Coding Theory Homework

Week 14 (Section 3.8 - 3.9)

Exercise 3.8.5

> Find the generator matrix G(2, 3)

$$G(2,3) = \begin{bmatrix} G(2,2) & G(2,2) \\ 0 & G(1,2) \end{bmatrix}$$

$$G(2,2) = \begin{bmatrix} G(1,2) \\ 0001 \end{bmatrix}$$

$$G(1,2) = \begin{bmatrix} G(1,1) & G(1,1) \\ 0 & G(0,1) \end{bmatrix}$$

$$G(1,1) = \begin{bmatrix} G(0,1) \\ 01 \end{bmatrix}$$

$$G(0,1) = [11]$$

$$G(1,1) = \begin{bmatrix} 11 \\ 01 \end{bmatrix}$$

$$G(1,2) = \begin{bmatrix} 11 & 11 \\ 01 & 01 \\ 00 & 11 \end{bmatrix}$$

$$G(2,2) = \begin{bmatrix} 1111 \\ 0101 \\ 0011 \\ 0001 \end{bmatrix}$$

$$G(2,3) = \begin{bmatrix} 1111 & 1111 \\ 0101 & 0101 \\ 0011 & 0011 \\ 0001 & 0001 \\ 0000 & 1111 \\ 0000 & 0101 \\ 0000 & 0011 \end{bmatrix}$$

Exercise 3.8.8 (only for m=3)

> Show that Theorem 3.8.7 holds for the codes $RM(r, m), 1 \leq m \leq 4$, constructed in Examples 3.8.1, 3.8.3, 3.8.4 and Exercises 3.8.5, 3.8.6.
>
> 1. Length $n = 2^m$
> 2. Distance $d = 2^{m-r}$
> 3. Dimension $k = \sum_{i=0}^{r} \binom{m}{i}$
> 4. $RM(r-1, m)$ is contained in $RM(r, m), r > 0$
> 5. Dual code $RM(m-1-r, m), r < m$

> r = 1
>
> m = 3

$$RM(1,3) = \begin{bmatrix} 1111 & 1111 \\ 0101 & 0101 \\ 0011 & 0011 \\ 0000 & 1111 \end{bmatrix}$$

> 1. Length $n = 2^m$

$n = 2^3 = 8$

> 2. Distance $d = 2^{m-r}$

$d = 2^{3-1} = 2^2 = 4$

> 3. Dimension $k = \sum_{i=0}^{r} \binom{m}{i}$

$k = \sum_{i=0}^{1} \binom{3}{i} = 4$

> 4. $RM(r-1, m)$ is contained in $RM(r, m), r > 0$

$RM(0,3) = \begin{bmatrix} 11111111 \end{bmatrix}$ Which is the first word in $GM(1,3)$

> 5. Dual code $RM(m-1-r, m), r < m$

> r = 2
> m = 3

$$RM(2,3) = \begin{bmatrix} 1111 & 1111 \\ 0101 & 0101 \\ 0011 & 0011 \\ 0001 & 0001 \\ 0000 & 1111 \\ 0000 & 0101 \\ 0000 & 0011 \end{bmatrix}$$

> 1. Length $n = 2^m$

$n = 2^3 = 8$

> 2. Distance $d = 2^{m-r}$

$d = 2^{3-2} = 2^1 = 2$

> 3. Dimension $k = \sum_{i=0}^{r} \binom{m}{i}$

$k = \sum_{i=0}^{2} \binom{3}{i} = 7$

> 4. $RM(r-1, m)$ is contained in $RM(r, m), r > 0$

$$RM(1,3) = \begin{bmatrix} 1111 & 1111 \\ 0101 & 0101 \\ 0011 & 0011 \\ 0000 & 1111 \end{bmatrix}$$ Which are the first four word in $GM(2,3)$

Exercise 3.8.10 (b)

> Let $G(1,3)$ be the generator for the $RM(1,3)$ code, decode the following received words.

> b. $w = 0110\ 0111$

$$G(1,3) = \begin{bmatrix} 1111 & 1111 \\ 0101 & 0101 \\ 0011 & 0011 \\ 0000 & 1111 \end{bmatrix}$$

$$d(0110\ 0111, 1111\ 1111) = 3$$
$$d(0110\ 0111, 0101\ 0101) = 3$$
$$d(0110\ 0111, 0011\ 0011) = 3$$
$$d(0110\ 0111, 0000\ 1111) = 3$$

Exercise 3.9.6 (b)

> Decode the received words in Exercise 3.8.10 using Algorithm 3.9.4 (and Example 3.9.2)

> b. $w = 0110\ 0111$

$$\overline{w} = \begin{bmatrix} -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} w_1 &= \overline{w} H_3^1 = \begin{bmatrix} 0 & -2 & 0 & 2 & 0 & -2 & 2 & 0 \end{bmatrix} \\ w_2 &= w_1 H_3^2 = \begin{bmatrix} 0 & 0 & 0 & -4 & 2 & -2 & -2 & -2 \end{bmatrix} \\ w_3 &= w_2 H_3^3 = \begin{bmatrix} 2 & -2 & -2 & -6 & -2 & 2 & 2 & -2 \end{bmatrix} \end{aligned}$$

The largest component of w is -6 occuring in position 3. Since $v(3) = 110$ and $-6 < 0$ the presumed message is 0110