1. Introduction to vectors

1.1 Vectors & linear combinations

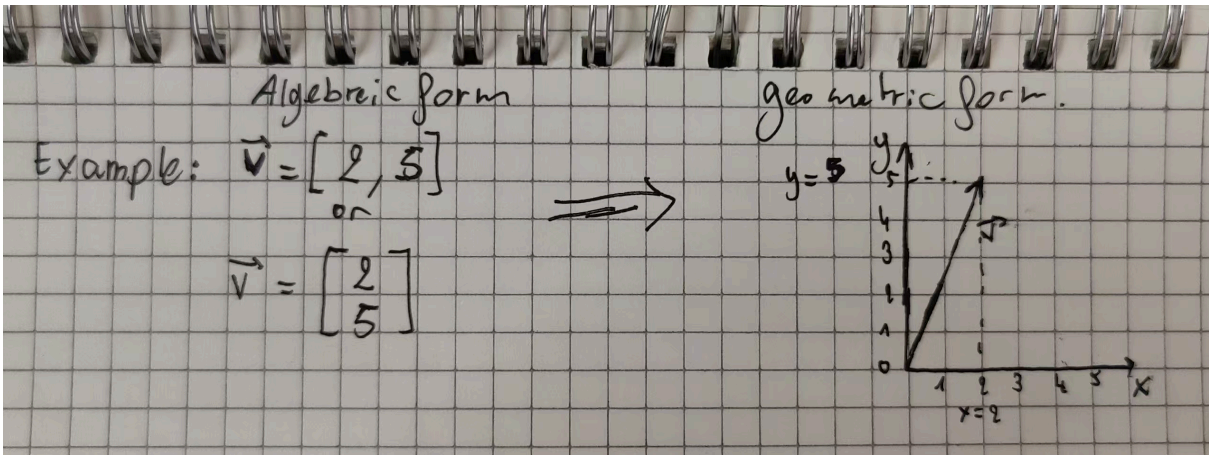
a. **Vector:** Mathematical object that has both a **magnitude** (length) and **direction**. geometrically it's an arrow, algebraically it's an ordered list of numbers, we can write it in two ways:

Row vectors $\quad \mathbf{a}_{row} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$

Column vectors $\quad \mathbf{a}_{col} = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix}$

a_1, a_2, ... a_n: are called components or coordinates of the vector a.

- The algebraic form tells us the geometric form: the coordinates tell you how to move along each axis to draw the arrow (vector).



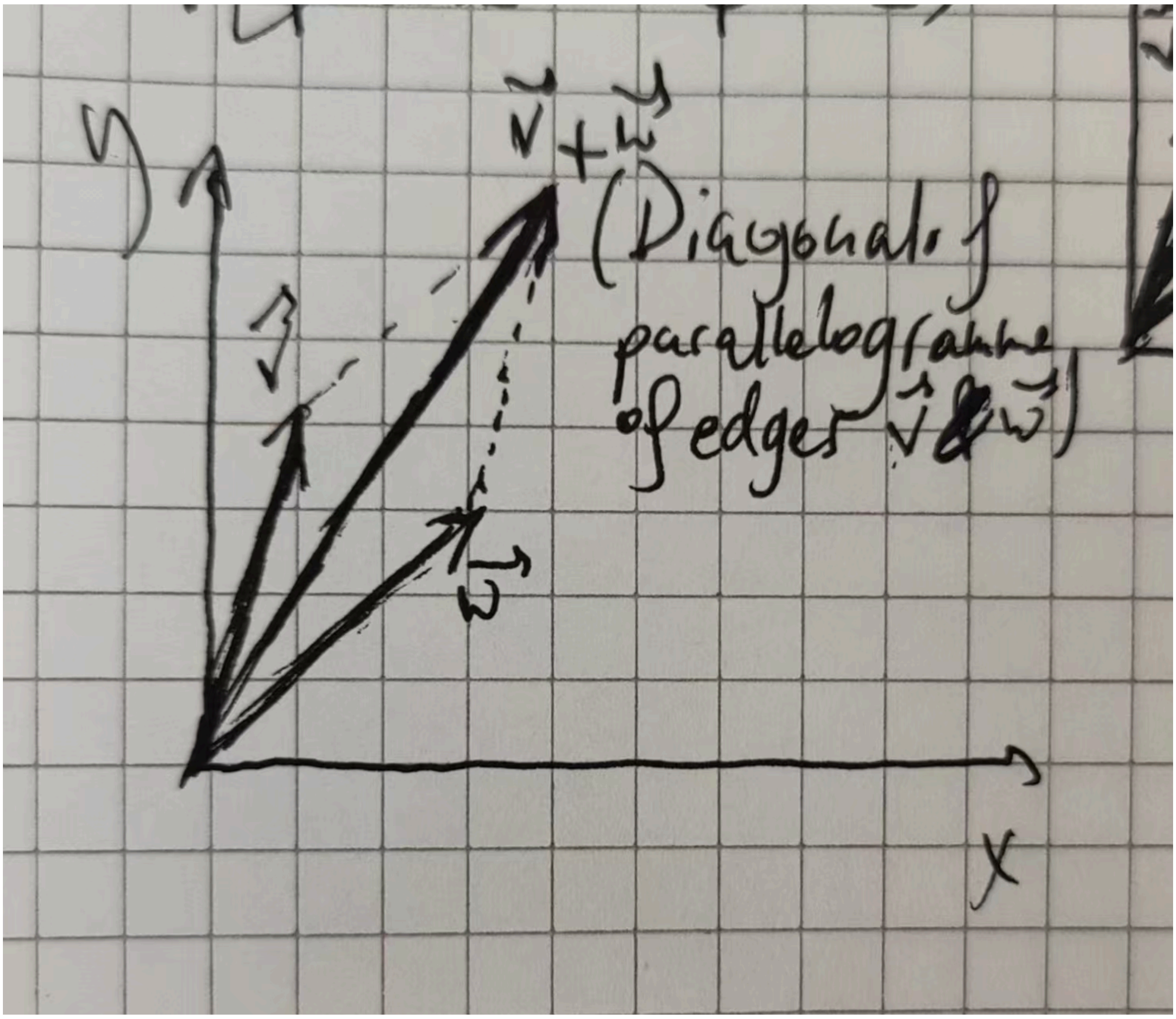
b. Linear combination: Combining vectors togethers using two fundamental operations >
+ scalar multiplication: multiplying a vector by a scalar = constant

$$C.\vec{V} = C\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} CV_1 \\ CV_2 \end{bmatrix}$$

+ Vector addition: Adding two vectors together

$$\vec{v} + \vec{w} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix}$$

Visual representation of vector addition for 2 non Parallele, non-zero vectors in 2D



+Generally, a linear combination is written as:
$c_1v_1 + c_2v_2 + ... + c_nv_n$ (it gives a whole new vector)

where: v_1, v_2, ..., v_n are your original vectors. c_1, c_2, ..., c_n are scalars.
You're basically scaling each vector by a number then adding their scaled versions together.

c. why we need linear combinations?
Linear combinations help us figure out and understand the following concepts:

- **Span**: Set of all possible linear combinations of a set of vectors.
  + If two vectors in a 2D plane have different directions, their span is the entire 2D plane (their possible linear combinations can fill up the entire 2D plane)
  + If two vectors are parallel, their span is just a line (all their linear combinations lead to the same line)

- **Linear dependence/independence**: a set is linearly independent if no vector can be written as a linear combination of the other. Otherwise, if it's possible, it's dependent.
- **Solving systems of equations**:

Every system of linear equations can be written as a vector equation asking a question about linear combinations.

The system:

2x + 3y = 7

1x + 1y = 3

Can be re-written as:

x * [2, -1] + y * [3, 1] = [7, 3]

The question becomes: "**What linear combination** of the vectors [2, -1] and [3, 1] results in the vector [7, 3]?" The scalars `x` and `y` are the solutions.

1.2 Lengths and dots product

1.3 Matrices