COMP110: Principles of Computing

5: Computational Complexity

Learning outcomes

- **Explain** the notion of computability
- **Use** “big *O*” notation to express computational complexity
- **Apply** appropriate algorithms to achieve efficiency

Search

Anderson, Martha
Parker, Debra
Russell, Mildred
Stewart, Howard
White, Amanda
Perez, Diana
Lewis, Rose
Scott, Michelle
Davis, Marilyn
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- We have a list of names, each with some data associated

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- We have a list of names, each with some data associated
- We want to find one of them

Linear search

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procedure FIND(name, list)
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How long does it take?

Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list

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- In the **best case**, how many items do we need to visit before finding the one we want?

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Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?
- How about in the **worst case**?

How long does it take?

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- If there are 25 items in the list, the **worst case** number of items visited is 25

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- If there are 25 items in the list, the **worst case** number of items visited is 25
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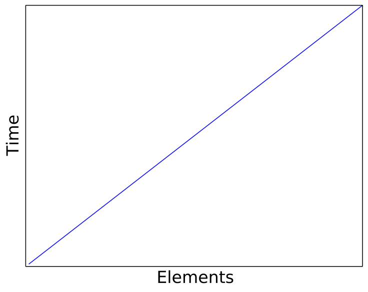
- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?

How long does it take?

Socrative room code: `FALCOMPED`

- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?
- If the number of items **doubles**, what happens to the amount of time the search takes?

Linear time



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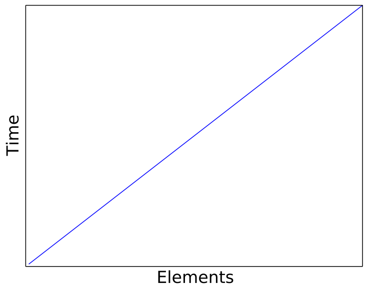
Linear time



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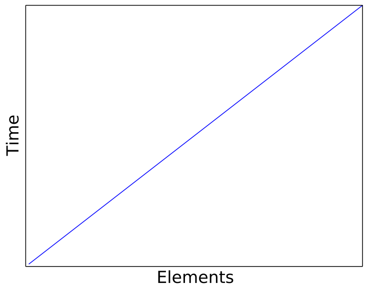

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- Linear search is said to have **linear time complexity**
- Also written as $O(n)$ **time complexity**

Searching a sorted list

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- If the list is **sorted** in alphabetical order, we can do better than linear...

Binary search

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procedure FIND(name, list)
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Binary search

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    if name = mid.name then
        return mid
```

Binary search

```
procedure FIND(name, list)
    if list is empty then
        throw "Not found"
    end if
    mid $\leftarrow$ the "middle" item of the list
    if name $=$ mid.name then
        return mid
    else if name $<$ mid.name then
        return FIND(name, first half of list)
```

Binary search

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Find “Lopez, Jeffrey“

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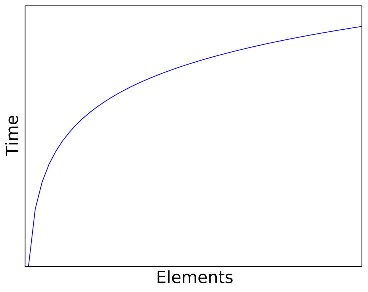

Hidden complexity

```
if name $<$ mid.name then
    return FIND(name, first half of list)
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    return FIND(name, second half of list)
end if
```

Hidden complexity

```
if name < mid.name then
    return FIND(name, first half of list)
else if name > mid.name then
    return FIND(name, second half of list)
end if
```

- Careful how you implement this!

Hidden complexity

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if name < mid.name then
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- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$

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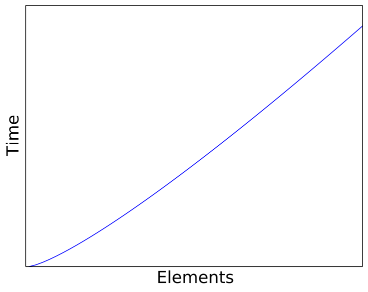

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**if** name $<$ mid.name **then**
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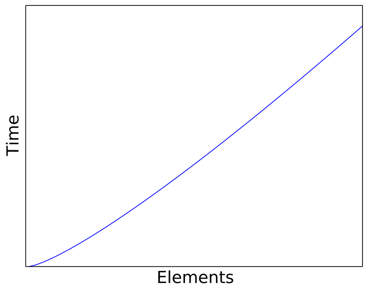

- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$
- The actual running time would be $O(n \log n)$
- Use **pointers** into the list instead of copying

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Binary search done wrong

```
def binary_search(name, mylist):
    if mylist == []:
        raise ValueError("Not found")

    mid_index = len(mylist) / 2
    mid = mylist[mid_index]

    if name == mid:
        return mid
    elif name < mid:
        return binary_search(name, mylist[:mid_index])
    else:
        return binary_search(name, mylist[mid_index ↩
            +1:])
```

Binary search done right

```
def binary_search(name, mylist, start, end):
    if end <= start:
        raise ValueError("Not found")

    mid_index = (start + end) / 2
    mid = mylist[mid_index]

    if name == mid:
        return mid
    elif name < mid:
        return binary_search(name, mylist, start, mid)
    else:
        return binary_search(name, mylist, mid+1, end)
```

Binary search vs linear search

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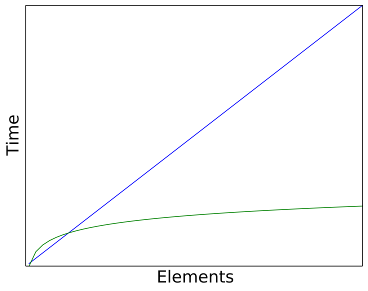

- So binary search is better than linear search... right?

Binary search vs linear search

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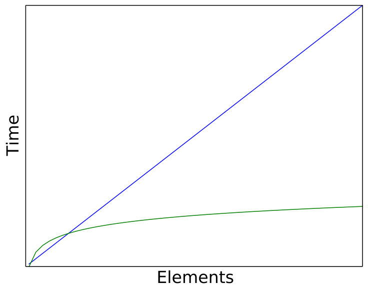
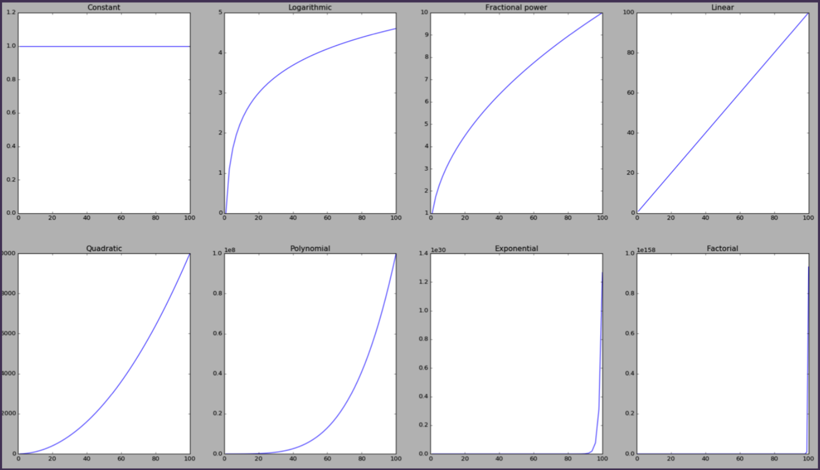


- So binary search is better than linear search... right?
- Discuss in **pairs**
- On Socrative, post **one reason** why, or **one situation** where, linear search may be a better choice than binary search

More complexity classes



Complexity in games

- Example: collision detection between *n* objects

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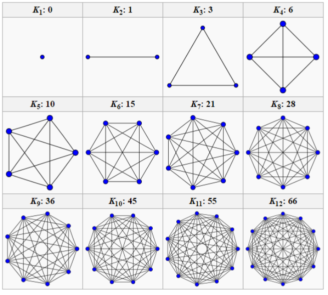

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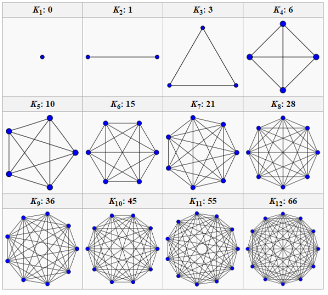

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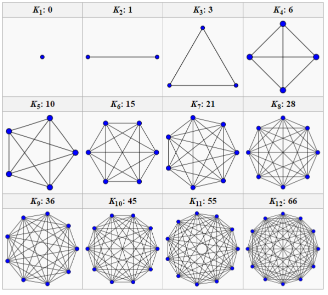

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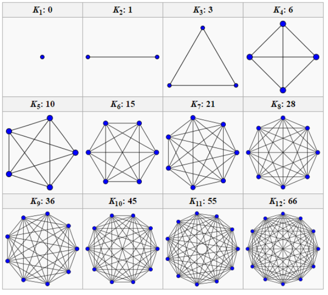

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  - Further reading: spatial hashing, quadtrees, octrees, Verlet lists

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- We've used search as an example, but don't reinvent the wheel!
  - Most languages have existing implementations of linear and binary search on arrays or lists
  - Most languages have a **dictionary** data structure based on **hashing**, which is generally better for this kind of key $\rightarrow$ value look-up

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- Software development is all about choosing **the right tool for the job**
    - If you need scalability, choose a scalable algorithm
    - Otherwise, choose simplicity

Summary

- Time complexity tells us how the running time of an algorithm **scales** with the size of the data it is given

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- Time complexity tells us how the running time of an algorithm **scales** with the size of the data it is given
- Choice of data structures and algorithms can have a large impact on the efficiency of your software
- ... but only if scalability is actually a factor