Coding Theory Homework

Week 6 (Section 2.3 - 2.4)

Exercise 2.3.4 (c, f)

> Test each of the following sets for linear independence. If the set is lineary dependent, extract from *S* a largest lineary independent subset.

> C. *S* = {1101, 0111, 1100, 0011}

a(1101) + b(0111) + c(1100) + d(0011) = 0000

```
a + c = 0
a + b + c = 0
b + d = 0
a + b + d = 0

a + b + d = 0
    a + 0 = 0
        a = 0

a + c = 0
0 + c = 0
    c = 0

a + b + c = 0
0 + b + 0 = 0
        b = 0

b + d = 0
0 + d = 0
d = 0

a = b = c = d = 0
```

S is linearly independent

> F. *S* = {1100, 1010, 1001, 0101}

a(1100) + b(1010) + c(1001) + d(0101) = 0000 = 0000

```
a + b + c = 0
a + d = 0
b = 0
c + d = 0

a + b + c = 0
a + 0 + c = 0
a + c = 0
a = c = 1

a + d = 0
1 + d = 0
d = 1

b = 0
a = c = d = 1
```

S is linearly dependent

The first word dependent on is 0101.

The new subset *S* is {1100, 1010, 1001}

a(1100) + b(1010) + c(1001) = 0000

```
a + b + c = 0
a = 0
b = 0
c = 0

a = b = c = 0
```

The new subset *S* is linearly independent.

Exercise 2.3.7 (b, d)

> For each set in Exercise 2.2.3 find a basis *B* for the code *C* = <*S*> and a basis B^{1} for the dual code C^{1}

> b. *S* = {1010, 0101, 1111}

> > $C = \langle S \rangle = \{0000, 1010, 0101, 1111\}$
> > $C^1 = S^1 = \{0000, 0101, 1010, 1111\}$

a(1010) + b(0101) + c(1111) = 0000

```
a + c = 0
b + c = 0

a = b = c = 1
```

S is linearly dependent

The first word dependent is 1111

The new subset S^1 is {1010, 0101}

a(1010) + b(0101) = 0000

```
a = 0
b = 0

a = b = 0
```

Subset S^1 is linearly independent

```
1010 = 1·x + 0·y + 1·z + 0·w
     = x + z = 0

0101 = 0·x + 1·y + 0·z + 1+w
     = y + w

1111 = 1·x + 1·y + 1·z + 1+w
     = x + y + z + w
```

$B^1 = S^1 = \{0000, 0101, 1010, 1111\}$

> d. $S = \{1000, 0100, 0010, 0001\}$
>
> > $C = \langle S \rangle = \{0000, 1000, 0100, 0010, 0001, 1100, 1010, 1001, 0110, 0101, 0011, 1110, 1101, 1011, 0111, 1111\}$
> > $C^1 = S^1 = \{0000\}$

a(1000) + b(0100) + c(0010) + d(0001) = 0000 = 0000

```
a = 0
b = 0
c = 0
d = 0

a = b = c = d = 0
```

S is linearly independent

```
1000 = 1·x + 0·y + 0·z + 0·w
     = x = 0

0100 = 0·x + 1·y + 0·z + 0·w
     = x = 0

0010 = 0·x + 0·y + 1·z + 0·w
     = x = 0

0001 = 0·x + 0·y + 0·z + 1·w
     = x = 0
```

$B^1 = S^1$ = {0000}

Exercise 2.3.8 (b, d)

> Find the dimensions of each code *C* = <*S*> and its dual code C^1 in Exercise 2.2.3

> b. *S* = {1010, 0101, 1111}

The dimension of *S* is 3.

> d. *S* = {1000, 0100, 0010, 0001}

the dimension of *S* is 4.

Exercise 2.3.10 (b)

> Write each of the following words in K^4 as a unique linear combination of the words in the basis {1000, 1100, 1110, 1111}

> b. 1010

a(1000) + b(1100) + c(1110) + d(1111) = 1010

```
? = 1
a = 0
a + b = 0
a + b + c = 0
```

Exercise 2.3.12 (b)

> Extend {101010, 010101} to a basis for K^6

101010, 010101, 000001, 000010, 000100, 001000, 010000, 100000

Exercise 2.3.15 (b, d)

> Check your answers in Exercise 2.3.8 with the equation in Theorem 2.3.14

Exercise 2.3.17

> Let *S* be a subset of K^s and assume that {11110000, 00001111, 10000001} is a basis for C^1. Find the number of words in *C* = <*S*>

Exercise 2.3.18

> Theorem 2.3.14 also holds in R^n. In R^n every vector can be written uniquely as the sum of a vector in <*S*> and a vector in S^1, and the zero vector is the only vector <*S*> and *S* have in commmon. (For example in R^3 take <*S*> to be the xy-plane and S^1 the axis). Use *S* = {000, 101} in K^3 to show that this is not the case in general in K^n

Exercise 2.3.23 (b)

> Find the number of different bases for $K^{n</sup}$ for each code *C* = <*S*> for

> b. *S* = {1010, 0101, 1111}

Exercise 2.4.1 (b, c)

> Find the product of each pair of the following matrices whenever the product is defined.

```
| 0101 |   | 110110 |   | 110000 |
| 1001 | x | 011011 | = | 011101 |
| 1100 |   | 101101 |   | 101101 |
           | 101011 |
```

Exercise 2.4.2

> Find 2 x 2 matrices A and B over K such that AB != BA

```
| 01 |   | 10 |   | 00 |
| 11 | x | 00 | = | 10 |
```

```
| 10 |   | 01 |   | 01 |
| 00 | x | 11 | = | 00 |
```

Exercise 2.4.3

> Find 2 x 2 matrices A and B over K, both different from the zero matrix 0, such that AB = 0

```
| 01 |   | 10 |   | 00 |
| 00 | x | 00 | = | 00 |
```

Exercise 2.4.4

> Find 2 x 2 matrices A, B and C over K such that AB = AC but B != C

```
| 11 |   | 10 |   | 11 |
| 01 | x | 01 | = | 01 |
```

```
| 11 |   | 01 |   | 11 |
| 01 | x | 01 | = | 01 |
```

Exercise 2.4.6 (b)

```
| 0101 |
| 1001 |
| 1100 |

0101 + 1001 = 1100
0101 + 1100 = 1001
1001 + 0101 = 1100
1001 + 1100 = 0101
1100 + 0101 = 1001
1100 + 1001 = 0101
```