Theory of Automata

Assignment 3

Question 1:

1- $L = \{0^n 1^m \mid 2n \le m \le 3n\}$

$S \rightarrow \lambda$

$S \rightarrow 0S11 \mid 0S111$

2- $L = \{a^i b^j c^k \mid i,j,k \ge 0 \text{ AND } i=j \text{ OR } i=k\}$

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow XY$

$X \rightarrow aXb \mid \lambda$

$Y \rightarrow cY \mid \lambda$

$S_2 \rightarrow aS_2c \mid Z$

$Z \rightarrow bZ \mid \lambda$

3. $L = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ AND } i+j=k \}$

$S \rightarrow aSc \mid Z$

$Z \rightarrow bZc \mid \lambda$

4. L = { Strings not of form $0^i 1^j$, $i, j \geq 0$, forms:

i. Contain 10
ii. Only 0's
iii. Only 1's }

$S \rightarrow X10X \mid Y \mid Z$

$Y \rightarrow 1Y \mid 1$

$Z \rightarrow 0Z \mid 0$

$X \rightarrow 1X \mid 0X \mid \lambda$

Question 2:

i. $S \rightarrow SS \mid a$

As the starting state 's' is on right side, introduce a new start state i.e S_0

$$S_0 \rightarrow S$$
$$S \rightarrow SS \mid a$$

There are no null productions.

There exist a unit production ($S_0 \rightarrow S$)
Eliminating this, we get:

$$S_0 \rightarrow SS \mid a$$
$$S \rightarrow SS \mid a$$

There doesnot exist any useless Symbol.

Final, CNF form is.

$$S_0 \rightarrow SS \mid a$$
$$S \rightarrow SS \mid a$$

ii. $S \rightarrow aSa \mid SSa \mid a$

As the start state 'S' exist on right side, introduce new start variable S_0.

$$S_0 \rightarrow S$$
$$S \rightarrow aSa \mid SSa \mid a$$

There are no null productions,

There exist a unit production ($S_0 \rightarrow S$). Eliminating this:

$$S_0 \rightarrow aSa \mid SSa \mid a$$
$$S \rightarrow aSa \mid SSa \mid a$$

No useless symbols exist.

Final CNF form is :

$$S_0 \rightarrow XY \mid ZY \mid a$$
$$S \rightarrow XY \mid ZY \mid a$$
$$Z \rightarrow SS$$
$$X \rightarrow YS$$
$$Y \rightarrow a$$

iii.

$S \rightarrow aXX$

$X \rightarrow aS \mid bS \mid a$

No null productions exist

No unit productions exist

No useless symbols present.

Final CNF is,

$S \rightarrow AZ$

$X \rightarrow AS \mid BS \mid a$

$Z \rightarrow XX$

$A \rightarrow a$

$B \rightarrow b$

Question 3 :

i- $S_0 \to SS \mid a$

$S \to SS \mid a$

First we number the variables S_0 and S as 1 & 2 respectively.

① $S_0 \to SS \mid a$

② $S \to SS \mid a$

There exist a left recursion $(S \to SS)$. Eliminating this,

$S_0 \to SS \mid a$

$S \to a \mid aR_1$

$R_1 \to S \mid SR_1$

'S' variable is in GNF. Substituting its value in S_0,

$S_0 \to a \mid aS \mid aR_1S$

$S \to a \mid aR_1$

$R_1 \to S \mid SR_1$

Substituting in R_1, we get

$S_0 \to a \mid aS \mid aR_1S$

$S \to a \mid aR_1$

$R_1 \to a \mid aR_1 \mid aR_1R_1$

Resulting grammar is in GNF.

ii-

$S_0 \to XY \mid ZY \mid a$

$S \to XY \mid ZY \mid a$

$Z \to SS$

$X \to YS$

$Y \to a$

We number the variables S_0, S, Z, X, Y as $1, 2, 3, 4, 5$ respectively.

① $S_0 \to XY \mid ZY \mid a$

② $S \to XY \mid ZY \mid a$

③ $Z \to SS$

④ $X \to YS$

⑤ $Y \to a$

As $(Z \to SS)$, Z is going to a lower numbered variable, substitute value of S in Z

$S_0 \to XY \mid ZY \mid a$

$S \to XY \mid ZY \mid a$

$Z \to XYS \mid ZYS \mid aS$

$X \to YS$

$Y \to a$

$(Z \to ZYS)$ is a left recursion, removing this we get

$S_0 \to XY \mid ZY \mid a$

$S \to XY \mid ZY \mid a$

$Z \to XYS \mid aS \mid XYSR_1 \mid aSR_1$

$X \to YS$

$Y \to a$

$R_1 \to YS \mid YSR_1$

Y is in GNF, substitute its value in ~~Z~~ X.

$S_0 \to XY \mid ZY \mid a$

$S \to XY \mid ZY \mid a$

$Z \to XYS \mid aS \mid XYSR_1 \mid aSR_1$

$X \to aS$

$Y \to a$

$R_1 \to YS \mid YSR_1$

ively.

Substitute 'X' in Z

$S_0 \to XY \mid ZY \mid a$

$S \to XY \mid ZY \mid a$

$Z \to aSYS \mid aS \mid aSYSR_1 \mid aSR_1$

$X \to aS$

$Y \to a$

$R_1 \to YS \mid YSR_1$

Substituting X and Z in S_0 & S and R_1

$S_0 \to a \mid aSY \mid aSYSY \mid aSYSR_1Y \mid aSR_1Y$

$S \to a \mid aSY \mid aSYSY \mid aSYSR_1Y \mid aSR_1Y$

$Z \to aSYS \mid aS \mid aSYSR_1 \mid aSR_1$

$X \to aS$

$Y \to a$

$R_1 \to aS \mid aSR_1$

iii.

$S \to AZ$

$X \to AS \mid BS \mid a$

$Z \to XX$

$A \to a$

$B \to b$

First number the variables S, X, Z, A, B as 1,2,3,4,5 respectively.

1. $S \to AZ$
2. $X \to AS \mid BS \mid a$
3. $Z \to XX$
4. $A \to a$
5. $B \to b$

As in (Z → XX), Z goes to lower number variable, substitute X in Z

$S \to AZ$

$X \to AS \mid BS \mid a$

$Z \to ASX \mid BSX \mid aX$

$A \to a$

$B \to b$

Substituting 'A' and 'B' in Z,

$S \to AZ$

$X \to AS \mid BS \mid a$

$Z \to aSX \mid bSX \mid aX$

$A \to a$

$B \to b$

Substituting A and B in X

$S \to AZ$

$X \to aS \mid bS \mid a$

$Z \to aSX \mid bSX \mid aX$

$A \to a$

$B \to b$

Substituting A in S,

$S \to aZ$

$X \to aS \mid bS \mid a$

$Z \to aSX \mid bSX \mid aX$

$A \to a$

$B \to b$