

# AA: TP2

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On remarque une amélioration globale des résultats avec les contraintes du TP2 par rapport à ceux du TP1 (certains résultats du TP1 semblent marginalement meilleurs). En particulier, la non relaxation de  $i, s, v$  rend le problème wheel trop compliqué pour être résolu par lpsolve en temps raisonnable.

Une version n'utilisant pas les ensembles mais des matrices est également fournie pour SF et MTZ. La traduction a été faite mécaniquement et peut abriter quelques erreurs (version du TP1).

## 1 Initial model

1 Cf Vanilla

2 We compute

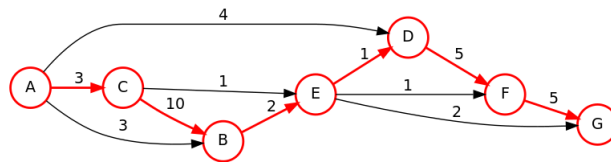


Figure 1: G1 vanilla

*Presolve eliminates 1 constraint.*  
*Adjusted problem:*  
*25 variables, all binary*  
*15 constraints, all linear; 67 nonzeros*  
*9 equality constraints*  
*6 inequality constraints*  
*1 linear objective; 11 nonzeros.*

*LP-SOLVE 4.0.1.0: optimal, objective 26*  
*21 simplex iterations*  
*totallength = 26*

3 Cf Vanilla

*Presolve eliminates 1 constraint.*  
*Adjusted problem:*  
*25 variables, all linear*

15 constraints, all linear; 67 nonzeros  
 9 equality constraints  
 6 inequality constraints  
 1 linear objective; 11 nonzeros.

LP\_SOLVE 4.0.1.0: optimal, objective 26  
 21 simplex iterations  
 totallength = 26

Same result. The constraint matrix is the node arc matrix of a directed graph which means it is TUM, therefore, the LP problem gives an integer solution.

5 We compute

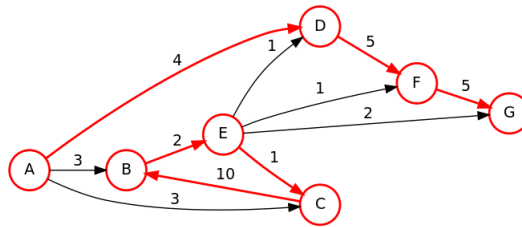


Figure 2: G2 vanilla

Presolve eliminates 1 constraint.  
 Adjusted problem:  
 25 variables, all binary  
 15 constraints, all linear; 67 nonzeros  
 9 equality constraints  
 6 inequality constraints  
 1 linear objective; 11 nonzeros.

LP\_SOLVE 4.0.1.0: optimal, objective 27  
 22 simplex iterations  
 totallength = 27

## 1.1 MTZ

1 Cf MTZ

2 We compute

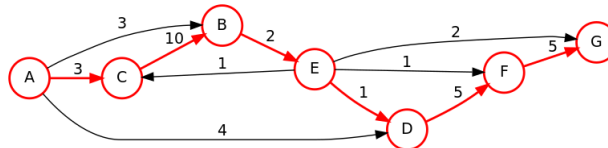


Figure 3: MTZ G2

*Presolve eliminates 1 constraint.*  
*Adjusted problem:*  
*32 variables:*  
     *25 binary variables*  
     *7 integer variables*  
*26 constraints, all linear; 100 nonzeros*  
     *9 equality constraints*  
     *17 inequality constraints*  
*1 linear objective; 11 nonzeros.*  
  
*LP\_SOLVE 4.0.1.0: optimal, objective 26*  
*94 simplex iterations*  
*9 branch & bound nodes: depth 4*  
*totallength = 26*

### 3 Cf MTZ

*Presolve eliminates 1 constraint.*  
*Adjusted problem:*  
*32 variables:*  
     *7 integer variables*  
     *25 linear variables*  
*26 constraints, all linear; 100 nonzeros*  
     *9 equality constraints*  
     *17 inequality constraints*  
*1 linear objective; 11 nonzeros.*  
  
*LP\_SOLVE 4.0.1.0: optimal, objective 26.625*  
*29 simplex iterations*  
*totallength = 26.625*

Non-integer solution, the matrix is not TUM anymore. Therefore we have no guarantee the solution will be integer. On the plus side, it is much faster.

**5** There should be  $|E| + 2|V|$  binary variables (arc matrix) and  $|V|$  integer variables (indexes). Moreover, there should be  $|V| + 2$  equalities and  $|E| + |V|$  inequalities.

This fits with what AMPL computes, though 1 inequalities seems to be eliminated by presolve.

## 1.2 SF

### 1 Cf SF

*Presolve eliminates 1 constraint.*  
*Adjusted problem:*  
*57 variables:*  
     *32 binary variables*  
     *25 linear variables*

*LP\_SOLVE 4.0.1.0: optimal, objective 26  
120 simplex iterations  
7 branch & bound nodes: depth 4  
totallength = 26*

**3** There should be  $|E| + 3|V|$  binary variables (arc matrix and indexes), and  $|E| + 2|V|$  real variables (flow). Moreover, there should be  $3|V| + 5$  equalities and  $|E| + 3|V|$  inequalities.

4 MTZ seems more interesting in terms of simplex iterations but causes more branch&bound (cf Figures 7,8,9)

The graph consists of 10 nodes labeled 0 through 9. The edges and their weights are as follows:

- 0 → 1 (red, weight 7)
- 0 → 2 (black, weight 8)
- 0 → 4 (black, weight 4)
- 0 → 5 (black, weight 2)
- 0 → 8 (black, weight 9)
- 1 → 4 (black, weight 1)
- 1 → 5 (black, weight 0)
- 2 → 0 (black, weight 9)
- 2 → 4 (black, weight 4)
- 2 → 5 (black, weight 6)
- 2 → 8 (black, weight 4)
- 2 → 9 (black, weight 7)
- 4 → 0 (black, weight 4)
- 4 → 1 (black, weight 1)
- 4 → 5 (black, weight 7)
- 4 → 8 (black, weight 9)
- 4 → 9 (black, weight 9)
- 5 → 0 (black, weight 8)
- 5 → 1 (black, weight 0)
- 5 → 2 (black, weight 6)
- 5 → 8 (black, weight 6)
- 5 → 9 (black, weight 7)
- 8 → 0 (black, weight 8)
- 8 → 2 (black, weight 9)
- 8 → 5 (black, weight 2)
- 8 → 9 (black, weight 7)
- 9 → 0 (black, weight 9)
- 9 → 2 (black, weight 7)
- 9 → 5 (black, weight 7)
- 6 → 2 (black, weight 10)
- 6 → 3 (red, weight 9)
- 3 → 6 (black, weight 9)

The red path highlighted is: 0 → 1 → 5 → 2 → 6 → 3.

4

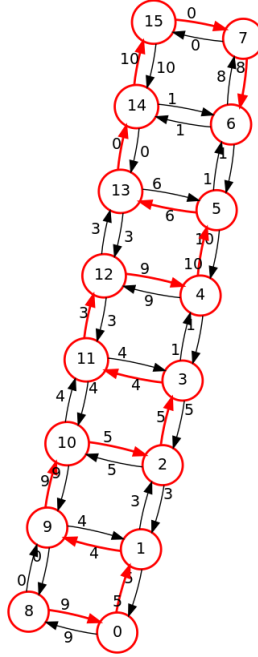


Figure 5: ladder\_8

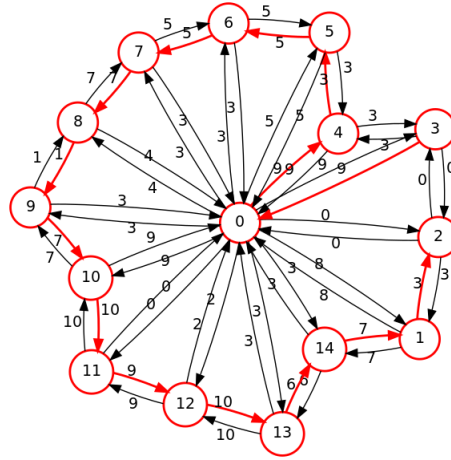


Figure 6: wheel\_15

## 2 GST

1 cf GST

2 figure

*Presolve eliminates 5 constraints and 4 variables.  
Adjusted problem:  
39 variables:*

fast_10	MTZ	SF	GST
Variables	54	118	98
Constraints	56	99	86
Simplex iterations	5289	11419	10832
Branch&Bound(depth)	451(21)	455(34)	377(33)

Figure 7: Results on fast\_10

ladder_8	MTZ	SF	GST
Variables	92	168	136
Constraints	78	145	126
Simplex iterations	38612	64508	98867
Branch&Bound(depth)	2547(32)	1933(42)	2819(42)

Figure 8: Results on ladder\_8

*11 binary variables*  
*28 linear variables*  
*43 constraints, all linear; 154 nonzeros*  
*14 equality constraints*  
*29 inequality constraints*  
*1 linear objective; 11 nonzeros.*  
  
*LP\_SOLVE 4.0.1.0: optimal, objective 26*  
*80 simplex iterations*  
*5 branch & bound nodes: depth 3*  
*totallength = 26*

**3** There should be  $|E|$  binary variables (arc matrix) and  $|E| + 3|V|$  integer variables (indexes and flow). Moreover, there should be  $2|V| + 2$  equalities and  $|E| + 3|V|$  inequalities.

This fits with what AMPL computes, though 2 equalities, 3 inequalities and 4 real variables seems to be eliminated by presolve.

**4** We experimentally verify that relaxing  $i \leq v$  does not change results and that  $i \leq v$  always take integer values in the relaxed case. It seems a bit faster to take relaxed values.

**5** GST seems inferior to either MTZ or GST on every aspect

Wheel_15	MTZ	SF	GST
Variables	101	187	157
Constraints	88	151	133
Simplex iterations	24761	19752	84939
Branch&Bound(depth)	1153(32)	413(25)	1031(44)

Figure 9: Results on wheel\_15

### 3 Data dump for binary/relaxed i s v comparison

binary i s v on g2

*Presolve eliminates 5 constraints and 4 variables. Adjusted problem: 39 variables: 28 binary variables 11 linear variables 43 constraints, all linear; 154 nonzeros 14 equality constraints 29 inequality constraints 1 linear objective; 11 nonzeros.*

*LP\_SOLVE 4.0.1.0: optimal, objective 26 78 simplex iterations 5 branch & bound nodes: depth 3 totallength = 26*

relaxed on fast

*98 variables: 34 binary variables 64 linear variables 86 constraints, all linear; 378 nonzeros 22 equality constraints 64 inequality constraints 1 linear objective; 32 nonzeros.*

*LP\_SOLVE 4.0.1.0: optimal, objective 63 10832 simplex iterations 377 branch & bound nodes: depth 33 totallength = 63*

binary on fast

*98 variables: 64 binary variables 34 linear variables 86 constraints, all linear; 378 nonzeros 22 equality constraints 64 inequality constraints 1 linear objective; 32 nonzeros.*

*LP\_SOLVE 4.0.1.0: optimal, objective 63 10969 simplex iterations 403 branch & bound nodes: depth 34 totallength = 63*

relaxed on ladder

*136 variables: 44 binary variables 92 linear variables 126 constraints, all linear; 532 nonzeros 34 equality constraints 92 inequality constraints 1 linear objective; 38 nonzeros.*

*LP\_SOLVE 4.0.1.0: optimal, objective 87 98867 simplex iterations 2819 branch & bound nodes: depth 42 totallength = 87*

binary on ladder

*136 variables:  
92 binary variables  
44 linear variables  
126 constraints, all linear; 532 nonzeros  
34 equality constraints  
92 inequality constraints  
1 linear objective; 38 nonzeros.*

*LP\_SOLVE 4.0.1.0: optimal, objective 87  
104125 simplex iterations  
3081 branch & bound nodes: depth 43  
totallength = 87*

relaxed on wheel

*157 variables: 56 binary variables 101 linear variables 133 constraints, all*

*linear; 602 nonzeros 32 equality constraints 101 inequality constraints 1  
linear objective; 50 nonzeros.*

*LP-SOLVE 4.0.1.0: optimal, objective 91 84939 simplex iterations 1031 branch &  
bound nodes: depth 44 totallength = 91*

binary on wheel

*157 variables: 101 binary variables 56 linear variables 133 constraints, all  
linear; 602 nonzeros 32 equality constraints 101 inequality constraints 1  
linear objective; 50 nonzeros.*

*LP-SOLVE 4.0.1.0: optimal, objective 91 62932 simplex iterations 839 branch &  
bound nodes: depth 44 totallength = 91*