MODEL

1. Consider the following transfer function noise model

$$y_t = \frac{\omega(B)}{\delta(B)} B^b x_t + N_t,$$

   where we assume N_t follows an ARMA (p, q) model and satisfies

$$\phi(B) N_t = \theta(B) a_t, \qquad a_t \sim NID(0, \sigma_a^2).$$

   Note that N_t can be estimated from

$$\hat{N}_t = y_t - \frac{\hat{\omega}(B)}{\hat{\delta}(B)} x_{t-b} = y_t - \hat{v}(B) x_t,$$

   where $\hat{v}(B)$ is the estimated transfer function and $v(B) \cong \frac{\hat{\omega}(B)}{\hat{\delta}(B)} B^b$

2. $\{\hat{a}_t\}$ can be obtained from

$$\hat{a}_t = \frac{\phi(B)}{\theta(B)} \hat{N}_t,$$

   where $\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$ and $\theta(B) = 1 + \theta_1 B + \cdots + \theta_q B^q$.

DIAGNOSTIC CHECKING ON TRANSFER FUNCTION NOISE MODEL

1. **Cross-correlation check**: to check whether the noise $\{a_t\}$ and the input series $\{x_t\}$ are uncorrelated.

$$Q_0 = m(m+2) \sum_{j=0}^{K} (m-j)^{-1} \hat{\rho}_{\hat{\alpha}\hat{a}}^2(j) \sim \chi_{K+1-M}^2$$

   where $\{\hat{\alpha}_t\}$ is obtained from the prewhitening process (see Eqn. (5) in the TFN course note), and $\{\hat{a}_t\}$ denotes the error terms of the N_t process, $m = n - t_0 + 1$ is the number of residuals $\hat{a}_t$ calculated, and M is the number of parameters δ_i and ω_i estimated in the transfer function $v(B) = \omega(B)/\delta(B)$. Note that the number of degrees of freedom for Q_0 is independent of the number of parameters estimated in the noise model.

2. **Autocorrelation check**: to check whether the noise model is adequate. A portmanteau test for ARMA models can be used.

$$Q_1 = m(m+2) \sum_{j=1}^{K} (m-j)^{-1} \hat{\rho}_{\hat{a}}^2(j).$$

   The Q_1 statistic approximately follows a χ^2 distribution with $(k - p - q)$ degrees of freedom depending only on the number of parameters in the noise model.