COMP270: Mathematics for 3D Worlds and Simulations

WEEK 2: GEOMETRY I
PART 3: FUNCTIONS AND DISCRETISATION

Objectives

- **Recall** the mathematical definition of a function
- **Understand** what it means to discretise a continuous function, e.g. for implementation in code

Recap: sets

- Definition: a **set** is a collection of objects (called **elements**) in which order has no significance, e.g.
$$A = \{1,7,3,24,999\}$$
  - The elements often have some shared characteristics, e.g. $B = \{r \in \mathbb{R} : r > 3\}$ is the set of all real numbers greater than 3
- The **Cartesian product** of n sets is the set of all ***n*-tuples** with one component from each set.

Functions

- In mathematics, a **function** is a mapping from one set to another
- If S and T are sets, then a function
  $$f : S \to T$$
  maps each element of S to an element of T
  $$s \in S, \qquad f(s) \in T$$
- S is called the **domain** of f, and T is the **codomain**
- Note: f maps each element of S to one and only one element of T; however it could map multiple elements of S to the same element of T (a many-to-one mapping).

Functions vs functions

- In mathematics:

$$f : S \to T$$

- In code:

```
class S {…};
class T {…};
T f(S s) {…}
```

- (Under the assumption that `f` is implemented to always give the same return value given the same argument – e.g. no internal or external state)

Functions vs functions

- In mathematics:

$$f : \mathbb{R} \to \mathbb{Z}$$

- In code:

```
int f(float x) {…}
```

- OK, so $\mathbb{Z}$ and `int` aren’t really the same, nor are $\mathbb{R}$ and `float`, but close enough for computing…

Multiple arguments

- The domain of a function could be a Cartesian product:

$$f : A \times B \to C$$

- In code:

```
C f(A a, B b) {…}
```

Continuous vs. discrete

- Much of traditional mathematics is **continuous**: functions can vary smoothly across the entire domain
- Computer science uses **discrete mathematics**, for objects that can only assume distinct, separate values
  - Computers can't represent every value exactly, e.g. floats vs. $\mathbb{R}$
  - Even if they could, we couldn't evaluate a function for e.g. every value of $\mathbb{R}$…
- Need to **discretise** mathematical functions by evaluating them at representative intervals

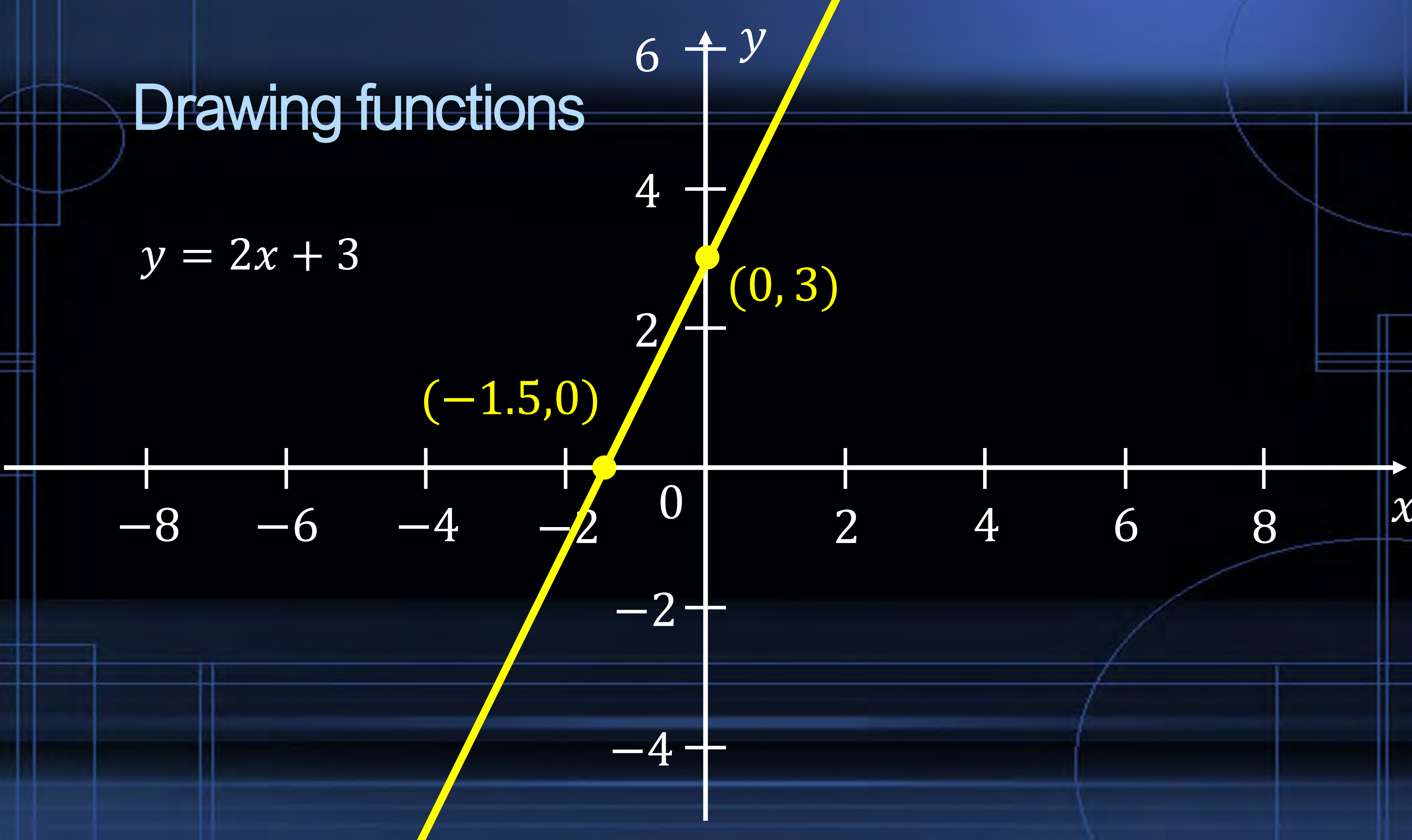
Similar to **sampling** in signal processing

Drawing functions

- For a function $f : \mathbb{R} \to \mathbb{R}$ we can plot the graph
$$y = f(x)$$
- Formed of the points $(x, f(x))$ for x in some range
- Note: only one point per x value, since f maps each x to one and **only one** y value
- Build a picture of a continuous shape by considering the value of $f(x)$ for values of x at various intervals…

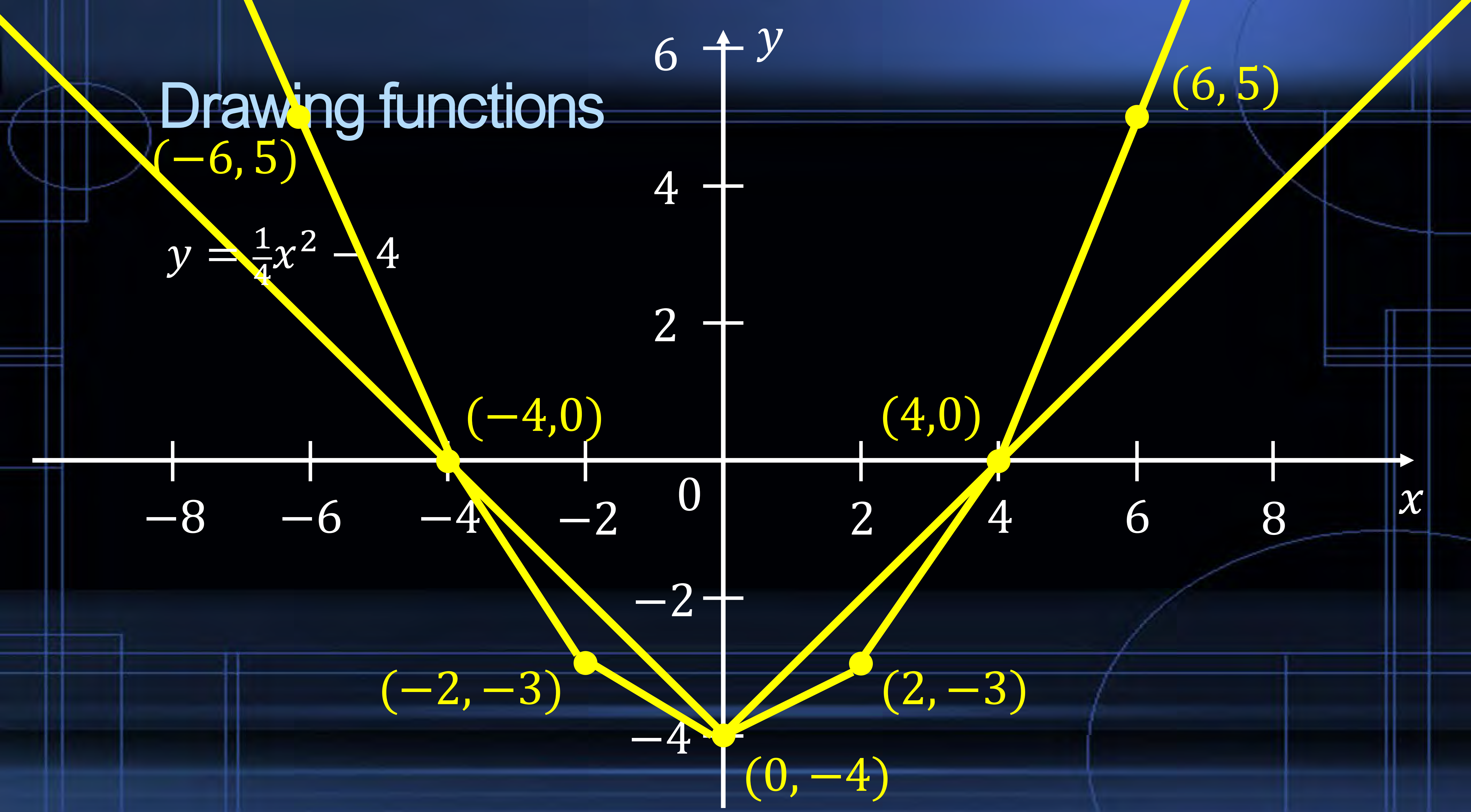
Drawing functions

$y = 2x + 3$



Drawing functions

$y = \frac{1}{4}x^2 - 4$



Butterflies and beyond