9: CONSTRAINT SATISFACTION

COMP702: CLASSICAL ARTIFICIAL INTELLIGENCE

PAPER CLUB

For next week's seminar, please read:

A.M. Smith, M. Mateas. Answer Set Programming for Procedural Content Generation: A Design Space Approach, IEEE Transactions on Computational Intelligence and AI in Games, 2011.

(PDF Link on LearningSpace)

CONSTRAINT SATISFACTION PROBLEMS

CONSTRAINT SATISFACTION PROBLEMS (CSP)

- A Constraint Satisfaction Problem (CSP) is defined by:
  - A set $\{X_1, \dots, X_n\}$ of variables
  - A set $\{D_1, \dots, D_n\}$ of domains, specifying what values each variable can take
  - A set $\{C_1, \dots, C_m\}$ of constraints that the variables must satisfy
- A solution is an assignment of a value to every variable which satisfies all the constraints

SUDOKU

- A 9x9 grid
- Each square contains a number 1-9
- A number cannot appear more than once in any given row, column, or 3x3 square outlined in bold
- Some numbers are given – the puzzle is to fill in the rest
- A properly designed Sudoku puzzle has exactly one solution

4				8		6		
	5	9	4		7	1		
			9		1	5	2	
9		3						
	6						1	
						9		8
	3	2	1		5			
		8	2		6	7	3	
		6		7				9

SUDOKU AS A CSP

- Variables: a subset of $\{X_{1,1}, X_{1,2}, \dots, X_{9,9}\}$
  - One variable per empty square
- All have the same domain: $D_{i,j} = \{1, \dots, 9\}$
- Constraints:
  - $X_{i,j} \neq X_{i,k}$ for all i, j, k with $j \neq k$
  - $X_{j,i} \neq X_{k,i}$ for all i, j, k with $j \neq k$
  - $X_{i,j} \neq X_{k,l}$ for i, j and k, l in the same 3x3 square

4				8		6		
	5	9	4		7	1		
			9		1	5	2	
9		3						
	6						1	
						9		8
	3	2	1		5			
		8	2		6	7	3	
		6		7				9

SOLVING SUDOKU BACKTRACKING

- Pick an empty square
- Try a possible value for that empty square
- Try to solve the rest of the puzzle (recursively)
- If we run into a dead end, go back and try a different value

SOLVING SUDOKU BACKTRACKING

```
Procedure SolveBacktracking(square)
    For n = 1, …, 9
        If square can have value n
            Set value of square to n
            If square is the last empty square or SolveBacktracking(next_empty_square)
                Return true
            End If
        End If
    End For
    Clear the value of square
    Return false
End Procedure
```

SOLVING SUDOKU
CONSTRAINT PROPAGATION

- Each empty square keeps track of which numbers it could possibly contain
- Update every square
- If a square has only one possible number, fill it in
- Repeat until stuck

SOLVING SUDOKU
CONSTRAINT PROPAGATION

```
Procedure SolveConstraintPropagation()
    Repeat
        For each empty square
            Determine the list of possible numbers that could go in the square
            If the list is empty
                Return false // unsolvable
            Else if the list has a single element
                Fill in the number
            End If
        End For
    Until no changes are made
    Return true
End Procedure
```

BACKTRACKING VS CONSTRAINT PROPAGATION

- Backtracking always finds a solution, but is inefficient
- Constraint propagation is efficient, but doesn't always find a solution
- More sophisticated constraint propagation algorithms do exist
- We could also combine the two
  - Use backtracking to "unstick" constraint propagation
  - Use constraint propagation to narrow down the options that backtracking needs to consider

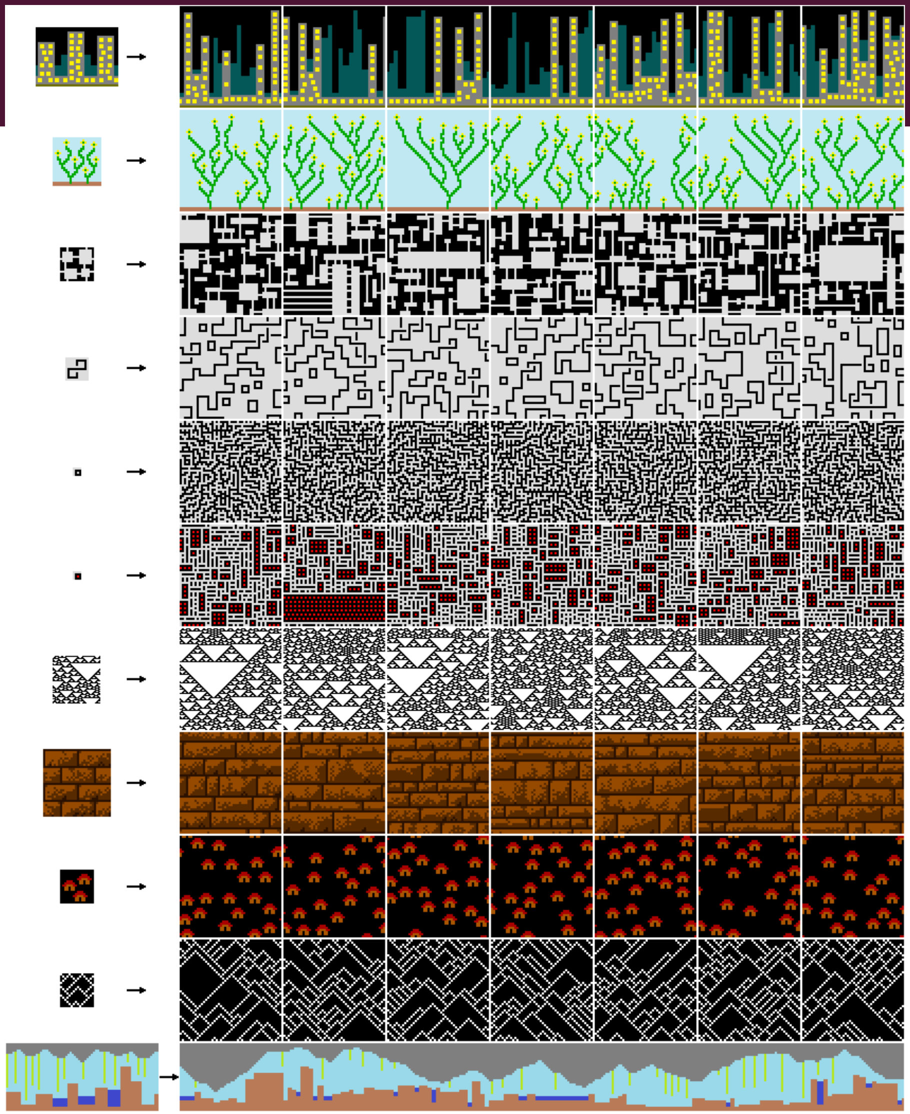
SOLVING SUDOKU
BACKTRACKING + CONSTRAINT PROPAGATION

```
Procedure SolveConstraintPropagation()
    Repeat
        For each empty square
            Determine the list of possible numbers
            If the list is empty
                Return false // unsolvable
            Else if the list has a single element
                Fill in the number
            End If
        End For
    Until no changes are made
    If the puzzle is solved
        Return true
    Else
        Return SolveBacktracking(first empty square)
End Procedure
```

```
Procedure SolveBacktracking(square)
    For n = 1, ..., 9
        If square can have value n
            Save the state of the board
            Set value of square to n
            If SolveConstraintPropagation()
                Return true
            End If
            Restore the saved state of the board
        End If
    End For
    Clear the value of square
    Return false
End Procedure
```

WAVE FUNCTION COLLAPSE

WAVE FUNCTION COLLAPSE



- Procedural Content Generation algorithm
- Invented by Maxim Gumin in 2016
- Named after a concept from quantum mechanics

WAVE FUNCTION COLLAPSE
THE IDEA

- Space is divided into cells, each of which has a state
  - E.g. pixels with a colour; voxels with a 3D mesh from a modular kit
- Constraints on what cell states can neighbour each other are extracted from examples
  - A basic form of machine learning
  - Constraints are probabilistic – not just what is allowed next to a given cell, but what is more or less likely
- Each cell has a probability distribution over states
- A cell is chosen, its state is set according to its probabilities, neighbouring cells are updated according to constraints
- The algorithm backtracks if it gets stuck

CHOOSING THE NEXT CELL

Named after Claude Shannon (1916-2001), the "father of Information Theory"

- A cell is chosen which has minimal Shannon entropy

$$S = -\sum_i P_i \log P_i$$

- Lower Shannon entropy = less uncertainty about the possible value of the cell

WAVE FUNCTION COLLAPSE RESOURCES

- https://github.com/mxgmn/WaveFunctionCollapse
- http://www.procjam.com/tutorials/wfc/
- http://oskarstalberg.com/game/wave/wave.html
- https://twitter.com/exutumno