Coding Theory Homework

Week 11 (Section 3.1)

Exercise 3.1.2

> Illustrate Theorem 3.1.1 for v = `10110` and $t = 3$ by listing all words in K^5 of distance of at most 3 from v, and then check that Theorem 3.1.1 does give the correct number of such words.

Word	Distance from *v*	Word	Distance from *v*
00000	**3**	**10000**	**2**
00001	4	**10001**	**3**
00010	**2**	**10010**	**1**
00011	**3**	**10011**	**2**
00100	**2**	**10100**	**1**
00101	**3**	**10101**	**2**
00110	**1**	**10110**	**0**
00111	**2**	**10111**	**1**
01000	4	**11000**	**3**
01001	5	11001	4
01010	**3**	**11010**	**3**
01011	4	**11011**	**3**
01100	**3**	**11100**	**2**
01101	4	**11101**	**3**

01110	**2**	**11110**	**1**
01111	**3**	**11111**	**2**

26 words in k^5 of a distance of at most 3 from *v*.

Checking this with theorem 3.1.1 we get the same result.

$$\binom{5}{0}+\binom{5}{1}+\binom{5}{2}+\binom{5}{3}=26$$

Exercise 3.1.5 (b)

> Find an upper bound for the size or dimension of a linear code with the given values of *n* and *d*.

> b. $n = 7$, $d = 3$

$t = (d-1)/2 = 1$

$$|C| \le \frac{2^7}{\binom{7}{0}+\binom{7}{1}} == \frac{128}{8} == 16$$

Exercise 3.1.6 (b)

> Verify the Hamming bound for the linear code *C* with the given generator matrix.

> b.

$$G = \begin{pmatrix} 100111 \\ 010101 \\ 001011 \end{pmatrix}$$

Exercise 3.1.10

> Columns 2,3 and 5 of the generator matrix *G* below are linearly dependent. Find a codeword which has zeros in position 2,3 and 5.

$$G = \begin{pmatrix} 11001 \\ 01110 \\ 00101 \end{pmatrix} \times 111 = 10010$$

Exercise 3.1.11

> Show that if a *k* x *n* generator matrix has *k* linearly depedent columns then there is a nonzero codeword with zeroes in those *k* position.

Exercise 3.1.18 (b)

> For each part of Exercise 3.1.5, let *k* = 2_d_ and decide, if possible, whether or not a linear code with the given parameters exists. Find a lower and upper bound for the maximum number of codewords such a code can have, assuming that *k* is unrestricted.

> b. *n* = 7, *d* = 3

t = (d-1)/2 = 1
k = 2_d_ = 14

Lower bound:

$$|C| \geq \frac{2^{7-1}}{\binom{6}{0}+\binom{6}{1}} == \frac{2^6}{1+7} == \frac{64}{8} == 8$$

Upper bound:

$$|C| \leq \frac{2^{7}}{\binom{7}{0}+\binom{7}{1}} == \frac{128}{1+7} == \frac{128}{8} == 16$$

Exercise 3.1.19 (b)

> Find a lower and upper bound for the maximum number of codewords in a linear code of length *n* and distance *d* where:

> b. *n* = 15, *d* = 3

t = (d-1)/2 = 1

Lower bound:

$$|C| \geq \frac{2^{15-1}}{\binom{14}{0}+\binom{14}{1}} == \frac{2^{14}}{1+15} == \frac{16\,384}{16} == 1024$$

Upper bound:

$$|C| \leq \frac{2^{15}}{\binom{15}{0}+\binom{15}{1}} == \frac{32\,768}{1+16} == \frac{32\,768}{16} == 2048$$

Exercise 3.1.20

> Is it possible to have a linear code with parameters (8, 3, 5)