



FALMOUTH
UNIVERSITY



COMP250 Artificial Intelligence

8: Evolutionary Algorithms

Optimisation



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- ▶ **Higher** scores are **better**
- ▶ We are exploring a **fitness landscape**

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- ▶ Want to generate a map where there is a path from start to goal, and that path is as long as possible
- ▶ Fitness measure:

$$f(x) = \begin{cases} \text{path length} & \text{if a path exists} \\ 0 & \text{otherwise} \end{cases}$$

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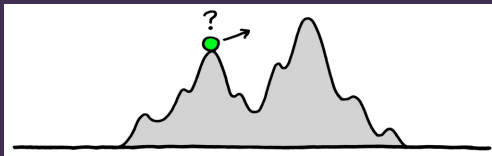
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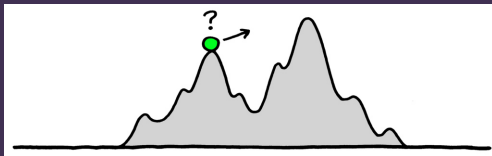
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- ▶ Repeat

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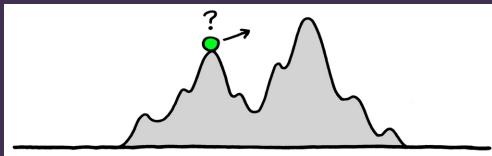


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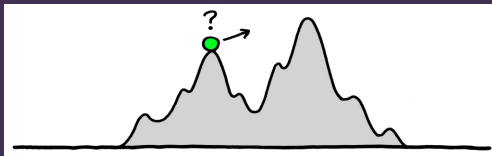
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- ▶ Hillclimbing tends to get stuck at a **local optimum**
- ▶ This may be much worse than the **global optimum**
- ▶ Have to let the solution get worse before it gets better — hillclimbing doesn't allow this

Escaping the local optimum

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 - ▶ This probability decreases as search progresses

Evolutionary algorithms



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- ▶ Generation 0: choose N solutions at random
- ▶ Generation $i + 1$: choose N new solutions based on the **fittest** individuals from generation i

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- ▶ Simple method: **tournament selection**
 - ▶ Randomly choose t individuals
 - ▶ Select the fittest out of those t

Mutation

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- ▶ Add the changed individual to the new population

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Elitism

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- ▶ Take the top $x\%$ of generation i , and pass it straight through to generation $i + 1$

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- ▶ **Linear combination:**

$$w_1 h_1 + w_2 h_2 + \dots + w_n h_n$$

where w_1, w_2, \dots, w_n are constants: **weights**

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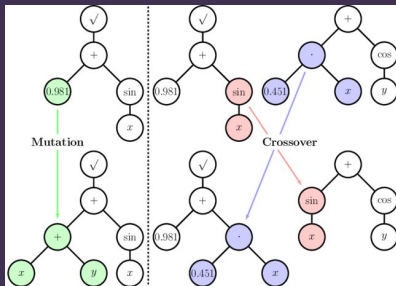
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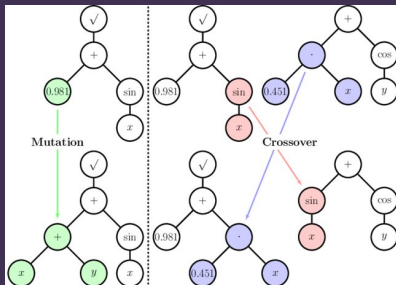
- ▶ What value to choose for the weights? This is an optimisation problem!

Genetic programming

Genetic programming

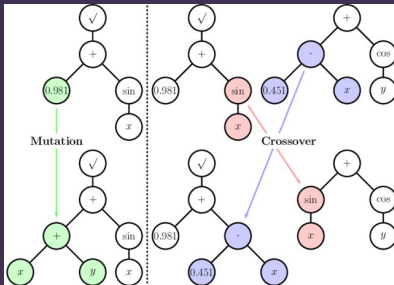


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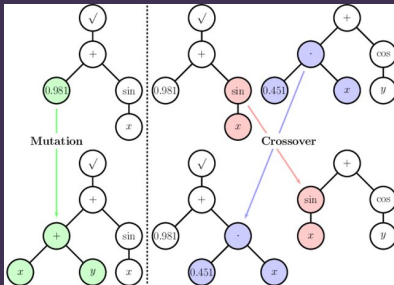
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- ▶ Typically uses a **tree-based** representation of code
- ▶ Other approaches exist e.g. template-based

Neural networks



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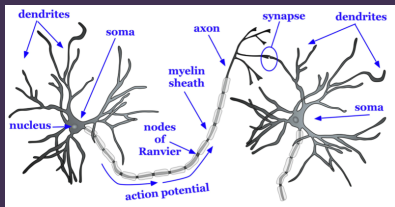
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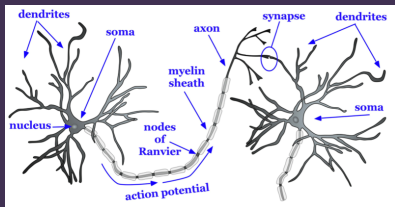
Artificial Neural Networks (ANNs)

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- ▶ Recent resurgence of interest: today's powerful CPUs and GPUs allow much larger ANNs to be used

Real neurons

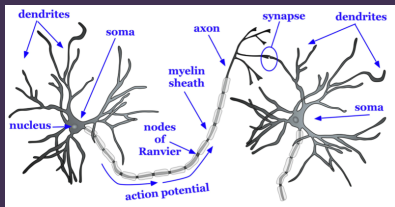


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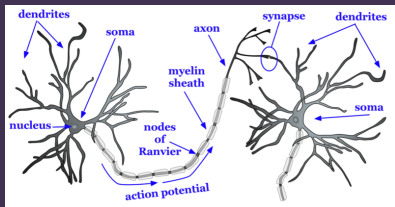
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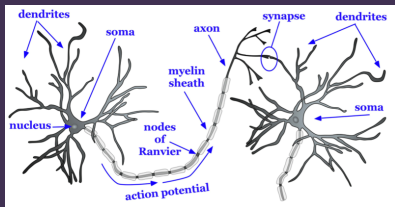
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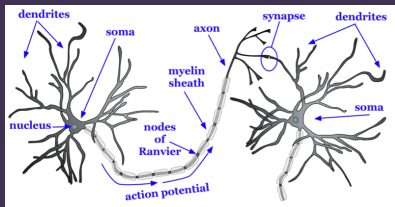
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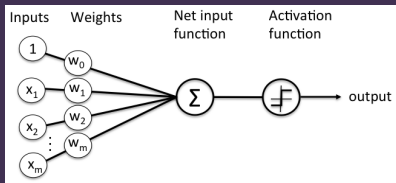
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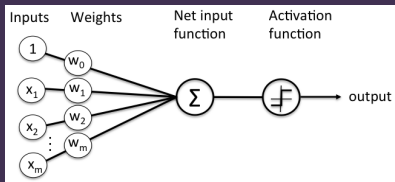
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- ▶ Human brain contains approximately **100 billion** neurons

An artificial neuron

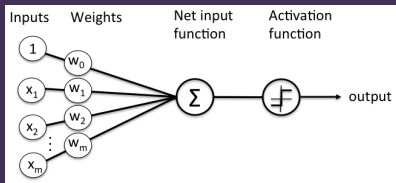


An artificial neuron

► A perceptron

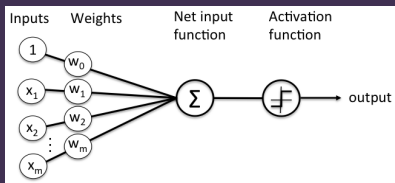


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- ▶ Each input has a **weight** w_i between -1 and $+1$

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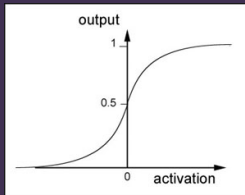
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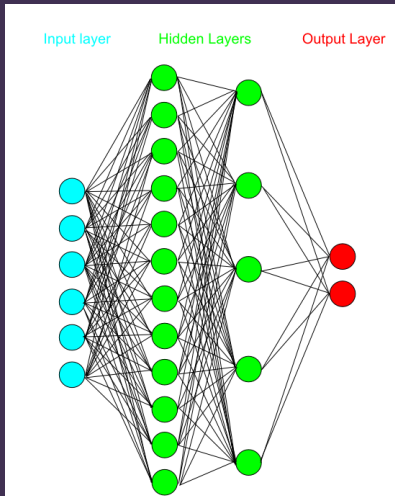
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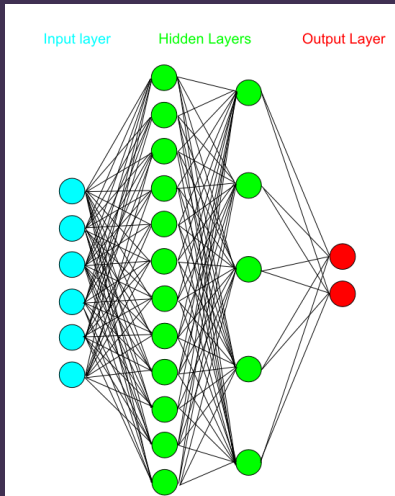
- ▶ More common: **sigmoid function**



An artificial neural network

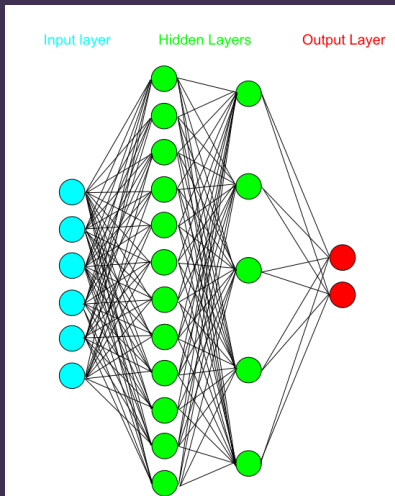


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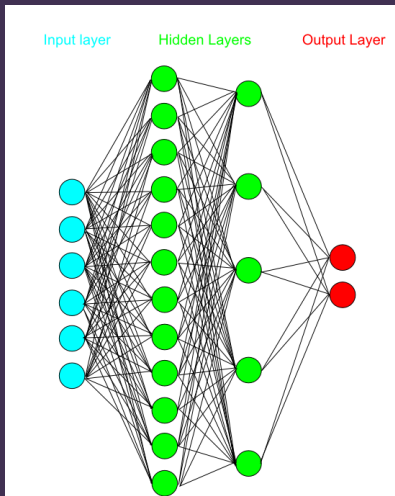
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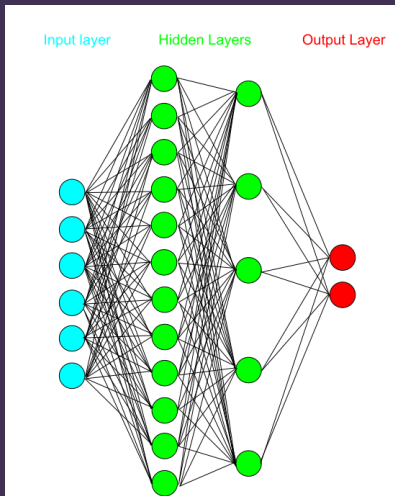
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Image classification



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- ▶ Classic example:
**handwritten digit
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Image classification



- ▶ Classic example: **handwritten digit recognition**
- ▶ Given a **raster image**, which of the digits 0 to 9 does it represent?



<https://twitter.com/NaughtThought/status/846262063827730432>

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- ▶ **Weights:** ???

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- ▶ Note: this requires a large amount of training data that is **tagged**, i.e. for which we already know the correct answer

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- ▶ Each epoch uses a randomly sampled **subset** of the training data
- ▶ This reduces computation time, and helps to escape local optima

ANN example

<http://playground.tensorflow.org>

Overfitting

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- ▶ Insufficient training data might result in the network learning “patterns” that are actually random anomalies