



COMP220: Graphics & Simulation

# 3: Vertices, Transforms & Projections



# Learning outcomes

By the end of this week, you should be able to:

- ▶ **Recall** alternative ways to represent mesh vertices in memory.
- ▶ **Apply** basic transforms using the GLM library.
- ▶ **Explain** the constituents of the model-view-projection matrix and how it can be used to create a first-person camera controller.

# Agenda

# Agenda

- ▶ Lecture (async):
  - ▶ **Compare** different ways to store vertex data in memory.
  - ▶ **Review** the transforms required to display 3D objects on a 2D screen.

# Agenda

- ▶ Lecture (async):
  - ▶ **Compare** different ways to store vertex data in memory.
  - ▶ **Review** the transforms required to display 3D objects on a 2D screen.
- ▶ Workshop (sync):
  - ▶ **Adapt** our basic triangle implementation to draw meshes with multiple triangles efficiently.
  - ▶ **Experiment** with creating transforms using GLM and using them to move objects and the camera.

# Vertices and Elements



# Vertex Data

# Vertex Data

- ▶ Vertices always have a **position**, but may also have other data associated, e.g. colour.

# Vertex Data

- ▶ Vertices always have a **position**, but may also have other data associated, e.g. colour.
- ▶ One way we could store this data is to use a separate Vertex Buffer for each **attribute**:

# Vertex Data

- ▶ Vertices always have a **position**, but may also have other data associated, e.g. colour.
- ▶ One way we could store this data is to use a separate Vertex Buffer for each **attribute**:

```
struct Vertex { float x,y,z; };
Vertex v[]={{-0.5f,-0.5f,0.0f},
            {0.5f,-0.5f,0.0f},
            {0.0f,0.5f,0.0f}};
```

```
struct Colour { float r,g,b,a; };
Colour c[]={{{1.0f,0.0f,0.0f,1.0f},
              {0.0f,1.0f,0.0f,1.0f},
              {0.0f,0.0f,1.0f,1.0f}}};
```

# Using Separate Vertex Buffers

```
GLuint vertexbuffers[2];
glGenBuffers(2, vertexbuffers);

// Set up vertex position buffer
glBindBuffer(GL_ARRAY_BUFFER, vertexbuffers[0]);
glBufferData(GL_ARRAY_BUFFER, sizeof(v), v, GL_STATIC_DRAW);
glEnableVertexAttribArray(0);
glVertexAttribPointer(0, 3, GL_FLOAT, GL_FALSE,
                     3 * sizeof(GL_FLOAT), (void*)0);

// Set up vertex colour buffer
glBindBuffer(GL_ARRAY_BUFFER, vertexbuffers[1]);
glBufferData(GL_ARRAY_BUFFER, sizeof(c), c, GL_STATIC_DRAW);
glEnableVertexAttribArray(1);
glVertexAttribPointer(1, 4, GL_FLOAT, GL_FALSE,
                     4 * sizeof(GL_FLOAT), (void*)0);
```

# Interleaved Vertices

# Interleaved Vertices

- ▶ Alternatively, we can create a **C structure** which represents a Vertex, which has member variables which represent positions, colours, normals etc.

# Interleaved Vertices

- ▶ Alternatively, we can create a **C structure** which represents a Vertex, which has member variables which represent positions, colours, normals etc.
- ▶ This is known as **Interleaved Vertices** and in **MOST** cases is more efficient.

# Interleaved Vertices

- ▶ Alternatively, we can create a **C structure** which represents a Vertex, which has member variables which represent positions, colours, normals etc.
- ▶ This is known as **Interleaved Vertices** and in **MOST** cases is more efficient.

```
struct Vertex
{
    float x, y, z;
    float r, g, b, a;
};

Vertex v[] = { {-0.5f, -0.5f, 0.0f, 1.0f, 0.0f, 0.0f, 1.0f},
               {0.5f, -0.5f, 0.0f, 0.0f, 1.0f, 0.0f, 1.0f},
               {0.0f, 0.5f, 0.0f, 0.0f, 0.0f, 1.0f, 1.0f} };
```

# Using a Single Buffer

# Using a Single Buffer

- We need to update the Vertex Attribute Array to interpret the data correctly:

# Using a Single Buffer

- We need to update the Vertex Attribute Array to interpret the data correctly:
  - Increase the **stride** for all attributes

# Using a Single Buffer

- ▶ We need to update the Vertex Attribute Array to interpret the data correctly:
  - ▶ Increase the **stride** for all attributes
  - ▶ Include the **offset** for everything but the position

# Using a Single Buffer

- We need to update the Vertex Attribute Array to interpret the data correctly:
  - Increase the **stride** for all attributes
  - Include the **offset** for everything but the position

```
GLuint vertexbuffer;
 glGenBuffers(1, &vertexbuffer);
 glBindBuffer(GL_ARRAY_BUFFER, vertexbuffer);
 glBufferData(GL_ARRAY_BUFFER, sizeof(v), v, GL_STATIC_DRAW);

 glEnableVertexAttribArray(0);    // Position attribute
 glVertexAttribPointer(0, 3, GL_FLOAT, GL_FALSE,
                      sizeof(Vertex), (void*)0);

 glEnableVertexAttribArray(1);    // Colour attribute
 glVertexAttribPointer(1, 4, GL_FLOAT, GL_FALSE,
                      sizeof(Vertex), (void*)(3 * sizeof(GL_FLOAT)));
```

# Vertices in Memory

	X	Y	Z	R	G	B	A
$v_0$	-0.5	-0.5	0.0	1.0	0.0	0.0	1.0
$v_1$	0.5	-0.5	0.0	0.0	1.1	0.0	1.0
$v_2$	0.0	0.2	0.0	0.0	0.0	1.0	1.0

# Vertices in Memory: Stride

A diagram illustrating vertex memory layout. A blue double-headed arrow at the top is labeled "Stride". Below it is a 3x7 grid of numbers. The columns are labeled with indices 0 through 6. The rows are labeled  $v_0$  (top),  $v_1$  (middle), and  $v_2$  (bottom). The values in the grid are:

	0	1	2	3	4	5	6
$v_0$	-0.5	-0.5	0.0	1.0	0.0	0.0	1.0
$v_1$	0.5	-0.5	0.0	0.0	1.1	0.0	1.0
$v_2$	0.0	0.2	0.0	0.0	0.0	1.0	1.0

# Vertices in Memory: Offset

$v_0 \quad v_1 \quad v_2 \quad \rightarrow$

Offset = 3 \* sizeof(float)

$v_0$	-0.5	-0.5	0.0	1.0	0.0	0.0	1.0
$v_1$	0.5	-0.5	0.0	0.0	1.1	0.0	1.0
$v_2$	0.0	0.2	0.0	0.0	0.0	1.0	1.0

# Multiple Triangles

# Multiple Triangles

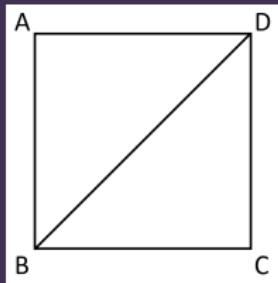
- ▶ For the triangle, we listed each vertex's data in the buffer.

# Multiple Triangles

- ▶ For the triangle, we listed each vertex's data in the buffer.
- ▶ What if we have shapes made of multiple triangles (known as **meshes**)?

# Multiple Triangles

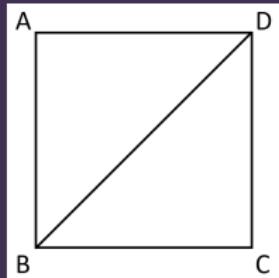
- ▶ For the triangle, we listed each vertex's data in the buffer.
- ▶ What if we have shapes made of multiple triangles (known as **meshes**)?



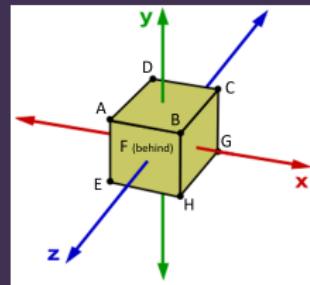
Square: 4 vertices  
2 triangles: 6 vertices

# Multiple Triangles

- ▶ For the triangle, we listed each vertex's data in the buffer.
- ▶ What if we have shapes made of multiple triangles (known as **meshes**)?



Square: 4 vertices  
2 triangles: 6 vertices



Cube: 8 vertices  
12 triangles: 36 vertices  
(6 square faces)

# Element Buffer

# Element Buffer

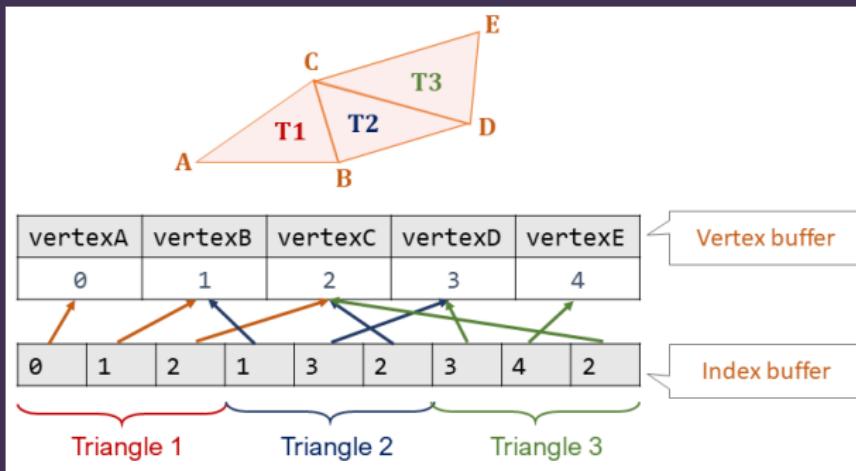
- We can use an **Element Buffer** to optimise our drawing and avoid storing duplicate vertices.

# Element Buffer

- ▶ We can use an **Element Buffer** to optimise our drawing and avoid storing duplicate vertices.
- ▶ An Element Buffer holds an integer which is an **offset** (or index) into a Vertex Buffer.

# Element Buffer

- ▶ We can use an **Element Buffer** to optimise our drawing and avoid storing duplicate vertices.
- ▶ An Element Buffer holds an integer which is an **offset** (or index) into a Vertex Buffer.



# Vertex Order

# Vertex Order

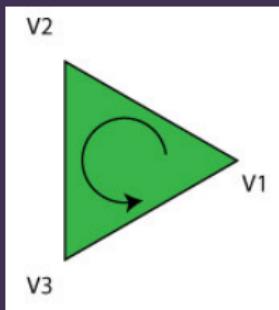
- ▶ It is sometimes important to know which side of a triangle is the “front” and which is the “back”

# Vertex Order

- ▶ It is sometimes important to know which side of a triangle is the “front” and which is the “back”
- ▶ OpenGL determines this by **winding order**

# Vertex Order

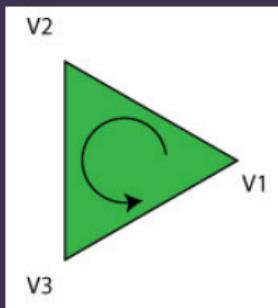
- ▶ It is sometimes important to know which side of a triangle is the “front” and which is the “back”
- ▶ OpenGL determines this by **winding order**



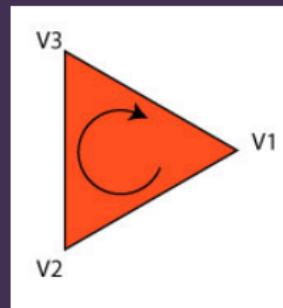
If the vertices go  
**anticlockwise**, you are  
looking at the **front**

# Vertex Order

- ▶ It is sometimes important to know which side of a triangle is the “front” and which is the “back”
- ▶ OpenGL determines this by **winding order**



If the vertices go **anticlockwise**, you are looking at the **front**



If the vertices go **clockwise**, you are looking at the **back**

# Backface culling

# Backface culling

```
glEnable(GL_CULL_FACE);
```

# Backface culling

```
glEnable(GL_CULL_FACE);
```

- ▶ This will cause only the front faces of triangles to be drawn

# Backface culling

```
glEnable(GL_CULL_FACE);
```

- ▶ This will cause only the front faces of triangles to be drawn
- ▶ Triangles whose front face is not visible will be **culled**

# Backface culling

```
glEnable(GL_CULL_FACE);
```

- ▶ This will cause only the front faces of triangles to be drawn
- ▶ Triangles whose front face is not visible will be **culled**
- ▶ Culled faces are not passed through the rasteriser or fragment shader

# Backface culling

```
glEnable(GL_CULL_FACE);
```

- ▶ This will cause only the front faces of triangles to be drawn
- ▶ Triangles whose front face is not visible will be **culled**
- ▶ Culled faces are not passed through the rasteriser or fragment shader
- ▶ Saves time, and should make no difference to appearance — as long as all meshes are closed and have correct winding

# When backface culling goes bad?



# Transforms & Projections



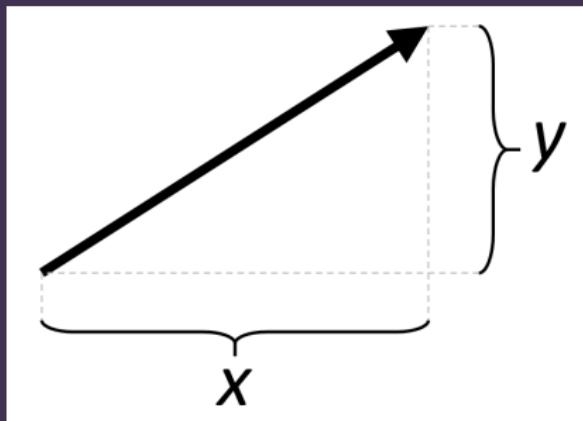
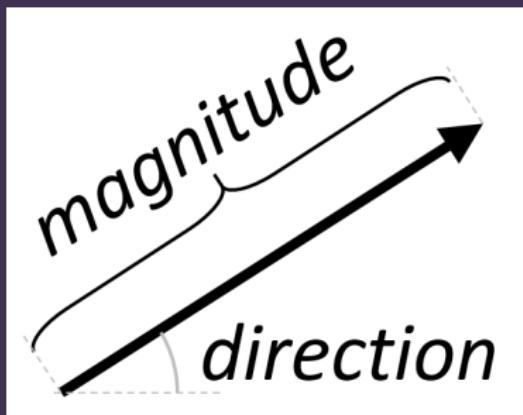
# Recap: Vectors

# Recap: Vectors

- A **vector** has a **direction** and a **magnitude**, represented by its **components**:

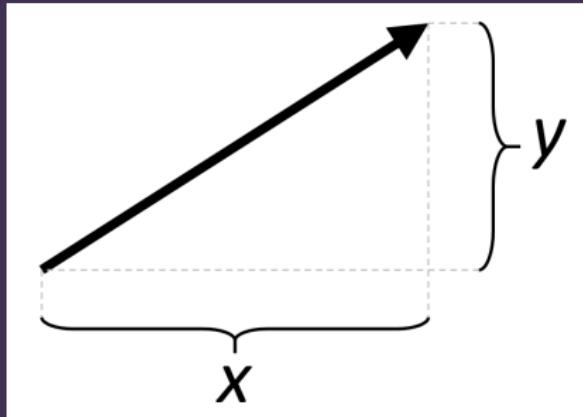
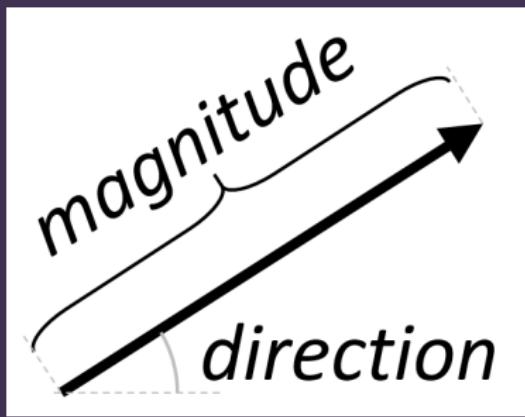
# Recap: Vectors

- A **vector** has a **direction** and a **magnitude**, represented by its **components**:



# Recap: Vectors

- ▶ A **vector** has a **direction** and a **magnitude**, represented by its **components**:



- ▶ A vector can represent a position in space as an offset from the origin.



# Recap: Matrices and Transforms

# Recap: Matrices and Transforms

- ▶ A vector can be transformed (moved in space) by multiplying with a **matrix**:

# Recap: Matrices and Transforms

- ▶ A vector can be transformed (moved in space) by multiplying with a **matrix**:
  - ▶ **Translation** applies a linear offset to each component (equivalent to vector addition).



# Recap: Matrices and Transforms

- ▶ A vector can be transformed (moved in space) by multiplying with a **matrix**:
  - ▶ **Translation** applies a linear offset to each component (equivalent to vector addition).
  - ▶ **Rotation** applies an angular displacement around the origin.



# Recap: Matrices and Transforms

- ▶ A vector can be transformed (moved in space) by multiplying with a **matrix**:
  - ▶ **Translation** applies a linear offset to each component (equivalent to vector addition).
  - ▶ **Rotation** applies an angular displacement around the origin.
  - ▶ **Scale** multiplies each component by a constant (equivalent to scalar multiplication for uniform scale).

# Recap: Matrices and Transforms

- ▶ A vector can be transformed (moved in space) by multiplying with a **matrix**:
  - ▶ **Translation** applies a linear offset to each component (equivalent to vector addition).
  - ▶ **Rotation** applies an angular displacement around the origin.
  - ▶ **Scale** multiplies each component by a constant (equivalent to scalar multiplication for uniform scale).
- ▶ Transforms can be combined by **multiplying** the matrices together.

# Recap: Matrices and Transforms

- ▶ A vector can be transformed (moved in space) by multiplying with a **matrix**:
  - ▶ **Translation** applies a linear offset to each component (equivalent to vector addition).
  - ▶ **Rotation** applies an angular displacement around the origin.
  - ▶ **Scale** multiplies each component by a constant (equivalent to scalar multiplication for uniform scale).
- ▶ Transforms can be combined by **multiplying** the matrices together.
  - ▶ Matrix multiplication is **NOT commutative**: the order matters.

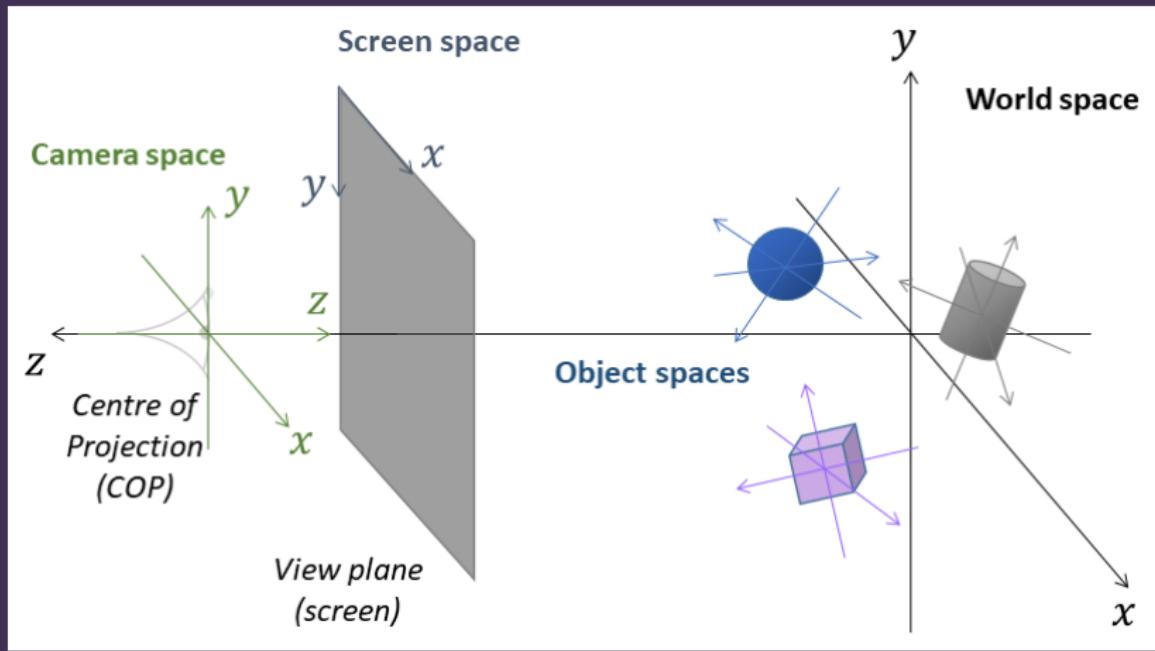
# Recap: Matrices and Transforms

- ▶ A vector can be transformed (moved in space) by multiplying with a **matrix**:
  - ▶ **Translation** applies a linear offset to each component (equivalent to vector addition).
  - ▶ **Rotation** applies an angular displacement around the origin.
  - ▶ **Scale** multiplies each component by a constant (equivalent to scalar multiplication for uniform scale).
- ▶ Transforms can be combined by **multiplying** the matrices together.
  - ▶ Matrix multiplication is **NOT commutative**: the order matters.
  - ▶ Standard order is **right to left**.

# Recap: Matrices and Transforms

- ▶ A vector can be transformed (moved in space) by multiplying with a **matrix**:
  - ▶ **Translation** applies a linear offset to each component (equivalent to vector addition).
  - ▶ **Rotation** applies an angular displacement around the origin.
  - ▶ **Scale** multiplies each component by a constant (equivalent to scalar multiplication for uniform scale).
- ▶ Transforms can be combined by **multiplying** the matrices together.
  - ▶ Matrix multiplication is **NOT commutative**: the order matters.
  - ▶ Standard order is **right to left**.
  - ▶ It's generally best to apply scale first, then rotation, then finally translation - otherwise results can be unexpected...

# Recap: Coordinate Spaces



# Model, View, Projection

# Model, View, Projection

Drawing a 3D object on screen generally involves **three** transformations:

# Model, View, Projection

Drawing a 3D object on screen generally involves **three** transformations:

- ▶ **Model:** translate, rotate and scale the object into its place in the scene

# Model, View, Projection

Drawing a 3D object on screen generally involves **three** transformations:

- ▶ **Model**: translate, rotate and scale the object into its place in the scene
- ▶ **View**: translate and rotate the scene to put the observer at the origin

# Model, View, Projection

Drawing a 3D object on screen generally involves **three** transformations:

- ▶ **Model**: translate, rotate and scale the object into its place in the scene
- ▶ **View**: translate and rotate the scene to put the observer at the origin
- ▶ **Projection**: convert points in 3D space to points on the 2D screen

# Model, View, Projection

Drawing a 3D object on screen generally involves **three** transformations:

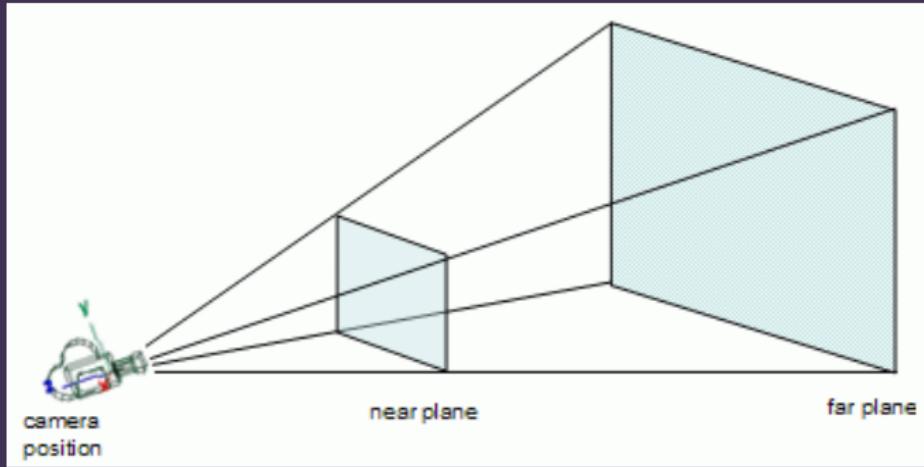
- ▶ **Model**: translate, rotate and scale the object into its place in the scene
- ▶ **View**: translate and rotate the scene to put the observer at the origin
- ▶ **Projection**: convert points in 3D space to points on the 2D screen

The **model-view-projection (MVP) matrix**:

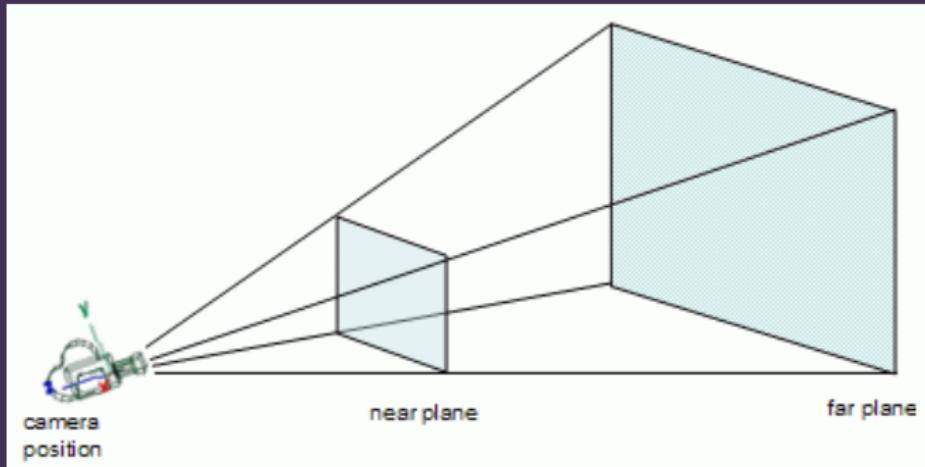
$$M_{MVP} = M_{\text{projection}} \times M_{\text{view}} \times M_{\text{model}}$$

# The view frustum

# The view frustum

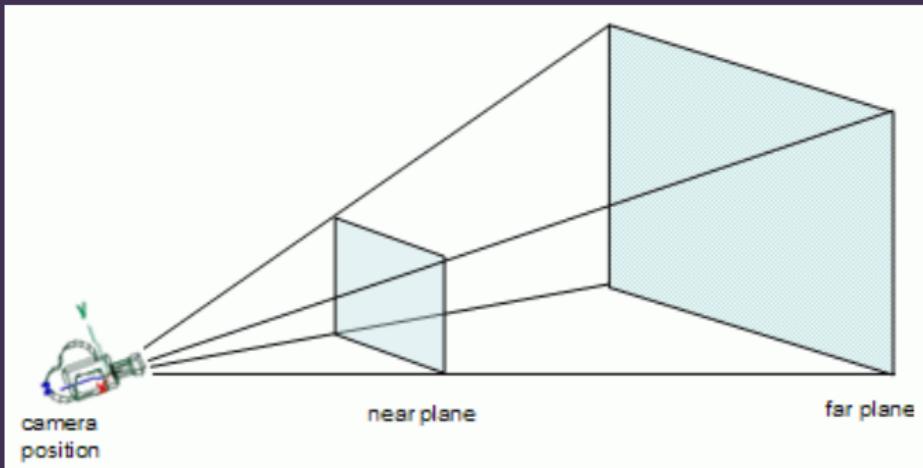


# The view frustum



- ▶ Defined by the **near and far clipping planes** and the **edges of the screen**

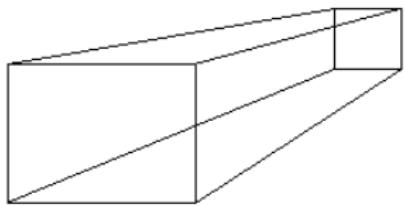
# The view frustum



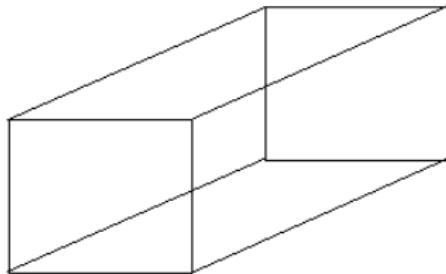
- ▶ Defined by the **near and far clipping planes** and the **edges of the screen**
- ▶ **Nothing outside** the view frustum is visible

# Types of projection

# Types of projection

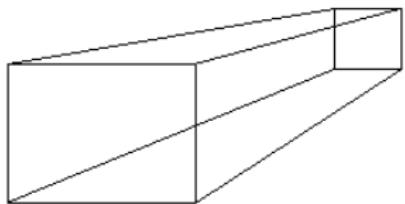


Perspective projection

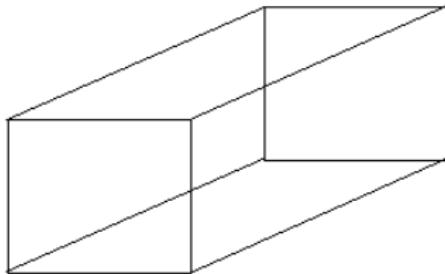


Orthographic projection

# Types of projection



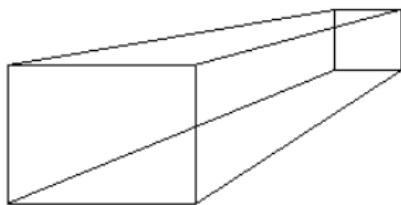
Perspective projection



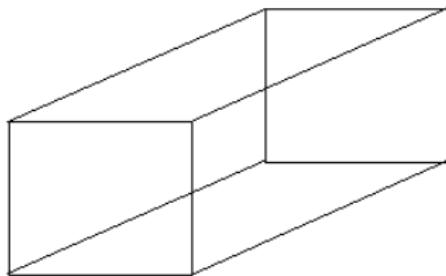
Orthographic projection

- Generally use **perspective** for 3D graphics

# Types of projection



Perspective projection



Orthographic projection

- ▶ Generally use **perspective** for 3D graphics
- ▶ **Orthographic** is useful for 2D or pseudo-2D graphics (e.g. isometric projection for engineering or technical drawings)

First person camera control



# The plan

# The plan

- ▶ Represent the player's **position** by a 3D vector

# The plan

- ▶ Represent the player's **position** by a 3D vector
- ▶ Represent the player's **orientation** by Euler angles

# The plan

- ▶ Represent the player's **position** by a 3D vector
- ▶ Represent the player's **orientation** by Euler angles
- ▶ Mouse events change these angles

# The plan

- ▶ Represent the player's **position** by a 3D vector
- ▶ Represent the player's **orientation** by Euler angles
- ▶ Mouse events change these angles
- ▶ View matrix is calculated using position and orientation

# The plan

- ▶ Represent the player's **position** by a 3D vector
- ▶ Represent the player's **orientation** by Euler angles
- ▶ Mouse events change these angles
- ▶ View matrix is calculated using position and orientation
- ▶ To move forwards, use the Euler angles to find the "forward" vector, and offset the position by this vector

# Euler angles

# Euler angles

- ▶ Any orientation of an object in 3D space can be described by **three** rotations around:

# Euler angles

- ▶ Any orientation of an object in 3D space can be described by **three** rotations around:
  - ▶ The x-axis (1, 0, 0)

# Euler angles

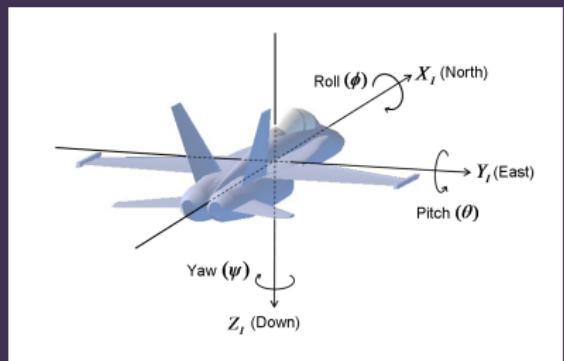
- ▶ Any orientation of an object in 3D space can be described by **three** rotations around:
  - ▶ The x-axis  $(1, 0, 0)$
  - ▶ The y-axis  $(0, 1, 0)$

# Euler angles

- ▶ Any orientation of an object in 3D space can be described by **three** rotations around:
  - ▶ The x-axis (1, 0, 0)
  - ▶ The y-axis (0, 1, 0)
  - ▶ The z-axis (0, 0, 1)

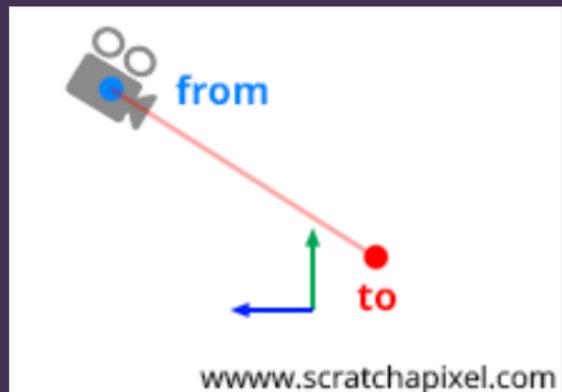
# Euler angles

- ▶ Any orientation of an object in 3D space can be described by **three** rotations around:
  - ▶ The x-axis (1, 0, 0)
  - ▶ The y-axis (0, 1, 0)
  - ▶ The z-axis (0, 0, 1)
- ▶ These angles are sometimes called **roll**, **pitch** and **yaw**



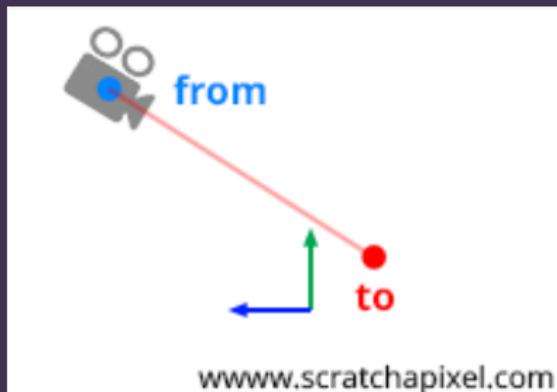
# Representing look direction

# Representing look direction

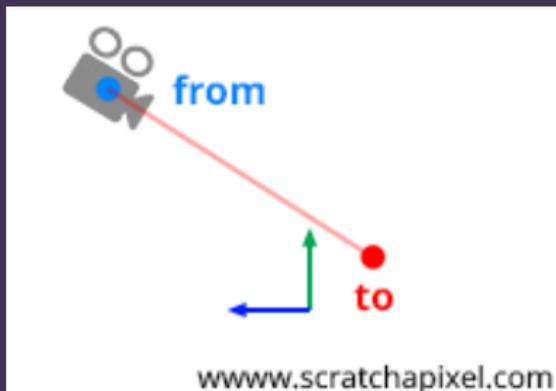


# Representing look direction

- ▶ Don't need roll, just pitch and yaw to look up/down and left/right.

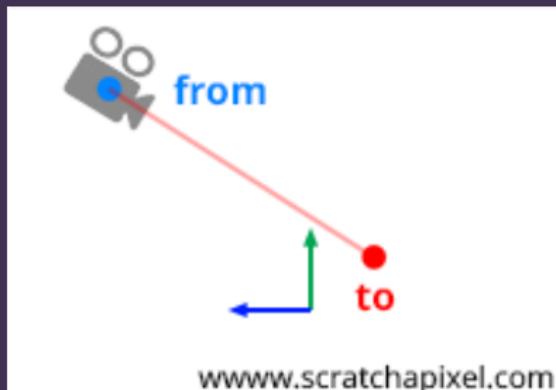


# Representing look direction



- ▶ Don't need roll, just pitch and yaw to look up/down and left/right.
- ▶ Eliminates the gimbal lock problem! :D

# Representing look direction



- ▶ Don't need roll, just pitch and yaw to look up/down and left/right.
- ▶ Eliminates the gimbal lock problem! :D
- ▶ **Forward vector / look vector** can be obtained by appropriate rotation of a **unit vector**.

# Unit vectors

# Unit vectors

- A **unit vector** is a vector of length 1

# Unit vectors

- ▶ A **unit vector** is a vector of length 1
- ▶ I.e.  $\sqrt{x^2 + y^2 + z^2} = 1$  (Pythagoras)

# Unit vectors

- ▶ A **unit vector** is a vector of length 1
- ▶ I.e.  $\sqrt{x^2 + y^2 + z^2} = 1$  (Pythagoras)
- ▶ Useful to represent **direction**

# Unit vectors

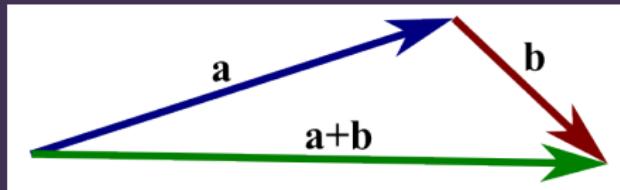
- ▶ A **unit vector** is a vector of length 1
- ▶ I.e.  $\sqrt{x^2 + y^2 + z^2} = 1$  (Pythagoras)
- ▶ Useful to represent **direction**
- ▶ Multiplying a vector of length  $a$  by a number  $b$  gives a vector of length  $a \times b$ , parallel to the original vector

# Unit vectors

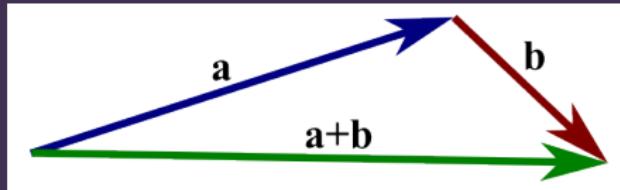
- ▶ A **unit vector** is a vector of length 1
- ▶ I.e.  $\sqrt{x^2 + y^2 + z^2} = 1$  (Pythagoras)
- ▶ Useful to represent **direction**
- ▶ Multiplying a vector of length  $a$  by a number  $b$  gives a vector of length  $a \times b$ , parallel to the original vector
- ▶ So multiplying a unit vector by  $b$  gives a vector of length  $b$ , parallel to the unit vector

# Moving forwards

# Moving forwards

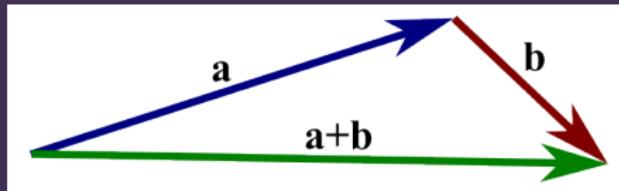


# Moving forwards



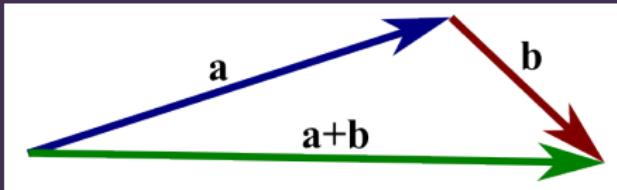
- ▶ E.g. if  $a$  is current position, and

# Moving forwards



- ▶ E.g. if  $a$  is current position, and
- ▶  $b$  is the distance and direction we want to move, then

# Moving forwards



- ▶ E.g. if  $a$  is current position, and
- ▶  $b$  is the distance and direction we want to move, then
- ▶  $a + b$  is the new position

# Addition in homogeneous coordinates

# Addition in homogeneous coordinates

- ▶ Recall: homogeneous coordinates have an extra  $w$  component, i.e.  $(x, y, z, w)$

# Addition in homogeneous coordinates

- ▶ Recall: homogeneous coordinates have an extra  $w$  component, i.e.  $(x, y, z, w)$
- ▶  $w = 1$  for positions,  $w = 0$  for offsets

# Addition in homogeneous coordinates

- ▶ Recall: homogeneous coordinates have an extra  $w$  component, i.e.  $(x, y, z, w)$
- ▶  $w = 1$  for positions,  $w = 0$  for offsets

$$\text{offset} + \text{offset} = \text{offset} \quad w = 0 + 0 = 0$$

# Addition in homogeneous coordinates

- ▶ Recall: homogeneous coordinates have an extra  $w$  component, i.e.  $(x, y, z, w)$
- ▶  $w = 1$  for positions,  $w = 0$  for offsets

$$\begin{array}{lll} \text{offset} & + & \text{offset} \\ \text{position} & + & \text{offset} \end{array} \quad = \quad \begin{array}{lll} \text{offset} & & w = 0 + 0 = 0 \\ \text{position} & & w = 1 + 0 = 1 \end{array}$$

# Addition in homogeneous coordinates

- ▶ Recall: homogeneous coordinates have an extra  $w$  component, i.e.  $(x, y, z, w)$
- ▶  $w = 1$  for positions,  $w = 0$  for offsets

$$\begin{array}{lllll} \text{offset} & + & \text{offset} & = & \text{offset} & w = 0 + 0 = 0 \\ \text{position} & + & \text{offset} & = & \text{position} & w = 1 + 0 = 1 \\ \text{position} & + & \text{position} & = & \text{???} & w = 1 + 1 = 2 \end{array}$$

# Next steps

- ▶ **Review** the additional asynchronous material for more background on representing mesh vertices and calculating the model, view and projection matrices.
- ▶ **Attend** the workshop to see how to implement an Element Buffer in OpenGL and use GLM to create and apply transforms.