Probability of No Consecutive Loss

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1. **Analytical Approach**

   This problem can be solved by analyzing a recurrence relationship.

   We can let P_n denote the probability that a team has no consecutive loss in a season of n games. This probability can be further decomposed into two scenarios, P_n^0 and P_n^1, where P_n^0 is the probability of the case where no consecutive loss occurs **and** the last game is a win, and P_n^1 is the probability of the case where no consecutive loss occurs **and** the last game is a loss **and** the second-to-last game is a win. Thus,

   $$P_n = P_n^0 + P_n^1$$

   We extend the case to P_{n+1}^0. As from P_n, the team just has to win one more game to achieve P_{n+1}^0, we obtain that

   $$P_{n+1}^0 = 0.8 \cdot P_n = 0.8 \cdot P_n^0 + 0.8 \cdot P_n^1$$

   Similarly, from P_n^0, the team just has to lose one more game to achieve P_{n+1}^1. Thus,

   $$P_{n+1}^1 = 0.2 \cdot P_n^0$$

   Combining the equations, we can get

   $$P_n = P_{n+1}^0/0.8 = P_n^0 + 0.2 \cdot P_{n-1}^0 = 0.8 \cdot P_{n-1} + 0.8 \cdot 0.2 \cdot P_{n-2}$$

   This forms a recurrence relationship that can be solved easily. The boundary cases are $P_0 = 1$ and $P_1 = 1$. I used a simple Python program (attached) to build up the recurrence list from bottom up to get $\mathbf{P_{82} = 0.0588 = 5.88\%}$ (rounded to fourth decimal place) Thus the probability of no consecutive loss in a 82-game season, given independent winning probability of 0.8, is 5.88%.

2. **Simulation Approach**

   As we are given the probability distribution of independent games, we can use a Monte Carlo simulation to simulate a large number of seasons and approximate the probability of no consecutive loss based on the proportion of seasons with no consecutive loss. The results are tabulated as follows:

Number of trials	Proportion
1000	0.0540
10000	0.0628
100000	0.0590

   Based on simulation, the probability that the prediction would be true (i.e. that the Warriors would not suffer any consecutive loss in 82 games) is around **5.8%**, similarly to the analytical approach.