COMP110: Principles of Computing

11: **Data Structures II**

Learning outcomes

- **Define** the key concepts of graph theory
- **Distinguish** advanced data structures such as trees, DAGs and graphs
- **Determine** the complexity of accessing and manipulating data in these data structures
- **Choose** the correct data structure for a given task

Stacks and queues

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- ▶ A **queue** is a **first-in first-out (LIFO)** data structure
- ▶ Items can be **enqueued** to the **back** of the queue
- ▶ Items can be **dequeued** from the **front** of the queue

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  - ▶ All of which are $O(1)$

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Linked lists

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Lists in Python

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- Deleting is $O(n)$
- Changing size sometimes requires the entire array to be reallocated and copied

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item	next
first	→

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second	→

item	next
third	×
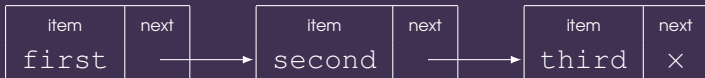
Linked lists vs arrays

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Implementing a linked list

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Graphs
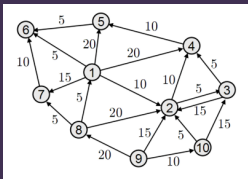
Graphs

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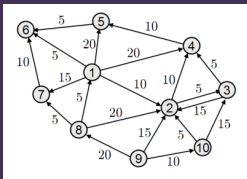
- A **graph** is defined by:
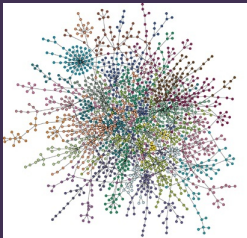
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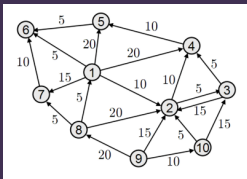
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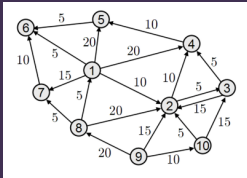 - A collection of **nodes** or **vertices** (points)
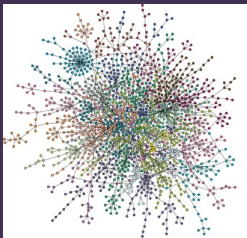
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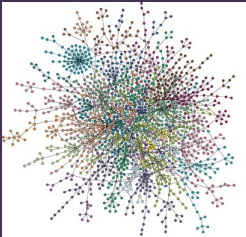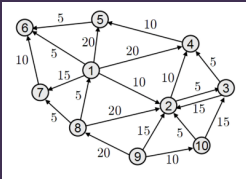
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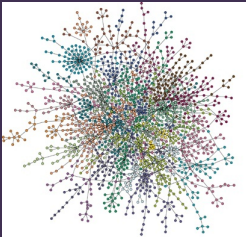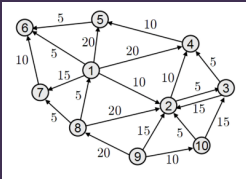 - A collection of **edges** or **arcs** (lines or arrows between points)

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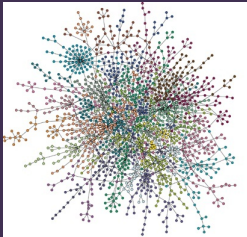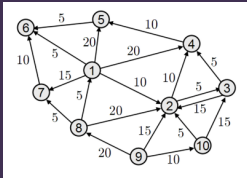
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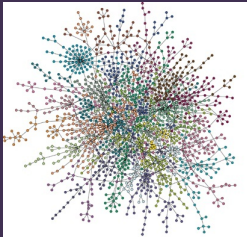
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- **Directed** graph: edges are arrows
- **Undirected** graph: edges are lines

Implementing graphs

Implementing graphs

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Implementing graphs

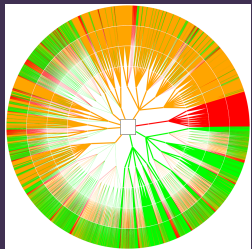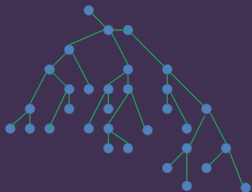- A graph is a **collection of nodes**
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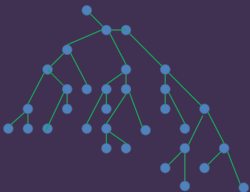
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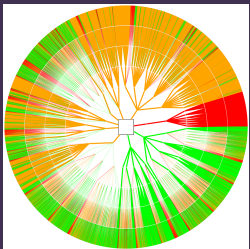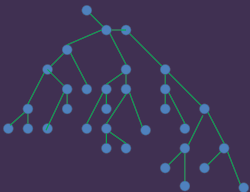- Each edge has exactly **two nodes** associated with it
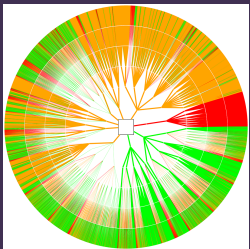
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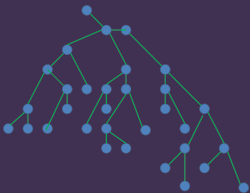- Edges go from **parent** to **child**

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- Edges go from **parent** to **child**
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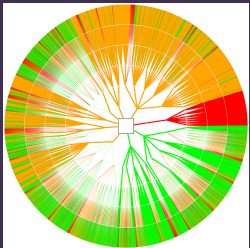
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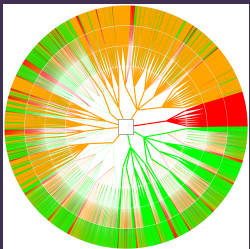 - One node (the **root**) has no incoming edges
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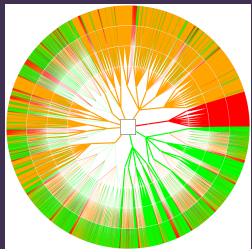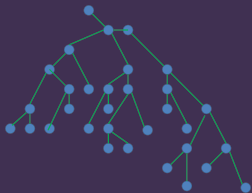 - Nodes can have 0, 1 or many children

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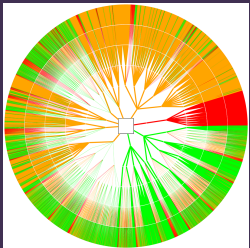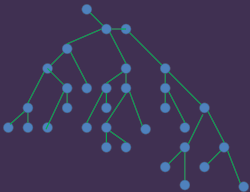
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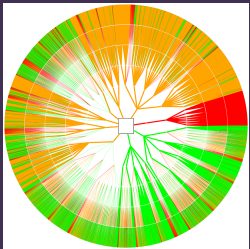 - All other nodes have exactly 1 incoming edge
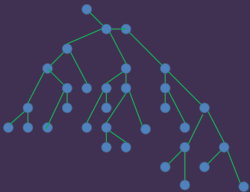- Edges go from **parent** to **child**
  - All nodes except the root have exactly one parent
  - Nodes can have 0, 1 or many children
- Used to model **hierarchies** (e.g. file systems, object inheritance, scene graphs, state-action trees, ...)

Implementing trees

Implementing trees

- A graph has a **root node**

Implementing trees

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- Each node has a **collection of children**

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- A graph has a **root node**
- Each node has a **collection of children**
- Each node other than the root has a **single parent**

Tree traversal

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Tree traversal

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  - Breadth first

Tree traversal

Tree traversal

procedure DEPTHFIRSTSEARCH

Tree traversal

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    let *S* be a stack

Tree traversal

procedure DEPTHFIRSTSEARCH
  let *S* be a stack
  push root node onto *S*

Tree traversal

procedure DEPTHFIRSTSEARCH
    let *S* be a stack
    push root node onto *S*
    while *S* is not empty **do**

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack
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    let *S* be a stack
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    while *S* is not empty **do**
        pop *n* from *S*
        print *n*

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack
    push root node onto S
    while S is not empty **do**
        pop n from S
        print n
        push children of n onto S

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack
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    while S is not empty **do**
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    end while
end procedure

Tree traversal

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procedure BREADTHFIRSTSEARCH

Tree traversal

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    end while
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procedure BREADTHFIRSTSEARCH
    let *Q* be a queue

Tree traversal

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    end while
end procedure

procedure BREADTHFIRSTSEARCH
    let Q be a queue
    enqueue root node into Q

Tree traversal

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end procedure
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Recursive depth first search

Recursive depth first search

procedure DEPTHFIRSTSEARCH(n)

Recursive depth first search

procedure DEPTHFIRSTSEARCH(*n*)
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Recursive depth first search

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    print *n*
    for each child *c* of *n* **do**

Recursive depth first search

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    print n
    for each child c of n **do**
        DEPTHFIRSTSEARCH(c)
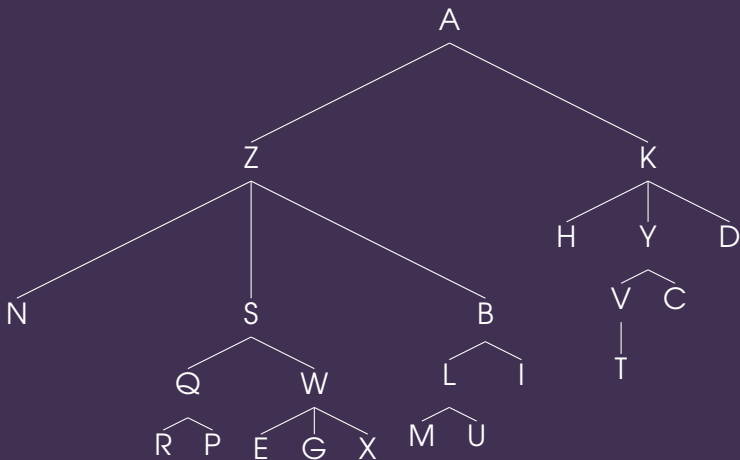
Recursive depth first search

procedure DEPTHFIRSTSEARCH(*n*)

   print *n*

   for each child *c* of *n* **do**

      DEPTHFIRSTSEARCH(*c*)

   end for

end procedure

Recursive depth first search

procedure DEPTHFIRSTSEARCH(*n*)
    print *n*
    for each child *c* of *n* **do**
        DEPTHFIRSTSEARCH(*c*)
    end for
end procedure

- Compare to the pseudocode on the previous slide. Where is the stack?

Tree traversal example

Worksheet D