Photons: Light Waves Behaving as Particles

I. *Concepts*

1. Photon: A fundamental particle of light that is considered a quantum of electromagnetic radiation. It exhibits both wave-like and particle-like properties and is responsible for carrying energy and momentum in the form of light.
2. Photoelectric Effect: The phenomenon where electrons are emitted from a material when light shines on it. This effect was crucial in establishing the particle nature of light as the classical wave theory of light could not fully explain it.
3. X-Ray Production: The process by which X-rays are generated, often through the acceleration of electrons and their interaction with a target. This can occur when high-energy electrons are decelerated or when electrons in an atom transition to lower energy levels.
4. Compton Scattering: The interaction in which a photon scatters off an electron, resulting in a transfer of energy and momentum between the photon and the electron. This causes a change in the photon's wavelength and provides evidence for the particle nature of photons.
5. Wave-Particle Duality: The principle that light and matter can exhibit both wave-like and particle-like characteristics. For example, photons can behave as particles in some interactions (like the photoelectric effect and Compton scattering) and as waves in others (showing interference and diffraction patterns).
6. Heisenberg Uncertainty Principle: States that it is impossible to precisely and simultaneously know the exact position and momentum of a particle. It reflects the probabilistic nature of quantum mechanics and sets a fundamental limit on our ability to measure and understand the microscopic world.

II. *Formulas*

1. Photon Energy and Momentum:

$$E = hf = \frac{hc}{\lambda}$$

The energy E of a photon is related to its frequency f and wavelength λ. Planck's constant $h = 6.62606896(33) \times 10^{-34} J \cdot s$ and the speed of light $c = 3.0 \times 10^{8} m/s$. Higher frequency or shorter wavelength photons have more energy.

$$p = \frac{E}{c} = \frac{h}{\lambda}$$

The momentum p of a photon is related to its energy and wavelength. Photons, despite having no rest mass, possess momentum, with shorter wave length photons having greater momentum.

2. Photoelectric Effect

$$K_{max} = \frac{1}{2} m v_{max}^2 = hf - \phi$$

The maximum kinetic energy K_{max} of the emitted electrons in the photoelectric effect. m is the mass of the electron, v_{max} is the maximum velocity of the emitted electrons, hf is the energy of the incident photon, and ϕ is the work function of the material (the minimum energy required to remove an electron).

$$eV_0 = hf - \phi$$

The relationship between the stopping potential V_0 and the maximum kinetic energy of the electrons. $e = 1.60 \times 10^{-19}C$ is the elementary charge. The stopping potential is the potential difference needed to stop the most energetic emitted electrons and is related to the frequency of the incident light.

3. X-Ray Production:

$$\lambda_{min} = \frac{hc}{eV_{AC}}$$

The minimum wavelength λ_{min} of X-rays produced. V_{AC} is the accelerating voltage. Higher accelerating voltages result in shorter wavelength (more energetic) X-rays, based on energy conservation as the energy of the accelerated electrons is converted into the energy of the X-ray photons.

4. Compton Scattering:

$$\lambda' - \lambda = \frac{h}{mc}(1 - cos\phi)$$

The change in wavelength $\Delta\lambda$ during Compton scattering. λ' is the wavelength of the scattered photon, λ is the wavelength of the incident photon, m is the mass of the electron, and ϕ is the scattering angle. $\frac{h}{mc} = 2.426 \times 10^{-12}m$ (Compton wavelength). The change in wavelength depends on the scattering angle, with larger angles resulting in greater changes, demonstrating the particle nature of photons in interaction with electrons.

5. Heisenberg Uncertainty Principle: shows the uncertainty relationship between the position $\boldsymbol{\Delta x}$ and momentum $\boldsymbol{\Delta p_x}$ of a particle. It sets a fundamental limit on the precision with which we can know both simultaneously, reflecting the probabilistic nature of the quantum world.

$$\Delta x \Delta p_x \geq \hbar/2 \text{ (where } \hbar = \frac{h}{2\pi}\text{)}$$

III. Problems

1. A photon has a frequency of $7.5 \times 10^{14}Hz$. Calculate its energy and momentum.

2. In a photoelectric experiment, the work function of the metal is $3.5eV$. What is the maximum kinetic energy of the electrons when light with a wavelength of $3.0 \times 10^{-7}m$ is incident on the metal?