LAB # 1: Mathematical Modelling of Mechanical, Electrical, Electronic and Electromechanical Systems using MATLAB/LabVIEW

1.1 Objectives

- To Construct mathematical models of various systems such as, Electrical, Electronic and Mechanical.
- To implement the developed models on LabVIEW/MATLAB to Display their characteristics such as stability, step response and impulse response.
- To Assemble the model of an electro-mechanical (Permanent Magnet DC Motor) system to Sketch its speed and position characteristics

1.2 Lab Instructions

- ✓ This lab activity comprises of three parts: Pre-lab, Lab Exercises, and Post-Lab Viva session.
- ✓ The students should perform and demonstrate each lab task separately for step-wise evaluation
- ✓ Only those tasks that completed during the allocated lab time will be credited to the students. Students are however encouraged to practice on their own in spare time for enhancing their skills.

1.3 Theory and Pre-Lab Exercises:

1.3.1 Introduction

"All models are wrong but some are useful" **(George Box)**

Mathematical Modelling means describing a physical system using mathematical concepts and language. It is the first step in studying the behaviour of the system at hand. These mathematical models help the designers in not only analysing an existing system but also designing new systems with more desirable behaviour as per the specific needs of an application. Unfortunately, we can never make a completely precise model of a physical system. There are always phenomena which we will not be able to model. Thus, there will always be model errors or model uncertainties. But even if a model describes just a part of the reality it can be very useful for analysis and design — if it describes the dominating dynamic properties of the system.

There may be various representations of a mathematical model for a system each with their own inherent advantages. After modelling a system, we will learn two representations of the developed model. Either representation gives us insight into the system from a different perspective. Two of the most important types of mathematical modelling that we are going to explore in this course are, Transfer Function and State Space Modelling. In this lab, we will study only transfer function implementation of various physical systems. We will study three types of systems which include Mechanical system, Electrical System and Electronic System. First of all, we will derive the transfer function of these systems on paper, and after that we will learn to implement these models on MATLAB/LabVIEW and finally we will see how to analyse the systems through their mathematical models. For details, you can study chapter two of Control systems by Ogata.

1.3.2 Transfer Function Representation

A transfer function is a mathematical representation of the relationship between the input and output of a system. This function allows separation of the input, system, and the output into three separate and distinct parts. The general form of a transfer function, H(s), is:

$$\mathrm{H(s)} = \frac{\mathrm{R(s)}}{\mathrm{E(s)}}$$

Where $R(s)$ and $E(s)$ are Laplace Transforms of Response (output) and Excitation (input) respectively.

1.3.3 Modelling a Translational Mechanical System

A shock absorber, commonly found in motorbikes and automobile, can be represented by a mass-spring system shown in Figure 1.1. This system can be mathematically modelled by Newton's second law of motion which states that the acceleration $'a'$ of a body is parallel and directly proportional to the net force F acting on it and inversely proportional to the mass M of the body. (Here in below figure x is displacement).

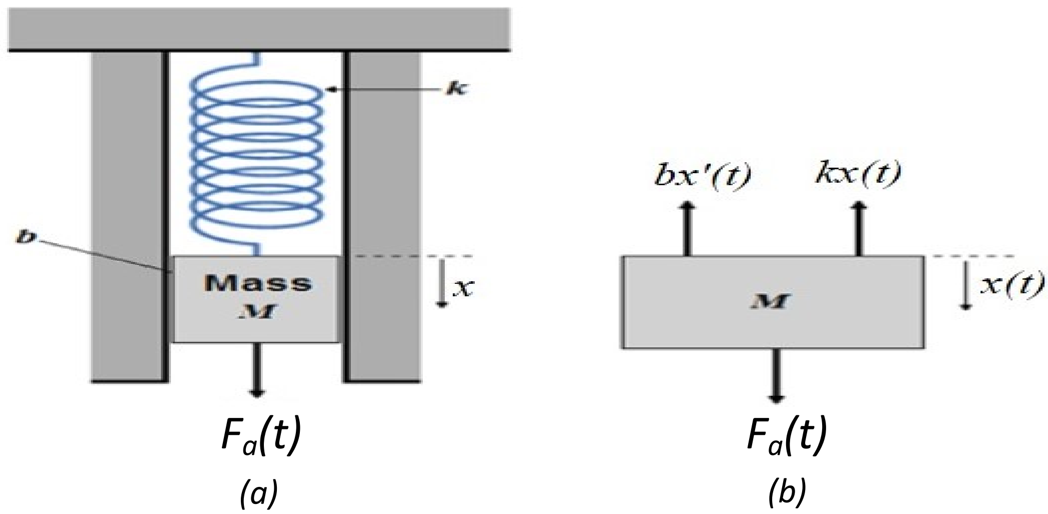

Figure 1.1

Let we apply a force $F_a(t)$ on the mass M and displace it from its mean position, say x_0, to a new position, say x_1. At this point there will be other forces acting on the mass M besides our applied force.

One of these three forces is Friction Force, $F_f(t)$. For the simplicity we assume here that the wall friction b is a viscous damper, that is, the friction force is linearly proportional to the velocity of the mass. In reality, however, this friction force may behave as a coulomb damper, also known as dry friction, which is a nonlinear function of the mass velocity. For a well-lubricated, sliding surface, the viscous friction is an appropriate approximation. Hence for friction force, $F_f(t)$., we can write that

$$F_f(t) = bv(t) \tag{1.1}$$

This friction force will obviously be in the reverse direction of applied force.

Another force acting on the mass is the spring force which is also in the opposite direction to the applied force. According to Hook's Law the extension of a spring is directly proportional to the applied load as long as the load does not exceed spring's elastic limits. Mathematically Hook's law can be stated as

$$F_s(t) = kx(t) \tag{1.2}$$

Here $F_s(t)$ is spring force, k is the spring constant and x is the displacement as a result of applied force.

Finally, from Newton's second law of motion we know that

$$\sum F = M\boldsymbol{a} \tag{1.3}$$

from Figure 1.1(b) we note that there are a total of three forces acting on the mass M. So,

$$\sum F = F_a - F_f - F_s \tag{1.4}$$

Putting (1.1) and (1.2) in (1.4) and then putting the resulting equation in (1.3) and rearranging the term, we get

$$M\frac{d^2x(t)}{dt^2} + b\frac{dx(t)}{dt} + kx(t) = F_a(t) \tag{1.5}$$

Equation (1.5) is a second-order linear constant-coefficient differential equation which completely models the behaviour of the given shock absorber system. Converting it into transfer function representation greatly eases the manipulation of the model and gives extended liberty to investigate various hidden aspects of the system in rather greater detail. To make transfer function out of equation (1.5) first we need to identify input and output variables from the model and then we need to take Laplace Transform of the equation.

For the system given above we take applied force Fa to be input to the system and let displacement $x(t)$ be output. So after taking Laplace transform we get following expression assuming zero initial conditions.

$$Ms^2X(s) + bsX(s) + kX(s) = F_a(s) \tag{1.6}$$

$$(Ms^2 + bs + k)X(s) = F_a(s) \tag{1.7}$$

Hence the transfer function $\frac{X(s)}{F_a(s)}$ will be,

$$\frac{X(s)}{F_a(s)} = \frac{1}{Ms^2 + bs + k} \tag{1.8}$$

a) Transfer Function Representation in LabVIEW

To represent the transfer function of Mechanical System in LabVIEW, go to block diagram and press right click than programming >> Structures >>Mathscript and we need to store its numerator and denominator coefficients in two separate vectors like

```
>> M = 200; b = 25; k = 10;
>> num = [1];
>> den = [M b k];
```

Now use tf command as follows

```
>> sys = tf(num, den)
```

You will get the following output

```
Transfer function:
       1
-------------------
200 s^2 + 25 s + 10
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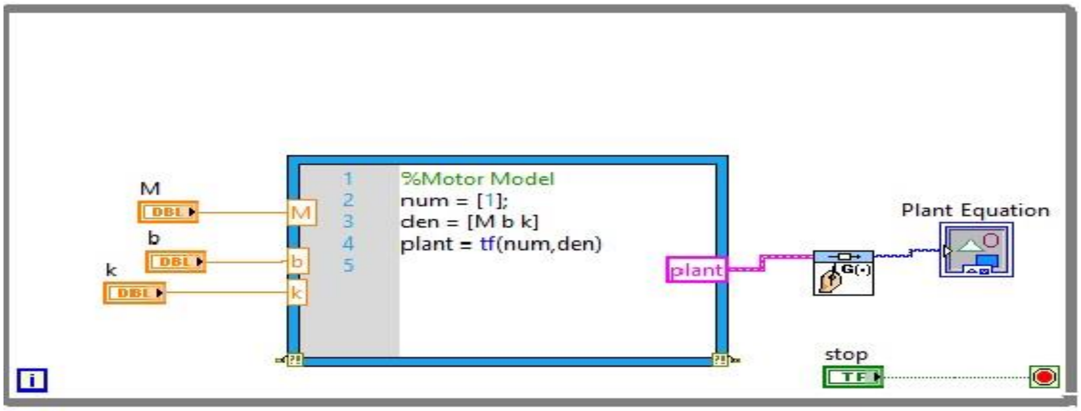

Figure 1.2

Note that we have also used while loop for our VI. Go to programming >> Structures >> while loop and place it on your block diagram. If we have a transfer function object sys and we want to find the numerator and denominator polynomials we can use the following command

>> [num, den] = tfdata(sys)

Please note that in transfer function model you have direct control over coefficients or weights of each and every term involved in the system. By changing these parameters, we can observe the change in the system response. You may remember from your DSP course that these weights in a filter transfer function are of utmost importance and having direct access to these coefficients means that you can easily change the gain response of the filter as per your requirement.

b) Pole-Zero-Gain Representation in LabVIEW

While transfer function allows us to examine the effects of coefficients of a system, it does not give any explicit information about the poles and zeros of the system. Poles and zeros are vital parameters for determining system's stability and they can never be overlooked. We can find the poles and zeros of any system either by applying roots() function on numerator and denominator polynomials separately or by using pole() and zero() functions as

>> p = pole(sys)
>> z = zero(sys)

We can also use pzmap() function to find both poles and zeros in one go as

>> [p, z] = pzmap(sys)

When used without left-hand-side arguments, pzmap() command plots the poles and zeros in s-plane where poles are represented by cross (×) and zeros are shown by circles (o).

We can also store the complete system in pole-zero-gain form in a single object. Explore the following command

>> sys2 = zpk(z,p,k)

This command will create and store a mathematical model by using given locations of zeros (stored in z vector), poles (stored in p vector) and a scalar value k.

We can infer from the above command that this model gives direct access to the poles' and zeros' locations and designer can easily change these locations to study the response and stability of a system.

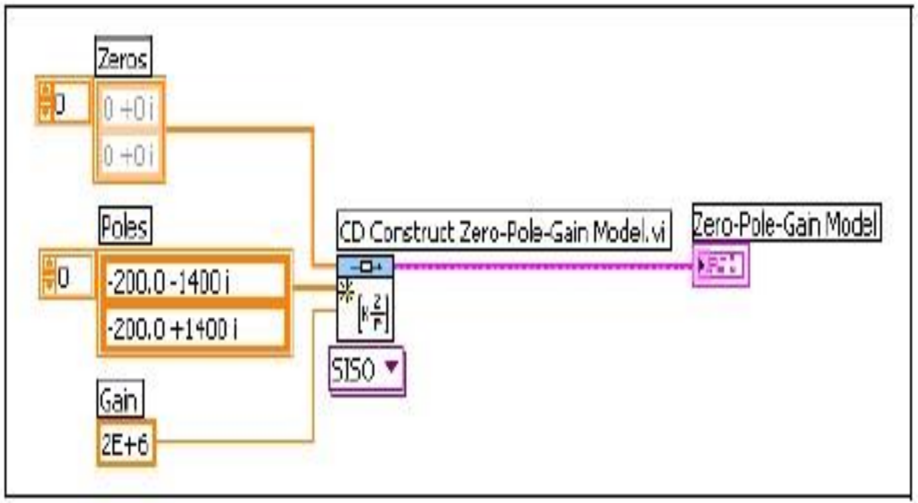

Figure 1.3

As with transfer function model we have a reciprocal command for zpk() function that takes system object as an input and returns zeros, poles and gain.

>> [z,p,k] = zpkdata(sys1)

However, there are commands that can transform one model representation to the other. Explore the following commands

[z,p,k] = tf2zp(num, den)
[num,den] = zp2tf(z,p,k)

You can now describe this system in LabVIEW either by tf() command or zpk() command. One very simple way is shown in Figure 1.4 to get Transfer Function in LabVIEW.

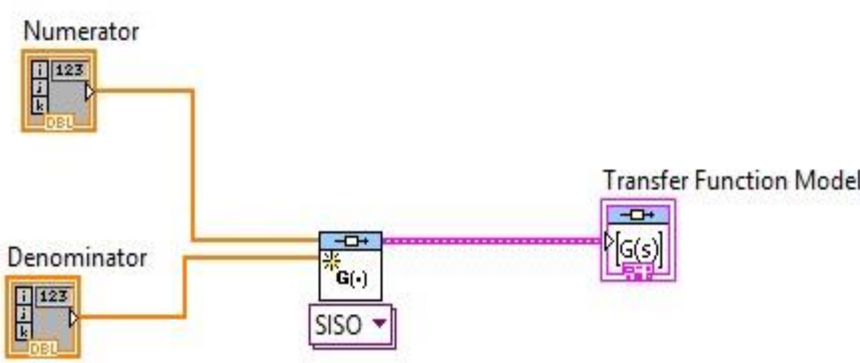

Figure 1.4

1.3.4 Modelling an Electrical System

Now let's take an example of commonly used electrical circuit and try to develop its mathematical model. Given below is a series RLC circuit. By applying Kirchhoff's voltage law in the single loop, we obtain

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = e_i \tag{1.9}$$

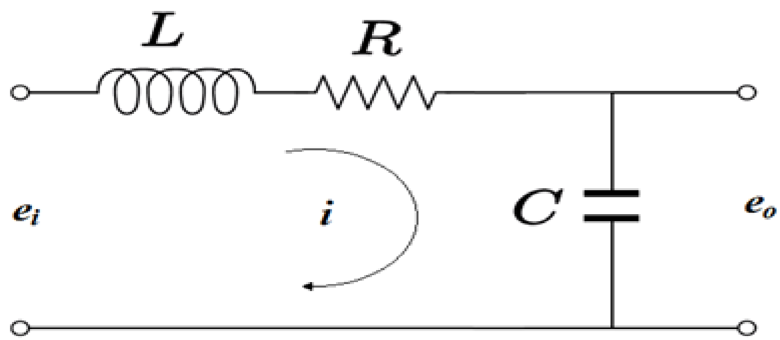

Figure 1.5

If we are interested in observing the capacitor voltage, then our output will be

$$e_o = \frac{1}{C}\int i\,dt \tag{1.10}$$

Taking Laplace transform of (1.9) and (1.10) will yield,

$$LsI(s) + RI(s) + \frac{1}{Cs}I(s) = E_i(s) \tag{1.11}$$

$$\frac{1}{Cs}I(s) = E_o(s) \tag{1.12}$$

Now substituting (1.12) in (1.11), we will get the transfer function of this RLC Circuit

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{LCs^2 + RCs + 1} \tag{1.13}$$

1.3.5 Modelling an Electronic System

In today's industry almost all of the controllers are electronic in nature. Operational Amplifiers are the most common circuit elements that are used in these controllers. Besides controllers, many of the control system elements are implemented using Op-Amps. In this section we shall discuss electronic controllers using operational amplifiers.

Operational Amplifiers are frequently used to amplify signals in sensor circuits. They are also used in filters for compensation purpose. Another beauty of Op-Amps is that they can be easily cascaded with other circuits because of their relatively high (near to infinity) input impedance.

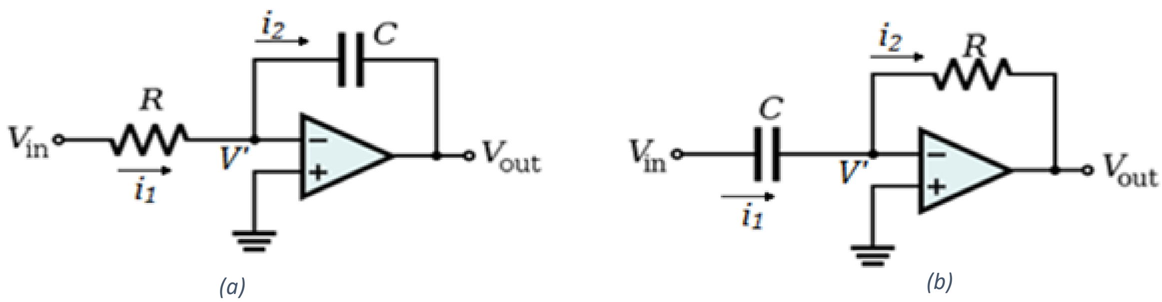

Figure 1.6

Consider simple Op-Amp circuits shown above in Figure 1.6. The configuration in Figure 1.6(a) is known as an integrator because it integrates the given signal over time and the circuit in 1.6(b) is a differentiator. These two circuits along with the most basic configuration of Op-Amp as an amplifier constitute majority of the electronic controllers commonly referred to as PID controllers (PID stands for Proportional plus Integral plus Differential). PID controllers will be studied in depth in the forthcoming labs. For now, let's find transfer function for these simple controllers.

An important fact about Op-Amps to remember is that as their input impedance is very high so the current going into the Op-Amp is normally negligible. Also the voltage difference at the terminals of Op-Amp is nearly zero. Considering these characteristics, we can write input and output equations for Op-Amp as follows,

$$i_1 = \frac{v_{in} - v'}{R} \tag{1.14}$$

$$i_2 = C\frac{d(v' - v_{out})}{dt}$$

As no current flows into the Op-Amp, we can say that $i_1 = i_2$, hence

$$\frac{v_{in} - v'}{R} = C\frac{d(v' - v_{out})}{dt} \tag{1.15}$$

Since$v' = 0$, so

$$\frac{v_{in}}{R} = C\frac{d(-v_{out})}{dt} \tag{1.16}$$

Taking Laplace transform and rearranging the terms gives the transfer function.

$$\frac{V_{in}}{R} = -CsV_{out} \tag{1.17}$$

$$\frac{V_{out}}{V_{in}} = -\frac{1}{RCs} \tag{1.18}$$

Similarly, we can find the transfer function of Differentiator as

$$\frac{V_{out}}{V_{in}} = -RCs \tag{1.19}$$

Derivation of this differentiator is left as a home assignment.

1.3.6 Modelling an Electro-Mechanical System (Armature Controlled DC Motor)

DC motors that are used in feedback controlled devices are called DC servomotors. Applications of DC servomotors abound, e.g. in robotics, computer disk drives, printers, aircraft flight control systems, machine tools, flexible manufacturing systems, automatic steering control etc. DC motors are classified as armature-controlled DC motors and field-controlled DC motors.

This experiment will focus on modelling, identification, and position control of an armature-controlled DC servomotor. In particular, we will first develop the governing differential equations and the Laplace domain transfer function model of an armature controlled DC motor. Next, we will tend to the identification of the unknown system parameters that appear in the transfer function model of the DC servomotor. Finally, we will develop and implement a position-plus-velocity feedback controller to ensure that the DC motor angular position response tracks a step command.

The motor, we would be discussing, is a permanent-magnet type and has a single armature winding. Current flow through the armature is controlled by power amplifiers as in Figure 1.7 so that rotation in both directions is possible by using one, or both of the inputs.

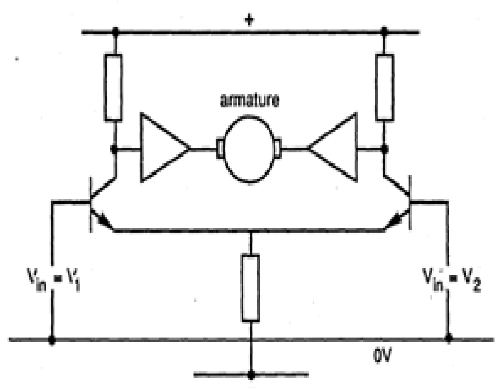

Figure 1.7: Armature Control of Permanent Magnet DC Motor

As the motor accelerates, the armature generates an increasing back-emf, V_b, which tends to oppose the driving voltage V_a. The armature current is thus roughly proportional to $V_a - V_b$. If the speed drops (due to loading) V_b reduces, the current increases and so does the motor torque. This tends to oppose the speed drop. That is why this mode of control is called 'armature-control' and gives a speed proportional to V_a.

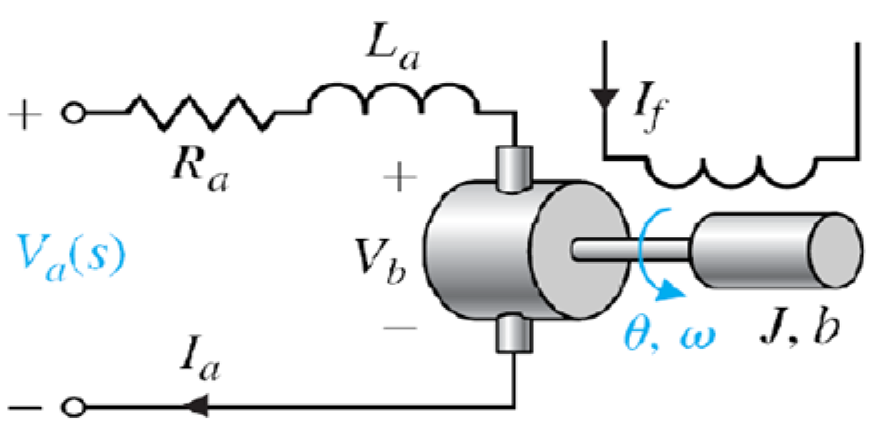

Figure 1.8: Separately Excited DC Motor

To derive a mathematical model for the motor under discussion, consider the schematic shown in Figure 1.8. As we know that a motor is an electro-mechanical device so its mathematic model consists of two differential equations each representing one of its aspects.

First, we observe that the torque of a permanent-magnet DC motor is directly proportional to its armature current. Stating mathematically, we say,

$$T_m = K_t i_a(t) \tag{1.20}$$

Here K_t is motor torque constant.

We also note that the back-emf produced in armature-controlled DC motor is directly proportional to the angular velocity of armature. Thus,

$$V_b = K_b \omega_a(t) \tag{1.21}$$

Here K_b is motor constant. In SI units, however, the value of $K_b$and K_t is equal.

When the motor rotates and produces a torque as given in equation 1.22, there emerges a retarding torque, because of friction, in response toT_m. If we assume the friction to be viscous friction, as we assumed for the case of mass-spring model in previous labs, we get

$$T_r = b\omega_a(t) \tag{1.22}$$

Newton's second law of motion, when translated into angular mechanics, takes the following form.

$$\sum T = J\alpha(t) \tag{1.23}$$

Here J is moment of inertia and $\alpha(t)$ is angular acceleration. Putting value, we get

$$T_m - b.\omega(t) = J\frac{d\omega(t)}{dt} \tag{1.24}$$

Or,

$$\frac{d\omega(t)}{dt} = \frac{1}{J}(K_t i_a(t) - b.\omega(t)) \tag{1.25}$$

Equation 1.25 is the mechanical equation for an armature-controlled DC motor.

Now we turn to the electrical aspect of this motor. Applying Kirchhoff's Voltage Law to Figure 1.8, we get,

$$V_a = Ri_a(t) + L\frac{di_a(t)}{dt} + V_b \tag{1.26}$$

Substituting equation 1.21 and rearranging the terms gives,

$$\frac{di_a(t)}{dt} = \frac{1}{L}\left(V_a - Ri_a(t) - K_b\omega_a(t)\right) \tag{1.27}$$

Equation 1.27 gives electrical equation of the motor. Combining these two equations results in a complete model of armature-controlled DC motor.

1.4 In-Lab Experiments

1.4.1 Analysing the Electronic Systems

Using Mathematical Models of Integrator and Differentiator Circuits described in pre lab session, do the following. Assume the following parameters

$$C = 0.1\ uF; \quad \mathrm{L} = 0.01H; \quad R = 47K\Omega;$$

a) Implement the models in MATLAB/LabVIEW. Write down the expression below

Integrator	Differentiator

b) Represent the system in Pole-Zero-Gain Form. Write down the expression below

Integrator	Differentiator

c) Evaluate and plot poles and zeros of the systems. Write numeric value below

Integrator	Differentiator

d) Find out Step Response of Systems

Integrator	Differentiator

e) Comment on stability

Integrator	Differentiator

1.4.2 Analysing the Electrical System

Using Mathematical Model of RLC Circuit described in pre lab session, do the following. Assume the following parameters

$$C = 0.1\,uF; \quad \mathrm{L} = 0.01H; \quad R = 47K\Omega;$$

a) Implement the model in MATLAB/LabVIEW. Write down the expression below

b) Represent the system in Pole-Zero-Gain Form. Write down the expression below

c) Evaluate and plot poles and zeros of the system. Write numeric value below

d) Find out Step Response of System

e) Comment on stability

1.4.3 Analysing the Electromechanical System

Using Mathematical Models of DC Motor derived in pre lab session and assuming following parameters.

R_a=Armature Resistance=8 Ω, L_a=Armature Inductance=1.2 x 10^{-3} H

K_t=Motor torque Constant=0.04 N.mA^{-1}, K_b=Motor Back Emf constant=0.04 $\frac{v}{rad.s^{-1}}$

b=Motor viscous friction constant=0.00002 N/m.s^{-1}, J=Moment of Inertia=4 x 10^{-6}Kg.m^2

a) Implement the Simulink Model for Speed of the System on Matlab Simulink

b) Find out Step Response of the System.

c) Implement the Simulink Model for Position of the System on Matlab Simulink

d) Find out Step Response of the System.

(Please Note that Recommended stop time for simulink simulation is 3s at given parameter values)

1.5 Post-Lab Exercise

Exercise 1:

Consider the Mass Spring System given in Pre Lab section. Find its step response using Simulink.

Rubric for Lab Assessment

The student performance for the assigned task during the lab session was:			
Excellent	The student completed assigned tasks without any help from the instructor, adhered to health and safety standards, and presented the results appropriately.	4	
Good	The student completed assigned tasks with minimal help from the instructor, adhered to health and safety standards, and presented the results appropriately.	3	
Average	The student could not complete all assigned tasks, adhered to health and safety standards, and presented partial results.	2	
Worst	The student did not complete assigned tasks.	1	

Instructor Signature: ______________________________ Date: ____________________

LAB # 2: Block Diagram Reduction Techniques and Measuring Time-Domain Performance Parameters & Effect of Disturbance in Open/Closed-Loop Control System using MATLAB

2.1 Objectives

- To Construct equivalent mathematical model of a complicated system using MATLAB
- To Measure performance of the system through time domain performance parameters using MATLAB
- To compare the performance of an open-loop and closed-loop system by measuring the Effect of Disturbance, using LabVIEW

2.2 Lab Instructions

- ✓ This lab activity comprises of three parts: Pre-lab, Lab Exercises, and Post-Lab Viva session.
- ✓ The students should perform and demonstrate each lab task separately for step wise evaluation
- ✓ Only those tasks that completed during the allocated lab time will be credited to the students. Students are however encouraged to practice on their own in spare time for enhancing their skills.

2.3 Theory and Pre-Lab Exercises:

In this exercise our prime objective is to simulate Linear Time-Invariant Systems. We will start by understanding the interconnection of various system components and methods to reduce a rather complex-looking system to a simple transfer function. These techniques facilitate the control engineers to analyse the whole system as a single unit and study its overall behaviour.

Primary concerns in control system design are performance and stability. In order to design and analyse control systems, we must first establish adequate performance specifications. Performance specifications can be presented in the time domain or the frequency domain. Time-domain specifications generally take the form of settling time, percent overshoot, rise time, and steady-state error. Stability and frequency-domain specifications are addressed in coming labs.

As we have learned so far, the advantages and disadvantages of open loop control systems over the closed loop control systems. One of the major advantage of closed loop control system over the open loop control system is that the closed loop control system is less sensitive to disturbance. In this pre lab session we are going to prove it mathematically. We will also observe the same during the simulation in lab.

In general, we use feedback in a control system to

1. Decrease the sensitivity of the system to plant variations,
2. Enable adjustment of the system’s transient response,
3. Reject disturbance, and
4. Reduce steady-state tracking error.

The aforementioned advantages of feedback come at the cost of increased number of components and system complexity, reduced closed-loop gain, and the introduction of possible instabilities. However, the advantages of feedback outweigh the disadvantages to such an extent that feedback control systems are found everywhere. In this lab, the advantages of feedback are illustrated with two examples. As always, our prime focus would be to emphasis the use of Matlab in the control system analysis.

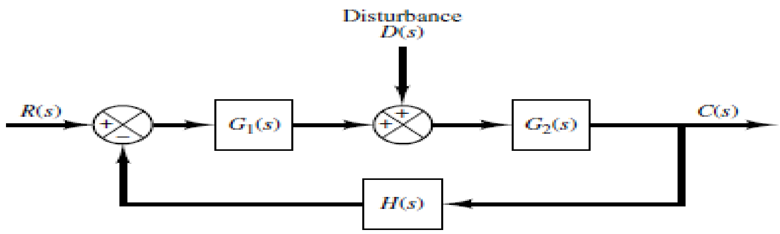

Figure 2.1

2.3.1 The block diagrams

A block diagram is a specialized, high-level type of flow chart. Its highly structured form presents a bird's eye view of major process steps and key process participants, as well as their relationships and interfaces involved. Various components in a system may be related to one another in different fashions, like series, parallel, or feedback. We shall first learn how to deal with these relations in MATLAB.

2.3.2 Series Function

Two or more blocks are said to be connected in series only if the output of one block is not affected by the next following block. Any number of cascaded blocks representing non-loading components can be replaced by a single block, the transfer function of which is simply the product of the individual transfer functions. Figure 2.2 shows a system comprising of two subsystems, represented by their transfer functions G1(s) and G2(s), connected in series.

Let the $G_1(s)$ and $G_2(s)$ is of the form,

$$\mathrm{G_1(s)} = \frac{\mathrm{num1}}{\mathrm{den1}} \qquad , \qquad \mathrm{G_2(s)} = \frac{\mathrm{num2}}{\mathrm{den2}}$$

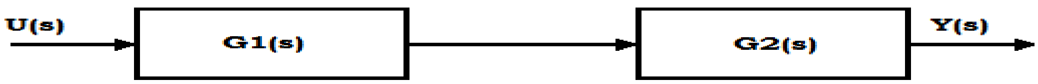

Figure 2.2: Series Connection

Then the overall transfer function, T(s), can be represented by,

$$\mathrm{T(s)} = \frac{\mathrm{num}}{\mathrm{den}}$$

If we know the transfer functions of the subsystems, $G_1(s)$ and $G_2(s)$, then we can find the transfer function of $T(s)$ by using the following Mathscript command.

>>$sys\ =\ series\ (sys1, sys2)$

2.3.3 Parallel Function

Block diagrams quite often have transfer functions in parallel configuration as shown in Figure 2.3.

To solve this problem, we use the following Math script command,

>>$sys\ =\ parallel(sys1, sys2)$

It will consume both transfer functions and the summing junction and give a single block between $U(s)\ and\ Y(s)$ equivalent to the consumed components. A similar way can also be performed for parallel connection.

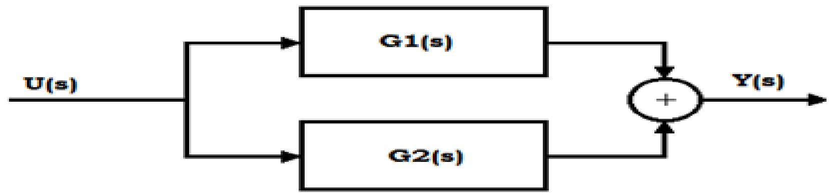

Figure 2.3: Parallel Combination

2.3.4 Feedback System

Feedback paths are present in every other control system. They enable us to continuously monitor the response of a system and make necessary changes in the reference input to get the desired output. Figure 2.4 shows a typical control system with G2(s) in feedback path.

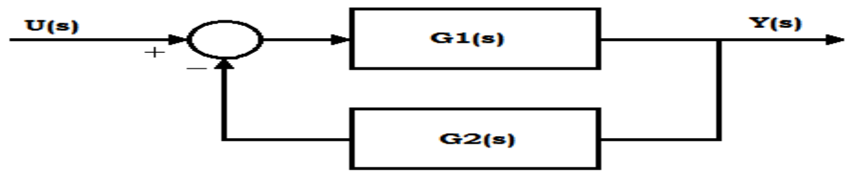

Figure 2.4: Feedback Combination

The overall transfer function of this system can be calculated as,

$$\mathrm{T(s)} = \frac{\mathrm{Y(s)}}{\mathrm{U(s)}} = \frac{\mathrm{G_1(s)}}{1 \mp \mathrm{G_1(s)G_2(s)}}$$

Matlab command feedback is used to resolve transfer functions connected in feedback style.

>>$sys\ = feedback(sys1, sys2)$

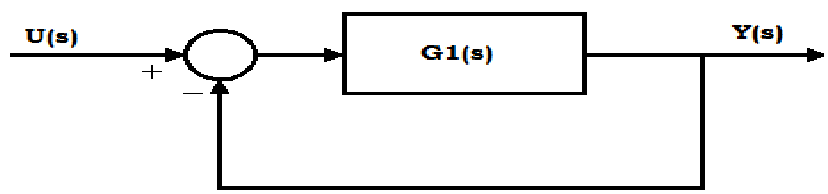

Figure 2.5: Unity Feedback Combination

By default feedback $(sys1, sys2)$ assumes negative feedback and is equivalent to $feedback(sys1, sys2, -1)$. For positive feedback $+1$ has to be written explicitly as third argument.

Now assume that $G_2(s)\ = 1$, that means we have connected output back to the input directly without any transfer function involved. It would result in a special feedback loop, called unity feedback, as shown in Figure 2.5. Total transfer function, T(s), in this case would then be,

$$\mathrm{T(s)} = \frac{\mathrm{Y(s)}}{\mathrm{U(s)}} = \frac{\mathrm{G_1(s)}}{1 \mp \mathrm{G_1(s)}}$$

Now we can use the same feedback command to calculate T(s),

>>$sys\ = feedback(sys1, 1)$

Here 1 represents unity feedback.

2.3.5 Time-domain specifications

Time-domain performance parameters are generally given in terms of the transient response of a system to a given input signal. Since the actual input, which the system will be subjected to in practice, is generally unknown, a standard test input signal is used. The test signals are of the general form

$$\mathrm{r(t) = t^n}$$

And the corresponding Laplace transform is

$$\mathrm{R(s) = \frac{n!}{s^{n+1}}}$$

When $n = 0, 1\ and\ 2$, we have step, ramp, and parabolic inputs respectively. An impulse function may also be used as a test signal.

The standard performance measures are usually defined in terms of the step response and the impulse response. The most common step response performance measures are percent overshoot $(P.O)$, rise time (t_r), peak time (t_p), settling time (t_s) and steady-state error (e_{ss}) as shown in figure 2.6.

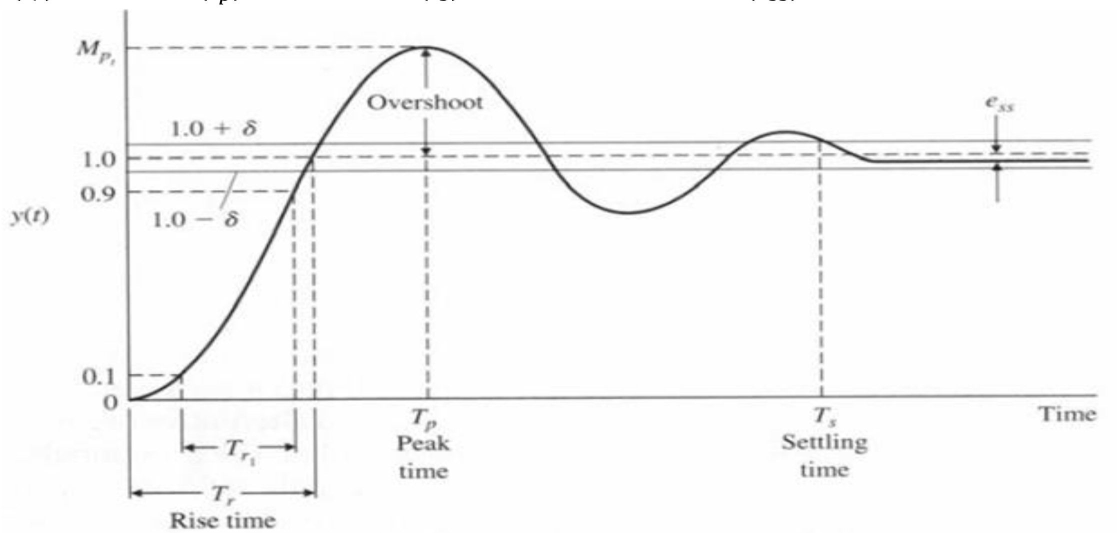

Figure 2.6: Step Response of a second order system

The performance parameters depicted in the Figure 2.6 above can be described as follows.

a) Rise Time

Rise time refers to the time required for a signal to rise from a specified low value to a specified high value. Typically, these values are 10% and 90% of the step height. The $0 - 100\%$ rise time, T_r, measures the time to reach 100% of the input magnitude. Alternatively, T_{r1}, may measure the time from $10\% - 90\%$ of the response to the step input.

b) Peak Time

The peak time is the time required for the response to reach the highest peak of the overshoot. Peak time is inversely proportional to the amount of overshoot.

c) Settling Time

The time required for the system's output to settle within a certain percentage of the input amplitude (which is usually taken as 2%) is called settling time. Settling time, T_s, is calculated as

$$\mathrm{T_s = \frac{4}{\zeta\omega_o}}$$

d) Overshoot

Overshoot is the maximum peak value of the response curve measured from the desired response of the system. It is calculated as

$$\mathrm{P.O} = \frac{\mathrm{M_p} - \mathrm{fv}}{\mathrm{fv}} \times 100\%$$

Where M_p is the highest peak magnitude of output and f_v is the final value of the output.

e) Steady-State Error

It is the difference between the desired final output and the actual one. Practically this difference can never be reduced to zero. However, a suitable tolerance range may be defined to assume that the system has reached its desired value. It is calculated as $abs(dcgain(1 - system))$.

2.3.6 Closed Loop Control System Subject to Disturbance

Figure 2.1 shows closed loop control systems subject to disturbance. When two inputs (reference and the disturbance) are present in an LTI system, each input can be treated separately according to super position theorem and the output can be calculated by adding individual outputs because of both inputs.

Examining the effect of disturbance, we get

$$\frac{C_D(S)}{D(S)} = \frac{G_2(S)}{1 + G_1(S)G_2(S)H(S)}$$

Now examining the effect of reference input

$$\frac{C_R(S)}{R(S)} = \frac{G_1(S)G_2(S)}{1 + G_1(S)G_2(S)H(S)}$$

The response to the simultaneous application of the reference input and disturbance can be calculated by adding the individual responses

$$C(S) = C_R(S) + C_D(S) = \frac{G_2(S)}{1 + G_1(S)G_2(S)H(S)}[G_1(S)R(S) + D(S)]$$

Consider now the case where $|G_1(s)H(s)| >> 1$ and $|G_1(s)G_2(s)H(s)| >> 1$. In this case, the closed-loop transfer function $C_D(s)/D(s)$ becomes almost zero, and the effect of the disturbance is suppressed. This is an advantage of the closed-loop system. On the other hand, the closed-loop transfer function $C_R(s)/R(s)$ approaches $1/H(s)$as the gain of $G_1(s)G_2(s)H(s)$ increases.This means that if $|G_1(s)G_2(s)H(s)| >> 1$, then the closed-loop transfer function $C_R(s)/R(s)$ becomes independent of $G_1(s)$ and $G_2(s)$and inversely proportional to H(s), so that the variations of $G_1(s)$ and $G_2(s)$do not affect the closed-loop transfer function $C_R(s)/R(s)$. This is another advantage of the closed-loop system. It can easily be seen that any closed-loop system with unity feedback, $H(s) = 1$, tends to equalize the input and output.

2.3.7 Electric Traction Motor Control

If you ever have visited Khewra Salt Mines, you would definitely have caught sight of a little electric train which was once used to draw salt out of the mine but today it serves as a tourist transit train. The basic principle of such an electric train is given in Figure 2.7

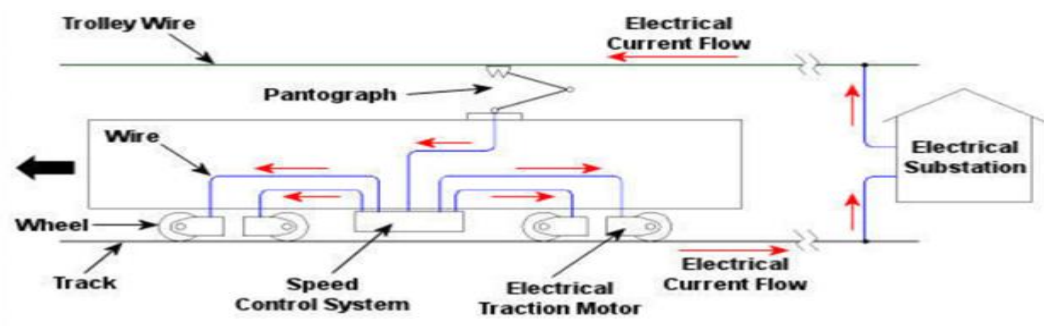

Figure 2.7: Electric Train Propulsion System

The substation at the right of Figure 2.7 performs the task of taking the power provided by an electric utility power plant and converting it into a form of power that the electric rail car can use to operate its traction motors. The two terminals of this substation are connected via a trolley wire and the two rails that the train runs on. The electric train has a spring-loaded arm called a pantograph on its roof that touches the trolley wire, allowing electrical current to flow into a speed control system housed under the train. This speed control system performs the task of varying the flow of electrical current to the traction motors, enabling the car to move, before it eventually exits the motor through its wheels, then back to the substation where it originated, thus completing an electrical circuit.

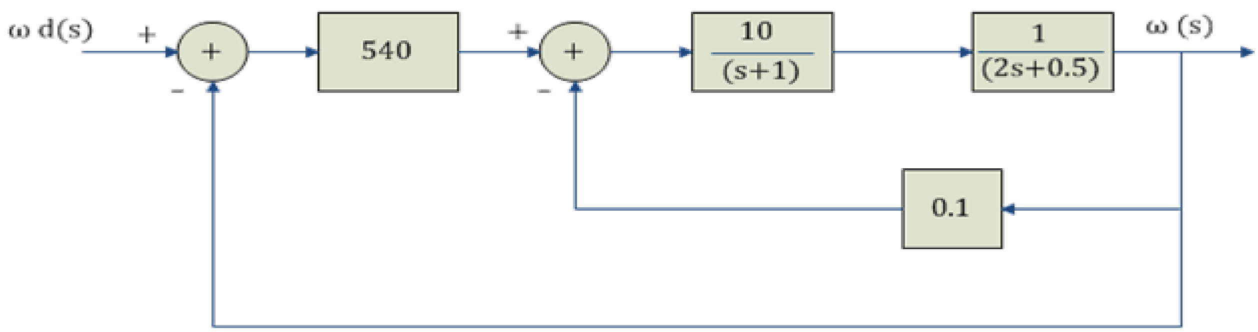

Figure 2.8: Block Diagram of the System shown in Figure 2.7

The speed control system of an electric train is shown in Figure 2.8. Our goal in this problem is to find the closed-loop transfer function and investigate the response of ω to a commanded ω_d. The first step in this direction would be to apply block diagram reduction techniques learnt in previous labs to compute the closed-loop transfer functionω / ω_d. We observe that closed-loop characteristic equation is second-order with $\omega_n = 52$ and $\xi = 0.012$. Since the damping is low, we might expect the response to be highly oscillatory as shown in Figure 2.9.

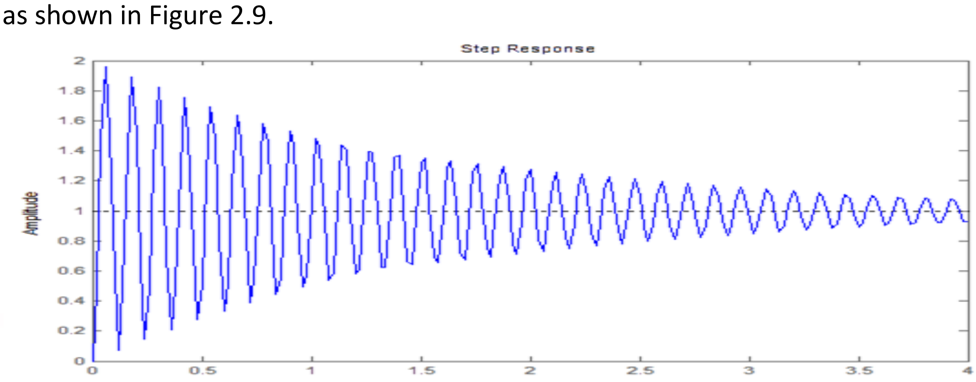

Figure 2.9: Step Response of an Electric Traction Motor

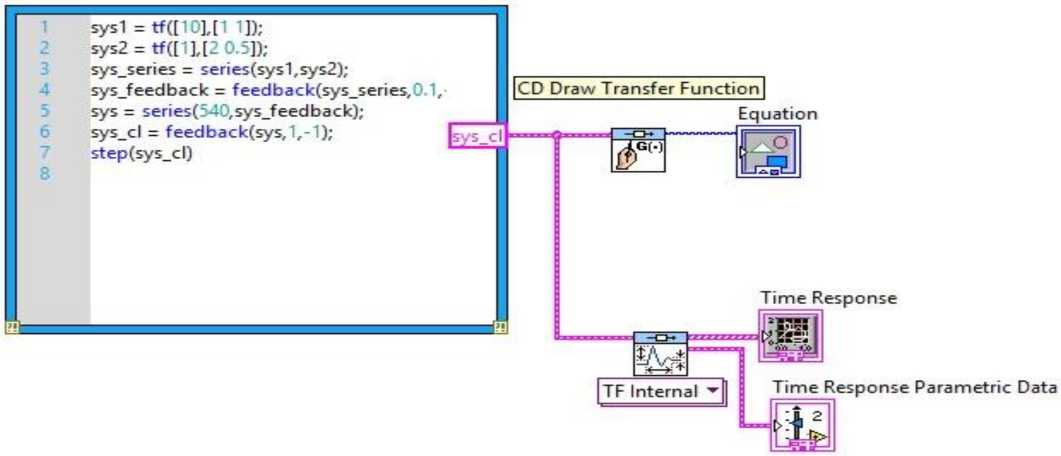

Figure 2.10: Implementation of Electric Traction System on LabVIEW

To quantitatively observe step response parameters of this system we may use CD Parametric Time Response block which, in this case, yields,

RiseTime: 0.0202
SettlingTime: 6.2294
SettlingMin: 0.0728
SettlingMax: 1.9624
Overshoot: 96.2920
Undershoot: 0
Peak: 1.9624
PeakTime: 0.0605

2.4 In-Lab Experiments

The open-loop block diagram description of the armature-controlled DC motor with a load torque disturbance, $T_d(s)$, is shown in Figure 2.11. The values for various parameters are given below.

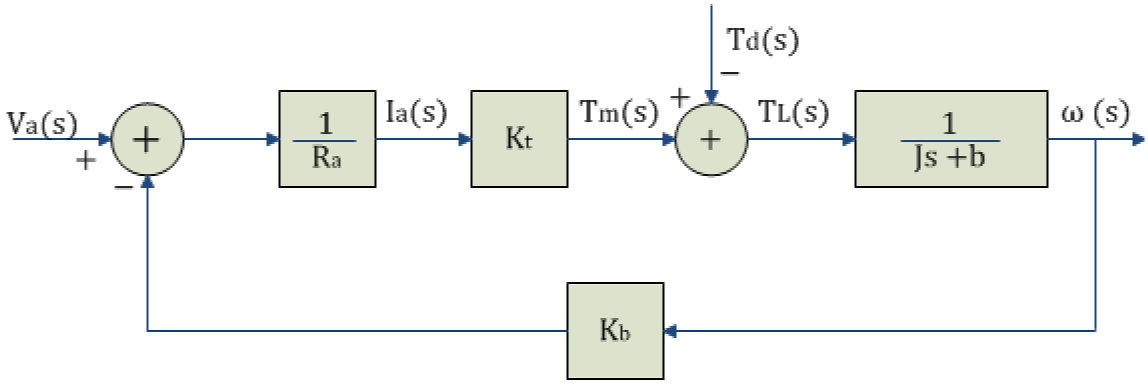

Figure 2.11: Speed Control Tachometer System

$R_a = 1$
$K_t = 10$
$K_b = 0.1$
$J = 2$
$b = 0.5$

2.4.1 Disturbance Analysis

1. Calculate the steady state speed of the motor due to disturbance (ideally it should be zero) in the above system, assume the system to be linear (which means you can ignore the input while calculating the effect of disturbance, as per superposition principle). Note this value in Table 3.1. Attach snapshots
2. Now apply a feedback path to the system with transfer function of the feedback block to be $K_{tac} = 1$ and calculate the steady state speed due to disturbance. Again Note this value in Table 2.1. Attach snapshots
3. Compare these values. What conclusion can be drawn based on these values?

2.4.2 Performance Analysis

1. Now apply step input to the system used in part 1 of 2.5.1(Assume disturbance to be zero) and find out the values performance parameters in the system. Note these values in Table 2.1. Attach snapshot
2. Calculate the performance parameters of the system used in Part 2 of 2.5.1. Note these values in Table 2.1. Attach snapshot
3. Compare the results obtained in Part 1 and 2 of this section.

2.4.3 Results

Table 2-1

Parameter	Symbol	Value	Unit
Rise Time of OL System	T_r		s
Peak Time of OL System	T_p		s
Settling Time of OL System	T_s		s
Percentage Overshoot of OL System	$P.O$		$-$
Steady State Error of OL System	e_{ss}		V
Steady State Value of OL System	V_{ss}		V
Rise Time of CL System	T_r		s
Peak Time of CL System	T_p		s
Settling Time of CL System	T_s		s
Percentage Overshoot of CL System	$P.O$		$-$
Steady State Error of CL System	e_{ss}		V
Steady State Value of CL System	V_{ss}		V

2.5 Post-Lab Exercise

Exercise 1

Implement given block diagram on Simulink and verify that the plot shown on scope is same as the step response of given transfer function which verifies that the given transfer function is simplifed form of given block diagram, obtained after applying block diagram reduction techniques.

$$\frac{X(s)}{R(s)} = \frac{1}{200s^2 + 25s + 10}$$

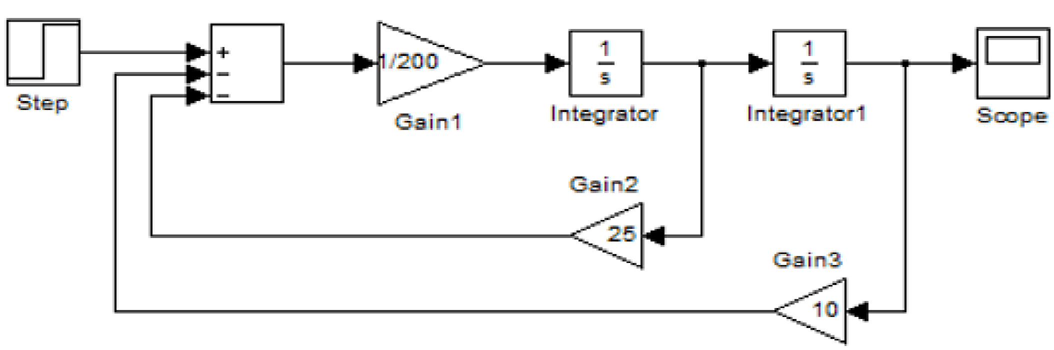

Figure 2.12

Rubric for Lab Assessment

The student performance for the assigned task during the lab session was:			
Excellent	The student completed assigned tasks without any help from the instructor, adhered to health and safety standards, and presented the results appropriately.	4	
Good	The student completed assigned tasks with minimal help from the instructor, adhered to health and safety standards, and presented the results appropriately.	3	
Average	The student could not complete all assigned tasks, adhered to health and safety standards, and presented partial results.	2	
Worst	The student did not complete assigned tasks.	1	

Instructor Signature: ________________________________ Date: _____________________

LAB # 3: Design of Proportional-Integral-Derivative (PID) Controller using MATLAB

3.1 Objectives

- To Design PID Controller for different systems

3.2 Lab Instructions

- ✓ This lab activity comprises of three parts: Pre-lab, Lab Exercises, and Post-Lab Viva session.
- ✓ The students should perform and demonstrate each lab task separately for step-wise evaluation
- ✓ Only those tasks that completed during the allocated lab time will be credited to the students. Students are however encouraged to practice on their own in spare time for enhancing their skills.

3.3 Pre-Lab Theory and Exercises:

A proportional–integral–derivative controller (PID controller) is the most commonly used feedback controller in industrial control systems. A PID controller calculates an error value as the difference between measured output and a desired set point. The controller attempts to minimize the error by adjusting the process control inputs.

The popularity of PID controller can be attributed partly to their robust performance in a wide range of operating conditions and partly to their functional simplicity, which allows engineers to operate them in a simple and straightforward manner.

3.3.1 PID Controller

The transfer function of the PID controller looks like the following;

$$\mathrm{K_p} + \frac{\mathrm{K_i}}{\mathrm{s}} + \mathrm{K_d s} = \frac{\mathrm{K_d s^2} + \mathrm{K_p s} + \mathrm{K_i}}{\mathrm{s}}$$

here,

$K_p \;=\; Proportional\ Gain$
$K_i \;=\; Integral\ Gain$
$K_d \;=\; Derivative\ Gain$

First, let's take a look at how the PID controller works in a closed-loop system using the schematic shown below.

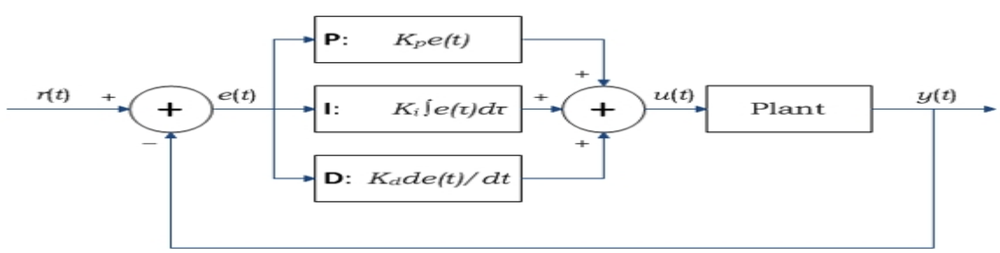

Figure 3.1:A block Diagram of a PID Controller

The signal $e(t)$ represents the tracking error, the difference between the reference input $r(t)$ and the actual output $y(t)$. This error signal will be sent to the PID controller, and the controller computes both the derivative and integral of this error signal. The signal $u(t)$ just past the controller is now equal to the proportional gain (K_p) times the magnitude of the error signal plus the integral gain (K_i) times the integral of the error signal plus the derivative gain (K_d) times the derivative of the error signal.

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt}$$

This signal, $u(t)$, will now be sent to the plant, and the new output, $y(t)$, will be obtained. This new output will be sent back to the sensor again to find the new error signal. The controller takes this new error signal and computes its derivative and it's integral again. This process goes on and on.

a) Characteristics of P, I, and D Controller

A proportional controller (K_p) will have the effect of reducing the rise time and will reduce but never eliminate the steady-state error. An integral control (K_i) will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative control (K_d) increases the stability of the system, reduces the overshoot, and improves the transient response. Effects of each controller on closed-loop system are summarized in the table below.

Table 3-1

CL Response	Rise Time	Overshoot	Settling Time	S-S Error
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

Note that these correlations may not be exactly accurate, because $K_p, K_i, and\ K_d$ are dependent on each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for $K_i, K_p\ and\ K_d$.

b) General Tips for Designing a PID Controller

When you are designing a PID controller for a given system, follow the steps shown below to obtain a desired response.

1. Obtain an open-loop response and determine what needs to be improved
2. Add a proportional control to improve the rise time
3. Add a derivative control to improve the overshoot
4. Add an integral control to eliminate the steady-state error
5. Adjust each of $K_p, K_i, and\ K_d$ until you obtain a desired overall response. You can always refer to the table shown above to find out which controller controls what characteristics.

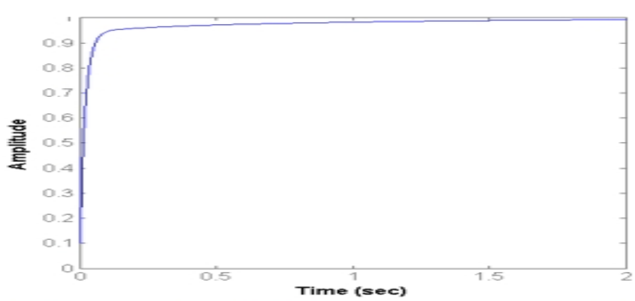

Figure 3.2 Step Response of the System with K_p=350,K_i=300 and K_d=50:

Lastly, keep in mind that you do not need to implement all three controllers (proportional, derivative, and integral) into a single system, if not necessary. For example, if a PI controller gives a good enough response (like the above example), then you don't need to implement a derivative controller on the system. Keep the controller as simple as possible.

3.4 In-Lab Experiments

3.4.1 Mass-Spring-Damper Model

Consider the Mass Spring System as discussed in Lab 1, with transfer function

$$\frac{X(s)}{F_a(s)} = \frac{1}{Ms^2 + bs + k}$$

Let the system parameters be,

$M = 1\,kg$
$b = 10\,N.s/m$
$k = 20\,N/m$
$F_a = 1N$

1. Design a PID controller using any technique learned in this course to achieve the following
   - ⇨ Rise Time $\leq 2\,s$
   - ⇨ Overshoot $\leq 5\%$
   - ⇨ No Steady-State Error

   Note down the values of proportional, integral and derivative control gains calculated

2. Implement the designed controller in MATLAB and analyse the response of your system before and after designing the controller. Attach Matlab code and Results

3.4.2 Design Problem: Ball & Beam System

The open-loop transfer function of the plant for a ball and beam System is:

$$\frac{X(s)}{\theta\ (s)} = \frac{-mgd}{L(\frac{J}{R^2} + m)}\frac{1}{s^2}$$

Where $m = 0.111$, $R = 0.015$, $g = -9.8$, $L = 1.0$, $d = 0.03$, $J = 9.99 \times 10^{-6}$

The design criteria for this problem are:

- ⇨ $T_s \leq 3\,s$
- ⇨ $PO \leq 5\%$

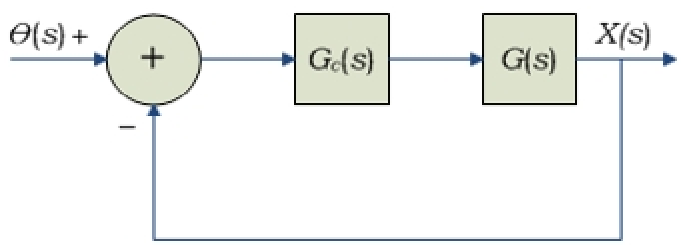

Figure 3.3: Ball and Beam Control System

1. Design a PID Controller to achieve the above criteria. Note down the values of proportional, integral and derivative control gains calculated

2. Implement the designed controller in MATLAB and analyse the response of your system before and after designing the controller. Attach Matlab code and Results

3.5 Post-Lab Exercises

Exercise 1:

The open-loop transfer function of the DC Motor speed is

$$\frac{\dot{\Theta}}{U} = \frac{K}{(Js + b)(Ls + R) + K^2}$$

For a 1 rad/sec step input, design a PID controller that satisfies the given criteria:

- Settling time less than 2 seconds
- Overshoot less than 5%
- Steady-stage error less than 1%

Exercise 2:

The open-loop transfer function of the DC Motor position is

$$\frac{\Theta}{U} = \frac{K}{s((Js + b)(Ls + R) + K^2)}$$

For a 1 rad/sec step input, design a PID controller

- Settling time less than 0.04 seconds
- Overshoot less than 16%
- No steady-state error
- No steady-state error due to a disturbance

Note: For exercise 1 and 2 refer to the Figure below.

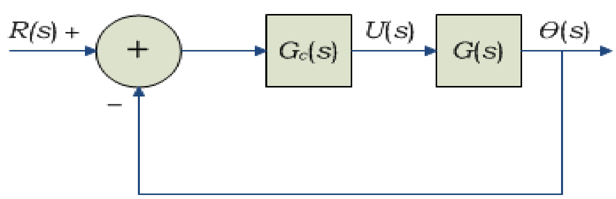

Figure 3.4

Rubric for Lab Assessment

The student performance for the assigned task during the lab session was:			
Excellent	The student completed assigned tasks without any help from the instructor, adhered to health and safety standards, and presented the results appropriately.	4	
Good	The student completed assigned tasks with minimal help from the instructor, adhered to health and safety standards, and presented the results appropriately.	3	
Average	The student could not complete all assigned tasks, adhered to health and safety standards, and presented partial results.	2	
Worst	The student did not complete assigned tasks.	1	

Instructor Signature: ______________________________ Date: ____________________

LAB # 4: Introduction to Vertical Take-off and landing (VTOL) and Design of a Current Controller using LabVIEW

4.1 Objectives

- To introduce the VTOL system and open-loop and closed-loop operations on the system
- Design of a current controller to control the amount of current drawn by the propeller motor.

4.2 Lab Instructions

- ✓ This lab activity comprises of three parts: Pre-lab, Lab Exercises, and Post-Lab Viva session.
- ✓ The students should perform and demonstrate each lab task separately for step-wise evaluation
- ✓ Only those tasks that completed during the allocated lab time will be credited to the students. Students are however encouraged to practice on their own in spare time for enhancing their skills.

4.3 Theory and Pre-Lab Exercises

4.3.1 Introduction

The QNET vertical take-off and landing (VTOL) trainer is shown in Figure 4.1. The system consists of a variable speed fan with a safety guard mounted on an arm. At the other end of the arm, an adjustable counterweight is attached. This allows the position of the weight to be changed, which in turn affects the dynamics of the system. The arm assembly pivots about a rotary encoder shaft. The VTOL pitch position can be acquired from this setup. Some examples of real-world VTOL devices are helicopters, rockets, balloons, and harrier jets. Aerospace devices are typically more difficult to model. Usually this will involve using software system identification tools to determine parameters or actual dynamics. Due to their inherent complexity, flight systems are usually broken down into different subsystems to make it more manageable. These subsystems can be dealt with individually and then integrated to provide an overall solution.

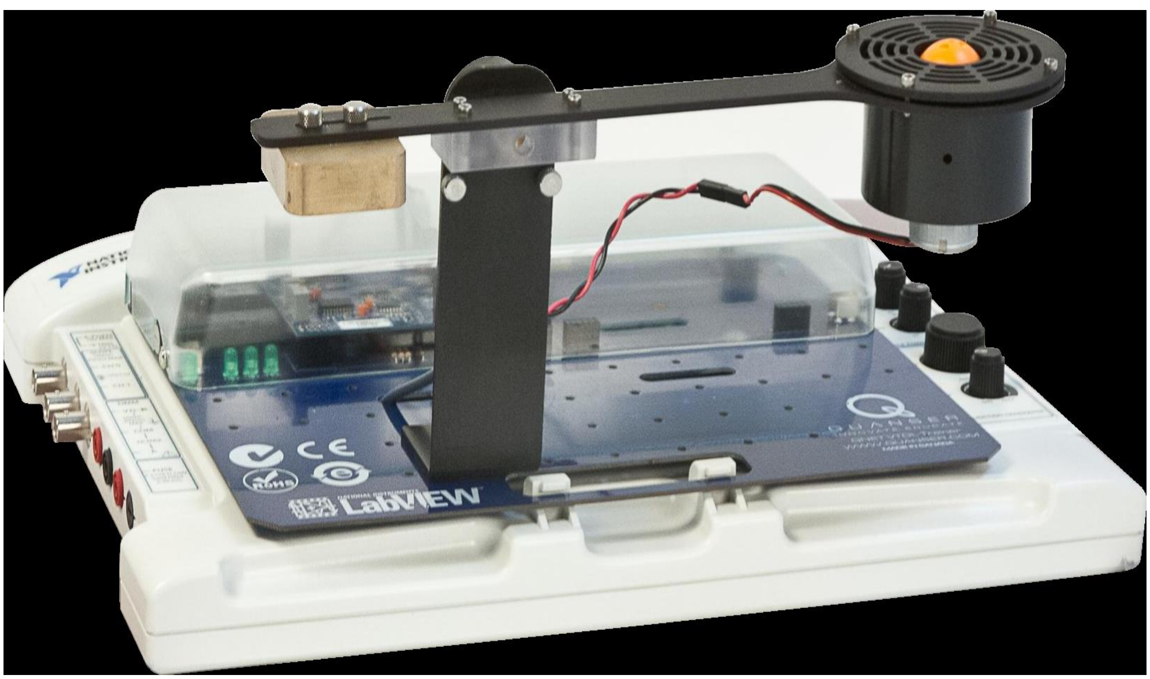

Figure 4.1

4.3.2 Current Control

a) Block Diagram and Mathematics

The VTOL device is broken down into two subsystems: the voltage-current dynamics of the motor and the current position dynamics of the VTOL body. The cascade control implemented in the VTOL trainer is depicted in Figure 4.2 below. A PI current controller, the inner loop, is designed to regulate the current inside the motor according to a desired current reference. This current reference is generated from the outer-loop controller: a PID compensator that controls the pitch of the VTOL trainer.

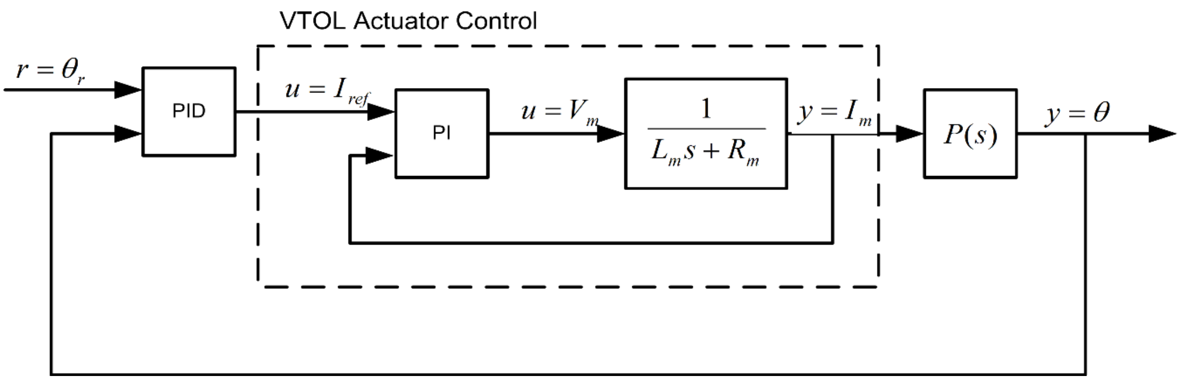

Figure 4.2

In cases where the actuator has relatively slow dynamics, such as an electromagnet with a large inductance, it is favourable to design a current controller. Typically, a proportional-integral compensator is used to regulate the current flowing in the load. This basically makes the actuator dynamics negligible and simplifies the control design of the outer-loop. In this case, the voltage-current relationship of the VTOL trainer motor can be described, in the time-domain, by the equation

$$V_m = -R_m i_m + L_m i_m$$

And by transfer function

$$I_m(s) = \frac{V_m(s)}{R_m + L_m s}$$

Figure 4.3 shows the VTOL current control system implemented. The PI compensator computes the voltage necessary to reach the desired current.

Using the PI controller

$$v_m(t) = k_{p,c}\left(i_{ref}(t) - i_m(t)\right) + k_{i,c}\int i_{ref}(t) - i_m(t)dt$$

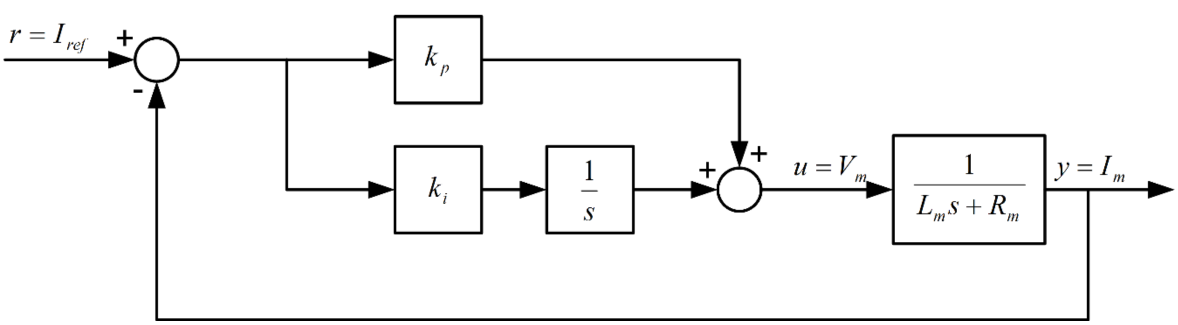

Figure 4.3

We obtain the following closed-loop transfer function

$$G_{I_{ref},I_m}(s) = \frac{k_{p,c} + k_{i,c}}{s^2 L_m + (k_{p,c} + R_m)s + k_{i,c}}$$

To match the standard second-order characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

We need a proportional gain of

$$k_{p,c} = -R_m + 2\xi\omega_n L_m$$

And an integral gain of

$$k_{i,c} = \omega_n^2 L_m$$

These gains can then be designed according to a desired natural frequency ω_n and damping ratio ξ

b) Introduction to Current Control VI

This VI is used to feed an open-loop voltage or current to the QNET-VTOL Trainer. The VI when in current mode is shown in Figure 4.4. In this mode, a current-controller is used to regulate the current in the motor, as discussed in previous section and the user chooses the reference current. In voltage mode, shown in Figure 4.5, the voltage chosen is applied directly to the QNET amplifier, which in turn drives the motor. As a quick VI description, Table 4.1 lists and describes the main elements of the QNET VTOL Current Control VI. Every element is uniquely identified by an ID number located in Figure 4.4 and Figure 4.5, for both current and voltage mode.

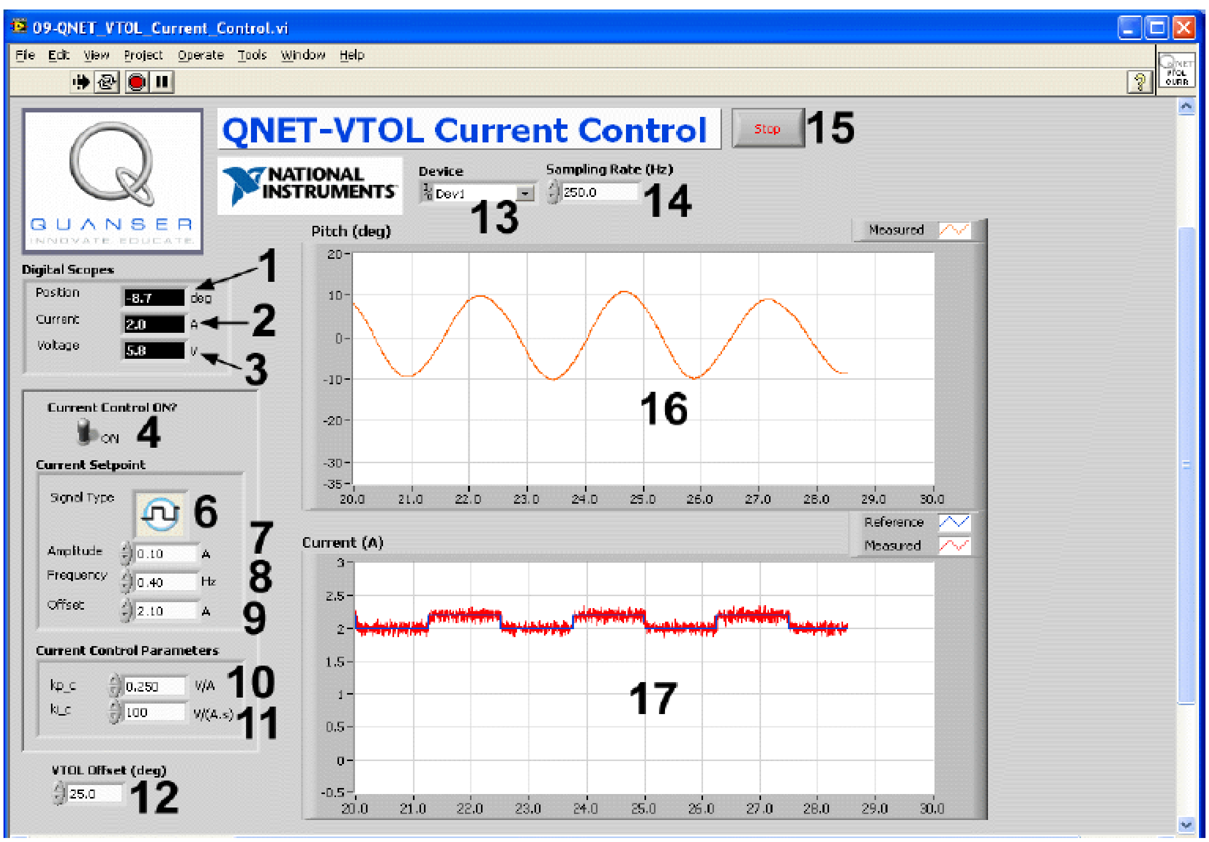

Figure 4.4

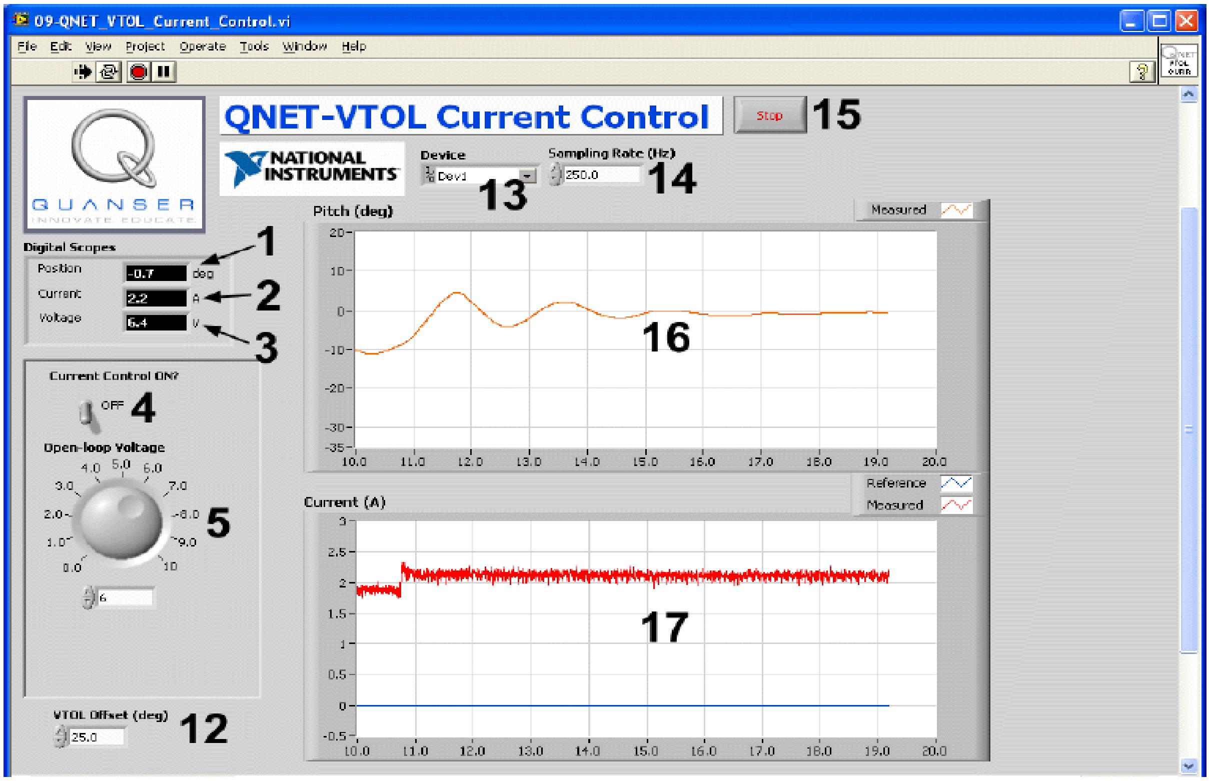

Figure 4.5

Table 4-1

ID #	Label	Parameter	Description	Unit
1	Position	θ	VTOL pitch position numeric display.	deg
2	Current	I_m	VTOL motor armature current numeric display.	A
3	Voltage	V_m	VTOL motor input voltage numeric display.	V
4	Current Control ON?		Turns current control on and off.	
5	Open-loop		Input motor voltage to be fed.	V
6	Signal Type		Type of signal generated for the current reference.	
7	Amplitude		Current setpoint amplitude input box.	A
8	Frequency		Current setpoint frequency input box.	Hz
9	Offset		Current setpoint offset input box.	A
10	kp_c	$k_{p,c}$	Current control proportional gain.	V/A
11	ki_c	$k_{i,c}$	Current control derivative gain.	V.s/A
12	VTOL Offset		Pitch calibration	deg
13	Device		Select the NI DAQ device.	
14	Sampling Rate		Sets the sampling rate of the VI.	Hz
15	Stop		Stops the LabVIEW VI from running.	
16	Pitch	θ	Scope with measured (in red) VTOL pitch position.	deg
17	Current	I_m	Scope with reference (in blue) and measured (in red) current.	A

4.4 In-Lab Experiments

4.4.1 Finding Resistance

1. Open the QNET_VTOL_Current_Control.vi Ensure the correct device is chosen
2. Run the VI
3. Set the Current Control ON switch to OFF
4. Set the Open-loop Voltage knob to 4.0 V. The VTOL Trainer propeller should begin turning as a voltage is applied to the motor.
5. Vary the voltage between 4.0 V and 8.0 V by steps of 1.0 V and measure the current at each voltage. Enter the results in the Table below and calculate the average resistance

Table 4-2

Sr.	Voltage (V)	Current (I)	Resistance (R=V/I)
1	4 V		
2	5 V		
3	6 V		
4	7 V		
5	8 V		
Average Resistance			

6. Click on Stop button to stop running the VI

4.4.2 Qualitative Current Control

1. Open the QNET_VTOL_Current_Control.vi
2. Ensure the correct device is chosen
3. Run the VI
4. Set the Current Control ON switch to ON
5. In the Current Set point section set:
   - ⇨ $Amplitude = 0.20\ A$
   - ⇨ $Frequency = 0.40\ Hz$
   - ⇨ $Offset = 0.90\ A$
6. In the Control Parameters section set the PI current gains to:
   - ⇨ $Kp_c = 0.250$
   - ⇨ $Ki_c = 10$

   The VTOL Trainer propeller should begin turning at various speeds according to the current command. Examine the reference and measured current response obtained in the Current (A) scope.
7. Show and explain the effects of having no integral gain. Attach a sample response

8. Show and explain the effects of having no proportional gain. Attach a sample response

9. Click on the Stop button to stop running the VI

4.4.3 Current Control Design

1. Calculate the PI gains, k_p and k_i, necessary to satisfy the following design criteria

⇨ $\omega_n = 42.5\ rad/s$
⇨ $\xi = 0.7$

Refer to section 5.4.2 (a) for calculations. Use the value of armature resistance as calculated in section 5.5.1 and inductance as given in the table below. Do all the calculations in the space provided below. Also fill the table below.

Table 4-3

Parameter	Value	Unit
R_m		ω
L_m	53.8	mH
ξ	0.7	
ω_n	42.5	rad/s
$k_{p,c}$		V/A
$k_{i,c}$		V/(A.s)

2. Open the QNET_VTOL_Current_Control.vi
3. Ensure the correct device is chosen
4. Run the VI
5. Set the Current Control ON switch to ON
6. In the Current Set point section set:
   ⇨ $Amplitude = 0.20\ A$
   ⇨ $Frequency = 0.40\ Hz$
   ⇨ $Offset = 0.90\ A$
7. In Current Control Parameters section, set the PI current gains to what you found above.
8. Run the VI. The VTOL Trainer propeller should begin turning at various speeds according to the current command. Examine the reference and measured current response obtained in the Current (A) scope. They should be tracking
9. Include a plot showing the current response with your designed PI gains. Does it satisfy the design criteria? Discuss below.

10. Click on Stop button to stop running the VI.

4.5 Post Lab Exercise

Exercise 1

Design a PI controller to control the current drawn by propeller motor using the criteria given below and validate the designed controller by using the calculated values of proportional and integral gain in current control VI in terms of performance specifications. Attach the responses.

Settling Time =5s
Overshoot = 5%

Rubric for Lab Assessment

The student performance for the assigned task during the lab session was:			
Excellent	The student completed assigned tasks without any help from the instructor, adhered to health and safety standards, and presented the results appropriately.	4	
Good	The student completed assigned tasks with minimal help from the instructor, adhered to health and safety standards, and presented the results appropriately.	3	
Average	The student could not complete all assigned tasks, adhered to health and safety standards, and presented partial results.	2	
Worst	The student did not complete assigned tasks.	1	

Instructor Signature: ______________________________ Date: ____________________

LAB # 5: Mathematical Modelling of the Vertical Take-Off and Landing System for the flight Control Operation and its validation using LabVIEW

5.1 Objectives

- To Create an open-loop mathematical model of the system for flight control operation, using the free body diagram
- To calculate the unknown parameters in the model
- To Design and Validate the model with the hardware prototype available in the lab using LabVIEW

5.2 Lab Instructions

- ✓ This lab activity comprises of three parts: Pre-lab, Lab Exercises, and Post-Lab Viva session.
- ✓ The students should perform and demonstrate each lab task separately for step-wise evaluation
- ✓ Only those tasks that completed during the allocated lab time will be credited to the students. Students are however encouraged to practice on their own in spare time for enhancing their skills.

5.3 Theory and Pre-Lab Exercises

5.3.1 Introduction and Background

Unlike a DC motor, this system has to be characterized with at least one second-order mode. The equation of motion is derived from first principles and then used to obtain the transfer function representing the current to position VTOL dynamics.

Various methods can be used to find the modelling parameters. In the laboratory, the parameters are first found manually by performing a few experiments and taking measurements. Thereafter, the LabVIEW ® Systems Identification Toolkit is used to automatically find the model. This demonstrates how to use software tools to identify parameters or even entire models (especially important for higher order systems). The model is then validated by running it in parallel with the actual system.

5.3.2 Torque Acting on VTOL

The free-body diagram of a 1-DOF Vertical Take-Off and landing device that pivots about the pitch axis is shown in Figure 5.1

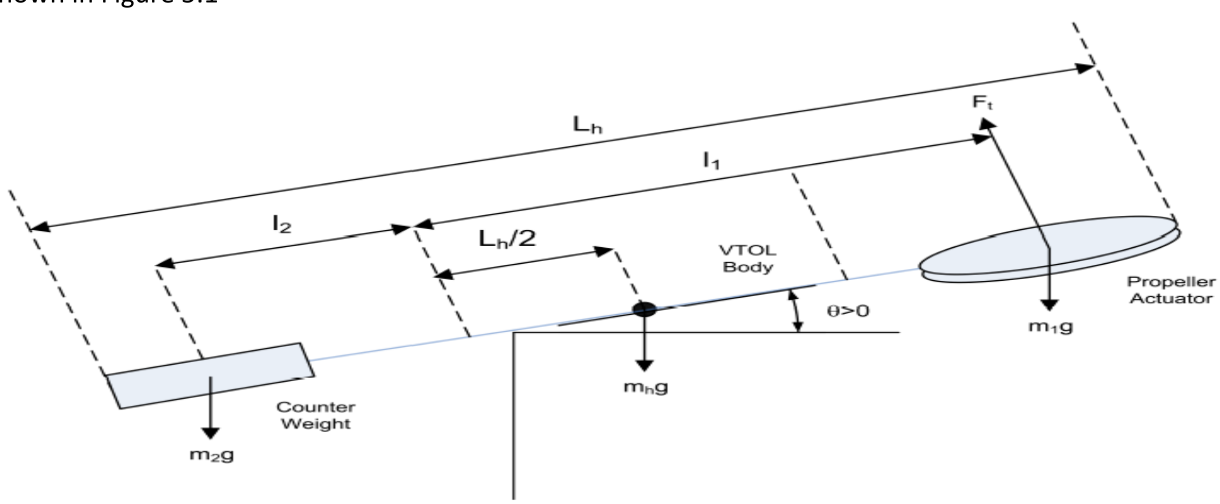

Figure 5.1:Free Body Diagram of 1-DOF VTOL

As shown in Figure 5.1, the torque acting on the rigid body system can be described by the equation

$$\tau_t + m_2 g l_2 cos\theta(t) - m_1 g l_1 cos\theta(t) - \frac{1}{2} m_h g L_h cos\theta(t) = 0$$

The thrust force, F_t, is generated by propeller and acts perpendicular to the fan assembly. The thrust torque is given by

$$\tau_t = F_t l_1$$

Where l_1 is the length between the pivot and centre of the propeller, as depicted in Figure 5.1. In terms of current, the thrust torque equals

$$\tau_t = K_t I_m$$

Where K_t is the thrust current-torque constant. With respect to current, the torque equation becomes

$$K_t I_m + m_2 g l_2 \cos\theta(t) - m_1 g l_1 \cos\theta(t) - \frac{1}{2} m_h g L_h \cos\theta(t) = 0$$

The torque generated the propeller and the gravitational toque acting of the counter-weight act in the same direction and oppose the gravitational torque on the helicopter body and propeller assembly.

We define the VTOL trainer as being in equilibrium when the thrust is adjusted until the VTOL is horizontal and parallel to the ground. At equilibrium, the torques acting on the system are described by the equation

$$K_t I_{eq} + m_2 g l_2 - m_1 g l_1 - \frac{1}{2} m_h g L_h = 0$$

Where I_{eq} is the current required to reach the equilibrium.

5.3.3 Equation of Motion

The angular motions of the VTOL trainer with respect to a thrust torque, τ_t, can be expressed by the equation

$$J\theta'' + B\theta' + K\theta = \tau_t$$

Where θ is the pitch angle, J is the equivalent moment of inertia acting about the pitch axis, B is the viscous damping, and K is the stiffness. With respect to current, this becomes

$$J\theta'' + B\theta' + K\theta = K_t I_m$$

As opposed to finding the moment of inertia by integrating over a continuous body, when finding the moment of inertia of a composite body with n point masses its easiest to use formula

$$J = \sum_{i=i}^{n} m_i r_i^2$$

5.3.4 Process Transfer Function Model

The transfer function representing the current to position dynamics of the VTOL trainer is

$$P(s) = \frac{K_t}{J\left(s^2 + \frac{B}{J}s + \frac{K}{J}\right)}$$

This is obtained by taking the Laplace Transform of Equation for angular motion and thrust torque of VTOL given in previous section and solving for $\Omega(s)/I_m(s)$. Notice the denominator $s^2 + \frac{B}{J}s + \frac{K}{J}$ matches the characteristic second order transfer function. By determining the natural frequency of the system, one can find the stiffness using

$$K = \omega_n^2 J$$

5.3.5 Modelling Virtual Instrument

The virtual instrument used to validate a transfer function model on the QNET VTOL trainer is shown in Figure 5.2. This VI can also be used to find the VTOL device transfer function using the system identification toolkit

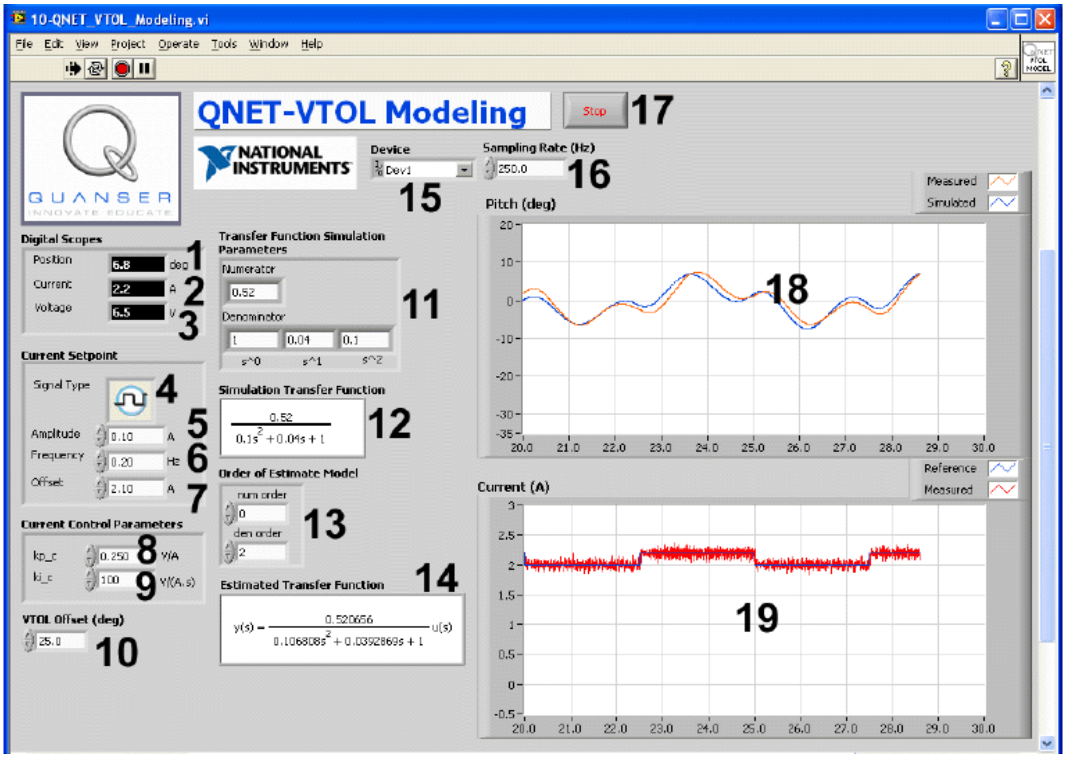

Figure 5.2:LabVIEW virtual instrument used to find and validate a model for the QNET VTOL trainer

Table 5-1

ID #	Label	Parameter	Description	Unit
1	Position	θ	VTOL pitch position numeric display.	deg
2	Current	I_m	VTOL motor armature current numeric display.	A
3	Voltage	V_m	VTOL motor input voltage numeric display.	V
4	Signal Type		Type of signal generated for the current reference.	
5	Amplitude		Current setpoint amplitude input box.	A
6	Frequency		Current setpoint signal frequency input box.	Hz
7	Offset		Current setpoint signal offset input box.	A
8	kp_c	$k_{p,c}$	Current control proportional gain.	V/A
9	ki_c	$k_{i,c}$	Current control derivative gain.	V.s/A
10	VTOL Offset		Pitch calibration	deg
11	Transfer Function Simulation Parameters		Transfer function used for simulation.	
12	Simulation Transfer Function		Displays the transfer function currently begin simulated.	
13	Order of Estimated Model		Order of transfer function to be estimated using the *System Identification Toolkit*.	
14	Estimated Transfer Function		Transfer function estimated using the *System Identification Toolkit*.	
15	Device		Selects the NI DAQ device.	
16	Sampling Rate		Sets the sampling rate of the VI.	Hz
17	Stop		Stops the LabVIEW VI from running.	
18	Pitch	θ	Scope with simulated position (in blue) and measured VTOL pitch position (in red).	deg
19	Current	I_m	Scope with reference (in blue) and measured (in red) current.	A

5.4 In Lab Experiments

5.4.1 Measuring the Equilibrium Current

1. Open the QNET_VTOL_Current_Control.vi
2. Ensure that the correct device is chosen
3. Make sure that the VTOL counter-weight is places as far from the propeller assembly as possible without lifting the propeller itself. The base of the propeller assembly should reset lightly on the surface of the QNET board as shown in Figure 5.3

Figure 5.3: VTOL Initial Position

4. Set the Current Control ON switch to ON
5. Run the VI
6. In the Current Control Parameters section, set the PI current gains found in previous experiment (section 4.4.3)
7. In the Current Set point section, set:
   - ⇨ Amplitude = 0.00 A
   - ⇨ Frequency = 0.40 Hz
   - ⇨ Offset = 1.00 A
8. Gradually increase the offset current until the VTOL Trainer is horizontal
9. The pitch should read 0 degrees when the VTOL is horizontal. You may need to adjust the pitch offset by varying the VTOL Offset control. By default, this is set to 25.0 degrees.
10. The current required to make the VTOL Trainer horizontal is called the equilibrium current, I_{eq}. Capture the pitch and current response and record this current. Note down this value here.

11. Click on Stop button to stop running the VI

5.4.2 Finding Natural Frequency

1. Ensure the QNET VTOL Current Control VI is open. Make sure the correct device is chosen
2. Make sure that the VTOL counter weight is placed as far from the propeller assembly as possible without lifting the propeller itself. The base of the propeller assembly should rest lightly on the surface of the QNET board as shown in Figure 5.3
3. Set the Current Control ON? switch to ON
4. In the Current Control parameters, set the PI current gains found in previous experiment (Section 4.4.3)
5. In the current set point section, set:
   - ➩ $Amplitude = 0.00\ A$
   - ➩ $Frequency = 0.40\ Hz$
   - ➩ $Offset = I_{eq}$(equilibrium current found in section 5.4.1)
6. Run the VI
7. When the VI starts and the equilibrium current step is applied, the VTOL Trainer will shoot upward quickly and then oscillate about its horizontal. Capture this response and measure the natural frequency. Note down this value here

8. Click on Stop button to stop running the VI

5.4.3 Model Validation

a) Derivation of Model

1. Using the VTOL Trainer Model given in section 5.3.4 and the specifications enlisted in Table 5.2, compute the moment of inertia acting about the pitch axis. Enter this value in Table 6.3. Show all the calculations in the space provided below

Table 5-2

Symbol	Description	Value	Unit
m_1	Propeller mass	0.068	kg
m_2	Counter-weight mass	0.27	kg
m_h	VTOL body mass	0.048	kg
l_1	Length from pivot to propeller center	15.6	cm
l_2	Length from pivot to center of counter-weight	5.6	cm
L_h	Total length of helicopter body	28.4	cm
B	Estimated viscous damping of VTOL	0.002	N·m/(rad/s)

2. Based on the natural frequency found in section 5.4.2 and the moment of inertia calculated above, find the stiffness of the VTOL Trainer. Enter this value in Table 5.3. Show all the calculations below

3. Using the equations presented in section 5.3.4 and equilibrium current found in section 5.4.1, calculate the thrust current-torque constant K_t. Enter this value in Table 5.3. Show all the calculations below

4. Compute the VTOL Trainer Transfer Function based on the previously found parameters: $K_t, J, B, and\ K$. Show all the calculations below

b) Testing the Derived Model

1. Ensure the QNET VTOL Modelling VI is open. Make sure the correct device is chosen
2. Make sure that the VTOL counter weight is placed as far from the propeller assembly as possible without lifting the propeller itself. The base of the propeller assembly should rest lightly on the surface of the QNET board as shown in Figure 5.3
3. In the Current Control parameters, set the PI current gains found in previous experiment (Section 4.5.2)
4. In the current set point section, set:
   - ⇨ $Amplitude = 0.00\ A$
   - ⇨ $Frequency = 0.20\ Hz$
   - ⇨ $Offset = I_{eq}$(equilibrium current found in section 5.4.1)
5. Run the VI
6. Let the VTOL Trainer stabilizes about the horizontal
7. In the Current set point section set
   - ⇨ $Amplitude = 0.10\ A$
8. In the Transfer Function Simulation Parameters section, enter the parameters computed in section 5.4.3(a)**.** Is the simulation matching the measured signal? Capture the response and Also compute steady state error

9. Click on the Stop button to stop running the VI

5.4.4 Using the System Identification Tool

1. Ensure the QNET VTOL Modelling VI is open. Make sure the correct device is chosen
2. Make sure that the VTOL counter weight is placed as far from the propeller assembly as possible without lifting the propeller itself. The base of the propeller assembly should rest lightly on the surface of the QNET board as shown in Figure 5.3
3. In the current set point section, set:
   - ⇨ $Amplitude = 0.10\ A$
   - ⇨ $Frequency = 0.20\ Hz$
   - ⇨ $Offset = I_{eq}$(equilibrium current found in section 5.4.1)
     Which should perturb the VTOL Trainer is about its horizontal equilibrium point with a current amplitude of ± 0.10 A
4. Let the VI run for at least 20 seconds
5. Click on stop button to stop running the VI. When the VI is stopped, the Estimated Transfer Function displays a newly identified transfer function VTOL system based on the 20 seconds of current (i.e. stimulus signal) and pitch angle (i.e. response signal) data
6. Write the identified transfer function below

7. Enter the identified TF parameters into the Transfer Function Simulation Parameters section
8. Go through steps 7-9 in section 5.4.3(b)**.** That is, bring the VTOL Trainer up to 0 degrees and then feed ± 0.1 A

9. How is the simulation matching the measured signal compared to the transfer function with the manually estimated parameters? Capture the response

10. Click on the Stop button to stop the VI
11. Assume the moment of inertia is as calculated in section 6.4.3(a). Then from the identified transfer function, find the stiffness($K_i d$), the viscous damping($B_i d$), and the current-torque constant($K_{t,id}$). Note these values in Table 5.3. How do they compare with the parameters you estimated manually?

5.4.5 Results

Table 5-3

Parameters	Symbol	Value	Units
Equilibrium Current	I_{eq}		A
Torque Thrust Constant	K_t		$(N.m)/A$
Moment of Inertia	J		$Kg.m^2$
Viscous Damping	B		$(N.m.s)/rad$
Natural Frequency	ω_n		rad
Stiffness	K		$(N.m)/rad$
Sys ID: Torque Thrust Constant	$K_{t,id}$		$(N.m)/A$
Sys ID: Viscous Damping	B_{id}		$(N.m.s)/rad$
Sys ID: Stiffness	K_{id}		$(N.m)/rad$

5.5 Post Lab Exercise

Exercise Question 1

Describe the major difference between VTOL setup and an actual helicopter, in terms of degree of freedom. Does the system provide good realization of helicopter?

Rubric for Lab Assessment

The student performance for the assigned task during the lab session was:			
Excellent	The student completed assigned tasks without any help from the instructor, adhered to health and safety standards, and presented the results appropriately.	4	
Good	The student completed assigned tasks with minimal help from the instructor, adhered to health and safety standards, and presented the results appropriately.	3	
Average	The student could not complete all assigned tasks, adhered to health and safety standards, and presented partial results.	2	
Worst	The student did not complete assigned tasks.	1	

Instructor Signature: ______________________________ Date: ____________________

LAB # 6: Design of a PID Controller for Pitch Control operation of VTOL

6.1 Objectives

- To Design a PID Controller on VTOL Flight
- To Tell the effects of a PID controller on System

6.2 Lab Instructions

- ✓ This lab activity comprises of three parts: Pre-lab, Lab Exercises, and Post-Lab Viva session.
- ✓ The students should perform and demonstrate each lab task separately for step-wise evaluation
- ✓ Only those tasks that completed during the allocated lab time will be credited to the students. Students are however encouraged to practice on their own in spare time for enhancing their skills.

6.3 Theory and Pre-Lab Exercises

6.3.1 Introduction and Background

It is expected at this point that after passing through two comprehensive lab sessions on VTOL which include closed loop current control operation, open loop voltage control operation, determination of various system parameters and system identification toolkit, students would be feeling easy on the apparatus. In flight control operation, all we are interested in is to control the pitch of the VTOL. We will start off with developing the mathematical model of the system which will lead to the steady state error analysis and finally controller design.

6.3.2 Steady-state Error Analysis

Steady-state error is the difference between the reference and output signals after the system response has settled. Thus for a time t when the system is in steady-state, the steady-state error equals

$$e_{ss} = r_{ss}(t) - y_{ss}(t)$$

Where r_{ss} is the value of the steady-state reference and y_{ss} is the steady-state value of the process output

The block diagram shown in Figure 6.1 is general unity feedback system with a compensator $C(s)$ and a transfer function representing the plant, $P(s)$. The measured output, $Y(s)$, is supposed to track the reference signal $R(s)$ and the tracking has to yield certain specifications.

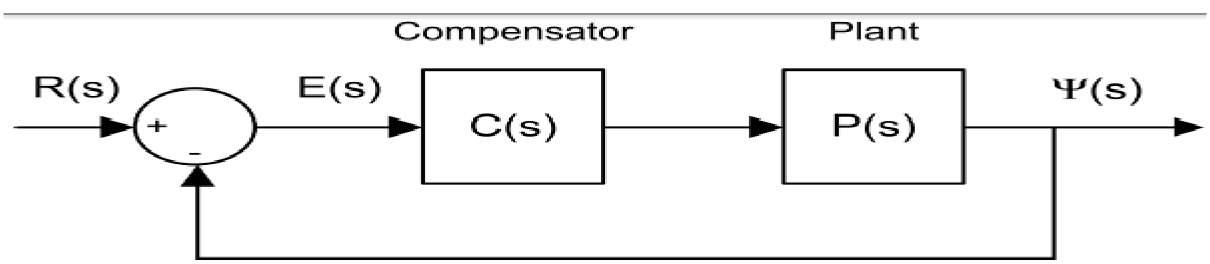

Figure 6.1:Unity Feedback System

The error of the system shown in Figure 6.1 is

$$E(s) = R(s) - Y(s)$$

And by solving for $E(s)$ the resulting closed-loop transfer function is obtained.

$$E(s) = \frac{R(s)}{1 + C(s)P(s)}$$

The error transfer function of the VTOL trainer when subject to step of

$$R(s) = \frac{R(0)}{s}$$

and using the PID compensator

$$C(s) = k_p + k_d s + \frac{k_i}{s}$$

$$E(s) = \frac{R_0}{s\left(1 + \frac{\left(k_p + k_d s + \frac{k_i}{s}\right)K_t}{J\left(s^2 + \frac{B}{J}s + \frac{K}{J}\right)}\right)}$$

If transfer function is stable, then the steady-state error can be found using the final value theorem (FVT)

$$e_{ss} = \lim_{s \to 0} sE(s)$$

Using FVT, the steady-state error of the VTOL trainer closed-loop PID step response is

$$e_{ss} = R_0 \left(\lim_{s \to 0} \frac{s(s^2 J + Bs + K)}{s^3 J + Bs^2 + K_t k s^2 + sK + K_t k_p s + K_t k_i} \right)$$

6.3.3 PID Control Design

The PID control loop used for the VTOL device is depicted in Figure 6.2

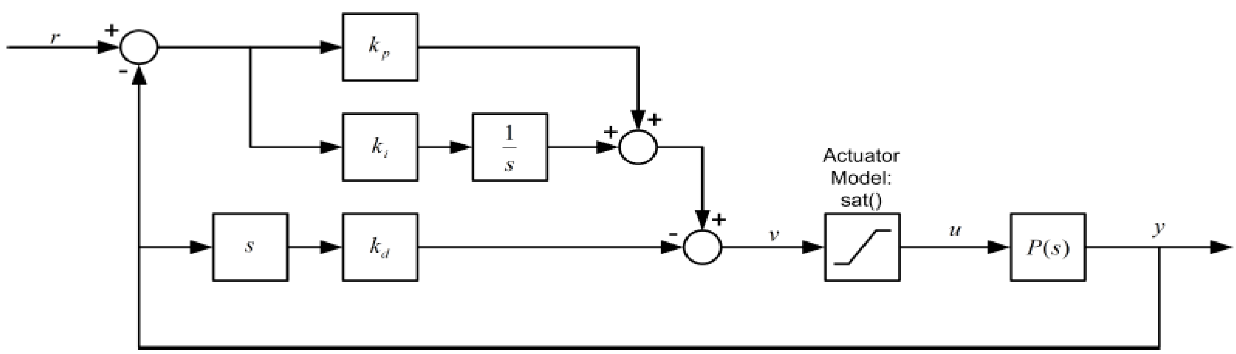

Figure 6.2: VTOL PID Control Loop

The transfer function representing the VTOL trainer position-current relation in previous lab session is used to design the PID controller. The input-output relation in the time-domain for a PID controller is

$$u = k_p(\theta_d - \theta) + k_i \int (\theta_d - \theta)dt - k_v \frac{d}{dt}(\theta)$$

Where k_p is the proportional gain, k_i is the integral gain and k_v is the velocity gain. Remark that only the measured velocity is used, i.e. instead of using the derivation of the error. The close-loop transfer function from the position reference, r, to the angular VTOL position output, θ, is

$$G_{\theta,r}(s) = \frac{K_t(k_p s + k_i)}{Js^3 + (B + K_t k_v)s^2 + (K + K_t k_p)s + K_t k_i}$$

The prototypic third order polynomial is

$$(s^2 + 2\xi\omega_n s + \omega_n^2)(s + p_0) = s^3 + (2\xi\omega_n + p_0)s^2 + (\omega_n^2 + 2\xi\omega_n p_0)s + \omega_n^2 p_0$$

Where ω_n is the natural frequency, ξ is the damping ratio, and p_0 is a zero.

Comparing the characteristic polynomial in closed-loop transfer function of the VTOL with the above equation yields the following expressions of gains

$$k_p = \frac{-K + 2p_0\xi\omega_n J + \omega_n^2 J}{K_t}$$

$$k_i = \frac{p_0\omega_n^2 J}{K_t}$$

$$k_v = \frac{-B + p_0 J + 2\xi\omega_n J}{K_t}$$

6.3.4 Flight Control Virtual Instrument

This VI runs the PID based cascade control system to control the position of the VTOL pitch. As a quick VI description, Table 6.1 lists and describes the main elements of the QNET VTOL Flight Control VI and every element is uniquely identified by an ID number in Figure 6.3

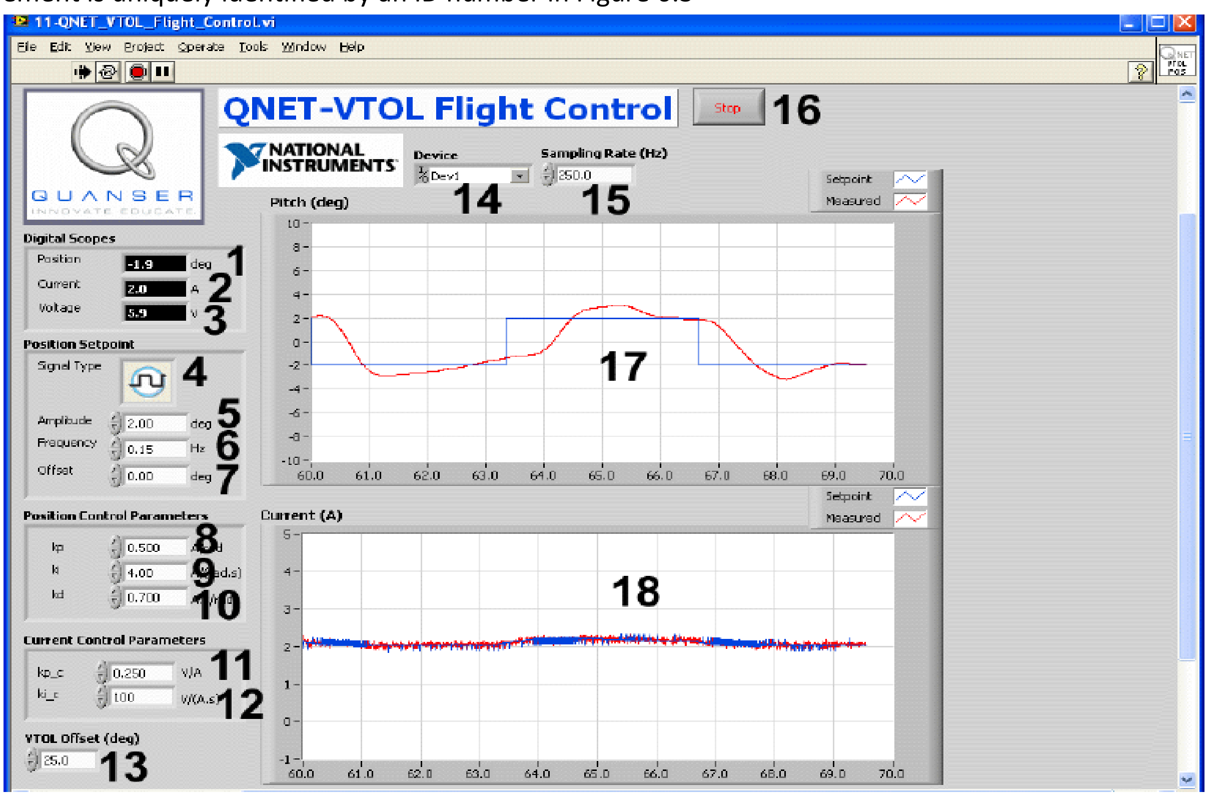

Figure 6.3

Table 6-1

ID #	Label	Parameter	Description	Unit
1	Position	θ	VTOL pitch position numeric display.	deg
2	Current	I_m	VTOL motor armature current numeric display.	A
3	Voltage	V_m	VTOL motor input voltage numeric display.	V
4	Signal Type		Type of signal generated for the current reference.	
5	Amplitude		Pitch setpoint signal amplitude input box.	A
6	Frequency		Pitch setpoint signal frequency input box.	Hz
7	Offset		Pitch setpoint signal offset input box.	A
8	kp	k_p	Position control proportional gain.	A/rad
9	ki	k_i	Position control integral gain.	A/(rad.s)
10	kd	k_d	Position control derivative gain.	A.s/rad
11	kp_c	$k_{p,c}$	Current control proportional gain.	V/A
12	ki_c	$k_{i,c}$	Current control derivative gain.	V.s/A
13	VTOL Offset		Pitch calibration	deg
14	Device		Selects the NI DAQ device.	
15	Sampling Rate		Sets the sampling rate of the VI.	Hz
16	Stop		Stops the LabVIEW VI from running.	
17	Pitch	θ	Scope with reference position (in blue) and measured VTOL pitch position (in red).	deg
18	Current	I_m	Scope with reference current (in blue) and measured current (in red).	A

6.4 In Lab Experiments

6.4.1 PD Steady State Analysis

a) Theoretical Prediction

Calculate the theoretical VTOL Trainer steady-state error when using a PD control with $k_p = 2$ and $k_d = 1$ and a step amplitude of $R_0 = 4.0\ degrees$. Use thrust current-torque and stiffness found in previous Lab Session. Show all the calculation in the space provided below. Enter the value of calculated steady-state error in Table 6.2

b) Experimental Observation

1. Open the QNET_VTOL_Flight_Control.vi. Make sure the correct device is chosen
2. Make sure that the VTOL counter-weight is placed as far from the propeller assembly as possible without lifting the propeller itself. The base of the propeller assembly should rest lightly on the surface of the QNET board.
3. Run the VI
4. In Position set point Section set:
   - ⇨ $Amplitude = 0.0\ deg$
   - ⇨ $Frequency = 0.15\ Hz$
   - ⇨ $Offset = 0.0\ deg$
5. In the Position Control Parameters section set:
   - ⇨ $k_p = 1.0\ A/rad$
   - ⇨ $k_i = 2\ A/(rad.s)$
   - ⇨ $k_d = 1.0\ A.s/rad$
6. Let the VTOL system stabilize about the 0.0 rad set point. Examine if the VTOL Trainer body is horizontal. If not, then you can adjust the pitch offset by varying the VTOL offset control. By default this is set to 25.0 degrees.
7. To use a PD control, in the Position Control Parameters section set:
   - ⇨ $k_p = 2.0\ A/rad$
   - ⇨ $k_i = 0\ A/(rad.s)$
   - ⇨ $k_d = 1.0\ A.s/rad$
8. In Position set point Section set:
   - ⇨ $Amplitude = 2.0\ deg$
   - ⇨ $Frequency = 0.40\ Hz$
   - ⇨ $Offset = 2.0\ deg$

   The VTOL trainer should be going up and down and tracking the square wave set point.
9. Capture the VTOL device step response when using this PD controller and measure the steady-state error. Enter this measured value in table 6.2. How does it compare with the computed value in section a?

10. In the Signal Generator section set Amplitude (rad) to 0 rad and slowly decrement Offset (rad) t0 -8.0 rad.
11. Click on the Stop button to stop running the VI.

6.4.2 PID Steady State Analysis

a) Theoretical Prediction

Calculate the theoretical VTOL Trainer steady-state error when using a PD control with $k_p = 2$, $k_i = 4.0\ A/(rad.s)$ and $k_d = 1$ and a step amplitude of $R_0 = 4.0\ degrees$. Use thrust current-torque, stiffness and moment of inertia as found in previous Lab Session. For viscous damping refer to Table 5.2. Show all the calculation in the space provided below. Enter the value of calculated steady-state error in Table 6.2

b) Experimental Observation

1. Go through steps 1-8 in Section 6.5.1 (b) to run the PD controller
2. In the Position Control Parameters section, increment the integral gain until you reach $k_i = 4.0\ A/(rad.s)$.
3. Capture the VTOL device step response when using a PID controller and measure the steady-state error. Enter this measured value in table 6.2. How does it compare with the computed value in section a?

4. In the Signal Generator section set Amplitude (rad) to 0 rad and slowly decrement Offset (rad) t0 -8.0 rad.
5. Click on the Stop button to stop running the VI.

6.4.3 PID Control Design

a) Theoretical Design

1. Find the natural frequency, ω_n, and damping ratio, ξ, required to meet a peak time of 1.0 seconds and a percent overshoot of 20%. Show all the necessary calculations below. Enter these values in Table 6.2.

2. Calculate the corresponding values of PID gains $k_p, k_i\ and\ k_v$, needed to meet the VTOL specifications. Enter these values in Table 6.2. Show all the calculations below.

b) Experimental Validation

1. Open the QNET_VTOL_Flight_Control.vi. Make sure the correct device is chosen
2. Make sure that the VTOL counter-weight is placed as far from the propeller assembly as possible without lifting the propeller itself. The base of the propeller assembly should rest lightly on the surface of the QNET board.
3. Run the VI
4. In Position set point Section set:
   - ⇨ $Amplitude = 0.0\ deg$
   - ⇨ $Frequency = 0.15\ Hz$
   - ⇨ $Offset = 0.0\ deg$
5. In the Position Control Parameters section, enter PID gains found in Section 6.5.3(a)
6. Let the VTOL system stabilize about the 0.0 rad set point. Examine if the VTOL Trainer body is horizontal. If not, then you can adjust the pitch offset by varying the VTOL offset control. By default this is set to 25.0 degrees.
7. In Position set point Section set:
   - ⇨ $Amplitude = 2.0\ deg$
   - ⇨ $Frequency = 0.40\ Hz$
   - ⇨ $Offset = 2.0\ deg$

   The VTOL trainer should be going up and down and tracking the square wave set point.
8. Capture the response of the VTOL system when using your designed controller
9. Measure the peak time and percent overshoot of the measured response. Show all the steps below. Enter these values in Table 6.2. Are the VTOL Trainer response specifications satisfied?

10. If the specifications were not given, what could be done to improve the response?

11. In the Signal Generator section set Amplitude (rad) to 0 rad and slowly decrement Offset (rad) t0 -8.0 rad.
12. Click on the Stop button to stop running the VI.

6.4.4 Results

Table 6-2

Parameters	Symbol	Value	Units
PD steady-state error	$e_{ss,pd}$		deg
Measured PD steady-state error	$e_{ss,meas,pd}$		deg
PID steady-state error	$e_{ss,pid}$		deg
Measured PID steady-state error	$e_{ss,meas,pid}$		deg
Desired peak time	t_p		s
Desired percentage overshoot	PO		%
Desired pole location	p_0		rad/s
Natural frequency	ω_n		rad/s
Damping ratio	ξ		-
Proportional gain	k_p		A/rad
Integral gain	k_i		$A/(rad.s)$
Derivative gain	k_v		$(A.s)/rad$
Measured peak time	$t_{p,meas}$		s
Measured percentage overshoot	PO_{meas}		%

6.5 Post Lab Exercise

Exercise Question 1

Design a PID controller to meet the following design criteria of VTOL pitch

- ⇨ Settling Time = 5 seconds
- ⇨ Percentage Overshoot = 5%

Validate the designed controller on experimental setup. Does it satisfy the design criteria?