

COMP270: Mathematics for 3D Worlds and Simulations

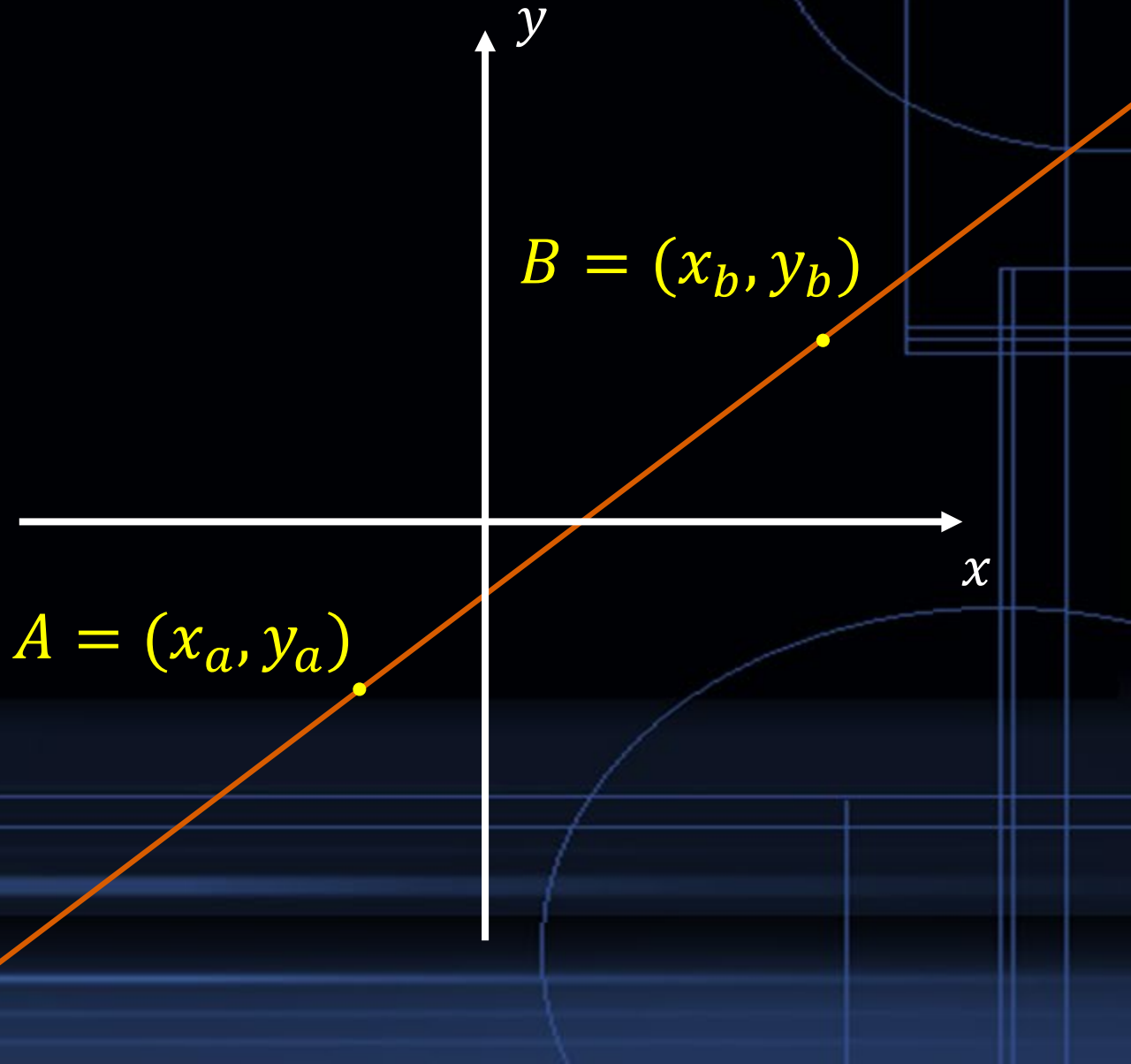
WEEK 2: GEOMETRY I
PART 2: VECTOR BASICS

Objectives

- **Recall** the definition of a vector and how it differs from a point or a line
- **Apply** basic arithmetic operations to vectors in 2D

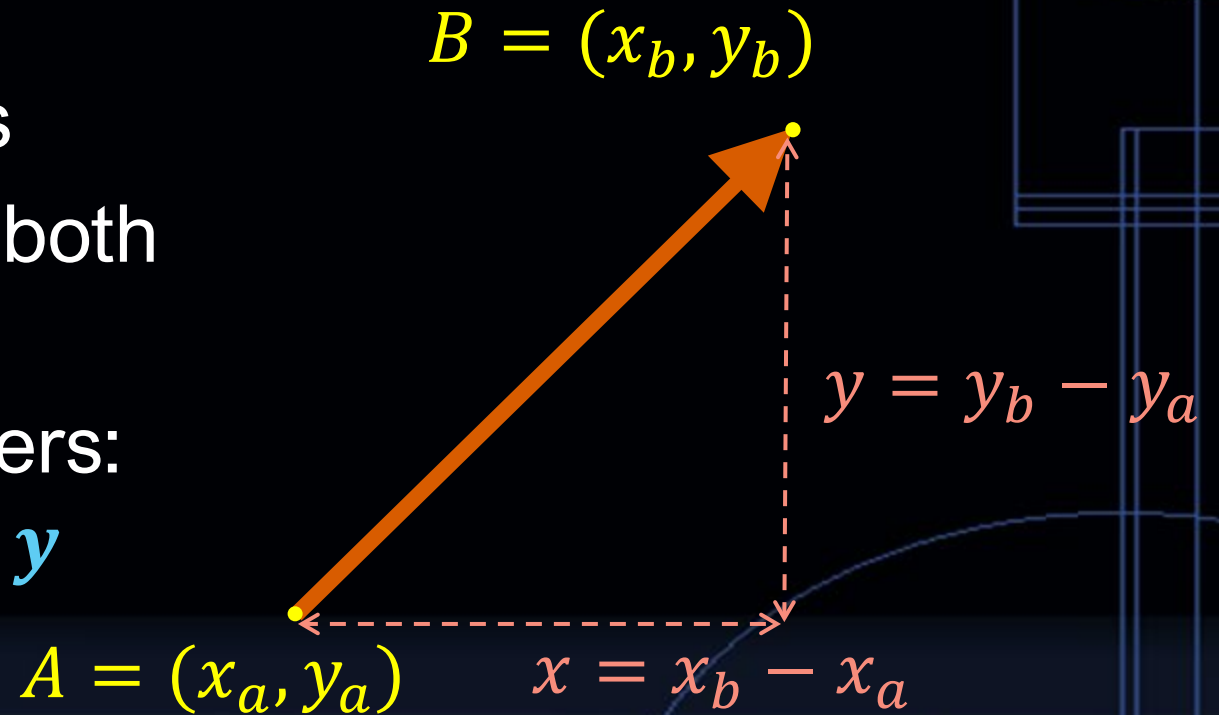
Recap

- A **point** in 2D is a pair of values, or **coordinates**, specifying a position relative to a pair of perpendicular axes.
- A **line** extends infinitely and can be uniquely defined by two **points** through which it passes.



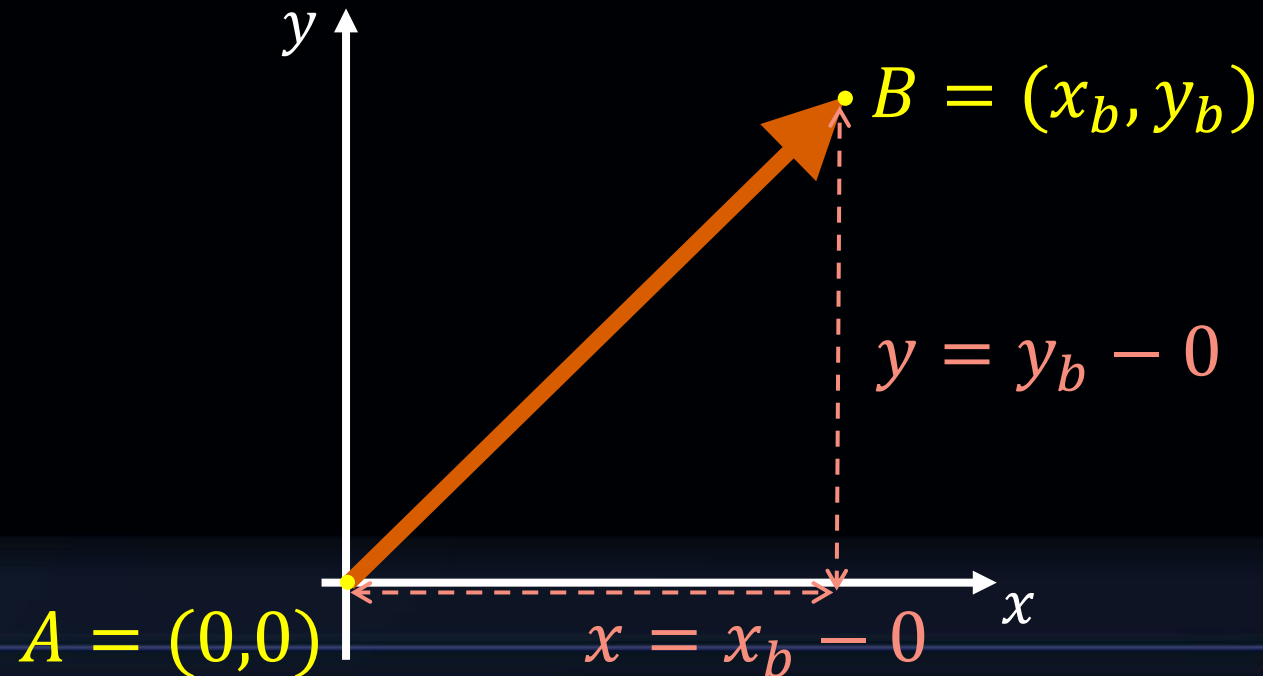
2D Vectors

- A **vector** is a directed line segment between 2 points
- Unlike a line, a vector has both **direction** and **length**
- Defined by a pair of numbers: the **x component** and the **y component**



Vectors and points

- A vector can represent a point relative to the origin:



- ... but a point is not the same as a vector!

(Except in
some code...)

Writing vectors

- As a pair of numbers: (x, y)

For COMP270: this is a **point**

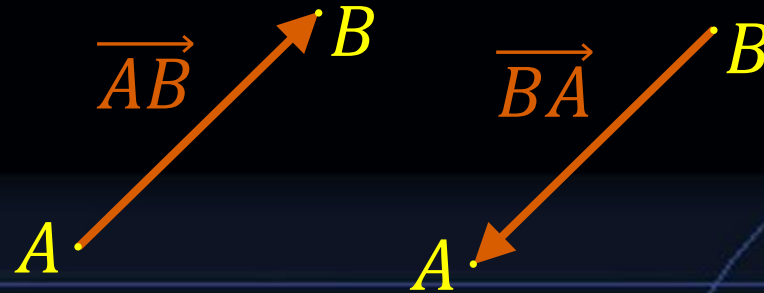
- As a column vector: $\begin{pmatrix} x \\ y \end{pmatrix}$

- Variable representing a vector: written in bold i.e. $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

- Other notations: \vec{v}, \underline{v}

- Vector from A to B : \overrightarrow{AB}

Vector from B to A : \overrightarrow{BA}



- Note: these are **not the same**, as the components will have opposite signs

Vector magnitude and direction

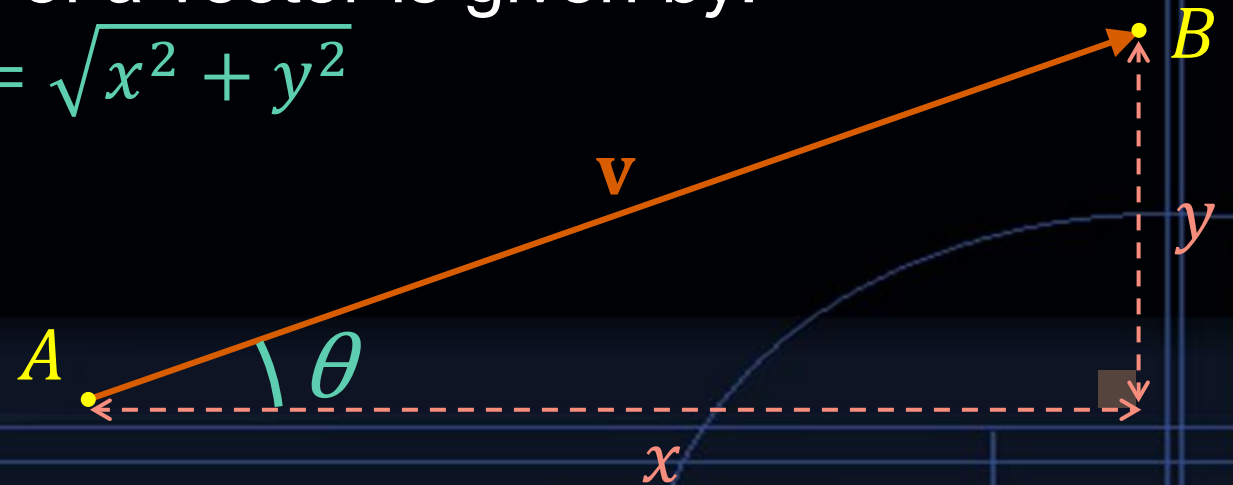
- Recall: a vector is the **hypotenuse** of the triangle formed with lines through its end points, parallel to the axes
 - The other side lengths are the vector components, x and y
- The **magnitude** (or length) of a vector is given by:

$$\|\mathbf{v}\| = \sqrt{x^2 + y^2}$$

- The angle between \mathbf{v} and the positive x -axis is given by

$$\tan \theta = \frac{y}{x}$$

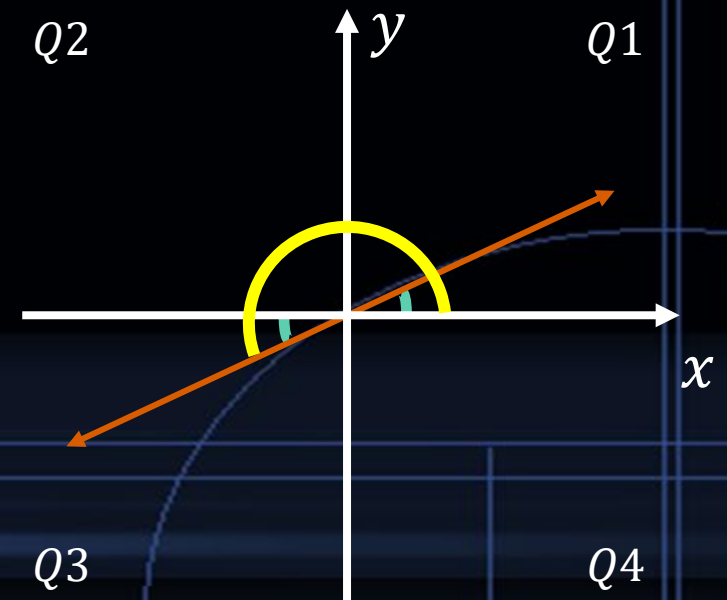
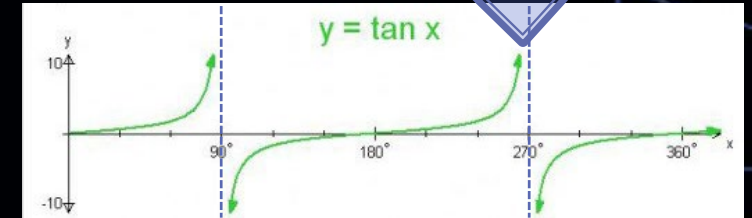
$$(\text{with } \sin \theta = \frac{y}{\|\mathbf{v}\|} \text{ and } \cos \theta = \frac{x}{\|\mathbf{v}\|})$$



Inverse tangent

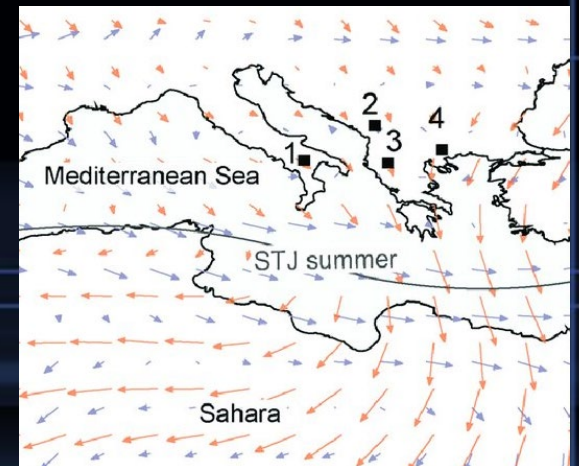
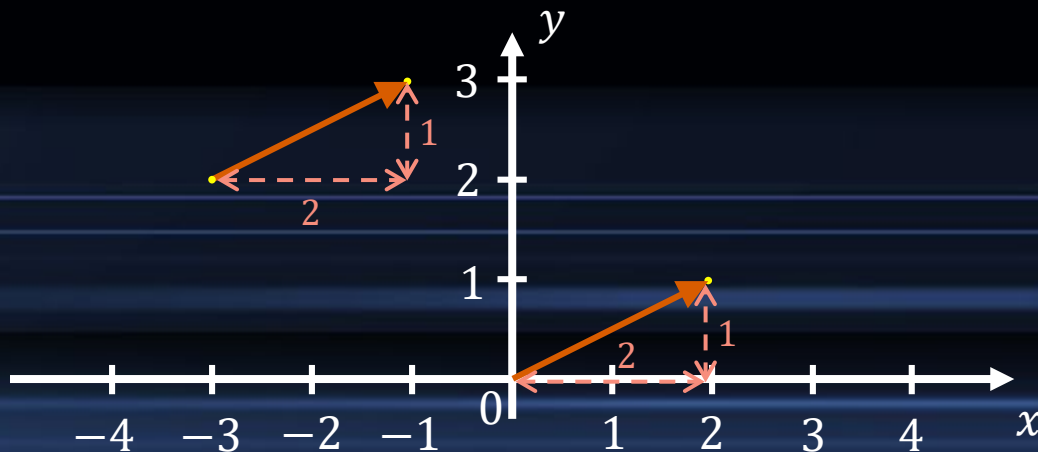
- Care is needed when using \tan^{-1} :
- $\tan^{-1} \frac{y}{x} = \tan^{-1} \frac{-y}{-x}$ so we need to be aware of which quadrant the vector is in (otherwise the result may be out by 180°)
- If $x = 0$ (the vector points vertically along the y -axis) then we're dividing by zero
- Most programming languages have an $\text{atan2}(y, x)$ function which handles all of these cases for you

An asymptote is a line or curve that approaches a given curve arbitrarily closely



Vector equivalence

- Vectors are defined by their **magnitude** and **direction**, but **not by their position** in space
 - e.g. the vector from the origin to the point $(2, 1)$ is the same as the vector from the point $(-3, 2)$ to the point $(-1, 3)$
- Used to represent more abstract quantities than just the difference between two points, e.g. wind



Vector addition

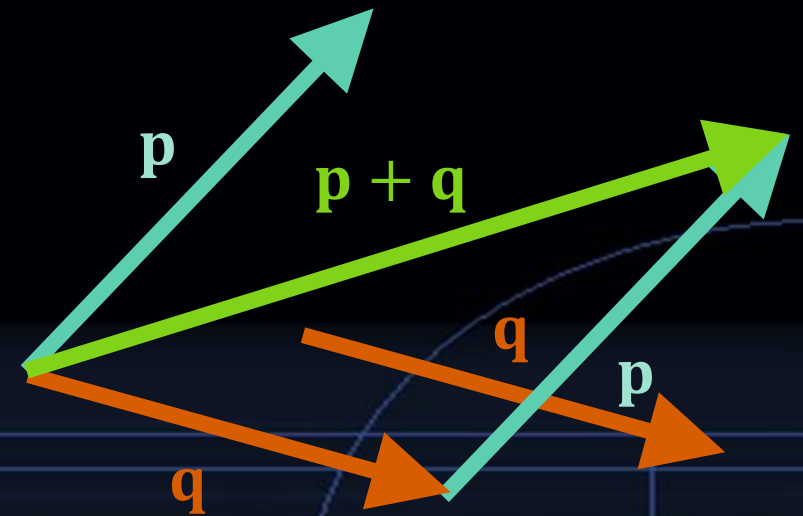
If \mathbf{p} and \mathbf{q} are vectors,

- $\mathbf{p} + \mathbf{q}$ is obtained by putting \mathbf{p} and \mathbf{q} end to end
- Performed **component-wise**,

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$

- Note: addition is **commutative**:

$$\mathbf{p} + \mathbf{q} = \mathbf{q} + \mathbf{p}$$



Vector subtraction

If \mathbf{p} and \mathbf{q} are vectors representing points on the plane,

- $\mathbf{q} - \mathbf{p}$ represents the vector *from* \mathbf{p} *to* \mathbf{q}

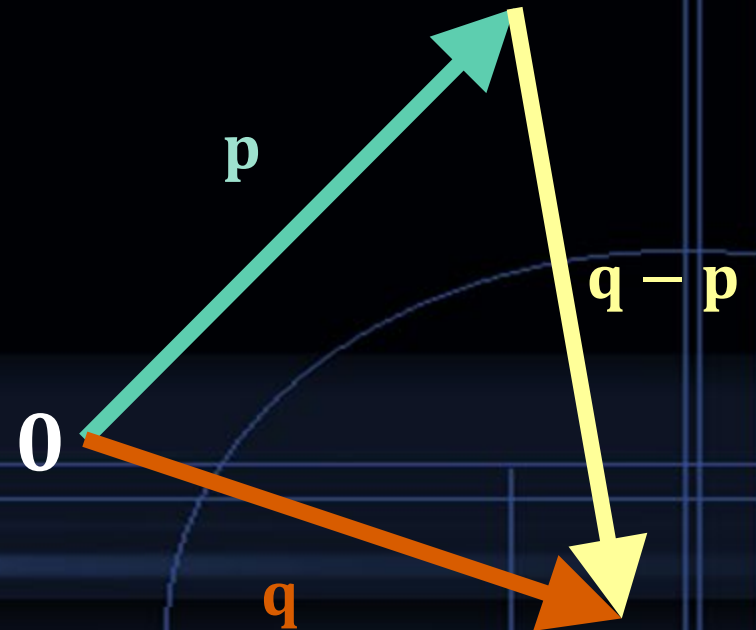
- Note the order of the subtraction!

- $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}$

- Note: subtraction is anticommutative:

$$\mathbf{q} - \mathbf{p} = -(\mathbf{p} - \mathbf{q})$$

- i.e. $\overrightarrow{BA} = -\overrightarrow{AB}$



Scalar multiplication

If \mathbf{v} is a vector and c is a positive number

- $c\mathbf{v}$ is a vector with the same direction as \mathbf{v} , but c times the magnitude
- $c \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cx \\ cy \end{pmatrix}$
- If c is negative then $c\mathbf{v}$ has the opposite direction to \mathbf{v}

