COMP110: Principles of Computing

2: Basic Principles for Computation

Learning outcomes

By the end of this week's sessions, you should be able to:

- **Use** binary, decimal and hexadecimal notation to represent and operate on numerical values
- **Explain** the basic architecture of a computer
- **Distinguish** the most common programming languages and paradigms in use today

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  - With reference to appropriate academic sources

Marking rubric

See assignment brief on LearningSpace/GitHub

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- Finding and reading academic papers takes time and effort — don't leave it until the last minute!

Binary notation

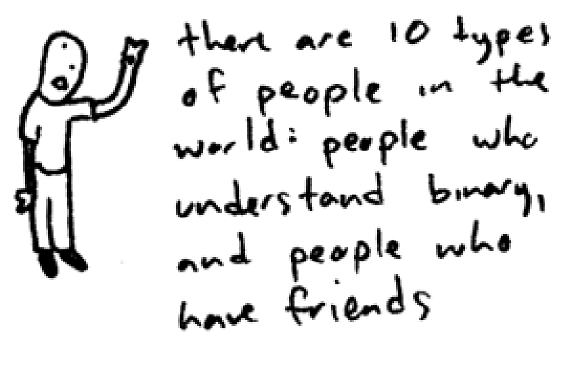

Image credit: `http://www.toothpastefordinner.com`

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  - $\left(6 \times 10^3\right) + \left(3 \times 10^2\right) + \left(9 \times 10^1\right) + \left(7 \times 10^0\right)$

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  $$= 139 \text{ (base 10)}$$

Converting to binary

https://www.youtube.com/watch?v=OezK_zTyvAQ

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Addition with carry

In base 10:

$$\begin{array}{ccccc} & 1 & 2 & 3 & 4 \\ + & 5 & 6 & 7 & 8 \\ \hline \end{array}$$

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\hline
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$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

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  - $4 - 7 = 13$

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- I.e. represent $-a$ as $2^n - a$
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- E.g. 16-bit number ranges from -32768 to $+32767$
- Note that the left-most bit can be interpreted as a **sign** bit: 1 if negative, 0 if positive or zero

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- (This is equivalent to subtracting the number from 2^n... why?)
- This is also the process for converting back from 2's complement, i.e. doing it twice should give the original number

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- In fact, subtraction can just be done as addition
- I.e. $a - b$ is the same as $a + (-b)$, where a and $-b$ are just n-bit numbers

Exercise Sheet i

Due next Tuesday!