COMP110: Principles of Computing

4: Logic and memory

Learning outcomes

- **Distinguish** the basic types of logic gate
- **Use** logic gates to build simple circuits
- **Explain** how computer memory works

FALMOUTH UNIVERSITY

Logic gates

Boolean logic

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- Representing as 1 and 0 leads to **binary notation**
- One boolean value = one **bit** of information
- Programmers use boolean logic for conditions in `if` and `while` statements

Simulating logic circuits

http://logic.ly/demo/

Not

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NOT *A* is TRUE
if and only if
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A	NOT *A*
FALSE	TRUE
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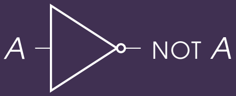

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Socrative FALCOMPED

What is the value of

$$A \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$(\text{NOT } A) \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative `FALCOMPED`

For what values of A, B, C, D is

$$A \text{ AND NOT } B \text{ AND NOT } (C \text{ OR } D) = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ OR NOT } A$$

?

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What is the value of

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What is the value of

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What is the value of

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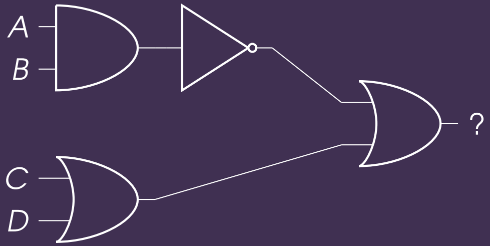
Socrative `FALCOMPED`

What expression is equivalent to this circuit?



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Other operators can be expressed by combining these

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How can A XOR B be written using the operations AND, OR, NOT?

Negative gates

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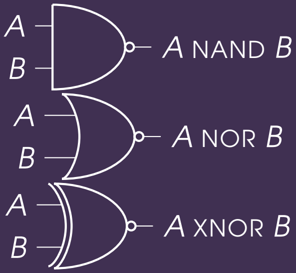

Binary notation

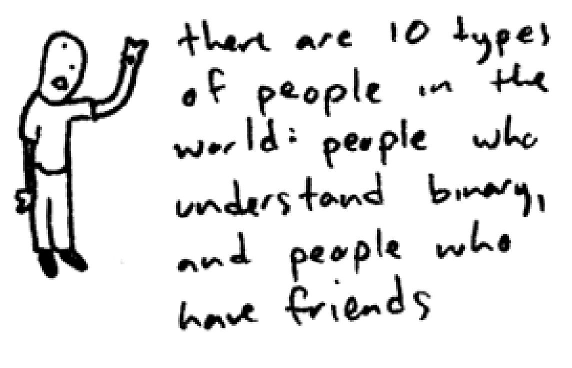

Image credit: http://www.toothpastefordinner.com

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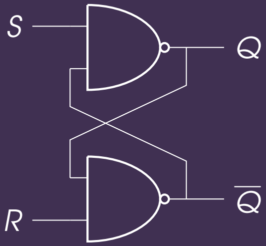
Computer memory

What does this circuit do?



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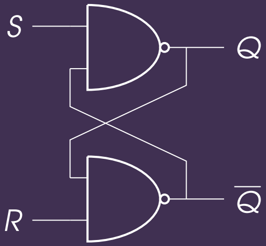

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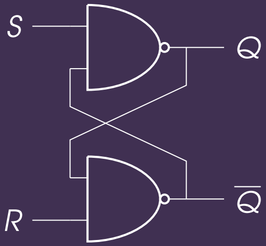

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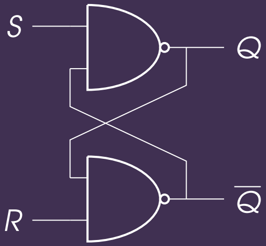

- This is called a **NAND latch**
- It "remembers" a single boolean value
- Put a few billion of these together (along with some control circuitry) and you've got **memory**!

Memory

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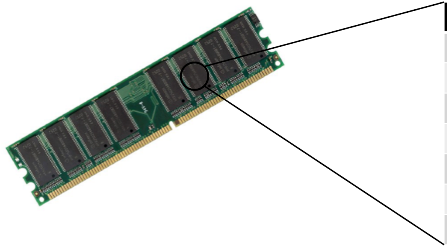

Address	Data
0	01001000
1	01100101
2	01101100
3	01101100
4	01101111
5	00100001
...	...

- Memory works like a set of **boxes**

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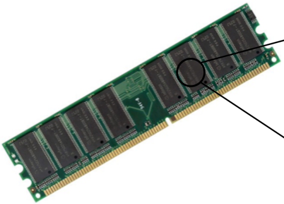

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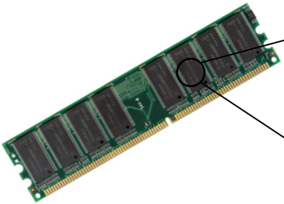

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- Each box contains a **byte** (8 bits)

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  - Executable: sequence of machine code operations

Arithmetic Logic Unit

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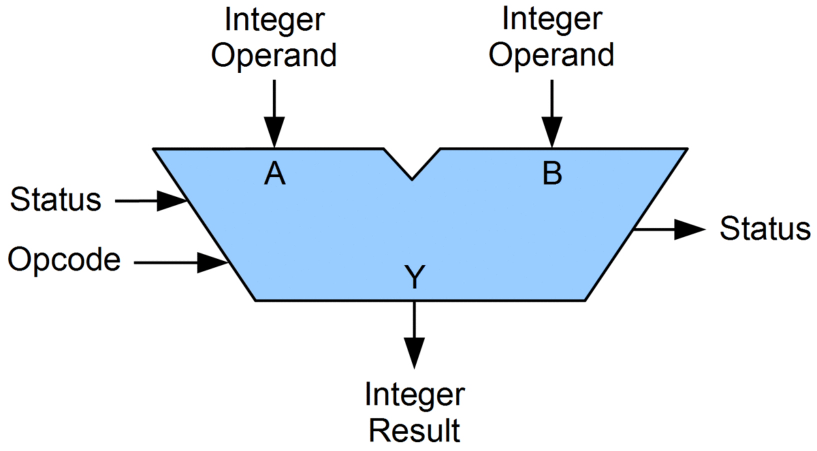

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- Opcode specifies how Y is calculated based on A and B

ALU operations

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- Increment, decrement
- Bitwise AND, OR, NOT, ...
- Bit shifts

Addition with carry

In base 10:

$$
\begin{array}{ccccc}
 & 1 & 2 & 3 & 4 \\
+ & 5 & 6 & 7 & 8 \\
\hline
\end{array}
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$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

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+ & 0_1 & 0_1 & 1 & 0_1 & 0_1 & 1_1 & 1 & 1 \\
\hline
 & & 0 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}$$

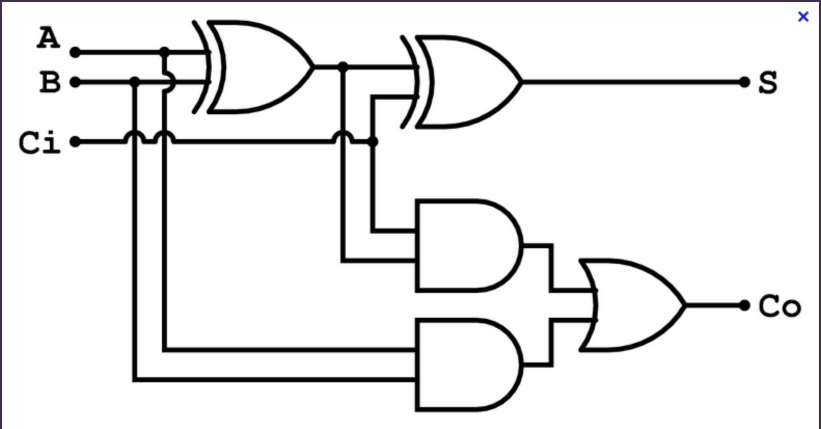
Addition with carry

In base 2:

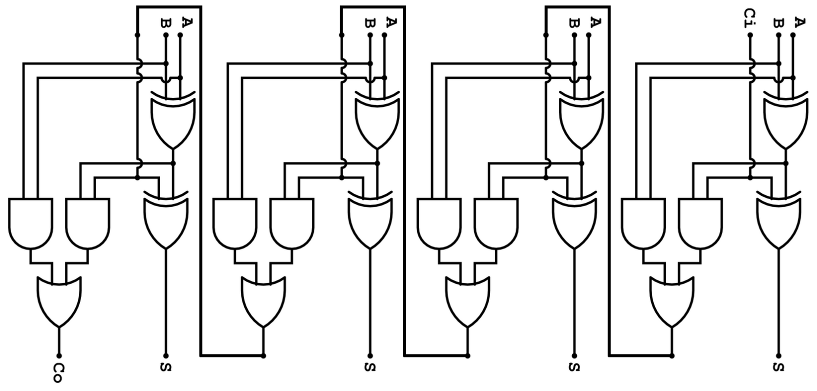
$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$\begin{array}{cccccccccc}
 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
+ & 0_1 & 0_1 & 1 & 0_1 & 0_1 & 1_1 & 1 & 1 \\
\hline
 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}$$

1-bit adder



n-bit adder



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Worksheet B