Coding Theory Homework

Week 2 (Section 1.5 - 1.8)

Exercise 1.6.2 (d, e, g)

> Calculate $\phi_{.97}$(v, w) for each fo the following pairs of *v* and *w*.
>
> d. v = 00000, w = 00000

$\phi_{.97}$(00000, 00000) = ($.97^5$) = 0.8587340257

> e. v = 1011010, w = 0000010

$\phi_{.97}$(1011010, 0000010) = ($.97^4$)*($.03^3$) = 0.00002390290587

> g. v = 111101, w = 000010

$\phi_{.97}$(111101, 000010) = ($.97^0$)*($.03^6$) = 0.000000000729

Exercise 1.6.9

> Which of the codewords 110110, 110101, 000111, 101000 is most likely to have been sent if w = 011001 is received.

v	d
110110	5
110101	4
000111	4
101000	3 <= smallest d

101000 has the smallest number of disagreements with w = 011001 and is thus the codeword that was most likely sent.

Exercise 1.6.10

> In Theorem 1.6.3 we assume that 1/2 < p < 1. What would change in the statement of Theorem 1.6.3 if we replace the assumption with
>
> a. 0 < p < 1/2
> b. p = 1/2

Exercise 1.7.1

> Show that if v is a word in K^n then v + v = 0

because both 0 + 0 = 0 and 1 + 1 = 0 in K. Any digit within v when added to itself will either be 0 + 0 or 1 + 1 both resulting in 0. This means that when v is added to itself all digits will have the result of 0 and thus v + v = 0.

Exercise 1.7.2

> Show that if v and w are words in K^n and v + w = 0 then v = w

As in the above exercise, if v and w are the same then the addition will result in 0. If any digit within v and w is not the same then the addition of those digits will be either 0 + 1 = 1, or 1 + 0 = 1. This means that if any digit within v or w does not match that the result will never be 0.

Exercise 1.8.1

> Compute the weight of each of the following words and the distance between each pair of them: v_1 = 1001010, v_2 = 0110101, v_3 = 0011110, $v_4 = v_2 + v_3$

	codeword	weight	distance	v_1	v_2	v_3	v_4
v_1	1001010	3		0	7	3	3
v_2	0110101	4		7	0	4	4
v_3	0011110	4		3	4	0	3
v_4	0110101 + 0011110 = 0101011	4		3	4	3	0

Exercise 1.8.2

> Let u = 01011, v = 11010, w = 01100. Compare each of the following pairs in quantities.
>
> a. wt(v + w), and wt(v) + wt(w)

wt(11010 + 01100) = wt(10110) = 3
wt(11010) + wt(01100) = 3 + 2 = 5

> b. d(v,w), and d(v,u) + d(u,w)

d(v,w) = d(11010, 01100) = 2
d(v,u) + d(u,w) = d(11010, 01011) + d(01011, 01100) = 2 + 3 = 5

Exercise 1.8.3

> Construct an example K^5 of each of the eleven rules above.
>
> 1. 0 <= wt(v) <= n

wt(00000) = 0
wt(10101) = 3
wt(11111) = 5

> 1. wt(0) = 0

wt(00000) = 0

> 1. if wt(v) = 0, then v = 0

wt(00000) = 0
wt(00001) = 1

> 1. 0 <= d(v,w) <= n

d(00000, 11111) = 5
d(10101, 01010) = 5
d(01101, 01101) = 0

> 1. d(v,v) = 0

d(00000, 00000) = 0
d(01111, 01111) = 0
d(10111, 10111) = 0

> 1. if d(v,w) = 0, then v = w

d(01010, 01010) = 0
d(10111, 10110) = 1

> 1. d(v,w) = d(w, v)

d(01101, 11011) = 3
d(11011, 01101) = 3

> 1. wt(v + w) <= wt(v) + wt(w)

wt(11100 + 11110) = wt(00010) = 1
wt(11100) + wt(11110) = 3 + 4 = 7

> 1. d(v, w) <= d(v, u) + d(u, w)

d(10101, 11001) = 2
d(10101, 00101) + d(00101, 11001) = 1 + 3 = 4

> 1. wt(av) = a * wt(v)
>
> 2. d(av,aw) = a * d(v,w)