COMP250: Artificial Intelligence

2: Designing AI behaviours

Reminder

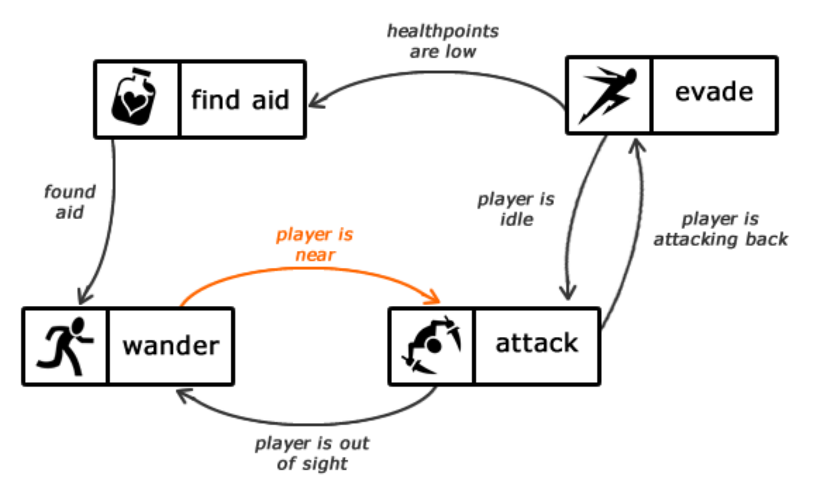
AI component proposal due **next week**!

AI architectures

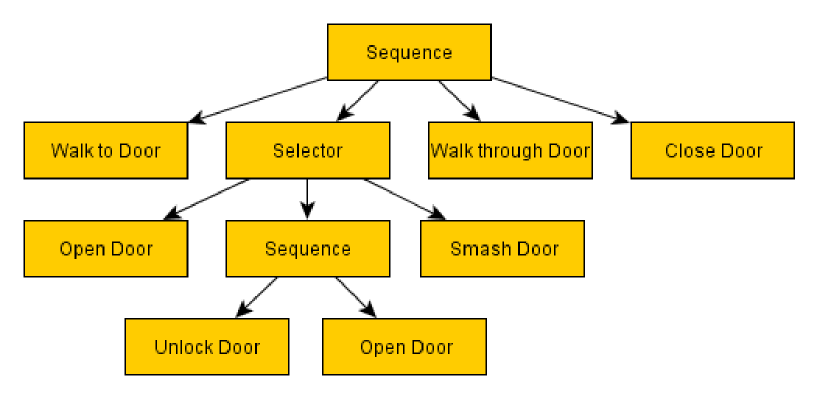
Rule-based AI

Generally implemented as `if` statements or event-based triggers

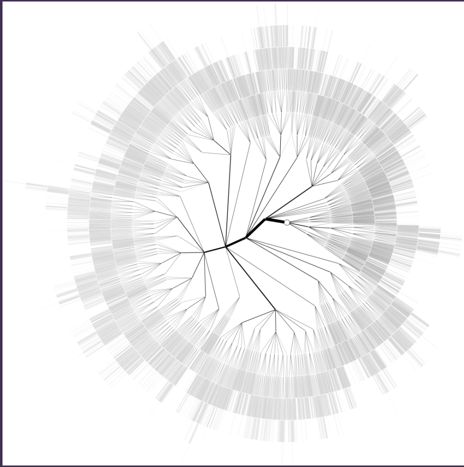
Finite state machines



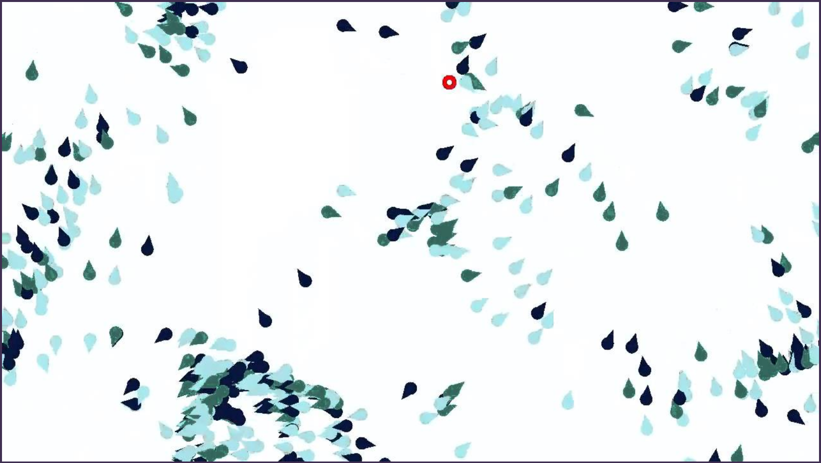
Behaviour trees



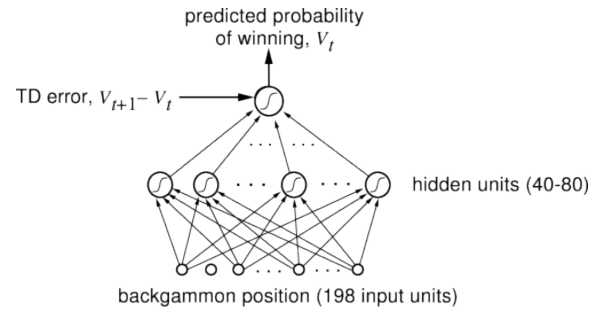
Game tree search



Multi-agent approaches (e.g. flocking)



Machine learning



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- Do you want to **design** the AI behaviours yourself, or do you want them to **emerge** from the system?

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- ... and **computational intelligence**
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- Do you want to **design** the AI behaviours yourself, or do you want them to **emerge** from the system?
- Predictability and authorial control versus unpredictability and novelty

Logic

Logical operations

Python	C family	Mathematics	Behaviour tree
`not a`	`!a`	$\neg A$ or $\overline{A}$	Inverter
`a and b`	`a && b`	$A \wedge B$	Sequence
`a or b`	`a || b`	$A \vee B$	Selector

The laws of thought

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- E.g. `LivesIn(Bob, Falmouth)` could be a predicate for "Bob lives in Falmouth"

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- $\forall x : \text{LivesIn}(x, \text{Falmouth}) \implies \text{LivesIn}(x, \text{Cornwall})$

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  - $\forall x : \text{InCornwall}(x) \wedge \text{IsCity}(x) \implies x = \text{Truro}$

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- Therefore if someone lives in Falmouth then they live in England

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- A **game** is a system where one or more **players** choose **actions**; the combination of these choices lead to each agent receiving a **payoff**
- Important applications in economics, ecology and social sciences as well as AI

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- If **Bob betrays Alice**, he receives an A whilst she gets expelled
- If **both betray each other**, both get an F

Payoff matrix

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	A silent	A betray
B silent	A: 50 B: 50	A: 70 B: -100
B betray	A: -100 B: 70	A: 0 B: 0

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Socrative `FALCOMPED`: what would you do?

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... and Bob's thought process is the same!

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- If all players are **rational** (in the sense of wanting to maximising payoff), they should converge upon a Nash equilibrium

Does every game have a Nash equilibrium?

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	A rock	A paper	A scissors
B rock	A: 0 B: 0	A: +1 B: -1	A: -1 B: +1
B paper	A: -1 B: +1	A: 0 B: 0	A: +1 B: -1
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- If we try to reason naïvely, we get stuck in a loop
  - If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...

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- The optimum strategy is to be **unpredictable**

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- If we try to reason naïvely, we get stuck in a loop
  - If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...
- The optimum strategy is to be **unpredictable**
- Choose rock with probability $\frac{1}{3}$, paper with probability $\frac{1}{3}$, scissors with probability $\frac{1}{3}$

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- In contrast to a **pure** or **deterministic strategy**, which always chooses the same action
- If we allow mixed strategies, **every game has at least one Nash equilibrium**

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- Socrative `FALCOMPED`: make your guesses!

The rational guess

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- So no rational player would guess greater than 29.629

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- So no rational player would guess greater than 44.444
- Which means the average can't possibly be greater than 44.444
- So no rational player would guess greater than 29.629
- ... and so on ad infinitum
- So the only **rational** guess is 0, as every rational player should guess 0 and $\frac{2}{3}$ of 0 is 0

Rationality

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- However it's important to remember that **humans aren't always rational**