COMP110: Principles of Computing

5: Computational Complexity

Worksheet 5

Due **Friday 8th November** (2 weeks tomorrow)

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Research presentations

Presentations

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- Prepare and deliver a **brief presentation** on your research journal

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- Maximum **four minutes**, maximum **three slides**

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- Outline the **contributions** of your chosen paper to the field of computing
- Describe the **influence** of your chosen paper on work that came since

Schedule

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- Presentations are timetabled for **week 7**

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Schedule

- Presentations are timetabled for **week 7**
- **Running order** will be released **next week**
  - Half of you will present Monday, the other half Friday
- **All students** are expected to attend **the full duration of both sessions** to listen and support their peers

Paper selection — please do ASAP!

Week 5: Computability & Complexity

- Session Introduction
- Session Plan
- Workshop Slides (Monday)
- Research Journal Paper Selection
- Additional Resources

Slides

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- Prepare your slides in **PowerPoint** or **PDF** format

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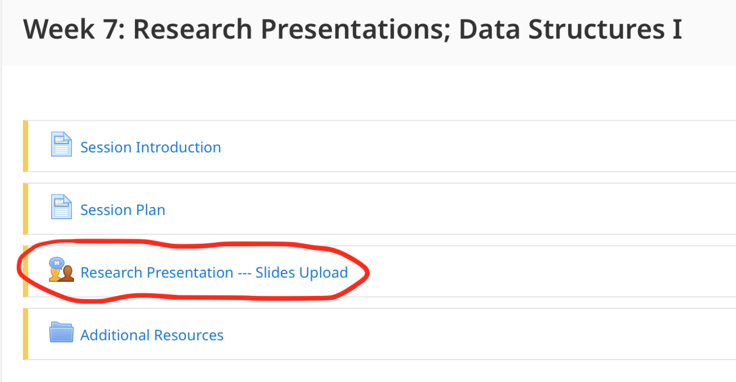
- Upload your slides to LearningSpace by **11:59pm on Friday 1st November**

Slides upload



Search

Search

- We have a list of names, each with some data associated

Search

- We have a list of names, each with some data associated
- We want to find one of them

Linear search

procedure FIND(name, list)

Linear search

```
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    for each item in list do
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Linear search

procedure FIND(name, list)
    for each item in list **do**
        if item.name = name **then**

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procedure FIND(name, list)
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            return item
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procedure FIND(name, list)
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    end for
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end procedure
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How long does it take?

Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list

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- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?

How long does it take?

Socrative room code: `FALCOMPED`

- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?
- How about in the **worst case**?

How long does it take?

Socrative room code: `FALCOMPED`

- If there are 25 items in the list, the **worst case** number of items visited is 25

How long does it take?

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- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?

How long does it take?

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- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?

How long does it take?

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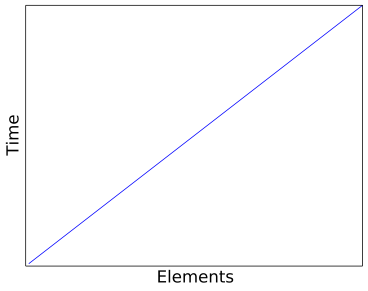
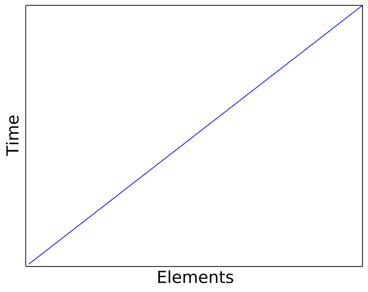
- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?
- If the number of items **doubles**, what happens to the amount of time the search takes?

Linear time



- The running time of linear search is **proportional** to the size *n* of the list

Linear time



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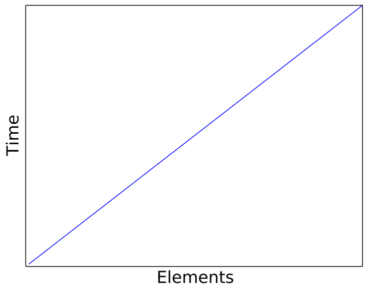
Linear time



- The running time of linear search is **proportional** to the size *n* of the list
- Linear search is said to have **linear time complexity**
- Also written as $O(n)$ **time complexity**

Searching a sorted list

- If the list is **sorted** in alphabetical order, we can do better than linear...

Binary search

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procedure FIND(name, list)
    if list is empty then
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    if name = mid.name then
        return mid
```

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    mid ← the "middle" item of the list
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        return FIND(name, first half of list)
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Binary search

procedure FIND(name, list)
    if list is empty **then**
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    end if
    mid $\leftarrow$ the "middle" item of the list
    if name $=$ mid.name **then**
        return mid
    else if name $<$ mid.name **then**
        return FIND(name, first half of list)
    else if name $>$ mid.name **then**
        return FIND(name, second half of list)

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    end if
end procedure
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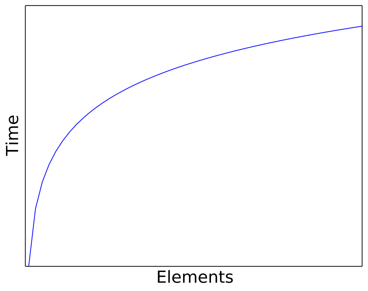

Hidden complexity

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if name < mid.name then
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- Careful how you implement this!

Hidden complexity

if name $<$ mid.name **then**
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- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$

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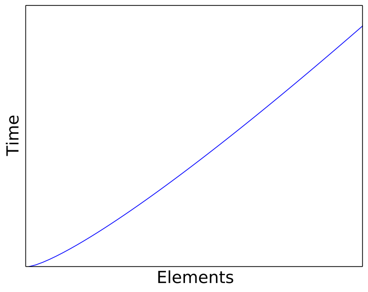

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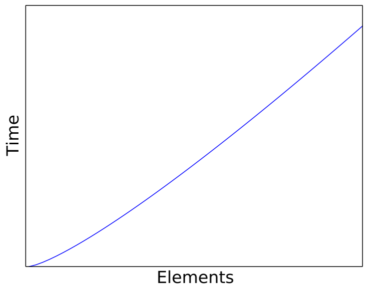

- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$
- The actual running time would be $O(n \log n)$
- Use **pointers** into the list instead of copying

Binary search done wrong

```
def binary_search(name, mylist):
    if mylist == []:
        raise ValueError("Not found")

    mid = len(mylist) / 2
    mid_name = mylist[mid_index].name

    if name == mid_name:
        return mid
    elif name < mid_name:
        return binary_search(name, mylist[:mid])
    else:
        return binary_search(name, mylist[mid+1:])
```

Binary search done right

```
def binary_search(name, mylist, start, end):
    if end <= start:
        raise ValueError("Not found")

    mid = (start + end) / 2
    mid_name = mylist[mid].name

    if name == mid_name:
        return mylist[mid]
    elif name < mid_name:
        return binary_search(name, mylist, start, mid)
    else:
        return binary_search(name, mylist, mid+1, end)
```

Binary search vs linear search

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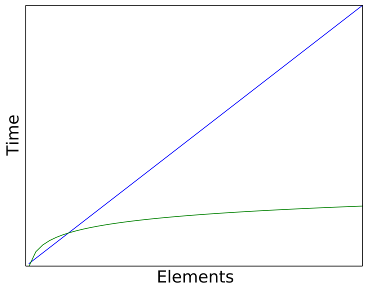

- So binary search is better than linear search... right?

Binary search vs linear search

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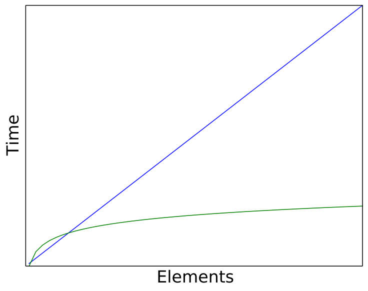

- So binary search is better than linear search... right?
- Discuss in **pairs**
- On Socrative, post **one reason** why, or **one situation** where, linear search may be a better choice than binary search

Hashing

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⋮	⋮
112	Ward, Jessica
113	Baker, Theresa
114	Collins, Jane
115	—
116	—
117	Hughes, Aaron
118	—
119	—
120	—
121	—
122	Brown, Janet
123	—
124	—
125	Gonzalez, Adam Lewis, Rose
126	—
127	—
128	—
129	—
130	—
131	—
132	Young, Frank
⋮	⋮

Hash look-up

98	Diaz, Harold
99	Parker, Debra Perez, Diana White, Amanda
112	Ward, Jessica
113	Baker, Theresa
114	Collins, Jane
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132	Young, Frank
135	Kelly, Philip
138	Cox, Shirley
142	Clark, Stephanie
144	Scott, Michelle
145	Miller, Jeremy
147	Davis, Marilyn
149	Lopez, Jeffrey
151	Anderson, Martha
158	Williams, Billy
162	Sanders, Phillip
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$$12 + 15 + 16 + 5 + 26 + 10 + 5 + 6 + 6 + 18 + 5 + 25 = 149$$

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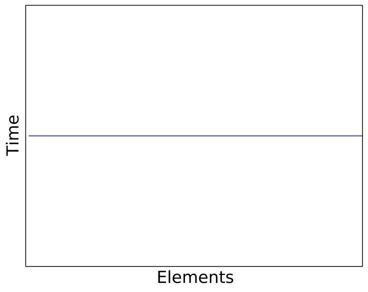

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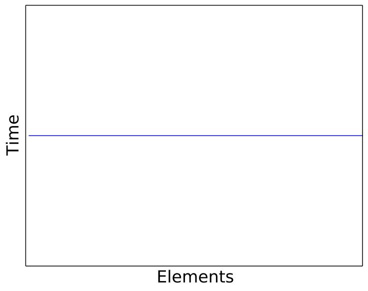

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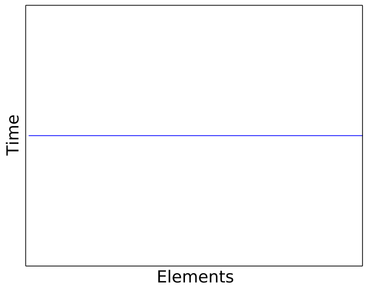

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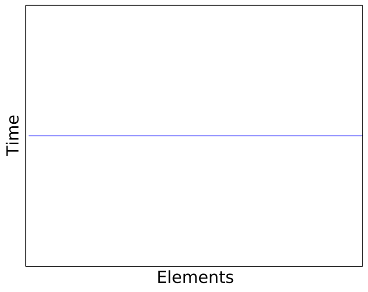

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- When there are collisions, need to fall back on something like linear or binary search within each bin

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- Hash tables in Python:
  - The `dict` (dictionary) data structure

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More on complexity

Common complexity classes

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$\mid$	Fractional power	$O(n^k)$, $k < 1$
$\mid$	Linear	$O(n)$

Common complexity classes

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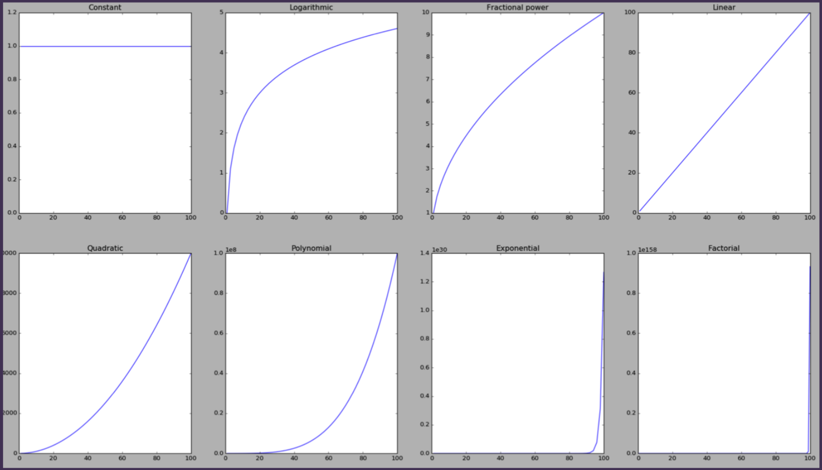
$\downarrow$	Exponential	$O(e^n)$
"Slower"	Factorial	$O(n!)$

Common complexity classes



Working with big O notation

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- Take only the **dominant term**
  - The term that is largest when n is large
  - If an algorithm takes $0.1n^3 + 300n^2 + 7000$ operations, it is $O(n^3)$

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- Multiply **compound** algorithms

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- Take only the **dominant term**
  - The term that is largest when n is large
  - If an algorithm takes $0.1n^3 + 300n^2 + 7000$ operations, it is $O(n^3)$
- Multiply **compound** algorithms
  - If an algorithm does n "things" and each "thing" is $O(n)$, then the overall algorithm is $O(n^2)$

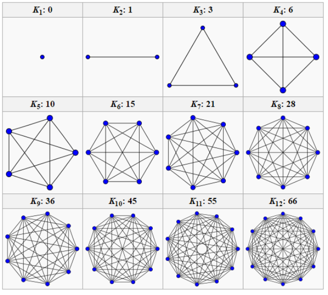
Quadratic complexity

- Collision detection between *n* objects

Quadratic complexity

- Collision detection between *n* objects
- The naïve way: check **each pair** of objects to see whether they have collided

Quadratic complexity



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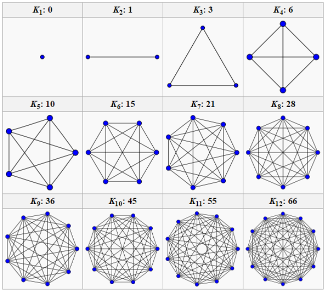

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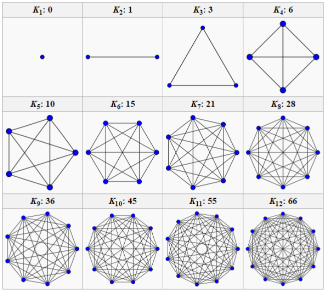

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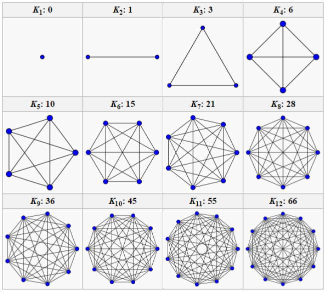

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  - Further reading: spatial hashing, quadtrees, octrees, Verlet lists

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- Are there any problems in NP but not in P?

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- ... but only if scalability is actually a factor

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  - Where to move the tape head: one space to the left, or one space to the right

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- A machine, language or system is **Turing complete** if it can simulate a Turing machine

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- Is this possible?