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SECTION : SE-R

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ASSIGNMENT # 04

QUESTION # 01

$n = 15$

$p = 0.25$

$q = 1 - 0.25 = 0.75$

$x = 0, 1, \ldots 15.$

for,

$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$P(x) = {}^{15}C_{x} (0.25)^{x} (0.75)^{15-x}$$

(a) $P(3 \le x \le 6) = P(x=3) + P(x=4) + P(x=5) + P(x=6)$

$= {}^{15}C_3 (0.25)^3 (0.75)^{15-3} + {}^{15}C_4 (0.25)^4 (0.75)^{15-4} +$

${}^{15}C_5 (0.25)^5 (0.75)^{15-5} + {}^{15}C_6 (0.25)^6 (0.75)^{15-6}$

$= 0.2252 + 0.2252 + 0.1651 + 0.0917$

$P(3 \le x \le 6) = 0.7073$

(b) $P(x < 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= {}^{15}C_0 (0.25)^0 (0.75)^{15} + {}^{15}C_1 (0.25)^1 (0.75)^{14} + {}^{15}C_2 (0.25)^2 (0.75)^{13}$

$+ {}^{15}C_3 (0.25)^3 (0.75)^{12}$.

$= 0.0134 + 0.0668 + 0.1559 + 0.2252$.

$P(x < 4) = 0.4613$

(c) $P(x > 5) = 1 - P(x \le 5)$

$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)]$

$= 1 - [0.4613 + {}^{15}C_4 (0.25)^4 (0.75)^{11} + {}^{15}C_5 (0.25)^5 (0.75)^{10}]$

$= 1 - [0.4613 + 0.2252 + 0.1651]$

$P(x > 5) = 0.1484$.

QUESTION # 02

$p = 0.40$

$q = 1 - 0.40 = 0.60$

$n = 6.$

$x = 4$

for,

$$P(X) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$P(X=4) = {}^{6}C_{4}\,(0.40)^{4}(0.60)^{6-4}$$

$$= {}^{6}C_{4}\,(0.40)^{4}(0.60)^{2}$$

$$P(x=4) = 0.1382$$

QUESTION # 03

$N = 50$, $N-K = 40$

No of defectives in shipment $= K = 50 \times \frac{20}{100} = 10.$

$n = 5$

$x = \{0, 1, 2\}$ ∴ no more then 2.

for, $P(X) = \dfrac{{}^{K}C_{x}\;{}^{N-K}C_{n-x}}{{}^{N}C_{n}}$ ∴ X follows Hypergeometric distribution

$$P(X \le 2) = P(x=0) + P(x=1) + P(x=2).$$

$$= \frac{{}^{10}C_{0} \cdot {}^{40}C_{5}}{{}^{50}C_{5}} + \frac{{}^{10}C_{1} \cdot {}^{40}C_{4}}{{}^{50}C_{5}} + \frac{{}^{10}C_{2} \cdot {}^{40}C_{3}}{{}^{50}C_{5}}$$

$P(x\le 2) = 0.3105 + 0.4313 + 0.2098$

$$P(x\le 2) = 0.9517$$

QUESTION # 04

$N = 150$

$n = 10$

$K = 150 - 30 = 120$

$p = \dfrac{\text{No of women worker}}{\text{Total workers}} = \dfrac{30}{150} = 0.2$

$q = 1 - 0.2 = 0.8$.

$N - K = 150 - 120 = 30$.

(a) Using bionomial approximation of hypergeometric distribution,

$P(X\geq 3) = 1 - P(x<3)$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - [{}^{10}C_0 (0.2)^0 (0.8)^{10} + {}^{10}C_1 (0.2)^1 (0.8)^9 + {}^{10}C_2 (0.2)^2 (0.8)^8]$

$= 1 - [0.10737 + 0.2684 + 0.30199]$

$$P(X\geq 3) = 0.3222$$

(3)

(b) Using hypergeometric distribution,

$$P(x \geq 3) = 1 - P(X < 3)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{{}^{120}C_0 \cdot {}^{30}C_{10-0}}{{}^{150}C_{10}} + \frac{{}^{120}C_1 \cdot {}^{30}C_9}{{}^{150}C_{10}} + \frac{{}^{120}C_2 \cdot {}^{30}C_8}{{}^{150}C_{10}} \right]$$

$$= 1 - [0.000000025 + 0.000001467 + 0.000035731]$$

$$= 1 - [0.000037223]$$

$$P(x \geq 3) = 0.99996$$

QUESTION 4 05

(b) Mean Number of Arrivals $= \lambda t$ $\quad \because t = 2$-hours, $\lambda = 7$

$$= 7 \times 2$$

$$\mu = 14$$

(a) $e = 14$

$t = 2$

$\lambda = 7$.

for, $P(x, \mu) = \dfrac{e^{-\mu} \mu^{x}}{x!}$

or, $P(\overset{10}{x}, \overset{7}{\mu}) = 1 - P(\dot{x} \leq 10)$

$$1-P(x\le 10) = 1-\left[\sum_{x=0}^{10} \frac{e^{-2\times7}(2\times7)^x}{x!}\right]$$

putting x from 0 to 10 in it,

$$= 1-[\,0+0.00001+0.00008+0.00038+0.00133+0.00373+0.00870+0.01739+0.03044+0.04734+0.06628\,]$$

$$= 1-0.1757$$

$$P(10,7) = 0.8243.$$

QUESTION # 06

$\lambda = 2.7$

(a) $P(x\le4) = P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)$

$$= \frac{e^{-2.7}\,2.7^0}{0!}+\frac{e^{-2.7}\,2.7^1}{1!}+\frac{e^{-2.7}\,2.7^2}{2!}+\frac{e^{-2.7}\,2.7^3}{3!}+\frac{e^{-2.7}\,2.7^4}{4!}$$

$$P(x\le4) = 0.8629$$

(b) $P(x\le1) = P(x=0)+P(x=1)$

$$= \frac{e^{-2.7}\,2.7^0}{0!}+\frac{e^{-2.7}\,2.7^1}{1!}$$

$$P(x\le1) = 0.2487$$

(c) $\lambda t = 2.7 \times 5 = 13.5$

$$P(x > 10) = 1 - P(x \leq 10) = 1 - \sum_{x=0}^{10} \frac{e^{-13.5}(13.5)^x}{x!}$$

$$= 1 - 0.2112$$

$$P(x > 10) = 0.7888.$$

QUESTION # 07

$\lambda = 100$

$t = \frac{3}{60} = 0.05$

$\mu = \lambda t = 100 \times 0.05 = 5$

(a)

$$P(x = 0) = \frac{e^{-5} 5^0}{0!}$$

$$P(x = 0) = 0.0067$$

(b)

$$P(x > 5) = 1 - P(x \leq 5)$$

$$= 1 - \left[\frac{5^0 e^{-5}}{0!} + \frac{5^1 e^{-5}}{1!} + \frac{5^2 e^{-5}}{2!} + \frac{5^3 e^{-5}}{3!} + \frac{5^4 e^{-5}}{4!} + \frac{5^5 e^{-5}}{5!} \right]$$

$$= 1 - 0.61596.$$

$$P(x > 5) = 0.3840$$

QUESTION # 08

$p = 0.01 \quad q = 0.99$

$n = 500$

$x = 15$

(a) $P(x=15) = {}^{n}C_{x} p^{x} q^{n-x}$

$= {}^{500}C_{15} (0.01)^{15} (0.99)^{485}$

$= 0.00014$

Hence, $\approx$ approximately zero (very negligble)

(b) $P(x=3) = {}^{500}C_{3} (0.01)^{3} (0.99)^{497}$

$= 0.1402$

(c) Using Poison's approximation,

(a)

$$P(x,\mu) = \frac{e^{-\mu}\mu^{x}}{x!} = \frac{e^{-5}(-5)^{15}}{15!}$$

$= 0.00015$

$$P(x,\mu) = \frac{e^{-5}(-5)^{3}}{3!} = 0.1403$$

QUESTION # 09

$X = 10.075$

$\mu = 10$

$\delta = 0.03$

(a)

$= P(X > 10.075)$

$= P\left(Z > \frac{10.075 - 10}{0.03}\right)$

$= P(Z > 2.5)$

$= 1 - 0.9938$

$= 0.0062$

Hence, 0.62% of rings have inside diameter of 10.075 cm.

(b)

$= P(9.97 < X < 10.03)$

$= P\left(\frac{9.97-10}{0.03}\right) < Z < P\left(\frac{10.03-10}{0.03}\right)$

$= P(-1.0 < Z < 1.0)$

$= 0.8413 - 0.1587$

$= 0.6826.$

(c) $P\left(Z < \frac{X-10}{0.03}\right) = 0.15$ — (i)

$P(Z < -1.036) = 0.15$ — (ii)

comparing (i) & (ii),

$\frac{X-10}{0.03} = -1.036$

$Z = 9.969cm.$

QUESTION # 10.

$n = 100$

$p = 0.9$

$q = 1 - p = 0.1.$ $\quad \delta = \sqrt{9} = 3$

$\mu = np = 100 \times 0.9 = 90$

$npq = 100 \times 0.9 \times 0.1 = 9$

(a)

$= P(83.5 < X < 95.5)$

$= P\left(\frac{83.5-90}{3} < Z < \frac{95.5-90}{3}\right)$

$= P(-2.17 < Z < 1.83)$

$= 0.9664 - 0.0150$

$= 0.9514$

(b)

$$P(x < 85.5) = P\left(z < \frac{85.5 - 70}{3}\right)$$

$$= P(z < -0.83)$$

$$= 0.2033.$$

QUESTION # 11

$$\mu = (180)\left(\frac{1}{6}\right) = 30$$

$$p = \frac{1}{6}$$

$$q = 1 - \frac{1}{6} \Rightarrow \frac{5}{6}$$

$$\sigma = \sqrt{(180)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} = 5.$$

(a)

$$P(x \geq 24.5) = P\left(z > \frac{24.5 - 30}{5}\right)$$

$$= P(z > -1.1)$$

$$= 1 - 0.1357$$

$$= 0.8643$$

(b)

$$= P(32.5 < x < 41.5) = P\left(\frac{32.5 - 30}{5} < z < \frac{41.5 - 30}{5}\right)$$

$$= P(0.5 < z < 2.3)$$

$$= 0.9893 - 0.6915$$

$$= 0.2978.$$

(c) $P(29.5 < X < 30.5)$

$= P\left(\frac{29.5-30}{5}\right) < Z < P\left(\frac{30.5-30}{5}\right)$

$= P(-0.1 < Z < 0.1)$

$= 0.5398 - 0.4602$

$= 0.0796$.