COMP110: Principles of Computing

asic Principles for Computation

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Research journal

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  - Maximum 1500 words
  - With reference to appropriate academic sources

Marking rubric

See assignment brief on LearningSpace/GitHub

Timeline

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- Finding and reading academic papers takes time and effort — don't leave it until the last minute!

Binary notation

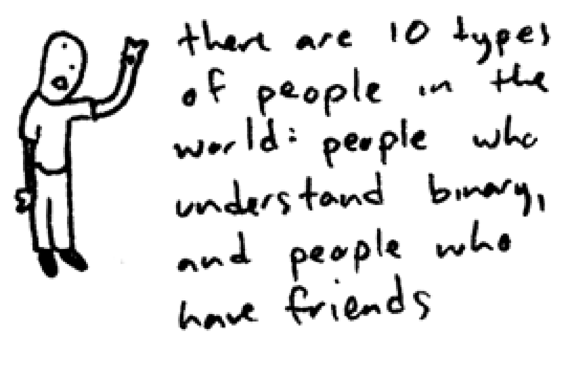

Image credit: `http://www.toothpastefordinner.com`

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Thousands	Hundreds	Tens	Units
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$$= 128 + 8 + 2 + 1 \text{ (base 10)}$$

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Converting to binary

https://www.youtube.com/watch?v=OezK_zTyvAQ

Bits, bytes and words

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  - $2^{64} - 1 = 18,446,744,073,709,551,615$

Addition with carry

In base 10:

$$\begin{array}{rrrrr} & 1 & 2 & 3 & 4 \\ + & 5 & 6 & 7 & 8 \\ \hline \end{array}$$

Addition with carry

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$$
\begin{array}{ccccc}
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+ & 5 & 6 & 7_1 & 8 \\
\hline
 & & & & 2
\end{array}
$$

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In base 10:

$$\begin{array}{ccccc} & 1 & 2 & 3 & 4 \\ + & 5 & 6_1 & 7_1 & 8 \\ \hline & & 9 & 1 & 2 \end{array}$$

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\begin{array}{ccccc}
 & 1 & 2 & 3 & 4 \\
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\hline
 & 6 & 9 & 1 & 2
\end{array}
$$

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$\begin{array}{cccccccccc} & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ + & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ \hline \end{array}$$

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  - $4-7=13$

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- E.g. 16-bit number ranges from -32768 to $+32767$

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- How can we represent negative numbers in binary?
- Represent them modulo 2^n (for n bits)
- I.e. represent $-a$ as $2^n - a$
- Instead of an n-bit number ranging from 0 to $2^n - 1$, it ranges from -2^{n-1} to $+2^{n-1} - 1$
- E.g. 16-bit number ranges from -32768 to $+32767$
- Note that the left-most bit can be interpreted as a **sign** bit: 1 if negative, 0 if positive or zero

Converting to 2's complement

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- Invert all the bits (i.e. change $0 \leftrightarrow 1$)
- Add 1
- (This is equivalent to subtracting the number from 2^n... why?)
- This is also the process for converting back from 2's complement, i.e. doing it twice should give the original number

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- Allows all addition and subtraction to be carried out modulo 2^n without caring whether numbers are positive or negative

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- Allows all addition and subtraction to be carried out modulo 2^n without caring whether numbers are positive or negative
- In fact, subtraction can just be done as addition
- I.e. $a - b$ is the same as $a + (-b)$, where a and $-b$ are just n-bit numbers

Exercise Sheet i

Due next Tuesday!

Turing machines

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- Introduced in 1936 by Alan Turing
- Theoretical model of a "computer"
  - I.e. a machine that carries out computations (calculations)

Turing machine

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- Has a finite number of **states**

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- Has an infinite **tape**
- Each space on the tape holds a **symbol** from a finite **alphabet**
- Has a **tape head** pointing at one space on the tape
- Has a transition table which, given:
  - The current state
  - The symbol under the tape head

  specifies:
  - A new state
  - A new symbol to write to the tape, overwriting the current symbol
  - Where to move the tape head: one space to the left, or one space to the right

Activity

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- In groups of 3-4

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- Line up 5-10 chocolates of different colours — this is your **tape**

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- Repeatedly apply the rules on the next slide

Activity

- In groups of 3-4
- Line up 5-10 chocolates of different colours — this is your **tape**
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- Repeatedly apply the rules on the next slide
- What computation does this machine perform?

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- Line up 5-10 chocolates of different colours — this is your **tape**
- Point your **Drumstick** lolly at the **leftmost** chocolate
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- Repeatedly apply the rules on the next slide
- What computation does this machine perform?
  - Hint: Milk = 0, White = 1...

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Current lolly	Current chocolate	New lolly	New chocolate	Move direction
Drumstick	Blank	Fruit	Blank	$\leftarrow$
Drumstick	Milk	Drumstick	White	$\rightarrow$
Drumstick	White	Drumstick	Milk	$\rightarrow$
Fruit	Blank	Swizzels	White	$\rightarrow$
Fruit	Milk	Swizzels	White	$\leftarrow$
Fruit	White	Fruit	Milk	$\leftarrow$
Swizzels	Blank	Stop	Blank	$\rightarrow$
Swizzels	Milk	Swizzels	Milk	$\leftarrow$
Swizzels	White	Swizzels	White	$\leftarrow$

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- If a calculation can be carried out by a mechanical process at all, then it can be carried out by a Turing machine
- I.e. a Turing machine is the most "powerful" computer possible, in terms of what is possible or impossible to compute
- A machine, language or system is **Turing complete** if it can simulate a Turing machine

Worksheet A review