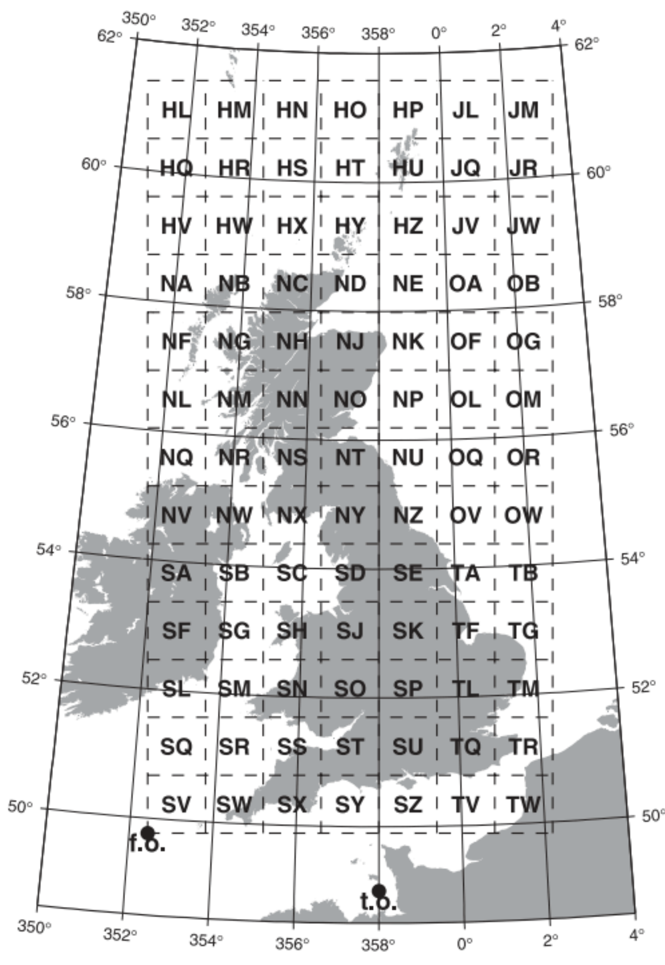
# Convert between Latitude/Longitude & OS National Grid References

Some people have asked me about converting between latitude/longitude & Ordnance Survey grid references. The maths seems extraordinarily complex (and way beyond me!), but the Ordnance Survey explain the resulting formulae very clearly in Annex C of their *Guide to coordinate systems in Great Britain*.

OS Grid References are based on 100km grid squares identified by letter-pairs, followed by digits which identify a sub-square within the grid square, as explained on the OS *Interactive Guide to the National Grid*. 6-digit references identify 100m grid squares; 8 digits identify 10m grid squares, and 10 digits identify 1m squares. TG51401317 represents a 10m box with its (south-west) origin 51.40km across, 13.17km up within the TG square.



OS National Grid, with true origin & false origin

*Functional demo*

*Enter OS grid references or latitude/longitude values into the test boxes to try out the calculations*:

| | | | |
|---|---|---|---|
| OS Grid Ref | TG 51409 13177 | ≡ | 651409,313177 |
| Lat/Lon (WGS84) | 52° 39′ 28.72″ N | | 001° 42′ 57.74″ E |
| (SW corner of grid square) | | | |
| Lat/Lon (OSGB36) | 52° 39′ 27.25″ N | | 001° 43′ 04.47″ E |
| no longer used (since 2014) | | | |

Display calculation results as: [✓ deg/min/sec] decimal degrees

An alternative way of expressing OS Grid References is as all-numeric eastings and northings, usually in metres. As square TG is six squares across, three squares up within the grid, grid reference TG 5140 1317 can also be expressed as 651400,313170.

As the surface of the earth is curved, and maps are flat, mapping involves a *projection* of the (ellipsoidal) curved surface onto a (plane) flat surface. The Ordnance Survey grid is a transverse Mercator projection.

## Latitude/longitudes require a datum

Before I started writing these geodesy scripts, I assumed a latitude/longitude point was a latitude/longitude point, and that was that. Since then, I have discovered that if you're being accurate, things get more complicated, and you need to know what *datum* you are working within.

Historically, cartographers worked at finding the ellipsoid which provided the closest mapping to the local *geoid*. The geoid is equivalent to the local mean-sea-level (or its equivalent in land-locked locales), and is determined by the local force of gravity. These ellipsoids were then fixed to local datums (fixed reference frames established through cartographic surveys). Simplifying enormously, this approach has now been broadly replaced by global geocentric ellipsoids fixed to global & continental datums (or reference frames) which can be readily mapped to each other to account for tectonic plate shifts. The best known global datum is WGS84, which is used by GPS systems; its more accurate siblings are (date-dependant) ITRFs. Continental datums include NAD-83 for North America, ETRS89 for Europe, GDA94 for Australia, etc.

Back in the 1930's, the UK Ordnance Survey defined 'OSGB-36' as the datum for the UK, based on the 'Airy 1830' ellipsoid. In 2014, they deprecated OSGB-36 in favour of WGS-84 for latitude/longitude coordinates, but OSGB-36 is still the basis for OS grid references. The Greenwich Observatory – historically the 'prime meridian' – is around 000°00′05″W on the WGS-84 datum (a difference of a bit over 100 metres): and moving every year. (I find the history of cartography facinating: I will put together a reading list sometime).

So to convert a (WGS84) latitude/longitude point to an OS grid reference, it must first be converted from the WGS84 datum to the OSGB36 datum, then have the transverse Mercator projection applied to transform it from a curved surface to a flat one.

## Datum conversions

To convert between datums, a ‘Helmert transformation’ is used. The Ordnance Survey explain the details in section 6 (and the annexes) of their *Guide to coordinate systems in Great Britain*.

The procedure is:

1. convert polar lat/long/height φ, λ, h to geocentric ‘ECEF’ cartesian co-ordinates x, y, z (using the source ellipsoid parameters *a*, *b*, *f*)
2. apply 7-parameter Helmert transformation; this applies a 3-dimensional shift & rotation, and a scale factor
3. convert cartesian co-ordinates back to polar lat/long/height (using the destination ellipsoid parameters)

**1 convert (geodetic) latitude/longitude to geocentric cartesian coordinates:**

| | |
|---|---|
| $e^2 = (a^2-b^2) / a^2$ | *eccentricity of source ellipsoid (= 2·f − f²)* |
| $\nu = a / \sqrt{(1 - e^2 \cdot \sin^2\varphi)}$ | *transverse radius of curvature* |
| $x = (\nu+h) \cdot \cos\varphi \cdot \cos\lambda$ | |
| $y = (\nu+h) \cdot \cos\varphi \cdot \sin\lambda$ | |
| $z = ((1-e^2)\cdot\nu + h) \cdot \sin\varphi$ | |

**2 apply Helmert transform**

The 7-parameter Helmert transform is given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}' = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \begin{bmatrix} 1+s & -r_z & r_y \\ r_z & 1+s & -r_x \\ -r_y & r_x & 1+s \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Hence:

$$x' = t_x + (1+s)\cdot x - r_z\cdot y + r_y\cdot z$$

$$y' = t_y + r_z\cdot x + (1+s)\cdot y - r_x\cdot z$$

$$z' = t_z - r_y\cdot x + r_x\cdot y + (1+s)\cdot z$$

**3 convert cartesian co-ordinates back to latitude/longitude**

While the conversion from geodetic to cartesian is straightforward, converting cartesian to geodetic is a complex problem.

There are a number of possible approaches: this uses Bowring’s 1985 method (*The Accuracy of Geodetic Latitude and Height Equations – Survey Review Vol 28, 218, Oct 1985*), which provides approximately μm precision on the earth ellipsoid, in a concise formulation. If you have more demanding requirements, you could compare e.g. Fukushima (2006), Vermeille (2011), Karney (2012), and others.

| | | |
|---|---|---|
| $e^2 = (a^2-b^2) / a^2$ | *1st eccentricity of ellipsoid (= 2·f − f²)* | |
| $\varepsilon^2 = (a^2-b^2) / b^2$ | *2nd eccentricity of ellipsoid (= e² / (1−e²))* | |
| $\nu = a / \sqrt{(1 - e^2 \cdot \sin^2\varphi)}$ | *length of normal terminated by minor axis* | |
| $p = \sqrt{(x^2 + y^2)}$ | *distance from minor axis* | |
| $R = \sqrt{(p^2 + z^2)}$ | *polar radius* | |
| $\tan\beta = (b\cdot z)/(a\cdot p) \cdot (1 + \varepsilon^2 \cdot b/R)$ | *parametric latitude* | (17) |
| $\tan\varphi = (z + \varepsilon^2\cdot b\cdot\sin^3\beta) / (p - e^2\cdot a\cdot\cos^3\beta)$ | *geodetic latitude* | (18) |
| $\tan\lambda = y / x$ | *longitude* | |
| $h = p\cdot\cos\varphi + z\cdot\sin\varphi - a^2/\nu$ | *height above ellipsoid* | (7) |

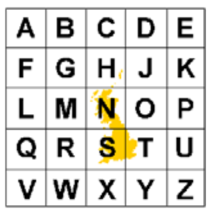
**Accuracy**: the Ordnance Survey say a single Helmert transformation between WGS84 and OSGB36 is accurate to within about 4–5 metres – for greater accuracy, a ‘rubber-sheet’ style transformation, which takes into account distortions in the ‘terrestrial reference frame’ (TRF), must be used (‘~~OSTN02~~’ ‘OSTN15’): this is a different level of complexity, if you require such accuracy, I expect you already know more about OSGB transformations than me.

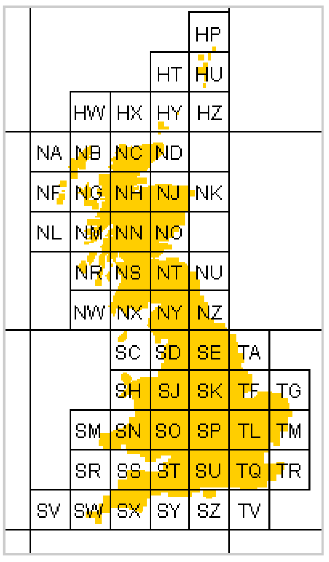
## Transverse Mercator projection

With the latitude/longitude in the OSGB-36 datum, a transverse Mercator projection is then applied to obtain a rectilinear grid. The OSGB uses the Lee–Redfearn implementation of the Gauss–Krüger projection.

The OSGB national grid uses a true origin of 49°N, 2°W. A false origin of −100 km north, 400 km east is then applied, so that eastings are always positive, and northings do not exceed 1,000 km.

## OSGB national grid references





Aside from the transformation maths, the other tricky bit of the script is converting grid letter-pairs to/from numeric eastings & northings. To follow what’s going on, it is worth noting that the letter-pairs define a 5x5 grid of 500km squares, each containing 5x5 sub-grids of 100km squares; the eastings & northings work from a ‘false origin’ at grid square SV, which is displaced from grid square AA by 10 squares E, 19 squares N, with the northing axis inverted; and letter ‘I’ is skipped.

OSGB Grid References apply to Great Britain only (Ireland and the Channel Islands have separate references).

## Example usage of OS-GridRef library

To convert an Ordnance Survey grid reference to a (WGS-84) latitude/longitude point:

```
<script type="module">
    import OsGridRef from 'https://cdn.jsdelivr.net/npm/geodesy@2/osgridref.js';
    const gridref = OsGridRef.parse('TL 44982 57869');
    const wgs84 = gridref.toLatLon();
    console.log(wgs84.toString('d', 2)); // '52.20°N, 000.12°E'
</script>
```

To convert a (WGS-84) latitude/longitude point to an Ordnance Survey grid reference:

```
<script type="module">
    import { LatLon } from 'https://cdn.jsdelivr.net/npm/geodesy@2/osgridref.js';
    const wgs84 = new LatLon(52.2, 0.12);
    const gridref = wgs84.toOsGrid();
    console.log(gridref.toString()); // 'TL 44982 57869'
</script>
```

See documentation for full details.

---

Distances between OS grid reference points are straightforward to calculate by pythagoras, once references are converted to numeric form. For UTM coordinates & MGRS grid references, see my UTM-MGRS page. For other scripts for calculating distances, bearings, etc between latitude/longitude points, see my Lat/Long page.

See below for the JavaScript source code of the transverse Mercator projection and the datum transformation, also available on GitHub. Full documentation is available, as well as a test suite.

Note I use Greek letters in variables representing maths symbols conventionally presented as Greek letters: I value the great benefit in legibility over the minor inconvenience in typing (if you encounter any problems, ensure your `<head>` includes `<meta charset="utf-8">`, and use UTF-8 encoding when saving files).

With its untyped C-style syntax, JavaScript reads remarkably close to pseudo-code: exposing the algorithms with a minimum of syntactic distractions. These functions should be simple to translate into other languages if required, though can also be used as-is in browsers and Node.js.

For convenience & clarity, I have extended the base JavaScript `Number` object with `toRadians()` and `toDegrees()` methods: I don’t see great likelihood of conflicts, as these are ubiquitous operations.

If you would like to show your appreciation and support continued development of these scripts, I would most gratefully accept donations.

If you need any advice or development work done, I am available for consultancy.

If you have any queries or find any problems, contact me at scripts-geo@movable-type.co.uk.

```
/* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */
/* Ordnance Survey Grid Reference functions                           (c) Chris Veness 2005-2021  */
/*                                                                                   MIT Licence  */
/* www.movable-type.co.uk/scripts/latlong-gridref.html                                            */
/* www.movable-type.co.uk/scripts/geodesy-library.html#osgridref                                  */
/* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */

import LatLonEllipsoidal, { Dms } from './latlon-ellipsoidal-datum.js';


/**
 * Ordnance Survey OSGB grid references provide geocoordinate references for UK mapping purposes.
 *
 * Formulation implemented here due to Thomas, Redfearn, etc is as published by OS, but is inferior
 * to Krüger as used by e.g. Karney 2011.
 *
 * www.ordnancesurvey.co.uk/documents/resources/guide-coordinate-systems-great-britain.pdf.
 *
 * Note OSGB grid references cover Great Britain only; Ireland and the Channel Islands have their
 * own references.
 *
 * Note that these formulae are based on ellipsoidal calculations, and according to the OS are
 * accurate to about 4–5 metres – for greater accuracy, a geoid-based transformation (OSTN15) must
 * be used.
 */

/*
 * Converted 2015 to work with WGS84 by default, OSGB36 as option;
 * www.ordnancesurvey.co.uk/blog/2014/12/confirmation-on-changes-to-latitude-and-longitude
 */


/* OsGridRef  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - */


const nationalGrid = {
    trueOrigin:  { lat: 49, lon: -2 },                 // true origin of grid 49°N,2°W on OSGB36 datum
    falseOrigin: { easting: -400e3, northing: 100e3 }, // easting & northing of false origin, metres from true origin
    scaleFactor: 0.9996012717,                         // scale factor on central meridian
    ellipsoid:   LatLonEllipsoidal.ellipsoids.Airy1830,
};
// note Irish National Grid uses t/o 53°30′N, 8°W, f/o 200kmW, 250kmS, scale factor 1.000035, on Airy 1830 Modified ellipsoid


/**
 * OS Grid References with methods to parse and convert them to latitude/longitude points.
 */
class OsGridRef {

    /**
     * Creates an OsGridRef object.
     *
     * @param {number} easting - Easting in metres from OS Grid false origin.
     * @param {number} northing - Northing in metres from OS Grid false origin.
     *
     * @example
     *   import OsGridRef from '/js/geodesy/osgridref.js';
     *   const gridref = new OsGridRef(651409, 313177);
     */
    constructor(easting, northing) {
        this.easting = Number(easting);
        this.northing = Number(northing);

        if (isNaN(easting)  || this.easting<0  || this.easting>700e3) throw new RangeError(`invalid easting ‘${easting}’`);
        if (isNaN(northing) || this.northing<0 || this.northing>1300e3) throw new RangeError(`invalid northing ‘${northing}’`);
    }


    /**
     * Converts ‘this’ Ordnance Survey Grid Reference easting/northing coordinate to latitude/longitude
     * (SW corner of grid square).
     *
     * While OS Grid References are based on OSGB-36, the Ordnance Survey have deprecated the use of
     * OSGB-36 for latitude/longitude coordinates (in favour of WGS-84), hence this function returns
     * WGS-84 by default, with OSGB-36 as an option. See www.ordnancesurvey.co.uk/blog/2014/12/2.
     *
     * Note formulation implemented here due to Thomas, Redfearn, etc is as published by OS, but is
     * inferior to Krüger as used by e.g. Karney 2011.
     *
     * @param   {LatLon.datum} [datum=WGS84] - Datum to convert grid reference into.
     * @returns {LatLon}       Latitude/longitude of supplied grid reference.
     *
     * @example
     *   const gridref = new OsGridRef(651409.903, 313177.270);
     *   const pWgs84 = gridref.toLatLon();                    // 52°39′28.723″N, 001°42′57.787″E
     *   // to obtain (historical) OSGB36 lat/lon point:
     *   const pOsgb = gridref.toLatLon(LatLon.datums.OSGB36); // 52°39′27.253″N, 001°43′04.518″E
     */
    toLatLon(datum=LatLonEllipsoidal.datums.WGS84) {
        const { easting: E, northing: N } = this;

        const { a, b } = nationalGrid.ellipsoid;            // a = 6377563.396, b = 6356256.909
        const φ0 = nationalGrid.trueOrigin.lat.toRadians(); // latitude of true origin, 49°N
        const λ0 = nationalGrid.trueOrigin.lon.toRadians(); // longitude of true origin, 2°W
        const E0 = -nationalGrid.falseOrigin.easting;       // easting of true origin, 400km
        const N0 = -nationalGrid.falseOrigin.northing;      // northing of true origin, -100km
        const F0 = nationalGrid.scaleFactor;                // 0.9996012717

        const e2 = 1 - (b*b)/(a*a);                         // eccentricity squared
        const n = (a-b)/(a+b), n2 = n*n, n3 = n*n*n;        // n, n², n³

        let φ=φ0, M=0;
        do {
            φ = (N-N0-M)/(a*F0) + φ;

            const Ma = (1 + n + (5/4)*n2 + (5/4)*n3) * (φ-φ0);
            const Mb = (3*n + 3*n*n + (21/8)*n3) * Math.sin(φ-φ0) * Math.cos(φ+φ0);
            const Mc = ((15/8)*n2 + (15/8)*n3) * Math.sin(2*(φ-φ0)) * Math.cos(2*(φ+φ0));
            const Md = (35/24)*n3 * Math.sin(3*(φ-φ0)) * Math.cos(3*(φ+φ0));
            M = b * F0 * (Ma - Mb + Mc - Md);                // meridional arc

        } while (Math.abs(N-N0-M) >= 0.00001);  // ie until < 0.01mm

        const cosφ = Math.cos(φ), sinφ = Math.sin(φ);
        const ν = a*F0/Math.sqrt(1-e2*sinφ*sinφ);            // nu = transverse radius of curvature
        const ρ = a*F0*(1-e2)/Math.pow(1-e2*sinφ*sinφ, 1.5); // rho = meridional radius of curvature
        const η2 = ν/ρ-1;                                    // eta = ?

        const tanφ = Math.tan(φ);
        const tan2φ = tanφ*tanφ, tan4φ = tan2φ*tan2φ, tan6φ = tan4φ*tan2φ;
        const secφ = 1/cosφ;
        const ν3 = ν*ν*ν, ν5 = ν3*ν*ν, ν7 = ν5*ν*ν;
        const VII = tanφ/(2*ρ*ν);
        const VIII = tanφ/(24*ρ*ν3)*(5+3*tan2φ+η2-9*tan2φ*η2);
        const IX = tanφ/(720*ρ*ν5)*(61+90*tan2φ+45*tan4φ);
        const X = secφ/ν;
        const XI = secφ/(6*ν3)*(ν/ρ+2*tan2φ);
        const XII = secφ/(120*ν5)*(5+28*tan2φ+24*tan4φ);
        const XIIA = secφ/(5040*ν7)*(61+662*tan2φ+1320*tan4φ+720*tan6φ);

        const dE = (E-E0), dE2 = dE*dE, dE3 = dE2*dE, dE4 = dE2*dE2, dE5 = dE3*dE2, dE6 = dE4*dE2, dE7 = dE5*dE2;
        φ = φ - VII*dE2 + VIII*dE4 - IX*dE6;
        const λ = λ0 + X*dE - XI*dE3 + XII*dE5 - XIIA*dE7;

        let point = new LatLon_OsGridRef(φ.toDegrees(), λ.toDegrees(), 0, LatLonEllipsoidal.datums.OSGB36);

        if (datum != LatLonEllipsoidal.datums.OSGB36) {
            // if point is required in datum other than OSGB36, convert it
            point = point.convertDatum(datum);
            // convertDatum() gives us a LatLon: convert to LatLon_OsGridRef which includes toOsGrid()
            point = new LatLon_OsGridRef(point.lat, point.lon, point.height, point.datum);
        }

        return point;
    }


    /**
     * Parses grid reference to OsGridRef object.
     *
     * Accepts standard grid references (eg 'SU 387 148'), with or without whitespace separators, from
     * two-digit references up to 10-digit references (1m × 1m square), or fully numeric comma-separated
     * references in metres (eg '438700,114800').
     *
     * @param   {string}    gridref - Standard format OS Grid Reference.
     * @returns {OsGridRef} Numeric version of grid reference in metres from false origin (SW corner of
     *   supplied grid square).
     * @throws  {Error}     Invalid grid reference.
     *
     * @example
     *   const grid = OsGridRef.parse('TG 51409 13177'); // grid: { easting: 651409, northing: 313177 }
     */
    static parse(gridref) {
        gridref = String(gridref).trim();

        // check for fully numeric comma-separated gridref format
        let match = gridref.match(/^(\d+),\s*(\d+)$/);
        if (match) return new OsGridRef(match[1], match[2]);

        // validate format
        match = gridref.match(/^[HNST][ABCDEFGHJKLMNOPQRSTUVWXYZ]\s*[0-9]+\s*[0-9]+$/i);
        if (!match) throw new Error(`invalid grid reference ‘${gridref}’`);

        // get numeric values of letter references, mapping A->0, B->1, C->2, etc:
        let l1 = gridref.toUpperCase().charCodeAt(0) - 'A'.charCodeAt(0); // 500km square
        let l2 = gridref.toUpperCase().charCodeAt(1) - 'A'.charCodeAt(0); // 100km square
        // shuffle down letters after 'I' since 'I' is not used in grid:
        if (l1 > 7) l1--;
        if (l2 > 7) l2--;

        // convert grid letters into 100km-square indexes from false origin (grid square SV):
        const e100km = ((l1 - 2) % 5) * 5 + (l2 % 5);
        const n100km = (19 - Math.floor(l1 / 5) * 5) - Math.floor(l2 / 5);

        // skip grid letters to get numeric (easting/northing) part of ref
        let en = gridref.slice(2).trim().split(/\s+/);
        // if e/n not whitespace separated, split half way
        if (en.length == 1) en = [ en[0].slice(0, en[0].length / 2), en[0].slice(en[0].length / 2) ];

        // validation
        if (en[0].length != en[1].length) throw new Error(`invalid grid reference ‘${gridref}’`);

        // standardise to 10-digit refs (metres)
        en[0] = en[0].padEnd(5, '0');
        en[1] = en[1].padEnd(5, '0');

        const e = e100km + en[0];
        const n = n100km + en[1];

        return new OsGridRef(e, n);
    }


    /**
     * Converts ‘this’ numeric grid reference to standard OS Grid Reference.
     *
     * @param   {number} [digits=10] - Precision of returned grid reference (10 digits = metres);
     *   digits=0 will return grid reference in numeric format.
     * @returns {string} This grid reference in standard format.
     *
     * @example
     *   const gridref = new OsGridRef(651409, 313177).toString(8); // 'TG 5140 1317'
     *   const gridref = new OsGridRef(651409, 313177).toString(0); // '651409,313177'
     */
    toString(digits=10) {
        if (![ 0,2,4,6,8,10,12,14,16 ].includes(Number(digits))) throw new RangeError(`invalid precision ‘${digits}’`); // eslint-disable-li

        let { easting: e, northing: n } = this;

        // use digits = 0 to return numeric format (in metres) - note northing may be >= 1e7
        if (digits == 0) {
            const format = { useGrouping: false,  minimumIntegerDigits: 6, maximumFractionDigits: 3 };
            const ePad = e.toLocaleString('en', format);
            const nPad = n.toLocaleString('en', format);
            return `${ePad},${nPad}`;
        }

        // get the 100km-grid indices
        const e100km = Math.floor(e / 100000), n100km = Math.floor(n / 100000);

        // translate those into numeric equivalents of the grid letters
        let l1 = (19 - n100km) - (19 - n100km) % 5 + Math.floor((e100km + 10) / 5);
        let l2 = (19 - n100km) * 5 % 25 + e100km % 5;

        // compensate for skipped 'I' and build grid letter-pairs
        if (l1 > 7) l1++;
        if (l2 > 7) l2++;
        const letterPair = String.fromCharCode(l1 + 'A'.charCodeAt(0), l2 + 'A'.charCodeAt(0));

        // strip 100km-grid indices from easting & northing, and reduce precision
        e = Math.floor((e % 100000) / Math.pow(10, 5 - digits / 2));
        n = Math.floor((n % 100000) / Math.pow(10, 5 - digits / 2));

        // pad eastings & northings with leading zeros
        e = e.toString().padStart(digits/2, '0');
        n = n.toString().padStart(digits/2, '0');
        return `${letterPair} ${e} ${n}`;
    }

}


/* LatLon_OsGridRef - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - */


/**
 * Extends LatLon class with method to convert LatLon point to OS Grid Reference.
 *
 * @extends LatLonEllipsoidal
 */
class LatLon_OsGridRef extends LatLonEllipsoidal {

    /**
     * Converts latitude/longitude to Ordnance Survey grid reference easting/northing coordinate.
     *
     * @returns {OsGridRef} OS Grid Reference easting/northing.
     *
     * @example
     *   const grid = new LatLon(52.65798, 1.71605).toOsGrid(); // TG 51409 13177
     *   // for conversion of (historical) OSGB36 latitude/longitude point:
     *   const grid = new LatLon(52.65798, 1.71605).toOsGrid(LatLon.datums.OSGB36);
     */
    toOsGrid() {
        // if necessary convert to OSGB36 first
        const point = this.datum == LatLonEllipsoidal.datums.OSGB36
            ? this
            : this.convertDatum(LatLonEllipsoidal.datums.OSGB36);

        const φ = point.lat.toRadians();
        const λ = point.lon.toRadians();

        const { a, b } = nationalGrid.ellipsoid;            // a = 6377563.396, b = 6356256.909
        const φ0 = nationalGrid.trueOrigin.lat.toRadians(); // latitude of true origin, 49°N
        const λ0 = nationalGrid.trueOrigin.lon.toRadians(); // longitude of true origin, 2°W
        const E0 = -nationalGrid.falseOrigin.easting;       // easting of true origin, 400km
        const N0 = -nationalGrid.falseOrigin.northing;      // northing of true origin, -100km
        const F0 = nationalGrid.scaleFactor;                // 0.9996012717

        const e2 = 1 - (b*b)/(a*a);                          // eccentricity squared
        const n = (a-b)/(a+b), n2 = n*n, n3 = n*n*n;         // n, n², n³

        const cosφ = Math.cos(φ), sinφ = Math.sin(φ);
        const ν = a*F0/Math.sqrt(1-e2*sinφ*sinφ);            // nu = transverse radius of curvature
        const ρ = a*F0*(1-e2)/Math.pow(1-e2*sinφ*sinφ, 1.5); // rho = meridional radius of curvature
        const η2 = ν/ρ-1;                                    // eta = ?

        const Ma = (1 + n + (5/4)*n2 + (5/4)*n3) * (φ-φ0);
        const Mb = (3*n + 3*n*n + (21/8)*n3) * Math.sin(φ-φ0) * Math.cos(φ+φ0);
        const Mc = ((15/8)*n2 + (15/8)*n3) * Math.sin(2*(φ-φ0)) * Math.cos(2*(φ+φ0));
        const Md = (35/24)*n3 * Math.sin(3*(φ-φ0)) * Math.cos(3*(φ+φ0));
        const M = b * F0 * (Ma - Mb + Mc - Md);              // meridional arc

        const cos3φ = cosφ*cosφ*cosφ;
        const cos5φ = cos3φ*cosφ*cosφ;
        const tan2φ = Math.tan(φ)*Math.tan(φ);
        const tan4φ = tan2φ*tan2φ;

        const I = M + N0;
        const II = (ν/2)*sinφ*cosφ;
        const III = (ν/24)*sinφ*cos3φ*(5-tan2φ+9*η2);
        const IIIA = (ν/720)*sinφ*cos5φ*(61-58*tan2φ+tan4φ);
        const IV = ν*cosφ;
        const V = (ν/6)*cos3φ*(ν/ρ-tan2φ);
        const VI = (ν/120) * cos5φ * (5 - 18*tan2φ + tan4φ + 14*η2 - 58*tan2φ*η2);

        const Δλ = λ-λ0;
        const Δλ2 = Δλ*Δλ, Δλ3 = Δλ2*Δλ, Δλ4 = Δλ3*Δλ, Δλ5 = Δλ4*Δλ, Δλ6 = Δλ5*Δλ;

        let N = I + II*Δλ2 + III*Δλ4 + IIIA*Δλ6;
        let E = E0 + IV*Δλ + V*Δλ3 + VI*Δλ5;

        N = Number(N.toFixed(3)); // round to mm precision
        E = Number(E.toFixed(3));

        try {
            return new OsGridRef(E, N); // note: gets truncated to SW corner of 1m grid square
        } catch (e) {
            throw new Error(`${e.message} from (${point.lat.toFixed(6)},${point.lon.toFixed(6)}).toOsGrid()`);
        }
    }
    /**
     * Override LatLonEllipsoidal.convertDatum() with version which returns LatLon_OsGridRef.
     */
    convertDatum(toDatum) {
        const osgbED = super.convertDatum(toDatum); // returns LatLonEllipsoidal_Datum
        const osgbOSGR = new LatLon_OsGridRef(osgbED.lat, osgbED.lon, osgbED.height, osgbED.datum);
        return osgbOSGR;
    }

}


/* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */

export { OsGridRef as default, LatLon_OsGridRef as LatLon, Dms };
```

```
/* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */
/* Geodesy tools for an ellipsoidal earth model                       (c) Chris Veness 2005-2022  */
/*                                                                                   MIT Licence  */
/* Core class for latlon-ellipsoidal-datum & latlon-ellipsoidal-referenceframe.                   */
/*                                                                                                */
/* www.movable-type.co.uk/scripts/latlong-convert-coords.html                                     */
/* www.movable-type.co.uk/scripts/geodesy-library.html#latlon-ellipsoidal                         */
/* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */

import Dms      from './dms.js';
import Vector3d from './vector3d.js';


/**
 * A latitude/longitude point defines a geographic location on or above/below the earth’s surface,
 * measured in degrees from the equator & the International Reference Meridian and in metres above
 * the ellipsoid, and based on a given datum.
 *
 * As so much modern geodesy is based on WGS-84 (as used by GPS), this module includes WGS-84
 * ellipsoid parameters, and it has methods for converting geodetic (latitude/longitude) points to/from
 * geocentric cartesian points; the latlon-ellipsoidal-datum and latlon-ellipsoidal-referenceframe
 * modules provide transformation parameters for converting between historical datums and between
 * modern reference frames.
 *
 * This module is used for both trigonometric geodesy (eg latlon-ellipsoidal-vincenty) and n-vector
 * geodesy (eg latlon-nvector-ellipsoidal), and also for UTM/MGRS mapping.
 *
 * @module latlon-ellipsoidal
 */



/*
 * Ellipsoid parameters; exposed through static getter below.
 *
 * The only ellipsoid defined is WGS84, for use in utm/mgrs, vincenty, nvector.
 */
const ellipsoids = {
    WGS84: { a: 6378137, b: 6356752.314245, f: 1/298.257223563 },
};


/*
 * Datums; exposed through static getter below.
 *
 * The only datum defined is WGS84, for use in utm/mgrs, vincenty, nvector.
 */
const datums = {
    WGS84: { ellipsoid: ellipsoids.WGS84 },
};


// freeze static properties
Object.freeze(ellipsoids.WGS84);
Object.freeze(datums.WGS84);


/* LatLonEllipsoidal - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */


/**
 * Latitude/longitude points on an ellipsoidal model earth, with ellipsoid parameters and methods
 * for converting points to/from cartesian (ECEF) coordinates.
 *
 * This is the core class, which will usually be used via LatLonEllipsoidal_Datum or
 * LatLonEllipsoidal_ReferenceFrame.
 */
class LatLonEllipsoidal {
    /**
     * Creates a geodetic latitude/longitude point on a (WGS84) ellipsoidal model earth.
     *
     * @param  {number} lat - Latitude (in degrees).
     * @param  {number} lon - Longitude (in degrees).
     * @param  {number} [height=0] - Height above ellipsoid in metres.
     * @throws {TypeError} Invalid lat/lon/height.
     *
     * @example
     *   import LatLon from '/js/geodesy/latlon-ellipsoidal.js';
     *   const p = new LatLon(51.47788, -0.00147, 17);
     */
    constructor(lat, lon, height=0) {
        if (isNaN(lat) || lat == null) throw new TypeError(`invalid lat '${lat}'`);
        if (isNaN(lon) || lon == null) throw new TypeError(`invalid lon '${lon}'`);
        if (isNaN(height) || height == null) throw new TypeError(`invalid height '${height}'`);

        this._lat = Dms.wrap90(Number(lat));
        this._lon = Dms.wrap180(Number(lon));
        this._height = Number(height);
    }


    /**
     * Latitude in degrees north from equator (including aliases lat, latitude): can be set as
     * numeric or hexagesimal (deg-min-sec); returned as numeric.
     */
    get lat()       { return this._lat; }
    get latitude()  { return this._lat; }
    set lat(lat) {
        this._lat = isNaN(lat) ? Dms.wrap90(Dms.parse(lat)) : Dms.wrap90(Number(lat));
        if (isNaN(this._lat)) throw new TypeError(`invalid lat '${lat}'`);
    }
    set latitude(lat) {
        this._lat = isNaN(lat) ? Dms.wrap90(Dms.parse(lat)) : Dms.wrap90(Number(lat));
        if (isNaN(this._lat)) throw new TypeError(`invalid latitude '${lat}'`);
    }

    /**
     * Longitude in degrees east from international reference meridian (including aliases lon, lng,
     * longitude): can be set as numeric or hexagesimal (deg-min-sec); returned as numeric.
     */
    get lon()       { return this._lon; }
    get lng()       { return this._lon; }
    get longitude() { return this._lon; }
    set lon(lon) {
        this._lon = isNaN(lon) ? Dms.wrap180(Dms.parse(lon)) : Dms.wrap180(Number(lon));
        if (isNaN(this._lon)) throw new TypeError(`invalid lon '${lon}'`);
    }
    set lng(lon) {
        this._lon = isNaN(lon) ? Dms.wrap180(Dms.parse(lon)) : Dms.wrap180(Number(lon));
        if (isNaN(this._lon)) throw new TypeError(`invalid lng '${lon}'`);
    }
    set longitude(lon) {
        this._lon = isNaN(lon) ? Dms.wrap180(Dms.parse(lon)) : Dms.wrap180(Number(lon));
        if (isNaN(this._lon)) throw new TypeError(`invalid longitude '${lon}'`);
    }

    /**
     * Height in metres above ellipsoid.
     */
    get height()       { return this._height; }
    set height(height) { this._height = Number(height); if (isNaN(this._height)) throw new TypeError(`invalid height '${height}'`); }


    /**
     * Datum.
     *
     * Note this is replicated within LatLonEllipsoidal in order that a LatLonEllipsoidal object can
     * be monkey-patched to look like a LatLonEllipsoidal_Datum, for Vincenty calculations on
     * different ellipsoids.
     *
     * @private
     */
    get datum()      { return this._datum; }
    set datum(datum) { this._datum = datum; }


    /**
     * Ellipsoids with their parameters; this module only defines WGS84 parameters a = 6378137, b =
     * 6356752.314245, f = 1/298.257223563.
     *
     * @example
     *   const a = LatLon.ellipsoids.WGS84.a; // 6378137
     */
    static get ellipsoids() {
        return ellipsoids;
    }

    /**
     * Datums; this module only defines WGS84 datum, hence no datum transformations.
     *
     * @example
     *   const a = LatLon.datums.WGS84.ellipsoid.a; // 6377563.396
     */
    static get datums() {
        return datums;
    }


    /**
     * Parses a latitude/longitude point from a variety of formats.
     *
     * Latitude & longitude (in degrees) can be supplied as two separate parameters, as a single
     * comma-separated lat/lon string, or as a single object with { lat, lon } or GeoJSON properties.
     *
     * The latitude/longitude values may be numeric or strings; they may be signed decimal or
     * deg-min-sec (hexagesimal) suffixed by compass direction (NSEW); a variety of separators are
     * accepted. Examples -3.62, '3 37 12W', '3°37′12″W'.
     *
     * Thousands/decimal separators must be comma/dot; use Dms.fromLocale to convert locale-specific
     * thousands/decimal separators.
     *
     * @param   {number|string|Object} lat|latlon - Latitude (in degrees), or comma-separated lat/lon, or lat/lon object.
     * @param   {number}               [lon]      - Longitude (in degrees).
     * @param   {number}               [height=0] - Height above ellipsoid in metres.
     * @returns {LatLon} Latitude/longitude point on WGS84 ellipsoidal model earth.
     * @throws  {TypeError} Invalid coordinate.
     *
     * @example
     *   const p1 = LatLon.parse(51.47788, -0.00147);              // numeric pair
     *   const p2 = LatLon.parse('51°28′40″N, 000°00′05″W', 17);   // dms string + height
     *   const p3 = LatLon.parse({ lat: 52.205, lon: 0.119 }, 17); // { lat, lon } object numeric + height
     */
    static parse(...args) {
        if (args.length == 0) throw new TypeError('invalid (empty) point');

        let lat=undefined, lon=undefined, height=undefined;

        // single { lat, lon } object
        if (typeof args[0]=='object' && (args.length==1 || !isNaN(parseFloat(args[1])))) {
            const ll = args[0];
            if (ll.type == 'Point' && Array.isArray(ll.coordinates)) { // GeoJSON
                [ lon, lat, height ] = ll.coordinates;
                height = height || 0;
            } else { // regular { lat, lon } object
                if (ll.latitude  != undefined) lat = ll.latitude;
                if (ll.lat       != undefined) lat = ll.lat;
                if (ll.longitude != undefined) lon = ll.longitude;
                if (ll.lng       != undefined) lon = ll.lng;
                if (ll.lon       != undefined) lon = ll.lon;
                if (ll.height    != undefined) height = ll.height;
                lat = Dms.wrap90(Dms.parse(lat));
                lon = Dms.wrap180(Dms.parse(lon));
            }
            if (args[1] != undefined) height = args[1];
            if (isNaN(lat) || isNaN(lon)) throw new TypeError(`invalid point ‘${JSON.stringify(args[0])}’`);
        }

        // single comma-separated lat/lon
        if (typeof args[0] == 'string' && args[0].split(',').length == 2) {
            [ lat, lon ] = args[0].split(',');
            lat = Dms.wrap90(Dms.parse(lat));
            lon = Dms.wrap180(Dms.parse(lon));
            height = args[1] || 0;
            if (isNaN(lat) || isNaN(lon)) throw new TypeError(`invalid point ‘${args[0]}’`);
        }

        // regular (lat, lon) arguments
        if (lat==undefined && lon==undefined) {
            [ lat, lon ] = args;
            lat = Dms.wrap90(Dms.parse(lat));
            lon = Dms.wrap180(Dms.parse(lon));
            height = args[2] || 0;
            if (isNaN(lat) || isNaN(lon)) throw new TypeError(`invalid point ‘${args.toString()}’`);
        }

        return new this(lat, lon, height); // 'new this' as may return subclassed types
    }
    /**
     * Converts ‘this’ point from (geodetic) latitude/longitude coordinates to (geocentric)
     * cartesian (x/y/z) coordinates.
     *
     * @returns {Cartesian} Cartesian point equivalent to lat/lon point, with x, y, z in metres from
     *   earth centre.
     */
    toCartesian() {
        // x = (ν+h)⋅cosφ⋅cosλ, y = (ν+h)⋅cosφ⋅sinλ, z = (ν⋅(1-e²)+h)⋅sinφ
        // where ν = a/√(1−e²⋅sinφ⋅sinφ), e² = (a²-b²)/a² or (better conditioned) 2⋅f-f²
        const ellipsoid = this.datum
            ? this.datum.ellipsoid
            : this.referenceFrame ? this.referenceFrame.ellipsoid : ellipsoids.WGS84;

        const φ = this.lat.toRadians();
        const λ = this.lon.toRadians();
        const h = this.height;
        const { a, f } = ellipsoid;

        const sinφ = Math.sin(φ), cosφ = Math.cos(φ);
        const sinλ = Math.sin(λ), cosλ = Math.cos(λ);

        const eSq = 2*f - f*f;                      // 1st eccentricity squared ≡ (a²-b²)/a²
        const ν = a / Math.sqrt(1 - eSq*sinφ*sinφ); // radius of curvature in prime vertical

        const x = (ν+h) * cosφ * cosλ;
        const y = (ν+h) * cosφ * sinλ;
        const z = (ν*(1-eSq)+h) * sinφ;

        return new Cartesian(x, y, z);
    }


    /**
     * Checks if another point is equal to ‘this’ point.
     *
     * @param   {LatLon} point - Point to be compared against this point.
     * @returns {bool} True if points have identical latitude, longitude, height, and datum/referenceFrame.
     * @throws  {TypeError} Invalid point.
     *
     * @example
     *   const p1 = new LatLon(52.205, 0.119);
     *   const p2 = new LatLon(52.205, 0.119);
     *   const equal = p1.equals(p2); // true
     */
    equals(point) {
        if (!(point instanceof LatLonEllipsoidal)) throw new TypeError(`invalid point ‘${point}’`);

        if (Math.abs(this.lat - point.lat) > Number.EPSILON) return false;
        if (Math.abs(this.lon - point.lon) > Number.EPSILON) return false;
        if (Math.abs(this.height - point.height) > Number.EPSILON) return false;
        if (this.datum != point.datum) return false;
        if (this.referenceFrame != point.referenceFrame) return false;
        if (this.epoch != point.epoch) return false;

        return true;
    }


    /**
     * Returns a string representation of ‘this’ point, formatted as degrees, degrees+minutes, or
     * degrees+minutes+seconds.
     *
     * @param   {string} [format=d] - Format point as 'd', 'dm', 'dms', or 'n' for signed numeric.
     * @param   {number} [dp=4|2|0] - Number of decimal places to use: default 4 for d, 2 for dm, 0 for dms.
     * @param   {number} [dpHeight=null] - Number of decimal places to use for height; default is no height display.
     * @returns {string} Comma-separated formatted latitude/longitude.
     * @throws  {RangeError} Invalid format.
     *
     * @example
     *   const greenwich = new LatLon(51.47788, -0.00147, 46);
     *   const d = greenwich.toString();                        // 51.4779°N, 000.0015°W
     *   const dms = greenwich.toString('dms', 2);              // 51°28′40″N, 000°00′05″W
     *   const [lat, lon] = greenwich.toString('n').split(','); // 51.4779, -0.0015
     *   const dmsh = greenwich.toString('dms', 0, 0);          // 51°28′40″N, 000°00′06″W +46m
     */
    toString(format='d', dp=undefined, dpHeight=null) {
        // note: explicitly set dp to undefined for passing through to toLat/toLon
        if (![ 'd', 'dm', 'dms', 'n' ].includes(format)) throw new RangeError(`invalid format ‘${format}’`);

        const height = (this.height>=0 ? ' +' : ' ') + this.height.toFixed(dpHeight) + 'm';
        if (format == 'n') { // signed numeric degrees
            if (dp == undefined) dp = 4;
            const lat = this.lat.toFixed(dp);
            const lon = this.lon.toFixed(dp);
            return `${lat}, ${lon}${dpHeight==null ? '' : height}`;
```

```javascript
        }

        const lat = Dms.toLat(this.lat, format, dp);
        const lon = Dms.toLon(this.lon, format, dp);

        return `${lat}, ${lon}${dpHeight==null ? '' : height}`;
    }

}


/* Cartesian  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - */


/**
 * ECEF (earth-centered earth-fixed) geocentric cartesian coordinates.
 *
 * @extends Vector3d
 */
class Cartesian extends Vector3d {

    /**
     * Creates cartesian coordinate representing ECEF (earth-centric earth-fixed) point.
     *
     * @param {number} x - X coordinate in metres (=> 0°N,0°E).
     * @param {number} y - Y coordinate in metres (=> 0°N,90°E).
     * @param {number} z - Z coordinate in metres (=> 90°N).
     *
     * @example
     *   import { Cartesian } from '/js/geodesy/latlon-ellipsoidal.js';
     *   const coord = new Cartesian(3980581.210, -111.159, 4966824.522);
     */
    constructor(x, y, z) {
        super(x, y, z); // arguably redundant constructor, but specifies units & axes
    }


    /**
     * Converts ‘this’ (geocentric) cartesian (x/y/z) coordinate to (geodetic) latitude/longitude
     * point on specified ellipsoid.
     *
     * Uses Bowring’s (1985) formulation for μm precision in concise form; ‘The accuracy of geodetic
     * latitude and height equations’, B R Bowring, Survey Review vol 28, 218, Oct 1985.
     *
     * @param   {LatLon.ellipsoids} [ellipsoid=WGS84] - Ellipsoid to use when converting point.
     * @returns {LatLon} Latitude/longitude point defined by cartesian coordinates, on given ellipsoid.
     * @throws  {TypeError} Invalid ellipsoid.
     *
     * @example
     *   const c = new Cartesian(4027893.924, 307041.993, 4919474.294);
     *   const p = c.toLatLon(); // 50.7978°N, 004.3592°E
     */
    toLatLon(ellipsoid=ellipsoids.WGS84) {
        // note ellipsoid is available as a parameter for when toLatLon gets subclassed to
        // Ellipsoidal_Datum / Ellipsoidal_Referenceframe.
        if (!ellipsoid || !ellipsoid.a) throw new TypeError(`invalid ellipsoid ‘${ellipsoid}’`);

        const { x, y, z } = this;
        const { a, b, f } = ellipsoid;

        const e2 = 2*f - f*f;           // 1st eccentricity squared ≡ (a²−b²)/a²
        const ε2 = e2 / (1-e2);         // 2nd eccentricity squared ≡ (a²−b²)/b²
        const p = Math.sqrt(x*x + y*y); // distance from minor axis
        const R = Math.sqrt(p*p + z*z); // polar radius

        // parametric latitude (Bowring eqn.17, replacing tanβ = z·a / p·b)
        const tanβ = (b*z)/(a*p) * (1+ε2*b/R);
        const sinβ = tanβ / Math.sqrt(1+tanβ*tanβ);
        const cosβ = sinβ / tanβ;

        // geodetic latitude (Bowring eqn.18: tanφ = z+ε²⋅b⋅sin³β / p−e²⋅cos³β)
        const φ = isNaN(cosβ) ? 0 : Math.atan2(z + ε2*b*sinβ*sinβ*sinβ, p - e2*a*cosβ*cosβ*cosβ);

        // longitude
        const λ = Math.atan2(y, x);

        // height above ellipsoid (Bowring eqn.7)
        const sinφ = Math.sin(φ), cosφ = Math.cos(φ);
        const ν = a / Math.sqrt(1-e2*sinφ*sinφ); // length of the normal terminated by the minor axis
        const h = p*cosφ + z*sinφ - (a*a/ν);

        const point = new LatLonEllipsoidal(φ.toDegrees(), λ.toDegrees(), h);

        return point;
    }
    /**
     * Returns a string representation of ‘this’ cartesian point.
     *
     * @param   {number} [dp=0] - Number of decimal places to use.
     * @returns {string} Comma-separated latitude/longitude.
     */
    toString(dp=0) {
        const x = this.x.toFixed(dp), y = this.y.toFixed(dp), z = this.z.toFixed(dp);
        return `[${x},${y},${z}]`;
    }

}


/* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */

export { LatLonEllipsoidal as default, Cartesian, Vector3d, Dms };
```

```
/* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */
/* Geodesy tools for conversions between (historical) datums          (c) Chris Veness 2005-2019  */
/*                                                                                   MIT Licence  */
/* www.movable-type.co.uk/scripts/latlong-convert-coords.html                                     */
/* www.movable-type.co.uk/scripts/geodesy-library.html#latlon-ellipsoidal-datum                  */
/* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */

import LatLonEllipsoidal, { Cartesian, Dms } from './latlon-ellipsoidal.js';


/**
 * Historical geodetic datums: a latitude/longitude point defines a geographic location on or
 * above/below the  earth’s surface, measured in degrees from the equator & the International
 * Reference Meridian and metres above the ellipsoid, and based on a given datum. The datum is
 * based on a reference ellipsoid and tied to geodetic survey reference points.
 *
 * Modern geodesy is generally based on the WGS84 datum (as used for instance by GPS systems), but
 * previously various reference ellipsoids and datum references were used.
 *
 * This module extends the core latlon-ellipsoidal module to include ellipsoid parameters and datum
 * transformation parameters, and methods for converting between different (generally historical)
 * datums.
 *
 * It can be used for UK Ordnance Survey mapping (OS National Grid References are still based on the
 * otherwise historical OSGB36 datum), as well as for historical purposes.
 *
 * q.v. Ordnance Survey ‘A guide to coordinate systems in Great Britain’ Section 6,
 * www.ordnancesurvey.co.uk/docs/support/guide-coordinate-systems-great-britain.pdf, and also
 * www.ordnancesurvey.co.uk/blog/2014/12/2.
 *
 * @module latlon-ellipsoidal-datum
 */


/*
 * Ellipsoid parameters; exposed through static getter below.
 */
const ellipsoids = {
    WGS84:         { a: 6378137,     b: 6356752.314245, f: 1/298.257223563 },
    Airy1830:      { a: 6377563.396, b: 6356256.909,    f: 1/299.3249646   },
    AiryModified:  { a: 6377340.189, b: 6356034.448,    f: 1/299.3249646   },
    Bessel1841:    { a: 6377397.155, b: 6356078.962818, f: 1/299.1528128   },
    Clarke1866:    { a: 6378206.4,   b: 6356583.8,      f: 1/294.978698214 },
    Clarke1880IGN: { a: 6378249.2,   b: 6356515.0,      f: 1/293.466021294 },
    GRS80:         { a: 6378137,     b: 6356752.314140, f: 1/298.257222101 },
    Intl1924:      { a: 6378388,     b: 6356911.946,    f: 1/297           }, // aka Hayford
    WGS72:         { a: 6378135,     b: 6356750.5,      f: 1/298.26        },
};


/*
 * Datums; exposed through static getter below.
 */
const datums = {
    // transforms: t in metres, s in ppm, r in arcseconds              tx       ty        tz       s        rx        ry        rz
    ED50:       { ellipsoid: ellipsoids.Intl1924,      transform: [   89.5,    93.8,    123.1,    -1.2,     0.0,      0.0,      0.156    ] },
    ETRS89:     { ellipsoid: ellipsoids.GRS80,         transform: [    0,       0,        0,       0,       0,        0,        0        ] },
    Irl1975:    { ellipsoid: ellipsoids.AiryModified,  transform: [ -482.530, 130.596, -564.557,  -8.150,   1.042,    0.214,    0.631    ] }
    NAD27:      { ellipsoid: ellipsoids.Clarke1866,    transform: [    8,     -160,     -176,      0,       0,        0,        0        ] },
    NAD83:      { ellipsoid: ellipsoids.GRS80,         transform: [    0.9956, -1.9103,  -0.5215, -0.00062, 0.025915, 0.009426, 0.011599 ] },
    NTF:        { ellipsoid: ellipsoids.Clarke1880IGN, transform: [  168,       60,     -320,      0,       0,        0,        0        ] },
    OSGB36:     { ellipsoid: ellipsoids.Airy1830,      transform: [ -446.448, 125.157, -542.060,  20.4894, -0.1502,  -0.2470,  -0.8421   ] },
    Potsdam:    { ellipsoid: ellipsoids.Bessel1841,    transform: [ -582,     -105,     -414,     -8.3,     1.04,     0.35,    -3.08     ] }
    TokyoJapan: { ellipsoid: ellipsoids.Bessel1841,    transform: [  148,     -507,     -685,      0,       0,        0,        0        ] }
    WGS72:      { ellipsoid: ellipsoids.WGS72,         transform: [    0,       0,       -4.5,    -0.22,    0,        0,        0.554    ] },
    WGS84:      { ellipsoid: ellipsoids.WGS84,         transform: [    0.0,     0.0,      0.0,     0.0,     0.0,      0.0,      0.0     ] },
};
/* sources:
 * - ED50:       www.gov.uk/guidance/oil-and-gas-petroleum-operations-notices#pon-4
 * - Irl1975:    www.osi.ie/wp-content/uploads/2015/05/transformations_booklet.pdf
 * - NAD27:      en.wikipedia.org/wiki/Helmert_transformation
 * - NAD83:      www.uvm.edu/giv/resources/WGS84_NAD83.pdf [strictly, WGS84(G1150) -> NAD83(CORS96) @ epoch 1997.0]
 *               (note NAD83(1986) ≡ WGS84(Original); confluence.qps.nl/pages/viewpage.action?pageId=29855173)
 * - NTF:        Nouvelle Triangulation Francaise geodesie.ign.fr/contenu/fichiers/Changement_systeme_geodesique.pdf
 * - OSGB36:     www.ordnancesurvey.co.uk/docs/support/guide-coordinate-systems-great-britain.pdf
 * - Potsdam:    kartoweb.itc.nl/geometrics/Coordinate%20transformations/coordtrans.html
 * - TokyoJapan: www.geocachingtoolbox.com?page=datumEllipsoidDetails
 * - WGS72:      www.icao.int/safety/pbn/documentation/eurocontrol/eurocontrol wgs 84 implementation manual.pdf
 *
 * more transform parameters are available from earth-info.nga.mil/GandG/coordsys/datums/NATO_DT.pdf,
 * www.fieldenmaps.info/cconv/web/cconv_params.js
 */
/* note:
 * - ETRS89 reference frames are coincident with WGS-84 at epoch 1989.0 (ie null transform) at the one metre level.
 */


// freeze static properties
Object.keys(ellipsoids).forEach(e => Object.freeze(ellipsoids[e]));
Object.keys(datums).forEach(d => { Object.freeze(datums[d]); Object.freeze(datums[d].transform); });


/* LatLon - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - */


/**
 * Latitude/longitude points on an ellipsoidal model earth, with ellipsoid parameters and methods
 * for converting between datums and to geocentric (ECEF) cartesian coordinates.
 *
 * @extends LatLonEllipsoidal
 */
class LatLonEllipsoidal_Datum extends LatLonEllipsoidal {

    /**
     * Creates a geodetic latitude/longitude point on an ellipsoidal model earth using given datum.
     *
     * @param {number} lat - Latitude (in degrees).
     * @param {number} lon - Longitude (in degrees).
     * @param {number} [height=0] - Height above ellipsoid in metres.
     * @param {LatLon.datums} datum - Datum this point is defined within.
     *
     * @example
     *   import LatLon from '/js/geodesy/latlon-ellipsoidal-datum.js';
     *   const p = new LatLon(53.3444, -6.2577, 17, LatLon.datums.Irl1975);
     */
    constructor(lat, lon, height=0, datum=datums.WGS84) {
        if (!datum || datum.ellipsoid==undefined) throw new TypeError(`unrecognised datum '${datum}'`);

        super(lat, lon, height);

        this._datum = datum;
    }


    /**
     * Datum this point is defined within.
     */
    get datum() {
        return this._datum;
    }


    /**
     * Ellipsoids with their parameters; semi-major axis (a), semi-minor axis (b), and flattening (f).
     *
     * Flattening f = (a−b)/a; at least one of these parameters is derived from defining constants.
     *
     * @example
     *   const a = LatLon.ellipsoids.Airy1830.a; // 6377563.396
     */
    static get ellipsoids() {
        return ellipsoids;
    }


    /**
     * Datums; with associated ellipsoid, and Helmert transform parameters to convert from WGS-84
     * into given datum.
     *
     * Note that precision of various datums will vary, and WGS-84 (original) is not defined to be
     * accurate to better than ±1 metre. No transformation should be assumed to be accurate to
     * better than a metre, for many datums somewhat less.
     *
     * This is a small sample of commoner datums from a large set of historical datums. I will add
     * new datums on request.
     *
     * @example
     *   const a = LatLon.datums.OSGB36.ellipsoid.a;                    // 6377563.396
     *   const tx = LatLon.datums.OSGB36.transform;                     // [ tx, ty, tz, s, rx, ry, rz ]
     *   const availableDatums = Object.keys(LatLon.datums).join(', '); // ED50, Irl1975, NAD27, ...
     */
    static get datums() {
        return datums;
    }


    // note instance datum getter/setters are in LatLonEllipsoidal


    /**
     * Parses a latitude/longitude point from a variety of formats.
     *
     * Latitude & longitude (in degrees) can be supplied as two separate parameters, as a single
     * comma-separated lat/lon string, or as a single object with { lat, lon } or GeoJSON properties.
     *
     * The latitude/longitude values may be numeric or strings; they may be signed decimal or
     * deg-min-sec (hexagesimal) suffixed by compass direction (NSEW); a variety of separators are
     * accepted. Examples -3.62, '3 37 12W', '3°37′12″W'.
     *
     * Thousands/decimal separators must be comma/dot; use Dms.fromLocale to convert locale-specific
     * thousands/decimal separators.
     *
     * @param   {number|string|Object} lat|latlon - Geodetic Latitude (in degrees) or comma-separated lat/lon or lat/lon object.
     * @param   {number}               [lon] - Longitude in degrees.
     * @param   {number}               [height=0] - Height above ellipsoid in metres.
     * @param   {LatLon.datums}        [datum=WGS84] - Datum this point is defined within.
     * @returns {LatLon} Latitude/longitude point on ellipsoidal model earth using given datum.
     * @throws  {TypeError} Unrecognised datum.
     *
     * @example
     *   const p = LatLon.parse('51.47736, 0.0000', 0, LatLon.datums.OSGB36);
     */
    static parse(...args) {
        let datum = datums.WGS84;

        // if the last argument is a datum, use that, otherwise use default WGS-84
        if (args.length==4 || (args.length==3 && typeof args[2] == 'object')) datum = args.pop();

        if (!datum || datum.ellipsoid==undefined) throw new TypeError(`unrecognised datum ‘${datum}’`);

        const point = super.parse(...args);

        point._datum = datum;

        return point;
    }


    /**
     * Converts ‘this’ lat/lon coordinate to new coordinate system.
     *
     * @param   {LatLon.datums} toDatum - Datum this coordinate is to be converted to.
     * @returns {LatLon} This point converted to new datum.
     * @throws  {TypeError} Unrecognised datum.
     *
     * @example
     *   const pWGS84 = new LatLon(51.47788, -0.00147, 0, LatLon.datums.WGS84);
     *   const pOSGB = pWGS84.convertDatum(LatLon.datums.OSGB36); // 51.4773°N, 000.0001°E
     */
    convertDatum(toDatum) {
        if (!toDatum || toDatum.ellipsoid==undefined) throw new TypeError(`unrecognised datum ‘${toDatum}’`);

        const oldCartesian = this.toCartesian();                 // convert geodetic to cartesian
        const newCartesian = oldCartesian.convertDatum(toDatum); // convert datum
        const newLatLon = newCartesian.toLatLon();               // convert cartesian back to geodetic

        return newLatLon;
    }


    /**
     * Converts ‘this’ point from (geodetic) latitude/longitude coordinates to (geocentric) cartesian
     * (x/y/z) coordinates, based on the same datum.
     *
     * Shadow of LatLonEllipsoidal.toCartesian(), returning Cartesian augmented with
     * LatLonEllipsoidal_Datum methods/properties.
     *
     * @returns {Cartesian} Cartesian point equivalent to lat/lon point, with x, y, z in metres from
     *   earth centre, augmented with reference frame conversion methods and properties.
     */
    toCartesian() {
        const cartesian = super.toCartesian();
        const cartesianDatum = new Cartesian_Datum(cartesian.x, cartesian.y, cartesian.z, this.datum);
        return cartesianDatum;
    }

}


/* Cartesian  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - */


/**
 * Augments Cartesian with datum the cooordinate is based on, and methods to convert between datums
 * (using Helmert 7-parameter transforms) and to convert cartesian to geodetic latitude/longitude
 * point.
 *
 * @extends Cartesian
 */
class Cartesian_Datum extends Cartesian {

    /**
     * Creates cartesian coordinate representing ECEF (earth-centric earth-fixed) point, on a given
     * datum. The datum will identify the primary meridian (for the x-coordinate), and is also
     * useful in transforming to/from geodetic (lat/lon) coordinates.
     *
     * @param  {number} x - X coordinate in metres (=> 0°N,0°E).
     * @param  {number} y - Y coordinate in metres (=> 0°N,90°E).
     * @param  {number} z - Z coordinate in metres (=> 90°N).
     * @param  {LatLon.datums} [datum] - Datum this coordinate is defined within.
     * @throws {TypeError} Unrecognised datum.
     *
     * @example
     *   import { Cartesian } from '/js/geodesy/latlon-ellipsoidal-datum.js';
     *   const coord = new Cartesian(3980581.210, -111.159, 4966824.522);
     */
    constructor(x, y, z, datum=undefined) {
        if (datum && datum.ellipsoid==undefined) throw new TypeError(`unrecognised datum ‘${datum}’`);

        super(x, y, z);

        if (datum) this._datum = datum;
    }


    /**
     * Datum this point is defined within.
     */
    get datum() {
        return this._datum;
    }
    set datum(datum) {
        if (!datum || datum.ellipsoid==undefined) throw new TypeError(`unrecognised datum ‘${datum}’`);
        this._datum = datum;
    }


    /**
     * Converts ‘this’ (geocentric) cartesian (x/y/z) coordinate to (geodetic) latitude/longitude
     * point (based on the same datum, or WGS84 if unset).
     *
     * Shadow of Cartesian.toLatLon(), returning LatLon augmented with LatLonEllipsoidal_Datum
     * methods convertDatum, toCartesian, etc.
     *
     * @returns {LatLon} Latitude/longitude point defined by cartesian coordinates.
     * @throws  {TypeError} Unrecognised datum
     *
     * @example
     *   const c = new Cartesian(4027893.924, 307041.993, 4919474.294);
     *   const p = c.toLatLon(); // 50.7978°N, 004.3592°E
     */
    toLatLon(deprecatedDatum=undefined) {
        if (deprecatedDatum) {
            console.info('datum parameter to Cartesian_Datum.toLatLon is deprecated: set datum before calling toLatLon()');
            this.datum = deprecatedDatum;
        }
        const datum = this.datum || datums.WGS84;
        if (!datum || datum.ellipsoid==undefined) throw new TypeError(`unrecognised datum ‘${datum}’`);

        const latLon = super.toLatLon(datum.ellipsoid); // TODO: what if datum is not geocentric?
        const point = new LatLonEllipsoidal_Datum(latLon.lat, latLon.lon, latLon.height, this.datum);
        return point;
    }
    /**
     * Converts ‘this’ cartesian coordinate to new datum using Helmert 7-parameter transformation.
     *
     * @param   {LatLon.datums} toDatum - Datum this coordinate is to be converted to.
     * @returns {Cartesian} This point converted to new datum.
     * @throws  {Error} Undefined datum.
     *
     * @example
     *   const c = new Cartesian(3980574.247, -102.127, 4966830.065, LatLon.datums.OSGB36);
     *   c.convertDatum(LatLon.datums.Irl1975); // [??,??,??]
     */
    convertDatum(toDatum) {
        // TODO: what if datum is not geocentric?
        if (!toDatum || toDatum.ellipsoid == undefined) throw new TypeError(`unrecognised datum ‘${toDatum}’`);
        if (!this.datum) throw new TypeError('cartesian coordinate has no datum');

        let oldCartesian = null;
        let transform = null;

        if (this.datum == undefined || this.datum == datums.WGS84) {
            // converting from WGS 84
            oldCartesian = this;
            transform = toDatum.transform;
        }
        if (toDatum == datums.WGS84) {
            // converting to WGS 84; use inverse transform
            oldCartesian = this;
            transform = this.datum.transform.map(p => -p);
        }
        if (transform == null) {
            // neither this.datum nor toDatum are WGS84: convert this to WGS84 first
            oldCartesian = this.convertDatum(datums.WGS84);
            transform = toDatum.transform;
        }

        const newCartesian = oldCartesian.applyTransform(transform);
        newCartesian.datum = toDatum;

        return newCartesian;
    }


    /**
     * Applies Helmert 7-parameter transformation to ‘this’ coordinate using transform parameters t.
     *
     * This is used in converting datums (geodetic->cartesian, apply transform, cartesian->geodetic).
     *
     * @private
     * @param   {number[]} t - Transformation to apply to this coordinate.
     * @returns {Cartesian} Transformed point.
     */
    applyTransform(t)   {
        // this point
        const { x: x1, y: y1, z: z1 } = this;

        // transform parameters
        const tx = t[0];                    // x-shift in metres
        const ty = t[1];                    // y-shift in metres
        const tz = t[2];                    // z-shift in metres
        const s  = t[3]/1e6 + 1;            // scale: normalise parts-per-million to (s+1)
        const rx = (t[4]/3600).toRadians(); // x-rotation: normalise arcseconds to radians
        const ry = (t[5]/3600).toRadians(); // y-rotation: normalise arcseconds to radians
        const rz = (t[6]/3600).toRadians(); // z-rotation: normalise arcseconds to radians

        // apply transform
        const x2 = tx + x1*s  - y1*rz + z1*ry;
        const y2 = ty + x1*rz + y1*s  - z1*rx;
        const z2 = tz - x1*ry + y1*rx + z1*s;

        return new Cartesian_Datum(x2, y2, z2);
    }
}


/* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  */

export { LatLonEllipsoidal_Datum as default, Cartesian_Datum as Cartesian, datums, Dms };
```