Assignment 5

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1 Introduction

In this document are the written assignments for Assignment 5 in Scientific computing - Bridging course presented. A brief presentation of the results of the Python coding exercises are also presented.

Workout 2.6

Suppose that the weather can be only sunny or cloudy and the weather conditions on successive mornings form a Markov chain with transition matrix

$$\begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix} \tag{1}$$

1. **If it is cloudy on a given day, what is the probability that it will also be cloudy the next day?**

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \tag{2}$$

   There will be a 60% chance of raining that next day.

2. **If it is sunny on a given day, what is the probability that it will be sunny on the next two days?**

$$\begin{aligned} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix} &= \begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \\ \begin{bmatrix} 0.7 & 0.3 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix} &= \begin{bmatrix} 0.67 & 0.33 \end{bmatrix} \end{aligned} \tag{3}$$

   sunny the two coming days : $0.6 \times 0.67 = 0.3960, \approx 40\%$

3. **If it is cloudy on a given day, what is the probability that it will be sunny on at least one of the next three days?**
   Same probability as: $1-$ P(cloudy three days in a row)

$$\begin{aligned}
\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.6 & 0.4\end{bmatrix}\\
\begin{bmatrix}0.6 & 0.4\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.66 & 0.34\end{bmatrix}\\
\begin{bmatrix}0.66 & 0.34\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.666 & 0.344\end{bmatrix}
\end{aligned} \tag{4}$$

$1 - 0.4 \times 0.34 \times 0.344 = 0.9546 \approx 95\%$

4. **If it is sunny on a certain Wednesday, what is the probability that it will be sunny on the following Saturday?**

$$\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} = \begin{bmatrix}0.667 & 0.333\end{bmatrix} \tag{5}$$

It is $\approx 67.7\%$ chance of sunny weather on Saturday

5. **If it is cloudy on a certain Wednesday, what is the probability that it will be sunny on the following Saturday?**

$$\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} = \begin{bmatrix}0.666 & 0.334\end{bmatrix} \tag{6}$$

It is $\approx 66.7\%$ chance of sunny weather on Saturday. Approaching a "steady state" after 3 days.

6. **If it is sunny on a certain Wednesday, what is the probability that it will be sunny on both the following Saturday and Sunday**

$$\begin{aligned}
\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.6 & 0.4\end{bmatrix}\\
\begin{bmatrix}0.7 & 0.3\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.67 & 0.33\end{bmatrix}\\
\begin{bmatrix}0.67 & 0.33\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.667 & 0.333\end{bmatrix}\\
\begin{bmatrix}0.667 & 0.333\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.6667 & 0.3333\end{bmatrix}
\end{aligned} \tag{7}$$

Probability for sun on both Saturday and Sunday: $0.667 \times 0.6667 \approx 44\%$

7. **If it is cloudy on a certain Wednesday, what is the probability that it will be sunny on both the following Saturday and Sunday?**

$$
\begin{aligned}
\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.6 & 0.4\end{bmatrix}\\
\begin{bmatrix}0.6 & 0.4\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.66 & 0.34\end{bmatrix}\\
\begin{bmatrix}0.66 & 0.34\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.666 & 0.344\end{bmatrix}\\
\begin{bmatrix}0.666 & 0.344\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} &= \begin{bmatrix}0.6666 & 0.3444\end{bmatrix}
\end{aligned} \tag{8}
$$

Probability for sun on both Saturday and Sunday: $0.666 \times 0.6666 \approx 44\%$

8. **Suppose that the probability that it will be sunny on a certain Wednesday is 0.2 and the probability that it will be cloudy is 0.8. Determine the probability that it will be cloudy on the next day, Thursday.**

$$
\begin{bmatrix}0.2 & 0.8\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} = \begin{bmatrix}0.62 & 0.38\end{bmatrix} \tag{9}
$$

Probability of cloudy weather on Thursday $\approx 38\%$

9. **With assumptions of item 8, determine the probability that it will be cloudy on Friday.**

$$
\begin{bmatrix}0.2 & 0.8\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.6 & 0.4\end{bmatrix} = \begin{bmatrix}0.665 & 0.335\end{bmatrix} \tag{10}
$$

Probability of cloudy weather on Friday $\approx 33.5\%$

Workout 4.5

How the inverse transform method can be applied to generate from Beta distributions Beta(α, 1) and Beta(1, β). Derive the formulation and implement the Python code

$$
\begin{aligned}
\mathrm{f}(x) &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma\beta}x^{\alpha-1}(1-x)^{\beta-1} \\
\beta = 1 \rightarrow \mathrm{f}(x) = x^{\alpha} &\rightarrow \mathrm{F}(x) = x^{\alpha} \\
\alpha = 1 \rightarrow \mathrm{f}(x) = (1-x)^{\beta-1} &\rightarrow \mathrm{F}(x) = -\frac{(1-x)^{\beta}}{\beta}
\end{aligned}
\tag{11}
$$

Implemented in Python

```
import numpy as np
import matplotlib.pyplot as plt


def UniformGen(a,b,N):
    U = np.random.uniform(a,b, N)
    return U


def BetaGen1(alpha, N):
        U1 = UniformGen(0,1,N)
        print(U1)
        X = U1**(alpha)
        print(X)
        return X

def BetaGen2(beta, N):
        U1 = UniformGen(0,1,N)
        X = - ((1-U1)**beta)/beta
        return X

plt.figure(figsize = (5,3))
N = 500
X = BetaGen2(10, N)
print(X)
plt.hist(X, bins = 30, histtype = 'bar', color = 'red'
    , density = True)
plt.show()

plt.figure(figsize = (5,3))
N = 500
X = BetaGen1(10, N)
print(X)
plt.hist(X, bins = 30, histtype = 'bar', color = 'red'
    , density = True)
plt.show()
```

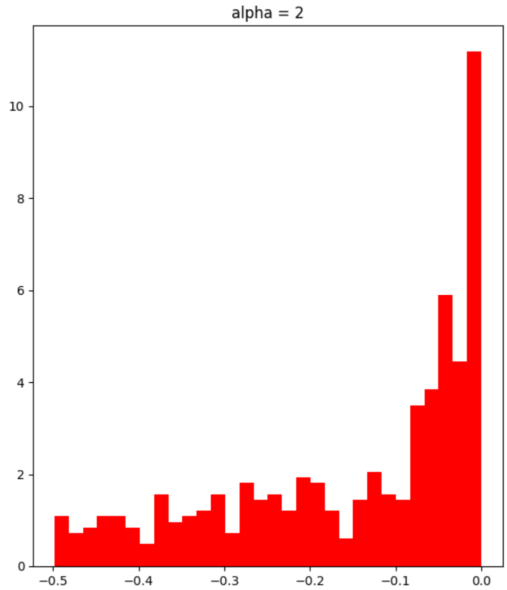


Figure 1: Beta distrubution, alpha = 2

Miniproject 4.3

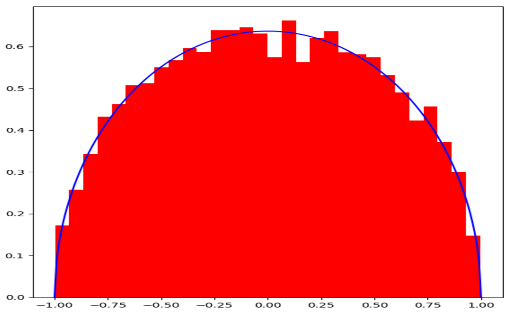
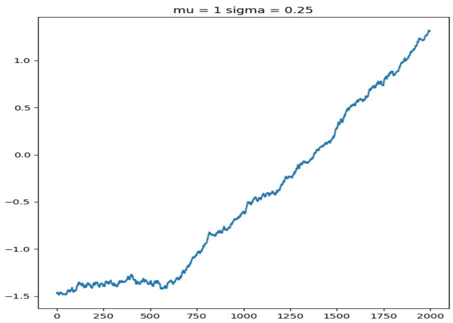
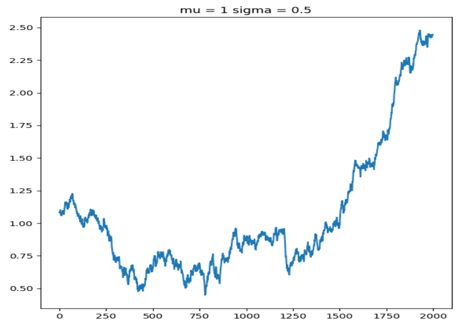


Figure 2: Semi circular pde

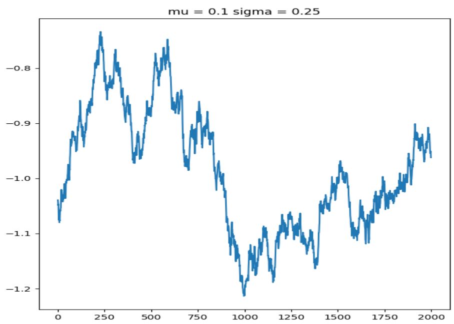
Miniproject 5.3



(a)



(b)



(c)

Miniproject 6.1

- **Output (N = 1000):** [14.2796957]
- **Output (N = 10000):** [12.41701961]
- **Output (N = 100000):** [11.9639898]

Miniproject 6.2

- **Output:** Approximation delta(0): [0.00041242]
- **Output:** Approximation delta(2): [3.1591858]
- **Output:** Approximation delta(4): [5.34118711]

2 Miniproject 7.1

Calculate expected value of h(X_1*, X_2) where h(x_1,x_2) = $x_1 x_2$. Changing the sum from X[0,:] to X[0,:]*X[1,:]).

```
Hhat = np.zeros(4)
for k in range(4):
        N = 10**(k+3)
        X0 = [0,0]
        X = McMcRandWalkGen(f, X0, Sigma, N)
        Hhat[k] = 1/N*np.sum(X[0,:]*X[1,:])
print("MCMC estimates = ",np.round(Hhat,4))
```

- **Output:** MCMC estimates = [1.0904 1.1598 1.113 1.1292]