
COMP110: Principles of Computing

# 4: Logic and memory

# Learning outcomes

- **Distinguish** the basic types of logic gate
- **Use** logic gates to build simple circuits
- **Explain** how computer memory works

# Logic gates

# Boolean logic

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- Works with two values: TRUE and FALSE
- Foundation of the **digital computer**: represented in circuits as **on** and **off**
- Representing as 1 and 0 leads to **binary notation**
- One boolean value = one **bit** of information
- Programmers use boolean logic for conditions in `if` and `while` statements

# Simulating logic circuits

`http://logic.ly/demo/`

# Not

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NOT *A* is TRUE
if and only if
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| *A* | NOT *A* |
|---|---|
| FALSE | TRUE |
| TRUE | FALSE |

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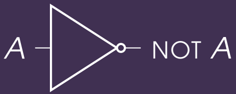

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if and only if
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|---|---|---|
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# Socrative `FALCOMPED`

What is the value of

$$A \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

# Socrative FALCOMPED

What is the value of

$$(\text{NOT } A) \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

# Socrative `FALCOMPED`

For what values of $A, B, C, D$ is

$$A \text{ AND NOT } B \text{ AND NOT } (C \text{ OR } D) = \text{TRUE}$$

?

# Socrative `FALCOMPED`

What is the value of

$$A \text{ OR NOT } A$$

?

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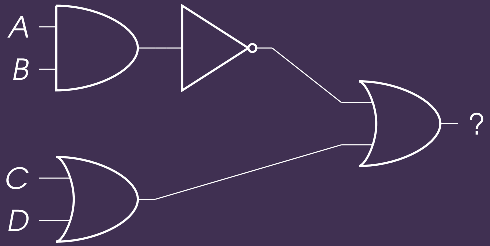
# Socrative `FALCOMPED`

## (not (A and B)) or (C or D)

What expression is equivalent to this circuit?



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Other operators can be expressed by combining these

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How can $A$ XOR $B$ be written using the operations AND , OR , NOT ?

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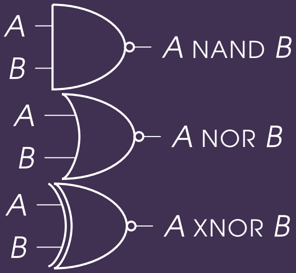

# Binary notation

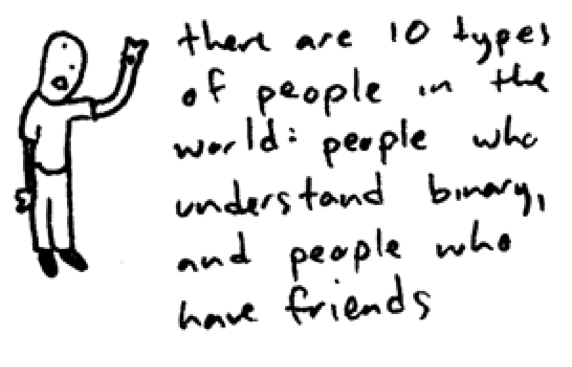


Image credit: http://www.toothpastefordinner.com

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  - $\left(6 \times 10^3\right) + \left(3 \times 10^2\right) + \left(9 \times 10^1\right) + \left(7 \times 10^0\right)$

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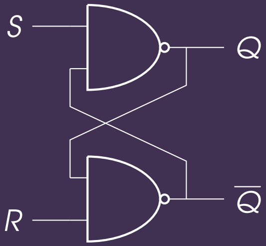
# Computer memory

# What does this circuit do?



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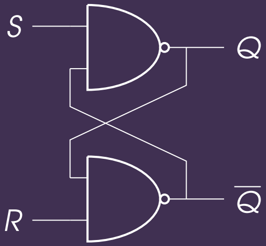


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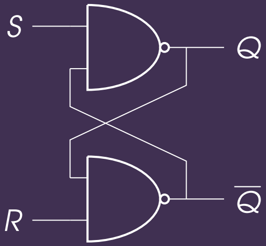


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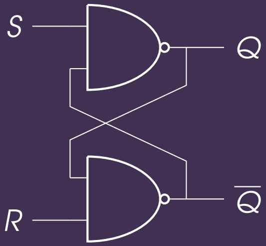


- This is called a **NAND latch**
- It "remembers" a single boolean value
- Put a few billion of these together (along with some control circuitry) and you've got **memory**!

# Memory

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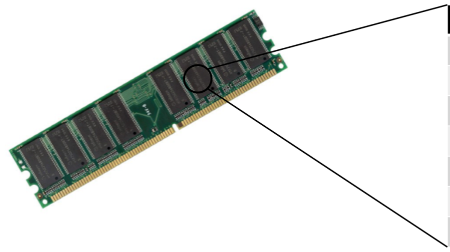

| Address | Data |
|---:|---|
| 0 | 01001000 |
| 1 | 01100101 |
| 2 | 01101100 |
| 3 | 01101100 |
| 4 | 01101111 |
| 5 | 00100001 |
| ... | ... |

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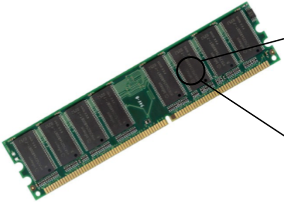

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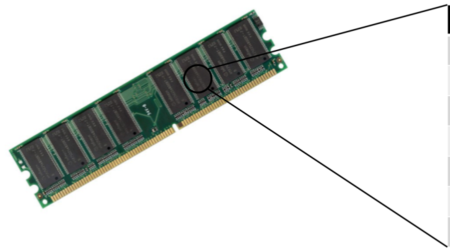

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- Each box contains a **byte** (8 bits)

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  - Executable: sequence of machine code operations

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# Worksheet B