

0.1 Functions and Relations

It is also important to define some types of relations and functions.

Definition (Relation):

A (binary) **relation** R on a set S is a subset of $S \times S$. If $(a, b) \in R$, we write aRb .

Example (of relations): 1. L "loves" is a relation on $P \times P$ (where P is a set of all people).

2. The set $R = \{(0, 0), (0, 1), (2, 2), (7, 18)\}$ is a relation on \mathbb{Z}^+ . We would write $0R0$, $0R1$, $2R2$, and $7R18$.

Definition (Equivalence Relation):

An equivalence relation on a set S is a relation s.t.:

1. Reflexive: For each $a \in S$, $a \sim a$.
2. Symmetric: For $a, b \in S$, if $a \sim b$, then $b \sim a$.
3. Transitive: For $a, b, c \in S$, if $a \sim b$ and $b \sim c$

Functions in the general sense are also a type of relation.

Definition (Function):

A **function**, F from a set A to a set B is a relation s.t.: if aFb and aFb' then $b = b'$.

This is a rule that assigns a unique $a \in A$ to a unique $b \in B$. Write $F(a) = b$.