Theory of Automata

Assignment 4

Question 1:

$$S \rightarrow SA \mid SB \mid a$$
$$A \rightarrow aA \mid BC \mid b$$
$$B \rightarrow CbC \mid b$$
$$C \rightarrow S \mid \Delta$$

First we convert the CFG to CNF
Introducing new start state S_0

$$S_0 \rightarrow S$$
$$S \rightarrow SA \mid SB \mid a$$
$$A \rightarrow aA \mid BC \mid b$$
$$B \rightarrow CbC \mid b$$
$$C \rightarrow S \mid \lambda$$

Removing the null productions. C is nullable

$S_0 \to S$

$S \to SA \mid SB \mid a$

$A \to aA \mid BC \mid b \mid B$

$B \to CbC \mid b \mid Cb \mid bC$

$C \to S$

Removing the unit productions : $S_0 \to S$, $A \to B$, $C \to S$

$S_0 \to SA \mid SB \mid a$

$S \to SA \mid SB \mid a$

$A \to aA \mid BC \mid b \mid CbC \mid Cb \mid bC$

$B \to CbC \mid b \mid Cb \mid bC$

$C \to SA \mid SB \mid a$

All variables are terminating (No useless variable to remove)

Writing Grammar in CNF:

$R_1 \to a$

$R_2 \to b$

$R_3 \to CR_2$

$S_0 \to SA \mid SB \mid a$

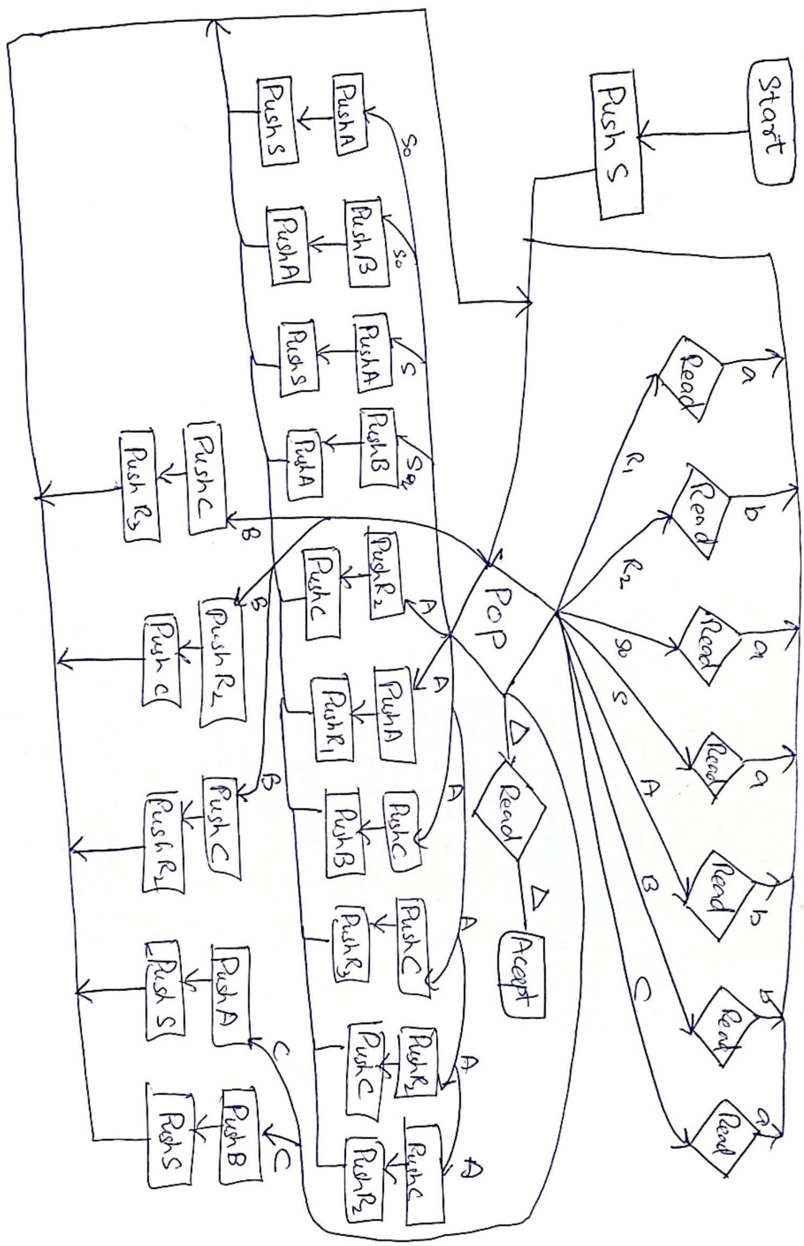
$S \to SA \mid SB \mid a$

$A \to R_1A \mid BC \mid b \mid R_3C \mid CR_2 \mid R_2C$

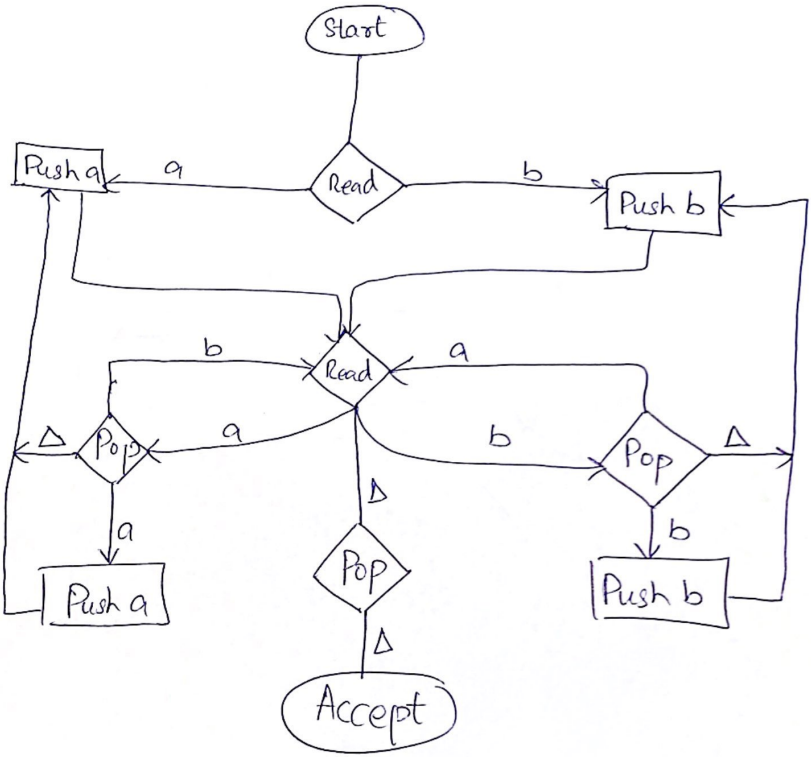
$B \to R_3C \mid b \mid CR_2 \mid R_2C$

$C \to SA \mid SB \mid a$

Converting this CNF to PDA, we get

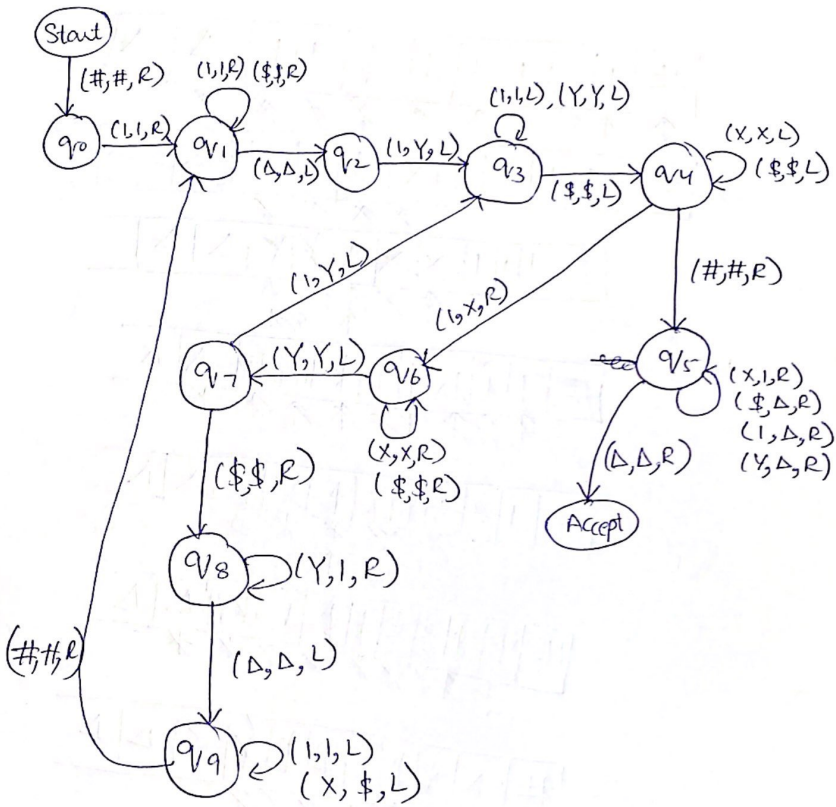


Question 2 1-PDA Equal Equal



Question 3 :

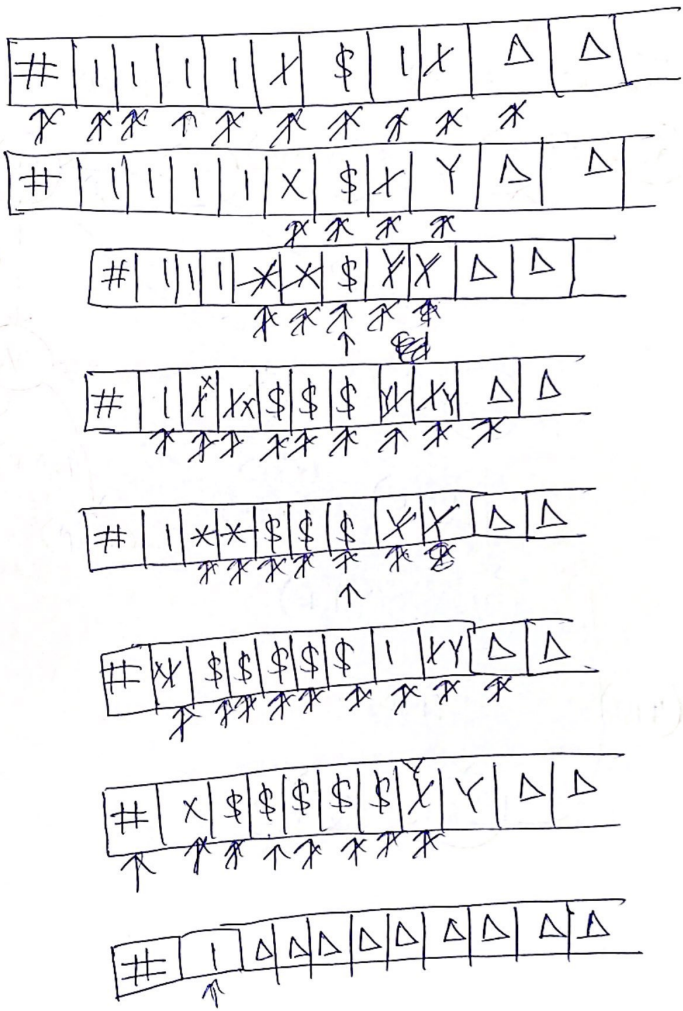
Turing machine mod (x, y)



Dry Run:

Consider numbers 5 and 2. Seperator is $

Input Tape :



Question 4:

$$A = \{ 0^n 1^m 0^k 1^{n+m} \mid n,m,k > 0 \}$$

Assume that the given language A is context free

If A is context free, assume a string w in A.

such that $w = uvxyz$, $|vxy| \leq p$

$|vx| \neq \varepsilon$

$|w| > p$

Let $u = \Lambda$

$vxy = 0^n 1^m$

$z = 0^k 1^{n+m}$.

$vxy = 0^n 1^m$

$\Rightarrow v = 0^a$, $x = 0^{n-a}$ $y = 1^m$.

If A is context free then $uv^i xy^i z \in A$

let $uv^2xy^2z = (\Lambda)(0^a)^2(0^{n-a})(1^m)^2 z$

$= 0^{a+n} 1^{2m} 0^k 1^{n+m} \in A$

But, we know $0^{a+n} 1^{2m} 0^k 1^{n+m} \notin A$.

↳ Since no. of ending 1's = sum of starting 0's & 1's

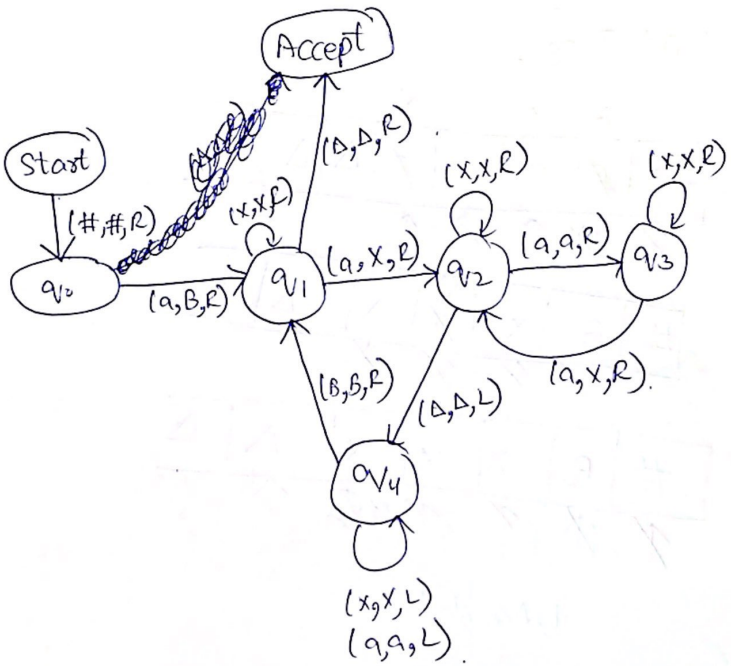
But $a+n+2m \neq n+m$.

Hence, we reach a contradiction,

so, A is not context free language.

Question 5:

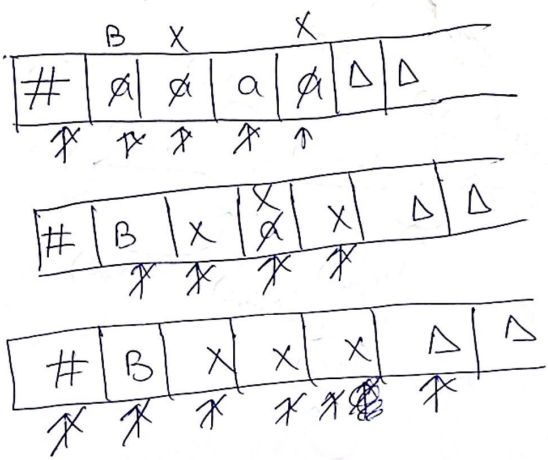
Turing machine for a^{2^n}



Dry Run :

Let string be aaaa

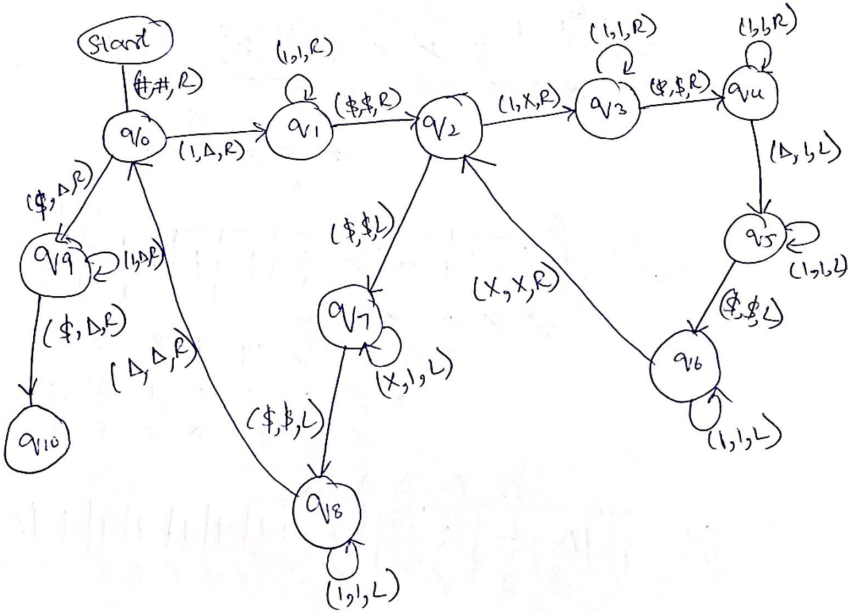
Input Tape :



↳ Accepted

Question 6:

Computational function Multiplication



Dry Run. : Suppose Numbers 2, 3.

Seperators : $

Input Tape :