COMP110: Principles of Computing

5: Complexity

Research Journal

Research journal presentations

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- Presentation sessions in **week 7** (week after next)

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- Prepare a **10 minute** presentation

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- Prepare a **10 minute** presentation
- Discussing your chosen paper, its context and influence
- Use any presentation software to prepare slides (e.g. Beamer, PowerPoint, Google Slides)

Research journal presentations — Why?

Research journal presentations — Why?

- Assessed on a **threshold** basis

Research journal presentations — Why?

- Assessed on a **threshold** basis
- A chance to share insights on what you've been reading

Research journal presentations — Why?

- Assessed on a **threshold** basis
- A chance to share insights on what you've been reading
- A chance for informal feedback

Research journal

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- You should also be working on the journal itself!

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- Peer review (draft needed) in **week 8**

Research journal

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- Peer review (draft needed) in **week 8**
- This week's worksheet is the last one for a while, to give you time to work on your research journal

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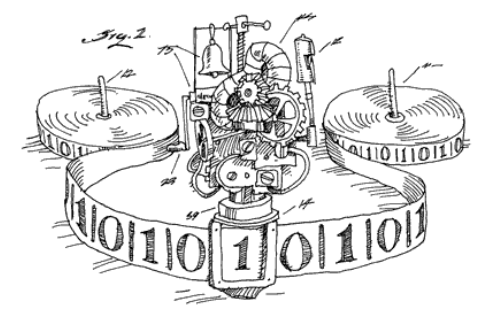
Turing machines

Turing machines



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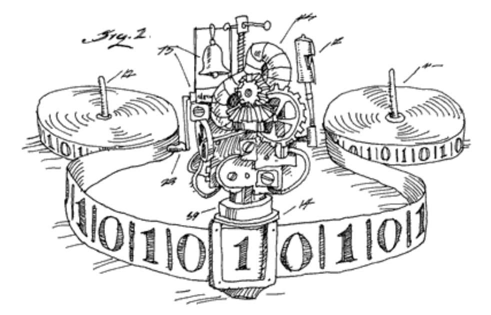

- Introduced in 1936 by Alan Turing

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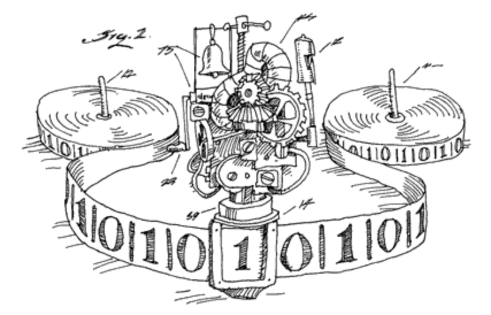

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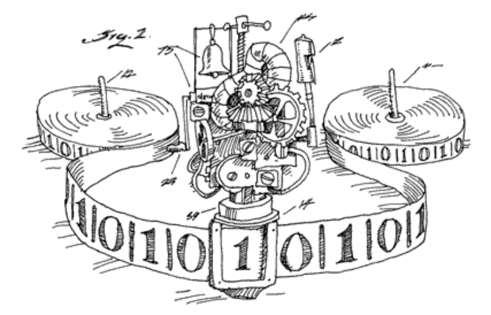

- Introduced in 1936 by Alan Turing
- Theoretical model of a "computer"
  - I.e. a machine that carries out computations (calculations)

Turing machine

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- Has a finite number of **states**

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  - Where to move the tape head: one space to the left, or one space to the right

The Church-Turing Thesis

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- A machine, language or system is **Turing complete** if it can simulate a Turing machine
- (In practice, nothing can simulate an infinite tape, so we just assume a large enough tape)

Examples of Turing complete systems

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- All general-purpose CPUs and programming languages

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Computability

Computability theory

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Computability theory

- Let A and B be **sets** of elements
  - NB: A may be **infinite**
- A function $f : A \to B$ is **computable** if there exists a Turing machine which computes f
  - I.e. given an encoding of $a \in A$ as input, the Turing machine outputs an encoding of $f(a)$

An uncomputable function

The **halting problem**

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- There is **no** Turing machine that computes f
- f is **uncomputable**

Computability and the Church-Turing Thesis

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- Church-Turing tells us that Turing machines are as powerful as any other computer

Computability and the Church-Turing Thesis

- Church-Turing tells us that Turing machines are as powerful as any other computer
- Therefore if a function is uncomputable, there is **no conceivable machine** that can compute it

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- Write a software tool that, given a C# program, predicts whether that program can go into an infinite loop
- Your tool must work for **all** C# programs, considering **all** possible inputs to the program
- This task is impossible!

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Search

Search

- We have a list of names, each with some data associated

Search

- We have a list of names, each with some data associated
- We want to find one of them

Linear search

procedure FIND(name, list)

Linear search

procedure FIND(name, list)
    for each item in list **do**

Linear search

```
procedure FIND(name, list)
    for each item in list do
        if item.name = name then
```

Linear search

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procedure FIND(name, list)
    for each item in list do
        if item.name = name then
            return item
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How long does it take?

Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list

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- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?

How long does it take?

Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?
- How about in the **worst case**?

How long does it take?

Socrative room code: `FALCOMPED`

- If there are 25 items in the list, the **worst case** number of items visited is 25

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- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?

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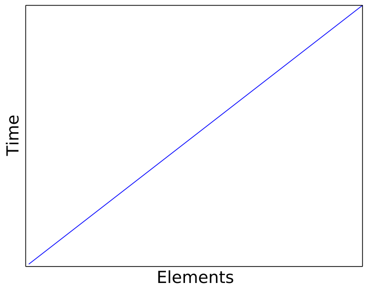
- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?

How long does it take?

Socrative room code: `FALCOMPED`

- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?
- If the number of items **doubles**, what happens to the amount of time the search takes?

Linear time



- The running time of linear search is **proportional** to the size *n* of the list

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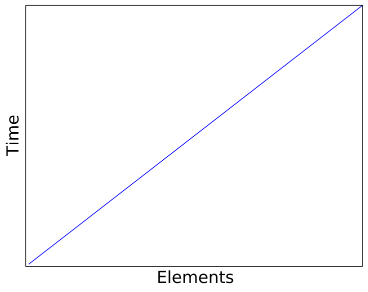

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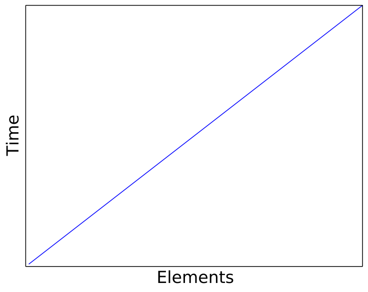

- The running time of linear search is **proportional** to the size *n* of the list
- Linear search is said to have **linear time complexity**
- Also written as $O(n)$ **time complexity**

Searching a sorted list

- If the list is **sorted** in alphabetical order, we can do better than linear...

Binary search

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    if list is empty **then**
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    mid $\leftarrow$ the "middle" item of the list

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    mid ← the "middle" item of the list
    if name = mid.name then
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```

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Binary search

procedure FIND(name, list)
    if list is empty **then**
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    end if
    mid $\leftarrow$ the "middle" item of the list
    if name $=$ mid.name **then**
        return mid
    else if name $<$ mid.name **then**
        return FIND(name, first half of list)

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```

Find “Lopez, Jeffrey”

Anderson, Martha
Baker, Theresa
Brown, Janet
Clark, Stephanie
Collins, Jane
Cox, Shirley
Davis, Marilyn
Diaz, Harold
Gonzalez, Adam
Henderson, Lawrence
Hughes, Aaron
Kelly, Philip
⟶ Lewis, Rose
Lopez, Jeffrey
Miller, Jeremy
Parker, Debra
Perez, Diana
Russell, Mildred
Sanders, Phillip
Scott, Michelle
Stewart, Howard
Ward, Jessica
White, Amanda
Williams, Billy
Young, Frank

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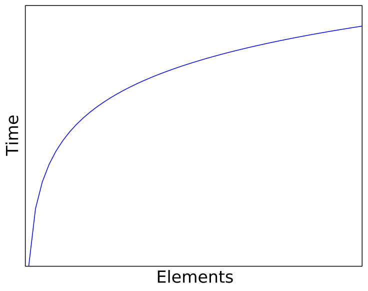

Hidden complexity

```
if name < mid.name then
    return FIND(name, first half of list)
else if name > mid.name then
    return FIND(name, second half of list)
end if
```

Hidden complexity

if name $<$ mid.name **then**
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- Careful how you implement this!

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- Careful how you implement this!
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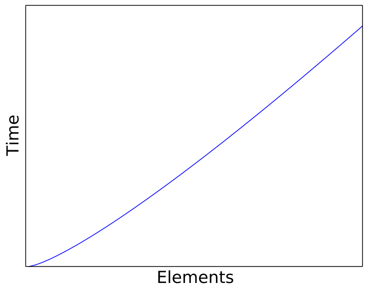

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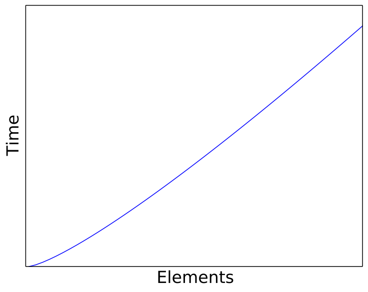
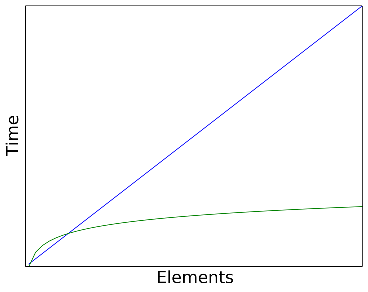


- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$
- The actual running time would be $O(n \log n)$
- Use **pointers** into the list instead of copying

Binary search vs linear search



- So binary search is better than linear search... right?

Hashing

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⋮	⋮
112	Ward, Jessica
113	Baker, Theresa
114	Collins, Jane
115	—
116	—
117	Hughes, Aaron
118	—
119	—
120	—
121	—
122	Brown, Janet
123	—
124	—
125	Gonzalez, Adam Lewis, Rose
126	—
127	—
128	—
129	—
130	—
131	—
132	Young, Frank
⋮	⋮

Hash look-up

98	Diaz, Harold
99	Parker, Debra Perez, Diana White, Amanda
112	Ward, Jessica
113	Baker, Theresa
114	Collins, Jane
117	Hughes, Aaron
122	Brown, Janet
125	Gonzalez, Adam Lewis, Rose
132	Young, Frank
135	Kelly, Philip
138	Cox, Shirley
142	Clark, Stephanie
144	Scott, Michelle
145	Miller, Jeremy
147	Davis, Marilyn
149	Lopez, Jeffrey
151	Anderson, Martha
158	Williams, Billy
162	Sanders, Phillip
171	Russell, Mildred
175	Stewart, Howard
183	Henderson, Lawrence

"Lopez, Jeffrey"

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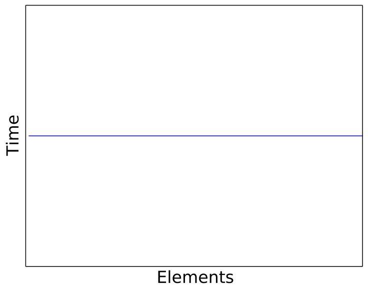
"Lopez, Jeffrey"

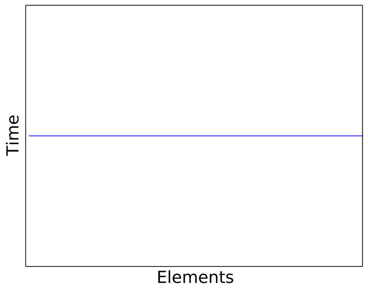
$12 + 15 + 16 + 5 + 26 + 10 + 5 + 6 + 6 + 18 + 5 + 25 = 149$

How long does it take?



- If there are no “collisions”, look-up time is **constant** or $O(1)$

How long does it take?



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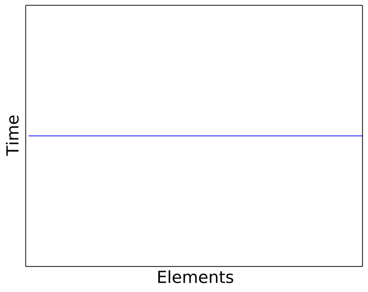

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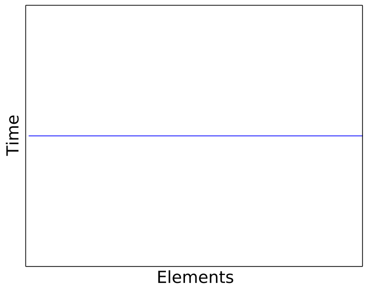

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More on complexity

Common complexity classes

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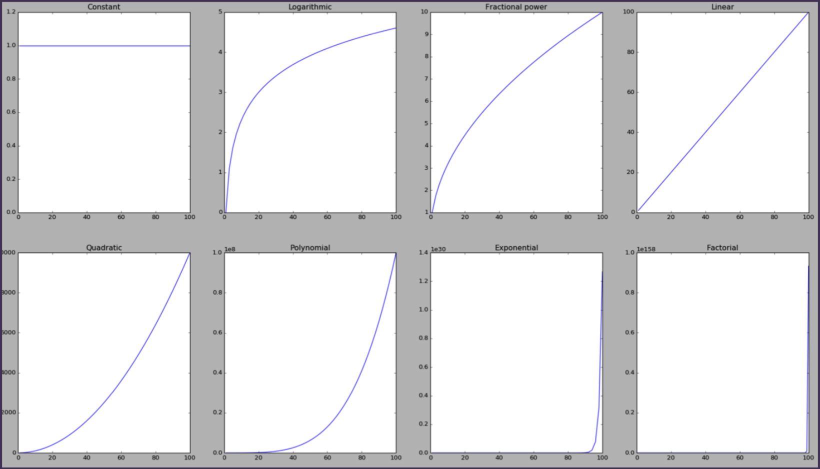

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  - If an algorithm does n "things" and each "thing" is $O(n)$, then the overall algorithm is $O(n^2)$

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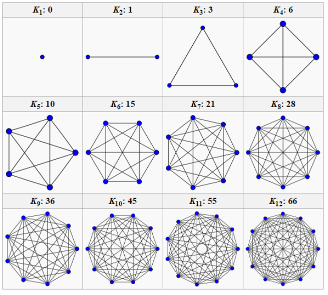

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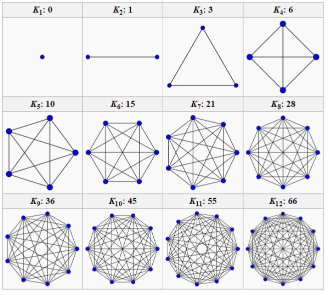

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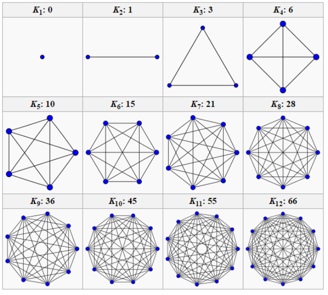

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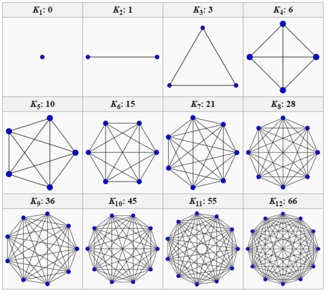

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  - Further reading: spatial hashing, quadtrees, octrees, Verlet lists

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- Are there any problems in NP but not in P?

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  - Quantum computers are "infinitely parallel" in a sense so *can* solve some large NP-hard problems

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- ... but only if scalability is actually a factor

FALMOUTH UNIVERSITY

Open workshop

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- Use the rest of this session to get on with your **research journal**
- Post in the chat if you have questions or need help!