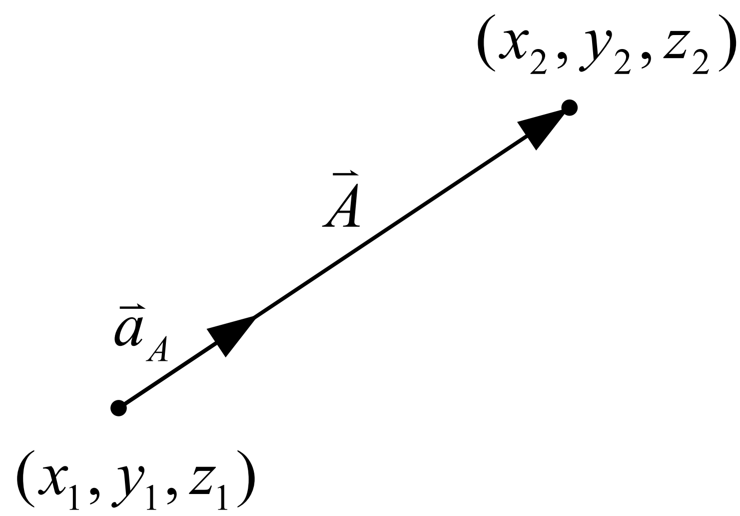
Vector analysis

Scalar and Vector

- Scalar
  - Magnitude
  - $L = 5 \text{ m}$
- Vector
  - Magnitude and direction
  - $\vec{E} = 2\vec{a}_x + 3\vec{a}_y - 4\vec{a}_z \text{ V/m}$

Vector Theory (1)

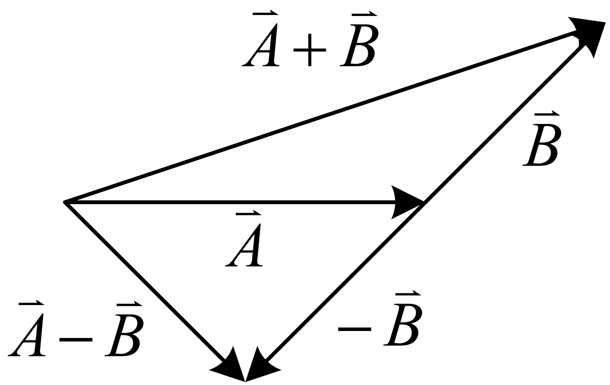


$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$
$$= (x_2 - x_1)\vec{a}_x + (y_2 - y_1)\vec{a}_y + (z_2 - z_1)\vec{a}_z$$

$$A = \left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{a}_A = \frac{\vec{A}}{A} = \frac{A_x}{A}\vec{a}_x + \frac{A_y}{A}\vec{a}_y + \frac{A_z}{A}\vec{a}_z$$

Vector Theory (2)



$$\vec{A} \pm \vec{B} = \left(A_x \pm B_x\right)\vec{a}_x + \left(A_y \pm B_y\right)\vec{a}_y + \left(A_z \pm B_z\right)\vec{a}_z$$

Vector Theory (3)

- Dot Product (Scalar Product)

$$\vec{A}\cdot\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

- Cross Product

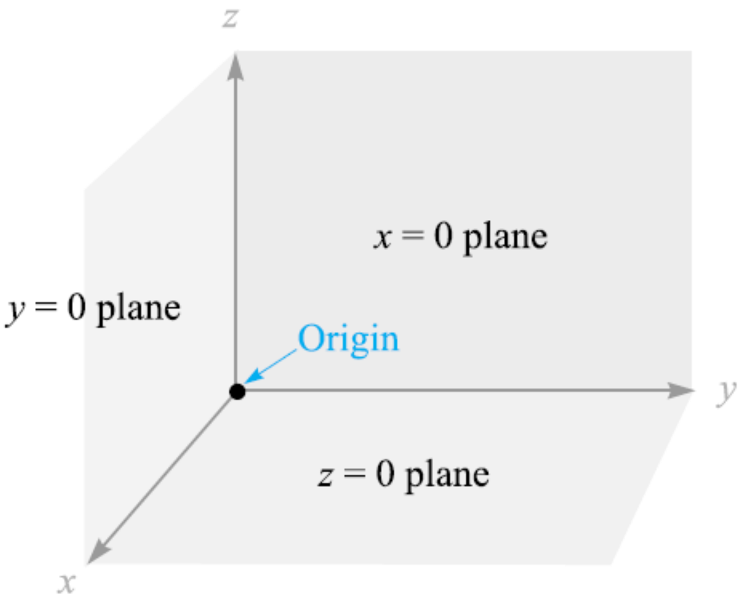
$$\vec{A}\times\vec{B} = \vec{a}_N\left|\vec{A}\right|\left|\vec{B}\right|\sin\theta = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

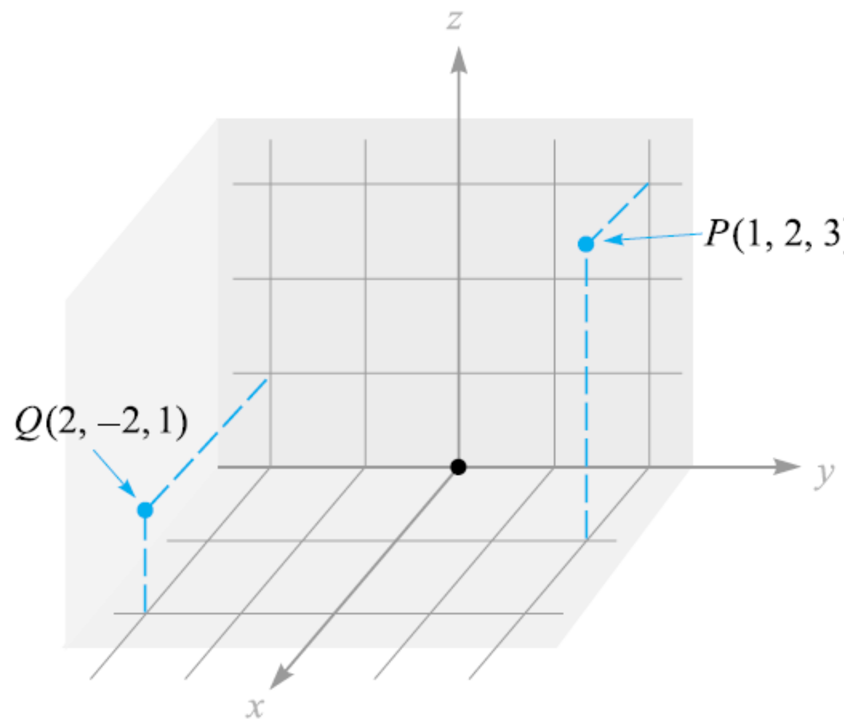
$$= \left(A_y B_z - A_z B_y\right)\vec{a}_x + \left(A_z B_x - A_x B_z\right)\vec{a}_y + \left(A_x B_y - A_y B_x\right)\vec{a}_z$$

$\vec{a}_N$: Right - Hand Rule

Rectangular Coordinate System (1)

Cartesian





Dot Product :

$$\vec{a}_x \cdot \vec{a}_x = \vec{a}_y \cdot \vec{a}_y = \vec{a}_z \cdot \vec{a}_z = 1$$

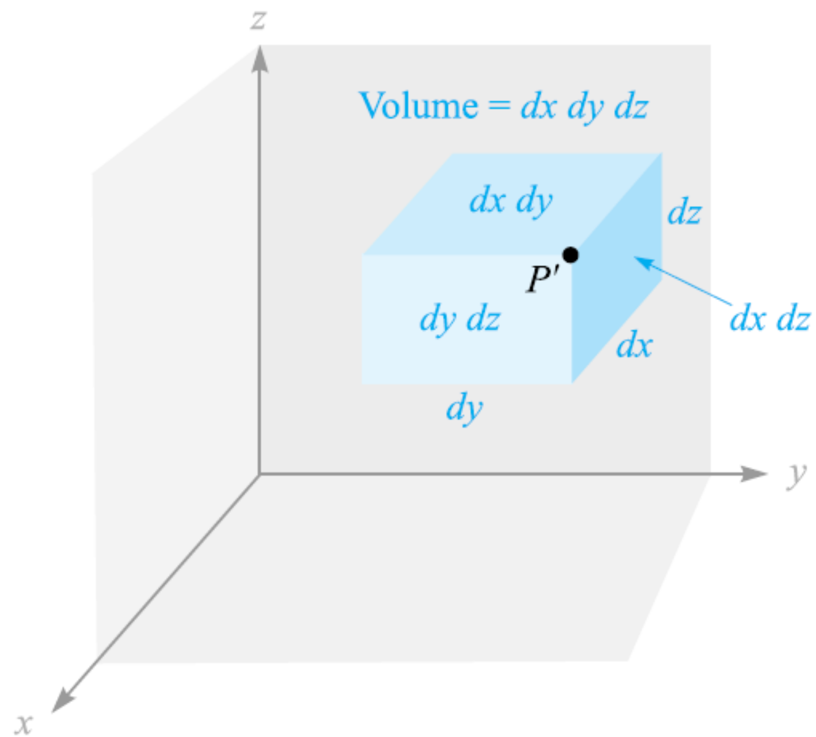
$$\vec{a}_x \cdot \vec{a}_y = \vec{a}_y \cdot \vec{a}_z = \vec{a}_z \cdot \vec{a}_x = 0$$

Cross Product :

$$\vec{a}_x \times \vec{a}_x = \vec{a}_y \times \vec{a}_y = \vec{a}_z \times \vec{a}_z = 0$$

$$\vec{a}_x \times \vec{a}_y = \vec{a}_z, \ \vec{a}_y \times \vec{a}_z = \vec{a}_x, \ \vec{a}_z \times \vec{a}_x = \vec{a}_y$$

Rectangular Coordinate System (2)

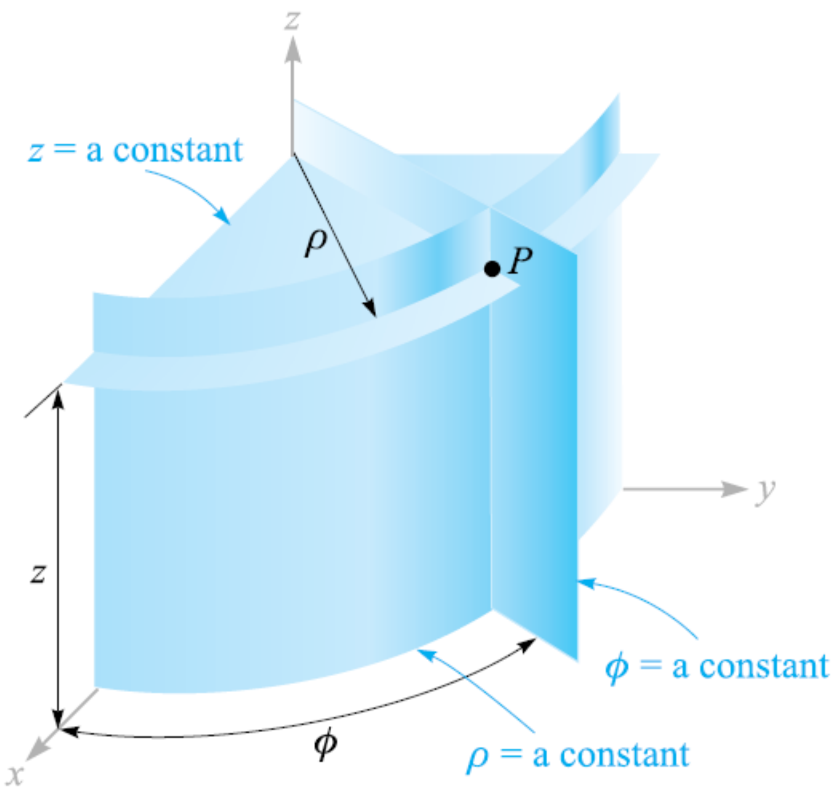


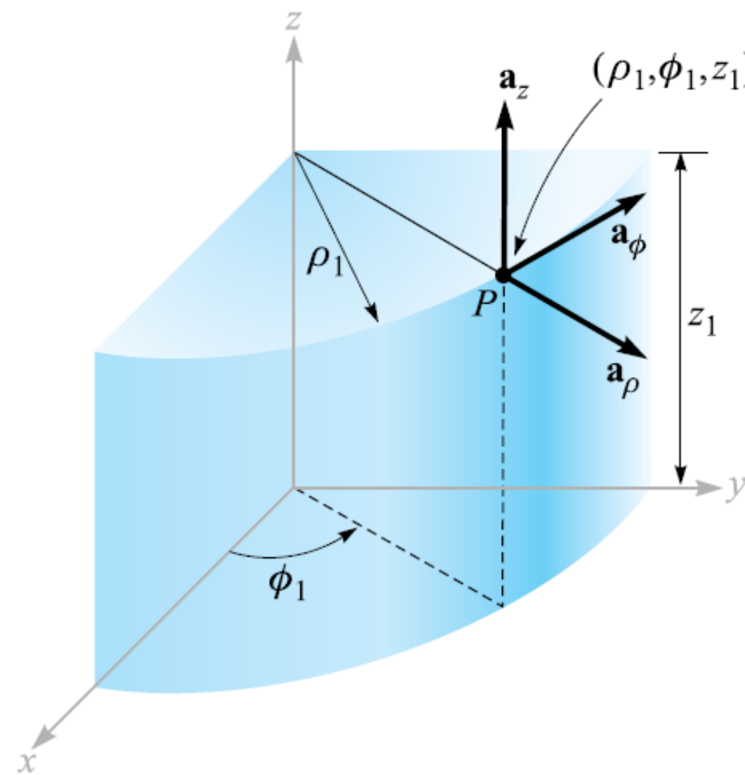
$$d\vec{L} = dx\vec{a}_x + dy\vec{a}_y + dz\vec{a}_z$$

$$d\vec{S} = \pm dxdy\vec{a}_z,\ \pm dxdz\vec{a}_y,\ \pm dydz\vec{a}_x$$

$$dv = dxdydz$$

Cylindrical Coordinate System (1)





Dot Product:

$$\vec{a}_\rho \cdot \vec{a}_\rho = \vec{a}_\phi \cdot \vec{a}_\phi = \vec{a}_z \cdot \vec{a}_z = 1$$

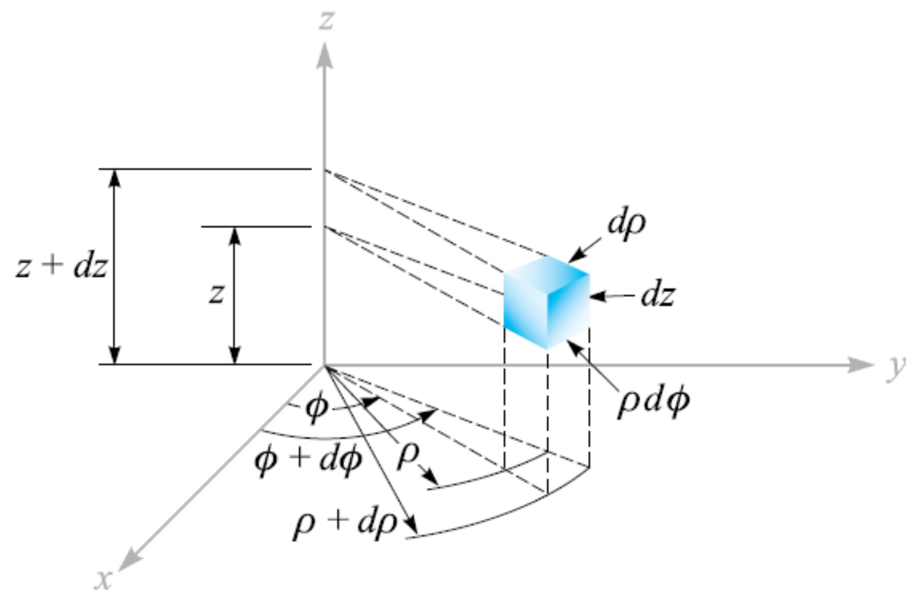
$$\vec{a}_\rho \cdot \vec{a}_\phi = \vec{a}_\phi \cdot \vec{a}_z = \vec{a}_z \cdot \vec{a}_\rho = 0$$

Cross Product:

$$\vec{a}_\rho \times \vec{a}_\rho = \vec{a}_\phi \times \vec{a}_\phi = \vec{a}_z \times \vec{a}_z = 0$$

$$\vec{a}_\rho \times \vec{a}_\phi = \vec{a}_z, \ \vec{a}_\phi \times \vec{a}_z = \vec{a}_\rho, \ \vec{a}_z \times \vec{a}_\rho = \vec{a}_\phi$$

Cylindrical Coordinate System (2)

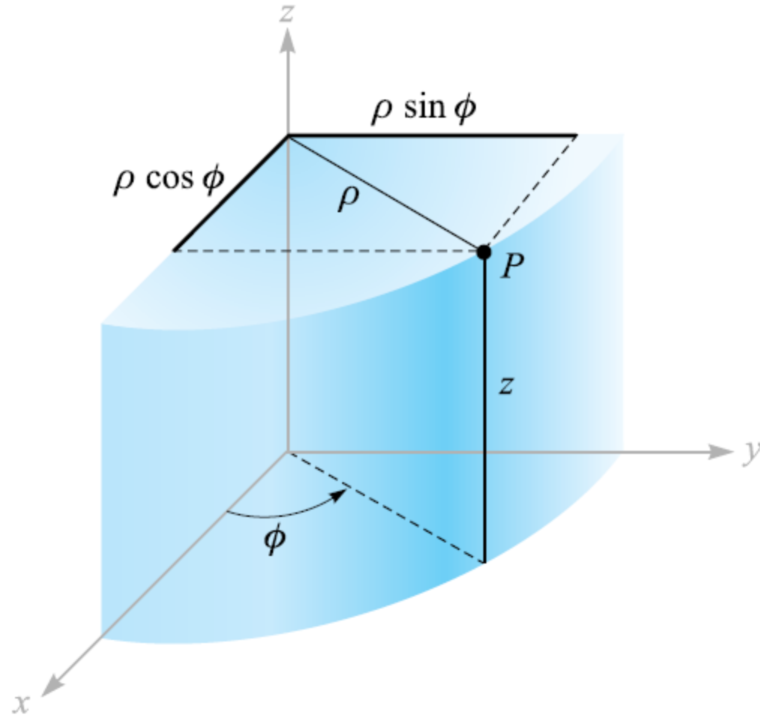


$$d\vec{L} = d\rho\vec{a}_\rho + \rho d\phi\vec{a}_\phi + dz\vec{a}_z$$

$$d\vec{S} = \pm\rho d\rho d\phi\vec{a}_z,\ \pm d\rho dz\vec{a}_\phi,\ \pm\rho d\phi dz\vec{a}_\rho$$

$$dv = \rho d\rho d\phi dz$$

Cylindrical Coordinate System (3)



$$x = \rho \cos(\phi)$$

$$y = \rho \sin(\phi)$$

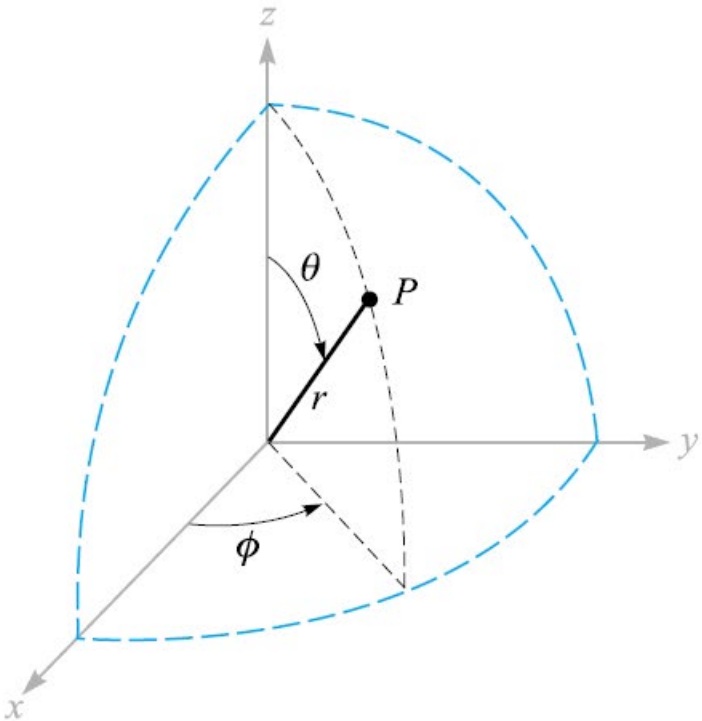
$$z = z$$

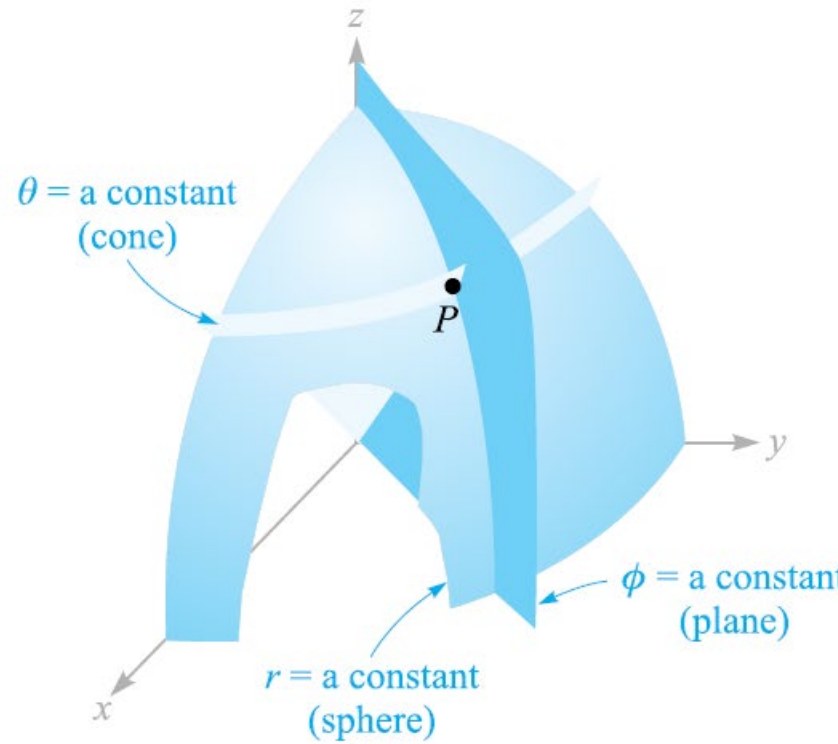
$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$z = z$$

Spherical Coordinate System (1)





Dot Product:

$$\vec{a}_r \cdot \vec{a}_r = \vec{a}_\theta \cdot \vec{a}_\theta = \vec{a}_\phi \cdot \vec{a}_\phi = 1$$

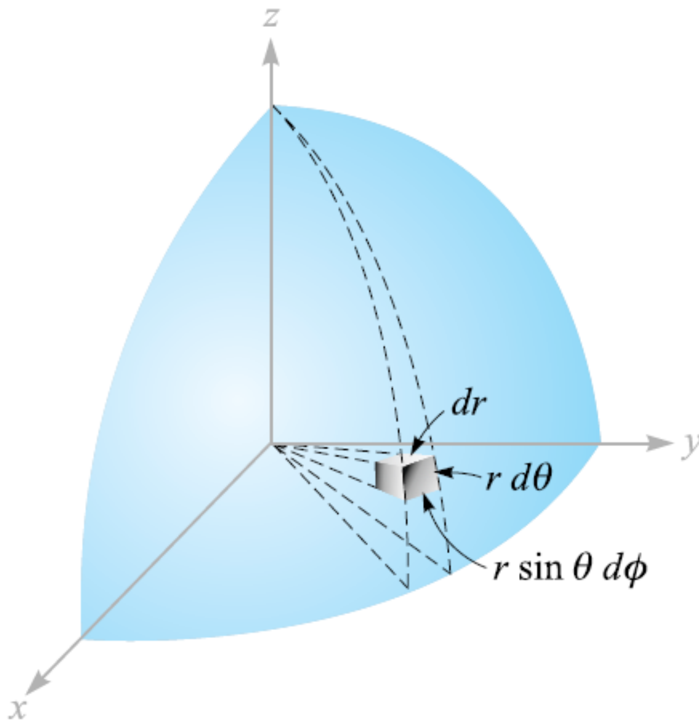
$$\vec{a}_r \cdot \vec{a}_\theta = \vec{a}_\theta \cdot \vec{a}_\phi = \vec{a}_\phi \cdot \vec{a}_r = 0$$

Cross Product:

$$\vec{a}_r \times \vec{a}_r = \vec{a}_\theta \times \vec{a}_\theta = \vec{a}_\phi \times \vec{a}_\phi = 0$$

$$\vec{a}_r \times \vec{a}_\theta = \vec{a}_\phi, \;\; \vec{a}_\theta \times \vec{a}_\phi = \vec{a}_r, \;\; \vec{a}_\phi \times \vec{a}_r = \vec{a}_\theta$$

Spherical Coordinate System (2)

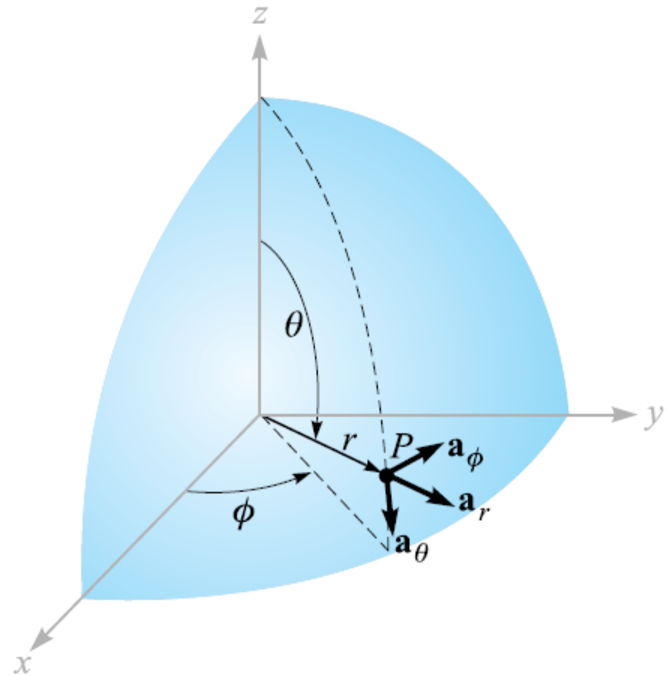


$$d\vec{L} = dr\vec{a}_r + rd\theta\vec{a}_\theta + r\sin(\theta)d\phi\vec{a}_\phi$$

$$d\vec{S} = \pm rdrd\theta\vec{a}_\phi,\ \pm r\sin\theta drd\phi\vec{a}_\theta,\ \pm r^2\sin\theta d\theta d\phi\vec{a}_r$$

$$dv = r^2\sin\theta drd\theta d\phi$$

Spherical Coordinate System (3)



$$x = r\sin(\theta)\cos(\phi) \qquad r = \sqrt{x^2 + y^2 + z^2}$$

$$y = r\sin(\theta)\sin(\phi) \qquad \theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

$$z = r\cos(\theta) \qquad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

Application (1)

- Circumference of Circle

$$dL = \rho d\phi$$

$$L = \rho \oint d\phi$$

$$= \rho \int_{0}^{2\pi} d\phi$$

$$= 2\pi\rho$$

Application (2)

- Area of Circle

$$dS = \rho d\rho d\phi$$

$$S = \int_0^{2\pi} \int_0^{r} \rho \, d\rho d\phi$$

$$= \int_0^{2\pi} \frac{\rho^2}{2}\bigg|_0^r \, d\phi$$

$$= \int_0^{2\pi} \frac{r^2}{2} d\phi$$

$$= \pi r^2$$

Application (3)

- Spherical Surface

$$dS = r^2 \sin\theta \, d\theta d\phi$$

$$S = r^2 \oiint \sin\theta \, d\theta d\phi$$

$$= r^2 \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta d\phi$$

$$= r^2 \int_0^{2\pi} \left(-\cos\theta \Big|_0^{\pi} \right) d\phi$$

$$= r^2 \int_0^{2\pi} 2 d\phi$$

$$= 4\pi r^2$$

Application (4)

- Spherical Volume

$$dv = r^2 \sin\theta \, dr d\theta d\phi$$

$$S = \int_0^{2\pi} \int_0^{\pi} \int_0^{a} r^2 \sin\theta \, dr d\theta d\phi$$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{a^3}{3} \sin\theta \, d\theta d\phi$$

$$= \int_0^{2\pi} \frac{2a^3}{3} d\phi$$

$$= \frac{4\pi a^3}{3}$$