COMP270: Mathematics for 3D Worlds and Simulations

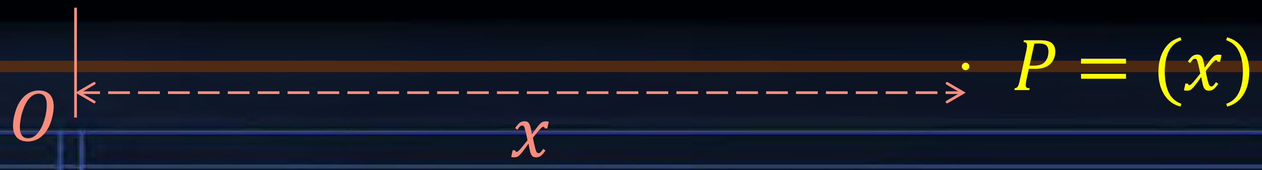
WEEK 2: GEOMETRY I
PART 1: POINTS, LINES AND TRIANGLES

Objectives

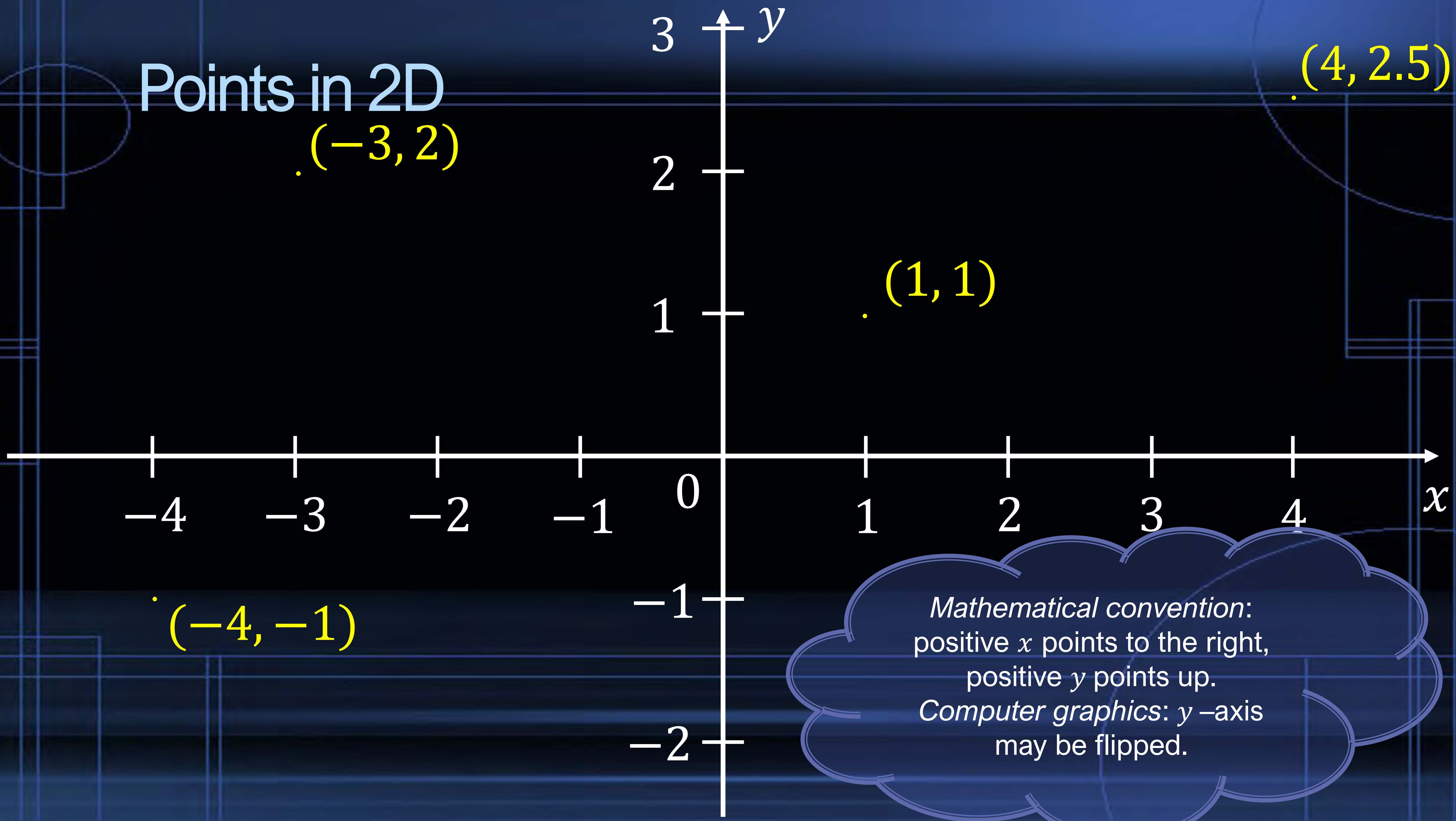
- **Define** the basic geometric primitives
- **Recall** the formulae that express relationships between the sides and angles of a right-angled triangle

What is a point?

- Definition: a **point** is a 0-dimensional mathematical object that can be specified in n-dimensional space using an n-tuple $(x_1, x_2, \dots, x_n)$ consisting of n **coordinates**.
- 0-dimensional because it has no measurements in any direction…
- In 1D space, its coordinate is just the distance from an origin:



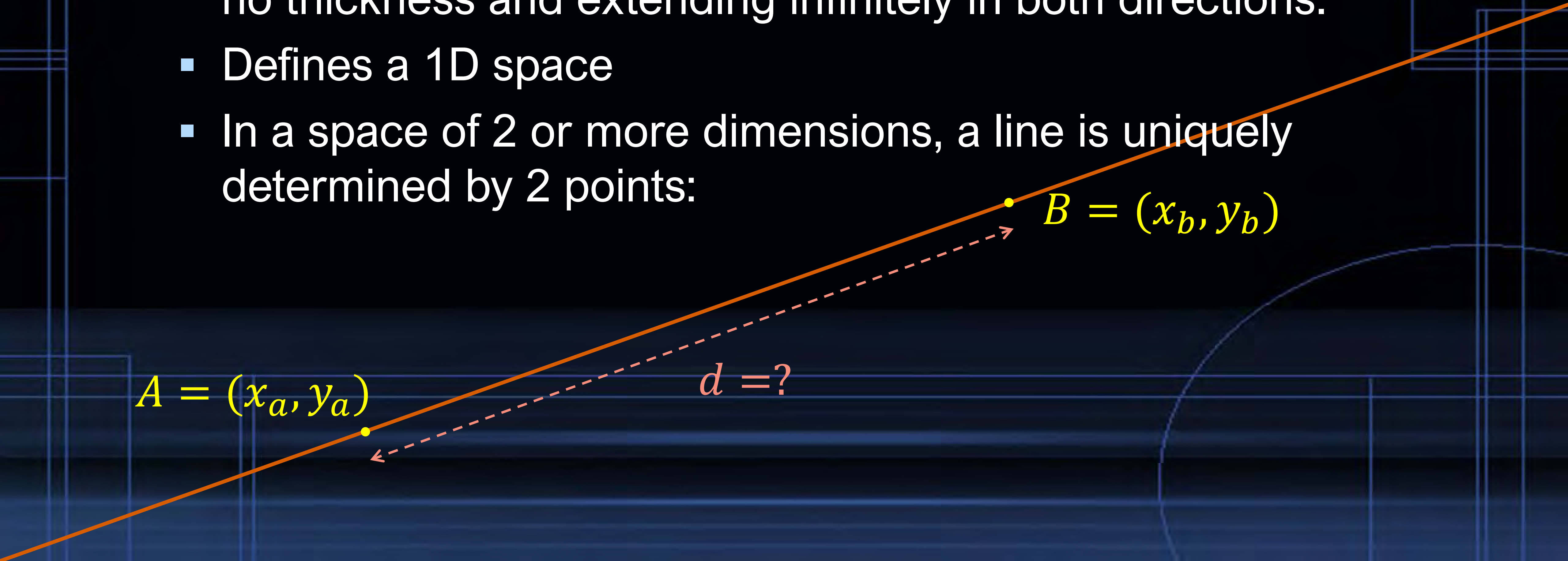
Points in 2D



$(-3, 2)$
$(4, 2.5)$
$(1, 1)$
$(-4, -1)$
x
y
3
2
1
0
-1
-2
-4
-3
-2
-1
1
2
3
4
Mathematical convention: positive x points to the right, positive y points up.
Computer graphics: y –axis may be flipped.

Lining up

- Definition: a **line** is a straight one-dimensional figure having no thickness and extending infinitely in both directions.
- Defines a 1D space
- In a space of 2 or more dimensions, a line is uniquely determined by 2 points:

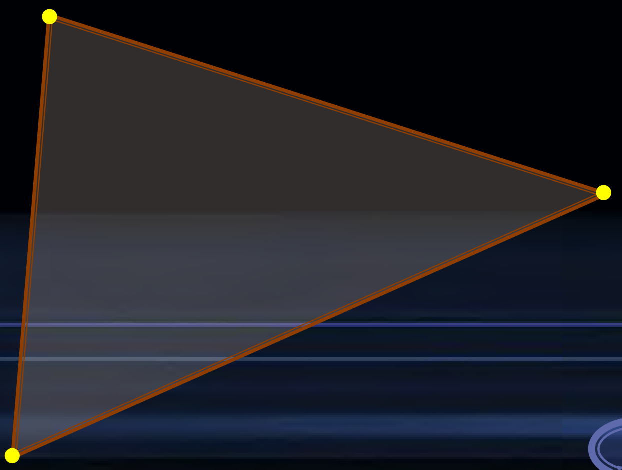


Let's try angles…

From Latin/Greek, "many-angled"; a shape with 3 or more straight sides.

- Definition: a **triangle** is a polygon with three sides and three angles, some of which may be the same.
- Uniquely determined by 3 points:

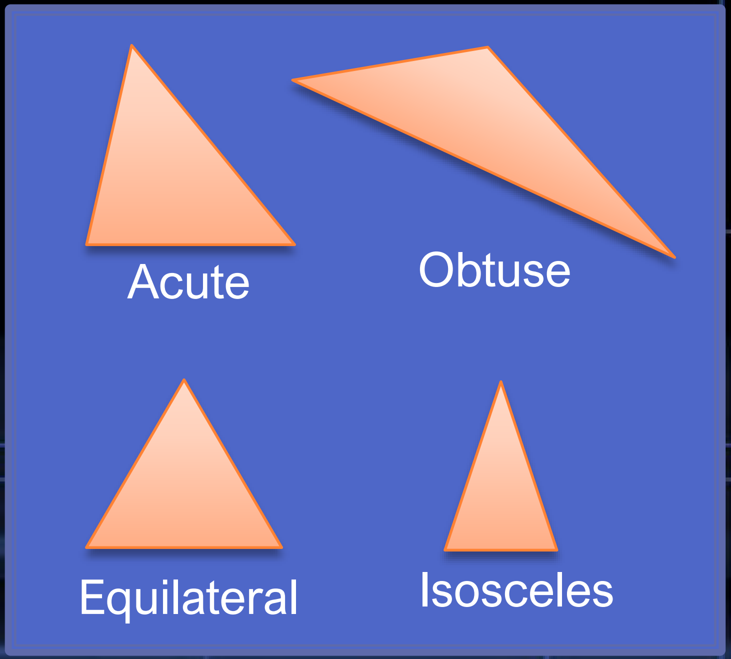
$A = (x_a, y_a)$



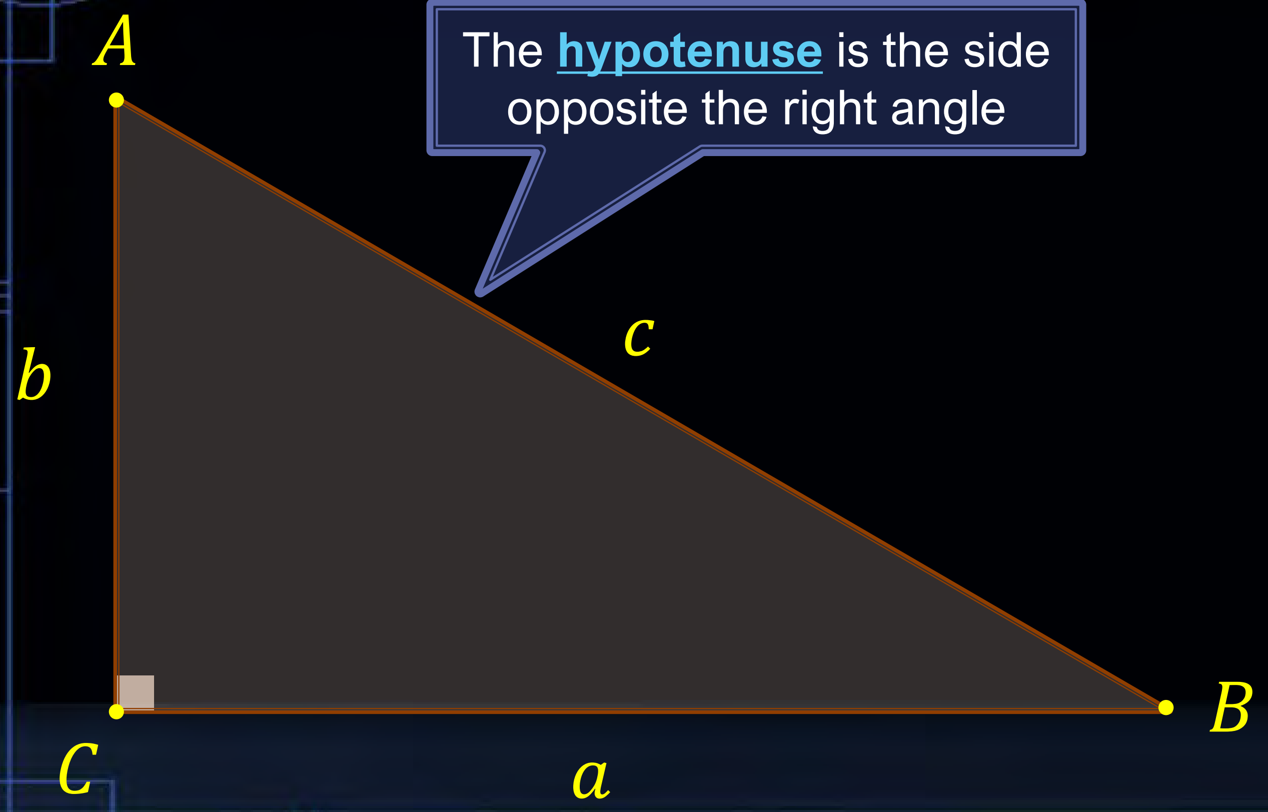
$B = (x_b, y_b)$

$C = (x_c, y_c)$

Angles add to 180°



Right-angled triangles



The **hypotenuse** is the side opposite the right angle

Named after Pythagoras of Samos (c570-c495BC), Greek philosopher

- **Pythagorean Theorem**:

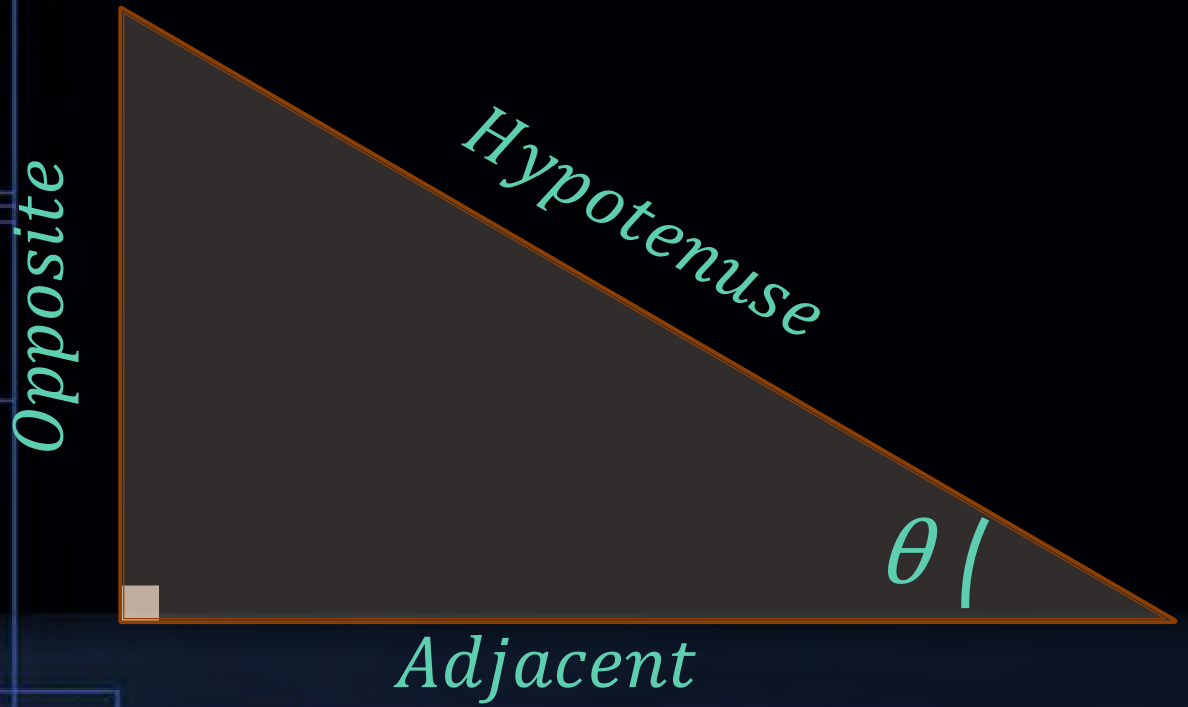
$$a^2 + b^2 = c^2$$

i.e.

$$c = \sqrt{a^2 + b^2}$$

Visual depiction:
www.youtube.com/watch?v=ANR4g0IPrEQ

The Trigonometric Functions



$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

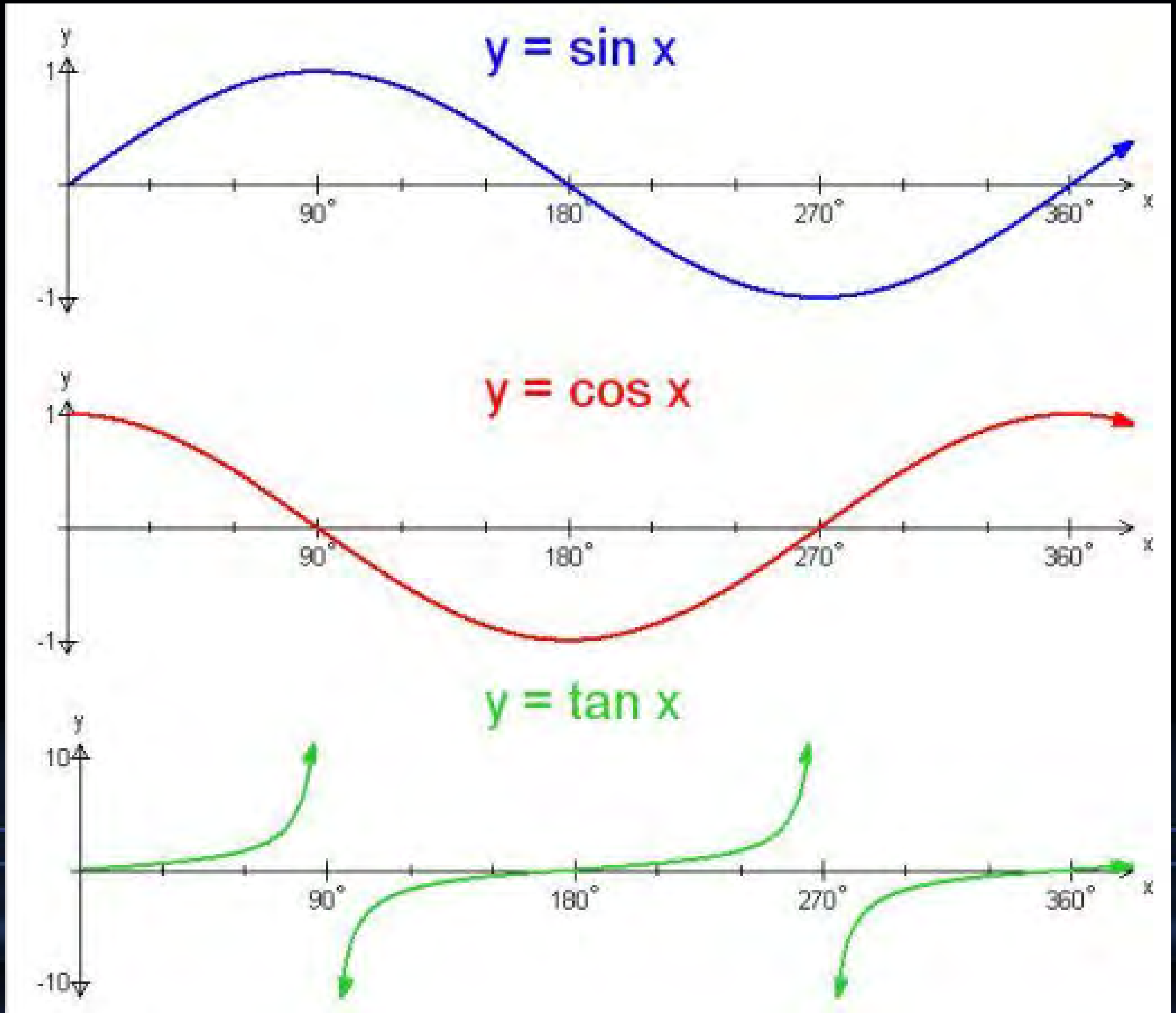
SOHCAHTOA

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{\sin\theta}{\cos\theta}$$

Silly Old Harry Caught A Herring
Trawling Off America

The Trigonometric Functions



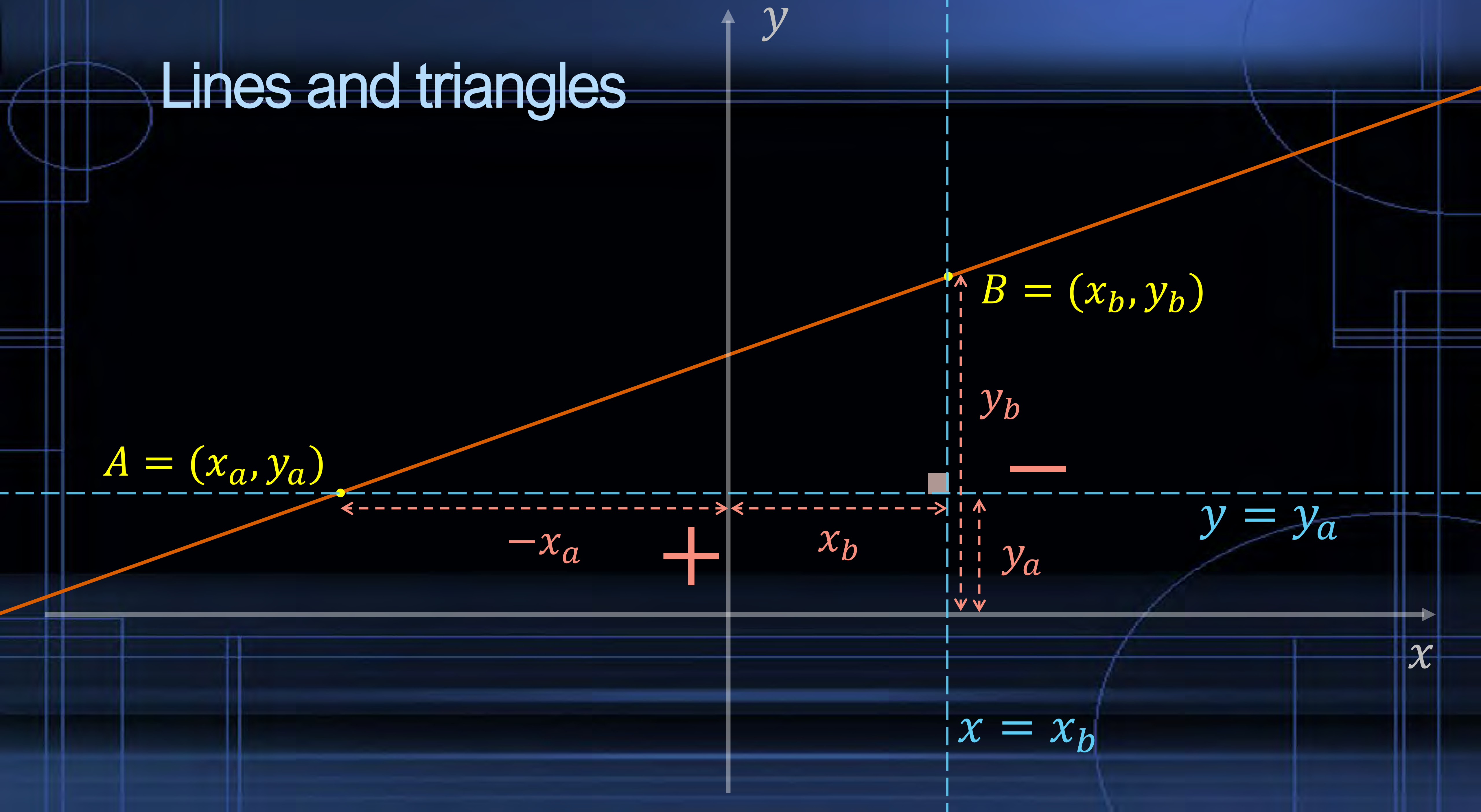
Inverse trig. functions

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\theta = \sin^{-1}\left(\frac{\text{Opposite}}{\text{Hypotenuse}}\right)$$

- Also known as **arcsin** / **arccos** / **arctan**
- In code: `asin()` / `acos()` / `atan()`

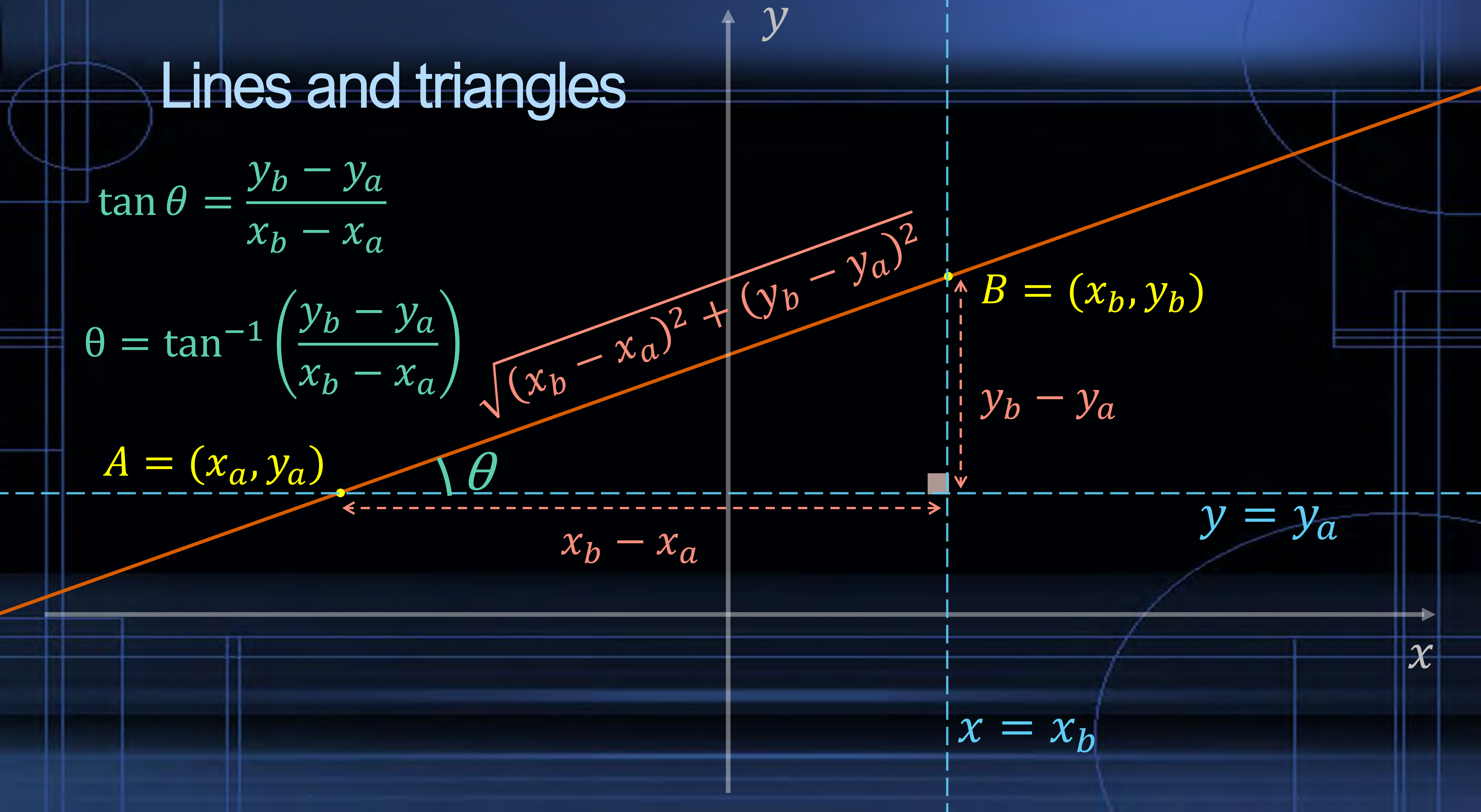
Lines and triangles



Lines and triangles

$$\tan\theta = \frac{y_b - y_a}{x_b - x_a}$$

$$\theta = \tan^{-1}\left(\frac{y_b - y_a}{x_b - x_a}\right)$$



Line direction

$$\tan\theta = \frac{y_a - y_b}{x_a - x_b} = \frac{-(y_b - y_a)}{-(x_b - x_a)}$$

$$= \frac{y_b - y_a}{x_b - x_a}$$