COMP110: Principles of Computing

7: Data Structures I

Assignments

- Peer review **next Thursday**
- Please come to the session with a **draft** of your research journal

- Worksheet 5 due **tomorrow**
- **No new worksheet** this week — work on your research journal instead

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Turing machines

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- Theoretical model of a "computer"
  - I.e. a machine that carries out computations (calculations)

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  - Where to move the tape head: one space to the left, or one space to the right

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The Church-Turing Thesis

- If a calculation can be carried out by a mechanical process at all, then it can be carried out by a Turing machine
- I.e. a Turing machine is the most "powerful" computer possible, in terms of what is possible or impossible to compute
- A machine, language or system is **Turing complete** if it can simulate a Turing machine

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Computability

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- A function $f : A \to B$ is **computable** if there exists a Turing machine which computes f
  - I.e. given an encoding of $a \in A$ as input, the Turing machine outputs an encoding of $f(a)$

An uncomputable function

The **halting problem**

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- f is **uncomputable**

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Halting revisited

- Write a software tool that, given a Python program, predicts whether that program can go into an infinite loop
- Your tool must work for **all** Python programs
- Is this possible?

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Arrays and lists

Memory allocation

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- Blocks can be allocated and deallocated at will, but can **never grow or shrink**

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  - Bundle together the data's representation in memory along with the algorithms for accessing it

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- Accessing an array element is **constant time** $O(1)$

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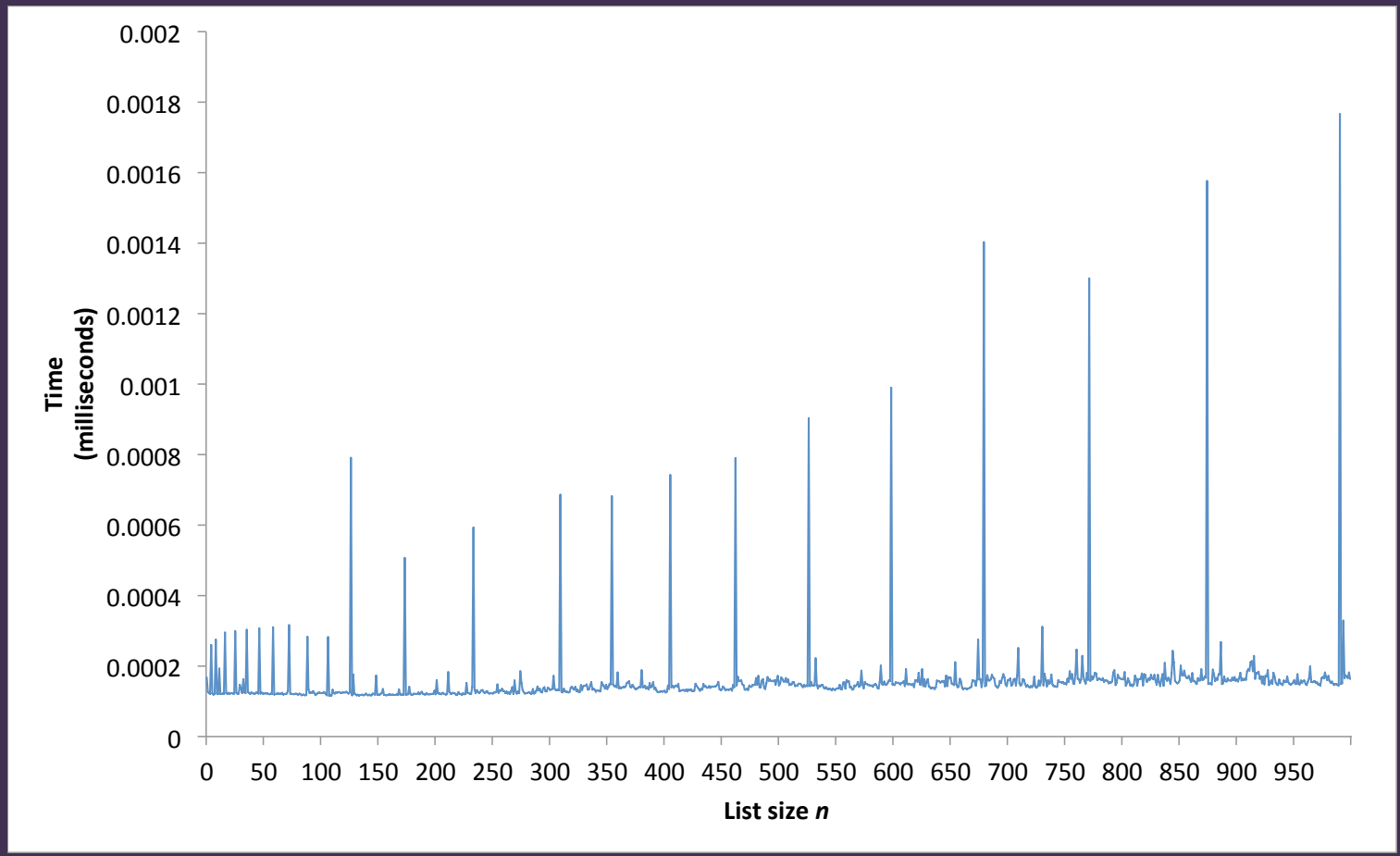
- When the list needs to change size, it **creates** a new array, **copies** the contents of the old array, and **deletes** the old array
- Implementation details: `http://www.laurentluce.com/posts/python-list-implementation/`

Time taken to append an element to a list of size *n*



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- **Inserting** anywhere other than the end is **linear time**
  - Can't just insert new bytes into a memory block — need to move all subsequent list elements to make room
- Similarly, **deleting** anything other than the last element is **linear time**

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Stacks and queues

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  - All of which are $O(1)$

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- Returning from a function **pops** the top frame off the stack

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- Our picture of a variable: a labelled box containing a value
- For "plain old data" (e.g. numbers), this is accurate
- For **objects** (i.e. instances of classes), variables actually hold **references** (a.k.a. **pointers**)
- It is possible (indeed common) to have **multiple references** to the same underlying object

The wrong picture

```
class Thing:
    def __init__(self,
                 a, b):
        self.a = a
        self.b = b

x = Thing(30, 40)
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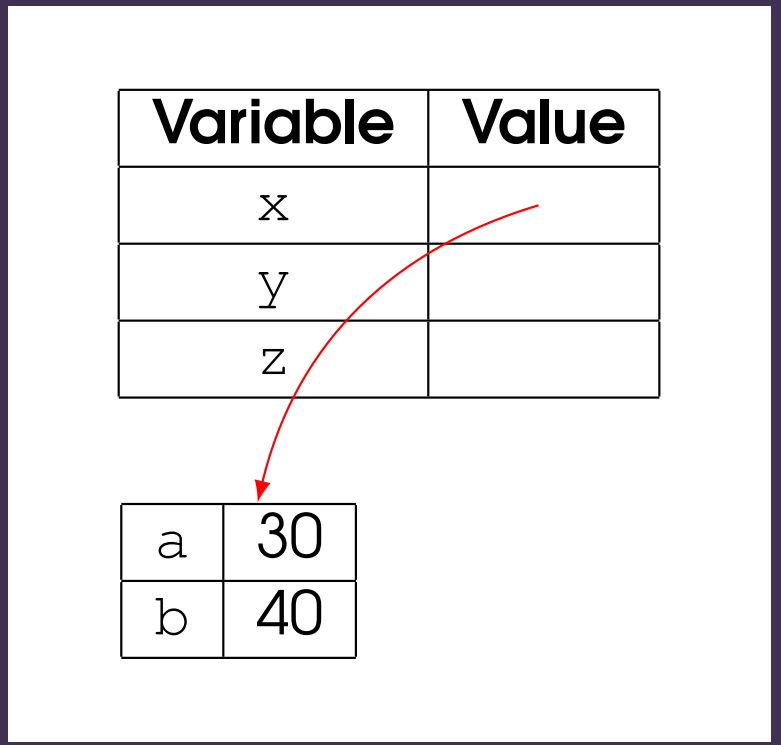

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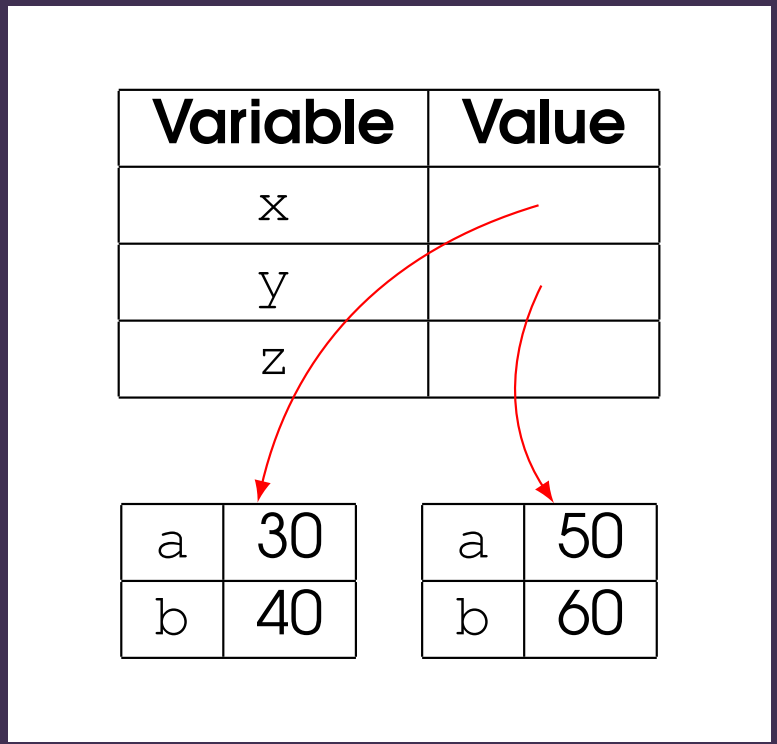

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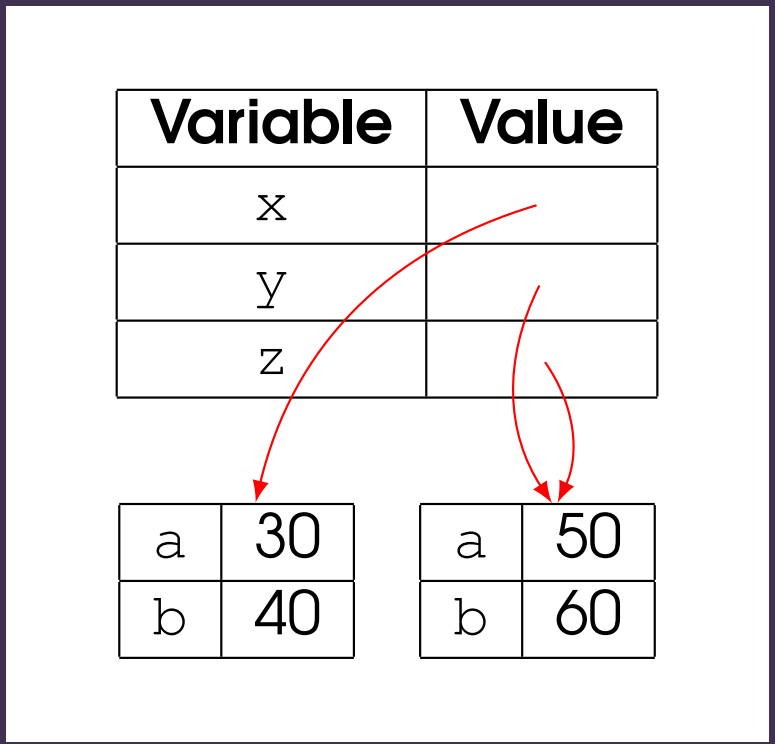

Values and references

Socrative room code: `FALCOMPED`

```
a = 10
b = a
a = 20
print("a:", a)
print("b:", b)
```

Values and references

Socrative room code: FALCOMPED

```
class X:
    def __init__(self, value):
        self.value = value

a = X(10)
b = a
a.value = 20
print("a:", a.value)
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`double` does not actually do anything, as `x` is just a local copy of whatever is passed in!

Pass by reference

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However, instances are passed by **reference**

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class Box:
    def __init__(self, v):
        self.value = v

def double(x):
    x.value *= 2

a = Box(7)
double(a)
print(a.value)
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class Box:
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def double(x):
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a = Box(7)
double(a)
print(a.value)
```

`double` now has an effect, as `x` gets a reference to the `Box` instance

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a = ["Hello"]
b = a
b.append("world")
print(a)   # ["Hello", "world"]
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print(a)   # ["Hello", "world"]
```

... which means you should be careful when passing lists into functions, because the function might actually change the list!

References can be circular

```
class X:
    pass

foo = X()
foo.x = foo
foo.y = "Hello"

print(foo.x.x.x.x.x.y)
```

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- References in other languages (e.g. C#, Python) are implemented using pointers
- C++ also has something called references, which are similar but different (pointers can be **retargeted** whilst references cannot)

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- Recall that memory is a series of 1-byte locations, each with a numeric **address**
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- Again this is not really possible in Python/C#, but a common source of bugs in C/C++