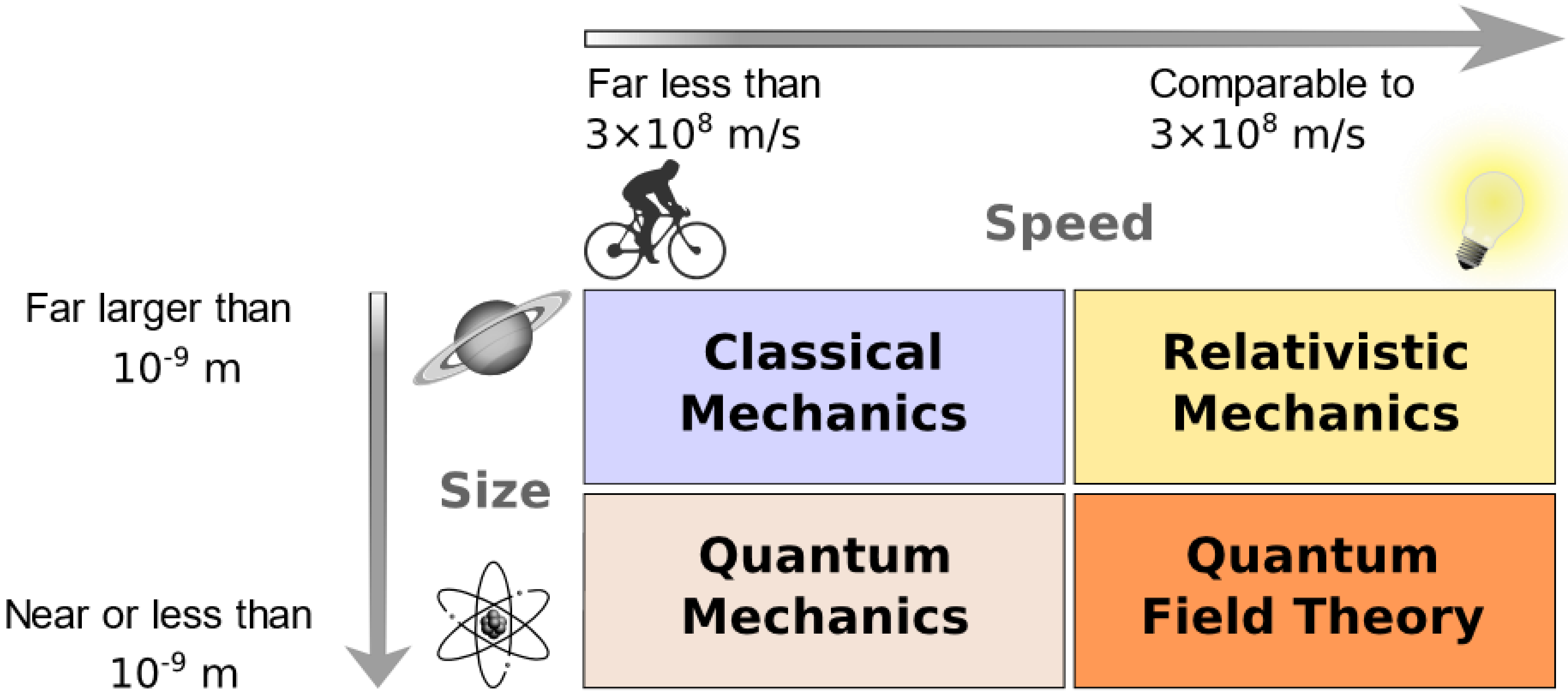
# PHYS1001B College Physics IB

Modern Physics I Photons: Light Waves Behaving as Particles
(Ch. 38)

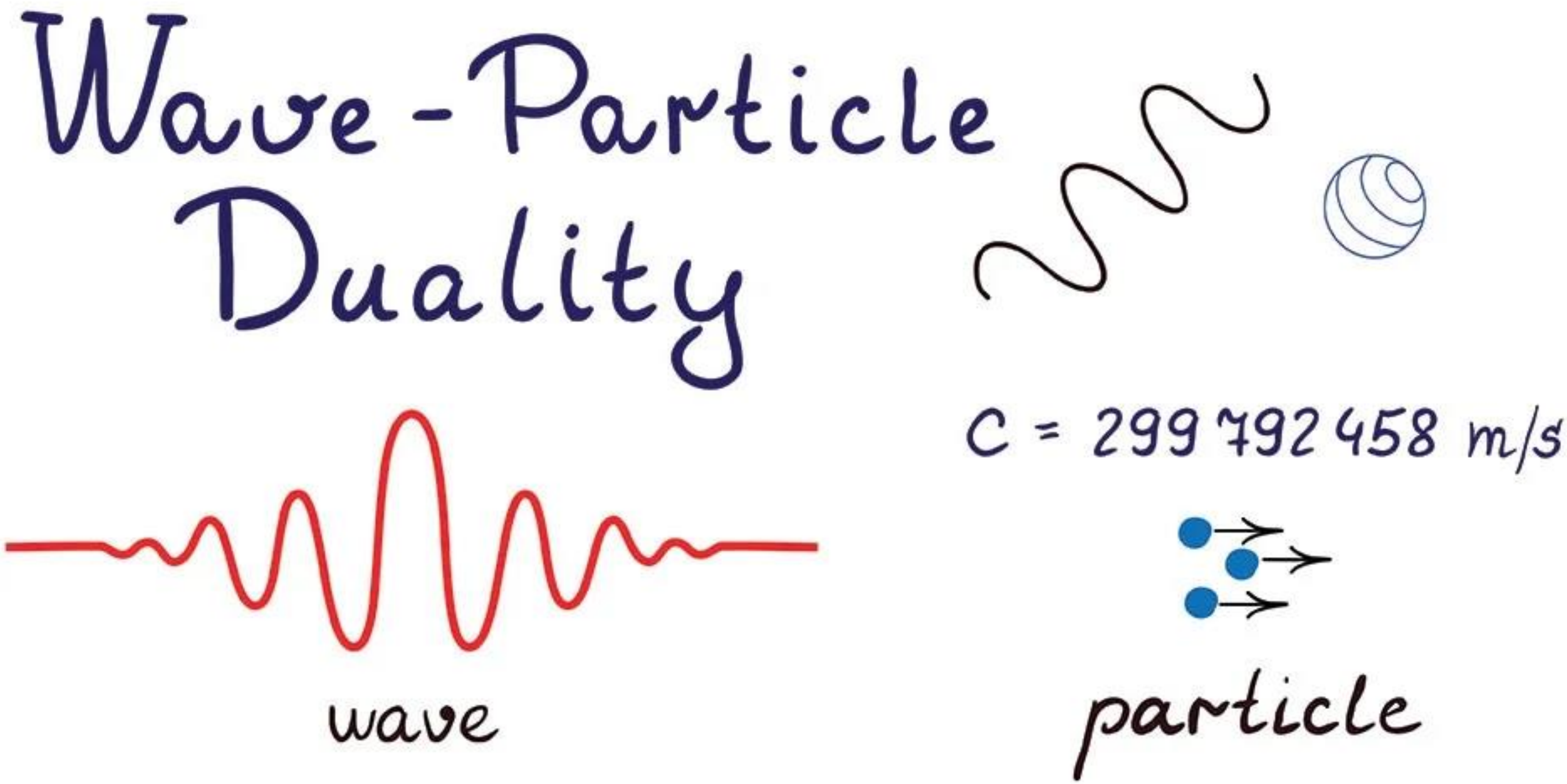
# Where quantum mechanics can be applied?




Source: wikipedia

# Outline

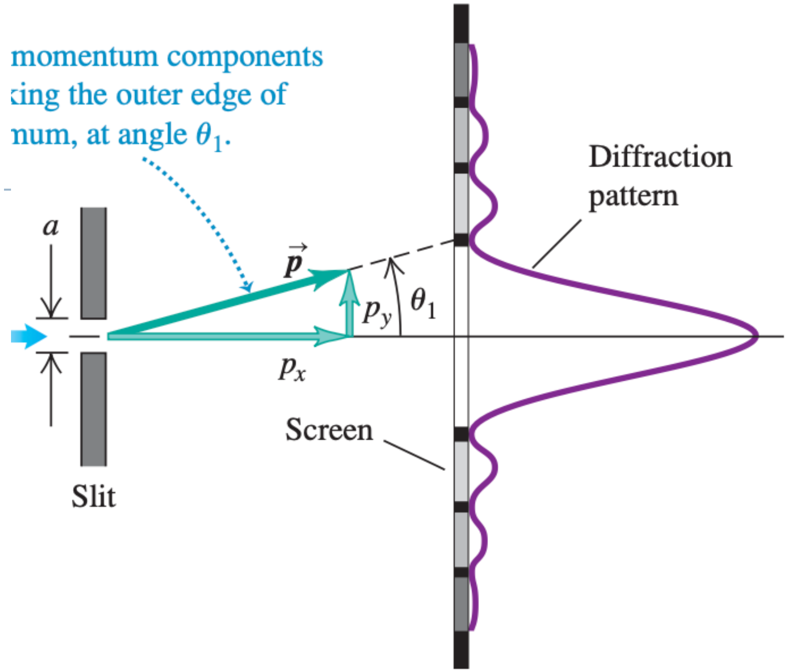
- 38.1 Light Absorbed as Photons: The Photoelectric Effect
- 38.2 Light Emitted as Photons: X-Ray Production
- 38.3 Light Scattered as Photons: Compton Scattering and Pair Production
- 38.4 Wave Particle Duality, Probability, and Uncertainty



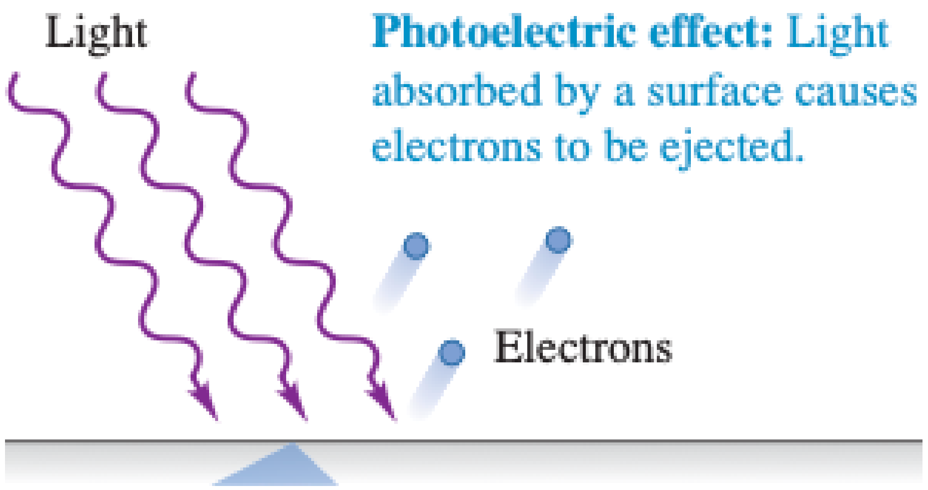

Source: Atman, Kazim. (2021, March 29). A Guide to Wave Particle Duality in Electron Diffraction. AZoQuantum

# Light

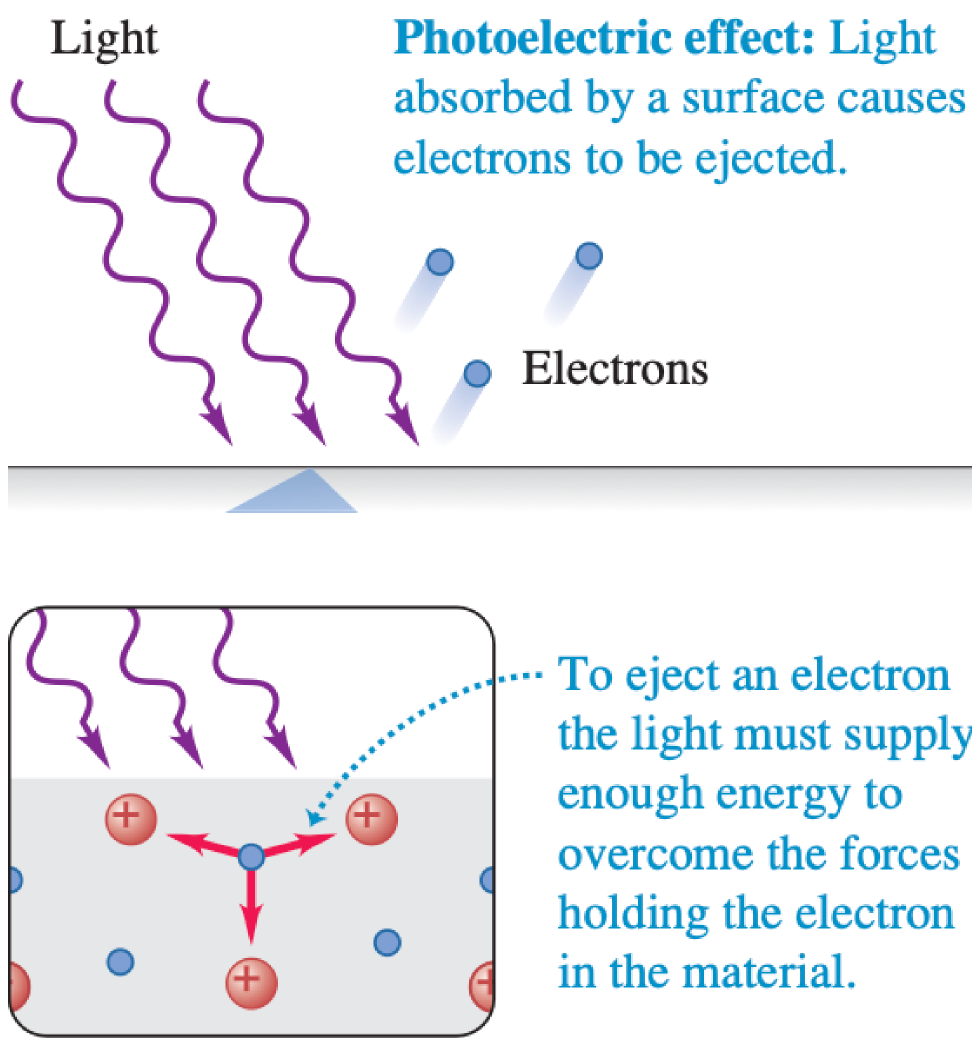
Wave nature



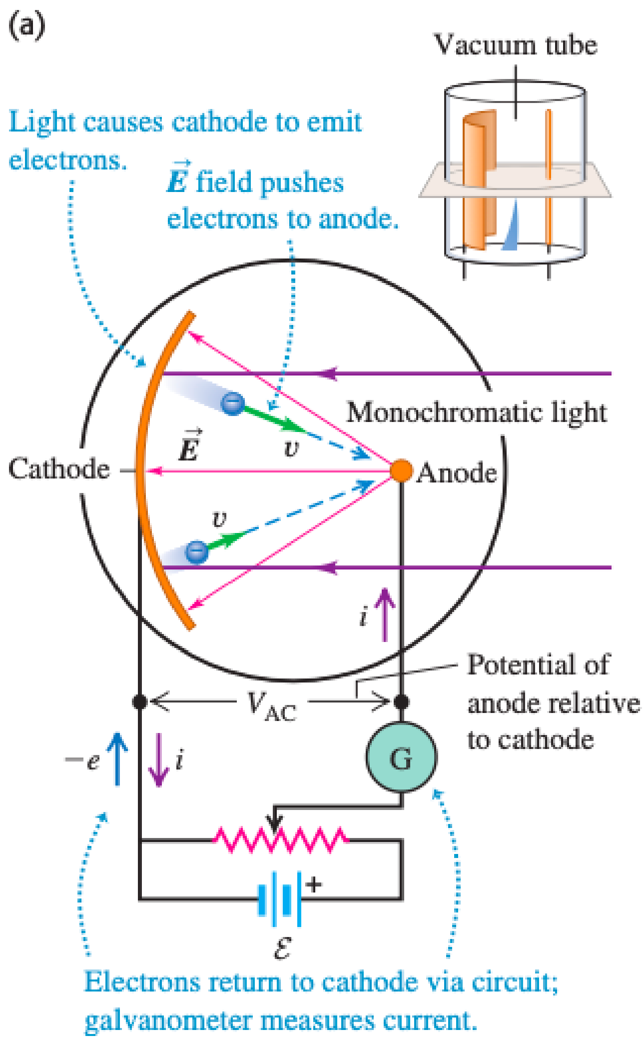

Particle nature



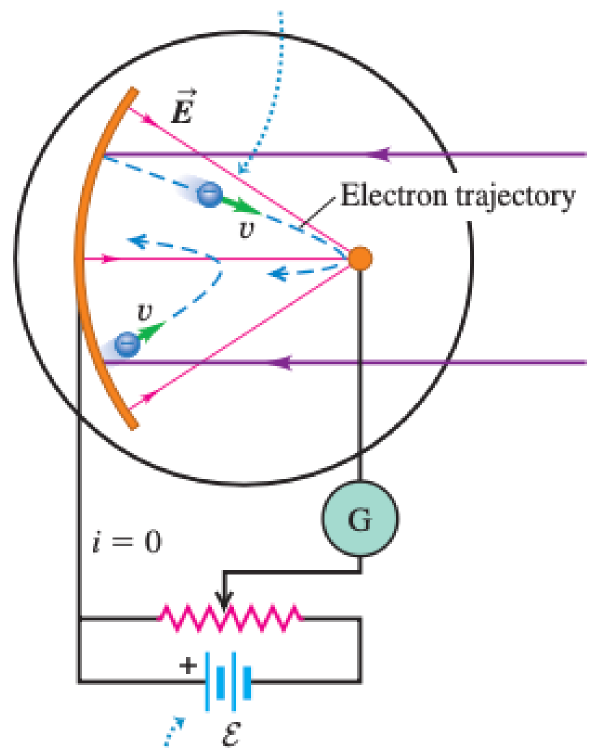
# 38.1 Light Absorbed as Photons: The Photoelectric Effect



# 38.1 Light Absorbed as Photons: The Photoelectric Effect




## Reverse electric field



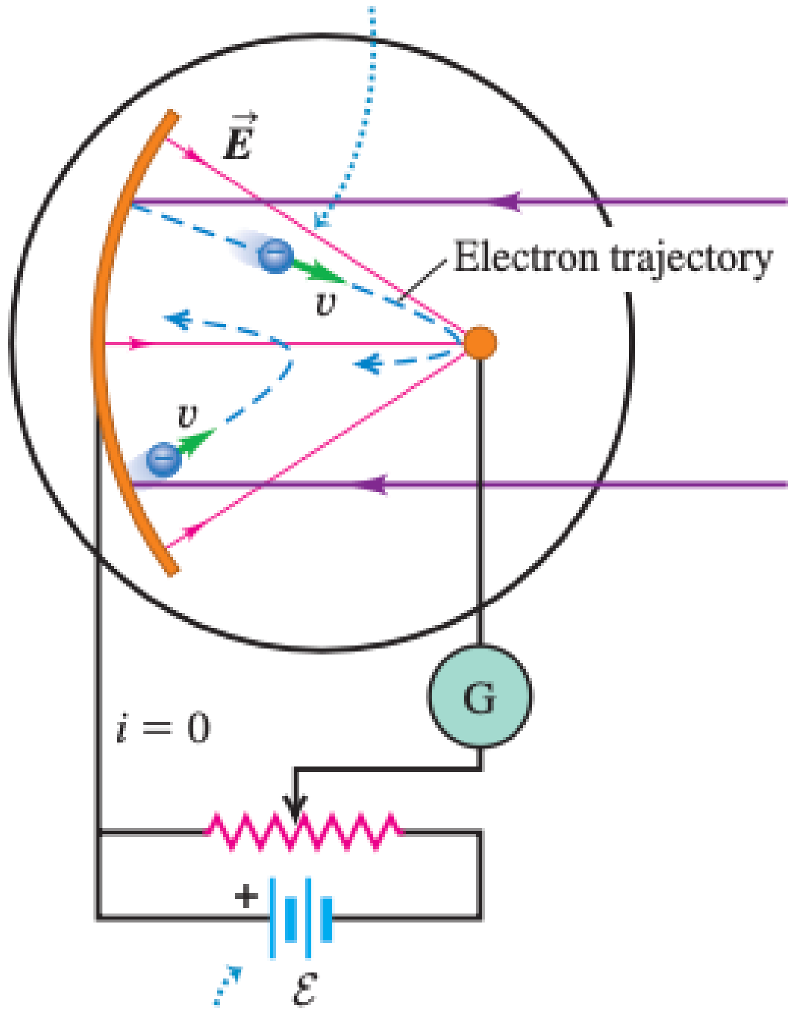

$V_0$ is called the **stopping potential**

# 38.1 Light Absorbed as Photons: The Photoelectric Effect

Maximum kinetic energy of electron

$$W_{tot} = -eV_0 = \Delta K = 0 - K_{max}$$

$$K_{max} = \frac{1}{2}mv_{max}^{\ 2} = eV_0$$



# 38.1 Light Absorbed as Photons: The Photoelectric Effect

***Wave-Model Prediction 1:*** We saw in Section 32.4 that the intensity of an electromagnetic wave depends on its amplitude but not on its frequency. So the photoelectric effect should occur for light of any frequency, and *the magnitude of the photocurrent should not depend on the frequency of the light*.

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}{}^2}{2\mu_0 c}$$

vs

***Experimental Result 1:*** *The photocurrent depends on the light frequency*. For a given material, monochromatic light with a frequency below a minimum **threshold frequency** produces *no* photocurrent, regardless of intensity. For most metals the threshold frequency is in the ultraviolet (corresponding to wavelengths $\lambda$ between 200 and 300 nm), but for other materials like potassium oxide and cesium oxide it is in the visible spectrum ($\lambda$ between 380 and 750 nm).

# 38.1 Light Absorbed as Photons: The Photoelectric Effect

**Wave-Model Prediction**

- the magnitude of the photocurrent should not depend on the frequency of the light.
- a time delay between when we switch on the light and when photo-electrons appear.
- the stopping potential should not depend on the frequency of the light

**Experimental results**

- The photocurrent depends on the light frequency.
- There is no measurable time delay between when the light is turned on and when the cathode emits photoelectrons
- The stopping potential does not depend on intensity, but does depend on frequency.

Contradict to Maxwell's description of light as an electro-magnetic wave
Albert Einstein in 1905 solved the dilemma. (Phonon)

# 38.1 Light Absorbed as Photons: The Photoelectric Effect

$$E = hf = \frac{hc}{\lambda} \qquad \text{(energy of a photon)} \qquad (38.2)$$

where $h$ is a universal constant called **Planck's constant.** The numerical value of this constant, to the accuracy known at present, is

$$h = 6.62606896(33) \times 10^{-34}\ \text{J} \cdot \text{s}$$

energy to find that the *maximum* kinetic energy $K_{\text{max}} = \frac{1}{2}mv_{\text{max}}{}^2$ for an emitted electron is the energy $hf$ gained from a photon minus the work function $\phi$:

$$K_{\text{max}} = \frac{1}{2}mv_{\text{max}}{}^2 = hf - \phi \qquad (38.3)$$

Substituting $K_{\text{max}} = eV_0$ from Eq. (38.1), we find

$$eV_0 = hf - \phi \qquad \text{(photoelectric effect)} \qquad (38.4)$$

# 38.1 Light Absorbed as Photons: The Photoelectric Effect

Furthermore, according to the special theory of relativity, every particle that has energy must also have momentum, even if it has no rest mass.

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \qquad \text{(momentum of a photon)}$$

# Sample Problem

## Example 38.1 Laser-pointer photons

A laser pointer with a power output of 5.00 mW emits red light ($\lambda = 650$ nm). (a) What is the magnitude of the momentum of each photon? (b) How many photons does the laser pointer emit each second?

**EXECUTE:** (a) We have $\lambda = 650 \text{ nm} = 6.50 \times 10^{-7}$ m, so from Eq. (38.5) the photon momentum is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{6.50 \times 10^{-7}\ \text{m}}$$
$$= 1.02 \times 10^{-27}\ \text{kg} \cdot \text{m/s}$$

(Recall that $1\ \text{J} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2$.)

(b) From Eq. (38.2), the energy of a single photon is

$$E = pc = (1.02 \times 10^{-27}\ \text{kg} \cdot \text{m/s})(3.00 \times 10^{8}\ \text{m/s})$$
$$= 3.06 \times 10^{-19}\ \text{J} = 1.91\ \text{eV}$$

The laser pointer emits energy at the rate of $5.00 \times 10^{-3}$ J/s, so it emits photons at the rate of

$$\frac{5.00 \times 10^{-3}\ \text{J/s}}{3.06 \times 10^{-19}\ \text{J/photon}} = 1.63 \times 10^{16}\ \text{photons/s}$$

# Sample Problem

## Example 38.2 A photoelectric-effect experiment

While conducting a photoelectric-effect experiment with light of a certain frequency, you find that a reverse potential difference of 1.25 V is required to reduce the current to zero. Find (a) the maximum kinetic energy and (b) the maximum speed of the emitted photoelectrons.

**EXECUTE:** (a) From Eq. (38.1),

$$K_{\max} = eV_0 = (1.60 \times 10^{-19}\ \mathrm{C})(1.25\ \mathrm{V}) = 2.00 \times 10^{-19}\ \mathrm{J}$$

(Recall that 1 V = 1 J/C.) In terms of electron volts,

$$K_{\max} = eV_0 = e(1.25\ \mathrm{V}) = 1.25\ \mathrm{eV}$$

since the electron volt (eV) is the magnitude of the electron charge $e$ times one volt (1 V).

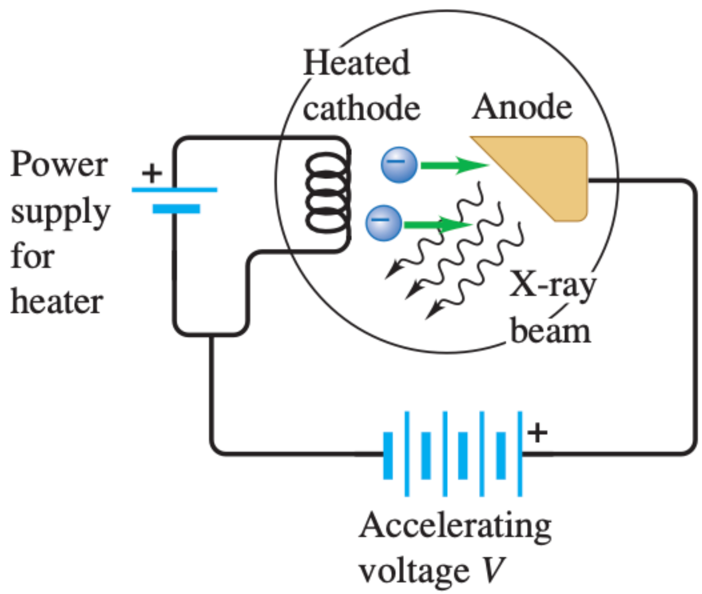
(b) From $K_{\max} = \frac{1}{2}mv_{\max}{}^2$ we get

$$v_{\max} = \sqrt{\frac{2K_{\max}}{m}} = \sqrt{\frac{2(2.00 \times 10^{-19}\ \mathrm{J})}{9.11 \times 10^{-31}\ \mathrm{kg}}}$$
$$= 6.63 \times 10^{5}\ \mathrm{m/s}$$

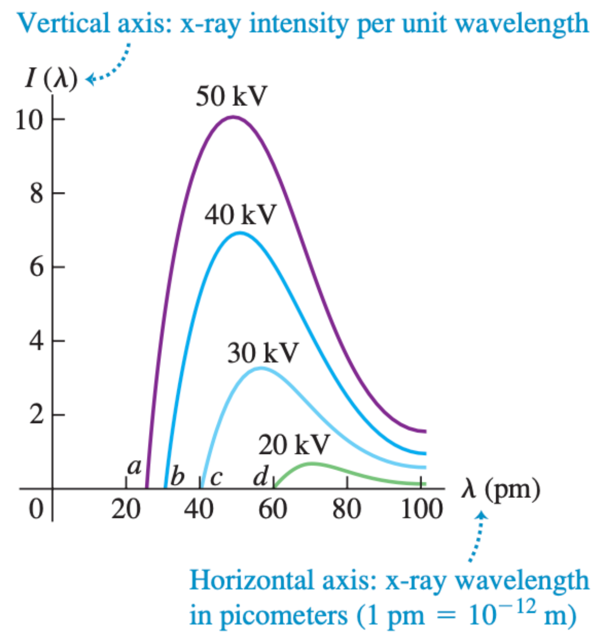
# 38.2 Light Emitted as Photons: X-Ray Production

**38.7** An apparatus used to produce x rays, similar to Röntgen's 1895 apparatus.

Electrons are emitted thermionically from the heated cathode and are accelerated toward the anode; when they strike it, x rays are produced.

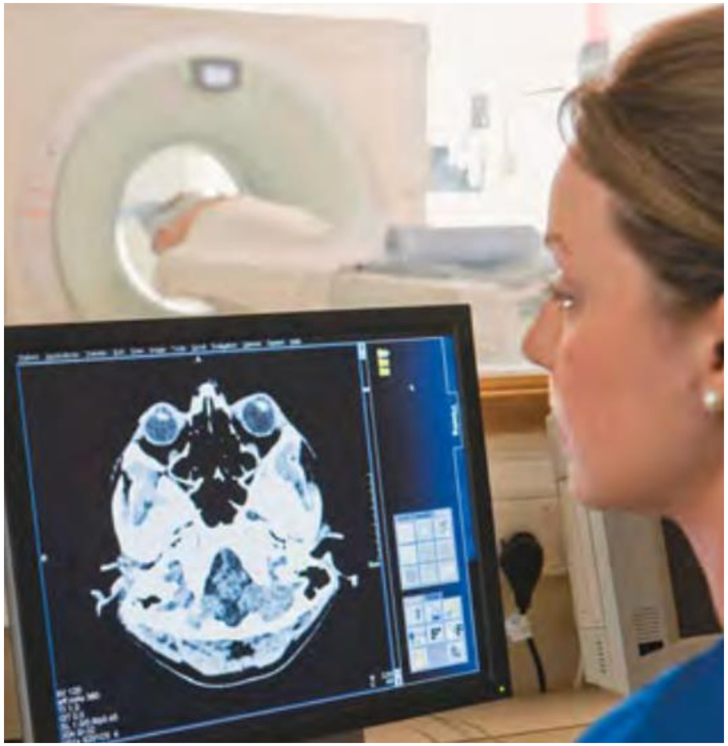



**38.8** The continuous spectrum of x rays produced when a tungsten target is struck by electrons accelerated through a voltage $V_{AC}$. The curves represent different values of $V_{AC}$; points $a$, $b$, $c$, and $d$ show the minimum wavelength for each voltage.




$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}} \qquad \text{(bremsstrahlung)}$$

# 38.2 Light Emitted as Photons: X-Ray Production



# Sample Problem

**Example 38.4** **Producing x rays**

Electrons in an x-ray tube accelerate through a potential difference of 10.0 kV before striking a target. If an electron produces one photon on impact with the target, what is the minimum wavelength of the resulting x rays? Find the answer by expressing energies in both SI units and electron volts.

**EXECUTE:** From Eq. (38.6), using SI units we have

$$\lambda_{\min} = \frac{hc}{eV_{AC}} = \frac{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})(3.00 \times 10^{8}\ \mathrm{m/s})}{(1.602 \times 10^{-19}\ \mathrm{C})(10.0 \times 10^{3}\ \mathrm{V})}$$

$$= 1.24 \times 10^{-10}\ \mathrm{m} = 0.124\ \mathrm{nm}$$

Using electron volts, we have

$$\lambda_{\min} = \frac{hc}{eV_{AC}} = \frac{(4.136 \times 10^{-15}\ \mathrm{eV \cdot s})(3.00 \times 10^{8}\ \mathrm{m/s})}{e(10.0 \times 10^{3}\ \mathrm{V})}$$

$$= 1.24 \times 10^{-10}\ \mathrm{m} = 0.124\ \mathrm{nm}$$

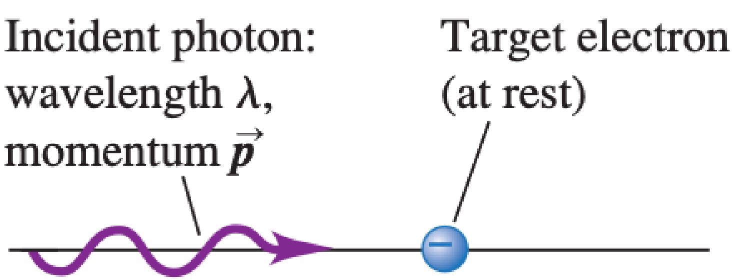
In the second calculation, the "*e*" for the magnitude of the electron charge cancels the "e" in the unit "eV," because the electron volt (eV) is the magnitude of the electron charge *e* times one volt (1 V).

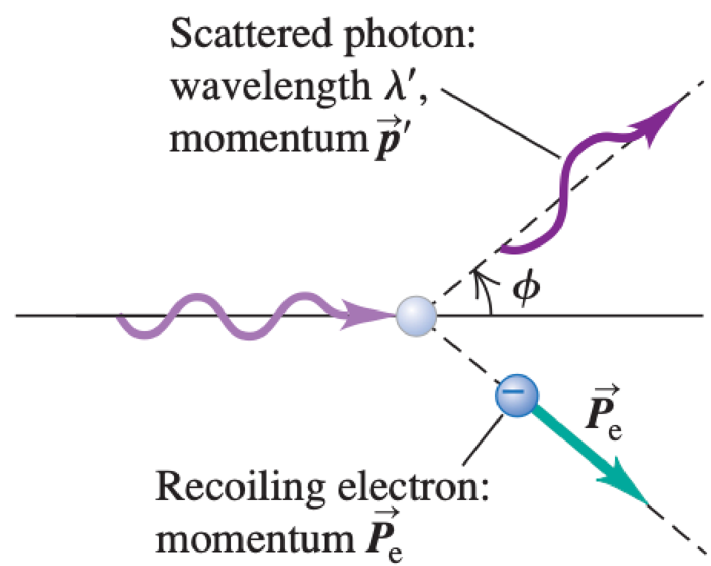
# 38.3 Light Scattered as Photons: Compton Scattering and Pair Production

## Compton-effect experiment

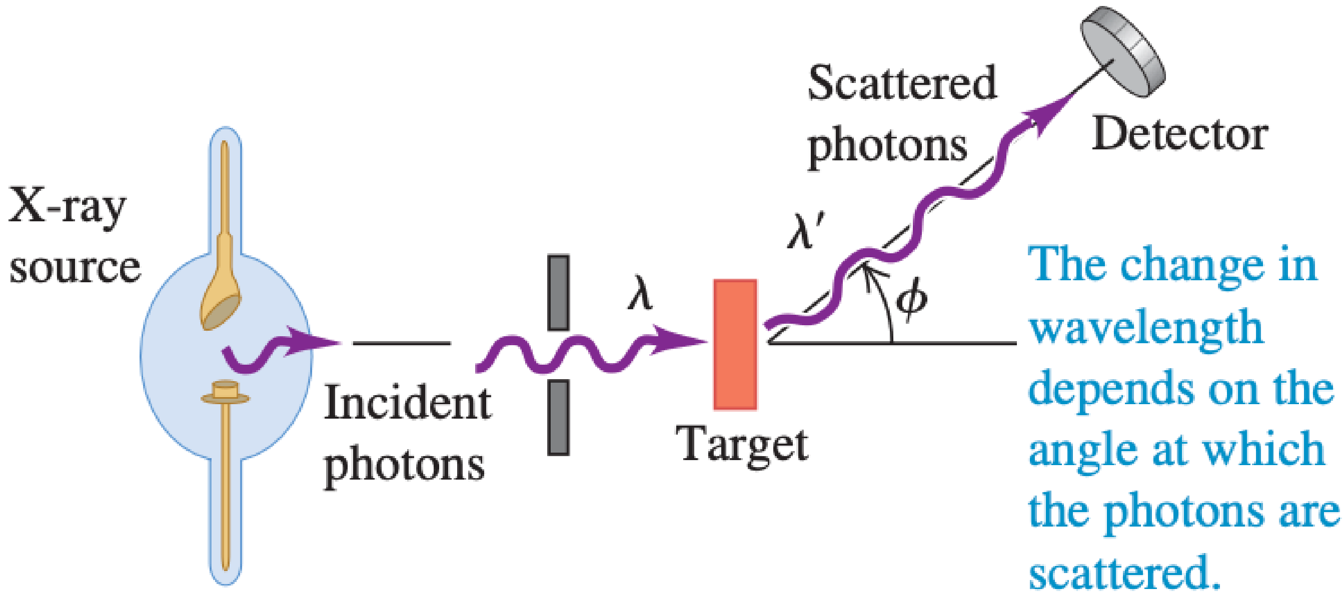
(b) After collision: The angle between the directions of the scattered photon and the incident photon is $\phi$.

(a) Before collision: The target electron is at rest.



# 38.3 Light Scattered as Photons: Compton Scattering and Pair Production




$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \qquad \text{(Compton scattering)}$$

$$\frac{h}{mc} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.109 \times 10^{-31}\ \text{kg})(2.998 \times 10^{8}\ \text{m/s})} = 2.426 \times 10^{-12}\ \text{m}$$

# Sample Problem

## Example 38.5 Compton scattering

You use 0.124-nm x-ray photons in a Compton-scattering experiment. (a) At what angle is the wavelength of the scattered x rays 1.0% longer than that of the incident x rays? (b) At what angle is it 0.050% longer?

**EXECUTE:** (a) In Eq. (38.7) we want $\Delta\lambda = \lambda' - \lambda$ to be 1.0% of 0.124 nm, so $\Delta\lambda = 0.00124\ \text{nm} = 1.24 \times 10^{-12}$ m. Using the value $h/mc = 2.426 \times 10^{-12}$ m, we find

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$$

$$\cos\phi = 1 - \frac{\Delta\lambda}{h/mc} = 1 - \frac{1.24 \times 10^{-12}\ \text{m}}{2.426 \times 10^{-12}\ \text{m}} = 0.4889$$

$$\phi = 60.7^\circ$$

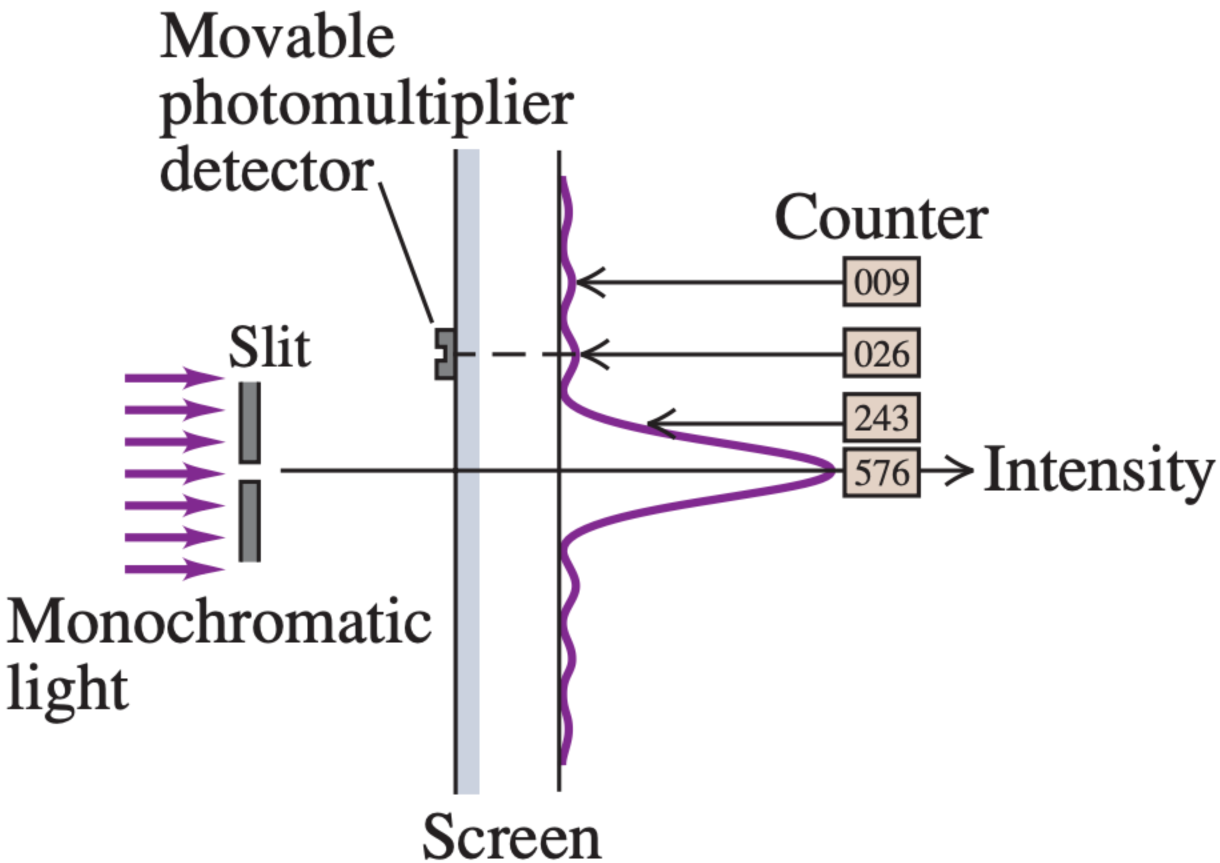
(b) For $\Delta\lambda$ to be 0.050% of 0.124 nm, or $6.2 \times 10^{-14}$ m,

$$\cos\phi = 1 - \frac{6.2 \times 10^{-14}\ \text{m}}{2.426 \times 10^{-12}\ \text{m}} = 0.9744$$

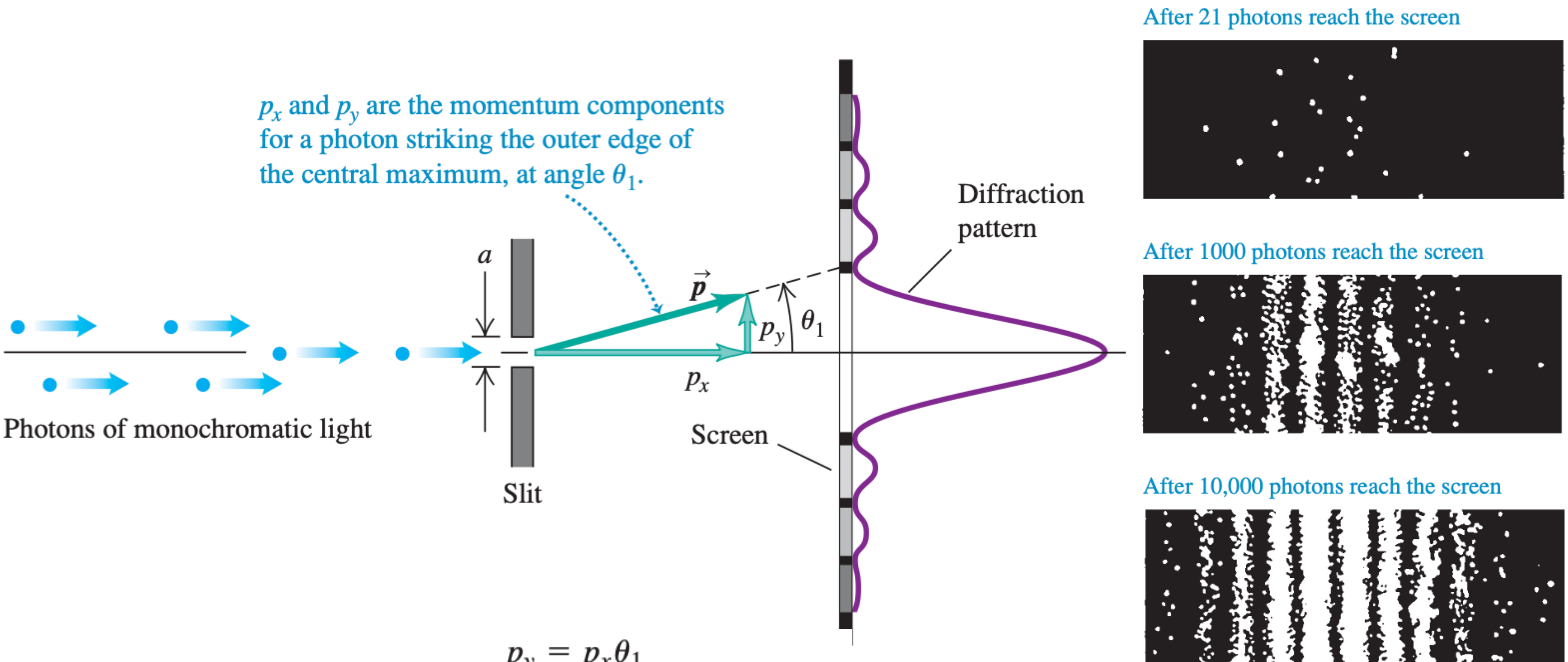
$$\phi = 13.0^\circ$$

# 38.4 Wave Particle Duality, Probability, and Uncertainty

Counting number of photons



# 38.4 Wave Particle Duality, Probability, and Uncertainty




$$p_y = p_x \theta_1$$

Substituting Eq. (38.12), $\theta_1 = \lambda / a$, into Eq. (38.13) gives

$$p_y = p_x \frac{\lambda}{a}$$

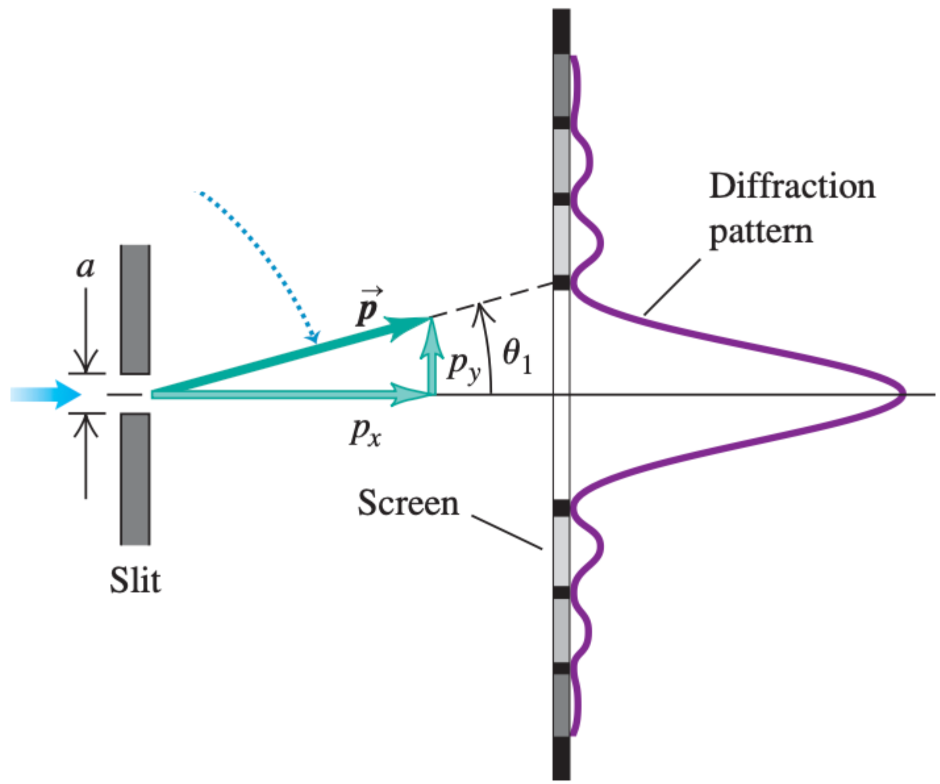
# 38.4 Wave Particle Duality, Probability, and Uncertainty

$$\Delta p_y \geq p_x \frac{\lambda}{a}$$

$$\lambda = h/p_x.$$

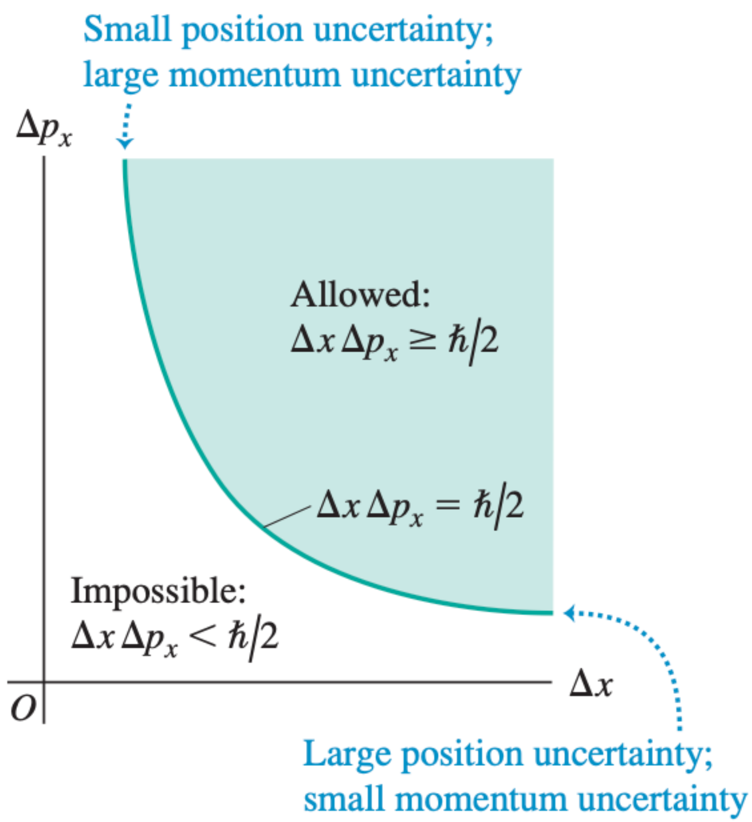
$$\Delta p_y \geq p_x \frac{h}{p_x a} = \frac{h}{a}$$

$$\Delta p_y a \geq h$$



# 38.4 Wave Particle Duality, Probability, and Uncertainty

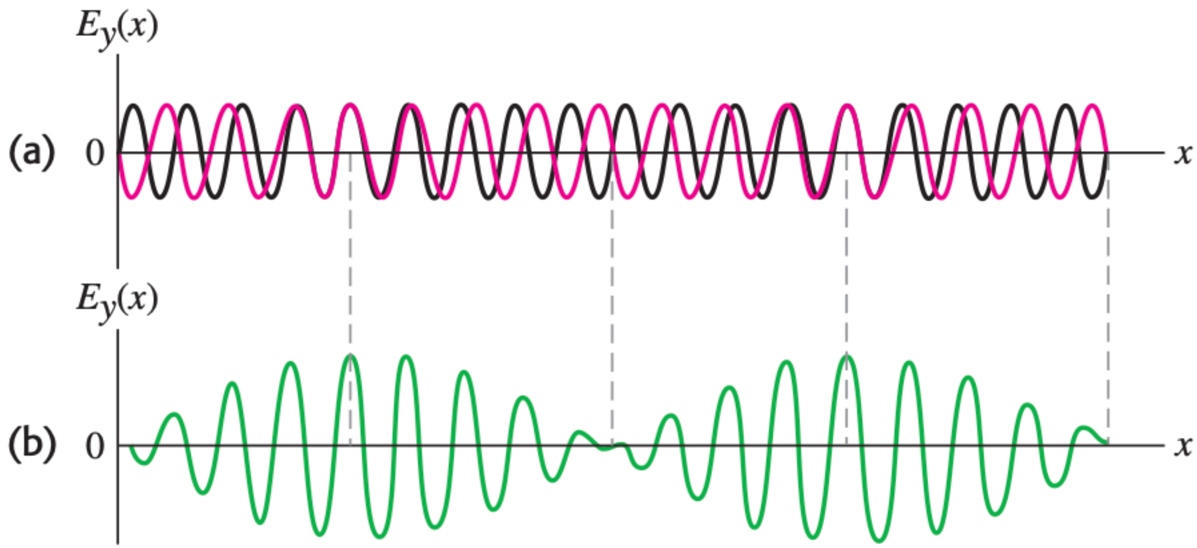
**38.18** The Heisenberg uncertainty principle for position and momentum components. It is impossible for the product $\Delta x \Delta p_x$ to be less than $\hbar/2 = h/4\pi$.




Heisenberg uncertainty principle

$$\Delta x \, \Delta p_x \geq \hbar/2$$

# 38.4 Wave Particle Duality, Probability, and Uncertainty




$$\Delta t \, \Delta E \geq \hbar/2$$

The photon is most likely to be found at the times when the amplitude is large. The price we pay for localizing the photon in time is that the wave does not have a definite energy.

# Sample Problem

**Example 38.7** **Ultrashort laser pulses and the uncertainty principle**

Many varieties of lasers emit light in the form of pulses rather than a steady beam. A tellurium–sapphire laser can produce light at a wavelength of 800 nm in ultrashort pulses that last only $4.00 \times 10^{-15}$ s (4.00 femtoseconds, or 4.00 fs). The energy in a single pulse produced by one such laser is $2.00\ \mu\text{J} = 2.00 \times 10^{-6}$ J, and the pulses propagate in the positive $x$-direction. Find (a) the frequency of the light; (b) the energy and minimum energy uncertainty of a single photon in the pulse; (c) the minimum frequency uncertainty of the light in the pulse; (d) the spatial length of the pulse, in meters and as a multiple of the wavelength; (e) the momentum and minimum momentum uncertainty of a single photon in the pulse; and (f) the approximate number of photons in the pulse.

# Sample Problem

**EXECUTE:** (a) From the relationship $c = \lambda f$, the frequency of 800-nm light is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \text{m/s}}{8.00 \times 10^{-7}\ \text{m}} = 3.75 \times 10^{14}\ \text{Hz}$$

(b) From Eq. (38.2) the energy of a single 800-nm photon is

$$\begin{aligned} E = hf &= (6.626 \times 10^{-34}\ \text{J} \cdot \text{s})(3.75 \times 10^{14}\ \text{Hz}) \\ &= 2.48 \times 10^{-19}\ \text{J} \end{aligned}$$

The time uncertainty equals the pulse duration, $\Delta t = 4.00 \times 10^{-15}\ \text{s}$. From Eq. (38.24) the minimum uncertainty in energy corresponds to the case $\Delta t\, \Delta E = \hbar/2$, so

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{2(4.00 \times 10^{-15}\ \text{s})} = 1.32 \times 10^{-20}\ \text{J}$$

This is 5.3% of the photon energy $E = 2.48 \times 10^{-19}$ J, so the energy of a given photon is uncertain by at least 5.3%. The uncertainty could be greater, depending on the shape of the pulse.

# Sample Problem

(c) From the relationship $f = E/h$, the minimum frequency uncertainty is

$$\Delta f = \frac{\Delta E}{h} = \frac{1.32 \times 10^{-20}\ \text{J}}{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}} = 1.99 \times 10^{13}\ \text{Hz}$$

This is 5.3% of the frequency $f = 3.75 \times 10^{14}$ Hz we found in part (a). Hence these ultrashort pulses do not have a definite frequency; the average frequency of many such pulses will be $3.75 \times 10^{14}$ Hz, but the frequency of any individual pulse can be anywhere from 5.3% higher to 5.3% lower.

(d) The spatial length $\Delta x$ of the pulse is the distance that the front of the pulse travels during the time $\Delta t = 4.00 \times 10^{-15}$ s it takes the pulse to emerge from the laser:

$$\begin{aligned}\Delta x = c\Delta t &= (3.00 \times 10^{8}\ \text{m/s})(4.00 \times 10^{-15}\ \text{s})\\ &= 1.20 \times 10^{-6}\ \text{m}\end{aligned}$$

$$\Delta x = \frac{1.20 \times 10^{-6}\ \text{m}}{8.00 \times 10^{-7}\ \text{m/wavelength}} = 1.50\ \text{wavelengths}$$

This justifies the term *ultrashort*. The pulse is less than two wavelengths long!

# Sample Problem

(e) From Eq. (38.5), the momentum of an average photon in the pulse is

$$p_x = \frac{E}{c} = \frac{2.48 \times 10^{-19}\ \text{J}}{3.00 \times 10^{8}\ \text{m/s}} = 8.28 \times 10^{-28}\ \text{kg} \cdot \text{m/s}$$

The spatial uncertainty is $\Delta x = 1.20 \times 10^{-6}$ m. From Eq. (38.17) minimum momentum uncertainty corresponds to $\Delta x\, \Delta p_x = \hbar/2$, so

$$\Delta p_x = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{2(1.20 \times 10^{-6}\ \text{m})} = 4.40 \times 10^{-29}\ \text{kg} \cdot \text{m/s}$$

This is 5.3% of the average photon momentum $p_x$. An individual photon within the pulse can have a momentum that is 5.3% greater or less than the average.

(f) To estimate the number of photons in the pulse, we divide the total pulse energy by the average photon energy:

$$\frac{2.00 \times 10^{-6}\ \text{J/pulse}}{2.48 \times 10^{-19}\ \text{J/photon}} = 8.06 \times 10^{12}\ \text{photons/pulse}$$

The energy of an individual photon is uncertain, so this is the *average* number of photons per pulse.