0.1 Multivariable Functions

These functions are maps from higher Euclidean dimensions to the real numbers, i.e. $\mathbb{R}^n$ to $\mathbb{R}$.

We describe the domain of these functions as all allowable inputs for each variable, the codomain as simply $\mathbb{R}$ and the range as all values or outputs.

Example:
Compute and sketch the domain and range of $f(x,y) = \frac{1}{\sqrt{x^2-y^2}}$.

We require $x^2 - y^2 > 0$ to stay in the real numbers. Thus, $x^2 > y^2$.

$$x > \pm y \text{ if } x > 0$$

$$-x > \pm y \text{ if } x < 0$$

Range: Get all $\frac{1}{\sqrt{x^2}} = \frac{1}{|x|} \therefore (0, \infty)$

More Examples:

$$f(x,y,z) = \ln(16 - 4x^2 - 4y^2 - z^2) \downarrow 16 - 4x^2 - 4y^2 - z^2 > 0$$

Thus, the domain is a ellipsoid and the range is $(-\infty, \ln 16)$.

0.2 Graphing Multivariable Functions

For a multivariable function in the form of To graph multivariable functions, we need a way to convey the higher dimensions.

An easy way to do this is using a