COMP110: Principles of Computing

4: Logic and memory

Worksheet 4

Due **next Friday!**

Logic gates

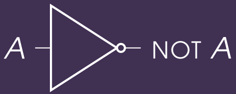
Boolean logic

- Works with two values: TRUE and FALSE
- Foundation of the **digital computer**: represented in circuits as **on** and **off**
- Representing as 1 and 0 leads to **binary notation**
- One boolean value = one **bit** of information
- Programmers use boolean logic for conditions in `if` and `while` statements

Not

NOT *A* is TRUE
if and only if
A is FALSE

A	NOT *A*
FALSE	TRUE
TRUE	FALSE



And

A AND *B* is TRUE
if and only if
both *A* **and** *B* are TRUE

A	*B*	*A* AND *B*
FALSE	FALSE	FALSE
FALSE	TRUE	FALSE
TRUE	FALSE	FALSE
TRUE	TRUE	TRUE

Or

A OR *B* is TRUE
if and only if
either *A* **or** *B*, **or both,** are TRUE

A	*B*	*A* AND *B*
FALSE	FALSE	FALSE
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE
TRUE	TRUE	TRUE

Socrative `FALCOMPED`

What is the value of

$$A \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$(\text{NOT } A) \text{ AND } (B \text{ OR } C)$$

when

$$A = \text{TRUE}$$
$$B = \text{FALSE}$$
$$C = \text{TRUE}$$

?

Socrative `FALCOMPED`

For what values of A, B, C, D is

$$A \text{ AND NOT } B \text{ AND NOT } (C \text{ OR } D) = \text{TRUE}$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ OR NOT } A$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ AND NOT } A$$

?

Socrative `FALCOMPED`

What is the value of

$$A \text{ OR } A$$

?

Socrative `FALCOMPED`

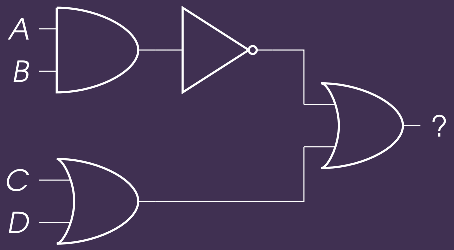
What is the value of

$$A \text{ AND } A$$

?

Socrative `FALCOMPED`

What expression is equivalent to this circuit?



Writing logical operations

Operation	Python	C family	Mathematics
NOT A	`not a`	`!a`	$\neg A$ or $\overline{A}$
A AND B	`a and b`	`a && b`	$A \wedge B$
A OR B	`a or b`	`a || b`	$A \vee B$

Other operators can be expressed by combining these

De Morgan's Laws

$$\text{NOT } (A \text{ OR } B) = (\text{NOT } A) \text{ AND } (\text{NOT } B)$$

$$\text{NOT } (A \text{ AND } B) = (\text{NOT } A) \text{ OR } (\text{NOT } B)$$

Proof: Worksheet 4, questions 3a and 3b

Truth tables

Enumeration

- Since booleans have only two possible values, we can often **enumerate** all possible values of a set of boolean variables
- For n variables there are 2^n possible combinations
- Essentially, all the n-bit binary numbers
- A **truth table** enumerates all the possible values of a boolean expression
- Can be used to prove that two expressions are equivalent

Truth table example

(A OR NOT B) AND C

A	B	C	NOT B	A OR NOT B	(A OR NOT B) AND C
FALSE	FALSE	FALSE	TRUE	TRUE	FALSE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	FALSE	FALSE	FALSE	FALSE
FALSE	TRUE	TRUE	FALSE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE	FALSE
TRUE	TRUE	TRUE	FALSE	TRUE	TRUE

Other logic gates

Exclusive Or

A XOR *B* is TRUE
if and only if
either *A* **or** *B*, **but not both,** are TRUE

A	*B*	*A* AND *B*
FALSE	FALSE	FALSE
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE
TRUE	TRUE	FALSE

Socrative `FALCOMPED`

How can A XOR B be written using the operations AND , OR , NOT ?

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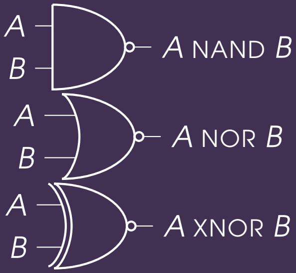
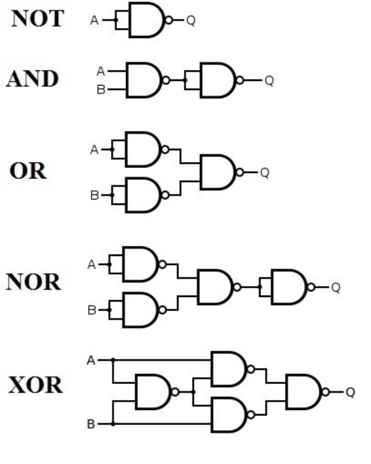
Negative gates

NAND , NOR , XNOR
are the **negations** of
AND , OR , XOR

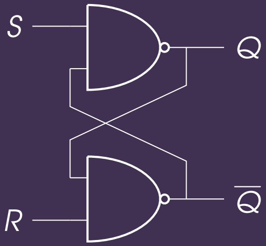
$$A \text{ NAND } B = \text{NOT } (A \text{ AND } B)$$
$$A \text{ NOR } B = \text{NOT } (A \text{ OR } B)$$
$$A \text{ XNOR } B = \text{NOT } (A \text{ XOR } B)$$



Any logic gate can be constructed from NAND gates



What does this circuit do?



- This is called a **NAND latch**
- It "remembers" a single boolean value
- Put a few billion of these together (along with some control circuitry) and you've got **memory**!

NAND gates

- All arithmetic and logic operations, as well as memory, can be built from NAND gates
- So an entire computer can be built just from NAND gates!
- Play the game: `http://nandgame.com`
- NAND gate circuits are **Turing complete**
- The same is true of NOR gates

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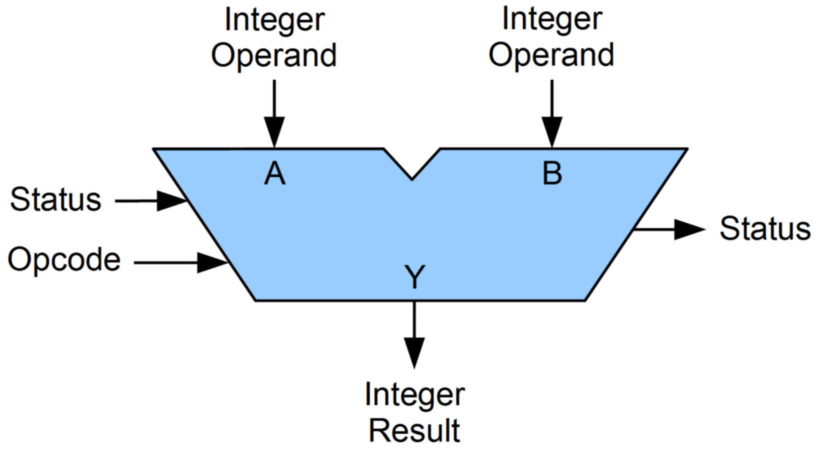
Arithmetic Logic Unit

Arithmetic Logic Unit



Arithmetic Logic Unit

- Important part of the CPU
- Inputs:
  - **Operand** words A, B
  - **Opcode**
  - **Status** bits
- Outputs:
  - **Result** word Y
  - **Status** bits
- Opcode specifies how Y is calculated based on A and B

ALU operations

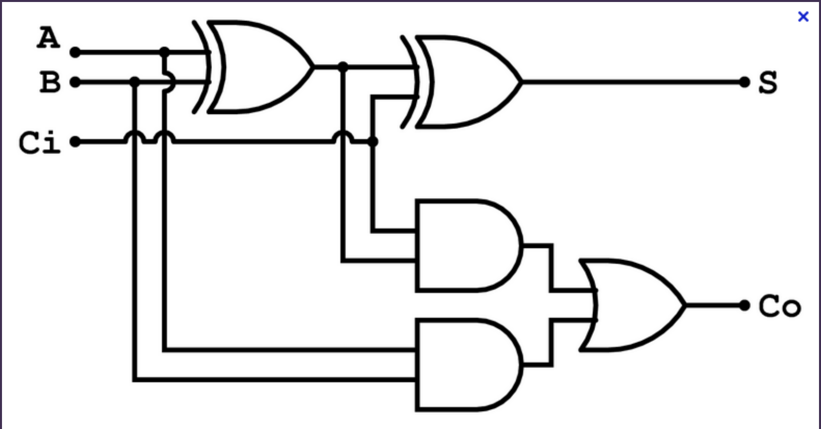
Typically include:

- Add with carry
- Subtract with borrow
- Negate (2's complement)
- Increment, decrement
- Bitwise AND, OR, NOT, ...
- Bit shifts

Adding 3 bits

A	B	C	$A + B + C$
0	0	0	00
0	0	1	01
0	1	0	01
0	1	1	10
1	0	0	01
1	0	1	10
1	1	0	10
1	1	1	11

1-bit adder



How does the 1-bit adder work?

Exercise:

- Write down the boolean expressions for *S* and *Co*
- Draw a truth table for these
- Compare the truth table to the addition table on a previous slide

n-bit adder