COMP702: Classical Artificial Intelligence

4: Theoretical models for games

Game theory

Game theory

- A branch of mathematics studying **decision making**
- A **game** is a system where one or more **players** choose **actions**; the combination of these choices lead to each agent receiving a **payoff**
- Important applications in economics, ecology and social sciences as well as AI

The ~~Prisoner's~~ Student's Dilemma

- Two students, **Alice** and **Bob**, are suspected of copying from each other
- Each is offered a deal in exchange for information
- Each can choose to **betray** the other or stay **silent** — but they **cannot communicate** before deciding what to do
- If **both stay silent**, both receive a C grade
- If **Alice betrays Bob**, she receives an A whilst he gets expelled
- If **Bob betrays Alice**, he receives an A whilst she gets expelled
- If **both betray each other**, both get an F

Payoff matrix

	A silent	A betray
B silent	A: 50 B: 50	A: 70 B: -100
B betray	A: -100 B: 70	A: 0 B: 0

Nash equilibrium

- Consider the situation where both have chosen to betray
- Neither person has anything to gain by switching to silence, assuming the other person doesn't also switch
- Such a situation is called a **Nash equilibrium**
- If all players are **rational** (in the sense of wanting to maximising payoff), they should converge upon a Nash equilibrium

Does every game have a Nash equilibrium?

	A rock	A paper	A scissors
B rock	A: 0 B: 0	A: +1 B: -1	A: -1 B: +1
B paper	A: -1 B: +1	A: 0 B: 0	A: +1 B: -1
B scissors	A: +1 B: -1	A: -1 B: +1	A: 0 B: 0

Nash equilibrium for Rock-Paper-Scissors

- Committing to any choice of action can be **exploited**
- E.g. if you always choose paper, I choose scissors
- If we try to reason naïvely, we get stuck in a loop
    - If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...
- The optimum strategy is to be **unpredictable**
- Choose rock with probability $\frac{1}{3}$, paper with probability $\frac{1}{3}$, scissors with probability $\frac{1}{3}$

Mixed strategies

- A **mixed strategy** assigns probabilities to actions and chooses one at random
- In contrast to a **pure** or **deterministic strategy**, which always chooses the same action
- If we allow mixed strategies, **every game has at least one Nash equilibrium**

Guess $\frac{2}{3}$ of the average

- Everyone guesses a real number (decimals are allowed) between 0 and 100 inclusive
- The winner is the person who guesses closest to $\frac{2}{3}$ of the mean of all guesses
- Example:
  - If the guesses are 30, 40 and 80...
  - ... then the mean is $\frac{30+40+80}{3} = 50$...
  - ... so the winning guess is 30, as this is closest to $\frac{2}{3} \times 50 = 33.333$

Rationality

- Rationality is a useful assumption for mathematics and AI programmers
- However it's important to remember that **humans aren't always rational**

Markov decision processes and games

Markov decision processes

- A **Markov decision process (MDP)** is defined by:
  - A finite set S of **states**;
  - A finite set A of **actions**;
  - $P(s, a, s')$ is the **probability** that action a in state s leads to state s';
  - $R(s, a, s')$ is the **reward** received from performing action a in state s and ending up in state s'.

The Markov property

- Given a state s and an action a, the next state s' is determined by $P(s, a, s')$
- The previous states before s have no effect
- Hence an MDP is “memoryless”
- (Or rather, any memory has to be contained within the state)

(Non)determinism

- MDP defines $P(s, a, s')$
- From state s and action a, there may be several possible states s'
- MDPs are **nondeterministic** i.e. **stochastic**
- If we have

$$P(s, a, s') = \begin{cases} 1 & \text{for some } s' \\ 0 & \text{for all other } s' \end{cases}$$

  then the MDP is **deterministic**
- In the deterministic case, the same state s and action a always leads to the same state s'

Markov decision problems

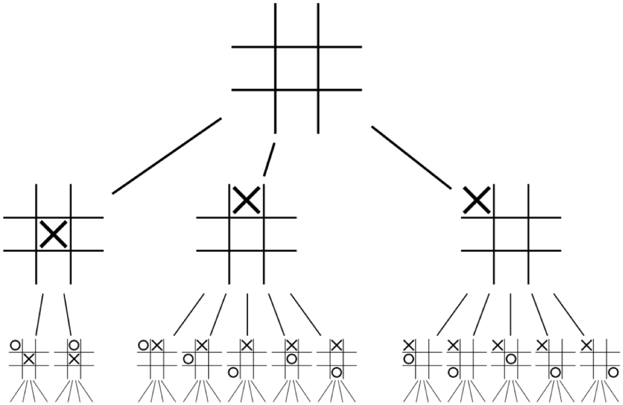
- A **policy** for an MDP is a function $\pi : S \to A$
- I.e. in state s, choose action $a = \pi(s)$
- Goal: find π which maximises the total reward over time

Multi-agent MDPs

- We have assumed a **single agent** choosing a policy π
- We can extend to n agents choosing policies $\pi_1, \dots, \pi_n$
- The "action" is now a combination $(a_1, \dots, a_n)$ of all the agents' actions
- Each agent has their own reward function and is trying to maximise it
- This is a **game**!

Minimax search

Game trees



Minimax

- Consider a 2-agent MDP with the following restrictions:
  - Deterministic
  - Only one agent chooses an action from a given state
  - Some states are **terminal** — these are the only ones with non-zero reward
  - The game is **zero sum**: $R_1(s, a, s') + R_2(s, a, s') = 0$
- I want to **maximise** my reward
- My opponent wants to maximise their reward, which is the same as **minimise** my reward
- Therefore I want to **maximise** the **minimum** value my opponent can achieve

Minimax search

- Recursively defines a **value** for non-terminal game states
- Consider each possible “next state”, i.e. each possible move
- If it’s my turn, the value is the **maximum** value over next states
- If it’s my opponent’s turn, the value is the **minimum** value over next states

Minimax search – example

Minimax search pseudocode

```
procedure MINIMAX(state, currentPlayer)
    if state is terminal then
        return value of state
    else if currentPlayer $= 1$ then
        bestValue $= -\infty$
        for each possible nextState do
            $v$ = MINIMAX(nextState, 3$-$ currentPlayer)
            bestValue = MAX(bestValue, $v$)
        return bestValue
    else if currentPlayer $= 2$ then
        bestValue $= +\infty$
        for each possible nextState do
            $v$ = MINIMAX(nextState, 3$-$ currentPlayer)
            bestValue = MIN(bestValue, $v$)
        return bestValue
```

Stopping early

```
**for each** possible nextState **do**
    $v$ = MINIMAX(nextState, 3− currentPlayer)
    bestValue = MAX(bestValue, $v$)
```

- State values are always between -1 and $+1$
- So if we ever have bestValue $= 1$, we can stop early
- Similarly when minimising if bestValue $= -1$
- There are techniques for smarter early stopping, e.g. alpha-beta pruning

Using minimax search

- To decide what move to play next...
- Calculate the minimax value for each move
- Choose the move with the maximum score
- If there are several with the same score, choose one at random

Minimax and game theory

- For a **two-player zero-sum** game with **perfect information** and **sequential moves**
- Minimax search will always find a **Nash equilibrium**
- I.e. a minimax player plays **perfectly**
- **But...**

Minimax for larger games

- The game tree for noughts and crosses has only a few thousand states
- Most games are too large to search fully
    - Connect 4 has $\approx 10^{13}$ states
    - Chess has $\approx 10^{47}$ states
- Generally need to **cut off** the tree at a certain depth and use **heuristics**
- Basically, define our own reward function that (hopefully) approximates the real one

Application to games

Application to board games

- The ideas of MDPs and game theory apply readily to board games
- State = state of the board
- Action = a move
- Reward = win or loss, score, etc.

Application to video games

- *Technically* a video game is an MDP
    - State = entire state of the game (memory?)
    - Action = controller input on each frame
- However this is intractable
- Generally necessary to **abstract** parts out of the game to treat as MDPs / game theory games