12: FUZZY LOGIC AND PROBABILITY

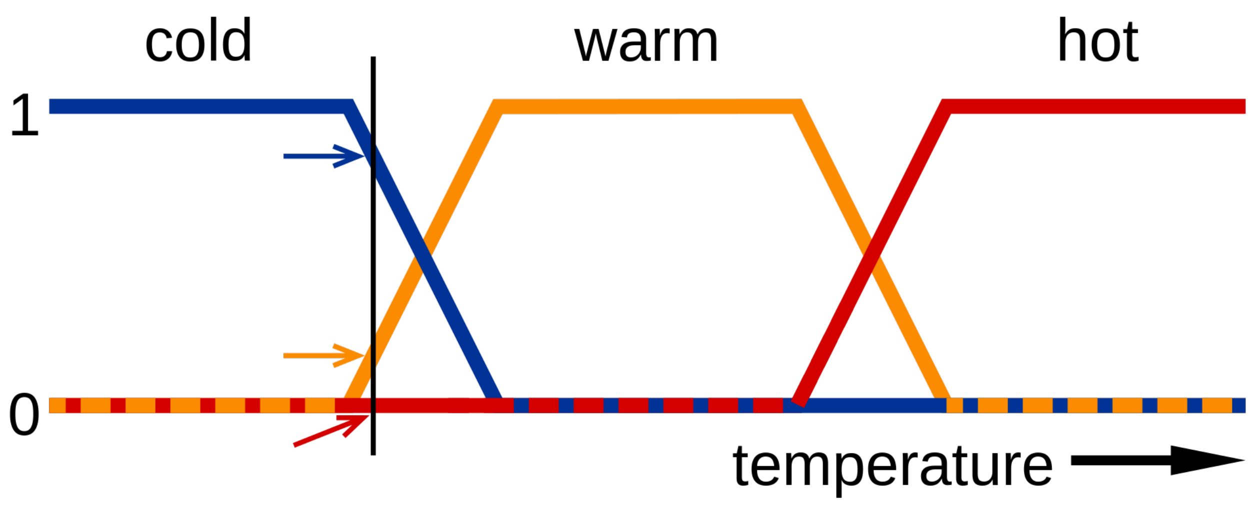
COMP702: CLASSICAL ARTIFICIAL INTELLIGENCE

FUZZY LOGIC

- **Boolean** logic: variables are either **true** or **false**
- Humans don't tend to reason with hard Boolean logic – we use information that is **vague**, **imprecise** and **uncertain**
- **Fuzzy logic** was introduced in 1960s to model this
- Variables have a **truth value** between **0 and 1**
- Related to **fuzzy set theory**: instead of elements being either **in or not in** a set, they have a **degree of membership** between 0 and 1

FUZZY LOGIC EXAMPLE

- "It is cold", "it is warm", "it is hot" are fuzzy statements



FUZZY LOGICAL OPERATORS

- A AND B → $\min(A, B)$
- A OR B → $\max(A, B)$
- NOT(A) → $1 - A$

PROBABILITY

- Similar to fuzzy logic: values between 0 and 1
- Fuzzy logic models **vagueness**
- Probability models **likelihood**

PROBABILITY "LOGIC"

- $P(A\ AND\ B) = P(A) \times P(B)$
- $P(A\ OR\ B) = P(A) + P(B) - P(A\ AND\ B)$
- $P(NOT\ A) = 1 - P(A)$

BELIEFS

- Recall: in a game of **imperfect information**, the **state** of the world is not known
- The **information set** is the set of all **possible states**
- Some states are **more likely** than others
- The agent's **beliefs** can be modelled as a **probability distribution** over the information set
  - A function assigning a probability to each state
  - Probabilities add up to 1

INFERENCE

- The agent's beliefs may **change** based on **observing** the game
- In particular, based on what **actions other players** choose
  - E.g. in Poker, inferring what cards other players are likely to have based on their choices of bids

BAYESIAN INFERENCE

- Given:
  - **Prior distribution**: beliefs about what state we were in before we saw action a
  - **Opponent model**: beliefs about how likely an agent is to choose a given action in a given state
- Can calculate:
  - **Posterior distribution**: beliefs about what state we are in now that we have seen action a

BAYESIAN INFERENCE

Posterior distribution:
Probability that we are in state s, given that we saw action a

Opponent model:
Probability that agent would choose action a if it could see state s

Prior distribution:
Probability that we were in state s

$$P(s|a) = \frac{P(a|s)P(s)}{P(a)}$$

Normalising term to make all probabilities add to 1

$$P(a) = \sum_{s'} P(a|s')P(s')$$

BLUFFING

- Bayesian inference relies on an **opponent model**
- Assumption of **rationality**
- However, if a rational agent knows that they are being observed and inferred upon, this might influence their behaviour…
- **Bluffing** refers to any behaviour deliberately meant to fool inference