Coding Theory Homework

Week 5 (Section 2.1 - 2.2)

Exercise 2.1.1b,c,g,h

> Determine which of the following codes are linear

> b. *C* = {000, 001, 010, 011}

000 + 001 = 001
000 + 010 = 010
000 + 011 = 011
001 + 010 = 011
001 + 011 = 010
010 + 011 = 001

C is linear.

> c. *C* = {0000, 0001, 1110}

0000 + 0001 = 0001
0000 + 1110 = 1110
0001 + 1110 = 1111 -- **Not in C**

C is not linear.

> g. *C* = {00000, 11110, 01111, 10001}

00000 + 11110 = 11110
00000 + 01111 = 01111
00000 + 10001 = 10001
11110 + 01111 = 10001
11110 + 10001 = 01111
01111 + 10001 = 11110

C is linear.

> h. *C* = {000000, 101010, 010101, 111111}

000000 + 101010 = 101010
000000 + 010101 = 010101
000000 + 111111 = 111111
101010 + 010101 = 111111
101010 + 111111 = 010101
010101 + 111111 = 101010

C is linear.

Exercise 2.1.3b,c,g,h

> Find the distance of each linear code in Exercise 2.1.1. Check answers with Exercise 1.11.12

> b. *C* = {000, 001, 010, 011}

The distance of *C* is d = 1.

> g. *C* = {00000, 11110, 01111, 10001}

The distance of *C* is d = 2.

> h. *C* = {000000, 101010, 010101, 111111}

The distance of *C* is d = 3.

Exercise 2.1.4

> Proof that the distance of a linear code is the weight of the nonzero codeword of least weight

C = {000000, 101010, 010101, 111111}

d(000000, 101010) = 3
d(000000, 010101) = 3
d(000000, 111111) = 6
d(101010, 010101) = 6
d(101010, 111111) = 3
d(010101, 111111) = 3

The distance of *C* is d = 3

Exercise 2.2.3b,d

> For each of the following sets *S*, list the elements of linear code for <S>

> b. *S* = {1010, 0101, 1111}

0000

1010

0101

1111

1010 + 0101 = 1111

1010 + 1111 = 0101

0101 + 1111 = 1010

1010 + 0101 + 1111 = 0000

C = <*S*> = {0000, 1010, 0101, 1111}

> d. *S* = {1000, 0100, 0010, 0001}

0000

1000

0100

0010

0001

1000 + 0100 = 1100

1000 + 0010 = 1010

1000 + 0001 = 1001

0100 + 0010 = 0110

0100 + 0001 = 0101

0010 + 0001 = 0011

1000 + 0100 + 0010 = 1110

1000 + 0100 + 0001 = 1101

0100 + 0010 + 0001 = 0111

1000 + 0100 + 0010 + 0001 = 1111

C = <*S*> = {0000, 1000, 0100, 0010, 0001, 1100, 1010, 1001, 0110, 0101, 0011, 1110, 1101, 0111, 1111}

Exercise 2.2.4

> Construct examples in K^S of each of the following rules

> a. *u · (v + w)* = *u · v* + *u · w*

u = 1110
v = 0010 *w* = 1000

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1110 · (0010 + 1000) = 1110 · 0010 + 1110 · 1000
                     = 0010        + 1000
                     = 1010
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> b. *a(v · w)* = *(av) · w* = *v · (aw)*

Exercise 2.2.5

Exercise 2.2.7b,d

> Find the dual code C^1 for each of the codes *C* = <*S*> in Exercise 2.2.3

> b. *S* = {1010, 0101, 1111}

C = <*S*> = {0000, 1010, 0101, 1111}

C^1 = _S^1 = {0000}

> d. *S* = {1000, 0100, 0010, 0001}

C = <*S*> = {0000, 1000, 0100, 0010, 0001, 1100, 1010, 1001, 0110, 0101, 0011, 1110, 1101, 0111, 1111}

C^1 = _S^1 = {0000}

Exercise 2.2.8

> Find an example of a nonzero word such that *v · v* = 0. What can say about the weight of such a word.

Such a word does not exist.