

Number Theory

Number Theory is the study of integers.

Prime Number: a positive integer that is divisible by exactly 2 positive integers: 1 and the number itself.

A Composite Number is a positive integer that is divisible by some positive integer besides 1 and the number itself.

Multiples: Let a and b be integers. We say that a is a multiple of b if a equals b times some integer. In other words, a is a multiple of b if there is an integer a such that a = b in . If a is a multiple of a and a is

These are useful divisibility tests:

Condition under which n is divisible by the number

- 2: Units digit of n is 0, 2, 4, 6, or 8
- 3: Sum of the digits of n is a multiple of 3
- 4: Number formed by last two digits of n is a multiple of 4
- 5: Units digit of n is 0 or 5
- 6: Divisible by 2 and by 3
- 9: Sum of the digits of n is a multiple of 9
- 10: Units digit of n is 0

If a number is a multiple of both a and b, then we say that the number is a **common multiple** of a and b.

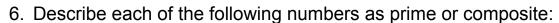
Questions:

Source: Art of Problem Solving, Prealgebra, Chapter 3

1. What number between 100 and 200 is both a perfect square and a multiple of 7?

Problem 3.2

2.	What is the greatest three-digit number that is a multiple of 13? Problem 3.3/MATHCOUNTS
3.	How many integers between 2 and 1004 are multiples of 5? Problem 3.4
4.	There are many positive two-digit multiples of 7, but only two of these multiples have a digit sum of 10. (The digit sum of an integer is the sum of its digits.) What are these two multiples of 7? Problem 3.1.2/AMC 8
5.	What is the greatest three-digit multiple of 33 that can be written using three different digits? Problem 3.1.7/MATHCOUNTS



- a) 61
- b) 91
- c) 143
- d) 157

Problem 3.16

7. What is the largest digit d for which the number 214, d07 is divisible by 3? Problem 3.2.7

8. Both ABC and 3D8 are three-digit numbers such that ABC - 3D8 = 269. If 3D8 is divisible by 9, then what number does ABC represent?

Problem 3.2.6

9.	What is the difference between the greatest positive factor of 121 and the	ne
	least positive factor of 6?	

Problem 3.68/MATHCOUNTS

10. What is the remainder when (99)(237) is divided by 9?

Problem 3.48

11. Find the largest two-digit composite number in which both digits are prime.

Problem 3.17

12. Suppose P and Q both represent prime numbers such that 5P + 7Q = 109

Find the value of the prime P.

Problem 3.3.7

	How many positive integers less than 20 have exactly two positive visors? oblem 3.63
14 .	If 661, 17A is a multiple of 12, what is A?
	The product of all prime numbers between 1 and 80 is divided by 10.7 hat is the remainder?