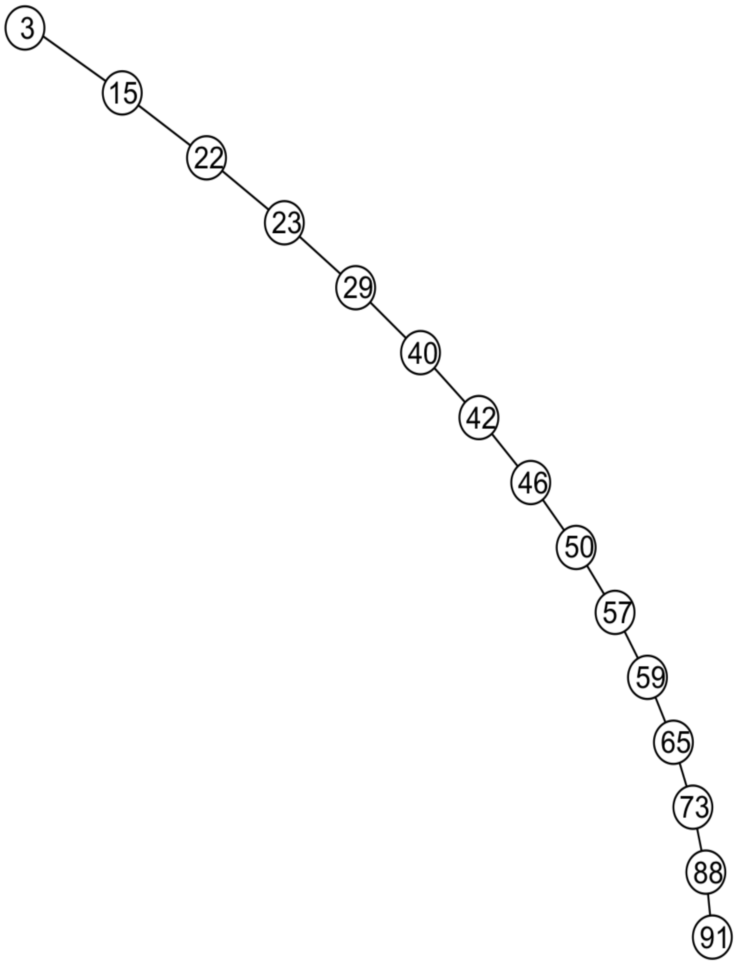
Insert 20 on left

# Inserting a Node in BST – Implementation

```
void IntBinaryTree::insertNode(int num) {

   TreeNode* newNode = new TreeNode; // Create a new node
   newNode->value = num;
   newNode->left = newNode->right = NULL;

   if (!root)  root = newNode; // If tree is empty.
   else { // Tree is not empty
      TreeNode* nodePtr = root; // create a pointer to traverse the tree
      while ( true ) {
         if (num < nodePtr->value) { // Left subtree
            if (nodePtr->left)  { nodePtr = nodePtr->left; }
            else { nodePtr->left = newNode; return;  }
         }
         else if (num > nodePtr->value) {  // Right subtree
            if (nodePtr->right)    nodePtr = nodePtr->right;
            else  {  nodePtr->right = newNode;   return; }
         }
         else {  cout << "Duplicate value found in tree.\n"; break; }
      }
   }
}
```

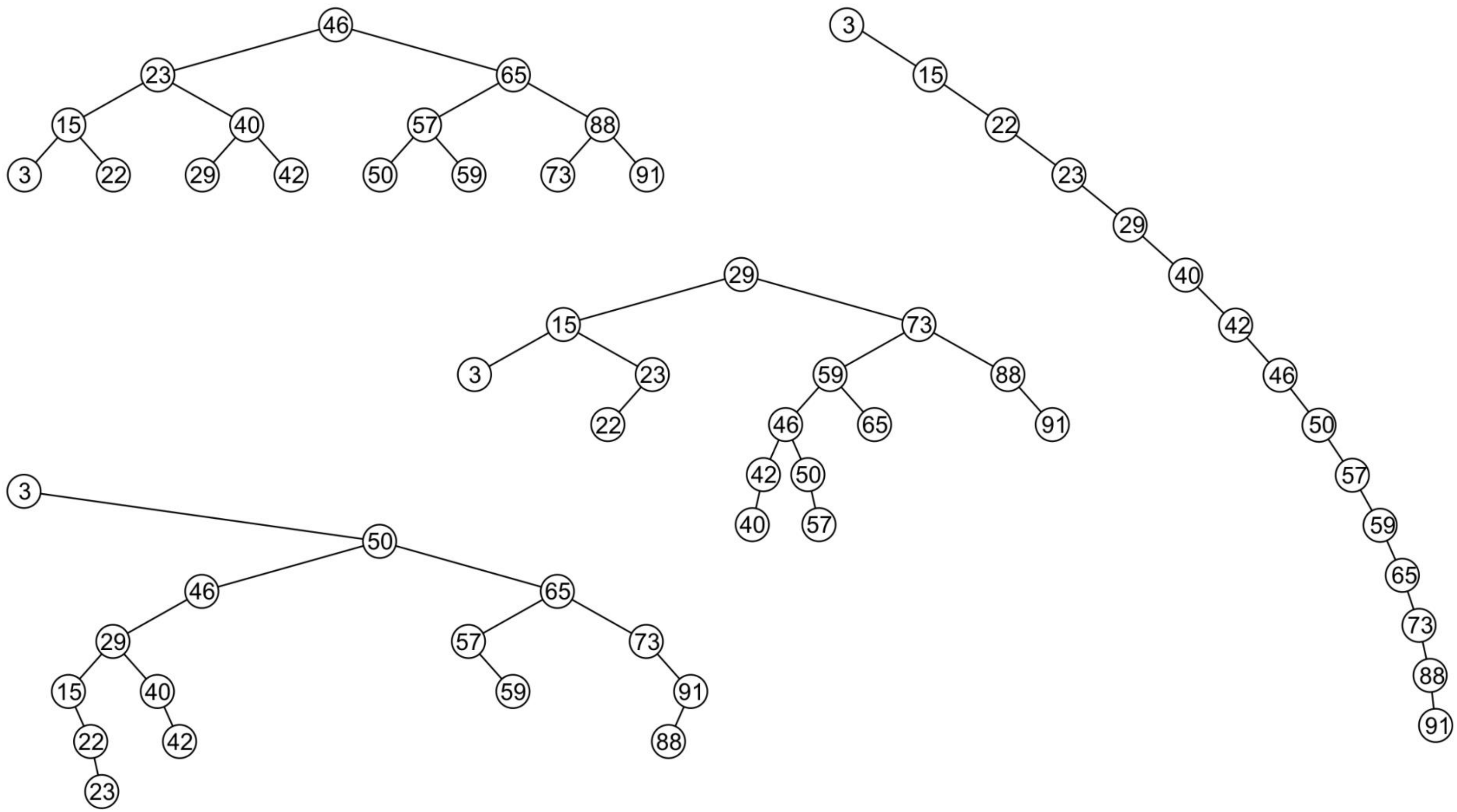
# Inserting a Node in BST – Observation (1)

- Insertion may unbalance the tree
- It is possible to construct degenerate BST
  - The example is equivalent to a linked list



# Inserting a Node in BST – Observation (2)

- All these binary search trees store the same data
  - Resultant tree depends on the order in which the values are inserted



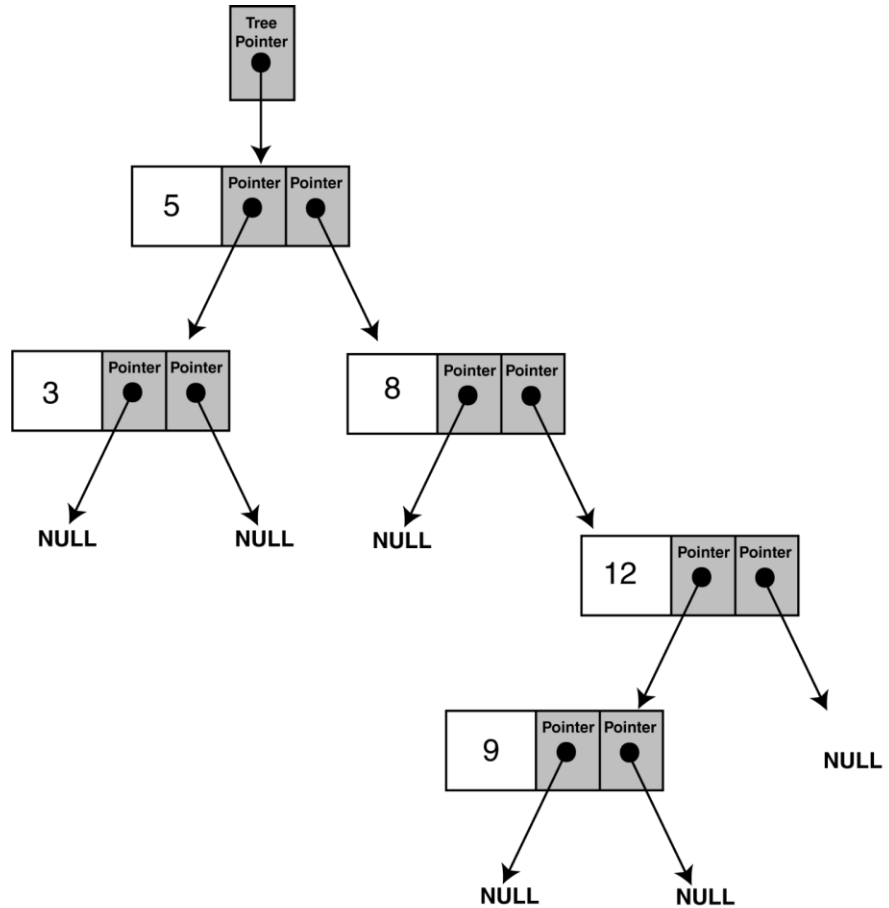
# Using BST (1)

```
// This program builds a binary tree with 5 nodes.
// The SearchNode function determines if the
// value 3 is in the tree.
#include <iostream.h>
#include "IntBinaryTree.h“

void main(void)
{
    IntBinaryTree tree;

    cout << "Inserting nodes.\n";
    tree.insertNode(5);
    tree.insertNode(8);
    tree.insertNode(3);
    tree.insertNode(12);
    tree.insertNode(9);
    if (tree.searchNode(3))
        cout << "3 is found in the tree.\n";
    else
        cout << "3 was not found in the tree.\n";
}
```

Output:
Inserting nodes.
3 is found in the tree.

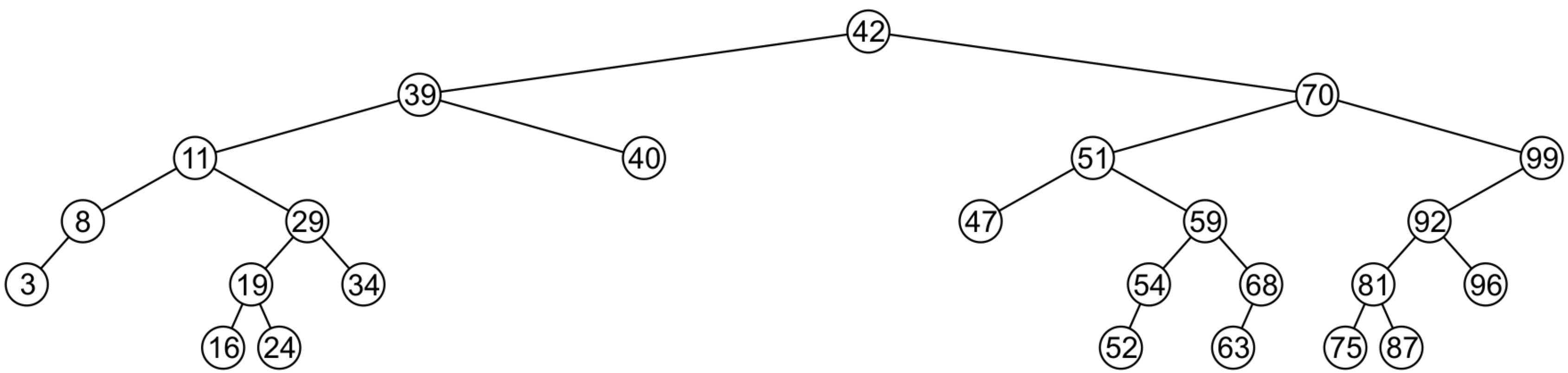
# Using BST (2)

- Structure of binary tree built by the program



# Deleting a Node

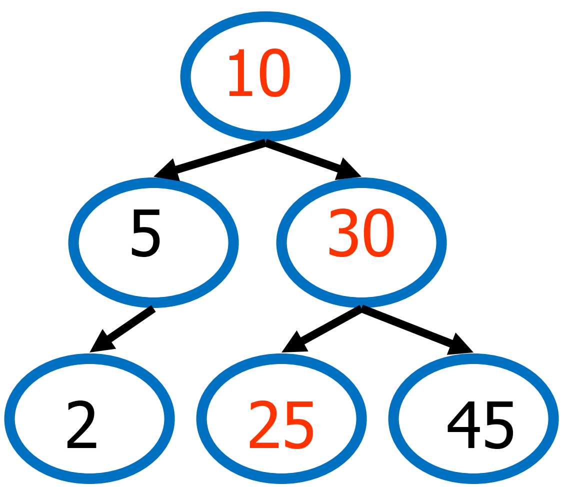
- A node being erased is not always going to be a leaf node
- There are three possible scenarios:
  - The node is a leaf node,
  - It has exactly one child, or
  - It has two children (it is a full node)



# Deleting a Node – Leaf Node

- Deleting a leaf node is easy
  - Find its parent
  - Set the child pointer that links to it to NULL
  - Free the node's memory

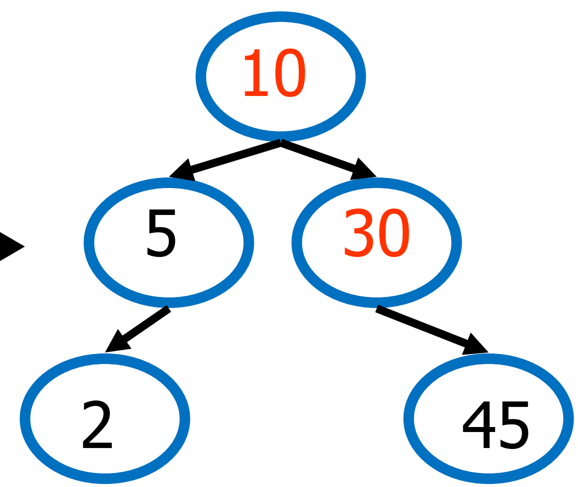
- Consider deleting node containing 25




10 < 25, right

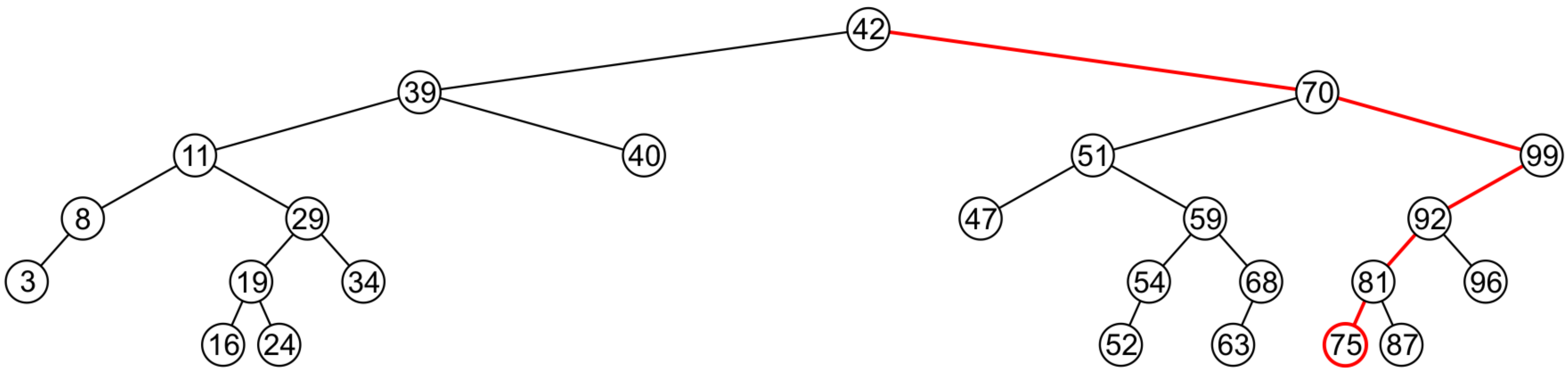
30 > 25, left

25 = 25, delete



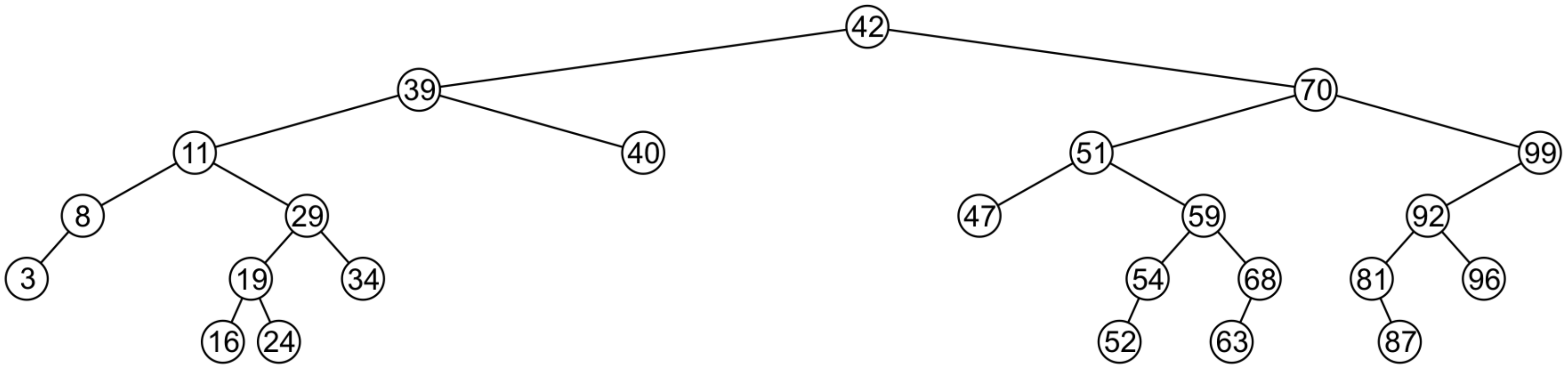
# Deleting a Node – Leaf Node

- Consider deleting node containing 75



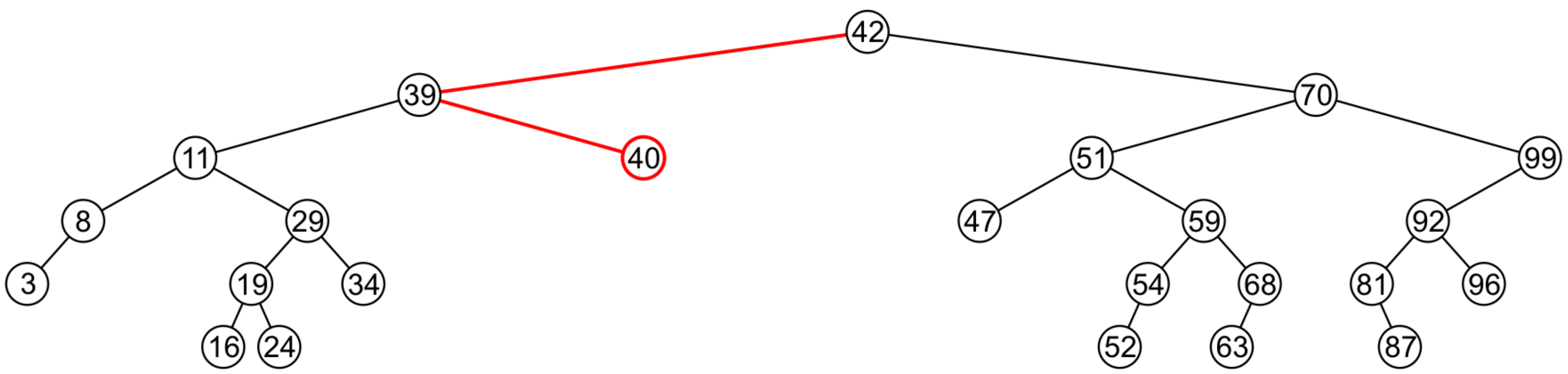
# Deleting a Node – Leaf Node

- Consider deleting node containing 75
  - The node is deleted and left child of 81 is set to NULL



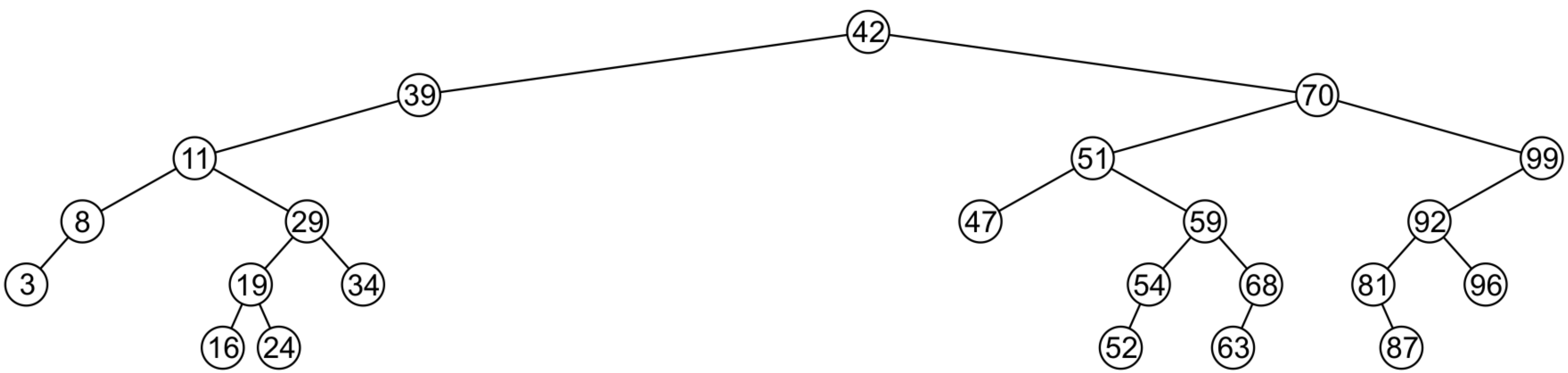
# Deleting a Node – Leaf Node

- Consider deleting node containing 40



# Deleting a Node – Leaf Node

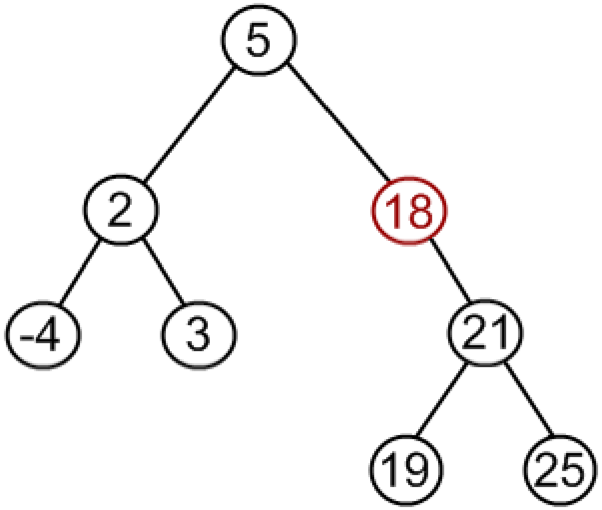
- Consider deleting node containing 40
  - Node is deleted and right child of 39 is set to NULL

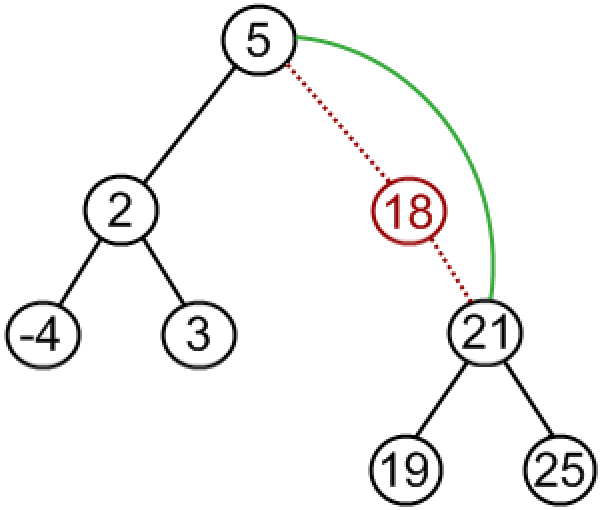


# Deleting a Node – Node With Child

- If a node has only one child (left or right)
  - Simply promote the subtree associated with the child

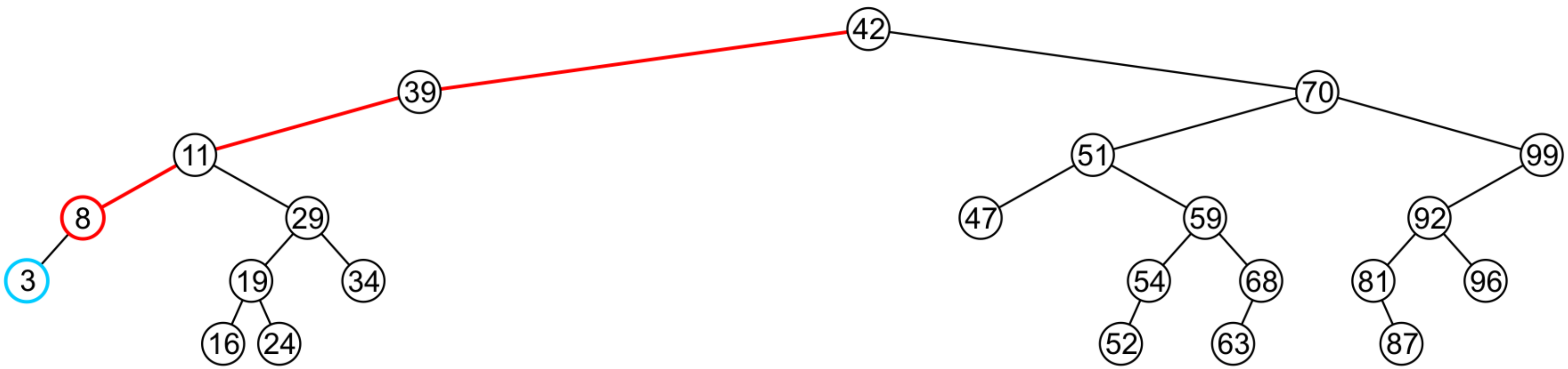
- Consider deleting 18 which has one right child
  - Node 18 is deleted and  right tree of node 5 is update to point to 21



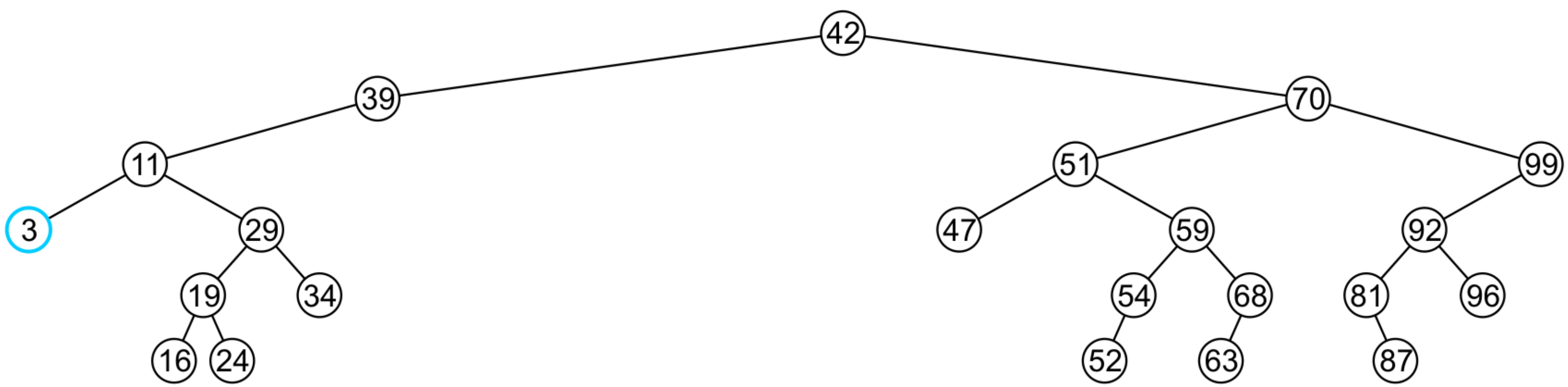
# Deleting a Node – Node With Child

- Consider deleting 8 which has one left child



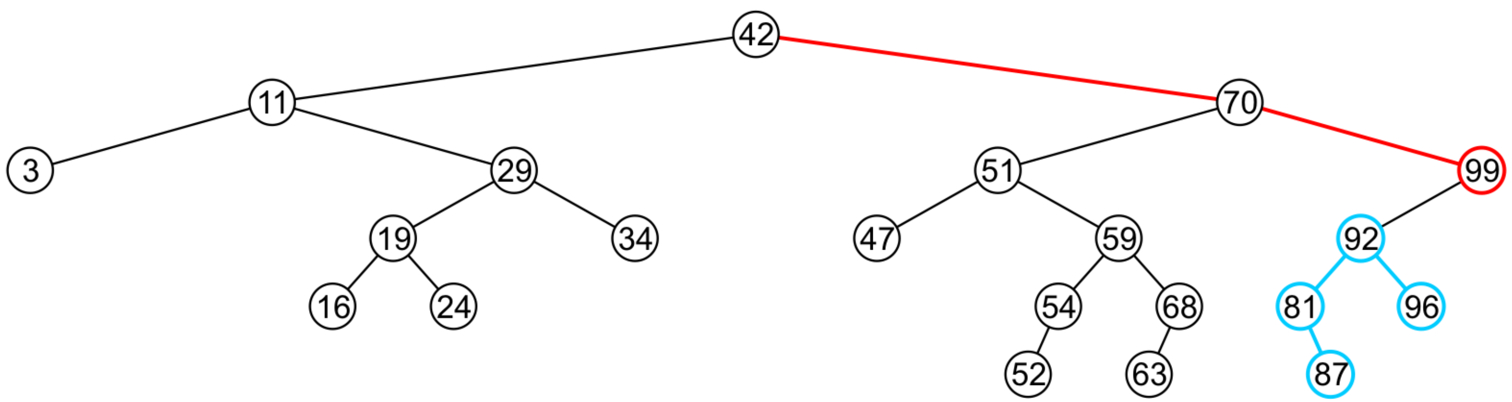
# Deleting a Node – Node With Child

- Consider deleting 8 which has one left child
  - Node 8 is deleted and the left tree of 11 is updated to point to 3



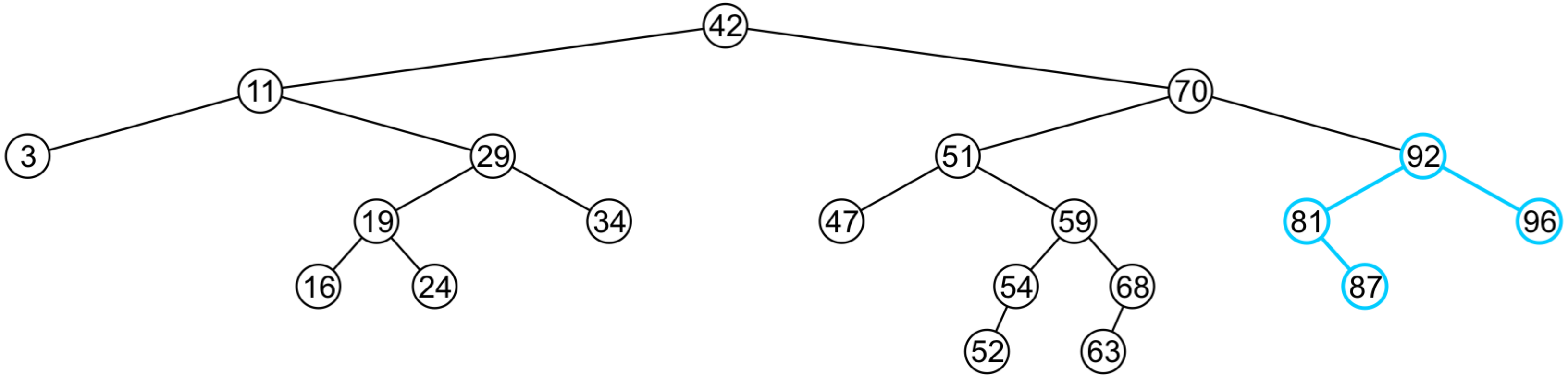
# Deleting a Node – Node With Child

- Consider deleting the node containing 99



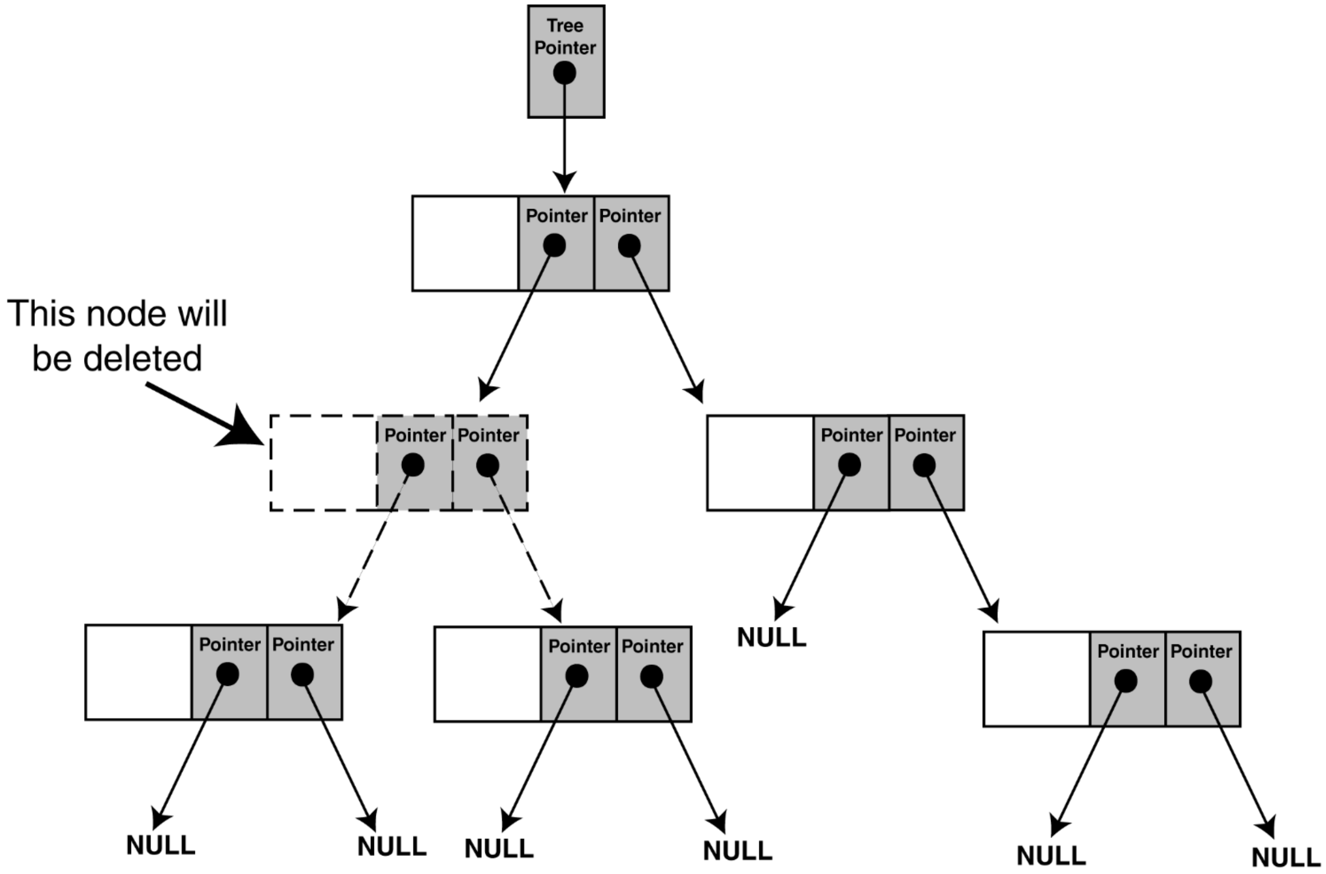
# Deleting a Node – Node With Child

- Consider deleting the node containing 99
  - The right tree of 70 is set to point to node 92
  - Again, the order of the tree is maintained



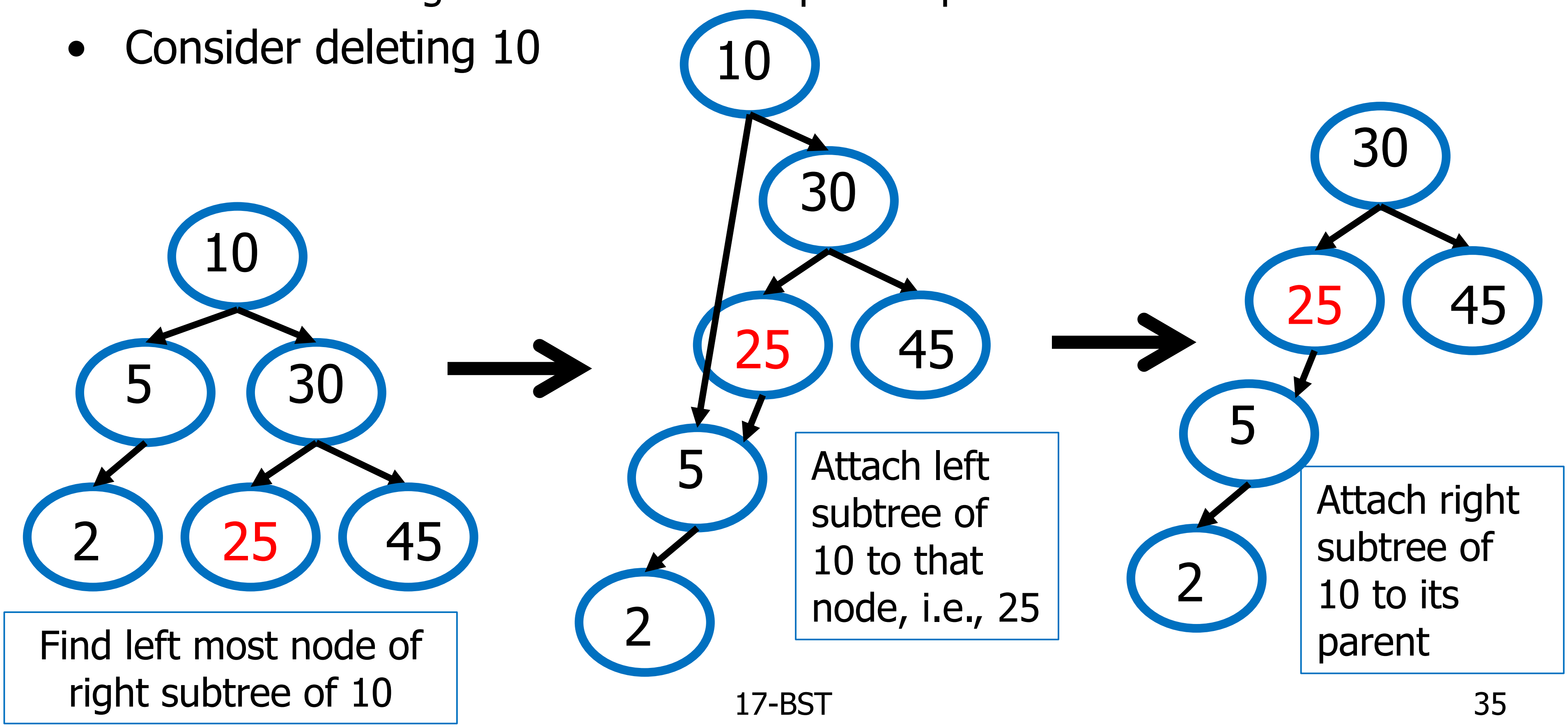
# Deleting a Node – Node With Two Children

- The problem is not as easily solved if the node has two children



# Deleting a Node – Node With Two Children

- Suppose node p with two children has to be deleted
  - Find a position in the right subtree of p to attach its left subtree
    - ➢ Left most node in the right subtree of node p
  - Attach the right subtree of node p to its parent
- Consider deleting 10



# Deleting a Node – Implementation

```
class IntBinaryTree {
   private:
      TreeNode *root;   // Pointer to the root of BST

      void destroySubTree(TreeNode *); //Recursively delete all tree nodes
      void deleteNode(int, TreeNode *&);
      void makeDeletion(TreeNode *&);
      void displayInOrder(TreeNode *);
      void displayPreOrder(TreeNode *);
      void displayPostOrder(TreeNode *);

   public:
      IntBinaryTree()                { root = NULL; }
      ~IntBinaryTree()               { destroySubTree(root); }
      void insertNode(int);
      bool find(int);
      void remove(int num);          { deleteNode( num, root)}
      void showNodesInOrder()        { displayInOrder(root);   }
      void showNodesPreOrder()       { displayPreOrder(root);  }
      void showNodesPostOrder()      { displayPostOrder(root); }
};
```

The argument passed to the remove function is the value of the node to be deleted.

# Deleting a Node – Implementation

```
void IntBinaryTree::deleteNode(int num, TreeNode *&nodePtr)
{
    if (nodePtr == NULL) // node does not exist in the tree
        cout << num <<“ not found.\n";
    else if (num < nodePtr->value)
        deleteNode(num, nodePtr->left);  // find in left subtree
    else if (num > nodePtr->value)
        deleteNode(num, nodePtr->right); // find in right subtree
    else // num == nodePtr->value i.e. node is found
        makeDeletion(nodePtr); // actually deletes node from BST
}
```

**Note:**

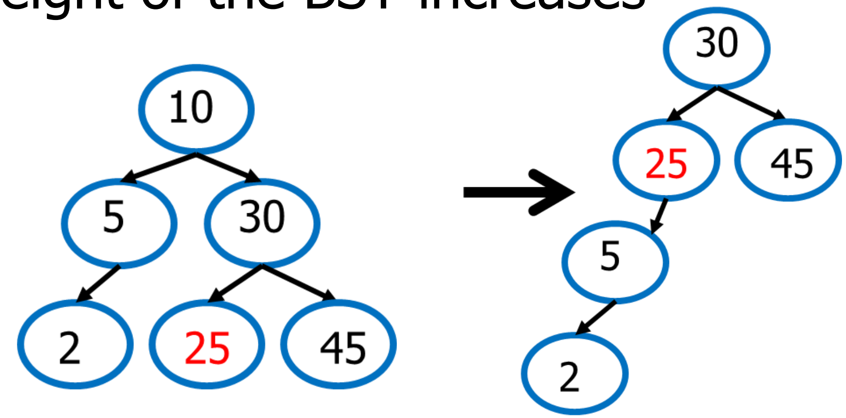
- The declaration of the `nodePtr` parameter: `TreeNode *&nodePtr;`
- `nodePtr` is a reference to a pointer to a `TreeNode` structure
  - Any action performed on `nodePtr` is actually performed on the argument passed into `nodePtr`

# Deleting a Node – Implementation

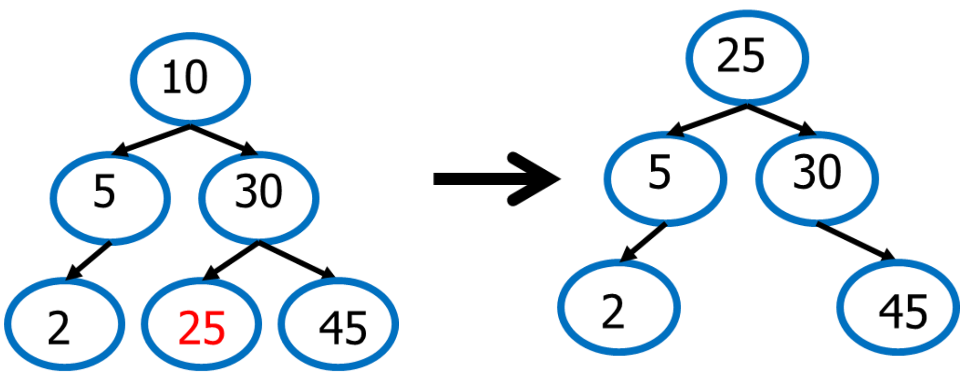
```
void IntBinaryTree::makeDeletion(TreeNode *&nodePtr) {
   TreeNode *tempNodePtr; // Temperary pointer
   if (nodePtr->right == NULL) { // case for leaf and one (left) child
      tempNodePtr = nodePtr;
      nodePtr = nodePtr->left; // Reattach the left child
      delete tempNodePtr;
   }
   else if (nodePtr->left == NULL) { // case for one (right) child
      tempNodePtr = nodePtr;
      nodePtr = nodePtr->right; // Reattach the right child
      delete tempNodePtr;
   }
   else {  // case for two children.
      tempNodePtr = nodePtr->right; // Move one node to the right
      while (tempNodePtr->left) { // Go to the extreme left node
         tempNodePtr = tempNodePtr->left;
      }
      tempNodePtr->left = nodePtr->left; // Reattach the left subtree
      tempNodePtr = nodePtr;
      nodePtr = nodePtr->right; // Reattach the right subtree
      delete tempNodePtr;
   }
}
```

# Deleting a Node – Node With Two Children

- Problem: Height of the BST increases

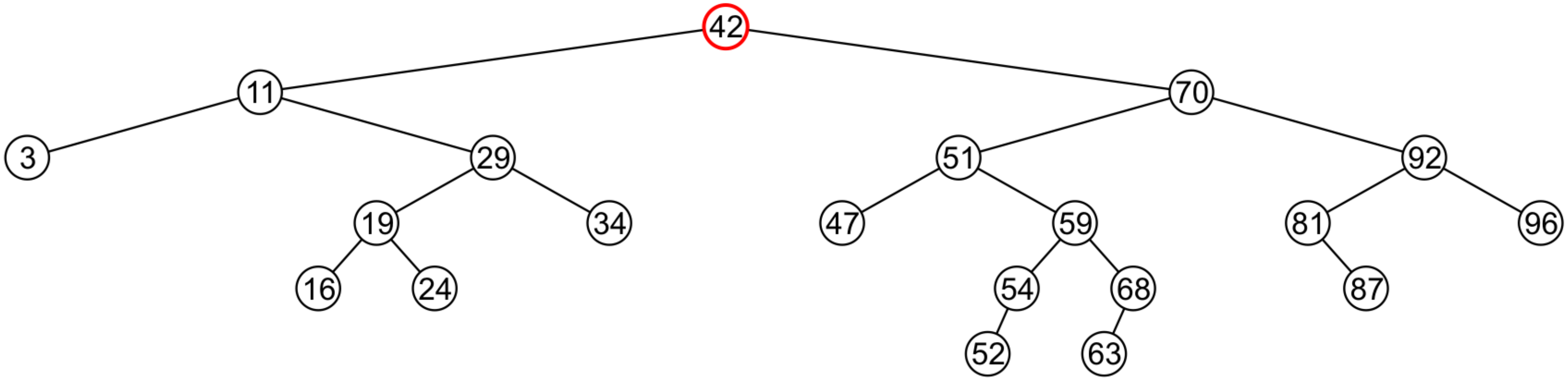



- A better Solution to delete node p with two children
  - Replace node p with the minimum object in the right subtree
  - Delete that object from the right subtree



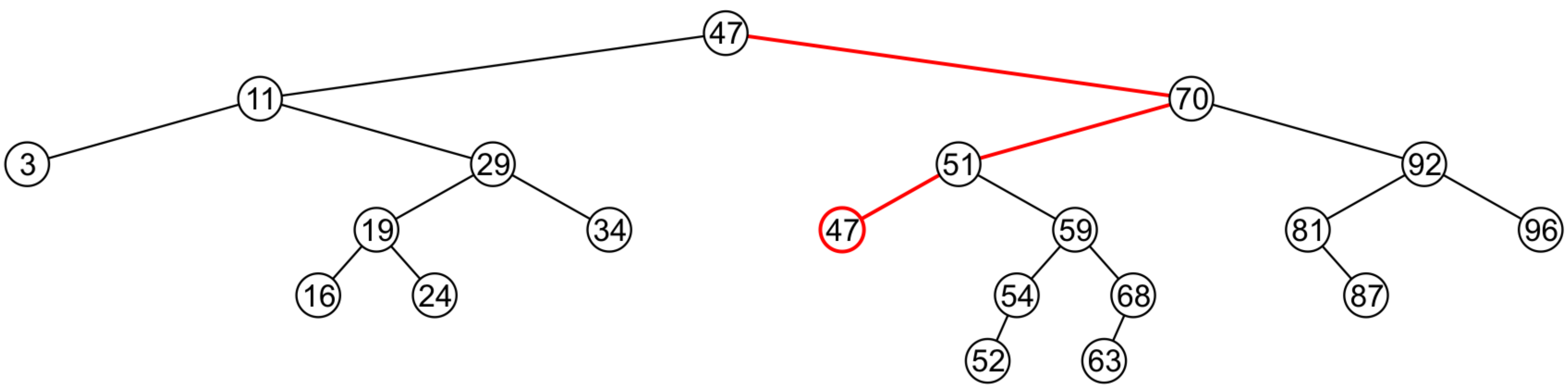
# Deleting a Node Two Children – Better Solution

- Consider the problem of deleting a full node, e.g., 42



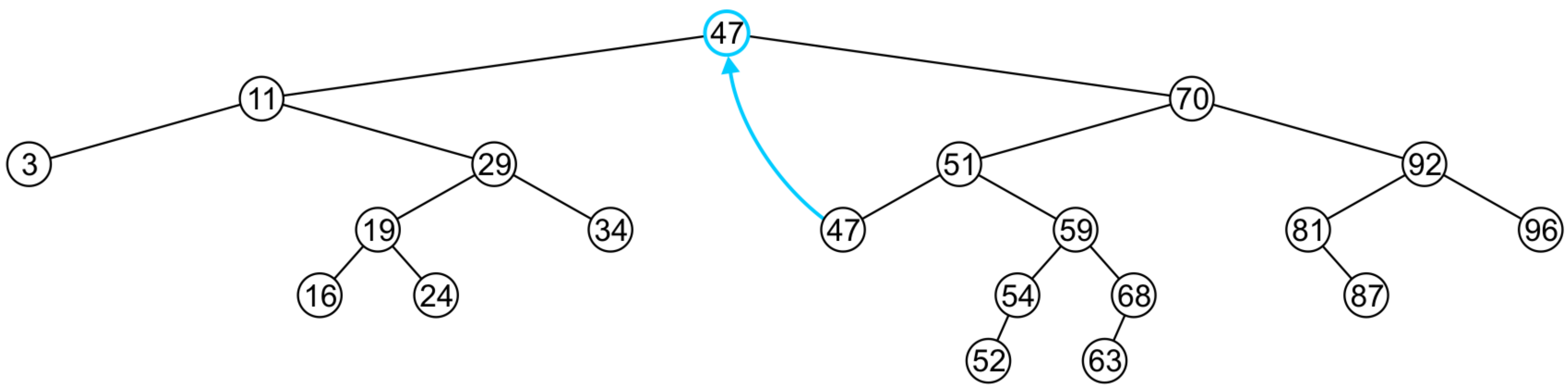
# Deleting a Node Two Children – Better Solution

- Consider the problem of deleting a full node, e.g., 42
  - Find minimum object in the right subtree, i.e., 47



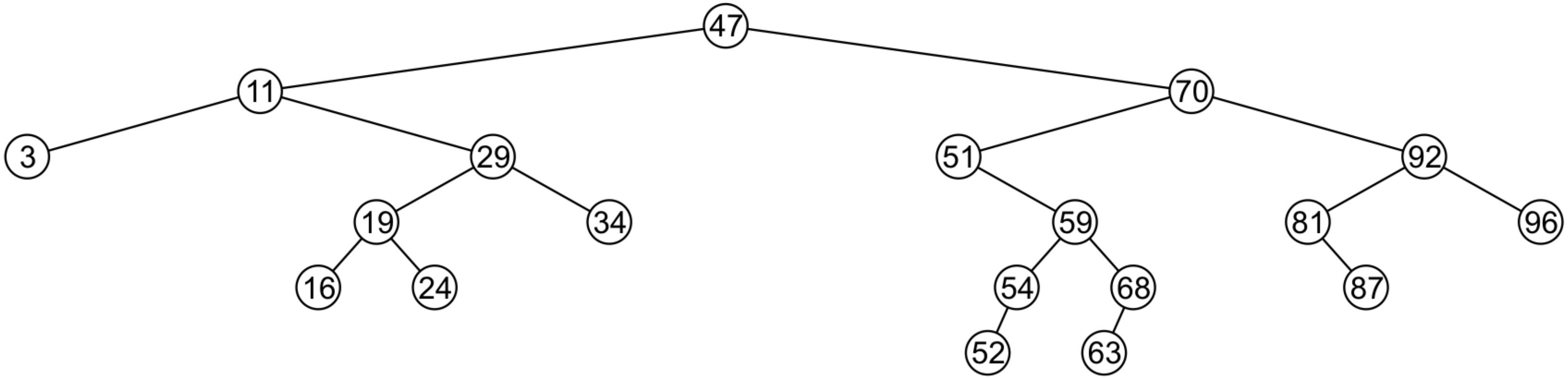
# Deleting a Node Two Children – Better Solution

- Consider the problem of deleting a full node, e.g., 42
  - Find minimum object in the right subtree, i.e., 47
  - Replace 42 with 47



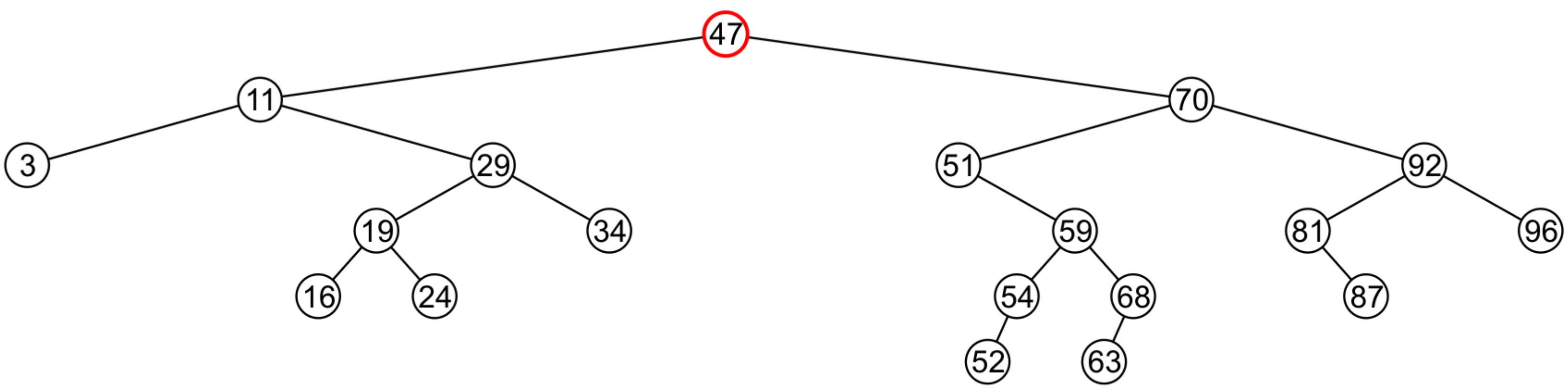
# Deleting a Node Two Children – Better Solution

- Consider the problem of deleting a full node, e.g., 42
  - Find minimum object in the right subtree, i.e., 47
  - Replace 42 with 47
  - Delete the leaf node 47



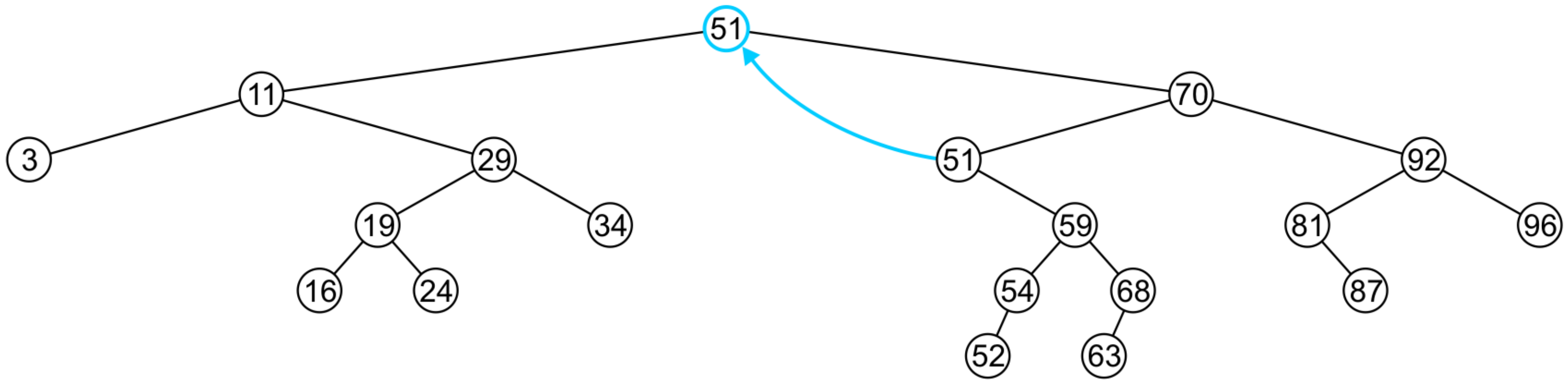
# Deleting a Node Two Children – Better Solution

- Consider the problem of deleting a full node, e.g., 47



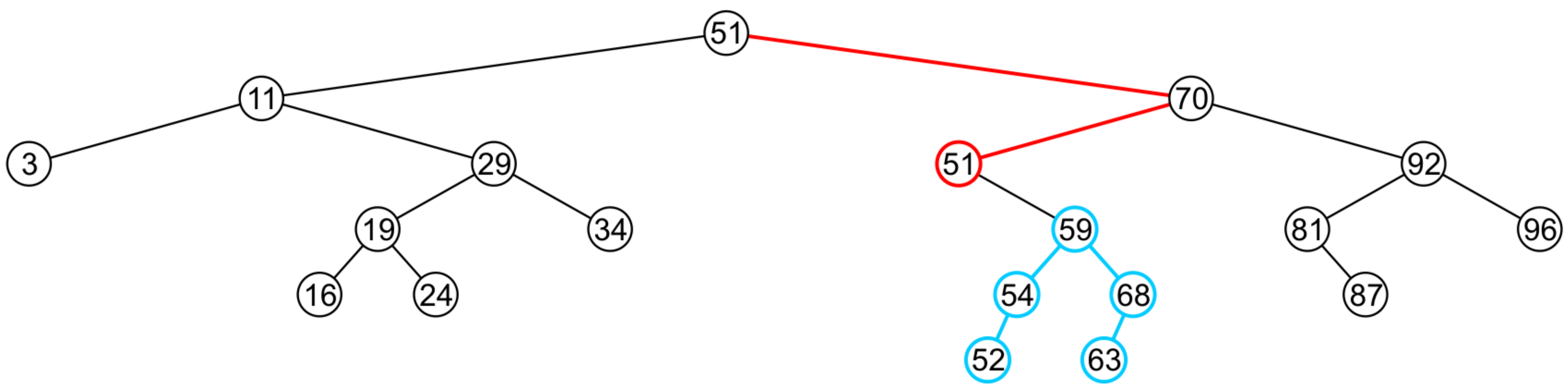
# Deleting a Node Two Children – Better Solution

- Consider the problem of deleting a full node, e.g., 47
  - Replace 47 with 51



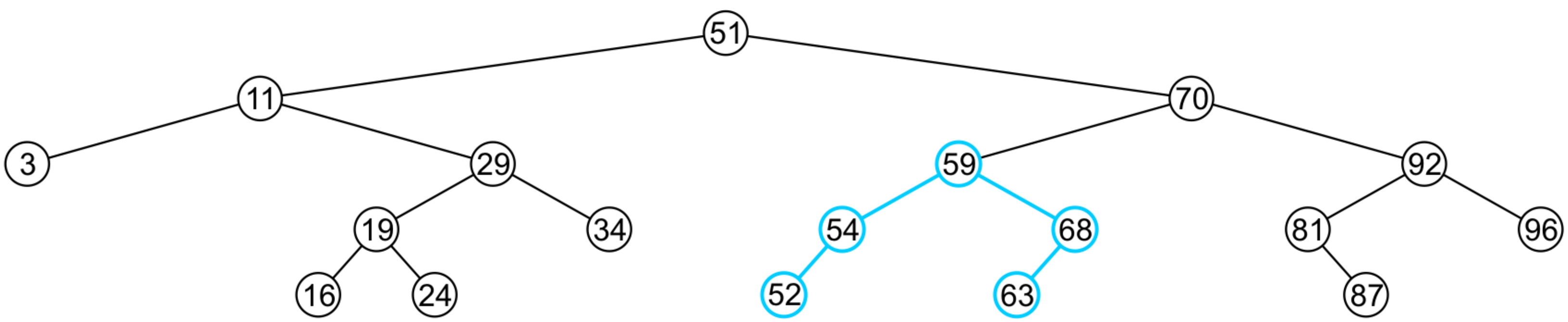
# Deleting a Node Two Children – Better Solution

- Consider the problem of deleting a full node, e.g., 47
  - Replace 47 with 51
  - Node 51 is not a leaf node



# Deleting a Node Two Children – Better Solution

- Consider the problem of deleting a full node, e.g., 47
  - Replace 47 with 51
  - Node 51 is not a leaf node
    - ➢ Assign the left subtree of 70 to point to 59



# Using BST

```
// This program builds a binary tree with 5 nodes.
// The DeleteNode function is used to remove two of them.
#include <iostream.h>
#include "IntBinaryTree.h"

void main(void) {
   IntBinaryTree tree;

   cout << "Inserting nodes.\n";
   tree.insertNode(5);
   tree.insertNode(8);
   tree.insertNode(3);
   tree.insertNode(12);
   tree.insertNode(9);

   cout << "Here are the values in the tree:\n";
   tree.showNodesInOrder();
   cout << "Deleting 8...\n";
   tree.remove(8);
   cout << "Deleting 12...\n";
   tree.remove(12);

   cout << "Now, here are the nodes:\n";
   tree.showNodesInOrder();
}
```

**Program Output:**
Inserting nodes.
Here are the values in the tree:
3
5
8
9
12

# Using BST

```
// This program builds a binary tree with 5 nodes.
// The DeleteNode function is used to remove two of them.
#include <iostream.h>
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void main(void) {
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   tree.insertNode(5);
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   tree.insertNode(3);
   tree.insertNode(12);
   tree.insertNode(9);

   cout << "Here are the values in the tree:\n";
   tree.showNodesInOrder();
   cout << "Deleting 8...\n";
   tree.remove(8);
   cout << "Deleting 12...\n";
   tree.remove(12);

   cout << "Now, here are the nodes:\n";
   tree.showNodesInOrder();
}
```

**Program Output:**
Inserting nodes.
Here are the values in the tree:
3
5
8
9
12
Deleting 8...
Deleting 12...
Now, here are the nodes:
3
5
9

# Traversing a Binary Search Tree

```
class IntBinaryTree {
   private:
      TreeNode *root;  // Pointer to the root of BST

      void destroySubTree(TreeNode *); //Recursively delete all tree nodes
      void deleteNode(int, TreeNode *&);
      void makeDeletion(TreeNode *&);
      void displayInOrder(TreeNode *);
      void displayPreOrder(TreeNode *);
      void displayPostOrder(TreeNode *);

   public:
      IntBinaryTree()            { root = NULL; }
      ~IntBinaryTree()           { destroySubTree(root); }
      void insertNode(int);
      bool find(int);
      void remove(int);
      void showNodesInOrder()    { displayInOrder(root);   }
      void showNodesPreOrder()   { displayPreOrder(root);  }
      void showNodesPostOrder()  { displayPostOrder(root); }
};
```

Recursive implementation as discussed in the slides of Tree Traversal chapter.

# Using BST

```
// This program builds a binary tree with 5 nodes.
// The nodes are displayed with inorder, preorder, and post
#include <iostream.h>
#include "IntBinaryTree.h"

void main(void)
{
   IntBinaryTree tree;

   cout << "Inserting nodes.\n";
   tree.insertNode(5);
   tree.insertNode(8);
   tree.insertNode(3);
   tree.insertNode(12);
   tree.insertNode(9);


   cout << "Inorder traversal:\n";
   tree.showNodesInOrder();
   cout << "\nPreorder traversal:\n";
   tree.showNodesPreOrder();
   cout << "\nPostorder traversal:\n";
   tree.showNodesPostOrder();
}
```

**Program output:**

Inserting nodes.
Inorder traversal:
3
5
8
9
12

Preorder traversal:
5
3
8
12
9

Postorder traversal:
3
9
12
8
5

# Any Question So Far?