



COMP220: Graphics & Simulation  
**10: Simulation & Animation**

# Learning outcomes

By the end of this week, you should be able to:

- ▶ **Recall** the key concepts involved in solving mechanics problems
- ▶ **Write** programs which feature realistic physics simulations
- ▶ **Describe** how a rigged model is transformed to produce animation

# Agenda

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- ▶ Lecture (async):
  - ▶ **Recap** the mathematical concepts of Newtonian physics.
  - ▶ **Explain** the role of rigging in 3D animation.
  - ▶ **Introduce** the scene graph and techniques for enhancing character animation.

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  - ▶ **Explain** the role of rigging in 3D animation.
  - ▶ **Introduce** the scene graph and techniques for enhancing character animation.
- ▶ Workshop (sync):
  - ▶ **Include** the Bullet Physics library in the OpenGL application.
  - ▶ **Explore** physics and animation techniques in code.

# Newtonian Mechanics



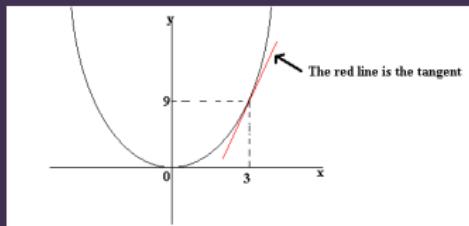
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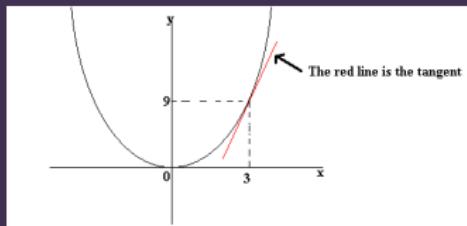
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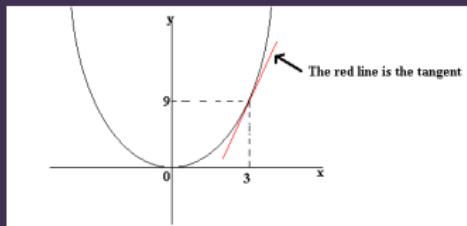
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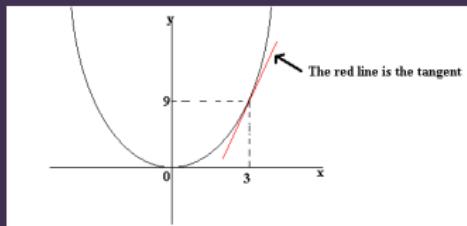
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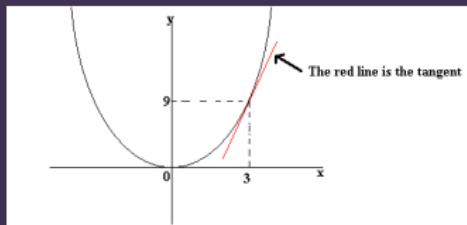
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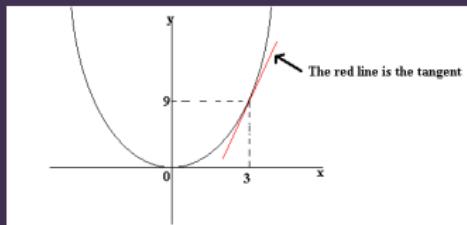
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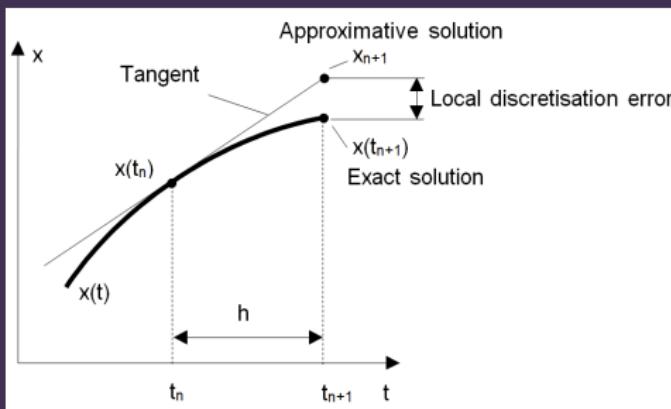
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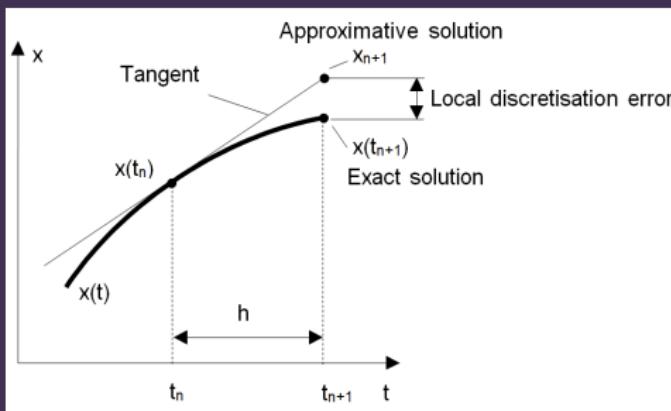
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$$x(t+h) \approx x(t) + h \times \frac{dx}{dt}(t)$$

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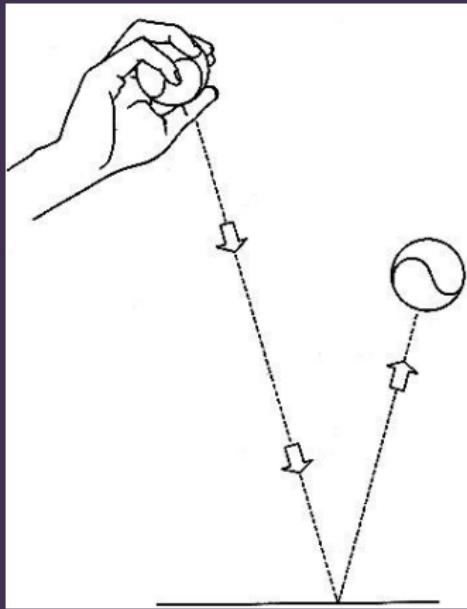
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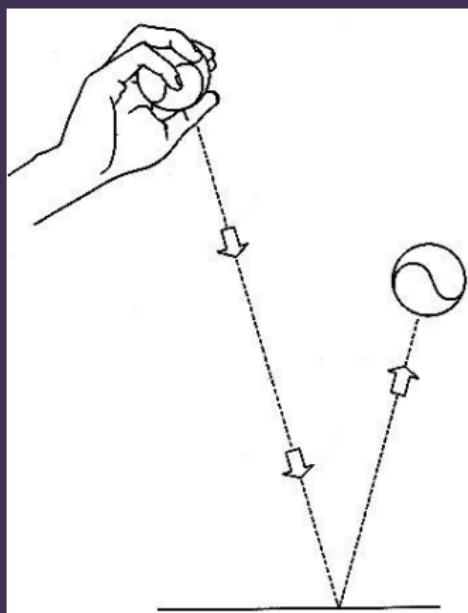
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When one body exerts a force on another, the second body exerts an equal and opposite force on the first

# Basic collision response

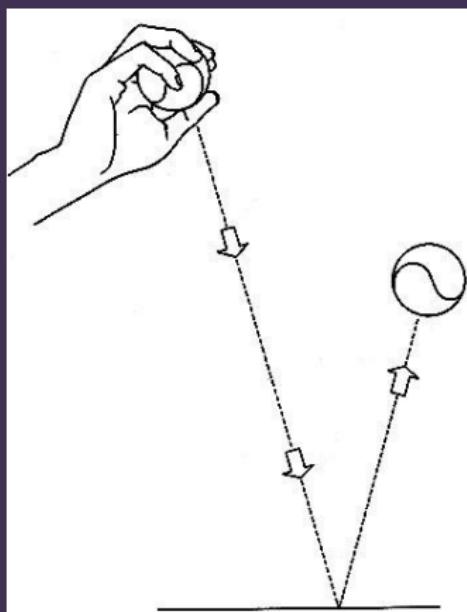


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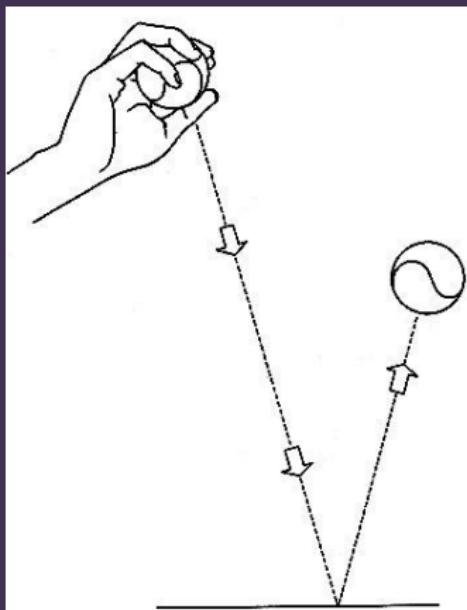
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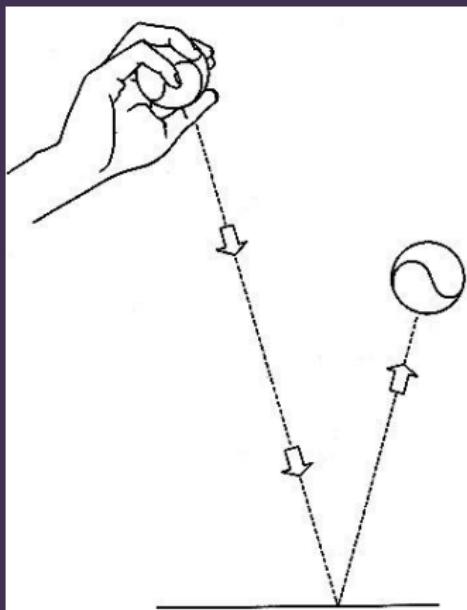
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- ▶ Flip the  $y$  component and multiply it by something between 0 and 1

# Skeletal Animation



# Coordinate spaces

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Model space

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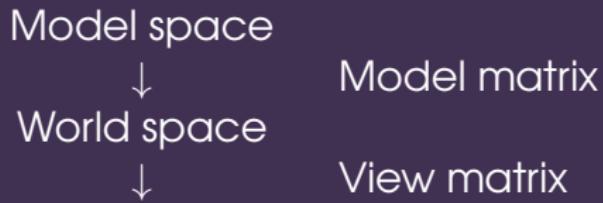


Model matrix

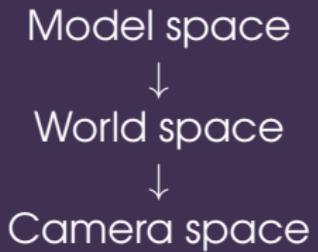
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Model matrix  
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- ▶ When performing calculations, **do not mix** vectors from **different coordinate spaces**
- ▶ E.g. when performing lighting calculations, ensure your fragment position, normal, light direction, eye direction are all in the **same** space

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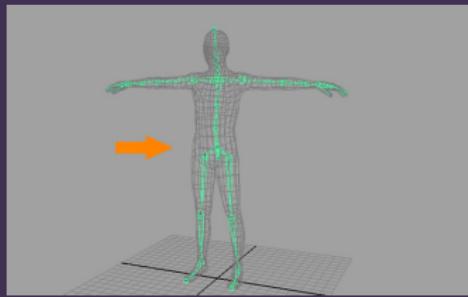
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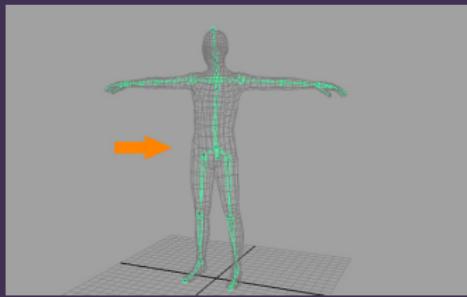
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- ▶ The model matrix is the **product** of model matrices for the node and its ancestors

# Rigging



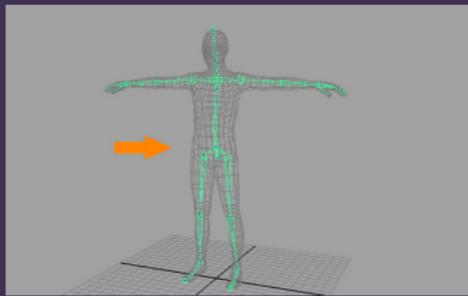
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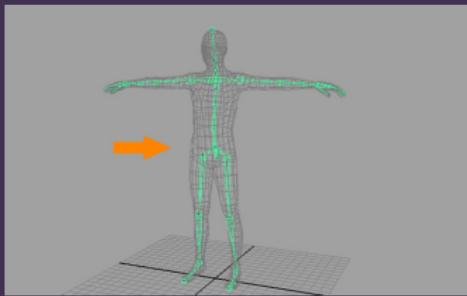
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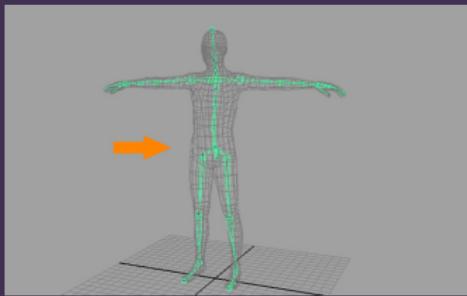


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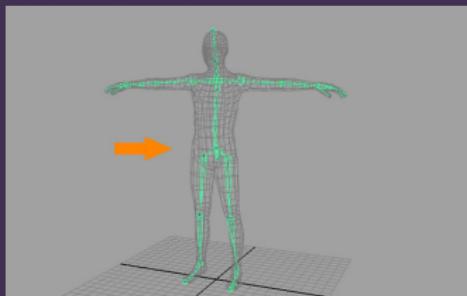


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  - ▶ 3D modelling software often represents bones as lines from parent bone to child bone

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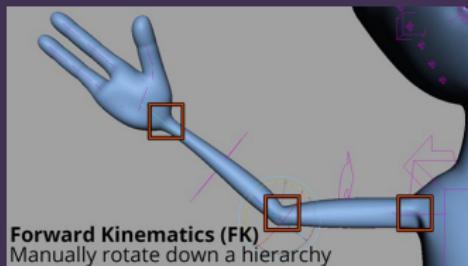
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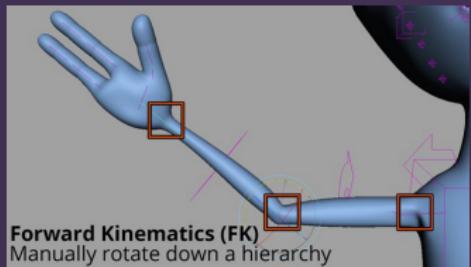
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  - ▶ E.g. bottom half plays “walk” animation, top half plays “fire weapon” animation

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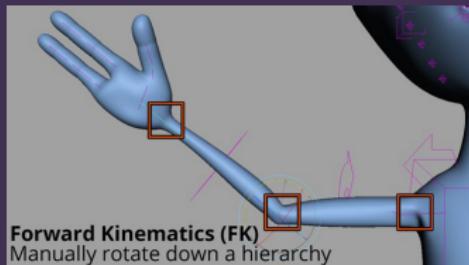


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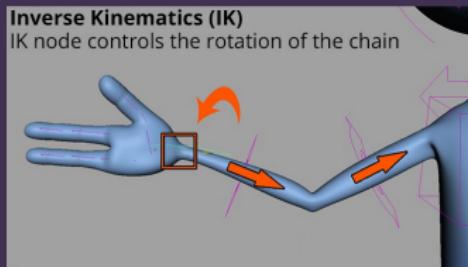
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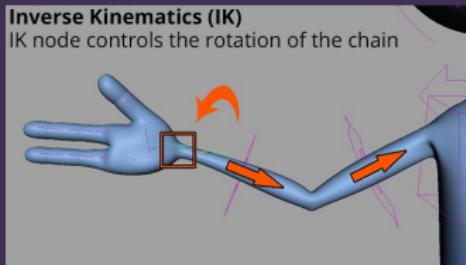


- ▶ Bone transformations are set **explicitly**
- ▶ Children are affected by parent transformations, e.g. if upper arm rotates, lower arm rotates with it

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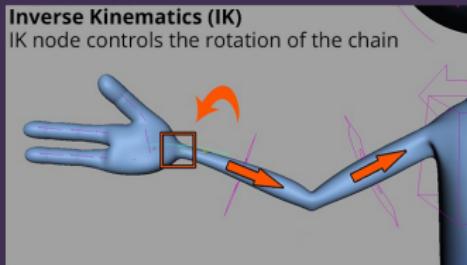


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- ▶ Bone transformations are calculated to reach a **target**
- ▶ E.g. we want character's hand to touch an object; IK calculates rotations of upper and lower arm to achieve this subject to constraints

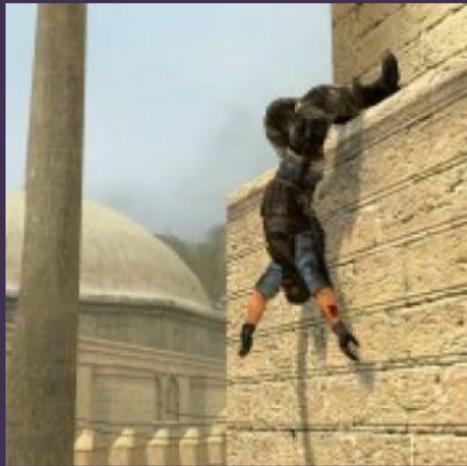
# The most common use for IK



# Ragdolls



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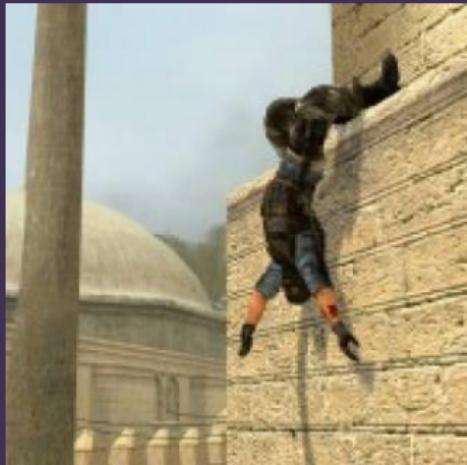
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- ▶ Attach a **rigid body** to each bone and run a **physics simulation**
- ▶ Often used for death animations
- ▶ Many games mix and blend some or all of keyframe animation, IK, and physics simulation

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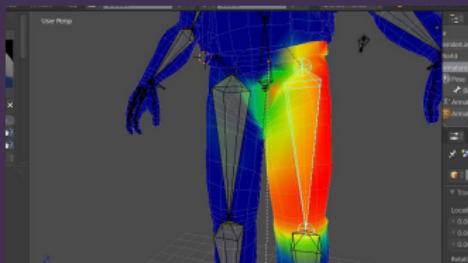
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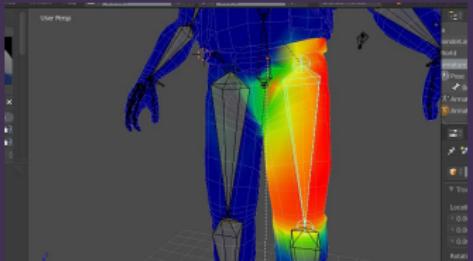
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- ▶ Generally handled by a **vertex shader**
- ▶ Uses **bone weights** to specify how much each vertex is affected by each bone's transformation

# Bone weights

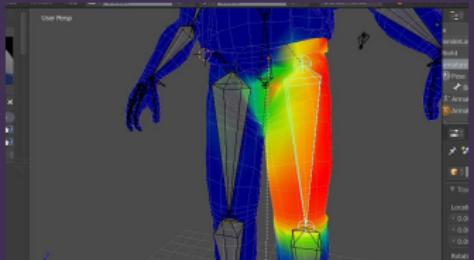


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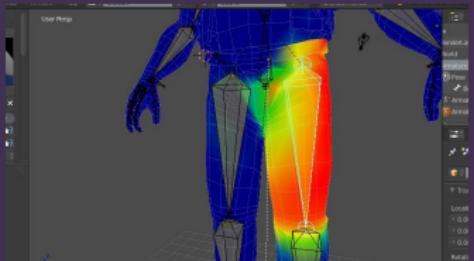


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- ▶ Vertex position is calculated as a weighted average of joint transforms

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- ▶ Can be **simulated** using a variety of techniques, though this is often expensive even for simplified methods
- ▶ A popular approach is to use **morph targets** (aka **blend shapes**)

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Image source: [https://antongerdelan.net/opengl/blend\\_shapes.html](https://antongerdelan.net/opengl/blend_shapes.html)

# Next steps

- ▶ **Review** the additional asynchronous material for more background on physics and animation techniques.
- ▶ **Attend** the workshop to learn how to use the Bullet Physics library in your own applications.