Week 5: Mechanics II
Part 3: Collision Response

COMP270: Mathematics for 3D Worlds and Simulations

Objectives

- **Apply** the physical laws that predict the behavior of colliding objects
- **Outline** methods to compensate for the fact that virtual objects aren't solid

Momentum and energy

- A moving object has **momentum** proportional to its mass m and velocity $\mathbf{v}$

$$\mathbf{p} = m\mathbf{v}$$

Unit: kgm/s or $kgms^{-1}$

- A moving object also has **kinetic energy** proportional to its mass and the square of its speed

$$E = \frac{1}{2}m\|\mathbf{v}\|^2$$

Unit: Joule (J), representing $kg(m/s)^2$ or $kg(ms)^{-2}$

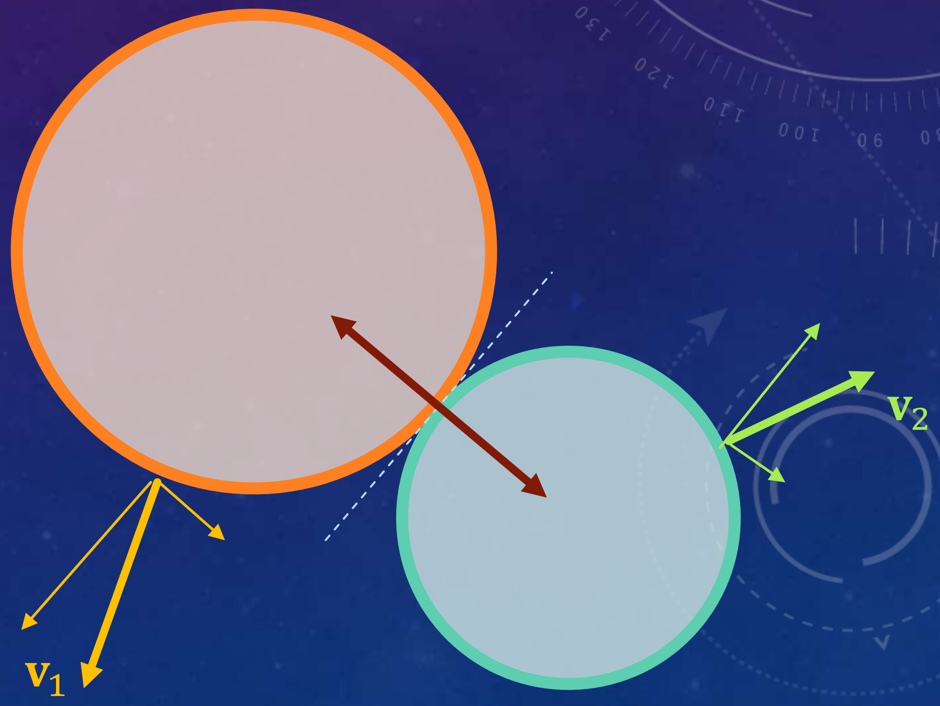
Conservation

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$$

- When two objects collide, the total momentum is conserved
- In an elastic collision, the total kinetic energy is also conserved
- In an inelastic collision, some kinetic energy is 'lost' (e.g. as sound, heat etc)
- These can be used to calculate the velocities of the objects after collision
  - Example here

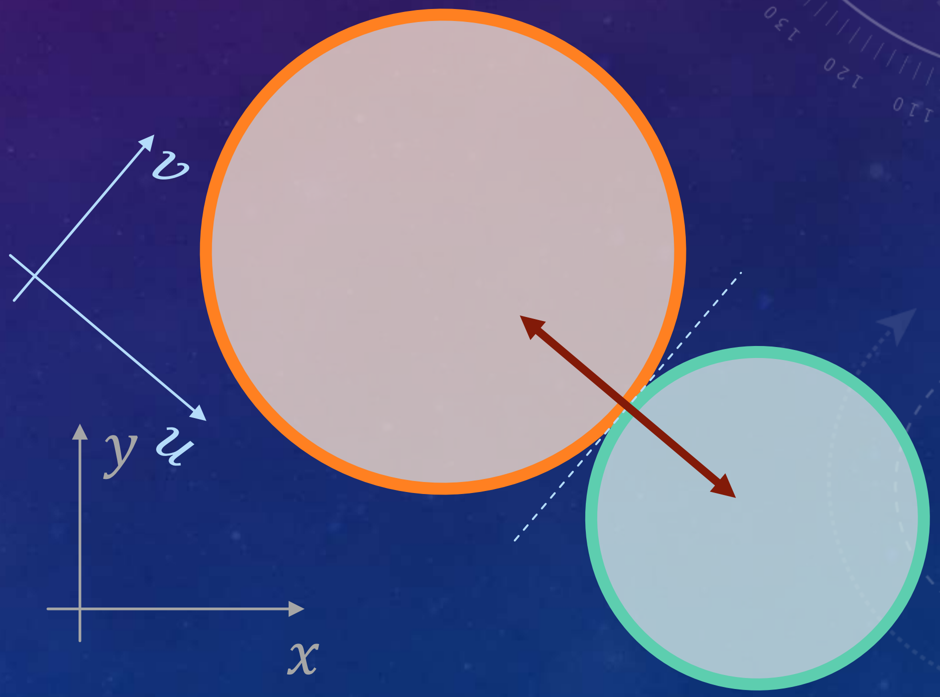
Contact normal

- Reaction force acts along the **contact normal** – perpendicular to both surfaces
- Component of velocity parallel to the normal changes; component perpendicular to the normal does not (disregarding friction)



Change in coordinate systems

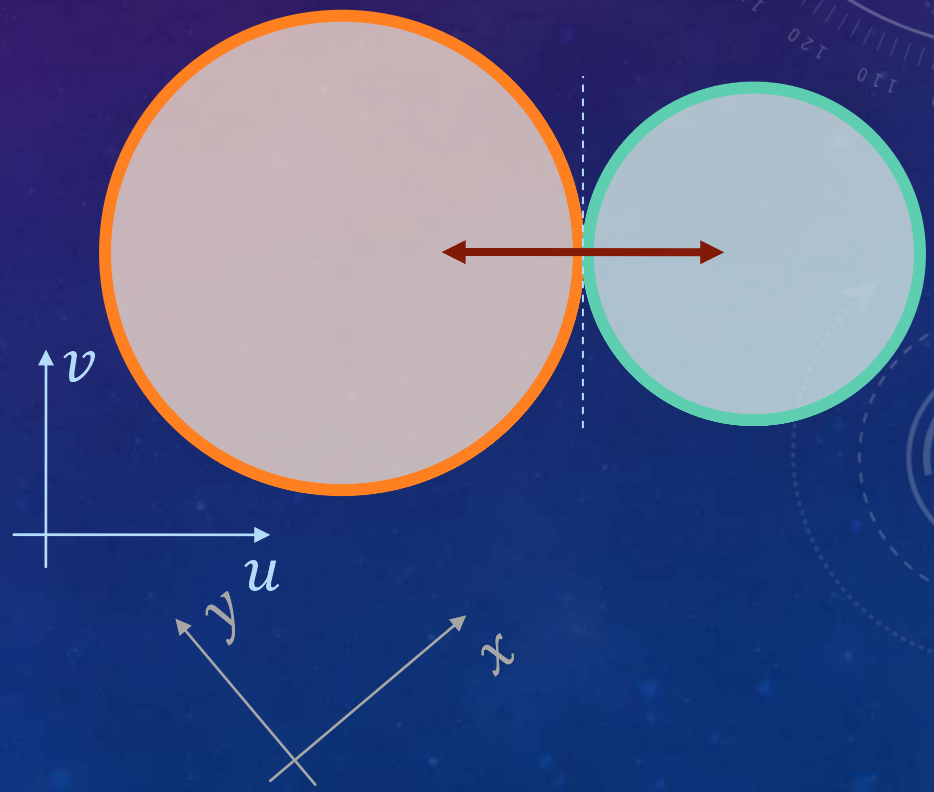
- Useful to consider the situation in a different coordinate system
- Instead of x- and y-axes, use u- and v-axes parallel and perpendicular to the normal
- Equivalent to rotation…



Change in coordinate systems

- Under the new coordinates, reaction force acts along the u axis
- So can calculate collision response just by using u components of vectors
- This is a common trick – solving problems by transforming to a more convenient coordinate system

Given by basis vector $\binom{1}{0}$



Simulating collisions

Position is computed before forces: could be moved to intersect another object

- For each object, store its position $\mathbf{x}$ and velocity $\mathbf{v}$
- On each time step:
  - Apply numerical integration to the velocity to determine the new position, $\mathbf{x}' = \mathbf{x} + \mathbf{v}\Delta t$
  - Calculate the forces acting upon the object, and thus the acceleration $\mathbf{a}$ from Newton's 2nd law
  - Apply numerical integration to the acceleration to determine the new velocity, $\mathbf{v}' = \mathbf{v} + \mathbf{a}\Delta t$

Correcting intersections

- Depth of penetration: measures how much of one object is inside another.
- Possible actions:
  - Adjust the positions
  - Adjust the velocities to achieve the required positions
  - Apply a penalty force to achieve the required velocities (to achieved the required positions)