# Laboratory №1
# Projectile Motion
# Section # 1

Kamilyam Nazakhanova
Symbat Demelkhan

**Objectives :**

- Calculate the average initial velocity at which the ball is fired from the projectile launcher
- Compare theoretical and experimental data regarding projectile motion

**Experimental Data:**

Table 1. Data recorded from 10 firings at 5 different angles

| Angle | d1,m | d2,m | d3,m | d4,m | d5, m | d6,m | d7,m | d8,m | d9 ,m | d10,m | $D_{average}$, m |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 25° | 0.847 | 0.848 | 0.863 | 0.863 | 0.866 | 0.867 | 0.869 | 0.873 | 0.880 | 0.882 | 0.8658 |
| 35° | 1.055 | 1.085 | 1.087 | 1.102 | 1.103 | 1.104 | 1.105 | 1.105 | 1.105 | 1.120 | 1.0971 |
| 45° | 1.194 | 1.198 | 1.201 | 1.202 | 1.204 | 1.206 | 1.207 | 1.208 | 1.211 | 1.226 | 1.2057 |
| 55° | 1.034 | 1.060 | 1.061 | 1.063 | 1.063 | 1.063 | 1.063 | 1.063 | 1.067 | 1.070 | 1.0607 |
| 65° | 0.872 | 0.872 | 0.873 | 0.873 | 0.874 | 0.875 | 0.879 | 0.881 | 0.884 | 0.888 | 0.8771 |

**Data Analysis**

**Part 1 :Theory (Derivation of the horizontal range formula)**

We derive the horizontal range formula using the kinematic equations [1]:

$a_x = 0$

$v_x = v_{0x}$

$\triangle x = v_{0x}t$

$a_y = -g$

$v_y = v_{0y} - gt$

$\triangle y = v_{0y}t - 12gt^2$

Where,

$v_{0x} = v_0 cos\theta$
$v_{0y} = v_0 sin\theta$
Suppose a projectile is thrown from the level of the ground, thus, the range is the distance between the launch point and landing point, where the projectile is hitting the ground. Further, when the projectile returns to the ground, the vertical displacement is zero, thus we have:

$0 = v_0 sin\theta t - 1/2gt2$

Solving for t, we have

$t = 0, 2v_0 sin\theta/g$

The first solution provides the time when the projectile is thrown and the second one is the time when it hits the ground. Plugging in the second solution into the displacement equation and using 2sinθcos θ = sin(2θ), we have

$R = \Delta x(t= 2v_0 sin\theta/g) = v_0\text{^}2\ sin(2\theta)/g\ (1)$

**Part 2: Calculation of initial velocity for each angle**

Now ,using the formula that we have derived in the theory part , we can solve for $v_0$

$v_0 = \sqrt{}gR/sin2\theta\ (2)$

Here , R (range) is equal to average distance .

Example calculation for 25° angle:
$v_0 = \sqrt{}\ 9.81*0.8658 / sin(25° * 2) = 3.328\ m/s$

Table 2. Initial velocities for each angle

| **Angle** | $v_0$ **, m/s** |
|---|---|
| **25°** | 3.328 |
| **35°** | 3.3825 |

| | |
|---|---|
| **45°** | 3.4374 |
| **55°** | 3.3259 |
| **65°** | 3.3497 |

As we can notice , since the projectile launcher fires with a stable force , the values of $v_0$ are rather similar for all angles.

**Part 3: Error analysis**

There is a need to find uncertainty of initial velocity. The uncertainty of a resultant is a function of the uncertainties of all the measurements used to calculate it. Accordingly , since the error of average initial velocity depends on the uncertainty of distance and angle , errors for distance (statistical and systematic) and angle must be calculated first.

We used = stdev() command in Excel to measure the statistical error of range measurement.

***ΔD1= stdv(d1,d2,d3,d4,d5,d6,d7,d8,d9,d10)***

The systematic error is the half of the scale's smallest division (0.5mm or 0.0005m)

***ΔD2 = 0.0005m***

Then , we combine statistical and systematic errors into a single error

$$\Delta D = \sqrt{\Delta D1^2 + \Delta D2^2}$$

Error for angle is 0.5°

***Δθ = 0.5°***

Finally , using the formula below , we calculated the error for initial velocity

$$\Delta v_0 = \sqrt{( dv_0/dD * \Delta D)^2 + ( dv_0/d\theta * \Delta\theta)^2} = \sqrt{ g*\Delta d^2 /4d\sin(2\theta) + d*g*\csc(2\theta) * \cot^2(2\theta) * \Delta\theta^2}$$

Example calculation for 25° angle:

$$\Delta v_0 = \sqrt{ (9.81*0.01101^2 / 4*\sin(50°)) + (0.01101*9.81*\csc(50°) * \cot^2(50°) * 0.5°^2} = 0.03229$$

Table 3 . All the measurements with errors

| ***θ*** | ***Δθ*** | ***$D_{ave}$,m*** | ***ΔD1, m*** | ***ΔD2,m*** | ***ΔD, m*** | ***$v_0$ , m/s*** | ***$\Delta v_0$,m/s*** |
|---|---|---|---|---|---|---|---|

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***25°*** | 0.5° | 0.8658 | 0.0110 | 0.0005 | 0.01101 | 3.328 | 0.03229 |
| ***35°*** | | 1.0971 | 0.0168 | | 0.0168 | 3.3825 | 0.02805 |
| ***45°*** | | 1.2057 | 0.0082 | | 0.0082 | 3.4374 | 0.01226 |
| ***55°*** | | 1.0607 | 0.0093 | | 0.0093 | 3.3259 | 0.01801 |
| ***65°*** | | 0.8771 | 0.0053 | | 0.0053 | 3.3497 | 0.02654 |

After finding mean of all initial velocities and mean of all errors for initial velocity , we get

***$v_0$ = 3.3647 ± 0.02343 m/s***

**Part 3: Theoretical and measured distances comparison**

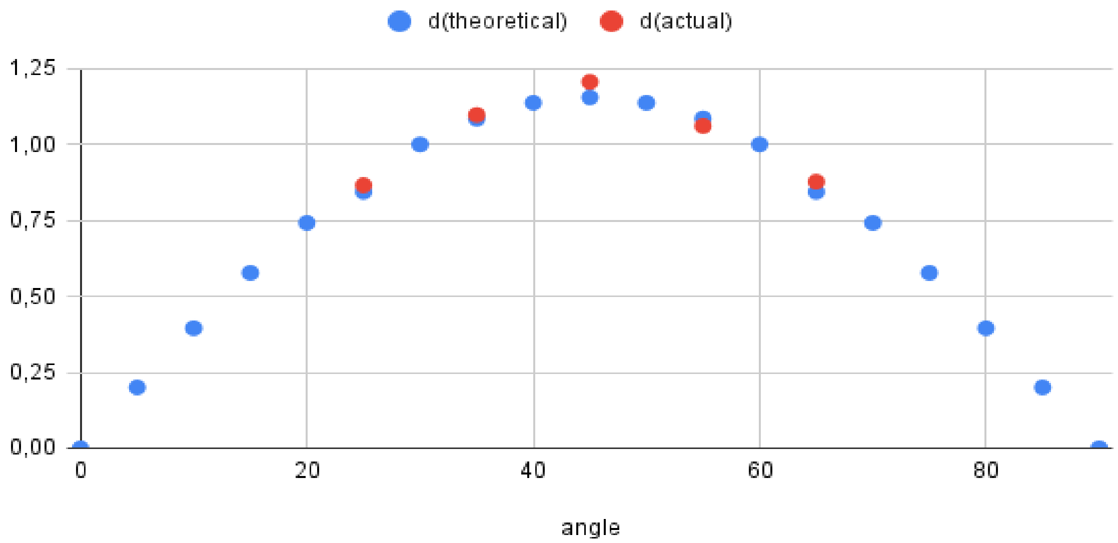
There is a relatively small difference between theoretical and actual distances of the ball fired from launcher at with an initial velocity of 3.3647m/s.

Table 4. Theoretical and measured distances

| ***$\theta$ , °*** | ***$v_0$, m/s*** | **d(theoretical) , m** | **d(actual) , m** |
|---|---|---|---|
| **0** | 3.3647 | 0 | |
| **5** | | 0.200 | |
| **10** | | 0.395 | |
| **15** | | 0.577 | |
| **20** | | 0.742 | |
| **25** | | 0.884 | 0.8658 |
| **30** | | 1.000 | |
| **35** | | 1.085 | 1.0971 |
| **40** | | 1.137 | |
| **45** | | 1.155 | 1.2057 |
| **50** | | 1.137 | |
| **55** | | 1.085 | 1.0607 |
| **60** | | 1.000 | |

| | | | |
|---|---|---|---|
| **65** | | 0.884 | 0.8771 |
| **70** | | 0.742 | |
| **75** | | 0.577 | |
| **80** | | 0.395 | |
| **85** | | 0.200 | |
| **90** | | 0 | |

**Graph 1. Theoretical and measured distances on scatter chart**




**Discussion**

As it is obvious from the scatter chart, the distance keeps increasing up until 45°, reaching its maximum range at this angle. Then it starts to decrease at the same pace as it was increasing , hitting the low of 0 m at an angle of 90°.

The initial velocity of the ball , at which it is launched from the projectile launcher , was determined to be 3.3647m/s with uncertainty of ± 0.89938 m/s. After simulating the projectile motion on the scatter chart and comparing theoretical and measured distances of the ball with such initial velocity , we noticed that distance points,in general, do match.The reasons for such relatively small errors might be that we used duck tape and tried not to move the stand. However , some unavoidable random errors definitely took place during the experiment , as points do not match perfectly .The most noticeable deviation from the normal curve is at an angle of 45°, which might be the result of the stand's movement or inaccuracy in measurement .

**Conclusion**

To conclude , we achieved all the objectives that we set at the beginning of the experiment. Our first objective was to calculate the average initial velocity. The average initial velocity of the ball , according to our calculations , is equal to ***3.3647 ± 0.02343 m/s.*** We compared theoretical and measured data and realized that the error is not that large .After discussing possible reasons for such a small difference between theoretical and actual data , we made the conclusion that it is due to the fact that we were using duck tape not to move the paper and we were in an area with few people , so that none could accidentally move our stand.

**Reference List**

[1] Toppr *Horizontal Range Formula : Projectile Motion Formula* , Examples[Online]. Available : https://www.toppr.com/guides/physics-formulas/horizontal-range-formula/