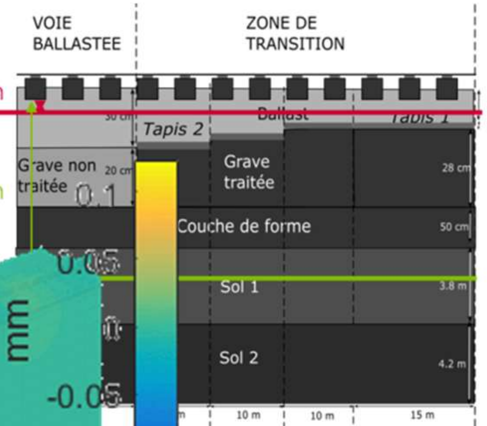
# Vibrations /Modal analysis

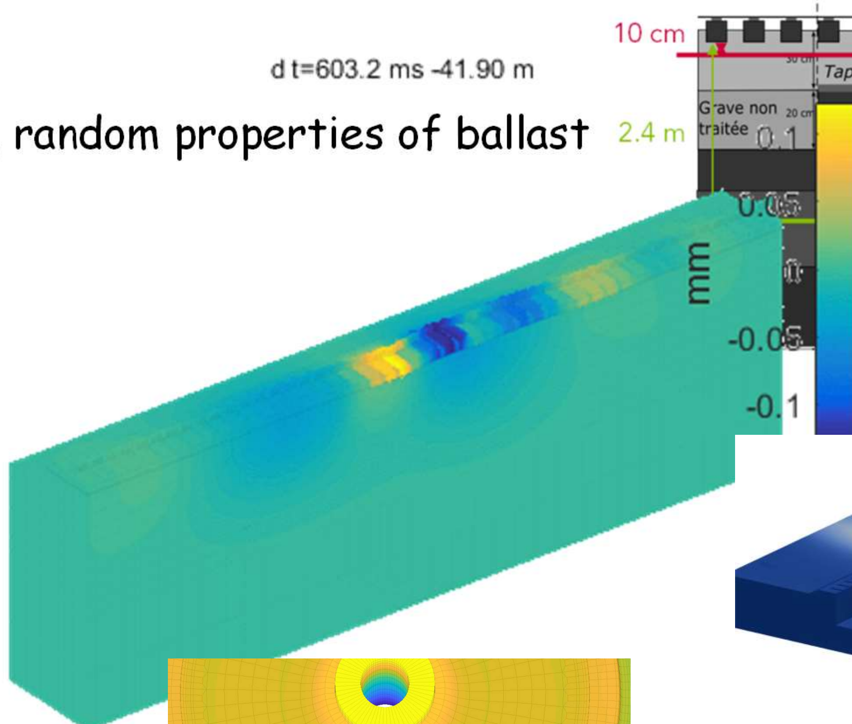
Etienne Balmes — Ensam/PIMM, SDTools
Mathieu Corus — CentraleSupelec, EDF

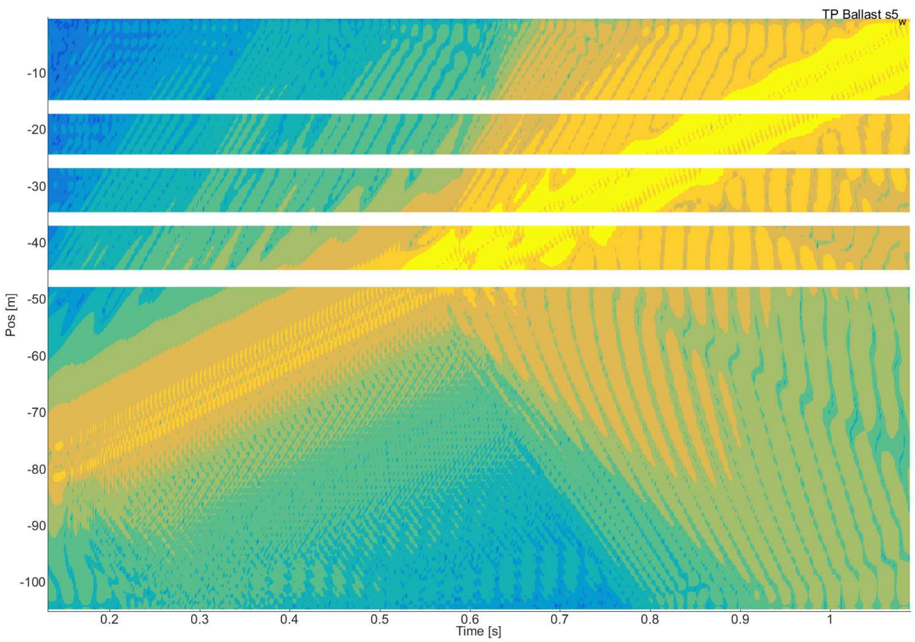
http://savoir.ensam.eu/moodle/course/view.php?id=1874

# Track/train interaction

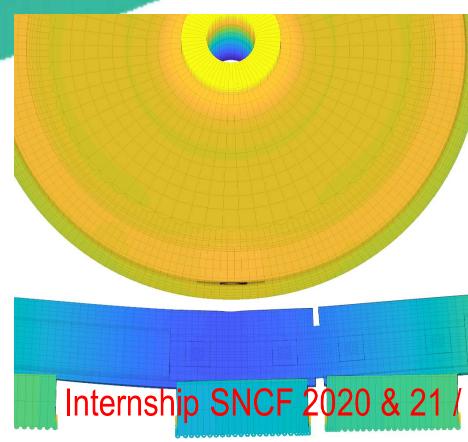
- Piece-wise periodic structure with 15e3 node per 60cm slice
  - Full 3D > 5h, 90 Gb
  - Reduced 23 min, 100 Mb
- Scientific issues
  - model reduction, wave propagation, random properties of ballast
  - Rail/wheel contact, fatique



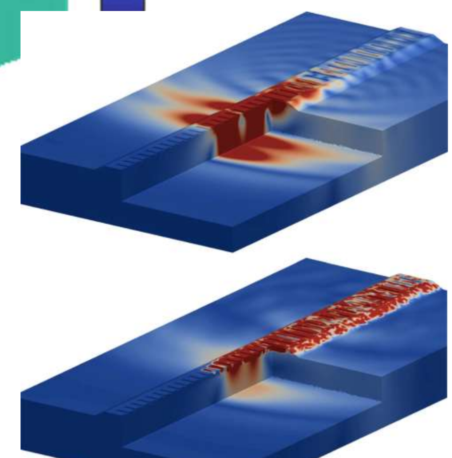




Ph.D. Elodie Arlaud (2014-16), Hadrien Pinault (2017-2020)



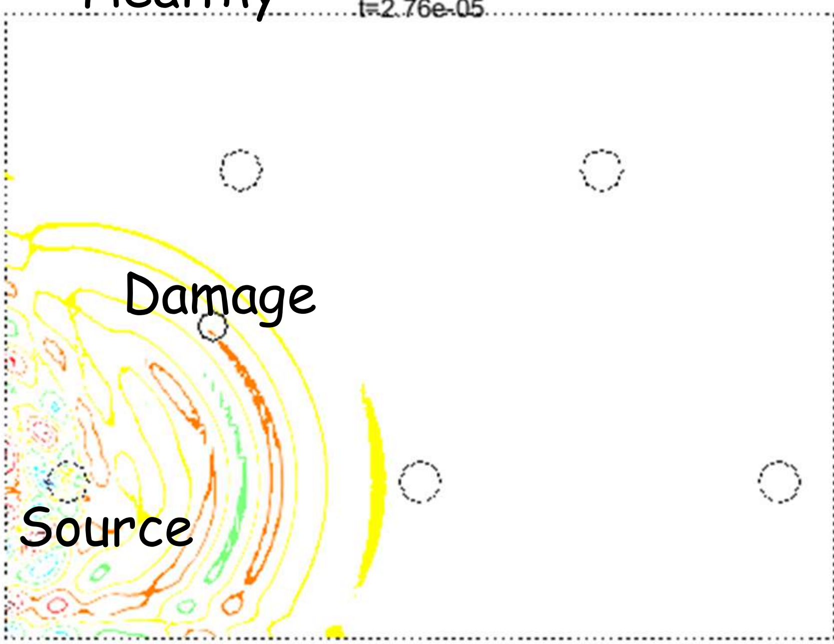
Internship SNCF 2020 & 21 / Olivier Vo Van



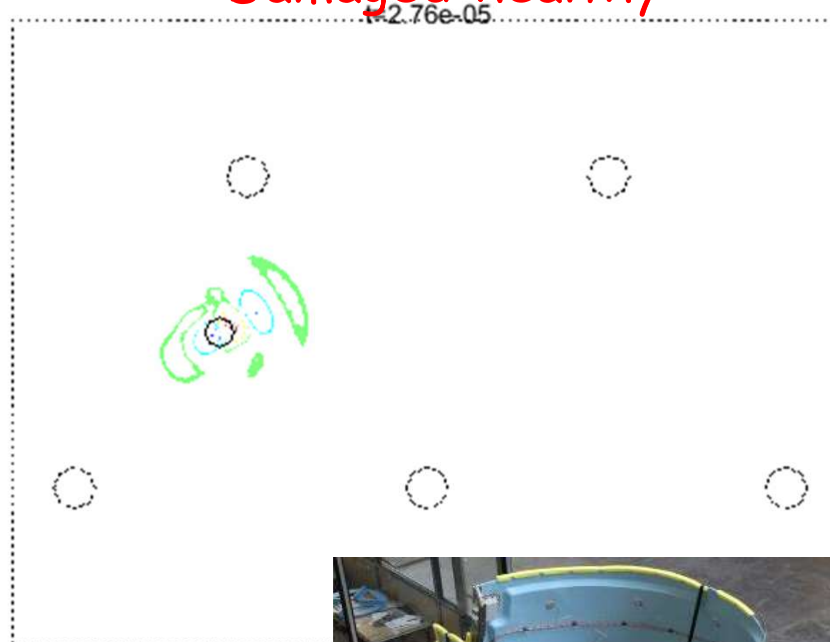
Lucio de Abreu Corrêa & al.

# SHM transients

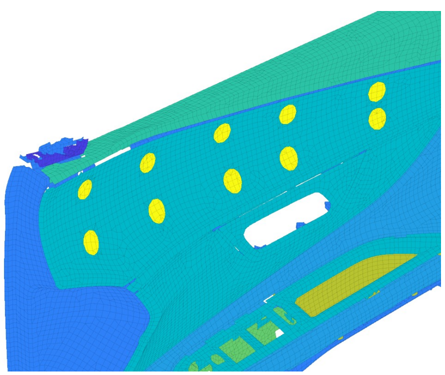
Healthy

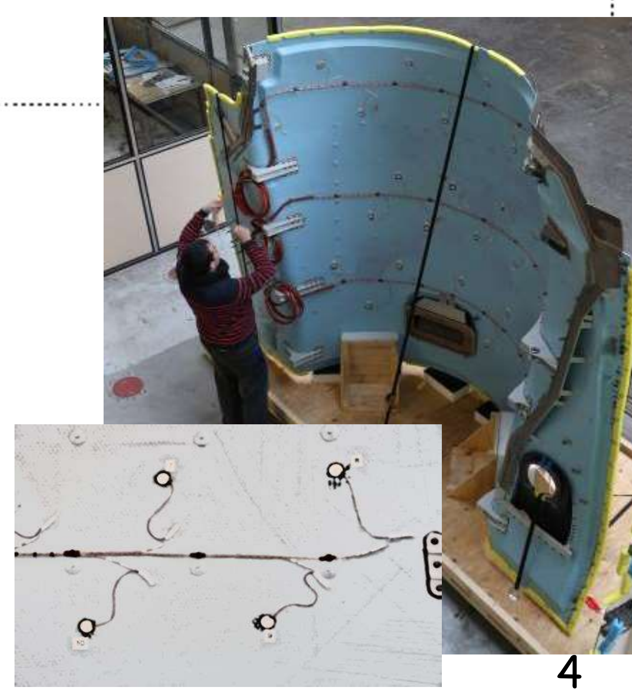



Damaged-healthy



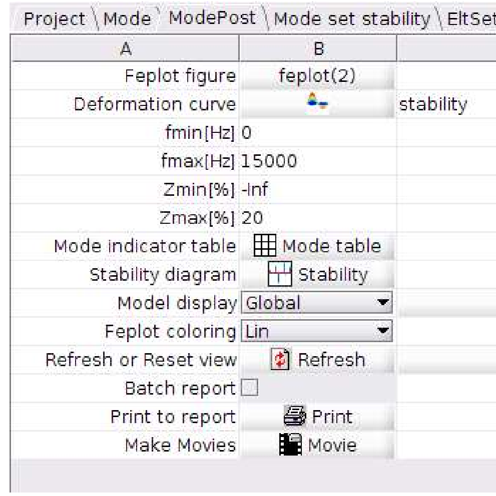

- Electrical signals used for measurement
- Damage acts as a source in healthy-damaged signal



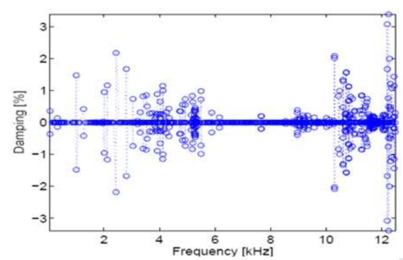


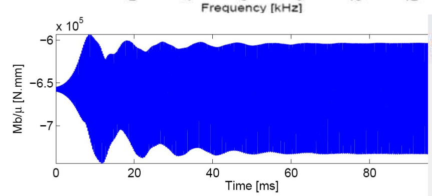
# Brake squeal simulation

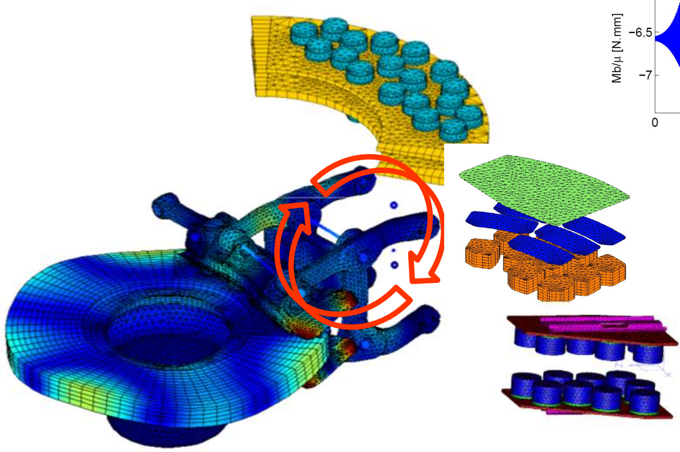
- 1 full time @ SDTools
  (Audi, Daimler, Stellantis, CBI, …)
- Advanced solvers in frequency & time
- Objectives
  - Industrial design tools
  - Parametric model reduction
  - Optimized transients

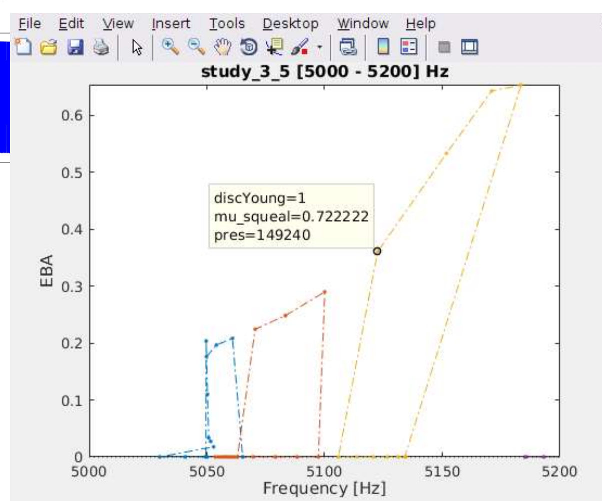


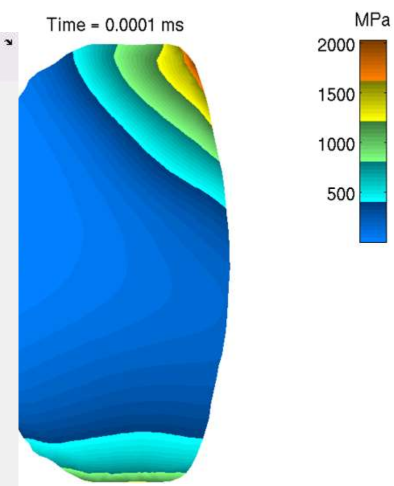






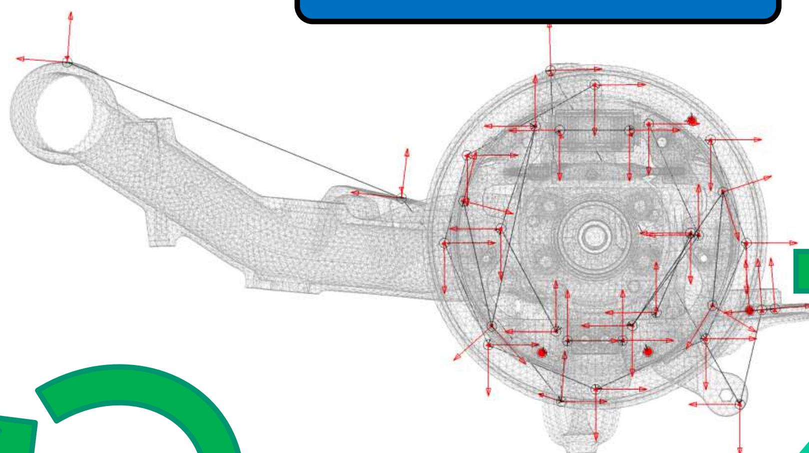



# Squeal testing : combined test/FEM

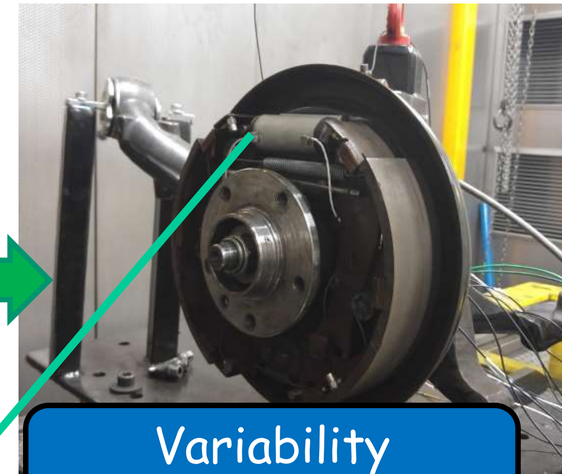
Full FEM

Update :
1. Geometry
2. Parameters
3. component contacts

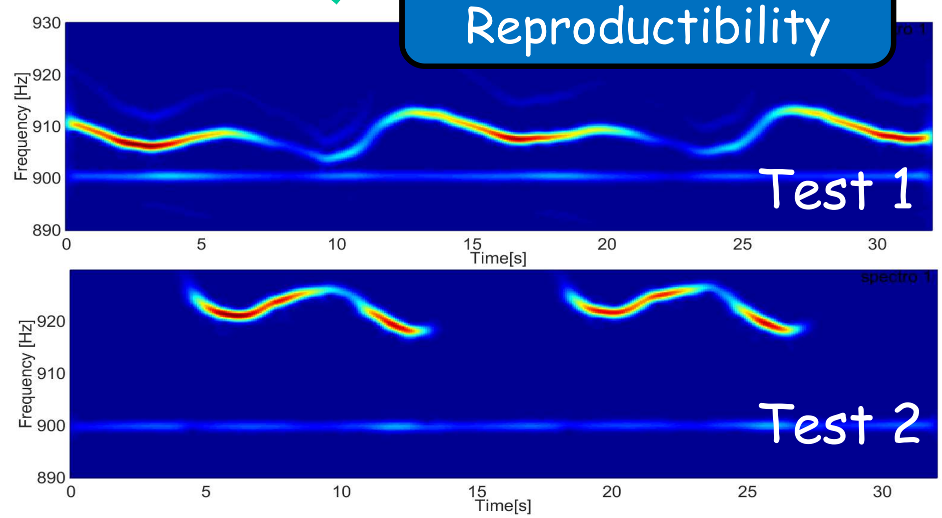
Sensor placement



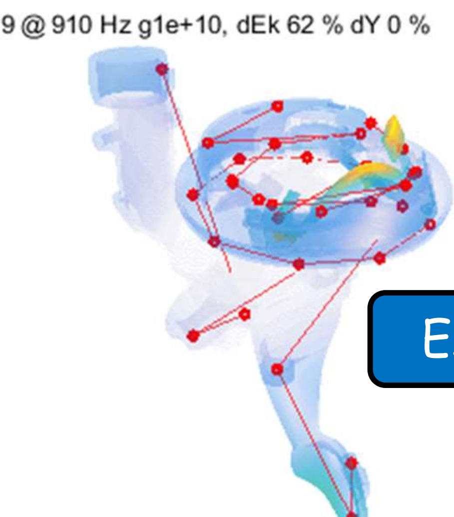
Real test



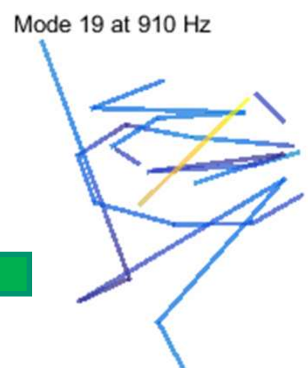
Variability Reproductibility




Squeal : time varying with wheel position




Expansion

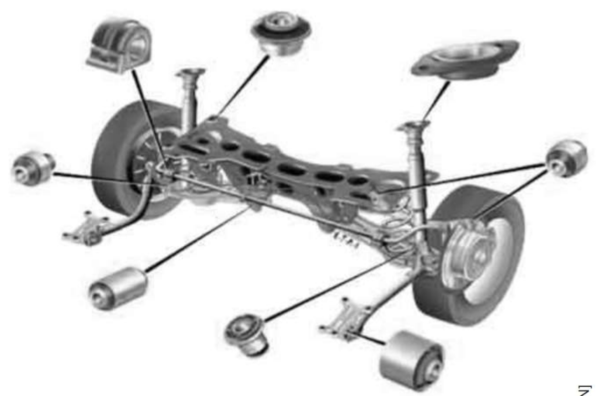


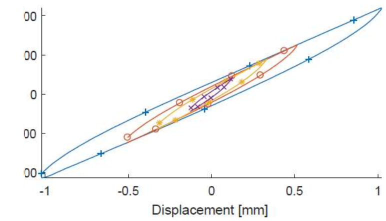

Shape extraction

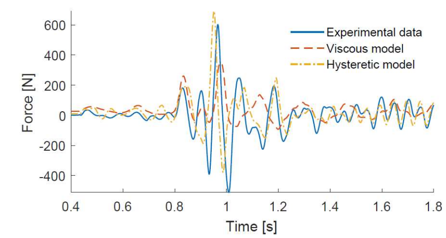
# Viscoelastic damping

## Scientific problems with damping

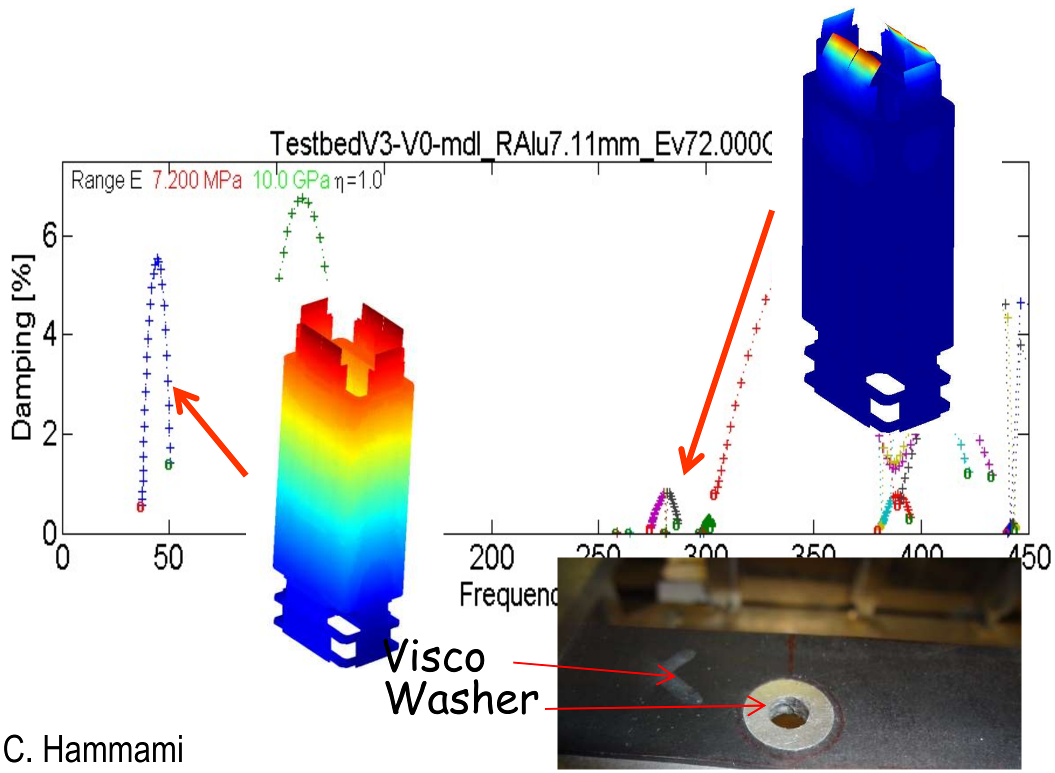
- 0D (structure) vs. 3D (material + geometry) modeling
- Damping in complex structures
- Homogeneization in composites



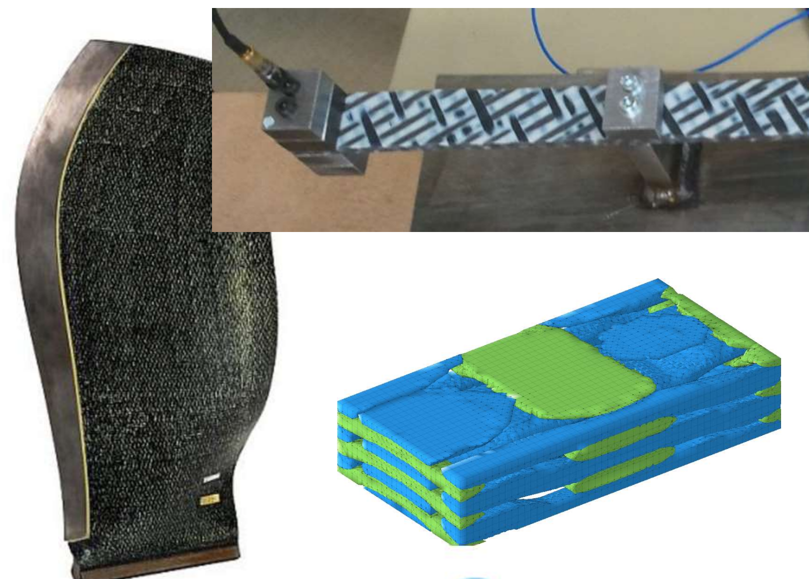



Ph.D. R. Penas 21




Ph.D. C. Hammami



Ph.D. F. Conejos 21

# SDT core focus

SDT (software since 1995, 700+ licenses)
SDTools (company since 2001
4 engineers (develop) + interact with clients & PhDs)

- FEM simulations
- System models (model reduction, state-space, active control)
- Experimental modal analysis
- Test/analysis correlation, model updating

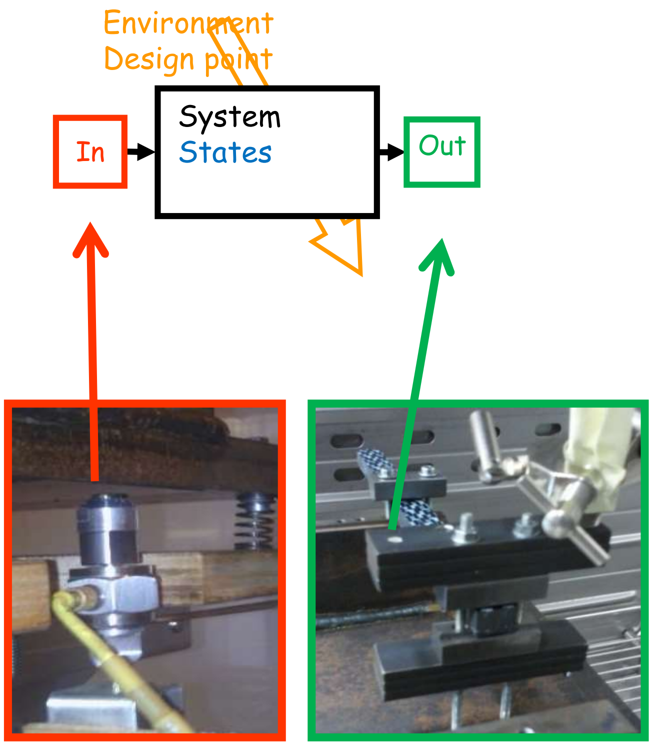
Simulation

Validation

| | |
|---|---|
| CAD/Meshing | CATIA, Workbench, … |
| FEM | NASTRAN, ABAQUS, ANSYS,... |
| Simulation | Adams, Simpack, Simulink,... |
| Testing | LMS TestLab, ME-Scope, … |

- Necessity: programmatic access to all steps
- Proposed solution: flexible toolbox & custom applications
- Base commercial library : for quality, durability, capitalization
- Consulting/research

# What is a system ?



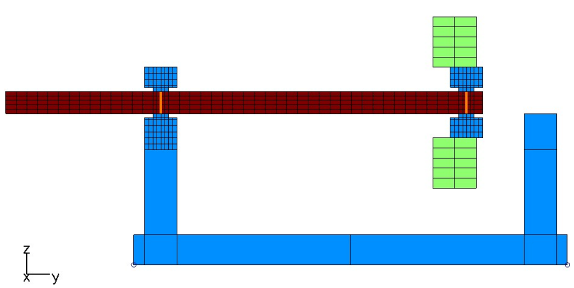

- **Inputs $u(t)$** : hammer with force measurement
- **Outputs $y(t)$**
  - Test : vibrometer on testbed
  - Computation : stresses
- **State $x(t)$**
  - Displacement & velocity field as function of time

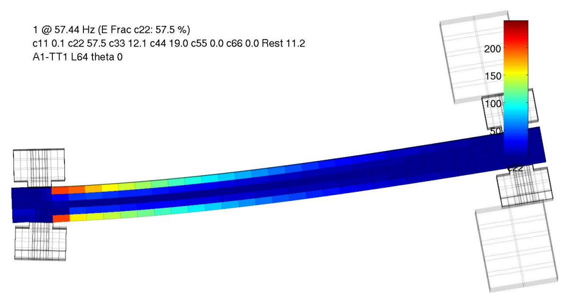
$$\{\dot{x}(t)\} = f(x(t), u(t), p, t) \qquad \text{evolution}$$

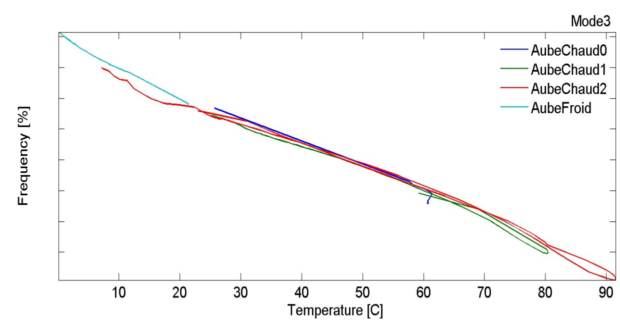
$$\{y(t)\} = g(x(t), u(t), p, t) \qquad \text{observation}$$

- **Environment variables $p$**
  - Dimensions, test piece (design point)
  - Temperature (value of constitutive law or state of thermo-viscoelastic)

- Feature : function of output (example modal frequency)

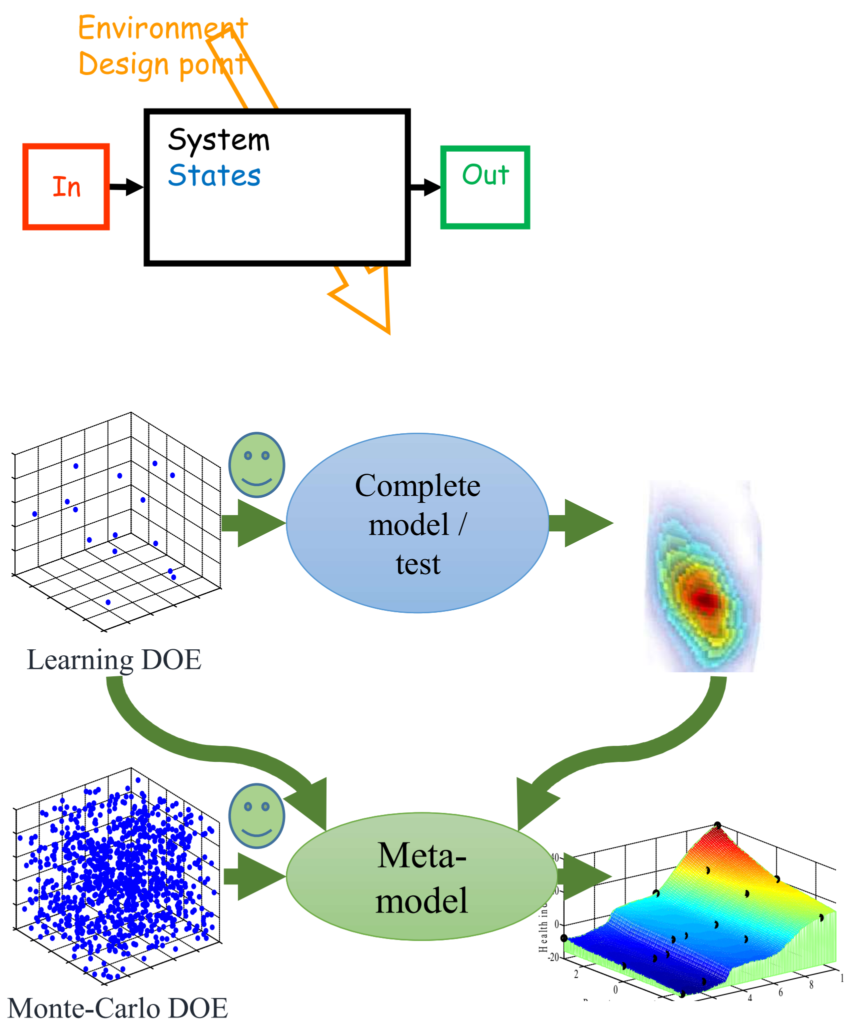





Simple example : modified Oberst test for 3D weaved composite test

# System models : nature & objectives?




## What is a model
- A function relating input and outputs
- For one or many parametric configurations

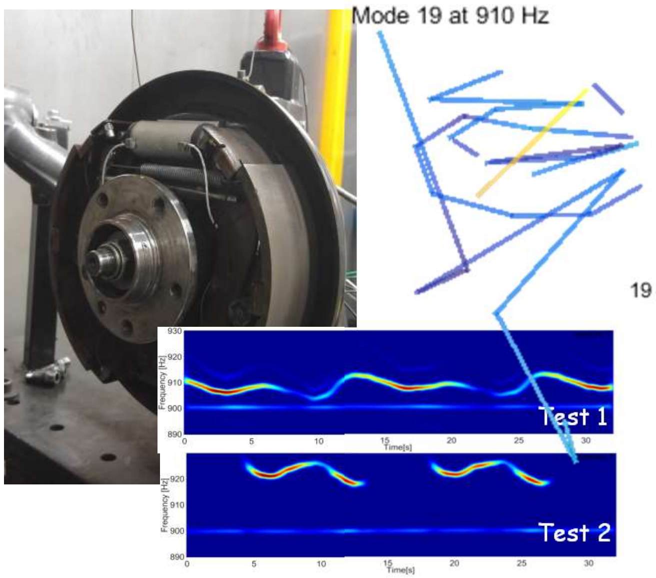
## Model categories
- Behavior models (meta-models)
  - Test, constitutive laws, Neural networks
  - Difficulties : choice of parametrization, domain of validity
- Knowledge models
  - Physical principles, low level meta-models

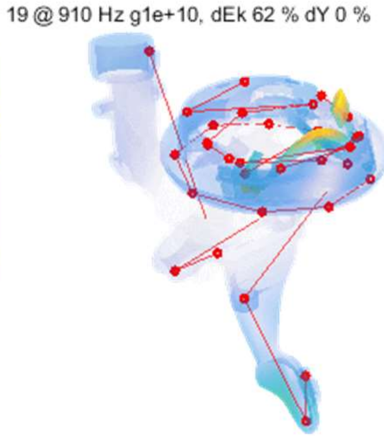
## Why do we need system models ?
**Design**
- Become predictive : understand, know limitations
- Perform sizing, optimize, deal with robustness

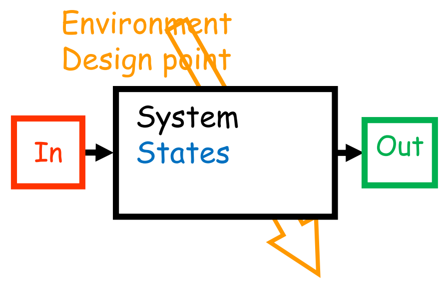
**Certify**
- Optimize tests : number, conditions
- Understand relation between real conditions and certification
- Account for variability

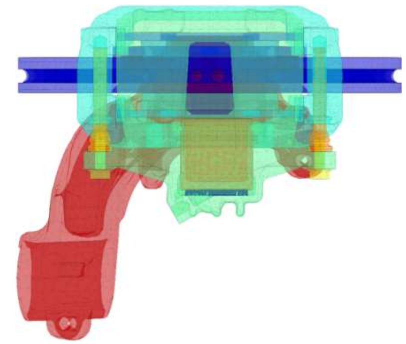
**Maintain during life**
- Design full life cycle (plan  maintenance)
- Use data for conditional maintenance (SHM)

# A system = I/O representation



Prototype

Virtual prototype

- ☺ all physics (no risk on validity)
- ☺ in operation response
- ☹ limited test inputs
- ☹ measurements only
- ☹ few designs

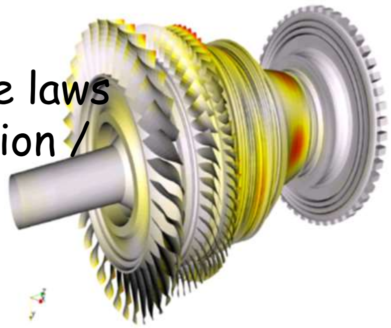
- 😐 Cost : build and operate

- ☹ limited physics (unknown & long CPU)
- ☹ design loads
- ☺ user chosen loads
- ☺ all states known
- ☺ multiple (but 1 hour, 1 night, several days, … thresholds)
- 😐 Cost : setup, manipulate

# FEM model / system model

$\sigma(x,t)$
$f(x,t)$

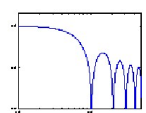
- Geometry
- Constitutive laws
- Discretization / idealization



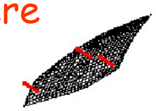
$u(x,t)$
$\sigma(x,t)$

Extensions : coupled, time variant, non-linear, …

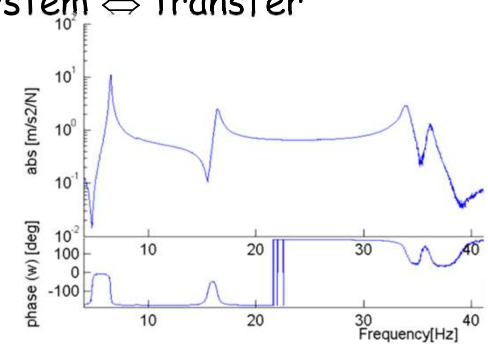
When



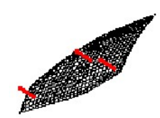
Where



Assumption : linear time invariant
System ⇔ transfer




Sensors

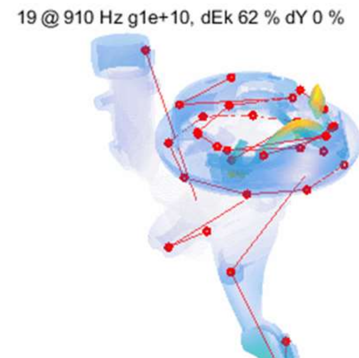


Extensions
- Coupling (structure, fluid, control, multi-body, …)
- Optimization, variability, damping, non linearity, …

# Model validation and verification

CAD Model

Experimental model




Continuous model +interfaces

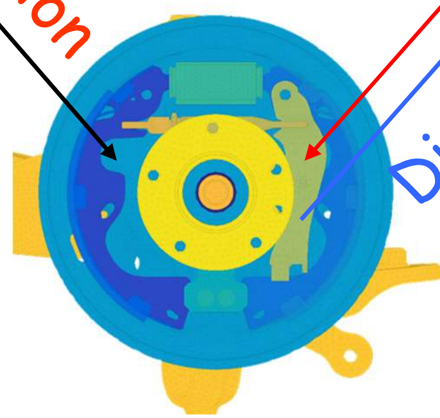
$$\text{On } \Omega \quad \mathrm{div}\sigma + F_v = \rho \ddot{u}$$
$$\text{On } \partial\Omega \quad u(x,t) = u_{given}$$
$$\{T\} = [\sigma]\{n\}$$

Verification

Design

Validation (Updating)

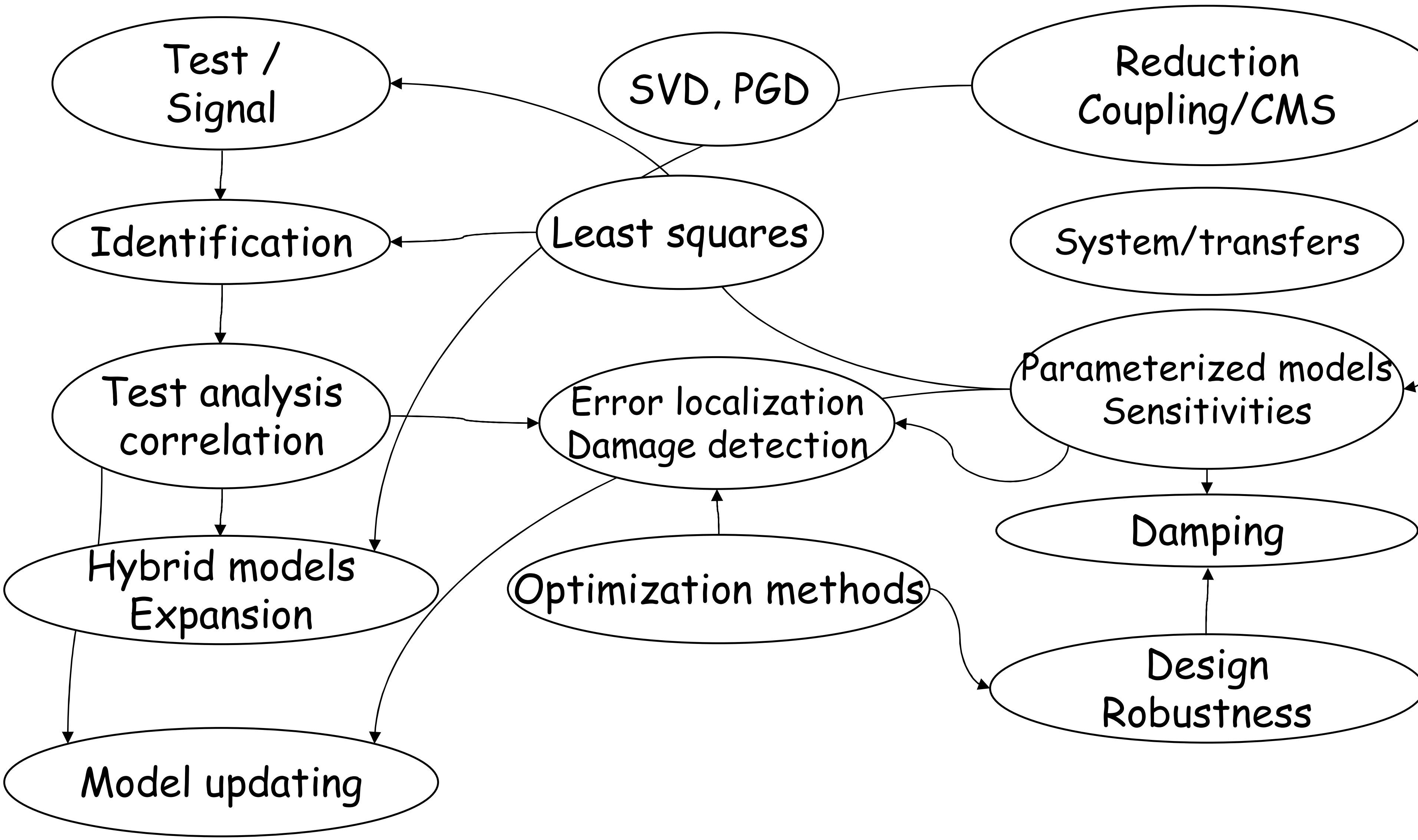
Dispersion



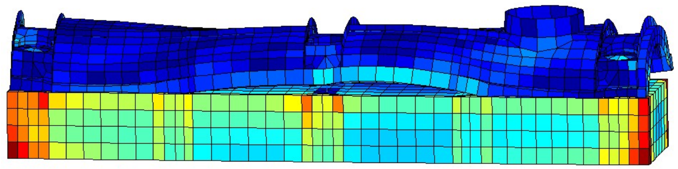
FE/numerical model

1990 : model updating
2000 : virtual prototype
2019 : digital twin

# Methods considered in the course



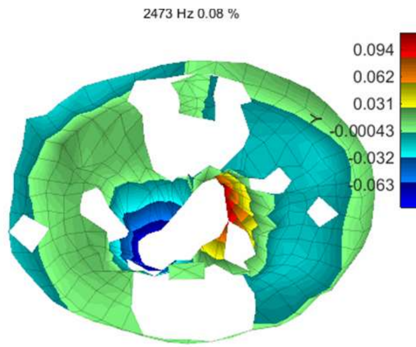
# Lab work / evaluation



Lab work (with Mathieu Corus )

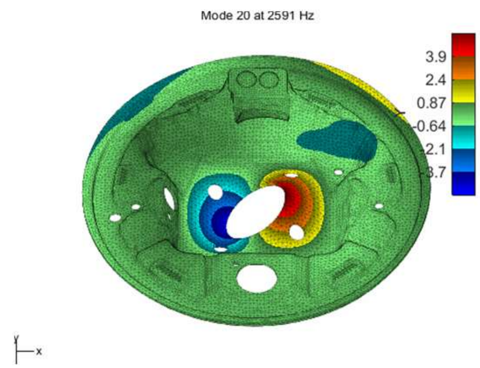
- 1 : code verification, signal, 1 DOF
- 2 : transfers, time/frequency
- 3 : identification, test/analysis correlation
- 4 : reduction and parametric models, updating

TP2-4 MATLAB+SDT : www.sdtools.com/sdtcur




Evaluation

- oral (PPT no interactive MATLAB), 30 mn (< 20 slides)
  equal weight for 1-2-3-4
- Work as pairs (not 3)
- Evaluation on how you expose & comment results
  (5 pt per lab)




Data : http://savoir.ensam.eu/moodle/course/view.php?id=1874

# 2022 Planning

- 20/9 Course1 : Intro, 1DOF, system
  C2 : modes & synthesis, base of reduction, spectral decomposition
- 27/9 C3 : signal for vibration (continuous vs. discrete, aliasing, windowing)
  C4 : Ritz and learning. Historical : McNeal, Craig-Bampton, …
- 4/10 Lab1 : 1 DOF, state-space, signal
- 11/10 C5 : experimental modal analysis : from test to a system model. Inverse problem.
  C6 : Model parameterization, sensitivity computations, validity and error control
- 18/10 C7 : Reduction for reanalysis. Topology correlation
  C8: Test/analysis correlation. Observation, MAC, expansion. Start of model updating.
- 25/10 Lab2 : modal base frequency domain, transfers, transient, signal processing
- 08/11 C9 : damping : devices, physical mechanisms, numeric tools
  C10 : CMS (Component Mode Synthesis), coupling models, reduction for coupling
- 15/11 : Lab3 : identification, sub-space, test/analysis
  22/11 : Lab4 : Parametric models, reduction, damping, updating
- 29/11 : C11 subspace methods, current issues with non-linear systems

http://savoir.ensam.eu/moodle/course/view.php?id=1874

# To go further

Course material (notes, slides)

https://savoir.ensam.eu/moodle/mod/folder/view.php?id=79532

[1] D. Inman, Engineering Vibration. Prentice-Hall, Englewood Clis, N.J., 1994.

[2] M. Geradin and D. Rixen, Mechanical Vibrations. Theory and Application to Structural Dynamics. John Wiley & Wiley and Sons, 1994, also in French, Masson, Paris, 1993.

[3] R. J. Craig and A. Kurdila, Fundamentals of Structural Dynamics. Wiley, 2006.

[4] W. Heylen, S. Lammens, and P. Sas, Modal Analysis Theory and Testing. KUL Press, Leuven, Belgium, 1997.

[5] D. Ewins, Modal Testing: Theory and Practice. John Wiley and Sons, Inc., New York, NY, 1984, 2009.

# Keywords : 1 DOF / signal

After intro course: (chapter1 and section 2.1, modal.pdf)

1. Transfer (time, Fourier $i\omega$/Laplace $s$, asymptotic prop, NL)
2. Poles, resonance, damping ratio -3 dB method
3. State-space, poles
4. 1 DOF time exponential, convolution, logarithmic decrement
5. Strategies for transient in time & frequency
6. Equivalent power

Measurement and signal processing (CM3, signal.pdf)

1. DFT $f_k$, relation $\delta t$, T, $\delta f$, linearity, dilatation, (section 3.1)
2. Aliasing : Shannon's theorem, when, mitigate, … (s3.2.1)
3. Leakage & windowing : continuous vs. DFT (s3.2.2)
4. Transfer function estimate (H1, coherence) (s3.3)
5. Technology : sensors, actuators, acquisition

TP1 : code verification for 1DOF, integration, signal

# Keywords : modes & synthesis, reduction

Modes & synthesis (CM2, see also second part of Modal.pdf slides)

1. Inputs/outputs, IO shape matrix, disp, resultants, … (s2.1)
2. Discrete modes (harmonic solution without input), orthogonality (section 2.2.1)
3. Ritz/Galerkin principles
4. Ritz/modal coordinates, PPV, series & state-space, time/freq strategies
5. Modal & Rayleigh damping, modal damping in physical coord (s 2.2.x)
6. Peak visibility, truncation, effective contributions (s 2.2.x)

Reduction (course 4, see also Reduction.pdf slides)

1. Ritz/Galerkin & learning
2. Modes + Residual flexibility (section 4.3)
3. McNeal= Ritz with "residual vectors", pre-filter low frequency modes
4. Residual vectors in presence of flexible modes
5. Guyan, Craig-Bampton = enforced displacement & bandwidth (s 4.3.2)

Left for other course : from vector set to basis

# Keywords : parametric models

Sensitivity / extended uses of modes (co. 5, SensitivityReanalysis.pdf)

1. Parametrization (s 9.1)
2. Sensitivity of static response, adjunct state (s 9.2)
3. Sensitivity of frequencies : relation with energy distribution (s 9.3)
4. Sens. of mode-shapes : modal crossing + numerical strategy for inverse of underdetermined problem

Parametric studies (course 6)

1. Reanalysis example in modal basis (start by continuous case of spring on tensioned wire). Generalization to $\Delta K$.
2. Multi-model and nominal + residual methods
3. Illustrations (damping/updating)
4. Error control, iterative basis refinement
5. Orthogonalization strategies, GS/GSM/IGSM, Mseq

TP2 : modes & synthesis, signal

# Keywords : experimental modal analysis

Identification (course 7, EMA.pdf)

1. Identification demo
2. Inverse problems : model forms, data (s 6.1)
3. Model forms for identification, discussion of residual terms
4. Frequency domain least-squares solution, implicit NL
5. Evaluation of results (ch 7)

Test/analysis correlation (course 8, Correlation.pdf)

1. Topology correlation (s 8.1)
2. Measuring distance between test & analysis. Shape correlation : MAC, pairing issues (s 8.2)
3. Static condensation/expansion, reduced mass, orthogonality on sensors (s 8.3)
4. Hybrid models

# Current trends 1

Damping (course 9, damping.pdf)

1. Sample damping devices
2. Notion of coupling & impact on damping
3. Viscoelasticity/complex modulus, MSE
4. Real modes/modal damping & separation
   complex modes & enhanced reduction
5. Internal states (// with friction)

Updating (course 10, updating.pdf)

1. Typical errors : property, geometry, contact, model
2. Physical and equivalent models
3. Least squares and conditioning, SVD (link with TP3)
4. Error localization
5. Sample applications

TP3 : parametric models, damping, identification, correlation

# Current trends 2

Substructuring (Component mode synthesis) (course 11)

1. Coupling conditions : energy or continuity
2. constraints : elimination, Lagrange, Penalization
3. contact & locking/stress concentration
4. Reduction for CMS : classical Craig-Bampton, CMT
5. The “problem” of large interfaces

Features in vibration behavior (course 12)

1. SVD for mechanics : principal loads, modal energy coordinates, interface DOFs
2. Modal coordinates, physical and macro-models models of junctions with contact/friction
3. Geometrically periodic systems (engine, track)