STA305/1004 - Design of Scientific Studies

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August 9, 2018

Thanks to Becky Wei Lin and Nathan Taback for the slides

Agenda Today

- Blocking in factorial design
- Fractional factorial design
- Split plot design

Blocking Factorial Design

Blocking Factorial Design

- In a trial conducted using a 2^3 design it might be desirable to use the same batch of raw material to make all 8 runs.
- Suppose that batches of raw material were only large enough to make 4 runs.

What can we do?

Blocking Factorial Design

- In a trial conducted using a 2^3 design it might be desirable to use the same batch of raw material to make all 8 runs.
- Suppose that batches of raw material were only large enough to make 4 runs.

What can we do? Blocking!

Blocking Factorial Design

Consider a 2^3 design.

Run	1	2	3	12	13	23	123
1	-1	-1	-1	1	1	1	-1
2	1	-1	-1	-1	-1	1	1
3	-1	1	-1	-1	1	-1	1
4	1	1	-1	1	-1	-1	-1
5	-1	-1	1	1	-1	-1	1
6	1	-1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1

Runs	Block
1, 4, 6, 7	I
2, 3, 5, 8	II

How are the runs assigned to the blocks?

Blocking Factorial Design

Consider a 2^3 design.

Run	1	2	3	12	13	23	123 4
1	-1	-1	-1	1	1	1	-1
2	1	-1	-1	-1	-1	1	1
3	-1	1	-1	-1	1	-1	1
4	1	1	-1	1	-1	-1	-1
5	-1	-1	1	1	-1	-1	1
6	1	-1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1

Runs	Block	4
1, 4, 6, 7	I	$-$
2, 3, 5, 8	II	$+$

- 4 is the block variable, denoted by B
- The blocking is said to be **generated** by the relationship $4 = 1 \times 2 \times 3$.

Blocking Factorial Design

- Any systematic differences between the two blocks of four runs will be eliminated from all the main effects and two factor interactions.
- However, the three factor interaction is confounded with any batch (block) difference.
- The ability to estimate the three factor interaction separately from the block effect is lost.

Blocking Factorial Design - Effect Hierarchy Principle

Effect hierarchy principle

- Lower-order effects are more likely to be important than higher-order effects.
- Effects of the same order are equally likely to be important.

Why effect hierarchy principle?

- Higher order interactions are more difficult to interpret or justify physically.
- Investigators are less interested in estimating the magnitudes of these effects even when they are statistically significant.

Orthogonal Blocks

Why is x_4 a good blocking variable?

Run	1	2	3	4=123
	x_1	x_2	x_3	x_4
1	-1	-1	-1	-1
2	1	-1	-1	1
3	-1	1	-1	1
4	1	1	-1	-1
5	-1	-1	1	1
6	1	-1	1	-1
7	-1	1	1	-1
8	1	1	1	1

- It can be shown that $\boldsymbol{x}_1 \cdot \boldsymbol{x}_4 = 0$, $\boldsymbol{x}_2 \cdot \boldsymbol{x}_4 = 0$ and $\boldsymbol{x}_3 \cdot \boldsymbol{x}_4 = 0$.
- 4 does not interact with any main effect, namely, 1, 2 or 3.

An example of how NOT to block

- Suppose a researcher wants to arrange the 2^3 design into four blocks.
- Two blocking variable: $B_1 = x_1x_2x_3$, $B_2 = x_2x_3$.

Run	1	2	3	4=123	5=23
	x_1	x_2	x_3	x_4	x_5
1	-1	-1	-1	-1	1
2	1	-1	-1	1	1
3	-1	1	-1	1	-1
4	1	1	-1	-1	-1
5	-1	-1	1	1	-1
6	1	-1	1	-1	-1
7	-1	1	1	-1	1
8	1	1	1	1	1

Exercise: What effects are B_1 and B_2 associated with?

An example of how NOT to block

Suppose a researcher wants to arrange the 2^3 design into four blocks.

Run	1	2	3	4=123	5=23
	x_1	x_2	x_3	x_4	x_5
1	-1	-1	-1	-1	1
2	1	-1	-1	1	1
3	-1	1	-1	1	-1
4	1	1	-1	-1	-1
5	-1	-1	1	1	-1
6	1	-1	1	-1	-1
7	-1	1	1	-1	1
8	1	1	1	1	1

Runs	Block	4	5
4, 6	I	$-$	$-$
1, 7	II	$-$	$+$
3, 5	III	$+$	$-$
2, 8	IV	$+$	$+$

Is there a problem with this block design? It is confounded with the main effect 1.

An example of how NOT to block

Arrange the 2^3 design into four blocks.

Run	1	2	3	4=123	5=23
	x_1	x_2	x_3	x_4	x_5
1	-1	-1	-1	-1	1
2	1	-1	-1	1	1
3	-1	1	-1	1	-1
4	1	1	-1	-1	-1
5	-1	-1	1	1	-1
6	1	-1	1	-1	-1
7	-1	1	1	-1	1
8	1	1	1	1	1

- This blocking is confounded with the interaction term 123 and 23 by design.
- $x_4 = x_1x_2x_3$, $x_5 = x_2x_3$, the blocking is also confounded with $x_4x_5 = x_1x_2x_3x_2x_3 = x_1$
- x_i's are indicators, that is, $x_i = 1$ or -1. $x_i^2 = 1$.

An example of how NOT to block

- Any blocking scheme that confounds main effects with blocks should not be used.
- This is based on the assumption:

The block-by-treatment interactions are negligible.

- This assumption states that treatment effects do not vary from block to block.
- Without this assumption estimability of the factorial effects will be very complicated.

An example of how to block

Two block variable $B_1 = 12$ and $B_2 = 13$.

Run	1	2	3	4=12	5=13
	x_1	x_2	x_3	x_4	x_5
1	-1	-1	-1	1	1
2	1	-1	-1	-1	-1
3	-1	1	-1	-1	1
4	1	1	-1	1	-1
5	-1	-1	1	1	-1
6	1	-1	1	-1	1
7	-1	1	1	-1	-1
8	1	1	1	1	1

Runs	Block	4	5
2, 7	I	$-$	$-$
3, 6	II	$-$	$+$
4, 5	III	$+$	$-$
1, 8	IV	$+$	$+$

Exercise: What effects is this blocking confounded with?

Generators and Defining Relations

- For a 2^k factorial design, to arrange it into 2^q blocks, we need q block variables and each block has 2^{k-1} factorial combinations.
- Let $B_1, B_2, \ldots, B_q$ be the block variables.
- Define $B_i = v_i$, where v_i is the factorial effect confounded with the ith blocking factor.
- The block design is confounded with all the possible products produced by $B_1, B_2, \ldots, B_q$.
- There are $(2^q - 1)$ possible combinations

In the arrange 2^3 factorial design into 4 blocks example:

- $k = 3$ and $q = 2$. Each block has $2^{3-2} = 2$ factorial combinations.
- $B_1 = 12$, $B_2 = 13$
- The block design is confounded with $B_1 = 12$, $B_2 = 13$ and $B_1 \cdot B_2 = 23$

Generators and Defining Relations

Exercise: Consider a 2^5 factorial design. To arrange it into 8 blocks, consider the following two blocking schemes:

- $B_1 = 135, B_2 = 235, B_3 = 1234$
- $B_1 = 12, B_2 = 13, B_3 = 45$

Questions:

1. What are the effects that are confounded with each blocking scheme?
2. Which design is better?

Generators and Defining Relations

Exercise: Consider a 2^5 factorial design. To arrange it into 8 blocks, consider the following two blocking schemes:

- $B_1 = 135, B_2 = 235, B_3 = 1234$
- $B_1 = 12, B_2 = 13, B_3 = 45$

Fractional Factorial Design

Fractional Factorial Design

- A 2^k full factorial requires 2^k runs.
- Full factorials are seldom used in practice for large k ($k >= 7$).
- For economic reasons, fractional factorial designs that consist of a fraction of the full factorial designs, are used.

Effect Aliasing and Design Resolution

- A chemist in an industrial development lab was trying to formulate a household liquid product using a new process.
- The liquid had good properties but was unstable.
- The chemist wanted to synthesize the product in hope of hitting conditions that would give stability, but without success.
- The chemist identified four important influences:
  - A: acid concentration
  - B: catalyst concentration
  - C: temperature
  - D: monomer concentration
- A full factorial design requires $2^4 = 16$ runs.
- The chemist can only afford 8 runs.

Effect Aliasing and Design Resolution

The chemist's 8 run fractional factorial design.

test	A	B	C	D	y
1	-1	-1	-1	-1	20
2	1	-1	-1	1	14
3	-1	1	-1	1	17
4	1	1	-1	-1	10
5	-1	-1	1	1	19
6	1	-1	1	-1	13
7	-1	1	1	-1	14
8	1	1	1	1	10

- The signs of the ABC interaction is used to accommodate factor D.
- The column for D is to estimate the main effect of D and also for interaction effect of ABC.
- The main factor D is said to be **aliased** with the ABC interaction.
- This is called a 2^{3-1} design. In general, a 2^{k-p} design is a $\frac{1}{2^p}$ fraction of a 2^k design using 2^{k-p} runs.

Effect Aliasing and Design Resolution

- The aliasing relation is denoted by

$$D = ABC$$

- Aliasing of the effects is a price one must pay for choosing a smaller design.
- The 2^{4-1} design has only 7 degrees of freedom for estimating factorial effects, it cannot estimate all 15 factorial effects among the factors A, B, C, D.

Effect Aliasing and Design Resolution

- A simple calculus is available to show the consequences of any proposed blocking arrangement.
- If any column in a 2^k design are multiplied by themselves a column of plus signs is obtained. This is denoted by the symbol I.

$$I = 11 = 22 = 33 = 44 = 55,$$

where, for example, 22 means the product of the elements of column 2 with itself.

- Any column multiplied by I leaves the elements unchanged. So, $I3 = 3$.

Effect Aliasing and Design Resolution

- The equation $I = ABCD$ is called the **defining relation** of the 2^{4-1} design.
- The design is said to have **resolution** IV because the defining relation consists of the "word" ABCD, which has "length" 4.
- Multiplying both sides of $I = ABCD$ by column A

$$A = A \times I = A \times ABCD = BCD,$$

  the relation $A = BCD$ is obtained.
- A is aliased with the BCD interaction. Following the same method all 7 aliasing relations can be obtained.

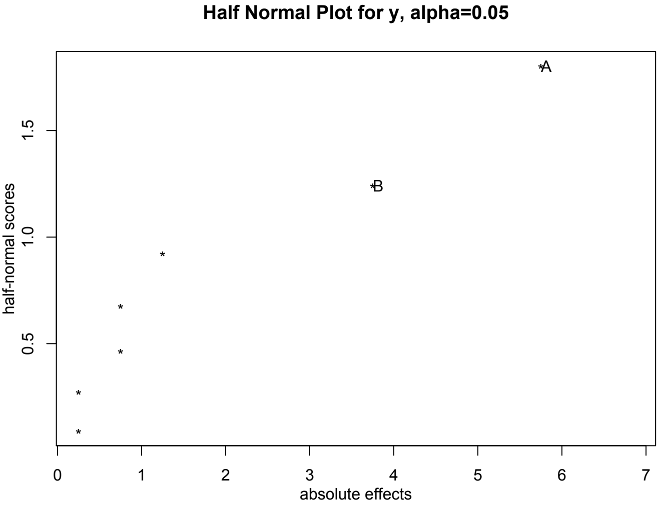
Example - 2^4 design for studying a chemical reaction

```
fact.prod <- lm(y~A*B*C*D,data=tab0602)
fact.prod1 <- aov(y~A*B*C*D,data=tab0602)
round(2*fact.prod$coefficients,2)

(Intercept)          A          B            C            D          A:B
      29.25      -5.75      -3.75        -1.25         0.75         0.25
        B:C        A:D        B:D          C:D        A:B:C        A:B:D
      -0.25         NA         NA           NA           NA           NA
      B:C:D    A:B:C:D
         NA         NA
```

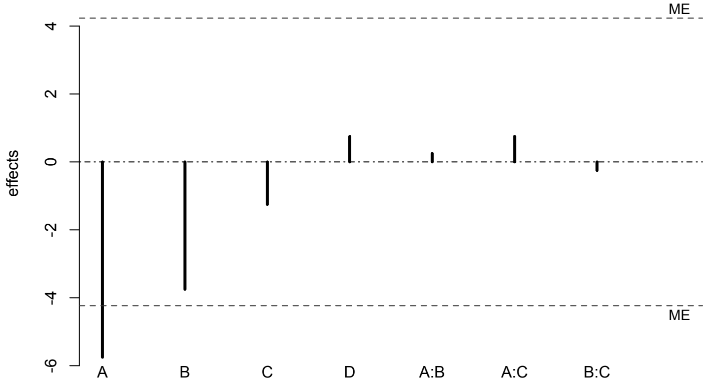
Example - 2^4 design for studying a chemical reaction

```
DanielPlot(fact.prod,half = T)
```



Example - 2^4 design for studying a chemical reaction

```
LenthPlot(fact.prod1)
   alpha       PSE        ME       SME
 0.050000  1.125000  4.234638 10.134346
```



Split Plot Design

Split Plot Design

Split plot designs were originally developed for agriculture by R.A. Fisher and F. Yates.

- Some factors need to be applied to larger plots as compared to other factors.
- For example, if type of irrigation method and the type of fertilizer used are two factors, then irrigation requires a larger plot.

Split Plot Design

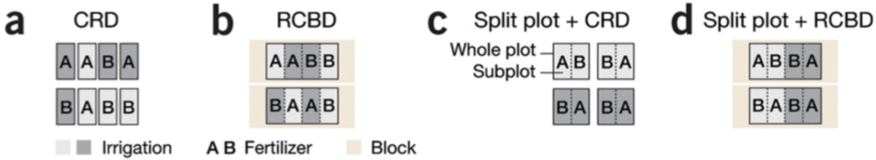
From

Points of Significance: Split plot design

Naomi Altman & **Martin Krzywinski**

Nature Methods **12**, 165–166 (2015) | doi:10.1038/nmeth.3293

Published online 26 February 2015



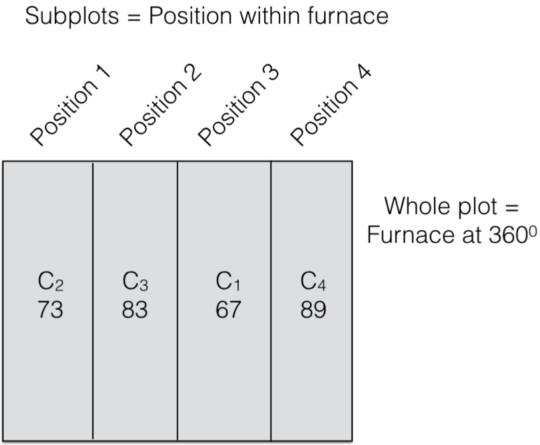
(**a**) In CRD, levels of irrigation and fertilizer are assigned to plots of land (experimental units) in a random and balanced fashion. (**b**) In RCBD, similar experimental units are grouped (for example, by field) into blocks and treatments are distributed in a CRD fashion within the block. (**c**) If irrigation is more difficult to vary on a small scale and fields are large enough to be split, a split plot design becomes appropriate. Irrigation levels are assigned to whole plots by CRD and fertilizer is assigned to subplots using RCBD (irrigation is the block). (**d**) If the fields are large enough, they can be used as blocks for two levels of irrigation. Each field is composed of two whole plots, each composed of two subplots. Irrigation is assigned to whole plots using RCBD (blocked by field) and fertilizer assigned to subplots using RCBD (blocked by irrigation).

Split plot designs - corrosion study

- An experiment of corrosion resistance of steel bars treated with four different coatings C_1, C_2, C_3, C_4 was conducted.
- Three furnace temperatures were investigated.
- Four differently coated bars randomly arranged in the furnace within each heat.
- Positions of the coated steel bars in the furnace randomized within each heat.
- Furnace temperature was difficult to change so heats were run in systematic order shown.
- The primary interests were the comparison of coatings and how they interacted with temperature.

Split plot designs - corrosion study

The split-plot experiment of corrosion resistance is shown for the first replicate at 360°.



Split plot designs - corrosion study

- There are $I = 3$ furnace temperatures arranged in $n = 2$ replications.
- Each furnace temperature is called a **whole plot**.
- The whole plot treatments are $T_1 = 360^\circ, T_2 = 370^\circ, T_3 = 380^\circ$.
- Within each furnace temperature there are $J = 4$ subplots.
- The four sub plot treatments C_1, C_2, C_3, C_4 are randomly applied to the sub plots within each whole plot.

Temperature	Position 1	Position 2	Position 3	Position 4
360°	C_2	C_3	C_1	C_4
370°	C_1	C_3	C_4	C_2
380°	C_3	C_1	C_2	C_4

Temperature	Position 1	Position 2	Position 3	Position 4
380°	C_4	C_3	C_2	C_1
370°	C_4	C_1	C_3	C_2
360°	C_1	C_4	C_2	C_3

Split plot designs - corrosion study

Analysis of the whole plots:

Source	df
Replications	$2-1$
Temperature	$3-1$
Whole plot Error	$(3-1)(2-1)$
Between whole plots	$3 \cdot 2 - 1$

Analysis of the sub plots:

Source	df
Between whole plots	$3 \cdot 2 - 1$
Coatings	$4-1$
Coatings $\times$ Temperature	$(4-1)(3-1)$
Sub plot Error	$3(2-1)(4-1)$
Total	$3 \cdot 2 \cdot 4 - 1$

Split plot designs - corrosion study

```
head(tab0901)
  run heats coating position replication resistance
1  r1  T360      C2        1           1         73
2  r1  T360      C3        2           1         83
3  r1  T360      C1        3           1         67
4  r1  T360      C4        4           1         89
5  r2  T370      C1        1           1         65
6  r2  T370      C3        2           1         87
```

ANOVA table - corrosion study

- The numerical calculations for the ANOVA of a split-plot design are the same as for other balanced designs (designs where all treatment combinations have the same number of observations) and can be performed in R or with other statistical software.
- Experimenters sometimes have difficulty identifying appropriate error terms.

```
spfurcoat1 <- aov(resistance~ replication*heats*coating,data=tab0901)
summary(spfurcoat1)
                         Df Sum Sq Mean Sq
replication               1    782     782
heats                     2  26519   13260
coating                   3   4289    1430
replication:heats         2  13658    6829
replication:coating       3    254      85
heats:coating             6   3270     545
replication:heats:coating 6    867     144
```

ANOVA table - corrosion study

```
spfurcoat1 <- aov(resistance~ replication*heats*coating,data=tab0901)
summary(spfurcoat1)
                          Df  Sum Sq Mean Sq
replication                1     782     782
heats                      2   26519   13260
coating                    3    4289    1430
replication:heats          2   13658    6829
replication:coating        3     254      85
heats:coating              6    3270     545
replication:heats:coating  6     867     144
```

The whole plot effects are 'replication' and 'replication:heats'. The ANOVA table for the whole plots is:

Source	DF	SS	MS
replication	1	782	782
heats	2	26519	13260
replication: heats(whole plot error)	2	13658	6829

The whole plot mean square error is 6829, which measures the differences between the replicated heats at the three different temperatures.

ANOVA table - corrosion study

```
spfurcoat1 <- aov(resistance~ replication*heats*coating,data=tab0901)
summary(spfurcoat1)
                          Df Sum Sq Mean Sq
replication                1    782     782
heats                      2  26519   13260
coating                    3   4289    1430
replication:heats          2  13658    6829
replication:coating        3    254      85
heats:coating              6   3270     545
replication:heats:coating  6    867     144
```

The subplot effects are:

Source	DF	SS	MS
coating	3	4289	1430
coating:heats	6	3270	545
Sub plot error	9	1121	124.6

- The subplot mean square error is $(254 + 867)/(3 + 6) = 124.6$.
- The subplot error measures to what extent the coatings give dissimilar results within each of the replicated temperatures.

ANOVA table - corrosion study

```
spfurcoat <- aov(resistance~ replication + heats + replication:heats
                 + coating + heats:coating
                 +Error(heats/replication),data=tab0901)
summary(spfurcoat)

Error: heats
      Df Sum Sq Mean Sq
heats  2  26519   13260

Error: heats:replication
                  Df Sum Sq Mean Sq
replication        1    782     782
replication:heats  2  13658    6829

Error: Within
              Df Sum Sq Mean Sq F value  Pr(>F)
coating        3   4289  1429.7  11.480 0.00198 **
heats:coating  6   3270   545.0   4.376 0.02407 *
Residuals      9   1121   124.5
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

ANOVA table - corrosion study

The values for the split plot experiment can be put into one ANOVA table.

Source	DF	SS	MS	F	$P(> F)$
Whole plot:					
replication	1	782	782	782/6829=0.12	0.77
heats	2	26519	13260	13260/6829=1.9	0.34
replication $\times$ heats	2	13658	6829		
(whole plot error)					
Subplot:					
coating	3	4289	1430	11.48	0.002
coating $\times$ heats	6	3270	545	4.376	0.02
Subplot error	9	1121	124.5		

ANOVA table - General Form

- Suppose that a split plot experiment is conducted with whole factor plot A with I levels and sub plot factor B with J levels.
- The experiment is replicated n times.

Source	DF	SS
Whole plot:		
replication	$n-1$	SS_{Rep}
A	$I-1$	SS_A
replication $\times$ A	$(n-1)(I-1)$	SS_W
(whole plot error)		
Subplot:		
B	$J-1$	SS_B
$A \times B$	$(I-1)(J-1)$	$SS_{A\times B}$
Subplot error	$I(J-1)(n-1)$	SS_S

Split plot - WRONG ANOVA

```
furcoatanova <- aov(resistance~heats*coating,data=tab0901)
summary(furcoatanova)
              Df Sum Sq Mean Sq F value  Pr(>F)
heats          2  26519   13260  10.226 0.00256 **
coating        3   4289    1430   1.103 0.38602
heats:coating  6   3270     545   0.420 0.85180
Residuals     12  15560    1297
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The two-way ANOVA shows that there is no evidence of a difference in the four coatings, evidence of a difference between temperatures, and no evidence of an interaction between temperature and coating.

Split plot - WRONG ANOVA

What happened?

- The two factors temperature and coating use different randomization schemes and the number of replicates is different for each factor.
- The subplot factor, coating, restricted randomization to the four positions within a given temperature (whole plot).
- Therefore, the error should consist of two parts: whole plot error and subplot error.
- In order to test the significance of the whole plot factor and the subplot factor, we need the mean squares with the whole plot error component and subplot error component, respectively.

Split plot - WRONG ANOVA

The (incorrect) two-way ANOVA model is

$$y_{ijk} = \eta + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk},\ \epsilon_{ijk} \sim N(0, \sigma^2)$$

y_{ijk} is the observation for the kth replicate of the ith level of factor A and the jth level of factor B. (adapted from Wu and Hamada)

Split plot ANOVA

The correct model is

$$y_{ijk} = \eta + \tau_k + \alpha_i + (\tau\alpha)_{ki} + \beta_j + (\alpha\beta)_{ij} + (\tau\beta)_{kj} + (\tau\alpha\beta)_{kij} + \epsilon'_{ijk},\ \epsilon'_{ijk} \sim N(0, \sigma^2)$$

$i = 1, ..., I;\ j = 1, ..., J;\ k = 1, ..., n.$

y_{ijk} is the observation for the kth replicate of the ith level of factor A and the jth level of factor B.

Whole plot effects

- τ_k is the effect of the kth replicate.
- α_i is the ith main effect for A
- $(\tau\alpha)_{ki}$ is the (k, i)th interaction effect between replicate and A. This is the whole plot error term.

Subplot effects

- β_j is the jth main effect of B
- $(\alpha\beta)_{ij}$ is the (i, j)th interaction between A and B.
- $(\tau\beta)_{kj}$ is the (k, j)th interaction between the replicate and B.
- $(\tau\alpha\beta)_{kij}$ is the (k, i, j)th interaction between the replicate, A, and B.
- ϵ'_{ijk} is the error term.

The term $\epsilon_{kij} = (\tau\beta)_{kj} + (\tau\alpha\beta)_{kij} + \epsilon'_{ijk}$ is the subplot error term.

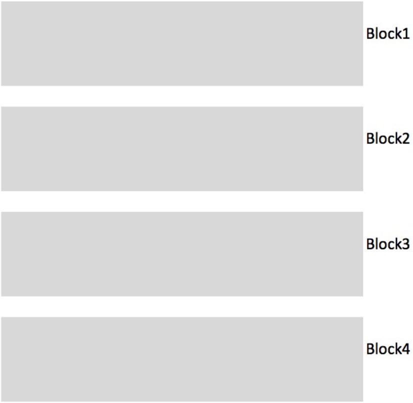
What is a split plot?

- A split-plot can be thought of as a blocked experiment where the blocks themselves serve as experimental units for a subset of the factors.
- Corresponding to two levels of experimental units are two levels of randomization.
- One randomization to determine assignment to whole plots.
- The randomization of treatments to split-plot experimental units occurs within each plot.

Randomizing a split plot experiment

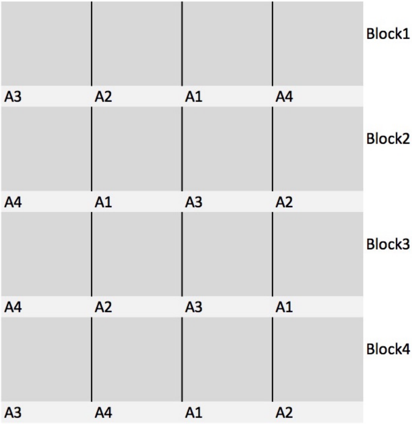
The three steps in randomizing a basic split-plot experiment consisting of 5 blocks (replicates), 4 levels of whole plot factor A, and 8 levels of split-plot factor B are:

1. Division of experimental area or material into five blocks



Randomizing a split plot experiment

2. Randomization of four levels of whole plot factor A to each of the five blocks.



Randomizing a split plot experiment

3. Randomization of eight levels of split plot factor B within each level of whole plot factor A.

B3	B6	B8	B8	
B7	B7	B7	B7	Block1
B6	B5	B1	B4	
B2	B3	B6	B3	
B4	B1	B3	B6	
B1	B2	B4	B5	
B5	B4	B2	B1	
B8	B8	B5	B2	
A3	A2	A1	A4	
B7	B7	B5	B4	
B8	B5	B2	B7	Block2
B3	B6	B1	B8	
B1	B1	B7	B1	
B2	B4	B4	B5	
B5	B8	B6	B2	
B6	B2	B8	B6	
B4	B3	B3	B3	
A4	A1	A3	A2	
B2	B4	B6	B8	
B5	B2	B3	B3	Block3
B3	B3	B5	B4	
B6	B1	B7	B2	
B8	B8	B4	B1	
B7	B5	B2	B5	
B1	B6	B8	B7	
B4	B7	B1	B6	

Split-plot designs versus Factorial Designs

How does the split-plot design compare with a $3 \cdot 4$ factorial design of coating and temperature?
In the factorial design:

- An oven temperature-coating combination would be randomly selected then we would obtain a corrosion resistance measure.
- Then randomly select another oven temperature-coating combination and obtain another corrosion resistance measure until we have a resistance measure for all 12 oven temperature-coating combinations.
- To run each combination in random order would require adjusting the furnace temperature up to 24 times (since there were two replicates) and would have resulted in a much larger variance.

Split plot

- is like a randomized block design (with whole plots as blocks) in which the opportunity is taken to introduce additional factors between blocks.
- In this design there is only one source of error influencing the resistance.

Split-plot designs versus Factorial Designs

There are two different experimental units:

- The six different furnace heats, called whole plots.
- The four positions within each furnace heat, called subplots, where the differently coated bars could be placed in the furnace.
- Misleading to treat as if only one error source and one variance.
- Two different experimental units: six furnace heats (whole plots); and four positions (subplots) where different coated bars are placed in furnace.
- Two different variances: σ_W^2 for whole plots and σ_S^2 for subplots.
- It would be misleading to treat as if only one error source and one variance.

Split-plot designs versus Factorial Designs

- Achieving and maintaining a given temperature in this furnace was very imprecise.
- The whole plot variance, measuring variation from one heat to another, was expected to be large.
- The subplot variance measuring variation from position to position, within a given heat, was expected to be small.
- The subplot effects and subplot-main plot interaction are estimated using with the same subplot error.

Why choose a split plot design?

- Two considerations important in choosing an experimental design are feasibility and efficiency.
- In industrial experimentation a split-plot design is often convenient and the only practical possibility.
- This is the case whenever there are certain factors that are **difficult to change** and others that are **easy to change**.
- In this example changing the **furnace temperature was difficult to change**; rearranging the **positions of the coated bars in the furnace was easy to change**.

Take-home Message

- Fractional factorial design
- Split plot
- Run and understand the R code in the notes
- Work on the exercises in the lecture notes and online notes.
- Online notes:
  `http://utstat.toronto.edu/~nathan/teaching/STA305/classnotes/week10/sta305-classnotes-week10.html`

  `http://utstat.toronto.edu/~nathan/teaching/STA305/classnotes/week11/sta305-week11-classnotes.html`

Information on the Final Exam

- Time: 19:00 - 22:00, Thursday, August 16, 2018
- Location: EX 100
- The final exam is cumulative, covering material from Lecture 1 - Lecture 11 inclusive.
- Five questions, each worth 30 marks.
- Aid sheet will be posted on Thursday (August 9) before lecture
- You will only be tested on the materials that Tiffany and I covered, including (but not limited to) concepts, formulas, proofs, derivations, R code, applications and exercises.

Information on the Final Exam

Office hours for the final:

- Location: SS 623B
- Lin: Tue 10a-1p (Aug 14); Thur 10a-12p (Aug 16).
- Jimmy: Wed 3-5p (Aug 15).
- Bo: Thur 4-6p (Aug 16).

Regular office hours on Tue and Thur for the week of August 13 are canceled.

Thank you and best luck on final!