# Unit - 3

* Predicate calculus.

Quantifiers refer to quantities such as 'some' or 'all'

i.e. for all (all, every)

there exists (some)

The two types can be expressed by:-

1) $\forall$: for all (all, every) [universal quantifiers]

2) $\exists$: there exists (some) [existential quantifiers]

e.g.- ① $2x$ is divisible by $2 \ \forall x$

② $\exists x$ such that $3x$ is not divisible by 9.

Q- Express the following using quantifiers.

① All human beings are mortal.

Soln $(\forall x) \quad P(x)$

[$P(x)$: $x$ is ~~immortal~~ mortal,

$x$: a human being]

② There exists a student.

③ Some students are clever.

④ Some students are not successfull.

Solⁿ $S(x)$: $x$ is a student.

$R(x)$: $x$ is clever

$T(x)$: $x$ is successfull.

for ② $(\exists x)\ S(x)$

for ③ $(\exists x)(S(x) \wedge R(x))$

for ④ $(\exists x)\ (S(x) \wedge \neg T(x))$

Q- Express the following using quantifiers.

① Every house is a physical object.

② Some physical objects are houses.

③ Every house is owned by somebody.

Solⁿ $S(x)$: $x$ is a house.

$R(x)$: is a physical object

~~$T(x)$: owned by somebody.~~

①

~~$R(x)\ S(x)\ (\forall x)$~~

$(\forall x)\ (S(x) \to R(x))$

② $(\exists x)\ (S(x) \wedge R(x))$

~~③ $(\forall x)\ (S(x) \vee T(x))$~~

③ $(\forall x)\ (S(x) \to (\exists y) T(x,y))$

$T(x,y)$: house $x$ is owned by person $y$

## * Scope and Binding

Let Y be a formula. A substring (a small expression) of Y is called a sub formula if and only if Z is a formula on its own.

The scope of an occurance of a quantifier occuring in Y is the sub-formula of Y starting with that occurance (from that place to the right).

The sub formulas of the formula

eg-1 $\forall x_1 \exists x_2 ((P_1'(x_3) \wedge P_2'(x_1)) \rightarrow P_1'(x_2))$

are:- $P_1'(x_3)$, $P_2'(x_1)$, $P_1'(x_2)$,

$P_1'(x_3) \wedge P_1'(x_1)$, $(P_1'(x_3) \wedge P_1'(x_1)) \rightarrow P_1'(x_2)$,

$\exists x_2((P_1'(x_3) \wedge P_1'(x_1)) \rightarrow P_1'(x_2))$

$\forall x_1, \exists x_2 ((P_1'(x_3) \wedge P_1'(x_1)) \rightarrow P_1'(x_2))$

The scope of $\forall$ is:

$\forall x_1 \exists x_2 ((---))$

The scope of $\exists$ is:

$\exists x_2((P_1'(x_3) \wedge P_1'(x_1)) \rightarrow P_1'(x_2))$

eg-2

$\exists x_2 (P_1^2(x_2, x_1) \wedge \forall x_1 P_3'(x_1))$

## * Bound and free variable

An occurance of a variable $x$ in $\psi$ (given formula) is a bound occurance if and only if this occurance is within the scope of an occurance of $\forall x$ or $\exists x$. A variable $x$ is a bound variable of $\psi$, if and only if there is atleast one bound occurance of $x$ in $\psi$.

An occurance of a variable in a formula is called a free ~~variable~~ occurance if and only if it is not bound. If their is a free occurance of a variable $x$ in a formula $\psi$, you say that $x$ is a free variable.

e.g. – $\exists x_2 (P_1^2(x_2, x_1) \wedge \forall x_1 P_3^1(x_1))$.

$x_1$ is a free variable w.r.t. '$\exists$'

$x_1$ is a bound variable w.r.t. '$\forall$'.

Process of finding bound and free variable is called binding.

## * Substitution :-

The expression $(x/t)$ is used to substitute the variable $x$ by the term $t$.

The expression $(X)[x/t]$ is used to express the formula obtained by replacing all the free occurances of $x$ with the term $t$ in the formula $T$.

e.g- 1) $(Px \rightarrow Qx)[x/t] = Pt \rightarrow Qt$.

2) $\forall x\,(Pxy[x/t]) = (\forall x)\,Pty$.

3) $\forall x\,\exists y\,((Px \wedge Qyx) \rightarrow Rzy\,[y/t])$

$= \forall x\,\exists y\,((Px \wedge Qyx) \rightarrow Rzt)$

4) $(\forall x\,\exists y\,(Px \wedge Qyx) \rightarrow Rxy)\,[x/t]$

$= (\forall x\,\exists y\,(Pt \wedge Qyt) \rightarrow Rty)$

If in a ~~state~~, $x$ has been assigned to $a$, then $P_x f(x)$ signifies '$a$ & $f(a)$' are related to $P$.

e.g- $f(a) \rightarrow$ father of rahul
$a \rightarrow$ Rahul

then, $f(a)$ & $a$ (father and Rahul are related to $P$)

Suppose $P_x f(x)$ signifies Rahul is younger than his father.

younger

'father of'

Let $l(x) =$ Rahul

$f(l(x)) =$ father of Rahul

Define $\phi$ by: $\phi(f) =$ father of

then $\boxed{l(f(x)) = \phi(f)\ l(x)}$

The symbol $l$ is for valuation.

$I = (D, \phi)$ where $\phi$ is called an associative function with relation on $D$ ($D \neq$ empty set)

A state $I_l$ ~~$(D, \phi, l)$~~ is a triplet

$(D, \phi, l)$

$I$ is the interpretation

$\phi$ is the associative relation

& $l$ is a valuation

If $X$ be any formula then $I_l \models X$ means '$I_l$ verifies $X$'. or '$I_l$ satisfies $X$'.

* Satisfiability and Validity

Let $X$ be a given formula, then $X$ is satisfiable if and only if some state satisfies it. $X$ is unsatisfiable if and only if each state ~~sta~~ satisfies it, X is valid if and only if each state satisfies it, X is invalid if and only if some state falsifies it.