

Question # 1 :-

prime numbers : 17, 19

encrypt the plain text = 13

$$p = 17, q = 19$$

$$n = p \times q = 17 \times 19 = 323$$

$$\phi(n) = (p-1)(q-1) = 16 \times 18 = 288$$

$$e = 1 < e < \phi(n)$$

$$\gcd(e, 288) = 1, \text{ so } e = 7$$

private key $\langle e, n \rangle = (7, 323)$
public

$$C = m^e \bmod n$$

$$C = 13^7 \bmod 323 = 276$$

$$M = C^d \bmod n$$

$$d \times e \bmod \phi(n) = 1 \Rightarrow d = e^{-1} \bmod \phi(n)$$

$$d = 247$$

$$M = 276^{247} \bmod 323$$

$$M = 13$$

Question # 2 :-

you are given RSA components

$$n = 2537 \quad n = 33$$

$$d = 937 \quad d = 7$$

$C = 855 \rightarrow$ encrypted using public key

$$C = 16$$

$$M = C^d \bmod n$$

$$M = 16^7 \bmod 33$$

$$M = 25$$

Question # 3:-

$$p = 17, q = 23$$

$$M = 88$$

$$n = p \times q = 17 \times 23 = 391$$

$$\phi(n) = (p-1)(q-1) = 16 \times 22 = 352$$

$$e \Rightarrow 1 < e < \phi(n)$$

$$\gcd(e, 352) = 1, \text{ so } e = 3$$

$$\text{public key } \langle e, n \rangle = \langle 3, 391 \rangle$$

$$C = m^e \bmod n$$

$$C = 88^3 \bmod 391$$

$$C = 350$$

$$d \times e \bmod \phi(n) = 1$$

$$d = 235$$

$$M = C^d \bmod n$$

$$M = 350^{235} \bmod 391$$

$$M = 88$$