PHYS1001B College Physics IB

Optics IV Diffraction (Ch. 36)

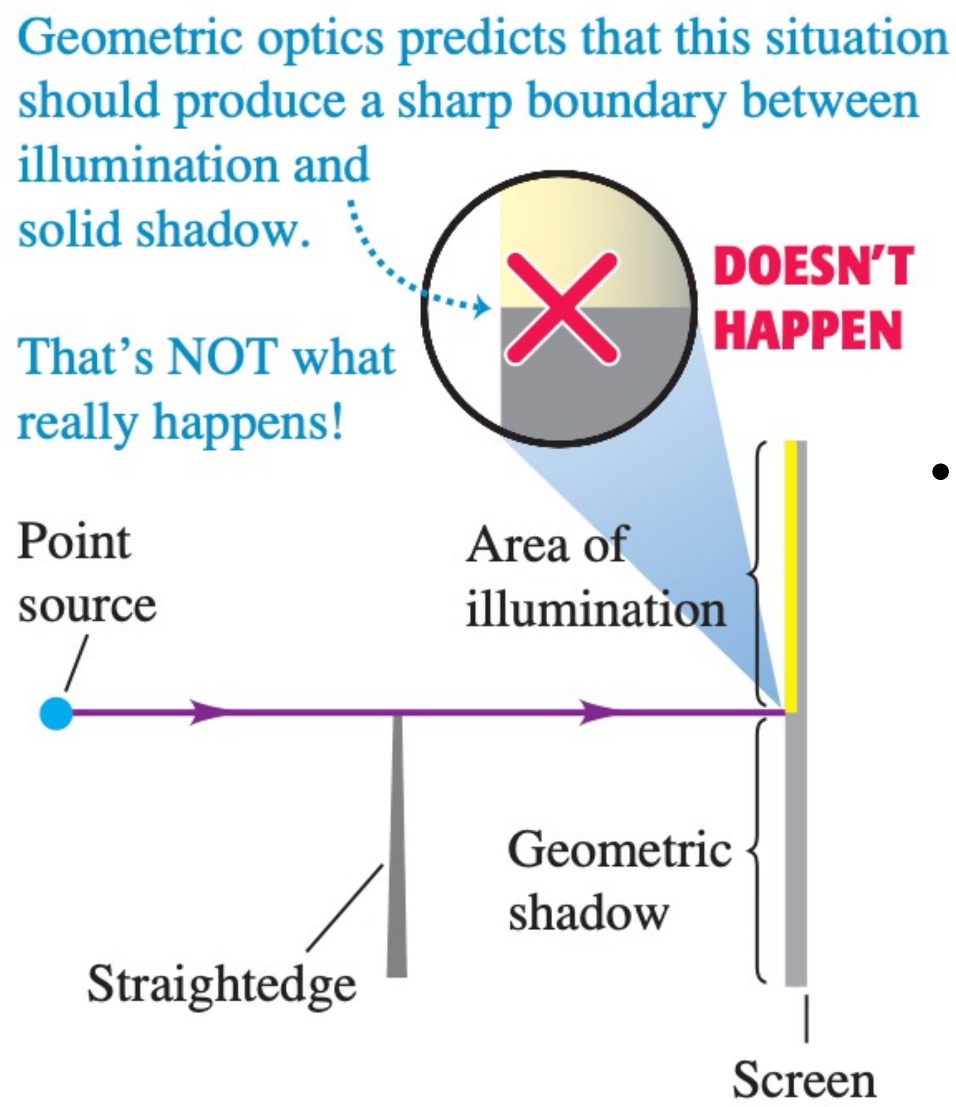
Introduction

The laser used to read a DVD has a wavelength of 650 nm, while the laser used to read a Blu-ray disc has a shorter 405-nm wavelength. How does this make it possible for a Blu-ray disc to hold more information than a DVD?

Outline

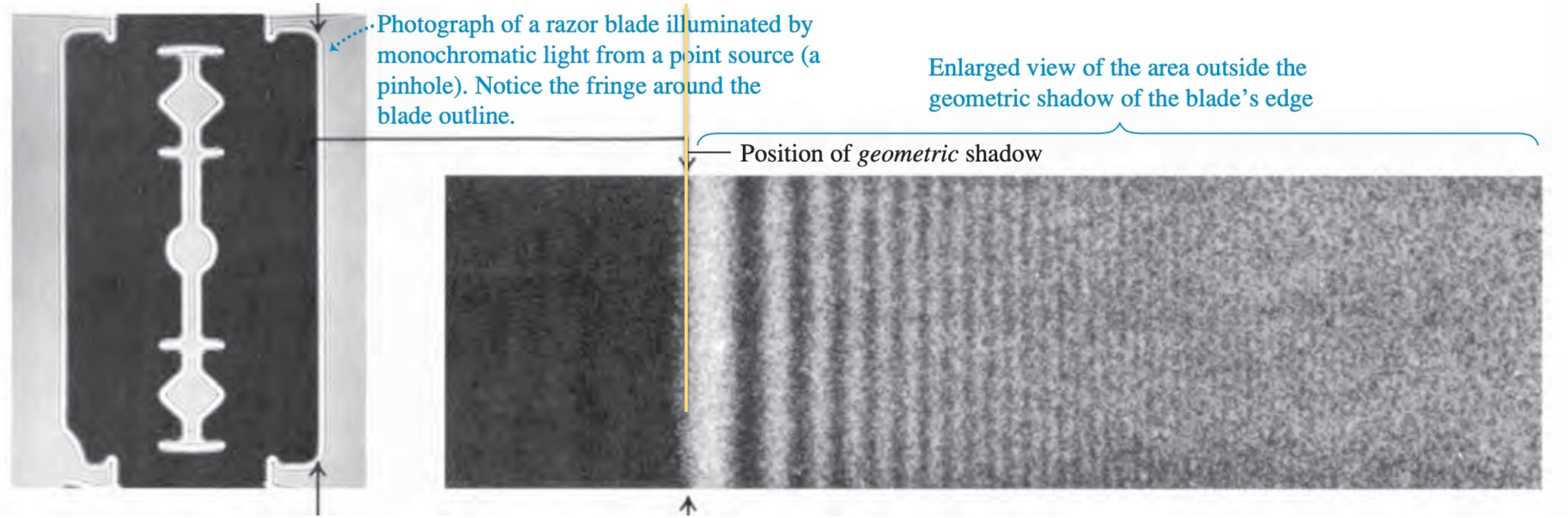
- 36-1 Fresnel and Fraunhofer Diffraction
- 36-2 Diffraction from a Single Slit
- 36-3 Intensity in the Single-Slit Pattern
- 36-4 Multiple Slits
- 36-5 The Diffraction Grating

36-1 Fresnel and Fraunhofer Diffraction



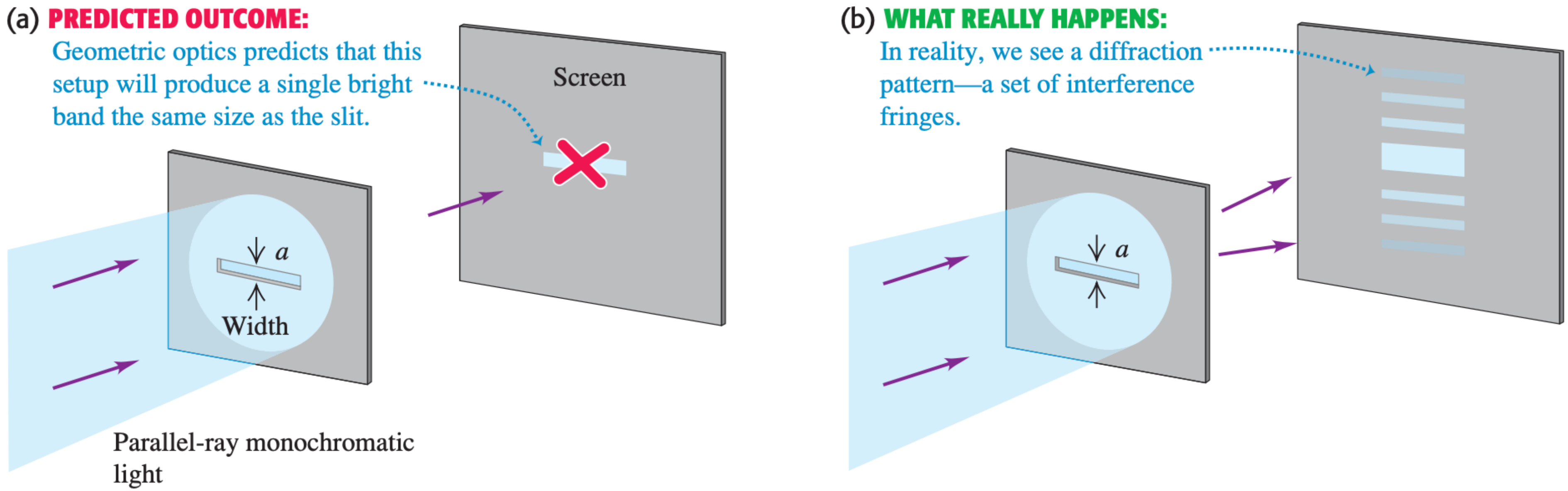
- Wave nature after they pass through an aperture

36-1 Fresnel and Fraunhofer Diffraction

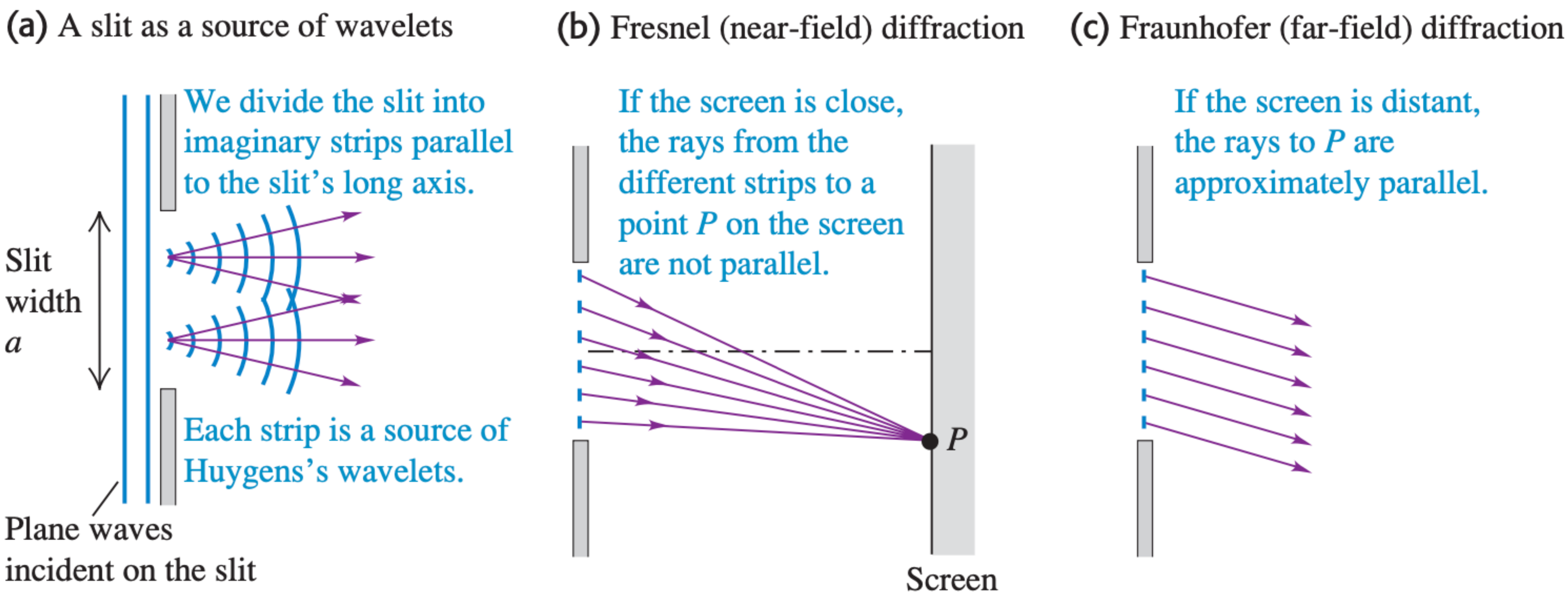


36-2 Diffraction from a Single Slit

Wave nature



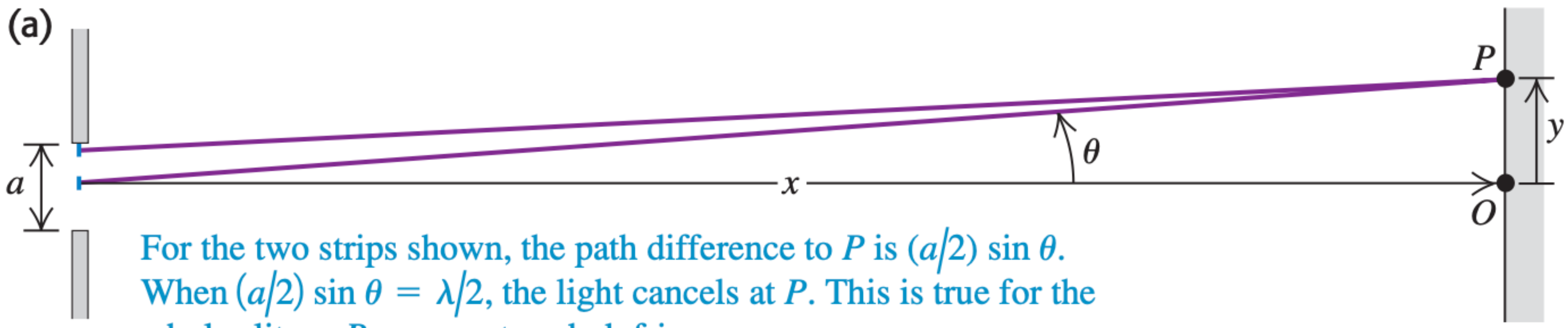
36-2 Diffraction from a Single Slit



Fresnel diffraction : the source, obstacle, and screen are close

Fraunhofer diffraction:the source, obstacle, and screen are far enough apart (Simpler, the case we studied), almost parallel light rays

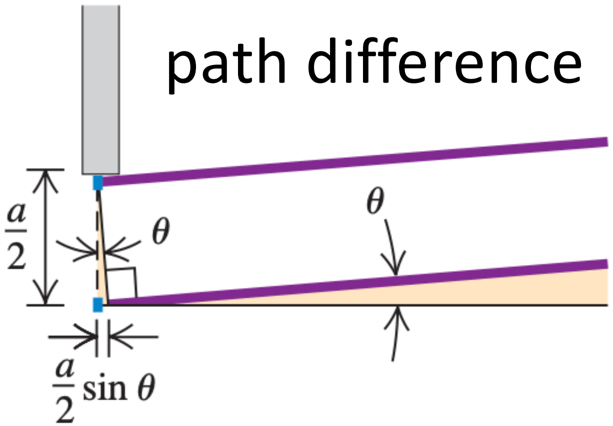
36-2 Diffraction from a Single Slit



For the two strips shown, the path difference to P is $(a/2)\sin\theta$. When $(a/2)\sin\theta = \lambda/2$, the light cancels at P. This is true for the whole slit, so P represents a dark fringe.

(b) Enlarged view of the top half of the slit

path difference



Dark fringes condition $$\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2}$$

θ is usually very small, so we can use the approximations $\sin\theta = \theta$ and $\tan\theta = \theta$. Then the condition for a dark band is

$$y_m = x\,\frac{m\lambda}{a}$$

$$\sin\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \pm 3, \dots) \qquad \text{(dark fringes in single-slit diffraction)}$$

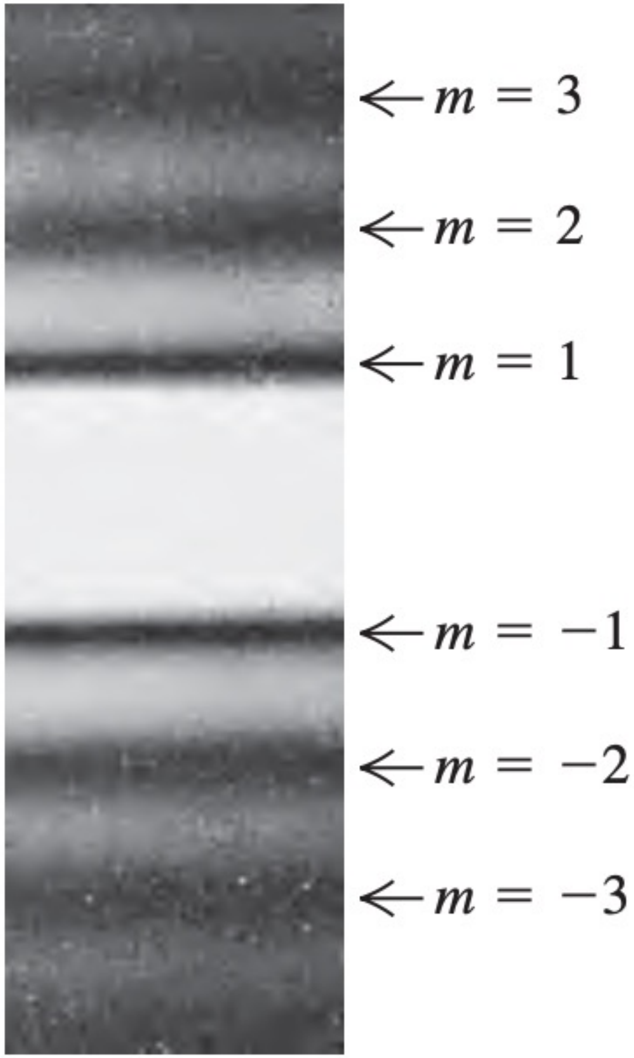
36-2 Diffraction from a Single Slit

Location of dark fringe

$$\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \pm 3, \ldots)$$

Small theta

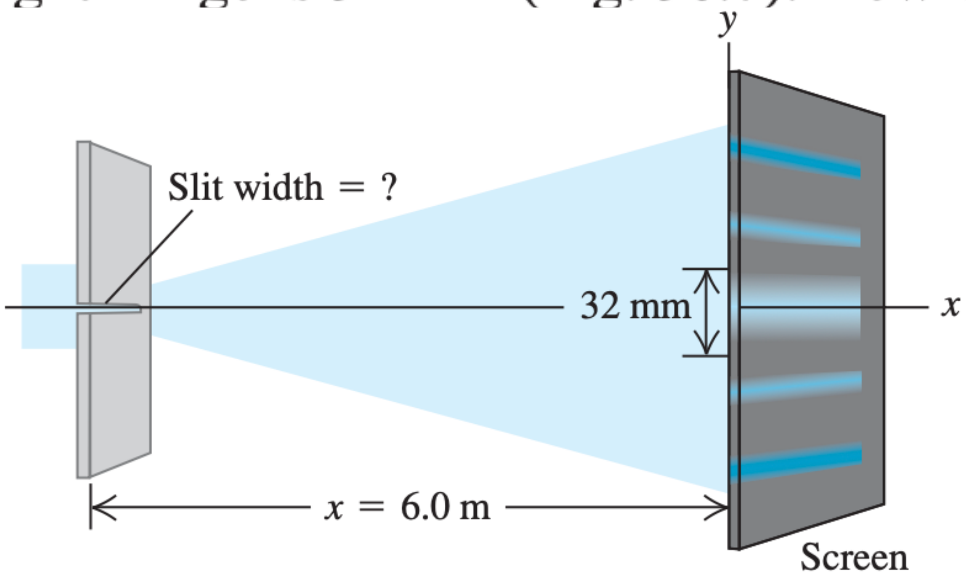
$$y_m = x\frac{m\lambda}{a} \qquad (\text{for } y_m \ll x)$$



Sample Problem

Example 36.1 **Single-slit diffraction**

You pass 633-nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm (Fig. 36.7). How wide is the slit?



Sample Problem

Example 36.1 Single-slit diffraction

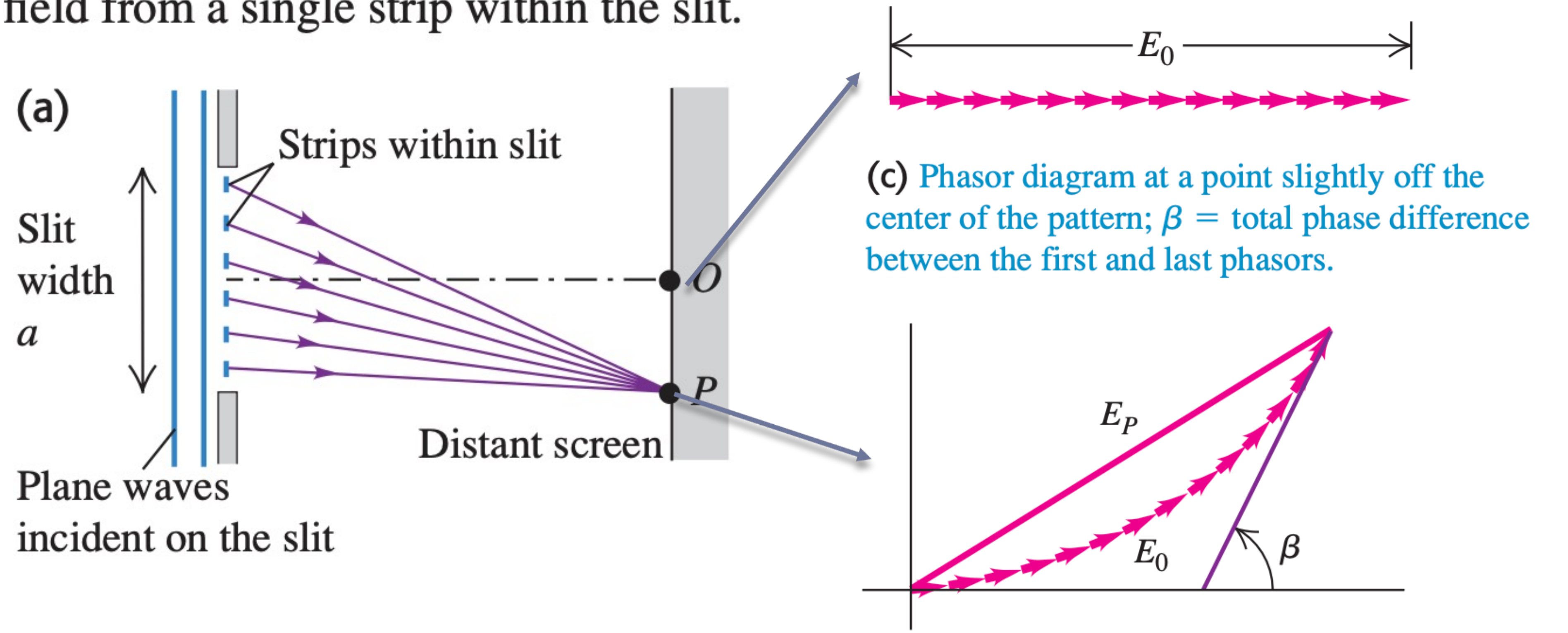
You pass 633-nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm (Fig. 36.7). How wide is the slit?

EXECUTE: The first minimum corresponds to $m = 1$ in Eq. (36.3). The distance y_1 from the central maximum to the first minimum on either side is half the distance between the two first minima, so $y_1 = (32\text{ mm})/2 = 16\text{ mm}$. Solving Eq. (36.3) for a, we find

$$a = \frac{x\lambda}{y_1} = \frac{(6.0\text{ m})(633 \times 10^{-9}\text{ m})}{16 \times 10^{-3}\text{ m}} = 2.4 \times 10^{-4}\text{ m} = 0.24\text{ mm}$$

36-3 Intensity in the Single-Slit Pattern

36.8 Using phasor diagrams to find the amplitude of the $\vec{E}$ field in single-slit diffraction. Each phasor represents the $\vec{E}$ field from a single strip within the slit.



(b) At the center of the diffraction pattern (point O), the phasors from all strips within the slit are in phase.

(c) Phasor diagram at a point slightly off the center of the pattern; β = total phase difference between the first and last phasors.

Path difference ➔ Phase difference of E-field

36-3 Intensity in the Single-Slit Pattern

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2} \qquad \text{(amplitude in single-slit diffraction)}$$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 \qquad \text{(intensity in single-slit diffraction)}$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$$I = I_0 \left\{ \frac{\sin[\pi a (\sin\theta)/\lambda]}{\pi a (\sin\theta)/\lambda} \right\}^2 \qquad \text{(intensity in single-slit diffraction)}$$

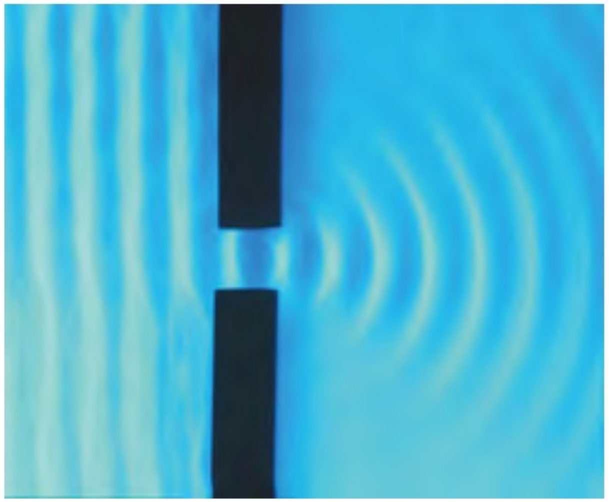
36-3 Intensity in the Single-Slit Pattern

$$\beta \approx \pm(2m+1)\pi \qquad (m = 0, 1, 2, \ldots)$$

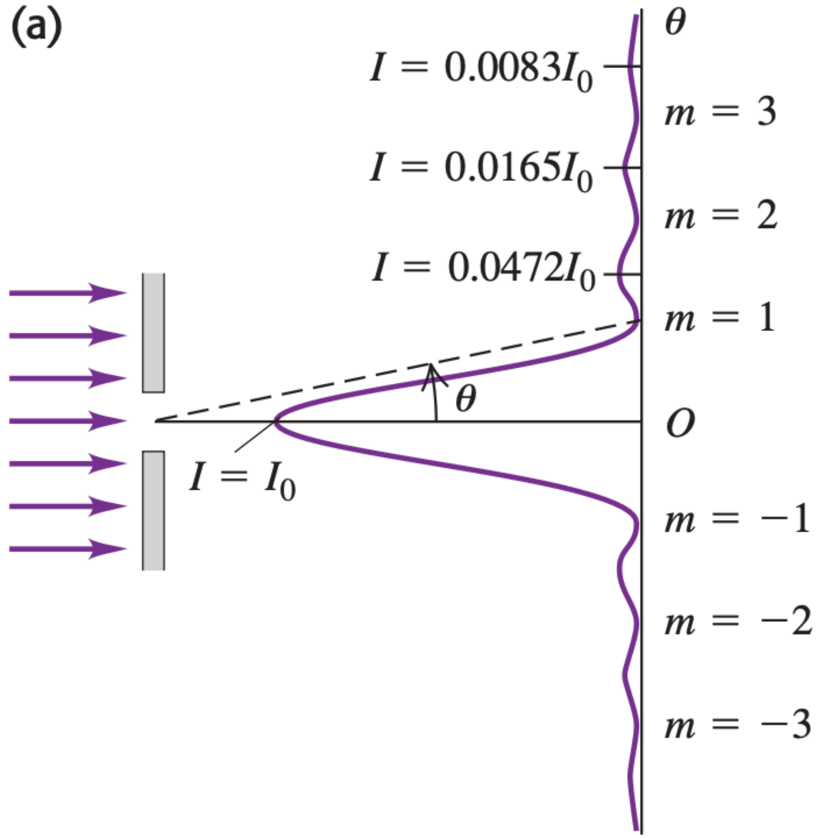
$$I_m \approx \frac{I_0}{\left(m+\frac{1}{2}\right)^2 \pi^2}$$

Approximate solution

$0.0450 I_0$ $\quad$ $0.0162 I_0$ $\quad$ $0.0083 I_0$



Exact solution



Sample Problem

Example 36.2 Single-slit diffraction: Intensity I

(a) The intensity at the center of a single-slit diffraction pattern is I_0. What is the intensity at a point in the pattern where there is a 66-radian phase difference between wavelets from the two edges of the slit? (b) If this point is 7.0° away from the central maximum, how many wavelengths wide is the slit?

EXECUTE: (a) We have $\beta/2 = 33$ rad, so from Eq. (36.5),

$$I = I_0\left[\frac{\sin(33\text{ rad})}{33\text{ rad}}\right]^2 = (9.2 \times 10^{-4})I_0$$

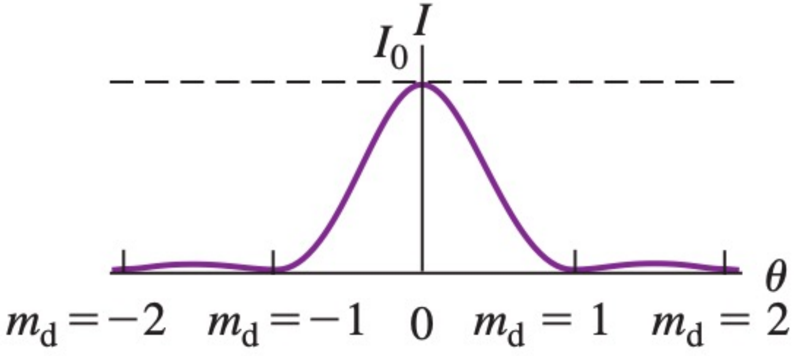
(b) From Eq. (36.6),

$$\frac{a}{\lambda} = \frac{\beta}{2\pi\sin\theta} = \frac{66\text{ rad}}{(2\pi\text{ rad})\sin 7.0°} = 86$$

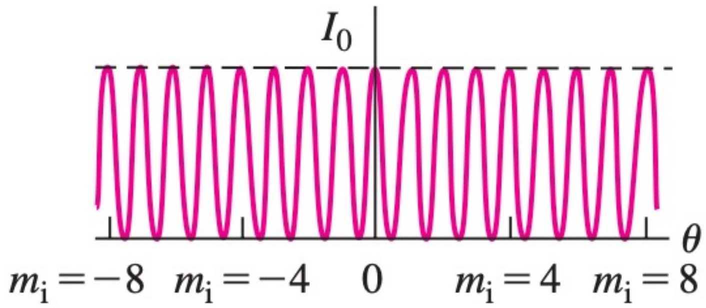
For example, for 550-nm light the slit width is $a = (86)(550\text{ nm}) = 4.7 \times 10^{-5}\text{ m} = 0.047\text{ mm}$, or roughly $\frac{1}{20}$ mm.

36-4 Multiple Slits

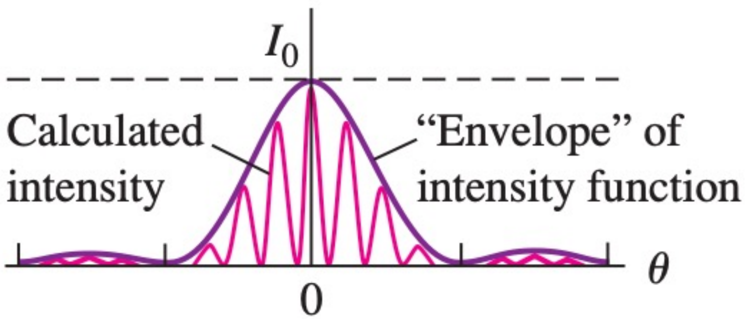
(a) Single-slit diffraction pattern for a slit width a



(b) Two-slit interference pattern for narrow slits whose separation d is four times the width of the slit in **(a)**

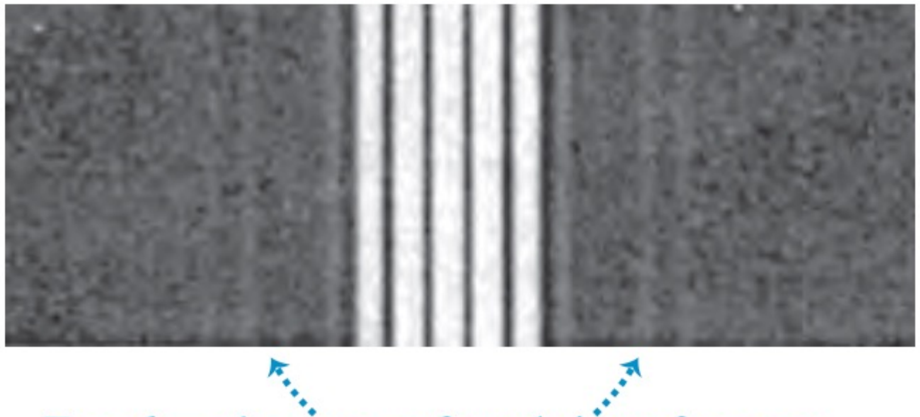


(c) Calculated intensity pattern for two slits of width a and separation $d = 4a$, including both interference and diffraction effects



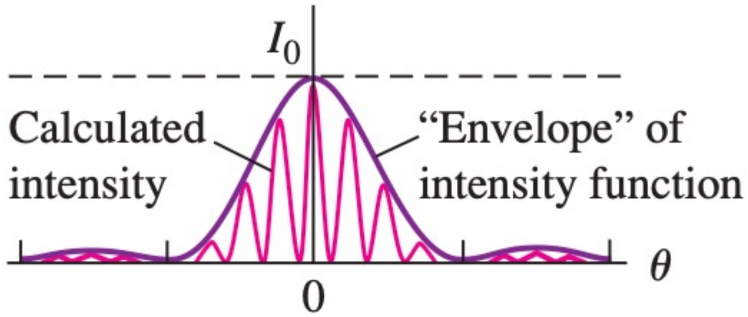
Two-slit: both interference and diffraction

36-4 Multiple Slits



For $d = 4a$, every fourth interference maximum at the sides ($m_i = \pm 4, \pm 8, \ldots$) is missing.

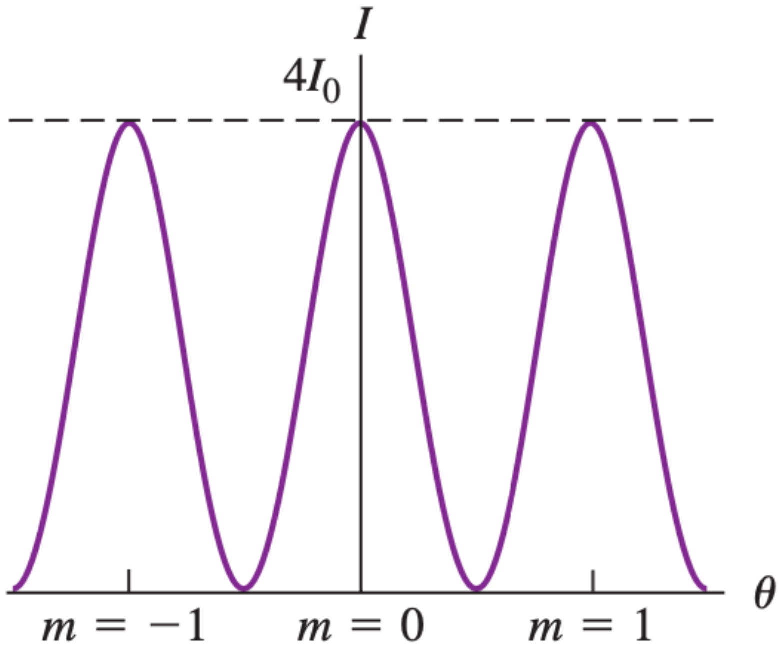
(c) Calculated intensity pattern for two slits of width a and separation $d = 4a$, including both interference and diffraction effects



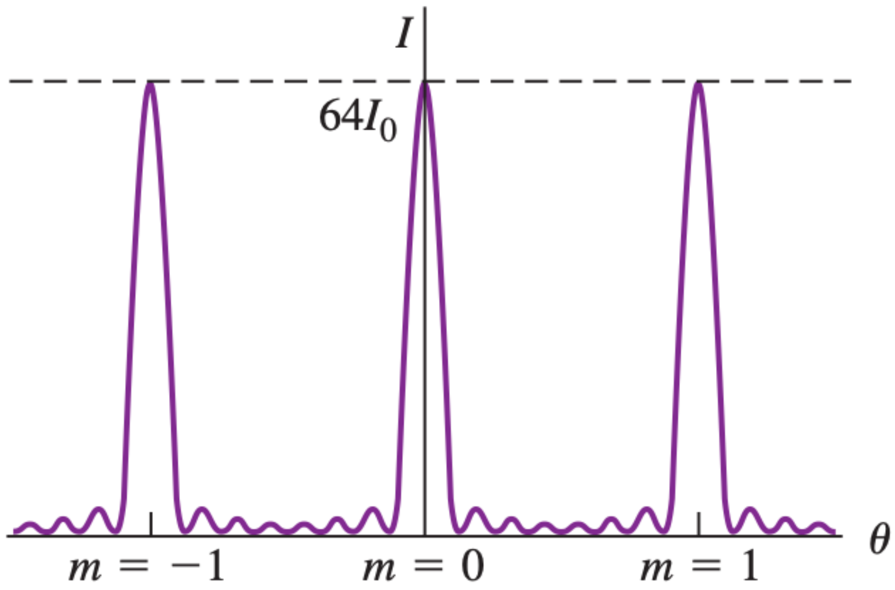
Two-slit: both interference and diffraction

36-4 Multiple Slits

(a) $N = 2$: two slits produce one minimum between adjacent maxima.



(b) $N = 8$: eight slits produce taller, narrower maxima in the same locations, separated by seven minima.

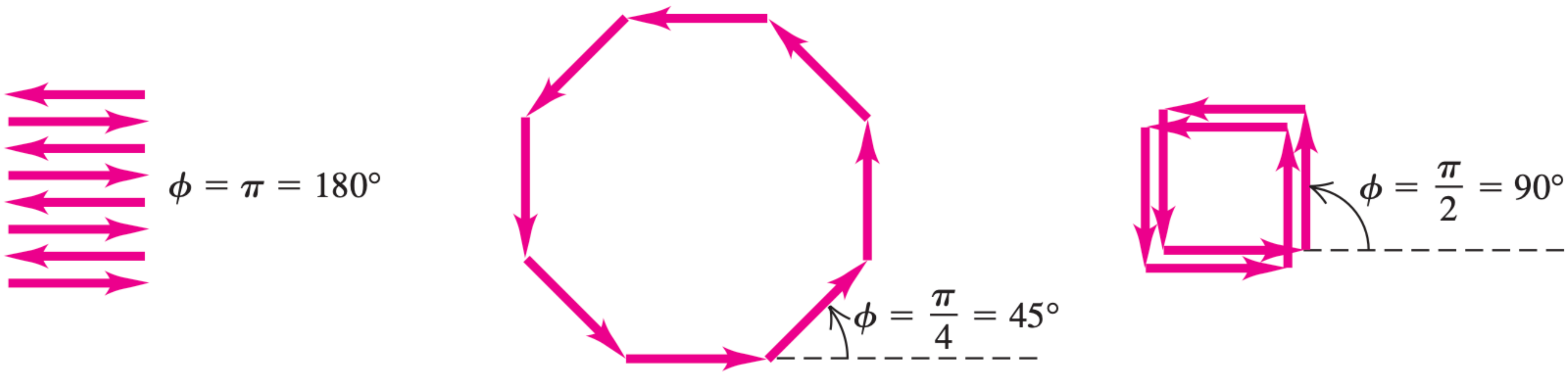


Why?

36-4 Multiple Slits

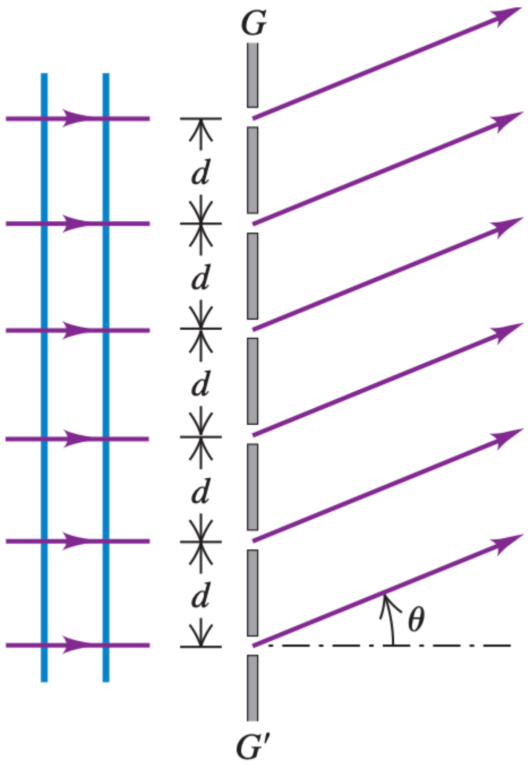
lot of minimum possible between two maximum peaks

(a) Phasor diagram for $\phi = \pi$ **(b)** Phasor diagram for $\phi = \frac{\pi}{4}$ **(c)** Phasor diagram for $\phi = \frac{\pi}{2}$



36-5 The Diffraction Grating

An array of a large number of parallel slits, all with the same width a and spaced equal distances d between centers, is called a **diffraction grating.**



Constructive interference

$$d \sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$

36-5 The Diffraction Grating

Reflection from the disc

- When a grating containing hundreds or thousands of slits is illuminated by a beam of parallel rays of monochromatic light, the pattern is a series of very sharp lines at angles
- The “grooves” are tiny pits 0.12 μm deep in the surface of the disc, with a uniform radial spacing of 0.74 μm = 740 nm. Information is coded on the DVD by varying the *length* of the pits.

Sample Problem

Example 36.4 Width of a grating spectrum

The wavelengths of the visible spectrum are approximately 380 nm (violet) to 750 nm (red). (a) Find the angular limits of the first-order visible spectrum produced by a plane grating with 600 slits per millimeter when white light falls normally on the grating.

(b) Do the first-order and second-order spectra overlap? What about the second-order and third-order spectra? Do your answers depend on the grating spacing?

Sample Problem

SOLUTION

IDENTIFY and SET UP: We must find the angles spanned by the visible spectrum in the first-, second-, and third-order spectra. These correspond to $m = 1, 2$, and 3 in Eq. (36.13).

EXECUTE: (a) The grating spacing is

$$d = \frac{1}{600 \text{ slits/mm}} = 1.67 \times 10^{-6}\text{ m}$$

We solve Eq. (36.13) for θ:

$$\theta = \arcsin\frac{m\lambda}{d}$$

Then for $m = 1$, the angular deviations θ_{v1} and θ_{r1} for violet and red light, respectively, are

$$\theta_{v1} = \arcsin\left(\frac{380 \times 10^{-9}\text{ m}}{1.67 \times 10^{-6}\text{ m}}\right) = 13.2°$$

$$\theta_{r1} = \arcsin\left(\frac{750 \times 10^{-9}\text{ m}}{1.67 \times 10^{-6}\text{ m}}\right) = 26.7°$$

That is, the first-order visible spectrum appears with deflection angles from $\theta_{v1} = 13.2°$ (violet) to $\theta_{r1} = 26.7°$ (red).

(b) With $m = 2$ and $m = 3$, our equation $\theta = \arcsin(m\lambda/d)$ for 380-mm violet light yields

$$\theta_{v2} = \arcsin\left(\frac{2(380 \times 10^{-9}\text{ m})}{1.67 \times 10^{-6}\text{ m}}\right) = 27.1°$$

$$\theta_{v3} = \arcsin\left(\frac{3(380 \times 10^{-9}\text{ m})}{1.67 \times 10^{-6}\text{ m}}\right) = 43.0°$$

For 750-nm red light, this same equation gives

$$\theta_{r2} = \arcsin\left(\frac{2(750 \times 10^{-9}\text{ m})}{1.67 \times 10^{-6}\text{ m}}\right) = 63.9°$$

$$\theta_{r3} = \arcsin\left(\frac{3(750 \times 10^{-9}\text{ m})}{1.67 \times 10^{-6}\text{ m}}\right) = \arcsin(1.35) = \text{undefined}$$

Hence the second-order spectrum extends from 27.1° to 63.9° and the third-order spectrum extends from 43.0° to 90° (the largest possible value of θ). The undefined value of θ_{r3} means that the third-order spectrum reaches $\theta = 90° = \arcsin(1)$ at a wavelength shorter than 750 nm; you should be able to show that this happens for $\lambda = 557$ nm. Hence the first-order spectrum (from 13.2° to 26.7°) does not overlap with the second-order spectrum, but the second- and third-order spectra do overlap. You can convince yourself that this is true for any value of the grating spacing d.