12: FUZZY LOGIC, PROBABILITY AND INFERENCE

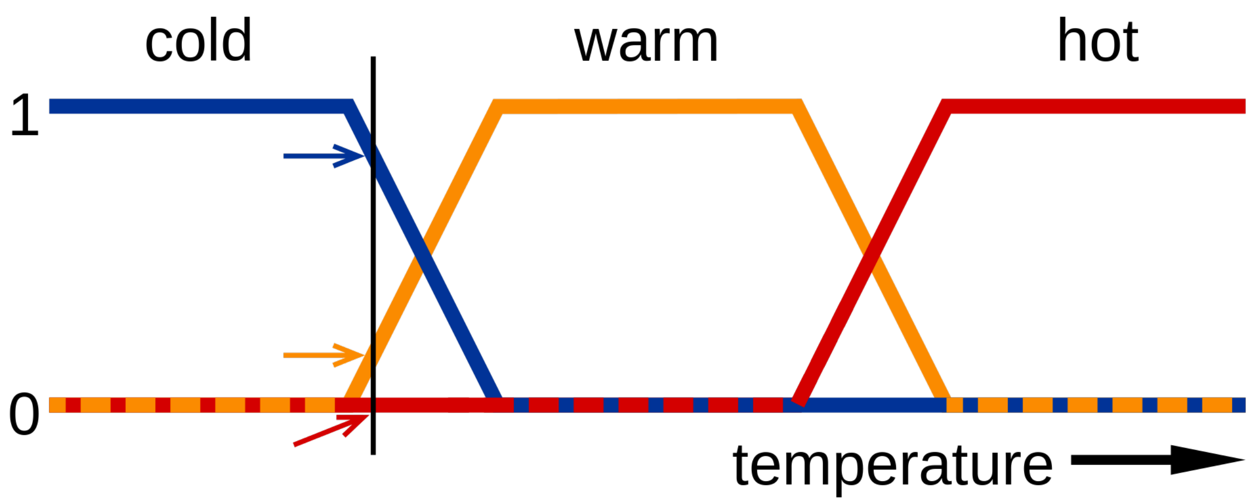
COMP702: CLASSICAL ARTIFICIAL INTELLIGENCE

FUZZY LOGIC

- **Boolean** logic: variables are either **true** or **false**
- Humans don't tend to reason with hard Boolean logic – we use information that is **vague**, **imprecise** and **uncertain**
- **Fuzzy logic** was introduced in 1960s to model this
- Variables have a **truth value** between **0 and 1**
- Related to **fuzzy set theory**: instead of elements being either **in or not in** a set, they have a **degree of membership** between 0 and 1

FUZZY LOGIC EXAMPLE

- "It is cold", "it is warm", "it is hot" are fuzzy statements



FUZZY LOGICAL OPERATORS

- A AND B → $\min(A, B)$
- A OR B → $\max(A, B)$
- NOT(A) → $1 - A$

PROBABILITY

- Similar to fuzzy logic: values between 0 and 1
- Fuzzy logic models **vagueness**
- Probability models **likelihood**

PROBABILITY "LOGIC"

- $P(A\ AND\ B) = P(A) \times P(B)$
- $P(A\ OR\ B) = P(A) + P(B) - P(A\ AND\ B)$
- $P(NOT\ A) = 1 - P(A)$

BELIEFS

- Recall: in a game of **imperfect information**, the **state** of the world is not known
- The **information set** is the set of all **possible states**
- Some states are **more likely** than others
- The agent's **beliefs** can be modelled as a **probability distribution** over the information set
  - A function assigning a probability to each state
  - Probabilities add up to 1

INFERENCE

- The agent's beliefs may **change** based on **observing** the game
- In particular, based on what **actions other players** choose
  - E.g. in Poker, inferring what cards other players are likely to have based on their choices of bids

BAYESIAN INFERENCE

- Given:
  - **Prior distribution**: beliefs about what state we were in before we saw action a
  - **Opponent model**: beliefs about how likely an agent is to choose a given action in a given state
- Can calculate:
  - **Posterior distribution**: beliefs about what state we are in now that we have seen action a

BAYESIAN INFERENCE

Posterior distribution:
Probability that we are in state s, given that we saw action a

Opponent model:
Probability that agent would choose action a if it could see state s

Prior distribution:
Probability that we were in state s

$$P(s|a) = \frac{P(a|s)P(s)}{P(a)}$$

Normalising term to make all probabilities add to 1

$$P(a) = \sum_{s'} P(a|s')P(s')$$

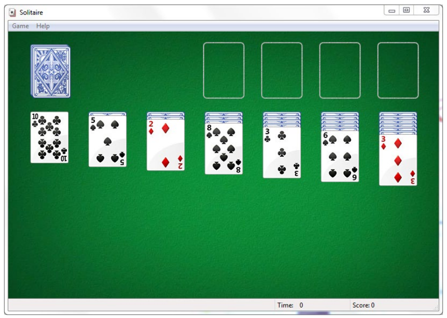
BLUFFING

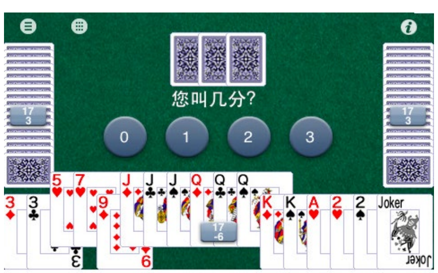
- Bayesian inference relies on an **opponent model**
- Assumption of **rationality**
- However, if a rational agent knows that they are being observed and inferred upon, this might influence their behaviour…
- **Bluffing** refers to any behaviour deliberately meant to fool inference

Emergent bluffing and inference with Monte Carlo Tree Search

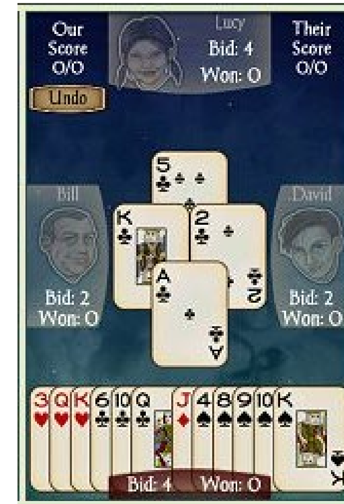
Peter Cowling, Daniel Whitehouse, Edward J. Powley. Proceedings of IEEE Conference on Computational Intelligence in Games, 2015.

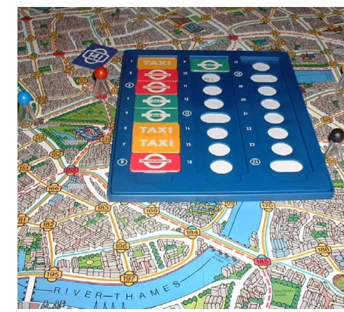
Motivation





- Many interesting games have **imperfect information**
- **MCTS** has successfully been applied to several such games
- MCTS is **robust** in the face of uncertainty, but bad at **information gathering and/or hiding**



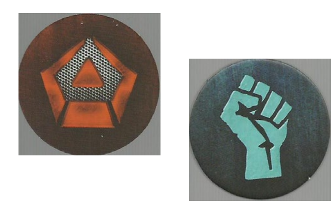


The Resistance

- Every player is either a **resistance** fighter or an imperial **spy**
- Spies know who the other spies are, resistance don't
- Resistance need to **infer** who the spies are, and spies need to **bluff** to appear un-spy-like
  - Resistance don't need to bluff, and spies don't need to infer

The Resistance

- Someone chooses a **team**
- Everyone **votes** (publicly, simultaneously)
- If a majority approves, the team goes on a **mission**
  - Spies can choose to **sabotage** (secretly)
- If **anyone sabotaged**, score 1 for the spies
- If not, score 1 for the resistance
- First team to score 3 wins



Information sets

- Observation gives a **set of possible states**, one of which is the actual state of the game

Actual state:

Observation:

Information set:

Information sets

- Spies have **perfect information**, so the information set is a **singleton**

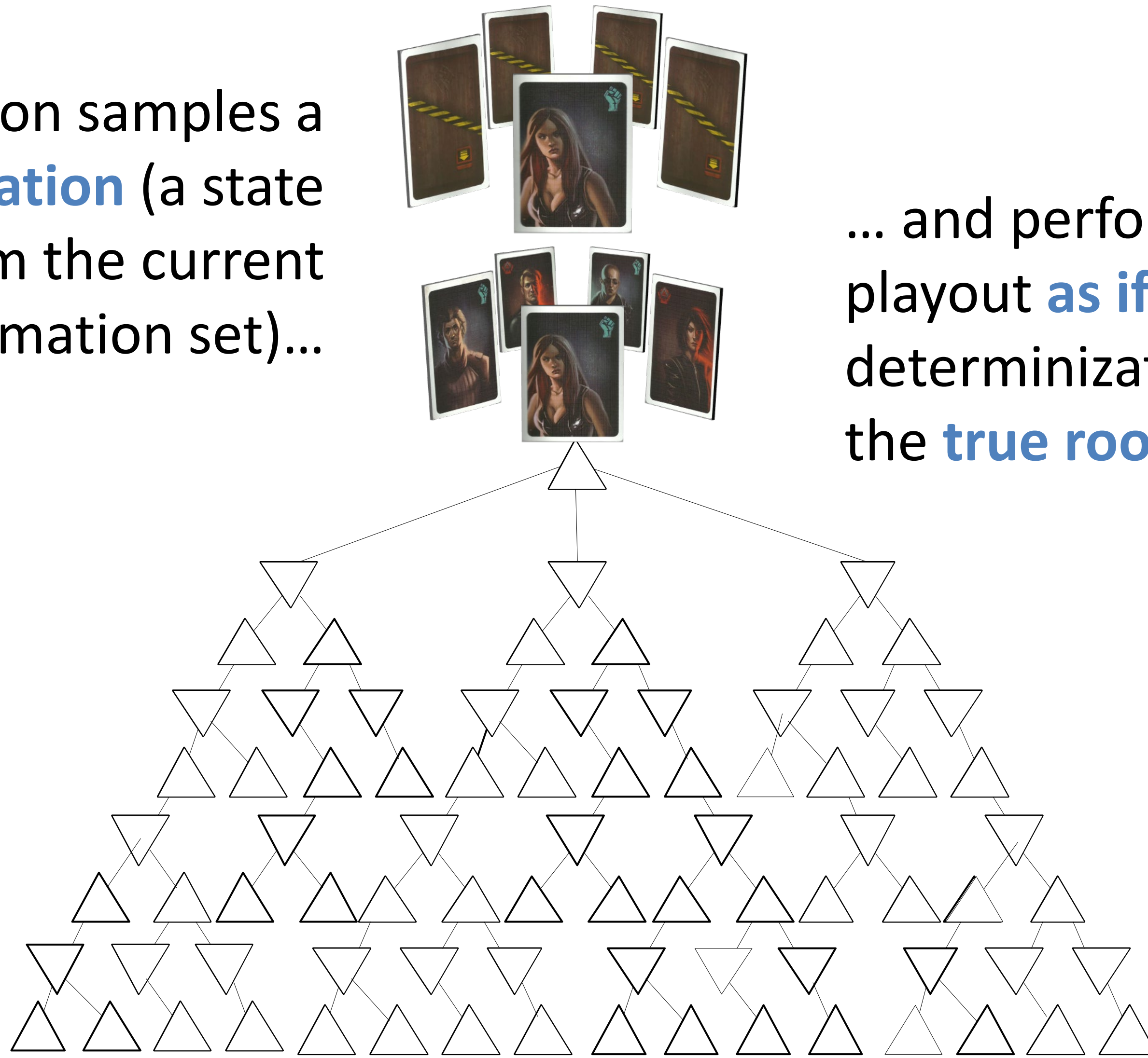
Information set:

Actual state:

Observation:

Information Set MCTS (ISMCTS)

Each iteration samples a **determinization** (a state from the current information set)…

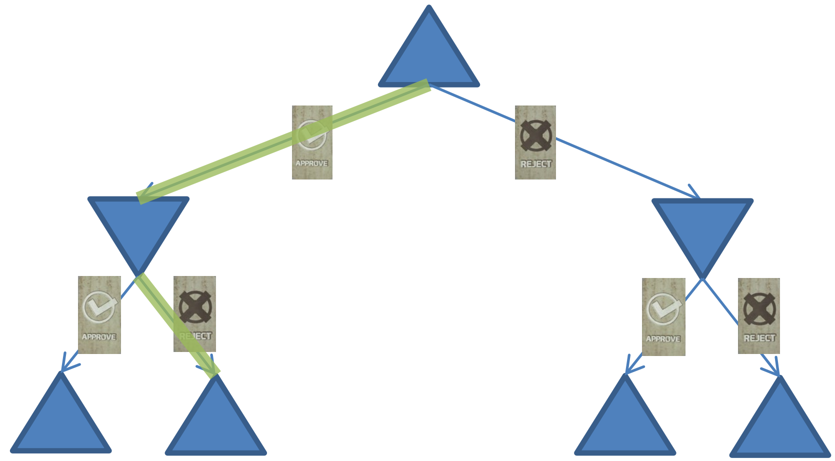


… and performs the playout **as if** that determinization was the **true root state**

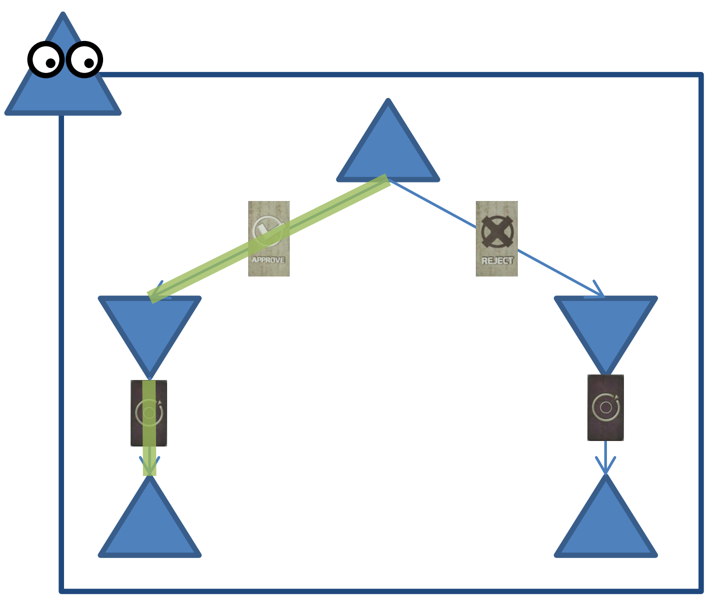
P. I. Cowling, E. J. Powley, D. Whitehouse. *Information Set Monte Carlo Tree Search*. IEEE Transactions on Computational Intelligence and AI in Games, 4(2):120-143, 2012.

Multiple trees in ISMCTS

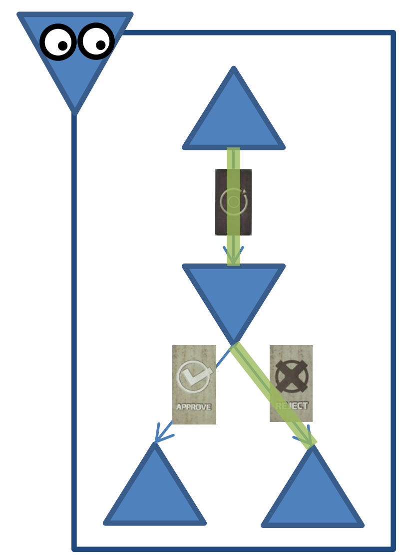
This tree doesn't capture the notion of **hidden moves**



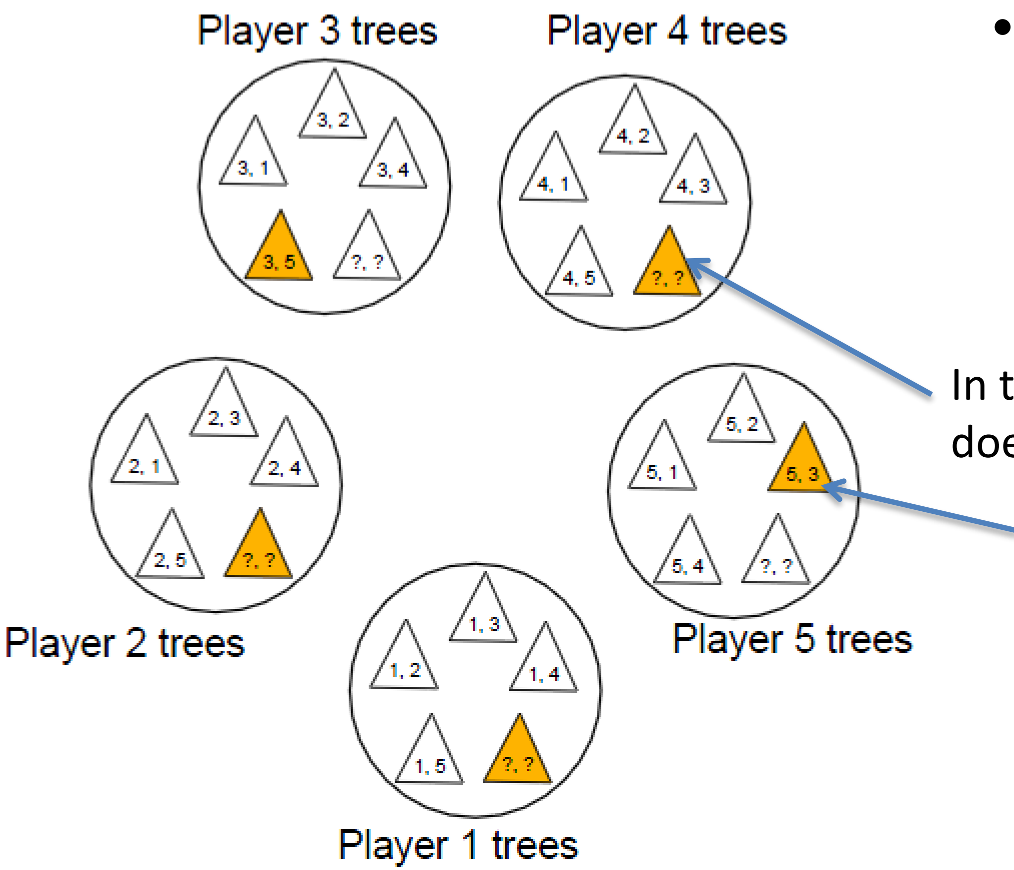
We need to consider **separate trees** from **each player's point of view**



Each **playout** descends **all trees in parallel** to keep track of the players' **differing viewpoints**



Multiple trees in ISMCTS



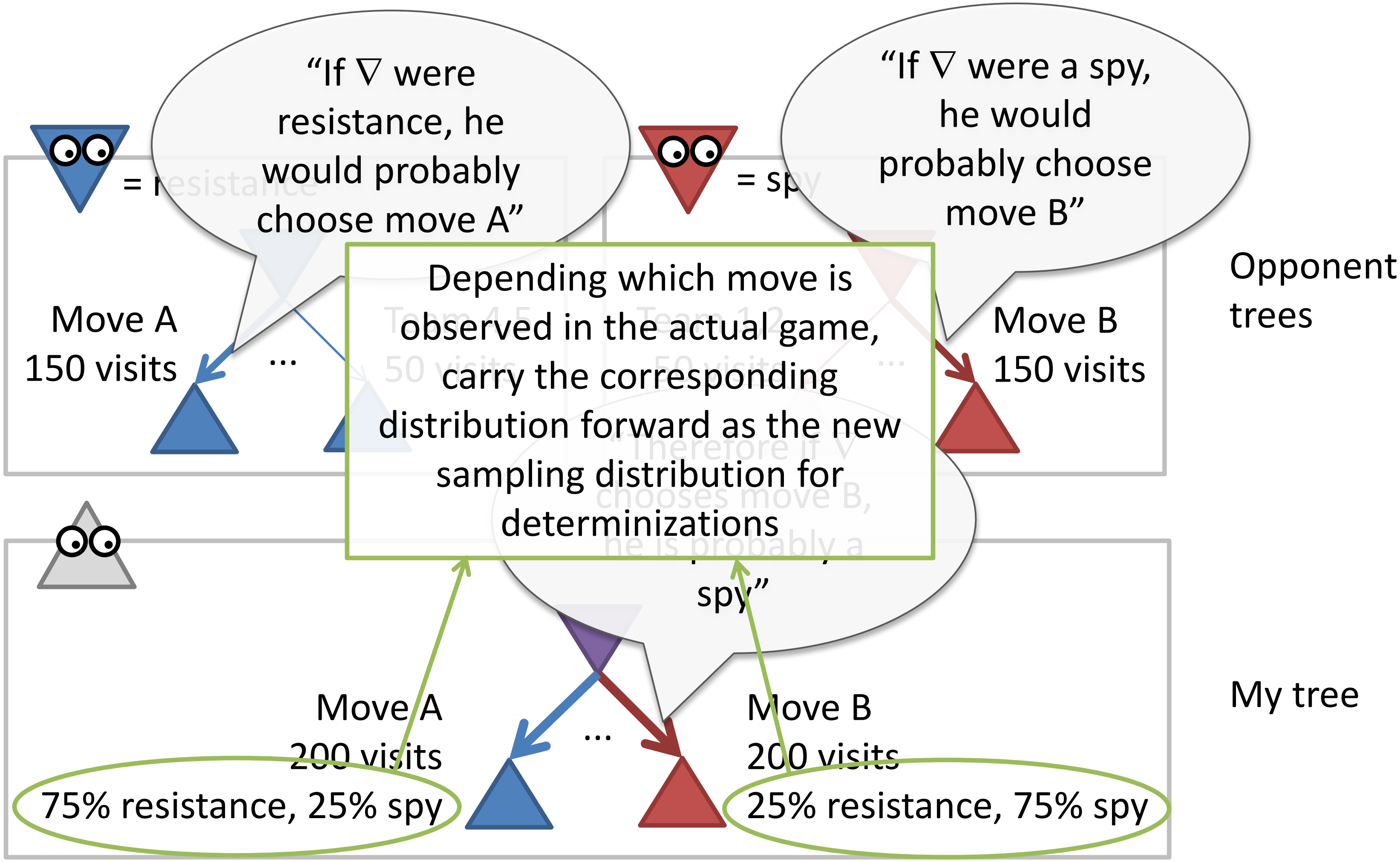
Determinization: "players 3 and 5 are spies"

- Each player has **multiple trees**, to model their observations of **different determinizations**

Particle filter inference

- Some **determinizations** reach some **parts of the tree** more than others
- By keeping track of this, hidden information can be **inferred** from **observed actions**
- Sampled frequencies are used to **update the sampling distribution** for determinizations for **future decisions**

Particle filter inference



Particle filter inference

- Bayesian inference usually requires an **opponent model**
- MCTS **implicitly generates** an opponent model
- Particle filter inference **uses this model** to perform inference
- Particle filter inference **estimates** $P(s|a)$ **from the MCTS playouts** – no need to go through Bayes’ rule

$$\xcancel{\overbrace{P(s|a)}^{\text{Posterior distribution}} = \frac{\overbrace{P(a|s)}^{\text{Opponent model}}\,\overbrace{P(s)}^{\text{Prior distribution}}}{\sum_{s'} \underbrace{P(a|s')}_{\text{Opponent model}}}}$$

Results

- 5 ISMCTS players, each using 20000 iterations (approx 0.2 seconds) per decision
- **Without inference**, resistance win **28.3%** of games
- When resistance use **particle filter inference**, resistance win **94.7%** of games
- With vanilla MCTS, both sides are stupid
  - Spies are blatant, but resistance don't notice
- Inference massively changes the balance of the game – spies can't afford to be blatant any more

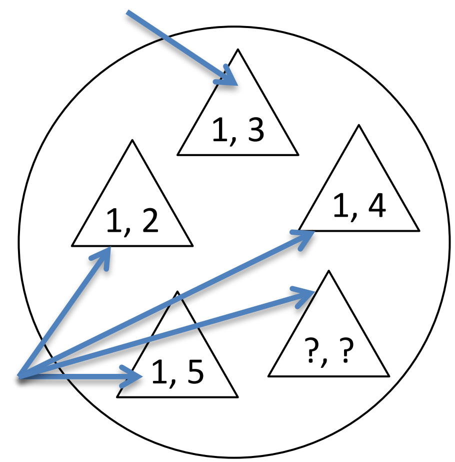
Bluffing

- Suppose I am a spy…
- To accurately **model what other players are thinking**, I need to bear in mind that **they don't know** I'm a spy
- Basic ISMCTS samples determinizations from the **current information set**
- If I know I'm a spy, then **100%** of the determinizations I sample have me as a spy
- Essentially I am assuming that **everyone knows I'm a spy**!
- To solve this, ISMCTS needs to **self-determinize**
  - Sample **determinizations where I am not a spy**, and allow those determinizations to **influence the opponent trees**

Bluffing

- Multi-tree ISMCTS can already handle self-determinization **without polluting** the tree used for the final decision

Tree corresponding to actual situation – used for final decision

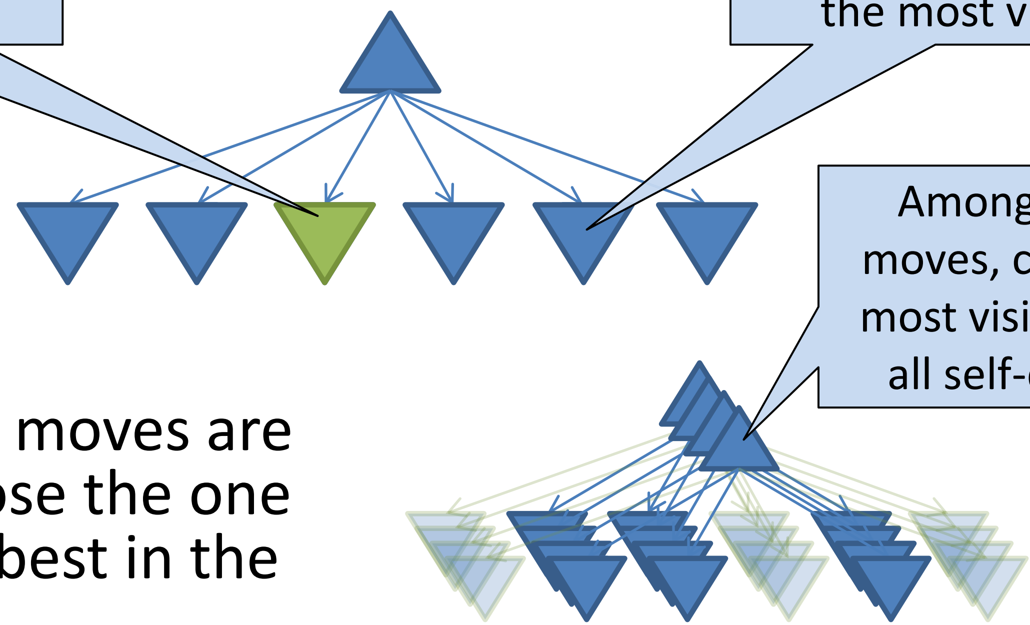


Trees corresponding to other situations – used to model "what I would do if..."

Bluffing

- Every ISMCTS iteration that self-determinizes is an iteration **not spent considering the true world state**
  - Only a fraction of MCTS iterations contribute to the final visit counts used to select a move; the rest are spent refining the opponent model
- We also tried **turning off** self-determinizations **partway through** the search
  - Increases the fraction of iterations used for move selection

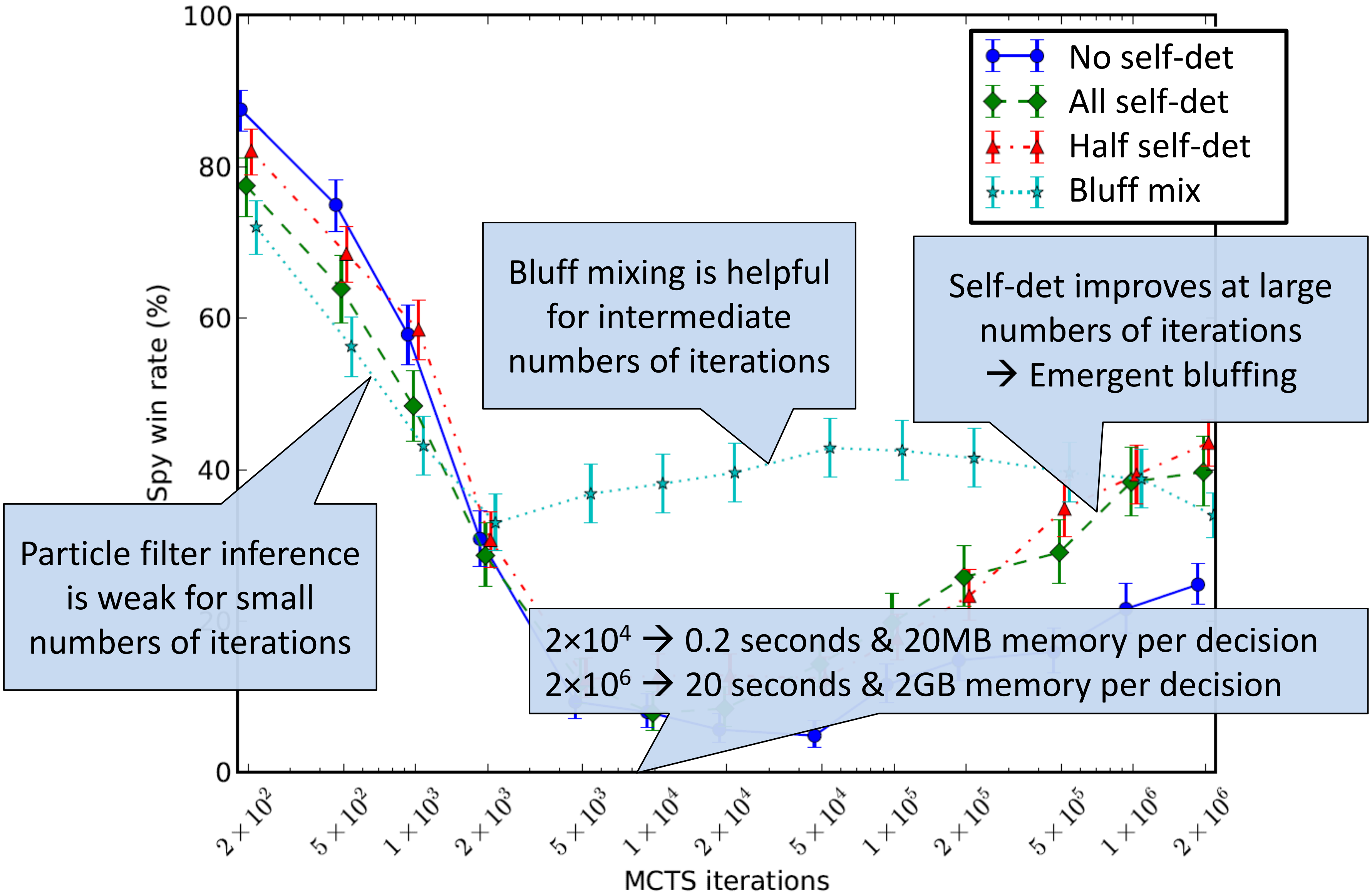
Bluff mixing



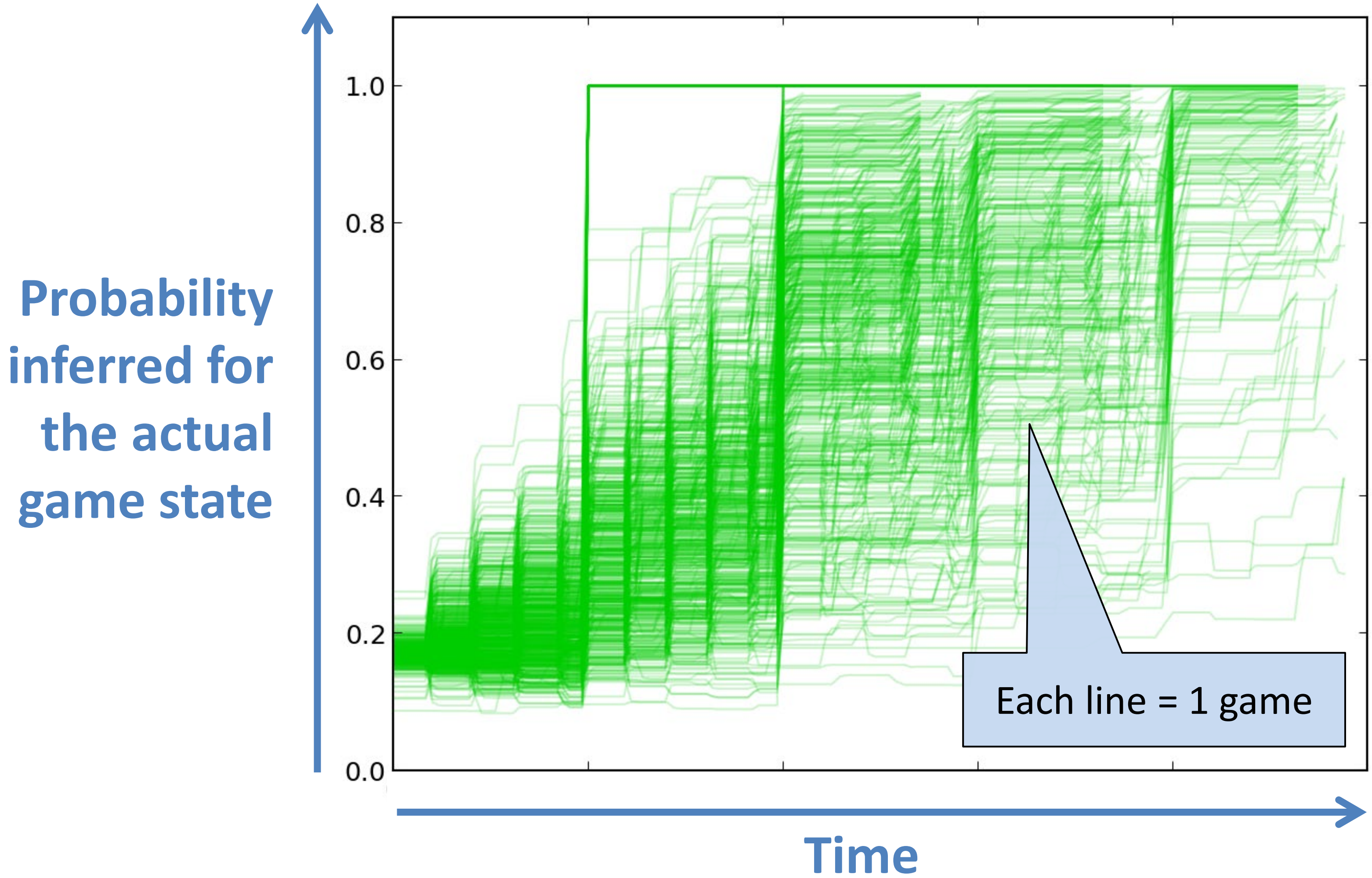
- If several good moves are available, choose the one that looks the best in the **average case**
- However, **don't play a bad move** for the sake of bluffing: if there is only one good move then play it

Results

5 ISMCTS players, all use specified iterations per decision
Resistance use particle filter inference



Explanation of the pictures you're about to see...

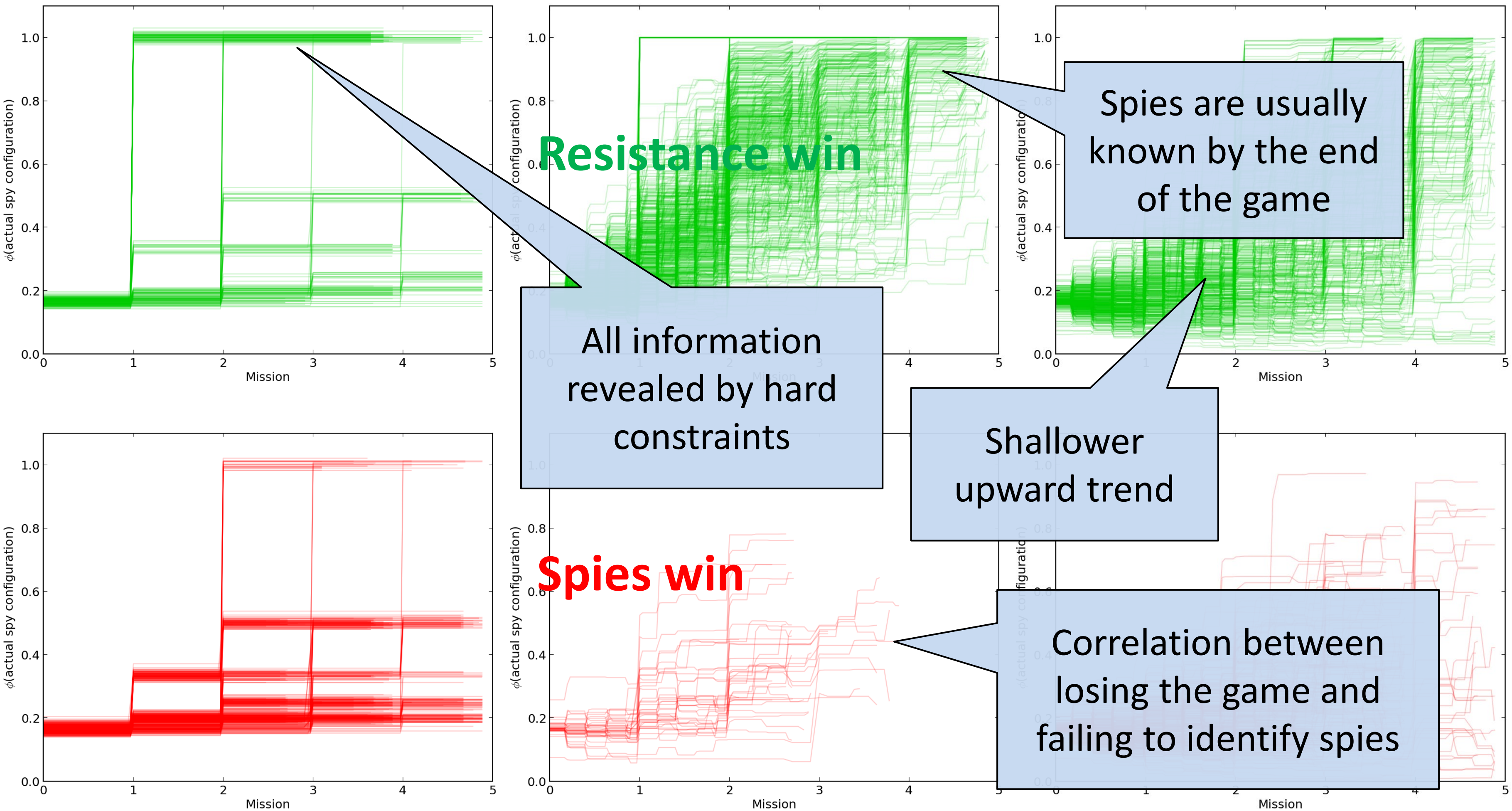


Results

Resistance: no inference
Spies: no self-det

Resistance: PF inference
Spies: no self-det

Resistance: PF inference
Spies: bluff mixing



Other games?

- Information sets in The Resistance are **small enough to enumerate** (≤6 states)
- This is not true of many other games
  - E.g. in Contract Bridge, information set size is $^{39}C_{13} \times {}^{26}C_{13} \approx 8.4 \times 10^{16}$
- Extending to other games will require more work
  - State **abstraction** and **bucketing**
  - **Particle reinvigoration**

Conclusion

- ISMCTS shows great promise for games of **imperfect information**
- ISMCTS can perform **inference** almost for free, reusing the opponent model it already generates
- **Bluffing** can emerge from ISMCTS, or can be introduced artificially for smaller CPU budgets