COMP110: Principles of Computing

3: Data Types

Binary notation

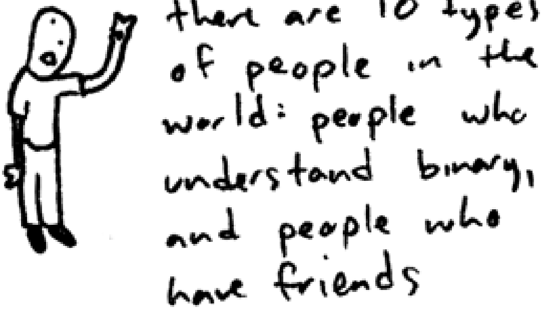


Image credit: http://www.toothpastefordinner.com

How we write numbers

- We write numbers in **base 10**
- We have 10 **digits**: $0, 1, 2, \ldots, 8, 9$
- When we write 6397, we mean:
  - Six thousand, three hundred and ninety seven
  - (Six thousands) and (three hundreds) and (nine tens) and (seven)
  - $(6 \times 1000) + (3 \times 100) + (9 \times 10) + (7)$
  - $(6 \times 10^3) + (3 \times 10^2) + (9 \times 10^1) + (7 \times 10^0)$
  -

Thousands	Hundreds	Tens	Units
6	3	9	7

Binary

- Binary notation works the same, but is **base 2** instead of **base 10**
- We have 2 **digits**: 0, 1
- When we write 10001011 in binary, we mean:
  $$(1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4)$$
  $$+ (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$
  $$= 2^7 + 2^3 + 2^1 + 2^0$$
  $$= 128 + 8 + 2 + 1 \text{ (base 10)}$$
  $$= 139 \text{ (base 10)}$$

Why binary?

- Modern computers are **digital**
- Based on the flow of current in a circuit being either **on** or **off**
- Hence it is natural to store and operate on numbers in base 2
- The binary digits 0 and 1 correspond to **off** and **on** respectively

Converting to binary

https://www.youtube.com/watch?v=OezK_zTyvAQ

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
  - The smallest possible unit of information
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary
- A **word** is the number of bits that the CPU works with at once
  - 32-bit CPU: 32 bits = 1 word
  - 64-bit CPU: 64 bits = 1 word
- An *n*-bit word can store a number between 0 and $2^n - 1$
  - $2^{16} - 1 = 65,535$
  - $2^{32} - 1 = 4,294,967,295$
  - $2^{64} - 1 = 18,446,744,073,709,551,615$

Other units

- A **nibble** is 4 **bits**
- A **kilobyte** is 1000 or 1024 **bytes**
  - $10^3 = 1000 \approx 1024 = 2^{10}$
- A **megabyte** is 1000 or 1024 **kilobytes**
- A **gigabyte** is 1000 or 1024 **megabytes**
- A **terabyte** is 1000 or 1024 **gigabytes**
- A **petabyte** is 1000 or 1024 **terabytes**
- An **exabyte** is 1000 or 1024 **petabytes**
- A **zettabyte** is 1000 or 1024 **exabytes**
- ...

Addition with carry

In base 10:

$$
\begin{array}{ccccc}
 & 1 & 2 & 3 & 4 \\
+ & 5 & 6_1 & 7_1 & 8 \\
\hline
 & 6 & 9 & 1 & 2
\end{array}
$$

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$\begin{array}{cccccccccc} & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ + & 0_1 & 0_1 & 1 & 0_1 & 0_1 & 1_1 & 1 & 1 \\ \hline & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}$$

Hexadecimal notation

- Other number bases than 2 and 10 are also useful
- Hexadecimal is **base 16**
- Uses extra digits:
  - `A`=10, `B`=11, ..., `F`=15

Hex	Dec	Hex	Dec	Hex	Dec
`00`	0	`10`	16	`F0`	240
`01`	1	`11`	17	`F1`	241
⋮	⋮	⋮	⋮	⋮	⋮
`09`	9	`19`	25	`F9`	249
`0A`	10	`1A`	26	`FA`	250
`0B`	11	`1B`	27	`FB`	251
`0C`	12	`1C`	28	`FC`	252
`0D`	13	`1D`	29	`FD`	253
`0E`	14	`1E`	30	`FE`	254
`0F`	15	`1F`	31	`FF`	255

Numeric types

Integers

- An **integer** is a whole number — positive, negative or zero
- Stored in memory using binary notation, with 2's complement for negative values
- In C#: `int` is a 32-bit integer, `long` is a 64-bit integer
- In Python: `int` is a "big integer" — expands number of bits automatically to fit the value to be stored

Floating point numbers

- What about storing non-integer numbers?
- Usually we use **floating point** numbers
- Details on in-memory representation later in the module
- In C#: **`float`** or **`double`**
- **`double`** is more precise, but uses twice as much memory (8 bytes vs 4 bytes)
- Python type: **`float`**, which has the same precision as C# **`double`**!

Integers vs floating point numbers

- Integers and floating point numbers are **different types**!
- `42` and `42.0` are technically different values
  - One is an **`int`**, the other is a **`double`**
  - They are stored differently in memory (completely different sequences of bytes)
  - However, `==` etc still know how to compare them sensibly

String types

Strings

- A **string** represents a sequence of textual characters
- E.g. `"Hello world!"`
- C# type: **`string`**
- Python type: **`str`**

String representation

- Stored as sequences of **characters** encoded as **integers**
- Often **null-terminated**
  - Character number 0 signifies the end of the string

What is a character?

- Broadly speaking, a single **printable symbol** or **glyph**
- (Actually a lot more complicated than this, but this will do for today)
- There are also some special **non-printable characters** e.g. line break

ASCII

- American Standard Code for Information Interchange
- Defines a standard set of 128 characters (7 bits per character)
- Originally developed in the 1960s for teletype machines, but survives in computing to this day
- 95 printable characters: upper and lower case English alphabet, digits, punctuation
- 33 non-printable characters

Hex	Value	Hex	Value	Hex	Value	Hex	Value	Hex	Value	Hex	Value	Hex	Value	Hex	Value
00	NUL	10	DLE	20	SP	30	0	40	@	50	P	60	`	70	p
01	SOH	11	DC1	21	!	31	1	41	A	51	Q	61	a	71	q
02	STX	12	DC2	22	"	32	2	42	B	52	R	62	b	72	r
03	ETX	13	DC3	23	#	33	3	43	C	53	S	63	c	73	s
04	EOT	14	DC4	24	$	34	4	44	D	54	T	64	d	74	t
05	ENQ	15	NAK	25	%	35	5	45	E	55	U	65	e	75	u
06	ACK	16	SYN	26	&	36	6	46	F	56	V	66	f	76	v
07	BEL	17	ETB	27	'	37	7	47	G	57	W	67	g	77	w
08	BS	18	CAN	28	(	38	8	48	H	58	X	68	h	78	x
09	HT	19	EM	29	)	39	9	49	I	59	Y	69	i	79	y
0A	LF	1A	SUB	2A	*	3A	:	4A	J	5A	Z	6A	j	7A	z
0B	VT	1B	ESC	2B	+	3B	;	4B	K	5B	[	6B	k	7B	{
0C	FF	1C	FS	2C	,	3C	<	4C	L	5C	\	6C	l	7C	|
0D	CR	1D	GS	2D	-	3D	=	4D	M	5D	]	6D	m	7D	}
0E	SO	1E	RS	2E	.	3E	>	4E	N	5E	^	6E	n	7E	~
0F	SI	1F	US	2F	/	3F	?	4F	O	5F	_	6F	o	7F	DEL

ASCII

- ASCII works OK for English
- Standards exist to add another 128 characters (taking us to 8 bits per character)
- E.g. accented characters for European languages, other Western alphabets e.g. Greek, Cyrillic, mathematical symbols
- However 256 characters isn't enough...

Unicode

- Standard character set developed from 1987 to present day
- Currently defines 144697 characters (Unicode 14.0)
- First 128 characters are the same as ASCII
- Covers most of the world's writing systems
- Also covers mathematical symbols and emoji

Encoding Unicode

- **UTF-32** encodes characters as 32-bit integers
- **UTF-8** encodes characters as 8, 16, 24 or 32-bit integers
  - 8-bit characters correspond to the first 128 ASCII characters $\Longrightarrow$ backwards compatible
  - More common Unicode characters are smaller $\Longrightarrow$ more efficient than UTF-32

UTF-8 representation

- For characters in ASCII, UTF-8 is the same:
  - a $\rightarrow$ [97]
- Other characters are encoded as multi-byte sequences:
  - ü $\rightarrow$ [195, 188]
  - 喂 $\rightarrow$ [229, 150, 130]
  - 😂 $\rightarrow$ [240, 159, 152, 130]
- `"Haha 😂"` encoded in UTF-8:

H	a	h	a	space	😂				null
72	97	104	97	32	240	159	152	130	0

Escape sequences

- Backslash \ has a special meaning in string literals — it denotes the start of an **escape sequence**
- Typically used to write **non-printable characters**
- Most useful: `"\n"` is a new line
- How to type a backslash character? Use `"\\"`

Text files

- Stored on disk as essentially one long string
- Line endings are denoted by non-printable characters
  - Unix format: line feed character (ASCII/UTF-8 character 10, `"\n"`)
  - Windows format: carriage return character (ASCII/UTF-8 character 13) followed by line feed, `"\r\n"`
  - Most text editors can handle and convert both formats
  - Most languages allow files to be opened in “text mode” which automatically converts

Booleans

Booleans

- A **boolean** can have one of two values: **true** or **false**
- Named after George Boole (1815–1864), British mathematician
- Type in both C# and Python: **`bool`**
- In Python, we have the keywords `True` and `False`
- In C#, we have **`true`** and **`false`**
- Could be represented by a single bit in memory...
- ... but since memory is addressed in bytes (or words of multiple bytes), usually represented as an **`int`** with 0 meaning **`false`** and any non-zero (e.g. 1) meaning **`true`**

Boolean values

- The `if` statement takes a boolean value as its condition:

```
if (x > 10)
{
    Debug.Log(x);
}
```

- Variables can also store boolean values:

```
bool result = x > 10;  // result now stores True or False
if (result)
{
    Debug.Log(x);
}
```

Boolean Logic

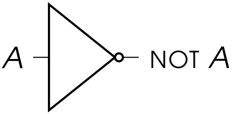
Boolean logic

- Works with two values: TRUE and FALSE
- Foundation of the **digital computer**: represented in circuits as **on** and **off**
- Representing as 1 and 0 leads to **binary notation**
- One boolean value = one **bit** of information
- Programmers use boolean logic for conditions in `if` and `while` statements

Not

NOT A is TRUE
if and only if
A is FALSE

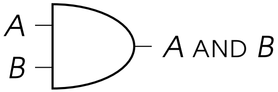
A	NOT A
FALSE	TRUE
TRUE	FALSE



And

A AND *B* is TRUE
if and only if
both *A* **and** *B* are TRUE

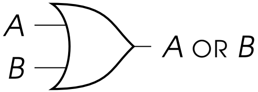
A	*B*	*A* AND *B*
FALSE	FALSE	FALSE
FALSE	TRUE	FALSE
TRUE	FALSE	FALSE
TRUE	TRUE	TRUE



Or

A OR *B* is TRUE
if and only if
either *A* **or** *B*, **or both,** are TRUE

A	*B*	*A* OR *B*
FALSE	FALSE	FALSE
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE
TRUE	TRUE	TRUE



Socrative `FALCOMPED`

What is the value of

$$A \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$(\text{NOT } A) \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative `FALCOMPED`

For what values of A, B, C, D is

$$A \text{ AND NOT } B \text{ AND NOT } (C \text{ OR } D) = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ OR NOT } A$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ AND NOT } A$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ OR } A$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ AND } A$$

?

Writing logical operations

Operation	Python	C#	Mathematics
NOT A	`not a`	`!a`	$\neg A$ or $\overline{A}$
A AND B	`a and b`	`a && b`	$A \wedge B$
A OR B	`a or b`	`a || b`	$A \vee B$

Other operators can be expressed by combining these

De Morgan's Laws

$$\text{NOT } (A \text{ OR } B) = (\text{NOT } A) \text{ AND } (\text{NOT } B)$$

$$\text{NOT } (A \text{ AND } B) = (\text{NOT } A) \text{ OR } (\text{NOT } B)$$

Proof: see this week's worksheet

Truth tables

Enumeration

- Since booleans have only two possible values, we can often **enumerate** all possible values of a set of boolean variables
- For n variables there are 2^n possible combinations
- Essentially, all the n-bit binary numbers
- A **truth table** enumerates all the possible values of a boolean expression
- Can be used to prove that two expressions are equivalent

Truth table example

(A OR NOT B) AND C

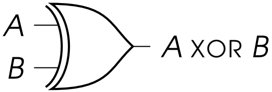
A	B	C	NOT B	A OR NOT B	(A OR NOT B) AND C
FALSE	FALSE	FALSE	TRUE	TRUE	FALSE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	FALSE	FALSE	FALSE	FALSE
FALSE	TRUE	TRUE	FALSE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE	FALSE
TRUE	TRUE	TRUE	FALSE	TRUE	TRUE

Other logic gates

Exclusive Or

A XOR *B* is TRUE
if and only if
either *A* **or** *B*, **but not both,** are TRUE

A	*B*	*A* XOR *B*
FALSE	FALSE	FALSE
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE
TRUE	TRUE	FALSE



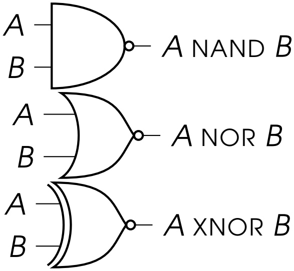
Socrative `FALCOMPED`

How can *A* XOR *B* be written using the operations AND , OR , NOT ?

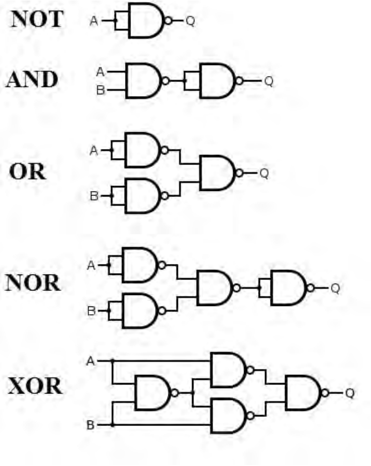
Negative gates

NAND , NOR , XNOR
are the **negations** of
AND , OR , XOR

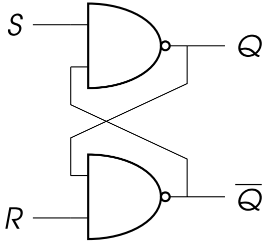
$$A \text{ NAND } B = \text{NOT } (A \text{ AND } B)$$
$$A \text{ NOR } B = \text{NOT } (A \text{ OR } B)$$
$$A \text{ XNOR } B = \text{NOT } (A \text{ XOR } B)$$



Any logic gate can be constructed from NAND gates



What does this circuit do?



- This is called a **NAND latch**
- It "remembers" a single boolean value
- Put a few billion of these together (along with some control circuitry) and you've got **memory**!

NAND gates

- All arithmetic and logic operations, as well as memory, can be built from NAND gates
- So an entire computer can be built just from NAND gates!
- NAND gate circuits are **Turing complete**
- The same is true of NOR gates

Workshop activity

- Split into **breakout groups**
- Play `http://nandgame.com`
- We'll reconvene and see how far everyone got!