Week 4: Mechanics I
Part 2: Quantities and variables

COMP270: Mathematics for 3D Worlds and Simulations

Objectives

- **Define** the basic quantities of mechanics and their units
- **Recall** Newton's Laws of Motion and the key steps to apply them in a physical simulation

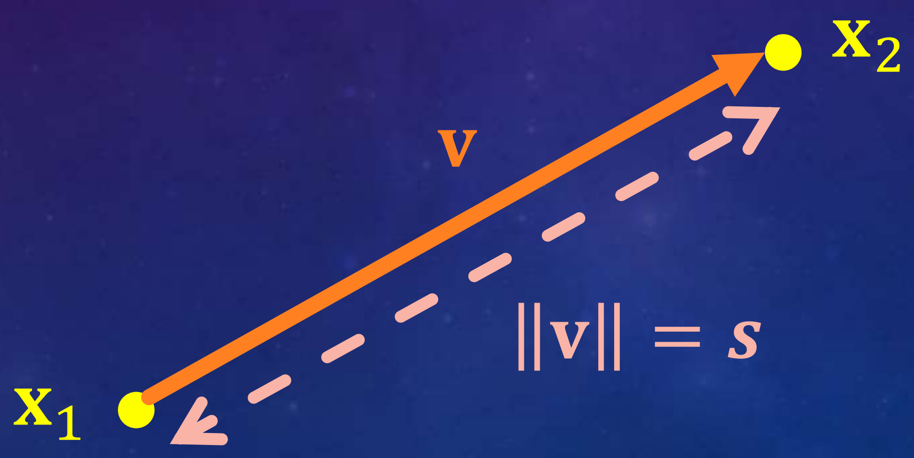
Basic quantities of mechanics

- **Position** describes an object's location in space: $\mathbf{x}$
- **Velocity** is rate of change of position: $\mathbf{v} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}$
- **Acceleration** is rate of change of velocity: $\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2}$
- **Jerk** is the rate of change of acceleration:

$$\mathbf{j} = \frac{\mathrm{d}\mathbf{a}}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{v}}{\mathrm{d}t^2} = \frac{\mathrm{d}^3\mathbf{x}}{\mathrm{d}t^3}$$

Velocity and speed

- Velocity is a vector quantity – has a magnitude and a direction
- We call the magnitude of velocity the **speed**



Units

Système international, or the International System of Units

- In SI units:
- Position is usually measured in metres (m)
- Velocity is measured in metres per second (m/s or ms^{-1})
- Acceleration is measured in metres per second per second (m/s^2 or ms^{-2})
- Other units are possible (e.g. pixels, miles, hours) but be consistent!

Force

- **Definition**: a **force** is a push or pull on an object resulting from its interaction with another object
  - Direct: e.g. friction, tension, air resistance
  - Distant: e.g. gravity, magnetism
- SI unit: Newtons (N)
- Linked to mass (kg)

Named after Isaac Newton (1642-1726/27), English mathematician

Newton's Laws of Motion

I. An object remains at rest or moves at constant velocity unless acted upon by an external force

II. The sum of forces acting upon an object is equal to its mass multiplied by its acceleration ($\mathbf{F} = m\mathbf{a}$)

III. When one body exerts a force on another, the second body exerts an equal and opposite force on the first

Force and acceleration

- **Definition:** 1 Newton of force is the force required to accelerate an object with a mass of 1 kilogram at 1 meter per second per second

$$\mathbf{F} = m\mathbf{a}$$

$$\Rightarrow 1\text{N} = 1\text{kg} \times 1\text{ms}^{-2}$$

- Alternative unit: N = kgms^{-2}

Gravity

$$F = G\frac{m_1 m_2}{d^2}$$

$G = 6.674 \times 10^{-11}\text{N}$

- A force which pulls all objects with mass towards each other
- Tiny unless one or both objects has huge mass (e.g. a planet)
- Near the surface of a planet, gravity pulls objects downwards (towards the centre of the planet) with a force called weight
- $w = mg$, where w is weight, m is mass and g is the acceleration due to gravity

$\mathbf{F} = ma$

- Near Earth's surface, $g \approx 9.81\text{ms}^{-2}$

Gravity and mass

- Gravity applies the same acceleration ($9.81\mathrm{ms}^{-2}$) to all objects on Earth, regardless of weight (or mass)
  - $\cancel{m}_o g = G\frac{\cancel{m}_o m_E}{d^2}$
- The only reason that some objects fall faster than others is air resistance – in a vacuum all objects fall at the same rate

Even a feather and a bowling ball!

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www.framestore.com/work/gravity

Simulating Newtonian physics

- For each object, store its position $\mathbf{x}$ and velocity $\mathbf{v}$
- On each time step:
  - Apply numerical integration to the velocity to determine the new position, $\mathbf{x}' = \mathbf{x} + \mathbf{v}\Delta t$
  - Calculate the forces acting upon the object, and thus the acceleration $\mathbf{a}$ from Newton's 2nd law
  - Apply numerical integration to the acceleration to determine the new velocity, $\mathbf{v}' = \mathbf{v} + \mathbf{a}\Delta t$