MPHY0030: Programming Foundations for Medical Image Analysis

Part2 Report

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1.Due to the properties of the conditional positive definition, polynomials are necessary to guarantee the non-singularity of the matrix. When the functions are positive, they are defined to ensure that the matrix is regular (invertible), the solvability of RBF registration function is always guaranteed.

2.The Gaussian are positive definite which ensures that the matrix **K** is invertible. Since an additional polynomial part f is not necessary, so the equation reduced to

$$k\alpha = q$$

The linear equation can be solved by using least square. The solution I used to solve this problem as below.

$$\begin{aligned}
&\|k\alpha - q\|_2^2 \\
&\text{let } k = U\Sigma \\
&= \left\|U\Sigma V^\top \alpha - q\right\|_2^2 \\
&= \left\|\Sigma V^\top \alpha - V^\top q\right\|_2^2 \\
&\Sigma V^\top \alpha - V^\top q = 0 \\
&V^\top \alpha = \Sigma^{-1} V^\top q \\
&\alpha = v\Sigma^{-1} v^\top q \\
&\text{when } (k + \lambda I)\alpha_k = q_k \\
&k + \lambda I = U\Sigma V^\top \\
&\alpha_k = U\Sigma^{-1} v^\top q_k \\
&\alpha = U\Sigma^{-1} V^\top q
\end{aligned}$$

3.I think singular value decomposition (SVD) is the best linear algebra algorithm to solve this spline fitting problem. SVD is a powerful tool for dimensionality reduction. We can use SVD to approximate the matrix and extract important features from it. For medical image registration, we can use SVD to extract relevant features from noisy data. At the same time, the size of

medical image is usually large, removing noise and redundant information is really helpful for saving memory.

4. We can't choose any control points in evaluating stage. The control points here should be as same as the points which were defined initially in fitting stage. It needs to be corresponding.

5.we don't need the weighting parameter lambda at evaluation stage, because we use it in fitting stage.

6. Vectorization can really help to save computing time, especially when the dataset is large. But at last, my code is only achieved partial vectorization by using matrix minus vector.

7. Sigma is a locality parameter, which represents the spatial range of influence induced by used control points.

8. In my opinion, the biophysically plausible deformation means the displacement of control points shouldn't be too large. Otherwise, there will be an extremely distortion in the image. So, we need to set the constraint of the deformation to prevent it. In order to prevent deformations in regions where no changes are desired, the control points should be well distributed placed. So, the distribution of the random transform should be a Gaussian distribution.

9. The interpolated voxel coordinates was driven by a set of transformed query point set, so if we chose the control point and transformation according to biophysically reasonable, theoretically, it will represent biophysically plausible deformation as well.

10. If we want to compute a warped 3D image, we need to take following operations. Firstly, the control points which should be fixed was selected from the original image. Secondly, using random transform generator to generate the coordinates of a set of moved control points. When it comes to fitting stage, alpha could be computed by using Gaussian Spline between fixed control points and moved control points. And then, using a evaluate function, we got the transformed query point set. At last, with all voxel coordinates interpolated by Gaussian spline using a set of transformed query points, a warped 3D image was computed.

11. I tried my best, but for part2. When it comes to the implementation, it still has many problems I can't fix. Most of problems in creating Free-form deformation class. So, I can't visualise the result. But according to my inference, I think the result depends, to a great extent, on the setting of relevant

parameters. But if the constraint of deformation is limited enough, the resulting distortion will be limited in the biophysically plausible range.

12. According to inference as well, the effect of three parameters was described as below:

1) The strength of randomness: it controls the degree of image distortion, with increasement of strength, the image will be more warped.
2) The number of control points: when the number of controls points increase, it will lead to more accurate interpolation result. Because it covers more points.
3) The Gaussian kernel parameter: As discussed above, the Gaussian kernel parameter represents the spatial range of influence induced by control points.