Number Theory

Number Theory is the study of integers.

Prime Number: a positive integer that is divisible by exactly 2 positive integers: 1 and the number itself.

A Composite Number is a positive integer that is divisible by some positive integer besides 1 and the number itself.

Multiples: Let *a* and *b* be integers. We say that *a* is a multiple of *b* if *a* equals *b* times some integer. In other words, *a* is a multiple o*f b* if there is an integer n such that *a* = *b*n. If *a* is a multiple of *b* and *b* is nonzero, then we say that *b* is a divisor, or factor, of *a*, and that *a* is divisible by *b*.

These are **useful divisibility tests**:
Condition under which n is divisible by the number
2: Units digit of n is 0, 2, 4, 6, or 8
3: Sum of the digits of n is a multiple of 3
4: Number formed by last two digits of n is a multiple of 4
5: Units digit of n is 0 or 5
6: Divisible by 2 and by 3
9: Sum of the digits of n is a multiple of 9
10: Units digit of n is 0

If a number is a multiple of both *a* and *b*, then we say that the number is a **common multiple** of *a* and *b.*

Questions:

Source: Art of Problem Solving's *Prealgebra* Chapter 3

1. What number between 100 and 200 is both a perfect square and a multiple of 7?
   MOEMS 8, AoPS question 3.2

2. What is the greatest three-digit number that is a multiple of 13?
   MATHCOUNTS, AoPS question 3.3

3. How many integers between 2 and 1004 are multiples of 5?
   AoPS question 3.4

4. There are many positive two-digit multiples of 7, but only two of these multiples have a digit sum of 10. (The digit sum of an integer is the sum of its digits.) What are these two multiples of 7?
   AMC 8, AoPS question 3.1.2

5. What is the greatest three-digit multiple of 33 that can be written using three different digits?
   MATHCOUNTS AoPS question 3.1.7

6. Describe each of the following numbers as prime or composite:
    a) 61
    b) 91
    c) 143
    d) 157

    AoPS question 3.16

7. What is the largest digit d for which the number $214, d07$ is divisible by 3?

    AoPS question 3.2.7

8. Both ABC and $3D8$ are three-digit numbers such that $ABC - 3D8 = 269$. If $3D8$ is divisible by 9, then what number does ABC represent?

    MOEMS, AoPS question 3.2.6

9. What is the difference between the greatest positive factor of 121 and the least positive factor of 6?
   MATHCOUNTS, AoPS question 3.68

10. What is the remainder when $(99)(237)$ is divided by 9?
    AoPS question 3.48

11. Find the largest two-digit composite number in which both digits are prime.
    AoPS question 3.17

12. Suppose P and Q both represent prime numbers such that
    $5P + 7Q = 109$
    Find the value of the prime P.
    AoPS question 3.3.7

13. How many positive integers less than 20 have exactly two positive divisors?

AoPS question 3.63

14. If $661,17A$ is a multiple of 12, what is A?

AoPS question 3.7.5

15. The product of all prime numbers between 1 and 80 is divided by 10. What is the remainder?

AoPS question 3.3.5