COMP110: Principles of Computing

10: Algorithm Strategies

Worksheets

- Worksheet 6: due **today**
- Worksheet 7: due **next Monday**

Recursion

Recursion

- A **recursive** function is a function that **calls itself**

```
int factorial(int n)
{
    if (n <= 1)
        return 1;
    else
        return n * factorial(n-1);
}
```

- Recursive functions need a **base case** where they stop recursing, otherwise they will go **forever**
- (Or rather, until a **stack overflow**)

Thinking recursively

- I want to solve a problem
- If I already had a function to solve smaller instances of the problem, I could use it to write my function
- I can solve the smallest possible problem
- Therefore I can write a recursive function

The call stack

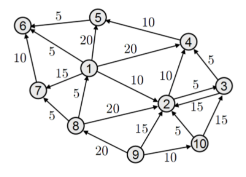
- Recall: nested function calls are handled using a **stack**
- Recursive functions are no different
- This means if a recursive function contains **local variables**, they are **independent** between instances of the function

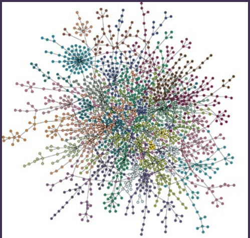
FALMOUTH UNIVERSITY

Graphs and trees

FALMOUTH UNIVERSITY

Graphs

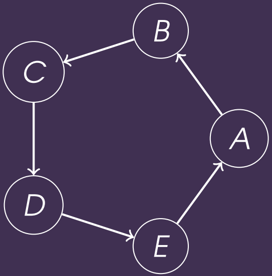


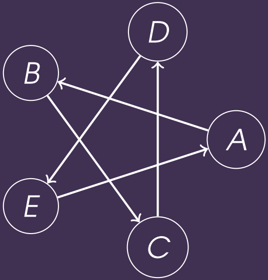


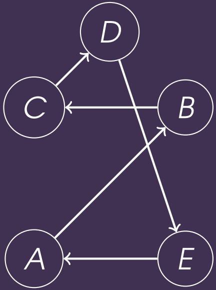
- A **graph** is defined by:
  - A collection of **nodes** or **vertices** (points)
  - A collection of **edges** or **arcs** (lines or arrows between points)
- Often used to model **networks** (e.g. social networks, transport networks, game levels, automata, ...)
- **Directed** graph: edges are arrows
- **Undirected** graph: edges are lines

Drawing graphs

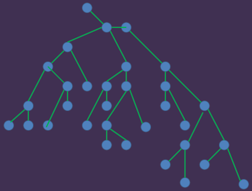
- A graph does not necessarily specify the physical **positions** of its nodes
- E.g. these are technically the same graph:

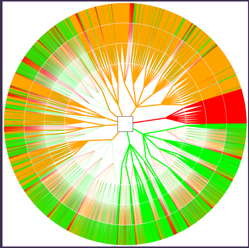






Trees





- A **tree** is a special type of directed graph where:
  - One node (the **root**) has no incoming edges
  - All other nodes have exactly 1 incoming edge
- Edges go from **parent** to **child**
  - All nodes except the root have exactly one parent
  - Nodes can have 0, 1 or many children
- Used to model **hierarchies** (e.g. file systems, object inheritance, scene graphs, state-action trees, behaviour trees, ...)

Tree traversal

Tree traversal

- **Traversal**: visiting all the nodes of the tree
- Two main types
  - Depth first
  - Breadth first

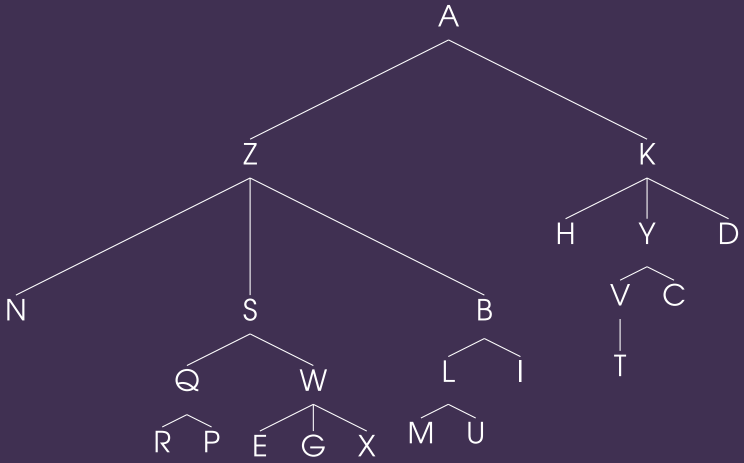
Tree traversal

```
procedure DepthFirstSearch
    let S be a stack
    push root node onto S
    while S is not empty do
        pop n from S
        print n
        push children of n onto S
    end while
end procedure

procedure BreadthFirstSearch
    let Q be a queue
    enqueue root node into Q
    while Q is not empty do
        dequeue n from Q
        print n
        enqueue children of n into Q
    end while
end procedure
```

Tree traversal example

Socrative `FALCOMPED`



Recursive depth first search

procedure DEPTHFIRSTSEARCH(n)
    print n
    for each child c of n **do**
        DEPTHFIRSTSEARCH(c)
    end for
end procedure

- Compare to the pseudocode on the previous slide. Where is the stack?

Algorithm strategies

The knapsack problem

- There is a set X of **items**
- Each item x has a weight $\mathrm{weight}(x)$ and a value $\mathrm{value}(x)$
- There is a maximum weight W
- What subset $S \subseteq X$ maximises the total value, whilst not exceeding the maximum weight?
- In other words: find $S \subseteq X$ to maximise

$$\sum_{x \in S} \mathrm{value}(x)$$

subject to

$$\sum_{x \in S} \mathrm{weight}(x) \leq W$$

Algorithm strategies

- Brute force
- Greedy
- Divide-and-conquer
- Dynamic programming

Brute force

- Try **every possible** solution and decide which is best

procedure KNAPSACK(X, W)
    $S_{\text{best}} \leftarrow \{\}$
    $v_{\text{best}} \leftarrow 0$
    for every subset $S \subseteq X$ **do**
        if weight$(S) \leq W$ and value$(S) > v_{\text{best}}$ **then**
            $S_{\text{best}} \leftarrow S$
            $v_{\text{best}} \leftarrow$ value(S)
        end if
    end for
    return S_{best}
end procedure

Socrative `FALCOMPED`

- If X contains n elements, how many subsets of X are there?
- Therefore what is the time complexity of the brute force algorithm?
- If we add one element to X, what happens to the running time of the algorithm?

Greedy algorithm

- At each stage of building a solution, take the **best** available option

procedure KNAPSACK(X, W)
    $S \leftarrow \{\}$
    for each $x \in X$, in descending order of $\mathsf{value}(x)$ **do**
        if $\mathsf{weight}(S) + \mathsf{weight}(x) \leq W$ **then**
            add x to S
        end if
    end for
    return S
end procedure

Greedy algorithm

- Time complexity is dominated by sorting X by value
- The rest of the algorithm runs in linear time
- In some problems an appropriately chosen greedy solution is **optimal**
  - A* pathfinding
  - Huffman coding
- **However** the greedy solution to the knapsack problem may not be optimal!

Divide and conquer

- Break the problem into smaller, easier to solve **subproblems**
- Requires that the solution to the original problem is composed of the solutions to the smaller problem
- Example from last time: **binary search**
  - Problem: find an element in a list
  - Subproblem: find the element in a list of half the size

Divide and conquer for the knapsack problem

- Consider an element $x \in X$ with $\mathrm{weight}(x) \leq W$
- Let X' be X with x removed
- The solution to the knapsack problem either includes x or it doesn't
- The solution is **either**:
  - The solution to the knapsack problem on X' with maximum weight W, **or**
  - The solution to the knapsack problem on X' with maximum weight $W - \mathrm{weight}(x)$, plus x
- ... whichever has the greater value
- Base case: the solution to the knapsack problem on the empty set **is** the empty set

Divide and conquer for the knapsack problem

procedure KNAPSACK(X, W, k)
    if $k < 0$ **then**
        return $\{\}$
    end if
    $S \leftarrow \text{KNAPSACK}(X, W, k-1)$
    if $\text{weight}(x_k) \leq W$ **then**
        $S' \leftarrow \text{KNAPSACK}(X, W - \text{weight}(x_k), k-1) \cup \{x_k\}$
        return whichever of S, S' has the larger value
    else
        return S
    end if
end procedure

Time complexity

- Each call to KNAPSACK has, in the worst case, **two** recursive calls to KNAPSACK
- Number of calls is

$$\underbrace{1 + 2 + 4 + 8 + \cdots + 2^i + \ldots}_{n \text{ terms}}$$

- Thus the worst case time complexity is $O(2^n)$ — still exponential!
- However in the **average** case many of the calls have only a single recursive call, so this is still more efficient than brute force

Overlapping subproblems

- Here we end up solving the **same subproblem multiple times**
- Can save time by **caching** (remembering) these sub-solutions
- This is called **memoization**
  - **Not** memorization!
- One of several techniques in the category of **dynamic programming**

Dynamic programming for the knapsack problem

procedure KNAPSACK(X, W, k)
    if KNAPSACK(X, W, k) has already been computed **then**
        return previously computed result
    end if
    if $k < 0$ **then**
        cache and return $\{\}$
    end if
    $S \leftarrow \text{KNAPSACK}(X, W, k-1)$
    if $\text{weight}(x_k) \leq W$ **then**
        $S' \leftarrow \text{KNAPSACK}(X, W - \text{weight}(x_k), k-1) \cup \{x_k\}$
        cache and return whichever of S, S' has the larger value
    else
        cache and return S
    end if
end procedure

Socrative `FALCOMPED`

- What is the maximum possible number of entries in the table of intermediate results?
- Therefore what is the time complexity of the dynamic programming algorithm?

Summary of algorithm strategies

- Brute force
  - Good enough for small/simple problems
- Greedy
  - Efficient for certain problems, but doesn't always give optimal solutions
- Divide-and-conquer
  - Good if the problem can be broken down into simpler subproblems
- Dynamic programming
  - Makes divide-and-conquer more efficient if subproblems often reoccur