COMP270: Mathematics for 3D Worlds and Simulations

WEEK 2: GEOMETRY I

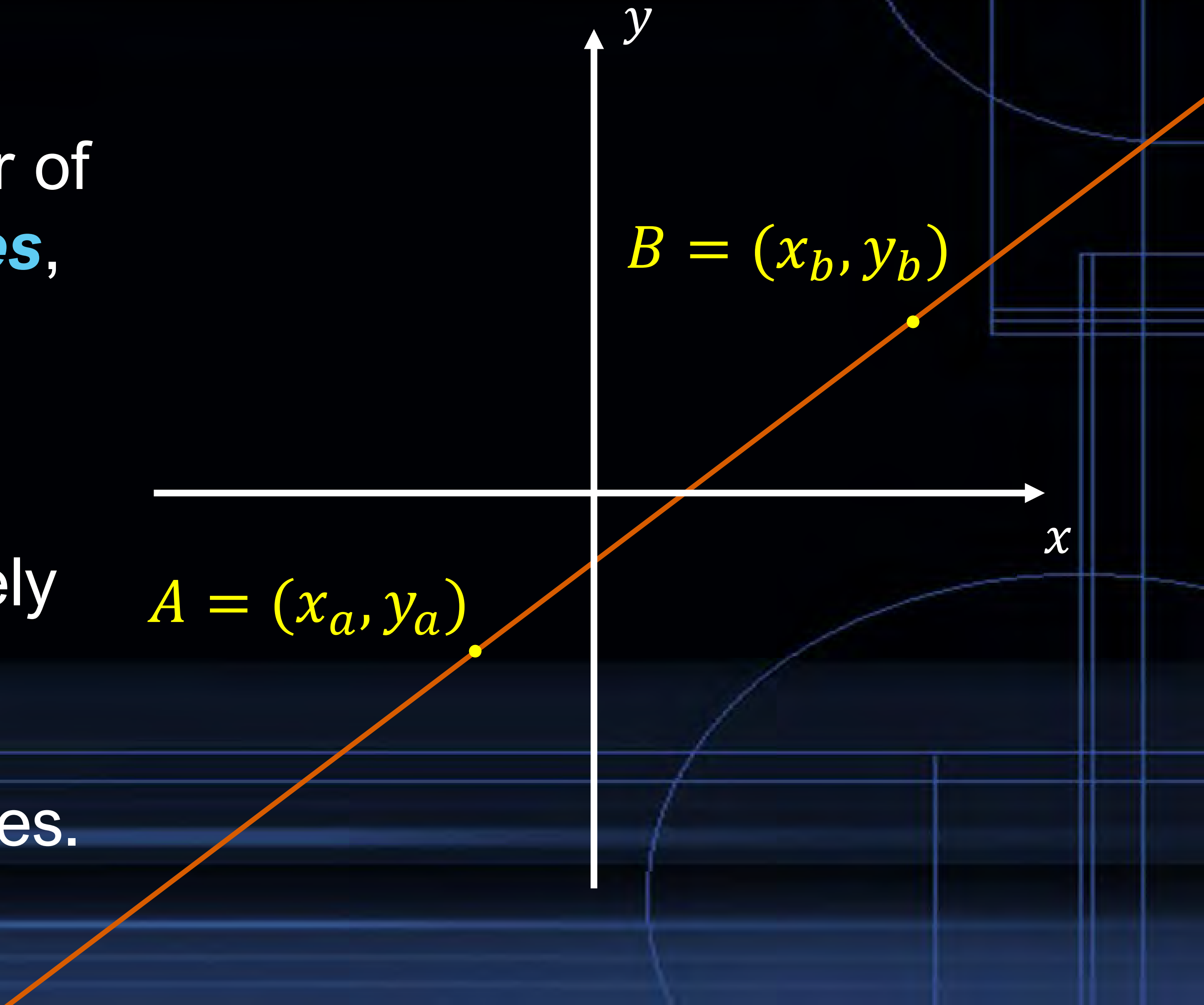
PART 2: VECTOR BASICS

Objectives

- **Recall** the definition of a vector and how it differs from a point or a line
- **Apply** basic arithmetic operations to vectors in 2D

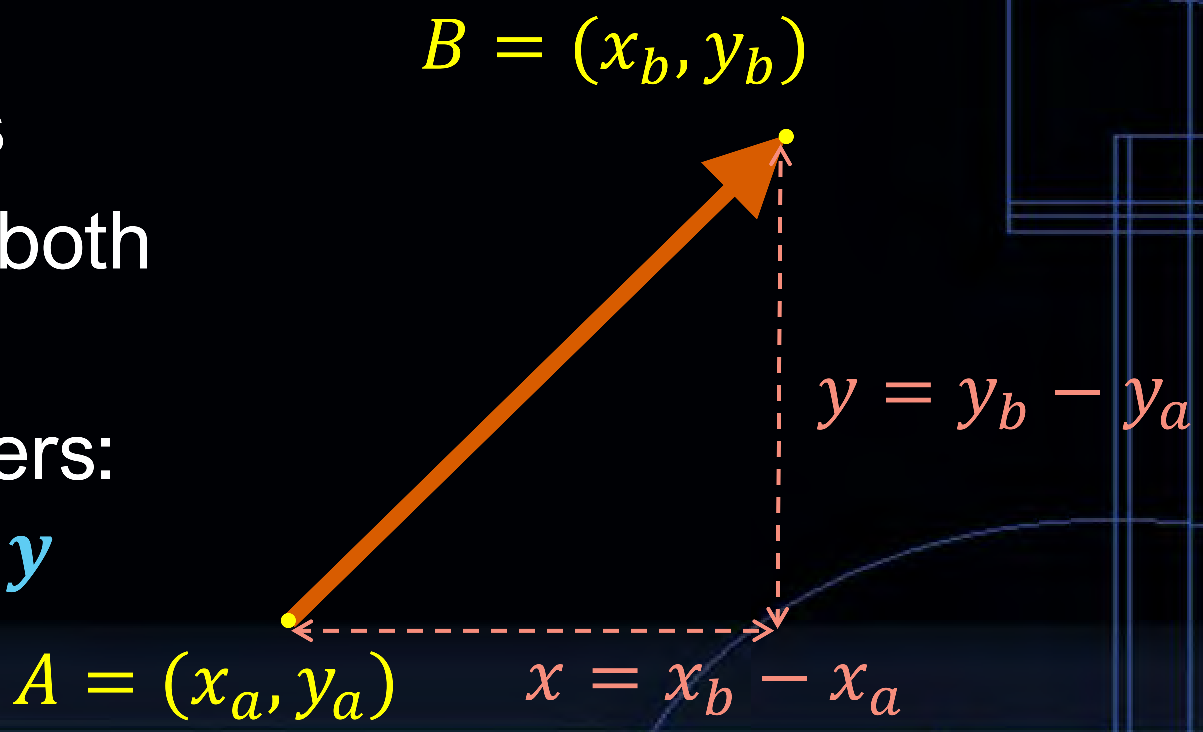
Recap

- A **point** in 2D is a pair of values, or ***coordinates***, specifying a position relative to a pair of perpendicular axes.
- A **line** extends infinitely and can be uniquely defined by two points through which it passes.



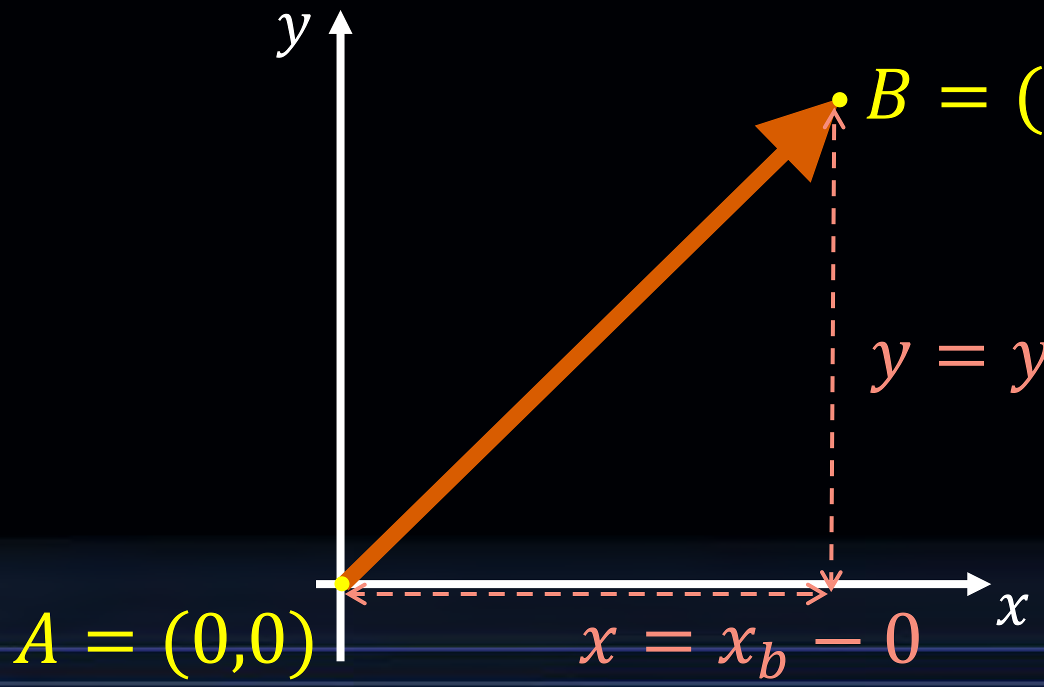
2D Vectors

- A **vector** is a directed line segment between 2 points
- Unlike a line, a vector has both **direction** and **length**
- Defined by a pair of numbers: the x **component** and the y **component**



Vectors and points

- A vector can represent a point relative to the origin:



- … but a point is not the same as a vector!

(Except in some code…)

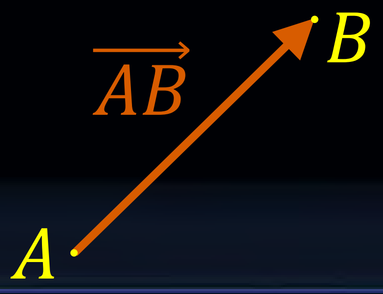
Writing vectors

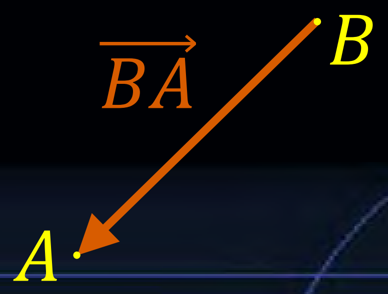
- As a pair of numbers: (x, y)

For COMP270: this is a **point**

- As a row or column vector: $[x \quad y]$, $\begin{pmatrix} x \\ y \end{pmatrix}$
- Variable representing a vector: written in bold i.e. $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$
  - Other notations: $\vec{\mathrm{v}}, \underline{\mathrm{v}}$
  - Vector from A to B: $\overrightarrow{AB}$

    Vector from B to A: $\overrightarrow{BA}$





    - Note: these are not the same, as the components will have opposite signs

Vector magnitude and direction

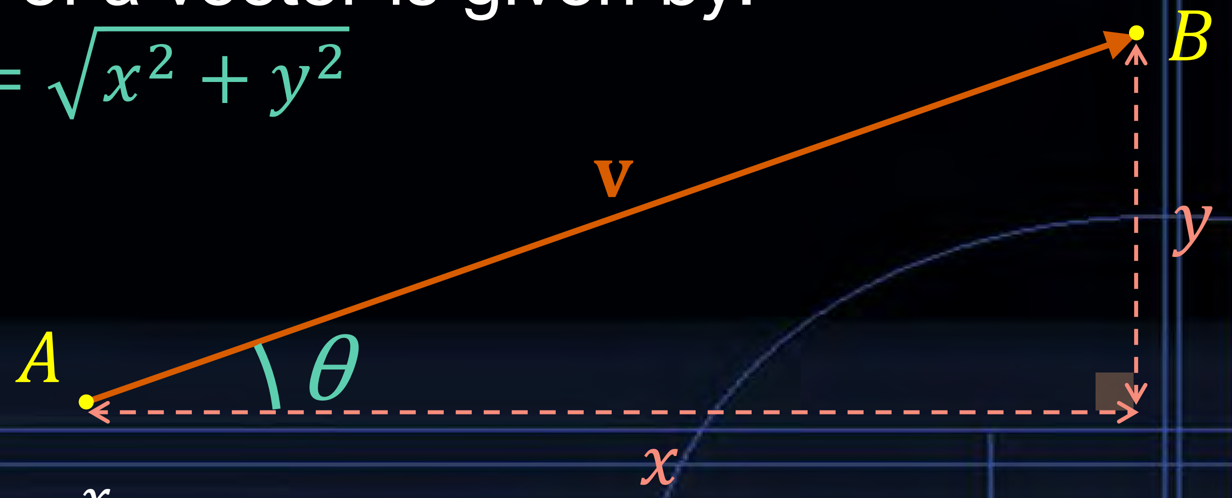
- Recall: a vector is the hypotenuse of the triangle formed with lines through its end points, parallel to the axes
  - The other side lengths are the vector components, x and y
- The **magnitude** (or length) of a vector is given by:

$$\|\mathbf{v}\| = \sqrt{x^2 + y^2}$$

- The angle between $\mathbf{v}$ and the positive x-axis is given by

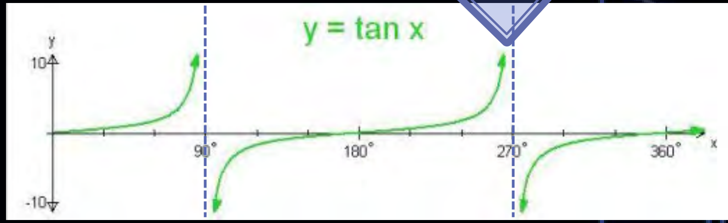
$$\tan\theta = \frac{y}{x}$$

(with $\sin\theta = \frac{y}{\|\mathbf{v}\|}$ and $\cos\theta = \frac{x}{\|\mathbf{v}\|}$)

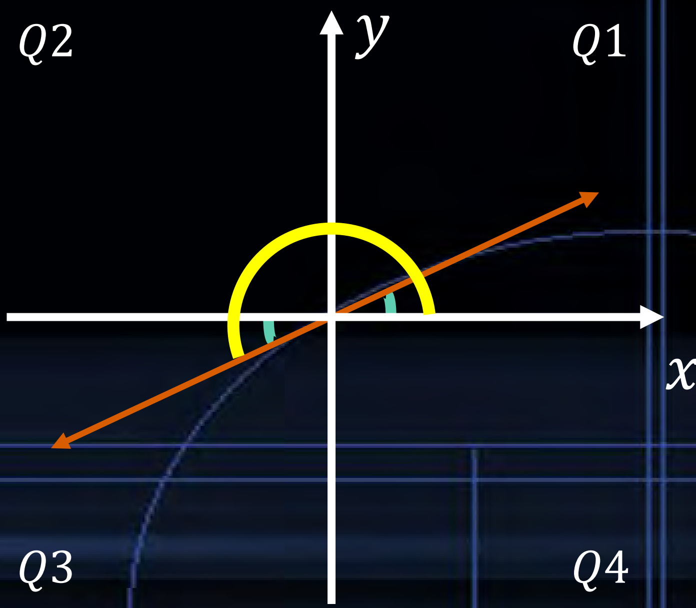


Inverse tangent

An **asymptote** is a line or curve that approaches a given curve arbitrarily closely

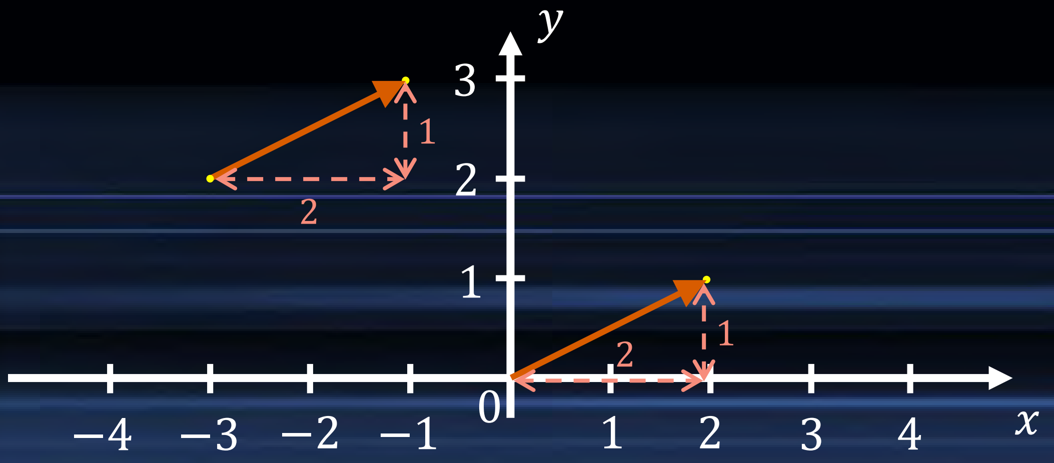


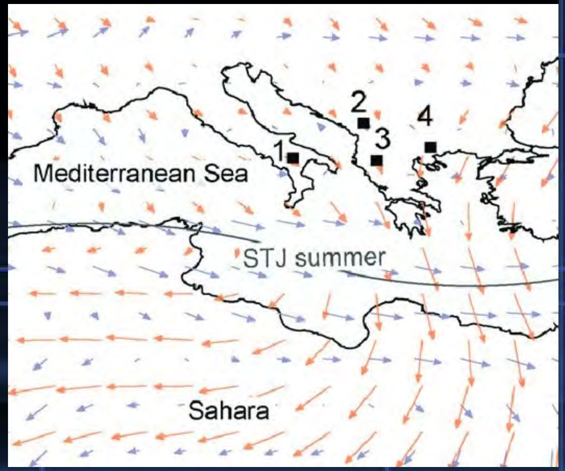
- Care is needed when using $\tan^{-1}$:
- $\tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-y}{-x}$ so we need to be aware of which quadrant the vector is in (otherwise the result may be out by 180°)
- If $x = 0$ (the vector points vertically along the y-axis) then we're dividing by zero
- Most programming languages have an `atan2(y, x)` function which handles all of these cases for you



Vector equivalence

- Vectors are defined by their magnitude and direction, but ***not*** **by their position** in space
  - e.g. the vector from the origin to the point $(2, 1)$ is the same as the vector from the point $(-3, 2)$ to the point $(-1, 3)$
- Used to represent more abstract quantities than just the difference between two points, e.g. wind

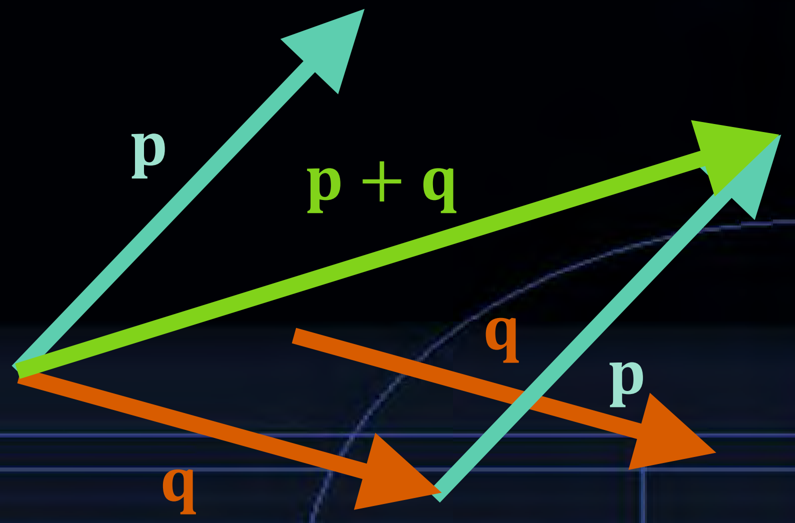




Vector addition

If $\mathbf{p}$ and $\mathbf{q}$ are vectors,

- $\mathbf{p} + \mathbf{q}$ is obtained by putting $\mathbf{p}$ and $\mathbf{q}$ end to end
- Performed component-wise,
$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$
- Note: addition is **commutative**:
$$\mathbf{p} + \mathbf{q} = \mathbf{q} + \mathbf{p}$$



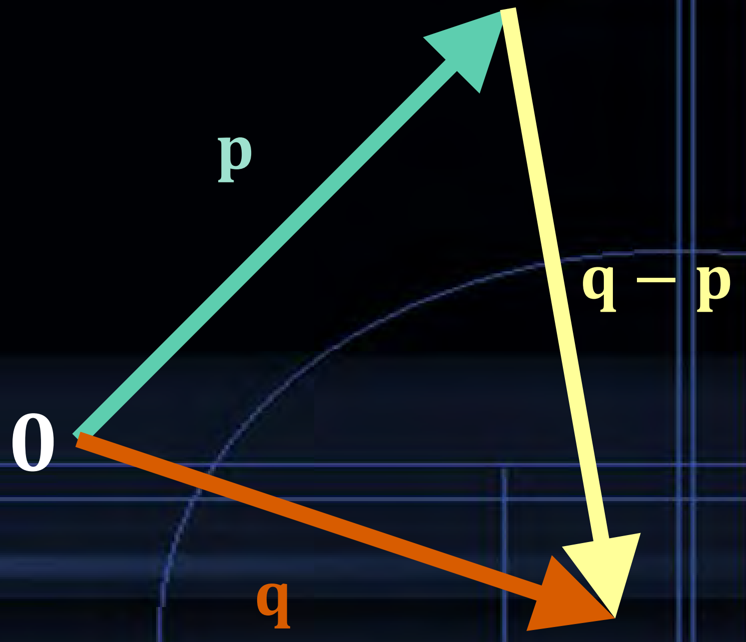
Vector subtraction

If $\mathbf{p}$ and $\mathbf{q}$ are vectors representing points on the plane,

- $\mathbf{q} - \mathbf{p}$ represents the vector *from* $\mathbf{p}$ *to* $\mathbf{q}$
  - Note the order of the subtraction!
- $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}$
- Note: subtraction is **anticommutative**:

$$\mathbf{q} - \mathbf{p} = -(\mathbf{p} - \mathbf{q})$$

  - i.e. $\overrightarrow{BA} = -\overrightarrow{AB}$



Scalar multiplication

If $\mathbf{v}$ is a vector and c is a positive number

- $c\mathbf{v}$ is a vector with the same direction as $\mathbf{v}$, but c times the magnitude
- $c\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cx \\ cy \end{pmatrix}$
- If c is negative then $c\mathbf{v}$ has the opposite direction to $\mathbf{v}$