Vector Analysis (MATH 304)

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Chapter 1

Introduction and Review

1.1 Determinants and Cross Products

Example (Computing areas)*:*
Suppose we have a paralellogram made by the vectors $(1, 4)$ and $(8, 3)$. Compute the area.

We could compute the area of the paralellogram using geometry, but we can easily see the area as 29 using the determinant.

$$A = \left|\det \begin{bmatrix} 1 & 8 \\ 4 & 3 \end{bmatrix}\right| = |1 \cdot 3 - 8 \cdot 4| = |-29| = 29$$

We can interpret this matrix as a linear transformation from the standard basis in $\mathbb{R}^2$, a change of basis.

$$e_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mapsto f_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mapsto f_2 = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

1.2 Cross Product