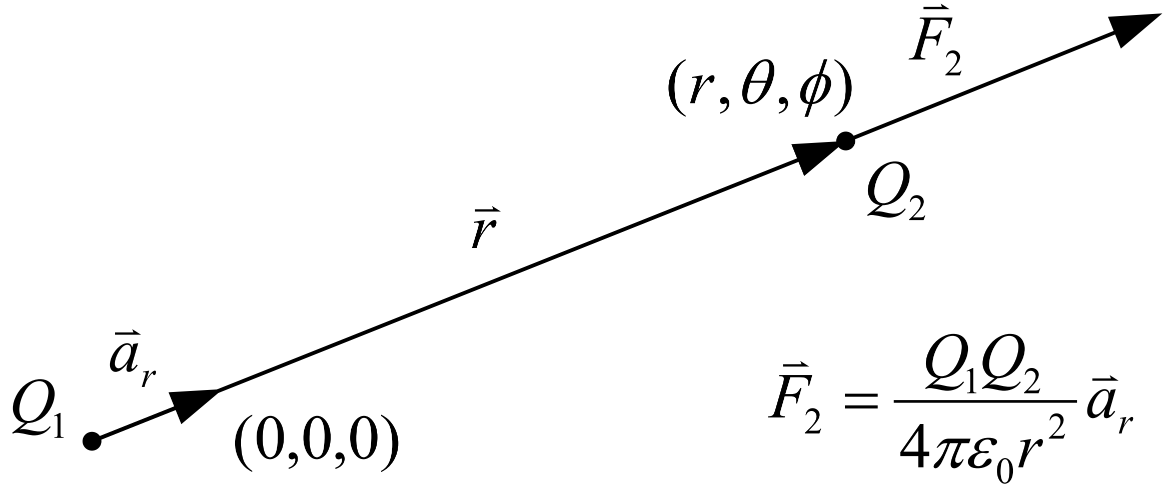
Electric Field Intensity

Coulomb's Law



$$\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

$\vec{F}_2$: Force on charge Q_2 (N)

Q_1 : Charge Q_1 (C)

Q_2 : Charge Q_2 (C)

ε_0 : Permittivity of free space $\approx \dfrac{1}{36\pi}\times 10^{-9}$ (F/m)

r : Distance between Q_1 and Q_2 (m)

Electric Field Intensity

ความเข้มสนามไฟฟ้า นิยามเป็น แรงที่กระทำต่อประจุ 1 C

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_r}$$

$\vec{E} = \vec{F}/Q$

$$\vec{F} = Q\vec{E}$$

$\vec{E}$: Electric Field Intensity (V/m)

Charge Distribution (1)

- Line Charge Density (C/m)

$$Q = \int_L \rho_L dL \qquad\qquad dQ = \rho_L dL$$

- Surface Charge Density (C/m²)

$$Q = \int_S \rho_S dS \qquad\qquad dQ = \rho_S dS$$

- Volume Charge Density (C/m³)

$$Q = \int_{vol} \rho_v dv \qquad\qquad dQ = \rho_v dv$$

Charge Distribution (2)

- Differential Electric Field

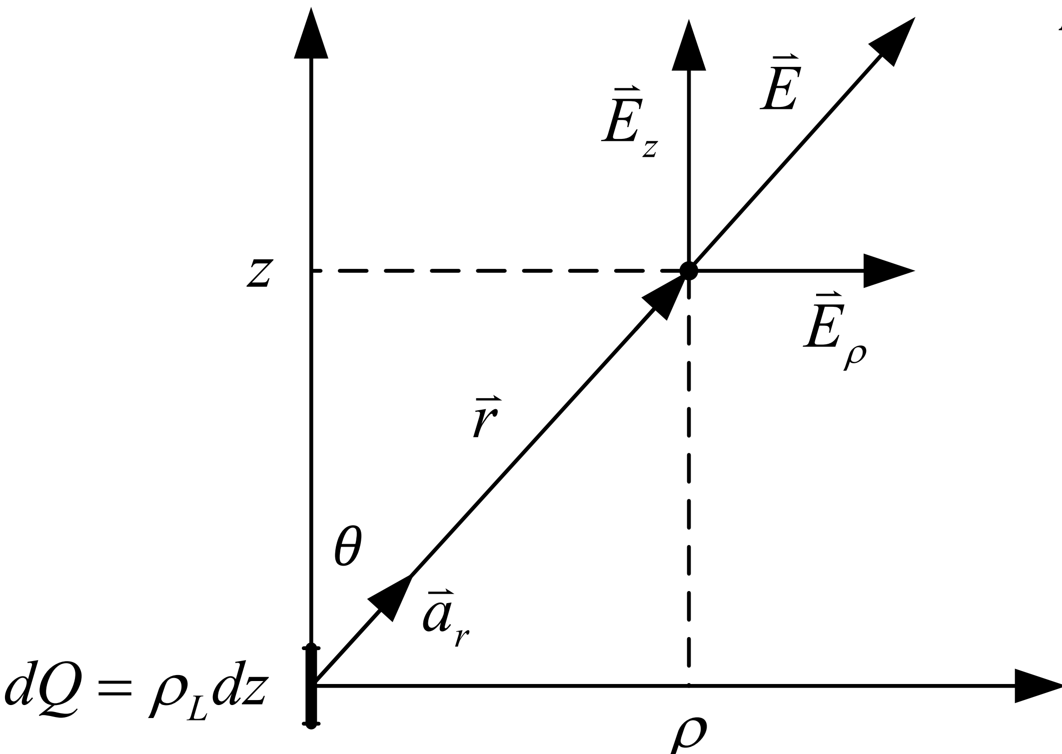
$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

- Electric Field

$$\vec{E} = \int \frac{dQ}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

Infinite Uniform Line Charge (1)

กำหนดให้มีประจุเชิงเส้นอยู่บนแกน z ของระบบพิกัดทรงกระบอก



$$\vec{r} = \rho\vec{a}_\rho + z\vec{a}_z,\ r = \sqrt{\rho^2 + z^2}$$

$$\vec{E} = \int \frac{dQ}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

$$= \int_{-\infty}^{\infty} \frac{\rho_L dz}{4\pi\varepsilon_0(\rho^2 + z^2)} \frac{\rho\vec{a}_\rho + z\vec{a}_z}{\sqrt{\rho^2 + z^2}}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{\rho dz}{(\rho^2 + z^2)^{3/2}}\vec{a}_\rho$$

$$+ \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{z dz}{(\rho^2 + z^2)^{3/2}}\vec{a}_z$$

Infinite Uniform Line Charge (2)

$$\frac{\rho}{z} = \tan\theta\,, \qquad z = \rho\cot\theta\,, \qquad dz = -\rho\,\mathrm{cosec}^2\theta d\theta = -\frac{\rho d\theta}{\sin^2\theta}$$

$$z \to -\infty \implies \theta \to \pi, \qquad z \to \infty \implies \theta \to 0$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\rho dz}{(\rho^2+z^2)^{3/2}}\vec{a}_\rho + \frac{\rho_L}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{z dz}{(\rho^2+z^2)^{3/2}}\vec{a}_z$$

$$= -\frac{\rho_L}{4\pi\varepsilon_0}\int_{\pi}^{0}\frac{\rho^2 d\theta}{(\rho^2+z^2)^{3/2}\sin^2\theta}\vec{a}_\rho - \frac{\rho_L}{4\pi\varepsilon_0}\int_{\pi}^{0}\frac{\rho z d\theta}{(\rho^2+z^2)^{3/2}\sin^2\theta}\vec{a}_z$$

$$= -\frac{\rho_L}{4\pi\varepsilon_0}\int_{\pi}^{0}\frac{\rho^3 d\theta}{\rho(\rho^2+z^2)^{3/2}\sin^2\theta}\vec{a}_\rho - \frac{\rho_L}{4\pi\varepsilon_0}\int_{\pi}^{0}\frac{\rho^2 z d\theta}{\rho(\rho^2+z^2)^{3/2}\sin^2\theta}\vec{a}_z$$

$$= -\frac{\rho_L}{4\pi\varepsilon_0\rho}\int_{\pi}^{0}\frac{\sin^3\theta}{\sin^2\theta}d\theta\vec{a}_\rho - \frac{\rho_L}{4\pi\varepsilon_0\rho}\int_{\pi}^{0}\frac{\sin^2\theta\cos\theta}{\sin^2\theta}d\theta\vec{a}_z$$

$$= -\frac{\rho_L}{4\pi\varepsilon_0\rho}\int_{\pi}^{0}\sin\theta\, d\theta\vec{a}_\rho - \frac{\rho_L}{4\pi\varepsilon_0\rho}\int_{\pi}^{0}\cos\theta\, d\theta\vec{a}_z$$

Infinite Uniform Line Charge (3)

$$\vec{E} = -\frac{\rho_L}{4\pi\varepsilon_0\rho}\int_{\pi}^{0} \sin\theta \, d\theta \vec{a}_\rho - \frac{\rho_L}{4\pi\varepsilon_0\rho}\int_{\pi}^{0} \cos\theta \, d\theta \vec{a}_z$$

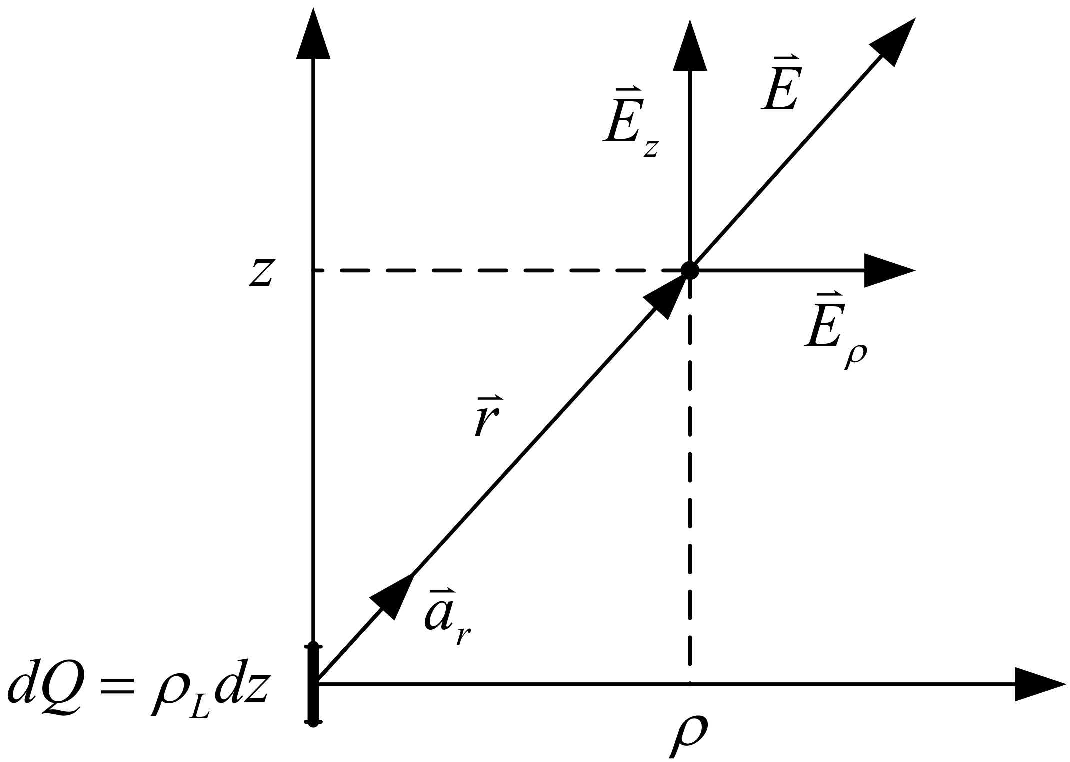
$$= \frac{\rho_L}{4\pi\varepsilon_0\rho}\cos\theta\Big|_{\theta=\pi}^{0} \vec{a}_\rho - \frac{\rho_L}{4\pi\varepsilon_0\rho}\sin\theta\Big|_{\theta=\pi}^{0} \vec{a}_z$$

$$= \frac{\rho_L}{4\pi\varepsilon_0\rho}(1+1)\vec{a}_\rho$$

$$= \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$$

Infinite Uniform Line Charge (4)

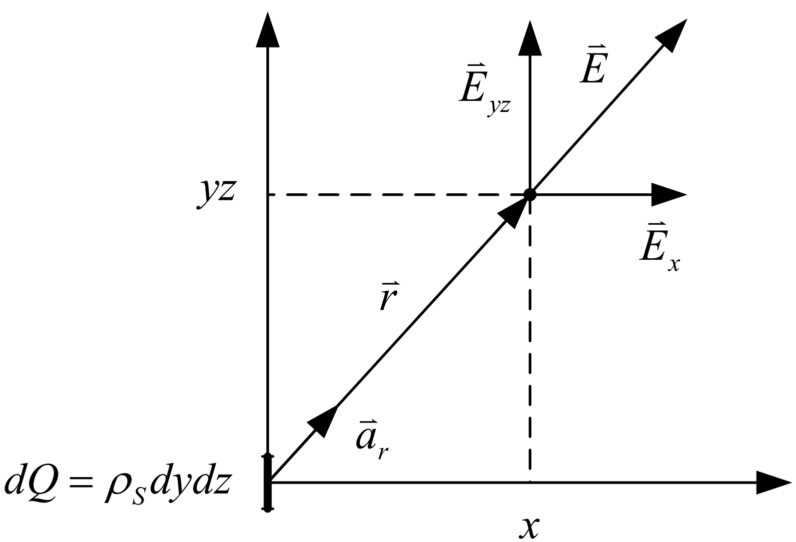
กำหนดให้มีประจุเชิงเส้นอยู่บนแกน z ของระบบพิกัดทรงกระบอก



$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho}$$

Infinite Uniform Surface Charge (1)

กำหนดให้มีประจุเชิงพื้นผิวอยู่บนระนาบ yz ของระบบพิกัดทรงสี่เหลี่ยมมุมฉาก



$$\vec{E} = \int \frac{dQ}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

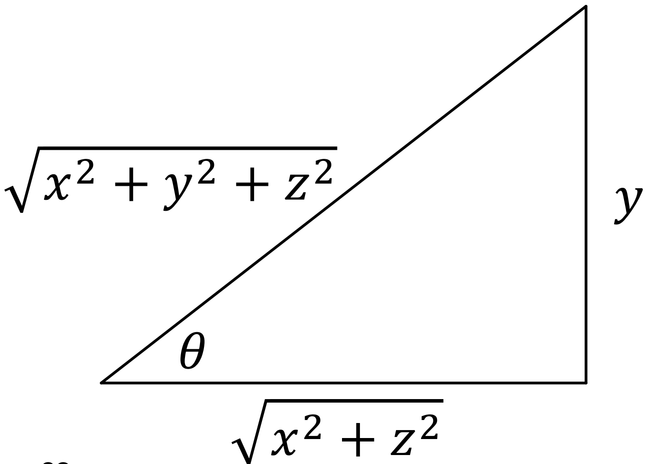
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\rho_s dydz}{4\pi\varepsilon_0(x^2+y^2+z^2)} \frac{x\vec{a}_x + y\vec{a}_y + z\vec{a}_z}{\sqrt{x^2+y^2+z^2}}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\rho_s x dydz\vec{a}_x}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}}$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\rho_s y dydz\vec{a}_y}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}}$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\rho_s z dydz\vec{a}_z}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}}$$

Infinite Uniform Surface Charge (2)



$$y = \sqrt{x^2+z^2}\tan\theta$$
$$dy = \sqrt{x^2+z^2}\sec^2\theta\, d\theta$$
$$y \to -\infty \Rightarrow \theta \to -\frac{\pi}{2},\ y \to \infty \quad \Rightarrow \theta \to \frac{\pi}{2}$$

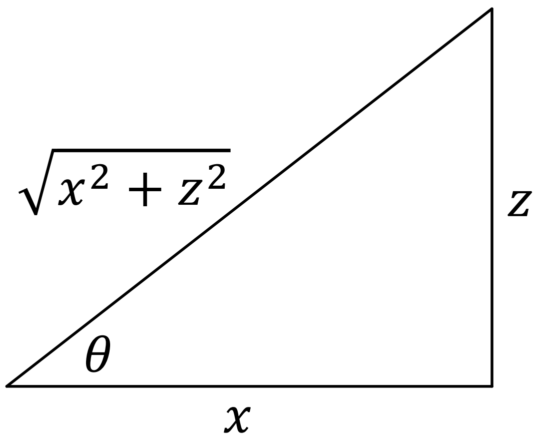
$$\int_{-\infty}^{\infty} \frac{\rho_s x dy}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}} = \int_{-\pi/2}^{\pi/2} \frac{\rho_s x (x^2+z^2)\sqrt{x^2+z^2}\sec^2\theta\, d\theta}{4\pi\varepsilon_0 (x^2+z^2)(x^2+y^2+z^2)^{3/2}}$$

$$= \frac{\rho_s x}{4\pi\varepsilon_0 (x^2+z^2)} \int_{-\pi/2}^{\pi/2} \cos^3\theta \sec^2\theta\, d\theta$$

$$= \frac{\rho_s x}{4\pi\varepsilon_0 (x^2+z^2)} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

$$= \frac{\rho_s x}{2\pi\varepsilon_0 (x^2+z^2)}$$

Infinite Uniform Surface Charge (3)



$$z = x\tan\theta$$
$$dz = x\sec^2\theta\, d\theta$$
$$z \to -\infty \Longrightarrow \theta \to -\frac{\pi}{2},\ z \to \infty \quad \Longrightarrow \theta \to \frac{\pi}{2}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\rho_s x dy dz \vec{a}_x}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}} = \int_{-\infty}^{\infty} \frac{\rho_s x dz \vec{a}_x}{2\pi\varepsilon_0 (x^2+z^2)}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\rho_s x^2 \sec^2\theta\, d\theta \vec{a}_x}{2\pi\varepsilon_0 (x^2+z^2)}$$

$$= \frac{\rho_s}{2\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} d\theta \vec{a}_x$$

$$= \frac{\rho_s}{2\varepsilon_0} \vec{a}_x$$

Infinite Uniform Surface Charge (4)

$$\int_{-\infty}^{\infty} \frac{\rho_s z dy}{4\pi\varepsilon_0 (x^2 + y^2 + z^2)^{3/2}} = \frac{\rho_s z}{2\pi\varepsilon_0 (x^2 + z^2)}$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\rho_s z dy dz \vec{a}_z}{4\pi\varepsilon_0 (x^2 + y^2 + z^2)^{3/2}} = \int_{-\infty}^{\infty} \frac{\rho_s z dz \vec{a}_z}{2\pi\varepsilon_0 (x^2 + z^2)}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\rho_s xz \sec^2 \theta \, d\theta \vec{a}_z}{2\pi\varepsilon_0 (x^2 + z^2)}$$

$$= \frac{\rho_s}{2\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \tan \theta \, d\theta \vec{a}_z$$

$$= -\frac{\rho_s}{2\pi\varepsilon_0} \ln|\cos \theta| \Big|_{\theta=-\pi/2}^{\pi/2}$$

$$= -\frac{\rho_s}{2\pi\varepsilon_0} \lim_{\theta \to \pi/2} \ln \left| \frac{\cos \theta}{\cos(-\theta)} \right|$$

$$= 0$$

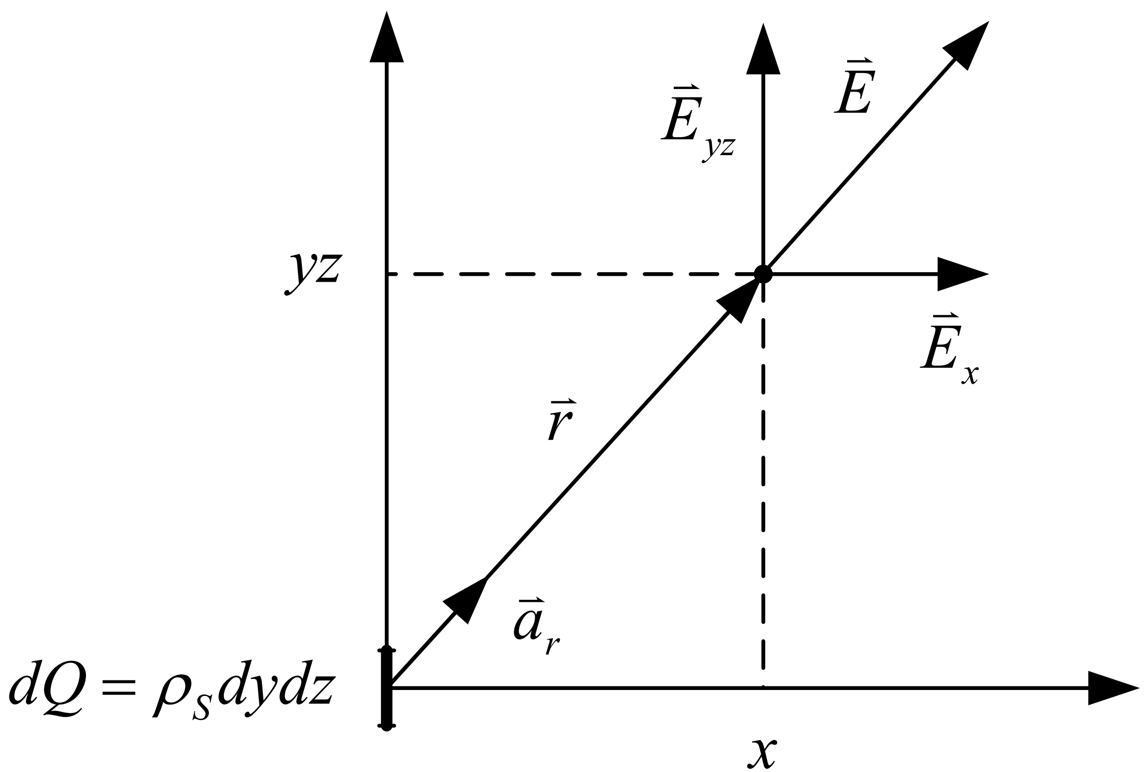
Infinite Uniform Surface Charge (5)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\rho_s y dy dz \vec{a}_y}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\rho_s z dy dz \vec{a}_z}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}} = 0$$

$$\begin{aligned}\vec{E} = & \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\rho_s x dy dz \vec{a}_x}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}} + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\rho_s y dy dz \vec{a}_y}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}} \\ & + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\rho_s z dy dz \vec{a}_z}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}} \\ = & \frac{\rho_s}{2\varepsilon_0}\vec{a}_x + 0 + 0 \\ = & \frac{\rho_s}{2\varepsilon_0}\vec{a}_x\end{aligned}$$

Infinite Uniform Surface Charge (6)

กำหนดให้มีประจุเชิงพื้นผิวอยู่บนระนาบ yz ของระบบพิกัดทรงสี่เหลี่ยมมุมฉาก



$$\boxed{\vec{E} = \frac{\rho_S}{2\varepsilon_0}\vec{a}_N}$$

Example

Q

กำหนดให้มีประจุ -12 nC อยู่ที่พิกัด (-3,-5,5) ความหนาแน่นประจุเชิงเส้น 4 nC/m ยาวอนันต์อยู่ในแนวแกน z ที่ ρ_L x=-4, ρ_L y=3 และความหนาแน่นประจุเชิงพื้นผิว -50 pc/m^2 ขนาดอนันต์อยู่ในระนาบ yz ที่ x=5 จงหาความเข้มสนามไฟฟ้าที่พิกัด (2,-2,2)

E

$$E = \frac{Q}{4\pi \varepsilon_0 r^2} a_r \qquad \varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$$

ประจุ $-12\,nC$ ที่ $(-3,-5,5)$

$$\underset{(-3,-5,5)}{Q} \xrightarrow{\vec r} \underset{(2,-2,2)}{\vec E}$$

$$\vec r = (2-(-3))\vec a_x + (-2-(-5))\vec a_y + (2-5)\vec a_z$$

$$\vec r = 5\vec a_x + 3\vec a_y - 3\vec a_z$$

$$r = \sqrt{(2+3)^2 + (-2+5)^2 + (2-5)^2}$$

$$= \sqrt{43}$$

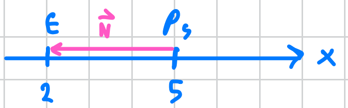
$$a_r = \frac{5}{\sqrt{43}}\vec a_x + \frac{3}{\sqrt{43}}\vec a_y - \frac{3}{\sqrt{43}}\vec a_z$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\vec a_\rho$$

$$\underset{\substack{x=-4 \\ y=3}}{\rho_L} \xrightarrow{\vec \rho} \underset{\substack{x=2 \\ y=-2}}{E}$$

$$\vec \rho = (2-(-4))\vec a_x + (-2-3)\vec a_y$$

$$\vec E = \frac{\rho_s}{2\varepsilon_0}\vec a_N$$



$$\vec a_n = -\vec a_x$$

Solution (1)

หา $\vec{E}_1$ ที่เกิดจากประจุ -12 nC

$$\vec{E}_1 = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

$$r = \sqrt{(2+3)^2 + (-2+5)^2 + (2-5)^2} = \sqrt{43}$$

$$\vec{a}_r = \frac{5}{\sqrt{43}}\vec{a}_x + \frac{3}{\sqrt{43}}\vec{a}_y - \frac{3}{\sqrt{43}}\vec{a}_z$$

$$\vec{E}_1 = \frac{-12\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}\times43}\left(\frac{5}{\sqrt{43}}\vec{a}_x + \frac{3}{\sqrt{43}}\vec{a}_y - \frac{3}{\sqrt{43}}\vec{a}_z\right)$$

$$= -1.92\vec{a}_x - 1.15\vec{a}_y + 1.15\vec{a}_z \quad \text{V/m}$$

Solution (2)

หา $\vec{E}_2$ ที่เกิดจากความหนาแน่นประจุเชิงเส้น 4 nC/m

$$\vec{E}_2 = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$$

$$\rho = \sqrt{(2+4)^2 + (-2-3)^2} = \sqrt{61}$$

$$\vec{a}_\rho = \frac{6}{\sqrt{61}}\vec{a}_x - \frac{5}{\sqrt{61}}\vec{a}_y$$

$$\vec{E}_2 = \frac{4\times10^{-9}}{2\pi\times\frac{1}{36\pi}\times10^{-9}\times\sqrt{61}}\left(\frac{6}{\sqrt{61}}\vec{a}_x - \frac{5}{\sqrt{61}}\vec{a}_y\right)$$

$$= 7.08\vec{a}_x - 5.90\vec{a}_y \quad \text{V/m}$$

Solution (3)

หา $\vec{E}_3$ ที่เกิดจากความหนาแน่นประจุเชิงพื้นผิว -50 pC/m^2

$$\vec{E}_3 = \frac{\rho_S}{2\varepsilon_0}\vec{a}_N$$

$$\vec{a}_N = -\vec{a}_x$$

$$\vec{E}_3 = \frac{-50\times 10^{-12}}{2\times \frac{1}{36\pi}\times 10^{-9}} \times -\vec{a}_x$$

$$= 2.83\vec{a}_x \quad \text{V/m}$$

Solution (4)

หา $\vec{E}$ ที่พิกัด (2,-2,2)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= 7.99\vec{a}_x - 7.05\vec{a}_y + 1.15\vec{a}_z \ \text{ V/m}$$

Quiz 1

Q_1 Q_2

กำหนดให้มีประจุ 50 nC อยู่ที่พิกัด (-2,1,0) ประจุ -30 nC

ρ_L

อยู่ที่พิกัด (3,2,-1) ความหนาแน่นประจุเชิงเส้น 10 nC/m ยาวอนันต์อยู่ในแนวแกน z ที่ x=-1, y=3 และความหนาแน่นประจุเชิงพื้นผิว -60 pc/m^2 ขนาดอนันต์อยู่ในระนาบ xy ที่ z=2 จงหาความเข้มสนามไฟฟ้าที่พิกัด (5,5,-5)

E

$$\vec{E}_1 = 3.69\vec{a}_x + 2.11\vec{a}_y - 2.64\vec{a}_z \ \text{V/m}$$
$$\vec{E}_2 = -3.46\vec{a}_x - 5.19\vec{a}_y + 6.92\vec{a}_z \ \text{V/m}$$
$$\vec{E}_3 = 27.00\vec{a}_x + 9.00\vec{a}_y \ \text{V/m}$$
$$\vec{E}_4 = 3.39\vec{a}_z \ \text{V/m}$$
$$\vec{E} = 27.23\vec{a}_x + 5.92\vec{a}_y + 7.67\vec{a}_z \ \text{V/m}$$

$50\,nC$ @ $(-2,1,0)$ $\quad$ $10\,nC/m$ (z) $\quad$ $-60\,pC/m^2$ $\quad$ @ $(5,5,-5)$?

$-30\,nC$ @ $(3,2,-1)$ $\quad$ $x=-1,\ y=3$ $\quad$ xy $\ z=2$

$$Q_1 \xrightarrow{\vec{r}} E \qquad (-2,1,0) \quad (5,5,-5)$$

$$\vec{r} = (5+2)\vec{a}_x + (5-1)\vec{a}_y + (-5-0)\vec{a}_z = 7\vec{a}_x + 4\vec{a}_y - 5\vec{a}_z$$

$$r = \sqrt{(5+2)^2 + (5-1)^2 + (-5-0)^2} = \sqrt{90}$$

$$\vec{a}_r = \frac{7}{\sqrt{90}}\vec{a}_x + \frac{4}{\sqrt{90}}\vec{a}_y - \frac{5}{\sqrt{90}}\vec{a}_z$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

$$= \frac{\cancel{50}^{5} \times \cancel{10^{-9}}}{\cancel{4\pi}\left(\frac{1}{\cancel{36\pi}}\times \cancel{10^{-9}}\right)\cancel{90}} \cdot \frac{7}{\sqrt{90}}\vec{a}_x + \frac{4}{\sqrt{90}}\vec{a}_y - \frac{5}{\sqrt{90}}\vec{a}_z$$

$$= 5\cdot\left(\frac{7}{\sqrt{90}}\vec{a}_x + \frac{4}{\sqrt{90}}\vec{a}_y - \frac{5}{\sqrt{90}}\vec{a}_z\right)$$

$$\underline{E_1 = 3.69\vec{a}_x + 2.11\vec{a}_y - 2.64\vec{a}_z \ V/m}$$

$$Q_2 \xrightarrow{\vec{r}} E \qquad (3,2,-1) \quad (5,5,-5)$$

$$\vec{r} = (5-3)\vec{a}_x + (5-2)\vec{a}_y + (-5+1)\vec{a}_z = 2\vec{a}_x + 3\vec{a}_y - 4\vec{a}_z$$

$$r = \sqrt{(5-3)^2 + (5-2) + (-5+1)^2} = \sqrt{29}$$

$$\vec{a}_r = \frac{2}{\sqrt{29}}\vec{a}_x + \frac{3}{\sqrt{29}}\vec{a}_y - \frac{4}{\sqrt{29}}\vec{a}_z$$

$$E_2 = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

$$= \frac{-30\times\cancel{10^{-9}}}{\cancel{4\pi}\left(\frac{1}{\cancel{36\pi}}\times\cancel{10^{-9}}\right)\cdot 29} \cdot \left(\frac{2}{\sqrt{29}}\vec{a}_x + \frac{3}{\sqrt{29}}\vec{a}_y - \frac{4}{\sqrt{29}}\vec{a}_z\right) \quad 9$$

$$= \frac{-30\cdot 9}{29}\cdot\left(\frac{2}{\sqrt{29}}\vec{a}_x + \frac{3}{\sqrt{29}}\vec{a}_x - \frac{4}{\sqrt{29}}\vec{a}_z\right)$$

$$\underline{E_2 = -3.46\vec{a}_x - 5.19\vec{a}_y + 6.92\vec{a}_z \ V/m}$$

$$\rho_L \xrightarrow{\vec{\rho}} E \qquad x=-1,\ y=3 \quad x=5,\ y=5$$

$$\vec{\rho} = (5+1)\vec{a}_x + (5-3)\vec{a}_y = 6\vec{a}_x + 2\vec{a}_y$$

$$\rho = \sqrt{(5+1)^2 + (5-3)^2} = \sqrt{40}$$

$$\vec{a}_\rho = \frac{6}{\sqrt{40}}\vec{a}_x + \frac{2}{\sqrt{40}}\vec{a}_y$$

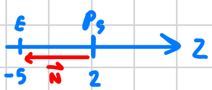
$$E_3 = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$$

$$= \frac{\overset{180}{10}\cdot\cancel{10^{-9}}}{\cancel{2\pi}\cdot\left(\frac{1}{\cancel{36\pi}}\cdot\cancel{10^{-9}}\right)\cdot\sqrt{40}} \cdot \left(\frac{6}{\sqrt{40}}\vec{a}_x + \frac{2}{\sqrt{40}}\vec{a}_y\right) \quad 18$$

$$= \frac{45}{\sqrt{40}}\cdot\left(\frac{6}{\sqrt{40}}\vec{a}_x + \frac{2}{\sqrt{40}}\vec{a}_y\right)$$

$$\underline{E_3 = 27.00\vec{a}_x + 9.00\vec{a}_y \ V/m}$$

$$\underline{\underline{E = 2.73\vec{a}_x + 5.92\vec{a}_y + 7.67\vec{a}_z \ V/m}}$$



$$E_4 = \frac{\rho_s}{2\varepsilon_0}\vec{a}_N$$

$$\vec{a}_n = -\vec{a}_z$$

$$\vec{N} = (2+5)\vec{a}_z = 7\vec{a}_z$$

$$N = \sqrt{(2+5)^2} = \sqrt{49}$$

$$\vec{a}_z = \frac{7}{\sqrt{49}}\vec{a}_z$$

$$\vec{a}_n = -\frac{7}{\sqrt{49}}\vec{a}_z$$

$$E_4 = \frac{-60\times10^{-12}}{2\left(\frac{1}{36\pi}\times10^{-9}\right)}\cdot\left(-\frac{7}{\sqrt{49}}\vec{a}_z\right)$$

$$\underline{E_4 = 3.39\vec{a}_z \ V/m}$$

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Assignment 1

กำหนดให้มีประจุ -63 nC อยู่ที่พิกัด (4,-3,-5) ประจุ 29 nC อยู่ที่พิกัด (-3,4,5) ความหนาแน่นประจุเชิงเส้น -5 nC/m ยาวอนันต์อยู่ในแนวแกน y ที่ x=4, z=-5 และความหนาแน่นประจุเชิงพื้นผิว -65 pc/m^2 ขนาดอนันต์อยู่ในระนาบ xz ที่ y=5 จงหาความเข้มสนามไฟฟ้าที่พิกัด (-1,-2,1)

$$\vec{E}_1 = 5.81\vec{a}_x - 1.16\vec{a}_y - 6.97\vec{a}_z \ \text{ V/m}$$

$$\vec{E}_2 = 1.25\vec{a}_x - 3.74\vec{a}_y - 2.49\vec{a}_z \ \text{ V/m}$$

$$\vec{E}_3 = 7.38\vec{a}_x - 8.85\vec{a}_z \ \text{ V/m}$$

$$\vec{E}_4 = 3.68\vec{a}_y \ \text{ V/m}$$

$$\vec{E} = 14.44\vec{a}_x - 1.22\vec{a}_y - 18.31\vec{a}_z \ \text{ V/m}$$

$$E = \frac{Q}{4\pi \varepsilon_0 r^2} a_r$$

$$\vec{r}_1 = (-1-4)\vec{a}_x + (-2+3)\vec{a}_y + (1+5)\vec{a}_z$$
$$= -5\vec{a}_x + \vec{a}_y + 6\vec{a}_z$$

$$r_1 = \sqrt{(-1-4)^2 + (-2+3)^2 + (1+5)^2}$$
$$= \sqrt{62}$$

$$\vec{a}_r = \frac{-5}{\sqrt{62}}\vec{a}_x + \frac{1}{\sqrt{62}}\vec{a}_y + \frac{6}{\sqrt{62}}\vec{a}_z$$

$$E_1 = \frac{\overset{-567}{-63 \times \cancel{10^{-9}}}}{\underset{1}{\cancel{4\pi}} \cdot \left(\frac{1}{\underset{9}{\cancel{36\pi}}} \cdot \cancel{10^{-9}}\right) \cdot 62} \cdot \left(\frac{-5}{\sqrt{62}}\vec{a}_x + \frac{1}{\sqrt{62}}\vec{a}_y + \frac{6}{\sqrt{62}}\vec{a}_z\right)$$

$$E_1 = \frac{-567}{62} \cdot \left(\frac{-5}{\sqrt{62}}\vec{a}_x + \frac{1}{\sqrt{62}}\vec{a}_y + \frac{6}{\sqrt{62}}\vec{a}_z\right)$$

$$\underline{E_1 = 5.81\vec{a}_x - 1.16\vec{a}_y - 6.97\vec{a}_z \ \text{V/m}}$$

$$\vec{r}_2 = (-1+3)\vec{a}_x + (-2-4)\vec{a}_y + (1-5)\vec{a}_z$$
$$= 2\vec{a}_x - 6\vec{a}_y - 4\vec{a}_z$$

$$r_2 = \sqrt{(-1+3)^2 + (-2-4)^2 + (1-5)^2}$$
$$= \sqrt{56}$$

$$\vec{a}_r = \frac{2}{\sqrt{56}}\vec{a}_x - \frac{6}{\sqrt{56}}\vec{a}_y - \frac{4}{\sqrt{56}}\vec{a}_z$$

$$E_2 = \frac{\overset{261}{29 \cdot \cancel{10^{-9}}}}{\cancel{4\pi} \cdot \left(\frac{1}{\underset{9}{\cancel{36\pi}}} \cdot \cancel{10^{-9}}\right) \cdot 56} \cdot \left(\frac{2}{\sqrt{56}}\vec{a}_x - \frac{6}{\sqrt{56}}\vec{a}_y - \frac{4}{\sqrt{56}}\vec{a}_z\right)$$

$$E_2 = \frac{261}{56} \cdot \left(\frac{2}{\sqrt{56}}\vec{a}_x + \frac{-6}{\sqrt{56}}\vec{a}_y + \frac{-4}{\sqrt{56}}\vec{a}_z\right)$$

$$\underline{E_2 = 1.25\vec{a}_x - 3.74\vec{a}_y - 2.49\vec{a}_z \ \text{V/m}}$$

$$E = \frac{\rho_L}{2\pi \varepsilon_0 \rho}\vec{a}_\rho$$

$$\vec{\rho} = (-1-4)\vec{a}_x + (1+5)\vec{a}_z$$
$$= -5\vec{a}_x + 6\vec{a}_z$$

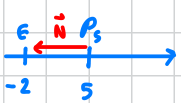
$$\rho = \sqrt{(-1-4)^2 + (1+5)^2}$$
$$= \sqrt{61}$$

$$\vec{a}_\rho = \frac{-5}{\sqrt{61}}\vec{a}_x + \frac{6}{\sqrt{61}}\vec{a}_z$$

$$E_3 = \frac{\overset{-90}{-5 \cdot \cancel{10^{-9}}}}{\underset{1}{\cancel{2\pi}} \left(\frac{1}{\underset{18}{\cancel{36\pi}}} \cdot \cancel{10^{-9}}\right) \sqrt{61}} \cdot \left(\frac{-5}{\sqrt{61}}\vec{a}_x + \frac{6}{\sqrt{61}}\vec{a}_z\right)$$

$$E_3 = \frac{-90}{\sqrt{61}} \cdot \left(\frac{-5}{\sqrt{61}}\vec{a}_x + \frac{6}{\sqrt{61}}\vec{a}_z\right)$$

$$\underline{E_3 = 7.38\vec{a}_x - 8.85\vec{a}_z \ \text{V/m}}$$



$$\vec{a}_n = -\vec{a}_y$$

$$\vec{N} = (5+2)\vec{a}_y$$
$$= 7\vec{a}_y$$

$$N = \sqrt{(5+2)^2}$$
$$= \sqrt{49}$$

$$\vec{a}_y = \frac{7}{\sqrt{49}}\vec{a}_y$$

$$\vec{a}_n = \frac{-7}{\sqrt{49}}\vec{a}_y$$

$$E = \frac{\rho_s}{2\varepsilon_0}\vec{a}_n$$

$$E_4 = \frac{-65 \times 10^{-12}}{2\left(\frac{1}{36\pi} \cdot 10^{-9}\right)} \cdot \left(\frac{-7}{\sqrt{49}}\vec{a}_y\right)$$

$$\underline{E_4 = 3.68\vec{a}_y \ \text{V/m}}$$

E_1 :	$5.81\vec{a}_x$	$-1.16\vec{a}_y$	$-6.97\vec{a}_z$	V/m
E_2 :	$1.25\vec{a}_x$	$-3.74\vec{a}_y$	$-2.49\vec{a}_z$	V/m
E_3 :	$7.38\vec{a}_x$		$-8.85\vec{a}_z$	V/m
E_4 :		$3.68\vec{a}_y$		V/m
E :	$14.44\vec{a}_x$	$-1.22\vec{a}_y$	$-18.31\vec{a}_z$	V/m

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