Assume the load factor is $\alpha = \frac{m}{n}$, the random variable X denotes the number of slots hit until an empty slot. Assume $n >= m+3$

$$P(X=k) = \frac{\binom{n-k}{m-k+1}}{\binom{n}{m}}$$

$$E(X) = \Sigma_{k=1}^{m+1} kP(X=k)$$

$$= \frac{\Sigma_{k=1}^{m+1} k\binom{n-k}{m-k+1}}{\binom{n}{m}}$$

$$= \frac{\Sigma_{l=1}^{m+1}\Sigma_{k=l}^{m+1}\binom{n-k}{m-k+1}}{\binom{n}{m}}$$

$$= \frac{\Sigma_{l=1}^{m+1}\Sigma_{k=l}^{m+1}\binom{n-k}{m-k+1}}{\binom{n}{m}}$$

$$= \frac{\Sigma_{l=1}^{m+1}\Sigma_{t=0}^{m+1-l}\binom{n-m-1+t}{t}}{\binom{n}{m}}$$

As we have

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$$

,

$$\Sigma_{t=0}^{m+1-l}\binom{n-m-1+t}{t}$$

$$= \binom{n-m-1}{0} + \binom{n-m}{1} + ... + \binom{n-l}{m-1+l}$$

$$= \binom{n-m-1}{0} - \binom{n-m-1}{1} + \binom{n-m-1}{1} + \binom{n-m}{1} + ... + \binom{n-l}{m-1+l}$$

$$= \binom{n-m-1}{0} - \binom{n-m-1}{1} + \binom{n-l}{m+2-l}$$

$$= m+2-n+\binom{n-l}{m+2-l}$$

Again, we use the same trick, and we get

$$\Sigma_{l=1}^{m+1}\Sigma_{t=0}^{m+1-l}\binom{n-m-1+t}{t}$$

$$= n(m+2-n) + 1 - \frac{(n-m-1)(n-m)}{2} + \binom{n-1}{m+2}$$

$$E(X) = O(\frac{\binom{n-1}{m+2}}{\binom{n}{m}})$$

$$= O(\frac{\frac{(n-1)!}{(m+2)!(n-m-3)!}}{\frac{n!}{(n-m)!m!}})$$

$$= O(\frac{(1-\alpha)n((1-\alpha)n-1)((1-\alpha)n-2)}{n(\alpha n+1)(\alpha n+2)})$$

$$= O(\frac{(1-\alpha)^2 n}{\alpha^2 n}) = O(1)$$