COMP110: Principles of Computing

4: Logic and memory

Learning outcomes

- **Distinguish** the basic types of logic gate
- **Use** logic gates to build simple circuits
- **Explain** how computer memory works

Logic gates

Boolean logic

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- Foundation of the **digital computer**: represented in circuits as **on** and **off**
- Representing as 1 and 0 leads to **binary notation**
- One boolean value = one **bit** of information
- Programmers use boolean logic for conditions in `if` and `while` statements

Simulating logic circuits

http://logic.ly/demo/

Not

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NOT *A* is TRUE
if and only if
A is FALSE

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A	NOT *A*
FALSE	TRUE
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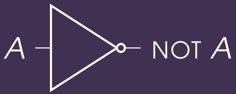

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Socrative `FALCOMPED`

What is the value of

$$A \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$(\textsc{not}\ A)\ \textsc{and}\ (B\ \textsc{or}\ C)$$

when

$$A = \textsc{True}$$
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$$C = \textsc{True}$$

?

Socrative `FALCOMPED`

For what values of A, B, C, D is

$$A \text{ AND NOT } B \text{ AND NOT } (C \text{ OR } D) = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ OR NOT } A$$

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What is the value of

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Socrative `FALCOMPED`

What is the value of

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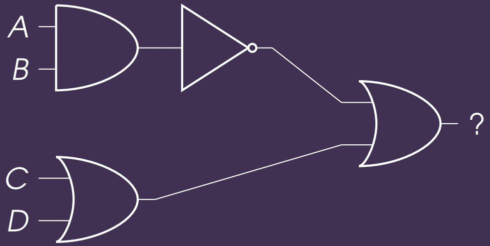
Socrative `FALCOMPED`

What expression is equivalent to this circuit?



Writing logical operations

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Other operators can be expressed by combining these

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How can A XOR B be written using the operations AND , OR , NOT ?

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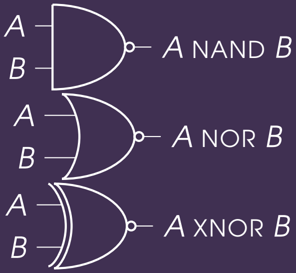

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Binary notation

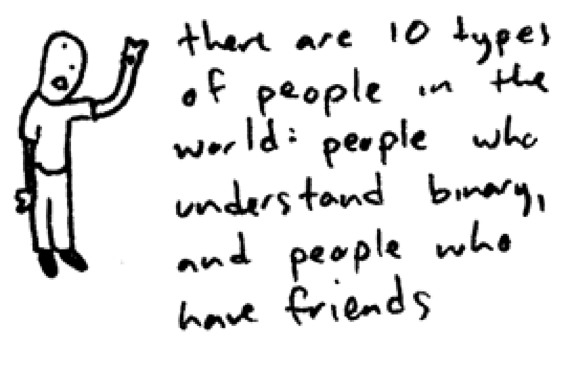

Image credit: http://www.toothpastefordinner.com

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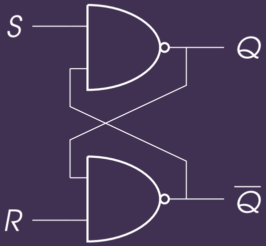
Computer memory

What does this circuit do?



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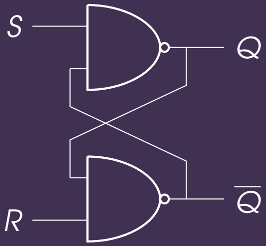

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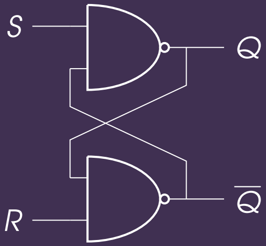

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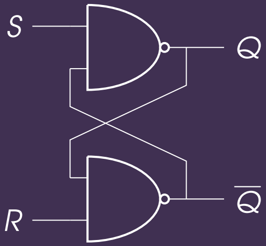

- This is called a **NAND latch**
- It "remembers" a single boolean value
- Put a few billion of these together (along with some control circuitry) and you've got **memory**!

Memory

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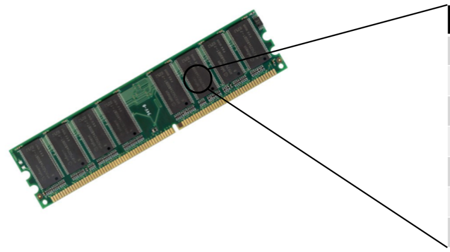

Address	Data
0	01001000
1	01100101
2	01101100
3	01101100
4	01101111
5	00100001
...	...

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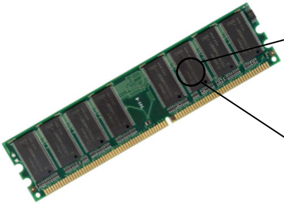

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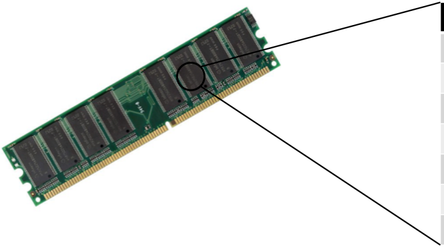

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- Each box contains a **byte** (8 bits)

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  - Executable: sequence of machine code operations

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Worksheet B