COMP220: Graphics & Simulation

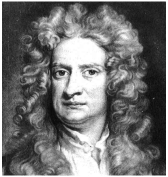
8: Newtonian mechanics

Learning outcomes

- **Recall** the definitions of key concepts such as position, velocity, acceleration, force, friction and restitution
- **Solve** simple mathematical problems involving these key concepts
- **Write** programs which feature realistic physics simulations

Calculus

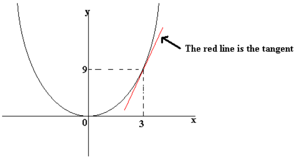
Isaac Newton (1643-1727)



- Invented **calculus**
  - Study of **rates of change**
- Developed **laws of motion**
  - "The" laws of motion until 20th Century (Einstein's theory of relativity, quantum mechanics)
  - Still useful for motion of "everyday" objects (size above quantum scale, speed much lower than speed of light)
- Developed **laws of gravitation**
  - Realised that falling objects and orbiting celestial bodies are governed by the same principles
- Many other contributions to mathematics and physics

Rates of change

- Consider a quantity that **changes over time**
- Rate of change $= \dfrac{\text{change in quantity}}{\text{change in time}}$



Same as the **gradient** of a graph (from GCSE maths):

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x}$$

- The **derivative** of a quantity x with respect to time t is **the rate of change** of x with respect to t
- Denoted $\dfrac{dx}{dt}$
- The mathematical process of finding $\dfrac{dx}{dt}$ given x is called **differentiation**

Derivatives – example

- A car is driving along a straight road at a constant speed
- In half an hour, it covers a distance of 20 miles
- Its average speed is $\dfrac{20 \text{ miles}}{0.5 \text{ hours}} = 40$ miles per hour
- In other words...
  - **Distance travelled** is a quantity varying with time
  - We call the rate of change of this quantity **speed**
  - If x is distance travelled and t is time, then we have

$$\frac{dx}{dt} = \frac{20}{0.5} = 40$$

Integration

- Given $\frac{dx}{dt}$, find x
- x is the **integral** of $\frac{dx}{dt}$
- The process of finding x is called **integration**, the opposite of differentiation
- We are interested in **numerical integration**
  - I.e. integration by computer calculation, not by mathematician with pen and paper...

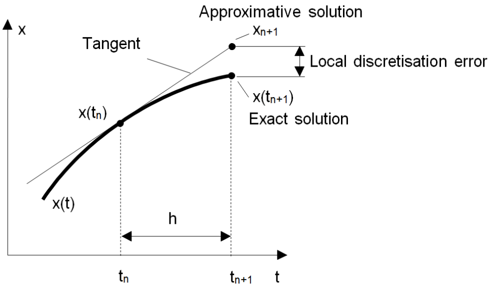
Euler method

- If we know values of x and $\frac{dx}{dt}$ at time t, we can **estimate** the value of x at time $t + h$
- Formula:

$$x(t + h) \approx x(t) + h \times \frac{dx}{dt}(t)$$

- $\frac{dx}{dt}$ is rate of change, i.e. how much x changes by if t changes by 1
- So $h \times \frac{dx}{dt}$ is how much x changes by if t changes by h

Euler method



- If $\frac{dx}{dt}$ does not change between t and $t + h$, this gives the **exact** answer; otherwise there will be an **error**
- If h is small enough, the error should also be small...
- There are more advanced forms of numerical integration which give smaller errors

Calculus with vectors

- Can talk about rate of change of vectors as well
- If x is an n-vector, then so is $\frac{dx}{dt}$
- Each component of $\frac{dx}{dt}$ is the rate of change of the corresponding component of x

Basic mechanics

Point masses

- For now we assume everything is a **point mass**, i.e. ignore the actual shape and size of objects
- **Mass** is measured in **kilograms**
- Not to be confused with **weight** (GCSE physics!)

Position, velocity and acceleration

- **Position** describes an object's location in space
  - Usually expressed as a 3-vector relative to the **origin**
  - Measured in **metres**
- **Velocity** is **rate of change of position**
  - Measured in **metres per second** (ms^{-1})
  - Velocity is a **vector**
  - Speed is a **scalar** (a number), the magnitude of velocity
  - speed : velocity :: distance : position
- **Acceleration** is **rate of change of velocity**
  - Measured in **metres per second per second** (ms^{-2})

Newton's Laws of Motion

An object remains at rest or moves at constant velocity unless acted upon by an external force

$F = ma$: The sum of forces acting upon an object is equal to its mass multiplied by its acceleration

When one body exerts a force on another, the second body exerts an equal and opposite force on the first

Force

- Measured in **Newtons** (N)
- $F = ma$: 1N of force causes a 1kg object to accelerate by 1ms^{-2}
- Forces occur when objects **interact**
- E.g. gravity, air resistance, friction
- E.g. reaction force: stops objects from passing through each other
- E.g. applied forces: car engine, rocket engine, launched projectile, human muscle, ...

Simulating Newtonian physics

- Each object needs to store its **position** and **velocity**
- On each frame:
  - Apply **numerical integration** to determine the new position from the current velocity
  - Calculate the **forces** acting upon the object and use these to calculate **acceleration**

$$F = ma \quad \Longrightarrow \quad a = \frac{F}{m}$$

  - Apply **numerical integration** again to determine the new velocity from the current acceleration

Gravity

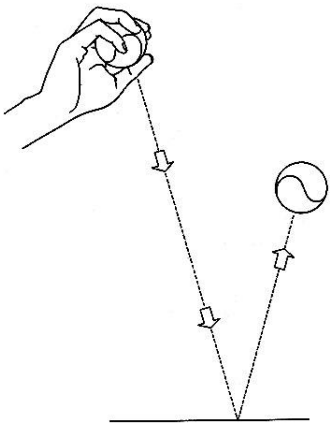
- Gravity pulls **all objects** with mass **towards each other**
- Gravitational force is tiny unless one or both objects has a huge mass (e.g. a planet...)
- Near the surface of a planet, gravity pulls objects **downwards** (i.e. towards the centre of the planet) with a force called **weight**
- $w = mg$, where w is weight, m is mass and g is the **gravitational constant**
- On Earth, $g \approx 9.81$ (often rounded to $g = 10$)

Gravity

$$
\begin{aligned}
F &= ma \\
F &= w = mg \\
\implies mg &= ma \\
\implies g &= a
\end{aligned}
$$

- So gravity applies **the same** acceleration (9.81 ms^{-2} downwards) to all objects **regardless** of weight!
- Famous experiment: in a **vacuum** (no air resistance), a bowling ball falls at the **same speed** as a feather

Basic collision response



- For an **elastic collision**, the component of velocity parallel to the **surface normal** is **reversed**
- E.g. if the surface is the *xz* plane, flip the *y* component
- For an **inelastic collision**, some velocity is lost
- Flip the *y* component and multiply it by something between 0 and 1

Sprint review