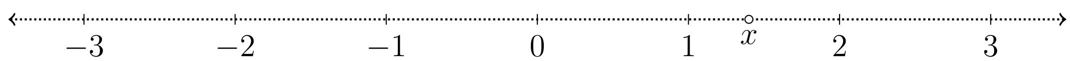
0.0.1 Constructing the Real Numbers

Dedekind Cuts

Now, we seen in the previous section that $\mathbb{Q}$ has "gaps". $x^2 = 2$ has no solution in $\mathbb{Q}$.



We need to fill in these gaps somehow while not knowing where the gaps and holes are.

Definition (Upper Bound)*:*
Let $E \subset S$ ordered. If there exists $\beta \in S$ such that for all $x \in E$, $x \leq \beta$, then β is an **upper bound (u.b.)** for E. We say E is bounded above.

A lower bound can be defined similarly with "greater than or equal to."

Example:
Consider the set $A = \{x \mid x^2 < 2\}$. 2 is an u.b. for A. $\dfrac{2}{3}$ is also an u.b. for A.

Definition:
If there exists an