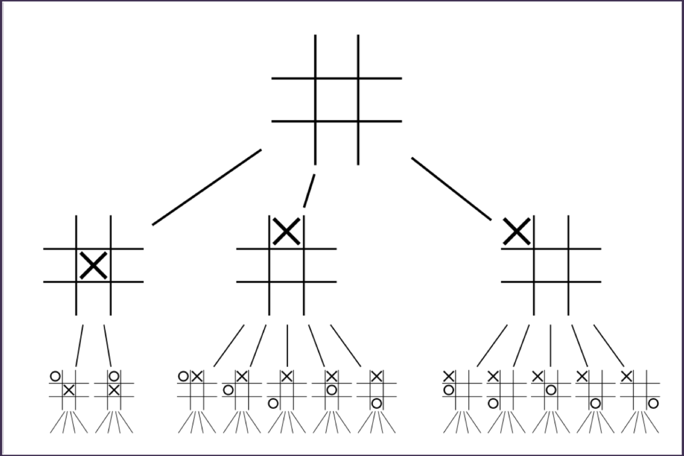
COMP250: Artificial Intelligence

5: Game Tree Search

Minimax search

Game trees



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- Therefore I want to **maximise** the **minimum** value my opponent can achieve
- This is generally only true for **two-player zero-sum** games

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Minimax search – example

Minimax search pseudocode

procedure MINIMAX(state, currentPlayer)

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procedure MINIMAX(state, currentPlayer)
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procedure MINIMAX(state, currentPlayer)
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    else if currentPlayer = 1 then
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        return bestValue
    else if currentPlayer $= 2$ **then**
        bestValue $= +\infty$
        for each possible nextState **do**
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        return bestValue

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- Similarly when minimising if bestValue $= -1$

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- **But...**

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  - Connect 4 has $\approx 10^{13}$ states
  - Chess has $\approx 10^{47}$ states

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Heuristics for search

Depth limiting

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- For nonterminal states at depth d, apply a heuristic evaluation instead of searching deeper
- Evaluation is a number between -1 and $+1$, estimating the probable outcome of the game

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- Often easier to design a “which state is better” heuristic than to directly design a “which move to play” heuristic

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- Thus ordering moves from **best to worst** means faster search
- How do we know which moves are "best" and "worst"? Use a heuristic!

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- Next time we will look at what we can do if we don't possess such knowledge

Monte Carlo evaluation

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- Applications in physics, engineering, finance, weather forecasting, graphics, ...

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- Seed is generally based on some source of **entropy**, e.g. system clock, mouse input, electronic noise

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  - Minimax assumes the evaluation is **deterministic**, but Monte Carlo is not
  - Not commonly used, mainly because there's something better...

Monte Carlo Tree Search

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- However, statistics from these rollouts are used to **bias** future rollouts
- Bias rollouts towards **plausible** lines of play, i.e. where each player is trying to play the best move

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  - **Expansion**: Choose a random legal move for which the current node does not have a child node. Add this new node to the tree.

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  - **Simulation**: Perform a Monte Carlo rollout, playing random moves until a terminal state is reached.
  - **Backpropagation**: For each node visited during **selection** and **expansion**, update the node's statistics based on the result of the simulation.
- Perform many rollouts, then use the statistics at the top level of the tree to choose the best move

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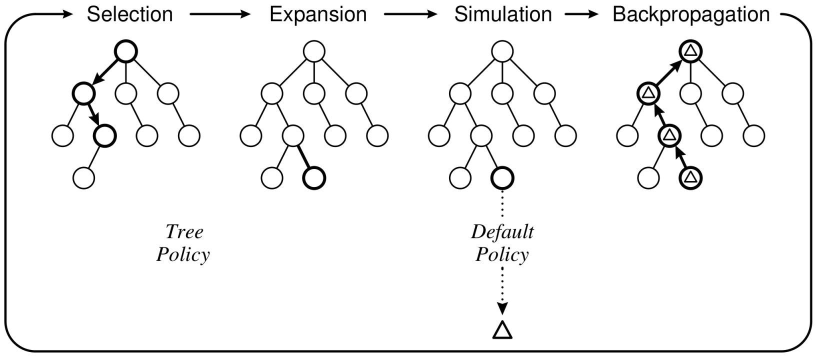

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- This can be modelled as a **multi-armed bandit problem**

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  - c is a parameter for adjusting the balance between exploitation and exploration

UCB demo

http://orangehelicopter.com/academic/bandits.html?ucb

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Upper Confidence Bound for Trees (UCT)

- Use UCB as the selection policy
- In each node x, record:
  - n_x: the number of visits to this node
  - V_x: the total value of rollouts through this node
- From node p, choose the child q such that

$$\frac{V_q}{n_q} + c\sqrt{\frac{\log n_p}{n_q}}$$

is largest

UCT demo

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