COMP250: Artificial Intelligence

5: Game Tree Search

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Game trees

State-action graphs

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  - Edges: actions, which cause transitions from one state to another
- Such graphs could contain **cycles** or **transpositions**
  - Cycle: a sequence of actions which takes us from a state s_0 back to the same state s_0
  - Transpositions: two different sequences of actions which both take us from a state s_1 to a state s_2

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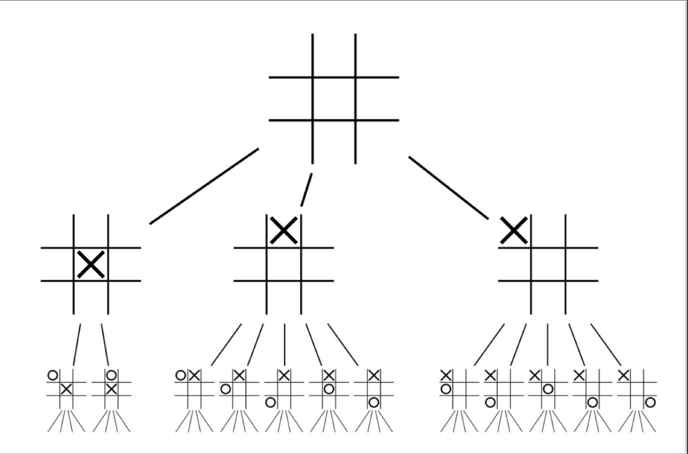
- Compare with pathfinding — needing to store a closed set of already searched nodes
- So we often use a **tree** representation instead
- Same state may appear multiple times in the tree
- However, there are guaranteed no cycles or transpositions

Example



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Minimax search

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- Therefore I want to **maximise** the **minimum** value my opponent can achieve
- This is generally only true for **two-player zero-sum** games

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Minimax search – example

`minimax.pptx`

Minimax search pseudocode

procedure MINIMAX(state, currentPlayer)

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    else if currentPlayer = 1 then
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  else if currentPlayer $= 1$ **then**
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    else if currentPlayer = 2 then
        bestValue = +∞
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            bestValue = MIN(bestValue, v)
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- Similarly when minimising if bestValue $= -1$

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- For a **two-player zero-sum** game with **perfect information** and **sequential moves**
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- **But...**

Minimax for larger games

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- Most games are too large to search fully
  - Connect 4 has $\approx 10^{13}$ states
  - Chess has $\approx 10^{47}$ states

Heuristics for search

Depth limiting

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- Still evaluate terminal states as $+1$ / 0 / -1
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- Evaluation is a number between -1 and $+1$, estimating the probable outcome of the game

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- This is essentially a **utility-based AI**

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- How do we know which moves are “best” and “worst”? Use a heuristic!

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- Good heuristic design requires **in-depth knowledge** of the tactics and strategy of the game
- What if we don't possess such knowledge?

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Monte Carlo evaluation

Monte Carlo methods

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- Applications in physics, engineering, finance, weather forecasting, graphics, ...

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- Seed is generally based on some source of **entropy**, e.g. system clock, mouse input, electronic noise

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- Higher expected value $=$ more chance of winning

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  - Minimax assumes the evaluation is **deterministic**, but Monte Carlo is not
  - Not commonly used, mainly because there's something better...

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Monte Carlo Tree Search

Monte Carlo Tree Search (MCTS)

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- First few rollouts are **random**
- However, statistics from these rollouts are used to **bias** future rollouts
- Bias rollouts towards **plausible** lines of play, i.e. where each player is trying to play the best move

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- Each rollout has four stages:
  - **Selection**: Starting from the root, descend the tree by choosing moves. Continue until we reach a node which does not yet have children for all legal moves.
  - **Expansion**: Choose a random legal move for which the current node does not have a child node. Add this new node to the tree.

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  - **Simulation**: Perform a Monte Carlo rollout, playing random moves until a terminal state is reached.

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  - **Backpropagation**: For each node visited during **selection** and **expansion**, update the node's statistics based on the result of the simulation.

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  - **Simulation**: Perform a Monte Carlo rollout, playing random moves until a terminal state is reached.
  - **Backpropagation**: For each node visited during **selection** and **expansion**, update the node's statistics based on the result of the simulation.
- Perform many rollouts, then use the statistics at the top level of the tree to choose the best move

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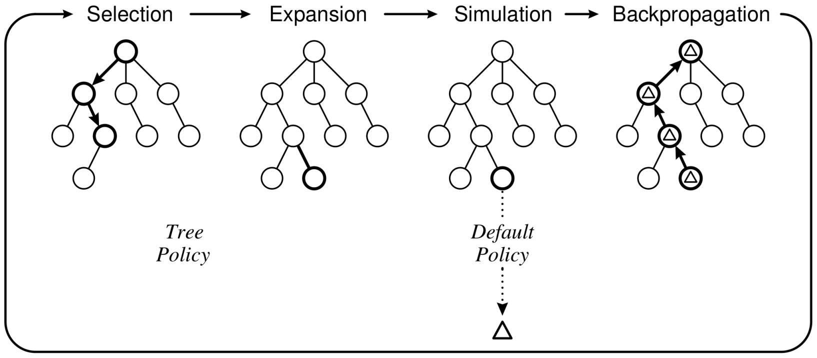

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- This can be modelled as a **multi-armed bandit problem**

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  - **Exploration** of machines that have not been tried often, to get a better estimate of their expected payout

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  - $\sqrt{\frac{\log n}{n_m}}$ is the **exploration** part: large if n_m is small

Upper Confidence Bound (UCB)

- For each machine m, record:
  - n_m: the number of plays of this machine
  - V_m: the total winnings from playing this machine
  - $n = \sum_m n_m$, total number of plays across all machines
- At each stage, play the machine for which

$$\frac{V_m}{n_m} + c\sqrt{\frac{\log n}{n_m}}$$

  is largest
  - $\frac{V_m}{n_m}$ is the **exploitation** part: average payout from this machine so far
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  - c is a parameter for adjusting the balance between exploitation and exploration

UCB demo

http://orangehelicopter.com/academic/bandits.html?ucb

Upper Confidence Bound for Trees (UCT)

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- From node p, choose the child q such that

$$\frac{V_q}{n_q} + c\sqrt{\frac{\log n_p}{n_q}}$$

is largest

UCT demo

`InteractiveDemo.exe`

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