Magnetic Field Intensity

Biot-Savart Law (1)

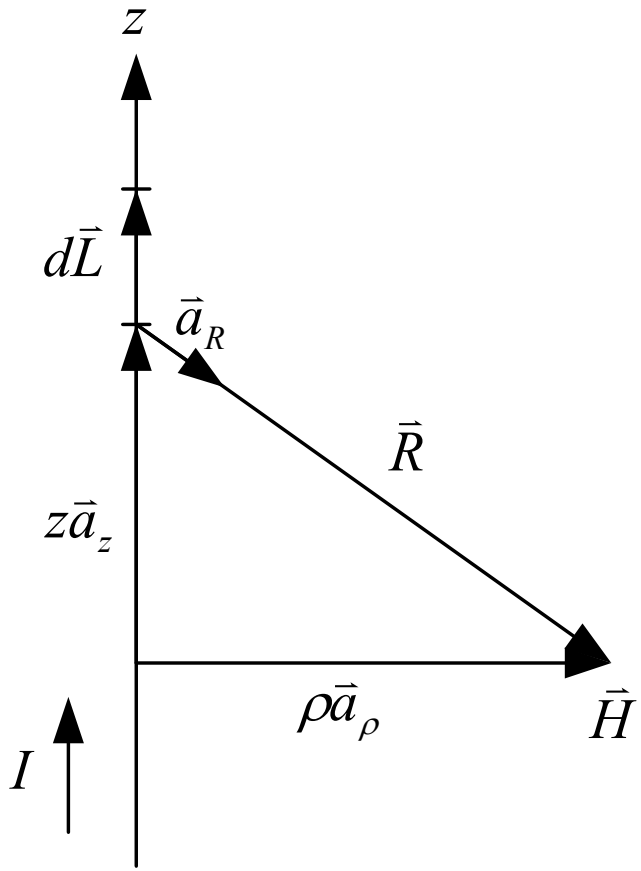
ความเข้มสนามแม่เหล็กเกิดจากกระแสไฟฟ้า

$$d\vec{H} = \frac{Id\vec{L} \times \vec{a}_R}{4\pi R^2}$$

$\vec{H}$: Magnetic Field Intensity (A/m)

Biot-Savart Law (2)

ความเข้มสนามแม่เหล็กที่เกิดจากกระแสบนเส้นลวดยาวอนันต์



$$d\vec{L} = dz\vec{a}_z$$

$$\vec{R} = \rho\vec{a}_\rho - z\vec{a}_z$$

$$R = (\rho^2 + z^2)^{1/2}$$

$$\vec{a}_R = \frac{1}{(\rho^2 + z^2)^{1/2}}(\rho\vec{a}_\rho - z\vec{a}_z)$$

Biot-Savart Law (3)

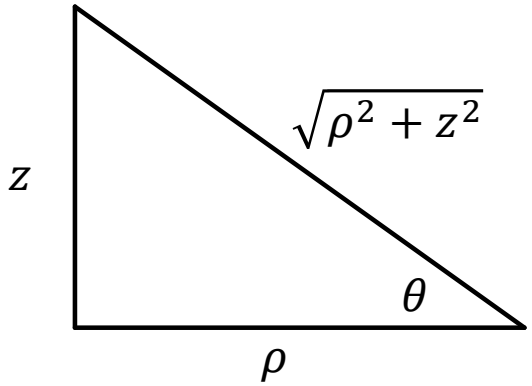
$$d\vec{H} = \frac{Id\vec{L} \times \vec{a}_R}{4\pi R^2}$$

$$= \frac{Idz\vec{a}_z}{4\pi(\rho^2 + z^2)} \times \frac{1}{(\rho^2 + z^2)^{1/2}} \left(\rho\vec{a}_\rho - z\vec{a}_z\right)$$

$$= \frac{I\rho dz}{4\pi(\rho^2 + z^2)^{3/2}} \vec{a}_\phi$$

$$\vec{H} = \frac{I}{4\pi} \int_{-\infty}^{\infty} \frac{\rho dz}{(\rho^2 + z^2)^{3/2}} \vec{a}_\phi$$

Biot-Savart Law (4)



$$z = \rho \tan\theta$$
$$dz = \rho \sec^2\theta \, d\theta$$
$$z \to -\infty \Longrightarrow \theta \to -\frac{\pi}{2}, \; z \to \infty \quad \Longrightarrow \theta \to \frac{\pi}{2}$$

$$\int_{-\infty}^{\infty} \frac{\rho dz}{(\rho^2 + z^2)^{3/2}} = \frac{1}{\rho^2} \int_{-\infty}^{\infty} \frac{\rho}{\sqrt{\rho^2 + z^2}} \frac{\rho}{\sqrt{\rho^2 + z^2}} \frac{\rho}{\sqrt{\rho^2 + z^2}} dz$$

$$= \frac{1}{\rho^2} \int_{-\pi/2}^{\pi/2} \cos^3\theta \, \rho \sec^2\theta \, d\theta$$

$$= \frac{1}{\rho} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= \frac{1}{\rho} \sin\theta \Big|_{\theta=-\pi/2}^{\pi/2}$$

$$= \frac{2}{\rho}$$

Biot-Savart Law (5)

$$\vec{H} = \frac{I}{4\pi}\int_{-\infty}^{\infty}\frac{\rho dz}{(\rho^2+z^2)^{3/2}}\vec{a}_\phi$$

$$= \frac{I}{4\pi}\left(\frac{2}{\rho}\right)\vec{a}_\phi$$

$$= \frac{I}{2\pi\rho}\vec{a}_\phi$$

$$\vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi \qquad (\mathrm{A/m})$$

Example

กำหนดให้ $I_1 = 2$ A ไหลในทิศทาง $\vec{a}_z$ อยู่ที่ตำแหน่ง $x = -3$ m, $y = 4$ m และ $I_2 = 1$ A ไหลในทิศทาง $-\vec{a}_z$ อยู่ที่ตำแหน่ง $x = -5$ m, $y = -3$ m จงหา $\vec{H}$ ที่พิกัด (2,-2,0)

Solution (1)

หา $\vec{H}_1$ ที่เกิดจาก $I_1 = 2$ A ได้

$$\vec{a}_{z'} = \vec{a}_z$$

$$\vec{\rho} = (2+3)\vec{a}_x + (-2-4)\vec{a}_y = 5\vec{a}_x - 6\vec{a}_y$$

$$\rho = \sqrt{5^2 + (-6)^2} = \sqrt{61}$$

$$\vec{a}_\rho = \frac{5}{\sqrt{61}}\vec{a}_x - \frac{6}{\sqrt{61}}\vec{a}_y$$

$$\vec{a}_\phi = \vec{a}_{z'} \times \vec{a}_\rho = \vec{a}_z \times \left(\frac{5}{\sqrt{61}}\vec{a}_x - \frac{6}{\sqrt{61}}\vec{a}_y \right) = \frac{6}{\sqrt{61}}\vec{a}_x + \frac{5}{\sqrt{61}}\vec{a}_y$$

$$\vec{H}_1 = \frac{I_1}{2\pi\rho}\vec{a}_\phi = \frac{2}{2\pi\sqrt{61}}\left(\frac{6}{\sqrt{61}}\vec{a}_x + \frac{5}{\sqrt{61}}\vec{a}_y \right)$$

$$= 31.31\vec{a}_x + 26.09\vec{a}_y \text{ mA/m } \#$$

Solution (2)

หา $\vec{H}_2$ ที่เกิดจาก $I_2 = 1$ A ได้

$$\vec{a}_{z'} = -\vec{a}_z$$

$$\vec{\rho} = (2+5)\vec{a}_x + (-2+3)\vec{a}_y = 7\vec{a}_x + \vec{a}_y$$

$$\rho = \sqrt{7^2 + 1^2} = \sqrt{50}$$

$$\vec{a}_\rho = \frac{7}{\sqrt{50}}\vec{a}_x + \frac{1}{\sqrt{50}}\vec{a}_y$$

$$\vec{a}_\phi = \vec{a}_{z'} \times \vec{a}_\rho = -\vec{a}_z \times \left(\frac{7}{\sqrt{50}}\vec{a}_x + \frac{1}{\sqrt{50}}\vec{a}_y \right) = \frac{1}{\sqrt{50}}\vec{a}_x - \frac{7}{\sqrt{50}}\vec{a}_y$$

$$\vec{H}_2 = \frac{I_2}{2\pi\rho}\vec{a}_\phi = \frac{2}{2\pi\sqrt{50}}\left(\frac{1}{\sqrt{50}}\vec{a}_x - \frac{7}{\sqrt{50}}\vec{a}_y \right)$$

$$= 3.18\vec{a}_x - 22.28\vec{a}_y \text{ mA/m } \#$$

Solution (3)

หา $\vec{H}$ ได้

$$\vec{H} = \vec{H}_1 + \vec{H}_2$$
$$= 34.49\vec{a}_x + 3.81\vec{a}_y \ \text{mA/m}$$

Quiz 6

กำหนดให้ $I_1 = 1$ A ไหลในทิศทาง $\vec{a}_x$ อยู่ที่ตำแหน่ง $y = 3$ m, $z = -5$ m และ $I_2 = 3$ A ไหลในทิศทาง $-\vec{a}_x$ อยู่ที่ตำแหน่ง $y = 5$ m, $z = 4$ m จงหา $\vec{H}$ ที่พิกัด (0,2,1)

$$\vec{H}_1 = -25.81\vec{a}_y - 4.30\vec{a}_z \text{ mA/m}, \quad \vec{H}_2 = -79.58\vec{a}_y + 79.58\vec{a}_z \text{ mA/m}, \quad \vec{H} = -105.39\vec{a}_y + 75.28\vec{a}_z \text{ mA/m}$$

H_1 $I_1 = 1A$ $y = 3m$ $(0, 2, 1)$
$\vec{a}_x$ $z = -5m$

$\vec{a}_{x'} = \vec{a}_x$

$\vec{\rho} = (2-3)\vec{a}_y + (1+5)\vec{a}_z = -\vec{a}_y + 6\vec{a}_z$

$\rho = \sqrt{(-1)^2 + (6)^2} = \sqrt{37}$

$\vec{a}_\rho = \frac{-1}{\sqrt{37}}\vec{a}_y + \frac{6}{\sqrt{37}}\vec{a}_z$

$\vec{a}_\phi = \vec{a}_{x'} \times \vec{a}_\rho = \vec{a}_x \times \left(\frac{-1}{\sqrt{37}}\vec{a}_y + \frac{6}{\sqrt{37}}\vec{a}_z\right)$

$= -\frac{6}{\sqrt{37}}\vec{a}_y - \frac{1}{\sqrt{37}}\vec{a}_z$

$\vec{H}_1 = \frac{I_1}{2\pi\rho}\vec{a}_\phi = \frac{1}{2\pi\sqrt{37}}\left(-\frac{6}{\sqrt{37}}\vec{a}_y - \frac{1}{\sqrt{37}}\vec{a}_z\right)$

$\vec{H}_1 = -25.81\vec{a}_y - 4.30\vec{a}_z$ mA/m

H_2 $I_2 = 3A$ $y = 5m$
$-\vec{a}_x$ $z = 4m$

$\vec{a}_{x'} = \vec{a}_x$

$\vec{\rho} = (2-5)\vec{a}_y + (1-9)\vec{a}_z = -3\vec{a}_y - 3\vec{a}_z$

$\rho = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$

$\vec{a}_\rho = \frac{3}{\sqrt{18}}\vec{a}_y - \frac{3}{\sqrt{18}}\vec{a}_z$

$\vec{a}_\phi = \vec{a}_{x'} \times \vec{a}_\rho = -\vec{a}_x \times \left(-\frac{3}{\sqrt{18}}\vec{a}_y - \frac{3}{\sqrt{18}}\vec{a}_z\right)$

$= -\frac{3}{\sqrt{18}}\vec{a}_y + \frac{3}{\sqrt{18}}\vec{a}_z$

$\vec{H}_2 = \frac{I_2}{2\pi\rho}\vec{a}_\phi = \frac{3}{2\pi\sqrt{18}}\left(-\frac{3}{\sqrt{18}}\vec{a}_y + \frac{3}{\sqrt{18}}\vec{a}_z\right)$

$\vec{H}_2 = -79.58\vec{a}_y + 79.58\vec{a}_z$ mA/m

$\vec{H}_1 = -25.81\vec{a}_y - 4.30\vec{a}_z$ mA/m
$\vec{H}_2 = -79.58\vec{a}_y + 79.58\vec{a}_z$ mA/m

$\vec{H} = -105.39\vec{a}_y + 75.28\vec{a}_z$ mA/m

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Assignment 6

กำหนดให้กระแส $I_1 = 2$ A ไหลในทิศทาง $-\bar{a}_y$ อยู่ตำแหน่ง $x = 4$ m, $z = 4$ m, กระแส $I_2 = 3$ A ไหลในทิศทาง $-\bar{a}_y$ อยู่ตำแหน่ง $x = 5$ m, $z = -4$ m, กระแส $I_3 = 1$ A ไหลในทิศทาง $\bar{a}_y$ อยู่ตำแหน่ง $x = -5$ m, $z = 3$ m และกระแส $I_4 = 4$ A ไหลในทิศทาง $\bar{a}_y$ อยู่ตำแหน่ง $x = -3$ m, $z = -4$ m จงหาความเข้มสนามแม่เหล็ก $\vec{H}$ ที่พิกัด (-1,0,1)

$\vec{H}_1 = 28.09\bar{a}_x - 46.81\bar{a}_z$ mA/m,
$\vec{H}_2 = -39.14\bar{a}_x - 46.96\bar{a}_z$ mA/m,
$\vec{H}_3 = -15.92\bar{a}_x - 31.83\bar{a}_z$ mA/m,
$\vec{H}_4 = 109.76\bar{a}_x - 43.90\bar{a}_z$ mA/m,
$\vec{H} = 82.79\bar{a}_x - 169.50\bar{a}_z$ mA/m

H (-1, 0, 1)

H_1 $I_1 = 2A$ $x = 9m$ $-\hat{a}_y$ $z = 9m$

$\hat{a}_{y'} = -\hat{a}_y$

$\vec{\rho} = (-1-9)\hat{a}_x + (1-9)\hat{a}_z = -5\hat{a}_x - 3\hat{a}_z$

$\rho = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34}$

$\hat{a}_\rho = -\frac{5}{\sqrt{34}}\hat{a}_x - \frac{3}{\sqrt{34}}\hat{a}_z$

$\hat{a}_\phi = \hat{a}_{y'} \times \hat{a}_\rho = -\hat{a}_y \times \left(-\frac{5}{\sqrt{34}}\hat{a}_x - \frac{3}{\sqrt{34}}\hat{a}_z\right)$

$= \frac{3}{\sqrt{34}}\hat{a}_x - \frac{5}{\sqrt{34}}\hat{a}_z$

$\vec{H}_1 = \frac{I_1}{2\pi\rho} = \frac{2}{2\pi\sqrt{34}} \times \left(\frac{3}{\sqrt{34}}\hat{a}_x - \frac{5}{\sqrt{34}}\hat{a}_z\right)$

$\vec{H}_1 = 28.09\hat{a}_x - 46.81\hat{a}_z$ mA/m

H_2 $I_2 = 3A$ $x = 5m$ $-\hat{a}_y$ $z = -9m$

$\hat{a}_{y'} = -\hat{a}_y$

$\vec{\rho} = (-1-5)\hat{a}_x + (1+9)\hat{a}_z = -6\hat{a}_x + 5\hat{a}_z$

$\rho = \sqrt{(-6)^2 + 5^2} = \sqrt{61}$

$\hat{a}_\rho = -\frac{6}{\sqrt{61}}\hat{a}_x + \frac{5}{\sqrt{61}}\hat{a}_z$

$\hat{a}_\phi = \hat{a}_{y'} \times \hat{a}_\rho = -\hat{a}_y \times \left(-\frac{6}{\sqrt{61}}\hat{a}_x + \frac{5}{\sqrt{61}}\hat{a}_z\right)$

$= -\frac{5}{\sqrt{61}}\hat{a}_x - \frac{6}{\sqrt{61}}\hat{a}_z$

$\vec{H}_2 = \frac{I_2}{2\pi\rho} = \frac{3}{2\pi\sqrt{61}} \times \left(-\frac{5}{\sqrt{61}}\hat{a}_x - \frac{6}{\sqrt{61}}\hat{a}_z\right)$

$\vec{H}_2 = -39.14\hat{a}_x - 46.96\hat{a}_z$ mA/m

H_3 $I_3 = 1A$ $x = -5m$ $\hat{a}_y$ $z = 3m$

$\hat{a}_{y'} = \hat{a}_y$

$\vec{\rho} = (-1+5)\hat{a}_x + (1-3)\hat{a}_z = 4\hat{a}_x - 2\hat{a}_z$

$\rho = \sqrt{4^2 + (-2)^2} = \sqrt{20}$

$\hat{a}_\rho = \frac{4}{\sqrt{20}}\hat{a}_x - \frac{2}{\sqrt{20}}\hat{a}_z$

$\hat{a}_\phi = \hat{a}_{y'} \times \hat{a}_\rho = \hat{a}_y \times \left(\frac{4}{\sqrt{20}}\hat{a}_x - \frac{2}{\sqrt{20}}\hat{a}_z\right)$

$= -\frac{2}{\sqrt{20}}\hat{a}_x - \frac{4}{\sqrt{20}}\hat{a}_z$

$\vec{H}_3 = \frac{I_3}{2\pi\rho} = \frac{1}{2\pi\sqrt{20}} \times \left(-\frac{2}{\sqrt{20}}\hat{a}_x - \frac{4}{\sqrt{20}}\hat{a}_z\right)$

$\vec{H}_3 = -15.92\hat{a}_x - 31.83\hat{a}_z$ mA/m

H_4 $I_4 = 4A$ $x = -3m$ $\hat{a}_y$ $z = -9m$

$\hat{a}_{y'} = \hat{a}_y$

$\vec{\rho} = (-1+3)\hat{a}_x + (1+9)\hat{a}_z = 2\hat{a}_x + 5\hat{a}_z$

$\rho = \sqrt{2^2 + 5^2} = \sqrt{29}$

$\hat{a}_\rho = \frac{2}{\sqrt{29}}\hat{a}_x + \frac{5}{\sqrt{29}}\hat{a}_z$

$\hat{a}_\phi = \hat{a}_{y'} \times \hat{a}_\rho = \hat{a}_y \times \left(\frac{2}{\sqrt{29}}\hat{a}_x + \frac{5}{\sqrt{29}}\hat{a}_z\right)$

$= \frac{5}{\sqrt{29}}\hat{a}_x - \frac{2}{\sqrt{29}}\hat{a}_z$

$\vec{H}_4 = \frac{I_4}{2\pi\rho} = \frac{4}{2\pi\sqrt{29}} \times \left(\frac{5}{\sqrt{29}}\hat{a}_x - \frac{2}{\sqrt{29}}\hat{a}_z\right)$

$\vec{H}_4 = 109.76\hat{a}_x - 43.90\hat{a}_z$ mA/m

$\vec{H}_1 = 28.09\hat{a}_x - 46.81\hat{a}_z$ mA/m

$\vec{H}_2 = -39.14\hat{a}_x - 46.96\hat{a}_z$ mA/m

$\vec{H}_3 = -15.92\hat{a}_x - 31.83\hat{a}_z$ mA/m

$\vec{H}_4 = 109.76\hat{a}_x - 43.90\hat{a}_z$ mA/m

$\vec{H} = 82.79\hat{a}_x - 169.50\hat{a}_z$ mA/m

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