

COMP110: Principles of Computing









Binary notation

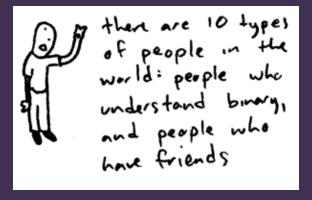


Image credit: http://www.toothpastefordinner.com

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Converting to binary

https://www.youtube.com/watch?v=OezK_zTyvAQ

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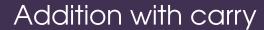
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Hex	Dec	Hex	Dec	Hex	Dec
00	0	10	16	F0	240
01	1	11	17	F1	241
		:		:	
09	9	19	25	F9	249
0A	10	1A	26	FA	250
0B	11	1в	27	FB	251
0C	12	1C	28	FC	252
0D	13	1D	29	FD	253
ΟE	14	1E	30	FE	254
OF	15	1F	31	FF	255





2's Complement

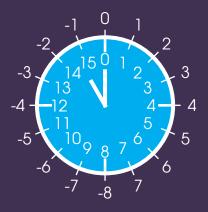




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Modular arithmetic



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▶
$$14 + 7 = 5$$

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- Useful for wrapping around e.g. loop indexes or screen coordinates

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- Note that the left-most bit can be interpreted as a sign bit: 1 if negative, 0 if positive or zero

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- This is also the process for converting back from 2's complement, i.e. doing it twice should give the original number

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- ▶ In fact, subtraction can just be done as addition
- ▶ I.e. a b is the same as a + (-b), where a and -b are just n-bit numbers





Worksheet 2

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Due next Friday!
Online quiz on LearningSpace





Algorithms

What is an algorithm?

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A sequence of instructions which can be followed step by step to perform a (computational) task.

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- Computers developed as machines for carrying out mathematical algorithms

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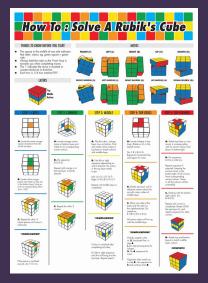
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 - E.g. it implements an algorithm for determining where to break a line of text, how much space to add to centre a line, etc.

Algorithms outside computing

- Preheat the oven to 180C, gas 4.
- 2 Beat together the eggs, flour, caster sugar, butter and baking powder until smooth in a large mixing bowl.
- Put the cocoa in separate mixing bowl, and add the water a little at a time to make a stiff paste. Add to the cake mixture.
- 4 Turn into the prepared tins, level the top and bake in the preheated oven for about 20-25 mins, or until shrinking away from the sides of the tin and springy to the touch.
- 5 Leave to cool in the tin, then turn on to a wire rack to become completely cold before icing.
- To make the icing: measure the cream and chocolate into a bowl and carefully melt over a pan of hot water over a low heat, or gently in the microwave for 1 min (600w microwave). Stir until melted, then set aside to cool a little and to thicken up.
- To ice the cake: spread the apricot jam on the top of each cake. Spread half of the ganache icing on the top of the jam on one of the cakes, then lay the other cake on top, sandwiching them together.
- 8 Use the remaining ganache icing to ice the top of the cake in a swirl pattern. Dust with icing sugar to serve.

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- Computability theory lets us reason about what computations are and are not possible