COMP220: Graphics & Simulation

3: Mathematics for graphics

Learning outcomes

- Outcome 1
- Outcome 2
- Outcome 3

Vectors

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A vector has **components**

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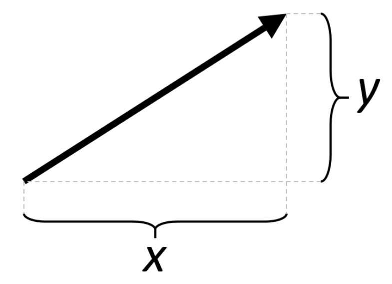

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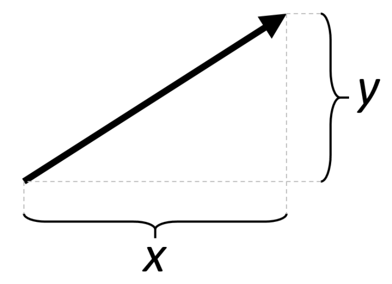

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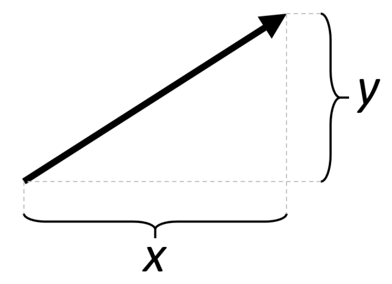

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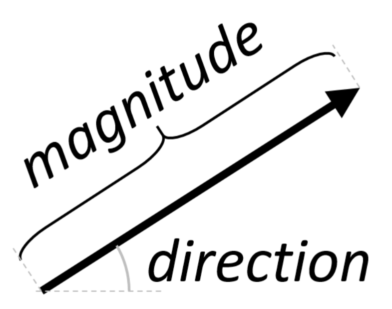

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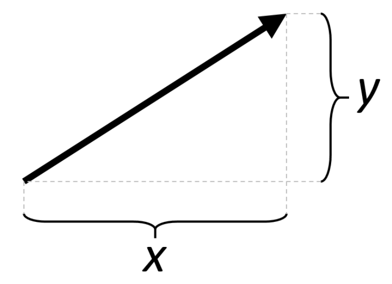

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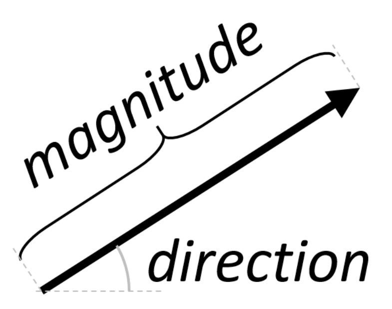

The **origin** is the point represented by the vector $(0, 0, \ldots)$

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- Careful! Some things in OpenGL work in **degrees**, others in **radians** (just to confuse you...)

Right hand rule

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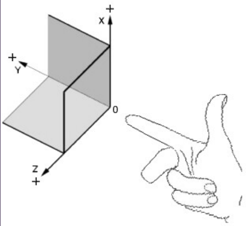

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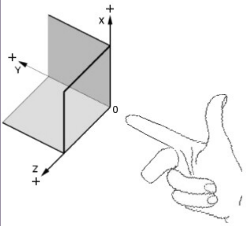

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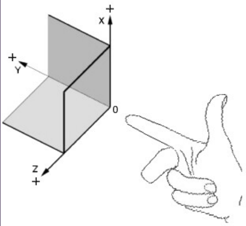

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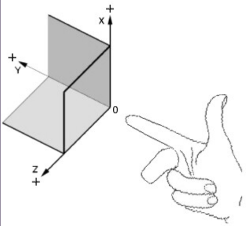

- The *x*-**axis** points towards the **right-hand side** of the screen
- The *y*-**axis** points towards the **top** of the screen
- The *z*-**axis** points **out** of the screen

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- In homogeneous coordinates, the origin is $(0, 0, 0, 1)$ not $(0, 0, 0, 0)$!

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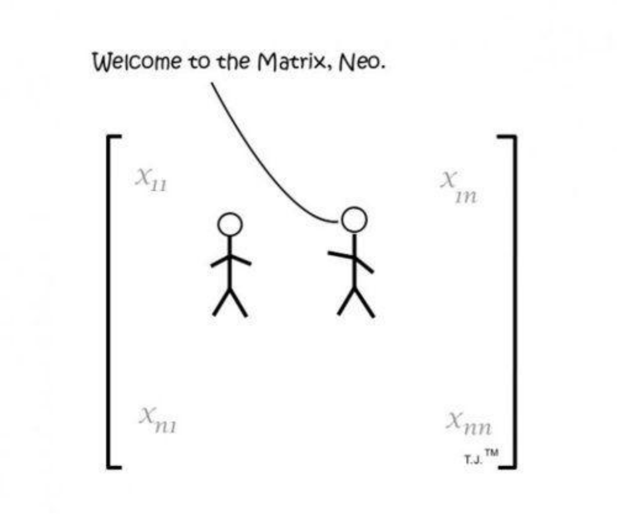
Matrices


Welcome to the Matrix, Neo.
x_{11}
x_{1n}
x_{n1}
x_{nn}
T.J.™

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- Note: the plural of **matrix** is **matrices**
- In computer graphics we mostly work with **square** matrices (number of rows = number of columns)

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- (But you don't really need to know how to calculate these manually...)

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- Multiplication of matrices is **not commutative**
  - In general, $A \times B \neq B \times A$
  - There may be some matrices where $A \times B = B \times A$, but they are the exception

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Transformations

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- Multiplying a vector by the matrix **applies** the transformation

GLM

We will use the **GLM** library to do matrix calculations for us

`http://glm.g-truc.net/`

GLM aims to mirror GLSL data types (`vec4`, `mat4` etc) in C++

Identity

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```
// Default constructor for glm::mat4 creates an ↩
    identity matrix
glm::mat4 transform;
```

Translation

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```
transform = glm::translate(transform, glm::vec3(0.3f,
    0.5f, 0.0f));
```

Scaling

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```
transform = glm::scale(transform, glm::vec3(1.2f, 0.5f
    , 1.0f));
```

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```
float angle = glm::pi<float>() * 0.5f;
glm::vec3 axis(0, 0, 1);
transform = glm::rotate(transform, angle, axis);
```

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- The order they are applied is the **reverse** of what you might think — i.e. the above rotates **then** translates

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  - The x-axis $(1, 0, 0)$
  - The y-axis $(0, 1, 0)$
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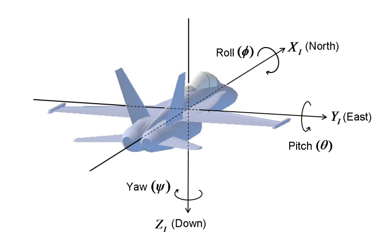
- These angles are sometimes called **roll**, **pitch** and **yaw**



Gimbal lock

https://youtu.be/rrUCBOlJdt4?t=1m55s