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SECTION : R

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ASSIGNMENT 03

QUESTION # 01

(a)

Finding the value of k,

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1$$

$$\int_{-\infty}^{\infty} k(x+2) = 1$$

$$\int_{-\infty}^{0} k(x+2) \cdot dx + \int_{0}^{1} k(x+2) \cdot dx + \int_{1}^{\infty} k(x+2) \cdot dx = 1$$

$$\int_0^1 k(x+2)\cdot dx = 1$$

applying integration,

$$k\left[\int_0^1 x\cdot dx - \int_0^1 2\,dx\right] = 1$$

$$k\left[\int_0^1 x\cdot dx - 2\int_0^1 x^0\,dx\right] = 1$$

$$k\left[\left|\frac{x^2}{2}\right|_0^1 + 2\left|x\right|_0^1\right] = 1$$

$$k\left[\frac{1}{2} - 0 + 2(1) - 2(0)\right] = 1$$

$$k\left[\frac{1}{2} + 2\right] \Rightarrow k\left[\frac{1+4}{2}\right] = 1$$

$$k\left[\frac{5}{2}\right] = 1$$

$$\boxed{k = 2/5}$$

(b)

$$P(1/4 < x < 1/2)$$

$$P\left(\frac{1}{4} < x < \frac{1}{2}\right) = \int_{1/4}^{1/2} k(x+2)\cdot dx$$

$$= K\int_{0.25}^{0.5} (x+2)\cdot dx$$

$$= (0.4)(0.5937)$$ ∴ putting value of K.

$$\boxed{P\left(\frac{1}{4} < x < \frac{1}{2}\right) = 0.2375}$$

(c)

(i) Mean (ii) Variance

(i) **Mean :-** $\mu = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$

$$= \int_{-\infty}^{0} x \cdot f(x) \cdot dx + \int_{0}^{1} x \cdot f(x) \cdot dx + \int_{1}^{\infty} x \cdot f(x) \cdot dx.$$

$$= \int_{0}^{1} x \cdot K(x+2) \cdot dx$$ ∴ putting value of f(x)

$$= K\int_{0}^{1} (x^2 + 2x)\, dx$$

By solving integration, putting value of K,

$$= \left(\frac{2}{5}\right)\left(\frac{4}{3}\right)$$

$$\mu = \frac{8}{15} = 0.533$$

(ii) **Variance :** $E(x^2) - \mu^2$

$$= \int_0^1 x^2 \cdot f(x) \cdot dx - \mu^2$$

$$= \int_0^1 x^2 \cdot k(x+2) \cdot dx - \mu^2$$

$$= k \int_0^1 x^3 + 2x^2 - \mu^2$$

Solving integration & putting value of k,

$$= \left(\frac{2}{5}\right)\left(\frac{11}{12}\right) - \left(\frac{8}{15}\right)^2$$

$$= \frac{22}{60} - \frac{64}{225}$$

Variance $= \frac{37}{450} = 0.0822$

QUESTION # 02

The die can land in 6 different ways with same probability of $\frac{1}{6}$

Hence, $f(x) = \frac{1}{6}$ for, $x = 1,2,3,4,5,6$

X	P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Mean :-

$$E(X) = \sum_{i=1}^{n} x_i P_i$$

$$= 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$

$$= \frac{1+2+3+4+5+6}{6}$$

$$E(x) = \frac{21}{6}$$

Variance :

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= \frac{1^2+2^2+3^2+4^2+5^2+6^2}{6} - \left[\frac{21}{6}\right]^2$$

$$= \frac{91}{6} - \left(\frac{21}{6}\right)^2 = \frac{35}{12}$$

QUESTION # 03

$K = 2 \qquad x = 0, 1, 2$

$N = 7$

$n = 3$

$N - K = 7 - 2 = 5$

$n - x = 3 - x$

$$f(x) = \frac{{}^{K}C_{x} \; {}^{N-K}C_{n-x}}{{}^{N}C_{n}}$$

$$f(x) = \frac{{}^{2}C_{x} \; {}^{5}C_{3-x}}{{}^{7}C_{3}} \quad \therefore x = 0, 1, 2$$

(i) $$P(x=0) = \frac{{}^{2}C_{0} \; {}^{5}C_{3}}{{}^{7}C_{3}}$$

$$= \frac{1 \times 10}{35}$$

$$P(x=0) = \frac{2}{7}$$

(ii) $P(x=1) = \dfrac{{}^2C_1 \; {}^5C_2}{{}^7C_3}$

$= \dfrac{2 \times 10}{35}$

$P(x=1) = \dfrac{4}{7}$

(iii) $P(x=2) = \dfrac{{}^2C_2 \; {}^5C_1}{{}^7C_3}$

$= \dfrac{1 \times 5}{35}$

$P(x=2) = \dfrac{1}{7}$

Mean :-

$\text{Mean} = \sum_{0}^{1} x \cdot f(x)$

$= 0 \cdot \left(\frac{2}{7}\right) + 1\left(\frac{4}{7}\right) + 2\left(\frac{1}{7}\right)$

$= \frac{4}{7} + \frac{2}{7}$

$\text{Mean} = \dfrac{6}{7}$

QUESTION # 04

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Finding mean μ,

$$\mu = E(x) = \int_0^1 x^2 \cdot dx + \int_1^2 x \cdot (2-x) \cdot dx$$

$$= \int_0^1 x^2 \cdot dx + \int_1^2 2x - x^2 \cdot dx$$

$$= \int_0^1 x^2 \cdot dx + \int_1^2 2x \cdot dx - \int_1^2 x^2 \cdot dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 + 2 \cdot \left|\frac{x^2}{2}\right|_1^2 - \left[\frac{x^3}{3}\right]_1^2$$

$$= \left[\frac{1}{3} - 0\right] + 2\left[\frac{4}{2} - \frac{1}{2}\right] - \left[\frac{8}{3} - \frac{1}{3}\right]$$

$$= \frac{1}{3} + 3 - \frac{7}{3} = \frac{3}{3}.$$

$$\mu = 1$$

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Hence,

Average number of hours per year $= \mu \cdot 100$

$= 1 \cdot 100$

$= 100$ hours.

QUESTION # 05

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{eleswhere} \end{cases}$$

Finding mean,

$$\mu = E(X) = \int_0^1 x \cdot 2(1-x) \cdot dx$$

$$= 2 \int_0^1 x - x^2 \cdot dx$$

$$= 2 \left[\int_0^1 x \cdot dx - \int_0^1 x^2 \cdot dx \right]$$

$$= 2 \left[\left[\frac{x^2}{2} \right]_0^1 - \left[\frac{x^3}{3} \right]_0^1 \right]$$

$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$

$= 2\left[\frac{1}{6}\right]$

$\mu = \frac{1}{3}$

also,

Average Profit per automobile $= \mu \cdot 5000$

$= \frac{1}{3} \cdot 5000$

$= \$1,667.67$

QUESTION # 06

$\mu_x = E(X) = \sum_{x=0}^{4} x \cdot f(x)$

$= 0(0.41) + 1(0.37) + 2(0.16) + 3(0.05) + 4(0.01)$

$\mu_x = E(x) = 0.88$

For Standard Deviation δ_x, we find variance,

$$\mu_x = E(x) = 0.88$$

$$\mu_x^2 = [E(x)]^2 = 0.7744$$

$$E(x^2) = (0^2)(0.41) + (1^2)(0.37) + (2^2)(0.16) + (3^2)(0.05) + (4^2)(0.01)$$

$$= 1.62$$

Standard Deviation :-

$$\delta_x^2 = E(x^2) - [E(x)]^2$$

$$= 1.62 - 0.7744$$

$$\delta_x^2 = 0.8456$$

$$\delta_x = \sqrt{0.8456}$$

$$\delta_x = 0.92$$