

COMP110: Principles of Computing

5: Computational Complexity

Learning outcomes

- ▶ **Explain** the notion of computability
- ▶ **Use** “big O ” notation to express computational complexity
- ▶ **Apply** appropriate algorithms to achieve efficiency

Computability



Computability theory

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 - ▶ NB: A may be **infinite**
- ▶ A function $f : A \rightarrow B$ is **computable** if there exists a Turing machine which computes f
 - ▶ I.e. given an encoding of $a \in A$ as input, the Turing machine outputs an encoding of $f(a)$

An uncomputable function

The **halting problem**

An uncomputable function

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- ▶ There is **no** Turing machine that computes f
- ▶ f is **uncomputable**

Church-Turing Thesis

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Church-Turing Thesis

- ▶ A system (e.g. a computer or programming language) is **Turing complete** if it can implement any given Turing machine
- ▶ If a function is **effectively calculable**, then it is **computable** by a Turing machine
- ▶ Effectively calculable = there is a method or algorithm for computing it
- ▶ So in terms of computability, Turing machines are as powerful as computers can be

Halting revisited

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- ▶ Write a software tool that, given a Python program, predicts whether that program can go into an infinite loop

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Halting revisited

- ▶ Write a software tool that, given a Python program, predicts whether that program can go into an infinite loop
- ▶ Your tool must work for **all** Python programs
- ▶ Is this possible?

Computation time



Resources

Resources

- ▶ All programs use **resources**

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 - ▶ Time

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- ▶ Often **time** is the resource we care about the most
 - ▶ Particularly in games: want to maintain a good **frame rate** free of **lag** or **stuttering**

Basic time measurement in Python

```
import time

start_time = time.clock()

... do something here ...

end_time = time.clock()
print("Computation took", end_time - start_time, " ←  
seconds")
```

Repeating for better accuracy

```
import time

start_time = time.clock()

repetition_count = 1000

for repetition in range(repetition_count):
    ... do something here ...

end_time = time.clock()
time_per = (end_time - start_time) / repetition_count
print("Computation took", time_per, "seconds")
```

Scaling

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- ▶ Timing is dependent on hardware and software issues

Scaling

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- ▶ We are often less interested in how many milliseconds a particular computation takes on today's hardware, and more interested in how the execution time **scales** with the problem size

Search



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- We have a list of names, each with some data associated

Search

- ▶ We have a list of names, each with some data associated
- ▶ We want to find one of them

Linear search

procedure FIND(name, list)

Linear search

```
procedure FIND(name, list)  
  for each item in list do
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Linear search

```
procedure FIND(name, list)
  for each item in list do
    if item.name = name then
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How long does it take?

Socrative room code: FALCOMPED

- Suppose there are 25 items in the list

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- ▶ Suppose there are 25 items in the list
- ▶ In the **best case**, how many items do we need to visit before finding the one we want?

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- ▶ Suppose there are 25 items in the list
- ▶ In the **best case**, how many items do we need to visit before finding the one we want?
- ▶ How about in the **worst case**?

How long does it take?

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- ▶ If there are 25 items in the list, the **worst case** number of items visited is 25

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- ▶ If there are 25 items in the list, the **worst case** number of items visited is 25
- ▶ How about if there are 50 items?

How long does it take?

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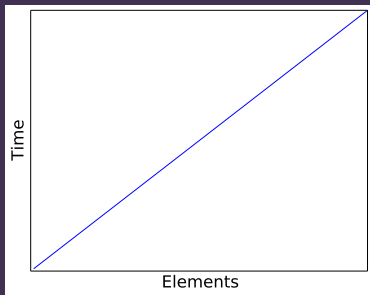
- ▶ If there are 25 items in the list, the **worst case** number of items visited is 25
- ▶ How about if there are 50 items?
- ▶ How about 100 items?

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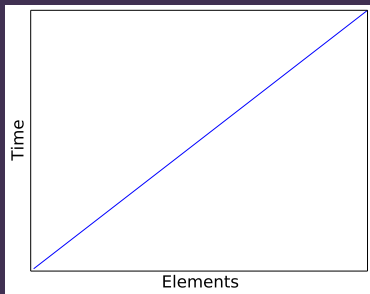
- ▶ If there are 25 items in the list, the **worst case** number of items visited is 25
- ▶ How about if there are 50 items?
- ▶ How about 100 items?
- ▶ If the number of items **doubles**, what happens to the amount of time the search takes?

Linear time



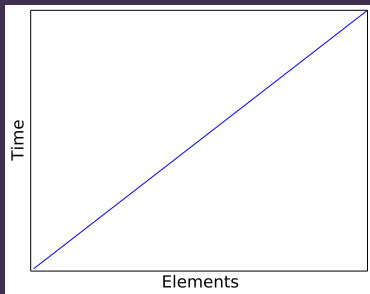
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Linear time



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- ▶ Linear search is said to have **linear time complexity**
- ▶ Also written as $O(n)$ **time complexity**

Searching a sorted list

- ▶ If the list is **sorted** in alphabetical order, we can do better than linear...

Binary search

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  if name = mid.name then
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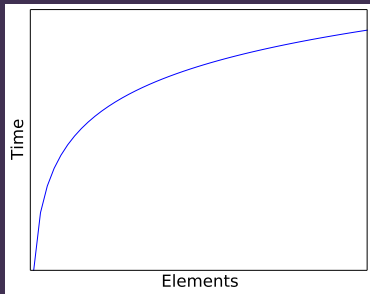
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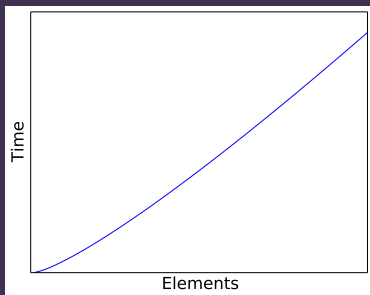
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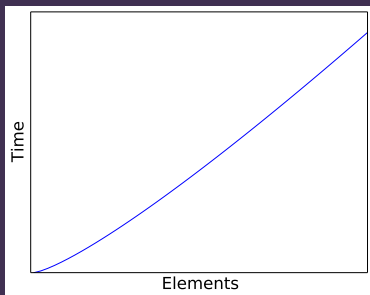
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- ▶ Careful how you implement this!
- ▶ **Copying** (half of) a list is **linear** $O(n)$
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- ▶ Use **pointers** into the list instead of copying

Binary search done wrong

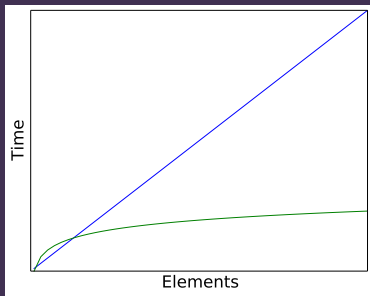
```
def binary_search(name, mylist):  
    if mylist == []:  
        raise ValueError("Not found")  
  
    mid = len(mylist) / 2  
    mid_name = mylist[mid_index].name  
  
    if name == mid_name:  
        return mid  
    elif name < mid_name:  
        return binary_search(name, mylist[:mid])  
    else:  
        return binary_search(name, mylist[mid+1:])
```

Binary search done right

```
def binary_search(name, mylist, start, end):  
    if end <= start:  
        raise ValueError("Not found")  
  
    mid = (start + end) / 2  
    mid_name = mylist[mid].name  
  
    if name == mid_name:  
        return mylist[mid]  
    elif name < mid_name:  
        return binary_search(name, mylist, start, mid)  
    else:  
        return binary_search(name, mylist, mid+1, end)
```

Binary search vs linear search

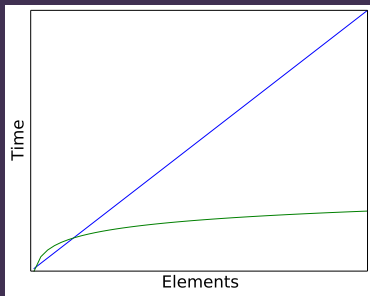
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- So binary search is better than linear search... right?

Binary search vs linear search

Socrative room code: FALCOMPED



- ▶ So binary search is better than linear search... right?
- ▶ Discuss in **pairs**
- ▶ On Socrative, post **one reason** why, or **one situation** where, linear search may be a better choice than binary search

Hashing

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:	:
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114	Collins, Jane
115	—
116	—
117	Hughes, Aaron
118	—
119	—
120	—
121	—
122	Brown, Janet
123	—
124	—
125	Gonzalez, Adam Lewis, Rose
126	—
127	—
128	—
129	—
130	—
131	—
132	Young, Frank
:	:
:	:

Hash look-up

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99	Parker, Debra Perez, Diana White, Amanda
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138	Cox, Shirley
142	Clark, Stephanie
144	Scott, Michelle
145	Miller, Jeremy
147	Davis, Marilyn
149	Lopez, Jeffrey
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158	Williams, Billy
162	Sanders, Phillip
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175	Stewart, Howard
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“Lopez, Jeffrey”

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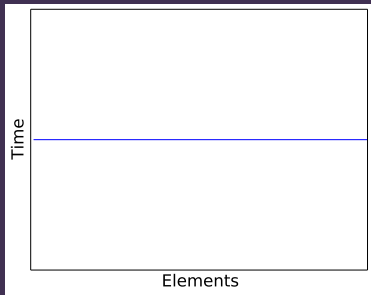
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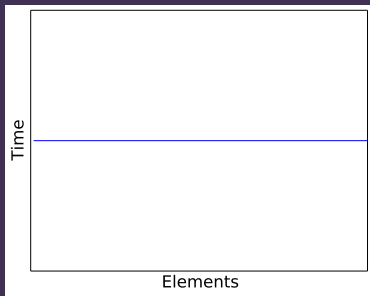
$$12 + 15 + 16 + 5 + 26 + 10 + 5 + 6 + 6 + 18 + 5 + 25 = 149$$

How long does it take?

- If there are no “collisions”, look-up time is **constant** or $O(1)$

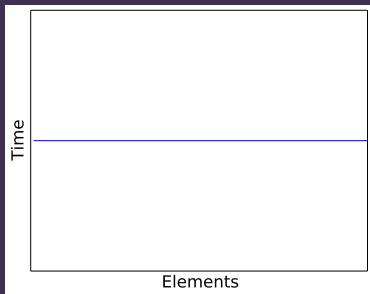


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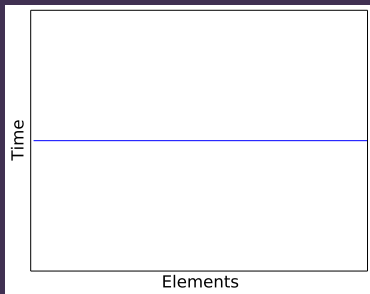
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- ▶ When there are collisions, need to fall back on something like linear or binary search within each bin

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[person for person in people if person.name  
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- ▶ Binary search in Python:
 - ▶ The `bisect` module
- ▶ Hash tables in Python:
 - ▶ The `dict` (dictionary) data structure

More on complexity



Common complexity classes

Common complexity classes

“Faster” Constant

$O(1)$

Common complexity classes

“Faster”



Constant
Logarithmic

$O(1)$
 $O(\log n)$

Common complexity classes

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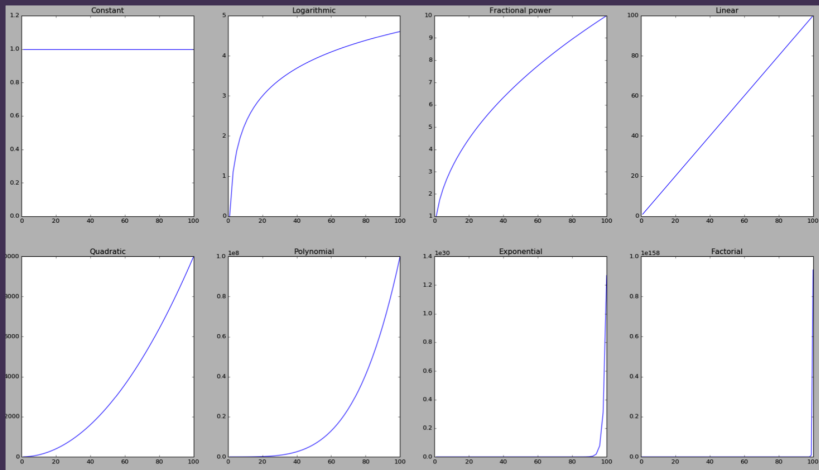
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	Exponential	$O(e^n)$

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		Polynomial	$O(n^k), k > 1$
		Exponential	$O(e^n)$
"Slower"		Factorial	$O(n!)$

Common complexity classes



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- ▶ Multiply **compound** algorithms
 - ▶ If an algorithm does n “things” and each “thing” is $O(n)$, then the overall algorithm is $O(n^2)$

Quadratic complexity

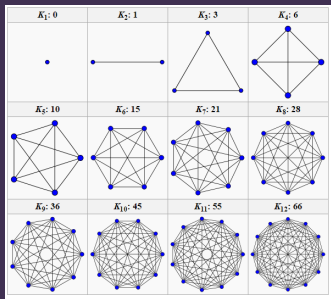
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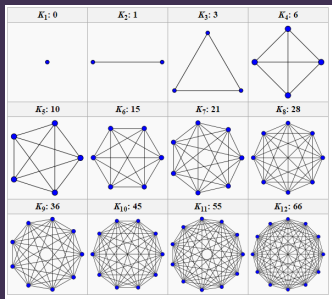
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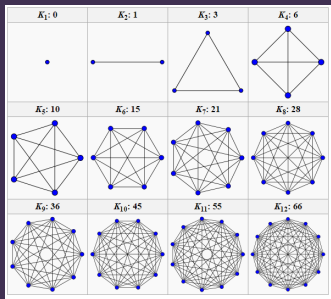
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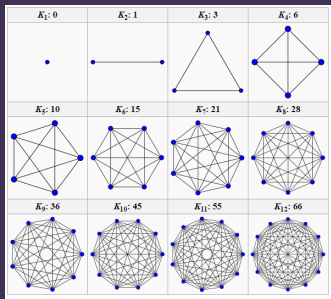
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 - ▶ Further reading: spatial hashing, quadtrees, octrees, Verlet lists



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- ▶ Adding 1 to n potentially **doubles** the running time!

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- ▶ Are there any problems in NP but not in P ?

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- ▶ It is believed that $P \neq NP$, so large instances of *NP*-hard problems are not solvable in a feasible amount of time
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 - ▶ Quantum computers are “infinitely parallel” in a sense so *can* solve some large *NP*-hard problems

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- ▶ Choice of data structures and algorithms can have a large impact on the efficiency of your software
- ▶ ... but only if scalability is actually a factor

Exercise Sheet ii

Due now

Worksheet C

Due in 2 weeks' time
(No class next week)