1. Introduction to vectors

1.1 Vectors & linear combinations

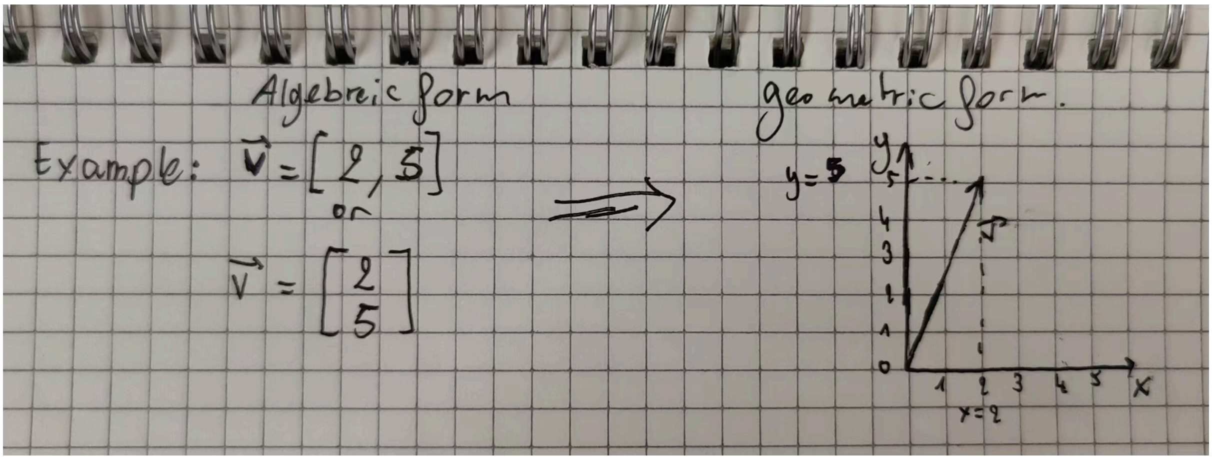
a. **Vector:** Mathematical object that has both a **magnitude** (length) and **direction**. geometrically it's an arrow, algebraically it's an ordered list of numbers, we can write it in two ways

Row vectors $\mathbf{a}_{row} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$

Column vectors $\mathbf{a}_{col} = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix}$

a_1, a_2, ... a_n: are called components or coordinates of the vector a.

- The algebraic form tells us the geometric form: the coordinates tell you how to move along each axis to draw the arrow (vector).



ps: the vector ***O* = [0, 0]** is called **the zero vector**
b. Linear combination: Combining vectors together using two fundamental operations >

+ scalar multiplication: multiplying a vector by a scalar = constant (can be any number)

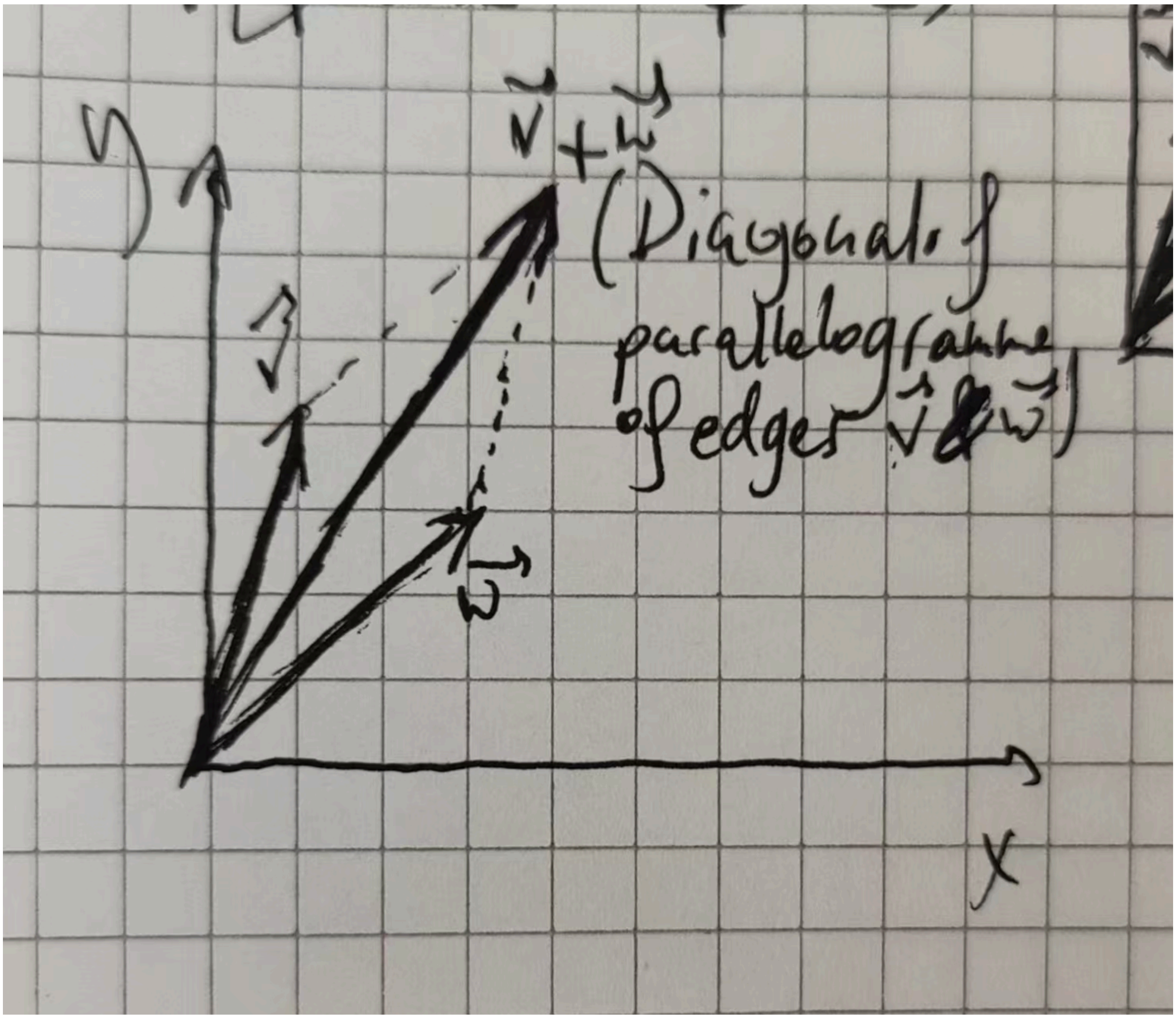
$$C.\vec{V} = C\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} CV_1 \\ CV_2 \end{bmatrix}$$

+ Vector addition: Adding two vectors together

ONLY VECTORS OF THE SAME DIMENTION COULD BE ADDED

$$\vec{v} + \vec{w} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix}$$

Visual representation of vector addition for 2 non Parallele, non-zero vectors in 2D, notice how v & w are edges of a parallelogram



+Generally, a linear combination is written as:
$c_1v_1 + c_2v_2 + \dots + c_nv_n$ (it gives a whole new vector)

where: $v_1, v_2, \dots, v_n$ are your original vectors. $c_1, c_2, \dots, c_n$ are scalars.
You're basically scaling each vector by a number then adding their scaled versions together.

ps: the zero vector is always a possible combination

c. why we need linear combinations?
Linear combinations help us figure out and understand the following concepts:

- **Span**: Set of all possible linear combinations of a set of vectors.
  + If two vectors in a 2D plane have different directions, their span is the entire 2D plane (their possible linear combinations can fill up the entire 2D plane)
  + If two vectors are parallel, their span is just a line (all their linear combinations lead to the same line)

- **Linear dependence/independence**: a set is linearly independent if no vector can be written as a linear combination of the other. Otherwise, if it's possible, it's dependent.
- **Solving systems of equations**:

Every system of linear equations can be written as a vector equation asking a question about linear combinations.

The system:

$2x + 3y = 7$

$1x + 1y = 3$

Can be re-written as:

$x * (2, 1) + y * (3, 1) = (7, 3)$

The question becomes: "**What linear combination** of the vectors [2, 1] and [3, 1] results in the vector (7, 3)?" The scalars `x` and `y` are the solutions.

Special linear combinations in the 2D plane

The basis of a 2D plane:

- **î** (i-hat) = `[1, 0]` (pointing purely in the x-direction)
- **ĵ** (j-hat) = `[0, 1]` (pointing purely in the y-direction)

they're independent and their linear combination can create any vector in the 2D plane

Linearly dependent vectors: the span of all their combinations is a line pointing in their direction (one dimensional); they have no unique combination (all their combinations can lead to the same vector)

Combinations in 3D:

Same concept, just an additional dimension.

if the 3 vectors are all independent, they fill the entire 3D space.

Special combinations in 3D:

The basis for 3D:

- **î** = `[1, 0, 0]` (x-direction)
- **ĵ** = `[0, 1, 0]` (y-direction)

- $\hat{\mathbf{k}}$ = `[0, 0, 1]` (z-direction)

+They're independent and form all the 3D span

+Any vector is a linear combination of these three

Spanning a Plane:

If a vector is a linear combination of two independent vectors (u=v + w), this vector lies in the plane made by the 2 vectors it depends on. All linear combinations of these 3 lie on the same plane (they span a 2D plane inside the 3D space)

Spanning a line:

If all 3 vectors are dependent on each other, then their span is the line on the same direction as these 3 vectors.

1.2 Lengths and angles from dot products

What is a dot product?

- The dot product of two vectors v = (v_1, v_2) and w = (w_1, w_2) is the **number** $v \cdot w = v_1 \times w_1 + v_2 \times w_2$.
- When the dot product equals 0, the vectors are perpendicular.
- The zero vector is perpendicular to all vectors.
- If u and v are two independent vectors, and the dot product of n with each of these vectors is 0, then n is perpendicular to the plane formed by u and v. Additionally, the dot product of n with any vector on this plane will also be 0.

The length?

$$\text{The length squared of } \boldsymbol{v} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \text{ is } \boldsymbol{v} \cdot \boldsymbol{v} = 1 + 9 + 4 = 14. \textbf{ The length is } ||\boldsymbol{v}|| = \sqrt{\mathbf{14}}.$$

from Gilbert Strang introduction to linear algebra

1.3 Matrices