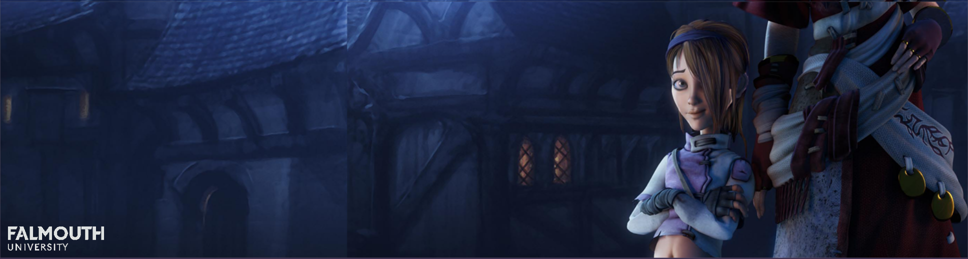

COMP220: Graphics & Simulation

11: Numerical accuracy

Next week

Catch-up tutorials

- Last chance for feedback on your CPD task

COMP210 and COMP220 vivas

- Timetable still being finalised
- Keep an eye on Slack / email

Deadlines?!?

- Check MyFalmouth

Representing numbers

Powers of 10

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$$10^6 = 1\underbrace{000000}_{\text{6 zeroes}}$$

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Multiplying by powers of 10 = shifting the decimal point left/right

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- Socrative `FALCOMPED`

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double lightYear = 9.461e15;
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double lightYear = 9.461e15;
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```

This also works in Python and many other programming languages

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- Exponent is a signed integer, stored with a **bias**

IEEE 754 floating point formats

Type	Sign	Exponent	Mantissa	Total
`float`	1 bit	8 bits	23 bits	32 bits
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Exponent is stored with a **bias**:

- Single precision: store exponent $+ 127$
- Double precision: store exponent $+ 1023$

Example

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```
0 10000001 10100000000000000000000
```

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- Exponent: $129 - 127 = 2$
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- $1 + \frac{1}{2} + \frac{1}{8} = 1.625$
- $1.625 \times 2^2 = 6.5$

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- Exponent: $129 - 127 = 2$
- Mantissa: binary 1.101
- $1 + \frac{1}{2} + \frac{1}{8} = 1.625$
- $1.625 \times 2^2 = 6.5$
- Alternatively: $1.101 \times 2^2 = 110.1$

Example

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- Exponent: $129 - 127 = 2$
- Mantissa: binary 1.101
- $1 + \frac{1}{2} + \frac{1}{8} = 1.625$
- $1.625 \times 2^2 = 6.5$
- Alternatively: $1.101 \times 2^2 = 110.1$
- $= 4 + 2 + \frac{1}{2} = 6.5$

Socrative `FALCOMPED`

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What is the value of this number expressed in IEEE 754 single precision format?

```
0 01111100 10011000000000000000000
```

You have **5 minutes**, and you **may** use a calculator!

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  - E.g. according to Python, `0.1 + 0.2 == 0.30000000000000004`

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- Due to rounding errors, using `==` or `!=` with floating point numbers is almost always a bad idea
- E.g. in Python, `0.1 + 0.2 == 0.3` evaluates to `False`
- Better to check for **approximate equality**: calculate the difference between the numbers, and check that it's smaller than some threshold

Numerical accuracy in simulations

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- Errors tend to **accumulate**
- Mixing **orders of magnitude** (i.e. mixing large and small numbers) is particularly bad

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- Euler integration: $x(t+h) \approx x(t) + h \times \frac{dx}{dt}(t)$
- If h varies, simulation becomes **non-deterministic** (or "random")
- This is bad!
- Better to use a **fixed time step** (we covered this in COMP150)

Fixed time step

```
bool running = true;
Uint32 lastUpdateTime = SDL_GetTicks();
const Uint32 timePerUpdate = 1000 / 60;

while (running)
{
    Uint32 currentTime = SDL_GetTicks();
    handleInput();

    while (currentTime - lastUpdateTime >=
        timePerUpdate)
    {
        update();
        lastUpdateTime += timePerUpdate;
    }

    render();
}
```

Further information on fixed time steps

- http://gafferongames.com/game-physics/fix-your-timestep/
- http://gameprogrammingpatterns.com/game-loop.html

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Advanced physics simulation

http://www.gdcvault.com.ezproxy.falmouth.ac.uk/play/1022143/Math-for-Game-Programmers-Game

http://www.gdcvault.com.ezproxy.falmouth.ac.uk/play/1017644/Physics-for-Game-Programmers-Continuous

http://www.gdcvault.com.ezproxy.falmouth.ac.uk/play/1020603/Physics-for-Game-Programmers-Understanding