COMP270 – Vector Revision

Answer the following questions on 2D vectors using pen(cil) and (graph) paper.

Pro tip: show your working – diagrams can be helpful!

1. State whether the following assertions are true or false (and if false, explain why):
    a. The vector from $(1, 2)$ to $(3, 5)$ is the same as the vector from $(99, -100)$ to $(101, -97)$.
    b. The vector from $(-1, -2)$ to $(-3, -5)$ is the same as the vector from $(-99, -100)$ to $(-101, -97)$.
    c. The size of a vector in a diagram doesn't matter; we just need to draw it in the right place.
    d. The displacement expressed by a vector can be visualised as a sequence of axially aligned displacements.
    e. These axially aligned displacements must occur in a specific order.
    f. The vector $\begin{pmatrix} x \\ y \end{pmatrix}$ gives the displacement from the point (x, y) to the origin.
2. Let $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$. Draw the following:
    a. $\mathbf{a}$
    b. $\mathbf{b}$
    c. $2\mathbf{a}$
    d. $-\frac{1}{2}\mathbf{b}$
    e. $\mathbf{a} + \mathbf{b}$
    f. $\mathbf{a} - \mathbf{b}$
    g. $\frac{1}{2}\mathbf{a} + 2\mathbf{b}$
    h. $-2\mathbf{a} - \frac{1}{3}\mathbf{b}$
3. Evaluate the following expressions:
    a. $2\begin{pmatrix} 9 \\ -3 \end{pmatrix}$
    b. $\frac{1}{2}\begin{pmatrix} 4 \\ 5 \end{pmatrix}$
    c. $-\begin{pmatrix} -7 \\ 1 \end{pmatrix}$
    d. $\left\|\begin{pmatrix} -12 \\ 5 \end{pmatrix}\right\|$
    e. $\left\|\frac{1}{3}\begin{pmatrix} 27 \\ -12 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -6 \\ 24 \end{pmatrix}\right\|$
    f. $\left\|\frac{-1}{\sqrt{2}}\begin{pmatrix} \sqrt{2} \\ 2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 \\ \frac{-2}{\sqrt{2}} \end{pmatrix}\right\|$
4. Write the end points of the following vectors (relative to the origin) in row format:
    a. $\mathbf{a} + \mathbf{b}$, where $\mathbf{a} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ -9 \end{pmatrix}$ describe displacements from the origin.
    b. $\mathbf{a} - \mathbf{b}$, where $\mathbf{a} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ -9 \end{pmatrix}$ describe displacements from the point $(-1, -1)$.
    c. The vector with length 5, starting at $(1, 2)$ and with direction pointing towards $(7, 10)$.
5. Find the angles between the following pairs of vectors $\mathbf{a}$ and $\mathbf{b}$ using trigonometry/`SOHCAHTOA':
    a. $\mathbf{a} = \begin{pmatrix} 3 \\ \sqrt{3} \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$
    b. $\mathbf{a} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

6. For each pair of vectors in question 5, evaluate their dot product using the algebraic definition

$$\mathbf{a}\cdot\mathbf{b} = a_x b_x + a_y b_y$$

and check the result against your answers to question 5 using the identity

$$\mathbf{a}\cdot\mathbf{b} = \|\boldsymbol{a}\|\|\boldsymbol{b}\|\cos\theta$$

7. Write down any two vectors that are (i) parallel and (ii) perpendicular to:
   a. $\binom{1}{1}$
   b. $\binom{-1}{-1}$
   c. $\binom{1}{2}$
   d. $\binom{0}{0}$

   Verify that your answers are correct using the dot product.

8. An NPC is standing at location $\mathbf{p}$ with a forward direction of $\mathbf{v}$.
   a. How can the dot product be used to determine whether the point $\mathbf{x}$ is in front of or behind the NPC?
   b. Let $\mathbf{p} = \binom{-3}{4}$ and v = $\binom{5}{-2}$. For each of the following points $\mathbf{x}$, determine whether $\mathbf{x}$ is in front of or behind the NPC:
      i. $\mathbf{x} = \binom{0}{0}$
      ii. $\mathbf{x} = \binom{1}{6}$
      iii. $\mathbf{x} = \binom{-6}{0}$
      iv. $\mathbf{x} = \binom{-4}{7}$
      v. $\mathbf{x} = \binom{5}{5}$
      vi. $\mathbf{x} = \binom{-3}{0}$
      vii. $\mathbf{x} = \binom{-6}{-3.5}$

9. Extending the concept from question 8, consider the case where the NPC has a limited field of view (FOV). If the total FOV angle is Φ, then the NPC can see to the left of the right of its forward direction by a maximum angle of $\Phi/2$.
   a. How can the dot product be used to determine whether the point $\mathbf{x}$ is visible to the NPC?
   b. For each of the points $\mathbf{x}$ in question 8, determine whether $\mathbf{x}$ is visible to the NPC if its FOV is 90°.
   c. Suppose that the NPC's viewing distance is also limited to a maximum distance of 7 units. Which points are visible to the NPC then?

<u>Reference/Further reading:</u>

3D Math Primer for Graphics and Game Development, Fletcher Dunn and Ian Parberry, CRC Press