ICE503 DSP-Homework#4

1. Let $X(e^{j\omega})$ denote the discrete-time Fourier transform (DTFT) of $x[n]$.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

   Proof that

   (a) the DTFT of $x[-n]$ is $X(e^{-j\omega})$.

   (b) the DTFT of $x^*[n]$ is $X^*(e^{-j\omega})$.

   (c) the DTFT of $x^*[-n]$ is $X^*(e^{j\omega})$.

   (d) the DTFT of $\text{Re}\{x[n]\}$ is $X_{cs}(e^{j\omega})$.

   (e) the DTFT of $x_{cs}[n]$ is $\text{Re}\{X(e^{j\omega})\}$.

   (f) the DTFT of $x[n-n_0]$ is $e^{-jn_0\omega}X(e^{j\omega})$

2. An LTI system is described as

$$h[n] = \left(\frac{1}{2}\right)^n \mu[n]$$

   (a) Determine the DTFT of the system $H(e^{j\omega})$.

   (b) If the input sequence is $x[n] = 2\delta[n] - \delta[n-1]$, determine the DTFT of the input sequence $X(e^{j\omega})$. The output sequence is $y[n] = x[n] * h[n]$, determine the DTFT of the output sequence $Y(e^{j\omega})$.

   (c) If the input sequence is $x[n] = \sin(\frac{\pi n}{2})$, determine the DTFT of the input sequence $X(e^{j\omega})$. The output sequence is $y[n] = x[n] * h[n]$, determine the DTFT of the output sequence $Y(e^{j\omega})$.

(There is MATLAB simulation in page 2.)

3. MATLAB simulation:

An LTI system is described as

$$h[n] = \left(\frac{1}{3}\right)^n \mu[n]$$

and the input sequence is described as

$$x[n] = \delta[n] - \frac{1}{2}\delta[n-1]$$

(a) Use stem function to plot $x[n]$ and $h[n]$ for $0 \leq \mathrm{n} \leq 99$. Plot $x[n]$ in subplot(2,1,1), plot $h[n]$ in subplot(2,1,2), and label each x-axis and y-axis clearly.

(b) Use the definition to calculate the DTFT of $x[n]$ and $h[n]$.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

where $\omega = 0{:}\frac{\pi}{100}{:}6\pi$.

(c) If the output sequence in time domain is $y[n] = x[n] * h[n]$, calculate $Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$ in frequency domain. Use plot function to plot the magnitude and angle part of $X(e^{j\omega})$, $H(e^{j\omega})$ and $Y(e^{j\omega})$ for $\omega = 0{:}\frac{\pi}{100}{:}6\pi$. Plot $|X(e^{j\omega})|$ in subplot(3,2,1), plot $\arg\{X(e^{j\omega})\}$ in subplot(3,2,2), plot $|H(e^{j\omega})|$ in subplot(3,2,3), plot $\arg\{H(e^{j\omega})\}$ in subplot(3,2,4), plot $|Y(e^{j\omega})|$ in subplot(3,2,5), plot $\arg\{Y(e^{j\omega})\}$ in subplot(3,2,6), and label each x-axis and y-axis clearly.

(d) Use the convolution to calculate the output sequence in time domain

$$y[n] = x[n] * h[n],$$

then use the definition to calculate the IDTFT of $Y(e^{j\omega})$.

$$\hat{y}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y\left(e^{j\omega}\right) e^{j\omega n} d\omega$$

Use stem function to plot $y[n]$ and $\hat{y}[n]$ for $0 \leq \mathrm{n} \leq 99$. Plot $y[n]$ in subplot(2,1,1), plot $\hat{y}[n]$ in subplot(2,1,2), and label each x-axis and y-axis clearly.