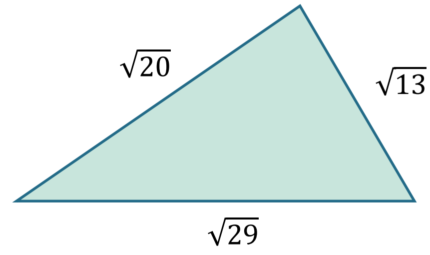
COMP270
Mathematics for 3D Worlds and Simulations

Week 3 Seminar: Vectors and the Dot Product

1. An NPC is standing at location $\mathbf{p}$ with a forward direction of $\mathbf{v}$.
   a. How can the dot product be used to determine whether the point $\mathbf{x}$ is in front of or behind the NPC?
   b. Let $\mathbf{p} = \binom{-3}{4}$ and $\mathbf{v} = \binom{5}{-2}$. For each of the following points $\mathbf{x}$, determine whether $\mathbf{x}$ is in front of or behind the NPC:
      i. $\mathbf{x} = \binom{0}{0}$
      ii. $\mathbf{x} = \binom{1}{6}$
      iii. $\mathbf{x} = \binom{-6}{0}$
      iv. $\mathbf{x} = \binom{-4}{7}$
      v. $\mathbf{x} = \binom{5}{5}$
      vi. $\mathbf{x} = \binom{-3}{0}$
      vii. $\mathbf{x} = \binom{-6}{-3.5}$
2. Extending the concept from question 3, consider the case where the NPC has a limited field of view (FOV). If the total FOV angle is Φ, then the NPC can see to the left or the right of its forward direction by a maximum angle of $\frac{\Phi}{2}$.
   a. How can the dot product be used to determine whether the point $\mathbf{x}$ is visible to the NPC?
   b. For each of the points $\mathbf{x}$ in question 3b, determine whether $\mathbf{x}$ is visible to the NPC if its FOV is 90°.
   c. Suppose that the NPC's viewing distance is also limited to a maximum distance of 7 units. Which points are visible to the NPC then?
3. A bus travels along a straight road, heading east-north-east through the origin, observed by Alex, who is standing two units east and one unit south of the origin.
   If the x-axis points east and the y-axis points north:
   a. Write the direction of the bus as a unit vector $\hat{\mathbf{b}}$ (magnitude 1).
      Hint: $\tan 22.5° = \sqrt{2} - 1$ *(proof here)*
   b. Write the displacement of Alex from the origin as a vector $\mathbf{a}$.
   c. Use the dot product to determine how far from the origin the bus has travelled when it is closest to Alex.
4. Use the dot product to find the area of this triangle:
   Hint: the area of a triangle is given by $\frac{1}{2} base \times height$ and it can be shown that for two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$,
   $$\| \mathbf{v}_1 - \mathbf{v}_2 \|^2 = \| \mathbf{v}_1 \|^2 + \| \mathbf{v}_2 \|^2 - 2\, \mathbf{v}_1 \cdot \mathbf{v}_2$$



Exercises may include some modified questions from:

Dunn, F & Parberry, I 2011, *3D Math Primer for Graphics and Game Development*, CRC Press, Boca Raton, FL
Southall, E & Pantaloni, V 2017, *Geometry Snacks*, Tarquin, St Albans UK
Povey, T 2015, *Professor Povey's Perplexing Problems*, Oneworld Publications, London UK

5. Give a vector proof that for a triangle inscribed within a semicircle, the included angle is always $\frac{\pi}{2}$ (90°).
   Hint: note that the dot product is both commutative ($\mathbf{v}_1 \cdot \mathbf{v}_2 = \mathbf{v}_2 \cdot \mathbf{v}_1$) and distributive ($\mathbf{v}_1 \cdot (\mathbf{v}_2 + \mathbf{v}_3) = \mathbf{v}_1 \cdot \mathbf{v}_2 + \mathbf{v}_1 \cdot \mathbf{v}_3$) – proof here.