COMP702: Classical Artificial Intelligence

4: Theoretical models for games

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Game theory

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- A **game** is a system where one or more **players** choose **actions**; the combination of these choices lead to each agent receiving a **payoff**

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- A **game** is a system where one or more **players** choose **actions**; the combination of these choices lead to each agent receiving a **payoff**
- Important applications in economics, ecology and social sciences as well as AI

The ~~Prisoner's~~ Student's Dilemma

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- If **both stay silent**, both receive a C grade
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- If **Bob betrays Alice**, he receives an A whilst she gets expelled
- If **both betray each other**, both get an F

Payoff matrix

Payoff matrix

	A silent	A betray
B silent	A: 50 B: 50	A: 70 B: -100
B betray	A: -100 B: 70	A: 0 B: 0

Alice's thought process

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... and Bob's thought process is the same!

Nash equilibrium

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- Such a situation is called a **Nash equilibrium**
- If all players are **rational** (in the sense of wanting to maximising payoff), they should converge upon a Nash equilibrium

Does every game have a Nash equilibrium?

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	A rock	A paper	A scissors
B rock	A: 0 B: 0	A: +1 B: -1	A: -1 B: +1
B paper	A: -1 B: +1	A: 0 B: 0	A: +1 B: -1
B scissors	A: +1 B: -1	A: -1 B: +1	A: 0 B: 0

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Nash equilibrium for Rock-Paper-Scissors

- Committing to any choice of action can be **exploited**
- E.g. if you always choose paper, I choose scissors
- If we try to reason naïvely, we get stuck in a loop
  - If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...

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- The optimum strategy is to be **unpredictable**

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- E.g. if you always choose paper, I choose scissors
- If we try to reason naïvely, we get stuck in a loop
  - If I choose paper, you'll choose scissors, so I should choose rock, but then you'll choose paper, so I'll choose scissors, so you'll choose rock, so I choose paper...
- The optimum strategy is to be **unpredictable**
- Choose rock with probability $\frac{1}{3}$, paper with probability $\frac{1}{3}$, scissors with probability $\frac{1}{3}$

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- In contrast to a **pure** or **deterministic strategy**, which always chooses the same action
- If we allow mixed strategies, **every game has at least one Nash equilibrium**

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- Example:
  - If the guesses are 30, 40 and 80...
  - ... then the mean is $\frac{30+40+80}{3} = 50$...
  - ... so the winning guess is 30, as this is closest to $\frac{2}{3} \times 50 = 33.333$

The rational guess

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- So no rational player would guess greater than 29.629
- ... and so on ad infinitum
- So the only **rational** guess is 0, as every rational player should guess 0 and $\frac{2}{3}$ of 0 is 0

Rationality

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- Rationality is a useful assumption for mathematics and AI programmers
- However it's important to remember that **humans aren't always rational**

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Markov decision processes and games

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  - A finite set A of **actions**;
  - $P(s, a, s')$ is the **probability** that action a in state s leads to state s';
  - $R(s, a, s')$ is the **reward** received from performing action a in state s and ending up in state s'.

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- Given a state s and an action a, the next state s' is determined by $P(s, a, s')$
- The previous states before s have no effect
- Hence an MDP is "memoryless"
- (Or rather, any memory has to be contained within the state)

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$$P(s, a, s') = \begin{cases} 1 & \text{for some } s' \\ 0 & \text{for all other } s' \end{cases}$$

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$$P(s, a, s') = \begin{cases} 1 & \text{for some } s' \\ 0 & \text{for all other } s' \end{cases}$$

  then the MDP is **deterministic**
- In the deterministic case, the same state s and action a always leads to the same state s'

Markov decision problems

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- Goal: find π which maximises the total reward over time

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- This is a **game**!

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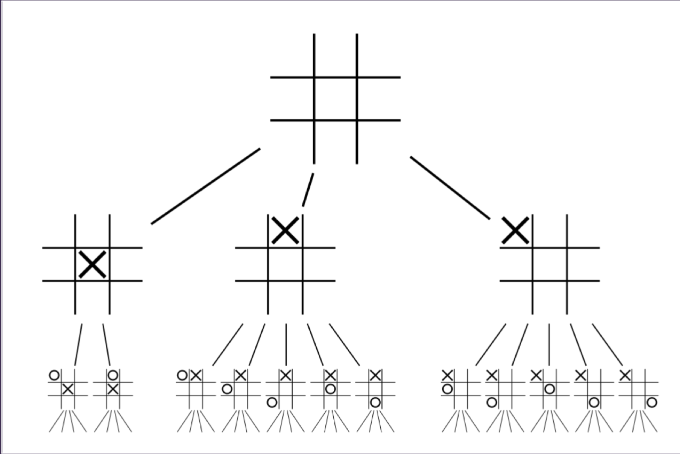
Minimax search

Game trees



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- My opponent wants to maximise their reward, which is the same as **minimise** my reward
- Therefore I want to **maximise** the **minimum** value my opponent can achieve

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Minimax search – example

Minimax search pseudocode

procedure MINIMAX(state, currentPlayer)

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procedure MINIMAX(state, currentPlayer)
    if state is terminal then
        return value of state
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procedure MINIMAX(state, currentPlayer)
    if state is terminal then
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    else if currentPlayer = 1 then
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Minimax search pseudocode

procedure MINIMAX(state, currentPlayer)
    if state is terminal **then**
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    else if currentPlayer $= 1$ **then**
        bestValue $= -\infty$

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        for each possible nextState do
```

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    if state is terminal **then**
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        for each possible nextState **do**
            v = MINIMAX(nextState, 3− currentPlayer)

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    else if currentPlayer = 2 then
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            bestValue = MAX(bestValue, v)
        return bestValue
    else if currentPlayer = 2 then
        bestValue = +∞
        for each possible nextState do
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            bestValue = MIN(bestValue, v)
        return bestValue
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    else if currentPlayer = 2 then
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        for each possible nextState do
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            bestValue = MIN(bestValue, v)
        return bestValue
```

Stopping early

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- Similarly when minimising if bestValue $= -1$
- There are techniques for smarter early stopping, e.g. alpha-beta pruning

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- Choose the move with the maximum score
- If there are several with the same score, choose one at random

Minimax and game theory

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Minimax and game theory

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- Minimax search will always find a **Nash equilibrium**
- I.e. a minimax player plays **perfectly**
- **But...**

Minimax for larger games

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  - Connect 4 has $\approx 10^{13}$ states
  - Chess has $\approx 10^{47}$ states
- Generally need to **cut off** the tree at a certain depth and use **heuristics**
- Basically, define our own reward function that (hopefully) approximates the real one

Application to games

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- State = state of the board
- Action = a move
- Reward = win or loss, score, etc.

Application to video games

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  - State = entire state of the game (memory?)

Application to video games

- *Technically* a video game is an MDP
  - State = entire state of the game (memory?)
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- Generally necessary to **abstract** parts out of the game to treat as MDPs / game theory games