COMP110: Principles of Computing

2: Basic Principles for Computation

Learning outcomes

By the end of this week's sessions, you should be able to:

- **Use** binary, decimal and hexadecimal notation to represent and operate on numerical values
- **Explain** the basic architecture of a computer
- **Distinguish** the most common programming languages and paradigms in use today

FALMOUTH UNIVERSITY

Research journal

Research journal

Research journal

- **Read** some seminal papers in computing (listed on the assignment brief)

Research journal

- **Read** some seminal papers in computing (listed on the assignment brief)
- **Choose** one of them

Research journal

- **Read** some seminal papers in computing (listed on the assignment brief)
- **Choose** one of them
- **Research** how this paper has influenced the field of computing

Research journal

- **Read** some seminal papers in computing (listed on the assignment brief)
- **Choose** one of them
- **Research** how this paper has influenced the field of computing
- **Write up** your findings

Research journal

- **Read** some seminal papers in computing (listed on the assignment brief)
- **Choose** one of them
- **Research** how this paper has influenced the field of computing
- **Write up** your findings
  - Maximum 1500 words

Research journal

- **Read** some seminal papers in computing (listed on the assignment brief)
- **Choose** one of them
- **Research** how this paper has influenced the field of computing
- **Write up** your findings
  - Maximum 1500 words
  - With reference to appropriate academic sources

Marking rubric

See assignment brief on LearningSpace/GitHub

Timeline

Timeline

- **Peer review** in week 11 (4th December)

Timeline

- **Peer review** in week 11 (4th December)
- **Deadline** shortly after (check MyFalmouth)

Timeline

- **Peer review** in week 11 (4th December)
- **Deadline** shortly after (check MyFalmouth)
- Finding and reading academic papers takes time and effort — don't leave it until the last minute!

FALMOUTH UNIVERSITY

Binary notation

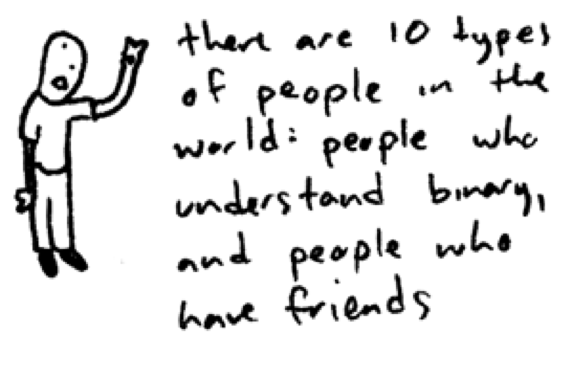

Image credit: http://www.toothpastefordinner.com

How we write numbers

How we write numbers

- We write numbers in **base 10**

How we write numbers

- We write numbers in **base 10**
- We have 10 **digits**: $0, 1, 2, \ldots, 8, 9$

How we write numbers

- We write numbers in **base 10**
- We have 10 **digits**: $0, 1, 2, \ldots, 8, 9$
- When we write 6397, we mean:

How we write numbers

- We write numbers in **base 10**
- We have 10 **digits**: $0, 1, 2, \ldots, 8, 9$
- When we write 6397, we mean:
  - Six thousand, three hundred and ninety seven

How we write numbers

- We write numbers in **base 10**
- We have 10 **digits**: $0, 1, 2, \ldots, 8, 9$
- When we write 6397, we mean:
  - Six thousand, three hundred and ninety seven
  - (Six thousands) and (three hundreds) and (nine tens) and (seven)

How we write numbers

- We write numbers in **base 10**
- We have 10 **digits**: $0, 1, 2, \ldots, 8, 9$
- When we write 6397, we mean:
  - Six thousand, three hundred and ninety seven
  - (Six thousands) and (three hundreds) and (nine tens) and (seven)
  - $(6 \times 1000) + (3 \times 100) + (9 \times 10) + (7)$

How we write numbers

- We write numbers in **base 10**
- We have 10 **digits**: $0, 1, 2, \ldots, 8, 9$
- When we write 6397, we mean:
  - Six thousand, three hundred and ninety seven
  - (Six thousands) and (three hundreds) and (nine tens) and (seven)
  - $(6 \times 1000) + (3 \times 100) + (9 \times 10) + (7)$
  - $\left(6 \times 10^3\right) + \left(3 \times 10^2\right) + \left(9 \times 10^1\right) + \left(7 \times 10^0\right)$

How we write numbers

- We write numbers in **base 10**
- We have 10 **digits**: $0, 1, 2, \ldots, 8, 9$
- When we write 6397, we mean:
  - Six thousand, three hundred and ninety seven
  - (Six thousands) and (three hundreds) and (nine tens) and (seven)
  - $(6 \times 1000) + (3 \times 100) + (9 \times 10) + (7)$
  - $(6 \times 10^3) + (3 \times 10^2) + (9 \times 10^1) + (7 \times 10^0)$
Thousands	Hundreds	Tens	Units
6	3	9	7

Binary

Binary

- Binary notation works the same, but is **base 2** instead of **base 10**

Binary

- Binary notation works the same, but is **base 2** instead of **base 10**
- We have 2 **digits**: 0, 1

Binary

- Binary notation works the same, but is **base 2** instead of **base 10**
- We have 2 **digits**: 0, 1
- When we write 10001011 in binary, we mean:

Binary

- Binary notation works the same, but is **base 2** instead of **base 10**
- We have 2 **digits**: 0, 1
- When we write 10001011 in binary, we mean:
  $$(1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

Binary

- Binary notation works the same, but is **base 2** instead of **base 10**
- We have 2 **digits**: 0, 1
- When we write 10001011 in binary, we mean:
  $(1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4)$
  $+ (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
  $= 2^7 + 2^3 + 2^1 + 2^0$

Binary

- Binary notation works the same, but is **base 2** instead of **base 10**
- We have 2 **digits**: 0, 1
- When we write 10001011 in binary, we mean:

$$
\begin{aligned}
&(1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) \\
&+ (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\
&= 2^7 + 2^3 + 2^1 + 2^0 \\
&= 128 + 8 + 2 + 1 \text{ (base 10)}
\end{aligned}
$$

Binary

- Binary notation works the same, but is **base 2** instead of **base 10**
- We have 2 **digits**: 0, 1
- When we write 10001011 in binary, we mean:

$$
\begin{aligned}
&(1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) \\
&+ (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\
&= 2^7 + 2^3 + 2^1 + 2^0 \\
&= 128 + 8 + 2 + 1 \text{ (base 10)} \\
&= 139 \text{ (base 10)}
\end{aligned}
$$

Converting to binary

https://www.youtube.com/watch?v=OezK_zTyvAQ

Bits, bytes and words

Bits, bytes and words

- A **bit** is a binary digit

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary
- A **word** is the number of bits that the CPU works with at once

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary
- A **word** is the number of bits that the CPU works with at once
  - 32-bit CPU: 32 bits = 1 word

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary
- A **word** is the number of bits that the CPU works with at once
  - 32-bit CPU: 32 bits = 1 word
  - 64-bit CPU: 64 bits = 1 word

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary
- A **word** is the number of bits that the CPU works with at once
  - 32-bit CPU: 32 bits = 1 word
  - 64-bit CPU: 64 bits = 1 word
- An n-bit word can store a number between 0 and $2^n - 1$

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary
- A **word** is the number of bits that the CPU works with at once
  - 32-bit CPU: 32 bits = 1 word
  - 64-bit CPU: 64 bits = 1 word
- An *n*-bit word can store a number between 0 and $2^n - 1$
  - $2^{16} - 1 = 65,535$

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary
- A **word** is the number of bits that the CPU works with at once
  - 32-bit CPU: 32 bits = 1 word
  - 64-bit CPU: 64 bits = 1 word
- An *n*-bit word can store a number between 0 and $2^n - 1$
  - $2^{16} - 1 = 65,535$
  - $2^{32} - 1 = 4,294,967,295$

Bits, bytes and words

- A **bit** is a binary digit
  - Can store a 0 or 1 (i.e. a boolean value)
- A **byte** is 8 **bits**
  - Can store a number between 0 and 255 in binary
- A **word** is the number of bits that the CPU works with at once
  - 32-bit CPU: 32 bits = 1 word
  - 64-bit CPU: 64 bits = 1 word
- An *n*-bit word can store a number between 0 and $2^n - 1$
  - $2^{16} - 1 = 65,535$
  - $2^{32} - 1 = 4,294,967,295$
  - $2^{64} - 1 = 18,446,744,073,709,551,615$

Addition with carry

In base 10:

$$\begin{array}{rrrrr} & 1 & 2 & 3 & 4 \\ + & 5 & 6 & 7 & 8 \\ \hline \end{array}$$

Addition with carry

In base 10:

$$
\begin{array}{ccccc}
 & 1 & 2 & 3 & 4 \\
+ & 5 & 6 & 7_1 & 8 \\
\hline
 & & & & 2
\end{array}
$$

Addition with carry

In base 10:

$$
\begin{array}{ccccc}
 & 1 & 2 & 3 & 4 \\
+ & 5 & 6_1 & 7_1 & 8 \\
\hline
 & & & 1 & 2
\end{array}
$$

Addition with carry

In base 10:

$$
\begin{array}{ccccc}
 & 1 & 2 & 3 & 4 \\
+ & 5 & 6_1 & 7_1 & 8 \\
\hline
 & & 9 & 1 & 2
\end{array}
$$

Addition with carry

In base 10:

$$\begin{array}{rcccc} & 1 & 2 & 3 & 4 \\ + & 5 & 6_1 & 7_1 & 8 \\ \hline & 6 & 9 & 1 & 2 \end{array}$$

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$\begin{array}{ccccccccc} & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ + & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ \hline \end{array}$$

FALMOUTH UNIVERSITY

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$\begin{array}{ccccccccc} & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ + & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ \hline & & & & & & & & 1 \end{array}$$

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$\begin{array}{ccccccccc}
 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
+ & 0 & 0 & 1 & 0 & 0 & 1_1 & 1 & 1 \\
\hline
 & & & & & & & 0 & 1
\end{array}$$

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$\begin{array}{ccccccccc} & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ + & 0 & 0 & 1 & 0 & 0_1 & 1_1 & 1 & 1 \\ \hline & & & & & & 1 & 0 & 1 \end{array}$$

Addition with carry

In base 2:

$$\boxed{1+1=10 \qquad 1+1+1=11}$$

$$\begin{array}{ccccccccc} & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ + & 0 & 0 & 1 & 0_1 & 0_1 & 1_1 & 1 & 1 \\ \hline & & & & & 0 & 1 & 0 & 1 \end{array}$$

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$
\begin{array}{cccccccccc}
 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
+ & 0 & 0 & 1 & 0_1 & 0_1 & 1_1 & 1 & 1 \\
\hline
 & & & & 1 & 0 & 1 & 0 & 1
\end{array}
$$

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$\begin{array}{ccccccccc}
 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
+ & 0 & 0_1 & 1 & 0_1 & 0_1 & 1_1 & 1 & 1 \\
\hline
 & & & 0 & 1 & 0 & 1 & 0 & 1
\end{array}$$

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$\begin{array}{cccccccccc}
 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
+ & 0_1 & 0_1 & 1 & 0_1 & 0_1 & 1_1 & 1 & 1 \\
\hline
 & & 0 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}$$

Addition with carry

In base 2:

$$\boxed{1 + 1 = 10 \qquad 1 + 1 + 1 = 11}$$

$$
\begin{array}{cccccccccc}
 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
+ & 0_1 & 0_1 & 1 & 0_1 & 0_1 & 1_1 & 1 & 1 \\
\hline
 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}
$$

Modular arithmetic

Modular arithmetic

- Arithmetic **modulo** N

Modular arithmetic

- Arithmetic **modulo** N
- Numbers "wrap around" between 0 and $N - 1$

Modular arithmetic

- Arithmetic **modulo** N
- Numbers "wrap around" between 0 and $N - 1$
- E.g. modulo 16:

Modular arithmetic

- Arithmetic **modulo** N
- Numbers “wrap around” between 0 and $N - 1$
- E.g. modulo 16:
  - $14 + 7 = 5$

Modular arithmetic

- Arithmetic **modulo** N
- Numbers "wrap around" between 0 and $N - 1$
- E.g. modulo 16:
  - $14 + 7 = 5$
  - $4 - 7 = 13$

2's complement

2's complement

- How can we represent negative numbers in binary?

2's complement

- How can we represent negative numbers in binary?
- Represent them modulo 2^n (for n bits)

2's complement

- How can we represent negative numbers in binary?
- Represent them modulo 2^n (for n bits)
- I.e. represent $-a$ as $2^n - a$

2's complement

- How can we represent negative numbers in binary?
- Represent them modulo 2^n (for n bits)
- I.e. represent $-a$ as $2^n - a$
- Instead of an n-bit number ranging from 0 to $2^n - 1$, it ranges from -2^{n-1} to $+2^{n-1} - 1$

2's complement

- How can we represent negative numbers in binary?
- Represent them modulo 2^n (for n bits)
- I.e. represent $-a$ as $2^n - a$
- Instead of an n-bit number ranging from 0 to $2^n - 1$, it ranges from -2^{n-1} to $+2^{n-1} - 1$
- E.g. 16-bit number ranges from -32768 to $+32767$

2's complement

- How can we represent negative numbers in binary?
- Represent them modulo 2^n (for n bits)
- I.e. represent $-a$ as $2^n - a$
- Instead of an n-bit number ranging from 0 to $2^n - 1$, it ranges from -2^{n-1} to $+2^{n-1} - 1$
- E.g. 16-bit number ranges from -32768 to $+32767$
- Note that the left-most bit can be interpreted as a **sign** bit: 1 if negative, 0 if positive or zero

Converting to 2's complement

Converting to 2's complement

- Convert the absolute value to binary

Converting to 2's complement

- Convert the absolute value to binary
- Invert all the bits (i.e. change $0 \leftrightarrow 1$)

Converting to 2's complement

- Convert the absolute value to binary
- Invert all the bits (i.e. change $0 \leftrightarrow 1$)
- Add 1

Converting to 2's complement

- Convert the absolute value to binary
- Invert all the bits (i.e. change $0 \leftrightarrow 1$)
- Add 1
- (This is equivalent to subtracting the number from 2^n... why?)

Why 2's complement?

Why 2's complement?

- Allows all addition and subtraction to be carried out modulo 2^n without caring whether numbers are positive or negative

Why 2's complement?

- Allows all addition and subtraction to be carried out modulo 2^n without caring whether numbers are positive or negative
- In fact, subtraction can just be done as addition

Why 2's complement?

- Allows all addition and subtraction to be carried out modulo 2^n without caring whether numbers are positive or negative
- In fact, subtraction can just be done as addition
- I.e. $a - b$ is the same as $a + (-b)$, where a and $-b$ are just n-bit numbers

Exercise Sheet i

Due next Tuesday!