COMP110: Principles of Computing

5: Complexity

FALMOUTH UNIVERSITY

Research Journal

Research journal presentations

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- Presentation sessions in **week 7** (week after next)

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- Prepare a **10 minute** presentation

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- Discussing your chosen paper, its context and influence
- Use any presentation software to provide slides (e.g. Beamer, PowerPoint, Google Slides)

Research journal presentations — Why?

Research journal presentations — Why?

- Assessed on a **threshold** basis

Research journal presentations — Why?

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- A chance to share insights on what you've been reading

Research journal presentations — Why?

- Assessed on a **threshold** basis
- A chance to share insights on what you've been reading
- A chance for informal feedback

Research journal

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- You should also be working on the journal itself!

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- Peer review (draft needed) in **week 8**

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- Peer review (draft needed) in **week 8**
- This week's worksheet is the last one for a while, to give you time to work on your research journal

Turing machines

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- Theoretical model of a "computer"
  - I.e. a machine that carries out computations (calculations)

Turing machine

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- Has a finite number of **states**

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  - Where to move the tape head: one space to the left, or one space to the right

The Church-Turing Thesis

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- A machine, language or system is **Turing complete** if it can simulate a Turing machine
- (In practice, nothing can simulate an infinite tape, so we just assume a large enough tape)

Examples of Turing complete systems

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- All general-purpose CPUs and programming languages

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Computability

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- Let A and B be **sets** of elements
  - NB: A may be **infinite**
- A function $f : A \to B$ is **computable** if there exists a Turing machine which computes f
  - I.e. given an encoding of $a \in A$ as input, the Turing machine outputs an encoding of $f(a)$

An uncomputable function

The **halting problem**

An uncomputable function

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- There is **no** Turing machine that computes f
- f is **uncomputable**

Computability and the Church-Turing Thesis

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- Church-Turing tells us that Turing machines are as powerful as any other computer

Computability and the Church-Turing Thesis

- Church-Turing tells us that Turing machines are as powerful as any other computer
- Therefore if a function is uncomputable, there is **no conceivable machine** that can compute it

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- Write a software tool that, given a C# program, predicts whether that program can go into an infinite loop
- Your tool must work for **all** C# programs, considering **all** possible inputs to the program
- This task is impossible!

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Search

Search

- We have a list of names, each with some data associated

Search

- We have a list of names, each with some data associated
- We want to find one of them

Linear search

procedure FIND(name, list)

Linear search

procedure FIND(name, list)
    for each item in list **do**

Linear search

```
procedure FIND(name, list)
    for each item in list do
        if item.name = name then
```

Linear search

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procedure FIND(name, list)
    for each item in list do
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    throw "Not found"
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How long does it take?

Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list

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- In the **best case**, how many items do we need to visit before finding the one we want?

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Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?
- How about in the **worst case**?

How long does it take?

Socrative room code: `FALCOMPED`

- If there are 25 items in the list, the **worst case** number of items visited is 25

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- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?

How long does it take?

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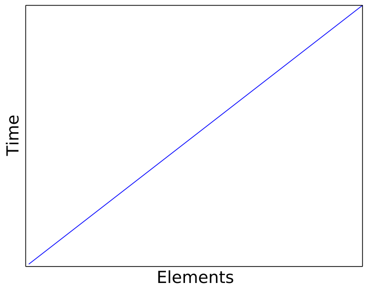
- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?

How long does it take?

Socrative room code: `FALCOMPED`

- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?
- If the number of items **doubles**, what happens to the amount of time the search takes?

Linear time



- The running time of linear search is **proportional** to the size *n* of the list

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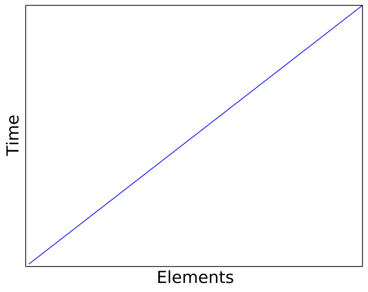

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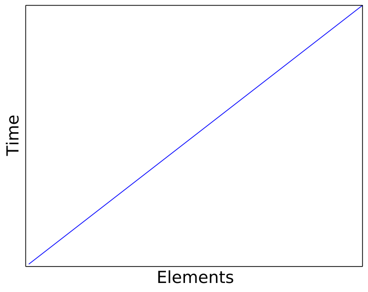

- The running time of linear search is **proportional** to the size *n* of the list
- Linear search is said to have **linear time complexity**
- Also written as $O(n)$ **time complexity**

Searching a sorted list

- If the list is **sorted** in alphabetical order, we can do better than linear...

Binary search

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    mid ← the "middle" item of the list
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    if list is empty **then**
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    mid $\leftarrow$ the "middle" item of the list
    if name $=$ mid.name **then**
        return mid

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Find “Lopez, Jeffrey”

Anderson, Martha
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Brown, Janet
Clark, Stephanie
Collins, Jane
Cox, Shirley
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Diaz, Harold
Gonzalez, Adam
Henderson, Lawrence
Hughes, Aaron
Kelly, Philip
⟶ Lewis, Rose
Lopez, Jeffrey
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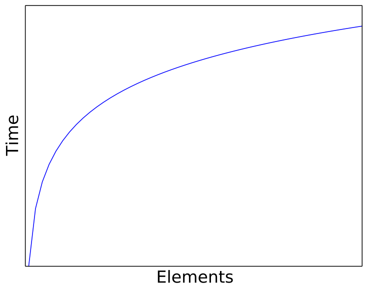

Hidden complexity

if name < mid.name **then**
    return FIND(name, first half of list)
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- Careful how you implement this!

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- Careful how you implement this!
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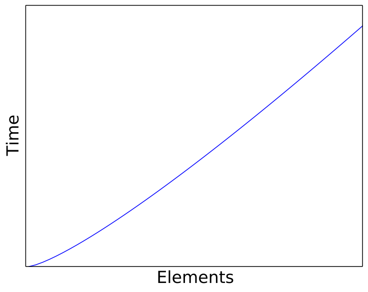

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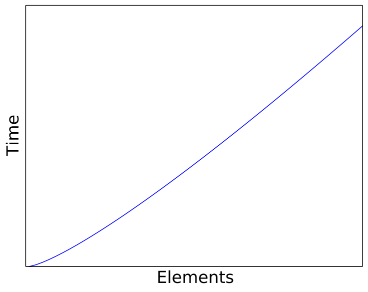


- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$
- The actual running time would be $O(n \log n)$
- Use **pointers** into the list instead of copying

Binary search done wrong

```
def binary_search(name, mylist):
    if mylist == []:
        raise ValueError("Not found")

    mid = len(mylist) / 2
    mid_name = mylist[mid_index].name

    if name == mid_name:
        return mid
    elif name < mid_name:
        return binary_search(name, mylist[:mid])
    else:
        return binary_search(name, mylist[mid+1:])
```

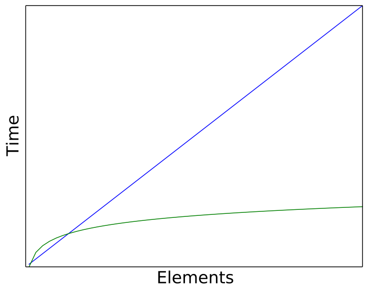
Binary search done right

```
def binary_search(name, mylist, start, end):
    if end <= start:
        raise ValueError("Not found")

    mid = (start + end) / 2
    mid_name = mylist[mid].name

    if name == mid_name:
        return mylist[mid]
    elif name < mid_name:
        return binary_search(name, mylist, start, mid)
    else:
        return binary_search(name, mylist, mid+1, end)
```

Binary search vs linear search



- So binary search is better than linear search... right?

Hashing

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⋮	⋮
112	Ward, Jessica
113	Baker, Theresa
114	Collins, Jane
115	—
116	—
117	Hughes, Aaron
118	—
119	—
120	—
121	—
122	Brown, Janet
123	—
124	—
125	Gonzalez, Adam Lewis, Rose
126	—
127	—
128	—
129	—
130	—
131	—
132	Young, Frank
⋮	⋮

Hash look-up

98	Diaz, Harold
99	Parker, Debra Perez, Diana White, Amanda
112	Ward, Jessica
113	Baker, Theresa
114	Collins, Jane
117	Hughes, Aaron
122	Brown, Janet
125	Gonzalez, Adam Lewis, Rose
132	Young, Frank
135	Kelly, Philip
138	Cox, Shirley
142	Clark, Stephanie
144	Scott, Michelle
145	Miller, Jeremy
147	Davis, Marilyn
149	Lopez, Jeffrey
151	Anderson, Martha
158	Williams, Billy
162	Sanders, Phillip
171	Russell, Mildred
175	Stewart, Howard
183	Henderson, Lawrence

"Lopez, Jeffrey"

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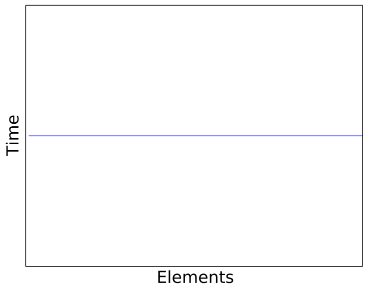
$$12 + 15 + 16 + 5 + 26 + 10 + 5 + 6 + 6 + 18 + 5 + 25 = 149$$

How long does it take?



- If there are no "collisions", look-up time is **constant** or $O(1)$

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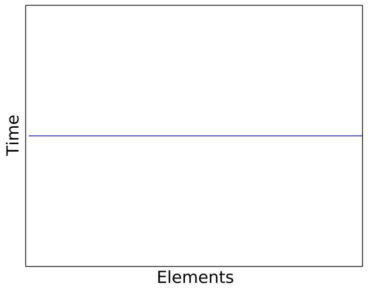

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  - (NB: constant **with respect to** n)

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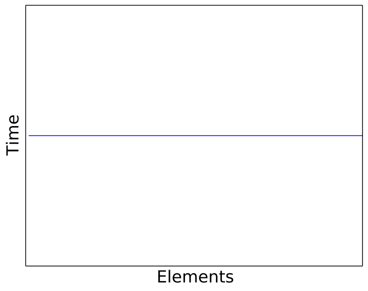

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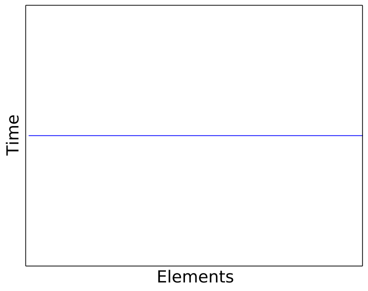

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More on complexity

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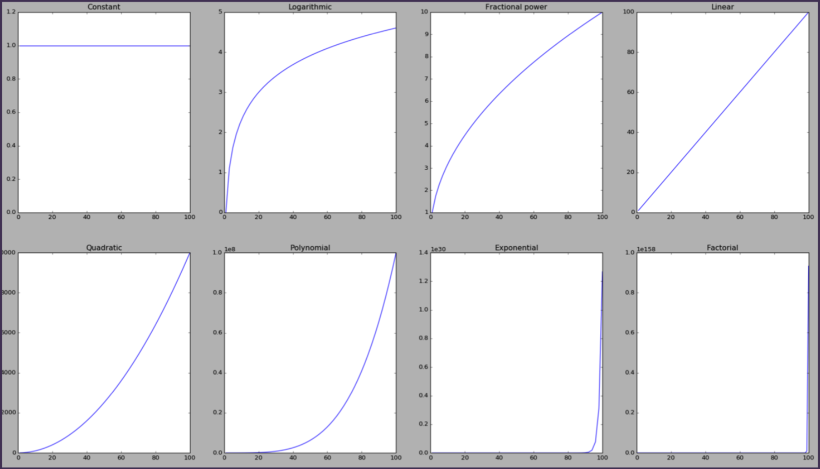

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- Multiply **compound** algorithms
  - If an algorithm does n "things" and each "thing" is $O(n)$, then the overall algorithm is $O(n^2)$

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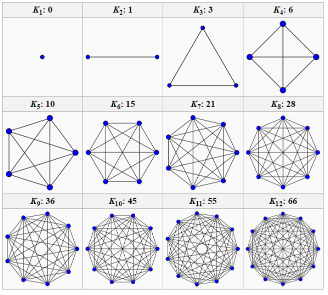

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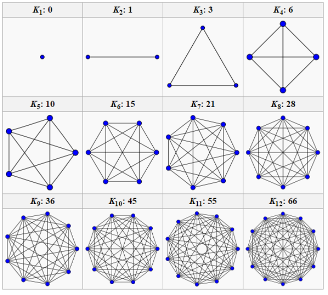

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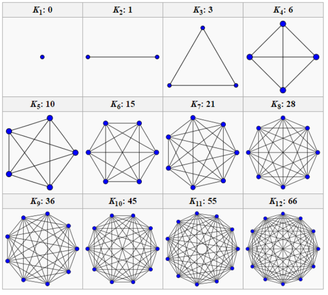

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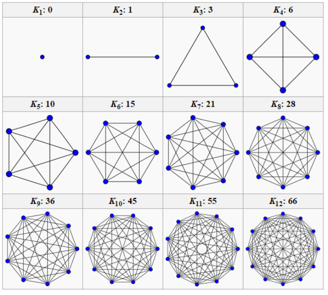

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  - Further reading: spatial hashing, quadtrees, octrees, Verlet lists

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- Adding 1 to n potentially **doubles** the running time!

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- Are there any problems in NP but not in P?

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  - Quantum computers are "infinitely parallel" in a sense so *can* solve some large NP-hard problems

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  - Otherwise, choose simplicity

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- Choice of data structures and algorithms can have a large impact on the efficiency of your software
- ... but only if scalability is actually a factor

Open workshop

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- Use the rest of this session to get on with your **research journal**
- Post in the chat if you have questions or need help!