COMP220: Graphics & Simulation

3: Mathematics for graphics

Learning outcomes

By the end of this session, you should be able to:

- **Explain** the role of vectors and matrices in computer graphics
- **Calculate** basic transformation matrices using the GLM library
- **Explain** the constituents of the model-view-projection matrix

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Transformations

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- Multiplying a vector by the matrix **applies** the transformation

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- GLM aims to mirror GLSL data types (`vec4`, `mat4` etc) in C++
- Lets us perform calculations with vectors and matrices in C++
- GLM types can be passed into shaders as uniforms, e.g.

```
// transformLocation points to a uniform of type mat4
glm::mat4 transform = ...;
glUniformMatrix4fv(transformLocation, 1, GL_FALSE, glm::value_ptr(transform));
```

Identity

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```
// Default constructor for glm::mat4 creates an identity matrix
glm::mat4 transform;
```

Translation

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```
transform = glm::translate(transform, glm::vec3(0.3f,
    0.5f, 0.0f));
```

Scaling

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```
transform = glm::scale(transform, glm::vec3(1.2f, 0.5f ↩
    , 1.0f));
```

Rotation

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```
float angle = glm::pi<float>() * 0.5f;
glm::vec3 axis(0, 0, 1);
transform = glm::rotate(transform, angle, axis);
```

Combining transformations

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- Transformations **do not commute** in general — changing the order will change the result
- The order they are applied is the **reverse** of what you might think — i.e. the above rotates **then** translates

Euler angles

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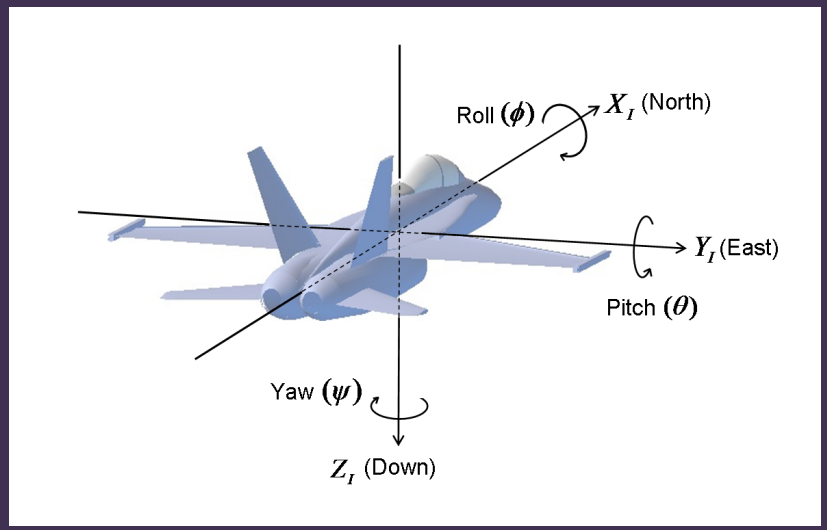
  - The y-axis $(0, 1, 0)$
  - The z-axis $(0, 0, 1)$
- These angles are sometimes called **roll**, **pitch** and **yaw**



Gimbal lock

https://youtu.be/rrUCBOlJdt4?t=1m55s

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The **model-view-projection (MVP) matrix**:

$$M_{MVP} = M_{\text{projection}} \times M_{\text{view}} \times M_{\text{model}}$$

(remember, multiplication goes in reverse order)

The model matrix

The model matrix

Exactly what we've been doing so far today...

The view matrix

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Need to translate and rotate the scene so that the "camera" is at $(0, 0, 0)$ and looking in the negative z direction

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Need to translate and rotate the scene so that the "camera" is at $(0, 0, 0)$ and looking in the negative z direction

```
glm::mat4 view = glm::lookAt(
  glm::vec3(2, 0, 2),     // eye
  glm::vec3(0, 0, 0),     // centre
  glm::vec3 up(0, 1, 0)   // up
);
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- `eye` is the position of the camera
- `centre` is a point for the camera to look at
- `up` is which direction is "up" for the camera (usually the positive y-axis)

Types of projection

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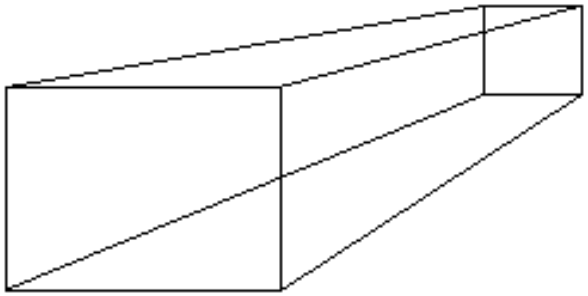
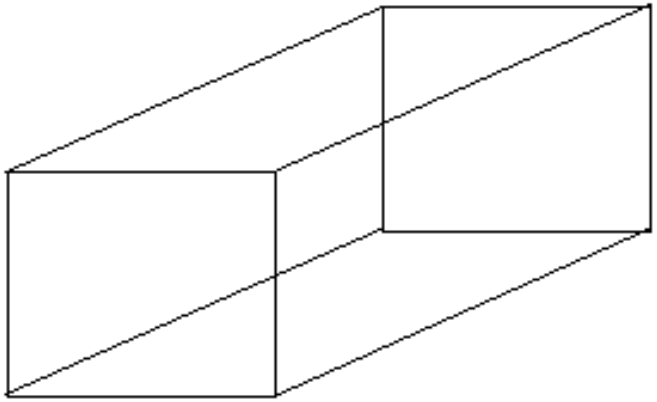


Perspective projection



Orthographic projection

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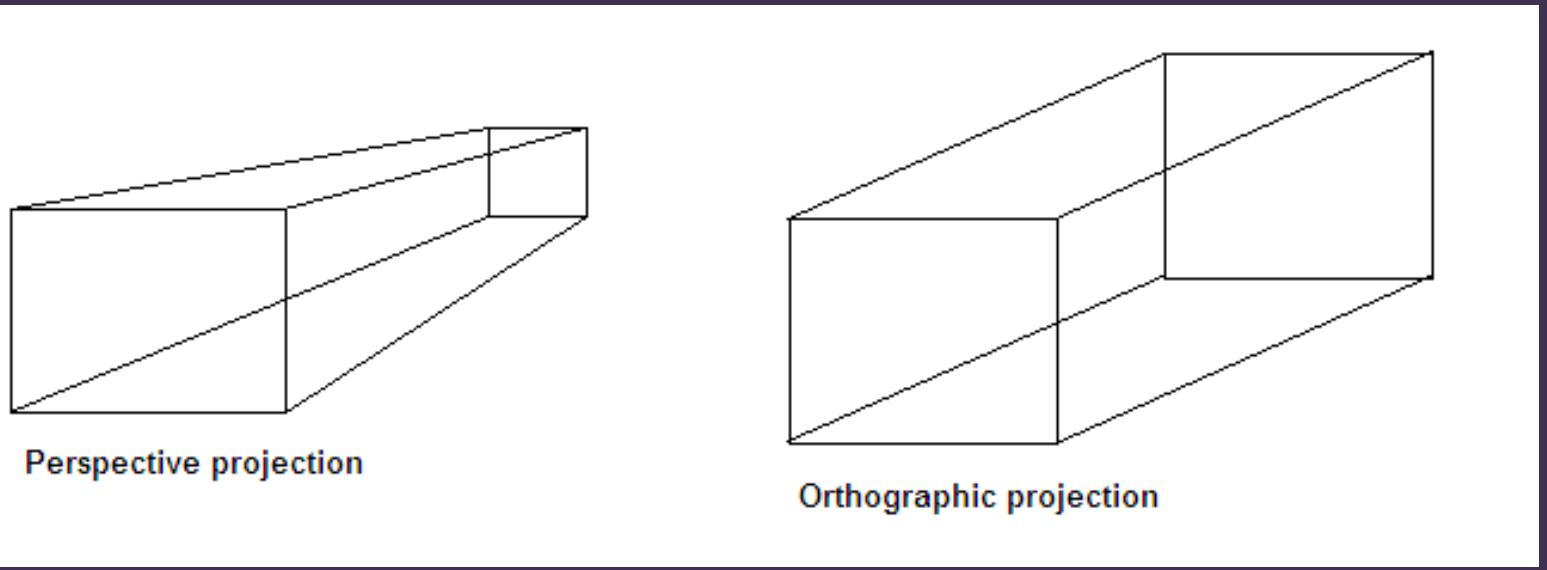

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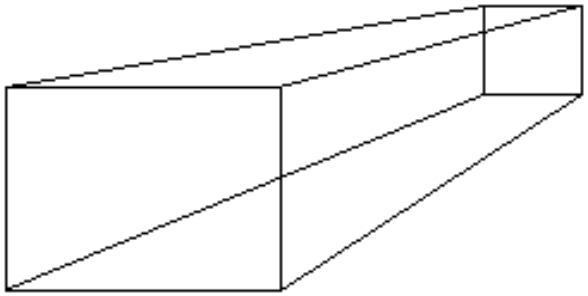

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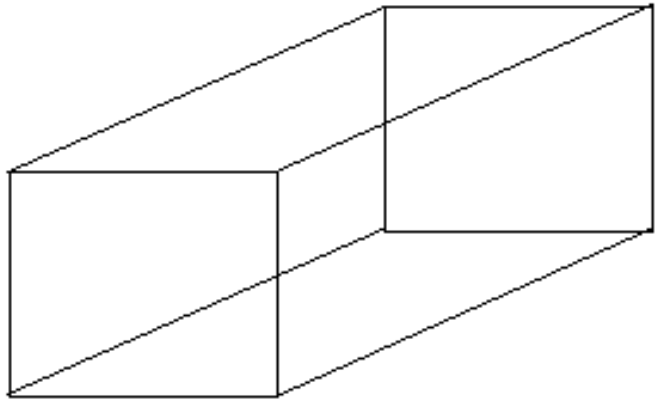

Orthographic projection

- Generally use **perspective** for 3D graphics
- **Orthographic** is useful for 2D or pseudo-2D graphics (e.g. isometric perspective)

The projection matrix

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glm::mat4 projection = glm::perspective(
    glm::radians(45.0f), // field of view
    4.0f / 3.0f,         // aspect ratio
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Also available: `glm::ortho` for orthographic projection

The view frustum

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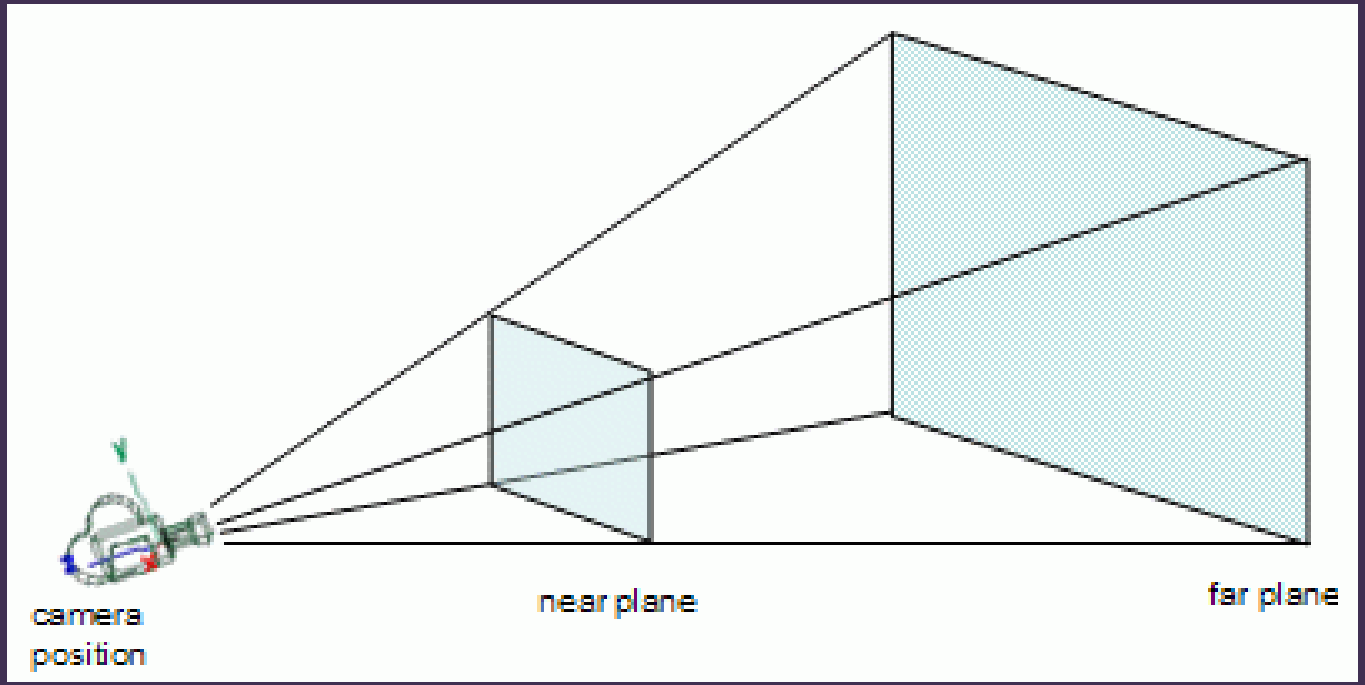

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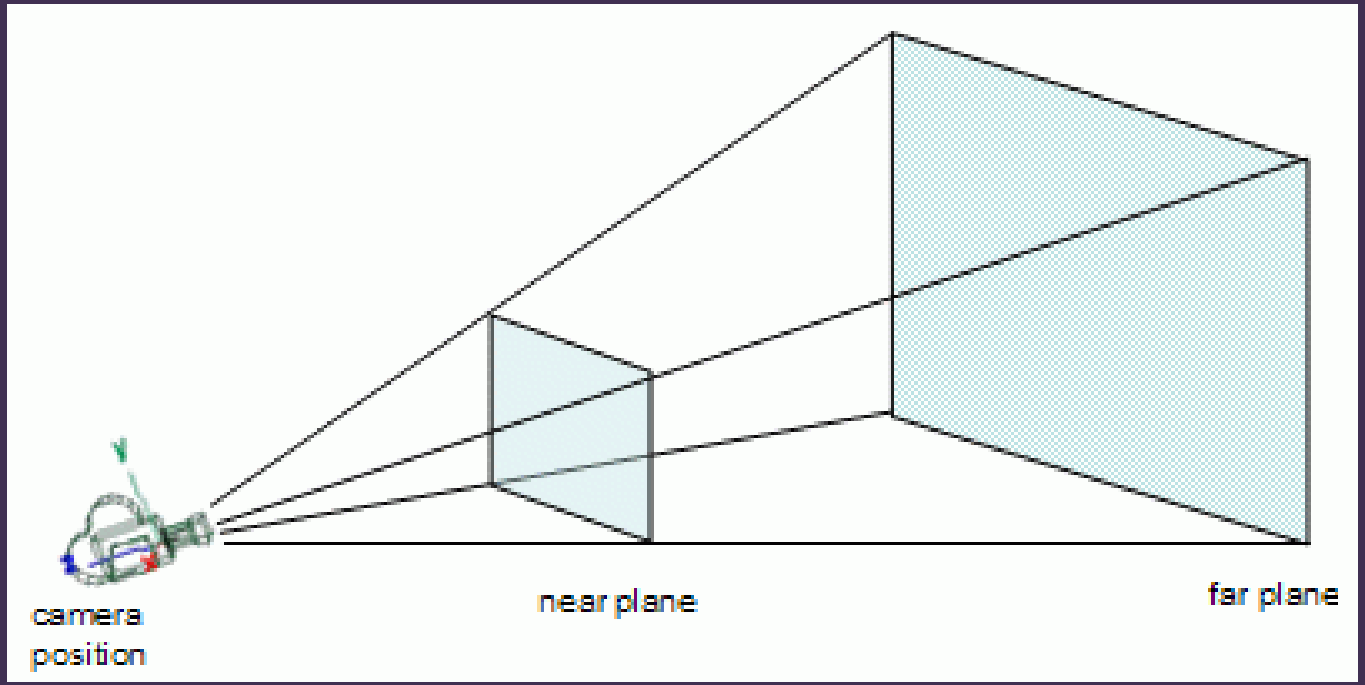

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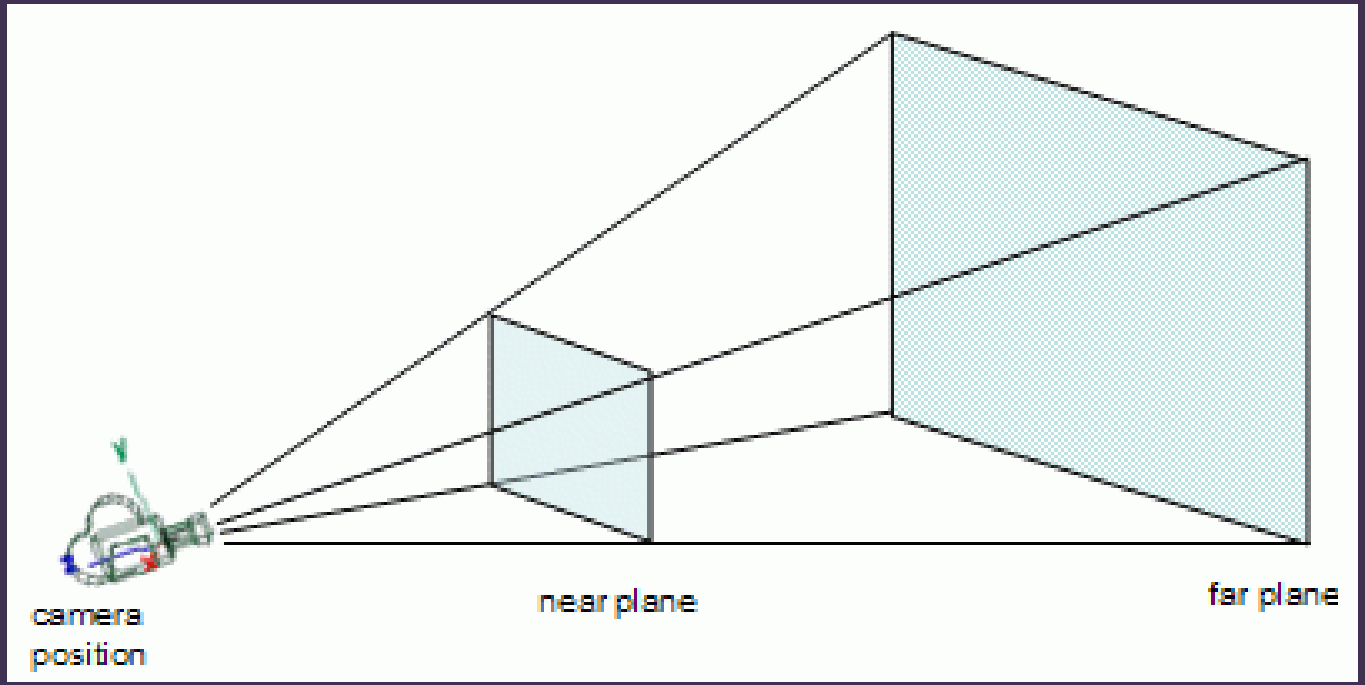

- Defined by the **near and far clipping planes** and the **edges of the screen**
- **Nothing outside** the view frustum is visible

Live Coding - Transformations

Exercise

- Using the following link, first implement model transformations and then view and projection
  - `http://www.opengl-tutorial.org/beginners-tutorials/tutorial-3-matrices/`
- Play around with the order of the Model Transformation, what happens when you do this?
- Use the keyboard to move, rotate and scale the triangle
- Use the Keyboard to move the camera
- **Stretch Goal:** Create an FPS camera using the mouse and keyboard