
COMP110: Principles of Computing

# 5: Computational Complexity

# Learning outcomes

- **Explain** the notion of computability
- **Use** "big $O$" notation to express computational complexity
- **Apply** appropriate algorithms to achieve efficiency

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# More on complexity

# Common complexity classes

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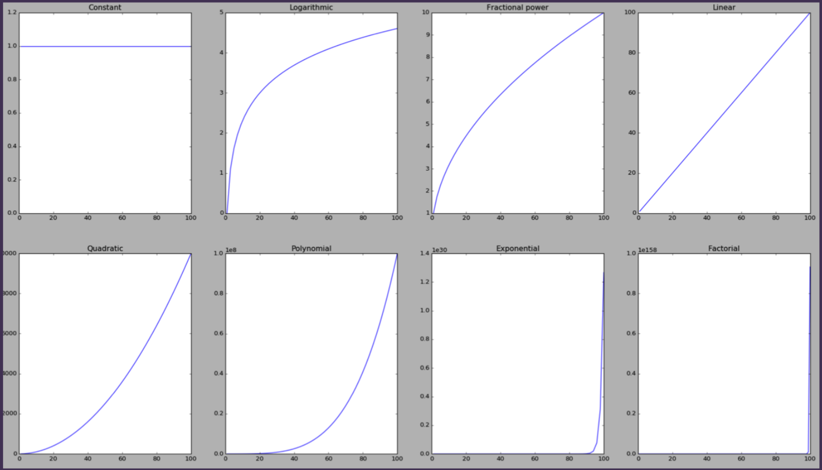
| Polynomial | $O(n^k)$, $k > 1$ |
| Exponential | $O(e^n)$ |
| Factorial | $O(n!)$ |

# Common complexity classes



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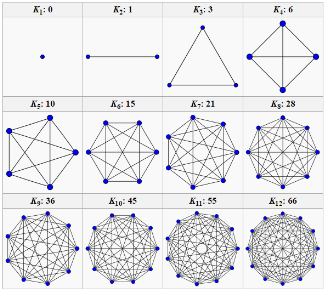


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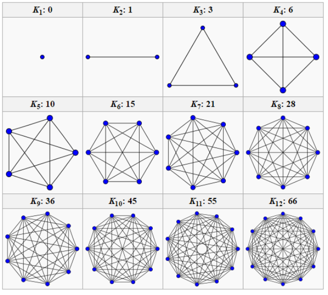


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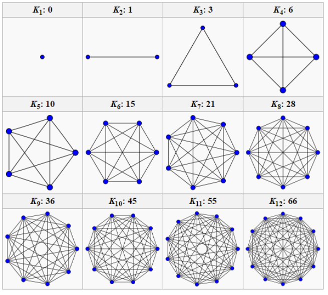


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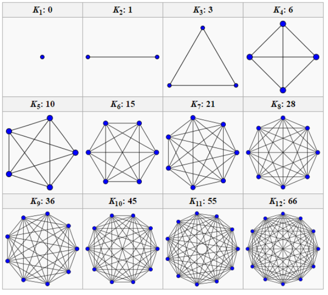


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  - Further reading: spatial hashing, quadtrees, octrees, Verlet lists

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- Are there any problems in *NP* but not in *P*?

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  - Many types of cryptography are based on this assumption
  - Quantum computers are "infinitely parallel" in a sense so *can* solve some large $NP$-hard problems

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  - Otherwise, choose simplicity

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- Choice of data structures and algorithms can have a large impact on the efficiency of your software
- ... but only if scalability is actually a factor