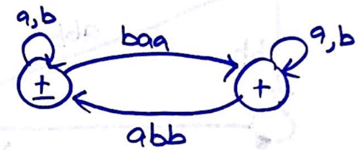
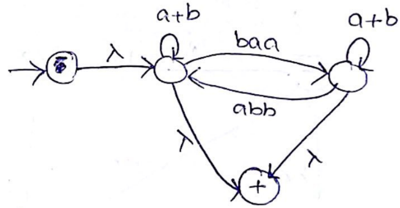
Theory of Automata

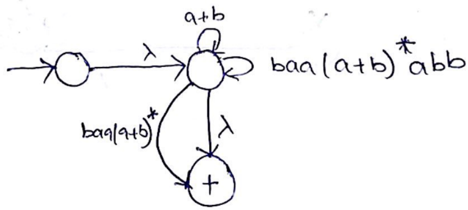
Assignment 2

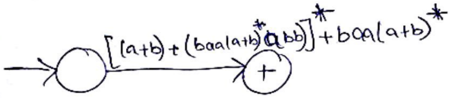
Chapter 7:

Question 4:



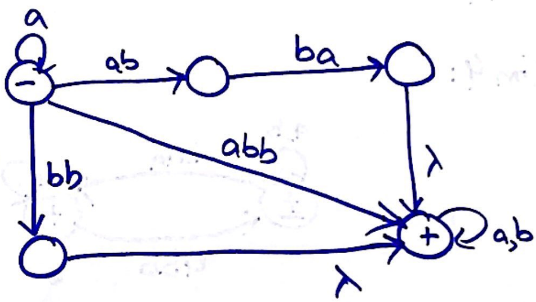


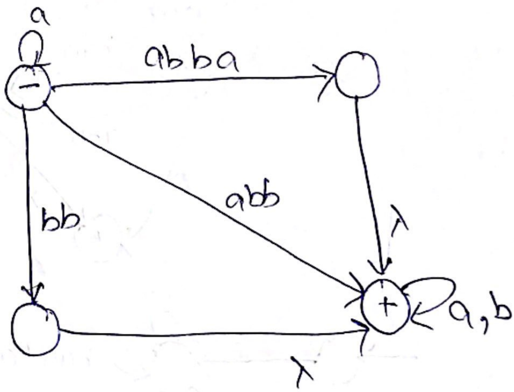


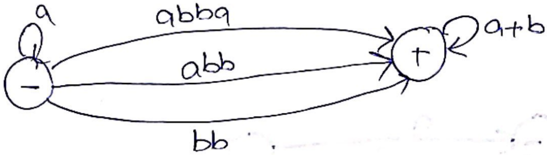


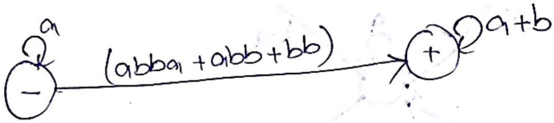
RE : $\left[(a+b) + (baa\,(a+b)^{*}\,abb)\right]^{*} + baa\,(a+b)^{*}$

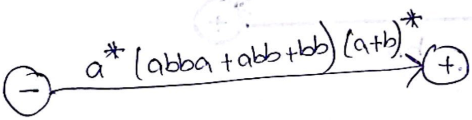
Question 5:





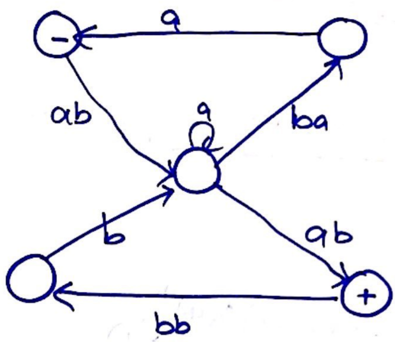


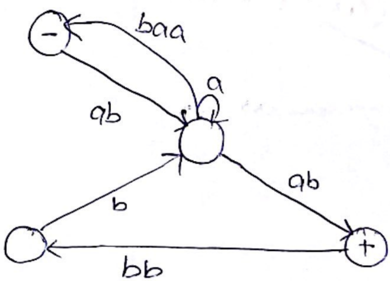


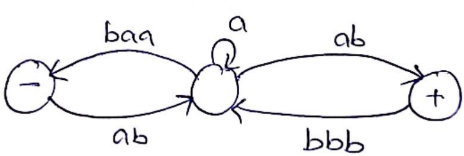


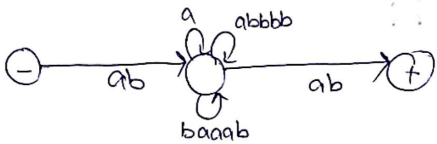
RE: $a^*(abba+abb+bb)(a+b)^*$

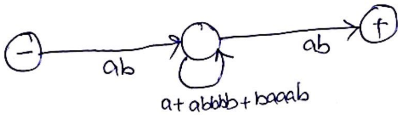
Question 6:

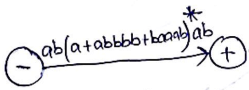








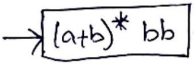


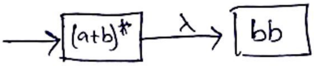


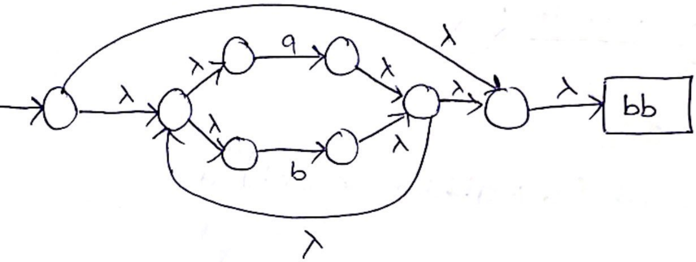
R.E : $ab(a + abbbb + baaab)^* ab$

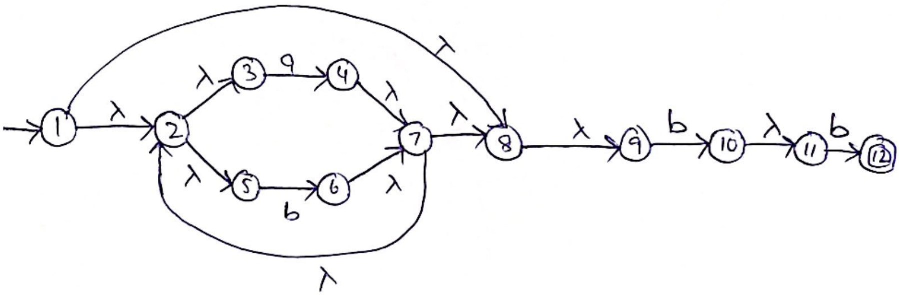
a- $(a+b)^* bb$

RE → λ NFA









λ- NFA $\rightarrow$ FA

A = λ-closure $\{1\}$ = $\{1, 2, 3, 5, 8, 9\}$

on input 'a' : $\{4\}$

on input 'b' : $\{6, 10\}$

B = λ-closure $\{4\}$ = $\{4, 7, 8, 9, 2, 3, 5\}$

on input 'a' : $\{4\} \Rightarrow B$

on input 'b' : $\{6, 10\}$

C = λ-closure $\{6, 10\}$ = $\{6, 10, 7, 2, 3, 5, 8, 9, 11\}$

on input 'a' : $\{4\} \Rightarrow B$

on input 'b' : $\{6, 10, 12\}$

D = λ-closure $\{6, 10, 12\}$ = $\{6, 10, 12, 7, 8, 9, 2, 3, 5, 11\}$

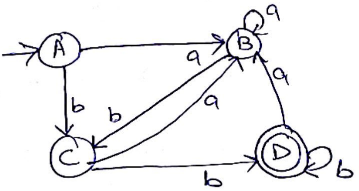
on input 'a' : $\{4\} \Rightarrow B$

on input 'b' : $\{6, 10, 12\} \Rightarrow D$

Transition Table

State	a	b
A	B	C
B	B	C
C	B	D
D	B	D

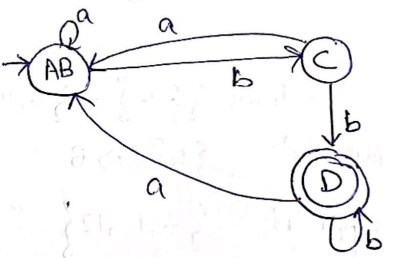
FA



Minimization of FA

states A, B are non-final and have same transitions.

Combining states A and B



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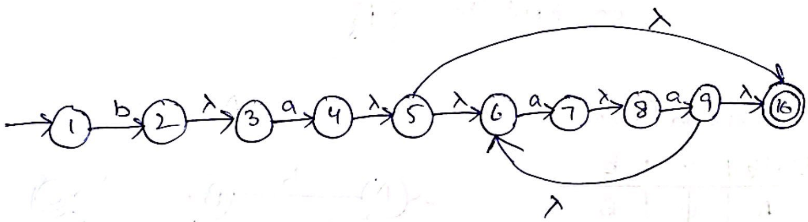
b- ba (aa)*

RE → λ -NFA

$\rightarrow \boxed{ba(aa)^*}$

$\rightarrow \boxed{ba} \xrightarrow{\lambda} \boxed{(aa)^*}$



λ-NFA → FA

A = λ-closure {1} = {1}

on input 'a' : {φ}

on input 'b' : {2}

B = λ-closure {2} = {2,3}

on input 'a' : {4}

on input 'b' : {φ}

C = λ-closure {4} = {4,5,6,10}

on input 'a' : {7}

on input 'b' : {φ}

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D = λ-closure $\{7\} = \{7, 8\}$

on input 'a' : $\{9\}$

on input 'b' : $\{\phi\}$

E = λ-closure $\{9\} = \{9, 6, 10\}$

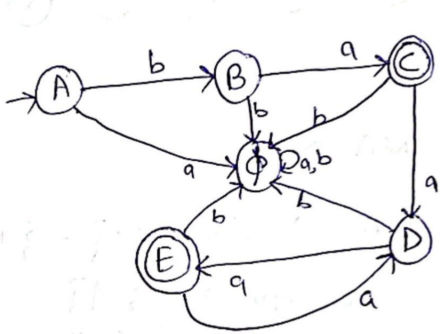
on input 'a' : $\{7\} \Rightarrow D$

on input 'b' : $\{\phi\}$

Transition Table

State	a	b
A	ϕ	B
B	C	ϕ
C	D	ϕ
D	E	ϕ
E	D	ϕ

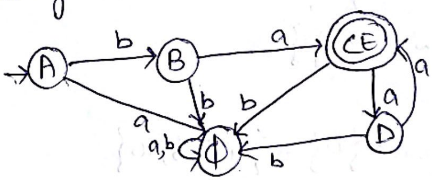
FA



Minimization of FA:

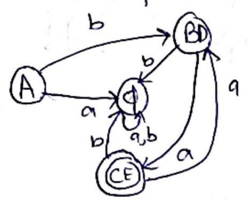
States C, E ~~are~~ ~~not~~ are both final with same transitions

Combining states C and E



States B and D are non final states with same transitions

Combining B & D

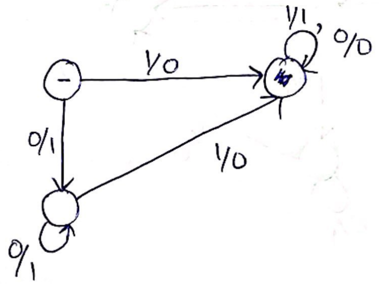


Chapter 8:

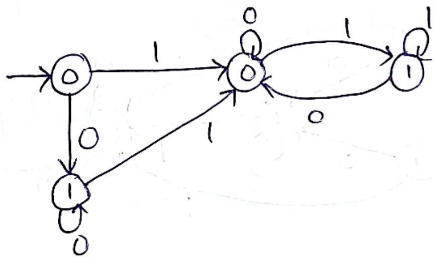
Question 1

a- decrement binary no. by 1

Mealy Machine

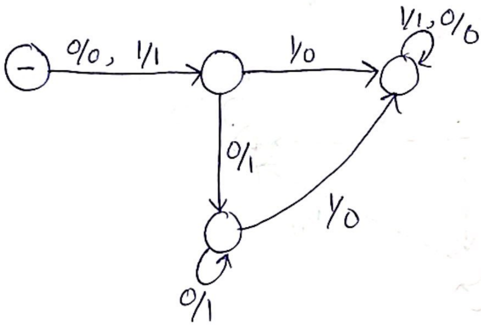


Moore Machine:

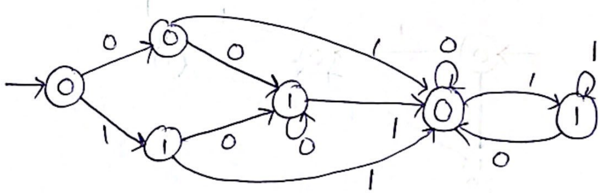


b- decrement binary no. by 2

Mealy Machine:

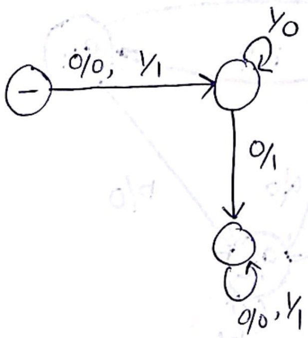


Moore Machine:

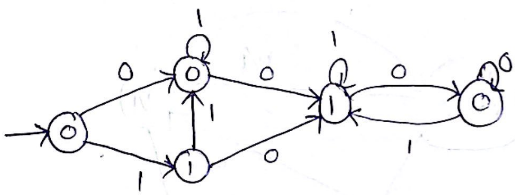


c_ increment binary no. by 2

Mealy Machine

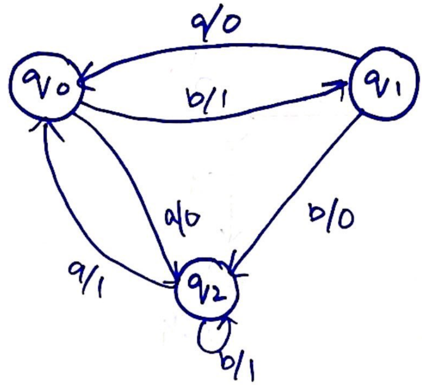


Moore Machine

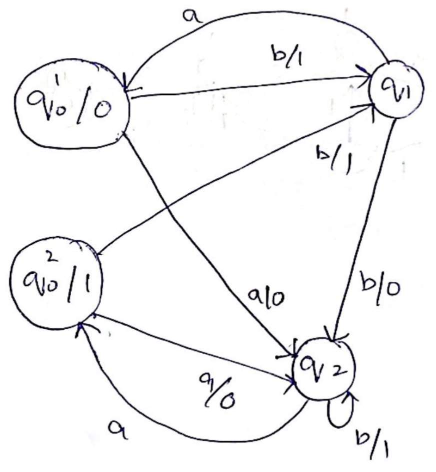


Question 2:

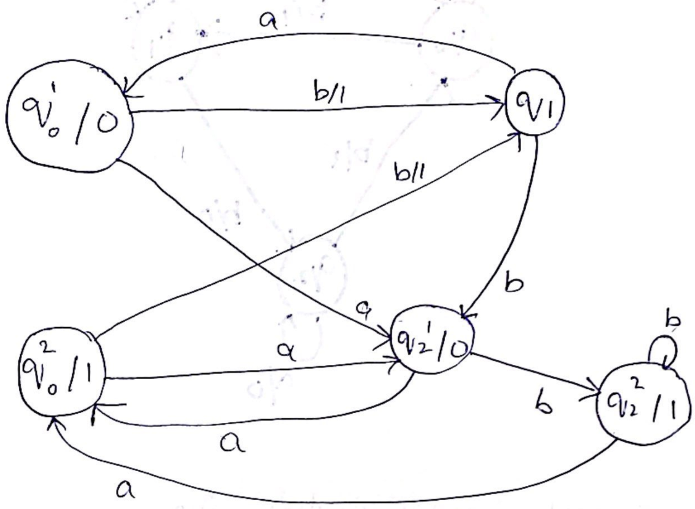
a)



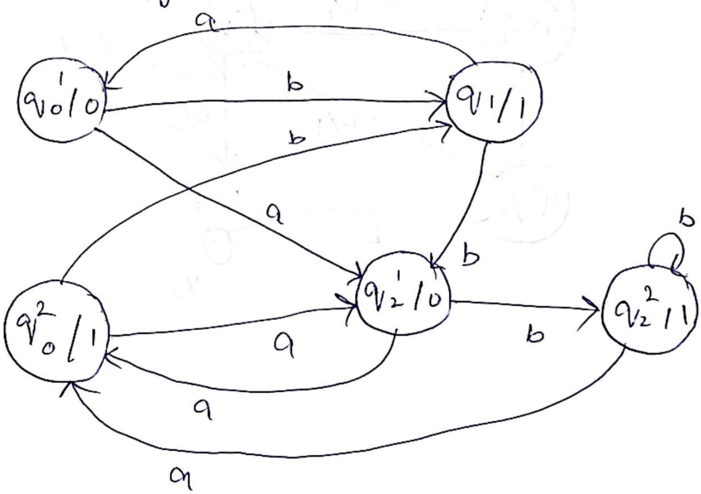
Firstly, we need 2 copies of state q_0



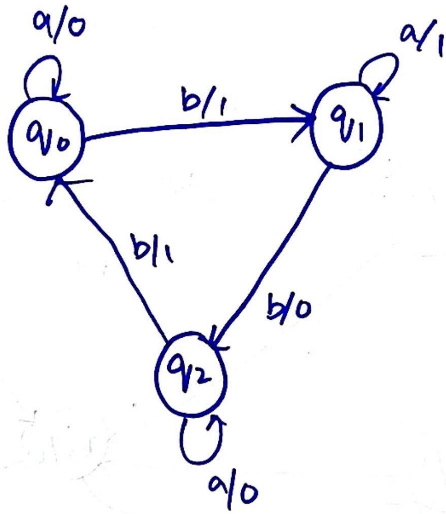
Now, we need 2 copies of state q_2.



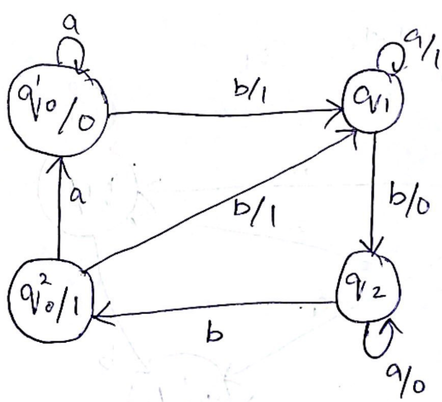
No need to copy state q_1



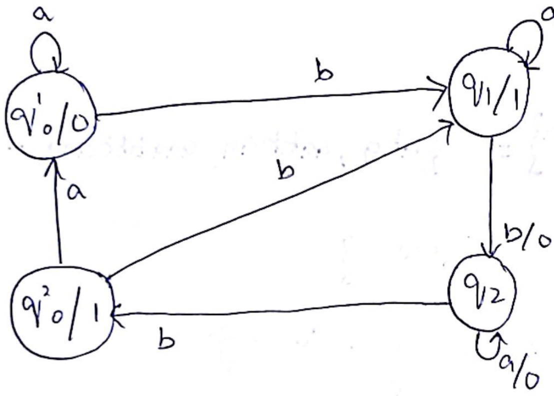
b)



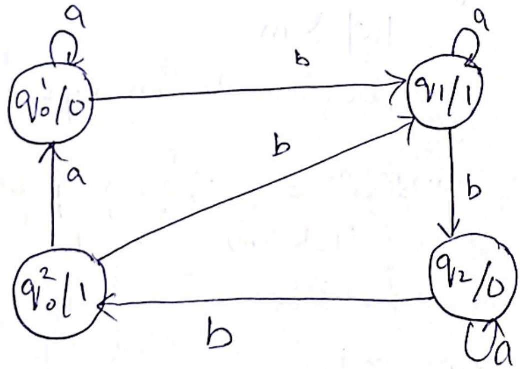
we need 2 copies of state q0



For state q_1, we donot need any copy



For state q_2, we donot need any copy



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Chapter 11

Question 5:

$\{a^n b^n a^n\} = \{aba, aabbaa, aaabbbaaa, \dots\}$

Assume $L = \{a^n b^n a^n\}$

Suppose L is a regular language

Let 'm' be critical length for L

Let a string $w = a^m b^m a^m \in L$

$$|w| \geq m$$

As L is regular, we can write w as xyz

$$w = xyz = a^m b^m a^m = \overbrace{\underbrace{a \dots a}_{x} \dots \underbrace{a \dots a}_{y}}^{m} \dots \overbrace{\underbrace{b \dots b \dots b \dots \overbrace{a \dots a \dots a}^{m}}_{z}}^{m}$$

$$\Rightarrow y = a^k \quad (1 \leq k \leq m)$$

As L is regular $\Rightarrow xy^iz \in L$ (for $i = 0, 1, 2, \dots$)

for $i = 2 \Rightarrow xy^2z \in L$

$$xy^2z = \overbrace{\underbrace{a \dots a}_{x} \dots \underbrace{a \dots a}_{y} \underbrace{\dots a}_{y}}^{m+k} \dots \underbrace{\overbrace{b \dots b \dots b}^{m} \dots \overbrace{a \dots a \dots a}^{m}}_{z}$$

$xy^2z \in L$ so $a^{m+k} b^m a^m \in L$

But $L = \{a^n b^n a^n : n \geq 0\}$ so, $a^{m+k} b^m a^m \notin L$

L is a non regular language

Question 6:

i_ $a^n b^{2n}$

$L = \{ a^n . b^{2n}, n \geq 0 \}$

Assume that L is a regular language

Let m be critical length for L

$w = a^m b^{2m} \in L$

$|w| \geq m$

If $w \in L$, it can be written as:

$w = xyz = a^m b^{2m}$

$$\overbrace{\underbrace{aa}_{x} \ldots \underbrace{aa}_{y}}^{m} \ldots \underbrace{\overbrace{bb \ldots bb \ldots bb \ldots bb}^{2m}}_{z}$$

$\Rightarrow y = a^k \quad (1 \leq k \leq m)$

If L is regular language then $xy^iz \in L$ (for $i = 0, 1, 2, \ldots$)

for $i = 2$, $xy^2z \in L$

$$\overbrace{\underbrace{aa}_{x} \ldots \underbrace{aa}_{y} \ldots \underbrace{aa}_{y}}^{m+k} \ldots \overbrace{bb \ldots bb \ldots bb \ldots bb}^{2m}$$

So, $a^{m+k} . b^{2m} \in L$

But $L = \{ a^n b^{2n}, n \geq 0 \}$ so, $a^{m+k} . b^{2m} \notin L$

L is a non-regular language

ii) $\{a^n b a^n\} = \{aba, aabaa, aaabaaa, \ldots\}$

$L = \{a^n b a^n : n \geq 0\}$

Suppose L is a regular language
Let m be critical length for L.

$w = a^m b a^m \in L, \ |w| \geq m$

If L is regular, we can write :

$w = xyz = a^m b a^m$

$$\overbrace{\underbrace{aa}_{x}\cdots\underbrace{aa}_{y}}^{m}\;\underbrace{b\,\overbrace{aa\cdots aa}^{m}}_{z}$$

$\Rightarrow y = a^k \quad (1 \leq k \leq m)$

If L is a regular language then $xy^iz \in L$ (for $i = 0, 1, 2, \ldots$)

for $i = 2$: $xy^2z \in L$

$$\underbrace{\overbrace{aa}^{x}\cdots\overbrace{aa}^{y}\cdots\overbrace{aa}^{y}}_{m+k}\;b\;\underbrace{\overbrace{aa\cdots aa}^{m}}_{z}$$

So, $a^{m+k} b a^m \in L$

But $L = \{a^n b a^n : n \geq 0\}$ So, $a^{m+k} b a^m \notin L$

L is not a regular language.

Question 9:

ODDPALINDROME = { all words in PALINDROME that have odd length }

O = Odd Palindrome = { aba, aabaa, bab, abaab, ... }

Suppose O is regular language
Assume a string w that belongs to Odd Palindrome

$w = a^n b a^n \in O$

If O is regular, w can be written as

$w = xyz = a^n b a^n$

$$\overbrace{\underbrace{a \cdot a}_{x}\ \underbrace{.a}_{y}}^{n}\ \underbrace{b\,\overbrace{a..a..a}^{n}}_{z} \Rightarrow y = a^k$$

If O is regular then $w \in O$

$xy^iz \in O$ (for $i = 0, 1, 2, \dots$)

for $i = 2$: $xy^2z \in O$

$$\overbrace{\underbrace{a...a}_{x}..\underbrace{a..}_{y}\underbrace{a}_{y}}^{n+k}\ \underbrace{b\,.\,\overbrace{a..a..a}^{n}}_{z}$$

So, $a^{n+k} b a^n \in O$

But $a^{n+k} b a^n$ is not palindrome string.

O is not a regular language

Question 17

ii) $PRIME' = \{a^n, n \text{ is not prime}\}$

Suppose PRIME' is regular language

let $w = xyz \in PRIME'$

$w = a^m \in PRIME'$

$w = xyz = a^m \quad (x = \Lambda, y = a^m, z = \Lambda)$

If PRIME' is regular

$xy^iz \in PRIME'$ (for $i = 0, 1, 2, 3, \ldots$)

$i = 1 = xyz = a^m$

$i = 2 = xyz = a^{m^2}$

$i = 3 = xyz = a^{m^3}$

$\vdots$

$i = k = xyz = a^{m^k} \in PRIME'$

So, PRIME' does pass the pumping lemma.

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