

Coding Theory Homework

Week 10 (Section 2.11 - 2.12)

Exercise 2.11.2(b)

Let C be the code of Example 2.10.5. Use the procedure for CMLD just outlined to decode each of the following received words.

(b) $w = 001001$

001001 occurs in the second coset of the the eight cosets listed in Example 2.10.5. this coset consists of:

```
100000
000110
110011
101111
010101
001001
111100
011010
```

The word of the least weight in this cosets is 100000

$$v = 001001 + 100000 = 101001$$

We can conclude that 101001 is a most likely codeword sent.

Exercise 2.11.8(b)

Construct an SDA assuming *IMLD* for each of the codes in Exercise 2.10.6

(b) $C = \{ 0000, 1010, 1101, 0111 \}$

```
| 0000 |  
| 1010 |  
| 1101 |  
| 0111 |
```

Move row 2 to position 1

```
| 1010 |  
| 0000 |  
| 1101 |  
| 0111 |
```

Add row 1 to row 3

```
| 1010 |  
| 0000 |  
| 0111 |  
| 0111 |
```

Move row 3 to position 2

```
| 1010 |  
| 0111 |  
| 0000 |  
| 0111 |
```

Add row 2 to row 4

```
| 1010 |  
| 0111 |  
| 0000 |  
| 0000 |
```

RREF

```
| 1010 |  
| 0111 |
```

Matrix G

```
| 10 | 10 |  
| 01 | 11 |
```

Matrix X

```
| 10 |
| 11 |
```

Matrix H

```
| 10 |
| 11 |
| 10 |
| 01 |
```

Cosets

```
0000    1000    0100    0001
1010    0010    1110    1011
1101    0101    1001    1100
0111    1111    0011    0110
```

SDA

Error patterns	Syndrome uH
0000	00
1000*	10*
0100	11
0001	01

Exercise 2.11.17(b)

Repeat the decoding in Exercise 2.11.2 using the SDA in Example 2.11.7

(b) $w = 001001$

SDA

Error patterns	Syndrome uH
000000	000

100000	110
010000	011
001000	111
000100	100
000010	010
000001	001
000101	101

Matrix H

```
| 110 |
| 011 |
| 111 |
| 100 |
| 010 |
| 001 |
```

$$wH = 110$$

$$\text{coset leader } u = 100000$$

$$v = 001001 + 100000 = 101001$$

Exercise 2.11.19

only for code from 2.11.8(b) using words w from 2.11.19(a)

For each of the following codes, use the SDA to decode the given received words.
(The SDA's for these codes were constructed in Exercises 2.11.8 and 2.11.9)

(b) $C = \{0000, 1010, 1101, 0111\}$

SDA

Error patterns	Syndrome uH
0000	00

1000*	10*
0100	11
0001	01

Matrix H

| 10 |
| 11 |
| 10 |
| 01 |

(i) $w = 1110$

$$wH = 11$$

$$\text{coset leader } u = 0100$$

$$v = 1110 + 0100 = 1010$$

(ii) $w = 1001$

$$wH = 11$$

$$\text{coset leader } u = 0100$$

$$v = 1001 + 0100 = 1101$$

(iii) $w = 0101$

$$wH = 10$$

Syndrome $uH = 10$ has an astrisk so we should ask for retransmission.

~~$$\text{coset leader } u = 1000$$~~

~~$$v = 0101 + 1000 = 1101$$~~

☞ Exercise 2.12.2 (only for 2.10.6(b))

Calculate $\Phi_p(C)$ for each of the codes in Exercises 2.10.6, 2.10.7, 2.10.8.

(b) $C = \{ 0000, 1010, 1101, 0111 \}$

$$\text{coset leaders} = \{ 0000, 1000, 0100, 0001 \}$$

one coset leader of weight 0 and three of weight 1.

$$\Phi_p(C) = p^3 + 3p^2(1 - p)$$