

COMP270: 3D Computational Geometry Worksheet 1

1. Calculate $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$ for the following vectors:

a. $\mathbf{a} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

b. $\mathbf{a} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$

c. $\mathbf{a} = \begin{pmatrix} 3 \\ 10 \\ 7 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 8 \\ -7 \\ 4 \end{pmatrix}$

2. A nonplayer character (NPC) is standing at a location \mathbf{p} with a forward direction of \mathbf{v} . Consider three points \mathbf{a} , \mathbf{b} and \mathbf{c} in the xz plane of a left-handed coordinate system, which represent waypoints on the NPC's path.
- How can the cross product be used to determine whether, when moving from \mathbf{a} to \mathbf{b} to \mathbf{c} , the NPC makes a clockwise or anticlockwise turn at \mathbf{b} , when viewing the path from above?
 - For each of the following sets of three points, determine whether the NPC is turning clockwise or anticlockwise when moving from \mathbf{a} to \mathbf{b} to \mathbf{c} :
 - $\mathbf{a} = (2, 0, 3), \mathbf{b} = (-1, 0, 5), \mathbf{c} = (-4, 0, 1)$
 - $\mathbf{a} = (-3, 0, -5), \mathbf{b} = (4, 0, 0), \mathbf{c} = (3, 0, 3)$
 - $\mathbf{a} = (1, 0, 4), \mathbf{b} = (7, 0, -1), \mathbf{c} = (-5, 0, -6)$
 - $\mathbf{a} = (-2, 0, 1), \mathbf{b} = (1, 0, 2), \mathbf{c} = (4, 0, 4)$
3. Consider a triangle defined by the vertices $(6, 10, -2)$, $(3, -1, 17)$ and $(-9, 8, 0)$.
- What is the equation of the plane containing this triangle?
 - Is the point $(3, 4, 5)$ on the front or back side of this plane?
How far is this point from the plane?
4. Consider the set of five points $(7, 11, -5)$, $(2, 3, 8)$, $(-3, 3, 1)$, $(-5, -7, 0)$ and $(6, 3, 4)$. An *axis aligned bounding box (AABB)* is the smallest box whose edges are aligned with the coordinate axes that contains all the points, defined by its minimum and maximum vertices \mathbf{p}_{min} and \mathbf{p}_{max} .
- What are \mathbf{p}_{min} and \mathbf{p}_{max} for the above five points?
 - List all eight vertices of the AABB.
 - Determine the centre point \mathbf{c} of the AABB.
 - Multiply the five points by the following matrix (a 45° rotation about the z -axis):
$$\begin{pmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
 - What is the AABB of these transformed points?

- f. What is the AABB we get by transforming the original AABB? (i.e. the bounding box of the transformed corner points).
5. A robot is at the position (1, 10, 3) and her right, up and forward vectors (expressed in world space) are $\begin{pmatrix} 0.866 \\ 0 \\ -0.5 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0.5 \\ 0 \\ 0.866 \end{pmatrix}$ respectively (note that these vectors form an orthonormal basis).
The following points are expressed in object space; calculate their coordinates in world space:
- a. (-1, 2, 0)
 - b. (1, 2, 0)
 - c. (0, 0, 0)
 - d. (1, 5, 0.5)
 - e. (0, 5, 10)

The coordinates below are in world space; find their positions relative to the robot:

- f. (1, 10, 3)
- g. (0, 0, 0)
- h. (2.732, 10, 2)
- i. (2, 11, 4)
- j. (1, 20, 3) [p758, 3.6.6]