Coding Theory Homework

Week 3 (Section 1.5 - 1.8)

Exercise 1.9.5

> |M| = 2, n = 3, and *C* = {001, 101}. If v = 001 is sent, when will IMLD conclude this correctly, and when will IMLD incorrectly conclude that 101 was sent.

w	001 + w	101 + w	v
000	001*	101	001
001	000*	100	001
010	011*	111	001
011	010*	110	001
100	101	001*	101
101	100	000*	101
110	111	011*	101
111	110	010*	101

Exercise 1.9.6

> Let |M| = 3 and n = 3. For each word w in K^3 that could be received, find the word v in de code C = {000, 001, 110} which IMLD will conclude was sent.

w	000 + w	001 + w	110 + w	v
000	000*	001	110	000
001	001	000*	111	001
010	010	011	100	---
011	011	010*	101	001
100	100	101	010	---
101	101	100*	011	001
110	110	111	000*	110
111	111	110	001*	110

Exercise 1.9.7b

> Construct the IMLD table for each of the following codes.
>
> \b. C = {000, 001, 010, 011}

w	000 + w	001 + w	010 + w	011 + w	v
000	000*	001	010	011	000
001	001	000*	011	010	001
010	010	011	000*	001	010
011	011	010	001	000*	011
100	100*	101	110	111	000
101	101	100*	111	110	001
110	110	111	100*	101	010
111	111	110	101	100*	011

Exercise 1.10.2b

> Suppose p = .90, |M| = 2, n = 3, C = {001, 101}, as in Exercise 1.9.5
>
> \b. if v = 101 is sent, find the probability that IMLD will correctly conclude this after one transaction.

$L(101) = \{100, 101, 110, 111\}$

$\Phi_p(C, 101) = \Phi_p(101,100) + \Phi_p(101, 101) + \Phi_p(101, 110) + \Phi_p(101, 111)$
$= p^2(1 - p) + p^3 + p(1 - p)^2 + p^2(1 - p)$
$= p^3 + 2p^2(1 - p) + p(1 - p)^2$
$= .90$ (assuming p = .90)

Exercise 1.10.4

> Suppose p = .90 and C = {000, 001, 110}, as in Exercise 1.9.6. If v = 110 is sent, find the probability that IMLD will correctly conclude this, and the probability that IMLD will incorrectly conclude that 000 was sent.

$L(110) = \{110, 111\}$

$\Phi_p(C, 110) = \Phi_p(110, 110) + \Phi_p(110, 111)$
$= p^3 + p^2(1 - p)$
$= 0.81$ (assuming p = 0.90)

$\Phi_p(110, 000)$
$= p(1 - p)^2$
$= 0.009$ (assuming p = 0.90)

Exercise 1.10.5

> For each of the following codes *C* calculate $\Phi_p(C, v)$ for each v in C using p = .90. (The IMLD tables for these codes were constructed in Exercise 1.9.7)
>
> \b. C = {000, 001, 010, 011}

$L(000) = \{000, 100\}$

$\Phi_p(C, 000) = \Phi_p(000, 000) + \Phi_p(000, 100)$
$= p^3 + p^2(1 - p)$

= 0.81 (assuming p = 0.90)

$L(001) = \{001, 101\}$

$\Phi_p(C, 001) = \Phi_p(001, 001) + \Phi_p(001, 101)$
$= p^3 + p^2(1 - p)$
= 0.81 (assuming p = 0.90)

$L(010) = \{010, 110\}$

$\Phi_p(C, 010) = \Phi_p(010, 010) + \Phi_p(010, 110)$
$= p^3 + p^2(1 - p)$
= 0.81 (assuming p = 0.90)

$L(011) = \{011, 111\}$

$\Phi_p(C, 011) = \Phi_p(011, 011) + \Phi_p(011, 111)$
$= p^3 + p^2(1 - p)$
= 0.81 (assuming p = 0.90)