0.1 Set Theory Foundation

This is a review of some introductory proof concepts that are important for laying the foundations of both rational numbers and real analysis.

Definition:
A **set** is a collection of objects called elements of the set.

Example:

1. $S = \{1, 2, 3\}$ $(= \{1, 2, 3, 3\})$
2. $E = \{$Even integers $\}$
3. $\{$College students$\}$

Notation:

- $x \in S$ means x is in S.
- $x \notin S$ means x is not in S.
- The empty set $\emptyset$ is the set with no elements.
- $A \subseteq B$ means A is a subset of B (i.e. if $x \in A$, then $x \in B$).
- If $A \subseteq B$ but $B \subsetneq A$ A is a proper subset.

If $A \subseteq B$ and $B \subseteq A$ then $A = B$. Otherwise $A \neq B$.

We can define more sets in terms of other sets. *Set Operations:* Let A and B be sets.

- Union: $A \cup B = \{x \,|\, x \in A \text{ or } x \in B\}$
- Intersection: $A \cap B = \{x \,|\, x \in A \text{ and } x \in B\}$
- Compliment: $B - A = \{x \,|\, x \in B \text{ and } x \notin A\}$
- Product: $A \times B = \{(a, b) \,|\, a \in A \text{ and } b \in B\}$

If U is a universal set (set of everything in context), we write $\bar{A} = U - A$
$= \{x \,|\, x \in U \text{ and } x \notin A\}$.