

Instructions. (16 points) *Part II: Discrete Distributions*

Recall that a Binomial distribution $\text{Bin}(n, p)$ can be approximated by a Poisson distribution

if: $n \geq 100$, $p \leq .01$ and $np \leq 20$.

A factory with 2647 employees reports on average 3.5 injured workers per day. We suppose that the injuries are independent of each other. Let X denote the random number of injuries across the factory on a randomly selected day.

- (4pts) 1. What is the probability that nobody is injured on this day?

Solution:

- (4pts) 2. What is the probability that at least one worker is injured on this day?

Solution:

- (4pts) 3. What is the probability that more than four workers are injured on this day?

Solution:

- (4pts) 4. Having a random sample of 6 employees with $p = 0.1$, what is the probability that more than four workers are injured? (Write the corresponding formulas in case you could not find an answer in the attached tables)

Solution:

Table 1: Binomial distribution — probability mass function

	x	p											
		0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	
$n=1$	0	.9900	.9500	.9000	.8500	.8000	.7500	.7000	.6500	.6000	.5500	.5000	1
	1	.0100	.0500	.1000	.1500	.2000	.2500	.3000	.3500	.4000	.4500	.5000	0
$n=2$	0	.9801	.9025	.8100	.7225	.6400	.5625	.4900	.4225	.3600	.3025	.2500	2
	1	.0198	.0950	.1800	.2550	.3200	.3750	.4200	.4550	.4800	.4950	.5000	1
	2	.0001	.0025	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500	0
$n=3$	0	.9703	.8574	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664	.1250	3
	1	.0294	.1354	.2430	.3251	.3840	.4219	.4410	.4436	.4320	.4084	.3750	2
	2	.0003	.0071	.0270	.0574	.0960	.1406	.1890	.2389	.2880	.3341	.3750	1
	3		.0001	.0010	.0034	.0080	.0156	.0270	.0429	.0640	.0911	.1250	0
$n=4$	0	.9606	.8145	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625	4
	1	.0388	.1715	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500	3
	2	.0006	.0135	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750	2
	3		.0005	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500	1
	4			.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625	0
$n=5$	0	.9510	.7738	.5905	.4437	.3277	.2373	.1681	.1160	.0778	.0503	.0313	5
	1	.0480	.2036	.3281	.3915	.4096	.3955	.3602	.3124	.2592	.2059	.1563	4
	2	.0010	.0214	.0729	.1382	.2048	.2637	.3087	.3364	.3456	.3369	.3125	3
	3		.0011	.0081	.0244	.0512	.0879	.1323	.1811	.2304	.2757	.3125	2
	4			.0005	.0022	.0064	.0146	.0284	.0488	.0768	.1128	.1563	1
	5				.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0313	0
$n=6$	0	.9415	.7351	.5314	.3771	.2621	.1780	.1176	.0754	.0467	.0277	.0156	6
	1	.0571	.2321	.3543	.3993	.3932	.3560	.3025	.2437	.1866	.1359	.0938	5
	2	.0014	.0305	.0984	.1762	.2458	.2966	.3241	.3280	.3110	.2780	.2344	4
	3		.0021	.0146	.0415	.0819	.1318	.1852	.2355	.2765	.3032	.3125	3
	4		.0001	.0012	.0055	.0154	.0330	.0595	.0951	.1382	.1861	.2344	2
	5			.0001	.0004	.0015	.0044	.0102	.0205	.0369	.0609	.0938	1
	6					.0001	.0002	.0007	.0018	.0041	.0083	.0156	0
$n=7$	0	.9321	.6983	.4783	.3206	.2097	.1335	.0824	.0490	.0280	.0152	.0078	7
	1	.0659	.2573	.3720	.3960	.3670	.3115	.2471	.1848	.1306	.0872	.0547	6
	2	.0020	.0406	.1240	.2097	.2753	.3115	.3177	.2985	.2613	.2140	.1641	5
	3		.0036	.0230	.0617	.1147	.1730	.2269	.2679	.2903	.2918	.2734	4
	4		.0002	.0026	.0109	.0287	.0577	.0972	.1442	.1935	.2388	.2734	3
	5			.0002	.0012	.0043	.0115	.0250	.0466	.0774	.1172	.1641	2
	6				.0001	.0004	.0013	.0036	.0084	.0172	.0320	.0547	1
	7						.0001	.0002	.0006	.0016	.0037	.0078	0
$n=8$	0	.9227	.6634	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084	.0039	8
	1	.0746	.2793	.3826	.3847	.3355	.2670	.1977	.1373	.0896	.0548	.0313	7
	2	.0026	.0515	.1488	.2376	.2936	.3115	.2965	.2587	.2090	.1569	.1094	6
	3	.0001	.0054	.0331	.0839	.1468	.2076	.2541	.2786	.2787	.2568	.2188	5
	4		.0004	.0046	.0185	.0459	.0865	.1361	.1875	.2322	.2627	.2734	4
	5			.0004	.0026	.0092	.0231	.0467	.0808	.1239	.1719	.2188	3
	6				.0002	.0011	.0038	.0100	.0217	.0413	.0703	.1094	2
	7					.0001	.0004	.0012	.0033	.0079	.0164	.0313	1
	8							.0001	.0002	.0007	.0017	.0039	0
		0.99	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	x
		p											

Table 2a: Standard Normal distribution**STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.**

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

Table 2b: Standard Normal distribution

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Table 3: Poisson distribution — probability mass function

x	λ										x
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
0	.9048	.8187	.7408	.6703	.6065	.5488	.4966	.4493	.4066	.3679	0
1	.0905	.1637	.2222	.2681	.3033	.3293	.3476	.3595	.3659	.3679	1
2	.0045	.0164	.0333	.0536	.0758	.0988	.1217	.1438	.1647	.1839	2
3	.0002	.0011	.0033	.0072	.0126	.0198	.0284	.0383	.0494	.0613	3
4		.0001	.0003	.0007	.0016	.0030	.0050	.0077	.0111	.0153	4
5				.0001	.0002	.0004	.0007	.0012	.0020	.0031	5
6							.0001	.0002	.0003	.0005	6
7									.0001		7
x	λ										x
	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	
0	.3329	.3012	.2725	.2466	.2231	.2019	.1827	.1653	.1496	.1353	0
1	.3662	.3614	.3543	.3452	.3347	.3230	.3106	.2975	.2842	.2707	1
2	.2014	.2169	.2303	.2417	.2510	.2584	.2640	.2678	.2700	.2707	2
3	.0738	.0867	.0998	.1128	.1255	.1378	.1496	.1607	.1710	.1804	3
4	.0203	.0260	.0324	.0395	.0471	.0551	.0636	.0723	.0812	.0902	4
5	.0045	.0062	.0084	.0111	.0141	.0176	.0216	.0260	.0309	.0361	5
6	.0008	.0012	.0018	.0026	.0035	.0047	.0061	.0078	.0098	.0120	6
7	.0001	.0002	.0003	.0005	.0008	.0011	.0015	.0020	.0027	.0034	7
8			.0001	.0001	.0001	.0002	.0003	.0005	.0006	.0009	8
9							.0001	.0001	.0001	.0002	9
x	λ										x
	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	
0	.1225	.1108	.1003	.0907	.0821	.0743	.0672	.0608	.0550	.0498	0
1	.2572	.2438	.2306	.2177	.2052	.1931	.1815	.1703	.1596	.1494	1
2	.2700	.2681	.2652	.2613	.2565	.2510	.2450	.2384	.2314	.2240	2
3	.1890	.1966	.2033	.2090	.2138	.2176	.2205	.2225	.2237	.2240	3
4	.0992	.1082	.1169	.1254	.1336	.1414	.1488	.1557	.1622	.1680	4
5	.0417	.0476	.0538	.0602	.0668	.0735	.0804	.0872	.0940	.1008	5
6	.0146	.0174	.0206	.0241	.0278	.0319	.0362	.0407	.0455	.0504	6
7	.0044	.0055	.0068	.0083	.0099	.0118	.0139	.0163	.0188	.0216	7
8	.0011	.0015	.0019	.0025	.0031	.0038	.0047	.0057	.0068	.0081	8
9	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0018	.0022	.0027	9
10	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0008	10
11						.0001	.0001	.0001	.0002	.0002	11
12									.0001		12
x	λ										x
	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	
0	.0450	.0408	.0369	.0334	.0302	.0273	.0247	.0224	.0202	.0183	0
1	.1397	.1304	.1217	.1135	.1057	.0984	.0915	.0850	.0789	.0733	1
2	.2165	.2087	.2008	.1929	.1850	.1771	.1692	.1615	.1539	.1465	2
3	.2237	.2226	.2209	.2186	.2158	.2125	.2087	.2046	.2001	.1954	3
4	.1733	.1781	.1823	.1858	.1888	.1912	.1931	.1944	.1951	.1954	4
5	.1075	.1140	.1203	.1264	.1322	.1377	.1429	.1477	.1522	.1563	5
6	.0555	.0608	.0662	.0716	.0771	.0826	.0881	.0936	.0989	.1042	6
7	.0246	.0278	.0312	.0348	.0385	.0425	.0466	.0508	.0551	.0595	7
8	.0095	.0111	.0129	.0148	.0169	.0191	.0215	.0241	.0269	.0298	8
9	.0033	.0040	.0047	.0056	.0066	.0076	.0089	.0102	.0116	.0132	9
10	.0010	.0013	.0016	.0019	.0023	.0028	.0033	.0039	.0045	.0053	10
11	.0003	.0004	.0005	.0006	.0007	.0009	.0011	.0013	.0016	.0019	11
12	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006	12
13					.0001	.0001	.0001	.0001	.0002	.0002	13
14									.0001		14