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Workshop 1: 2D Vector revision

COMP270: Mathematics for 3D Worlds and Simulation
BSc (Hons) Computing for Games

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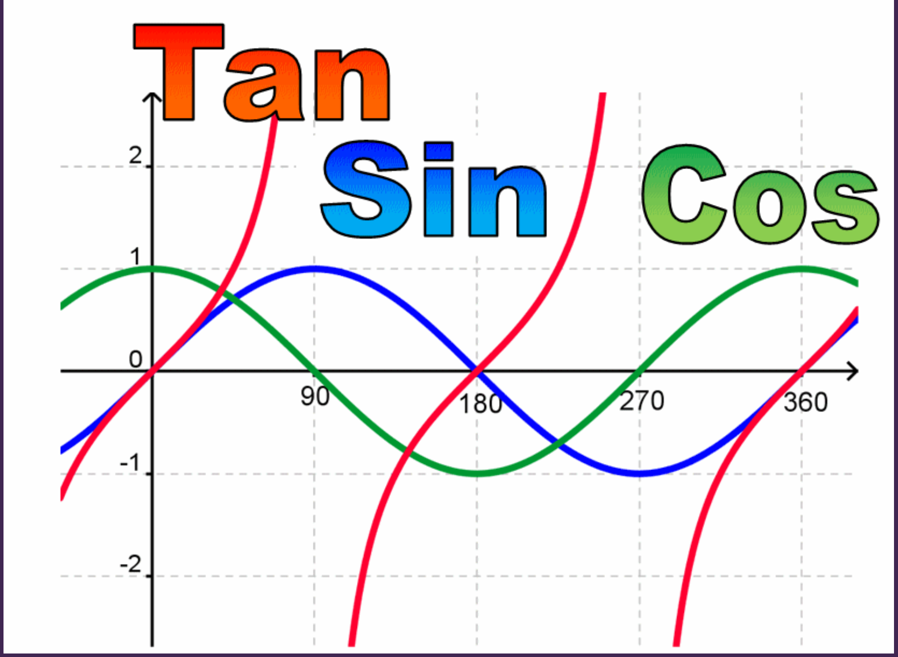
Trigonometric Values

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$



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Answers: Question 1

a. True:

$$\begin{pmatrix}3\\5\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

$$\begin{pmatrix}101\\-97\end{pmatrix} - \begin{pmatrix}99\\-100\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

b. False:

$$\begin{pmatrix}-3\\-5\end{pmatrix} - \begin{pmatrix}-1\\-2\end{pmatrix} = \begin{pmatrix}-2\\-3\end{pmatrix}$$

$$\begin{pmatrix}-101\\-97\end{pmatrix} - \begin{pmatrix}-99\\-100\end{pmatrix} = \begin{pmatrix}-2\\3\end{pmatrix}$$

… The length is the same, but the direction isn’t.

Answers: Question 1 cont.

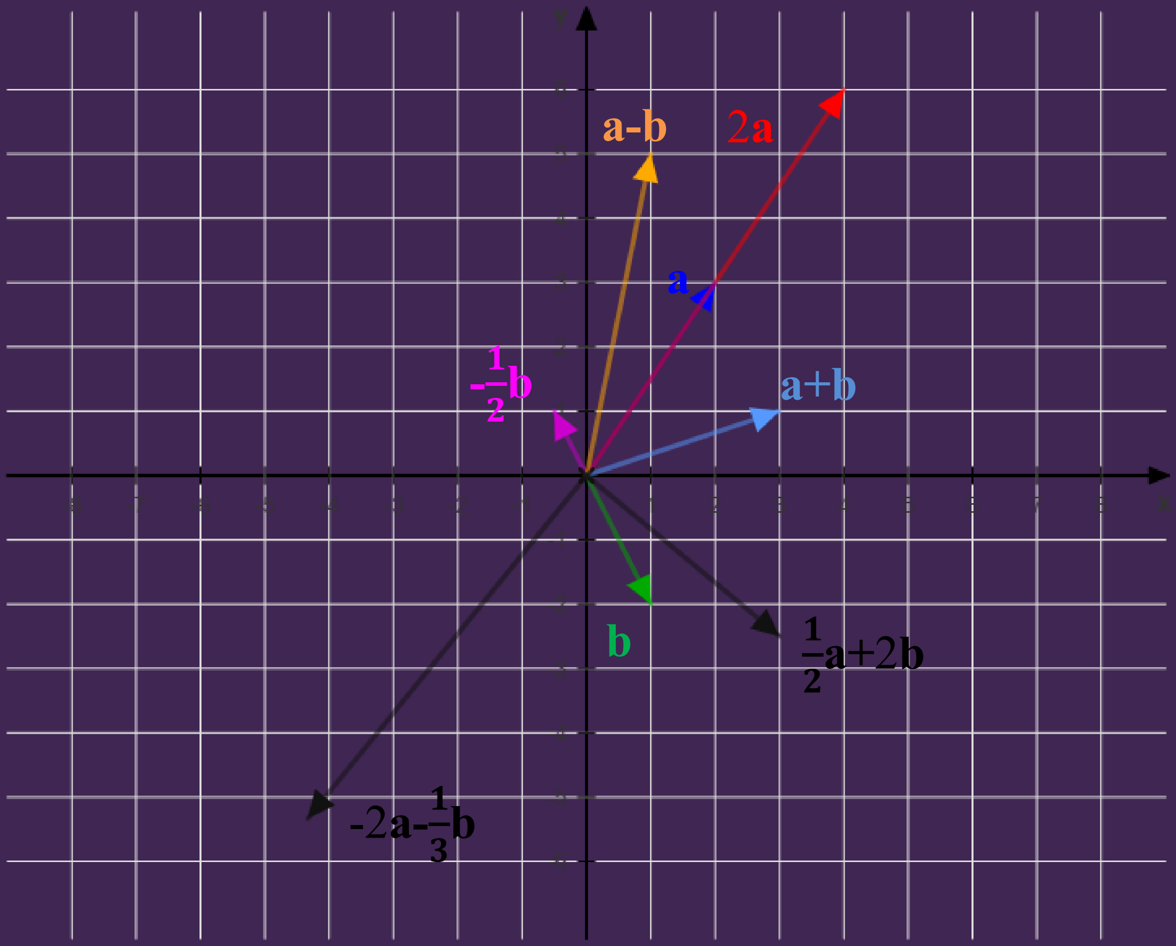
c. False: vectors have both direction **<u>and</u>** length.

d. True

e. False; addition is commutative

f. False! $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is the displacement *from* the origin *to* (x, y).

Answers: Question 2



Answers: Question 3

a. $\begin{pmatrix} 18 \\ -6 \end{pmatrix}$

b. $\begin{pmatrix} 2 \\ 2.5 \end{pmatrix}$

c. $\begin{pmatrix} 7 \\ -1 \end{pmatrix}$

d. $\sqrt{(-12)^2+5^2} = 13$

e. $\left\| \begin{pmatrix} 9 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ -12 \end{pmatrix} \right\| = \sqrt{12^2 + (-16)^2} = 20$

f. $\left\| \begin{pmatrix} -1 \\ \frac{-2}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \right\| = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{-3}{\sqrt{2}}\right)^2} = \frac{\sqrt{19}}{2}$

Answers: Question 4

a. (2, -2)

$$\begin{pmatrix}-3\\7\end{pmatrix}+\begin{pmatrix}5\\-9\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}2\\-2\end{pmatrix}$$

b. (-9, 15)

$\boldsymbol{a}-\boldsymbol{b}=\binom{-3}{7}-\binom{5}{-9}=\binom{-8}{16}$ describes a displacement anywhere in space, but in this case we want to find the point relative to the origin we end up at if we travel along $\boldsymbol{a}-\boldsymbol{b}$ starting from (-1, -1):

$$\begin{pmatrix}-1\\-1\end{pmatrix}+\begin{pmatrix}-8\\16\end{pmatrix}=\begin{pmatrix}-9\\15\end{pmatrix}$$

c. (4, 6)

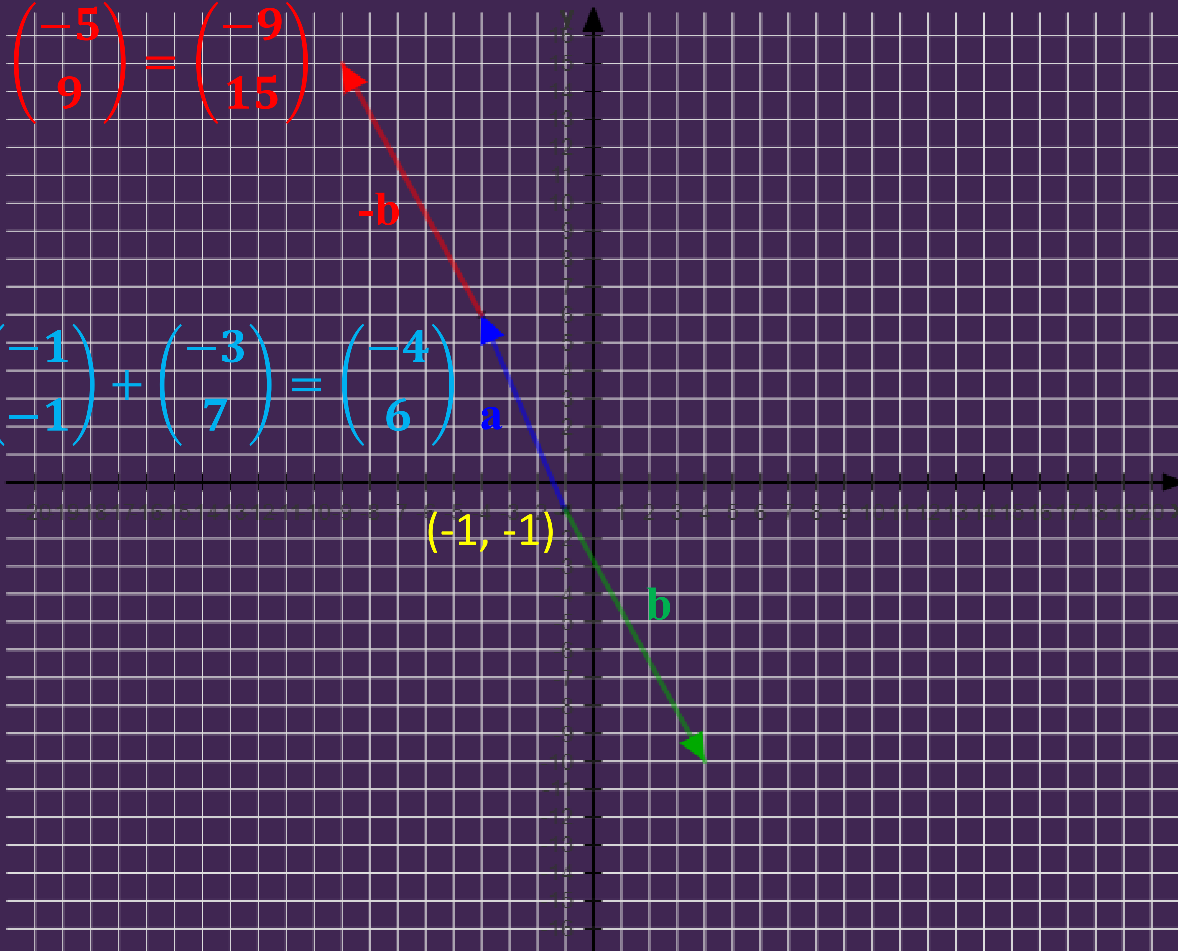
The length of the vector from $(1, 2)$ to $(7, 10)$ is $\left\|\binom{7}{10}-\binom{1}{2}\right\|=\left\|\binom{6}{8}\right\|=\sqrt{6^2+8^2}=10$

We want the vector that is half as long in the same direction, $\binom{3}{4}$, added to the starting point.

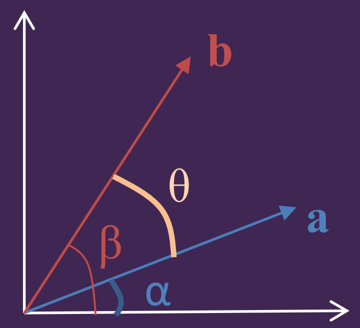
Answers: Question 4b visualised!

$$\begin{pmatrix}-4\\6\end{pmatrix}+\begin{pmatrix}-5\\9\end{pmatrix}=\begin{pmatrix}-9\\15\end{pmatrix}$$

$$\begin{pmatrix}-1\\-1\end{pmatrix}+\begin{pmatrix}-3\\7\end{pmatrix}=\begin{pmatrix}-4\\6\end{pmatrix}$$



Answers: Question 5



a. $\alpha = \tan^{-1}\frac{\sqrt{3}}{3} = 30°$

$\beta = \tan^{-1}\frac{\sqrt{3}}{1} = 60°$

$\theta = 60° - 30° = 30°$

b. $\alpha = \tan^{-1}\frac{3}{3} = 45°$

$\beta = \tan^{-1}\frac{2}{-2} = 135°$

$\theta = 135° - 45° = 90°$

Answers: Question 6

a. $\begin{pmatrix} 3 \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} = 3 + 3 = 6$

$$\left\| \begin{pmatrix} 3 \\ \sqrt{3} \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \right\| = \sqrt{12} \times \sqrt{4} = 4\sqrt{3}$$

$$\theta = \cos^{-1}\frac{6}{4\sqrt{3}} = \cos^{-1}\frac{\sqrt{3}}{2} = 30°$$

b. $\begin{pmatrix} 3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 2 \end{pmatrix} = -6 + 6 = 0$

$$\theta = \cos^{-1}0 = 90°$$

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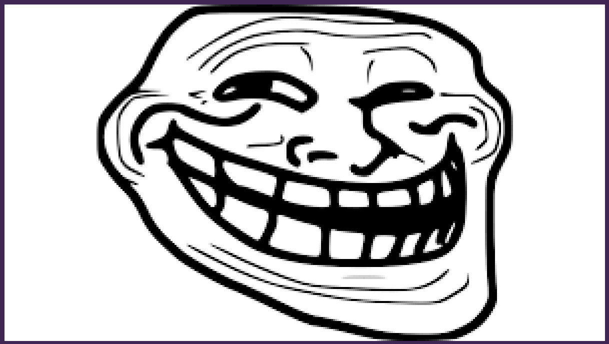
Answers: Question 7

a. (i) $n\binom{1}{1}$ (ii) $n\binom{-1}{1}$

b. (i) $n\binom{1}{1}$ (ii) $n\binom{1}{-1}$

c. (i) $n\binom{1}{2}$ (ii) $n\binom{2}{-1}$

d. 

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Answers: Question 7 cont.

For any vector $\begin{pmatrix} x \\ y \end{pmatrix}$, the vector $n\begin{pmatrix} x \\ y \end{pmatrix}$ is parallel:

$$\begin{pmatrix} x \\ y \end{pmatrix} \cdot n\begin{pmatrix} x \\ y \end{pmatrix} = nx^2 + ny^2 = n(x^2 + y^2)$$

$$\left\|\begin{pmatrix} x \\ y \end{pmatrix}\right\| \left\|n\begin{pmatrix} x \\ y \end{pmatrix}\right\| = \sqrt{x^2 + y^2} \times \sqrt{n^2(x^2 + y^2)}$$

$$= n(x^2 + y^2)$$

$$\theta = cos^{-1} 1 = 0$$

Answers: Question 7 cont.

For any vector $\begin{pmatrix} x \\ y \end{pmatrix}$, the vector $n\begin{pmatrix} y \\ -x \end{pmatrix}$ is perpendicular:

$$\begin{pmatrix} x \\ y \end{pmatrix} \cdot n \begin{pmatrix} y \\ -x \end{pmatrix} = nxy = nyx = 0$$

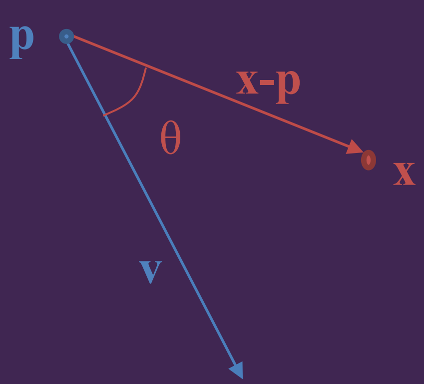
$$\theta = cos^{-1} 0 = 90°$$

Answers: Question 8a

Use the sign of the dot product between $\boldsymbol{v}$ and $\boldsymbol{x} - \boldsymbol{p}$. This follows from the geometric interpretation of the dot product:

$$\boldsymbol{v} \cdot (\boldsymbol{x} - \boldsymbol{p}) = \|\boldsymbol{v}\| \|\boldsymbol{x} - \boldsymbol{p}\| cos\theta$$

where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{x} - \boldsymbol{p}$.



Answers: Question 8a cont.

$$\boldsymbol{v} \cdot (\boldsymbol{x} - \boldsymbol{p}) = \|\boldsymbol{v}\| \|\boldsymbol{x} - \boldsymbol{p}\| cos\theta$$

Both $\|\boldsymbol{v}\|$ and $\|\boldsymbol{x} - \boldsymbol{p}\|$ are always positive, leaving the sign of the dot product entirely up to the value of $cos\theta$. If $cos\theta$ > 0 then θ is less than 90° and x is *in front* of the NPC. Similarly, if $cos\theta$ < 0 then θ is greater than 90 ° and x is *behind* the NPC.

The special case of $\boldsymbol{v} \cdot (\boldsymbol{x} - \boldsymbol{p}) = 0$ means that x lies either directly to the left or the right of the NPC. If this case does not need to be handled explicitly, it can be assigned arbitrarily to mean either in front or behind.

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Answers: Question 8b

i. x is in front of the NPC.

$$\begin{pmatrix}5\\-2\end{pmatrix}\cdot\left[\begin{pmatrix}0\\0\end{pmatrix}-\begin{pmatrix}-3\\4\end{pmatrix}\right]=\begin{pmatrix}5\\-2\end{pmatrix}\cdot\begin{pmatrix}3\\-4\end{pmatrix}=15+8=23$$

ii. x is in front of the NPC.

$$\begin{pmatrix}5\\-2\end{pmatrix}\cdot\left[\begin{pmatrix}1\\6\end{pmatrix}-\begin{pmatrix}-3\\4\end{pmatrix}\right]=\begin{pmatrix}5\\-2\end{pmatrix}\cdot\begin{pmatrix}4\\2\end{pmatrix}=20-4=16$$

iii. x is behind the NPC.

$$\begin{pmatrix}5\\-2\end{pmatrix}\cdot\left[\begin{pmatrix}-6\\0\end{pmatrix}-\begin{pmatrix}-3\\4\end{pmatrix}\right]=\begin{pmatrix}5\\-2\end{pmatrix}\cdot\begin{pmatrix}-3\\-4\end{pmatrix}=-15+8=-7$$

iv. x is behind the NPC.

$$\begin{pmatrix}5\\-2\end{pmatrix}\cdot\left[\begin{pmatrix}-4\\7\end{pmatrix}-\begin{pmatrix}-3\\4\end{pmatrix}\right]=\begin{pmatrix}5\\-2\end{pmatrix}\cdot\begin{pmatrix}-1\\3\end{pmatrix}=-5-6=-11$$

v. x is in front of the NPC.

$$\begin{pmatrix}5\\-2\end{pmatrix}\cdot\left[\begin{pmatrix}5\\5\end{pmatrix}-\begin{pmatrix}-3\\4\end{pmatrix}\right]=\begin{pmatrix}5\\-2\end{pmatrix}\cdot\begin{pmatrix}8\\1\end{pmatrix}=40-2=38$$

vi. x is in front of the NPC.

$$\begin{pmatrix}5\\-2\end{pmatrix}\cdot\left[\begin{pmatrix}-3\\0\end{pmatrix}-\begin{pmatrix}-3\\4\end{pmatrix}\right]=\begin{pmatrix}5\\-2\end{pmatrix}\cdot\begin{pmatrix}0\\-4\end{pmatrix}=0+8=8$$

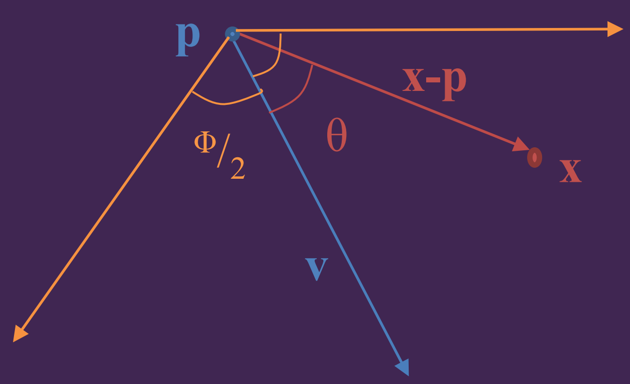
vii. x is in directly to the side of the NPC.

$$\begin{pmatrix}5\\-2\end{pmatrix}\cdot\left[\begin{pmatrix}-6\\-3.5\end{pmatrix}-\begin{pmatrix}-3\\4\end{pmatrix}\right]=\begin{pmatrix}5\\-2\end{pmatrix}\cdot\begin{pmatrix}-3\\-7.5\end{pmatrix}=-15+15=0$$

Answers: Question 9a

To determine whether the point x is visible to the NPC, compare $\cos\theta$ to $\cos(^{\Phi}/_{2})$:

If $\cos\theta \geq \cos(^{\Phi}/_{2})$, then x is visible to the NPC.



Answers: Question 9b

$\cos({}^{\Phi}/_{2})$ = $\cos(45°)$ ≈ 0.707

i. x is visible to the NPC ($\cos\theta$ ≈ 0.854)

ii. x is *not* visible to the NPC ($\cos\theta$ ≈ 0.664)

iii. x is *not* visible to the NPC ($\cos\theta$ ≈ -0.260)

iv. x is *not* visible to the NPC ($\cos\theta$ ≈ -0.646)

v. x is visible to the NPC ($\cos\theta$ ≈ 0.875)

vi. x is *not* visible to the NPC ($\cos\theta$ ≈ 0.371)

vii. x is *not* visible to the NPC ($\cos\theta = 0$)

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Answers: Question 9c

i. x is visible to the NPC:

$$\left\| \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} \right\| = 5 < 7$$

ii. [x is outside the FOV]

iii. [x is outside the FOV]

iv. [x is outside the FOV]

v. x is *not* visible to the NPC:

$$\left\| \begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} \right\| = \sqrt{65} > 7$$

vi. [x is outside the FOV]

vii. [x is outside the FOV]