COMP110: Principles of Computing

4: Logic and memory

Learning outcomes

- **Distinguish** the basic types of logic gate
- **Use** logic gates to build simple circuits
- **Explain** how computer memory works

Logic gates

Boolean logic

- Works with two values: TRUE and FALSE
- Foundation of the **digital computer**: represented in circuits as **on** and **off**
- Representing as 1 and 0 leads to **binary notation**
- One boolean value = one **bit** of information
- Programmers use boolean logic for conditions in `if` and `while` statements

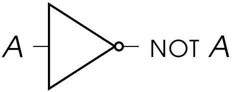
Simulating logic circuits

`http://logic.ly/demo/`

Not

NOT A is TRUE
if and only if
A is FALSE

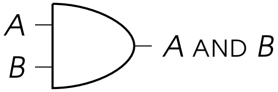
A	NOT A
FALSE	TRUE
TRUE	FALSE



And

A AND *B* is TRUE
if and only if
both *A* **and** *B* are TRUE

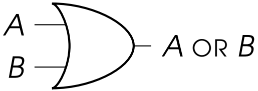
A	*B*	*A* AND *B*
FALSE	FALSE	FALSE
FALSE	TRUE	FALSE
TRUE	FALSE	FALSE
TRUE	TRUE	TRUE



Or

A OR *B* is TRUE
if and only if
either *A* **or** *B*, **or both,** are TRUE

A	*B*	*A* AND *B*
FALSE	FALSE	FALSE
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE
TRUE	TRUE	TRUE



Socrative `FALCOMPED`

What is the value of

$$A \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative FALCOMPED

What is the value of

$$(\textsc{not}\ A)\ \textsc{and}\ (B\ \textsc{or}\ C)$$

when

$$A = \textsc{True}$$
$$B = \textsc{False}$$
$$C = \textsc{True}$$

?

Socrative FALCOMPED

For what values of A, B, C, D is

$$A \text{ AND NOT } B \text{ AND NOT } (C \text{ OR } D) = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ OR NOT } A$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ AND NOT } A$$

?

Socrative FALCOMPED

What is the value of

$$A \text{ OR } A$$

?

Socrative `FALCOMPED`

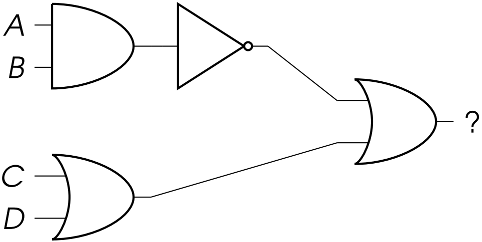
What is the value of

$$A \text{ AND } A$$

?

Socrative `FALCOMPED`

What expression is equivalent to this circuit?



Writing logical operations

Operation	Python	C family	Mathematics
NOT A	`not a`	`!a`	$\neg A$ or $\overline{A}$
A AND B	`a and b`	`a && b`	$A \wedge B$
A OR B	`a or b`	`a || b`	$A \vee B$

Other operators can be expressed by combining these

De Morgan's Laws

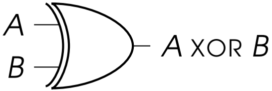
$$\text{NOT } (A \text{ OR } B) = (\text{NOT } A) \text{ AND } (\text{NOT } B)$$

$$\text{NOT } (A \text{ AND } B) = (\text{NOT } A) \text{ OR } (\text{NOT } B)$$

Exclusive Or

A XOR *B* is TRUE
if and only if
either *A* **or** *B*, **but not both,** are TRUE

A	*B*	*A* AND *B*
FALSE	FALSE	FALSE
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE
TRUE	TRUE	FALSE



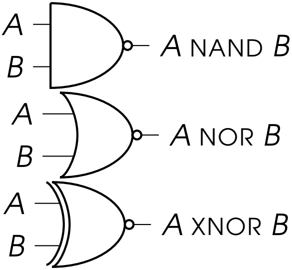
Socrative `FALCOMPED`

How can A XOR B be written using the operations AND , OR , NOT ?

Negative gates

NAND , NOR , XNOR
are the **negations** of
AND , OR , XOR

$$A \text{ NAND } B = \text{NOT } (A \text{ AND } B)$$
$$A \text{ NOR } B = \text{NOT } (A \text{ OR } B)$$
$$A \text{ XNOR } B = \text{NOT } (A \text{ XOR } B)$$



Binary notation

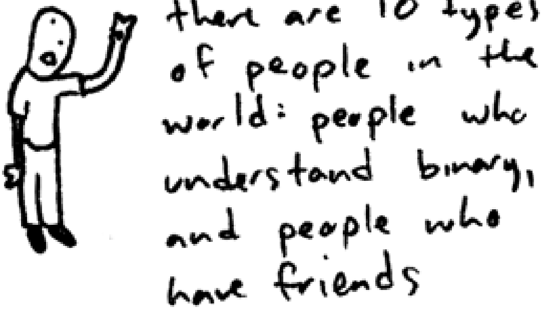


Image credit: `http://www.toothpastefordinner.com`

How we write numbers

- We write numbers in **base 10**
- We have 10 **digits**: $0, 1, 2, \ldots, 8, 9$
- When we write 6397, we mean:
  - Six thousand, three hundred and ninety seven
  - (Six thousands) and (three hundreds) and (nine tens) and (seven)
  - $(6 \times 1000) + (3 \times 100) + (9 \times 10) + (7)$
  - $\left(6 \times 10^3\right) + \left(3 \times 10^2\right) + \left(9 \times 10^1\right) + \left(7 \times 10^0\right)$

Binary

- Binary notation works the same, but is **base 2** instead of **base 10**
- We have 2 **digits**: 0, 1
- When we write 10001011 in binary, we mean:

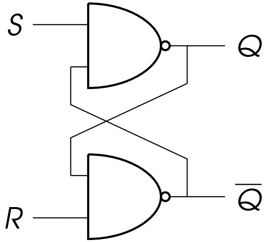
$$\begin{aligned}
&(1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) \\
&+ (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\
&= 2^7 + 2^3 + 2^1 + 2^0 \\
&= 128 + 8 + 2 + 1 \text{ (base 10)} \\
&= 139 \text{ (base 10)}
\end{aligned}$$

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary
- A **word** is the number of bits that the CPU works with at once
  - 32-bit CPU: 32 bits = 1 word
  - 64-bit CPU: 64 bits = 1 word
- An n-bit word can store a number between 0 and $2^n - 1$
  - $2^{16} - 1 = 65,535$
  - $2^{32} - 1 = 4,294,967,295$
  - $2^{64} - 1 = 18,446,744,073,709,551,615$

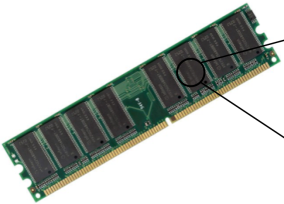
Computer memory

What does this circuit do?



- This is called a **NAND latch**
- It “remembers” a single boolean value
- Put a few billion of these together (along with some control circuitry) and you’ve got **memory**!

Memory



Address	Data
0	01001000
1	01100101
2	01101100
3	01101100
4	01101111
5	00100001
...	...

- Memory works like a set of **boxes**
- Each box has a number, its **address**
- Each box contains a **byte** (8 bits)

Data representation

- Memory stores **sequences of numbers**
- Therefore, any data stored by a computer must be represented as a sequence of numbers
  - Text: sequence of ASCII (or Unicode etc) character codes
  - Image: sequence of pixel colour values
  - 3D model: sequence of vertex coordinates
  - Audio: sequence of displacements
  - Executable: sequence of machine code operations

Worksheet B