# Introduction

The questions below explore some applications of the cross product in games and graphics situations.

# Exercises

1. A nonplayer character (NPC) is standing at a location with a forward direction of .  
   Consider three points , and in the plane of a left-handed coordinate system, which represent waypoints on the NPC’s path.
   1. How can the cross product be used to determine whether, when moving from to to , the NPC makes a clockwise or anticlockwise turn at , when viewing the path from above?  
      There are essentially two possible configurations in which the three points may be arranged:  
        
        
        
        
        
        
        
        
        
        
        
      The NPC’s path consists of two vectors, and . In the configuration on the left – turning clockwise – taking the cross product will give a vector pointing the positive direction (“out of the page”, since it’s a left-handed coordinate system), while in the second configuration, turning anticlockwise, the direction will be reversed.  
        
      We can simplify the calculation by observing that all the points and vectors in question have zero coordinates, i.e.  
       , and The direction the NPC turns in can therefore be determined by checking the sign of the value : positive for clockwise; negative for anticlockwise.  
      If the value happens to be zero, then the points are colinear and the NPC either continues straight or doubles back on itself:
   2. For each of the following sets of three points, determine whether the NPC is turning clockwise or anticlockwise when moving from to to :
      1. , ; the NPC turns anticlockwise.
      2. ,   
         ; the NPC turns anticlockwise.
      3. , ; the NPC turns clockwise.
      4. , ; the NPC turns anticlockwise.
2. Consider a triangle defined by the vertices (6, 10, -2), (3, -1, 17) and (-9, 8, 0).
   1. What is the (implicit) equation of the plane containing this triangle?  
      The standard equation of a plane is the one that expresses the idea that any vector lying along the plane is perpendicular to the plane’s normal, i.e. if is known point on the plane, then for any point on the plane, , or where is the perpendicular distance from the origin to the plane.  
        
      Since all the vertices of the triangle lie on the plane, its edges do too, and the cross product of any two of the edges will give a vector perpendicular to the plane (in either possible direction, depending on the order; in some situations, for instance rendering, it’s important to note which direction should be the “front” of the triangle).  
        
      Choosing our two edges to follow the order the vertices were given in:  
      we get the following cross product:  
      Though it’s not always necessary to normalise it (if we’re only interested in the sign of the direction, say), it’s often useful for later calculations – first, we need the magnitude:  
      which we can divide by to get the unit normal,   
        
      Now we can compute with any of the three triangle vertices as ; let’s use the first one:  
      , or in geometric () form,
   2. Is the point (3, 4, 5) on the front or back side of this plane (relative to the direction of the normal)?  
      How far is this point from the plane?  
      Both parts of this question can be answered with the same calculation. In the lecture, we saw that the distance from the point to the plane is given by , where is the point in question and is the distance from the origin to the plane as above. This gives the distance as  
      This is a positive distance, therefore the point is on the ‘front’ side of the plane (i.e. the direction the normal is pointing in).