0.1 Defining Partial Derivatives

Definition (Partial Derivative)*:*
The **partial derivative of** $f(x, y)$ **with respect to x** is the usual x-derivative, but with y held constant, written as $\frac{\partial f}{\partial x}$ or f_x.

Similarly, $\frac{\partial f}{\partial y}$ or f_y is the usual y-derivative, but with x held constant.

We can interpret $\frac{\partial f}{\partial x}(p_0)$ as the rate of change (r.o.c.) of f at p_0 in the +x-direction and $\frac{\partial f}{\partial y}(p_0)$ as the r.o.c. of f at p_0 in the +y-direction.

The limit definition of the partial derivative is a more rigorous statement of the first defintion. However, this can help extend the partial derivative to functions of more variables in an obvious way.

$$f_x(a, b) = \lim_{h->0} \frac{f(a + h, b) - f(a, b)}{h}$$

$$f_y(a, b) = \lim_{h->0} \frac{f(a, b + h) - f(a, b)}{h}$$