COMP110: Principles of Computing

5: Computational Complexity

Learning outcomes

- **Explain** the notion of computability
- **Use** "big O" notation to express computational complexity
- **Apply** appropriate algorithms to achieve efficiency

Worksheet C

- Computational complexity
- Due in class on **Monday 24th October** (next week)

Reading

E. G. Gilbert, D. W. Johnson, and S. S. Keerthi, 1988. A Fast Procedure for Computing the Distance Between Complex Objects in Three-Dimensional Space. *IEEE Journal of Robotics and Automation*, 4(2):193–203.

Computation time

Resources

Resources

- All programs use **resources**

Resources

- All programs use **resources**
  - Time

Resources

- All programs use **resources**
  - Time
  - Memory

Resources

- All programs use **resources**
  - Time
  - Memory
  - Network bandwidth

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- Often **time** is the resource we care about the most

Resources

- All programs use **resources**
  - Time
  - Memory
  - Network bandwidth
  - Power
  - ...
- Often **time** is the resource we care about the most
  - Particularly in games: want to maintain a good **frame rate** free of **lag** or **stuttering**

Basic time measurement in Python

```
import time

start_time = time.clock()

... do something here ...

end_time = time.clock()
print "Computation took", end_time - start_time, " ↩
    seconds"
```

Repeating for better accuracy

```
import time

start_time = time.clock()

repetition_count = 1000

for repetition in xrange(repetition_count):
    ... do something here ...

end_time = time.clock()
time_per = (end_time - start_time) / repetition_count
print "Computation took", time_per, "seconds"
```

Scaling

Scaling

- Timing is dependent on hardware and software issues

Scaling

- Timing is dependent on hardware and software issues
- We are often less interested in how many milliseconds a particular computation takes on today's hardware, and more interested in how the execution time **scales** with the problem size

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Search

Search

Anderson, Martha
Parker, Debra
Russell, Mildred
Stewart, Howard
White, Amanda
Perez, Diana
Lewis, Rose
Scott, Michelle
Davis, Marilyn
Cox, Shirley
Young, Frank
Collins, Jane
Kelly, Philip
Miller, Jeremy
Clark, Stephanie
Brown, Janet
Diaz, Harold
Hughes, Aaron
Sanders, Phillip
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Baker, Theresa
Gonzalez, Adam
Lopez, Jeffrey
Ward, Jessica

- We have a list of names, each with some data associated

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- We have a list of names, each with some data associated
- We want to find one of them

Linear search

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procedure FIND(name, list)

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    for each item in list **do**

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        end if
    end for
    throw "Not found"
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How long does it take?

Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list

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- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?

How long does it take?

Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?
- How about in the **worst case**?

How long does it take?

Socrative room code: `FALCOMPED`

- If there are 25 items in the list, the **worst case** number of items visited is 25

How long does it take?

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- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?

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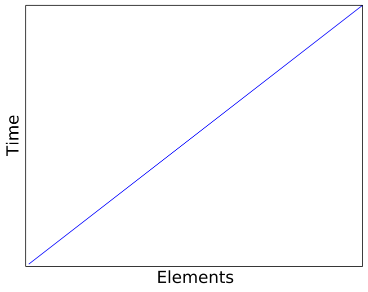
- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?

How long does it take?

Socrative room code: `FALCOMPED`

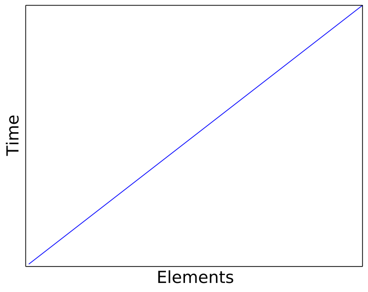
- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?
- If the number of items **doubles**, what happens to the amount of time the search takes?

Linear time



- The running time of linear search is **proportional** to the size *n* of the list

Linear time



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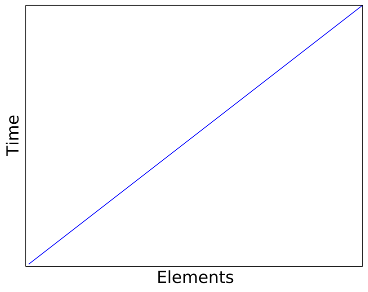
- Linear search is said to have **linear time complexity**

Linear time



- The running time of linear search is **proportional** to the size n of the list
- Linear search is said to have **linear time complexity**
- Also written as $O(n)$ **time complexity**

Searching a sorted list

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Baker, Theresa
Brown, Janet
Clark, Stephanie
Collins, Jane
Cox, Shirley
Davis, Marilyn
Diaz, Harold
Gonzalez, Adam
Henderson, Lawrence
Hughes, Aaron
Kelly, Philip
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Sanders, Phillip
Scott, Michelle
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- If the list is **sorted** in alphabetical order, we can do better than linear...

Binary search

procedure FIND(name, list)

Binary search

```
procedure FIND(name, list)
    if list is empty then
        throw "Not found"
    end if
```

Binary search

procedure FIND(name, list)
    if list is empty **then**
        throw "Not found"
    end if
    mid $\leftarrow$ the "middle" item of the list

Binary search

```
procedure FIND(name, list)
    if list is empty then
        throw "Not found"
    end if
    mid ← the "middle" item of the list
    if name = mid.name then
        return mid
```

Binary search

```
procedure FIND(name, list)
    if list is empty then
        throw "Not found"
    end if
    mid ← the "middle" item of the list
    if name = mid.name then
        return mid
    else if name < mid.name then
        return FIND(name, first half of list)
```

Binary search

procedure FIND(name, list)
  if list is empty **then**
    throw "Not found"
  end if
  mid $\leftarrow$ the "middle" item of the list
  if name $=$ mid.name **then**
    return mid
  else if name $<$ mid.name **then**
    return FIND(name, first half of list)
  else if name $>$ mid.name **then**
    return FIND(name, second half of list)

Binary search

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    end if
    mid ← the "middle" item of the list
    if name = mid.name then
        return mid
    else if name < mid.name then
        return FIND(name, first half of list)
    else if name > mid.name then
        return FIND(name, second half of list)
    end if
end procedure
```

Find “Lopez, Jeffrey”

- Anderson, Martha
- Baker, Theresa
- Brown, Janet
- Clark, Stephanie
- Collins, Jane
- Cox, Shirley
- Davis, Marilyn
- Diaz, Harold
- Gonzalez, Adam
- Henderson, Lawrence
- Hughes, Aaron
- Kelly, Philip
- $\longrightarrow$ Lewis, Rose
- Lopez, Jeffrey
- Miller, Jeremy
- Parker, Debra
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- Russell, Mildred
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Socrative room code: `FALCOMPED`

- Each iteration cuts the list in **half**

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- If the size of the list is **doubled**, what happens to the worst-case **number of iterations** required?

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- The running time is **logarithmic** or $O(\log n)$

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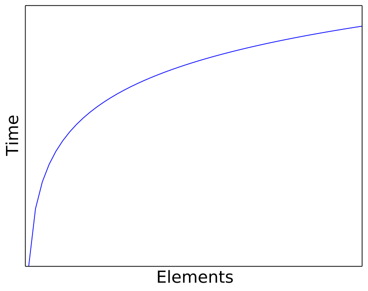

Hidden complexity

```
if name < mid.name then
    return Find(name, first half of list)
else if name > mid.name then
    return Find(name, second half of list)
end if
```

Hidden complexity

if name $<$ mid.name **then**
    return FIND(name, first half of list)
else if name $>$ mid.name **then**
    return FIND(name, second half of list)
end if

- Careful how you implement this!

Hidden complexity

if name $<$ mid.name **then**
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- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$

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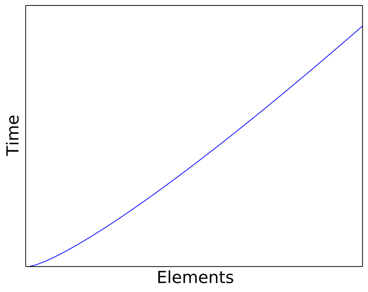

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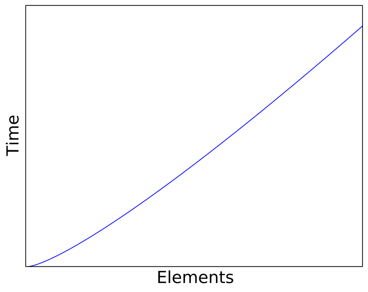


- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$
- The actual running time would be $O(n \log n)$
- Use **pointers** into the list instead of copying

Binary search done wrong

```
def binary_search(name, mylist):
    if mylist == []:
        raise ValueError("Not found")

    mid = len(mylist) / 2
    mid_name = mylist[mid_index].name

    if name == mid_name:
        return mid
    elif name < mid_name:
        return binary_search(name, mylist[:mid])
    else:
        return binary_search(name, mylist[mid+1:])
```

Binary search done right

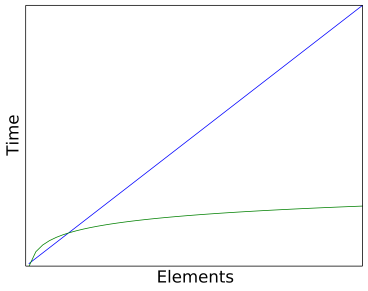
```
def binary_search(name, mylist, start, end):
    if end <= start:
        raise ValueError("Not found")

    mid = (start + end) / 2
    mid_name = mylist[mid].name

    if name == mid_name:
        return mylist[mid]
    elif name < mid_name:
        return binary_search(name, mylist, start, mid)
    else:
        return binary_search(name, mylist, mid+1, end)
```

Binary search vs linear search

Socrative room code: `FALCOMPED`



- So binary search is better than linear search... right?

Binary search vs linear search

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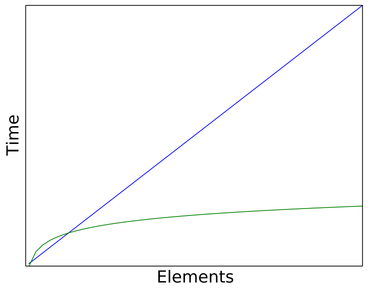

- So binary search is better than linear search... right?
- Discuss in **pairs**
- On Socrative, post **one reason** why, or **one situation** where, linear search may be a better choice than binary search

Hashing

- Come up with a **hashing function** which maps elements to numbers

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⋮	⋮
112	Ward, Jessica
113	Baker, Theresa
114	Collins, Jane
115	—
116	—
117	Hughes, Aaron
118	—
119	—
120	—
121	—
122	Brown, Janet
123	—
124	—
125	Gonzalez, Adam Lewis, Rose
126	—
127	—
128	—
129	—
130	—
131	—
132	Young, Frank
⋮	⋮

Hash look-up

98	Diaz, Harold
99	Parker, Debra Perez, Diana White, Amanda
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114	Collins, Jane
117	Hughes, Aaron
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132	Young, Frank
135	Kelly, Philip
138	Cox, Shirley
142	Clark, Stephanie
144	Scott, Michelle
145	Miller, Jeremy
147	Davis, Marilyn
149	Lopez, Jeffrey
151	Anderson, Martha
158	Williams, Billy
162	Sanders, Phillip
171	Russell, Mildred
175	Stewart, Howard
183	Henderson, Lawrence

"Lopez, Jeffrey"

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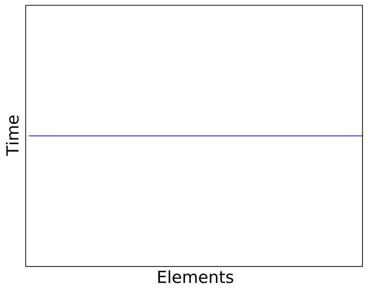
$$12 + 15 + 16 + 5 + 26 + 10 + 5 + 6 + 6 + 18 + 5 + 25 = 149$$

How long does it take?



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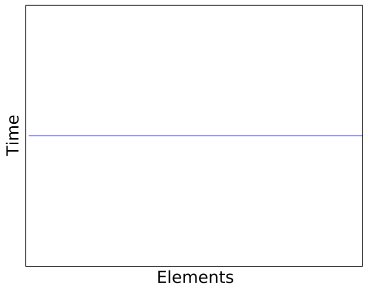
How long does it take?



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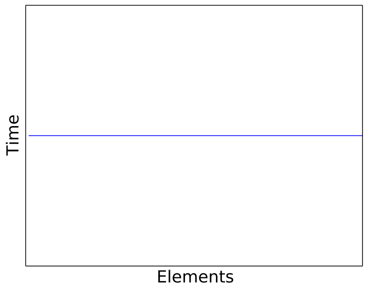

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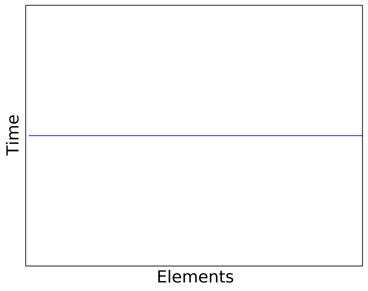

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- I.e. doubling the size of the list **does not change** the look-up time
- When there are collisions, need to fall back on something like linear or binary search within each bin

Don't reinvent the wheel!

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- We are using search as an **example**, to learn the **principles** — in practice you should hardly ever implement your own search

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    - **List comprehension**, e.g.

```
[person for person in people if person.name ↩
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- Binary search in Python:
  - The `bisect` module
- Hash tables in Python:

Don't reinvent the wheel!

- We are using search as an **example**, to learn the **principles** — in practice you should hardly ever implement your own search
- Linear search in Python:
  - `list.index()` method
  - **List comprehension**, e.g.

```
[person for person in people if person.name
    == "Lopez, Jeffrey"]
```

- Binary search in Python:
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- Hash tables in Python:
  - The `dict` (dictionary) data structure

More on complexity

Common complexity classes

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"Slower"	Factorial	$O(n!)$

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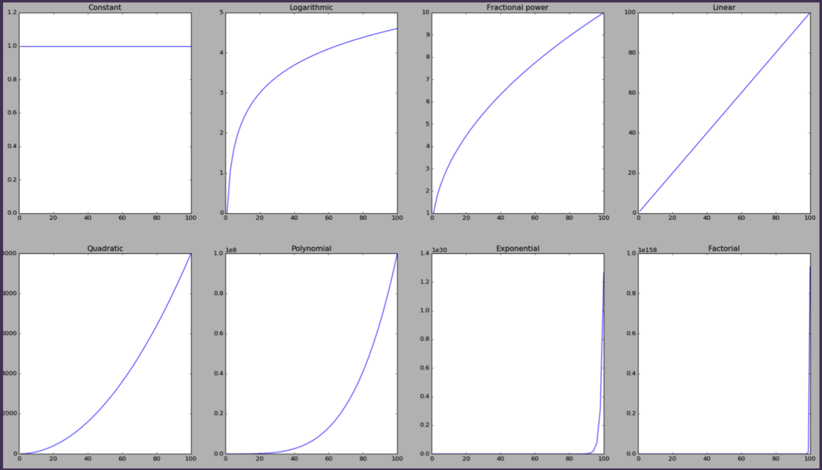

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  - If an algorithm does n "things" and each "thing" is $O(n)$, then the overall algorithm is $O(n^2)$

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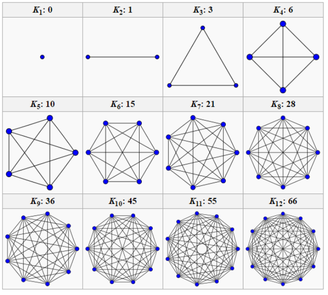

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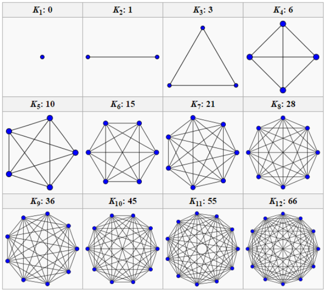

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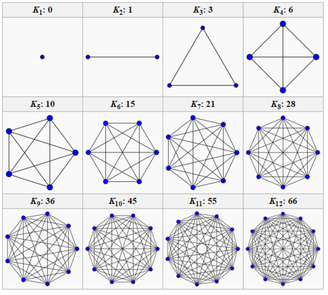

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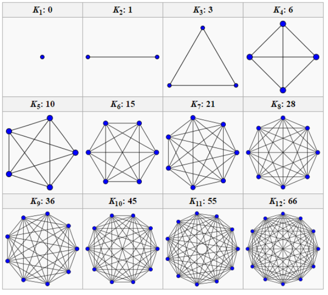

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  - Further reading: spatial hashing, quadtrees, octrees, Verlet lists

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- Adding 1 to n potentially **doubles** the running time!

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- Are there any problems in *NP* but not in *P*?

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  - Many types of cryptography are based on this assumption
  - Quantum computers are "infinitely parallel" in a sense so *can* solve some large NP-hard problems

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  - Otherwise, choose simplicity

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- Choice of data structures and algorithms can have a large impact on the efficiency of your software
- ... but only if scalability is actually a factor