COMP110: Principles of Computing

5: Computational Complexity

Learning outcomes

- **Explain** the notion of computability
- **Use** "big O" notation to express computational complexity
- **Apply** appropriate algorithms to achieve efficiency

Worksheet C

- Computational complexity
- Due in **2 weeks' time**

Quiz B — bonus question

$$\sum_{i=M}^{N} \sum_{j=M}^{N} (i+j)(i-j)$$

Quiz B — bonus question

$$\sum_{i=M}^{N}\sum_{j=M}^{N}(i+j)(i-j)=\sum_{i=M}^{N}\sum_{j=M}^{N}\left(i^2+ij-ij-j^2\right)$$

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$$\sum_{i=M}^{N}\sum_{j=M}^{N}(i+j)(i-j) = \sum_{i=M}^{N}\sum_{j=M}^{N}\left(i^2+ij-ij-j^2\right)$$

$$= \sum_{i=M}^{N}\sum_{j=M}^{N}\left(i^2-j^2\right)$$

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$$\sum_{i=M}^{N}\sum_{j=M}^{N}(i+j)(i-j) = \sum_{i=M}^{N}\sum_{j=M}^{N}\left(i^2 + ij - ij - j^2\right)$$

$$= \sum_{i=M}^{N}\sum_{j=M}^{N}\left(i^2 - j^2\right)$$

$$= \left(\sum_{i=M}^{N}\sum_{j=M}^{N} i^2\right) - \left(\sum_{i=M}^{N}\sum_{j=M}^{N} j^2\right)$$

Quiz B — bonus question

$$
\begin{aligned}
\sum_{i=M}^{N}\sum_{j=M}^{N}(i+j)(i-j) &= \sum_{i=M}^{N}\sum_{j=M}^{N}\left(i^2+ij-ij-j^2\right)\\
&= \sum_{i=M}^{N}\sum_{j=M}^{N}\left(i^2-j^2\right)\\
&= \left(\sum_{i=M}^{N}\sum_{j=M}^{N} i^2\right)-\left(\sum_{i=M}^{N}\sum_{j=M}^{N} j^2\right)\\
&= (N-M+1)\left(\sum_{i=M}^{N} i^2\right)-(N-M+1)\left(\sum_{j=M}^{N} j^2\right)
\end{aligned}
$$

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&= \sum_{i=M}^{N}\sum_{j=M}^{N}\left(i^2-j^2\right) \\
&= \left(\sum_{i=M}^{N}\sum_{j=M}^{N} i^2\right) - \left(\sum_{i=M}^{N}\sum_{j=M}^{N} j^2\right) \\
&= (N-M+1)\left(\sum_{i=M}^{N} i^2\right) - (N-M+1)\left(\sum_{j=M}^{N} j^2\right) \\
&= (N-M+1)\left(\sum_{i=M}^{N} i^2 - \sum_{j=M}^{N} j^2\right) = 0
\end{aligned}
$$

Computation time

Resources

Resources

- All programs use **resources**

Resources

- All programs use **resources**
  - Time

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- All programs use **resources**
  - Time
  - Memory

Resources

- All programs use **resources**
  - Time
  - Memory
  - Network bandwidth

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- Often **time** is the resource we care about the most

Resources

- All programs use **resources**
  - Time
  - Memory
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  - ...
- Often **time** is the resource we care about the most
  - Particularly in games: want to maintain a good **frame rate** free of **lag** or **stuttering**

Basic time measurement in Python

```
import time

start_time = time.clock()

... do something here ...

end_time = time.clock()
print "Computation took", end_time - start_time, " ↩
    seconds"
```

Repeating for better accuracy

```
import time

start_time = time.clock()

repetition_count = 1000

for repetition in xrange(repetition_count):
    ... do something here ...

end_time = time.clock()
time_per = (end_time - start_time) / repetition_count
print "Computation took", time_per, "seconds"
```

Scaling

Scaling

- Timing is dependent on hardware and software issues

Scaling

- Timing is dependent on hardware and software issues
- We are often less interested in how many milliseconds a particular computation takes on today's hardware, and more interested in how the execution time **scales** with the problem size

Search

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- We have a list of names, each with some data associated

Search

- We have a list of names, each with some data associated
- We want to find one of them

Linear search

procedure FIND(name, list)

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    for each item in list **do**
        if item.name = name **then**

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            return item

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How long does it take?

Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list

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- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?

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- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?
- How about in the **worst case**?

How long does it take?

Socrative room code: `FALCOMPED`

- If there are 25 items in the list, the **worst case** number of items visited is 25

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- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?

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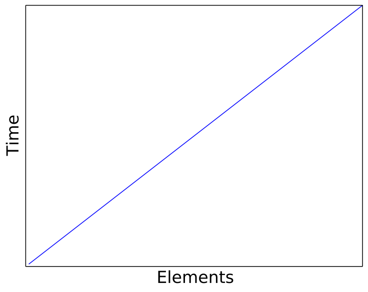
- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?

How long does it take?

Socrative room code: `FALCOMPED`

- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?
- If the number of items **doubles**, what happens to the amount of time the search takes?

Linear time



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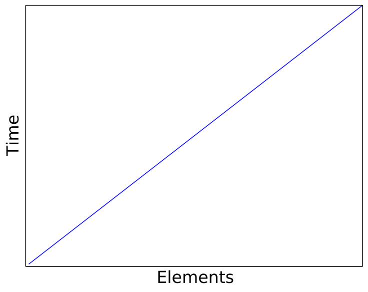
Linear time



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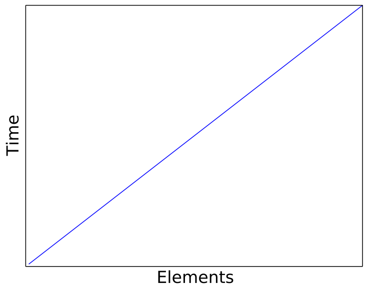

- The running time of linear search is **proportional** to the size n of the list
- Linear search is said to have **linear time complexity**
- Also written as $O(n)$ **time complexity**

Searching a sorted list

- If the list is **sorted** in alphabetical order, we can do better than linear...

Binary search

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    end if
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Binary search

procedure FIND(name, list)
    if list is empty **then**
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    end if
    mid $\leftarrow$ the "middle" item of the list

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procedure FIND(name, list)
    if list is empty then
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    mid ← the "middle" item of the list
    if name = mid.name then
        return mid
```

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procedure FIND(name, list)
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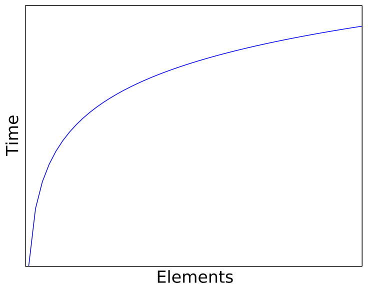

Hidden complexity

if name $<$ mid.name **then**
    return FIND(name, first half of list)
else if name $>$ mid.name **then**
    return FIND(name, second half of list)
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- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$

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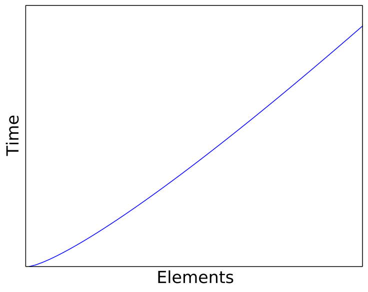

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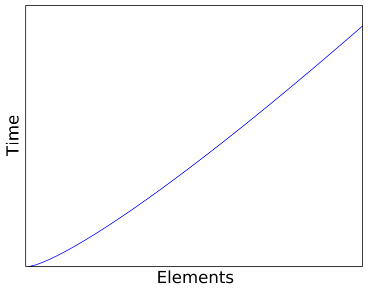

- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$
- The actual running time would be $O(n \log n)$
- Use **pointers** into the list instead of copying

Binary search done wrong

```
def binary_search(name, mylist):
    if mylist == []:
        raise ValueError("Not found")

    mid = len(mylist) / 2
    mid_name = mylist[mid_index].name

    if name == mid_name:
        return mid
    elif name < mid_name:
        return binary_search(name, mylist[:mid])
    else:
        return binary_search(name, mylist[mid+1:])
```

Binary search done right

```
def binary_search(name, mylist, start, end):
    if end <= start:
        raise ValueError("Not found")

    mid = (start + end) / 2
    mid_name = mylist[mid].name

    if name == mid_name:
        return mylist[mid]
    elif name < mid_name:
        return binary_search(name, mylist, start, mid)
    else:
        return binary_search(name, mylist, mid+1, end)
```

Binary search vs linear search

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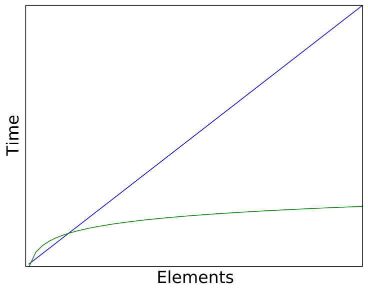

- So binary search is better than linear search... right?

Binary search vs linear search

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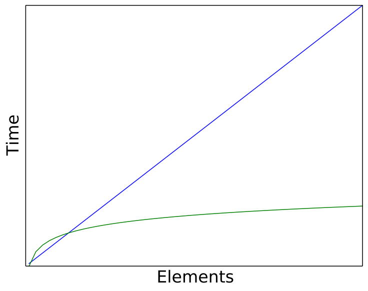

- So binary search is better than linear search... right?
- Discuss in **pairs**
- On Socrative, post **one reason** why, or **one situation** where, linear search may be a better choice than binary search

Hashing

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⋮	⋮
112	Ward, Jessica
113	Baker, Theresa
114	Collins, Jane
115	—
116	—
117	Hughes, Aaron
118	—
119	—
120	—
121	—
122	Brown, Janet
123	—
124	—
125	Gonzalez, Adam Lewis, Rose
126	—
127	—
128	—
129	—
130	—
131	—
132	Young, Frank
⋮	⋮

Hash look-up

98	Diaz, Harold
99	Parker, Debra Perez, Diana White, Amanda
112	Ward, Jessica
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138	Cox, Shirley
142	Clark, Stephanie
144	Scott, Michelle
145	Miller, Jeremy
147	Davis, Marilyn
149	Lopez, Jeffrey
151	Anderson, Martha
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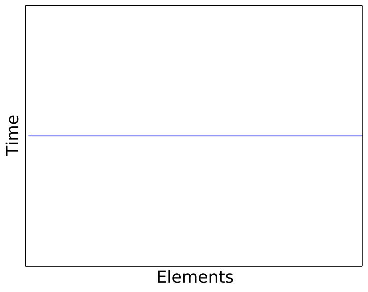
$$12 + 15 + 16 + 5 + 26 + 10 + 5 + 6 + 6 + 18 + 5 + 25 = 149$$

How long does it take?



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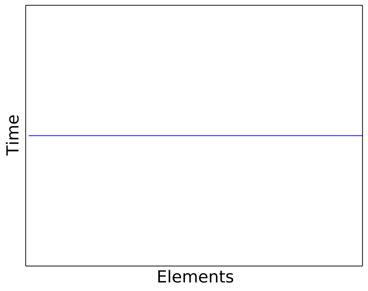

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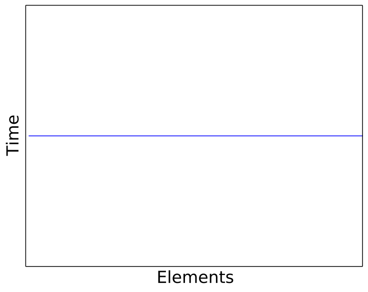

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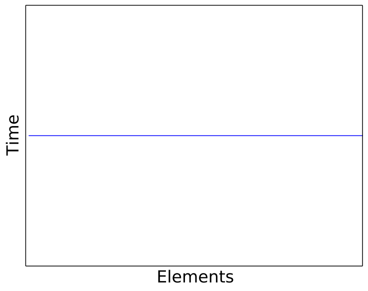

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- When there are collisions, need to fall back on something like linear or binary search within each bin

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- Hash tables in Python:
  - The `dict` (dictionary) data structure

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More on complexity

Common complexity classes

Common complexity classes

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Common complexity classes

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Common complexity classes

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$\vert$	Fractional power	$O(n^k), k < 1$
$\vert$	Linear	$O(n)$

Common complexity classes

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"Slower"	Factorial	$O(n!)$

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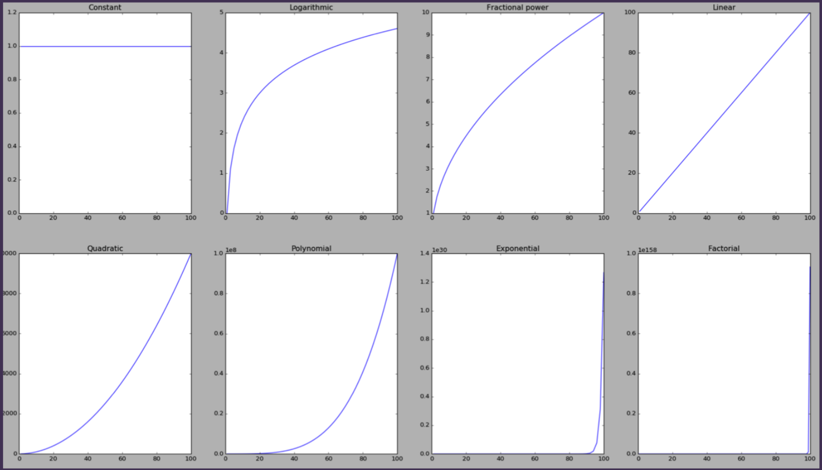

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  - If one algorithm takes n^2 operations, another takes $500n^2$ and a third takes $0.00000001n^2$, all three are $O(n^2)$
- Take only the **dominant term**
  - The term that is largest when n is large

Working with big O notation

- Can ignore **leading constants**
  - If one algorithm takes n^2 operations, another takes $500n^2$ and a third takes $0.00000001n^2$, all three are $O(n^2)$
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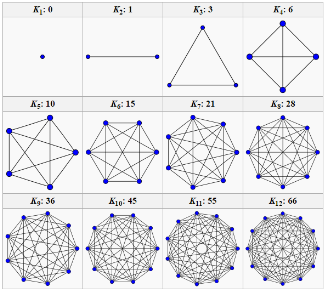

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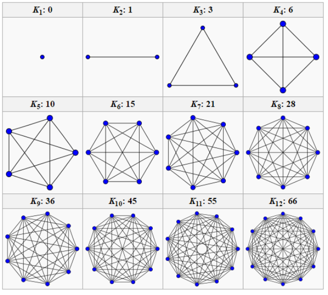

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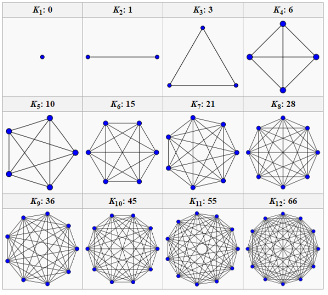

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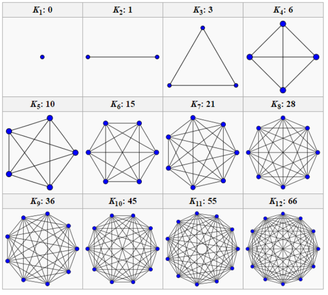

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  - Further reading: spatial hashing, quadtrees, octrees, Verlet lists

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- Adding 1 to n potentially **doubles** the running time!

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- Are there any problems in NP but not in P?

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  - Quantum computers are "infinitely parallel" in a sense so *can* solve some large *NP*-hard problems

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  - Otherwise, choose simplicity

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- Choice of data structures and algorithms can have a large impact on the efficiency of your software
- ... but only if scalability is actually a factor

Computability

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- A function $f : A \to B$ is **computable** if there exists a Turing machine which computes f
  - I.e. given an encoding of $a \in A$ as input, the Turing machine outputs an encoding of $f(a)$

An uncomputable function

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- f is **uncomputable**

Turing completeness

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- A system (e.g. a computer or programming language) is **Turing complete** if it can implement any given Turing machine

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- So in terms of computability, Turing machines are as powerful as computers can be

Halting revisited

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- Your tool must work for **all** Python programs
- Is this possible?