

COMP110: Principles of Computing









Binary notation

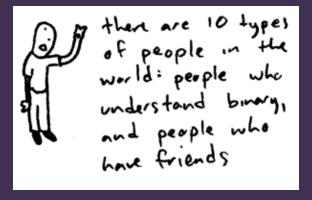


Image credit: http://www.toothpastefordinner.com

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- ► The binary digits 0 and 1 correspond to off and on respectively



Converting to binary

https://www.youtube.com/watch?v=OezK_zTyvAQ

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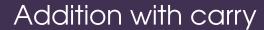
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Hex	Dec	Hex	Dec	Hex	Dec
00	0	10	16	F0	240
01	1	11	17	F1	241
		:		:	
09	9	19	25	F9	249
0A	10	1A	26	FA	250
0B	11	1в	27	FB	251
0C	12	1C	28	FC	252
0D	13	1D	29	FD	253
ΟE	14	1E	30	FE	254
OF	15	1F	31	FF	255





2's Complement

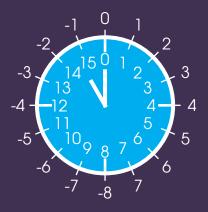




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Modular arithmetic



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▶
$$14 + 7 = 5$$

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- Useful for wrapping around e.g. loop indexes or screen coordinates

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- \blacktriangleright E.g. 16-bit number ranges from -32768 to +32767
- Note that the left-most bit can be interpreted as a sign bit: 1 if negative, 0 if positive or zero

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- This is also the process for converting back from 2's complement, i.e. doing it twice should give the original number

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- ► In fact, subtraction can just be done as addition
- ▶ I.e. a b is the same as a + (-b), where a and -b are just n-bit numbers





Worksheet 2

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Due next Friday!
Online quiz on LearningSpace





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 - I.e. a machine that carries out computations (calculations)

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- Has a tape head pointing at one space on the tape
- ► Has a transition table which, given:
 - ► The current state
 - The symbol under the tape head

specifies:

- A new state
- A new symbol to write to the tape, overwriting the current symbol
- Where to move the tape head: one space to the left, or one space to the right

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- Repeatedly apply the rules on the next slide
- What computation does this machine perform?
 - ► Hint: Milk = 0, White = 1...

Current	Current	New	New	Move
lolly	chocolate	lolly	chocolate	direction
Drumstick	Blank	Fruit	Blank	\leftarrow
Drumstick	Milk	Drumstick	White	\rightarrow
Drumstick	White	Drumstick	Milk	\rightarrow
Fruit	Blank	Swizzels	White	\rightarrow
Fruit	Milk	Swizzels	White	\leftarrow
Fruit	White	Fruit	Milk	\leftarrow
Swizzels	Blank	Stop	Blank	\rightarrow
Swizzels	Milk	Swizzels	Milk	\leftarrow
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- ► A machine, language or system is **Turing complete** if it can simulate a Turing machine