

AAE2001 – Introduction to Aircraft Design and Aviation Systems

Atmosphere

Dr. Yiping Jiang, AAE

Ground Rules

- For students:

Open mind; speak English; participate activities assigned; ask questions

- For teachers:

Arrive on time; reply emails on time; answer questions related to the subject;

Be curious. Be active. Be inspired.

Study further by yourself.

About Myself

- Dr. Yiping Jiang
- Experience: Sydney, Toulouse, Shenzhen, HK
UNSW, ENAC, DJI, PolyU
- Contact: QR823, yiping.jiang@polyu.edu.hk
- Expertise:
 - Satellite Navigation for Civil Aviation
 - Augmentation Systems for Civil Aviation

Intended Learning Outcomes

- Demonstrate good understanding of the principles of key systems in civil transport aircraft (e.g., control system, fuel system, engine system, electrical system, environmental control system and emergency system).
- Gain the basic knowledge of aviation systems and their functions in the aviation industry including the roles.
- Understand the interrelationships among civil aviation administration, airlines and airport operations; air traffic control; maintenance scheduling and aviation associated environmental issues.

Contents

1. Atmosphere
2. Fundamentals and Structure of Aviation System
3. Air Traffic Control
4. Electrical System
5. Quiz

Purpose

- How does the air make flight possible?
- What are the properties of the atmosphere?
- How will the properties air affect the airplane's performance?
- How do the properties change?

Space Jump. Joe Kittinger

https://www.youtube.com/watch?v=sbVQ33ujzFw&ab_channel=NationalMuseumoftheU.S.AirForce



Purpose

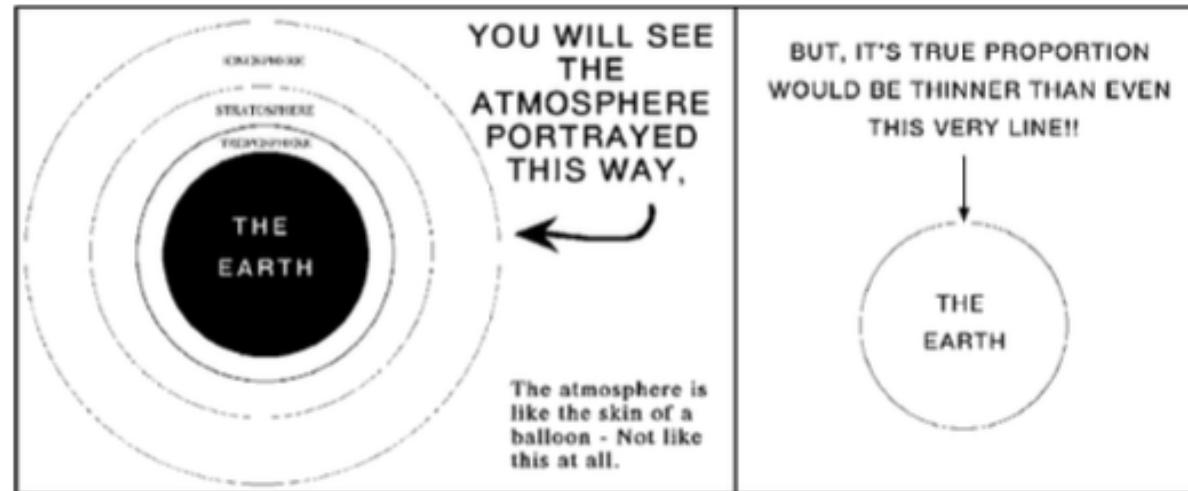
- Introduce general composition and structure of the atmosphere
- The effect of temperature and atmospheric pressure
 - Temperature affecting many aspects of weather
 - Pressure enables meteorologists to track weather phenomena as they move across the surface of the Earth. Additionally, pressure is important to the aviation community since one of the most basic flight instruments, the **barometric altimeter**, operates from the action of atmospheric pressure upon its sensors.

Imagine you are a pilot, why you need to have this knowledge?

The Atmosphere

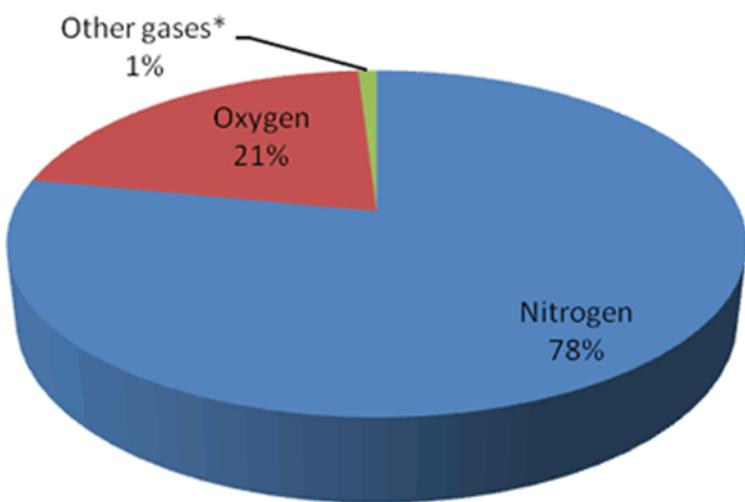
The atmosphere is the space around the Earth which is filled by a mixture of gasses held against the Earth by the force of gravity. This envelope of air rotates with the Earth but also has a continuous motion relative to the Earth's surface, called circulation.

Circulation is created primarily by the large temperature difference between the tropics and polar regions, and is complicated by uneven heating of land and water areas by the Sun.



Composition of the Atmosphere

The composition and the structure of the atmosphere makes possible for life to thrive and survive on Earth. The composition of the atmosphere consists of various gases. The major gases in the atmosphere are nitrogen and oxygen. Nitrogen makes up 78% of the gases in the atmosphere and oxygen is about 21%. Other gases in the atmosphere make up the trace gases. The following is the list of major gases that make up the atmosphere:



- Nitrogen (N₂)
- Oxygen (O₂)
- Argon (Ar)
- Carbon dioxide (CO₂)
- Neon (Ne)
- Helium (He)
- Methane (CH₄)
- Krypton (Kr)
- Hydrogen (H₂)
- Nitrous oxide (N₂O)
- Carbon monoxide (CO)
- Xenon (Xe)
- Ozone (O₃)
- Nitrogen dioxide (NO₂)
- Iodine (I₂)
- Ammonia (NH₃)

The Atmosphere

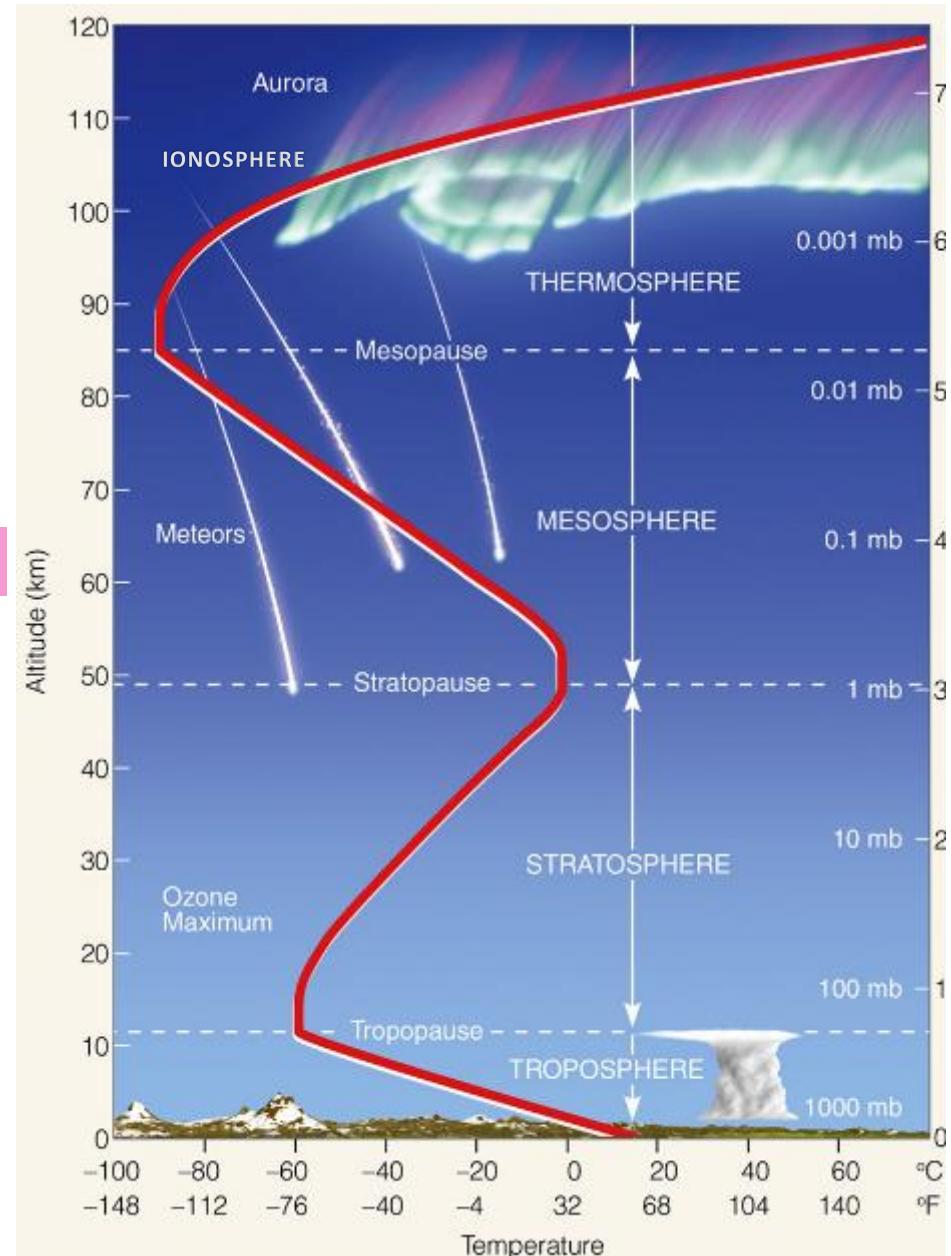
The Earth's atmosphere that we know today is divided into five layers. In order of descending altitude, they are:

- Exosphere
- Thermosphere
- Mesosphere
- Stratosphere
- Troposphere

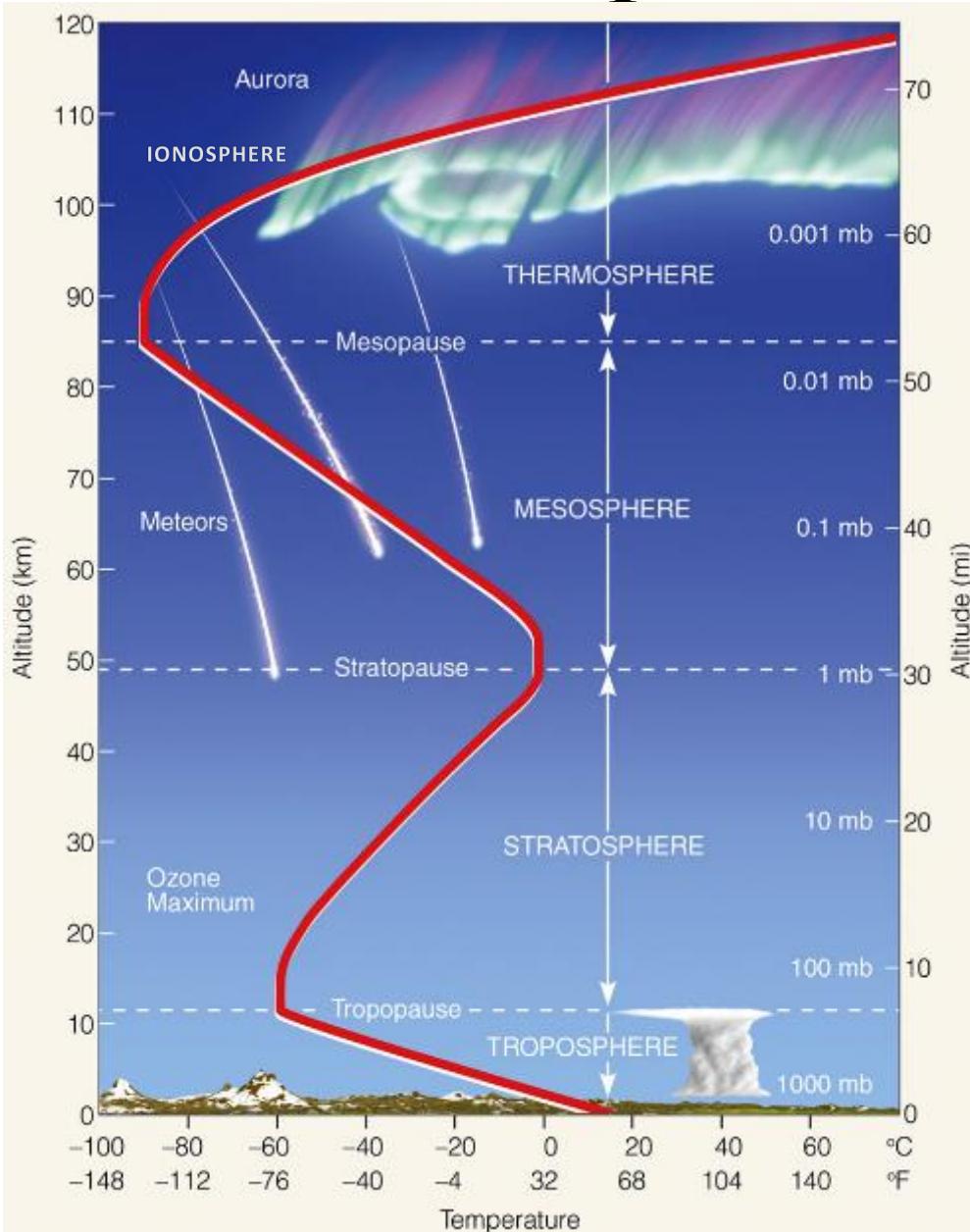
The different layers are typically defined by a change in properties, variations in temperature and pressure with altitude.

The change in properties in each layer can be linked to specific functions that help protect the earth and form the different weather around the world.

The atmosphere extends to 10,000 km above the Earth's surface, where it merges seamlessly with space at continuously diminishing air density.



The Atmosphere

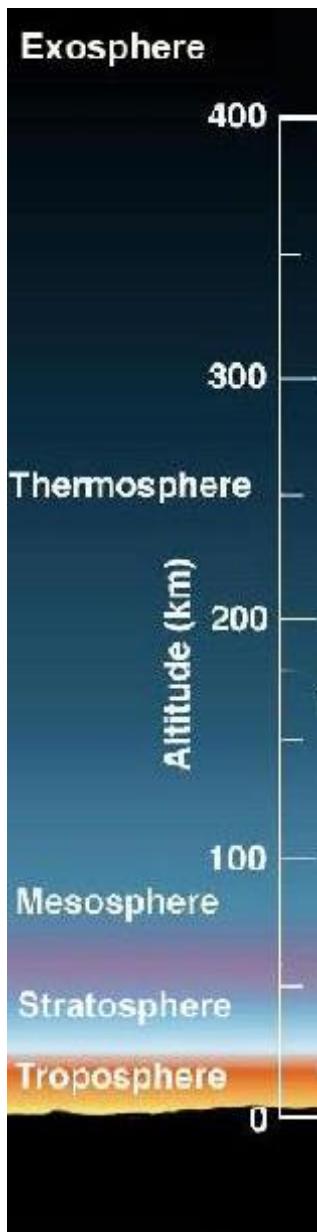


The invisible boundaries between the various layers maybe characterised by abrupt changes in properties, temperature and/or behaviour. The interfaces between the layers are called:

- Thermopause
- Mesopause
- Stratopause
- Tropopause

The decrease in atmospheric temperature with increasing altitude is called the temperature lapse rate.

Atmospheric Layers

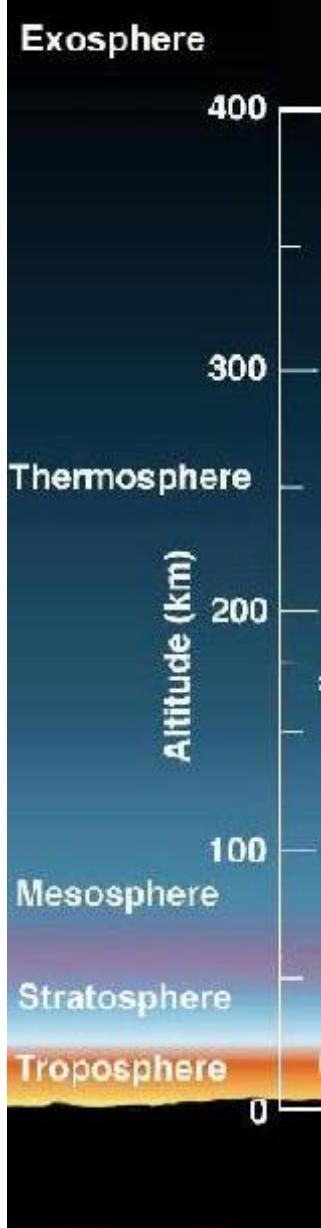


Troposphere 0–17 km: The troposphere is the layer of the atmosphere that supports life. Most of the weather occurs in the troposphere. It contains about *80% of the air and 99% of the water vapours* in the atmosphere.

The troposphere is mostly heated from the surface, where sunlight warms the ground and ocean, which in turn radiates the heat into the air right above it. Warm air rises, it keeps the air in the troposphere "stirred up". The troposphere is **warmest at the bottom and becomes cooler with increasing altitude**. It extends to a height of about 17 km at the Equator and somewhat less at the Poles.



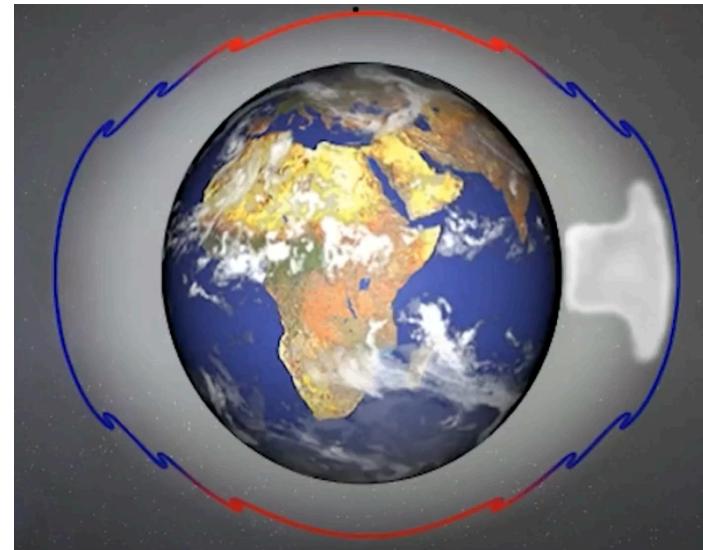
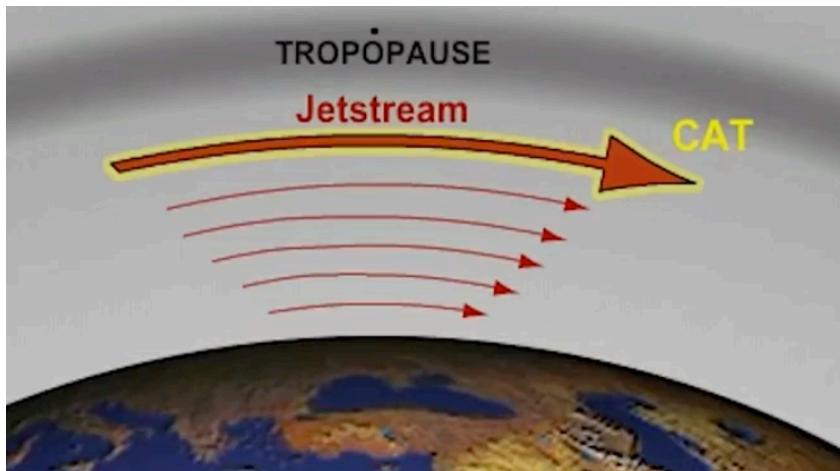
Atmospheric Layers



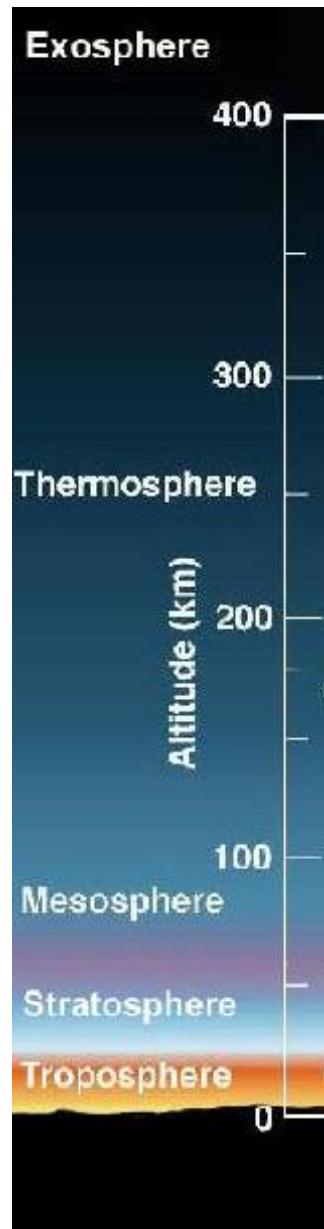
Why is Tropopause so important to us?

1. It signifies the start of temperature inversion, which in turn limits the vertical air movement in clouds, and also limits the cloud development.
2. The lower density of air in the upper atmosphere causes the upper wind to be very strong.

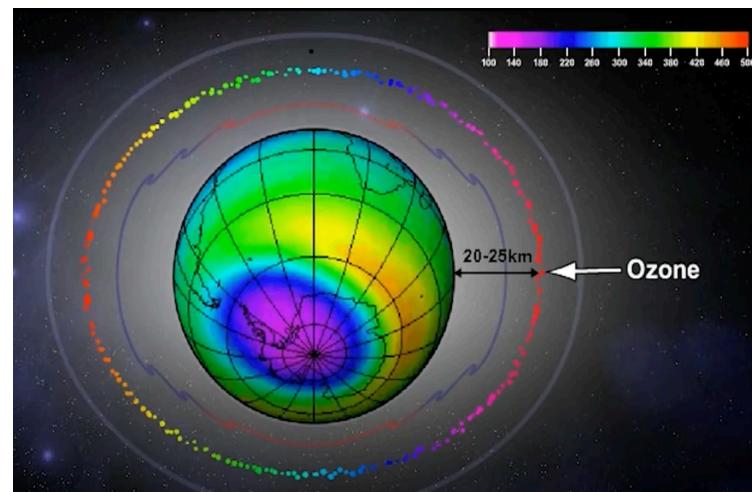
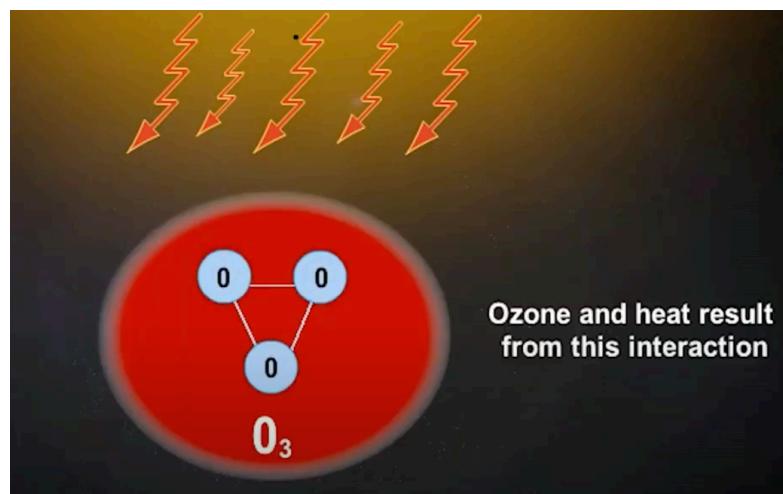
The “Jetstream” can cause turbulence



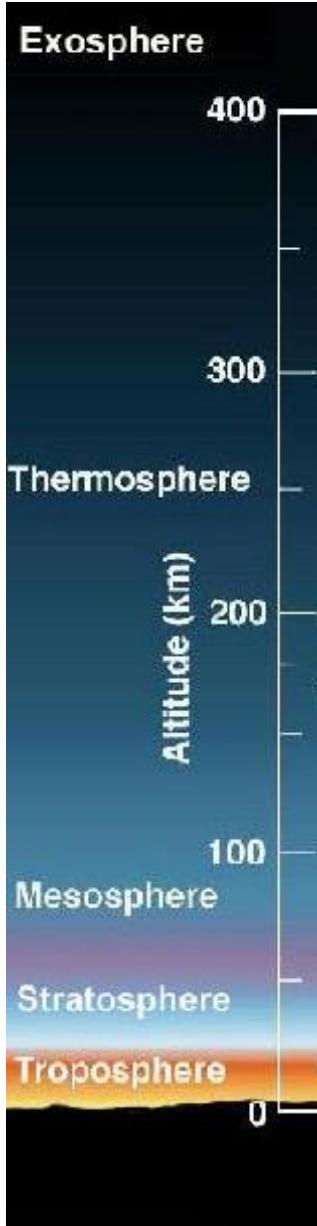
Atmospheric Layers



Stratosphere 16–50 km: This layer of the atmosphere contains about 19% of the atmospheric gases. It is very dry - containing very little water vapor. For this reason, the stratosphere is very stable, with essentially no weather effects. *The temperature rises within the stratosphere*, but they still remain below freezing point. The temperature rise is due to the absorption of solar radiation by the ozone. **The Ozone layer** is characterized by a relatively high concentration of O_3 . The ozone layer lies within the stratosphere and absorbs the UV radiation from the Sun, which is harmful to all living creators on Earth.



Atmospheric Layers

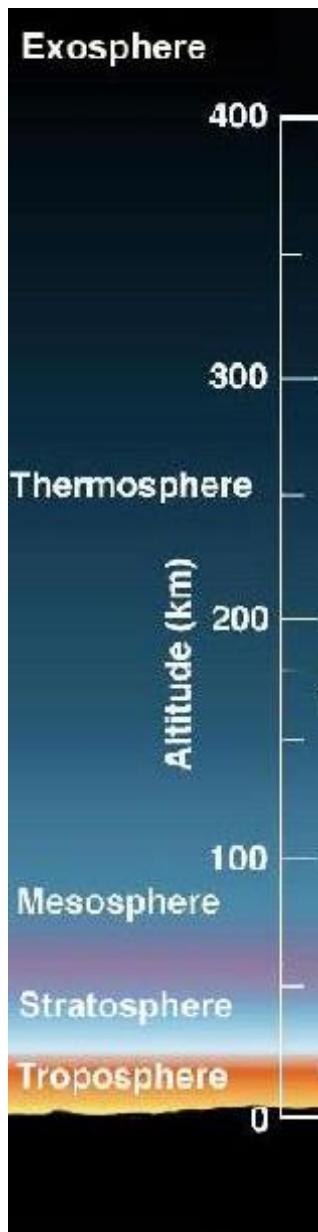


Mesosphere 50–85 km: The density of air at this layer is sufficient to cause meteors from space to burn up as they enter this layer. This gives the Earth sufficient protection against most small space rocks (meteoroids) from making a sizable impact on the Earth's surface.



The temperature decreases with the height. The Mesosphere is **the coldest layer**, any water vapor is frozen into crystal ice clouds. Studying the mesosphere is essential to understanding long-term changes in the Earth's atmosphere and how these changes affect climate.

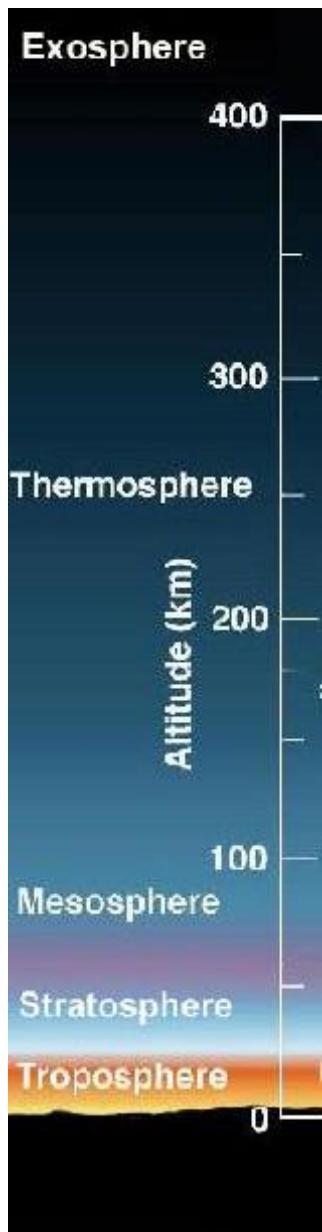
Atmospheric Layers



Thermosphere 85–600 km: The air density increases with decreasing altitude. The high energy UV rays and X-ray solar radiation from the sun bombards with and are absorbed by gaseous molecules in this layer. It causes molecules, such as oxygen and nitrogen, to ionize, split into their component atoms and creating heat. This is the reason for the rise in temperature in this layer. Even though the temperature can be really high, up to 2500°C, one would still feel the cold in this layer because the air in this layer of the atmosphere is very thin and there are very few gaseous molecules that will conduct the heat.

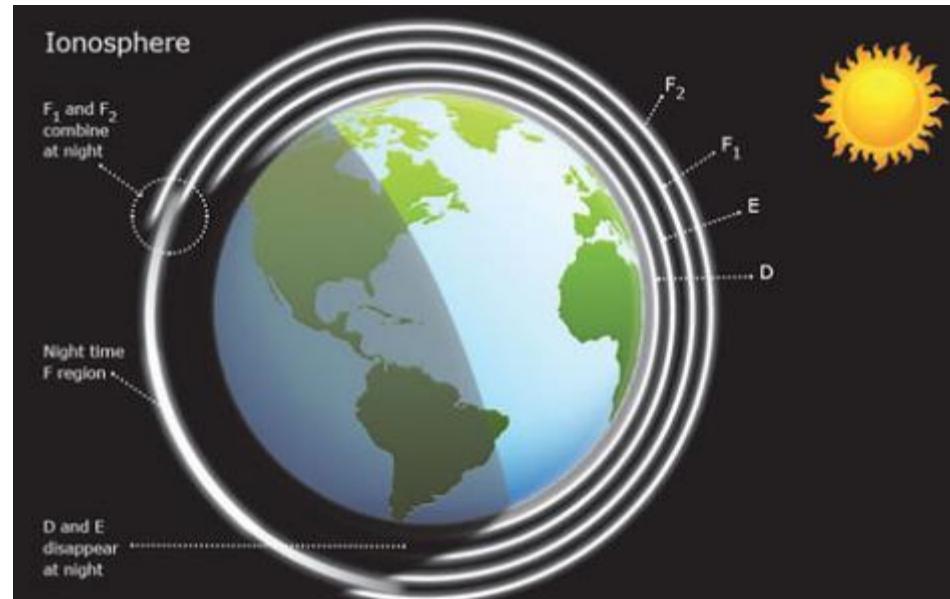
Exosphere 600–10,000 km: The exosphere is the upper limit of the gaseous atmosphere. Starting at 600 km, the exosphere merges with space without a clear bound. This area contains the lightest gases, such as ions of hydrogen, helium, CO₂ and oxygen atoms. Air molecules are constantly escaping to space and entering the atmosphere from the exosphere. Many satellites orbit the Earth at this layer.

Atmospheric Layers

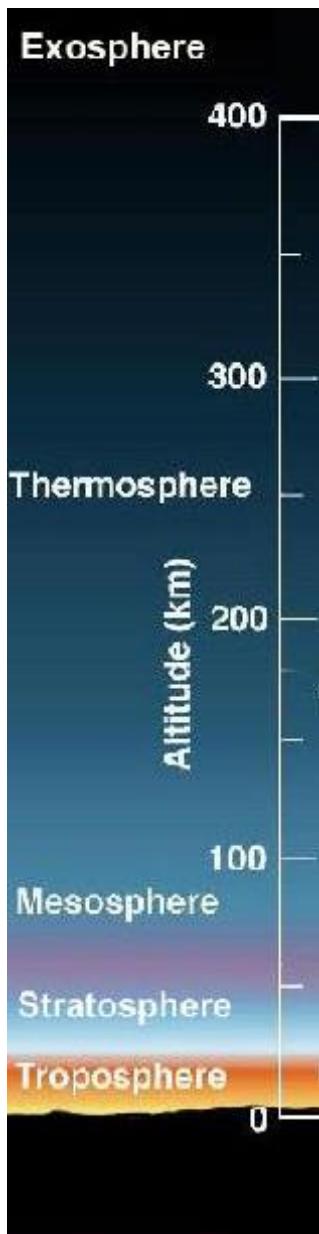


Ionosphere 85 – 600 km: In the thermosphere, the interactions of UV and X-ray solar radiations from the Sun with the gaseous molecules creates a shell of charges, and ionized atoms and molecules. This process occurs over the entire height of the thermosphere and into parts of the mesosphere and exosphere. This layer of ionized particles is the **Ionosphere**. The extent of this layer varies with solar radiation, which cycles daily. On the day side, exposure to solar radiation causes ionized particles to extend down as far as the mesosphere.

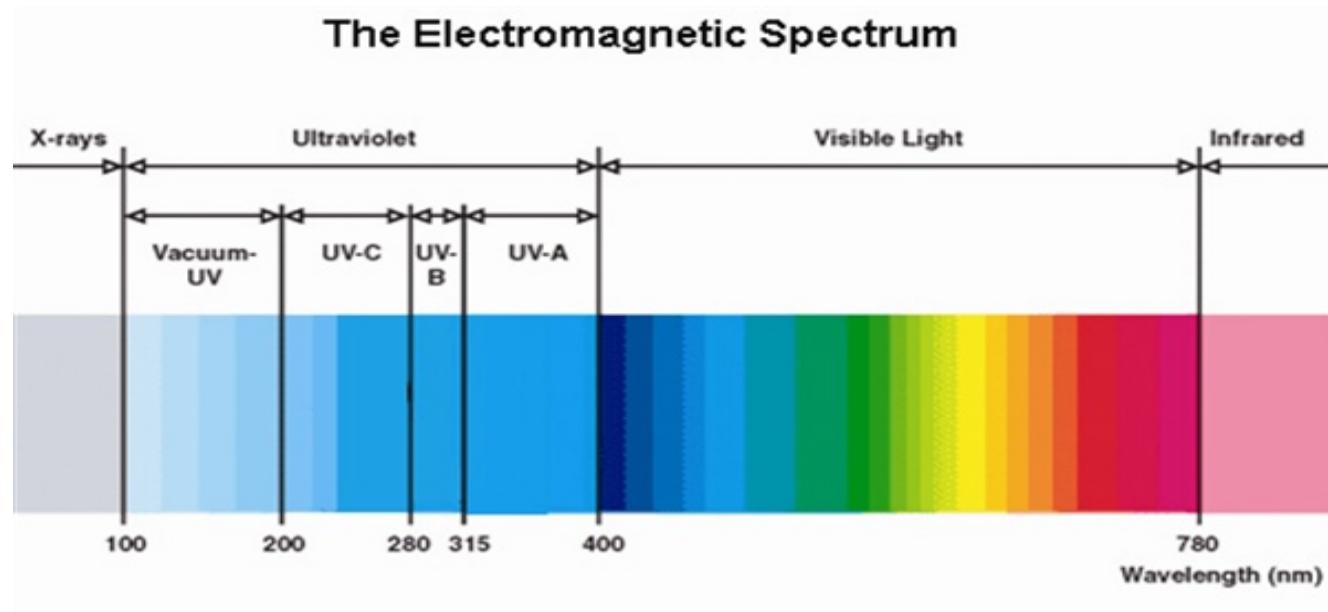
On night side, the ionosphere collapses as the Sun's radiation ceases to interact with the particles in the thermosphere.



Atmospheric Layers



The solar X-ray and extreme ultraviolet radiation (XUV) at wavelengths < 170 nm is almost completely absorbed by the thermosphere.



Solar radiation wavelength of > 175 nm is absorbed in the Mesosphere and Stratosphere by O₂ and O₃.

Exosphere

400

300

Thermosphere

200

100

0

Thermosphere

53–375 Miles

In the thermosphere, molecules of oxygen and nitrogen are bombarded by radiation and energetic particles from the Sun, causing the molecules to split into their component atoms and creating heat. The thermosphere increases in temperature with altitude because the atomic oxygen and nitrogen cannot radiate the heat from this absorption.

Mesosphere

31–53 Miles

Studying the mesosphere is essential to understanding long-term changes in the Earth's atmosphere and how these changes affect climate. Since the mesosphere is responsive to small changes in atmospheric chemistry and composition, it could provide clues for scientists, such as how added greenhouse gases may contribute to a change in temperature or water composition in the atmosphere.

Stratosphere

10–31 Miles

The ozone layer lies within the stratosphere and absorbs ultraviolet radiation from the Sun.

Troposphere

0–10 Miles

The troposphere is the layer of the Earth's atmosphere where all human activity takes place.

https://www.nasa.gov/mission_pages/hubble/main/index.html

HUBBLE SPACE TELESCOPE

370 Miles

WEATHER SATELLITES

250 Miles

INTERNATIONAL SPACE

STATION

250 Miles

SOUNDING ROCKET

50–1,500 Miles

BARREL, NASA

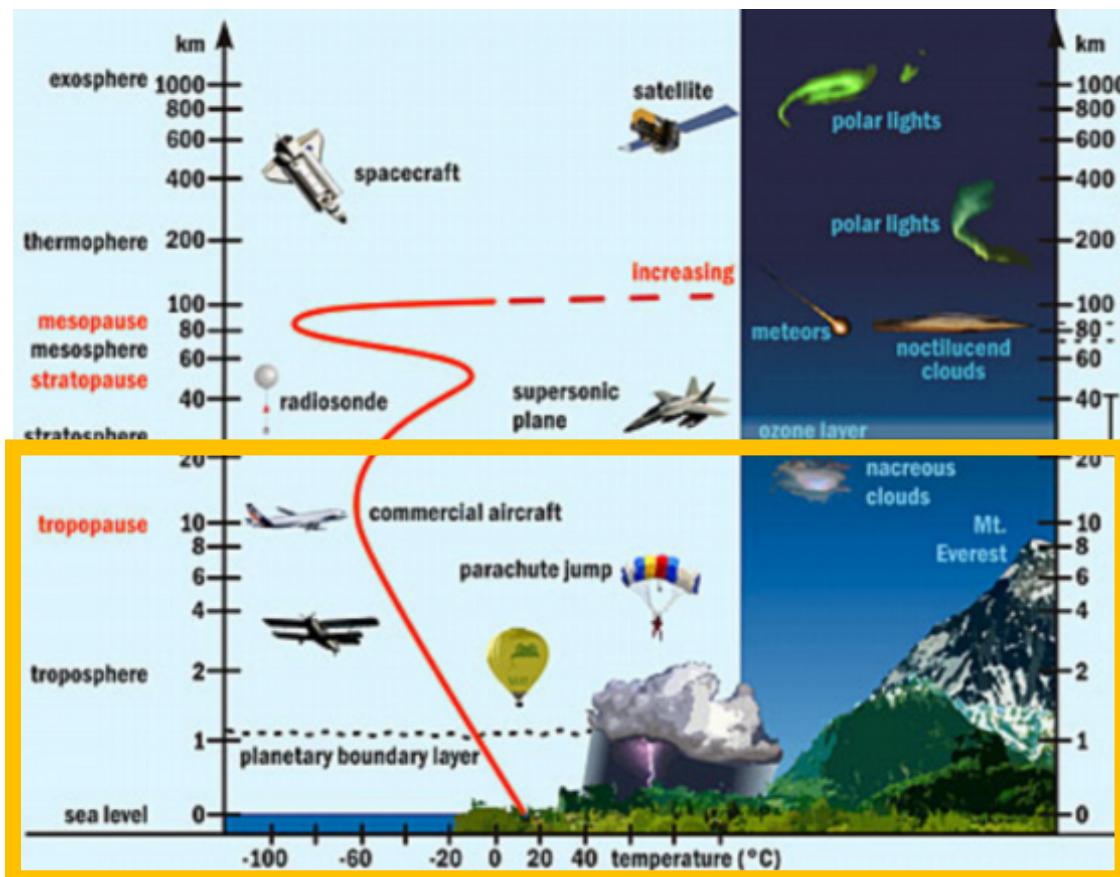
SUPER-PRESSURE BALLOON

20.8 Miles



Limits of Air-Breathing-Engined Aircraft

Each airframe-engine combination are designed to operate only within a certain range of altitudes and Mach numbers (velocities). Air-breathing engine aircraft (as opposite to non air-breathing spacecraft) can only operate within the stratosphere and the troposphere.



At higher altitudes, the air in the atmosphere is too thin for air-breathing combustion engines to be feasible. At such altitudes, rocket-type engines are necessary to generate the propulsion needed for flight.

For airplanes designed for subsonic flights operate within the lower stratosphere and troposphere. A good understanding of the atmospheric properties in this region is needed.

Further Study

- **Research By yourself:** other characteristics of different atmospheric layers that are relevant for civil aviation
- **NASA's eyes:** monitor our planet's vital signs, such as sea level height, atmospheric carbon dioxide concentration and Antarctic ozone. Explore the geo-located satellite images of recent Earth events, such as super storms.

<https://eyes.nasa.gov/eyes-on-the-earth.html>

The Atmosphere

- It is unquestionable that the atmosphere is not at a standstill - it is in constant state of flux. The atmospheric condition, e.g. *temperature, pressure, density and wind velocity* are changing all the time, and are influenced by geography, time of the day, season, moon, the Sun's solar cycle etc.
- Being someone involved in aviation and aeronautics, we need to recognize that atmospheric condition is one of the most important factors that affects flight and flight safety. Before we can design an airplane, or study the performance of airplane flying through the air, we need to know something about the air that it flies in, and answer some fundamental questions.

Atmospheric Condition

Atmospheric condition is a set of physical properties that describe the condition of the atmosphere. These properties include:

- Pressure, P
- Density, ρ
- Temperature, T
- Dynamic viscosity, μ

Atmospheric Pressure, P – is essentially the weight of all the air particles above over a unit area at various altitude. The higher the altitude, the lesser the amount air particles remain above, the lower the pressure.

A typical value of atmospheric pressure at sea level is 101,325 Pa (14.7 lbs per square inch or 1013.25 milli-bars, or 29.92 inches of mercury)

Temperature in Kelvin scale, T – is known as the absolute temperature or temperature in Kelvin scale. The melting point of ice is 0 °C, which corresponds to 273.15 Kelvin. Temperature is the kinetic energy of the molecular particles.

Density ρ – The density is the mass of the molecules over its volume. In the same volume, if the mass of molecules decrease, the density is decreased. The density of water is 1000 kg/m³ at standard sea level condition. The density of air is 1.225 kg/m³ at SSL.

Viscosity, μ - is due to the friction between neighboring particles in a fluid that are moving at different velocities. $\mu = 1.789 \times 10^{-5}$ kg/m/s at sea-level.

Standard Atmosphere

The atmospheric condition is constantly fluctuating and subject to a wide variation. Taking atmospheric measurements at different geographical locations, at different altitude, the results will be very different. The figure below shows this variation in temperature as a function of altitude.

It does not make sense for aeronautical engineers to use their “local” atmospheric conditions for design and engineering evaluations. It is also very difficult to compare or to take into account all the variations for flight or in the design and performance analysis of air vehicles.

Instead, **a common standard is needed for reference in cases such as equipment calibration and flight testing.**

This necessity led to the development of the standard atmosphere.

The purpose of the standard atmosphere is to provide a standardized table of reference of atmospheric conditions to aeronautics engineer everywhere.

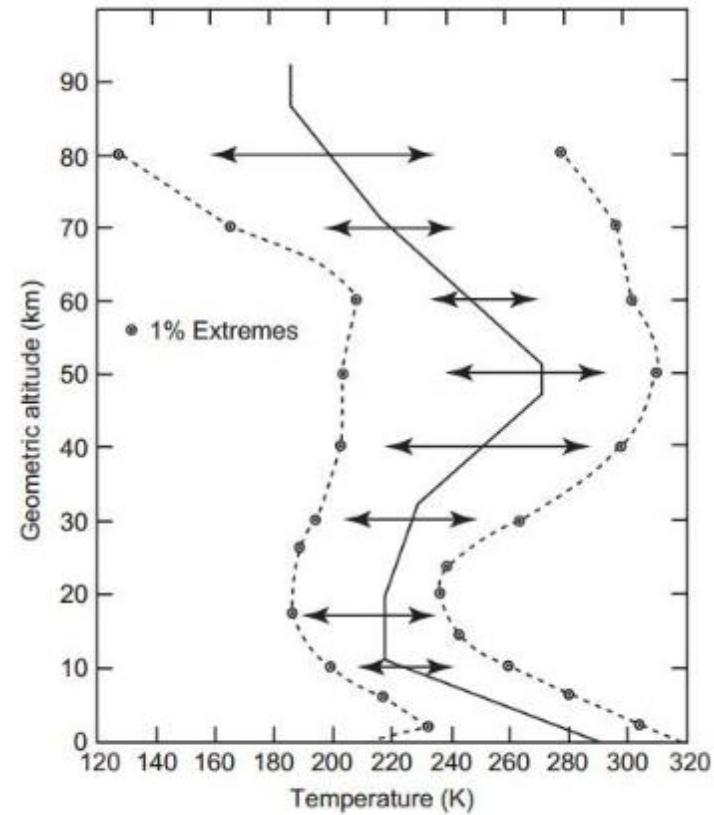


Figure (From Sissenwine *et al.* (1976).)

International Standard Atmosphere

A number of standard atmospheres were developed at different parts of the world over the history of aviation.

The 3 most common standards are:

- International Standard Atmosphere (ISA) by ISO
- International Civil Aviation Organization (ICAO) Standard Atmosphere.
- U.S. Standard Atmosphere by the U.S. Government

The Standard Atmosphere reflects the mean values of pressure, temperature, density and other properties as a function of altitude. We will focus on the ISA, which is used to calibrate current avionics

Atmospheric models were developed based on atmospheric measurement that were averaged, then curve fit to produce the model's equations. A defined temperature profile as a function of altitude forms the basis of the standard atmosphere. The figure on the RHS shows the temperature profile and the variation in temperature around a standard atmosphere model up to 90 km.

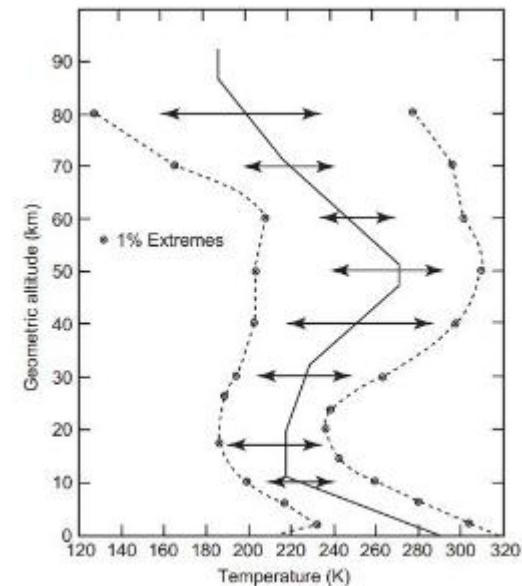


Figure (From Sissenwine et al. (1976).)

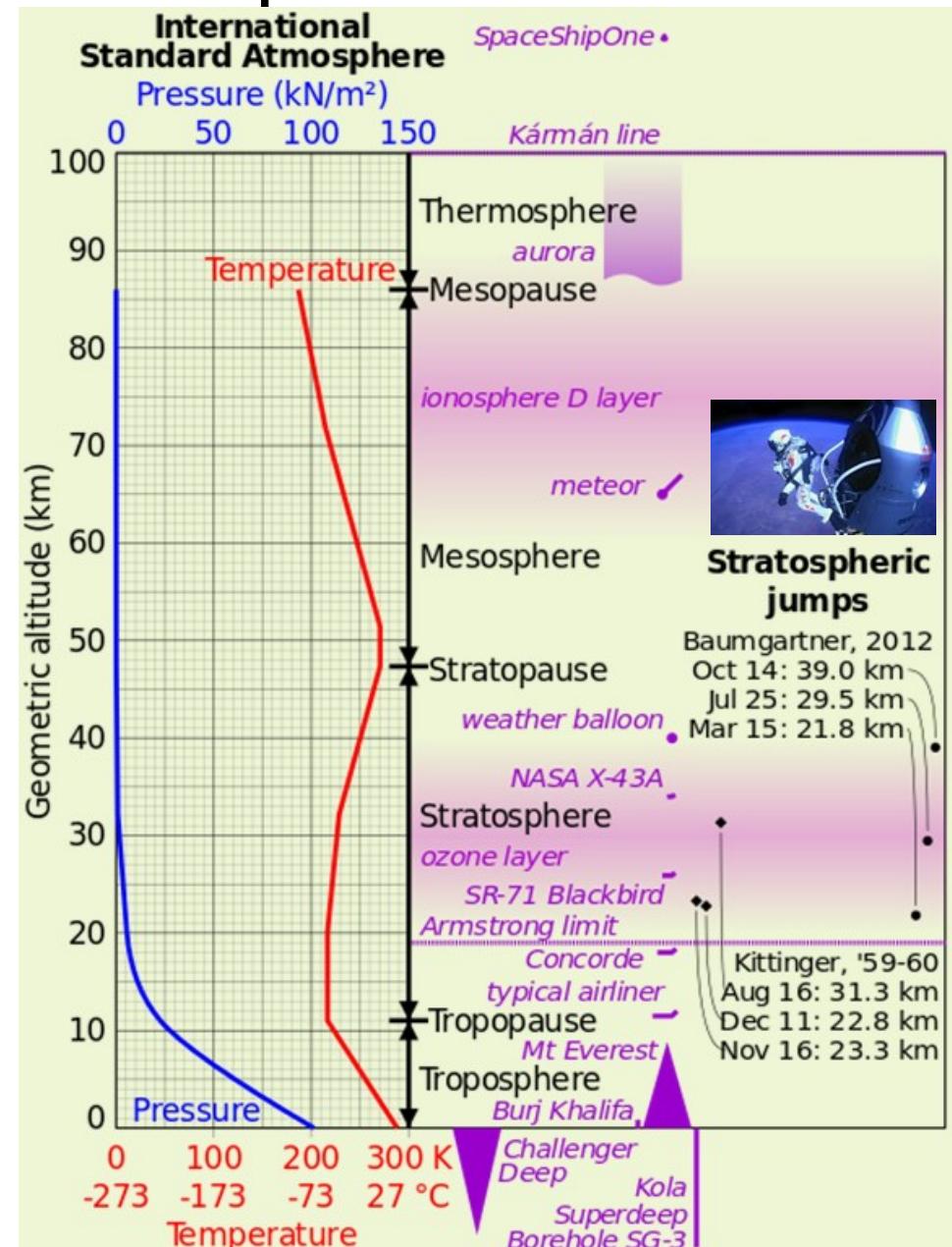
International Standard Atmosphere

The International Standard Atmosphere

Atmosphere (ISA) is an idealized atmosphere with specific vertical profile of pressure, temperature, and density prescribed by international agreement

- ▶ the standard atmosphere is used for several aerospace applications, such as determining altitude from pressure altimeters
- ▶ The air in the model is assumed to be dry and at rest with respect to the ground – no wind, no turbulence, no moisture.
- ▶ The air is assumed a perfect gas.

The lower stratosphere and troposphere is where the majority of aircraft operations take place. We will look at the mathematical model of the ISA in this region.



International Standard Atmosphere

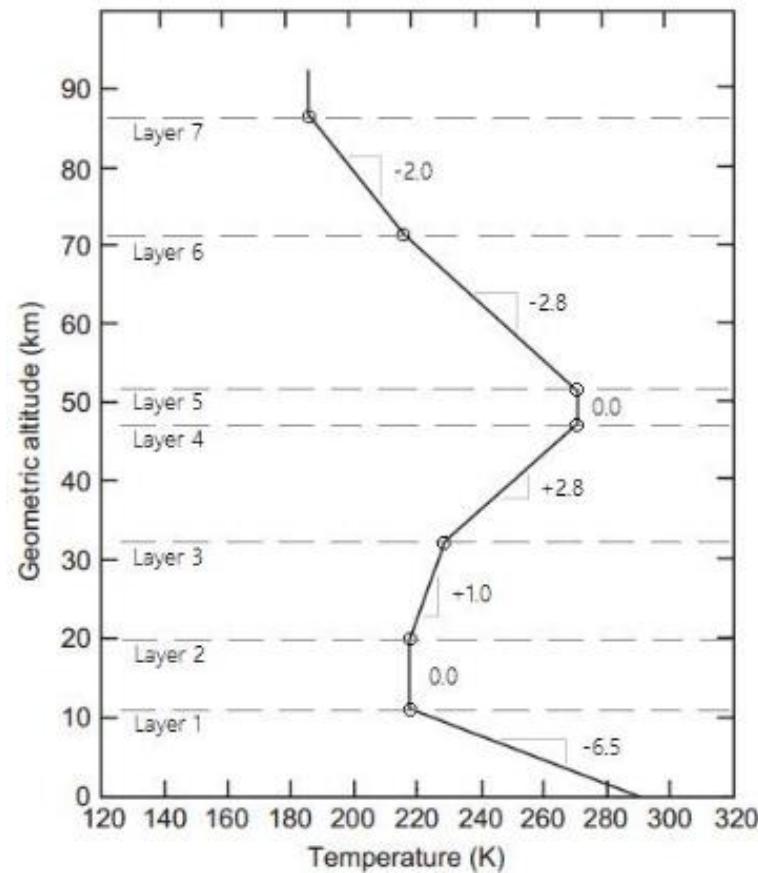
Pressure	$p_0 = 101325 \text{ N/m}^2 (\text{Pa})$
Density	$\rho_0 = 1.225 \text{ kg/m}^3$
Temperature	$T_0 = 288.15 \text{ K (15}^\circ\text{C)}$
Speed of sound	$a_0 = 340.294 \text{ m/sec}$
Acceleration of gravity	$g_0 = 9.80665 \text{ m/sec}^2$
Gas constant	$R = 8.314462 \text{ J/K/mol}$
Viscosity	$\mu = 1.789 \times 10^{-5} \text{ kg/m/s}$
Radius of the Earth	$R_{\text{Earth}} = 6356.766 \text{ km (ISA 1976)}$

A defined temperature profile as a function of altitude forms the basis of the ISA (also for all other standard atmosphere models). The model starts at sea level (altitude = 0), where the standard sea level conditions as follow.

The temperature model is made up of layers, each is a straight line, some inclined and some vertical. The gradients are called **lapse rates**. The temperature profile of each layer is modelled as,

$$T^{(i)}(h) = T_0^{(i)} - L_R^{(i)}(h - h_0^{(i)})$$

where (i) is the layer index, h is the geopotential altitude.



The **lapse rate** L_R is defined as the decrease in atmospheric temperature with increasing altitude.

ISA – Temperature Profile

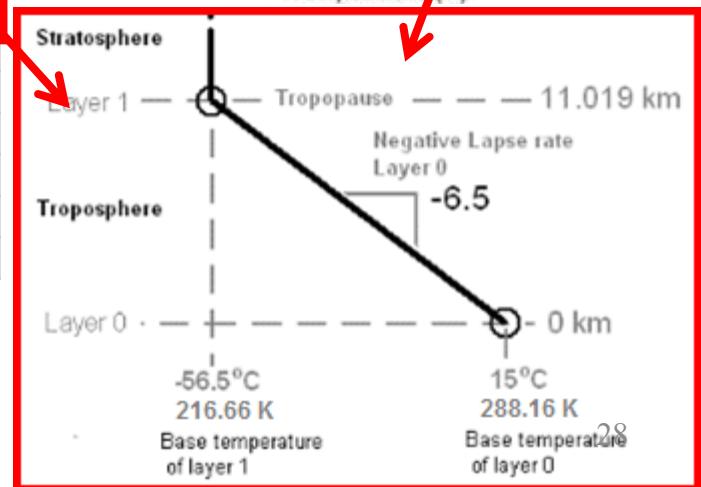
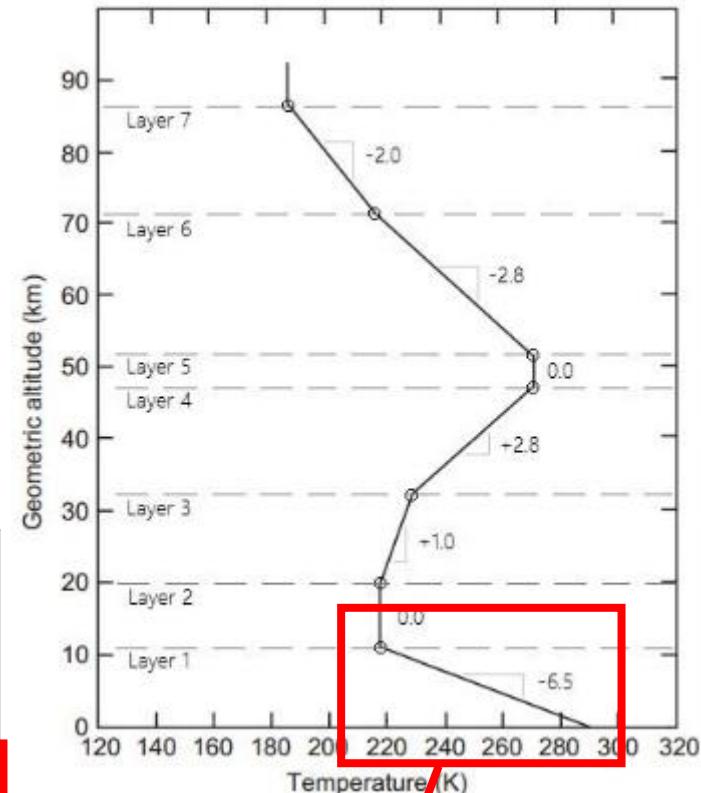
The lapse rates that define the international standard atmosphere are summarized in the table and figure below.

Below 20 km, where most airplanes fly, in the troposphere, the lapse rate is, on average, 6.5 °C per km. In the stratosphere, between 11-20 km, the lapse rate is zero, i.e., the atmospheric temperature does not change with altitude.



Level	Level Name	Base Geopotential Altitude above MSL h (in km)	Base Geometric Altitude above MSL z (in km)	Lapse Rate (in - °C/km)	Base Temp T (in °C / K)	Base Atmospheric Pressure [Pa]
0	Troposphere	0.0	0.0	6.5	15 / 288.16	101325
1	Tropopause	11.000	11.019	0.0	-56.5 / 216.66	22632
2	Stratosphere	20.000	20.063	-1.0	-56.5 / 216.66	5474.9
3	Stratosphere	32.000	32.162	-2.8	-44.5 / 228.66	868.02
4	Stratopause	47.000	47.350	0.0	-2.5 / 270.66	110.91
5	Mesosphere	51.000	51.413	2.8	-2.5 / 270.66	66.939
6	Mesosphere	71.000	71.802	2.0	-58.6 / 214.56	3.9564
7	Mesopause	84.852	86.000	—	-86.28 / 186.88	0.3734

We will develop the pressure and density equations for these various layer segments.



Equation of State for Perfect Gas

- **What is a perfect gas?**

A perfect gas is one in which intermolecular forces are negligible.

- Real gases experience intermolecular forces. At significantly low pressure, high temperature, real gases behave qualitatively like an ideal gas.
- Ideal gases undergo perfectly elastic collisions. This implies that intermolecular interactions contribute negligibly to the energetics of the gas molecules.

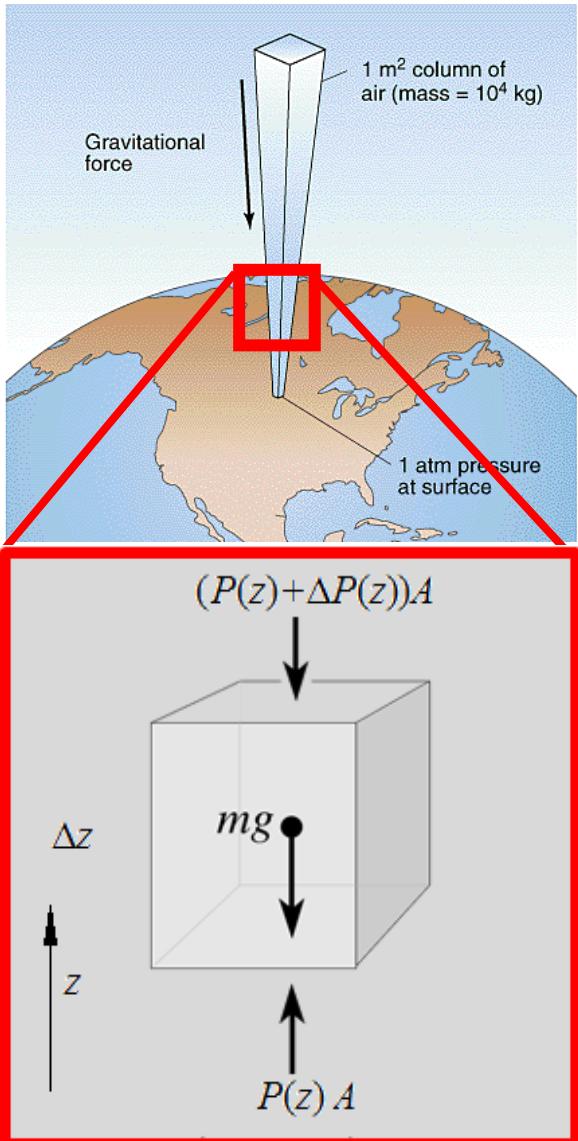
- **Ideal / Perfect Gas Law**

 $P = \rho R_s T$

where P is the absolute pressure of the gas, ρ is the density of the gas, R_s is the specific gas constant and T is the absolute temperature of the gas.

Specific gas constant $R_s = 286.97 \text{ m}^2/(\text{s}^2\text{K})$

Hydrostatic Equilibrium of the Atmosphere



Consider a thin vertical slice, Δz , of the atmosphere of cross-sectional area, A , that starts at height, z , above sea level. Note: z is the geometric altitude

For static equilibrium in still air, the difference between the upward and downward pressure force exerted on this slice from the air must be balanced by the weight of the slice of atmosphere.

$$\sum F_z = 0 = P(z)A - [P(z) + \Delta P(z)]A - \rho_{air}(z)g(z)\Delta z A$$

The weight of the slice is, $\rho A \Delta z g$, where ρ is the density of air, and g is the acceleration due to gravity. It follows that the force balance is:

$$\frac{\Delta P(z)}{\Delta z} = -\rho_{air}(z)g(z)$$

At the limit of $\Delta z \rightarrow 0$

$$\frac{dP}{dz} = -\rho_{air}(z)g(z)$$

General Equation of the Atmosphere

Combining the perfect gas law and the hydrostatic equilibrium equation, the general equation of the atmosphere is given as,

$$\frac{dP}{P} = -\frac{1}{R_s} \frac{g(z)}{T(z)} dz$$

The equation for pressure could be obtained by integrating on the above equation. But this could be quite messy - depending on the function of $g(z)$ and $T(z)$. To simplify this, let us introduce the Geopotential altitude h . Based on Newton's law of gravitation, the acceleration due to gravity, g , on an object at an altitude z above the Earth's surface is given by,

$$g(z) = \frac{GM_{\text{Earth}}}{(r_{\text{Earth}} + z)^2} = g_0 \frac{r_{\text{Earth}}^2}{(r_{\text{Earth}} + z)^2}$$

where g_0 is the acceleration due to gravity at sea-level, r_{Earth} is Earth's radius at sea-level.

Then, $g(z)dz = g_0 dh$ where $dh = \frac{r_{\text{Earth}}^2}{(r_{\text{Earth}} + z)^2} dz$

Integrating for h gives the definition of the Geopotential altitude with respect to the geometric altitude z . The ISA is defined on h .

$$h = \frac{r_{\text{Earth}}}{(r_{\text{Earth}} + z)} z$$

ISA Pressure Model

The general equation of the atmosphere can be rewritten as, $\frac{dP}{P} = -\frac{g_0}{R_s} \frac{dh}{T(h)}$

Integration of the RHS with respect to the Geopotential altitude, h , is significantly less difficult.

For constant T, i.e. the lapse rate = 0,

$$\ln(P) = -\frac{g_0}{R_s T} h + C \rightarrow P = D \exp\left(-\frac{g_0}{R_s T} h\right)$$

For linear variation of T with respect to geopotential altitude, i.e. the non-zero lapse rate.

$$\ln(P) = \frac{g_0}{R_s L_R} \ln(T_i - L_R(h - h_i)) + C \rightarrow P_i = D(T_i)^{\frac{g_0}{R_s L_R}}$$

where C and D are constants

Zero Lapse Rate Segments

At the base of each layer of the international standard atmosphere, the base pressure P_i , the base temperature T_i and the base Geopotential altitude h_i are known. The pressure equation of the layers with zero lapse rate, or constant temperature at the base is,

$$P_i = D \exp\left(-\frac{g_0}{R_s T_i} h_i\right)$$

Then, the ratio of pressure P at a Geopotential altitude h with respect to the base values is,

$$\frac{P}{P_i} = \exp\left(-\frac{g_0}{R_s T_i} (h - h_i)\right)$$

Using the relationship between density and pressure defined by the ideal gas law, The ratio of density of air at a Geopotential altitude h with respect to the base is,

$$\frac{\rho}{\rho_i} = \frac{P}{P_i} = \exp\left(-\frac{g_0}{R_s T_i} (h - h_i)\right)$$

Constant Lapse Rate Segments

Similarly, the pressure equation of the layers with constant lapse rate, or linear temperature profile, the pressure at the base is,

$$P_i = D(T_i)^{\frac{g_0}{R_s L_R}}$$

Then, the ratio of pressure P at a Geopotential altitude h with respect to the base values is,

$$\frac{P}{P_i} = \left(\frac{T}{T_i} \right)^{\frac{g_0}{R_s L_R}}$$

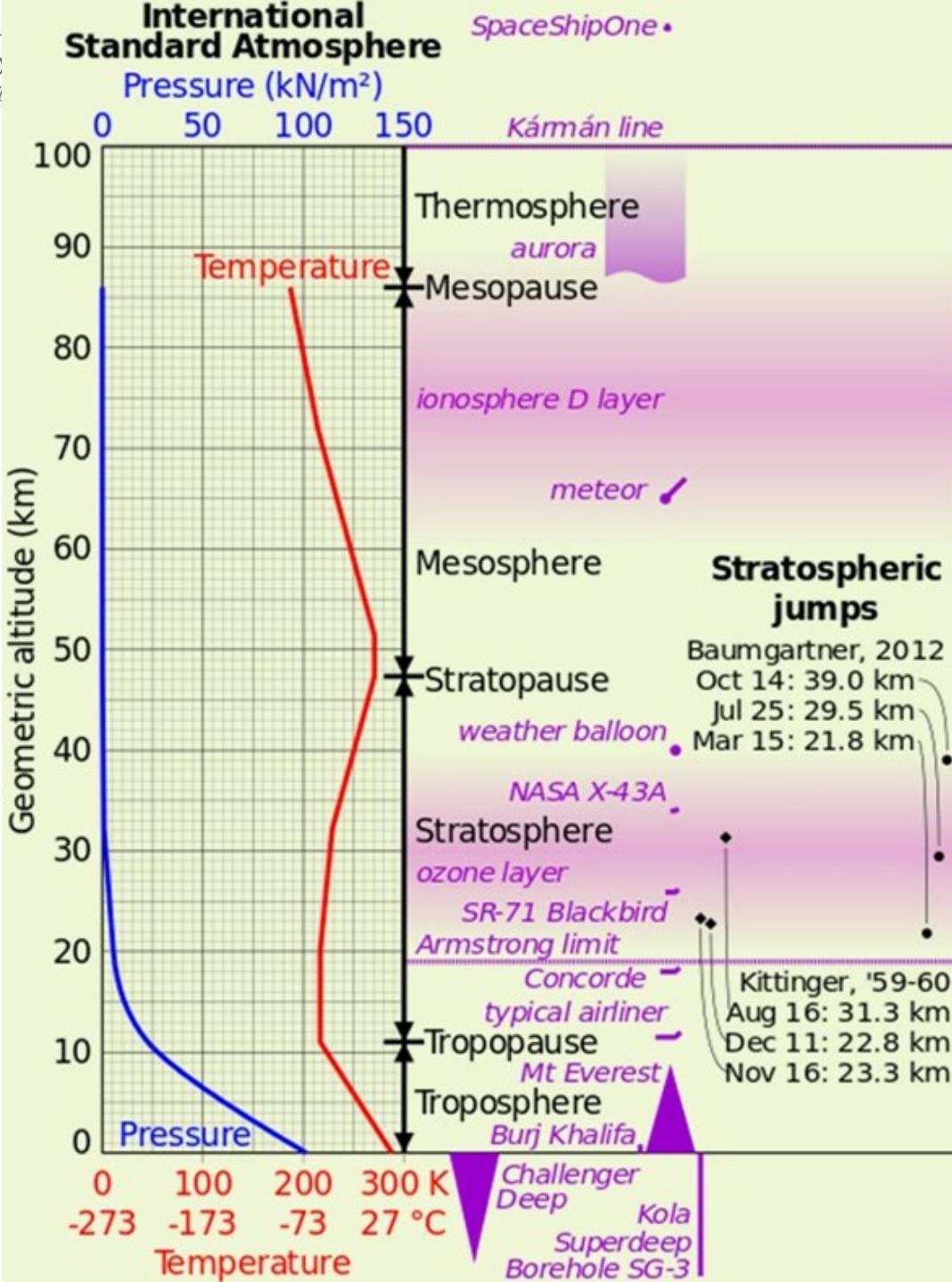
Using the relationship between density and pressure defined by the ideal gas law, The ratio of density of air at a Geopotential altitude h with respect to the base is,

$$\frac{\rho}{\rho_i} = \left(\frac{T}{T_i} \right)^{\frac{g_0}{R_s L_R} - 1}$$

International Standard Atmosphere

International Standard Atmosphere

SpaceShipOne •



Geopotential and Geometric altitudes

$$h = \frac{r_{\text{Earth}}}{(r_{\text{Earth}} + z)} z$$

Geopotential altitude h [m]	Geometric altitude z [m]
0	0
2000	2001
4000	4003
6000	6006
8000	8010
10000	10016
12000	12023
14000	14031
16000	16040
18000	18051
20000	20063
22000	22076
24000	24091
26000	26106
28000	28123
30000	30142
32000	32161

Exercise 1

Calculate the atmospheric properties at a Geopotential altitude at 8000 m.

Reminder: Geopotential altitude of 8000 m is within the Troposphere, hence it's within layer 0 of the ISA. The base Geopotential altitude is 0, the base temperature is 288.15 K, $L_R = 0.0065 \text{ K/m}$, and the base pressure is 101325 Pa.

Exercise 1

Calculate the atmospheric properties at a Geopotential altitude at 8000 m.

Reminder: Geopotential altitude of 8000 m is within the Troposphere, hence it's within layer 0 of the ISA. The base Geopotential altitude is 0, the base temperature is 288.15 K, $L_R = 0.0065 \text{ K/m}$, and the base pressure is 101325 Pa.

Temperature

$$\begin{aligned} T^{(0)}(8000) &= T_0^{(0)} - L_R (8000 - h_0^{(0)}) \\ &= 288.15 - 0.0065(8000 - 0) \\ &= 236.15 \text{ K} \end{aligned}$$

Pressure

$$\begin{aligned} P^{(0)}(8000) &= P_0^{(0)} \left(\frac{T^{(0)}(8000)}{T_0^{(0)}} \right)^{\frac{g_0}{L_R R_s}} \\ &= 101325 \left(\frac{236.15}{288.15} \right)^{5.2574} \\ &= 35599.13 \text{ Pa} \end{aligned}$$

Density

$$\begin{aligned} \rho &= \frac{P}{RT} = \frac{35599.13}{286.97 \times 236.16} \quad \text{or} \\ &= \rho_0 \left(\frac{T}{T_0} \right)^{\frac{g_0}{L_R R_s} - 1} = 1.225(0.81954)^{4.2574} \\ &= 0.52516 \text{ kg/m}^3 \end{aligned}$$

Exercises 2

Calculate the atmospheric properties at a Geopotential altitude at 14 km.

Other Exercises

1. Calculate the atmospheric properties at a Geopotential altitude at 6500 m.
2. Calculate the atmospheric properties at a Geometric altitude at 6500 m.
3. Calculate the atmospheric properties at a Geopotential altitude at 12 km.
4. Calculate the atmospheric properties at a Geometric altitude at 12 km.

Exercises 2

Calculate the atmospheric properties at a Geopotential altitude at 14 km.

Note:

Geopotential altitude of 14000 m is within the Stratosphere, within layer 1 of the ISA. The base Geopotential altitude is 11000 m, the base temperature is 216.65 K, $L_R = 0 \text{ K/m}$, and the base pressure is 22632 Pa. 0.3640 kg/m^3 .

Temperature

$$\begin{aligned} T^{(1)}(14000) &= T_0^{(1)} - L_R^{(1)}(14000 - h_0^{(1)}) \\ &= 216.65 - 0(14000 - 11000) \\ &= 216.65 \text{ K} \end{aligned}$$

Pressure

$$\begin{aligned} P^{(1)}(14000) &= P_0^{(1)} \exp\left(-\frac{g_0(h - h_0^{(1)})}{R_s T_0^{(1)}}\right) \\ &= 22632 \exp\left(-\frac{29419.95}{62189.393}\right) \\ &= 14101.69 \text{ Pa} \end{aligned}$$

Density

$$\begin{aligned} \rho &= \frac{P}{RT} = \frac{14101.69}{286.97 \times 216.65} \quad \text{or} \\ &= \rho_0 \exp\left(-\frac{g_0(h - h_0^{(1)})}{R_s T_0^{(1)}}\right) \\ &= 0.3639(0.6230) \\ &= 0.2268 \text{ kg/m}^3 \end{aligned}$$

Exercises

1. Calculate the atmospheric properties at a Geopotential altitude at 6500 m.
2. Calculate the atmospheric properties at a Geometric altitude at 6500 m.
3. Calculate the atmospheric properties at a Geopotential altitude at 12 km.
4. Calculate the atmospheric properties at a Geometric altitude at 12 km.

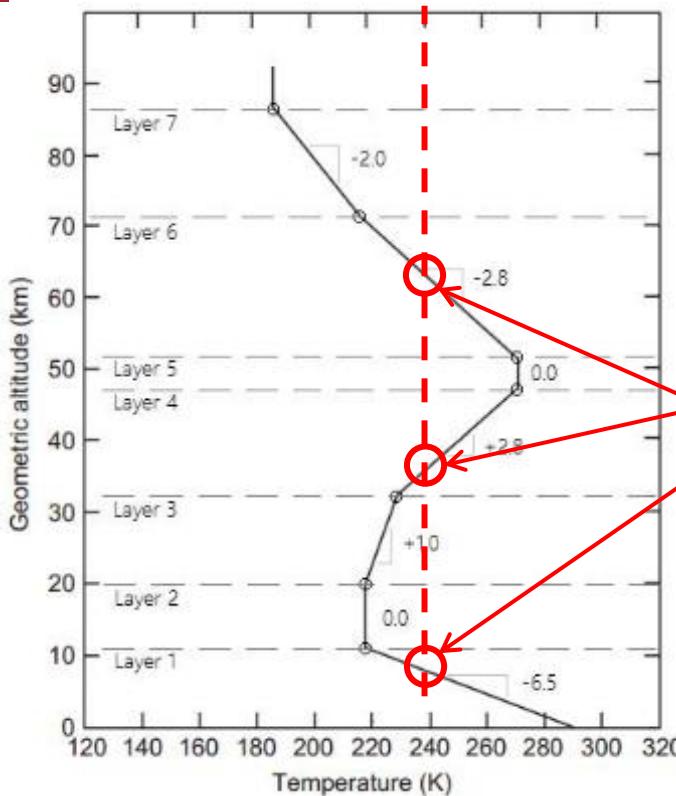
Different Altitudes

The pressure, density and temperature on a normal at a given altitude (the real altitude) will different from that of the standard atmosphere.

- *Pressure altitude* is the Geopotential altitude corresponding to the measured atmospheric pressure assuming the international standard atmosphere.
- *Temperature altitude* is the Geopotential altitude corresponding to the measured atmospheric temperature assuming the international standard atmosphere. Temperature altitude is only usable in low Troposphere.
- *Density altitude* is the Geopotential altitude corresponding to the measured air density assuming the international standard atmosphere.

Exercises 3

At the altitude the airplane is flying, the actual pressure and temperature were measured to be 4.72×10^4 Pa and 255.7 K. Calculate the pressure altitude, temperature altitude and density altitude.



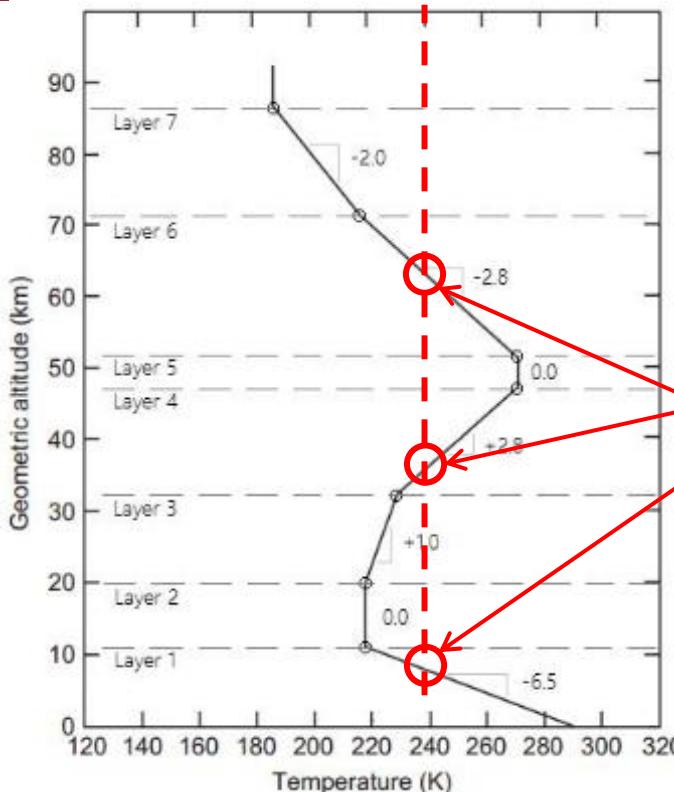
Keep in mind that when considering temperature alone, several solutions of the temperature altitude are possible. For conventional airplane, only the lowest altitude is possible. Pressure and density on the other hand decrease monotonically, hence only one altitude is possible for a given pressure and density.

Exercises 3

At the altitude the airplane is flying, the actual pressure and temperature were measured to be 4.72×10^4 Pa and 255.7 K. Calculate the pressure altitude, temperature altitude and density altitude.

Solution:

The atmospheric pressure is between the base pressure of Layers 0 and 1, hence the altitude must be within the Troposphere. The base temperature is 288.15 K, $L_R = 0.0065$ K/m, and the base pressure is 101325 Pa.



Keep in mind that when considering temperature alone, several solutions of the temperature altitude are possible. For conventional airplane, only the lowest altitude is possible. Pressure and density on the other hand decrease monotonically, hence only one altitude is possible for a given pressure and density.

Temperature altitude

$$T(h) = T_0 - L_R(h - h_0)$$

$$255.7 = 288.15 - 0.0065(h - 0)$$

$$h = 4992.31 \text{ m}$$

Pressure altitude

$$\frac{P(h)}{P_0} = \left(\frac{T(h)}{T_0} \right)^{\frac{g}{L_R R_s}}$$

$$\frac{47200}{101325} = \left(\frac{288.15 - 0.0065(h - h_0)}{288.15} \right)^{5.257}$$

$$h = 5995.88 \text{ m}$$

Density altitude

$$\frac{P}{RT} = \rho_0 \left(\frac{T(h)}{T_0} \right)^{\frac{g}{L_R R_s} - 1}$$

$$\frac{47200}{286.97 \times 255.7} = 1.225 \left(\frac{288.15 - 0.0065(h - 0)}{288.15} \right)^{4.257}$$

$$h = 6225.30 \text{ m}$$

Further Reading

- Weather for Aircrews, AFH 11-203, Volume 1, Chapters 1, 3, and 4.
- Aviation Weather for Pilots and Flight Operations Personnel, Chapters 1-3.