



FALMOUTH  
UNIVERSITY



COMP110: Principles of Computing

# Basic Principles for Computation

# Worksheet 1



# Worksheet 1

- ▶ Due **tomorrow!**
- ▶ Support workshop **later this afternoon**

# Worksheet submission

- ▶ <https://github.com/falmouth-games-academy/comp110-worksheet-1>
- ▶ If the YouTube uploader within SpaceChem does not work, save the video to disk and upload to YouTube manually
- ▶ If this also doesn't work, use e.g. OBS to record videos and upload them (I only need to see your solutions running)
- ▶ If **this** doesn't work, record your screen with your phone!

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- ▶ <https://github.com/falmouth-games-academy/comp110-worksheet-1>
- ▶ Don't forget to upload your **save file**, add your **playlist link**, and open a **pull request**
- ▶ Please make sure I can figure out who you are! If it's not obvious from your username, please put your **name or student number** in the title of your pull request
- ▶ Any problems, send me a message via email or slack

# Binary notation



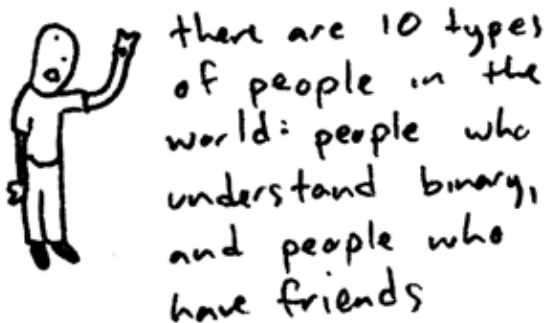


Image credit: <http://www.toothpastefordinner.com>

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- ▶ The binary digits 0 and 1 correspond to **off** and **on** respectively

# Converting to binary

[https://www.youtube.com/watch?v=OezK\\_zTyvAQ](https://www.youtube.com/watch?v=OezK_zTyvAQ)

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$$1 + 1 = 10 \quad 1 + 1 + 1 = 11$$

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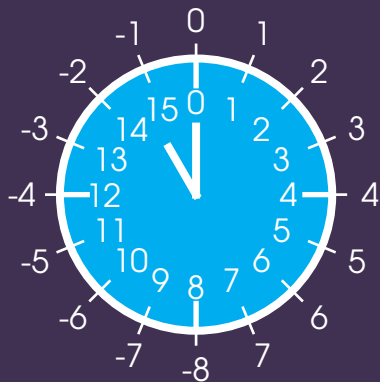
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Hex	Dec	Hex	Dec	Hex	Dec
00	0	10	16	F0	240
01	1	11	17	F1	241
⋮	⋮	⋮	⋮	⋮	⋮
09	9	19	25	F9	249
0A	10	1A	26	FA	250
0B	11	1B	27	FB	251
0C	12	1C	28	FC	252
0D	13	1D	29	FD	253
0E	14	1E	30	FE	254
0F	15	1F	31	FF	255

# 2's Complement

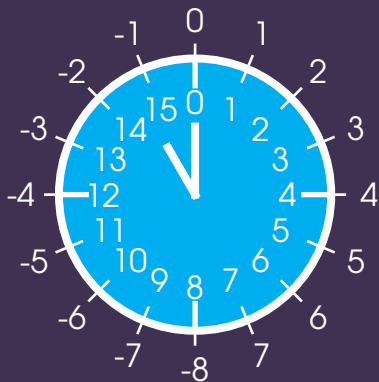


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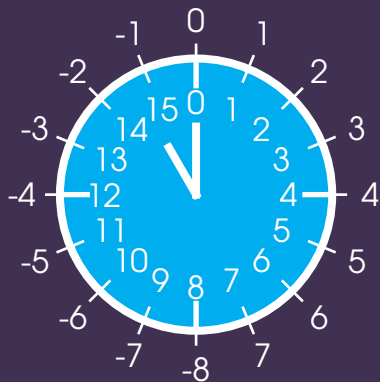


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- ▶ `a % b` gives the **remainder** of `a` divided by `b`
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- ▶ Useful for wrapping around e.g. loop indexes or screen coordinates

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- ▶ E.g. 16-bit number ranges from  $-32768$  to  $+32767$
- ▶ Note that the left-most bit can be interpreted as a **sign** bit: 1 if negative, 0 if positive or zero

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- ▶ This is also the process for converting back from 2's complement, i.e. doing it twice should give the original number

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- ▶ I.e.  $a - b$  is the same as  $a + (-b)$ , where  $a$  and  $-b$  are just  $n$ -bit numbers

# Worksheet 2



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Due next Friday!  
Online quiz on LearningSpace

# Algorithms



# What is an algorithm?

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A **sequence of instructions** which can be followed **step by step** to perform a **(computational) task**.



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- ▶ Used in mathematics to describe steps for calculations
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- ▶ Computers developed as machines for carrying out mathematical algorithms

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  - ▶ E.g. it implements an algorithm for determining where to break a line of text, how much space to add to centre a line, etc.

# Algorithms outside computing

- 1 Preheat the oven to 180C, gas 4.
- 2 Beat together the eggs, flour, caster sugar, butter and baking powder until smooth in a large mixing bowl.
- 3 Put the cocoa in separate mixing bowl, and add the water a little at a time to make a stiff paste. Add to the cake mixture.
- 4 Turn into the prepared tins, level the top and bake in the preheated oven for about 20-25 mins, or until shrinking away from the sides of the tin and springy to the touch.
- 5 Leave to cool in the tin, then turn on to a wire rack to become completely cold before icing.
- 6 To make the icing: measure the cream and chocolate into a bowl and carefully melt over a pan of hot water over a low heat, or gently in the microwave for 1 min (600w microwave). Stir until melted, then set aside to cool a little and to thicken up.
- 7 To ice the cake: spread the apricot jam on the top of each cake. Spread half of the ganache icing on the top of the jam on one of the cakes, then lay the other cake on top, sandwiching them together.
- 8 Use the remaining ganache icing to ice the top of the cake in a swirl pattern. Dust with icing sugar to serve.

[illegible]

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- ▶ Can reason about the **complexity** (time, space etc) of an algorithm — and place **lower bounds** on the best possible algorithm
- ▶ **Computability** theory lets us reason about what computations are and are not possible