COMP250: Artificial Intelligence

4: Planning

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Logic

Logical operations

Python	C family	Mathematics	Behaviour tree
`not a`	`!a`	$\neg A$ or $\overline{A}$	Inverter
`a and b`	`a && b`	$A \wedge B$	Sequence
`a or b`	`a || b`	$A \vee B$	Selector

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- E.g. `LivesIn(Bob, Falmouth)` could be a predicate for "Bob lives in Falmouth"

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  - $\forall x : \mathrm{InCornwall}(x) \wedge \mathrm{IsCity}(x) \implies x = \mathrm{Truro}$

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- Therefore if someone lives in Falmouth then they live in England

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- $\exists x : \mathrm{LivesIn}(x, \mathrm{Cornwall}) \wedge \neg \mathrm{Likes}(x, \mathrm{Cider})$

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- Problem: find a sequence of actions that leads to the goal

STRIPS planning

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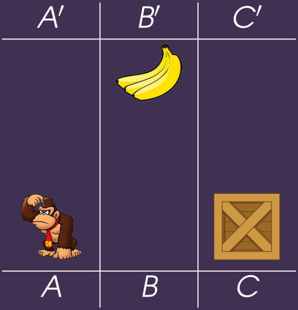
    - Postconditions (specifying what predicates are made true or false by this action)

STRIPS example



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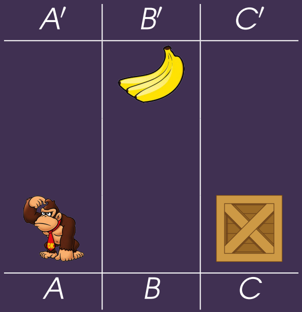

Initial state:

```
At(A),
BoxAt(C),
BananasAt(B')
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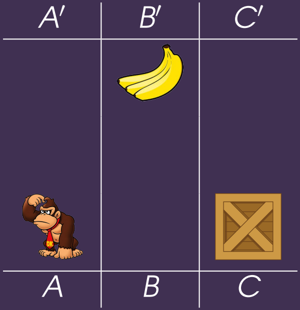

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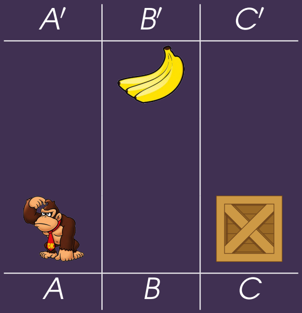
Goal:

```
HasBananas
```

STRIPS example — Actions




```
Move(x, y)
  Pre:   At(x)
  Post: !At(x), At(y)

ClimbUp(x)
  Pre:   At(x), BoxAt(x)
  Post: !At(x), At(x′)

ClimbDown(x′)
  Pre:   At(x′), BoxAt(x)
  Post: !At(x′), At(x)

PushBox(x, y)
  Pre:   At(x), BoxAt(x)
  Post: !At(x), At(y),
        !BoxAt(x), BoxAt(y)

TakeBananas(x)
  Pre:   At(x), BananasAt(x)
  Post: !BananasAt(x), HasBananas
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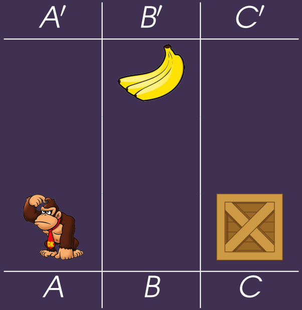
STRIPS example — Solution



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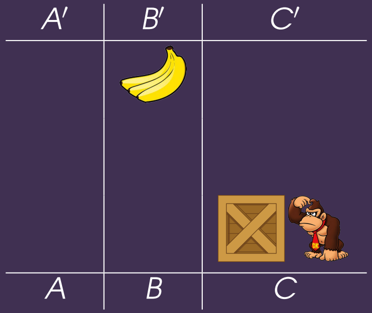

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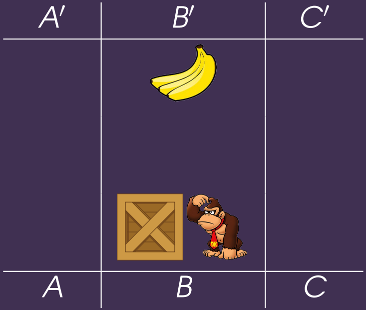

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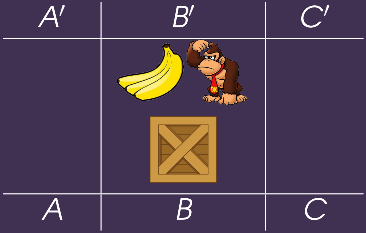

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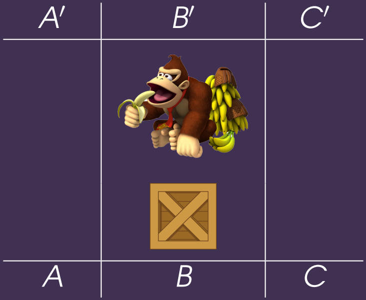

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- We can construct a **tree** of states and actions
- We can then **search** this tree to find a goal state

Tree traversal

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