Rules of the game

Participation in this practical work is compulsory to acquire the right to take the exam of the course ‘Modeling and simulation of composite structures up to rupture’. The involvement in the work, as well as the results obtained, will be evaluated and will form part of the final grade of the course. To answer the questions in this subject, you have the right to use your course notes.

Evaluation

You can work alone or in couple.

Your report is expected to consist of:

- Code Matlab;
- A synthetic report including figures or results requested in the subject.

Attention: as future engineers or researchers, you are responsible for your results, and not only for the process! The results (units, values) are therefore very important, double check your orders of magnitude carefully.

A first version of the report will be given to the supervisor at the end of this practical session to assess the progress of the work. The final version is expected one week after the date of the session (by e-mail to federica.daghia@ens-paris-saclay.fr).

1 Behavior of a single ply and change of basis

We are interested in the behavior of a single ply, described as an orthotropic material, under the plane stress assumption. The material properties in the basis associated with the main directions of orthotropy $(1, 2, 3)$ are known. We propose to determine the material properties in a basis (x, y, z), rotated by an angle θ with respect to the basis $(1, 2, 3)$ (Figure 1).

We define

$$\boldsymbol{\sigma}_m = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}, \quad \boldsymbol{\sigma}_p = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}, \quad \boldsymbol{\varepsilon}_m = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix}, \quad \boldsymbol{\varepsilon}_p = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{bmatrix}$$

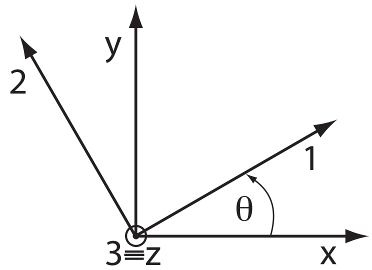

Figure 1 • Rotation of the basis $(1, 2, 3)$ to the basis (x, y, z) (direction $3 \equiv z$ out of the plane of the figure).

Question 1 Write a function in Matlab that gives the plane stress stiffness matrix $\mathbf{C}_m$ in the basis $(1, 2, 3)$ in function of the elastic proprieties of the single ply E_1, E_2, ν_{12}, G_{12}.

Question 2 Write a function in Matlab that gives the strain related to thermal effects $\boldsymbol{\varepsilon}_m^{th}$ in the basis $(1, 2, 3)$ in function of coefficients of thermal expansion α_1 et α_2 the temperature difference ΔT from the reference temperature.

Question 3 Write a function in Matlab that gives the stiffness matrix $\mathbf{C}_p$ in the basis (x, y, z) in function of the matrix $\mathbf{C}_m$ defined in Question 1 and the angle θ.

Question 4 Write a function in Matlab that draws, in polar coordinates, the elastic modulus E_x in direction x in function of the angle θ. The modulus is defined by $E_x(\theta) = \frac{1}{S_{xxxx}(\theta)}$ where $\mathbf{S}_p(\theta) = \text{inv}(\mathbf{C}_p(\theta))$.

Question 5 Draw $E_x(\theta)$ for the following materials:

- Material 1 (Unidirectional ply):

$$E_1 = 100 \text{ GPa}, \quad E_2 = 10 \text{ GPa}, \quad \nu_{12} = 0.3, \quad G_{12} = 5 \text{ GPa}$$

- Material 2 (Plain-Woven ply):

$$E_1 = 50 \text{ GPa}, \quad E_2 = 50 \text{ GPa}, \quad \nu_{12} = 0.3, \quad G_{12} = 5 \text{ GPa}$$

- Material 3 (Isotropic):

$$E = 50 \text{ GPa}, \quad \nu = 0.3, \quad G = \frac{E}{2\,(1+\nu)}$$

Comment.

2 Constitutive behavior of a composite plate

We are now interested in the behavior of a laminate, shown in Figure 2, described by the Classical Laminates' Theory.

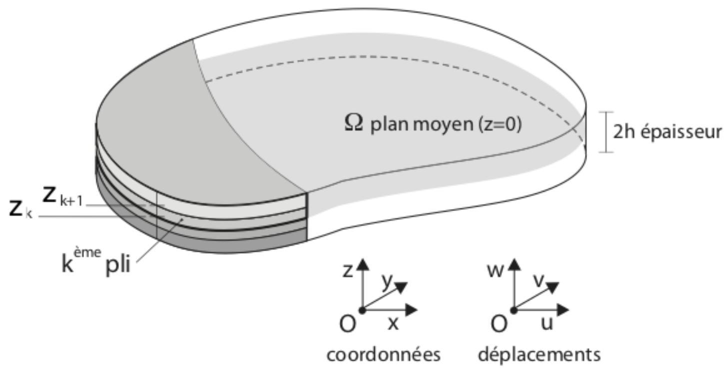

Figure 2 • Laminate under consideration

The displacement field is assumed as follows:

$$u(x,y,z) = u_0(x,y) - z w_{0,x}(x,y)$$

$$v(x,y,z) = v_0(x,y) - z w_{0,y}(x,y)$$

$$w(x,y,z) = w_0(x,y)$$

The vectors $\boldsymbol{\mu}$ and $\boldsymbol{\chi}$ are defined as:

$$\boldsymbol{\varepsilon}_p = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{bmatrix} + z \begin{bmatrix} -\frac{\partial w_0^2}{\partial x^2} \\ -\frac{\partial w_0^2}{\partial y^2} \\ -2\frac{\partial w_0^2}{\partial x \partial y} \end{bmatrix} = \boldsymbol{\mu} + z\boldsymbol{\chi}$$

The vectors $\mathbf{N}$ and $\mathbf{M}$ contain the resultants and the moments associated with the in-plane stresses (generalized stresses).

$$\mathbf{N} = \begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \int_{-h}^{+h} \boldsymbol{\sigma}_p \mathrm{d}z = \int_{-h}^{+h} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} \mathrm{d}z$$

$$\mathbf{M} = \begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = \int_{-h}^{+h} z\boldsymbol{\sigma}_p \mathrm{d}z = \int_{-h}^{+h} z \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} \mathrm{d}z$$

Question 6 Write a function in Matlab that gives the elastic behavior of the plate as a function of the properties of each ply and the information on the stacking sequence.

Question 7 Write a function in Matlab that gives the thermal contribution into the plate behavior as a function of the properties of each ply and the information on the stacking sequence.

Question 8 Write a function in Matlab that gives the local strains and stresses in the material basis at the given position of z as a function of the generalized strain state of the plate, the properties of each ply and the information on the stacking sequence.

Question 9 Using the material 1 given in the previous question, calculate the matrices of the elastic behavior **A**, **B** et **D** for the following stacking sequences :

- Stacking sequence 1 (cross-ply): $[0/90]_s$;
- Stacking sequence 2 (quasi-isotropic): $[0/45/90/-45]_s$;
- Stacking sequence 3 (non symmetric): $[0/90]$.

(We assume that the thickness of each ply is $h_p = 0.125$ mm).
Comment on the expected results.

Question 10 We now consider the stacking sequence 2, subjected to a uni-axial tensile test. One chooses to control the test in displacement in the direction x, and one imposes a constant strain $\mu_x = 0.01$.

- Determine the associated generalized stresses.
- Determine the local stresses in the material basis for each ply.
- Is the solution valid near a free edge? If not, explain how we can introduce a correction.

Question 11 We consider again the stacking sequence 2, subjected this time to a thermal load $\Delta T = -160°$ C (representative of the difference between the manufacturing temperature and the environmental temperature). Considering the thermal expansion coefficients $\alpha_1 = 0$ and $\alpha_2 = 30 \cdot 10^{-6} (°\mathrm{C})^{-1}$:

- Calculate the 'relaxed' generalized strains $\boldsymbol{\mu}_r$ and $\boldsymbol{\chi}_r$. Can this configuration be achieved in the real structure?
- Calculate the stresses due to the thermal loading in the material basis for each ply associated with the relaxed configuration.

3 Analytical solution of a simplified plate problem

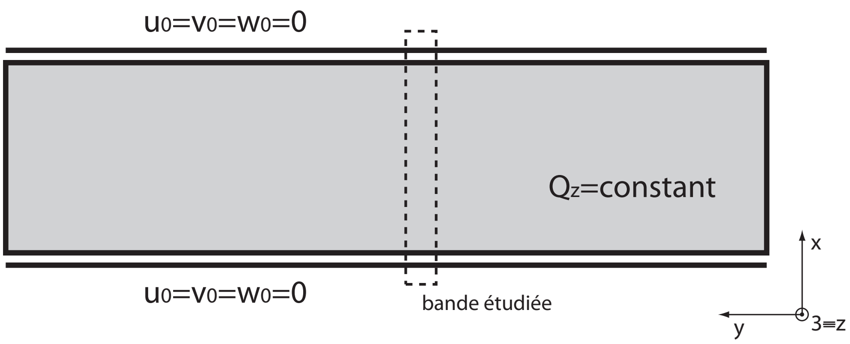

Figure 3 • Rectangular plate submitted to transverse loading.

Question 12 We consider a rectangular plate of dimensions 0.1×0.4 m (material 1, stacking sequence 2). It is supported ($u_0 = v_0 = w_0 = 0$) on the long sides and free on the short sides, and subjected to a pressure distributed on the surface $Q_z = 5 \cdot 10^4$ N/m^2 (see Figure 3). To determine the maximum deflection, we consider a band far enough from the free edges, which makes it possible to introduce the following simplifying assumptions:

$$f(x, y, z) = f(x, z) \quad \text{et} \quad \frac{\partial f}{\partial y} = 0$$

- Considering the plate equations and the proposed simplification, determine the distribution of moments $M_{xx}(x)$ along the band.
- Determine the displacement $w_0(x)$ and deduce from it the relation between the imposed loading and the maximum deflection.
- Compare the results obtained with the results of the finite element calculation (shell elements) given in Figure 4. Comment on and interpret any differences.
- Compare the results obtained with the results of the finite element calculations (3D elements) given in Figures 5 and 6. The two calculations differ in terms of the boundary conditions, specified in these figures. Comment on and interpret any differences.

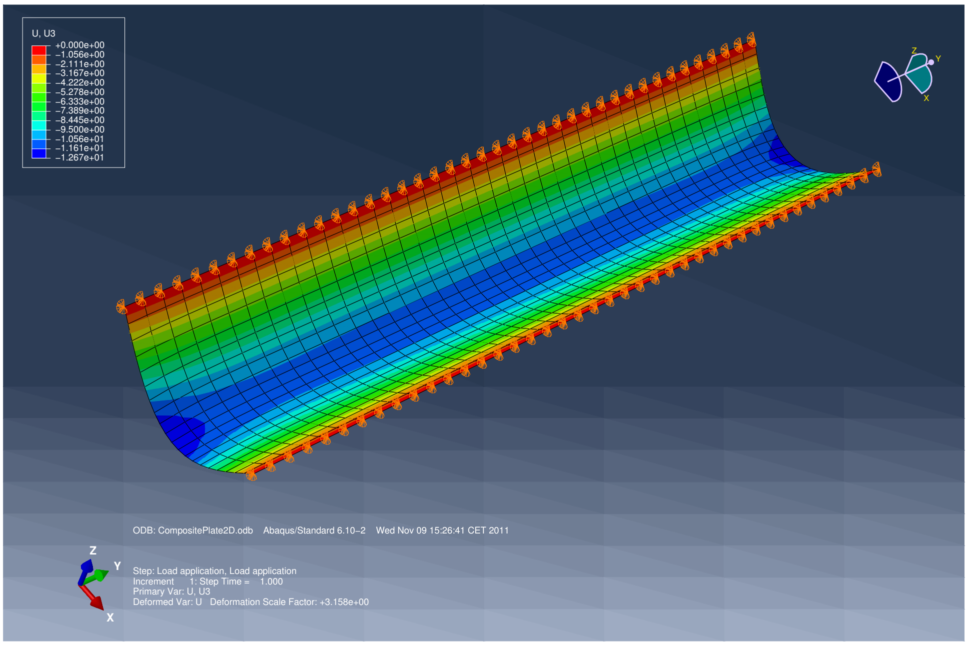

Figure 4 • 2D computation (shell elements)
Displacement at the center of the plate (Maximum deflection)
$= -11.11$mm

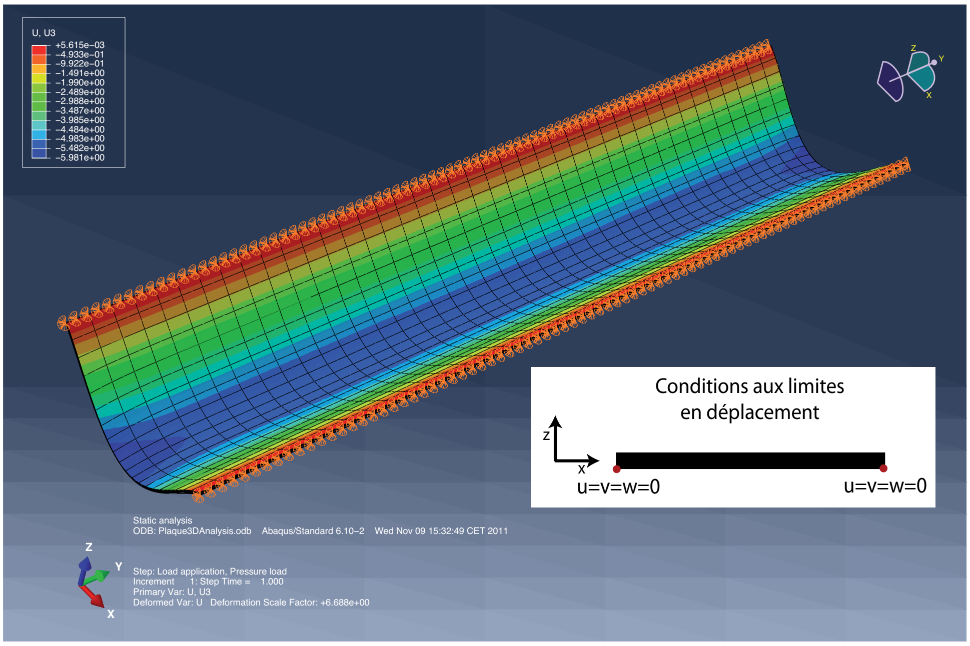

Figure 5 • 3D computation (volumic elements)
Displacement at the center of the plate (Maximum deflection)
$= -5.44$mm

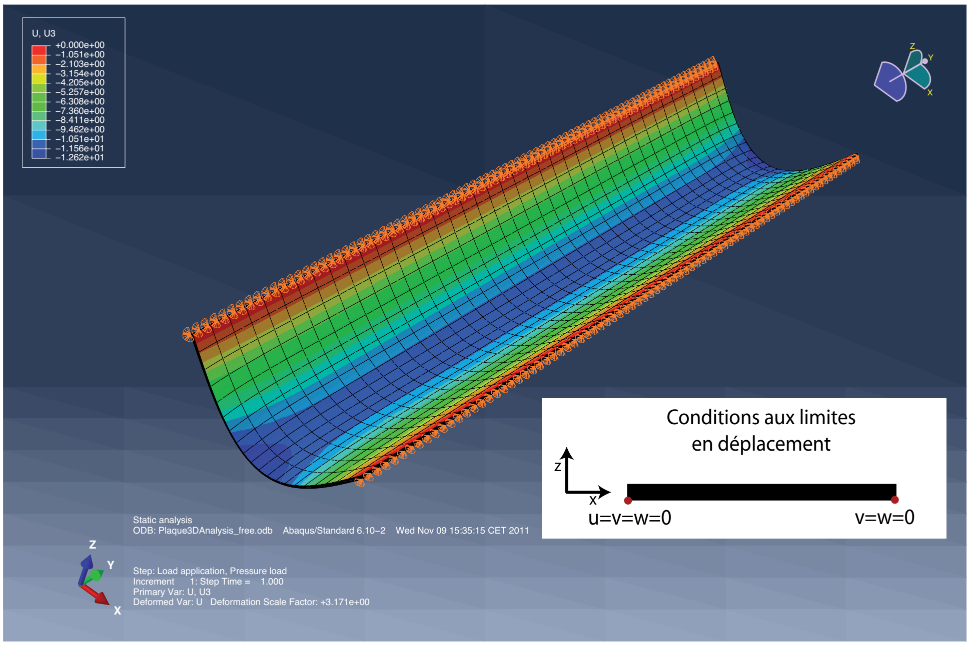

Figure 6 • 3D computation (volumic elements)
Displacement at the center of the plate (Maximum deflection)
$= -11.15$mm

4 Analysis of finite element results on plates

We consider a plate made up of plies all oriented in the same direction. In this plate, we cut different specimens of dimensions 150mm $*$ 30mm. We want to simulate uni-axial traction tests, which we model as in Figure 7. To reduce the computation time, one simulates only a quarter of each tested plate sample, which one models as in Figure 8. In particular, the composite is modeled with multi-layer composite plate elements. We propose to check if the calculations carried out using ABAQUS are correct and if they are representative of the uni-axial tensile tests. Material 1 is used in the simulations.

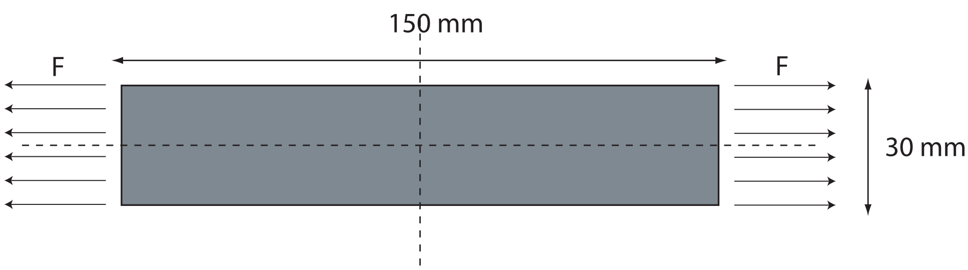

Figure 7 • Plate submitted to a tensile loading

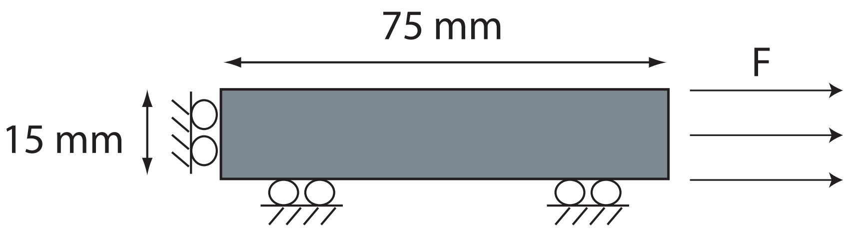

Figure 8 • A quarter of the plate submitted to a tensile loading

Question 13 We consider a specimen at 0°. From the html *HTMLreport.html* page in the *Report_0deg* directory, identify the Young's modulus E_1 and the Poisson's ratio ν_{12} (give the formulas used in the notation presented in the course and in the notation used in the Abaqus report). Is the calculation performed representative of the tensile test? Why ?

Question 14 We consider a specimen at 90°. From the html *HTMLreport.html* page in the *Report _90deg* directory, identify the Young modulus E_2, and check again the Poisson's coefficient ν_{12} (give the formulas used in the notation presented in the course and in the notation used in the Abaqus report). Is the calculation performed representative of the tensile test? Why ?

Question 15 We consider a specimen cut at 45°. From the html *HTMLreport.html* page in the *Report_45deg* directory, identify the Young's modulus G_{12} (give the formulas used in the notation presented in the course and in the notation used in the Abaqus report). Is the calculation performed representative of the tensile test? Why ?