▶**19.** The following ordinary difference table is for $f(x) = x + \sin(x)/3$. Use it to find

a. $f'(0.72)$ from a cubic polynomial.
b. $f'(1.33)$ from a quadratic.
c. $f'(0.50)$ from a fourth-degree polynomial.

In each part, choose the best starting i-value.

i	x_i	f_i	Δf_i	$\Delta^2 f_i$	$\Delta^3 f_i$	$\Delta^4 f_i$
0	0.30	0.3985	0.2613	−0.0064	−0.0022	0.0003
1	0.50	0.6598	0.2549	−0.0086	−0.0018	0.0004
2	0.70	0.9147	0.2464	−0.0104	−0.0014	0.0005
3	0.90	1.1611	0.2360	−0.0118	−0.0010	
4	1.10	1.3971	0.2241	−0.0128		
5	1.30	1.6212	0.2113			
6	1.50	1.8325				

43. Simpson's 3/8 rule cannot be applied directly to Exercise 41 because the number of panels is not divisible by three. Still, you can use it in combination with the 1/3 rule over two panels. There are several choices of where to use the 1/3 rule. Which of these choices gives the most accurate answer?

▶**41.** Repeat Exercise 35, but now use Simpson's 1/3 rule.

▶**35.** Use the data in the table to find the integral between $x = 1.0$ and 1.8, using the trapezoidal rule:

a. With $h = 0.1$.
b. With $h = 0.2$.
c. With $h = 0.4$.

x	$f(x)$
1.0	1.543
1.1	1.669
1.2	1.811
1.3	1.971
1.4	2.151
1.5	2.352
1.6	2.577
1.7	2.828
1.8	3.107

36. The function tabulated in Exercise 35 is $\cos h(x)$. What are the errors in parts (a), (b), and (c)? How closely are these proportional to h^2? What errors are present besides the truncation error?

42. Use the error term for Simpson's 1/3 rule to bound the errors in Exercise 41 for each application of the rule. What are the values for ξ for each value of h?

51. Compute the integral of $f(x) = \sin(x)/x$ between $x = 0$ and $x = 1$ using Simpson's $\frac{1}{3}$ rule with $h = 0.5$ and then with $h = 0.25$. (Remember that the limit of $\sin(x)/x$ at $x = 0$ is 1.) From these two results, extrapolate to get a better result. What is the order of the error after the extrapolation? Compare your answer with the true answer.

64. Repeat Example 5.10, but use the trapezoidal rule. At what value for h do the computations terminate? How many function evaluations are required compared to Simpson's 1/3 rule?

EXAMPLE 5.10 Integrate $f(x) = 1/x^2$ over the interval [0.2, 1] using Simpson's $\frac{1}{3}$ rule. Use a tolerance value of 0.02 to terminate the halving of $h = \Delta x$. From calculus, we know that the exact answer is 4.0.

We introduce a special notation that will be used throughout this section:

$S_n[a, b]$ = the computed value using Simpson's $\frac{1}{3}$ rule with $\Delta x = h_n$ over $[a, b]$.

If we use this notation, the composite Simpson rule becomes

$$\boxed{I(f) = S_n[a, b] - \frac{(b-a)}{180} h_n^4 f^{(4)}(\xi), \quad a < \xi < b.}$$

Using this with $h_1 = (1.0 - 0.2)/2 = 0.4$, we compute S_1 [0.2, 1.0]. We continue halving h, $h_{n+1} = h_n/2$, computing its corresponding $S_{n+1}[a, b]$ until $|S_{n+1} - S_n| < 0.02$, the tolerance value. The following table shows the results:

n	h_n	S_n	$\lvert S_{n+1} - S_n \rvert$
1	0.4	4.948148	
			0.761111
2	0.2	4.187037	
			0.162819
3	0.1	4.024218	
			0.022054
4	0.05	4.002164	
			0.002010
5	0.025	4.000154	

From the table we see that, at $n = 5$, we have met the tolerance criterion, because $|S_5 - S_4| < 0.02$. A Romberg extrapolation gives

$$RS[a, b] = S_5 + \frac{S_5 - S_4}{15} = 4.00002.$$

(We use $RS[a, b]$ to represent the Romberg extrapolation from Simpson's rule.)

73. What is the error if the integral of $\sin(x)/x$ over $x = [0, 2]$ is evaluated with a four-term Gaussian formula? How many intervals would be needed with Simpson's 1/3 rule to get the value with the same accuracy?

Ch4

21. Find Padé approximations for these functions, with numerators and denominators each of the third degree:

$$\cos^2(x),\ \sin(x^4 - x),\ xe^x.$$