1: Module Introduction

COMP270: MATHEMATICS FOR 3D WORLDS & SIMULATIONS

Module Introduction

Aim

To empower you to leverage mathematics and mathematical modelling in the design and implementation of real-time 3D worlds and simulations.

Summary

On this module, you learn the fundamental mathematics involved in the design, development and maintenance of real-time 3D worlds and simulations. In doing so, you will leverage mathematics practically to generate and manipulate worlds and simulations relevant to a range of creative computing contexts. Indicatively, content spans topics such as linear algebra (vectors, matrices and quaternions), geometry, trigonometry, 3D transformation, collision detection, Newtonian mechanics, numerical control, calculus, and efficiency and optimisation of numerical methods.

Learning Outcome

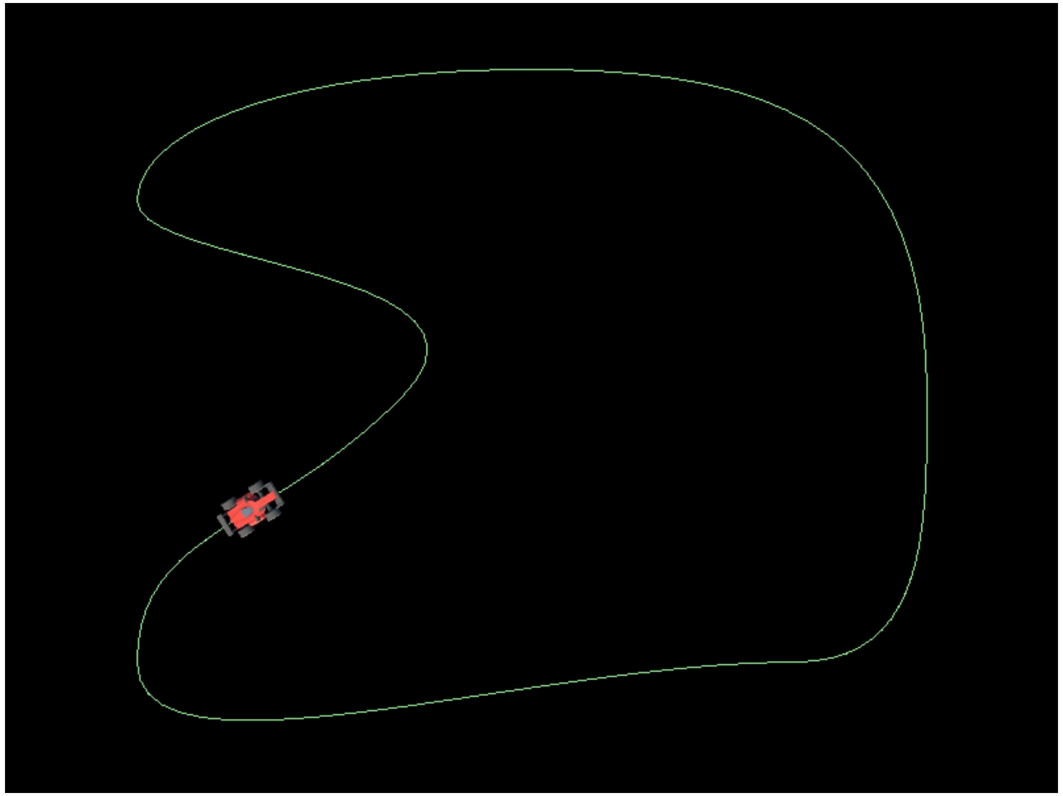
ID	NAME	DESCRIPTION	ASSESSMENT CRITERIA CATEGORY
3	Solve	Apply knowledge of algorithms, data structures, and mathematics to solve well-defined problems.	PROCESS

Assignments

- Assignment 1: Worksheet Tasks [100%]
- **Four** worksheets (roughly one every 2 weeks)
- Worksheets test your **mathematical problem solving** and **C++ programming**
- See LearningSpace for assignment brief, worksheets and formative deadlines
- See MyFalmouth for summative deadline

Worksheet A

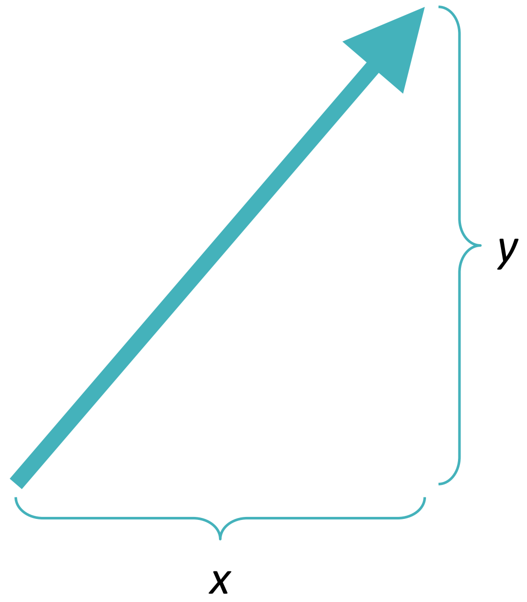
- Bézier Curves
- Due Monday week 4 (14th October)



Vectors (a refresher)

2D Vectors

- A 2D **vector** is an **arrow** on the 2D **plane**
- Represents a quantity with **direction** and **length**
- Can represent a **point** on the plane
  (relative to the **origin**)
- Defined by a pair of numbers:
  the **x component** and the **y component**
- By mathematical convention,
  positive x points to the **right**
  and positive y points **up**
- In computer graphics we sometimes have
  positive y point **down** instead



Writing vectors

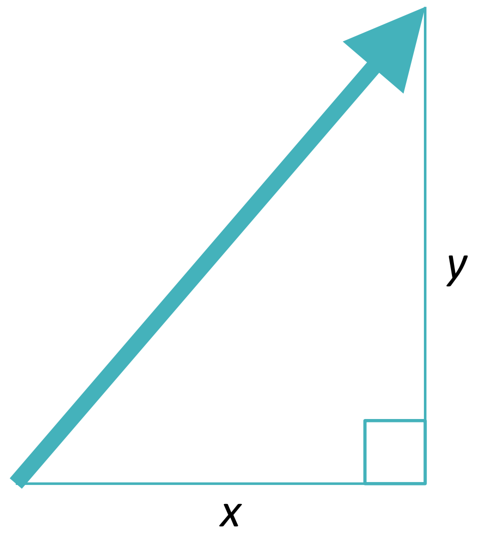
- As a pair of numbers: (x, y)
- As a column vector: $\begin{pmatrix} x \\ y \end{pmatrix}$
- Variable representing a vector: written in bold i.e. $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$
  - Other notations: $\vec{\mathrm{v}}, \underline{\mathrm{v}}$

2D vectors and triangles

- A 2D vector with positive x and y components defines a **right-angled triangle**
- The short sides have lengths x and y, and the hypotenuse corresponds to the vector
- This gives us the formula for the **magnitude** of the vector:

$$\|\mathbf{v}\| = \sqrt{x^2 + y^2}$$

- This also works if x and/or y are zero or negative (remember that $x^2 \geq 0$ for all x)

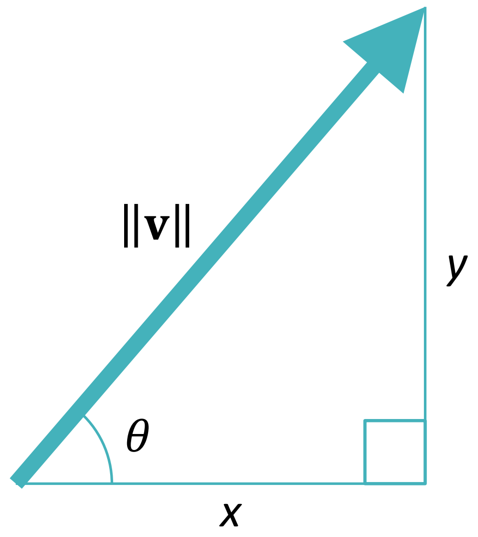


2D vectors and trigonometry

- Consider the angle θ that $\mathbf{v}$ makes with the positive x axis
- Then basic trigonometry ("SOH CAH TOA") tells us:

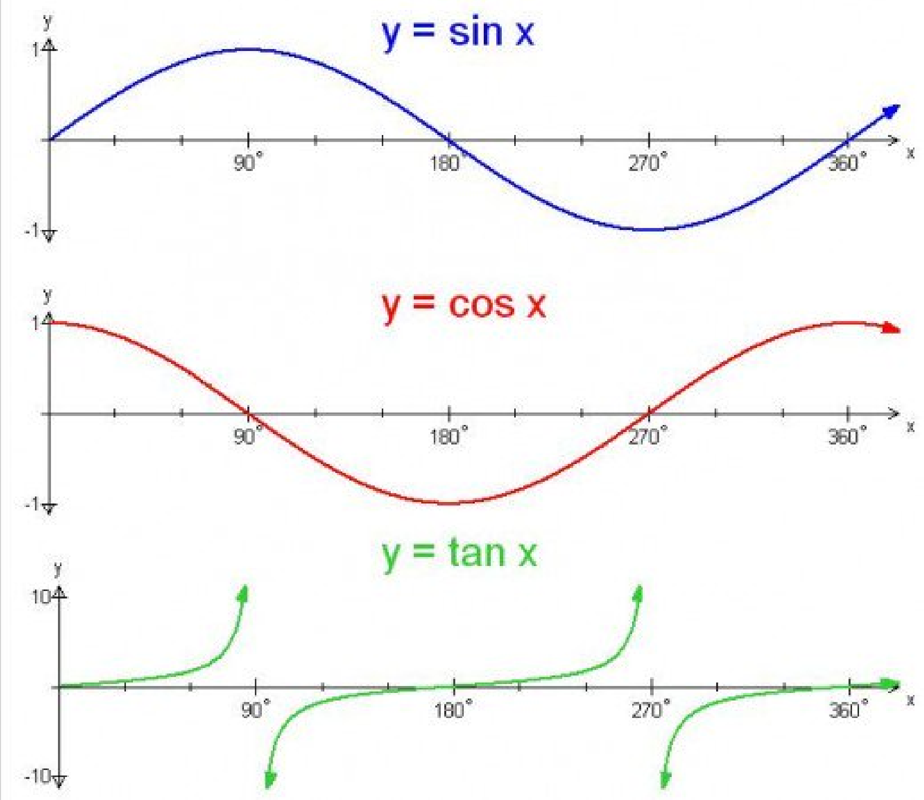
$$\sin\theta = \frac{y}{\|\mathbf{v}\|}$$
$$\cos\theta = \frac{x}{\|\mathbf{v}\|}$$
$$\tan\theta = \frac{y}{x}$$

- Again, this also works when x and/or y are non-positive



Finding the angle

- Given $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$, find the angle θ that $\mathbf{v}$ makes with the positive x axis
- Use the **inverse tan** function
$$\theta = \tan^{-1}\frac{y}{x}$$
- Most programming languages refer to inverse trig functions as **arc** functions (shortened to **a**): `asin`, `acos`, `atan`



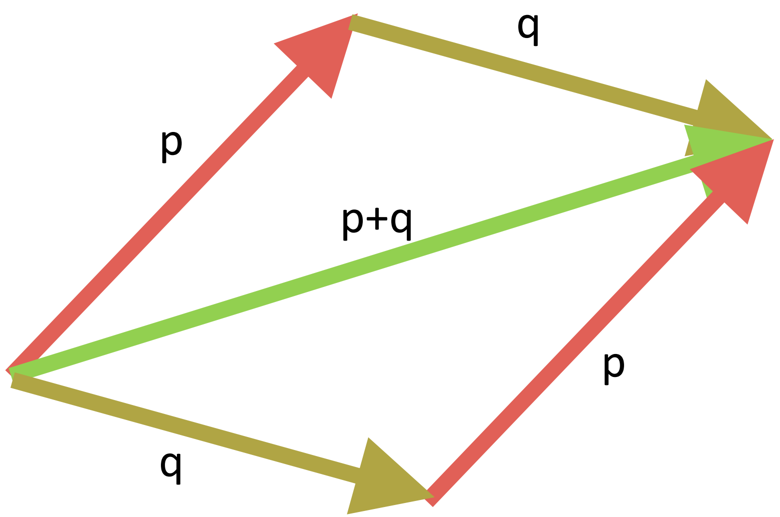
Inverse tangent

- Care is needed when using $\tan^{-1}$
- $\tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-y}{-x}$ so we need to be aware of which **quadrant** the vector is in (otherwise the result may be out by 180°)
- If $x = 0$ (the vector points vertically along the y axis) then we're dividing by zero (which is bad)
- Most programming languages have an `atan2(y, x)` function which takes two arguments and handles all of these cases for you

Vector addition

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$

- If $\mathbf{p}$ and $\mathbf{q}$ are vectors
- Then $\mathbf{p} + \mathbf{q}$ is obtained by putting $\mathbf{p}$ and $\mathbf{q}$ end to end as shown here
- Note addition is **commutative**: $\mathbf{p} + \mathbf{q} = \mathbf{q} + \mathbf{p}$

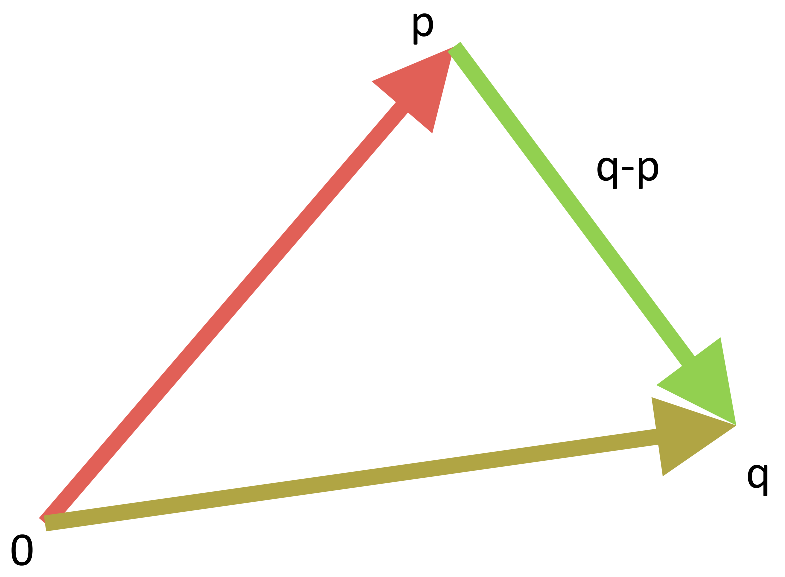


Vector subtraction

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}$$

- If $\mathbf{p}$ and $\mathbf{q}$ are vectors representing points on the plane
- Then $\mathbf{q} - \mathbf{p}$ represents the vector from $\mathbf{p}$ to $\mathbf{q}$
- Note the order of the subtraction!
- Note subtraction is **anticommutative**:

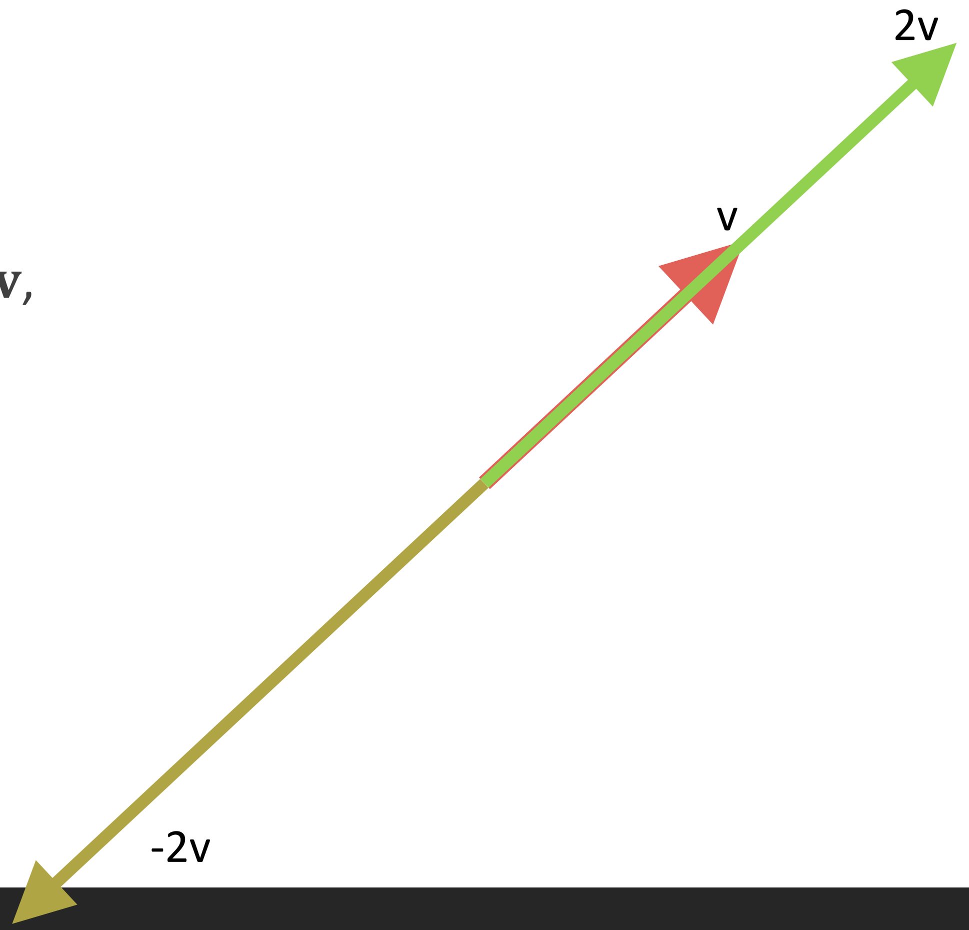
$$\mathbf{q} - \mathbf{p} = -(\mathbf{p} - \mathbf{q})$$



Scalar multiplication

$$c\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cx \\ cy \end{pmatrix}$$

- If $\mathbf{v}$ is a vector and c is a positive number
- Then $c\mathbf{v}$ is a vector with the same direction as $\mathbf{v}$, but c times the magnitude
- If c is negative then $c\mathbf{v}$ has the opposite direction to $\mathbf{v}$



Dot product

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \cdot \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = x_1 x_2 + y_1 y_2$$

- If $\mathbf{p}$ and $\mathbf{q}$ are vectors
- Then $\mathbf{p} \cdot \mathbf{q} = \|\mathbf{p}\| \|\mathbf{q}\| \cos\theta$ where θ is the angle between $\mathbf{p}$ and $\mathbf{q}$
- $\mathbf{p}$ and $\mathbf{q}$ are perpendicular **if and only if** $$\mathbf{p} \cdot \mathbf{q} = 0$$