Properties of Arithmetic

Review

Order of Operations (PEMDAS)

1. Evaluate expressions inside parentheses first
2. Compute powers
3. Multiply and divide from left to right
4. Add and subtract left to right

Addition and Multiplication

1. They are commutative $a + b = b + a$ and $ab = ba$
2. They are associative $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$
3. Multiplication distributes over addition $a(b + c) = ab + ac$

Subtraction

1. It is not commutative or associative
2. $a - b = a + (-b)$

Division

1. $0 \div a = 0$
2. Remember - You can't divide by zero! $a \div 0 \neq no\ number!\ It\ is\ undefined.$
3. The quotient is the result of dividing one number by another

Reciprocal

1. For the number a the reciprocal is $\frac{1}{a}$ so that $a \cdot \frac{1}{a} = 1$.

Questions:

Source: The Art of Problem Solving, *Prealgebra*, Chapter 1

1. Add $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

2. Compute $99 + 99 + 99 + 101 + 101 + 101$.

   Problem 1.2.1

3. Compute $1999 + 2001 + 1999 + 2001 + 1999 + 2001 + 1999 + 2001$
   Problem 1.2.2

4. Compute $(3 + 13 + 23 + 33 + 43) + (7 + 17 + 27 + 37 + 47)$.
   Problem 1.2.3

5. Compute $(1 + 2 + 3 + \ldots + 49 + 50) + (99 + 98 + 97 + \ldots + 51 + 50)$
   Problem 1.2.4

6. Compute $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

   Problem 1.3.3

7. Compute $1 \cdot 100 \cdot 2 \cdot 50 \cdot 4 \cdot 25 \cdot 5 \cdot 20$

   Problem 1.3.2

8. What is the value of the product $25 \cdot 17 \cdot 4 \cdot 20$?

   Problem 1.3.1

9. Using the distributive property, evaluate the following expression
   $11 \cdot 43 + 11 \cdot 57$
   Problem 1.3.6a

10. Compute $-631 + (114 + 631)$
    Problem 1.4.1

11. What is the sum of all the negative integers that are greater than -5
    Problem 1.4.2

12. What is the sum of $-10 + (-9) + (-8) + \ldots + 9 + 10 + 11 + 12$?

Problem 1.4.3

13. If x and y are numbers such that $y - x = 7$, what is the value of $x - y$?

Problem 1.5.5

14. Compute

$1901 + 1902 + 1903 + \ldots + 1993) - (101 + 102 + 103 + \ldots + 193)$.

Problem 1.5.7/AMC 8

15. By how much does the sum
$19 + 28 + 37 + 46 + 55 + 64 + 73 + 82 + 91$ exceed the sum
$18 + 27 + 36 + 45 + 54 + 63 + 72 + 81 + 90$?
Problem 1.5.8

16. Compute $1990 \cdot 1991 - 1989 \cdot 1990$.
Problem 1.5.9

17. The sum of the first 10,000 positive even numbers is how much more than the sum of the first 10,000 positive odd numbers?
Problem 1.5.11

18. What is the product of any nonzero number and twice its reciprocal?

Problem 1.6.3

19. Compute $(2 \cdot 3 \cdot 4)(\frac{1}{2} + \frac{1}{3} + \frac{1}{4})$

Problem 1.6.5/AMC 8

20. What is $20 \cdot 24 \cdot 28 \cdot 32 \div (10 \cdot 12 \cdot 14 \cdot 16)$?

Problem 1.39

21. Divide $205 \cdot 205$ by 205. What is the result?

Problem 1.7.2

22. What is $28 \div \frac{1}{7}$?

Problem 1.7.5

23. What number is 10 more than the quotient when 78 is divided by $\frac{1}{2}$?

Problem 1.7.6/MATHCOUNTS

24. What is the value of $\frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2}$?

Problem 1.7.7

25. $90 + 91 + 92 + 93 + 94 + 95 + 96 + 97 + 98 + 99$

Problem 1.42/AMC 8

26. Compute

$3(101 + 103 + 105 + 107 + 109 + 111 + 113 + 115 + 117 + 119)$.

Problem 1.44

27. Express in simplest form: $6((24 - 98) - (19 - 98))$

Problem 1.49/MATHCOUNTS

28. Compute

$1 - 3 + 5 - 7 + 9 - 11 + 13 - 15 + 17 - 19 + 21 - 23 + 25$

Problem 1.50

29. Express in simplest form $(6 \div (-3))(4 - 12)$.

Problem 1.55/MATHCOUNTS

30. What is the value of the sum $5 + 10 + 15 + \ldots + 95 + 100$?

Problem 1.64