COMP250: Artificial Intelligence

5: Monte Carlo Tree Search

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Assignment check-in

AI component

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- **Peer review** will run during **development week** (next week)

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- Complete your reviews by **11:59PM Friday 9th March**

Research wiki

MicroRTS bot

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- The server is now live!

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- See the `comp250-bot` repository on GitHub for details

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Monte Carlo evaluation

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- Designing a good heuristic requires in-depth knowledge of the game
- What if you don't have such knowledge?

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- Then the **expected value** of X is

$$\sum_x x \cdot p(x)$$

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- What this means: if you play the slot machine N times, on average you will win $N \times$ £0.40

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- Seed is generally based on some source of **entropy**, e.g. system clock, mouse input, electronic noise

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- Applications in physics, engineering, finance, weather forecasting, graphics, ...

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- Higher expected value $=$ more chance of winning

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  - Not commonly used, mainly because there's something better...

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- First few rollouts are **random**
- However, statistics from these rollouts are used to **bias** future rollouts
- Bias rollouts towards **plausible** lines of play, i.e. where each player is trying to play the best move

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  - **Backpropagation**: For each node visited during **selection** and **expansion**, update the node's statistics based on the result of the simulation.
- Perform many rollouts, then use the statistics at the top level of the tree to choose the best move

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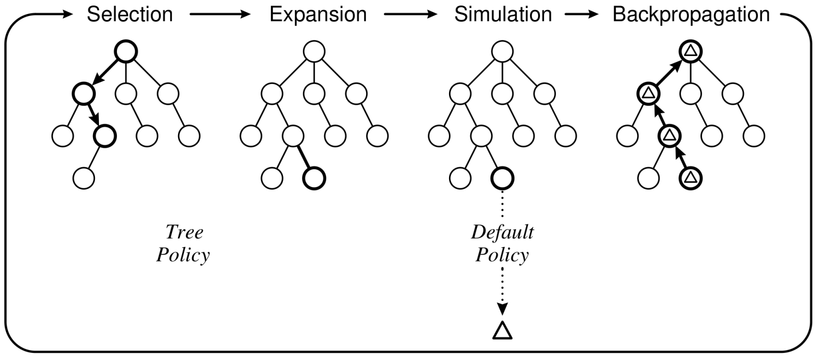

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- This can be modelled as a **multi-armed bandit problem**

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  - c is a parameter for adjusting the balance between exploitation and exploration

UCB demo

http://orangehelicopter.com/academic/bandits.html?ucb

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- From node p, choose the child q such that

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UCT demo

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  - Selects which parts of the tree to expand more deeply

MCTS for games of imperfect information