COMP250: Artificial Intelligence

7: Monte Carlo Tree Search

Heuristics for search

From session 2: Minimax search

```
procedure MINIMAX(state, currentPlayer)
    if state is terminal then
        return value of state
    else if currentPlayer is maximising then
        bestValue = −∞
        for each possible nextState do
            v = MINIMAX(nextState, 3− currentPlayer)
            bestValue = MAX(bestValue, v)
            if bestValue ≥ 1 then
                break
        return bestValue
    else if currentPlayer is minimising then
        bestValue = +∞
        for each possible nextState do
            v = MINIMAX(nextState, 3− currentPlayer)
            bestValue = MIN(bestValue, v)
            if bestValue ≤ −1 then
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  - Chess has $\approx 10^{47}$ states

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- Evaluation is a number between -1 and $+1$, estimating the probable outcome of the game

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- Often easier to design a "which state is better" heuristic than to directly design a "which move to play" heuristic

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- What if we don't have that knowledge? ...

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- Then the **expected value** of X is

$$\sum_x x \cdot p(x)$$

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- What this means: if you play the slot machine N times, on average you will win $N \times \$0.40$

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- Seed is generally based on some source of **entropy**, e.g. system clock, mouse input, electronic noise

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- Applications in physics, engineering, finance, weather forecasting, graphics, ...

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- Higher expected value $=$ more chance of winning

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  - Minimax assumes the evaluation is **deterministic**, but Monte Carlo is not
  - Not commonly used, mainly because there's something better...

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- First few rollouts are **random**
- However, statistics from these rollouts are used to **bias** future rollouts
- Bias rollouts towards **plausible** lines of play, i.e. where each player is trying to play the best move

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  - **Backpropagation**: For each node visited during **selection** and **expansion**, update the node's statistics based on the result of the simulation.
- Perform many rollouts, then use the statistics at the top level of the tree to choose the best move

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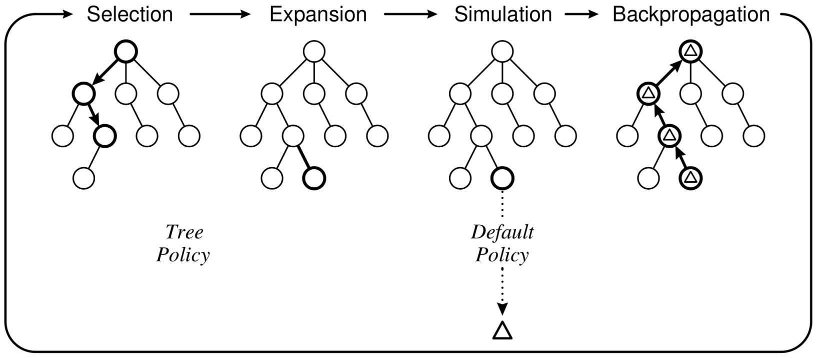

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- This can be modelled as a **multi-armed bandit problem**

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  - c is a parameter for adjusting the balance between exploitation and exploration

UCB demo

http://orangehelicopter.com/academic/bandits.html?ucb

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UCT demo

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  - Selects which parts of the tree to expand more deeply

MCTS for games of imperfect information