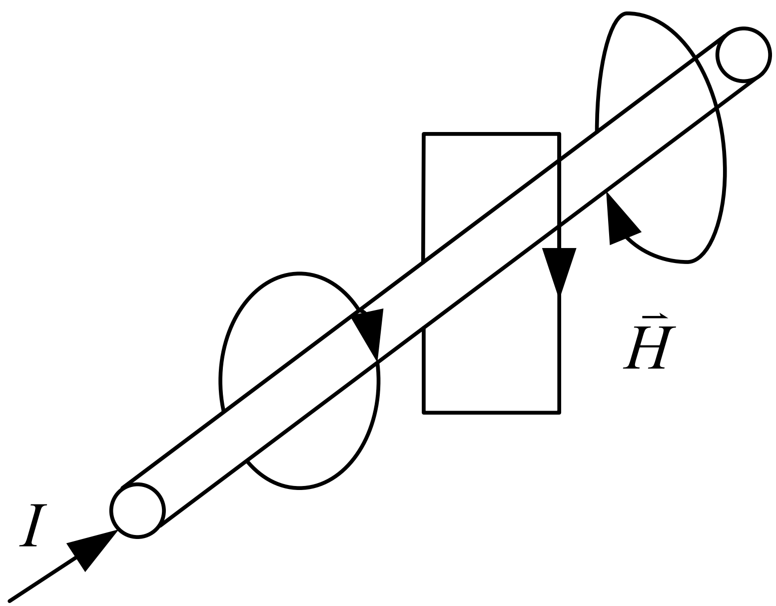
Magnetic Field Intensity (Cont.)

Ampere's Law (1)

กระแสไฟฟ้าเกิดจากความเข้มสนามแม่เหล็ก



$$I = \oint \vec{H} \cdot d\vec{L}$$

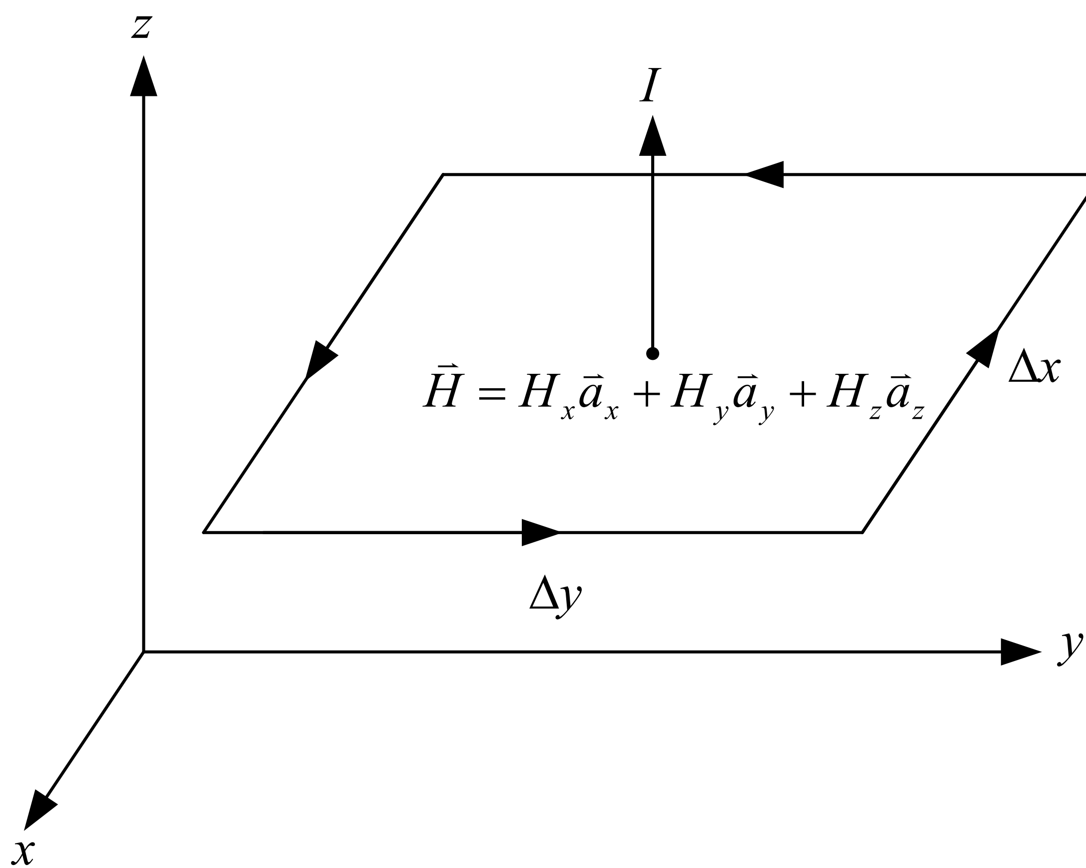
Ampere's Law (2)

กระแสที่เกิดจากความเข้มสนามแม่เหล็กในเส้นทางปิดวงกลม

$$
\begin{aligned}
I &= \oint \vec{H} \cdot d\vec{L} \\
&= \vec{H} \cdot 2\pi\rho\vec{a}_\phi \\
&= 2\pi\rho H_\phi
\end{aligned}
$$

$$
\begin{aligned}
H_\phi &= \frac{I}{2\pi\rho} \\
\vec{H} &= \frac{I}{2\pi\rho}\vec{a}_\phi
\end{aligned}
$$

Curl (1)



Curl (2)

$$I = \oint \vec{H} \cdot d\vec{L}$$

$$= \underbrace{\left(H_x - \frac{\Delta y}{2} \frac{\partial H_x}{\partial y} \right) \Delta x}_{\text{left}} + \underbrace{\left(H_y + \frac{\Delta x}{2} \frac{\partial H_y}{\partial x} \right) \Delta y}_{\text{front}} + \underbrace{\left(H_x + \frac{\Delta y}{2} \frac{\partial H_x}{\partial y} \right) (-\Delta x)}_{\text{right}} + \underbrace{\left(H_y - \frac{\Delta x}{2} \frac{\partial H_y}{\partial x} \right) (-\Delta y)}_{\text{back}}$$

$$= \Delta x \Delta y \frac{\partial H_y}{\partial x} - \Delta x \Delta y \frac{\partial H_x}{\partial y}$$

$$= \Delta x \Delta y \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right)$$

$$\frac{I}{\Delta x \Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$$

$$\vec{J}_z = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \vec{a}_z$$

Curl (3)

$$J_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$$

$$J_x = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$$

$$J_y = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}$$

$$\vec{J} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \vec{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \vec{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \vec{a}_z$$

$$= \nabla \times \vec{H}$$

Curl (4)

- Definition of Curl

$$\nabla \times \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \vec{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \vec{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \vec{a}_z$$

$$= \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) \vec{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right) \vec{a}_\phi + \frac{1}{\rho} \left(\frac{\partial \rho H_\phi}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi} \right) \vec{a}_z$$

$$= \frac{1}{r \sin\theta} \left(\frac{\partial \sin\theta H_\phi}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi} \right) \vec{a}_r + \frac{1}{r} \left(\frac{1}{\sin\theta} \frac{\partial H_r}{\partial \phi} - \frac{\partial r H_\phi}{\partial r} \right) \vec{a}_\theta$$

$$+ \frac{1}{r} \left(\frac{\partial r H_\theta}{\partial r} - \frac{\partial H_r}{\partial \theta} \right) \vec{a}_\phi$$

Curl (5)

- Curl in Determinant Form

$$\nabla \times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\vec{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\vec{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\vec{a}_z$$

Curl (6)

- Maxwell's Equation

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times \vec{E} = 0$$

Stokes' Theorem

จาก

$$\vec{J} = \nabla \times \vec{H}$$

$$I = \int_S \vec{J} \cdot d\vec{S}$$

$$I = \oint \vec{H} \cdot d\vec{L}$$

จะได้

$$\oint \vec{H} \cdot d\vec{L} = \int_S \left(\nabla \times \vec{H}\right) \cdot d\vec{S}$$

Magnetic Flux Density

$$\vec{B} = \mu_0 \vec{H}$$

$\vec{B}$: Magnetic flux density (T)

μ_0 : Permeability of free space $\approx 4\pi \times 10^{-7}$ H/m

Magnetic Flux

$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$

Φ : Magnetic flux (Wb)

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

$$\nabla \cdot \vec{B} = 0$$

Magnetic Potential

- Magnetic Potential

$$\vec{H} = -\nabla V_m \qquad \left(\vec{J} = 0\right)$$

$$V_m : \text{ Magnetic Potential } \; (\text{A})$$

- Magnetic Potential Difference

$$V_{m,AB} = -\int_B^A \vec{H} \cdot d\vec{L} \qquad \left(\text{Specificed Path}\right)$$

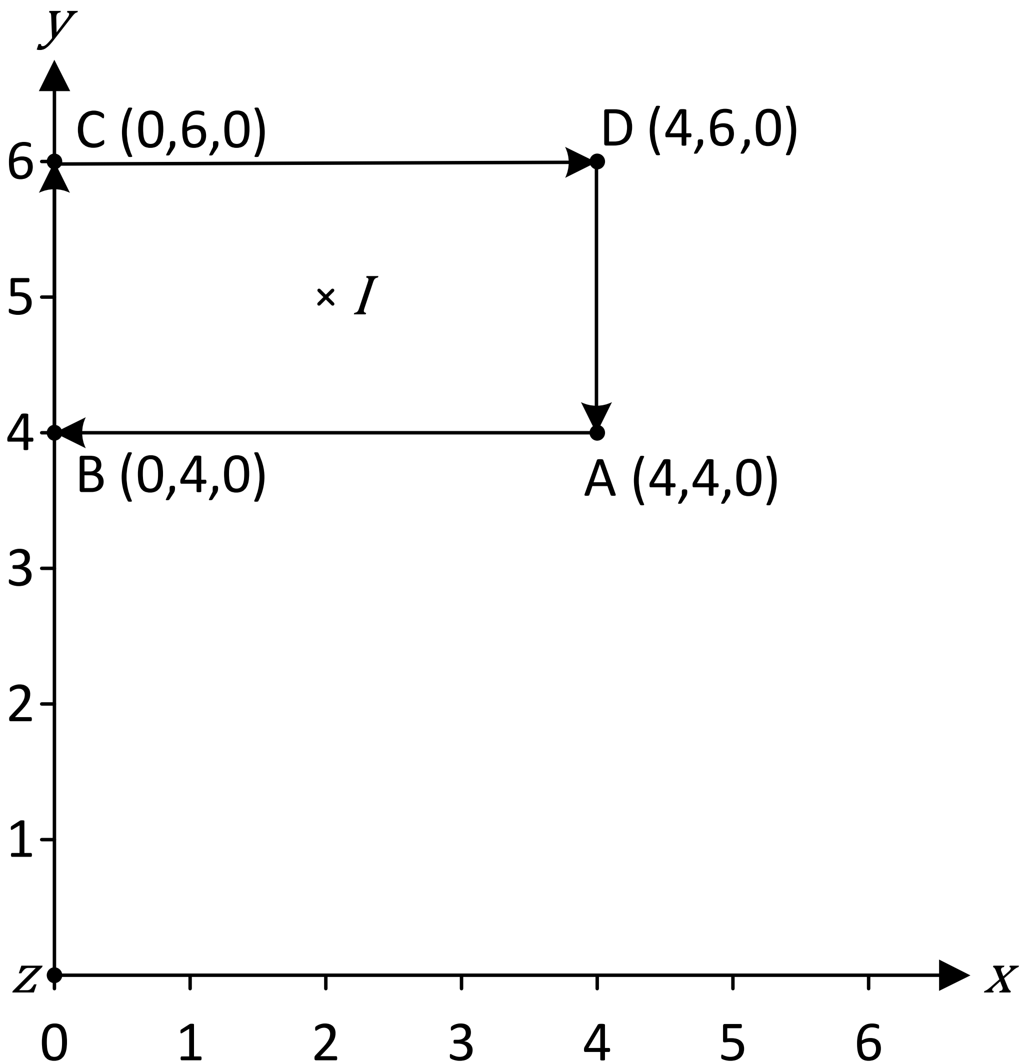
- Laplace's Equation

$$\nabla^2 V_m = 0 \qquad \left(\vec{J} = 0\right)$$

Example

กำหนดให้ $\vec{H} = xy\vec{a}_x + x\vec{a}_y$ A/m กำหนดเส้นทางปิดสี่เหลี่ยม A→B→ C→ D→A โดยที่จุด A, B, C และ D อยู่ที่พิกัด (4,4,0), (0,4,0), (0,6,0) และ (4,6,0) ตามลำดับ จงหา I โดยใช้ Ampere’s Law และ Curl

Solution (1)



Solution (2)

หา I โดยใช้ Ampere's Law

$$I = \oint \vec{H} \cdot d\vec{L}$$

$$= \int_A^B \vec{H}_x \cdot dx\vec{a}_x + \int_B^C \vec{H}_y \cdot dy\vec{a}_y + \int_C^D \vec{H}_x \cdot dx\vec{a}_x + \int_D^A \vec{H}_y \cdot dy\vec{a}_y$$

$$= \int_A^B xy\vec{a}_x \cdot dx\vec{a}_x + \int_B^C x\vec{a}_y \cdot dy\vec{a}_y + \int_C^D xy\vec{a}_x \cdot dx\vec{a}_x + \int_D^A x\vec{a}_y \cdot dy\vec{a}_y$$

$$= 4\int_4^0 xdx + 0\int_4^6 dy + 6\int_0^4 xdx + 4\int_6^4 dy$$

Solution (3)

$$= 4\int_4^0 x dx + 0\int_4^6 dy + 6\int_0^4 x dx + 4\int_6^4 dy$$

$$= 4\left.\frac{x^2}{2}\right|_{x=4}^{0} + 0 + 6\left.\frac{x^2}{2}\right|_{x=0}^{4} + 4\left.y\right|_{y=6}^{4}$$

$$= 4\left(0 - \frac{4^2}{2}\right) + 0 + 6\left(\frac{4^2}{2} - 0\right) + 4(4-6)$$

$$= -32 + 0 + 48 - 8$$

$$= 8\ \mathrm{A}$$

$I = 8\ \mathrm{A}$ ไหลในทิศทาง $-\vec{a}_z$

Solution (4)

หา I โดยใช้ Curl

$$\vec{J}_z = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\vec{a}_z$$

$$= \left(\frac{\partial x}{\partial x} - \frac{\partial xy}{\partial y}\right)\vec{a}_z$$

$$= (1 - x)\vec{a}_z \ \ \mathrm{A/m^2}$$

$$I = \int_S \vec{J} \cdot d\vec{S}$$

$$= \int_4^6 \int_0^4 (1 - x)\vec{a}_z \cdot dxdy\vec{a}_z$$

$$= \int_4^6 \int_0^4 (1 - x)dxdy$$

Solution (5)

$$= \int_4^6 \int_0^4 (1 - x)dxdy$$

$$= \left(x - \frac{x^2}{2}\right)\Bigg|_{x=0}^{4} \; y\Big|_{y=4}^{6}$$

$$= -4 \times 2$$

$$= -8 \text{ A}$$

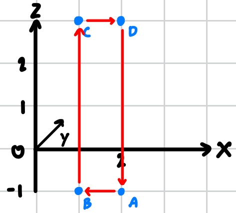
$I = 8 \text{ A}$ ไหลในทิศทาง $-\vec{a}_z$

Quiz 7

กำหนดให้ $\vec{H} = 3x^2z\vec{a}_x + 2xz\vec{a}_z$ A/m กำหนดเส้นทางปิดสี่เหลี่ยม A→B→ C→ D→A โดยที่จุด A, B, C และ D อยู่ที่พิกัด (2,0,-1), (1,0,-1), (1,0,3) และ (2,0,3) ตามลำดับ จงหา I โดยใช้ Ampere's Law และ Curl

$\vec{J}_y = (3x^2 - 2z)\vec{a}_y$ A / m^2 , $I = 20$ A ไหลในทิศทาง $\vec{a}_y$

A) 2, 0, -1
B) 1, 0, -1
C) 1, 0, 3
D) 2, 0, 3



$\vec{H} = 3x^2z\,\vec{a}_x + 2xz\,\vec{a}_z$ A/m

J_z $x-y$
J_x $y-z$
J_y $z-x$

$I = \oint \vec{H}\cdot d\vec{L}$

$= \int_A^B \vec{H}_x \cdot dx\vec{a}_x + \int_B^C \vec{H}_z\cdot dz\vec{a}_z + \int_C^D \vec{H}_x\cdot dx\vec{a}_x + \int_D^A \vec{H}_z\cdot dz\vec{a}_z$

$= \int_A^B 3x^2z\vec{a}_x\cdot dx\vec{a}_x + \int_B^C 2xz\vec{a}_z\cdot dz\vec{a}_z + \int_C^D 3x^2z\vec{a}_x\cdot dx\vec{a}_x + \int_D^A 2xz\vec{a}_z\cdot dx\vec{a}_z$

$= -1\int_2^1 3x^2dx + 1\int_{-1}^3 2z\,dz + 3\int_1^2 3x^2dx + 2\int_3^{-1} 2z\,dz$

$= 7 + 8 + 21 - 16$

$I = 20\,A$ ทิศ $\vec{a}_y$

$\vec{J}_y = \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\vec{a}_y$

$= \left(\frac{\partial 3x^2z}{\partial z} - \frac{\partial 2xz}{\partial x}\right)\vec{a}_y$

$\vec{J}_y = (3x^2 - 2z)\,\vec{a}_y$ A/m²

$I = \int_S \vec{J}\cdot d\vec{S}$

$= \int_{-1}^{3}\int_{1}^{2} (3x^2-2z)\vec{a}_y\cdot dx\,dz\,\vec{a}_y$

$= \int_{-1}^{3}\int_{1}^{2} (3x^2-2z)\,dx\,dz$

$= \int_{-1}^{3}\int_{1}^{2} 3x^2\,dx\,dz - \int_{-1}^{3}\int_{1}^{2} 2z\,dx\,dz$

$= 28 - 8$

$I = 20\,A$ ทิศ $\vec{a}_y$

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Assignment 7

กำหนดให้ $\vec{H} = 3y^2z^2\vec{a}_y + 4y^2z^3\vec{a}_z$ A/m กำหนดเส้นทางปิดสี่เหลี่ยม A→B→ C→ D→A โดยที่จุด A, B, C และ D อยู่ที่พิกัด (0,-2,-1), (0,1,-1), (0,1,-2) และ (0,-2,-2) ตามลำดับ จงหา I โดยใช้ Ampere’s Law และ Curl

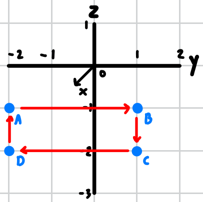
$\vec{J}_x = (8yz^3 - 6y^2z)\vec{a}_x$ A / m^2, $I = 72$ A ไหลในทิศทาง $\vec{a}_x$

A) 0,-2,-1

B) 0,1,-1

C) 0,1,-2

D) 0,-2,-2



$\vec{H} = 3y^2z^2\vec{a}_y + 4y^2z^3\vec{a}_z$ A/m

x y

y-z

$I = \oint \vec{H}\cdot d\vec{L}$

$= \int_A^B \vec{H}_y \cdot dy\vec{a}_y + \int_B^C \vec{H}_z \cdot dz\vec{a}_z + \int_C^D \vec{H}_y \cdot dy\vec{a}_y + \int_D^A \vec{H}_z \cdot dz\vec{a}_z$

$= \int_A^B 3y^2z^2 \cdot dy\vec{a}_y + \int_B^C 4y^2z^3 \cdot dz\vec{a}_z + \int_C^D 3y^2z^2 \cdot dy\vec{a}_y + \int_D^A 4y^2z^3 \cdot dz\vec{a}_z$

$= (-1)^2\int_{-2}^{1} 3y^2dy + 1^2\int_{-1}^{-2} 4z^3dz + (-2)^2\int_{1}^{-2} 3y^2dy + (-2)^2\int_{-2}^{-1} 4z^3dz$

$= 9+15-36-60$

$= -72$ A ทิศ $-\vec{a}_x$

$\underline{I = 72 \text{ A ทิศ } \vec{a}_x}$

$\vec{J}_x = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\vec{a}_x$

$= \left(\frac{\partial 4y^2z^3}{\partial y} - \frac{\partial 3y^2z^2}{\partial z}\right)\vec{a}_x$ A/m²

$\underline{\vec{J}_x = (8yz^3 - 6y^2z)\vec{a}_x \text{ A/m}^2}$

$I = \int_S \vec{J}\cdot d\vec{S}$

$= \int_{-2}^{-1}\int_{-2}^{1} (8yz^3 - 6y^2z)\vec{a}_x \cdot dydz\vec{a}_x$

$= \int_{-2}^{-1}\int_{-2}^{1} (8yz^3 - 6y^2z)\,dydz$

$= \int_{-2}^{-1}\int_{-2}^{1} 8yz^3\,dydz - \int_{-2}^{-1}\int_{-2}^{1} 6y^2z\,dydz$

$= 45 - (-27)$

$\underline{I = 72 \text{ A ทิศ } \vec{a}_x}$

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