Title

Christopher K. Walsh

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Preface

This document compiles all my notes on differential equations including any self-study. Some sections require content from other math subjects to be completed understood (such as linear algebra or other analysis topics). The corresponding topic and section will be referred to in-text.

If you need this for school and industry, I hope that you are able to do whatever you are trying to accomplish. If you are here because this interests you, I hope you find this as entertaining as I did.

- Christopher

Chapter 1

Sequences and Series

Definition (Infinite Series)*:*
An infinite series is an expression where we add an infinite number of elements together:

$$\sum_{n=1}^{\infty} a_k = a_1 + a_2 + a_3 + \dots$$

a_k

Definition (Partial Sum)*:*
We write the nth partial sum:

$$S_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots + a_n$$

We can define a sequence S_n. We say that:

- $\sum_{k=1}^{\infty} a_k$ converges if S_n converges.
- $\sum_{k=1}^{\infty} a_k$ diverges to $\pm\infty$ if S_n diverges to $\pm\infty$.
- $\sum_{k=1}^{\infty} a_k$ really diverges if S_n really diverges.

Example (Geometric Series (r=1/2))*:*
The following series converges to 1.

$$\sum_{k=1}^{\infty} \frac{1}{2^k} = 1/2 + 1/4 + 1/8 + \dots$$

This is a geometric series since $\frac{a_{k+1}}{a_k}$ equals some constant (independent of k). Here:

$$\frac{\frac{1}{2^{k+1}}}{\frac{1}{2^k}} = \frac{1}{2}$$