PHYS1001B College Physics IB

Optics III Interference (Ch. 35)

Introduction

Optical effects that depend on the wave nature of light are grouped under the heading **physical optics.**

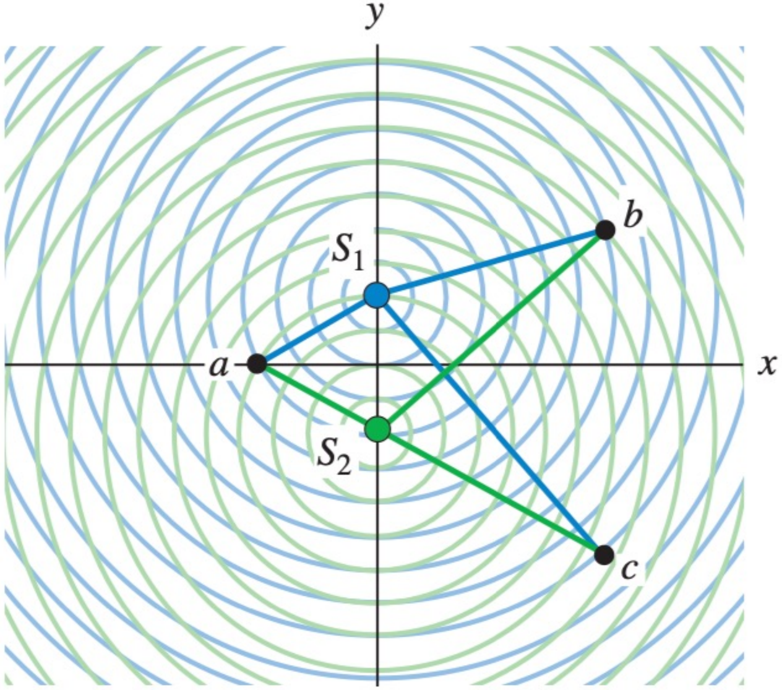
Interference/ diffraction

Outline

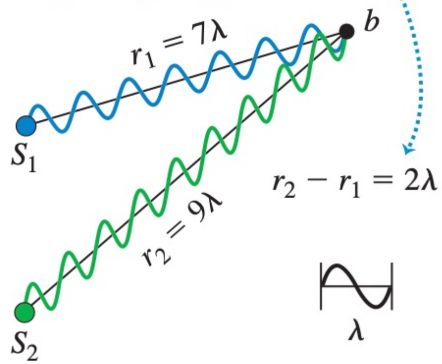
- 35-1 Interference and Coherent Sources
- 35-2 Two-Source Interference of Light
- 35-3 Intensity in Interference Patterns

35-1 Interference and Coherent Sources

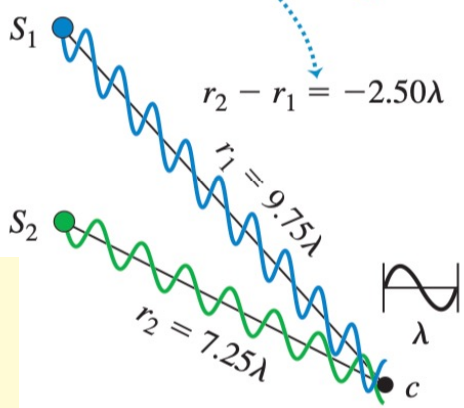
(a) Two coherent wave sources separated by a distance 4λ



(b) Conditions for constructive interference:
Waves interfere constructively if their path lengths differ by an integral number of wavelengths: $r_2 - r_1 = m\lambda$.



(c) Conditions for destructive interference:
Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths: $r_2 - r_1 = (m + \frac{1}{2})\lambda$.



principle of superposition

When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

35-1 Interference and Coherent Sources

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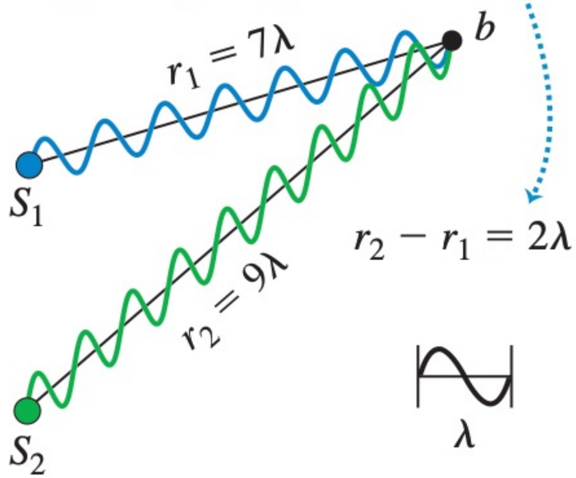
Waves interfere constructively if their path lengths differ by an integral number of wavelengths: $r_2 - r_1 = m\lambda$.



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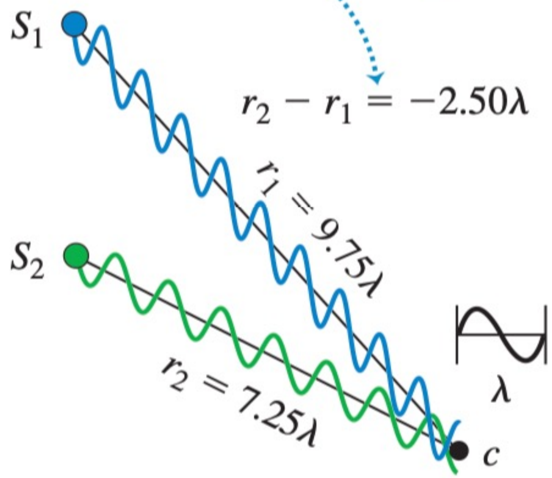


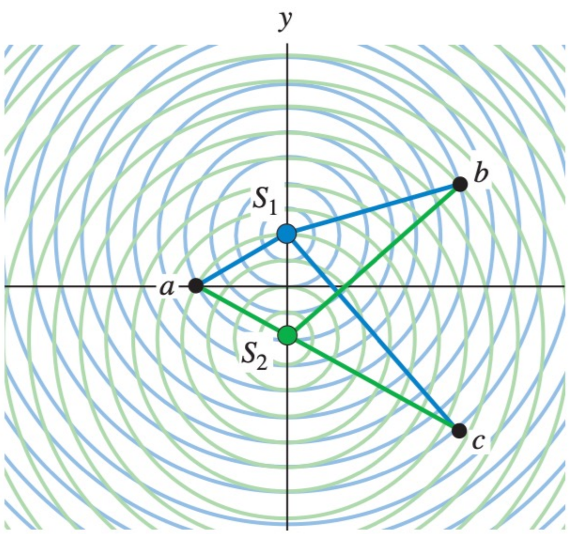
$$r_2 - r_1 = m\lambda \qquad (m = 0, \pm 1, \pm 2, \pm 3, \dots)$$

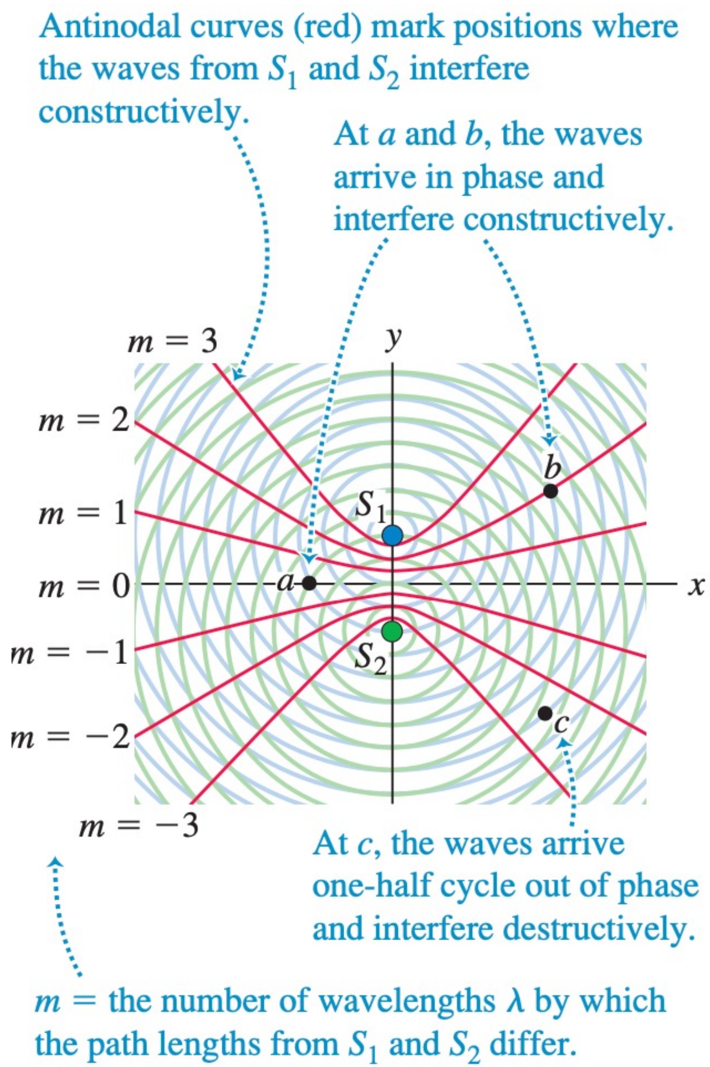
(constructive interference, sources in phase)

$$r_2 - r_1 = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \dots)$$

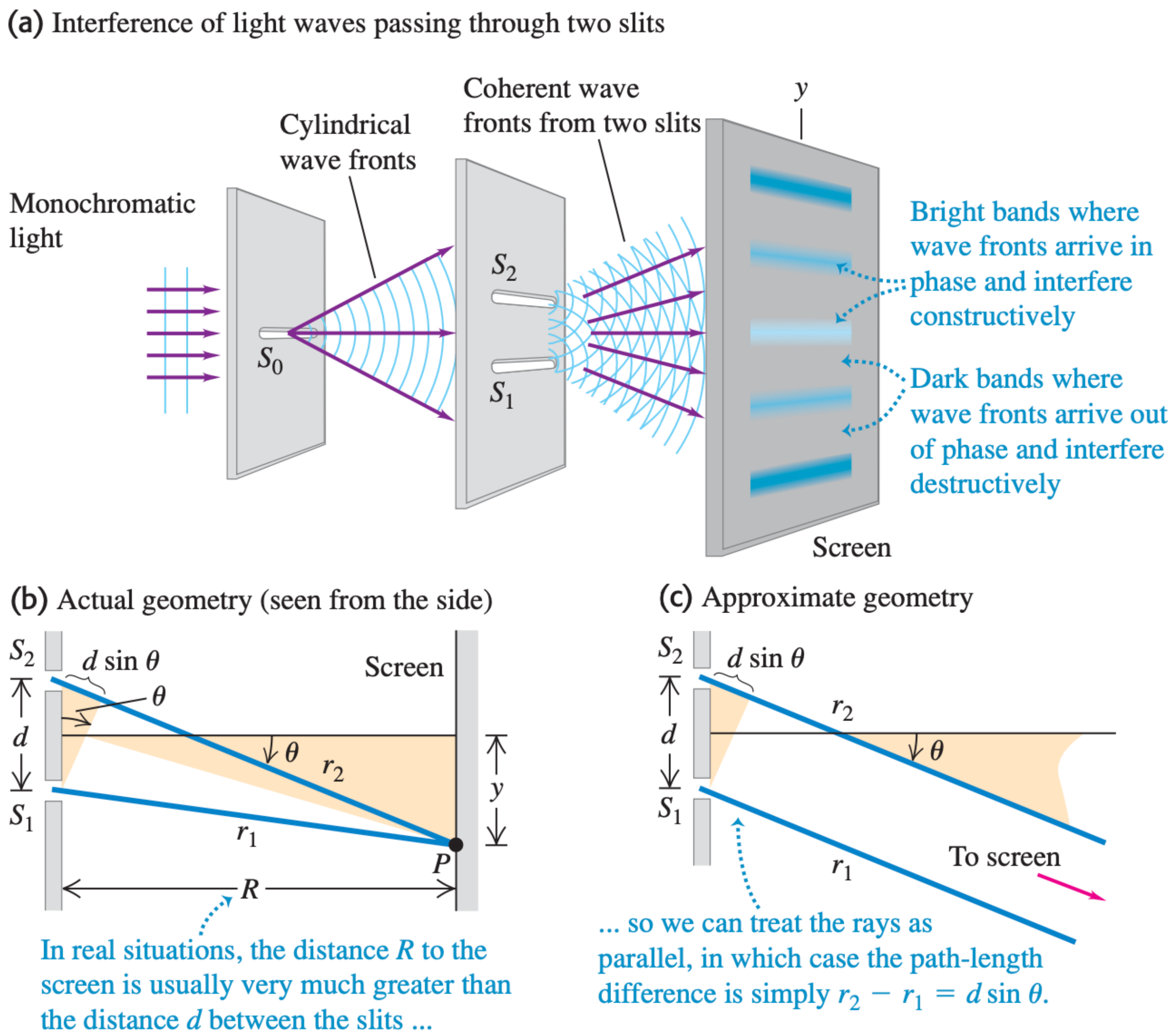
(destructive interference, sources in phase)

35-1 Interference and Coherent Sources

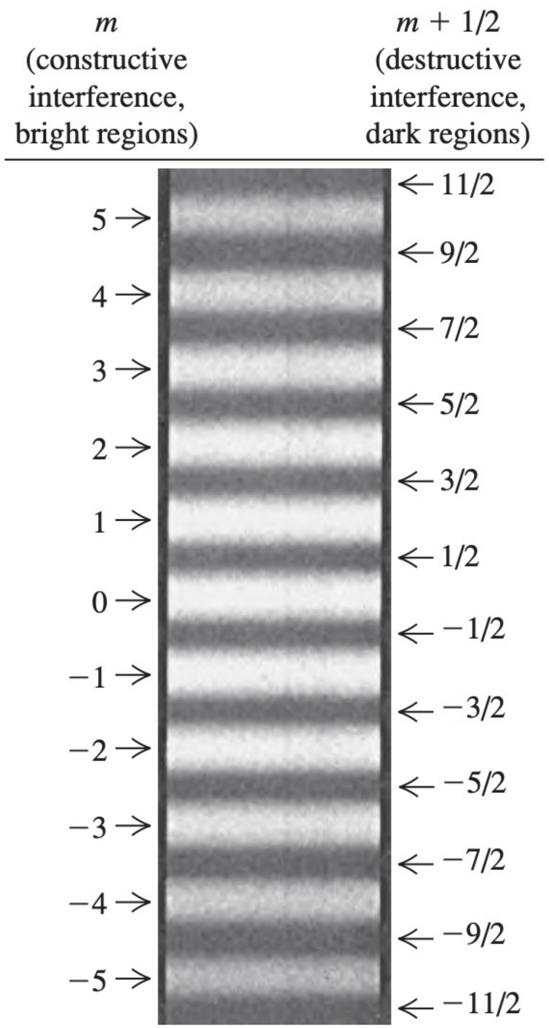




35-2 Two-Source Interference of Light



35-2 Two-Source Interference of Light



Young's double- slit

Constructive interference

$$d\sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \dots)$$

Constructive interference with small angle

$$y_m = R\frac{m\lambda}{d}$$

Destructive interference

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, \pm 1, \pm 2, \dots)$$

Sample Problem

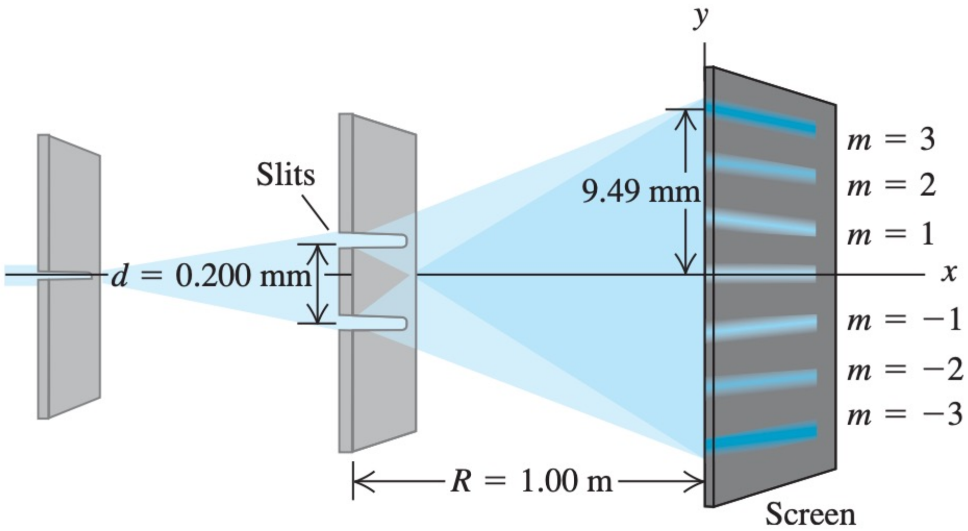
Example 35.1 **Two-slit interference**

Figure 35.7 shows a two-slit interference experiment in which the slits are 0.200 mm apart and the screen is 1.00 m from the slits. The $m = 3$ bright fringe in the figure is 9.49 mm from the central fringe. Find the wavelength of the light.

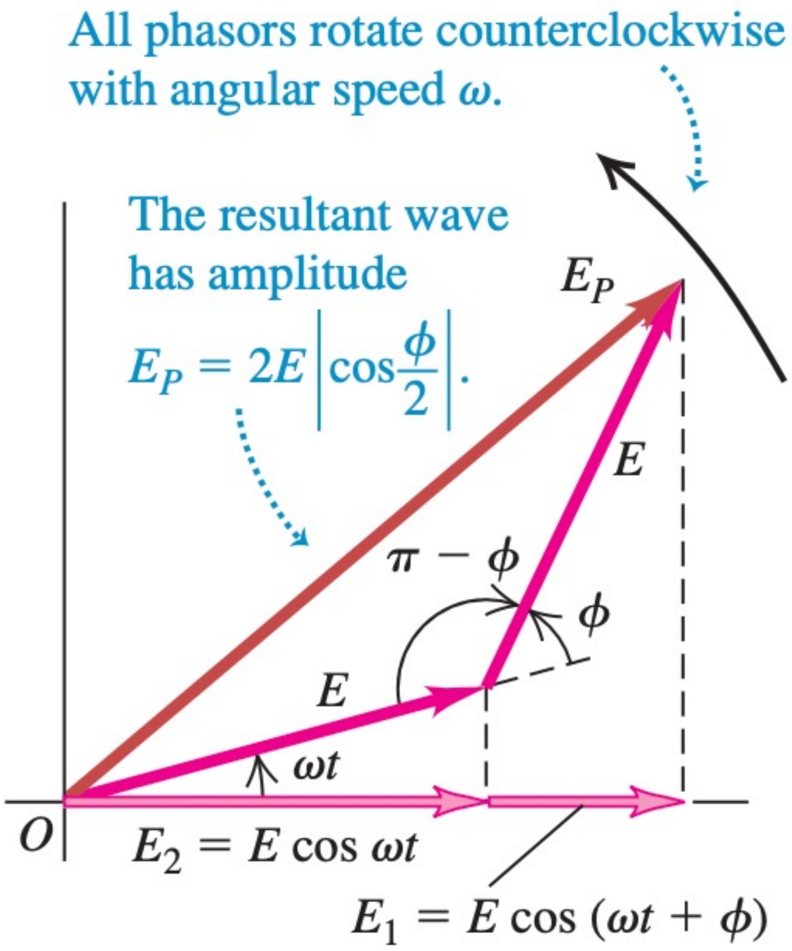
EXECUTE: We solve Eq. (35.6) for λ for the case $m = 3$:

$$\lambda = \frac{y_m d}{mR} = \frac{(9.49 \times 10^{-3}\ \text{m})(0.200 \times 10^{-3}\ \text{m})}{(3)(1.00\ \text{m})}$$

$$= 633 \times 10^{-9}\ \text{m} = 633\ \text{nm}$$



35-3 Intensity in Interference Patterns



$$E_P{}^2 = E^2 + E^2 - 2E^2\cos(\pi - \phi)$$
$$= E^2 + E^2 + 2E^2\cos\phi$$

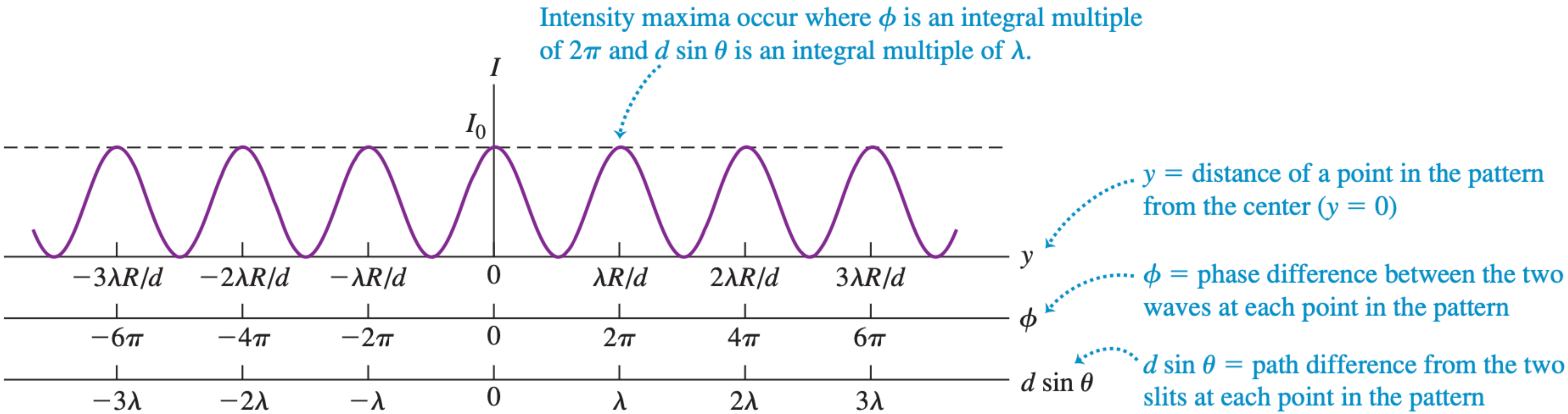
$$E_P{}^2 = 2E^2(1 + \cos\phi) = 4E^2\cos^2\left(\frac{\phi}{2}\right)$$

$$E_P = 2E\left|\cos\frac{\phi}{2}\right|$$

$$I = S_{av} = \frac{E_P{}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_P{}^2 = \frac{1}{2}\epsilon_0 c E_P{}^2$$

$$I = I_0\cos^2\frac{\phi}{2}$$

35-3 Intensity in Interference Patterns



$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1)$$ (phase difference related to path difference)

Sample Problem

Example 35.3 A directional transmitting antenna array

Suppose the two identical radio antennas of Fig. 35.8 are moved to be only 10.0 m apart and the broadcast frequency is increased to $f = 60.0$ MHz. At a distance of 700 m from the point midway between the antennas and in the direction $\theta = 0$ (see Fig. 35.8), the intensity is $I_0 = 0.020\ \mathrm{W/m^2}$. At this same distance, find (a) the intensity in the direction $\theta = 4.0°$; (b) the direction near $\theta = 0$ for which the intensity is $I_0/2$; and (c) the directions in which the intensity is zero.

EXECUTE: The wavelength is $\lambda = c/f = 5.00$ m. The spacing $d = 10.0$ m between the antennas is just twice the wavelength (as was the case in Example 35.2), so $d/\lambda = 2.00$ and Eq. (35.14) becomes

$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) = I_0 \cos^2[(2.00\pi \text{ rad})\sin\theta]$$

(a) When $\theta = 4.0°$,

$$I = I_0 \cos^2[(2.00\pi \text{ rad})\sin 4.0°] = 0.82 I_0$$
$$= (0.82)(0.020\ \mathrm{W/m^2}) = 0.016\ \mathrm{W/m^2}$$

(b) The intensity I equals $I_0/2$ when the cosine in Eq. (35.14) has the value $\pm 1/\sqrt{2}$. The smallest angles at which this occurs correspond to $2.00\pi \sin\theta = \pm\pi/4$ rad, so that $\sin\theta = \pm(1/8.00) = \pm 0.125$ and $\theta = \pm 7.2°$.

(c) The intensity is zero when $\cos[(2.00\pi \text{ rad})\sin\theta] = 0$. This occurs for $2.00\pi \sin\theta = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2, \ldots$, or $\sin\theta = \pm 0.250, \pm 0.750, \pm 1.25, \ldots$. Values of $\sin\theta$ greater than 1 have no meaning, so the answers are

$$\theta = \pm 14.5°, \pm 48.6°$$