COMP270: 3D Computational Geometry Worksheet 1

1. Calculate $\boldsymbol{a} \times \boldsymbol{b}$ and $\boldsymbol{b} \times \boldsymbol{a}$ for the following vectors:

   a. $\boldsymbol{a} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

   b. $\boldsymbol{a} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$

   c. $\boldsymbol{a} = \begin{pmatrix} 3 \\ 10 \\ 7 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 8 \\ -7 \\ 4 \end{pmatrix}$

2. A nonplayer character (NPC) is standing at a location $\boldsymbol{p}$ with a forward direction of $\boldsymbol{v}$. Consider three points $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ in the xz plane of a left-handed coordinate system, which represent waypoints on the NPC's path.

   a. How can the cross product be used to determine whether, when moving from $\boldsymbol{a}$ to $\boldsymbol{b}$ to $\boldsymbol{c}$, the NPC makes a clockwise or anticlockwise turn at $\boldsymbol{b}$, when viewing the path from above?

   b. For each of the following sets of three points, determine whether the NPC is turning clockwise or anticlockwise when moving from $\boldsymbol{a}$ to $\boldsymbol{b}$ to $\boldsymbol{c}$:

      i. $\boldsymbol{a}$ = (2, 0, 3), $\boldsymbol{b}$ = (-1, 0, 5), $\boldsymbol{c}$ = (-4, 0, 1)

      ii. $\boldsymbol{a}$ = (-3, 0, -5), $\boldsymbol{b}$ = (4, 0, 0), $\boldsymbol{c}$ = (3, 0, 3)

      iii. $\boldsymbol{a}$ = (1, 0, 4), $\boldsymbol{b}$ = (7, 0, -1), $\boldsymbol{c}$ = (-5, 0, -6)

      iv. $\boldsymbol{a}$ = (-2, 0, 1), $\boldsymbol{b}$ = (1, 0, 2), $\boldsymbol{c}$ = (4, 0, 4)

3. Consider a triangle defined by the vertices (6, 10, -2), (3, -1, 17) and (-9, 8, 0).

   a. What is the equation of the plane containing this triangle?

   b. Is the point (3, 4, 5) on the front or back side of this plane?
   How far is this point from the plane?

4. Consider the set of five points (7, 11, -5), (2, 3, 8), (-3, 3, 1), (-5, -7, 0) and (6, 3, 4).
   An *axis aligned bounding box (AABB)* is the smallest box whose edges are aligned with the coordinate axes that contains all the points, defined by its minimum and maximum vertices $\boldsymbol{p}_{min}$ and $\boldsymbol{p}_{max}$.

   a. What are $\boldsymbol{p}_{min}$ and $\boldsymbol{p}_{max}$ for the above five points?

   b. List all eight vertices of the AABB.

   c. Determine the centre point $\boldsymbol{c}$ of the AABB.

   d. Multiply the five points by the following matrix (a 45° rotation about the z-axis):

$$\begin{pmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   e. What is the AABB of these transformed points?

f. What is the AABB we get by transforming the original AABB? (i.e. the bounding box of the transformed corner points).

5. A robot is at the position $(1, 10, 3)$ and her right, up and forward vectors (expressed in world space) are $\begin{pmatrix} 0.866 \\ 0 \\ -0.5 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0.5 \\ 0 \\ 0.866 \end{pmatrix}$ respectively (note that these vectors form an orthonormal basis).
The following points are expressed in object space; calculate their coordinates in world space:

   a. (-1, 2, 0)

   b. (1, 2, 0)

   c. (0, 0, 0)

   d. (1, 5, 0.5)

   e. (0, 5, 10)

   The coordinates below are in world space; find their positions relative to the robot:

   f. (1, 10, 3)

   g. (0, 0, 0)

   h. (2.732, 10, 2)

   i. (2, 11, 4)

   j. (1, 20, 3) [p758, 3.6.6]