



# Number Theory

**Number Theory** is the study of integers.

**Prime Number:** a positive integer that is divisible by exactly 2 positive integers: 1 and the number itself.

**A Composite Number** is a positive integer that is divisible by some positive integer besides 1 and the number itself.

**Multiples:** Let  $a$  and  $b$  be integers. We say that  $a$  is a multiple of  $b$  if  $a$  equals  $b$  times some integer. In other words,  $a$  is a multiple of  $b$  if there is an integer  $n$  such that  $a = bn$ . If  $a$  is a multiple of  $b$  and  $b$  is nonzero, then we say that  $b$  is a divisor, or factor, of  $a$ , and that  $a$  is divisible by  $b$ .

These are **useful divisibility tests**:

Condition under which  $n$  is divisible by the number

2: Units digit of  $n$  is 0, 2, 4, 6, or 8

3: Sum of the digits of  $n$  is a multiple of 3

4: Number formed by last two digits of  $n$  is a multiple of 4

5: Units digit of  $n$  is 0 or 5

6: Divisible by 2 and by 3

9: Sum of the digits of  $n$  is a multiple of 9

10: Units digit of  $n$  is 0

If a number is a multiple of both  $a$  and  $b$ , then we say that the number is a **common multiple** of  $a$  and  $b$ .

## Questions:

Source: Art of Problem Solving, *Prealgebra*, Chapter 3

1. What number between 100 and 200 is both a perfect square and a multiple of 7?

Problem 3.2

2. What is the greatest three-digit number that is a multiple of 13?

Problem 3.3/MATHCOUNTS

3. How many integers between 2 and 1004 are multiples of 5?

Problem 3.4

4. There are many positive two-digit multiples of 7, but only two of these multiples have a digit sum of 10. (The digit sum of an integer is the sum of its digits.) What are these two multiples of 7?

Problem 3.1.2/AMC 8

5. What is the greatest three-digit multiple of 33 that can be written using three different digits?

Problem 3.1.7/MATHCOUNTS

6. Describe each of the following numbers as prime or composite:

a) 61

b) 91

c) 143

d) 157

Problem 3.16

7. What is the largest digit  $d$  for which the number  $214, d07$  is divisible by 3?

Problem 3.2.7

8. Both  $ABC$  and  $3D8$  are three-digit numbers such that  $ABC - 3D8 = 269$ .

If  $3D8$  is divisible by 9, then what number does  $ABC$  represent?

Problem 3.2.6

9. What is the difference between the greatest positive factor of 121 and the least positive factor of 6?

Problem 3.68/MATHCOUNTS

10. What is the remainder when  $(99)(237)$  is divided by 9?

Problem 3.48

11. Find the largest two-digit composite number in which both digits are prime.

Problem 3.17

12. Suppose  $P$  and  $Q$  both represent prime numbers such that

$$5P + 7Q = 109$$

Find the value of the prime  $P$ .

Problem 3.3.7

13. How many positive integers less than 20 have exactly two positive divisors?

Problem 3.63

14. If  $661,17A$  is a multiple of 12, what is  $A$ ?

Problem 3.7.5

15. The product of all prime numbers between 1 and 80 is divided by 10. What is the remainder?

Problem 3.3.5