COMP250: Artificial Intelligence

3: Game Theory and Planning

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Game theory

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- Important applications in economics, ecology and social sciences as well as AI

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- If **Bob betrays Alice**, he receives an A whilst she gets expelled
- If **both betray each other**, both get an F

Payoff matrix

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	A silent	A betray
B silent	A: 50 B: 50	A: 70 B: -100
B betray	A: -100 B: 70	A: 0 B: 0

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... and Bob's thought process is the same!

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- If all players are **rational** (in the sense of wanting to maximising payoff), they should converge upon a Nash equilibrium

Does every game have a Nash equilibrium?

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	A rock	A paper	A scissors
B rock	A: 0 B: 0	A: +1 B: -1	A: -1 B: +1
B paper	A: -1 B: +1	A: 0 B: 0	A: +1 B: -1
B scissors	A: +1 B: -1	A: -1 B: +1	A: 0 B: 0

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- E.g. if you always choose paper, I choose scissors
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  - If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...

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- If we try to reason naïvely, we get stuck in a loop
  - If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...
- The optimum strategy is to be **unpredictable**
- Choose rock with probability $\frac{1}{3}$, paper with probability $\frac{1}{3}$, scissors with probability $\frac{1}{3}$

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- In contrast to a **pure** or **deterministic strategy**, which always chooses the same action
- If we allow mixed strategies, **every game has at least one Nash equilibrium**

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- Example:
  - If the guesses are 30, 40 and 80...
  - ... then the mean is $\frac{30+40+80}{3} = 50$...
  - ... so the winning guess is 30, as this is closest to $\frac{2}{3} \times 50 = 33.333$

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- ... and so on ad infinitum
- So the only **rational** guess is 0, as every rational player should guess 0 and $\frac{2}{3}$ of 0 is 0

Rationality

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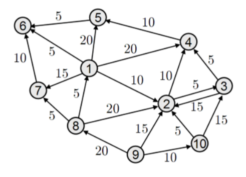
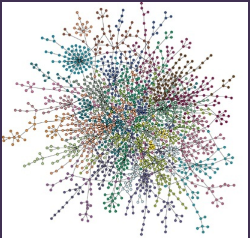
- However it's important to remember that **humans aren't always rational**

Graphs and trees

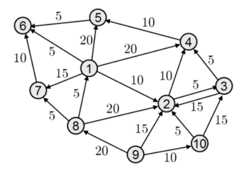
Graphs

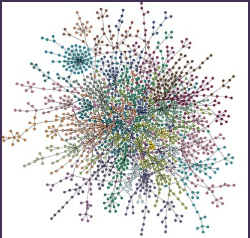
Graphs





Graphs





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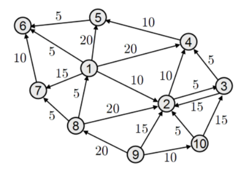

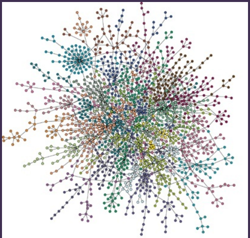

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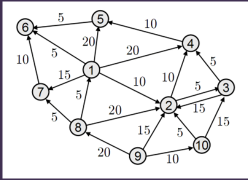

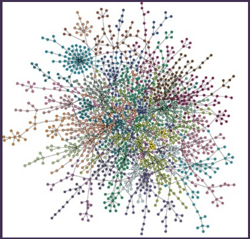

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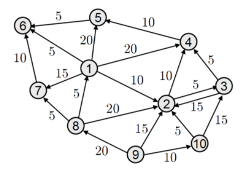

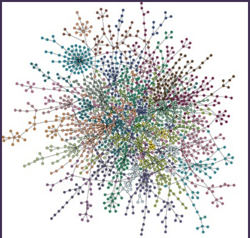

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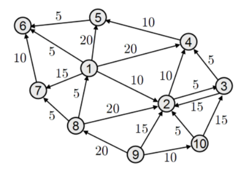

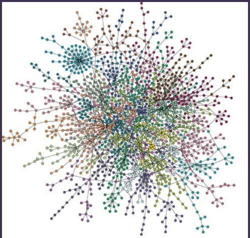

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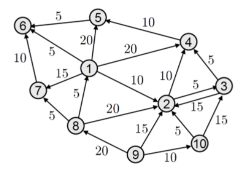

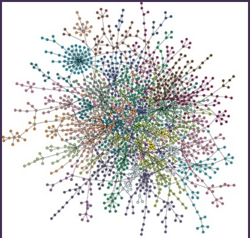

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- **Directed** graph: edges are arrows
- **Undirected** graph: edges are lines

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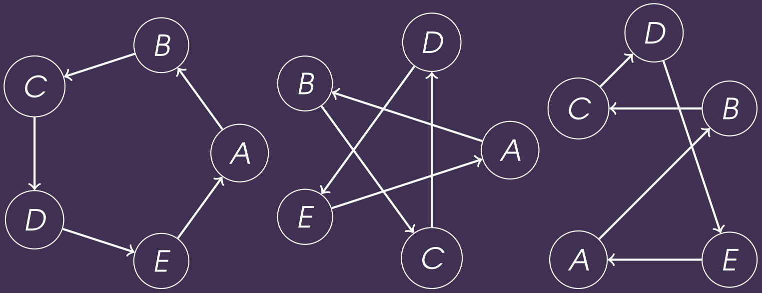
- E.g. these are technically the same graph:



Trees

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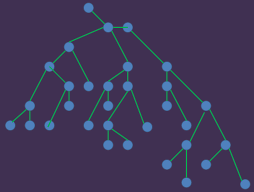

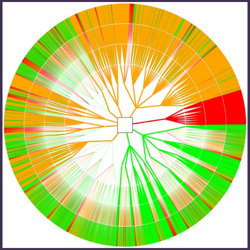

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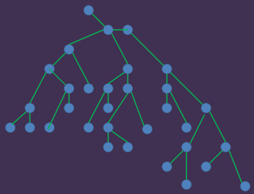

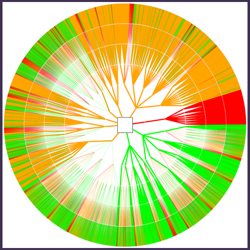

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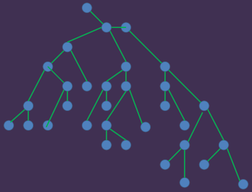

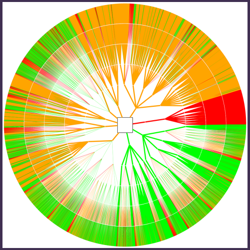

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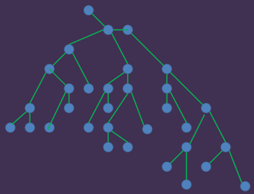

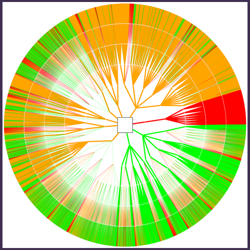

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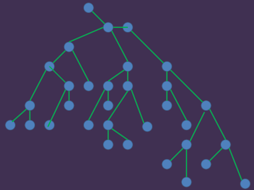

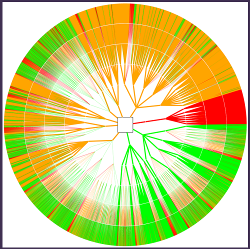

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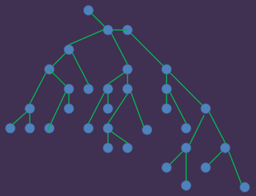

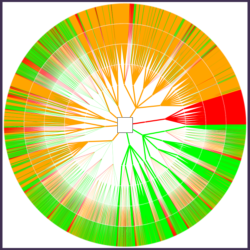

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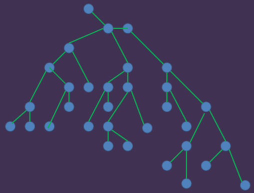

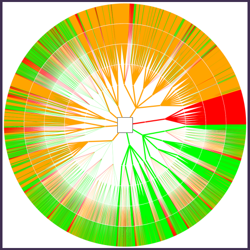

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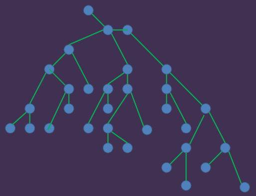

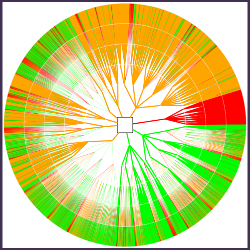

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  - Nodes can have 0, 1 or many children
- Used to model **hierarchies** (e.g. file systems, object inheritance, scene graphs, state-action trees, behaviour trees, ...)

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- The agent wants to change the state so as to achieve a **goal**
- Problem: find a sequence of actions that leads to the goal

STRIPS planning

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  - The **goal state** (a set of predicates, specifying whether each should be true or false)
  - The set of **actions**, each specifying:

STRIPS planning

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- Describes the state of the environment by a set of **predicates** which are true
- Models a problem as:
    - The **initial state** (a set of predicates which are true)
    - The **goal state** (a set of predicates, specifying whether each should be true or false)
    - The set of **actions**, each specifying:
        - Preconditions (a set of predicates which must be satisfied for this action to be possible)

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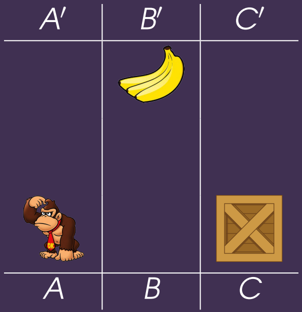
    - Postconditions (specifying what predicates are made true or false by this action)

STRIPS example



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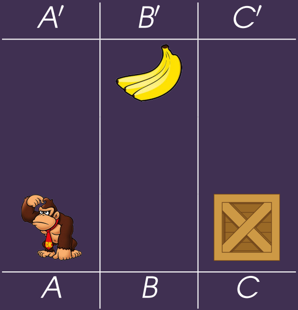

Initial state:

```
At(A),
BoxAt(C),
BananasAt(B')
```

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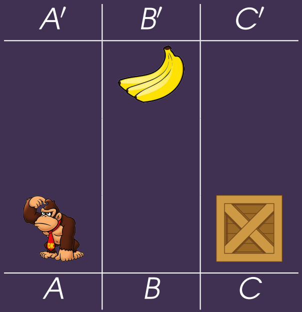

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Goal:

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HasBananas
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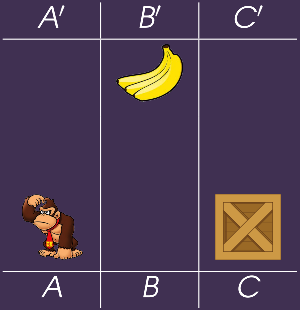
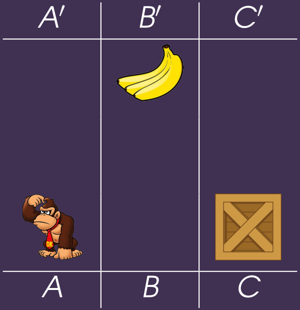
STRIPS example — Actions




```
Move(x, y)
  Pre:   At(x)
  Post: !At(x), At(y)

ClimbUp(x)
  Pre:   At(x), BoxAt(x)
  Post: !At(x), At(x′)

ClimbDown(x′)
  Pre:   At(x′), BoxAt(x)
  Post: !At(x′), At(x)

PushBox(x, y)
  Pre:   At(x), BoxAt(x)
  Post: !At(x), At(y),
        !BoxAt(x), BoxAt(y)

TakeBananas(x)
  Pre:   At(x), BananasAt(x)
  Post: !BananasAt(x), HasBananas
```

STRIPS example — Solution



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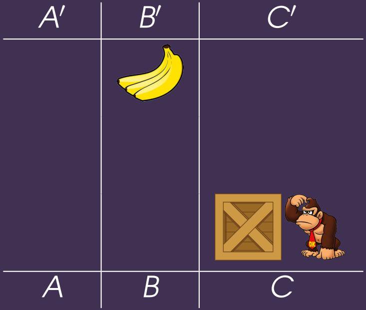

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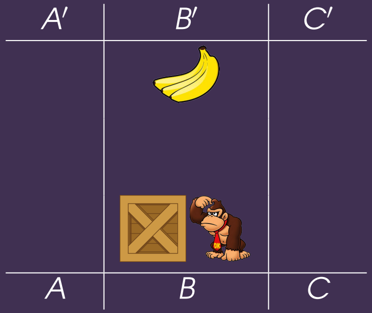

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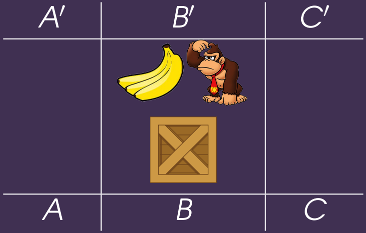

STRIPS example — Solution

A'	B'	C'
		
		
A	B	C

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- We can construct a **state-action graph**
  - Nodes: environment states
  - Edges: actions
- We can then **search** this tree to find a goal state

Searching for the solution

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  - Dijkstra's algorithm
  - A* (if we have a suitable **heuristic**)