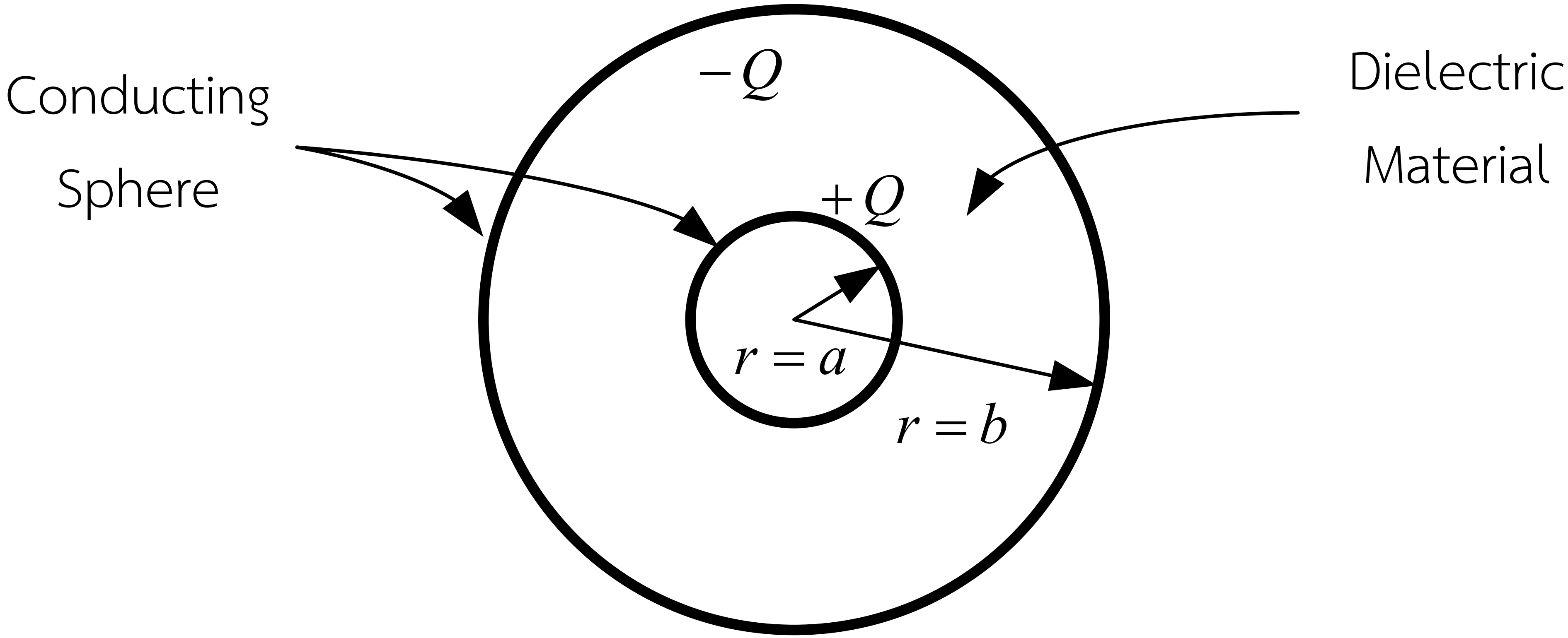
Electric Flux Density

Faraday's Experiment



$$\Psi = Q$$

Ψ : Electric Flux (C)

Electric Flux Density

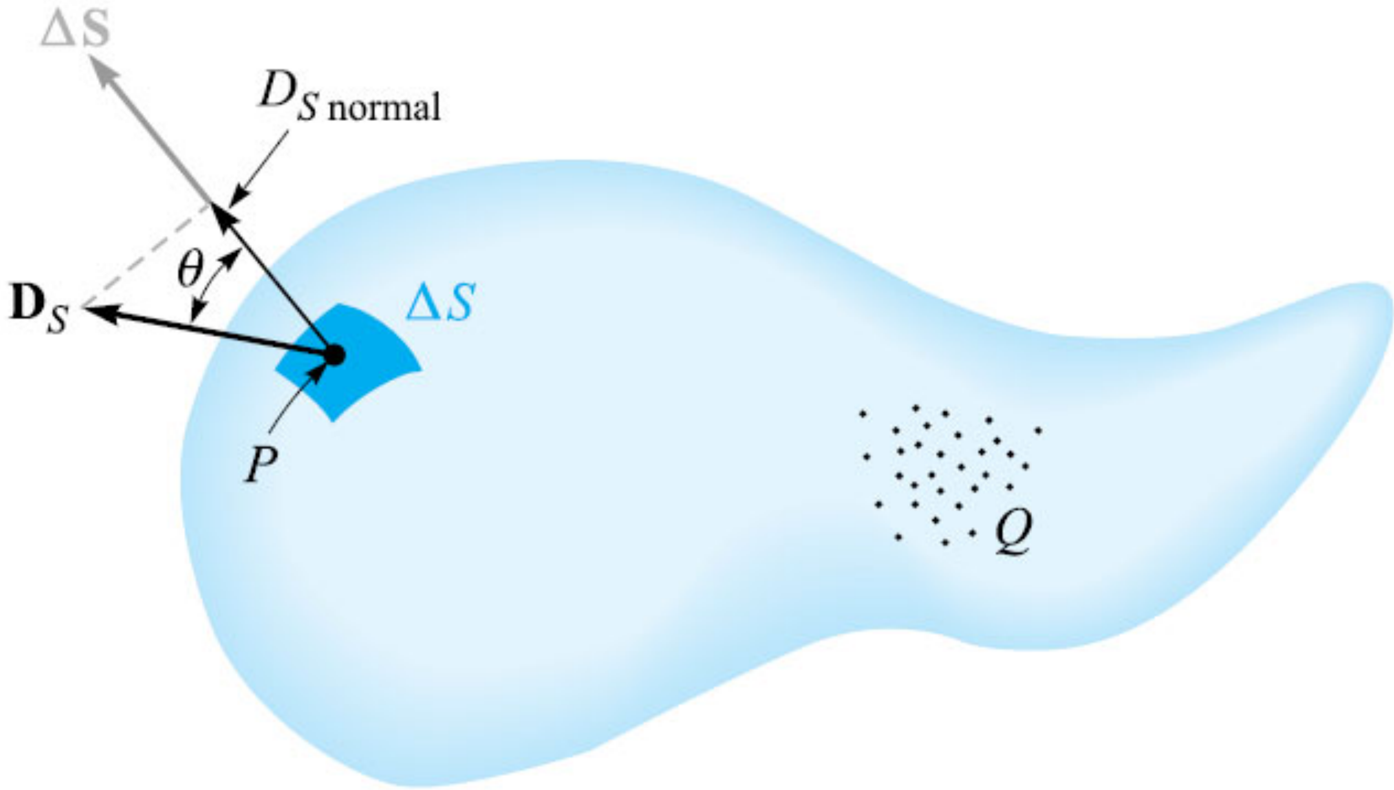
$$\vec{D} = \frac{Q}{4\pi r^2}\vec{a}_r$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

$$\boxed{\vec{D} = \varepsilon_0 \vec{E}}$$

$\vec{D}$: Electric flux density $(\mathrm{C/m^2})$

Gauss's Law



$$\boxed{Q = \oint_S \vec{D} \cdot d\vec{S}}$$

Divergence Theorem (1)

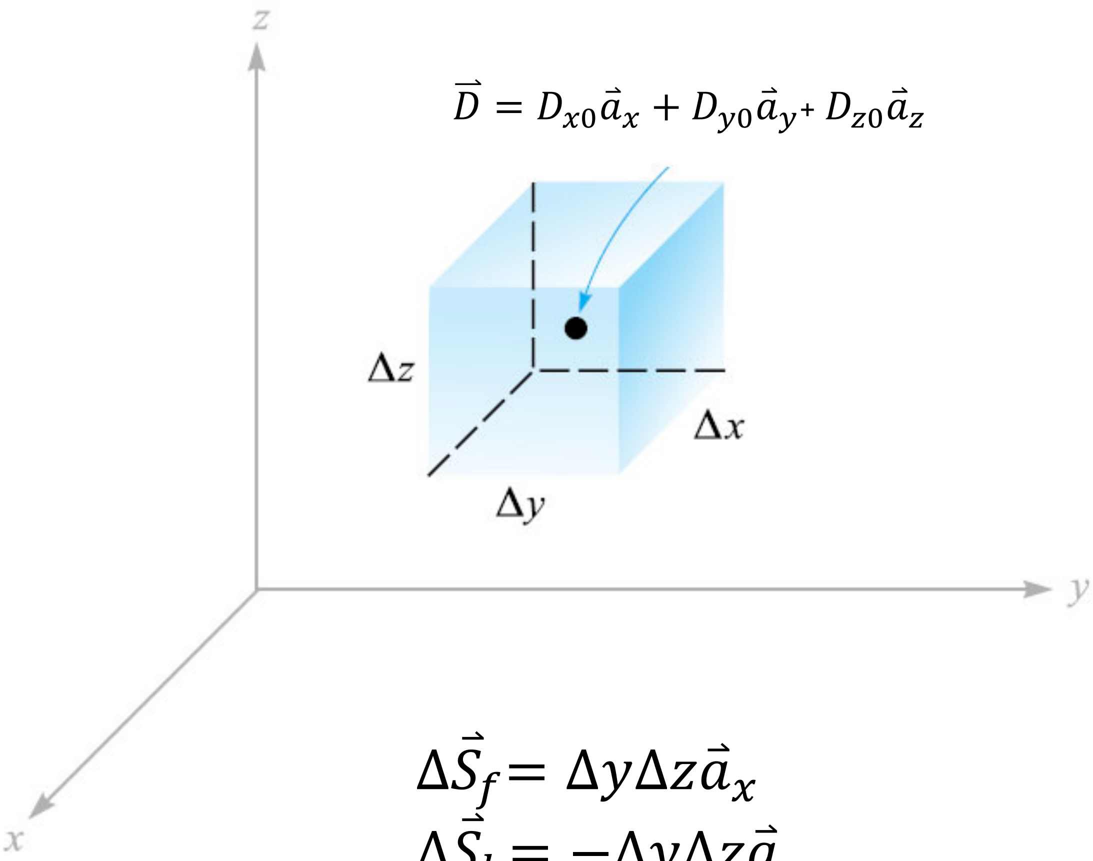
$$Q = \oint_S \vec{D} \cdot d\vec{S}$$

$$= \int_{front} \vec{D}_f \cdot d\vec{S}_f + \int_{back} \vec{D}_b \cdot d\vec{S}_b + \int_{left} \vec{D}_l \cdot d\vec{S}_l$$

$$+ \int_{right} \vec{D}_r \cdot d\vec{S}_r + \int_{top} \vec{D}_t \cdot d\vec{S}_t + \int_{bottom} \vec{D}_{b'} \cdot d\vec{S}_{b'}$$

$$= \vec{D}_f \cdot \Delta\vec{S}_f + \vec{D}_b \cdot \Delta\vec{S}_b + \vec{D}_l \cdot \Delta\vec{S}_l + \vec{D}_r \cdot \Delta\vec{S}_r + \vec{D}_t \cdot \Delta\vec{S}_t + \vec{D}_{b'} \cdot \Delta\vec{S}_{b'}$$

Divergence Theorem (2)



$$\vec{D}_f = \left(D_{x0} + \frac{\Delta x}{2}\frac{\partial}{\partial x}D_x\right)\vec{a}_x$$

$$\vec{D}_b = \left(D_{x0} - \frac{\Delta x}{2}\frac{\partial}{\partial x}D_x\right)\vec{a}_x$$

$$\vec{D}_l = \left(D_{y0} - \frac{\Delta y}{2}\frac{\partial}{\partial y}D_y\right)\vec{a}_y$$

$$\vec{D}_r = \left(D_{y0} + \frac{\Delta y}{2}\frac{\partial}{\partial y}D_y\right)\vec{a}_y$$

$$\vec{D}_t = \left(D_{z0} + \frac{\Delta z}{2}\frac{\partial}{\partial z}D_z\right)\vec{a}_z$$

$$\vec{D}_{b'} = \left(D_{z0} - \frac{\Delta z}{2}\frac{\partial}{\partial z}D_z\right)\vec{a}_z$$

$$\Delta\vec{S}_f = \Delta y\Delta z\vec{a}_x$$
$$\Delta\vec{S}_b = -\Delta y\Delta z\vec{a}_x$$
$$\Delta\vec{S}_l = -\Delta x\Delta z\vec{a}_y$$
$$\Delta\vec{S}_r = \Delta x\Delta z\vec{a}_y$$
$$\Delta\vec{S}_t = \Delta x\Delta y\vec{a}_z$$
$$\Delta\vec{S}_{b'} = -\Delta x\Delta y\vec{a}_z$$

Divergence Theorem (3)

$$
\begin{aligned}
Q &= \vec{D}_f \cdot \Delta\vec{S}_f + \vec{D}_b \cdot \Delta\vec{S}_b + \vec{D}_l \cdot \Delta\vec{S}_l + \vec{D}_r \cdot \Delta\vec{S}_r + \vec{D}_t \cdot \Delta\vec{S}_t + \vec{D}_{b'} \cdot \Delta\vec{S}_{b'} \\
&= \left(D_{x0} + \frac{\Delta x}{2}\frac{\partial}{\partial x}D_x\right)\vec{a}_x \cdot \Delta y\Delta z\vec{a}_x + \left(D_{x0} - \frac{\Delta x}{2}\frac{\partial}{\partial x}D_x\right)\vec{a}_x \cdot (-\Delta y\Delta z\vec{a}_x) \\
&\quad + \left(D_{y0} - \frac{\Delta y}{2}\frac{\partial}{\partial y}D_y\right)\vec{a}_y \cdot (-\Delta x\Delta z\vec{a}_y) + \left(D_{y0} + \frac{\Delta y}{2}\frac{\partial}{\partial y}D_y\right)\vec{a}_y \cdot \Delta x\Delta z\vec{a}_y \\
&\quad + \left(D_{z0} + \frac{\Delta z}{2}\frac{\partial}{\partial z}D_z\right)\vec{a}_z \cdot \Delta x\Delta y\vec{a}_z + \left(D_{z0} - \frac{\Delta z}{2}\frac{\partial}{\partial z}D_z\right)\vec{a}_z \cdot (-\Delta x\Delta y\vec{a}_z) \\
&= \frac{\partial}{\partial x}D_x\Delta x\Delta y\Delta z + \frac{\partial}{\partial y}D_y\Delta x\Delta y\Delta z + \frac{\partial}{\partial z}D_z\Delta x\Delta y\Delta z \\
&= \left(\frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z\right)\Delta v
\end{aligned}
$$

$$\frac{Q}{\Delta v} = \frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z$$

$$\rho_v = \nabla \cdot \vec{D}$$

Divergence Theorem (4)

$$\nabla \cdot \vec{D} = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$= \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho D_\rho \right) + \frac{1}{\rho} \frac{\partial}{\partial \phi} D_\phi + \frac{\partial}{\partial z} D_z$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_r \right) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta D_\theta \right) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} D_\phi$$

∇ : Del operator

$\nabla \cdot \vec{D}$: Diveregence of $\vec{D}$

$$\boxed{\rho_v = \nabla \cdot \vec{D}} \qquad \boxed{Q = \int_{vol} \rho_v dv}$$

Example

กำหนดให้ $\vec{D} = 2xy\vec{a}_x + x^2\vec{a}_y$ C/m^2 กำหนดพื้นผิวปิดเป็น $x = 0$ และ $1,$ $y = 0$ และ $2,$ $z = 0$ และ 3 จงหา Q โดยใช้ Gauss's Law และ Divergence Theorem

Solution (1)

หา Q โดยใช้ Gauss's Law

$$Q = \oint_S \vec{D} \cdot d\vec{S}$$

$$= \int_0^3 \int_0^2 2xy\vec{a}_x \cdot dydz\vec{a}_x + \int_0^3 \int_0^2 2xy\vec{a}_x \cdot (-dydz\vec{a}_x)$$

$$+ \int_0^3 \int_0^1 x^2\vec{a}_y \cdot dxdz\vec{a}_y + \int_0^3 \int_0^1 x^2\vec{a}_y \cdot \left(-dxdz\vec{a}_y\right)$$

$$= \int_0^3 \int_0^2 2(1)y\vec{a}_x \cdot dydz\vec{a}_x - \int_0^3 \int_0^2 2(0)y\vec{a}_x \cdot dydz\vec{a}_x$$

$$= (3)\, y^2\Big|_{y=0}^{2}$$

$$= 12 \text{ C}$$

Solution (2)

หา Q โดยใช้ Divergence Theorem

$$\rho_v = \nabla \cdot \vec{D}$$
$$= \frac{\partial}{\partial x} 2xy + \frac{\partial}{\partial y} x^2$$
$$= 2y \ \text{C/m}^3$$

$$\mathrm{Q} = \int_v \rho_v dv$$
$$= \int_0^3 \int_0^2 \int_0^1 2y dx dy dz$$
$$= (3) \left. y^2 \right|_{y=0}^{2} (1)$$
$$= 12 \ \text{C}$$

Quiz 2

กำหนดให้ $\vec{D} = xy\vec{a}_x + xz\vec{a}_z$ C/m^2 กำหนดพื้นผิวปิดเป็น $x = 0$ และ 1, $y = 0$ และ 2, $z = 0$ และ 3 จงหา Q โดยใช้ Gauss’s Law และ Divergence Theorem

$$Q = 9 \text{ C}$$

$\vec{D} = xy\vec{a}_x + xz\vec{a}_z \quad C/m^2$

$x = 0,1 \quad y = 0,2 \quad z = 0,3$

Gauss's Law

$Q = \oint \vec{D} \cdot d\vec{s}$

Front ↔ Back Top ↕ Bottom

$= \int_0^3 \int_0^2 \cancel{x}^{1} y\hat{a}_x \cdot (dy\,dz\,\hat{a}_x) + \int_0^3 \int_0^2 \cancel{x}^{0} y\hat{a}_x \cdot (-dy\,dz\,\hat{a}_x) + \int_0^2 \int_0^1 x\cancel{z}^{3}\hat{a}_z \cdot (dx\,dy\,\hat{a}_z) + \int_0^2 \int_0^1 x\cancel{z}^{0}\hat{a}_z \cdot (-dx\,dy\,\hat{a}_z)$

$= \int_0^3 \int_0^2 y\hat{a}_x \cdot dy\,dz\,\hat{a}_x + \int_0^2 \int_0^1 3x\hat{a}_z \cdot dx\,dy\,\hat{a}_z$

$= \int_0^3 \frac{y^2}{2}\Big|_0^2 \hat{a}_x \, dz\,\hat{a}_x + \int_0^2 \frac{3x^2}{2}\Big|_0^1 \hat{a}_z \, dy\,\hat{a}_z$

$= \int_0^3 [2-0]\hat{a}_x \, dz\,\hat{a}_x + \int_0^2 [\frac{3}{2}-0]\hat{a}_z \, dy\,\hat{a}_z$

$= \int_0^3 2\hat{a}_x \, dz\,\hat{a}_x + \int_0^2 \frac{3}{2}\hat{a}_z \, dy\,\hat{a}_z$

$= 2z\Big|_0^3 + \frac{3}{2}y\Big|_0^2$

$= 2(3-0) + \frac{3}{2}(2-0)$

$= 2(3) + \frac{3}{\cancel{2}}\cancel{2}$

$= 6 + 3$

$= 9\ C$

$\rho_v = \nabla \cdot \vec{D}$

Divergence Theorem

$= \frac{\partial}{\partial x} xy + \frac{\partial}{\partial z} xz$

$= y + x \quad C/m^3$

$Q = \int_v \rho_v \, dv$

$= \int_0^3 \int_0^2 \int_0^1 y + x \, dx\,dy\,dz$

$= \int_0^3 \int_0^2 yx\Big|_0^1 + \frac{x^2}{2}\Big|_0^1 dy\,dz$

$= \int_0^3 \int_0^2 y[1-0] + [\frac{1^2}{2}-0] \, dy\,dz$

$= \int_0^3 \int_0^2 y + \frac{1}{2} \, dy\,dz$

$= \int_0^3 \frac{y^2}{2}\Big|_0^2 + \frac{1}{2}y\Big|_0^2 \, dz$

$= \int_0^3 [\frac{2^2}{2}-0] + \frac{1}{2}[2-0] \, dz$

$= \int_0^3 2+1 \, dz = \int_0^3 3 \, dz$

$= 3z\Big|_0^3$

$= 3[3-0]$

$= 9\ C$

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Assignment 2

กำหนดให้ $\vec{D} = 3xyz^2\vec{a}_x + 3x^2y^2z^2\vec{a}_y + 3x^2yz\vec{a}_z$ C/m^2 กำหนดพื้นผิวปิดเป็น $x = -2$ และ $2,$ $y = -1$ และ $2,$ $z = -1$ และ 1 จงหา Q โดยใช้ Gauss's Law และ Divergence Theorem

$$Q = 92 \ \mathrm{C}$$

Gauss's Law

$$Q = \oint_S \vec{D} \cdot d\vec{s}$$

$$= \int_{-1}^{1}\int_{-1}^{2} 3\cancelto{2}{x}yz^2\vec{a}_x \cdot dydz\vec{a}_x + \int_{-1}^{1}\int_{-1}^{2} 3\cancelto{-2}{x}yz^2 \cdot (-dydz\vec{a}_x) + \int_{-1}^{1}\int_{-2}^{2} 3x^2\cancelto{2}{y}^2z^2\vec{a}_y \cdot dxdz\vec{a}_y + \int_{-1}^{1}\int_{-2}^{2} 3x^2\cancelto{-1}{y}^2z^2\vec{a}_y \cdot (-dxdz\vec{a}_y) + \int_{-1}^{2}\int_{-2}^{2} 3x^2y\cancelto{1}{z}\vec{a}_z \cdot dxdy\vec{a}_z + \int_{-1}^{2}\int_{-2}^{2} 3x^2y\cancelto{-1}{z}\vec{a}_z \cdot (-dxdy\vec{a}_z)$$

$$= \int_{-1}^{1}\int_{-1}^{2} 6yz^2\vec{a}_x \cdot dydz\vec{a}_x - \int_{-1}^{1}\int_{-1}^{2} -6yz^2\, dydz\vec{a}_x + \int_{-1}^{1}\int_{-2}^{2} 12x^2z^2\vec{a}_y \cdot dxdz\vec{a}_y - \int_{-1}^{1}\int_{-2}^{2} 3x^2z^2\vec{a}_y \cdot dxdz\vec{a}_y + \int_{-1}^{2}\int_{-2}^{2} 3x^2y\vec{a}_z \cdot dxdy\vec{a}_z - \int_{-1}^{2}\int_{-2}^{2} -3x^2y\vec{a}_z \cdot dxdy\vec{a}_z$$

$$= 6 - (-6) + \frac{128}{3} - \frac{32}{3} + 24 - (-24)$$

$$\underline{= 92\ c}$$

Divergence Theorem

$$\rho_v = \nabla \cdot \vec{D}$$

$$= \frac{\partial}{\partial x}3xyz^2 + \frac{\partial}{\partial y}3x^2y^2z^2 + \frac{\partial}{\partial z}3x^2yz$$

$$= 3yz^2 + 6x^2yz^2 + 3x^2y$$

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$$Q = \int_v \rho_v\, dv$$

$$= \int_{-1}^{1}\int_{-1}^{2}\int_{-2}^{2} 3yz^2 + 6x^2yz^2 + 3x^2y\ dxdydz$$

$$\underline{= 92\ c}$$

$$\int_{-1}^{1}\int_{-1}^{2}\int_{-2}^{2} 3yz^2\, dxdydz + \int_{-1}^{1}\int_{-1}^{2}\int_{-2}^{2} 6x^2yz^2 dxdydz + \int_{-1}^{1}\int_{-1}^{2}\int_{-2}^{2} 3x^2y\ dxdydz$$

$$= 12 + 32 + 48$$

$$= 92c$$