# Introduction

This worksheet is split into two sections; the first deals with matrix transformations between coordinate spaces, while the second contains some exercises involving quaternion calculations.

You may find the Symbolab [matrix multiplication calculator](https://www.symbolab.com/solver/matrix-multiply-calculator) useful for finding/checking your results.

# Matrix Transformations

1. An object initially had its axes and origin coincident with the world axes and origin. It was first rotated 30° about the -axis, and then -22° about the world -axis.
   1. What is the matrix that can be used to transform column vectors from object space to world space?
   2. What about the matrix to transform vectors from world space to object space?
   3. Express the object’s -axis using world coordinates.
2. A robot is at the position (1, 10, 3) and her right, up and forward vectors (expressed in world space) are , and respectively (note that these vectors form an orthonormal basis).
   1. The following points are expressed in object space; calculate their coordinates in world space:
      1. (-1, 2, 0)
      2. (1, 2, 0)
      3. (0, 0, 0)
      4. (1, 5, 0.5)
      5. (0, 5, 10)
   2. The coordinates below are in world space; find their positions relative to the robot:
      1. (1, 10, 3)
      2. (0, 0, 0)
      3. (2.732, 10, 2)
      4. (2, 11, 4)
      5. (1, 20, 3)

# Quaternions

1. A quaternion to rotate through an angle is written as .
   1. Construct a quaternion to rotate 30° about the -axis.
   2. What is the magnitude of this quaternion?
   3. What is its conjugate, \*?
   4. Assume the quaternion is used to rotate points from object space to world space. What would the position of the point be under this rotation?
2. Compute a quaternion that performs twice the rotation of the quaternion .
3. Consider the quaternions:  
   1. Compute the dot product , given by the formula
   2. Compute the quaternion product , given by the Hamilton product
   3. Compute the difference from to , given by the quaternion (with \*).

Exercises may include some modified questions from  
Dunn, F & Parberry, I 2011, *3D Math Primer for Graphics and Game Development*, CRC Press, Boca Raton, FL