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Predictive Maintenance of Machine Tool Systems Using Artificial Intelligence Techniques Applied to Machine Condition Data

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Abstract

Often, manufacturing equipment is utilized without a planned maintenance approach. Such a strategy frequently results in unplanned downtime, owing to unexpected failures. Scheduled maintenance replaces components frequently to avoid unexpected equipment stoppages, but increases the time associated with machine non-operation and maintenance cost. The emergence of Industry 4.0 and smart systems is leading to increasing attention to predictive maintenance (PdM) strategies that can decrease the cost of downtime and increase the availability (utilization rate) of manufacturing equipment. PdM also has the potential to foster sustainable practices in manufacturing by maximizing the useful lives of components. In this paper, the AI-based algorithms for predictive maintenance are presented, and are applied to monitor two critical machine tool system elements: the cutting tool and the spindle motor. A data-driven modeling approach will be described, and it will be utilized to investigate the tool wear and the bearing failures.

1. Introduction

In the 21st century, the smart manufacturing (Industry 4.0) is empowered by integration of cyber- and physical- system with the evolution of computing infrastructures; artificial intelligence, big data, data analytics, cloud computing, IoT platform, etc. The integration of the systems with the help of ICT enables to construct an integrative and collaborative system that responses in real time to meet changing conditions in the factory, supply network, and customers demand.

In the complex manufacturing field where many elements (e.g., human and tangible and intangible resources) interact with each other [1], a large amount of data is collected and accumulated during manufacturing operations. A computing infrastructure informed by the processed manufacturing data can be controlled by pre-trained AI algorithms.

To extract useful information from manufacturing data, AI techniques have been widely used. The techniques infuse intelligence into the systems to automatically learn and adapt to the changing environment using historical experience through training [2]. In addition, the ability to handle high-dimensional data, reduce complexity, improve existing knowledge, and identify relevant process relations are highlighted to demonstrate the applicability of the techniques in the manufacturing industry [3]. These abilities enable to forecast the topic of manufacturer's interest to possibly reduce variation in their production line and improve productivity and product quality. Therefore, the future behavior of the manufacturing system can be approximated by applying AI algorithms to the system, and this created knowledge may help decision making.

Extracted meaningful knowledge provides insights to make a better decision, which can assist the transformation toward

sustainable practices in the manufacturing industry (e.g., reducing waste [4], increasing energy and resource efficiency [5], and predictive maintenance [6]). To achieve improved industrial sustainability using the smart manufacturing platform, one approach is to develop a communication tool between machinery and reliability/maintenance engineers to optimize machinery maintenance tasks. In the manufacturing plant, optimal maintenance strategies are necessary to ensure system reliability, reduce cost, avoid downtime, and maximize the useful life of a component [6]. According to the recent article, unplanned downtime caused by a poor maintenance strategy reduces a plant's overall productive capacity by up to 20 percent and costs around $50 Billion each year [7].

The earliest maintenance strategy is unplanned maintenance (run to failure), in which no maintenance will occur until a machine breakdown happens [8]. In this situation, the utilization of a machine component may be increased to some extent, but the unplanned downtime is unavoidable. Preventative maintenance, more widely used strategy in the industry, inspects and maintains the components with periodic intervals to prevent unexpected machine breakages. However, the regular inspection/maintenance practice can incur long suspension time and high maintenance cost. Because of these pros and cons, a maintenance engineer often confronts with the tradeoff situation: they need to choose between maximizing the useful life of a component (unplanned maintenance) and maximizing uptime (preventive maintenance) [7].

While unplanned and preventive maintenances have the tradeoff scenario, predictive maintenance (PdM) is a promising technique that has an ability to break the tradeoff by maximizing the useful life of a component and uptime simultaneously. It is designed to monitor the condition of in-service equipment, and then predict when equipment will fail. It means that the future behavior/condition of machine components can be approximated, which will help to optimize maintenance tasks (e.g., prognostic health monitoring). Accordingly, the machine downtime and maintenance cost can be reduced significantly while making the maintenance frequency as low as possible.

In this paper, the two critical machine tool system elements, cutting tool and the spindle motor, are selected to be monitored using the AI algorithms. The algorithms are trained for predicting the systems' failure events. Several predictive modeling techniques are explained and are applied to the manufacturing data to study their performance. The simulation results are illustrated using the confusion matrix to display prediction accuracy and error together.
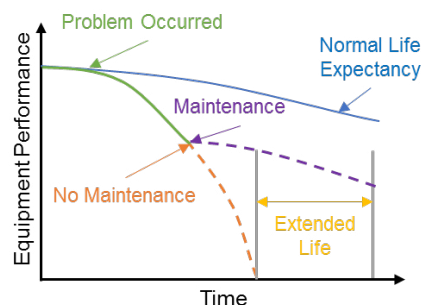

Fig. 1. Extending life with predictive maintenance (modified from [9]).

2. Predictive maintenance of machine tool systems

2.1. Predictive maintenance of the cutting tool

In a milling process, a rotating cutting tool removes materials from a workpiece to obtain the desired shape. Over the machining time, a geometry of the tool changes as a result of the interaction between the tool and workpiece. In the process, a material is deformed plastically, and energy is expended in overcoming friction between the tool and workpiece [10]; a tool gradually wears due to the generation of heat and stress during the process. Consequently, it will degrade the performance of the cutting tool, which will affect a surface finish. A surface finish is considered as a critical measure of the product quality.

To ensure the product quality, the condition of the cutting tool is necessarily monitored and controlled. Failure to monitor the condition of cutting tool could generate a poor-quality product, which will turn out to be a scrap. Therefore, a PdM of cutting tool in the machining processes cannot only inform when a tool needs to be replaced (when the length of tool wear reaches its wear limit), but also enable to estimate the remaining useful life (RUL) of the tool.

The conditions of the cutting tool can be described by the lengths of wear on the different faces of the cutting tool as shown in Fig. 2. Since a common practice to define the condition of the tool is measuring the abrasive wear length on a flank face of cutting tool [11], a flank wear limit is used as a metric to define the condition (normal, warning, and failure) of cutting tool in the simulation, which will be presented in section 5.
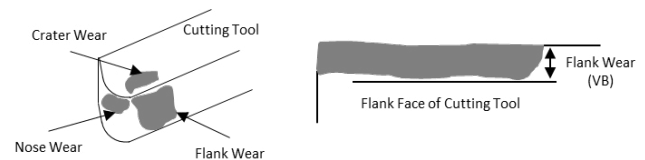

Fig. 2. Types of cutting tool wear.

2.2. Predictive maintenance of the spindle motor

A spindle is a rotating mechanical element, and an important component in manufacturing because it directly affects the quality and productivity of manufacturing processes [12, 13]. Since the power is transmitted to machine tools through spindles, static and dynamic forces are constantly applied in the rolling elements. Continuously applied forces gradually wear the components (e.g., bearing, rotor, and shaft), and it could result in a mechanical breakage at the extreme cases.

Once a spindle is damaged, replacing the parts and calibrating accuracies such as tool runout are difficult tasks. Spare components can be stocked to be replaced during maintenance schedule. However, it is difficult to know the current condition of a component (especially, in slight wear condition) and to predict the remaining useful life of a component [13]. Therefore, a PdM of the spindle is a valuable method to optimize the maintenance jobs ahead while

maintaining the process quality and productivity through preventing unexpected downtime.

Table 1. Parameters used in calculations of the characteristic frequencies.

Parameter	Pitch diameter	Ball diameter	Contact angle	Spindle speed
Description	D_p	D_b	α	f_s

Among many possible defects in rotating components, bearing defects are the main cause for the spindle damage [12]. For a PdM of spindles, therefore, piezo-electric force measurement sensors [14] and accelerometers [15] are used to measure the vibration due to the geometric changes of a ball bearing's inner and outer race. For the local defects of a ball bearing, characteristic frequencies can be calculated mathematically using the geometry of a rolling element bearing as shown in Table 1 and Fig. 3 [16]. In Fig. 3, the four different kinds of faults and their corresponding characteristic frequencies are explained: Inner race fault (f_{IR}), Outer race fault (f_{OR}), Rolling element fault (f_{ball}), and Bearing cage fault (f_{cage}). So, the spindle conditions can be evaluated using the variation of these four frequencies. In addition to the time and frequency domain analyses, AI techniques such as ANN (Artificial Neural Network), fuzzy logic, and Bayesian classification were used for finding the bearing faults in spindles [13].
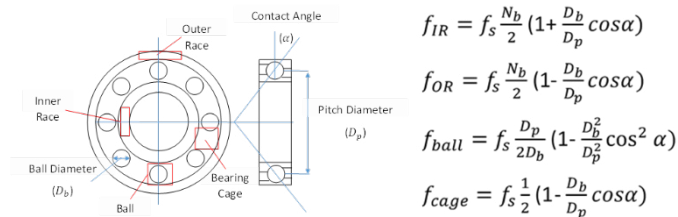

$$f_{IR} = f_s \frac{N_b}{2} (1 + \frac{D_b}{D_p} cos\alpha)$$

$$f_{OR} = f_s \frac{N_b}{2} (1 - \frac{D_b}{D_p} cos\alpha)$$

$$f_{ball} = f_s \frac{D_p}{2D_b} (1 - \frac{D_b^2}{D_p^2} cos^2\alpha)$$

$$f_{cage} = f_s \frac{1}{2} (1 - \frac{D_b}{D_p} cos\alpha)$$

Fig. 3. The geometry of rolling element bearing and derivation of characteristic frequencies (modified from [16]).

3. A procedure to build a condition monitoring

The conditions of the in-service machine tools may be translated by signals obtained from externally mounted sensors (e.g., accelerometer, microphone, dynamometer, and thermometer). To extract meaningful information from raw analog signals, first, they are necessarily processed to filter out unwanted frequency spectrums. Next step is to extract features from the processed signal to generate condition-related information as well as to compress data. Features can be extracted in either time domain or frequency domain depending on the tool characteristics (for example, Fourier transform of vibration signal from a spindle indicates a local defect of ball bearings clearly than the signal in the time domain). Then, the extracted feature can be used to train AI algorithms.

AI techniques can be divided into supervised, unsupervised, and reinforcement learning. Supervised learning trains extracted features with their corresponding labels. For example, if the features from spindle monitoring are tied to normal or fault state of the spindle, supervised learning algorithms can be used. It includes regression models, support vector machine (SVM), decision tree, ANN, etc. Unsupervised

learning has no labels for each dataset but generates estimation models. K-means clustering and principal component analysis (PCA) are in the category of the unsupervised learning. Reinforced learning model learns itself from rewards and penalties, then the policy is generated to achieve the goal. After an AI model is trained, the result can be estimated from the model. For example, once a spindle health model (predictive model) is trained from processed accelerometer signals and conditions of a spindle (labels), its condition (health) can be estimated from current accelerometer signals. Fig. 4 illustrates the general procedure of PdM using supervised learning algorithms.

Fig. 4. The procedure of PdM using supervised learning algorithms.

4. Manufacturing data

To study the capability of the AI algorithms in a PdM system, the manufacturing dataset from the milling experiment [17, 18] and the bearing experiment [19] were used to construct the monitoring systems (PdM systems). In both experiments, sensors are mounted on the in-service equipment to collect data in a real-time, and the experiments are run until the failure occurs. The data collected during the failure events are used to test the monitoring systems' capability to detect the failure after the training stage.

4.1. Milling dataset

In the experiment, a cast iron workpiece with the dimensions of 483 mm x 178 mm x 51 mm was machined through the face milling operations. The operations were performed at a cutting speed of 200 m/min (=826 rev/min), two depths of cut (0.75 mm and 1.5 mm), and two feeds (0.25 mm/rev and 0.5 mm/rev) to investigate the amount of flank wear (VB) after each cutting run.

During the operation, six different signals, (1) the DC spindle motor current, (2) the AC spindle motor current, (3) the table vibration (VBtable), (4) the spindle vibration (VBspindle), (5) the acoustic emission at the table (AEtable), and (6) the acoustic emission at the spindle (AEspinlde), were collected. The sampling rate of 250 Hz was used for all sensors, and 3500 data points between entry and exit cuts were used for each run in this paper. Other experiment details can be found in [17, 18].

4.2. Bearing dataset

In the experiment, a test rig was set up to access the performance of the bearings. In the setup, four bearings were
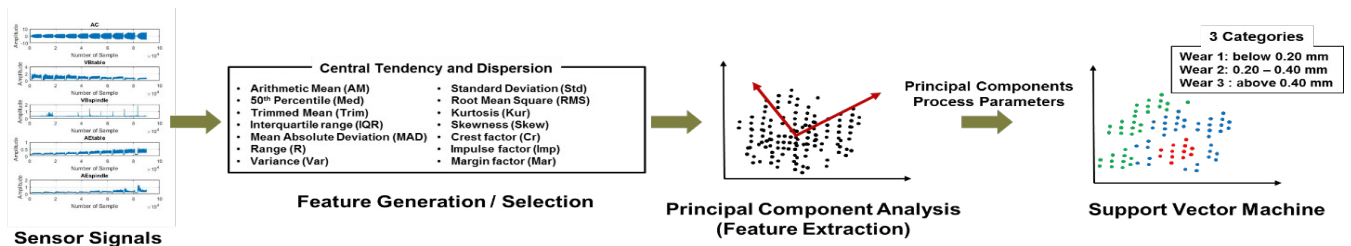
Fig. 5. Schematic diagram of the SVM-based tool condition classification.

installed on a shaft, and it was kept constantly rotating at 2000 RPM by the AC motor, which was coupled via rub belt.

The shaft and bearings were put under a radial load of 6000 lb by a spring mechanism. A high-sensitivity accelerometer was mounted on each bearing housing with 20 kHz sampling rate. The experiment was a run-to-fail type, in which bearings were tested until failure occurred. Three tests were implemented (test1, test2, test3), and other experiment details can be found in [19].

5. Simulation Results

In this section, the two experimental datasets (milling and bearing) are used to build and test a predictive model, which can be used in a PdM system. Two powerful classification techniques, support vector machine and artificial neural networks (recurrent neural network and convolutional neural network) are trained and tested to monitor and predict the conditions of the cutting tool and the bearing, respectively.

To evaluate the performances of the classification techniques, a confusion matrix is drawn. A confusion matrix is known to be a popular visualization method to present the performance of a classification technique. In the matrix, true and model-predicted values are presented in each row and column, and prediction accuracies (green boxes) and errors are shown in each box. By displaying the accuracies and errors together in the boxes, the matrix can show which classifier is confused.

5.1. Monitoring the conditions of the cutting tool

The condition of the cutting tool should be monitored to replace the tool at the right time in the machining process. To classify the conditions of the tool, the multi-class classification technique, support vector machine (SVM), is used here. The SVM is a machine learning method to solve multi-class classification problems [20]. In the algorithm, several hyperplanes are generated first. And then, the optimal hyperplanes which can separate different class most effectively are identified by the features (support vectors) [11]. This process (training) is performed by solving a linearly constrained quadratic optimization problem.

To better classify the non-linear signals, a non-linear function (kernel function) is incorporated in the method to transform the original input space to a higher-dimensional feature space; this is called a non-linear SVM. According to the VC (Vapnik-Chervonenkis) statistical learning theory, transforming to a higher-dimensional space enable to separate a dataset linearly, which is not be linearly separable in the

original space [20]. To test sensitivity of different kernel functions, three kernel functions, linear ($K(x_i, x_k)= x_i^T x_k$), polynomial ($K(x_i, x_k)= (x_i^T x_k+1)^d$), and Gaussian ($K(x_i, x_k)= exp(-\|x_i - x_k\|^2 / 2\sigma^2)$)) are selected ($d$ is a coefficient) to be compared. Further details of the method can be found in [21].

For classification of the tool condition, the flank wear is used as mentioned above. Three conditions (tool wear below 0.2 mm, between 0.2 mm – 0.4 mm, and above 0.4 mm) are used to represent for normal, warning, and failure states, respectively.

To generate the input dataset for the SVM algorithms, several features should be extracted from raw signals [22]. Features representing central tendency or dispersion of the collected dataset may describe the conditions of the tool more appropriately. Here, the 14 features characterizing the central tendency and the dispersion of the machining dataset ((1) arithmetic mean, (2) 50th percentile, (3) trimmed mean, (4) interquartile range, (5) mean absolute deviation, (6) range, (7) variance, (8) standard deviation, (9) root mean square, (10) kurtosis, (11) skewness, (12) crest factor, (13) impulse factor, and (14) margin factor) are generated. After generating the 14 features, the meaningful features are selected, showing a distinguishable difference in a performance as the tool wears after each cutting operation. Since the dimensionality of the selected features dataset is too large to be used as input dataset, the dimensionality reduction technique, principal component analysis (PCA), is applied to extract new effective features (principal components) from the feature dataset. PCA is a mathematical procedure for mapping multi-dimensional dataset onto new axes (lower dimensions) while preserving global information [23]. The mapped dataset can be represented by principle component vectors (e.g., first principal component, second principal component, and so on), and these will be used as an input for the SVM algorithms.

To train and test the SVM algorithm, the two principal components (the first and second principal components) and two milling process parameters (depth of cut and feed) are used to classify the condition of the tool. A schematic diagram of the SVM-based condition classification is shown in Fig. 5.

Fig. 6. Confusion matrix for the classification of cutting tool conditions using SVM.
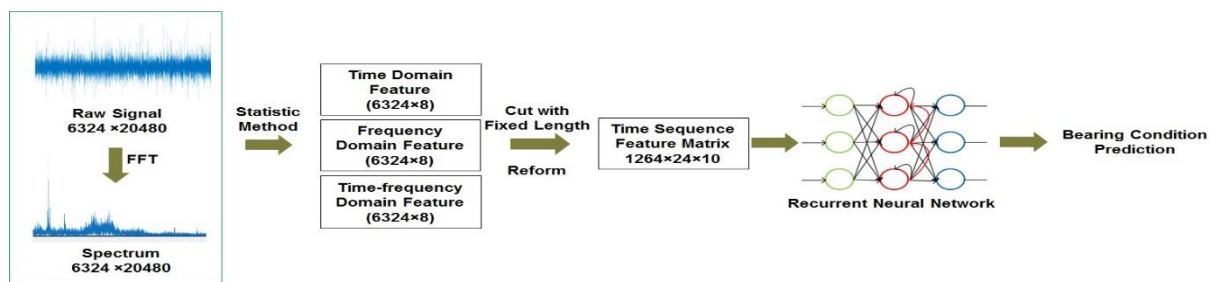
Fig. 7. Schematic diagram of the RNN-based bearing condition classification.

As mentioned above, three different kernel functions are tested to find the best-fitted function, and the polynomial kernel with d=3 (Cubic SVM) shows the highest average accuracy of 87% as presented in Fig. 6.

5.2. Monitoring the conditions of bearing

To monitor the condition of bearing, two AI algorithms, recurrent neural network (RNN) convolutional neural network (CNN), are tested. Here, the raw signals are processed depending on the characteristic of the algorithms. The bearing conditions are labeled into three states: normal, warning, and failure, corresponding to the remaining useful life of 100 - 66.7%, 66.7 - 33.4%, and 33.4 - 0%, respectively.

The RNN is a connectionist model that can capture the dynamics of sequence data. Unlike feedforward neural networks, RNNs can use their internal state (memory) to process sequences of inputs. This would be a favorable feature for PdM monitoring system where the conditional data is always time-sequential.

Feature datasets were extracted from the raw accelerometer signals, and the features are cut in fixed-width windows with 50% overlap to form a time-series sample. The detail can be found in the schematic diagram as shown in Fig. 7.

Fig. 8. Confusion matrix for the classification of bearing conditions using RNN.
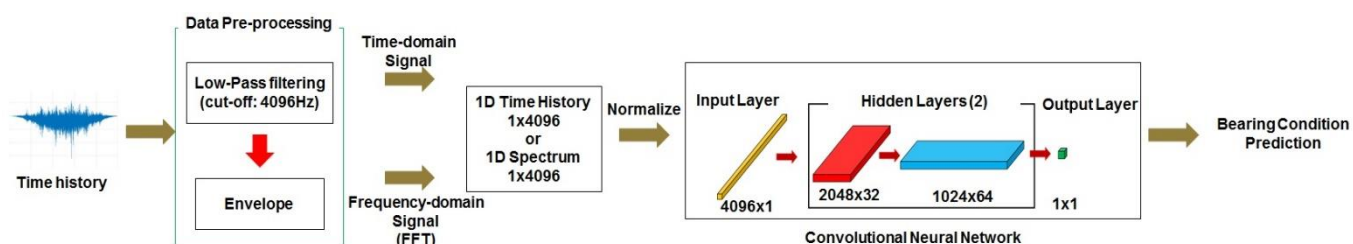
The samples are randomly divided into two groups (learning group and testing group), and the learning group is used to train the RNN model, and the testing group will be used to test the model accuracy. The testing results (average accuracy of 93%) are shown by the confusion matrix as shown Fig. 8. Although some confusion may happen in the early stages, for the critical state (failure), the model proves to be more accurate.

The CNN was originally devised for image classification (2D). Since the algorithm is optimized based upon normalized data from an image file, it can be used for classifying one-dimensional vibration signal without the feature extraction [20, 21, 22]. Therefore, the normalized signals in time- and frequency-domains can be used as an input to the CNN.

Several studies use a spectrum analysis by transforming enveloped time-domain vibration signal into the frequency domain to monitor a bearing performance [27, 28]. In these studies, the proposed model was able to track the characteristic frequencies, which are related to the physical attributes of bearing (e.g., rotational speed and pitch and ball diameter). Theoretically, each kind of the defect on bearing could have its own characteristic frequency. For this reason, a specific fault in bearing cause a change in the amplitude at some frequency bands, which are harmonics of the characteristic frequency.

In the simulation, the time history data and spectrum data of the bearing signals are applied separately to the same algorithm to compare a prediction accuracy with regard to the bearing conditions. In the data pre-processing stage, the envelope analysis over the time-domain signal is used to clarify the amplitudes of characteristic frequencies. After the pre-processing, the labeled signals are divided for the training and testing. The schematic of the simulation is described in Fig. 9.

The simulation results are shown in Fig. 10 using the confusion matrices. The average accuracies of the model with time- and frequency-domain signals are calculated as 84% and 98%, respectively. As expected, the frequency-domain signals show better performance when predicting the condition of the bearing.

Fig. 9. Schematic diagram of the CNN-based bearing condition classification.

Fig. 10. Confusion matrices for the classification of bearing conditions using CNN; time history data (left) and right: spectrum data (right).

6. Conclusion

Two machine tool system elements, the cutting tool and the spindle, are investigated to build PdM system using the AI algorithms. For the PdM of the cutting tool and the spindle, the flank wear and the bearing's RUL are used, respectively, as a metric to represent the component's conditions (normal, warning, and failure) in the systems. To classify the condition of the tools, the SVM and ANNs (RNN and CNN) methods are applied with the different feature extraction techniques.

Application of the algorithms on the features extracted from the experimental data shows that the conditions are effectively monitored to assess the degradation of the tools. The features from the frequency domain make the algorithm more accurate than the features from the time domain when evaluating the condition of the bearing. To enhance the recognition capabilities of an AI model, raw manufacturing dataset should be processed appropriately to extract meaningful features from a large amount of dataset.

To sum up, the PdM of machine tool systems can reduce the machine downtime and increase the RUL of a component. These will save a cost significantly by optimizing maintenance task and supply change management while ensuring machine safety.

Acknowledgements

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