

Inductor

# Inductance

- Self Inductance

$$L = \frac{N\Phi}{I}$$

$L$  : Inductance (H)

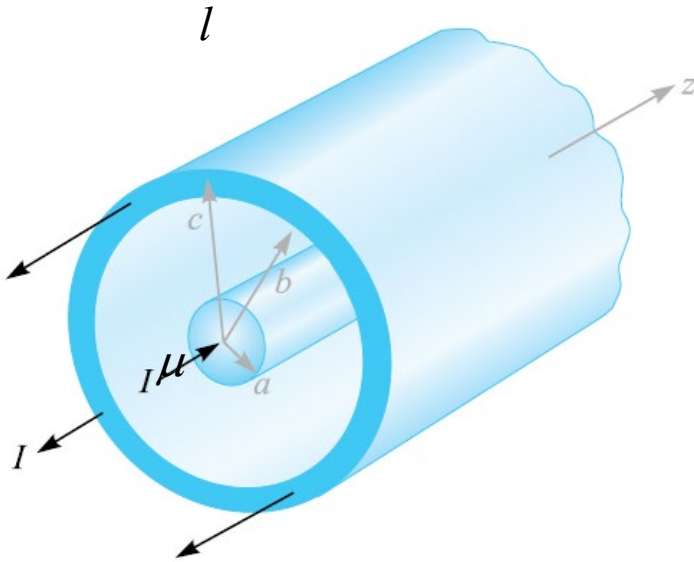
- Mutual Inductance

$$M_{12} = \frac{N_2\Phi_{12}}{I_1}$$

$M_{12}$  : Mutual Inductance (H)

# Coaxial Inductor

$$a < \rho < b$$



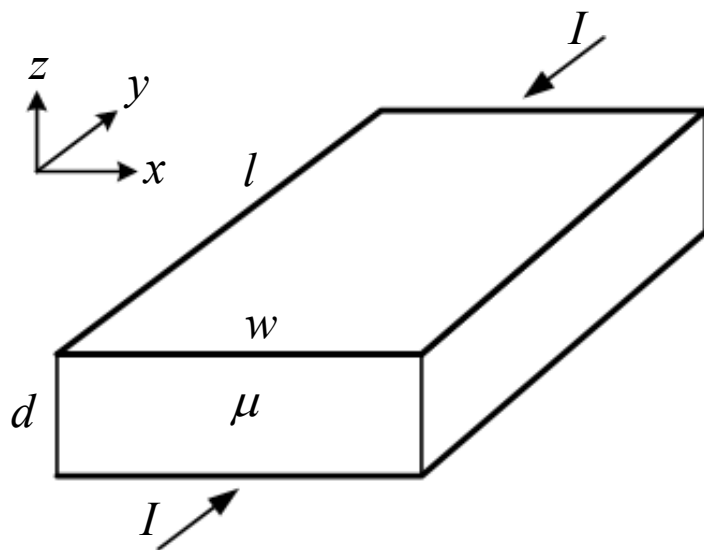
$$\vec{H} = \frac{I}{2\pi\rho} \vec{a}_\phi$$

$$\vec{B} = \mu\vec{H} = \frac{\mu I}{2\pi\rho} \vec{a}_\phi$$

$$\Phi = \int_s \vec{B} \cdot d\vec{S} = \frac{\mu l I}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$L = \frac{N\Phi}{I} = \frac{\mu l}{2\pi} \ln\left(\frac{b}{a}\right)$$

# Parallel Plate Inductor



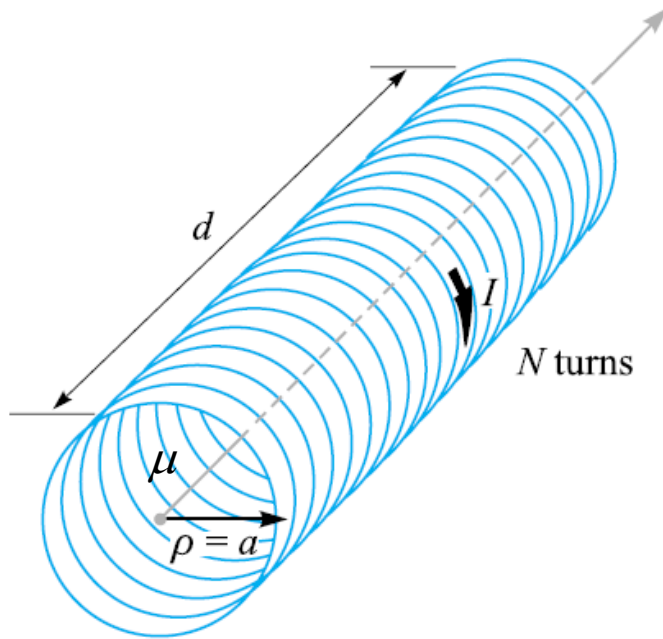
$$\vec{H} = \frac{I}{w} \vec{a}_x$$

$$\vec{B} = \mu \vec{H} = \frac{\mu I}{w} \vec{a}_x$$

$$\Phi = \int_S \vec{B} \cdot d\vec{S} = \frac{\mu d l I}{w}$$

$$L = \frac{N\Phi}{I} = \frac{\mu d l}{w}$$

# Solenoid Inductor



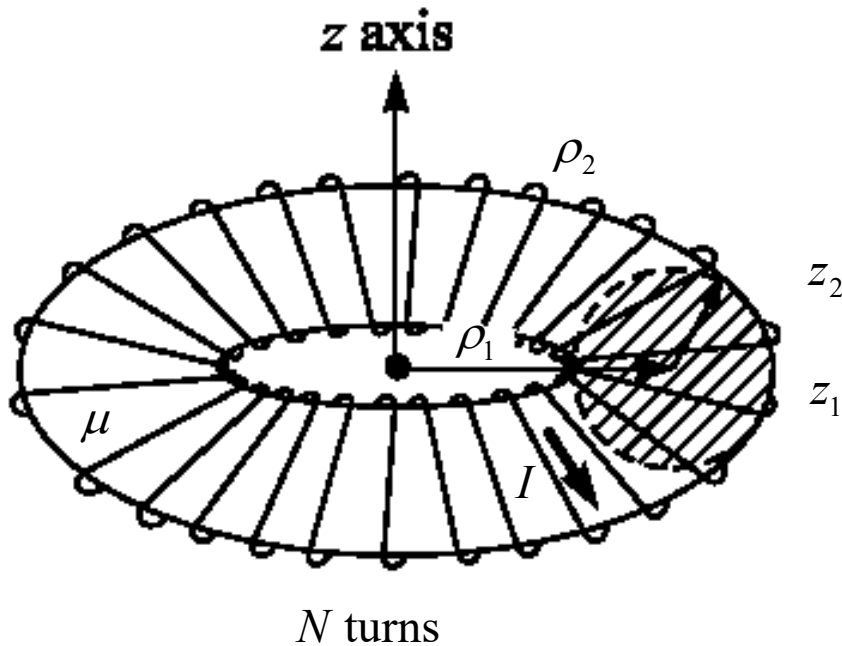
$$\vec{H} = \frac{NI}{d} \vec{a}_z$$

$$\vec{B} = \mu \vec{H} = \frac{\mu NI}{d} \vec{a}_z$$

$$\Phi = \int_s \vec{B} \cdot d\vec{S} = \frac{\pi \mu a^2 NI}{d}$$

$$L = \frac{N\Phi}{I} = \frac{\pi \mu a^2 N^2}{d}$$

# Toroid Inductor



$$\vec{H} = \frac{NI}{2\pi\rho} \vec{a}_\phi$$

$$\vec{B} = \mu\vec{H} = \frac{\mu NI}{2\pi\rho} \vec{a}_\phi$$

$$\Phi = \int_S \vec{B} \cdot d\vec{S} = \frac{\mu NI}{2\pi} (z_2 - z_1) \ln\left(\frac{\rho_2}{\rho_1}\right)$$

$$L = \frac{N\Phi}{I} = \frac{\mu N^2}{2\pi} (z_2 - z_1) \ln\left(\frac{\rho_2}{\rho_1}\right)$$

# Maxwell Equations (1)

- Time Invariant

Differential Form

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \times \vec{E} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

Integral Form

$$\oint_S \vec{D} \cdot d\vec{S} = Q$$

$$\oint \vec{E} \cdot d\vec{L} = 0$$

$$\oint \vec{H} \cdot d\vec{L} = I$$

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

# Maxwell Equations (2)

- Time Varying

Differential Form

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

Integral Form

$$\oint_S \vec{D} \cdot d\vec{S} = Q$$

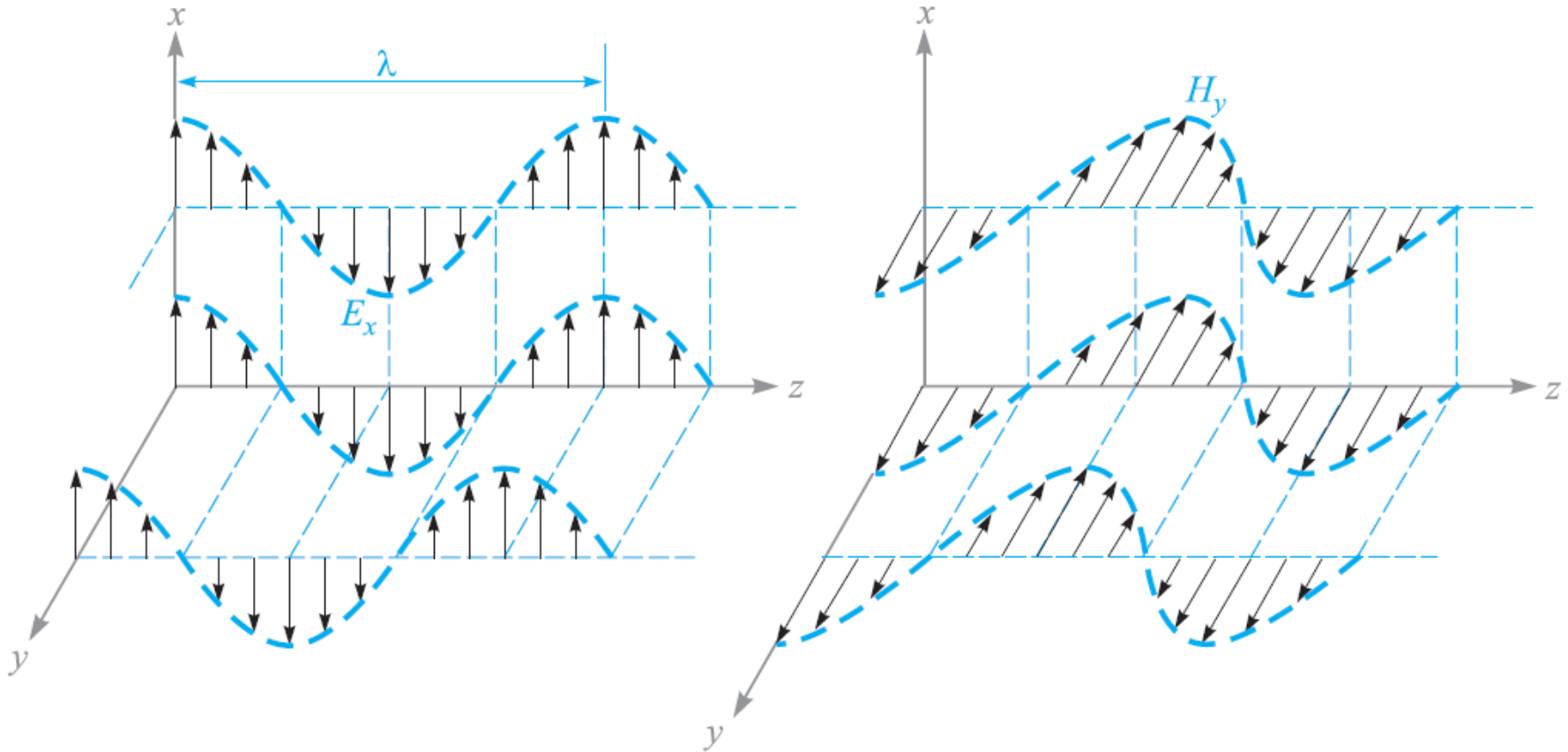
$$\oint \vec{E} \cdot d\vec{L} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\oint \vec{H} \cdot d\vec{L} = I + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$$

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$



# Plane Wave (1)



Direction of  $\vec{E} \perp$  Direction of  $\vec{H}$

Direction of  $\vec{v} =$  Direction of  $\vec{E} \times \vec{H}$

# Plane Wave (2)

$$E = \eta H \quad \eta = \sqrt{\frac{\mu}{\epsilon}} \quad v = \frac{1}{\sqrt{\epsilon\mu}}$$

$E$  : Electric Field Intensity (V/m)

$H$  : Magnetic Field Intensity (A/m)

$\epsilon$  : Permittivity,  $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$  H/m for Free Space

$\mu$  : Permeability,  $\mu_0 = 4\pi \times 10^{-7}$  H/m for Free Space

$\eta$  : Intrinsic Impedance,  $\eta_0 \approx 120\pi \, \Omega$  for Free Space

$v$  : Velocity,  $c \approx 3 \times 10^8$  m/s for Free space

# Example

Solenoid Inductor 2 ชุด อยู่ใกล้กันโดยมีแกนกลางเดียวกันอยู่ในแกน  $z$  กำหนดให้  $I_1 = 2$  mA ไหลในทิศทาง  $\vec{a}_\phi$ ,  $d = 5$  cm,  $a = 2$  cm  $N_1 = 50$  รอบ,  $N_2 = 100$  รอบ  $\mu_r = 10$  และฟลักซ์แม่เหล็กไปยัง Solenoid Inductor ชุดที่ 2 ได้ 80% จงหา  $\vec{H}_1$ ,  $\vec{B}_1$ ,  $\Phi_1$ ,  $L_1$ ,  $\Phi_{12}$ ,  $M_{12}$

# Solution (1)

หา  $\vec{H}_1$  ได้

$$\vec{H}_1 = \frac{N_1 I_1}{d} \vec{a}_z = \frac{50 \times 2 \times 10^{-3}}{5 \times 10^{-2}} \vec{a}_z = 2.00 \vec{a}_z \text{ A/m}$$

หา  $\vec{B}_1$  ได้

$$\vec{B}_1 = \mu \vec{H}_1 = 10 \times 4\pi \times 10^{-7} \times 2 \vec{a}_z = 25.13 \vec{a}_z \text{ } \mu\text{T}$$

หา  $\Phi_1$  ได้

$$\Phi_1 = \int_S \vec{B}_1 \cdot d\vec{S} = \pi a^2 B_1 = \pi \times (2 \times 10^{-2})^2 \times 25.13 \times 10^{-6} = 31.58 \text{ nWb}$$

## Solution (2)

หา  $L_1$  ได้

$$L_1 = \frac{N_1 \Phi_1}{I_1} = \frac{50 \times 31.58 \times 10^{-9}}{2 \times 10^{-3}} = 789.50 \text{ } \mu\text{H}$$

หา  $\Phi_{12}$  ได้

$$\Phi_{12} = \Phi_1 \times \frac{80}{100} = 31.58 \times 10^{-9} \times \frac{80}{100} = 25.26 \text{ nWb}$$

หา  $M_{12}$  ได้

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1} = \frac{100 \times 25.26 \times 10^{-9}}{2 \times 10^{-3}} = 1.26 \text{ mH}$$

# Quiz 10

Solenoid Inductor 2 ชุด อยู่ใกล้กันโดยมีแกนกลางเดียวกันอยู่ในแกน  $z$  กำหนดให้  $I_1 = 5$  mA ไหลในทิศทาง  $\vec{a}_\phi$ ,  $d = 2$  cm,  $a = 1$  cm  $N_1 = 80$  รอบ,  $N_2 = 120$  รอบ  $\mu_r = 4$  และฟลักซ์แม่เหล็กไปยัง Solenoid Inductor ชุดที่ 2 ได้ 90% จงหา  $\vec{H}_1$ ,  $\vec{B}_1$ ,  $\Phi_1$ ,  $L_1$ ,  $\Phi_{12}$ ,  $M_{12}$

$$\vec{H}_1 = 20.00\vec{a}_z \text{ A/m}, \vec{B}_1 = 100.53\vec{a}_z \text{ }\mu\text{T}, \Phi_1 = 31.58 \text{ nWb}, L_1 = 505.28 \text{ }\mu\text{H}, \Phi_{12} = 28.42 \text{ nWb}, M_{12} = 682.08 \text{ }\mu\text{H}$$

# Assignment 10

Solenoid Inductor 2 ชุด อยู่ใกล้กันโดยมีแกนกลางเดียวกันอยู่ในแกน  $z$  กำหนดให้  $I_1 = 3$  mA ไหลในทิศทาง  $\vec{a}_\phi$ ,  $d = 4$  cm,  $a = 3$  cm  $N_1 = 60$  รอบ,  $N_2 = 80$  รอบ  $\mu_r = 5$  และฟลักซ์แม่เหล็กไปยัง Solenoid Inductor ชุดที่ 2 ได้ 85% จงหา  $\vec{H}_1$ ,  $\vec{B}_1$ ,  $\Phi_1$ ,  $L_1$ ,  $\Phi_{12}$ ,  $M_{12}$

$$\vec{H}_1 = 4.50\vec{a}_z \text{ A/m}, \vec{B}_1 = 28.27\vec{a}_z \text{ }\mu\text{T}, \Phi_1 = 79.93 \text{ nWb}, L_1 = 1.60 \text{ mH}, \Phi_{12} = 67.94 \text{ nWb}, M_{12} = 1.81 \text{ mH}$$

$$\vec{H}_1 = \frac{N_1 I_1}{l} \vec{a}_2$$

$$= \frac{60 \times 3 \times 10^{-3}}{4 \times 10^{-2}} \vec{a}_2$$

$$\vec{H}_1 = 4.50 \vec{a}_2 \text{ A/m}$$

$$\vec{B}_1 = \mu \vec{H}_1$$

$$= 5 \times 4\pi \times 10^{-7} \times 4.50 \vec{a}_2$$

$$\vec{B}_1 = 28.27 \vec{a}_2 \mu\text{T}$$

$$\Phi_1 = \int_S \vec{B}_1 \cdot d\vec{S} = \mu a^2 \vec{B}_1$$

$$= \pi \times (3 \times 10^{-2})^2 \times 28.27 \vec{a}_2 \mu$$

$$\Phi_1 = 79.93 \text{ n Wb}$$

$$L_1 = \frac{N_1 \Phi_1}{I_1}$$

$$= \frac{60 \times 79.93 \text{ n}}{3 \times 10^{-3}}$$

$$L_1 = 1.60 \text{ mH}$$

$$\Phi_{12} = \Phi_1 \times \text{eff.}$$

$$= 79.93 \text{ n} \times \frac{85}{100}$$

$$\Phi_2 = 67.94 \text{ n Wb}$$

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$

$$= \frac{80 \times 67.94 \text{ n}}{3 \times 10^{-3}}$$

$$M_{12} = 1.81 \text{ mH}$$

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