COMP110: Principles of Computing

4: Logic and memory

Learning outcomes

- **Distinguish** the basic types of logic gate
- **Use** logic gates to build simple circuits
- **Explain** how computer memory works

Logic gates

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- Representing as 1 and 0 leads to **binary notation**
- One boolean value = one **bit** of information
- Programmers use boolean logic for conditions in `if` and `while` statements

Not

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NOT A is TRUE
if and only if
A is FALSE

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NOT *A* is TRUE
if and only if
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A	NOT *A*
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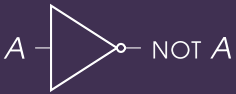

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Socrative `FALCOMPED`

What is the value of

$$A \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative FALCOMPED

What is the value of

$$(\textsc{not}\ A)\ \textsc{and}\ (B\ \textsc{or}\ C)$$

when

$$A = \textsc{True}$$
$$B = \textsc{False}$$
$$C = \textsc{True}$$

?

Socrative `FALCOMPED`

For what values of A, B, C, D is

$$A \text{ AND NOT } B \text{ AND NOT } (C \text{ OR } D) = \text{TRUE}$$

?

Socrative FALCOMPED

What is the value of

$$A \text{ OR NOT } A$$

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Socrative `FALCOMPED`

What is the value of

$$A \text{ AND NOT } A$$

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What is the value of

$$A \text{ OR } A$$

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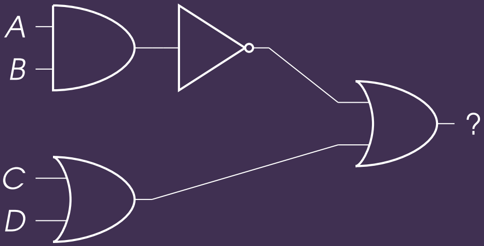
Socrative `FALCOMPED`

What expression is equivalent to this circuit?



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Other operators can be expressed by combining these

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Socrative `FALCOMPED`

How can A XOR B be written using the operations AND, OR, NOT?

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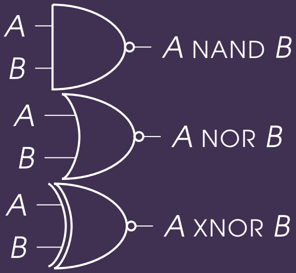
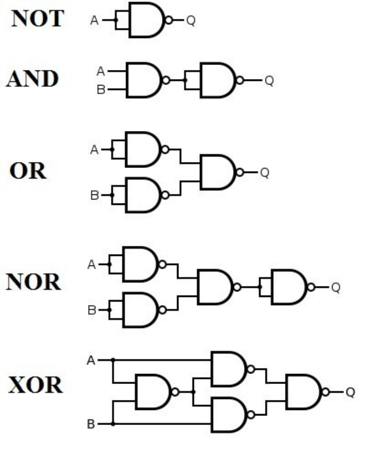
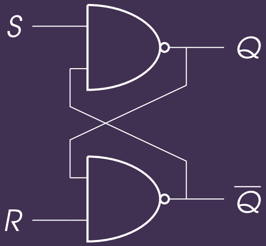


Any logic gate can be constructed from NAND gates



What does this circuit do?



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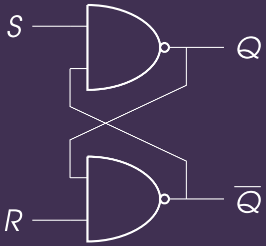

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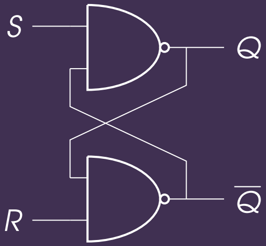

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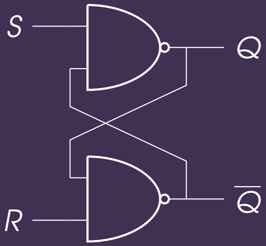

- This is called a **NAND latch**
- It "remembers" a single boolean value
- Put a few billion of these together (along with some control circuitry) and you've got **memory**!

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- Play the game: `http://nandgame.com`
- NAND gate circuits are **Turing complete**
- The same is true of NOR gates

Arithmetic Logic Unit

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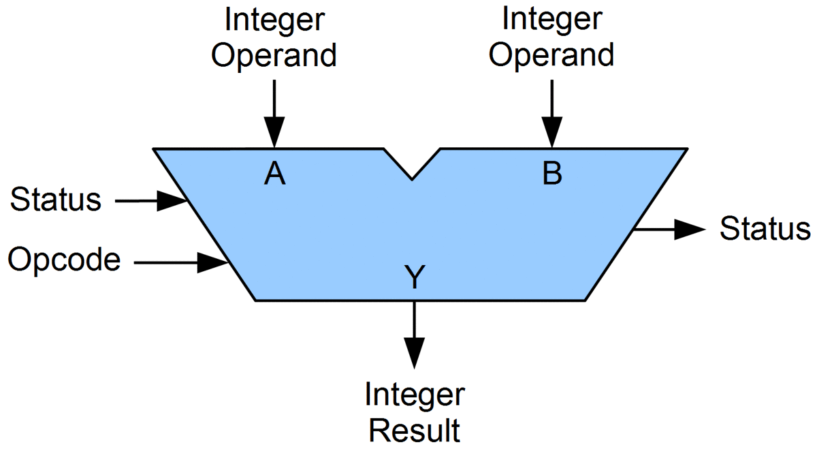

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  - **Operand** words A, B
  - **Opcode**
  - **Status** bits
- Outputs:
  - **Result** word Y
  - **Status** bits
- Opcode specifies how Y is calculated based on A and B

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ALU operations

Typically include:

- Add with carry
- Subtract with borrow
- Negate (2's complement)
- Increment, decrement
- Bitwise AND, OR, NOT, ...
- Bit shifts

Adding 3 bits

A	B	C	$A + B + C$
0	0	0	00
0	0	1	01
0	1	0	01
0	1	1	10
1	0	0	01
1	0	1	10
1	1	0	10
1	1	1	11

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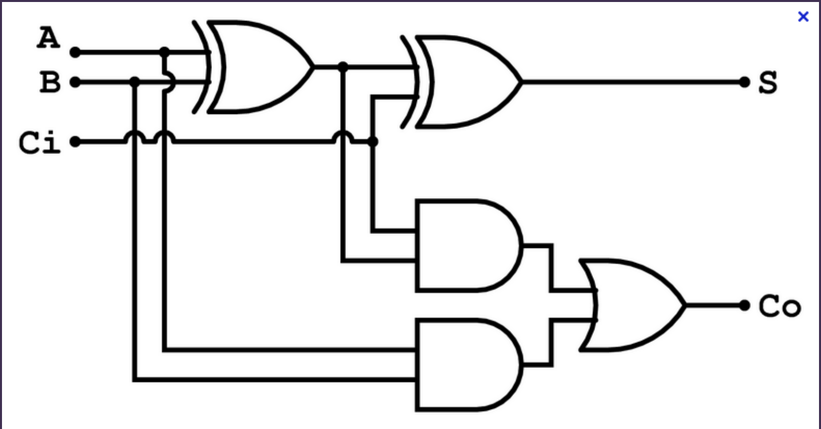
1-bit adder

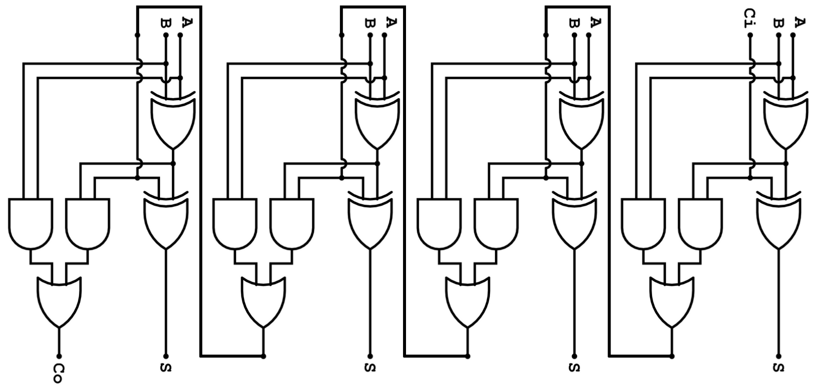


How does the 1-bit adder work?

Exercise:

- Write down the boolean expressions for S and Co
- Draw a truth table for these
- Compare the truth table to the addition table on a previous slide

n-bit adder



Exercise Sheet ii

Due **next Tuesday!**