COMP110: Principles of Computing

7: Data structures

Learning outcomes

- **Explain** the difference between pass-by-value and pass-by-reference
- **Distinguish** the basic data structures available in Python
- **Determine** the complexity of accessing and manipulating data in these data structures
- **Choose** the correct data structure for a given task

Worksheet D

- Data structures
- Due in class on **Monday 7th November** (next week)

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Pass by reference

References

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- For "plain old data" (e.g. numbers), this is accurate
- For **objects** (i.e. instances of classes), variables actually hold **references** (a.k.a. **pointers**)
- It is possible (indeed common) to have **multiple references** to the same underlying object

The wrong picture

```
class Thing:
    def __init__(self,
                 a, b):
        self.a = a
        self.b = b

x = Thing(30, 40)
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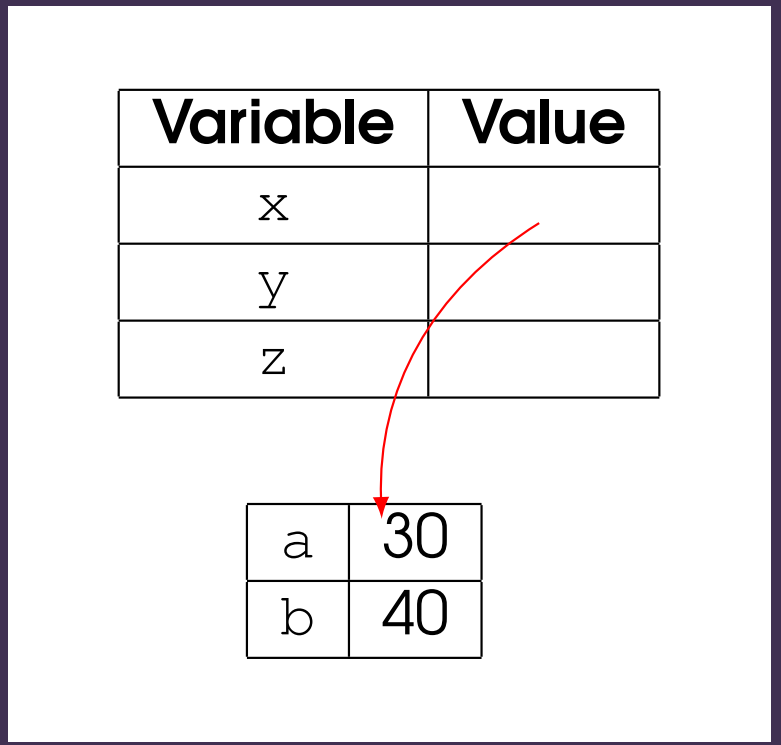

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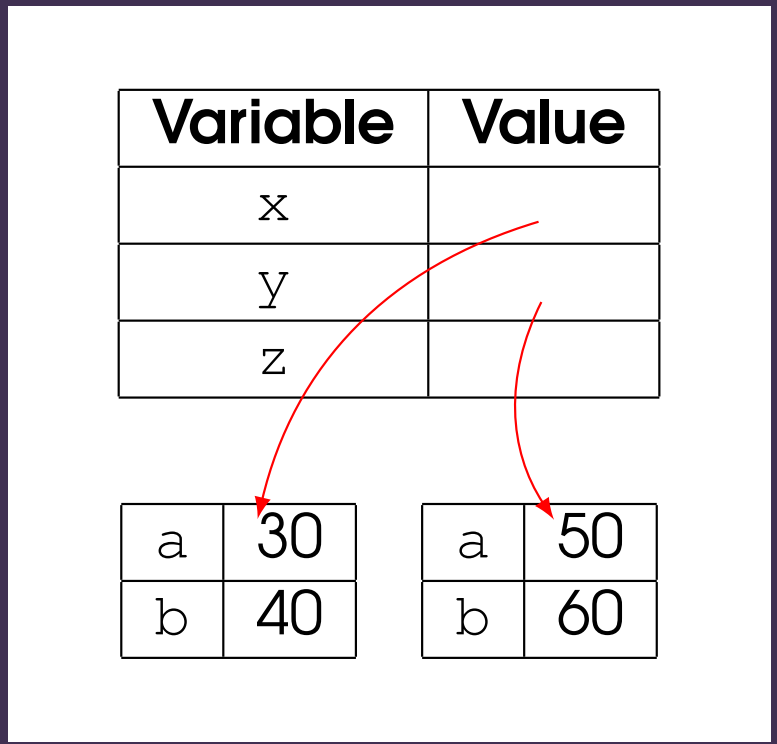

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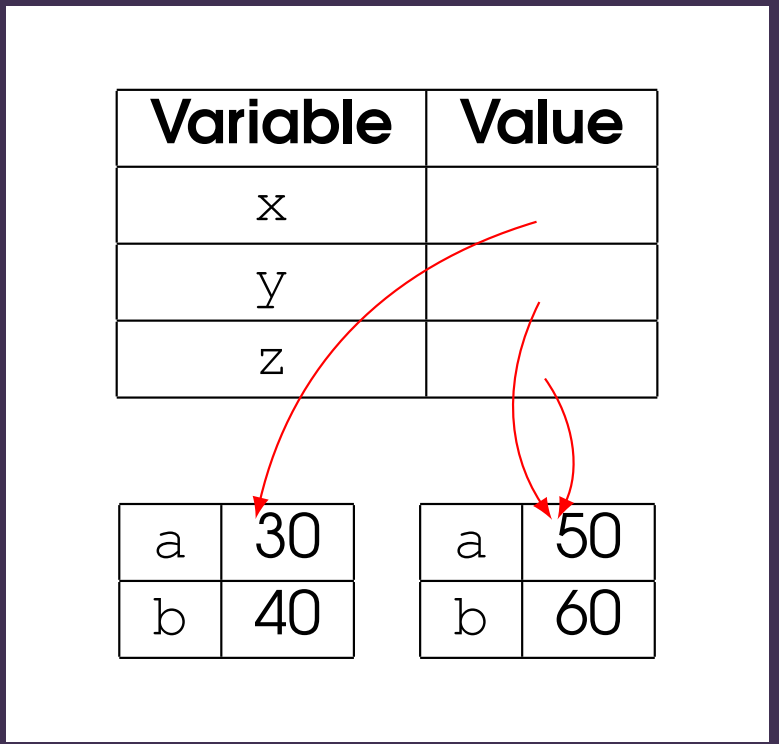

Values and references

Socrative room code: `FALCOMPED`

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a = 10
b = a
a = 20
print "a:", a
print "b:", b
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double(a)
print a
```

`double` does not actually do anything, as `x` is just a local copy of whatever is passed in!

Pass by reference

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However, instances are passed by **reference**

```
class Box:
    def __init__(self, v):
        self.value = v

def double(x):
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a = Box(7)
double(a)
print a.value
```

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print a.value
```

`double` now has an effect, as `x` gets a reference to the `Box` instance

Lists are objects too

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```
a = ["Hello"]
b = a
b.append("world")
print a  # ["Hello", "world"]
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... which means you should be careful when passing lists into functions, because the function might actually change the list!

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Basic containers in Python

Memory allocation

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- Blocks can be allocated and deallocated at will, but can **never grow or shrink**

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  - Hide the details of memory allocation, and allow the programmer to write simpler code
- Containers are an **encapsulation**
  - Bundle together the data's representation in memory along with the algorithms for accessing it

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- Accessing an array element is **constant time** $O(1)$

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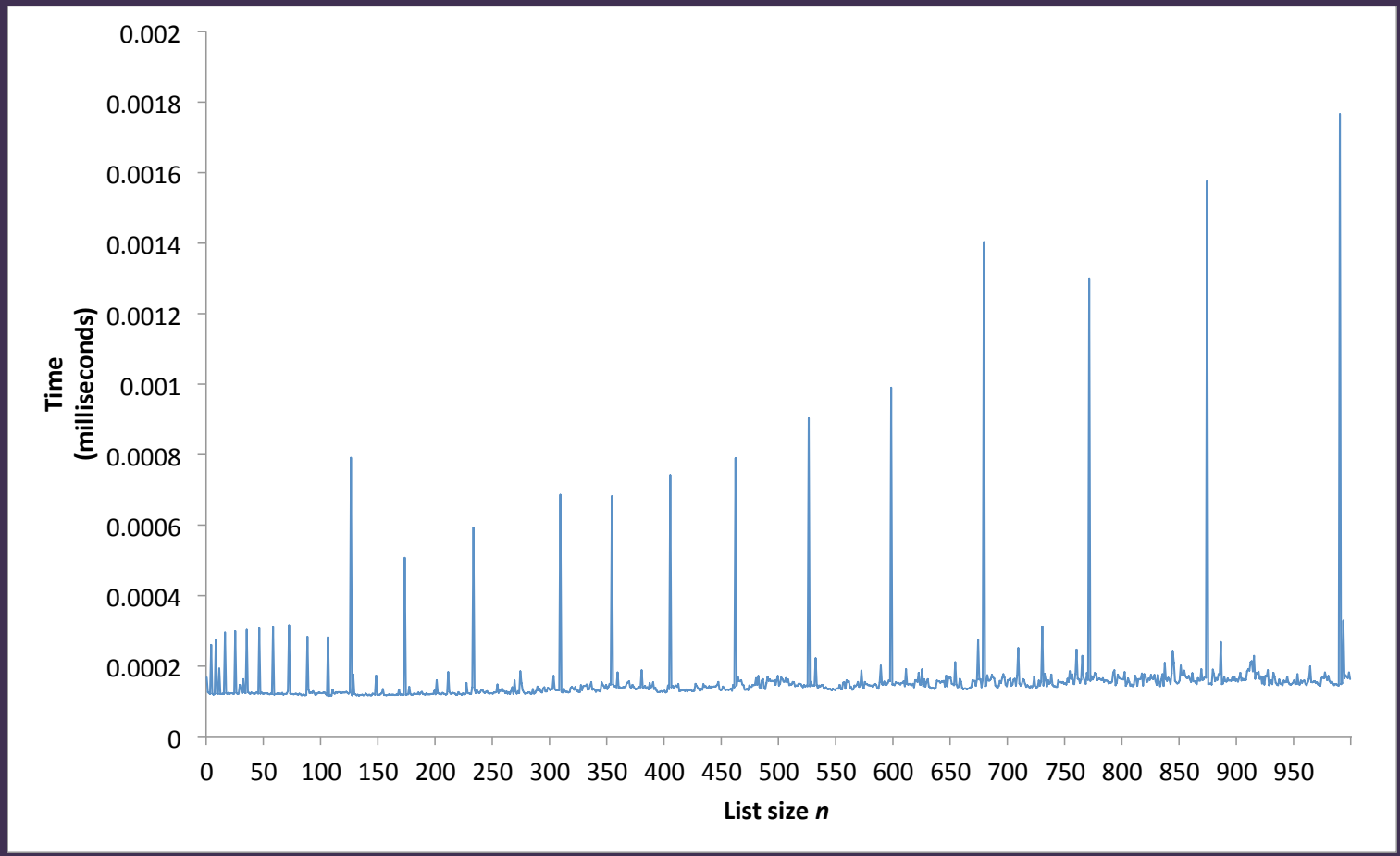
- When the list needs to change size, it **creates** a new array, **copies** the contents of the old array, and **deletes** the old array
- Implementation details: `http://www.laurentluce.com/posts/python-list-implementation/`

Time taken to append an element to a list of size *n*



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- **Inserting** anywhere other than the end is **linear time**
  - Can't just insert new bytes into a memory block — need to move all subsequent list elements to make room
- Similarly, **deleting** anything other than the last element is **linear time**

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- Create tuples with `()`, just as you create lists with `[]`
  - Exception: a single element tuple is created as `(foo,)` because `(foo)` would be interpreted as a bracketed expression
- Can often omit the parentheses entirely, e.g.

```
my_tuple = 1,2,3
```

Unpacking

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- Unpacking requires the number of elements to match exactly — if `foo` has more than 4 elements, the code on the left will give an error

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```
temp = a
a = b
b = temp
```

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- This isn't changing the string, it's creating a new one and throwing the old one away!
- Hence building a long string by appending can be slow (appending strings is $O(n)$)

Dictionaries

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- A dictionary maps **keys** to **values**
  - Keys must be immutable (numbers, strings, tuples etc)
  - Values can be anything (including dictionaries or other containers)
- A dictionary is implemented as a **hash table** (see Session 5)

Using dictionaries

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Create them using `{}`:

```
age = {"Alice": 23, "Bob": 36, "Charlie": 27}
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Access values using `[]`:

```
print age["Alice"]   # prints 23
age["Bob"] = 40      # overwriting an existing item
age["Denise"] = 21   # adding a new item
```

Iterating over dictionaries

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Iterating over a dictionary gives the **keys**:

```
for x in age:
    print x    # prints Alice, Bob, Charlie
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Use `iteritems` to get **key,value** pairs:

```
for key, value in age.iteritems():
    print key, "is", age, "years old"
```

Dictionaries are unordered

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What does this print?

```
square_root = {}
for i in xrange(30):
    square_root[i*i] = i

for key, value in square_root.iteritems():
    print "The square root of", key, "is", value
```

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Dictionaries are **unordered** — never rely on the order of their elements, because the order isn't guaranteed!

Sets

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Operation	**List**	**Set**
Add element	Append: $O(1)$ Insert: $O(n)$	$O(1)$
Delete element	$O(n)$	$O(1)$
Contains element?	$O(n)$	$O(1)$

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Common problem: we want to represent a **2-dimensional array** of values (i.e. a grid)

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$$\begin{matrix} v_{0,0} & v_{1,0} & \cdots & v_{w-1,0} \\ v_{0,1} & v_{1,1} & \cdots & v_{w-1,1} \\ \vdots & \vdots & \ddots & \vdots \\ v_{0,h-1} & v_{1,h-1} & \cdots & v_{w-1,h-1} \end{matrix}$$

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- E.g. $w = 5, h = 4$:

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19

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  - Each element of the "outer" list represents a column of the array
- The element in column x row y is accessed by `list[x][y]`, i.e. the yth element of the xth column

Approach 3: dictionary

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Approach 4: NumPy array

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- Requires NumPy or SciPy, and can only store numeric types
- However, highly optimised for intensive calculations (e.g. "tinkering" with image pixel colours...?)

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There is no single "best" approach — it depends how you use it

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More data structures

Stacks and queues

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- Items can be **dequeued** from the **front** of the queue

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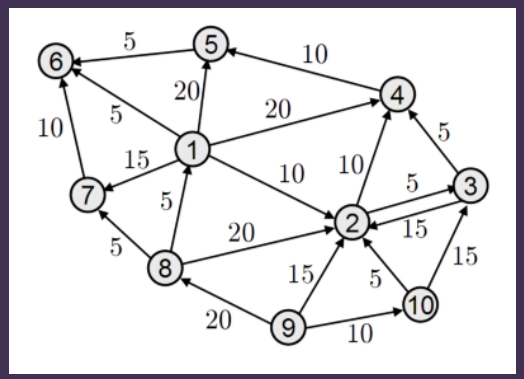
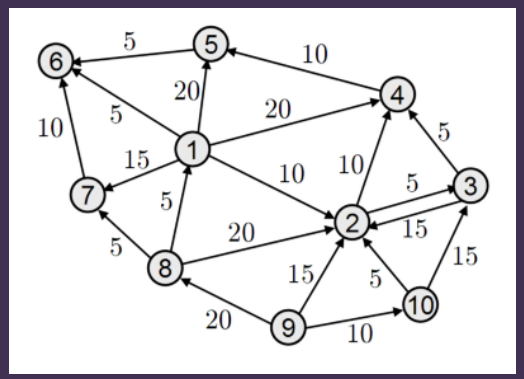
- Queues can be implemented as lists, but not efficiently
- `deque` (from the `collections` module) implements an efficient **double-ended queue**
- Inserting and removing elements from the start and end of a `deque` is $O(1)$

Graphs

Graphs



Graphs



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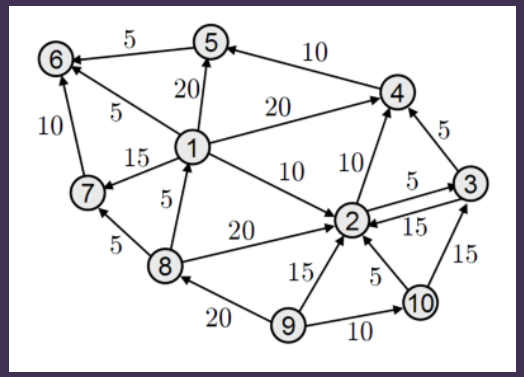
Graphs



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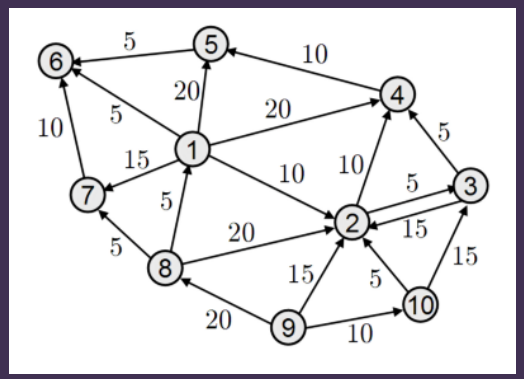

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  - A collection of **edges** or **arcs** (undirected lines or directed arrows between points)

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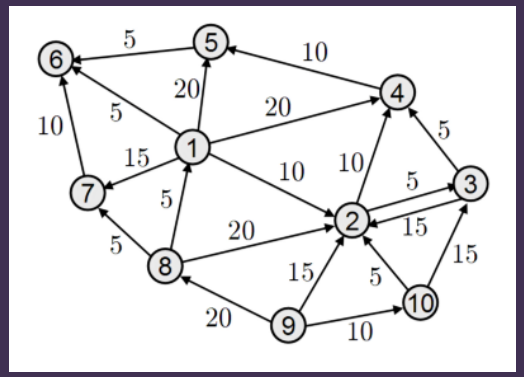

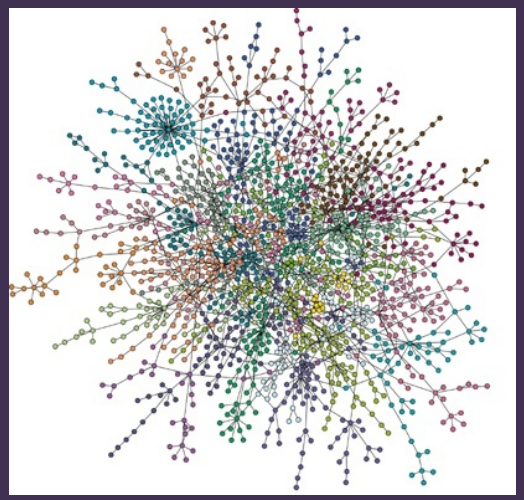

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- Often used to model **networks** (e.g. social networks, transport networks, game levels, finite state automata, ...)

Trees

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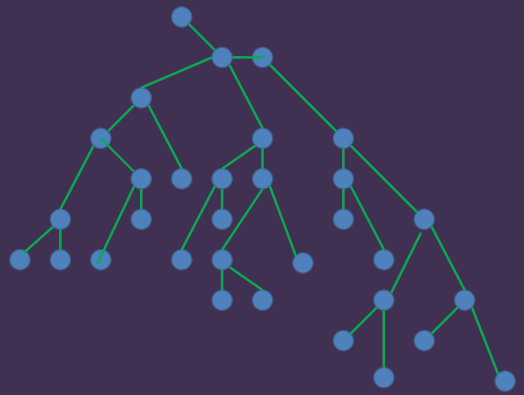

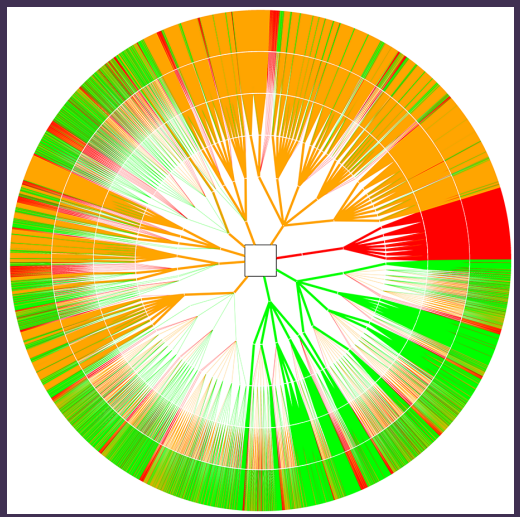

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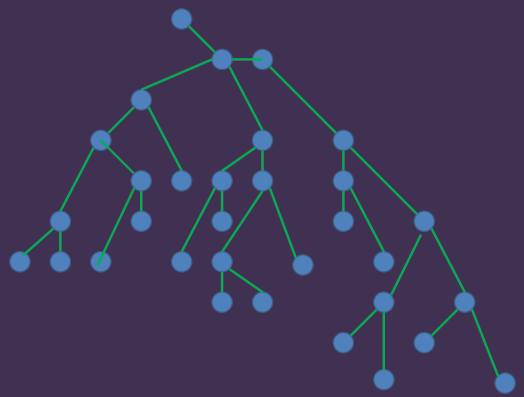
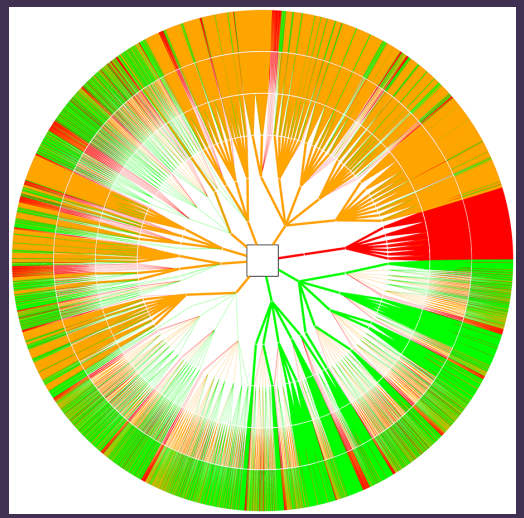


- A **tree** is a special type of directed graph where:



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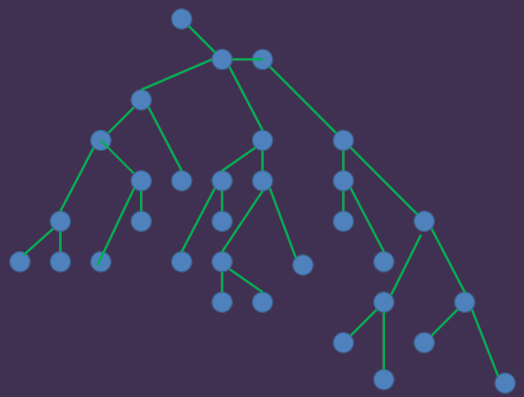

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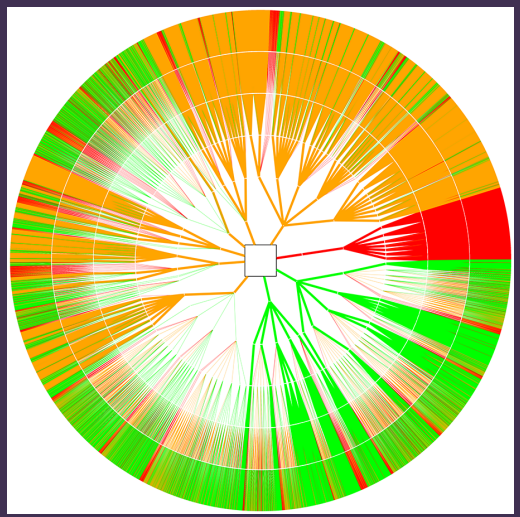

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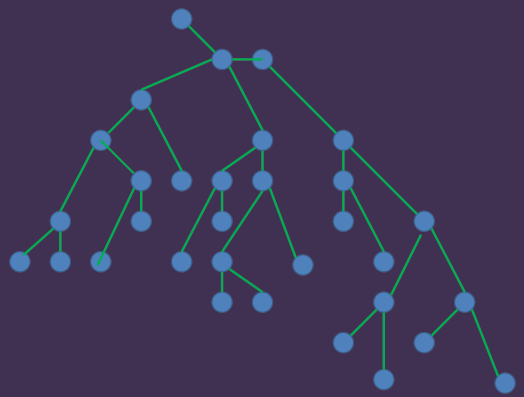

- A **tree** is a special type of directed graph where:
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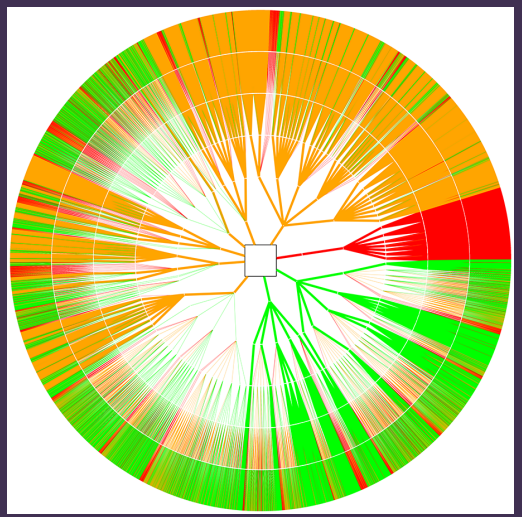

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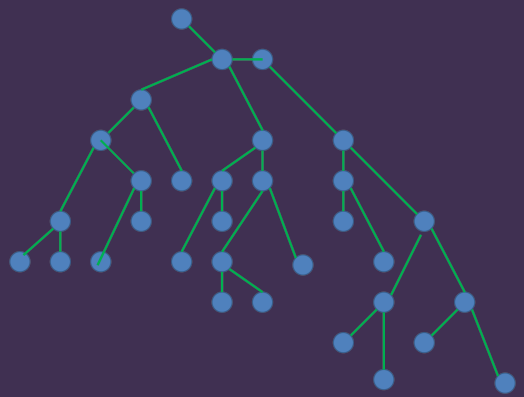

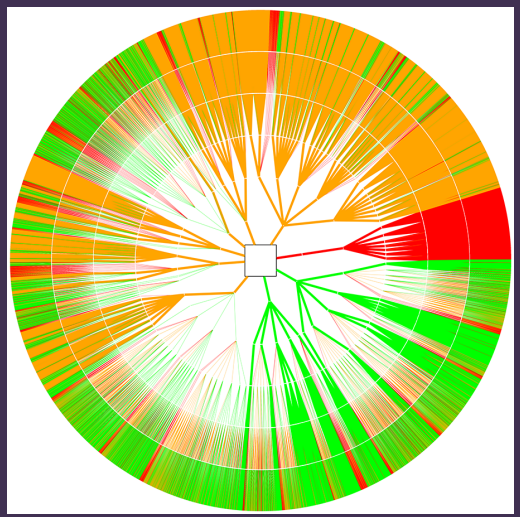

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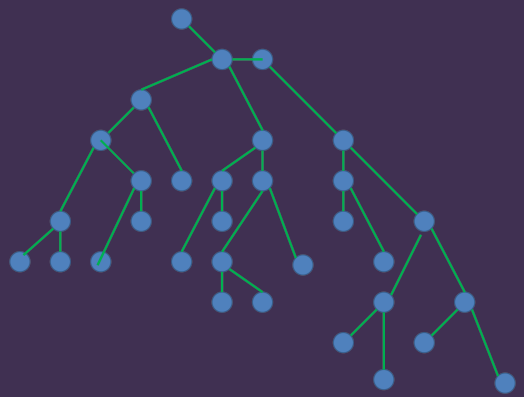

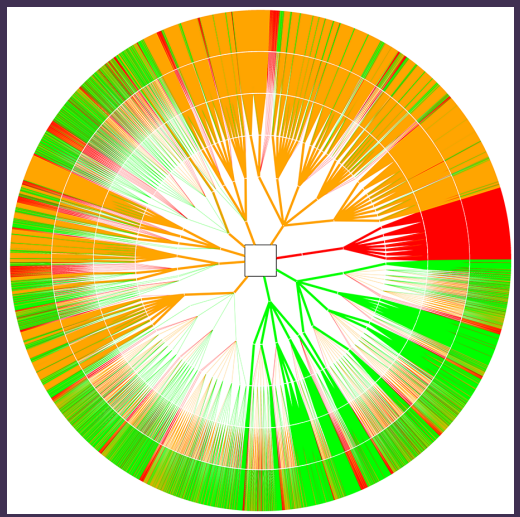

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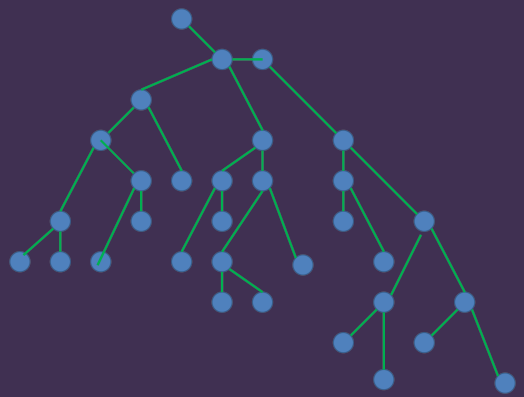

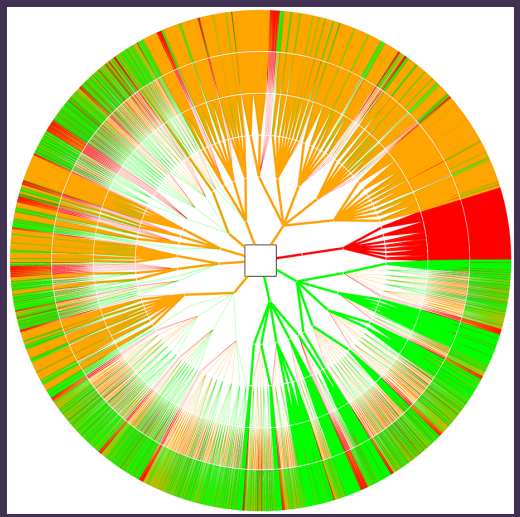

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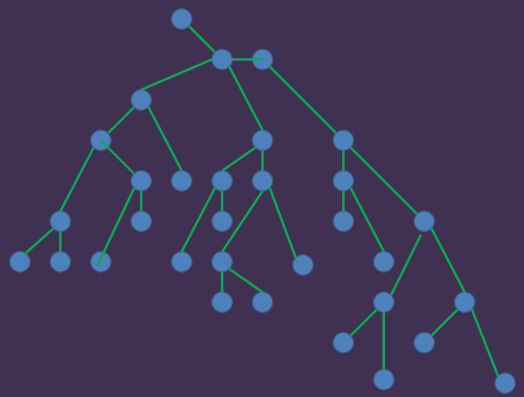

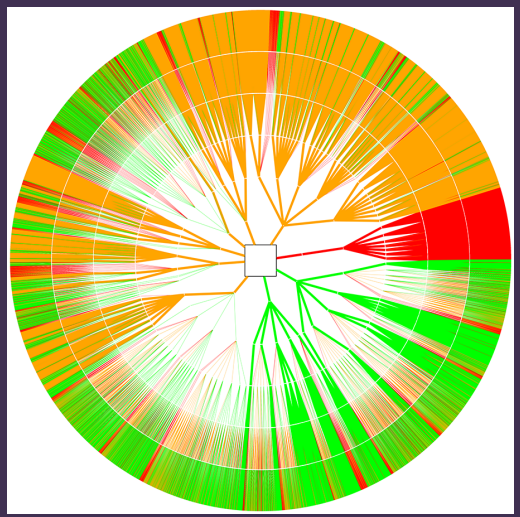

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- Used to model **hierarchies** (e.g. file systems, object inheritance, scene graphs, state-action trees, ...)

"Smart data structures and dumb code works a lot better than the other way around."
— Eric S. Raymond