
COMP110: Principles of Computing

# 8: Data Structures

# Learning outcomes

- **Define** the key concepts of graph theory
- **Distinguish** advanced data structures such as trees, DAGs and graphs
- **Determine** the complexity of accessing and manipulating data in these data structures
- **Choose** the correct data structure for a given task

# Exercise Sheet iii

Due **next week**

# Graphs

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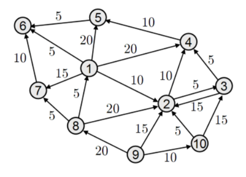


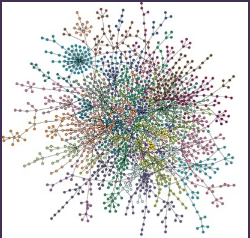

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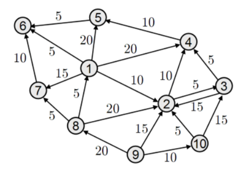


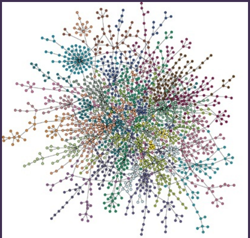

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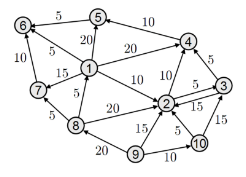

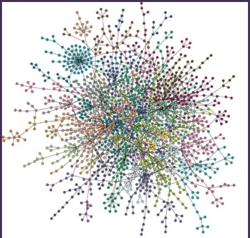

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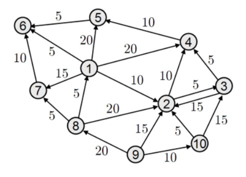


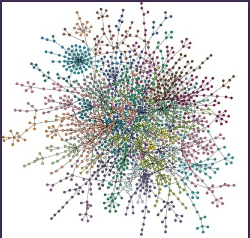

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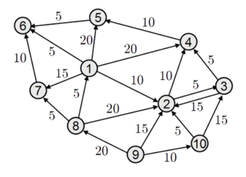


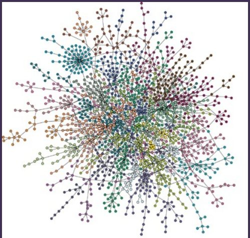

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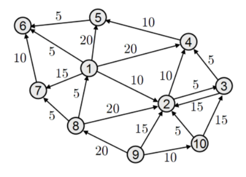


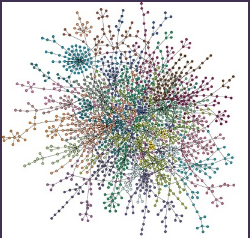

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- **Directed** graph: edges are arrows

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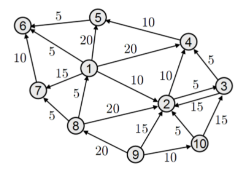


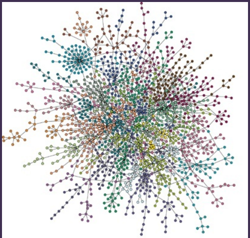

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  - A collection of **edges** or **arcs** (lines or arrows between points)
- Often used to model **networks** (e.g. social networks, transport networks, game levels, automata, ...)
- **Directed** graph: edges are arrows
- **Undirected** graph: edges are lines

# Implementing graphs

# Implementing graphs

- A graph has a **set of nodes** and a **set of edges**

# Implementing graphs

- A graph has a **set of nodes** and a **set of edges**
- Each edge has exactly **two nodes** associated with it (e.g. "from" and "to")

# Drawing graphs

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- A graph does not necessarily specify the physical **positions** of its nodes

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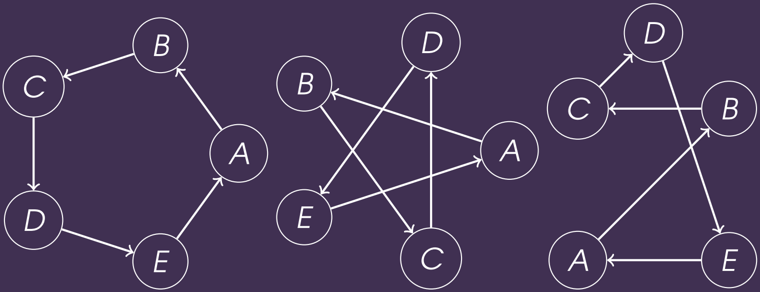
- E.g. these are technically the same graph:



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- A graph is **planar** if it can be drawn with no overlapping edges
- A region enclosed by edges is called a **faces**
- A connected planar graph obeys **Euler's formula**:

$$n_{\text{nodes}} - n_{\text{edges}} + n_{\text{faces}} = 2$$

# Trees

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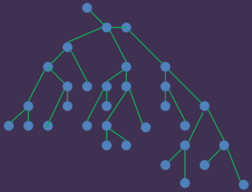

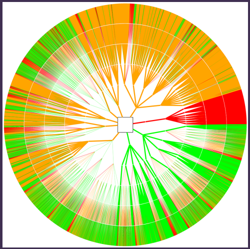

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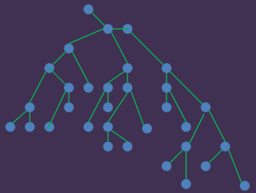

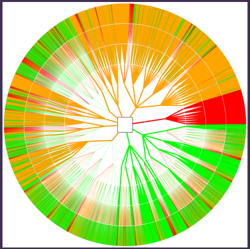

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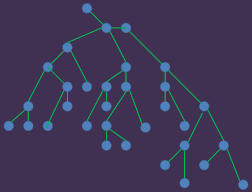

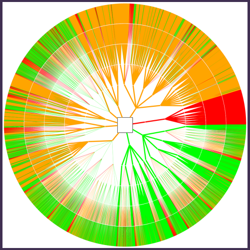

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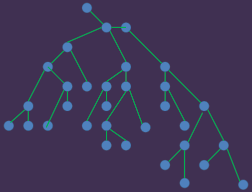

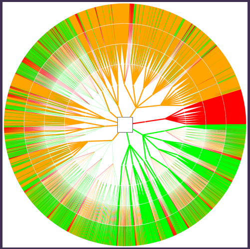

- A **tree** is a special type of directed graph where:
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  - All other nodes have exactly 1 incoming edge

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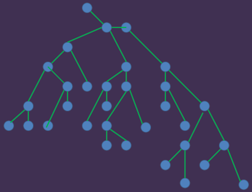

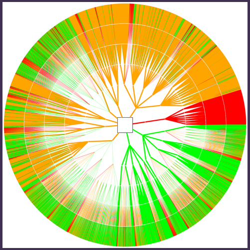

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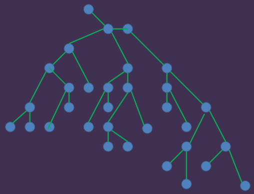

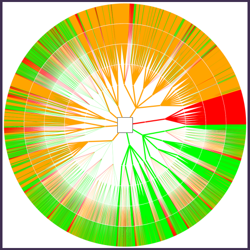

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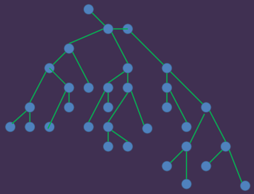

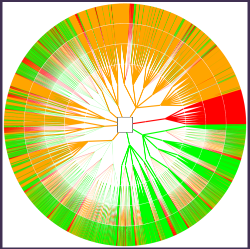

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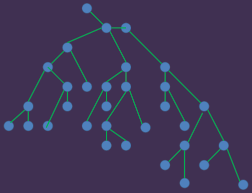

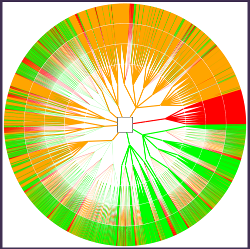

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- Edges go from **parent** to **child**
  - All nodes except the root have exactly one parent
  - Nodes can have 0, 1 or many children
- Used to model **hierarchies** (e.g. file systems, object inheritance, scene graphs, state-action trees, ...)

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- Each node has a **collection of children**
- Each node other than the root has a **single parent**

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- `deque` (from the `collections` module) implements an efficient **double-ended queue**
- Provides methods `append`, `appendleft`, `pop`, `popleft`
  - All of which are $O(1)$

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# Stacks and function calls

- Stacks are used to implement **nested function calls**
- Each invocation of a function has a **stack frame**
- This specifies information like **local variable values** and **return address**
- Calling a function **pushes** a new frame onto the stack
- Returning from a function **pops** the top frame off the stack

# Graph traversal

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  - Depth first
  - Breadth first

# Tree traversal

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**procedure** DEPTHFIRSTSEARCH

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    let $S$ be a stack

# Tree traversal

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    let $S$ be a stack
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# Tree traversal

**procedure** DepthFirstSearch
    let *S* be a stack
    push root node onto *S*
    **while** *S* is not empty **do**
        pop *n* from *S*
        print *n*

# Tree traversal

**procedure** DEPTHFIRSTSEARCH
    let *S* be a stack
    push root node onto *S*
    **while** *S* is not empty **do**
        pop *n* from *S*
        print *n*
        push children of *n* onto *S*

# Tree traversal

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procedure DepthFirstSearch
    let S be a stack
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    while S is not empty do
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        print n
        push children of n onto S
    end while
end procedure
```

# Tree traversal

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    let Q be a queue
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**end procedure**

**procedure** BREADTHFIRSTSEARCH
    let $Q$ be a queue
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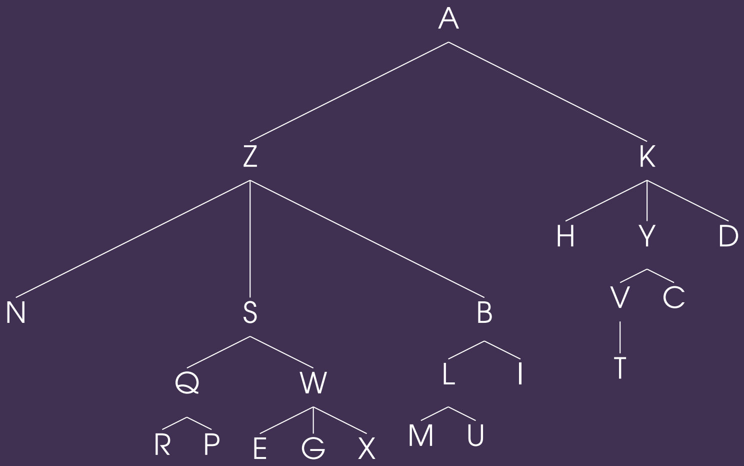
**end procedure**

# Tree traversal example



# Recursive depth first search

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**procedure** DEPTHFIRSTSEARCH($n$)

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**procedure** DEPTHFIRSTSEARCH($n$)
    print $n$

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**procedure** DEPTHFIRSTSEARCH($n$)
    print $n$
    **for each** child $c$ of $n$ **do**

# Recursive depth first search

**procedure** DEPTHFIRSTSEARCH($n$)
    print $n$
    **for each** child $c$ of $n$ **do**
        DEPTHFIRSTSEARCH($c$)

# Recursive depth first search

```
procedure DepthFirstSearch(n)
    print n
    for each child c of n do
        DepthFirstSearch(c)
    end for
end procedure
```

# Recursive depth first search

```
procedure DepthFirstSearch(n)
    print n
    for each child c of n do
        DepthFirstSearch(c)
    end for
end procedure
```

- Compare to the pseudocode on the previous slide. Where is the stack?