COMP250: Artificial Intelligence

5: Game Tree Search

Next few weeks

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- Week 5: now

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- Week 6: no session (development week)

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- Week 7: AI component peer review

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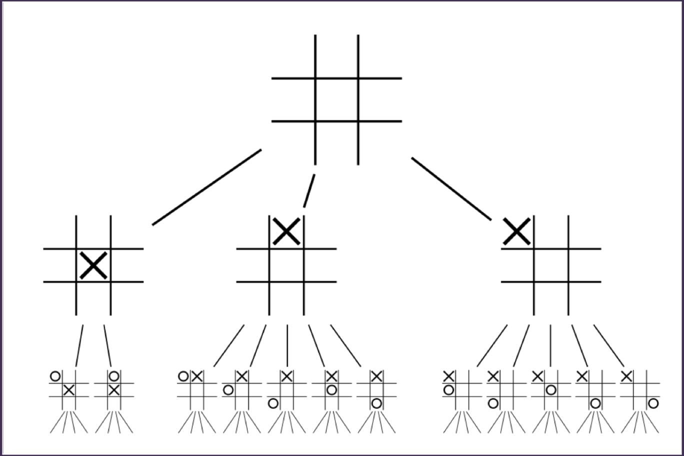
Minimax search

Game trees



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- My opponent wants to **minimise** the value
- Therefore I want to **maximise** the **minimum** value my opponent can achieve
- This is generally only true for **two-player zero-sum** games

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- If it's my opponent's turn, the value is the **minimum** value over next states

Minimax search – example

Minimax search pseudocode

procedure MINIMAX(state, currentPlayer)

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    if state is terminal **then**

Minimax search pseudocode

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procedure MINIMAX(state, currentPlayer)
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        return value of state
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procedure MINIMAX(state, currentPlayer)
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    else if currentPlayer = 1 then
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  else if currentPlayer $= 2$ **then**

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            bestValue = MAX(bestValue, v)
        return bestValue
    else if currentPlayer = 2 then
        bestValue = +∞
        for each possible nextState do
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            bestValue = MIN(bestValue, v)
        return bestValue
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    else if currentPlayer = 1 then
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        for each possible nextState do
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- Similarly when minimising if bestValue $= -1$

Using minimax search

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- Calculate the minimax value for each move

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- If there are several with the same score, choose one at random

Minimax and game theory

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- **But...**

Minimax for larger games

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  - Connect 4 has $\approx 10^{13}$ states
  - Chess has $\approx 10^{47}$ states

Heuristics for search

Depth limiting

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- For nonterminal states at depth d, apply a heuristic evaluation instead of searching deeper
- Evaluation is a number between -1 and $+1$, estimating the probable outcome of the game

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- Often easier to design a "which state is better" heuristic than to directly design a "which move to play" heuristic

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- Thus ordering moves from **best to worst** means faster search
- How do we know which moves are "best" and "worst"? Use a heuristic!

Designing heuristics

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- Next time we will look at what we can do if we don't possess such knowledge

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Monte Carlo evaluation

From last time

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- Designing a good heuristic requires in-depth knowledge of the game
- What if you don't have such knowledge?

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- Then the **expected value** of X is

$$\sum_x x \cdot p(x)$$

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  i.e. £0.40
- What this means: if you play the slot machine N times, on average you will win $N \times$ £0.40

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- Seed is generally based on some source of **entropy**, e.g. system clock, mouse input, electronic noise

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- Applications in physics, engineering, finance, weather forecasting, graphics, ...

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- Higher expected value $=$ more chance of winning

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- How about minimax with $d > 1$ and Monte Carlo evaluation?
  - Minimax assumes the evaluation is **deterministic**, but Monte Carlo is not
  - Not commonly used, mainly because there's something better...

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Monte Carlo Tree Search

Monte Carlo Tree Search (MCTS)

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- First few rollouts are **random**
- However, statistics from these rollouts are used to **bias** future rollouts
- Bias rollouts towards **plausible** lines of play, i.e. where each player is trying to play the best move

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  - **Expansion**: Choose a random legal move for which the current node does not have a child node. Add this new node to the tree.

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  - **Simulation**: Perform a Monte Carlo rollout, playing random moves until a terminal state is reached.

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  - **Backpropagation**: For each node visited during **selection** and **expansion**, update the node's statistics based on the result of the simulation.

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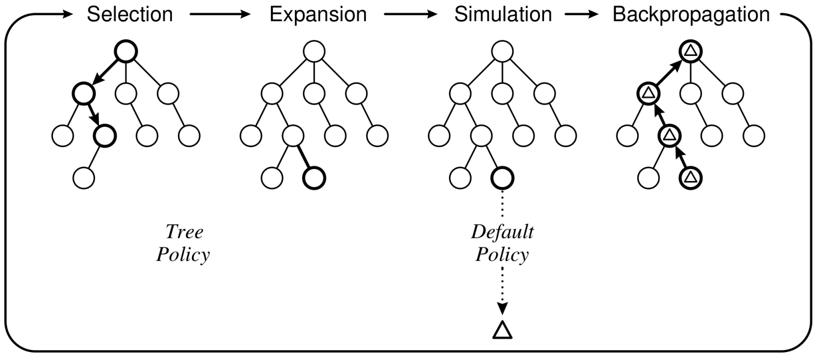
  - **Simulation**: Perform a Monte Carlo rollout, playing random moves until a terminal state is reached.
  - **Backpropagation**: For each node visited during **selection** and **expansion**, update the node's statistics based on the result of the simulation.
- Perform many rollouts, then use the statistics at the top level of the tree to choose the best move

The MCTS algorithm



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Selection policy

- Selection must balance:
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- This can be modelled as a **multi-armed bandit problem**

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  - **Exploration** of machines that have not been tried often, to get a better estimate of their expected payout

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$$\frac{V_m}{n_m} + c\sqrt{\frac{\log n}{n_m}}$$

is largest

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  - c is a parameter for adjusting the balance between exploitation and exploration

UCB demo

http://orangehelicopter.com/academic/bandits.html?ucb

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UCT demo

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  - Selects which parts of the tree to expand more deeply

Workshop

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- Clone `https://github.com/Falmouth-Games-Academy/comp250-workshop-5`
- This is an implementation of Connect-4 in Python 3 with PyGame
- Edit `ai_player.py` to implement MCTS
- Use `http://mcts.ai/code/python.html` as a guide