Definition (Matrices)*:*
*Let n, m be two integers ≥ 1. A **matrix** is an array of numbers with m rows and n columns (called a $m \times n$ matrix).*

Definition (ij-entry)*:*
*We call a_{ij} the **ij-entry** which is the entry in the ith row and the jth column. We write a matrix often as $A = (a_{ij})$ and define a_{ij}.*

Example (Identity Matrix)*:*
The Kronecker delta is defined as follows:

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 2, & i \neq j \end{cases}$$

Then we define the **identity matrix** as:

$$\mathbf{I_{n \times n}} = (\delta_{ij}) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

Definition (Column and Row Vectors)*:*
*Each column of an $m \times n$ matrix is a **column vector**.*

Definition (Row Vector)*:*
*Each row of an $m \times n$ matrix is a **row vector**.*

Example:
If we have a matrix, $A = \begin{bmatrix} 3 & 4 \\ 1 & -1 \\ 2 & 2 \end{bmatrix}$, the second column vector of A is $\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ and the second row vector of A is $\begin{bmatrix} 1 & -1 \end{bmatrix}$.

We can describe matrices using their column or row vectors. For example:

$$A = \begin{bmatrix} \vec{r} \\ \vec{s} \\ \vec{t} \end{bmatrix}$$

where

$$\vec{r} = \begin{bmatrix} 3 & 4 \end{bmatrix}, \vec{s} = \begin{bmatrix} 1 & -1 \end{bmatrix}, \vec{t} = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Or:

$$A = \begin{bmatrix} \vec{a} & \vec{b} \end{bmatrix}$$

where

$$\vec{a} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \vec{b} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

Definition (Square Matrix)*:*
Suppose we have a matrix, $A = [aij], i = 1, \ldots, m \, and \, j = 1, \ldots, n$*. If* $m = n$*, then* A *is a* ***square matrix****.*

Symmetric

skew-Symmetric square matrix diagonal

0.1 Matrix Algebra and Other Operations

We treat matricies the same way as numbers.

Let A be an $m \times n$ matrix and B be an $p \times q$ matrix.

- We can add A and B. If $m = p$ and $n = q$, then
- We can

0.2 Applications of Matricies

The following section covers different applications of matricies. Some topics are explored in later sections more throughly.

0.2.1 Linear Transformations

Matricies can be used to model transformations of vectors from $\mathbf{R}^n$ to $\mathbf{R}^m$. This is accomplished by having an $m \times n$ matrix, A, written as:

$$A : \mathbf{R}^n \leftarrow \mathbf{R}^m$$

Example ($\mathbf{R}^n$ to $\mathbf{R}^m$)*:*
Let $A = \begin{bmatrix} 3 & 4 \\ 1 & -1 \\ 2 & 2 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$,. Thus, $A : \mathbf{R}^2$ to $\mathbf{R}^3$ and:

$$A\vec{v} = \begin{bmatrix} 3 & 4 \\ 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \\ 6 \end{bmatrix}$$

Note. We are getting a linear combination of the column vectors of A. In other words, $A\vec{v} = x\vec{a} + y\vec{b}$.