深度學習

HW3

學號 : B103012002

姓名 ： 林凡皓

I. SVM Classifier :

- **svm_loss_naive :**

1. 解題思路 ：

(1) dW : 根據下圖的公式推導結果，將 dW 完成。

$$1^\circ \quad \nabla_{w_j} L_i = 1(w_j^T x_i - w_{y_i}^T x_i + 1 > 0) \cdot x_i$$

$$\nabla_{w_j} L_i = \frac{\partial L_i}{\partial w_j^T x_i} \frac{\partial w_j^T x_i}{\partial w_j}$$

$$\text{for } j \neq y_i, \quad j = 1 \Rightarrow \frac{\partial L_1}{\partial w_1^T x_i} = \frac{\partial w_1^T x_i - w_{y_i}^T x_i + 1}{\partial w_1^T x_i} = 1(w_1^T x_i - w_{y_i}^T x_i + 1 > 0)$$

$$j = 2 \Rightarrow \frac{\partial L_2}{\partial w_2^T x_i} = \frac{\partial w_2^T x_i - w_{y_i}^T x_i + 1}{\partial w_2^T x_i} = 1(w_2^T x_i - w_{y_i}^T x_i + 1 > 0)$$

$$\therefore \frac{\partial L_i}{\partial w_j^T x_i} = 1(w_j^T x_i - w_{y_i}^T x_i + 1 > 0) \quad \text{and} \quad \frac{\partial w_j^T x_i}{\partial w_j} = x_i$$

$$\Rightarrow \nabla_{w_j} L_i = 1(w_j^T x_i - w_{y_i}^T x_i + 1 > 0) \cdot x_i$$

$$2^\circ \quad \nabla_{w_{y_i}} L_i = -\left(\sum_{j \neq y_i} 1(w_j^T x_i - w_{y_i}^T x_i + 1 > 0)\right) \cdot x_i$$

$$\nabla_{w_{y_i}} L_i = \frac{\partial L_i}{\partial w_{y_i}^T x_i} \frac{\partial w_{y_i}^T x_i}{\partial w_{y_i}^T}$$

$$\frac{\partial L_i}{\partial w_{y_i}^T x_i} = \frac{\partial \sum_{j \neq y_i} \max(0, w_j^T x_i - w_{y_i}^T x_i + 1)}{\partial w_{y_i}^T x_i} = \sum_{j \neq y_i} -1(w_j^T x_i - w_{y_i}^T x_i + 1 > 0)$$

$$\frac{\partial w_{y_i}^T x_i}{\partial w_{y_i}^T} = x_i$$

$$\therefore \nabla_{w_{y_i}} L_i = -\left(\sum_{j \neq y_i} 1(w_j^T x_i - w_{y_i}^T x_i + 1 > 0)\right) \cdot x_i$$

(2) loss : 先分別計算出估計出來的分數以及真實答案的分數，接著可以利用 continue 來將 j = yi 時跳過。計算出(估計分數 − 真實分數 + 1)並判斷此數值是否大於 0，如果是就將 loss 加上計算出來的數值，否則 loss 不變。最後再將 loss 除以訓練樣本數以及加上 regularization term 即可。

2. 執行結果 ：

(1) Loss check :

```
loss: 9.000869
```

(2) gradient check :

without regularization term :

```
numerical: 0.031599 analytic: 0.031599, relative error: 3.887711e-07
numerical: 0.111444 analytic: 0.111444, relative error: 1.603834e-07
numerical: 0.011204 analytic: 0.011204, relative error: 1.003052e-06
numerical: -0.046128 analytic: -0.046128, relative error: 1.470228e-08
numerical: 0.071948 analytic: 0.071948, relative error: 1.000117e-07
numerical: 0.025688 analytic: 0.025688, relative error: 1.407617e-08
numerical: 0.185388 analytic: 0.185388, relative error: 4.086995e-08
numerical: -0.021740 analytic: -0.021740, relative error: 7.159385e-08
numerical: -0.159613 analytic: -0.159613, relative error: 9.199232e-08
numerical: 0.092690 analytic: 0.092690, relative error: 6.470382e-08
```

With regularization term :

```
numerical: 0.124849 analytic: 0.124849, relative error: 7.976456e-08
numerical: 0.168915 analytic: 0.168915, relative error: 9.920512e-08
numerical: 0.148752 analytic: 0.148752, relative error: 5.747575e-08
numerical: -0.024936 analytic: -0.024936, relative error: 6.470254e-08
numerical: -0.008570 analytic: -0.008570, relative error: 7.174549e-07
numerical: -0.103155 analytic: -0.103155, relative error: 3.462148e-08
numerical: -0.335573 analytic: -0.335573, relative error: 2.199511e-08
numerical: -0.222176 analytic: -0.222176, relative error: 1.731537e-08
numerical: 0.681163 analytic: 0.681163, relative error: 1.887528e-08
numerical: -0.004089 analytic: -0.004089, relative error: 1.101669e-06
```

- **svm_loss_vectorized :**

1. 解題思路 ：

(1) Loss : 先透過矩陣乘法取得所有類別的分數，以及對所有類別分數做 index operation 來取得正確類別的分數。接著透過 scores - correct_class_score + 1 來計算出每個類別的 loss，並將正確類別的 loss 設定成 0。最後將 loss 們相加、除以訓練樣本數並加上 regularization term。

(2) dW : 這題的整體概念與 svm_loss_navie 的 dW 計算一樣，都是將數學推導的結果寫成程式碼。

先創建一個 mask，將 margin 中大於 0 的數值設為 1，以及正確類別的分數設為該 column 的和。接著透過 torch.mm 將訓練後結果 X 和 mask 做矩陣乘法。最後再除以訓練樣本數以及加上 regularization term。

2. 執行結果：

(1) Loss :

```
Naive loss: 9.002106e+00 computed in 495.54ms
Vectorized loss: 9.002106e+00 computed in 5.00ms
Difference: -1.78e-15
Speedup: 99.17X
```

由上圖可以看到，有沒有使用 for loop 計算出來的 loss 是差不多的，但是透過 vectorization 可以讓運算速度快上 99.17 倍。

(2) Gradient :

```
Naive loss and gradient: computed in 483.28ms
Vectorized loss and gradient: computed in 5.00ms
Gradient difference: 1.82e-14
Speedup: 96.70X
```

由上圖可以看到，有沒有使用 for loop 計算出來的 gradient 是差不多的，但是透過 vectorization 可以讓運算速度快上 96.7 倍。

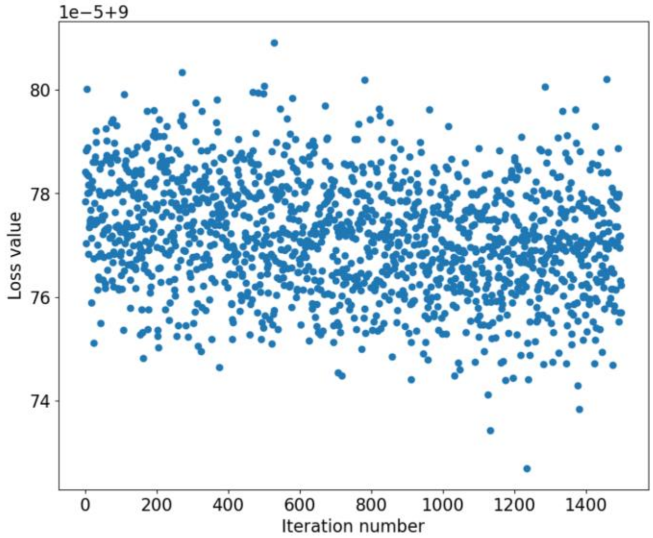
- **sample_batch :**

1. 解題思路：先利用 torch.randint 創建數值從 0 到訓練樣本數的 index(shape = (batch_size,))，然後透過 index 從 X 和 y 中抓取 batch。

- **train_linear_classifier :**

1. 解題思路：本題就是在做 training 的步驟。首先要初始化權重 W，接著在每一次迭代去計算 loss 的 gradient，並根據計算出的 gradient 去更新權重。
2. 執行結果：

```
iteration 0 / 1500: loss 9.000784
iteration 100 / 1500: loss 9.000764
iteration 200 / 1500: loss 9.000777
iteration 300 / 1500: loss 9.000768
iteration 400 / 1500: loss 9.000779
iteration 500 / 1500: loss 9.000771
iteration 600 / 1500: loss 9.000771
iteration 700 / 1500: loss 9.000769
iteration 800 / 1500: loss 9.000771
iteration 900 / 1500: loss 9.000771
iteration 1000 / 1500: loss 9.000771
iteration 1100 / 1500: loss 9.000789
iteration 1200 / 1500: loss 9.000787
iteration 1300 / 1500: loss 9.000769
iteration 1400 / 1500: loss 9.000777
That took 2.385910s
```



- **Predict_linear_classifier :**
  1. 解題思路 ： 本題要透過先前訓練好的 W 來做預測。首先要透過矩陣相乘計算 W*X，並從中選擇最大值的 index 即為預測結果。
  2. 執行結果 ：

```
Training accuracy: 9.35%
Validation accuracy: 9.11%
```

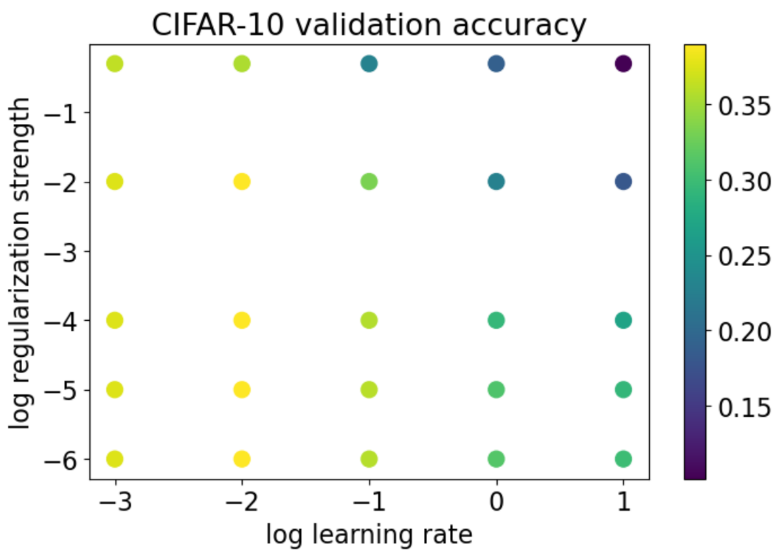
- **Get_search_params :**
  1. 解題思路 ： 本題要創建 learning_rate list 跟 regularization_strength list，分別用來存放要嘗試的 learning rate 和 regularization strength。因此只需要利用 list 的創建方法並在其中存放要嘗試的值即可。
- **Test_one_param_set :**
  1. 解題思路 ： 本題要根據 get_search_params 中的參數去計算訓練準確度與驗證準確度。由於方才訓練的模型屬於 LinearClassifier 這個 class，因此要得到預測結果我們可以透過.predict 來得到。有了預測結果後，只要將預測正確的數量除以總數量即可得到準確度。
  2. 執行結果 ：

```
Training SVM 1 / 25 with learning_rate=1.000000e-03 and reg=1.000000e-06
Training SVM 2 / 25 with learning_rate=1.000000e-03 and reg=1.000000e-05
Training SVM 3 / 25 with learning_rate=1.000000e-03 and reg=1.000000e-04
Training SVM 4 / 25 with learning_rate=1.000000e-03 and reg=1.000000e-02
Training SVM 5 / 25 with learning_rate=1.000000e-03 and reg=5.000000e-01
Training SVM 6 / 25 with learning_rate=1.000000e-02 and reg=1.000000e-06
Training SVM 7 / 25 with learning_rate=1.000000e-02 and reg=1.000000e-05
Training SVM 8 / 25 with learning_rate=1.000000e-02 and reg=1.000000e-04
Training SVM 9 / 25 with learning_rate=1.000000e-02 and reg=1.000000e-02
Training SVM 10 / 25 with learning_rate=1.000000e-02 and reg=5.000000e-01
Training SVM 11 / 25 with learning_rate=1.000000e-01 and reg=1.000000e-06
Training SVM 12 / 25 with learning_rate=1.000000e-01 and reg=1.000000e-05
Training SVM 13 / 25 with learning_rate=1.000000e-01 and reg=1.000000e-04
Training SVM 14 / 25 with learning_rate=1.000000e-01 and reg=1.000000e-02
Training SVM 15 / 25 with learning_rate=1.000000e-01 and reg=5.000000e-01
Training SVM 16 / 25 with learning_rate=1.000000e+00 and reg=1.000000e-06
Training SVM 17 / 25 with learning_rate=1.000000e+00 and reg=1.000000e-05
Training SVM 18 / 25 with learning_rate=1.000000e+00 and reg=1.000000e-04
Training SVM 19 / 25 with learning_rate=1.000000e+00 and reg=1.000000e-02
Training SVM 20 / 25 with learning_rate=1.000000e+00 and reg=5.000000e-01
Training SVM 21 / 25 with learning_rate=1.000000e+01 and reg=1.000000e-06
Training SVM 22 / 25 with learning_rate=1.000000e+01 and reg=1.000000e-05
Training SVM 23 / 25 with learning_rate=1.000000e+01 and reg=1.000000e-04
Training SVM 24 / 25 with learning_rate=1.000000e+01 and reg=1.000000e-02
Training SVM 25 / 25 with learning_rate=1.000000e+01 and reg=5.000000e-01
lr 1.000000e-03 reg 1.000000e-06 train accuracy: 0.388700 val accuracy: 0.374800
lr 1.000000e-03 reg 1.000000e-05 train accuracy: 0.388725 val accuracy: 0.374600
lr 1.000000e-03 reg 1.000000e-04 train accuracy: 0.388650 val accuracy: 0.375000
lr 1.000000e-03 reg 1.000000e-02 train accuracy: 0.388450 val accuracy: 0.374900
lr 1.000000e-03 reg 5.000000e-01 train accuracy: 0.376400 val accuracy: 0.362700
lr 1.000000e-02 reg 1.000000e-06 train accuracy: 0.414400 val accuracy: 0.390200
lr 1.000000e-02 reg 1.000000e-05 train accuracy: 0.413725 val accuracy: 0.389300
lr 1.000000e-02 reg 1.000000e-04 train accuracy: 0.413725 val accuracy: 0.389800
lr 1.000000e-02 reg 1.000000e-02 train accuracy: 0.410350 val accuracy: 0.389900
lr 1.000000e-02 reg 5.000000e-01 train accuracy: 0.362150 val accuracy: 0.355000
lr 1.000000e-01 reg 1.000000e-06 train accuracy: 0.401100 val accuracy: 0.358000
lr 1.000000e-01 reg 1.000000e-05 train accuracy: 0.401600 val accuracy: 0.358800
lr 1.000000e-01 reg 1.000000e-04 train accuracy: 0.399000 val accuracy: 0.357300
lr 1.000000e-01 reg 1.000000e-02 train accuracy: 0.355675 val accuracy: 0.331600
lr 1.000000e-01 reg 5.000000e-01 train accuracy: 0.230575 val accuracy: 0.229600
lr 1.000000e+00 reg 1.000000e-06 train accuracy: 0.346950 val accuracy: 0.313000
lr 1.000000e+00 reg 1.000000e-05 train accuracy: 0.341325 val accuracy: 0.309200
lr 1.000000e+00 reg 1.000000e-04 train accuracy: 0.325775 val accuracy: 0.293300
lr 1.000000e+00 reg 1.000000e-02 train accuracy: 0.232850 val accuracy: 0.226700
lr 1.000000e+00 reg 5.000000e-01 train accuracy: 0.183650 val accuracy: 0.188800
lr 1.000000e+01 reg 1.000000e-06 train accuracy: 0.329900 val accuracy: 0.299400
lr 1.000000e+01 reg 1.000000e-05 train accuracy: 0.324975 val accuracy: 0.292200
lr 1.000000e+01 reg 1.000000e-04 train accuracy: 0.287775 val accuracy: 0.268800
lr 1.000000e+01 reg 1.000000e-02 train accuracy: 0.178925 val accuracy: 0.179600
lr 1.000000e+01 reg 5.000000e-01 train accuracy: 0.099650 val accuracy: 0.101400
best validation accuracy achieved during cross-validation: 0.390200
```

由上圖可以看到最好的結果發生在 learning rate = 0.01，regularization strength = 0.000001。最好的結果為 39.02 %。

會有每一次的迭代並不是因為 test_one_param_set 中有 for loop，而是我們用 for loop 多次呼叫 test_one_param。

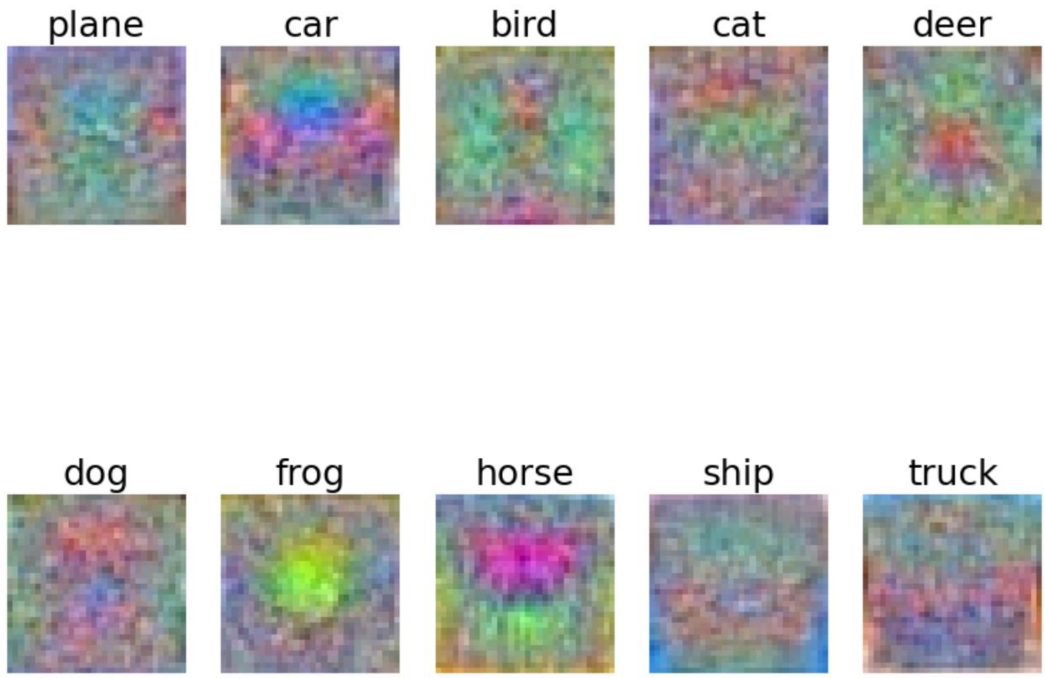
將結果視覺化：



對於測試資料來說，此模型有 38.71 %的準確度，如下圖，

```
linear SVM on raw pixels final test set accuracy: 0.387100
```

- **每個類別的模板：**



II. 額外嘗試：Softmax Classifier

- **Softmax_loss_vectorized :**

1. 解題思路：根據以下推導，將 dW 完成。

$$\begin{cases} P_{y_i} = \dfrac{e^{s_{y_i}}}{\sum_j e^{s_j}} \\ s_j = W_j^T x_j \end{cases} \qquad \frac{\partial P_{y_i}}{\partial s_j} = \begin{cases} P_{y_i}(1-P_j), & y_i = j \\ -P_{y_i} P_j, & y_i \neq j \end{cases}$$

$$\nabla_{W_j} L_i = \frac{\partial L_i}{\partial s_j} \frac{\partial s_j}{\partial W_j}$$

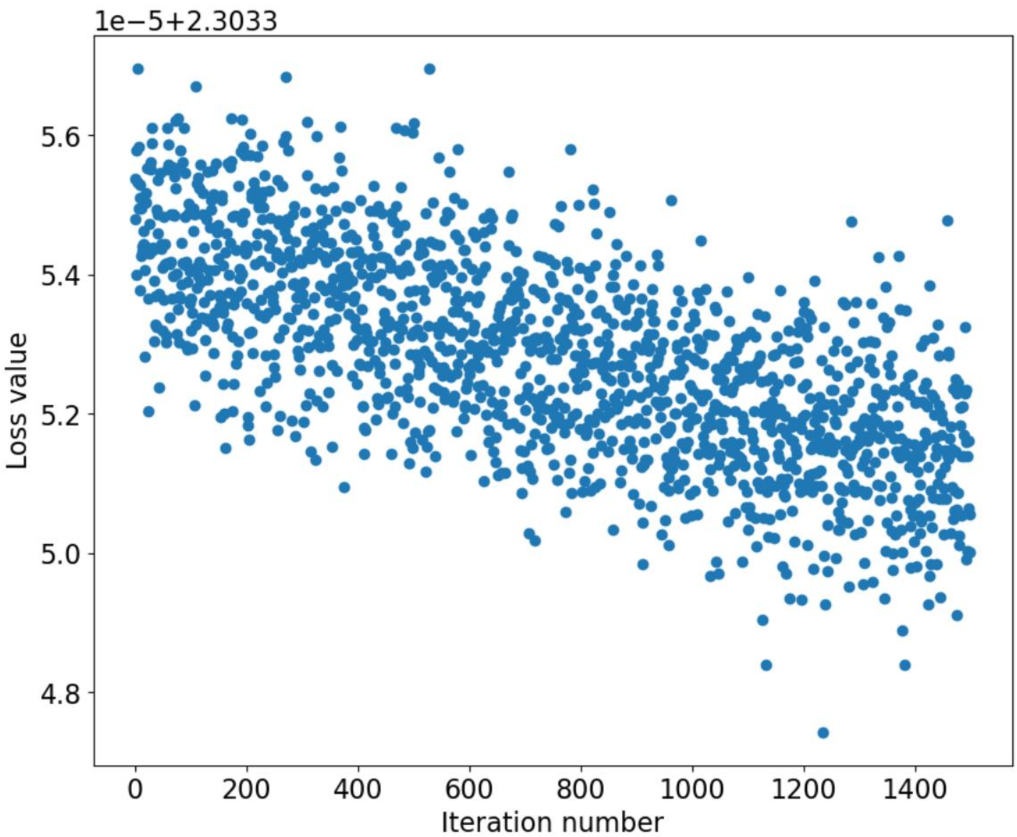
$$1^\circ \ \frac{\partial L_i}{\partial s_j} = \frac{\partial(-\log P_{y_i})}{\partial s_j} = -\frac{1}{P_{y_i}} \frac{\partial P_{y_i}}{\partial s_j} = \begin{cases} (P_j - 1), & y_i = j \\ P_j, & y_i \neq j \end{cases}$$

$$2^\circ \ \frac{\partial s_j}{\partial W_j} = x_j$$

$$\therefore \nabla_{W_j} L_i = \begin{cases} (P_j - 1) x_j, & y_i = j \\ P_j x_j, & y_i \neq j \end{cases}$$

2. 執行結果：

```
iteration 0 / 1500: loss 2.303355
iteration 100 / 1500: loss 2.303353
iteration 200 / 1500: loss 2.303354
iteration 300 / 1500: loss 2.303353
iteration 400 / 1500: loss 2.303354
iteration 500 / 1500: loss 2.303353
iteration 600 / 1500: loss 2.303353
iteration 700 / 1500: loss 2.303353
iteration 800 / 1500: loss 2.303353
iteration 900 / 1500: loss 2.303353
iteration 1000 / 1500: loss 2.303352
iteration 1100 / 1500: loss 2.303354
iteration 1200 / 1500: loss 2.303354
iteration 1300 / 1500: loss 2.303352
iteration 1400 / 1500: loss 2.303352
That took 2.229633s
```

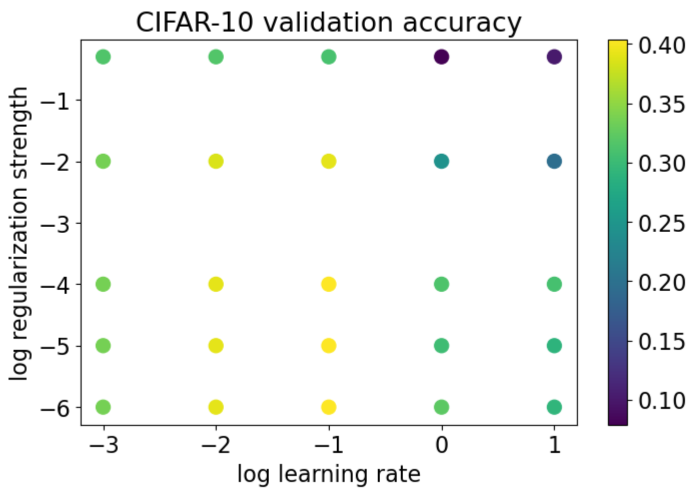


由此結果與 SVM loss 比較會發現到 softmax 的 loss 會比較大。對於這樣的現象，我認為並不是說 SVM loss 計算出來的值就會比 softmax 的值小，而是要考慮到更多的東西，像是資料分布。SVM loss 的值與 softmax 的值有所不同是合理的，畢竟兩個的計算公式就不相同，至於誰大誰小，我認為會根據不同的情況有所差別。

- **Training result :**

1. Training :

```
Training Softmax 1 / 25 with learning_rate=1.000000e-03 and reg=1.000000e-06
Training Softmax 2 / 25 with learning_rate=1.000000e-03 and reg=1.000000e-05
Training Softmax 3 / 25 with learning_rate=1.000000e-03 and reg=1.000000e-04
Training Softmax 4 / 25 with learning_rate=1.000000e-03 and reg=1.000000e-02
Training Softmax 5 / 25 with learning_rate=1.000000e-03 and reg=5.000000e-01
Training Softmax 6 / 25 with learning_rate=1.000000e-02 and reg=1.000000e-06
Training Softmax 7 / 25 with learning_rate=1.000000e-02 and reg=1.000000e-05
Training Softmax 8 / 25 with learning_rate=1.000000e-02 and reg=1.000000e-04
Training Softmax 9 / 25 with learning_rate=1.000000e-02 and reg=1.000000e-02
Training Softmax 10 / 25 with learning_rate=1.000000e-02 and reg=5.000000e-01
Training Softmax 11 / 25 with learning_rate=1.000000e-01 and reg=1.000000e-06
Training Softmax 12 / 25 with learning_rate=1.000000e-01 and reg=1.000000e-05
Training Softmax 13 / 25 with learning_rate=1.000000e-01 and reg=1.000000e-04
Training Softmax 14 / 25 with learning_rate=1.000000e-01 and reg=1.000000e-02
Training Softmax 15 / 25 with learning_rate=1.000000e-01 and reg=5.000000e-01
Training Softmax 16 / 25 with learning_rate=1.000000e+00 and reg=1.000000e-06
Training Softmax 17 / 25 with learning_rate=1.000000e+00 and reg=1.000000e-05
Training Softmax 18 / 25 with learning_rate=1.000000e+00 and reg=1.000000e-04
Training Softmax 19 / 25 with learning_rate=1.000000e+00 and reg=1.000000e-02
Training Softmax 20 / 25 with learning_rate=1.000000e+00 and reg=5.000000e-01
Training Softmax 21 / 25 with learning_rate=1.000000e+01 and reg=1.000000e-06
Training Softmax 22 / 25 with learning_rate=1.000000e+01 and reg=1.000000e-05
Training Softmax 23 / 25 with learning_rate=1.000000e+01 and reg=1.000000e-04
Training Softmax 24 / 25 with learning_rate=1.000000e+01 and reg=1.000000e-02
Training Softmax 25 / 25 with learning_rate=1.000000e+01 and reg=5.000000e-01
lr 1.000000e-03 reg 1.000000e-06 train accuracy: 0.343225 val accuracy: 0.336500
lr 1.000000e-03 reg 1.000000e-05 train accuracy: 0.343225 val accuracy: 0.336500
lr 1.000000e-03 reg 1.000000e-04 train accuracy: 0.343225 val accuracy: 0.336500
lr 1.000000e-03 reg 1.000000e-02 train accuracy: 0.342600 val accuracy: 0.336200
lr 1.000000e-03 reg 5.000000e-01 train accuracy: 0.314500 val accuracy: 0.313900
lr 1.000000e-02 reg 1.000000e-06 train accuracy: 0.406625 val accuracy: 0.390700
lr 1.000000e-02 reg 1.000000e-05 train accuracy: 0.406625 val accuracy: 0.390700
lr 1.000000e-02 reg 1.000000e-04 train accuracy: 0.406675 val accuracy: 0.390700
lr 1.000000e-02 reg 1.000000e-02 train accuracy: 0.403650 val accuracy: 0.385500
lr 1.000000e-02 reg 5.000000e-01 train accuracy: 0.316525 val accuracy: 0.315300
lr 1.000000e-01 reg 1.000000e-06 train accuracy: 0.435125 val accuracy: 0.403800
lr 1.000000e-01 reg 1.000000e-05 train accuracy: 0.435000 val accuracy: 0.403900
lr 1.000000e-01 reg 1.000000e-04 train accuracy: 0.434725 val accuracy: 0.403400
lr 1.000000e-01 reg 1.000000e-02 train accuracy: 0.410050 val accuracy: 0.390700
lr 1.000000e-01 reg 5.000000e-01 train accuracy: 0.311975 val accuracy: 0.308800
lr 1.000000e+00 reg 1.000000e-06 train accuracy: 0.361125 val accuracy: 0.322300
lr 1.000000e+00 reg 1.000000e-05 train accuracy: 0.344675 val accuracy: 0.303400
lr 1.000000e+00 reg 1.000000e-04 train accuracy: 0.349775 val accuracy: 0.313500
lr 1.000000e+00 reg 1.000000e-02 train accuracy: 0.250225 val accuracy: 0.244500
lr 1.000000e+00 reg 5.000000e-01 train accuracy: 0.077800 val accuracy: 0.079000
lr 1.000000e+01 reg 1.000000e-06 train accuracy: 0.322275 val accuracy: 0.292100
lr 1.000000e+01 reg 1.000000e-05 train accuracy: 0.310825 val accuracy: 0.288600
lr 1.000000e+01 reg 1.000000e-04 train accuracy: 0.322425 val accuracy: 0.307700
lr 1.000000e+01 reg 1.000000e-02 train accuracy: 0.193625 val accuracy: 0.195300
lr 1.000000e+01 reg 5.000000e-01 train accuracy: 0.099650 val accuracy: 0.101400
best validation accuracy achieved during cross-validation: 0.403900
```



由以上結果可以看到，當 learning rate = 0.1，regularization strength

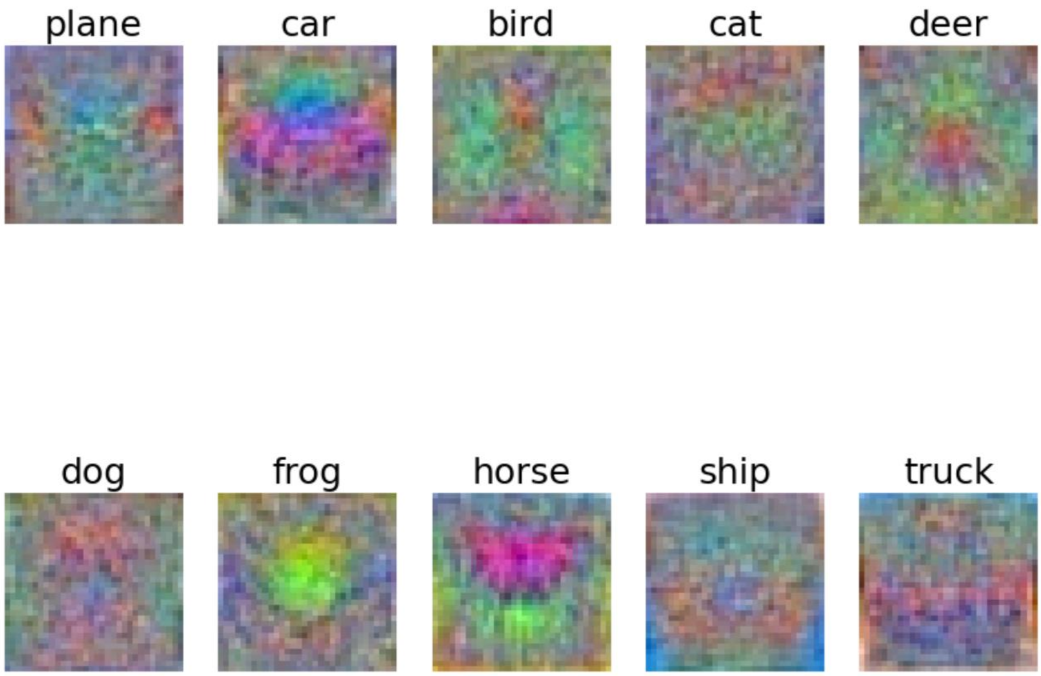
$= 0.00001$ 時，得到最佳的結果為 40.39 %。跟 SVM loss 比起來，結果稍微好一點。

2. Testing :

```
linear Softmax on raw pixels final test set accuracy: 0.403500
```

對於測試資料來說，此模型有 40.35 % 準確度。

- **每個類別的模板 ：**



與 SVM 的模板相比，我們會發現到結果大致相同。

III. 結果討論 ：

- CPU、GPU:

在這次作業中，我有遇到一個問題就是存在 CPU 的變數要和存在 GPU 中的變數一起做運算時會發生錯誤。還有將 tensor 轉換成 numpy 時，如果 tensor 在 GPU 上會出現錯誤如下圖，

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[14], line 6
      4 # plot training accuracy
      5 marker_size = 100
----> 6 colors = [results[x][0].numpy() for x in results]
      7 plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap='viridis')
      8 plt.colorbar()

Cell In[14], line 6, in <listcomp>(.0)
      4 # plot training accuracy
      5 marker_size = 100
----> 6 colors = [results[x][0].numpy() for x in results]
      7 plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap='viridis')
      8 plt.colorbar()

TypeError: can't convert cuda:0 device type tensor to numpy. Use Tensor.cpu() to copy the tensor to host memory first.
```

因此注意參數存放的位置是相當重要的。

- SVM loss v.s. Sigmoid:

  1. 計算損失方式 ：

     (1) SVM loss : 鼓勵正確分類，懲罰錯誤分類。

     (2) Softmax : 得分代表說屬於該類別的機率，並使正確類別的機率提高，錯誤類別的機率降低。

  2. 優化目標 ：

     (1) SVM loss : 最大化類別邊界的間隔。

     (2) Softmax : 最小化預測機率與正確類別之間的差異。

  3. 表現差異 ： 對於這次的線性分類器來說，兩個 loss function 的表現差異很小。但是我覺得整體表現上，softmax 會比 SVM loss 來的好一些，因為 softmax loss 考慮的是機率，也就是說他會將所有類別的機率都考慮進去，但是 SVM loss 只有在$w_j^T x_i - w_{y_i}^T x_i + 1 > 0$的情況才會考慮，這就會造成 SVM loss 少考慮一些情況的現象，造成其準確度會比 softmax loss 來的低。

- 心得總結 ：

  這次作業主要是讓我們從頭實作一個線性分類器，並根據在數學上的推導實作出 gradient。

  這次訓練的線性分類器的準確度比上一次作業 Knn 的表現還要好上不少，不管是預測所需的時間、預測準確度……等，唯獨在訓練上線性分類器需要花上比較多時間。

  再想辦法優化模型的過程，我嘗試了各種 learning rate、regularization strength 和 loss function，但是準確度的極限大約就是 40 %左右，我認為這應該就是線性分類器的極限。線性分類器在訓練上相對簡單，模型的複雜度也很低，但是準確度也就較低，如果要提升準確度，可能要使用更複雜的模型，或是增加一些非線性的參數讓模型可以更好的去擬和更複雜的曲線。

IV. Reference :

[1] Standford “CS231n Convolutional Neural Networks for Visual recognition – Linear Classification”. https://cs231n.github.io/linear-classify/

[2] OpenAI. (2023). ChatGPT (Mar 14 version) [Large language model]. https://chat.openai.com/