COMP110: Principles of Computing

4: Logic and memory

Worksheet 4

Due **next Friday!**

Logic gates

Boolean logic

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- Foundation of the **digital computer**: represented in circuits as **on** and **off**
- Representing as 1 and 0 leads to **binary notation**
- One boolean value = one **bit** of information
- Programmers use boolean logic for conditions in `if` and `while` statements

Not

Not

NOT *A* is TRUE
if and only if
A is FALSE

Not

NOT A is TRUE
if and only if
A is FALSE

A	NOT A
FALSE	TRUE
TRUE	FALSE

Not

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if and only if
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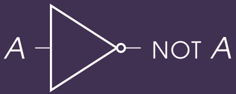

And

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A AND *B* is TRUE
if and only if
both *A* **and** *B* are TRUE

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A	*B*	*A* AND *B*
FALSE	FALSE	FALSE
FALSE	TRUE	FALSE
TRUE	FALSE	FALSE
TRUE	TRUE	TRUE

And

A AND B is TRUE
if and only if
both A **and** B are TRUE

A	B	A AND B
FALSE	FALSE	FALSE
FALSE	TRUE	FALSE
TRUE	FALSE	FALSE
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Or

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A OR *B* is TRUE
if and only if
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Socrative `FALCOMPED`

What is the value of

$$A \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative FALCOMPED

What is the value of

$$(\text{NOT } A) \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative `FALCOMPED`

For what values of A, B, C, D is

$$A \text{ AND NOT } B \text{ AND NOT } (C \text{ OR } D) = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ OR NOT } A$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ AND NOT } A$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ OR } A$$

?

Socrative `FALCOMPED`

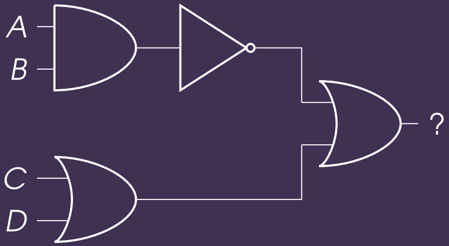
What is the value of

$$A \text{ AND } A$$

?

Socrative `FALCOMPED`

What expression is equivalent to this circuit?



Writing logical operations

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Operation	Python	C family	Mathematics
NOT A	`not a`	`!a`	$\neg A$ or $\overline{A}$

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Other operators can be expressed by combining these

De Morgan's Laws

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Proof: Worksheet 4, questions 3a and 3b

Truth tables

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- For n variables there are 2^n possible combinations
- Essentially, all the n-bit binary numbers
- A **truth table** enumerates all the possible values of a boolean expression
- Can be used to prove that two expressions are equivalent

Truth table example

(A OR NOT B) AND C

A	B	C	NOT B	A OR NOT B	(A OR NOT B) AND C

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FALSE	TRUE	TRUE	FALSE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE	FALSE
TRUE	TRUE	TRUE	FALSE	TRUE	TRUE

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Other logic gates

Exclusive Or

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Socrative `FALCOMPED`

How can A XOR B be written using the operations AND , OR , NOT ?


BOOLEAN HAIR LOGIC
A
B
AND
OR
XOR
Toothpaste For Dinner.com

Negative gates

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NAND , NOR , XNOR
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$$A \text{ NAND } B = \text{NOT } (A \text{ AND } B)$$
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$$A \text{ XNOR } B = \text{NOT } (A \text{ XOR } B)$$

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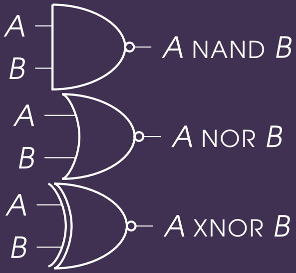
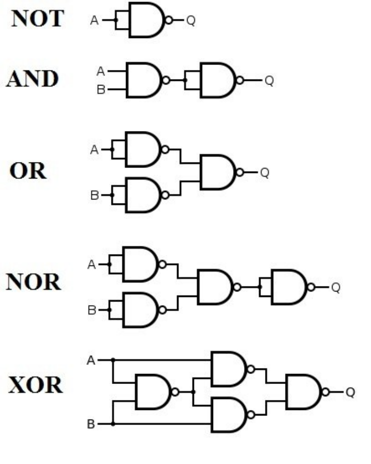
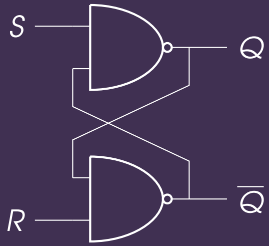


Any logic gate can be constructed from NAND gates



What does this circuit do?



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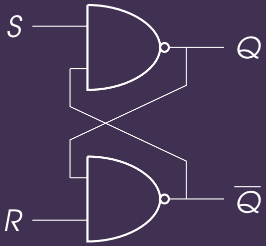

- This is called a **NAND latch**

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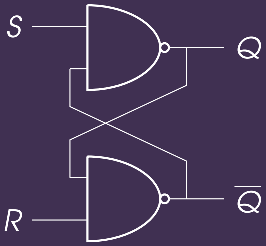

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- It "remembers" a single boolean value

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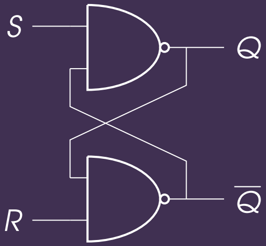

- This is called a **NAND latch**
- It "remembers" a single boolean value
- Put a few billion of these together (along with some control circuitry) and you've got **memory**!

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- The same is true of NOR gates

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Arithmetic Logic Unit

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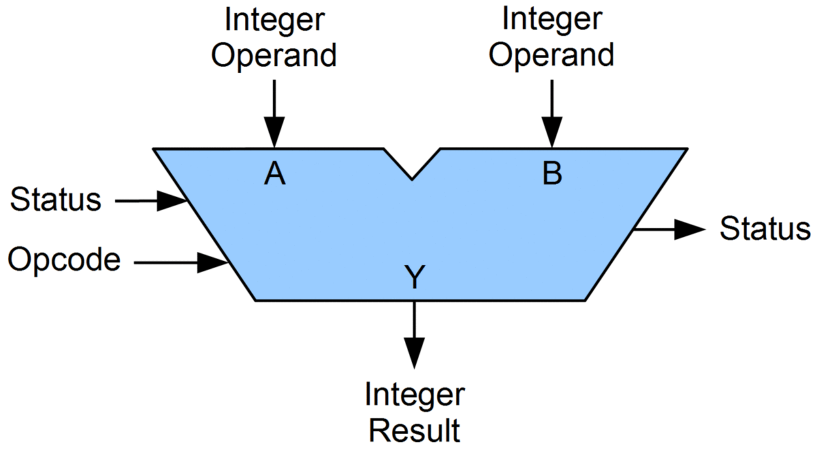

Arithmetic Logic Unit

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Arithmetic Logic Unit

- Important part of the CPU
- Inputs:
  - **Operand** words A, B
  - **Opcode**
  - **Status** bits
- Outputs:
  - **Result** word Y
  - **Status** bits
- Opcode specifies how Y is calculated based on A and B

ALU operations

Typically include:

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- Add with carry

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- Bitwise AND, OR, NOT, . . .

ALU operations

Typically include:

- Add with carry
- Subtract with borrow
- Negate (2's complement)
- Increment, decrement
- Bitwise AND, OR, NOT, . . .
- Bit shifts

Adding 3 bits

A	B	C	$A+B+C$
0	0	0	00
0	0	1	01
0	1	0	01
0	1	1	10
1	0	0	01
1	0	1	10
1	1	0	10
1	1	1	11

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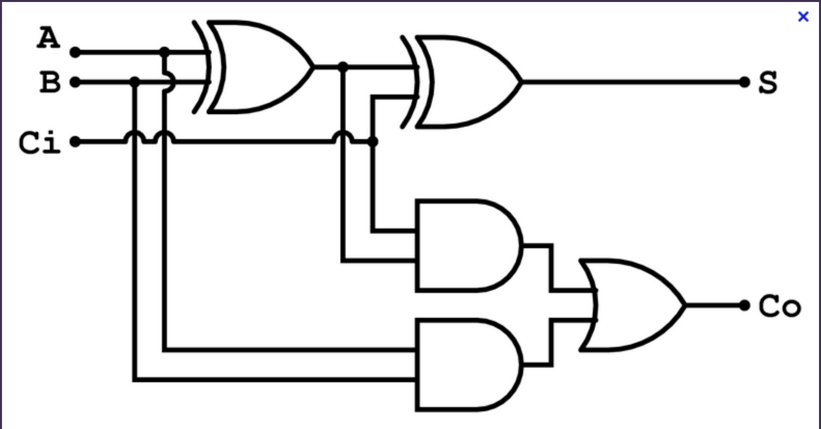
1-bit adder



How does the 1-bit adder work?

Exercise:

- Write down the boolean expressions for S and Co
- Draw a truth table for these
- Compare the truth table to the addition table on a previous slide

n-bit adder