2014

(First Semester)

MASTER OF COMPUTER APPLICATIONS

Paper No: MCA 101

(Discrete Mathematics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question No 1 and **any four** from the rest

1. i) Expand $(x-y)^n$

   ii) State De-Morgan's laws.

   iii) If a set has 10 elements, how many subsets will it have?

   iv) Let $A=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, find (A+2B).

   v) What is the generating function for the sequence 1,1,1,1,1,1?

   vi) Construct the truth table for $P \wedge \neg P$.

(2X6=12)

2. i) Using Principle of Mathematical Induction prove that

   $n(n+1)(n+2)$ is a multiple of 6. **[6]**

ii) Find the terms independent of x in the expansion of

(a) $\left(x^2+\frac{1}{x}\right)^9$ (b) $\left(2x-\frac{1}{x}\right)^{10}$

[3+3]

3. i) State the Mean Value theorem and hence verify $f(x)=x+\frac{1}{x}$ in [1,3]. [4]

ii) Evaluate: $\int Sin\sqrt{\phi}\, d\phi$ [4]

iii) Find the gcd of:

a) (1529,14038)

b) (9888,6060) [4]

4. i) If $A\cup B=A\cap B$, prove that A=B. [5]

ii) If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, show that $2x\frac{dy}{dx}+y=2\sqrt{x}$. [4]

iii) Let R be a relation on Q defined by $R=\{(a,b):a,b\in Q\,\&\,a-b\in Z\}$. Show that R is an equivalence relation. [3]

5. i) Apply Gauss elimination method to solve the given equations: [6]

$$x+4y-z=-5$$

$$x+y-6z=-12$$

$$3x-y-z=4$$

ii) Find the rank of the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ [3]

iii) Three persons enter a railway carriage where there are five (5) vacant seats. In how many ways can they seat themselves? [3]

6. i) Among a set of 12 books, 6 are novels, 7 were published in the year 1984 and 3 novels were published in 1984. Find the number of books, which are either novels or published in 1984. [6]

ii) Show that: $(P\vee Q)\wedge(\neg P\wedge(\neg P\wedge Q))\Leftrightarrow(\neg P\wedge Q)$ [6]

7. i) Define spanning tree. Show that there is one and only one path between every pair of vertices in a tree.

[2+4]

ii) What is chromatic number? Prove that every tree with two or more vertices is 2-chromatic. [1+5]

8. i) Using the Principle of Mathematical induction, prove that:

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

[6]

ii) Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

[6]