



COMP702: Classical Artificial Intelligence

4: Theoretical models for games



Game theory



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- ▶ A **game** is a system where one or more **players** choose **actions**; the combination of these choices lead to each agent receiving a **payoff**
- ▶ Important applications in economics, ecology and social sciences as well as AI

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- ▶ If **Bob betrays Alice**, he receives an A whilst she gets expelled
- ▶ If **both betray each other**, both get an F

Payoff matrix

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	A silent	A betray
B silent	A: 50 B: 50	A: 70 B: -100
B betray	A: -100 B: 70	A: 0 B: 0

Alice's thought process

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 - ▶ However if Bob stays silent, I can get a better mark by betraying him
 - ▶ And if Bob betrays me, I should betray him to avoid getting expelled
 - ▶ Therefore the rational choice is to betray
- ... and Bob's thought process is the same!

Nash equilibrium

Nash equilibrium

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Nash equilibrium

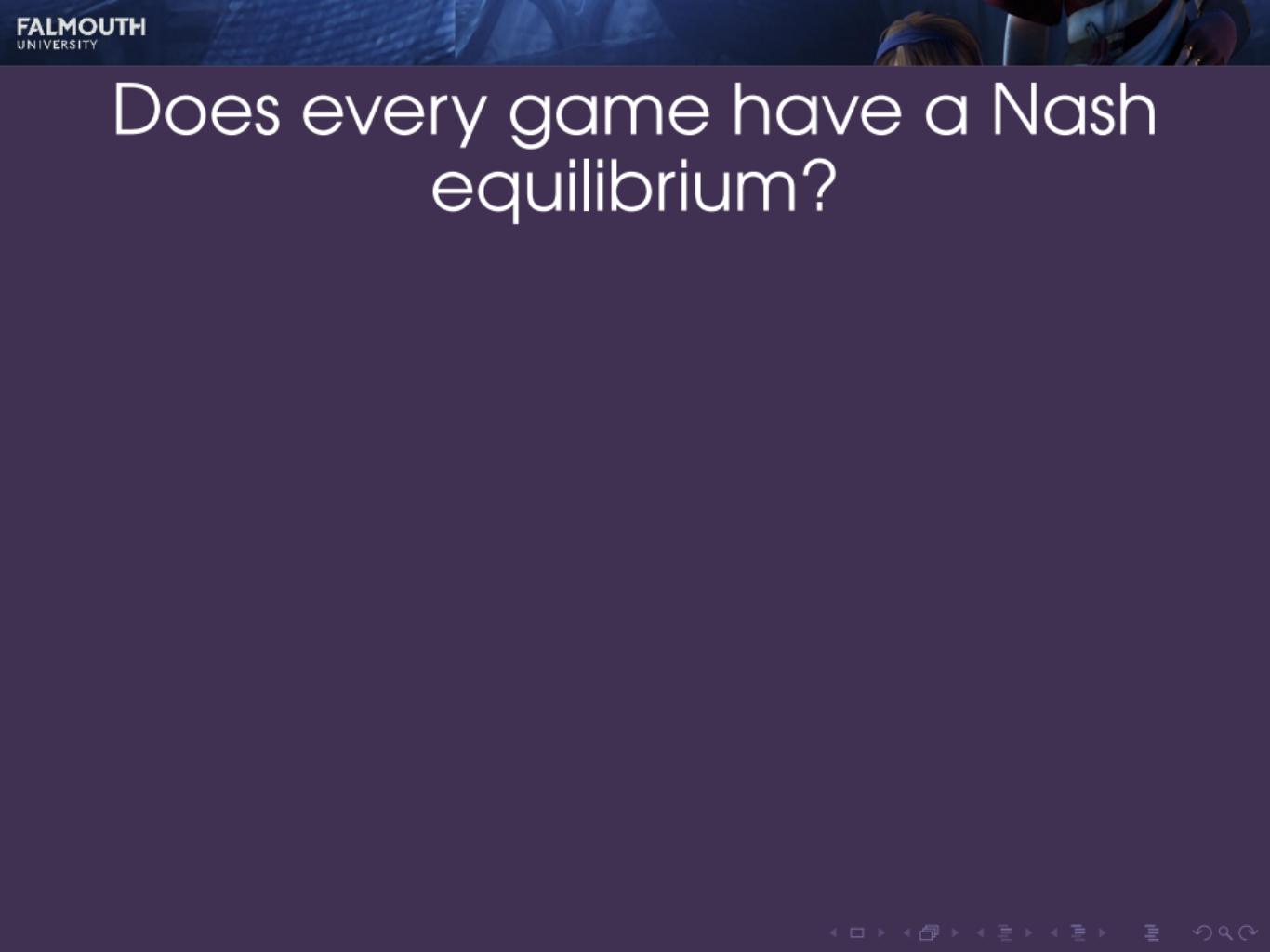
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- ▶ Neither person has anything to gain by switching to silence, assuming the other person doesn't also switch
- ▶ Such a situation is called a **Nash equilibrium**
- ▶ If all players are **rational** (in the sense of wanting to maximising payoff), they should converge upon a Nash equilibrium



Does every game have a Nash equilibrium?

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	A rock	A paper	A scissors
B rock	A: 0 B: 0	A: +1 B: -1	A: -1 B: +1
B paper	A: -1 B: +1	A: 0 B: 0	A: +1 B: -1
B scissors	A: +1 B: -1	A: -1 B: +1	A: 0 B: 0

Nash equilibrium for Rock-Paper-Scissors

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- ▶ Committing to any choice of action can be **exploited**
- ▶ E.g. if you always choose paper, I choose scissors
- ▶ If we try to reason naïvely, we get stuck in a loop
 - ▶ If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...

Nash equilibrium for Rock-Paper-Scissors

- ▶ Committing to any choice of action can be **exploited**
- ▶ E.g. if you always choose paper, I choose scissors
- ▶ If we try to reason naïvely, we get stuck in a loop
 - ▶ If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...
- ▶ The optimum strategy is to be **unpredictable**

Nash equilibrium for Rock-Paper-Scissors

- ▶ Committing to any choice of action can be **exploited**
- ▶ E.g. if you always choose paper, I choose scissors
- ▶ If we try to reason naïvely, we get stuck in a loop
 - ▶ If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...
- ▶ The optimum strategy is to be **unpredictable**
- ▶ Choose rock with probability $\frac{1}{3}$, paper with probability $\frac{1}{3}$, scissors with probability $\frac{1}{3}$

Mixed strategies

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- ▶ A **mixed strategy** assigns probabilities to actions and chooses one at random
- ▶ In contrast to a **pure** or **deterministic strategy**, which always chooses the same action
- ▶ If we allow mixed strategies, **every game has at least one Nash equilibrium**



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 - ▶ ... then the mean is $\frac{30+40+80}{3} = 50...$

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- ▶ The winner is the person who guesses closest to $\frac{2}{3}$ of the mean of all guesses
- ▶ Example:
 - ▶ If the guesses are 30, 40 and 80...
 - ▶ ... then the mean is $\frac{30+40+80}{3} = 50...$
 - ▶ ... so the winning guess is 30, as this is closest to $\frac{2}{3} \times 50 = 33.333$

The rational guess

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- The average can't possibly be greater than 100

The rational guess

- ▶ The average can't possibly be greater than 100
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- ▶ ... and so on ad infinitum

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- ▶ Which means the average can't possibly be greater than 44.444
- ▶ So no rational player would guess greater than 29.629
- ▶ ... and so on ad infinitum
- ▶ So the only **rational** guess is 0, as every rational player should guess 0 and $\frac{2}{3}$ of 0 is 0

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- ▶ However it's important to remember that **humans aren't always rational**

Markov decision processes and games



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- ▶ A **Markov decision process (MDP)** is defined by:
 - ▶ A finite set S of **states**;
 - ▶ A finite set A of **actions**;
 - ▶ $P(s, a, s')$ is the **probability** that action a in state s leads to state s' ;
 - ▶ $R(s, a, s')$ is the **reward** received from performing action a in state s and ending up in state s' .

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- ▶ Given a state s and an action a , the next state s' is determined by $P(s, a, s')$
- ▶ The previous states before s have no effect
- ▶ Hence an MDP is “memoryless”
- ▶ (Or rather, any memory has to be contained within the state)

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$$P(s, a, s') = \begin{cases} 1 & \text{for some } s' \\ 0 & \text{for all other } s' \end{cases}$$

then the MDP is **deterministic**

- ▶ In the deterministic case, the same state s and action a always leads to the same state s'

Markov decision problems

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- ▶ Goal: find π which maximises the total reward over time

Multi-agent MDPs

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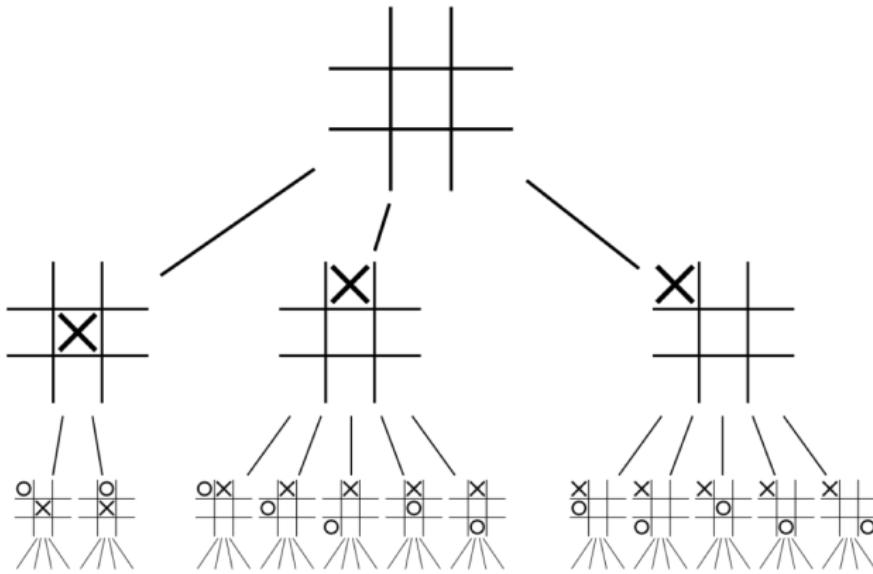
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- ▶ The “action” is now a combination (a_1, \dots, a_n) of all the agents’ actions
- ▶ Each agent has their own reward function and is trying to maximise it
- ▶ This is a **game!**

Minimax search



Game trees



Minimax

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- ▶ My opponent wants to maximise their reward, which is the same as **minimise** my reward

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 - ▶ Some states are **terminal** — these are the only ones with non-zero reward
 - ▶ The game is **zero sum**: $R_1(s, a, s') + R_2(s, a, s') = 0$
- ▶ I want to **maximise** my reward
- ▶ My opponent wants to maximise their reward, which is the same as **minimise** my reward
- ▶ Therefore I want to **maximise** the **minimum** value my opponent can achieve

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- ▶ Consider each possible “next state”, i.e. each possible move
- ▶ If it’s my turn, the value is the **maximum** value over next states
- ▶ If it’s my opponent’s turn, the value is the **minimum** value over next states

Minimax search – example

Minimax search pseudocode

```
procedure MINIMAX(state, currentPlayer)
```

Minimax search pseudocode

```
procedure MINIMAX(state, currentPlayer)
    if state is terminal then
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Minimax search pseudocode

```
procedure MINIMAX(state, currentPlayer)
    if state is terminal then
        return value of state
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procedure MINIMAX(state, currentPlayer)
    if state is terminal then
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procedure MINIMAX(state, currentPlayer)
    if state is terminal then
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        bestValue = -∞
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procedure MINIMAX(state, currentPlayer)
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    else if currentPlayer = 1 then
        bestValue = -∞
        for each possible nextState do
```

Minimax search pseudocode

```
procedure MINIMAX(state, currentPlayer)
    if state is terminal then
        return value of state
    else if currentPlayer = 1 then
        bestValue = -∞
        for each possible nextState do
            v = MINIMAX(nextState, 3 – currentPlayer)
```

Minimax search pseudocode

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procedure MINIMAX(state, currentPlayer)
    if state is terminal then
        return value of state
    else if currentPlayer = 1 then
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            v = MINIMAX(nextState, 3 – currentPlayer)
            bestValue = Max(bestValue, v)
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            bestValue = MAX(bestValue, v)
        return bestValue
    else if currentPlayer = 2 then
```

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    if state is terminal then
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    else if currentPlayer = 1 then
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            v = MINIMAX(nextState, 3 – currentPlayer)
            bestValue = MAX(bestValue, v)
        return bestValue
    else if currentPlayer = 2 then
        bestValue = +∞
        for each possible nextState do
            v = MINIMAX(nextState, 3 – currentPlayer)
            bestValue = MIN(bestValue, v)
        return bestValue
```

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    else if currentPlayer = 1 then
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            bestValue = MAX(bestValue, v)
        return bestValue
    else if currentPlayer = 2 then
        bestValue = +∞
        for each possible nextState do
            v = MINIMAX(nextState, 3 – currentPlayer)
            bestValue = MIN(bestValue, v)
        return bestValue
```

Stopping early

for each possible nextState **do**

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- ▶ State values are always between –1 and +1

Stopping early

for each possible nextState **do**

$v = \text{MINIMAX}(\text{nextState}, 3 - \text{currentPlayer})$
 $\text{bestValue} = \text{MAX}(\text{bestValue}, v)$

- ▶ State values are always between -1 and $+1$
- ▶ So if we ever have $\text{bestValue} = 1$, we can stop early

Stopping early

for each possible nextState **do**

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- ▶ State values are always between -1 and $+1$
- ▶ So if we ever have $\text{bestValue} = 1$, we can stop early
- ▶ Similarly when minimising if $\text{bestValue} = -1$
- ▶ There are techniques for smarter early stopping, e.g. alpha-beta pruning

Using minimax search

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- To decide what move to play next...

Using minimax search

- ▶ To decide what move to play next...
- ▶ Calculate the minimax value for each move

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- ▶ To decide what move to play next...
- ▶ Calculate the minimax value for each move
- ▶ Choose the move with the maximum score

Using minimax search

- ▶ To decide what move to play next...
- ▶ Calculate the minimax value for each move
- ▶ Choose the move with the maximum score
- ▶ If there are several with the same score, choose one at random

Minimax and game theory

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- ▶ For a **two-player zero-sum** game with **perfect information** and **sequential moves**

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- ▶ I.e. a minimax player plays **perfectly**

Minimax and game theory

- ▶ For a **two-player zero-sum** game with **perfect information** and **sequential moves**
- ▶ Minimax search will always find a **Nash equilibrium**
- ▶ I.e. a minimax player plays **perfectly**
- ▶ **But...**

Minimax for larger games

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- The game tree for noughts and crosses has only a few thousand states

Minimax for larger games

- ▶ The game tree for noughts and crosses has only a few thousand states
- ▶ Most games are too large to search fully

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- ▶ Basically, define our own reward function that (hopefully) approximates the real one

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- ▶ However this is intractable
- ▶ Generally necessary to **abstract** parts out of the game to treat as MDPs / game theory games

Workshop



Portfolio

For next Wednesday: please prepare a draft proposal for
Instance 2 (Computational Intelligence)