COMP110: Principles of Computing

5: Computational Complexity

Learning outcomes

- **Explain** the notion of computability
- **Use** "big O" notation to express computational complexity
- **Apply** appropriate algorithms to achieve efficiency

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Computability

Computability theory

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  - NB: A may be **infinite**
- A function $f : A \to B$ is **computable** if there exists a Turing machine which computes f
  - I.e. given an encoding of $a \in A$ as input, the Turing machine outputs an encoding of $f(a)$

An uncomputable function

The **halting problem**

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- There is **no** Turing machine that computes f
- f is **uncomputable**

Church-Turing Thesis

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- If a function is **effectively calculable**, then it is **computable** by a Turing machine
- Effectively calculable = there is a method or algorithm for computing it
- So in terms of computability, Turing machines are as powerful as computers can be

Halting revisited

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Halting revisited

- Write a software tool that, given a Python program, predicts whether that program can go into an infinite loop
- Your tool must work for **all** Python programs
- Is this possible?

Computation time

Resources

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- All programs use **resources**

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  - Time

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Resources

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- Often **time** is the resource we care about the most
  - Particularly in games: want to maintain a good **frame rate** free of **lag** or **stuttering**

Basic time measurement in Python

```
import time

start_time = time.clock()

... do something here ...

end_time = time.clock()
print("Computation took", end_time - start_time, " ↩
    seconds")
```

Repeating for better accuracy

```
import time

start_time = time.clock()

repetition_count = 1000

for repetition in range(repetition_count):
    ... do something here ...

end_time = time.clock()
time_per = (end_time - start_time) / repetition_count
print("Computation took", time_per, "seconds")
```

Scaling

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- We are often less interested in how many milliseconds a particular computation takes on today's hardware, and more interested in how the execution time **scales** with the problem size

Search

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- We want to find one of them

Linear search

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Socrative room code: `FALCOMPED`

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- In the **best case**, how many items do we need to visit before finding the one we want?

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- Suppose there are 25 items in the list
- In the **best case**, how many items do we need to visit before finding the one we want?
- How about in the **worst case**?

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- If there are 25 items in the list, the **worst case** number of items visited is 25

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- How about if there are 50 items?
- How about 100 items?

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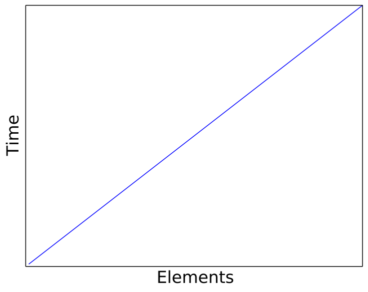
- If there are 25 items in the list, the **worst case** number of items visited is 25
- How about if there are 50 items?
- How about 100 items?
- If the number of items **doubles**, what happens to the amount of time the search takes?

Linear time



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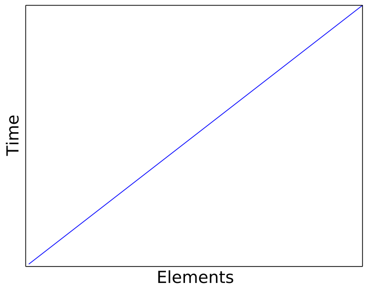

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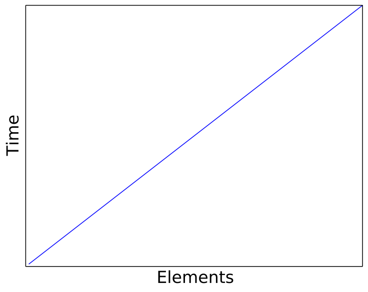

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- Also written as $O(n)$ **time complexity**

Searching a sorted list

- If the list is **sorted** in alphabetical order, we can do better than linear...

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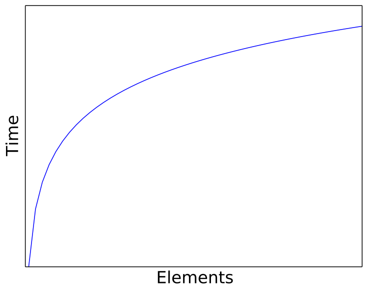

Hidden complexity

if name $<$ mid.name **then**
    return FIND(name, first half of list)
else if name $>$ mid.name **then**
    return FIND(name, second half of list)
end if

Hidden complexity

```
if name < mid.name then
    return FIND(name, first half of list)
else if name > mid.name then
    return FIND(name, second half of list)
end if
```

- Careful how you implement this!

Hidden complexity

if name $<$ mid.name **then**
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- Careful how you implement this!
- **Copying** (half of) a list is **linear** $O(n)$

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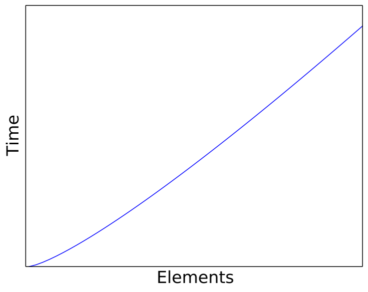

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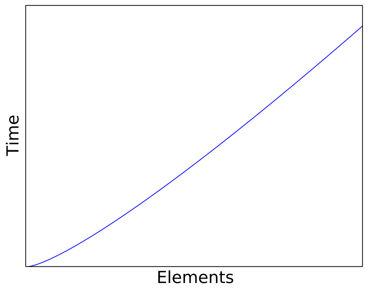

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- The actual running time would be $O(n \log n)$
- Use **pointers** into the list instead of copying

Binary search done wrong

```
def binary_search(name, mylist):
    if mylist == []:
        raise ValueError("Not found")

    mid = len(mylist) / 2
    mid_name = mylist[mid_index].name

    if name == mid_name:
        return mid
    elif name < mid_name:
        return binary_search(name, mylist[:mid])
    else:
        return binary_search(name, mylist[mid+1:])
```

Binary search done right

```
def binary_search(name, mylist, start, end):
    if end <= start:
        raise ValueError("Not found")

    mid = (start + end) / 2
    mid_name = mylist[mid].name

    if name == mid_name:
        return mylist[mid]
    elif name < mid_name:
        return binary_search(name, mylist, start, mid)
    else:
        return binary_search(name, mylist, mid+1, end)
```

Binary search vs linear search

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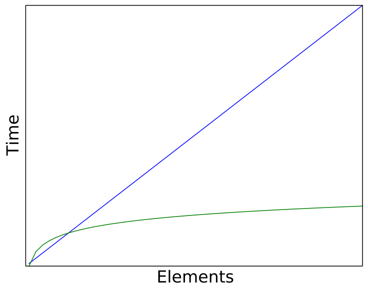

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Binary search vs linear search

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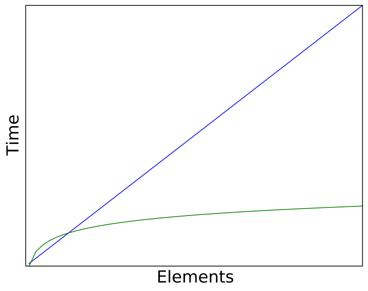

- So binary search is better than linear search... right?
- Discuss in **pairs**
- On Socrative, post **one reason** why, or **one situation** where, linear search may be a better choice than binary search

Hashing

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⋮	⋮
112	Ward, Jessica
113	Baker, Theresa
114	Collins, Jane
115	—
116	—
117	Hughes, Aaron
118	—
119	—
120	—
121	—
122	Brown, Janet
123	—
124	—
125	Gonzalez, Adam Lewis, Rose
126	—
127	—
128	—
129	—
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Hash look-up

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142	Clark, Stephanie
144	Scott, Michelle
145	Miller, Jeremy
147	Davis, Marilyn
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151	Anderson, Martha
158	Williams, Billy
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$$12 + 15 + 16 + 5 + 26 + 10 + 5 + 6 + 6 + 18 + 5 + 25 = 149$$

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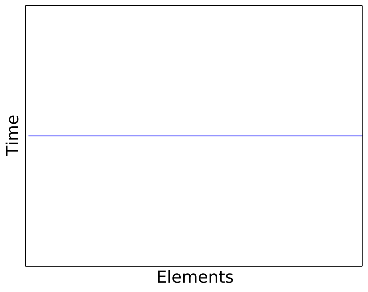

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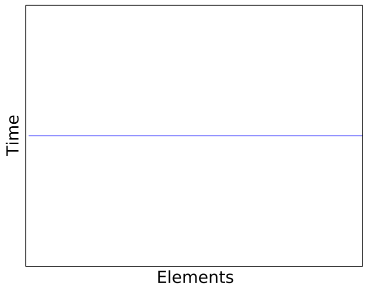

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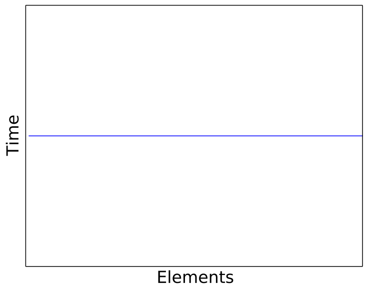

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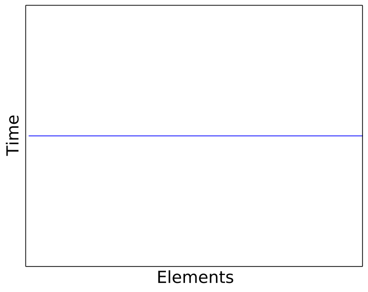

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- When there are collisions, need to fall back on something like linear or binary search within each bin

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- Hash tables in Python:
  - The `dict` (dictionary) data structure

More on complexity

Common complexity classes

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$\downarrow$	Exponential	$O(e^n)$

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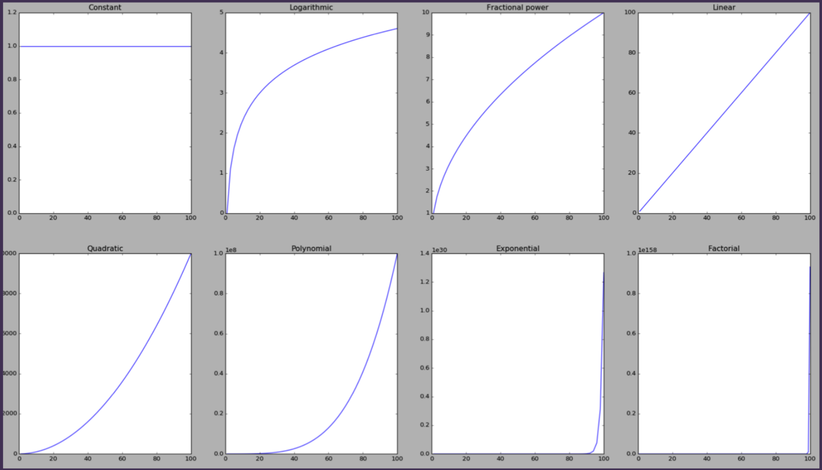
"Slower"	Factorial	$O(n!)$

Common complexity classes



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  - If an algorithm does n "things" and each "thing" is $O(n)$, then the overall algorithm is $O(n^2)$

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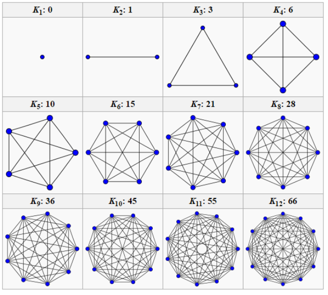

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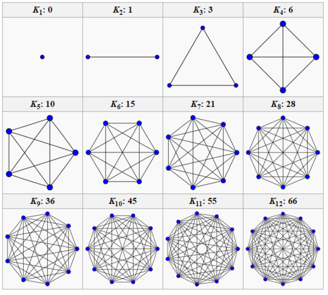

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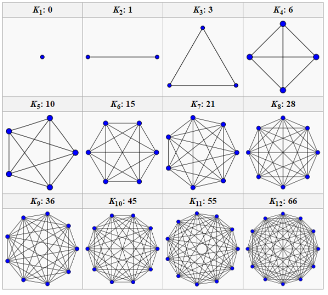

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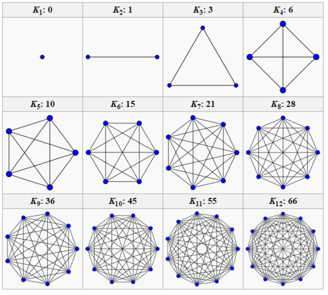

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  - Further reading: spatial hashing, quadtrees, octrees, Verlet lists

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- Adding 1 to n potentially **doubles** the running time!

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- Are there any problems in NP but not in P?

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    - Quantum computers are "infinitely parallel" in a sense so *can* solve some large NP-hard problems

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  - Otherwise, choose simplicity

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- Choice of data structures and algorithms can have a large impact on the efficiency of your software
- ... but only if scalability is actually a factor

Exercise Sheet ii

Due now

Worksheet C

Due in 2 weeks' time
(No class next week)