

COMP270: Mathematics for 3D Worlds and Simulations

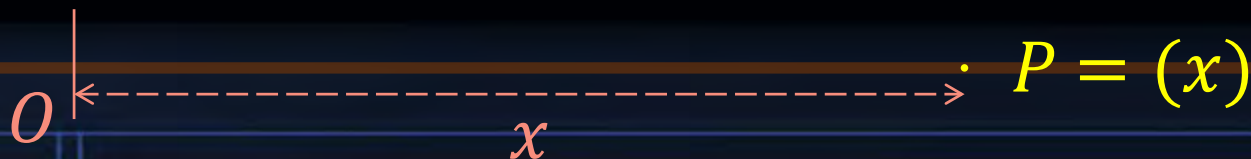
WEEK 2: GEOMETRY I
PART 1: POINTS, LINES AND TRIANGLES

Objectives

- **Define** the basic geometric primitives
- **Recall** the formulae that express relationships between the sides and angles of a right-angled triangle

What is a point?

- Definition: a point is a 0-dimensional mathematical object that can be specified in n -dimensional space using an n -tuple (x_1, x_2, \dots, x_n) consisting of n coordinates.
- 0-dimensional because it has no measurements in any direction...
- In 1D space, its coordinate is just the distance from an origin:



Points in 2D

$(-3, 2)$

$(4, 2.5)$

$(1, 1)$

$(-4, -1)$

Mathematical convention:
positive x points to the right,
positive y points up.
Computer graphics: y -axis
may be flipped.

Lining up

- Definition: a line is a straight one-dimensional figure having no thickness and extending infinitely in both directions.
- Defines a 1D space
- In a space of 2 or more dimensions, a line is uniquely determined by 2 points:

$$A = (x_a, y_a)$$

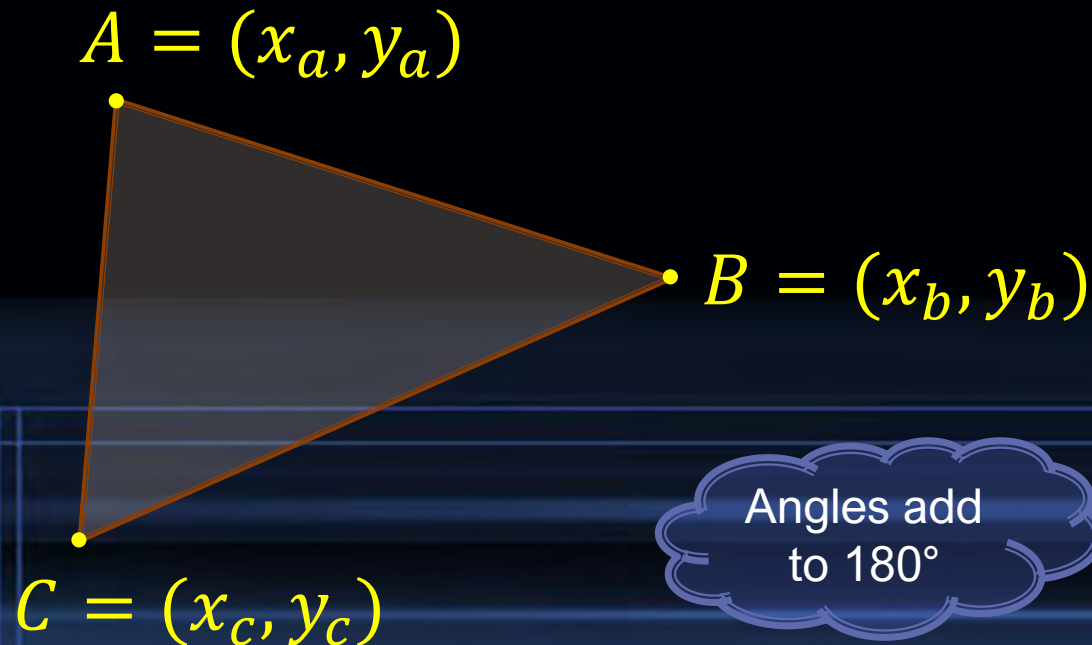
$$B = (x_b, y_b)$$

$$d = ?$$

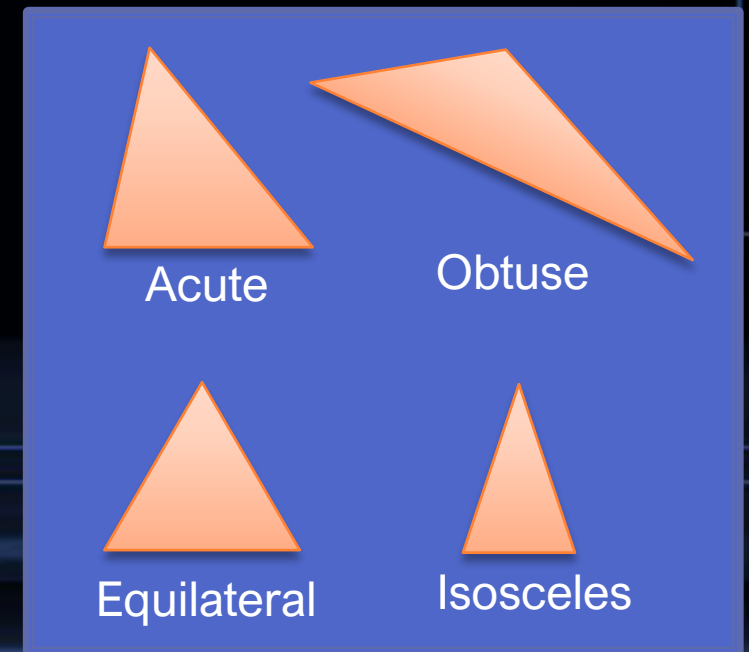
Let's try angles...

From Latin/Greek, “many-angled”; a shape with 3 or more straight sides.

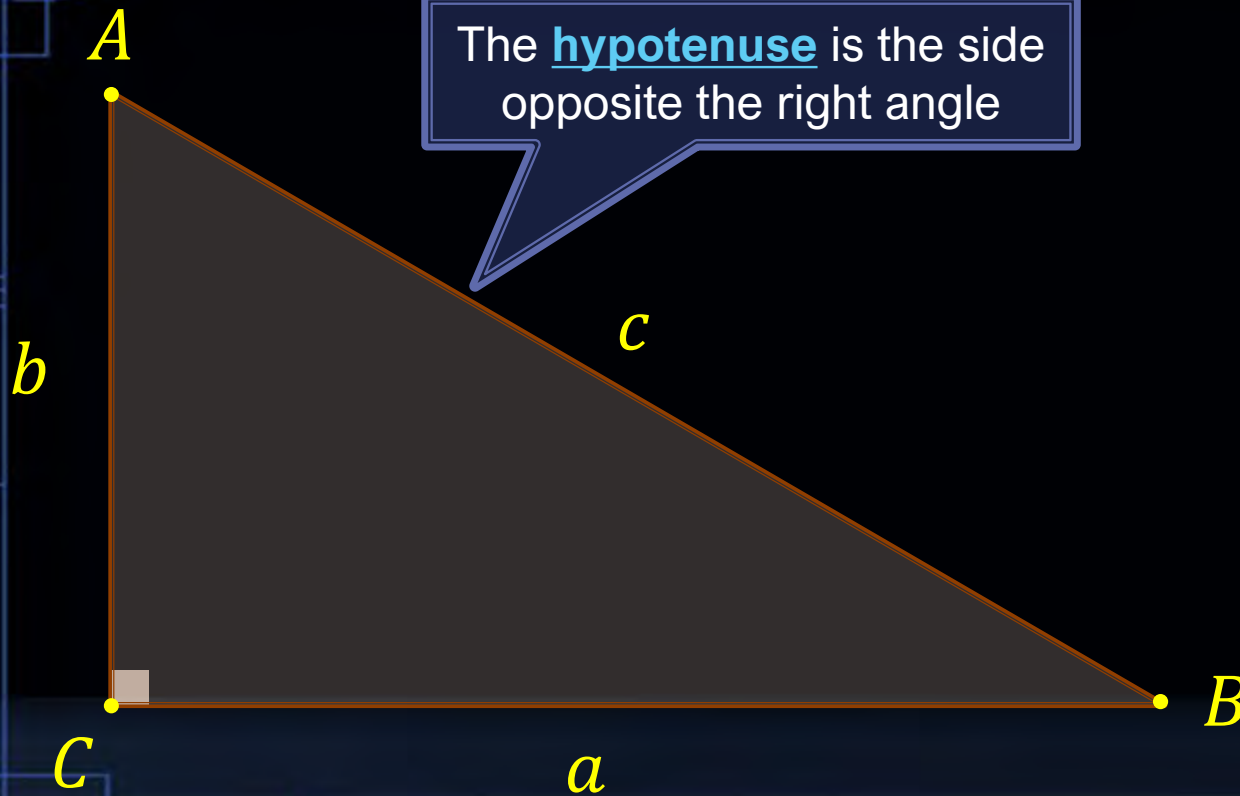
- Definition: a **triangle** is a **polygon with three sides** and three angles, some of which may be the same.
- Uniquely determined by 3 points:



Angles add
to 180°



Right-angled triangles



The hypotenuse is the side opposite the right angle

Named after Pythagoras of Samos (c570-c495BC), Greek philosopher

- **Pythagorean Theorem:**

$$a^2 + b^2 = c^2$$

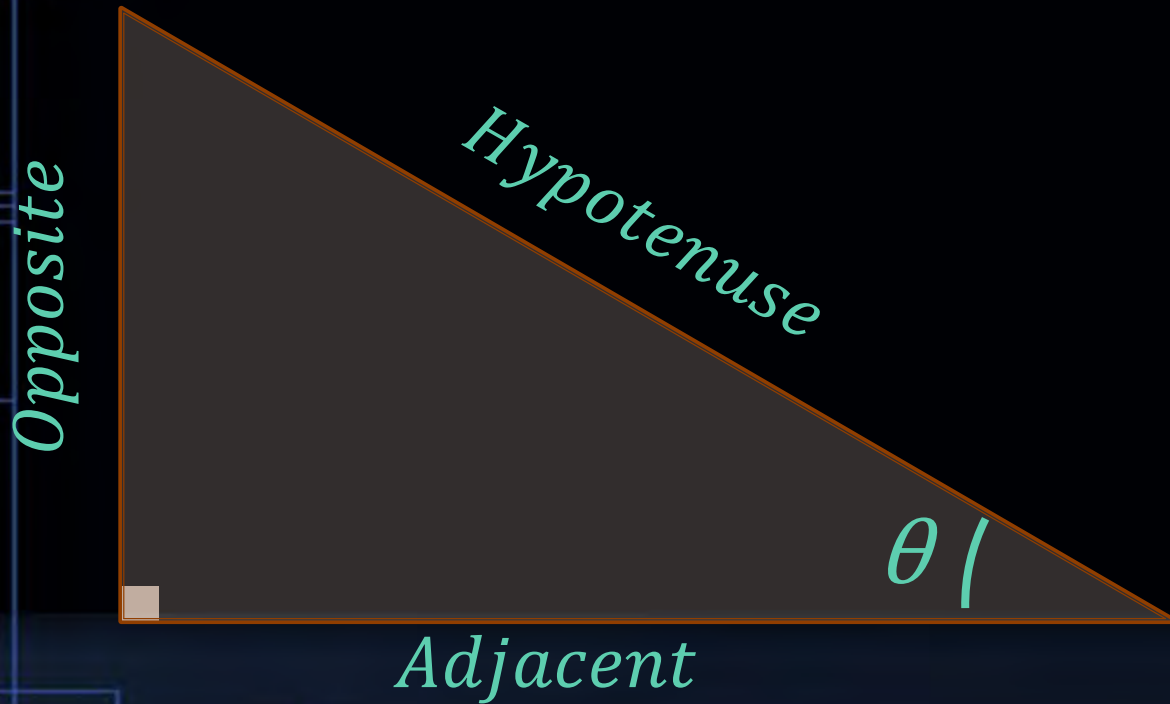
i.e.

$$c = \sqrt{a^2 + b^2}$$

Visual depiction:

www.youtube.com/watch?v=ANR4g0IPrEQ

The Trigonometric Functions



$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

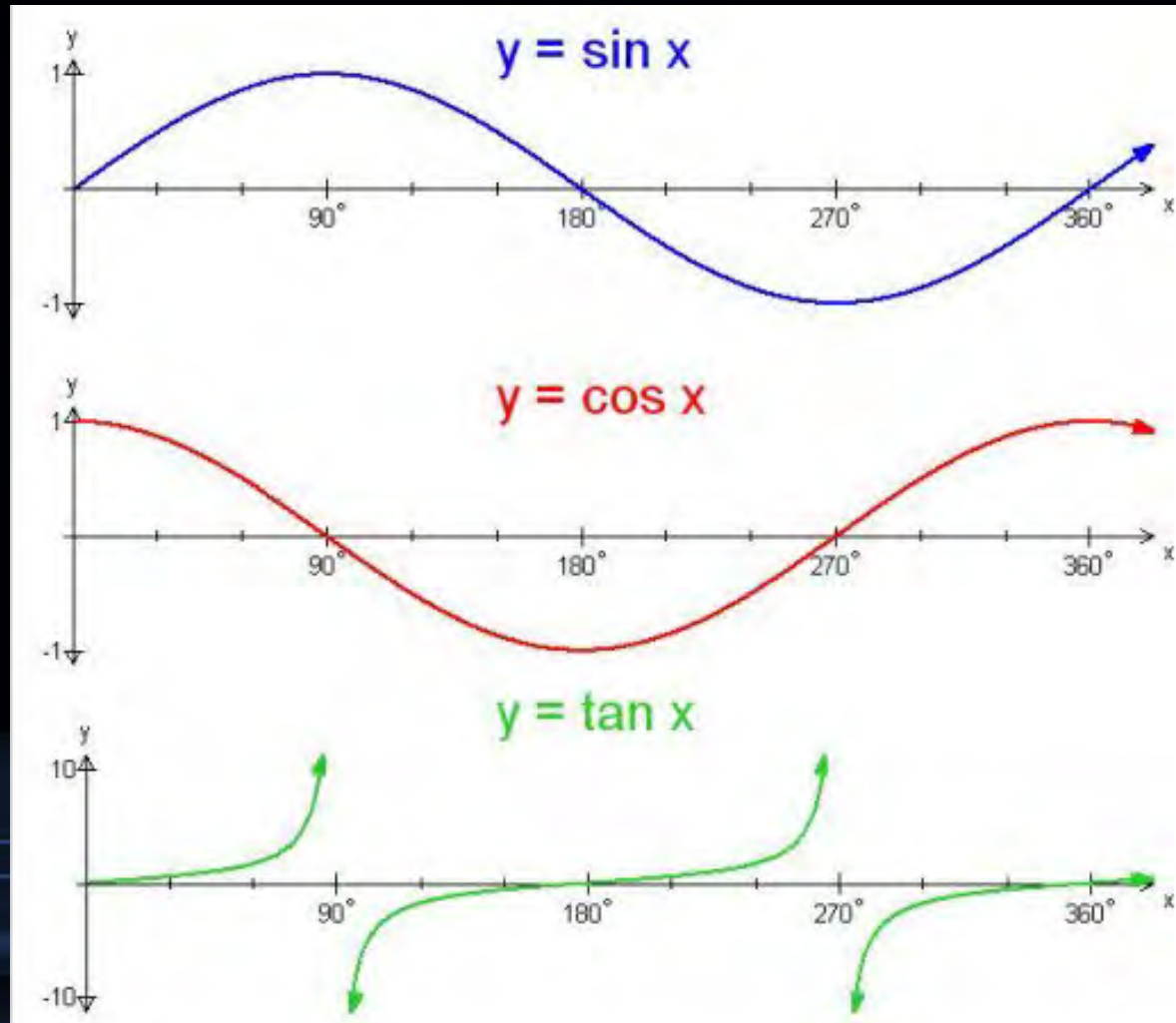
$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{\sin \theta}{\cos \theta}$$

SOHCAHTOA

Silly Old Harry Caught A Herring
Trawling Off America

The Trigonometric Functions



Inverse trig. functions

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\theta = \sin^{-1} \left(\frac{\text{Opposite}}{\text{Hypotenuse}} \right)$$

- Also known as arcsin / arccos / arctan
- In code: asin() / acos() / atan()

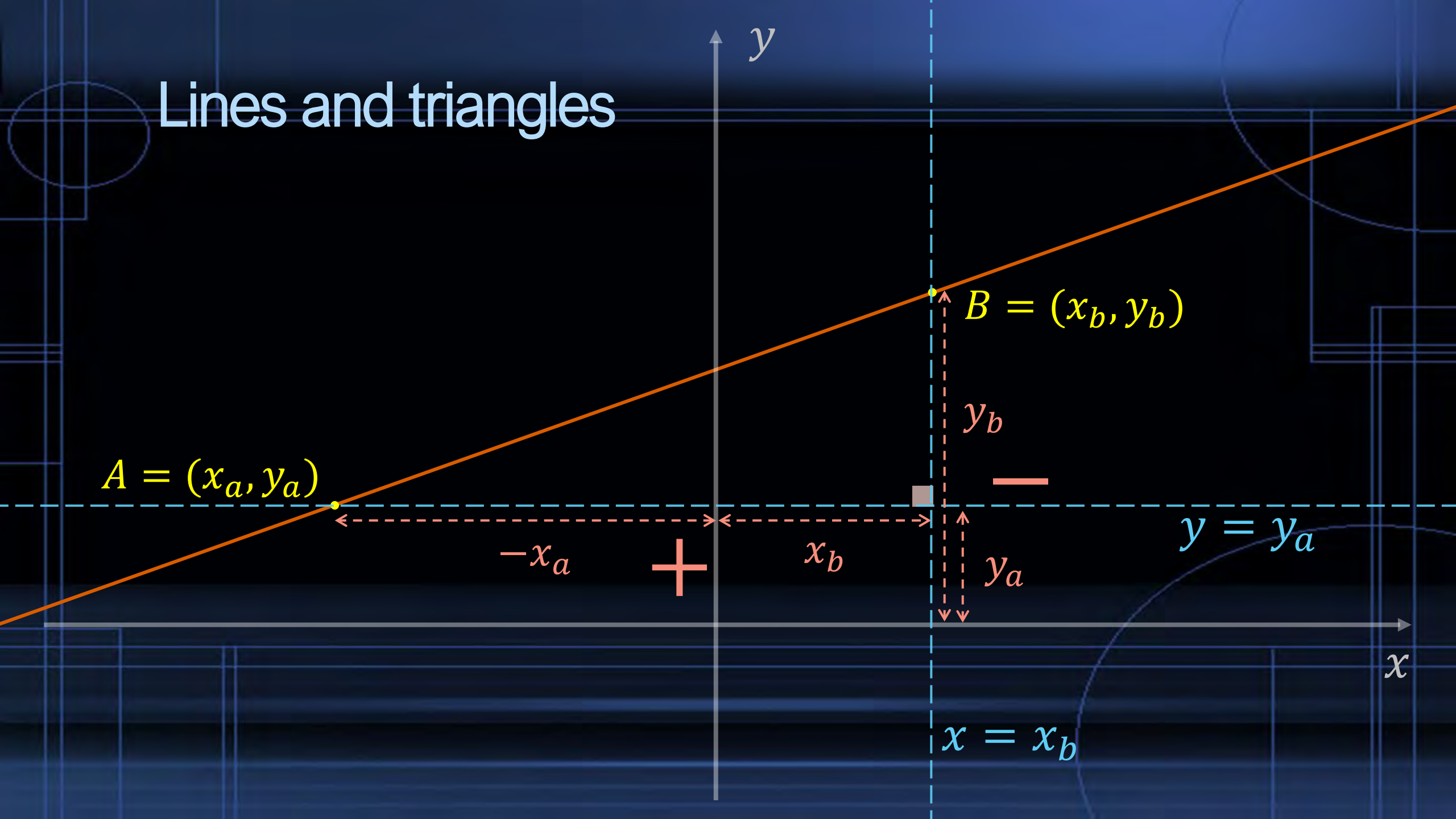
Lines and triangles

A 2D coordinate system with x and y axes. A blue line passes through two points, A and B .

Point A is labeled $A = (x_a, y_a)$. Point B is labeled $B = (x_b, y_b)$.

Dashed lines project these points onto the axes. The horizontal distance from the y -axis to point A is labeled $-x_a$, and the horizontal distance from the y -axis to point B is labeled x_b . The vertical distance from the x -axis to point A is labeled y_a , and the vertical distance from the x -axis to point B is labeled y_b .

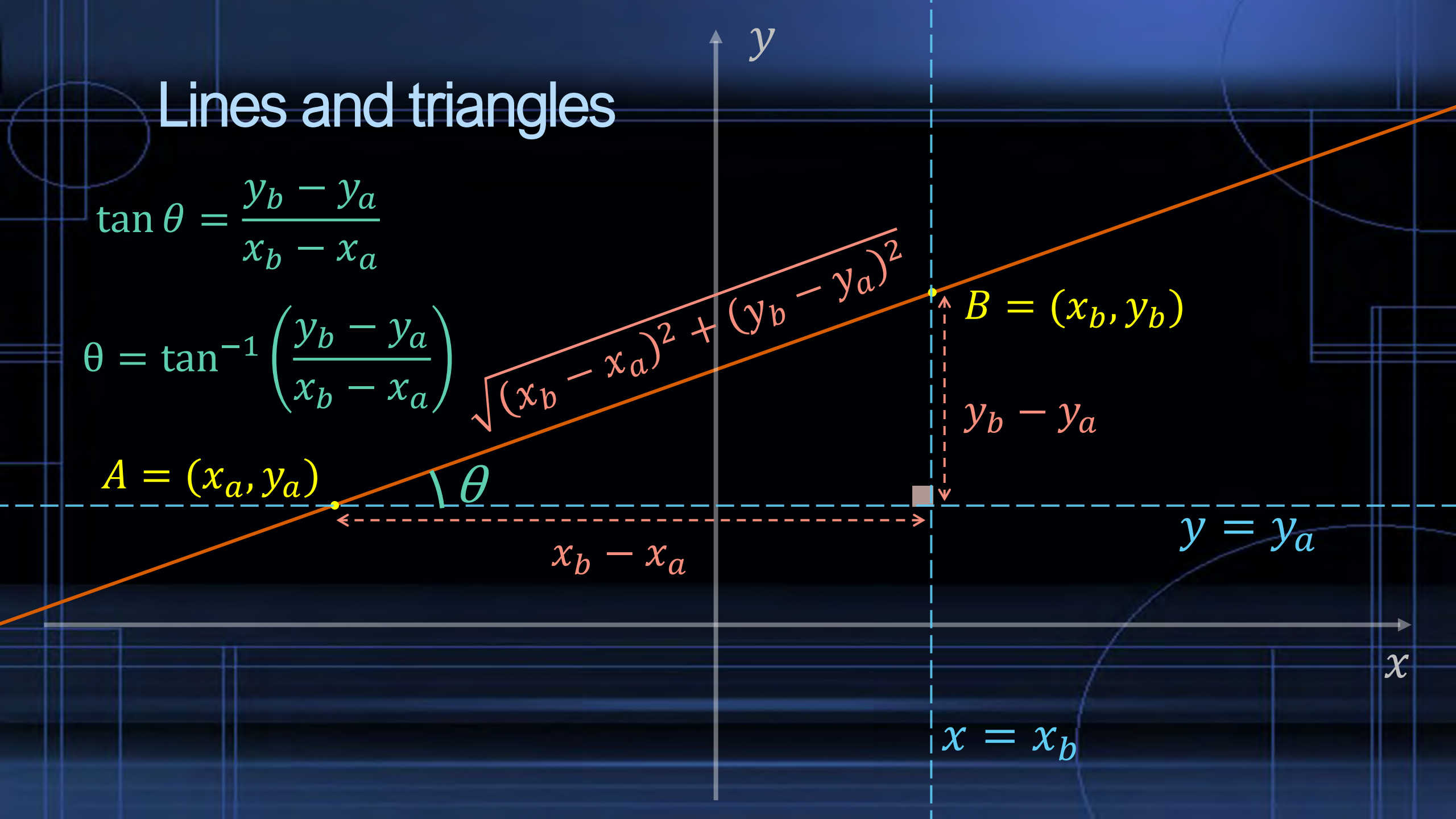
A red '+' sign is placed near the x -axis, and a red '-' sign is placed near the y -axis. A red square is placed at the intersection of the dashed lines $x = x_b$ and $y = y_a$.



Lines and triangles

$$\tan \theta = \frac{y_b - y_a}{x_b - x_a}$$

$$\theta = \tan^{-1} \left(\frac{y_b - y_a}{x_b - x_a} \right)$$



$$\sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}$$

$$B = (x_b, y_b)$$

$$y_b - y_a$$

$$A = (x_a, y_a)$$

$$\theta$$

$$x_b - x_a$$

$$y = y_a$$

$$x = x_b$$

x

y

Line direction

$$\begin{aligned}\tan \theta &= \frac{y_a - y_b}{x_a - x_b} = \frac{-(y_b - y_a)}{-(x_b - x_a)} \\ &= \frac{y_b - y_a}{x_b - x_a}\end{aligned}$$

