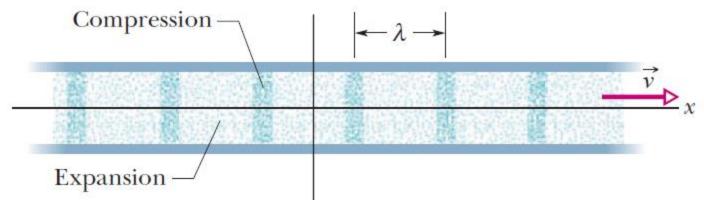
# PHYS1001B College Physics IB

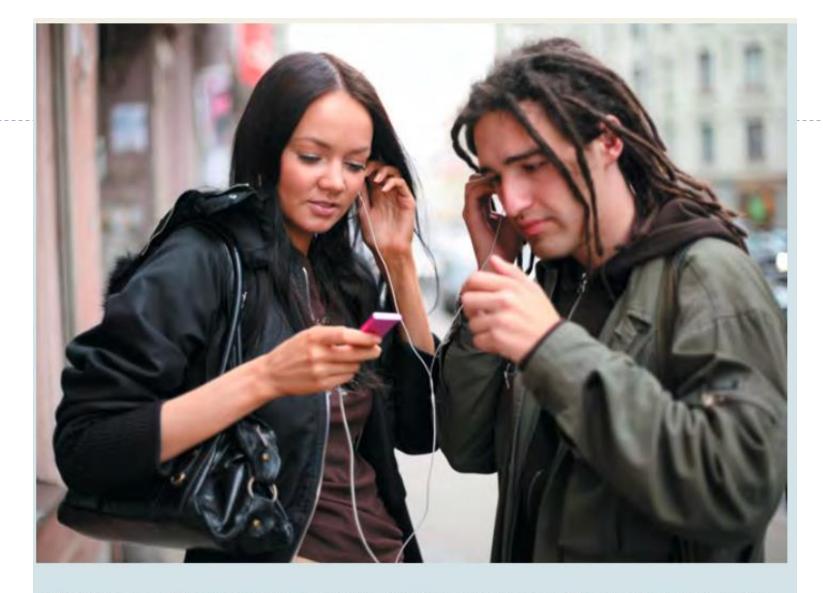
Waves II — Sound and Hearing (Ch. 16)

#### Introduction

This chapter focuses on sound waves which are <u>longitudinal waves</u> involving oscillations parallel to the direction of wave travel



- Similar to string waves, we will study their <u>interference</u> and <u>resonance</u>
- Doppler effect follows



Most people like to listen to music, but hardly anyone likes to listen to noise. What is the physical difference between musical sound and noise?

#### Outline

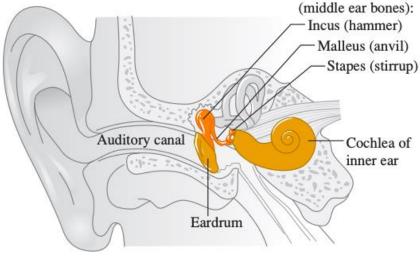
- ▶ 16-1 Sound Waves
- ▶ 16-2 Speed of Sound Waves
- ▶ 16-3 Sound Intensity
- ▶ 16-4 Standing Sound Waves and Normal Modes
- ▶ 16-6 Interference of Waves
- ▶ 16-7 Beats
- ▶ 16-8 The Doppler Effect
- ▶ 16-9 Shock Waves

Sound waves usually travel out in all directions from the source of sound, with an amplitude that depends on the direction and distance from the source

The human ear is sensitive to waves in the frequency range from about 20 to 20,000 Hz, called the **audible range** 

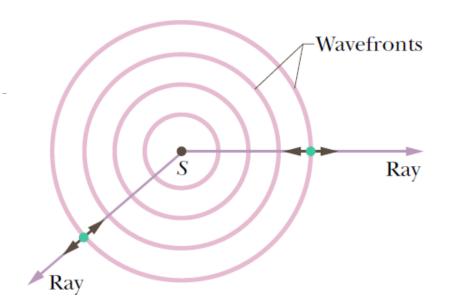
"sound" for similar waves with frequencies above (ultrasonic) and below

(infrasonic) the range of human hearing.



Ossicles

A sound wave travels from a point source *S* through a three-dimensional medium. The wavefronts form spheres centered on *S*; the rays are radial to *S*.

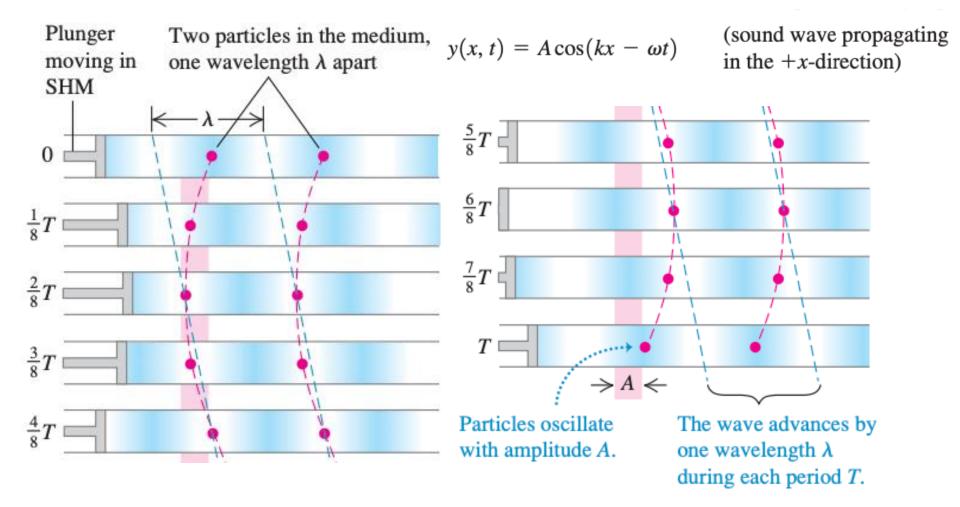


# ▶ Wavefronts (波面)

are surfaces over which the oscillations due to the sound wave have the <u>same value</u>; such surfaces are represented by whole or partial circles in a two-dimensional drawing

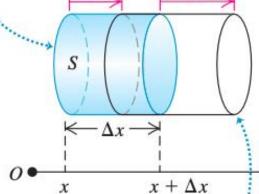
# ▶ Rays (波線)

are directed lines perpendicular to the wavefronts that indicate the *direction* of travel of the wavefronts



Undisturbed cylinder of fluid has cross-sectional area S, length  $\Delta x$ , and volume  $S\Delta x$ .

A sound wave displaces the left ... and the end of the cylinder by  $y_1 = y(x, t)$  ...  $y_2 = y(x + \Delta x, t)$ .



The change in volume of the disturbed cylinder of fluid is  $S(y_2 - y_1)$ .

$$\Delta V = S(y_2 - y_1) = S[y(x + \Delta x, t) - y(x, t)]$$

$$\frac{dV}{V} = \lim_{\Delta x \to 0} \frac{S[y(x + \Delta x, t) - y(x, t)]}{S\Delta x} = \frac{\partial y(x, t)}{\partial x}$$

$$B = -p(x, t)/(dV/V)$$

$$p(x,t) = -B \frac{\partial y(x,t)}{\partial x}$$

$$y(x, t) = A\cos(kx - \omega t)$$

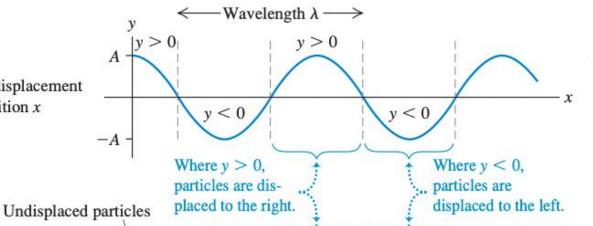
$$p(x,t) = BkA\sin(kx - \omega t)$$

 $p_{\text{max}} = BkA$  (sinusoidal sound wave)

#### Sound Waves As Pressure

**Fluctuations** 

(a) A graph of displacementy versus position xat t = 0



(b) A cartoon showing the displacement of individual particles in the fluid at t = 0

Displaced particles

Rarefaction:

particles pulled apart;

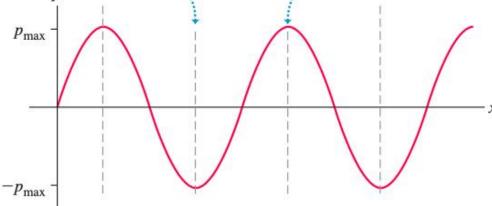
pressure is most negative.

Compression:

particles pile up;

pressure is most positive.

(c) A graph of pressure fluctuation p versus position x at t = 0

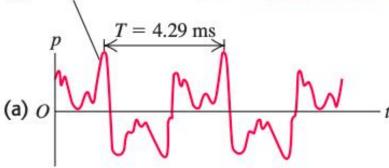


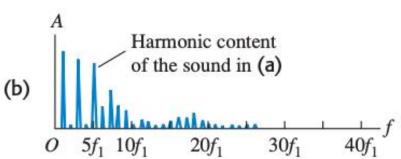
Pressure fluctuation versus time for a clarinet with fundamental frequency  $f_1 = 233 \text{ Hz}$ 

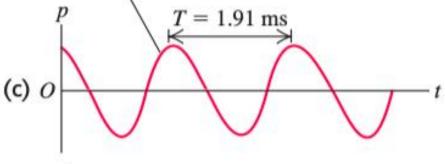


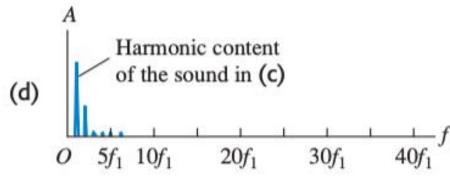
Pressure fluctuation versus time for an alto recorder with fundamental frequency  $f_1 = 523 \text{ Hz}$ 











#### Example 16.1 Amplitude of a sound wave in air

In a sinusoidal sound wave of moderate loudness, the maximum pressure variations are about  $3.0 \times 10^{-2}$  Pa above and below atmospheric pressure. Find the corresponding maximum displacement if the frequency is 1000 Hz. In air at normal atmospheric pressure and density, the speed of sound is 344 m/s and the bulk modulus is  $1.42 \times 10^5$  Pa.

**EXECUTE:** From Eq. (15.6),

$$k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{344 \text{ m/s}} = 18.3 \text{ rad/m}$$

Then from Eq. (16.5), the maximum displacement is

$$A = \frac{p_{\text{max}}}{Bk} = \frac{3.0 \times 10^{-2} \text{ Pa}}{(1.42 \times 10^5 \text{ Pa})(18.3 \text{ rad/m})} = 1.2 \times 10^{-8} \text{ m}$$

The speed of any mechanical wave depends on both an inertial property of the medium (kinetic energy) and an elastic property of the medium (potential energy)

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

Speed of sound in Fluid

$$v = \sqrt{\frac{B}{\rho}}$$

- ▶ *B* is the bulk modulus
  - $-\frac{\Delta p}{\Delta V/V}$ ; pressure change against fractional volume change
- $\rho$  is the density of medium

Medium	Speed (m/s)	
Gases		
Air (0°C)	331	
Air (20°C)	343	
Helium	965	
Hydrogen	1284	
Liquids		
Water (0°C)	1402	
Water (20°C)	1482	
Seawater <sup>b</sup>	1522	
Solids		
Aluminum	6420	
Steel	5941	
Granite	6000	

Speed of sound in solid rod

$$v = \sqrt{\frac{Y}{\rho}}$$

- Y is the Young's modulus
- $\rho$  is the density of medium
- Bulk solid: depends also on the shear modulus



sound waves of very high frequency and very short wavelength, called *ultrasound* More sensitive than x rays

$$v = \sqrt{\frac{\gamma RT}{M}}$$
 (speed of sound in an ideal gas)

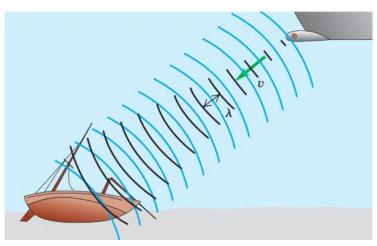
M is the molar mass R is the gas constant, has the same value for all gases  $\gamma$  is the ratio of heat capacities T is the temperature

$$R = 8.314472(15) \text{ J/mol} \cdot \text{K}$$

#### Example 16.3

# Wavelength of sonar waves

A ship uses a sonar system (Fig. 16.8) to locate underwater objects. Find the speed of sound waves in water using Eq. (16.7), and find the wavelength of a 262-Hz wave.



**EXECUTE:** In Example 16.2, we used Table 11.2 to find  $B = 2.18 \times 10^9$  Pa. Then

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.18 \times 10^9 \text{ Pa}}{1.00 \times 10^3 \text{ kg/m}^3}} = 1480 \text{ m/s}$$

and

$$\lambda = \frac{v}{f} = \frac{1480 \text{ m/s}}{262 \text{ s}^{-1}} = 5.65 \text{ m}$$

▶ The intensity I of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface

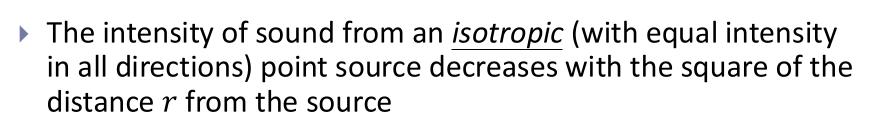
$$I = \frac{P}{A}$$

▶ where *P* is the time rate of energy transfer (the power) of the sound wave and *A* is the area of the surface intercepting the sound

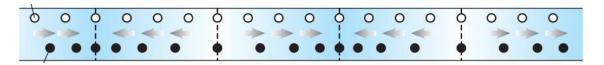
- A point source S emits sound waves uniformly in all directions. The waves pass through an imaginary sphere of radius r that is centered on S
- ▶ The intensity *I* at the sphere is

$$I = \frac{P_S}{4\pi r^2}$$





inverse-square law



A sound wave propagating in the +x-direction

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

$$p(x,t)v_y(x,t) = [BkA\sin(kx - \omega t)][\omega A \sin(kx - \omega t)]$$

$$= B\omega kA^2 \sin^2(kx - \omega t)$$

The intensity is the time average value of  $p(x, t) v_y(x, t)$ 

$$I = \frac{1}{2}B\omega kA^2$$

$$I = \frac{1}{2} \sqrt{\rho B} \omega^2 A^2$$
 (intensity of a sinusoidal sound wave)

#### Intensity in term of pressure

$$I = \frac{1}{2}B\omega kA^2 \qquad p_{\text{max}} = BkA$$

$$I = \frac{\omega p_{\text{max}}^2}{2Bk} = \frac{v p_{\text{max}}^2}{2B}$$

$$v^2 = B/\rho$$

$$I = \frac{p_{\text{max}}^2}{2\rho v} = \frac{p_{\text{max}}^2}{2\sqrt{\rho B}}$$



**16.10** By cupping your hands like this, you direct the sound waves emerging from your mouth so that they don't propagate to the sides. Hence the intensity decreases with distance more slowly than the inverse-square law would predict, and you can be heard at greater distances.

#### Example 16.5

#### Intensity of a sound wave in air

Find the intensity of the sound wave in Example 16.1, with  $p_{\text{max}} = 3.0 \times 10^{-2}$  Pa. Assume the temperature is 20°C so that the density of air is  $\rho = 1.20 \text{ kg/m}^3$  and the speed of sound is v = 344 m/s.

#### **EXECUTE:** From Eq. (16.14),

$$I = \frac{p_{\text{max}}^2}{2\rho v} = \frac{(3.0 \times 10^{-2} \text{ Pa})^2}{2(1.20 \text{ kg/m}^3)(344 \text{ m/s})}$$
$$= 1.1 \times 10^{-6} \text{ J/(s} \cdot \text{m}^2) = 1.1 \times 10^{-6} \text{ W/m}^2$$

Sound level

$$\beta = (10 \text{ dB}) \log \frac{I}{I_0}$$

- ▶ dB is the abbreviation for decibel (分貝), the unit of sound level
- I<sub>0</sub> is the standard reference intensity ( $I_0 = 10^{-12} \text{ W/m}^2$ ), which is the lower limit of the human range of hearing
- For  $I = I_0$ ,  $\beta = 10 \log 1 = 0$ , so our standard reference level corresponds to zero decibels

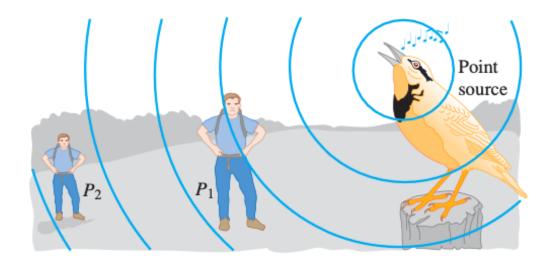
# Table 16.2 Sound Intensity Levels from Various Sources (Representative Values)

Source or Description of Soun		ntensity Level, 3 (dB)	Intensity, I (W/m²)
Military jet aircraft 30 m away	Immediate danger	140	$10^{2}$
Threshold of pain	miniculate danger	120	1
Riveter		95	$3.2 \times 10^{-3}$
Elevated train	Lana mania di danca sa	90	$10^{-3}$
Busy street traffic	Long period damage	70	$10^{-5}$
Ordinary conversation		65	$3.2 \times 10^{-6}$
Quiet automobile		50	$10^{-7}$
Quiet radio in home		40	$10^{-8}$
Average whisper		20	$10^{-10}$
Rustle of leaves		10	$10^{-11}$
Threshold of hearing at 1000 Hz	Z	0	$10^{-12}$

#### Example 16.9

# A bird sings in a meadow

Consider an idealized bird (treated as a point source) that emits constant sound power, with intensity obeying the inverse-square law (Fig. 16.11). If you move twice the distance from the bird, by how many decibels does the sound intensity level drop?



**EXECUTE:** The difference  $\beta_2 - \beta_1$  between any two sound intensity levels is related to the corresponding intensities by

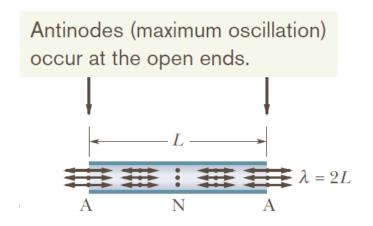
$$\beta_2 - \beta_1 = (10 \text{ dB}) \left( \log \frac{I_2}{I_0} - \log \frac{I_1}{I_0} \right)$$
  
=  $(10 \text{ dB}) \left[ (\log I_2 - \log I_0) - (\log I_1 - \log I_0) \right]$   
=  $(10 \text{ dB}) \log \frac{I_2}{I_1}$ 

For this inverse-square-law source, Eq. (15.26) yields  $I_2/I_1 = r_1^2/r_2^2 = \frac{1}{4}$ , so

$$\beta_2 - \beta_1 = (10 \text{ dB}) \log \frac{I_1}{I_2} = (10 \text{ dB}) \log \frac{1}{4} = -6.0 \text{ dB}$$

# 16-4 Standing Sound Waves and Normal Modes

- Musical sounds can be set up by oscillating strings (guitar), membranes (drum), air columns (pipe organ),...
- The advantage of setting up standing waves is that the air particles oscillate with a large, sustained amplitude, thus generating a noticeable sound at resonant frequencies



The simplest standing wave pattern of displacement for (longitudinal) sound waves in a pipe with both ends open and the corresponding standing wave pattern for (transverse) string waves

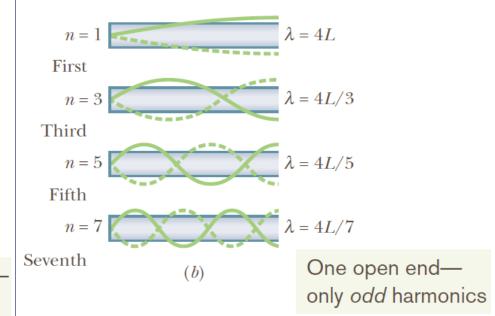
#### (a) Pipe opens at both ends

# First harmonic n = 2 $\lambda = 2L/2 = L$ Second n = 3 $\lambda = 2L/3$ Third n = 4 $\lambda = 2L/4 = L/2$ Fourth Two open ends—any harmonic

$$\lambda = \frac{2L}{n}, \qquad for \, n = 1, 2, 3, \dots$$

$$f = \frac{v}{\lambda} = n \frac{v}{2L}$$
, for  $n = 1, 2, 3, ...$ 

#### (b) Pipe opens at one end only



$$\lambda = \frac{4L}{n}$$
, for  $n = 1, 3, 5, ...$ 

$$f = \frac{v}{\lambda} = n \frac{v}{4L}$$
, for  $n = 1, 3, 5, ...$ 

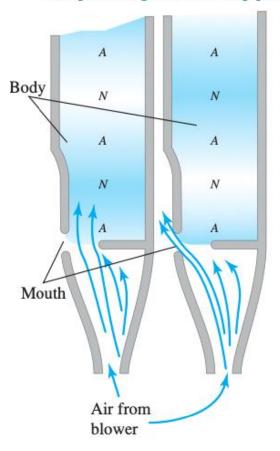
where *n* is the *harmonic number* 

# 16-4 Standing Sound Waves and Normal Modes



**16.15** Organ pipes of different sizes produce tones with different frequencies.

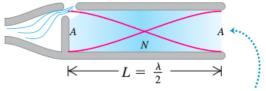
Vibrations from turbulent airflow set up standing waves in the pipe.





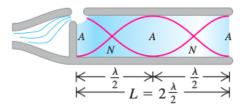
# 16-4 Standing Sound Waves and Normal Modes

(a) Fundamental: 
$$f_1 = \frac{v}{2L}$$

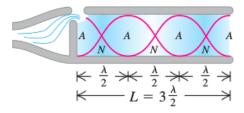


Open end is always a displacement antinode.

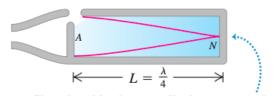
**(b)** Second harmonic: 
$$f_2 = 2\frac{v}{2L} = 2f_1$$



(c) Third harmonic: 
$$f_3 = 3\frac{v}{2L} = 3f_1$$

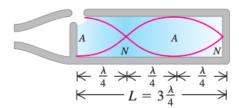


(a) Fundamental: 
$$f_1 = \frac{v}{4L}$$

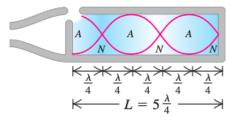


Closed end is always a displacement node.

**(b)** Third harmonic: 
$$f_3 = 3\frac{v}{4L} = 3f_1$$



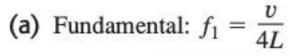
(c) Fifth harmonic: 
$$f_5 = 5\frac{v}{4L} = 5f_1$$

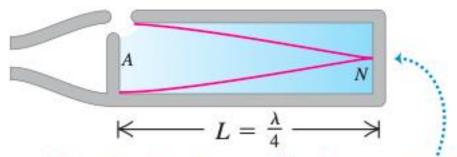




# Example 16.11 A tale of two pipes

On a day when the speed of sound is 345 m/s, the fundamental frequency of a particular stopped organ pipe is 220 Hz. (a) How long is this pipe? (b) The second *overtone* of this pipe has the same wavelength as the third *harmonic* of an *open* pipe. How long is the open pipe?





Closed end is always a displacement node.

**EXECUTE:** (a) For a stopped pipe  $f_1 = v/4L$ , so

$$L_{\text{stopped}} = \frac{v}{4f_1} = \frac{345 \text{ m/s}}{4(220 \text{ s}^{-1})} = 0.392 \text{ m}$$

(b) The frequency of the second overtone of a stopped pipe (the third possible frequency) is  $f_5 = 5f_1 = 5(220 \text{ Hz}) = 1100 \text{ Hz}$ . If the wavelengths for the two pipes are the same, the frequencies are also the same. Hence the frequency of the third harmonic of the open pipe, which is at  $3f_1 = 3(v/2L)$ , equals 1100 Hz. Then

1100 Hz = 
$$3\left(\frac{345 \text{ m/s}}{2L_{\text{open}}}\right)$$
 and  $L_{\text{open}} = 0.470 \text{ m}$ 

#### 16-6 Interference of Waves

Sound waves can undergo interference, like transverse waves. Let us consider, in particular, the interference between two identical sound waves travelling in the same direction

 $S_1$   $L_2$ The interference at P depends on the difference in the path lengths to reach P.

Path length difference,  $\Delta L = |L_2 - L_1|$ 

If the difference is equal to, say,  $2.0\lambda$ , then the waves arrive exactly in phase. This is how transverse waves would look.

If the difference is equal to, say,  $2.5\lambda$ , then the waves arrive exactly out of phase. This is how transverse waves would look.

#### 16-6 Interference of Waves

▶ Phase difference (相差)

▶ Fully constructive interference (相長干涉)

$$\phi = m(2\pi), \text{ for } m = 0,1,2,...$$

$$\frac{\Delta L}{\lambda} = 0,1,2,...$$

▶ Fully destructive interference (相消干涉)

$$\phi = (2m+1)\pi$$
, for  $m = 0,1,2,...$ 

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \dots$$

#### 16-6 Interference of Waves

(a) The path lengths from the speakers to the microphone differ by  $\lambda$  ...

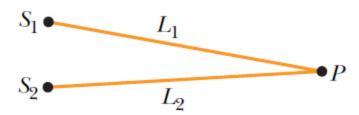


(b) The path lengths from the speakers to the microphone differ by  $\frac{\lambda}{2}$  ...



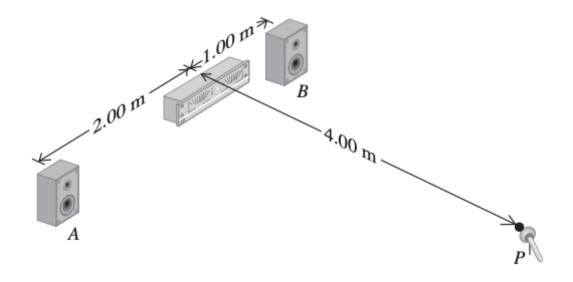
# Questions

Two point sources  $S_1$  and  $S_2$ , which are in phase, emit identical sound waves of wavelength 2.0 m. In terms of wavelengths, what is the phase difference between the waves arriving at point P if (a)  $L_1 = 38$  m and  $L_2 = 34$  m, and (b)  $L_1 = 39$  m and  $L_2 = 36$  m? (c) Assuming that the source separation is much smaller than  $L_1$  and  $L_2$ , what type of interference occurs at P in situations (a) and (b)?



# Example 16.13 Loudspeaker interference

Two small loudspeakers, A and B (Fig. 16.23), are driven by the same amplifier and emit pure sinusoidal waves in phase. (a) For what frequencies does constructive interference occur at point P? (b) For what frequencies does destructive interference occur? The speed of sound is 350 m/s.



**EXECUTE:** The distance from A to P is  $[(2.00 \text{ m})^2 + (4.00 \text{ m})^2]^{1/2} = 4.47 \text{ m}$ , and the distance from B to P is  $[(1.00 \text{ m})^2 + (4.00 \text{ m})^2]^{1/2} = 4.12 \text{ m}$ . The path difference is d = 4.47 m - 4.12 m = 0.35 m.

(a) Constructive interference occurs when  $d=0, \lambda, 2\lambda, \ldots$  or  $d=0, v/f, 2v/f, \ldots = nv/f$ . So the possible frequencies are

$$f_n = \frac{nv}{d} = n \frac{350 \text{ m/s}}{0.35 \text{ m}}$$
  $(n = 1, 2, 3, ...)$   
= 1000 Hz, 2000 Hz, 3000 Hz,...

(b) Destructive interference occurs when  $d = \lambda/2$ ,  $3\lambda/2$ ,  $5\lambda/2$ ,... or d = v/2f, 3v/2f, 5v/2f,.... The possible frequencies are

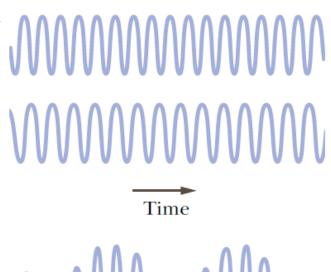
$$f_n = \frac{nv}{2d} = n \frac{350 \text{ m/s}}{2(0.35 \text{ m})}$$
  $(n = 1, 3, 5, ...)$   
= 500 Hz, 1500 Hz, 2500 Hz,...



### 16-7 Beats (拍)

#### Beat

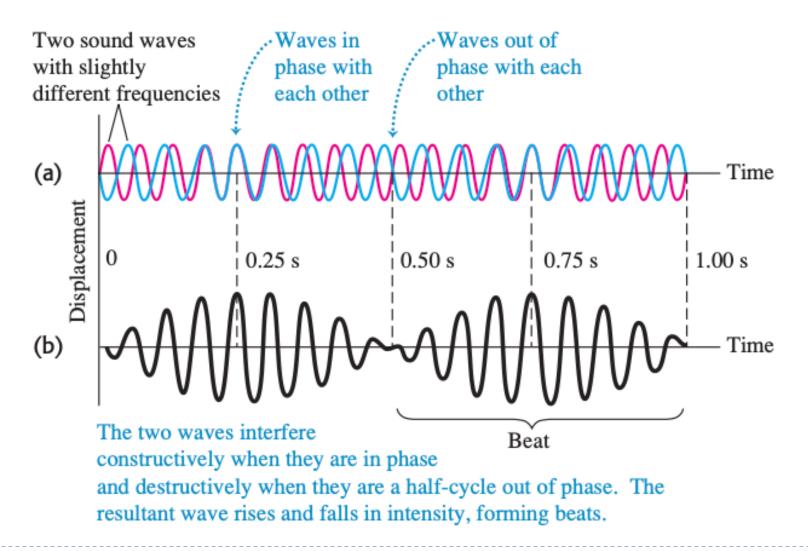
When two sound waves whose frequencies ( $f_1$  and  $f_2$ ) are close, but not the same, are superimposed, a variation in the intensity of the resultant sound wave is heard



#### Beat frequency

- At which the wavering of intensity occurs
  - $f_{beat} = |f_1 f_2|$

### 16-7 Beats (拍)



## 16-7 Beats (拍)

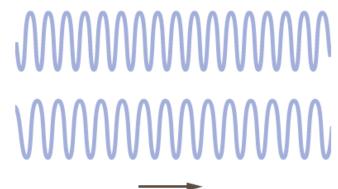
$$T_{\text{beat}} = nT_a$$
 and

$$T_{\text{beat}} = (n-1)T_b$$

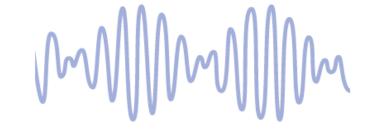
$$T_{\text{beat}} = \frac{T_a T_b}{T_b - T_a}$$

$$f_{\text{beat}} = \frac{T_b - T_a}{T_a T_b} = \frac{1}{T_a} - \frac{1}{T_b}$$

$$f_{\text{beat}} = f_a - f_b$$
 (beat frequency)







- A pipe with two open ends. Suppose that the frequency of the first harmonic produced by side A is  $f_{A1} = 432$  Hz and the frequency of the first harmonic produced by side B is  $f_{B1} = 371$  Hz. What is the beat frequency between those two first-harmonic frequencies and between the two second-harmonic frequencies?
- For the first-harmonic frequencies,

$$f_{\text{beat},1} = f_{A1} - f_{B1} = 432 \text{ Hz} - 371 \text{ Hz}$$
  
= 61 Hz.

For the second-harmonic frequencies,

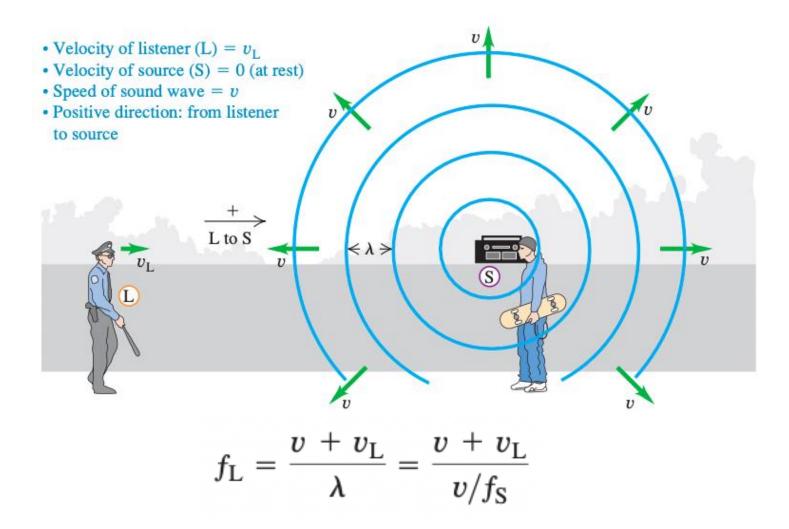
$$f_{\text{beat,2}} = f_{A2} - f_{B2} = 2f_{A1} - 2f_{B1}$$
  
= 2(432 Hz) - 2(371 Hz)  
= 122 Hz.

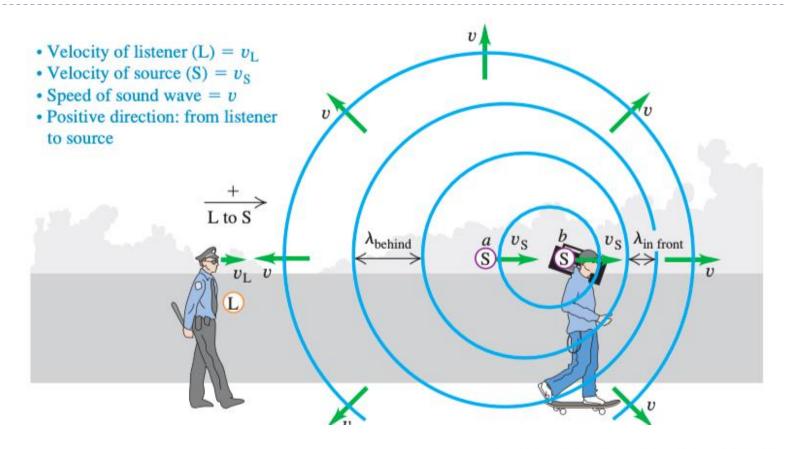


- The Doppler effect is the change in frequency of a wave for a detector (observer) moving relative to its source
- ▶ The *detected frequency* can be related by a general equation

$$f' = f \frac{v \pm v_D}{v \pm v_S}$$

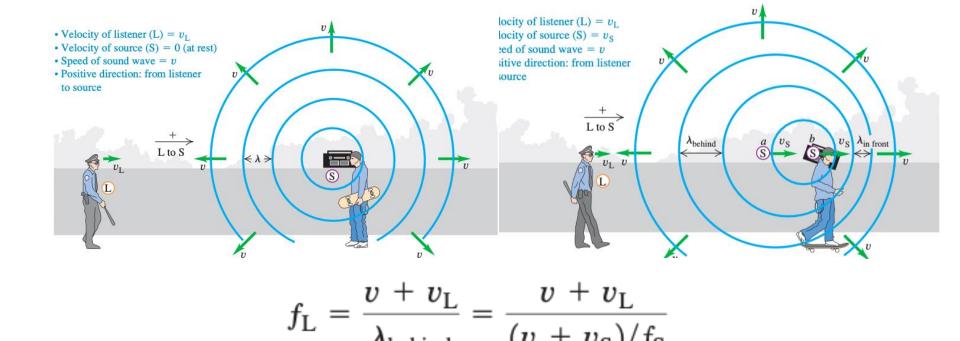
- ▶ *f* is the emitted frequency
- ightharpoonup v is the speed of sound through the medium
- $\triangleright$   $v_D$  is the detector's speed *relative to the medium*
- $\triangleright v_S$  is the source's speed *relative to the medium*
- When the motion of detector or source is <u>toward</u> the other, the sign on its speed must give an <u>upward shift</u> in frequency
- When the motion of detector or source is <u>away</u> from the other, the sign on its speed must give a <u>downward shift</u> in frequency





$$\lambda_{
m in \ front} = rac{v}{f_{
m S}} - rac{v_{
m S}}{f_{
m S}} = rac{v - v_{
m S}}{f_{
m S}}$$

$$\lambda_{\text{behind}} = \frac{v + v_{\text{S}}}{f_{\text{S}}}$$



$$f_{\rm L} = \frac{v + v_{\rm L}}{v + v_{\rm S}} f_{\rm S}$$

(Doppler effect, moving source and moving listener)

**16.28** The Doppler effect explains why the siren on a fire engine or ambulance has a high pitch  $(f_L > f_S)$  when it is approaching you  $(v_S < 0)$  and a low pitch  $(f_L < f_S)$  when it is moving away  $(v_S > 0)$ .



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#### Example 16.14 Doppler effect I: Wavelengths

A police car's siren emits a sinusoidal wave with frequency  $f_S = 300$  Hz. The speed of sound is 340 m/s and the air is still. (a) Find the wavelength of the waves if the siren is at rest. (b) Find the wavelengths of the waves in front of and behind the siren if it is moving at 30 m/s.

(a) When the source is at rest,

$$\lambda = \frac{v}{f_S} = \frac{340 \text{ m/s}}{300 \text{ Hz}} = 1.13 \text{ m}$$

(b) From Eq. (16.27), in front of the siren

$$\lambda_{\text{in front}} = \frac{v - v_{\text{S}}}{f_{\text{S}}} = \frac{340 \text{ m/s} - 30 \text{ m/s}}{300 \text{ Hz}} = 1.03 \text{ m}$$

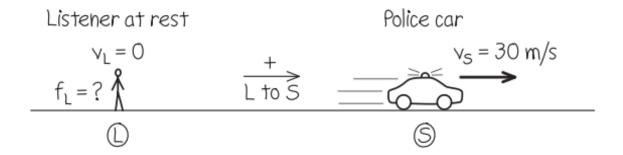
From Eq. (16.28), behind the siren

$$\lambda_{\text{behind}} = \frac{v + v_{\text{S}}}{f_{\text{S}}} = \frac{340 \text{ m/s} + 30 \text{ m/s}}{300 \text{ Hz}} = 1.23 \text{ m}$$

#### Example 16.15

#### **Doppler effect II: Frequencies**

If a listener L is at rest and the siren in Example 16.14 is moving away from L at 30 m/s, what frequency does the listener hear?

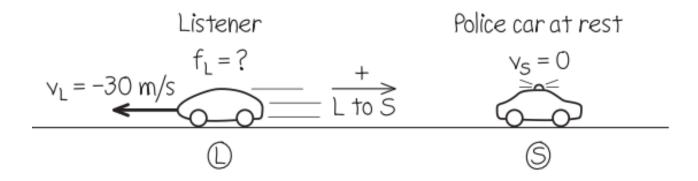


wavelength behind the source (where the listener in Fig. 16.30 is located) is 1.23 m. The wave speed relative to the stationary listener is v = 340 m/s even though the source is moving, so

$$f_{\rm L} = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{1.23 \text{ m}} = 276 \text{ Hz}$$

#### Example 16.16 Doppler effect III: A moving listener

If the siren is at rest and the listener is moving away from it at 30 m/s, what frequency does the listener hear?



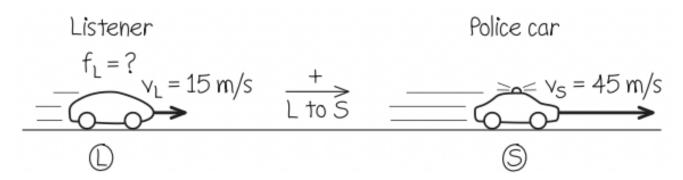
**EXECUTE:** From Eq. (16.29),

$$f_{\rm L} = \frac{v + v_{\rm L}}{v} f_{\rm S} = \frac{340 \text{ m/s} + (-30 \text{ m/s})}{340 \text{ m/s}} (300 \text{ Hz}) = 274 \text{ Hz}$$

#### Example 16.17

#### Doppler effect IV: Moving source, moving listener

The siren is moving away from the listener with a speed of 45 m/s relative to the air, and the listener is moving toward the siren with a speed of 15 m/s relative to the air. What frequency does the listener hear?

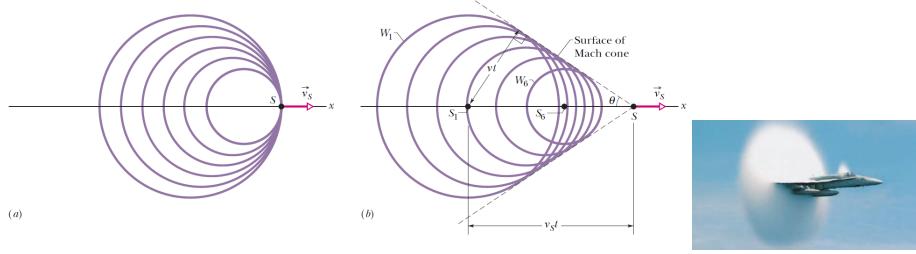


**EXECUTE:** From Eq. (16.29),

$$f_{\rm L} = \frac{v + v_{\rm L}}{v + v_{\rm S}} f_{\rm S} = \frac{340 \text{ m/s} + 15 \text{ m/s}}{340 \text{ m/s} + 45 \text{ m/s}} (300 \text{ Hz}) = 277 \text{ Hz}$$

#### 16-9 Shock Waves

When the speed of the source  $(v_S)$ , exceeds the speed of sound (v), the aforementioned equations no longer apply

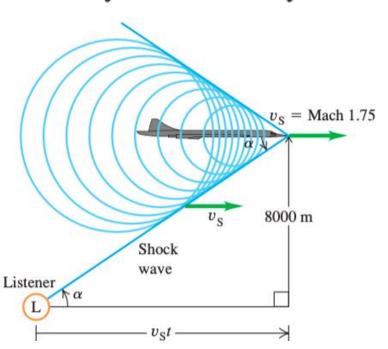


- A shock wave exists along the surface of this cone

  - Mach number,  $M = \frac{v_S}{v}$   $\begin{cases} < 1 \ (subsonic 亞/次音速) \\ = 1 \ (sonic 音速) \\ > 1 \ (supersonic 超音速) \end{cases}$

#### Example 16.19 Sonic boom from a supersonic airplane

An airplane is flying at Mach 1.75 at an altitude of 8000 m, where the speed of sound is 320 m/s. How long after the plane passes directly overhead will you hear the sonic boom?



**EXECUTE:** From Eq. (16.31) the angle  $\alpha$  of the shock cone is

$$\alpha = \arcsin \frac{1}{1.75} = 34.8^{\circ}$$

The speed of the plane is the speed of sound multiplied by the Mach number:

$$v_{\rm S} = (1.75)(320 \text{ m/s}) = 560 \text{ m/s}$$

From Fig. 16.37 we have

$$\tan \alpha = \frac{8000 \text{ m}}{v_S t}$$

$$t = \frac{8000 \text{ m}}{(560 \text{ m/s})(\tan 34.8^\circ)} = 20.5 \text{ s}$$