

MPHY0030: Programming Foundations for Medical Image Analysis

Part2 Report

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1. Due to the properties of the conditional positive definition, polynomials are necessary to guarantee the non-singularity of the matrix. When the functions are positive, they are defined to ensure that the matrix is regular (invertible), the solvability of RBF registration function is always guaranteed.

2. The Gaussian are positive definite which ensures that the matrix \mathbf{K} is invertible. Since an additional polynomial part f is not necessary, so the equation reduced to

$$k\alpha = q$$

The linear equation can be solved by using least square. The solution I used to solve this problem as below.

$$\begin{aligned} & \|k\alpha - q\|_2^2 \\ & \text{let } k = U\Sigma \\ & = \|U\Sigma V^\top \alpha - q\|_2^2 \\ & = \|\Sigma V^\top \alpha - V^\top q\|_2^2 \\ & \Sigma V^\top \alpha - V^\top q = 0 \\ & V^\top \alpha = \Sigma^{-1} V^\top q \\ & \alpha = v \Sigma^{-1} v^\top q \\ & \text{when } (k + \lambda I) \alpha_k = q_k \\ & k + \lambda I = U\Sigma V^\top \\ & \alpha_k = U\Sigma^{-1} v^\top q_k \\ & \alpha = U\Sigma^{-1} V^\top q \end{aligned}$$

3. I think singular value decomposition (SVD) is the best linear algebra algorithm to solve this spline fitting problem. SVD is a powerful tool for dimensionality reduction. We can use SVD to approximate the matrix and extract important features from it. For medical image registration, we can use SVD to extract relevant features from noisy data. At the same time, the size of

medical image is usually large, removing noise and redundant information is really helpful for saving memory.

4. The control points here are the same as the points which were fixed in the fitting stage. If we want to evaluate the performance of the algorithm, we can't choose any points as control points at the evaluation stage.

5. We don't need the weighting parameter λ at the evaluation stage, because we use it in the fitting stage.

6. Vectorization can really help to save computing time, especially when the dataset is large. But at last, my code only achieved partial vectorization by using matrix minus vector.

7. σ is a locality parameter, which represents the spatial range of influence induced by used control points.

8. In my opinion, the biophysically plausible deformation means the displacement of control points shouldn't be too large. Otherwise, there will be an extremely distortion in the image. So, we need to set the constraint of the deformation to prevent it. In order to prevent deformations in regions where no changes are desired, the control points should be well distributed. So, the distribution of the random transform should be a Gaussian distribution.

9. The interpolated voxel coordinates were driven by a set of transformed query point set, so if we chose the control point and transformation according to biophysically reasonable, it will represent biophysically plausible deformation as well.

10. If we want to compute a warped 3D image, we need to take the following operations. Firstly, the control points which should be fixed were selected from the original image. Secondly, using a random transform generator to generate the coordinates of a set of moved control points. When it comes to the fitting stage, α could be computed by using Gaussian Spline between fixed control points and moved control points. And then, using an evaluate function, we got the transformed query point set. At last, with all voxel coordinates interpolated by Gaussian spline using a set of transformed query points, a warped 3D image was computed.