COMP250: Artificial Intelligence

4: Utility-Based AI

Utility

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- Multiply by -1 and we have **cost**

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- Utility is a **single number** — we essentially have to put a monetary value on the longer wait time

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- The values used will influence the agent's behaviour and so must be carefully tuned by the designer

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- You have a moral objection to supporting large corporations, and would rather support local businesses
- Assuming everything else equal, where do you buy it from?
- To apply utility theory, we need to **quantify** everything — which may mean putting a numerical value on **intangible** things

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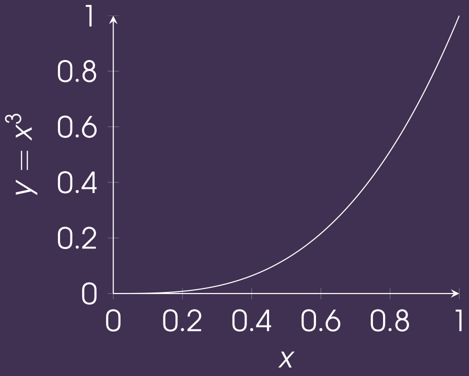
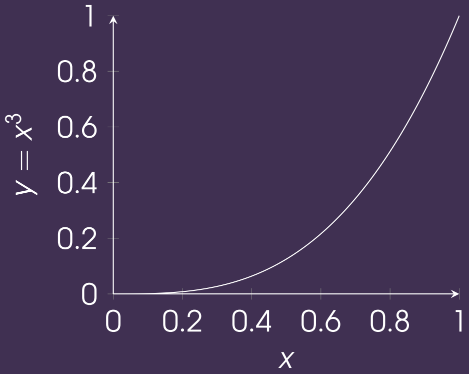
- Therefore we may want to apply a **curve** mapping to decision factors

Polynomial curve



Polynomial curve



- Formula: $y = x^k$

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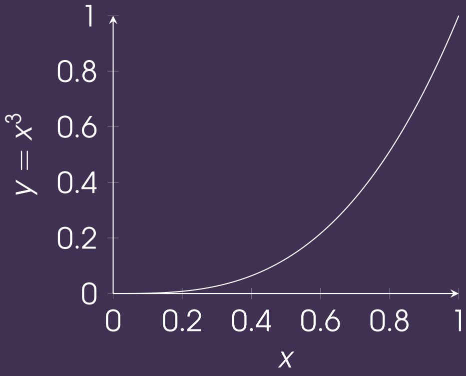

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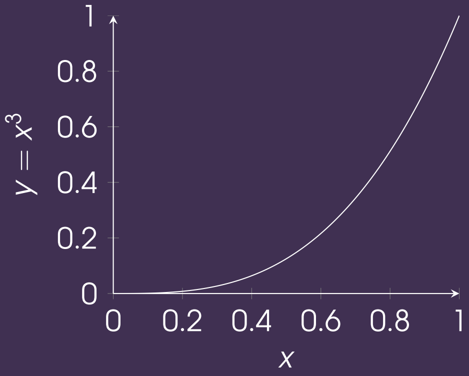

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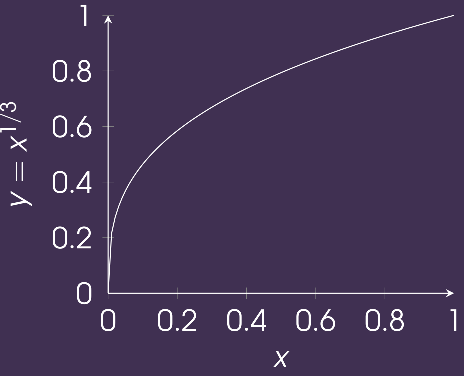

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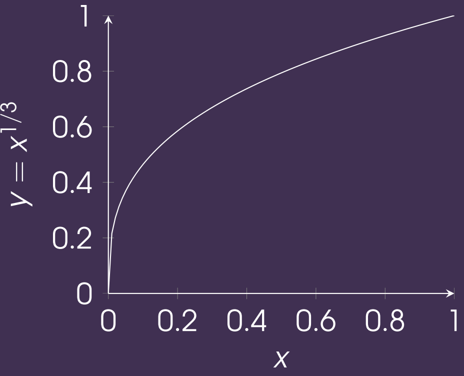

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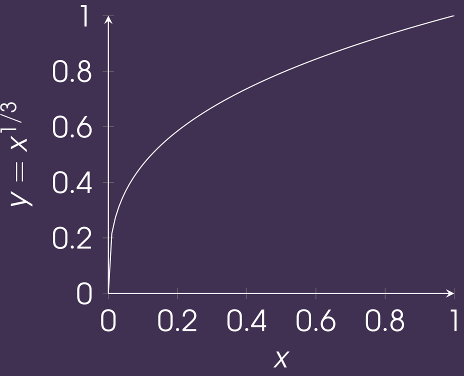
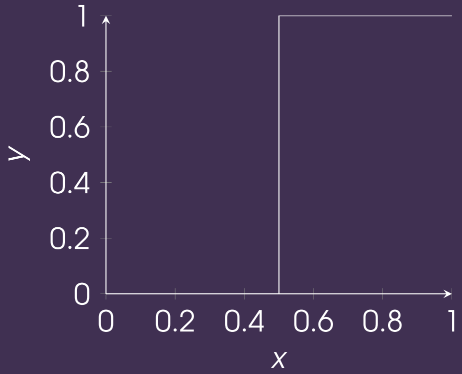


- Same formula as polynomial curve, but *k* is between 0 and 1
- *k* closer to 0 gives a steeper curve

Step function



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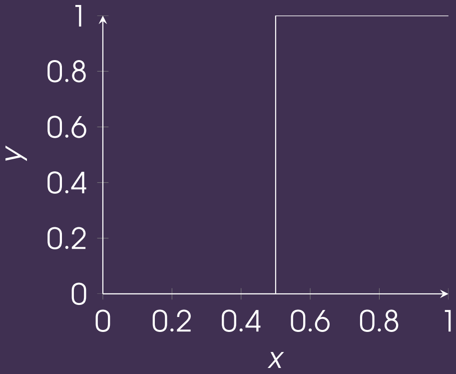
Step function



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$$y = \begin{cases} 0 & \text{if } x < 0.5 \\ 1 & \text{if } x \geq 0.5 \end{cases}$$

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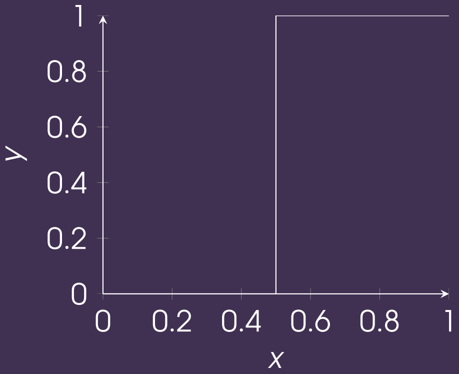

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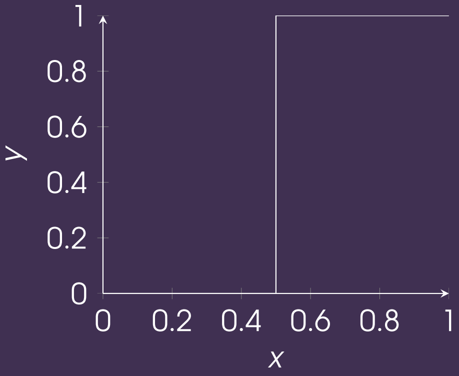

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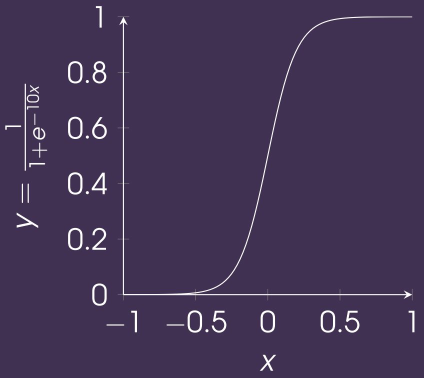
- Models a **threshold** or **if-then** rule

Logistic function



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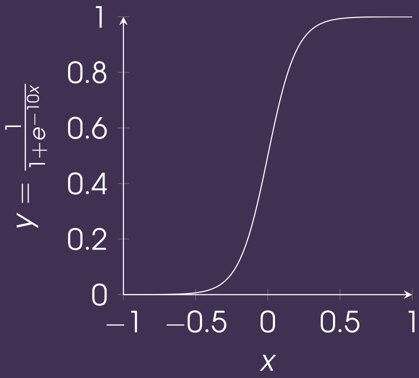

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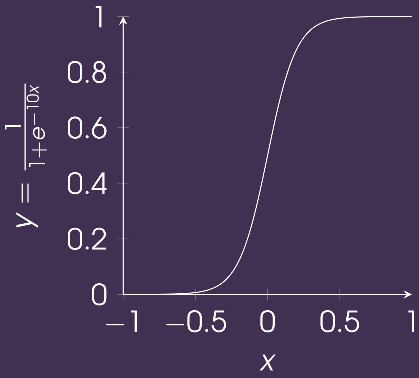

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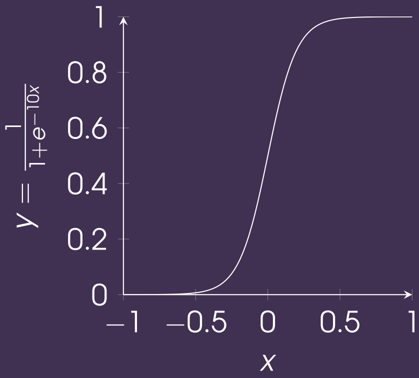

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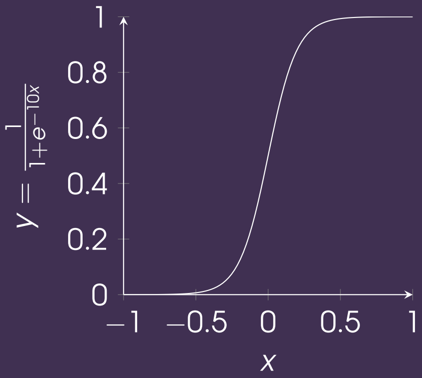

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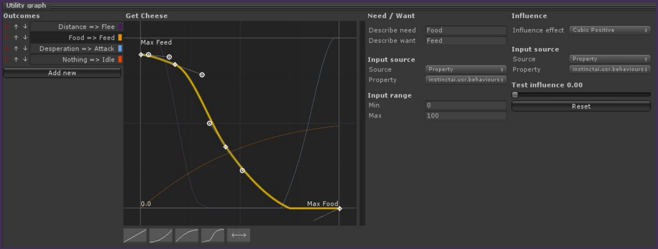
- **Graphical curve editors** are worth investigating
- E.g. `AnimationCurve` in Unity, "Curve Float" in UE4
- E.g. InstinctAI asset for Unity:



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- Can force the agent to finish its current action before evaluating and choosing another
- Can give a utility bonus to sticking to the current action — this still allows the agent to change its mind if the current action's utility becomes very bad

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1-ply Search

1-ply tree

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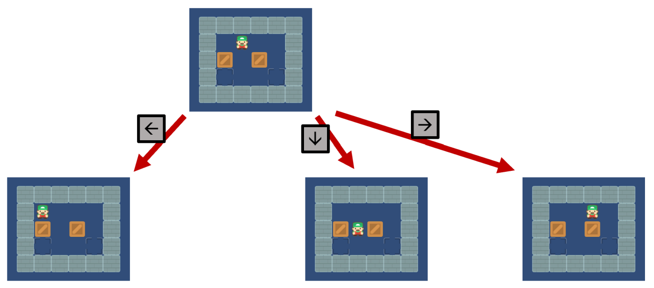
- Consider a tree with only one level or **ply**



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- Then we can "search" for the action leading to the best utility score

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end procedure

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    bestAction $\leftarrow$ null
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  - E.g. distance to goal — greedy search
  - E.g. material evaluation in chess
- Much faster than a full-blown BFS or game tree search, if depth of forward planning is not required

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- That is, the sum of utility values weighted by their probabilities

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- If it costs £1 to play the slot machine, the expected utility overall is $-$£0.60
- (Although the actual utility can range from $-$£1 to $+$£9)

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Multi-Objective Optimisation

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- An alternative is **multi-objective**: treat all decision factors as separate, and find an action that optimises all of them at once

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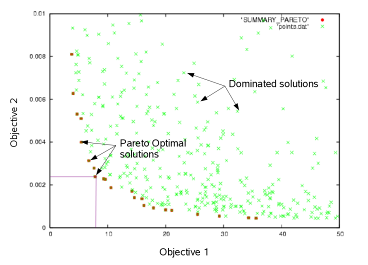
- A solution is **Pareto optimal** if it is not dominated by any other solution



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- ... which may boil down to weights (or at least priorities) anyway