COMP110: Principles of Computing

11: Data Structures II

Learning outcomes

- **Define** the key concepts of graph theory
- **Distinguish** advanced data structures such as trees, DAGs and graphs
- **Determine** the complexity of accessing and manipulating data in these data structures
- **Choose** the correct data structure for a given task

Quiz D

Due **this time next week**

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Stacks and queues

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- Items can be **enqueued** to the **back** of the queue
- Items can be **dequeued** from the **front** of the queue

Stacks in Python

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- Provides methods `append`, `appendleft`, `pop`, `popleft`
  - All of which are $O(1)$

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- Each invocation of a function has a **stack frame**
- This specifies information like **local variable values** and **return address**
- Calling a function **pushes** a new frame onto the stack
- Returning from a function **pops** the top frame off the stack

Linked lists

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Lists in Python

- Implemented as a (variable-sized) **array**
- Appending is $O(1)$
- Inserting is $O(n)$
- Deleting is $O(n)$
- Changing size sometimes requires the entire array to be reallocated and copied

Linked list

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- Composed of a number of **nodes**

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Linked lists vs arrays

Operation	Array	Linked list

[1] If we already have a reference to the last node
[2] If we already have a reference to the relevant node

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Append	$O(1)$	$O(1)$ [1]

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Operation	Array	Linked list
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Insert	$O(n)$	$O(1)$ [2]

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Insert	$O(n)$	$O(1)$ [2]
Delete	$O(n)$	$O(1)$ [2]

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Implementing a linked list

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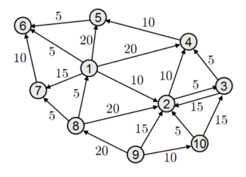
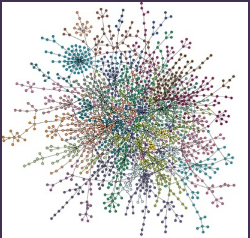
Graphs

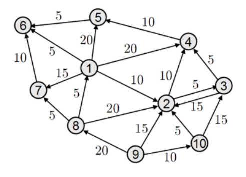
Graphs

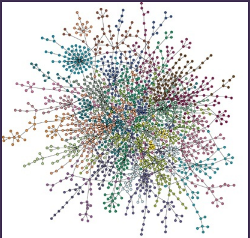
Graphs





Graphs





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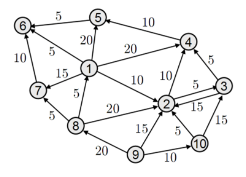
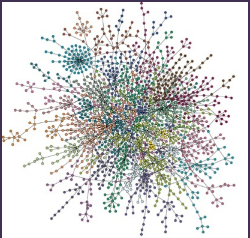
Graphs





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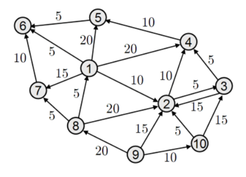
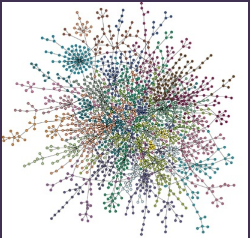
Graphs





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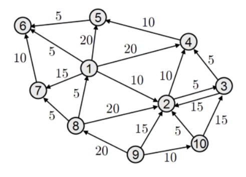
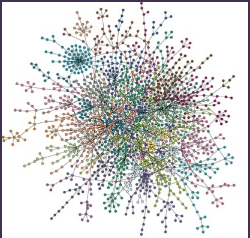
Graphs





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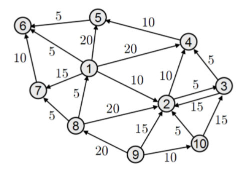

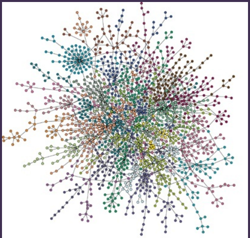

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- **Directed** graph: edges are arrows

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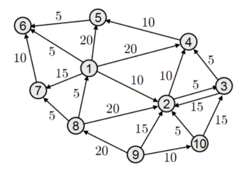

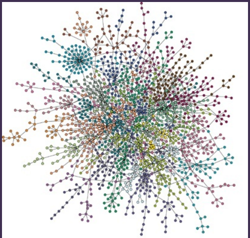

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- **Directed** graph: edges are arrows
- **Undirected** graph: edges are lines

Implementing graphs

Implementing graphs

- A graph is a **collection of nodes**

Implementing graphs

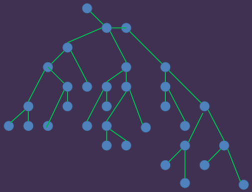
- A graph is a **collection of nodes**
- Each node has a **collection of edges**

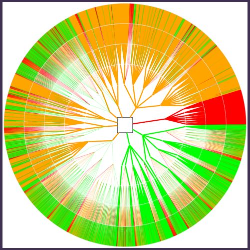
Implementing graphs

- A graph is a **collection of nodes**
- Each node has a **collection of edges**
- Each edge has exactly **two nodes** associated with it

Trees

Trees





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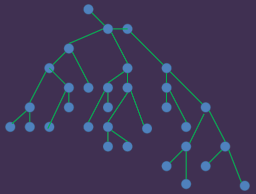

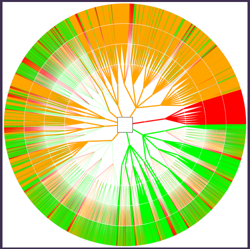

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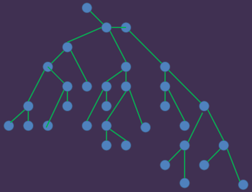
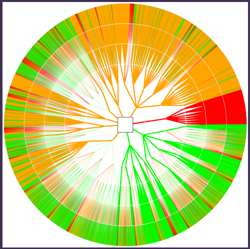
Trees





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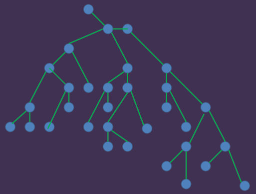
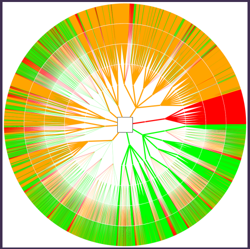
  - One node (the **root**) has no incoming edges

Trees





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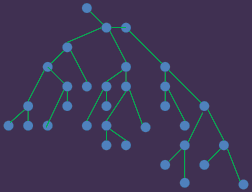
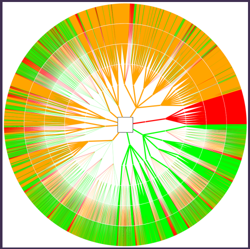
  - All other nodes have exactly 1 incoming edge

Trees





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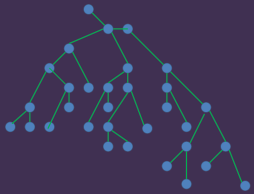

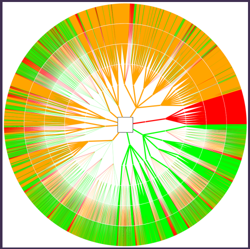

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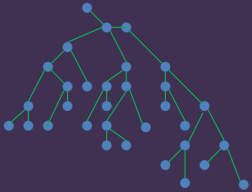
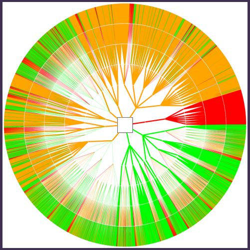
Trees





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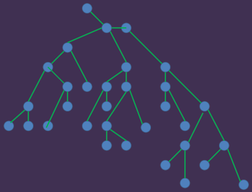

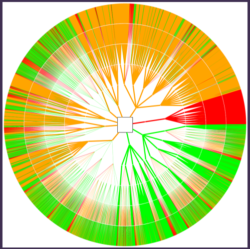

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- Edges go from **parent** to **child**
  - All nodes except the root have exactly one parent
  - Nodes can have 0, 1 or many children
- Used to model **hierarchies** (e.g. file systems, object inheritance, scene graphs, state-action trees, ...)

Implementing trees

Implementing trees

- A graph has a **root node**

Implementing trees

- A graph has a **root node**
- Each node has a **collection of children**

Implementing trees

- A graph has a **root node**
- Each node has a **collection of children**
- Each node other than the root has a **single parent**

Tree traversal

Tree traversal

- **Traversal**: visiting all the nodes of the tree

Tree traversal

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- Two main types

Tree traversal

- **Traversal**: visiting all the nodes of the tree
- Two main types
  - Depth first

Tree traversal

- **Traversal**: visiting all the nodes of the tree
- Two main types
  - Depth first
  - Breadth first

Tree traversal

Tree traversal

procedure DEPTHFIRSTSEARCH

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack

Tree traversal

procedure DEPTHFIRSTSEARCH

  let S be a stack

  push root node onto S

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack
    push root node onto S
    while S is not empty **do**

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack
    push root node onto S
    while S is not empty **do**
        pop n from S

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack
    push root node onto S
    while S is not empty **do**
        pop n from S
        print n

Tree traversal

procedure DEPTHFIRSTSEARCH
  let *S* be a stack
  push root node onto *S*
  while *S* is not empty **do**
    pop *n* from *S*
    print *n*
    push children of *n* onto *S*

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack
    push root node onto S
    while S is not empty **do**
        pop n from S
        print n
        push children of n onto S
    end while
end procedure

Tree traversal

procedure DEPTHFIRSTSEARCH
    let *S* be a stack
    push root node onto *S*
    while *S* is not empty **do**
        pop *n* from *S*
        print *n*
        push children of *n* onto *S*
    end while
end procedure

procedure BREADTHFIRSTSEARCH

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack
    push root node onto S
    while S is not empty **do**
        pop n from S
        print n
        push children of n onto S
    end while
end procedure

procedure BREADTHFIRSTSEARCH
    let Q be a queue

Tree traversal

procedure DEPTHFIRSTSEARCH
    let *S* be a stack
    push root node onto *S*
    while *S* is not empty **do**
        pop *n* from *S*
        print *n*
        push children of *n* onto *S*
    end while
end procedure

procedure BREADTHFIRSTSEARCH
    let *Q* be a queue
    enqueue root node into *Q*

Tree traversal

procedure DEPTHFIRSTSEARCH
    let *S* be a stack
    push root node onto *S*
    while *S* is not empty **do**
        pop *n* from *S*
        print *n*
        push children of *n* onto *S*
    end while
end procedure

procedure BREADTHFIRSTSEARCH
    let *Q* be a queue
    enqueue root node into *Q*
    while *Q* is not empty **do**

Tree traversal

```
procedure DepthFirstSearch
    let S be a stack
    push root node onto S
    while S is not empty do
        pop n from S
        print n
        push children of n onto S
    end while
end procedure

procedure BreadthFirstSearch
    let Q be a queue
    enqueue root node into Q
    while Q is not empty do
        dequeue n from Q
```

Tree traversal

procedure DEPTHFIRSTSEARCH
    let S be a stack
    push root node onto S
    while S is not empty **do**
        pop n from S
        print n
        push children of n onto S
    end while
end procedure

procedure BREADTHFIRSTSEARCH
    let Q be a queue
    enqueue root node into Q
    while Q is not empty **do**
        dequeue n from Q
        print n

Tree traversal

procedure DEPTHFIRSTSEARCH
    let *S* be a stack
    push root node onto *S*
    while *S* is not empty **do**
        pop *n* from *S*
        print *n*
        push children of *n* onto *S*
    end while
end procedure

procedure BREADTHFIRSTSEARCH
    let *Q* be a queue
    enqueue root node into *Q*
    while *Q* is not empty **do**
        dequeue *n* from *Q*
        print *n*
        enqueue children of *n* into *Q*

Tree traversal

```
procedure DepthFirstSearch
    let S be a stack
    push root node onto S
    while S is not empty do
        pop n from S
        print n
        push children of n onto S
    end while
end procedure

procedure BreadthFirstSearch
    let Q be a queue
    enqueue root node into Q
    while Q is not empty do
        dequeue n from Q
        print n
        enqueue children of n into Q
    end while
end procedure
```

Recursive depth first search

Recursive depth first search

procedure DepthFirstSearch(n)

Recursive depth first search

procedure DEPTHFIRSTSEARCH(n)
    print n

Recursive depth first search

procedure DEPTHFIRSTSEARCH(n)
    print n
    for each child c of n **do**

Recursive depth first search

```
procedure DepthFirstSearch(n)
    print n
    for each child c of n do
        DepthFirstSearch(c)
```

Recursive depth first search

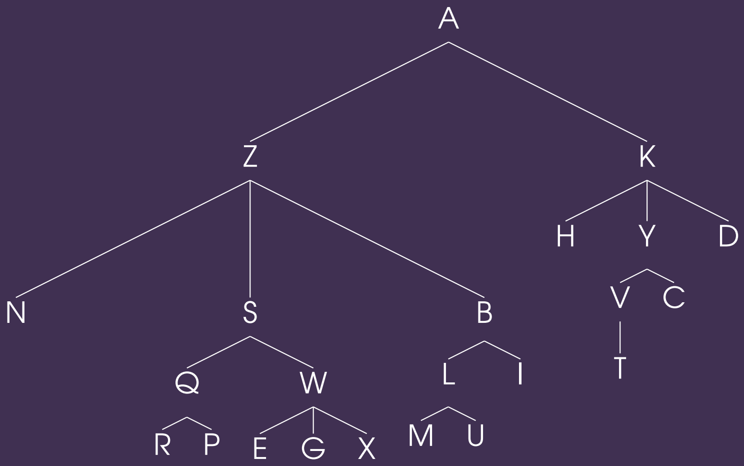
```
procedure DepthFirstSearch(n)
    print n
    for each child c of n do
        DepthFirstSearch(c)
    end for
end procedure
```

Recursive depth first search

```
procedure DEPTHFIRSTSEARCH(n)
    print n
    for each child c of n do
        DEPTHFIRSTSEARCH(c)
    end for
end procedure
```

- Compare to the pseudocode on the previous slide. Where is the stack?

Tree traversal example



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Worksheet D