Ch 39 - Particles Behaving as Waves

I. *Concepts*

1. De Broglie Wavelength: A free particle with rest mass m and moving at non-relativistic speed v has a wavelength λ related to its momentum $p = mv$ as $\lambda = \frac{h}{p} = \frac{h}{mv}$, where h is Planck's constant. This shows that particles can exhibit wave-like characteristics, similar to the wave-particle duality of light.
2. Electron Diffraction Experiment: Electrons emitted from a heated filament are accelerated and directed at a crystal. The scattered electrons are detected at various angles. When the scattered waves are in phase, a peak in the intensity of scattered electrons is observed, proving the wave nature of electrons. The similarity between electron diffraction and X-ray diffraction further demonstrates the wave property of electrons.
3. Atomic Spectra: Light from a heated filament contains all wavelengths and shows a continuous spectrum. Heated gas emits light only at certain discrete wavelengths, forming a line spectrum.
4. Rutherford Scattering Experiment: Alpha particles emitted by a radioactive element are collimated into a narrow beam by lead screens and directed at a gold foil. The scattered alpha particles are detected by a scintillation screen. This experiment shows that atoms have a positively charged nucleus and most of the atomic mass is concentrated in the nucleus.
5. Failure of Classical Physics in Atomic Structure: According to classical physics, an orbiting electron should radiate electromagnetic waves due to its acceleration, lose energy, and spiral inward. Atoms should collapse quickly and emit a continuous spectrum. In reality, atoms are stable and emit light only at specific frequencies, which led to the development of new theories.
6. Energy Level Transitions: When an atom drops from an initial energy level i to a lower-energy final level f, it emits a photon with energy equal to the difference between the two levels, i.e., $hf = E_i - E_f$, or $hf = \frac{hc}{\lambda} = E_i - E_f$.
7. Blackbody Radiation: A small aperture in a hollow box can be approximated as a blackbody. The intensity of blackbody radiation follows $I = \sigma T^4$ (Stefan - Boltzmann law), where $\sigma = 5.670400(40) \times 10^{-8} W/m^2 \cdot K^4$.
8. Wien's Displacement Law: As the temperature increases, the peak of the spectral emissivity curve shifts to shorter wavelengths, and $\lambda_m T = 2.90 \times 10^{-3} m \cdot K$, where λ_m is the peak wavelength.

II. *Formulas*

1. Quantization of Angular Momentum:

$$L_n = m v_n r_n = n \frac{h}{2\pi}$$

where n is the quantum number, determining that the angular momentum of the electron orbit is quantized.

2. Quantization of Orbital Radius:

$$r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2}$$

electrons can only move in specific, quantized orbits.

3. Quantization of Energy:

$$E_n = -\frac{hcR}{n^2}$$

where $R = \frac{me^4}{8\epsilon_0^2 h^3 c}$, the energy of the atom is discontinuous and can only take specific values.

4. Stefan - Boltzmann Law:

$$I = \sigma T^4$$

5. Planck's Radiation Law:

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

6. Wien's Displacement Law:

$$\lambda_m T = 2.90 \times 10^{-3} m \cdot K$$

III. *Problems*

1. Given the electron mass $m = 9.109 \times 10^{-31} kg$, Planck's constant $h = 6.626 \times 10^{-34} J \cdot s$, and the electron is accelerated by a voltage $V = 100V$, find the de Broglie wavelength of the electron.

2. A hydrogen atom transitions from the $n = 3$ energy level to the $n = 2$ energy level. Given $R = \frac{me^4}{8\epsilon_0^2 h^3 c} = 1.097 \times 10^7 m^{-1}$, $h = 6.626 \times 10^{-34} J \cdot s$, and $c = 3.00 \times 10^8 m/s$, find the wavelength of the emitted photon.