MPHY0030: Programming Foundations for Medical Image Analysis

Part2 Report

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1.Due to the properties of the conditional positive definition, polynomials are necessary to guarantee the non-singularity of the matrix. When the functions are positive, they are defined to ensure that the matrix is regular (invertible), the solvability of RBF registration function is always guaranteed.

2.The Gaussian are positive definite which ensures that the matrix **K** is invertible. Since an additional polynomial part f is not necessary, so the equation reduced to

$$k\alpha = q$$

The linear equation can be solved by using least square. The solution I used to solve this problem as below.

$$\begin{aligned}
&\|k\alpha - q\|_2^2\\
&\text{let } k = U\Sigma\\
&= \left\|U\Sigma V^\top \alpha - q\right\|_2^2\\
&= \left\|\Sigma V^\top \alpha - V^\top q\right\|_2^2\\
&\Sigma V^\top \alpha - V^\top q = 0\\
&V^\top \alpha = \Sigma^{-1} V^\top q\\
&\alpha = v\Sigma^{-1} v^\top q\\
&\text{when } (k + \lambda I)\alpha_k = q_k\\
&k + \lambda I = U\Sigma V^\top\\
&\alpha_k = U\Sigma^{-1} v^\top q_k\\
&\alpha = U\Sigma^{-1} V^\top q
\end{aligned}$$

3.I think singular value decomposition (SVD) is the best linear algebra algorithm to solve this spline fitting problem. SVD is a powerful tool for dimensionality reduction. We can use SVD to approximate the matrix and extract important features from it. For medical image registration, we can use SVD to extract relevant features from noisy data. At the same time, the size of medical image is usually large, removing noise and redundant information is really helpful for saving memory.

4.The control points here is as same as the points which were fixed in fitting stage. if we want to evaluate the performance of algorithm, we can't choose any points as control points at evaluation stage.

5.we don't need the weighting parameter lambda at evaluation stage, because we use it in fitting stage.

6. Vectorization can really help to save computing time, especially when the dataset is large. But at last, my code is only achieved partial vectorization by using matrix minus vector.

7. Sigma is a locality parameter, which represents the spatial range of influence induced by used control points.

8. Actually, I'm not sure the range of biophysically plausible deformation. In my opinion, it means the displacement of control points should be too large. Otherwise, there will be distortion in the image. So, we need to set the constraint of the deformation. In order to prevent deformations in regions where no changes are desired. So, the distribution of the random transform should be a Gaussian distribution.

9. The interpolated voxel coordinates was driven by a set of transformed query point set, so if we chose the control point and transformation according to biophysically reasonable, it will represent biophysically plausible deformation as well.

10. Although I didn't success to implement the whole algorithm, I tried to take a big picture of it by reading references. If we want to compute a warped 3D image, we need to take following operations. Firstly, the control points which should be fixed was selected from the original image. Secondly, using random transform generator to generate the coordinates of a set of moved control points. When it comes to fitting stage, alpha could be computed by using Gaussian Spline between fixed control points and moved control points. And then, using a evaluate function, we got the transformed query point set. At last, with all voxel coordinates interpolated by Gaussian spline using a set of transformed query points, a warped 3D image was computed.