Matlab Toolbox Heterogeneous Agents Dynamic Programming

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Contents

Preface

This is a work-in-progress Matlab package consisting of functions that facilitate Dynamic Programming and Related Tasks. Materials gathered from various projects in which Matlab code is used. Files are the MEconTools repository. Matlab files are linked below by section with livescript files. Tested with Matlab 2019a (The MathWorks Inc, 2019).

This bookdown file is a collection of mlx based vignettes for functions that are available from MEconTools. Each Vignette file contains various examples for invoking each function. The goal of this repository is to make it easier to find/re-use codes produced for various projects.

From other repositories: For dynamic borrowing and savings problems, see Dynamic Asset Repository; For code examples, see also R Example Code, Matlab Example Code, and Stata Example Code; For intro stat with R, see Intro Statistics for Undergraduates, and intro Math with Matlab, see Intro Mathematics for Economists. See here for all of Fan's public repositories.

The site is built using Bookdown (Xie, 2020).

Please contact FanWangEcon for issues or problems.

Chapter 1

Savings Dynamic Programming

1.1 FF_VFI_AZ_LOOP Dynamic Programming Asset Problem with Shocks Loop

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_vfi_az_loop** from the **MEconTools Package.** This function solves the dynamica programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon. This is the looped code, it is extremely slow for larger state-space problems.

1.1.1 Test FF_VFI_AZ_LOOP Defaults

Call the function with defaults. By default, shows the asset policy function summary.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_loop(mp_params);

Elapsed time is 0.438199 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum       mean      std     coefvari    min
          -    ---    ----    -----    ----    ----    ------    ------    -----    --------    ---

    ap    1     1      2       350      50      7      8427.6    24.079    14.27    0.59263      0

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
             c1        c2        c3        c4        c5        c6        c7
           ------    ------    ------    ------    ------    ------    ------

    r1          0         0         0         0         0         0    2.0408
    r2          0         0         0    1.0204    1.0204    1.0204    3.0612
    r3     1.0204    1.0204    1.0204    2.0408    2.0408    2.0408    4.0816
    r4     2.0408    2.0408    2.0408    2.0408    3.0612    3.0612     5.102
    r5     3.0612    3.0612    3.0612    3.0612    4.0816    4.0816    6.1224
    r46    43.878    43.878    43.878    43.878    43.878    44.898    45.918
```

```
    r47    44.898    44.898    44.898    44.898    44.898    45.918    46.939
    r48    45.918    45.918    45.918    45.918    45.918    46.939    47.959
    r49    46.939    46.939    46.939    46.939    46.939    47.959     48.98
    r50    47.959    47.959    47.959    47.959    47.959     48.98        50
```

1.1.2 Test FF_VFI_AZ_LOOP Control Outputs

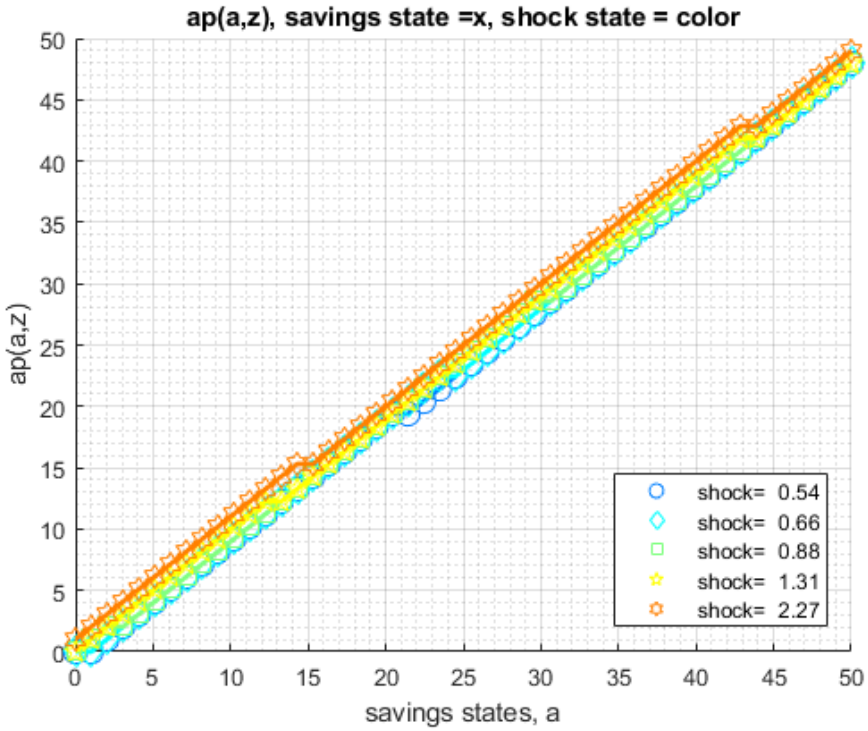
Run the function first without any outputs;

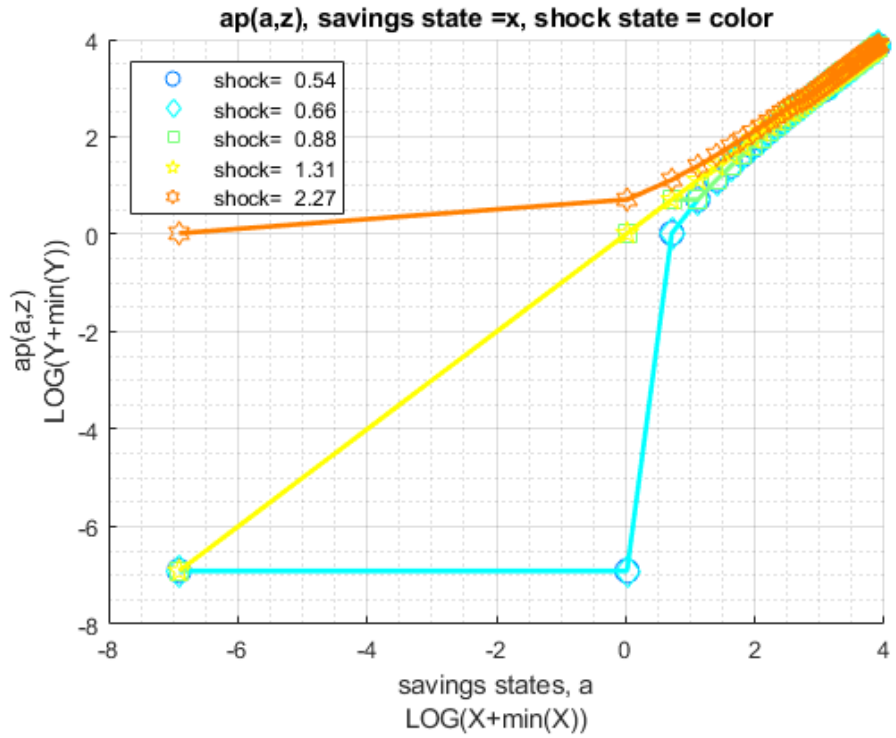
```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = false;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
```

Run the function and show policy function for savings choice:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {};
mp_support('ls_ffsna') = {'ap'};
mp_support('ls_ffgrh') = {'ap'};
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.245489 seconds.
xxx  ff_vfi_az_vec, outcome=ap  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group      a       mean_z_0_54195    mean_z_0_66401    mean_z_0_88162    mean_z_1_3095    mean_z
    -----    ------    --------------    --------------    --------------    -------------    ------

      1          0             0                 0                 0                0          1.0
      2     1.0204             0                 0            1.0204           1.0204          2.0
      3     2.0408        1.0204            1.0204            2.0408           2.0408          3.0
      4     3.0612        2.0408            2.0408            2.0408           3.0612          4.0
      5     4.0816        3.0612            3.0612            3.0612           4.0816           5.
      6      5.102        4.0816            4.0816            4.0816            5.102          6.1
      7     6.1224         5.102             5.102             5.102           6.1224          7.1
      8     7.1429        6.1224            6.1224            6.1224           7.1429          8.1
      9     8.1633        7.1429            7.1429            7.1429           8.1633          9.1
     10     9.1837        8.1633            8.1633            8.1633           9.1837          10.
     11     10.204        9.1837            9.1837            9.1837           10.204          11.
     12     11.224        10.204            10.204            10.204           11.224          12.
     13     12.245        11.224            11.224            11.224           12.245          13.
     14     13.265        12.245            12.245            12.245           12.245          14.
     15     14.286        13.265            13.265            13.265           13.265          15.
     16     15.306        14.286            14.286            14.286           14.286          15.
     17     16.327        15.306            15.306            15.306           15.306          16.
     18     17.347        16.327            16.327            16.327           16.327          17.
     19     18.367        17.347            17.347            17.347           17.347          18.
     20     19.388        18.367            18.367            18.367           18.367          19.
     21     20.408        19.388            19.388            19.388           19.388          20.
     22     21.429        19.388            20.408            20.408           20.408          21.
     23     22.449        20.408            21.429            21.429           21.429          22.
     24     23.469        21.429            22.449            22.449           22.449          23.
     25      24.49        22.449            22.449            23.469           23.469           24
     26      25.51        23.469            23.469             24.49            24.49           25
     27     26.531         24.49             24.49             25.51            25.51          26.
     28     27.551         25.51             25.51            26.531           26.531          27.
```

29	28.571	26.531	26.531	27.551	27.551	28.
30	29.592	27.551	27.551	28.571	28.571	29.
31	30.612	28.571	28.571	28.571	29.592	30.
32	31.633	29.592	29.592	29.592	30.612	31.
33	32.653	30.612	30.612	30.612	31.633	32.
34	33.673	31.633	31.633	31.633	32.653	33.
35	34.694	32.653	32.653	32.653	33.673	34.
36	35.714	33.673	33.673	33.673	34.694	35.
37	36.735	34.694	34.694	34.694	35.714	36.
38	37.755	35.714	35.714	35.714	36.735	37.
39	38.776	36.735	36.735	36.735	37.755	38.
40	39.796	37.755	37.755	37.755	38.776	39.
41	40.816	38.776	38.776	38.776	39.796	40.
42	41.837	39.796	39.796	39.796	40.816	41.
43	42.857	40.816	40.816	40.816	41.837	42.
44	43.878	41.837	41.837	41.837	41.837	42.
45	44.898	42.857	42.857	42.857	42.857	43.
46	45.918	43.878	43.878	43.878	43.878	44.
47	46.939	44.898	44.898	44.898	44.898	45.
48	47.959	45.918	45.918	45.918	45.918	46.
49	48.98	46.939	46.939	46.939	46.939	47.
50	50	47.959	47.959	47.959	47.959	48





Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 1.633068 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean      std       coefva
                    -    ---    ----    -----    ----    ----    ------    ------    ------    ------

    ap              1     1      2       900     100      9       21825     24.25    14.089      0.58
    savefraccoh     2     2      2       900     100      9      411.21    0.4569    0.2651    0.5802

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
               c1          c2          c3          c4          c5          c6          c7          c8          -
             -------     -------     -------     -------     -------     -------     -------     ------     -

    r1             0           0           0           0           0           0     0.50505     1.5152     3
    r2             0           0           0           0     0.50505     0.50505     1.0101     1.5152     3
    r3       0.50505     0.50505     0.50505     0.50505     0.50505      1.0101      1.5152     2.0202     4
    r4        1.0101      1.0101      1.0101      1.0101      1.0101      1.5152      2.0202     2.5253     4
    r5        1.5152      1.5152      1.5152      1.5152      1.5152      2.0202      2.5253     3.0303     5
    r96       45.455      45.455      45.455       45.96       45.96       45.96      46.465     47.475     4
    r97        45.96       45.96       45.96      46.465      46.465      46.465       46.97      47.98     4
    r98       46.465      46.465      46.465      46.465       46.97       46.97      47.475     48.485
    r99        46.97       46.97       46.97       46.97      47.475      47.475       47.98      48.99
```

```
    r100     47.475     47.475     47.475     47.475      47.98      47.98     48.485     49.495

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1           c2           c3           c4           c5           c6           c7
            ---------    ---------    ---------    ---------    --------    ---------    ---------

    r1              0            0            0            0           0            0    0.0094749
    r2              0            0            0            0    0.009643    0.0095804      0.01895
    r3      0.0097386    0.0097261    0.0097083    0.0096824    0.009643     0.019161     0.028425
    r4       0.019477     0.019452     0.019417     0.019365    0.019286     0.028741       0.0379
    r5       0.029216     0.029178     0.029125     0.029047    0.028929     0.038321     0.047374
    r96       0.87647      0.87535      0.87375       0.8811     0.87751      0.87181      0.87169
    r97       0.88621      0.88507      0.88346      0.89078     0.88716      0.88139      0.88116
    r98       0.89595       0.8948      0.89317      0.89078      0.8968      0.89097      0.89064
    r99       0.90569      0.90452      0.90287      0.90046     0.90644      0.90055      0.90011
    r100      0.91543      0.91425      0.91258      0.91014     0.91609      0.91013      0.90959
```

1.1.3 Test FF_VFI_AZ_LOOP Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.079084 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                  i    idx    ndim    numel    rowN    colN     sum       mean       std       coefv
                  -    ---    ----    -----    ----    ----    ------    ------    -------    -----

    savefraccoh   1     1      2       250      50      5      48.774    0.1951    0.23298     1.19

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4           c5
            -------    -------    -------    -------    ---------

    r1            0          0          0          0    0.0058555
    r2            0          0          0          0    0.0058555
    r3            0          0          0          0    0.0058555
    r4            0          0          0          0    0.0058555
    r5            0          0          0          0    0.0058555
    r46     0.62112    0.61921    0.61584    0.60931      0.59509
```

```
    r47    0.66655     0.6645    0.66088    0.65388      0.63861
    r48    0.71414    0.71195    0.70807    0.70057      0.68421
    r49    0.76395     0.7616    0.75745    0.74943      0.73193
    r50    0.81602    0.81351    0.80908    0.80051      0.78182

% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.290307 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean       std      coefv
                   -    ---    ----    -----    ----    ----    ------    ------    -------    -----

    savefraccoh    1     1      2       250      50      5      59.526    0.2381    0.27148     1.14

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
             c1          c2           c3           c4           c5
            -------    -------    ----------    ---------    --------

    r1            0          0    0.00051196     0.005772    0.021238
    r2            0          0    0.00051196     0.005772    0.021238
    r3            0          0    0.00051196     0.005772    0.021238
    r4            0          0    0.00099992     0.005772    0.021238
    r5            0          0    0.00099992    0.0079177    0.021238
    r46    0.73495    0.73278       0.72894       0.7215     0.70527
    r47    0.78505    0.78273       0.77862      0.77068     0.75334
    r48    0.83737    0.83489       0.83052      0.82204     0.80355
    r49    0.89196    0.88933       0.88466      0.87564     0.85594
    r50    0.94888    0.94608       0.94111      0.93151     0.91056
```

1.1.4 Test FF_VFI_AZ_LOOP Changing Risk Aversion

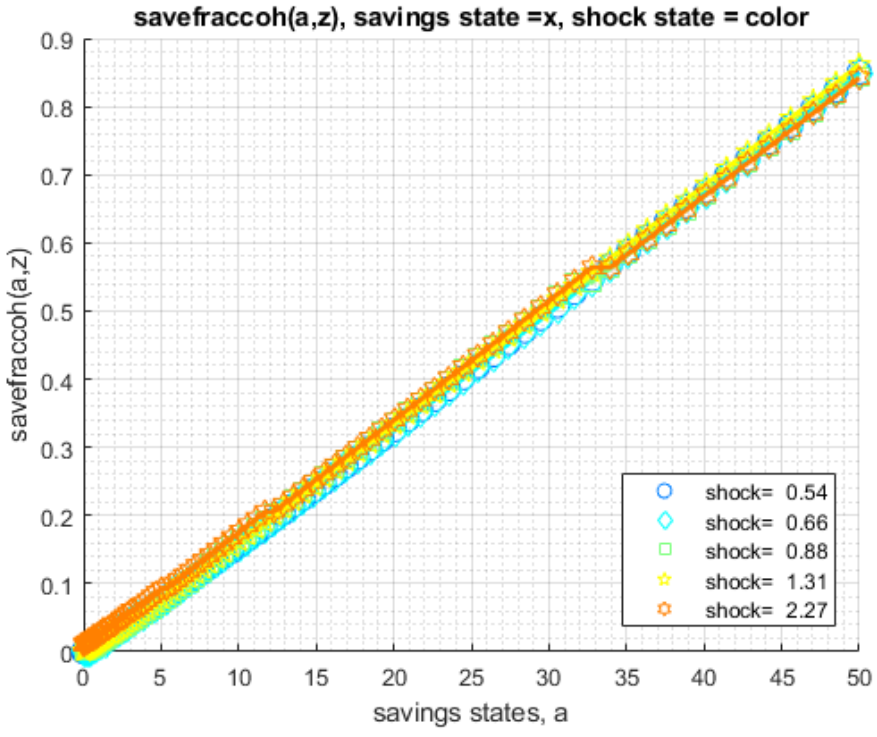
Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

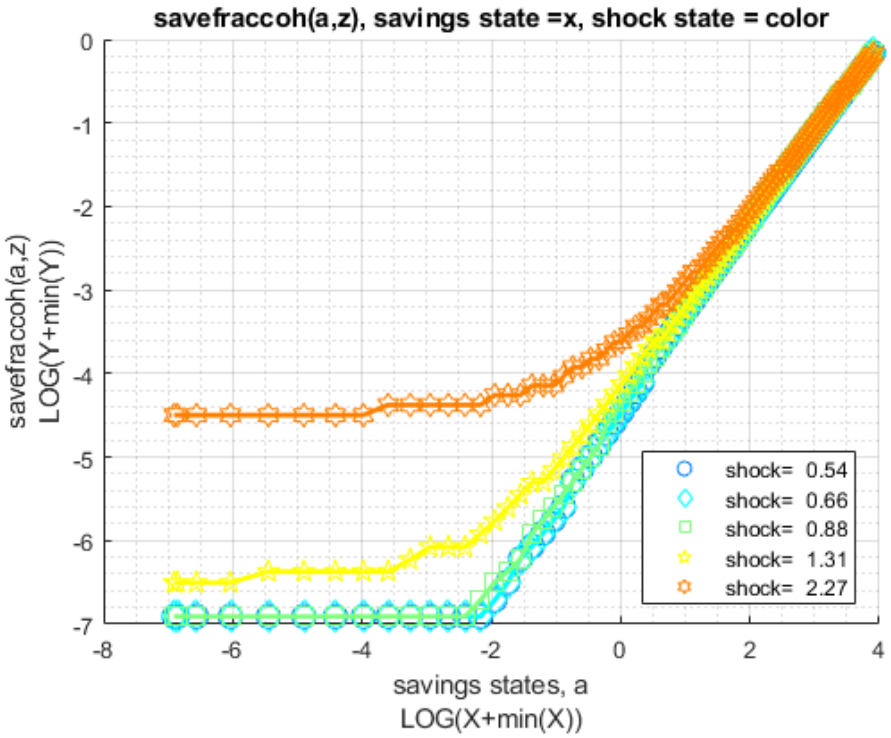
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.580227 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

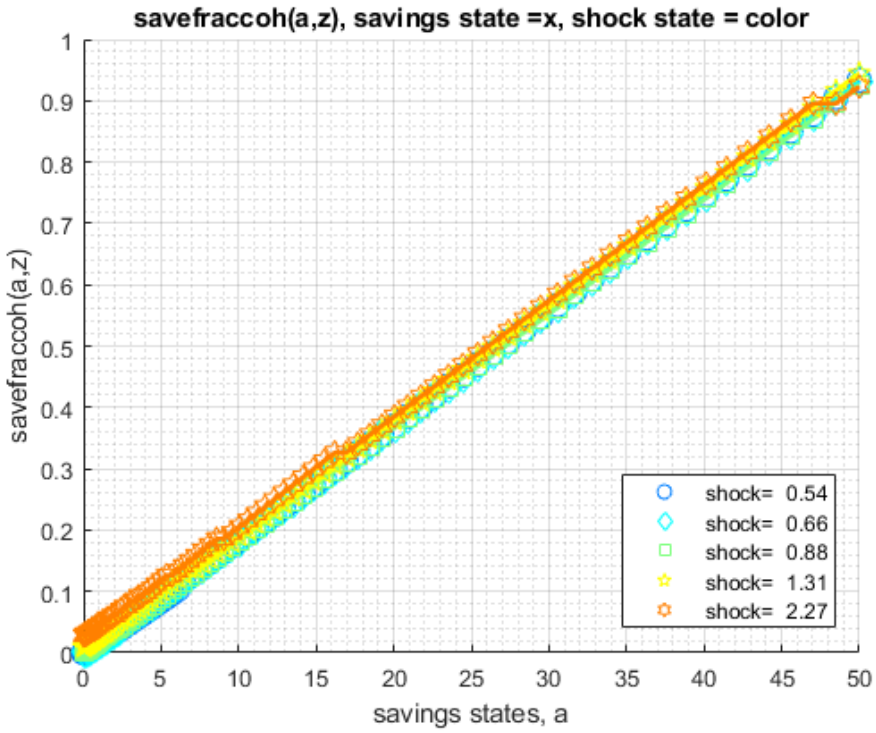
```
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum       mean        std       coef
                    -     ---     ----     -----     ----     ----     ------     -------     -------     ----

    savefraccoh     1      1       2        500      100       5       104.98     0.20996     0.24341      1.1

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1          c2          c3           c4           c5
            -------     -------     -------     ----------     --------

    r1            0           0           0      0.0004985     0.010131
    r2            0           0           0      0.0004985     0.010131
    r3            0           0           0      0.0004985     0.010131
    r4            0           0           0      0.0004985     0.010131
    r5            0           0           0     0.00070978     0.010131
    r96     0.74758     0.74533     0.74137        0.75815      0.74086
    r97     0.77249     0.77018     0.76608        0.78315      0.76529
    r98     0.79796     0.79557     0.79134        0.80868      0.79024
    r99     0.82398     0.82151     0.81714        0.83477      0.81573
    r100    0.85055       0.848     0.84349        0.86141      0.84176
```

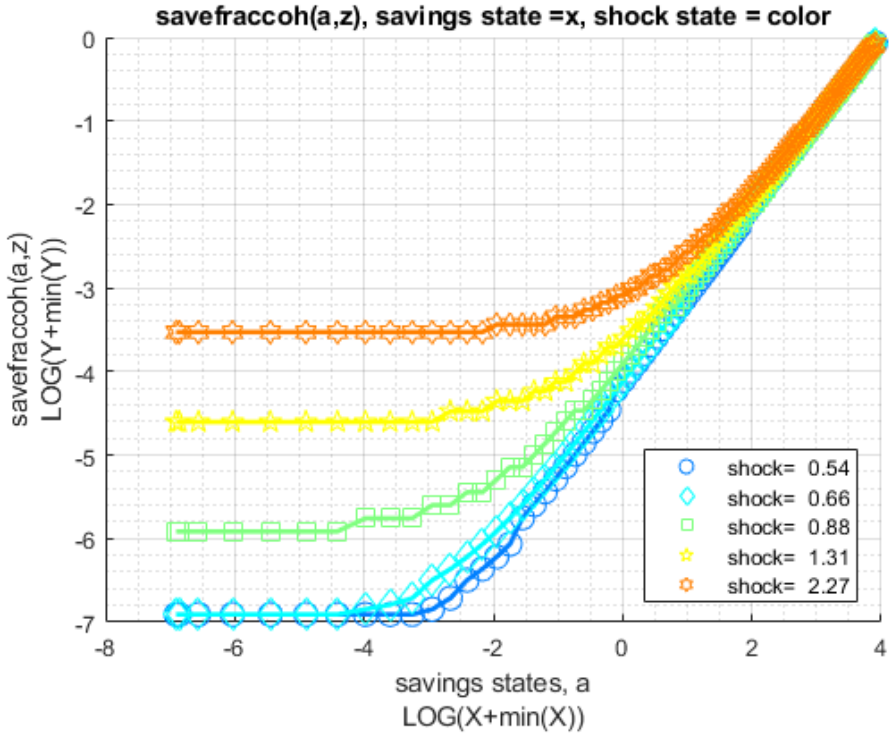




When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.906648 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1      1       2      500     100       5     119.58    0.23916    0.26719     1.1

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5
            -------    -------    -------    ---------    --------

    r1            0          0     0.0017    0.0090168    0.028344
    r2            0          0     0.0017    0.0090168    0.028344
    r3            0          0     0.0017    0.0090168    0.028344
    r4            0          0     0.0017    0.0090168    0.028344
    r5            0          0     0.0017    0.0090168    0.028344
    r96     0.82398    0.82151    0.81714      0.83477     0.84176
    r97     0.85055      0.848    0.84349      0.86141     0.86834
    r98      0.8777    0.87507    0.87041      0.88861     0.89548
    r99     0.90541     0.9027     0.8979      0.91637     0.89548
    r100    0.93371    0.93091    0.92595      0.94471     0.92317
```





1.1.5 Test FF_VFI_AZ_LOOP with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.05;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.942299 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum      mean        std      coefv
                    -    ---    ----    -----    ----    ----    -----    -------    -------    -----

    savefraccoh     1     1      2       500     100      5      112.7    0.22539    0.26207     1.16

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4           c5
            -------    -------    -------    -------    ----------

    r1            0          0          0          0    0.00049994
    r2            0          0          0          0    0.00049994
    r3            0          0          0          0    0.00049994
    r4            0          0          0          0    0.00049994
    r5            0          0          0          0    0.00049994
    r96     0.79191    0.79066    0.81492    0.81313       0.81102
    r97     0.81774    0.81644     0.8412    0.83936       0.83718
    r98     0.84411    0.84277    0.86805    0.86615       0.86389
    r99     0.87105    0.86967    0.89546     0.8935       0.89117
    r100    0.89855    0.89713    0.92344    0.92142       0.91902
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.25;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.908385 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum      mean        std      coefv
                    -    ---    ----    -----    ----    ----    -----    -------    -------    -----

    savefraccoh     1     1      2       500     100      5      115.6    0.23119    0.25857     1.11

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2           c3           c4           c5
            -------    -------    ----------    ---------    --------

    r1            0          0    0.00021288    0.0066707    0.033639
    r2            0          0    0.00021288    0.0066707    0.033639
```

r3	0	0	0.00021288	0.0066707	0.033639
r4	0	0	0.00021288	0.0066707	0.033639
r5	0	0	0.00021288	0.0066707	0.033639
r96	0.79959	0.79731	0.79275	0.80778	0.80256
r97	0.82566	0.82331	0.8186	0.83384	0.82817
r98	0.85229	0.84986	0.84501	0.86045	0.85432
r99	0.87949	0.87699	0.87197	0.88762	0.88101
r100	0.90726	0.90468	0.89951	0.91536	0.90826

1.2 FF_VFI_AZ_VEC Dynamic Programming Asset Problem with Shocks Vectorized

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_vfi_az_vec** from the **MEconTools Package.** This function solves (vectorized) the dynamica programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon. This is the vectorized code, its speed is much faster than the looped code.

1.2.1 Test FF_VFI_AZ_VEC Defaults

Call the function with defaults. By default, shows the asset policy function summary.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_vec(mp_params);

Elapsed time is 0.407936 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum      mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    -----    ------    ------    --------    ---

    ap    1     1      2       2100     300      7      50584    24.088    13.973    0.58008      0

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
            -------    -------    -------    -------    -------    -------    ------

    r1            0          0          0          0    0.16722     0.6689    2.0067
    r2            0          0          0    0.16722    0.33445    0.83612    2.1739
    r3      0.16722    0.16722    0.16722    0.16722    0.50167     1.0033    2.3411
    r4      0.33445    0.33445    0.33445    0.33445     0.6689     1.1706    2.5084
    r5      0.33445    0.33445    0.50167    0.50167    0.83612     1.3378    2.5084
    r296     46.823      46.99      46.99      47.157     47.492     48.161    49.498
    r297      46.99     47.157     47.157     47.324     47.659     48.328    49.666
    r298     47.157     47.324     47.324     47.492     47.826     48.495    49.833
    r299     47.324     47.492     47.492     47.659     47.993     48.662        50
    r300     47.492     47.659     47.659     47.826     48.161     48.829        50
```

1.2.2 Test FF_VFI_AZ_VEC Control Outputs

Run the function first without any outputs;

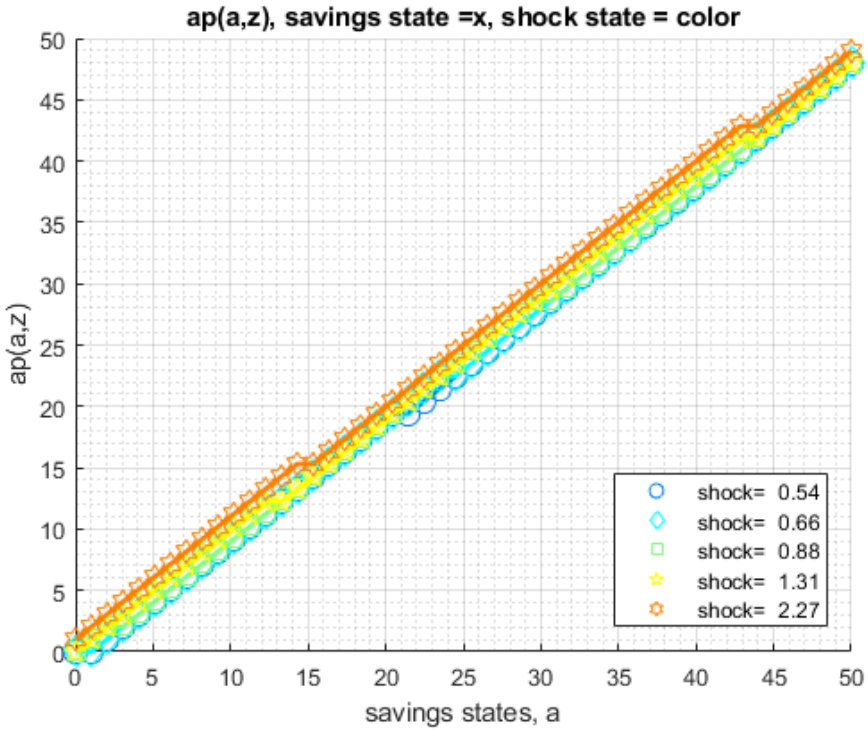
```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = false;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
```

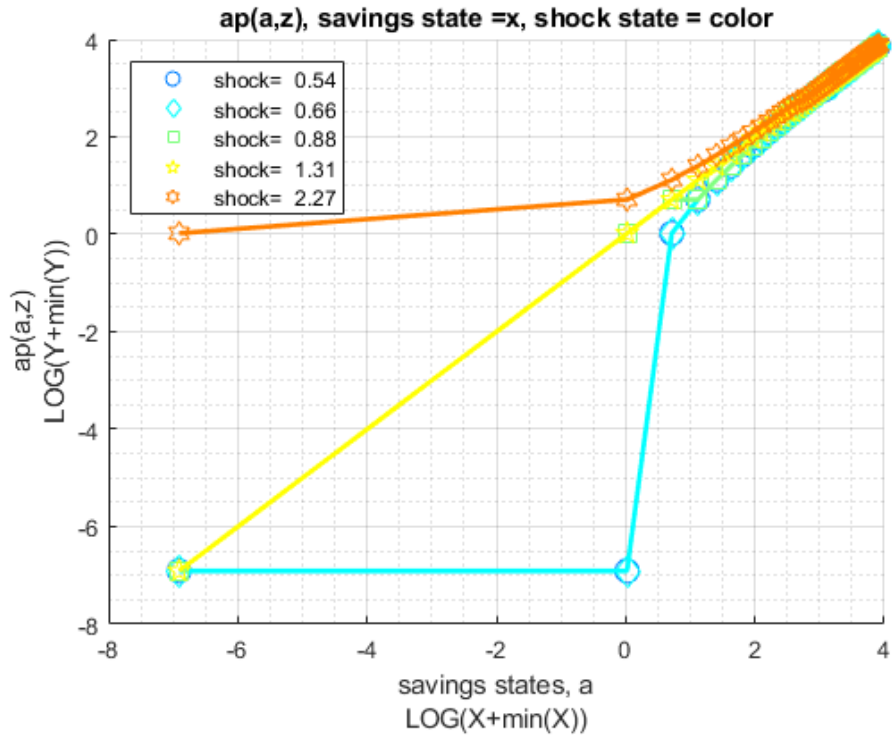
Run the function and show policy function for savings choice:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {};
mp_support('ls_ffsna') = {'ap'};
mp_support('ls_ffgrh') = {'ap'};
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.020296 seconds.
xxx  ff_vfi_az_vec, outcome=ap  xxxxxxxxxxxxxxxxxxxxxxxxxx
```

group	a	mean_z_0_54195	mean_z_0_66401	mean_z_0_88162	mean_z_1_3095	mean_z
1	0	0	0	0	0	1.0
2	1.0204	0	0	1.0204	1.0204	2.0
3	2.0408	1.0204	1.0204	2.0408	2.0408	3.0
4	3.0612	2.0408	2.0408	2.0408	3.0612	4.0
5	4.0816	3.0612	3.0612	3.0612	4.0816	5.
6	5.102	4.0816	4.0816	4.0816	5.102	6.1
7	6.1224	5.102	5.102	5.102	6.1224	7.1
8	7.1429	6.1224	6.1224	6.1224	7.1429	8.1
9	8.1633	7.1429	7.1429	7.1429	8.1633	9.1
10	9.1837	8.1633	8.1633	8.1633	9.1837	10.
11	10.204	9.1837	9.1837	9.1837	10.204	11.
12	11.224	10.204	10.204	10.204	11.224	12.
13	12.245	11.224	11.224	11.224	12.245	13.
14	13.265	12.245	12.245	12.245	12.245	14.
15	14.286	13.265	13.265	13.265	13.265	15.
16	15.306	14.286	14.286	14.286	14.286	15.
17	16.327	15.306	15.306	15.306	15.306	16.
18	17.347	16.327	16.327	16.327	16.327	17.
19	18.367	17.347	17.347	17.347	17.347	18.
20	19.388	18.367	18.367	18.367	18.367	19.
21	20.408	19.388	19.388	19.388	19.388	20.
22	21.429	19.388	20.408	20.408	20.408	21.
23	22.449	20.408	21.429	21.429	21.429	22.
24	23.469	21.429	22.449	22.449	22.449	23.
25	24.49	22.449	22.449	23.469	23.469	24
26	25.51	23.469	23.469	24.49	24.49	25
27	26.531	24.49	24.49	25.51	25.51	26.
28	27.551	25.51	25.51	26.531	26.531	27.
29	28.571	26.531	26.531	27.551	27.551	28.
30	29.592	27.551	27.551	28.571	28.571	29.
31	30.612	28.571	28.571	28.571	29.592	30.
32	31.633	29.592	29.592	29.592	30.612	31.
33	32.653	30.612	30.612	30.612	31.633	32.

34	33.673	31.633	31.633	31.633	32.653	33.
35	34.694	32.653	32.653	32.653	33.673	34.
36	35.714	33.673	33.673	33.673	34.694	35.
37	36.735	34.694	34.694	34.694	35.714	36.
38	37.755	35.714	35.714	35.714	36.735	37.
39	38.776	36.735	36.735	36.735	37.755	38.
40	39.796	37.755	37.755	37.755	38.776	39.
41	40.816	38.776	38.776	38.776	39.796	40.
42	41.837	39.796	39.796	39.796	40.816	41.
43	42.857	40.816	40.816	40.816	41.837	42.
44	43.878	41.837	41.837	41.837	41.837	42.
45	44.898	42.857	42.857	42.857	42.857	43.
46	45.918	43.878	43.878	43.878	43.878	44.
47	46.939	44.898	44.898	44.898	44.898	45.
48	47.959	45.918	45.918	45.918	45.918	46.
49	48.98	46.939	46.939	46.939	46.939	47.
50	50	47.959	47.959	47.959	47.959	48





Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.126640 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum       mean        std       coef
                    -     ---     ----     -----     ----     ----     ------     -------     -------     ----

    ap              1       1        2       900      100        9      21825      24.25      14.089       0.
    savefraccoh     2       2        2       900      100        9     752.38     0.83597     0.13497     0.16

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
               c1          c2          c3          c4          c5          c6          c7          c8          -

    r1            0           0           0           0           0           0     0.50505      1.5152      3
    r2            0           0           0           0     0.50505     0.50505      1.0101      1.5152      3
    r3      0.50505     0.50505     0.50505     0.50505     0.50505      1.0101      1.5152      2.0202      4
    r4       1.0101      1.0101      1.0101      1.0101      1.0101      1.5152      2.0202      2.5253      4
    r5       1.5152      1.5152      1.5152      1.5152      1.5152      2.0202      2.5253      3.0303      5
    r96      45.455      45.455      45.455       45.96       45.96       45.96      46.465      47.475      4
    r97       45.96       45.96       45.96      46.465      46.465      46.465       46.97       47.98      4
    r98      46.465      46.465      46.465      46.465       46.97       46.97      47.475      48.485
    r99       46.97       46.97       46.97       46.97      47.475      47.475       47.98       48.99
```

```
    r100     47.475     47.475     47.475     47.475      47.98      47.98     48.485     49.495

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c6         c7         c8
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0          0    0.24587    0.48182
    r2            0          0          0          0     0.3075    0.25444    0.39276    0.41371
    r3      0.30679    0.29486    0.27938    0.25939     0.2338    0.40362    0.49043     0.4833
    r4       0.4668    0.45285    0.43438    0.40981    0.37721    0.50166    0.56006    0.53755
    r5      0.56502    0.55132    0.53293    0.50802    0.47415    0.57101    0.61221    0.58103
    r96     0.91292     0.9117    0.90997    0.91752    0.91364    0.90746    0.90692    0.90732
    r97     0.91357    0.91236    0.91064    0.91812    0.91427    0.90815    0.90761    0.90799
    r98      0.9142      0.913     0.9113    0.90882    0.91489    0.90882    0.90828    0.90865
    r99     0.91482    0.91363    0.91195    0.90949    0.91549    0.90949    0.90894    0.90929
    r100    0.91543    0.91425    0.91258    0.91014    0.91609    0.91013    0.90959    0.90992
```

1.2.3 Test FF_VFI_AZ_VEC Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.711562 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                  i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                  -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh   1     1      2       6750     750      9      3291.4    0.48762    0.27804    0.57

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c6         c7         c8
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0          0    0.01987    0.12517
    r2            0          0          0          0          0          0    0.01987    0.12517
    r3            0          0          0          0          0          0    0.01987    0.12517
    r4            0          0          0          0          0          0    0.01987    0.12517
    r5            0          0          0          0          0          0    0.01987    0.12517
    r746    0.80538    0.80084    0.79932     0.7971    0.79372    0.79177    0.78608    0.77969
```

```
    r747    0.80218    0.80112     0.7996    0.79739    0.79402    0.79208    0.78643    0.78008
    r748    0.80245    0.80139    0.79988    0.79767    0.79432     0.7924    0.78677    0.78046
    r749    0.80272    0.80167    0.80016    0.79796    0.79462    0.79271    0.78711    0.78085
    r750    0.80299    0.80194    0.80044    0.79825    0.79492    0.79303    0.78745    0.78124
```

```
% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 2.436171 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2      6750     750      9      4491.9    0.66547    0.28771    0.43

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4          c5         c6         c7         c8
            -------    -------    -------    -------    --------    -------    -------    -------

    r1            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r2            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r3            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r4            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r5            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r746    0.92742       0.93     0.9283    0.92581     0.92578    0.92349    0.92443    0.91686
    r747     0.9275    0.93007    0.92838     0.9259     0.92588    0.92361    0.92457    0.91706
    r748    0.92757    0.93014    0.92846    0.92599     0.92598    0.92373    0.92472    0.91359
    r749    0.92764    0.93022    0.92854    0.92608     0.92608    0.92384    0.92115    0.91014
    r750    0.92772    0.93029    0.92862    0.92617     0.92618    0.92396     0.9213    0.90671
```

1.2.4 Test FF_VFI_AZ_VEC Changing Risk Aversion

Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

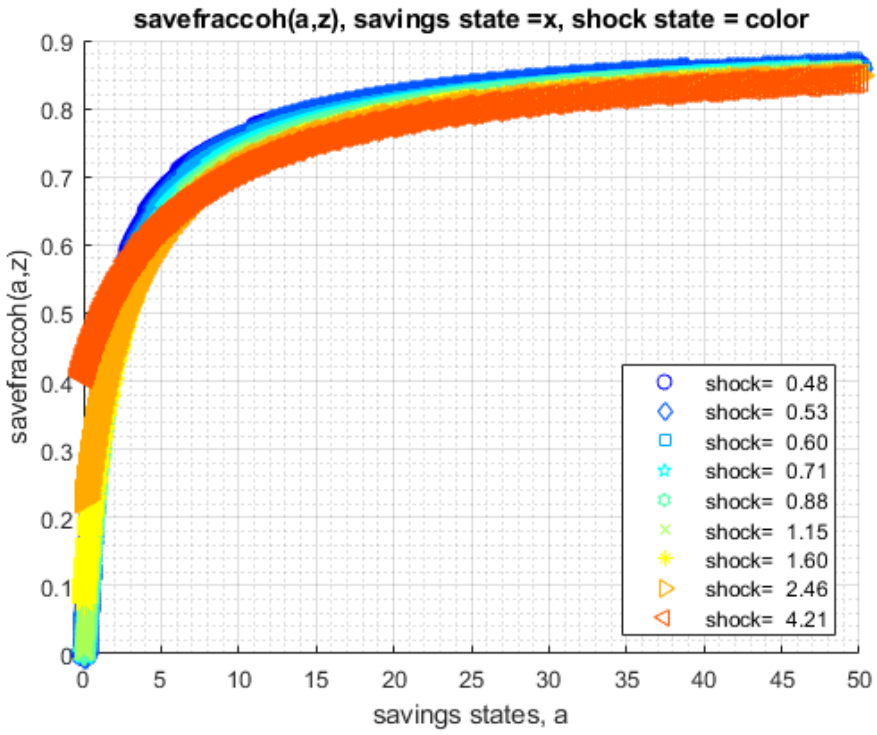
```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 2.064222 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

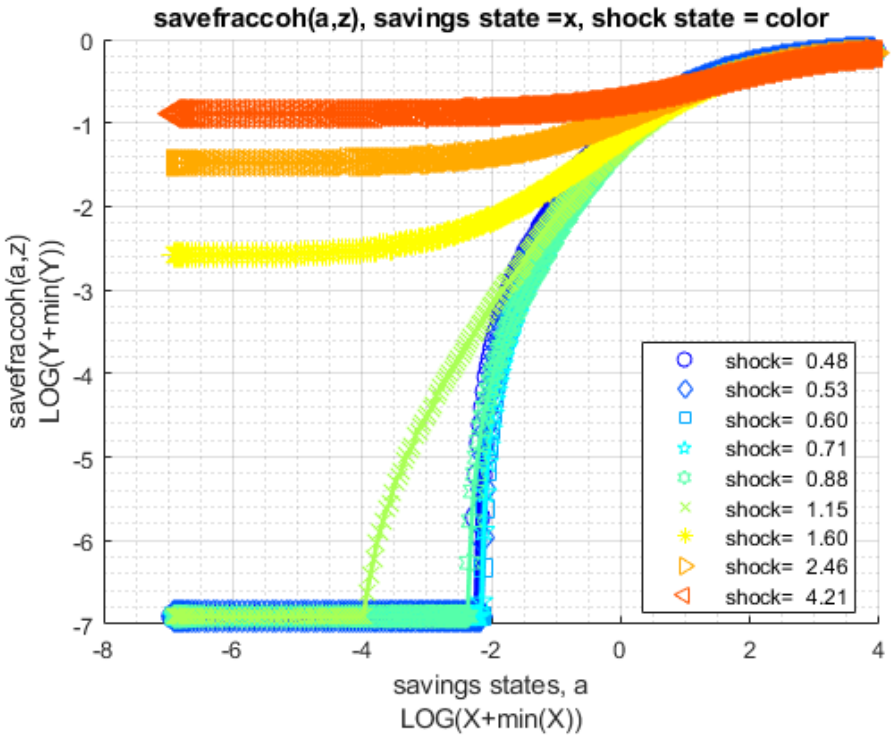
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefv
savefraccoh	1	1	2	6750	750	9	3735.9	0.55347	0.2897	0.523

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx

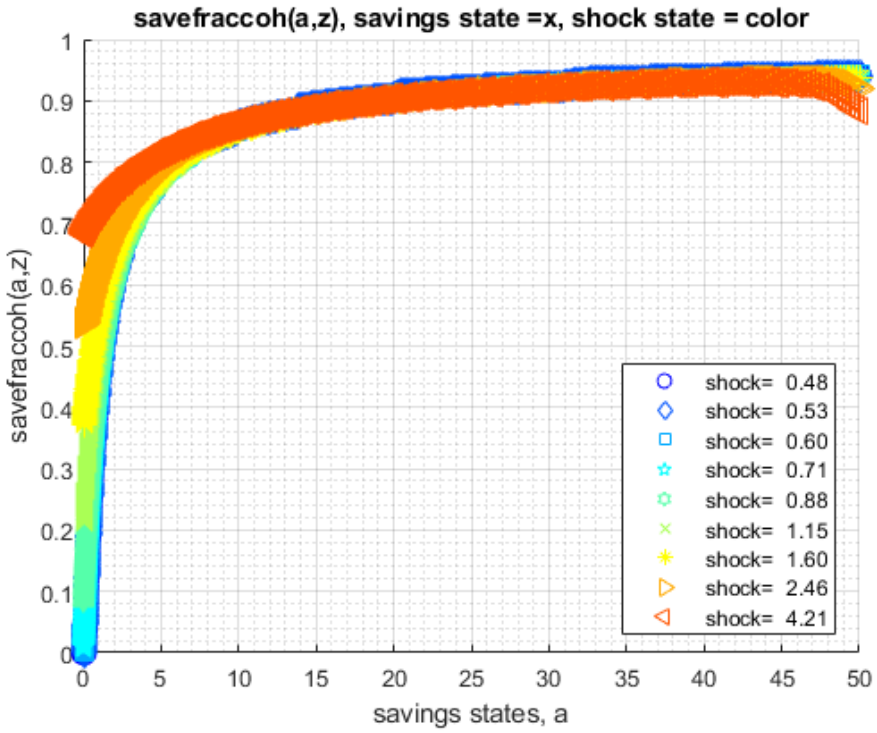
	c1	c2	c3	c4	c5	c6	c7	c8
r1	0	0	0	0	0	0	0.075021	0.22812
r2	0	0	0	0	0	0	0.075021	0.22812
r3	0	0	0	0	0	0	0.075021	0.22812
r4	0	0	0	0	0	0	0.075021	0.22812
r5	0	0	0	0	0	0	0.075021	0.22812
r746	0.85928	0.85816	0.85657	0.85425	0.85428	0.8522	0.84972	0.84635
r747	0.85946	0.85834	0.85676	0.85444	0.85449	0.85242	0.84997	0.84665
r748	0.85963	0.85852	0.85694	0.85464	0.85469	0.85264	0.85021	0.84694
r749	0.85981	0.8587	0.85713	0.85483	0.85489	0.85286	0.85046	0.84723
r750	0.85998	0.85888	0.85731	0.85502	0.85509	0.85307	0.8507	0.84752

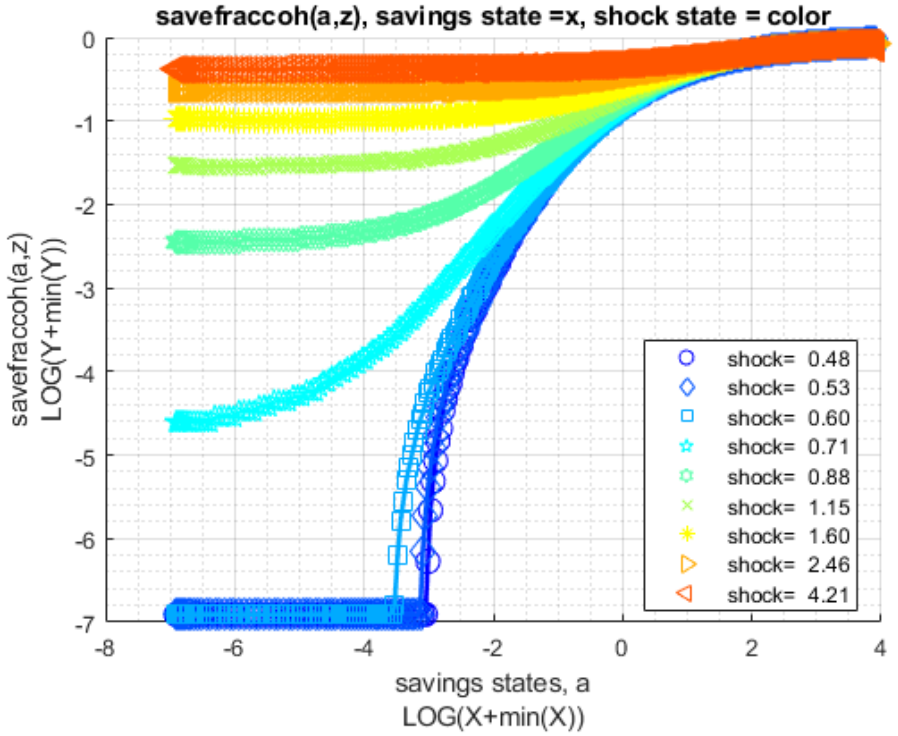




When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 1.900222 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum       mean       std       coefv
                    -     ---     ----     -----     ----     ----     ------     ------    -------     -----

    savefraccoh     1      1       2       6750      750       9      4639.3     0.6873    0.28204     0.410

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1          c2          c3          c4           c5          c6          c7          c8
            -------     -------     -------     ---------    --------    -------     -------     -------

    r1            0           0           0     0.008995     0.085095    0.21314     0.37277     0.53628
    r2            0           0           0     0.008995     0.085095    0.21314     0.37277     0.53628
    r3            0           0           0     0.008995     0.085095    0.21314     0.37277     0.53628
    r4            0           0           0     0.008995     0.085095    0.21314     0.37277     0.53628
    r5            0           0           0    0.0089949     0.085094    0.21314     0.37277     0.53628
    r746    0.94083      0.9396     0.94168      0.93912      0.93904     0.94041     0.93743     0.92949
    r747    0.94091     0.93969     0.94176      0.93921      0.93914     0.93674     0.93758     0.92969
    r748    0.94098     0.93977     0.94184      0.93931      0.93924     0.93686     0.93772     0.92618
    r749    0.94106     0.93985     0.94192       0.9394      0.93934     0.93699     0.93787     0.92269
    r750    0.94113     0.93993       0.942      0.93949      0.93944     0.93711     0.93801     0.91921
```





1.2.5 Test FF_VFI_AZ_VEC with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.05;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 2.123001 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       6750     750      9      3935.8    0.58309    0.32813    0.56

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7           c8
            -------    -------    -------    -------    -------    -------    ---------    --------

    r1            0          0          0          0          0          0    0.0035419    0.022183
    r2            0          0          0          0          0          0    0.0035419    0.022183
    r3            0          0          0          0          0          0    0.0035419    0.022183
    r4            0          0          0          0          0          0    0.0035419    0.022182
    r5            0          0          0          0          0          0    0.0035419    0.022182
    r746    0.91062    0.90972    0.91245    0.91134    0.91009    0.91241      0.91083     0.90905
    r747    0.91075    0.90986    0.91259    0.91148    0.91024    0.91256      0.91099     0.90921
    r748    0.91088       0.91    0.91272    0.91162    0.91038     0.9127      0.91114     0.90937
    r749    0.91102    0.91013    0.91286    0.91176    0.91053    0.91285      0.91129     0.90952
    r750    0.91115    0.91027    0.91299     0.9119    0.91067    0.90929      0.91144     0.90968
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.25;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 1.968323 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       6750     750      9      4429.3    0.65619    0.28387    0.43

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7          c8
            -------    -------    -------    -------    --------    -------    -------    -------

    r1            0          0          0          0    0.011319    0.12886    0.32464    0.53487
    r2            0          0          0          0    0.011319    0.12886    0.32464    0.53487
```

r3	0	0	0	0	0.011319	0.12886	0.32464	0.53487
r4	0	0	0	0	0.011319	0.12886	0.32464	0.53487
r5	0	0	0	0	0.011319	0.12886	0.32464	0.53487
r746	0.91612	0.91885	0.9173	0.91484	0.91448	0.91454	0.91098	0.90731
r747	0.91622	0.91896	0.91741	0.91496	0.9146	0.91469	0.91117	0.90394
r748	0.91633	0.91906	0.91751	0.91507	0.91473	0.91483	0.91136	0.90422
r749	0.91643	0.91916	0.91762	0.91519	0.91486	0.91498	0.91154	0.90449
r750	0.91653	0.91926	0.91773	0.91531	0.91498	0.91512	0.91173	0.90115

Chapter 2

Summarize Policy and Value

2.1 FF_SUMM_ND_ARRAY Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_summ_nd_array** from the **MEconTools Package**. This function summarizes policy and value functions over states.

2.1.1 Test FF_SUMM_ND_ARRAY Defaults

Call the function with defaults.

```
ff_summ_nd_array();
```

```
xxx  Summ over (a,z), condi age as cols, kids/marriage as rows  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group    marry    kids    mean_age_18    mean_age_19    mean_age_20    mean_age_21
    _____    _____    ____    ___________    ___________    ___________    ___________

      1        0        1        0.52456        0.51689        0.48412        0.54526
      2        1        1        0.49355        0.52906         0.5583        0.47342
      3        0        2        0.49085        0.51315        0.45158        0.43201
      4        1        2        0.58096        0.50596        0.47985        0.58791
      5        0        3        0.57811         0.6068        0.55221        0.50677
      6        1        3        0.53023        0.49258        0.48728        0.43352
      7        0        4        0.50339        0.48449        0.53618        0.45993
      8        1        4        0.44418         0.5223        0.55657        0.48583
```

2.1.2 Test FF_SUMM_ND_ARRAY with Random 2 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random 2D dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(5,4);
bl_print_table = true;
ar_st_stats = ["mean"];
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'a', linspace(0,1,size(mn_polval,1))});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'z', linspace(-1,1,size(mn_polval,2))});
disp(mn_polval);
```

```
    0.6965    0.4231    0.3432    0.7380
    0.2861    0.9808    0.7290    0.1825
    0.2269    0.6848    0.4386    0.1755
    0.5513    0.4809    0.0597    0.5316
    0.7195    0.3921    0.3980    0.5318
```

Second, show the entire matrix (no labels):

```
it_aggd = 0;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim2    mean_vardim1_1    mean_vardim1_2    mean_vardim1_3    mean_vardim1_4    mean
    -----    -------    --------------    --------------    --------------    --------------    ----

      1          1          0.69647           0.28614           0.22685            0.55131          0
      2          2          0.42311           0.98076           0.68483            0.48093          0
      3          3          0.34318           0.72905           0.43857           0.059678          0
      4          4            0.738           0.18249           0.17545            0.53155          0
```

Third, rotate row and column, and now with labels:

```
it_aggd = 0;
bl_row = 1;
ar_permute = [2,1];
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group     a      mean_z__1    mean_z__0_33333    mean_z_0_33333    mean_z_1
    -----    ----    ---------    ---------------    --------------    --------

      1         0      0.69647          0.42311            0.34318         0.738
      2      0.25      0.28614          0.98076            0.72905       0.18249
      3       0.5      0.22685          0.68483            0.43857       0.17545
      4      0.75      0.55131          0.48093           0.059678       0.53155
      5         1      0.71947          0.39212            0.39804       0.53183
```

Fourth, dimension one as columns, average over dim 2:

```
it_aggd = 1;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x    mean_z__1    mean_z__0_33333    mean_z_0_33333    mean_z_1
    -----    -    ---------    ---------------    --------------    --------

      1      1     0.49605          0.59235             0.3937        0.43186
```

Fifth, dimension one as rows, average over dim 2:

```
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group       z         sum       mean        std      coefvari      min         max
```

1	-1	2.4802	0.49605	0.22895	2.1666	0.22685	0.71947
2	-0.33333	2.9617	0.59235	0.24524	2.4154	0.39212	0.98076
3	0.33333	1.9685	0.3937	0.23907	1.6468	0.059678	0.72905
4	1	2.1593	0.43186	0.24575	1.7573	0.17545	0.738

Sixth, dimension two as rows, average over dim 1:

```
ar_permute = [2,1];
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);
```

```
xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxx
    group     a       sum       mean        std      coefvari      min         max
    -----    ----    ------    -------    -------    --------    --------    -------

      1        0     2.2007    0.55019    0.19636     2.8019      0.34318      0.738
      2      0.25    2.1784    0.54461    0.37514     1.4518      0.18249    0.98076
      3       0.5    1.5257    0.38143    0.23212     1.6432      0.17545    0.68483
      4      0.75    1.6235    0.40587    0.23269     1.7443     0.059678    0.55131
      5        1     2.0415    0.51036    0.15361     3.3226      0.39212    0.71947
```

2.1.3 Test FF_SUMM_ND_ARRAY with Random 6 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random ND dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(8,7,6,5,4,3);
bl_print_table = true;
ar_st_stats = ["mean"];
```

Second, summarize over the first four dimensions, row group others:

```
it_aggd = 4;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);
```

```
xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim5    vardim6     sum       mean        std      coefvari       min          max
    -----    -------    -------    ------    -------    -------    --------    ----------    -------

      1         1          1       836.78    0.49808    0.29255     1.7026     8.1888e-05    0.99964
      2         2          1       842.15    0.50128    0.28968     1.7305     6.7838e-05    0.99936
      3         3          1       831.45    0.49491    0.28851     1.7154     0.00091373    0.99989
      4         4          1        843.9    0.50232    0.28154     1.7842     0.00012471    0.99731
      5         1          2       838.99     0.4994     0.2911     1.7156     0.00029749    0.99938
      6         2          2       830.81    0.49453    0.28634     1.7271     0.00027113     0.9992
      7         3          2       832.59    0.49559    0.28682     1.7279     0.00035994    0.99936
      8         4          2       820.42    0.48835    0.29032     1.6821     0.00096259    0.99896
      9         1          3       870.56    0.51819    0.29111     1.7801      0.0010616    0.99951
     10         2          3       854.68    0.50874    0.28458     1.7877       0.001884    0.99965
     11         3          3       838.29    0.49898     0.2891      1.726       0.0019192    0.99945
     12         4          3       842.83    0.50169     0.2877     1.7438     0.00016871    0.99963
```

Third, summarize over the first four dimensions, column group 5th, and row group others:

```
it_aggd = 4;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ["sum"], it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim6    sum_vardim5_1    sum_vardim5_2    sum_vardim5_3    sum_vardim5_4
    -----    -------    -------------    -------------    -------------    -------------

      1          1           836.78           842.15           831.45            843.9
      2          2           838.99           830.81           832.59           820.42
      3          3           870.56           854.68           838.29           842.83
```

Fourth, summarize over the first five dimensions, column group 6th, no row groups:

```
it_aggd = 5;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ["mean", "std"], it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x    mean_vardim6_1    mean_vardim6_2    mean_vardim6_3    std_vardim6_1    std_vardim6
    -----    -    --------------    --------------    --------------    -------------    -----------

      1      1        0.49915           0.49447            0.5069           0.28805          0.28862
```

Fifth, summarize over all six dimensions, summary statistics over the entire dataframe:

```
it_aggd = 6;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x     sum      mean        std      coefvari       min          max
    -----    -    -----    -------    -------    --------    ----------    -------

      1      1    10083    0.50017    0.28831     1.7349     6.7838e-05    0.99989
```

2.1.4 Test FF_SUMM_ND_ARRAY with Random 7 Dimensional Matrix with All Parameters

Given a random seven dimensional matrix, average over the 2nd, 4th and 5th dimensionals. Show as row groups the 3, 6 and 7th dimensions, and row groups the 1st dimension. Show Coefficient of Variation only.

```
st_title = "avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim.";
rng(123)
mn_polval = rand(3,10,2,10,10,2,3);
ar_permute = [2,4,5,1,3,6,7];
bl_print_table = true;
ar_st_stats = ["coefvari"];
it_aggd = 3; % mean over 3 dims
bl_row = 1; % one var for row group
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'age', [18, 19, 20]});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'savings', linspace(0,1,10)});
cl_mp_datasetdesc{3} = containers.Map({'name', 'labval'}, ...
    {'borrsave', [-1,+1]});
cl_mp_datasetdesc{4} = containers.Map({'name', 'labval'}, ...
    {'shocka', linspace(-5,5,10)});
cl_mp_datasetdesc{5} = containers.Map({'name', 'labval'}, ...
```

```
    {'shockb', linspace(-5,5,10)});
cl_mp_datasetdesc{6} = containers.Map({'name', 'labval'}, ...
    {'marry', [0,1]});
cl_mp_datasetdesc{7} = containers.Map({'name', 'labval'}, ...
    {'region', [1,2,3]});
% call function
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, cl_mp_datasetdes

xxx  avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim.  xxxxxxxxxxxxxxxxxxxx
    group    borrsave    marry    region    cv_age_18    cv_age_19    cv_age_20
    -----    --------    -----    ------    ---------    ---------    ---------

      1         -1         0        1        1.7607       1.7534       1.7065
      2          1         0        1        1.6566       1.7501       1.7042
      3         -1         1        1        1.6608       1.7658       1.7291
      4          1         1        1         1.756       1.7479       1.7606
      5         -1         0        2        1.7314       1.7506        1.786
      6          1         0        2        1.7347        1.728        1.738
      7         -1         1        2        1.7811        1.755       1.7568
      8          1         1        2        1.7445       1.7398       1.7746
      9         -1         0        3        1.7025       1.7286         1.69
     10          1         0        3          1.74       1.7549       1.7356
     11         -1         1        3        1.7147       1.7287       1.7341
     12          1         1        3        1.7919       1.7313       1.7452
```

Chapter 3

Distributional Analysis

3.1 FF_SIMU_STATS Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_simu_stats** from the **MEconTools Package.** This is a gate-way function that computes mean, percentiles, covariance etc between several variables.

3.1.1 Test FF_SIMU_STATS Defaults

Call the function with defaults.

```
ff_simu_stats();
```

```
xxx tb_outcomes: all stats xxx
    OriginalVariableNames     cl_mt_pol_a    cl_mt_pol_c
    ---------------------     -----------    -----------

    {'mean'              }     -0.11081         8.8423
    {'sd'                }       4.1239         6.5845
    {'coefofvar'         }      -37.215        0.74466
    {'min'               }           -7        -6.3772
    {'max'               }            9         21.786
    {'pYis0'             }     0.064259              0
    {'pYls0'             }      0.54867       0.027329
    {'pYgr0'             }      0.38707        0.97267
    {'pYisMINY'          }     0.051764       0.015232
    {'pYisMAXY'          }     0.027329       0.046484
    {'p1'                }           -7        -6.3772
    {'p10'               }           -6        0.27238
    {'p25'               }           -3         5.2138
    {'p50'               }           -1         6.5321
    {'p75'               }            3         13.799
    {'p90'               }            5         16.887
    {'p99'               }            9         21.786
    {'fl_cov_cl_mt_pol_a'}       17.007        -22.084
    {'fl_cor_cl_mt_pol_a'}            1       -0.81327
    {'fl_cov_cl_mt_pol_c'}      -22.084         43.356
    {'fl_cor_cl_mt_pol_c'}     -0.81327              1
    {'fracByP1'          }       3.2699      -0.010985
    {'fracByP10'         }       5.9889      -0.013362
    {'fracByP25'         }       14.165       0.041007
    {'fracByP50'         }       16.208         0.1893
```

```
    {'fracByP75'         }       12.702         0.59539
    {'fracByP90'         }       6.6611          0.8307
    {'fracByP99'         }            1               1
```

3.1.2 Test FF_SIMU_STATS Four States-Points Matrix

Over some (a,z) states that is 3 by 3, c matrix, generate all stats

```
% Set Parameters
mt_x_of_s = [1, 2,  3.0;...
             3, 1,  1.5;...
             4, 3,  2.0];
mt_y_of_s = [2, -10, 9.0;...
             5, 1.1,3.0;...
             1, 3,  -1.5];
mt_z_of_s = [1.1, 2,3.3;...
             2.3, 1,1.5;...
             4, 2.5,2.0];
mp_cl_mt_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
% Mass
rng(123);
mt_f_of_s = rand(size(mt_x_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s, mp_cl_mt_xyz_of_s);

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_mt_x_of_s    cl_mt_y_of_s    cl_mt_z_of_s
    ______________________    ____________    ____________    ____________

    {'mean'               }        2.0763          1.9323          2.0668
    {'sd'                 }        0.9071          5.2239          0.9042
    {'coefofvar'          }       0.43688          2.7034         0.43749
    {'min'                }             1             -10               1
    {'max'                }             4               9               4
    {'pYis0'              }             0               0               0
    {'pYls0'              }             0         0.20441               0
    {'pYgr0'              }             1         0.79559               1
    {'pYisMINY'           }       0.28039         0.10917         0.14247
    {'pYisMAXY'           }      0.044922         0.19422        0.044922
    {'p1'                 }             1             -10               1
    {'p10'                }             1             -10               1
    {'p25'                }             1             1.1             1.1
    {'p50'                }             2               2               2
    {'p75'                }             3               5             2.5
    {'p90'                }             3               9             3.3
    {'p99'                }             4               9               4
    {'fl_cov_cl_mt_x_of_s'}       0.82282           1.589         0.78646
    {'fl_cor_cl_mt_x_of_s'}             1         0.33534         0.95887
    {'fl_cov_cl_mt_y_of_s'}         1.589          27.289          1.8353
    {'fl_cor_cl_mt_y_of_s'}       0.33534               1         0.38856
    {'fl_cov_cl_mt_z_of_s'}       0.78646          1.8353         0.81758
    {'fl_cor_cl_mt_z_of_s'}       0.95887         0.38856               1
    {'fracByP1'           }       0.13504        -0.56498        0.068934
    {'fracByP10'          }       0.13504        -0.56498        0.068934
```

```
    {'fracByP25'          }       0.13504        -0.53456         0.14234
    {'fracByP50'          }       0.42991        -0.39181         0.43856
    {'fracByP75'          }       0.91346        0.095425         0.60296
    {'fracByP90'          }       0.91346               1         0.91306
    {'fracByP99'          }             1               1               1
```

3.1.3 Test FF_SIMU_STATS Four States-Points Matrix Single Column Inputs

Same as before, but now inputs are single column, should have identical results:

```
% Array Inputs
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_mt_xyz_of_s);

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_mt_x_of_s    cl_mt_y_of_s    cl_mt_z_of_s
    ______________________    ____________    ____________    ____________

    {'mean'               }        2.0763          1.9323          2.0668
    {'sd'                 }        0.9071          5.2239          0.9042
    {'coefofvar'          }       0.43688          2.7034         0.43749
    {'min'                }             1             -10               1
    {'max'                }             4               9               4
    {'pYis0'              }             0               0               0
    {'pYls0'              }             0         0.20441               0
    {'pYgr0'              }             1         0.79559               1
    {'pYisMINY'           }       0.28039         0.10917         0.14247
    {'pYisMAXY'           }      0.044922         0.19422        0.044922
    {'p1'                 }             1             -10               1
    {'p10'                }             1             -10               1
    {'p25'                }             1             1.1             1.1
    {'p50'                }             2               2               2
    {'p75'                }             3               5             2.5
    {'p90'                }             3               9             3.3
    {'p99'                }             4               9               4
    {'fl_cov_cl_mt_x_of_s'}       0.82282           1.589         0.78646
    {'fl_cor_cl_mt_x_of_s'}             1         0.33534         0.95887
    {'fl_cov_cl_mt_y_of_s'}         1.589          27.289          1.8353
    {'fl_cor_cl_mt_y_of_s'}       0.33534               1         0.38856
    {'fl_cov_cl_mt_z_of_s'}       0.78646          1.8353         0.81758
    {'fl_cor_cl_mt_z_of_s'}       0.95887         0.38856               1
    {'fracByP1'           }       0.13504        -0.56498        0.068934
    {'fracByP10'          }       0.13504        -0.56498        0.068934
    {'fracByP25'          }       0.13504        -0.53456         0.14234
    {'fracByP50'          }       0.42991        -0.39181         0.43856
    {'fracByP75'          }       0.91346        0.095425         0.60296
    {'fracByP90'          }       0.91346               1         0.91306
    {'fracByP99'          }             1               1               1
```

3.1.4 Test FF_SIMU_STATS Print Many Details

The Same As before, but now control which percentiles and other details to display.

```
% Array Inputs
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_ar_xyz_of_s('cl_ar_x_of_s') = {mt_x_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('cl_ar_z_of_s') = {mt_z_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('ar_st_y_name') = ["cl_ar_x_of_s", "cl_ar_z_of_s"];

% controls
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_display_detail') = false;
mp_support('bl_display_final') = true;
mp_support('bl_display_drvm2outcomes') = false;
mp_support('ar_fl_percentiles') = [25 50 75];
mp_support('bl_display_drvstats') = true;
mp_support('bl_display_drvm2covcor') = false;

% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_ar_xyz_of_s, mp_support);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: cl_ar_x_of_s
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    2.0763

fl_choice_sd
    0.9071

fl_choice_coefofvar
    0.4369

fl_choice_prob_zero
     0

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
     1

fl_choice_prob_max
    0.0449

tb_disc_cumu
    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

                1                          0.28039                28.039     0.13504
              1.5                          0.13561                 41.6      0.23301
                2                          0.20441                62.041     0.42991
                3                          0.33466                95.508     0.91346
                4                         0.044922                   100           1

    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

                1                          0.28039                28.039     0.13504
```

```
            1.5                        0.13561                41.6     0.23301
              2                        0.20441              62.041     0.42991
              3                        0.33466              95.508     0.91346
              4                       0.044922                 100           1

tb_prob_drv
    percentiles    cl_ar_x_of_sDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ---------------------------------------    --------------------------------

        25                          1                                  0.13504
        50                          2                                  0.42991
        75                          3                                  0.91346

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: cl_ar_z_of_s
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    2.0668

fl_choice_sd
    0.9042

fl_choice_coefofvar
    0.4375

fl_choice_prob_zero
     0

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
     1

fl_choice_prob_max
    0.0449

tb_disc_cumu
    cl_ar_z_of_sDiscreteVal    cl_ar_z_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

              1                          0.14247                 14.247     0.068934
            1.1                          0.13792                 28.039      0.14234
            1.5                          0.13561                   41.6      0.24076
              2                          0.20441                 62.041      0.43856
            2.3                          0.056663                67.708      0.50162
            2.5                          0.083786                76.086      0.60296
            3.3                           0.19422                95.508      0.91306
              4                         0.044922                   100           1

    cl_ar_z_of_sDiscreteVal    cl_ar_z_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

              1                          0.14247                 14.247     0.068934
            1.1                          0.13792                 28.039      0.14234
            1.5                          0.13561                   41.6      0.24076
```

```
             2                    0.20441          62.041      0.43856
           2.3                   0.056663          67.708      0.50162
           2.5                   0.083786          76.086      0.60296
           3.3                    0.19422          95.508      0.91306
             4                   0.044922             100            1

tb_prob_drv
    percentiles    cl_ar_z_of_sDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ---------------------------------------    --------------------------------

        25                          1.1                                0.14234
        50                            2                                0.43856
        75                          2.5                                0.60296

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_ar_x_of_s    cl_ar_z_of_s
    ----------------------    ------------    ------------

    {'mean'               }        2.0763          2.0668
    {'sd'                 }        0.9071          0.9042
    {'coefofvar'          }       0.43688         0.43749
    {'min'                }             1               1
    {'max'                }             4               4
    {'pYis0'              }             0               0
    {'pYls0'              }             0               0
    {'pYgr0'              }             1               1
    {'pYisMINY'           }       0.28039         0.14247
    {'pYisMAXY'           }      0.044922        0.044922
    {'p25'                }             1             1.1
    {'p50'                }             2               2
    {'p75'                }             3             2.5
    {'fl_cov_cl_ar_x_of_s'}       0.82282         0.78646
    {'fl_cor_cl_ar_x_of_s'}             1         0.95887
    {'fl_cov_cl_ar_z_of_s'}       0.78646         0.81758
    {'fl_cor_cl_ar_z_of_s'}       0.95887               1
    {'fracByP25'          }       0.13504         0.14234
    {'fracByP50'          }       0.42991         0.43856
    {'fracByP75'          }       0.91346         0.60296
```

3.2 FF_DISC_RAND_VAR_STATS Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_stats** from the **MEconTools Package.** This function summarizes statistics of matrixes stored in a container map, as well as scalar, string, function and other values stored in container maps.

3.2.1 Test FF_DISC_RAND_VAR_STATS Defaults

Call the function with defaults.

```
ff_disc_rand_var_stats();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: binom
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
```

```
fl_choice_mean
   -1.0000

fl_choice_sd
    2.5100

fl_choice_coefofvar
   -2.5100

fl_choice_prob_zero
    0.1416

fl_choice_prob_below_zero
    0.5888

fl_choice_prob_above_zero
    0.2696

fl_choice_prob_max
   2.0589e-16

tb_disc_cumu
    binomDiscreteVal    binomDiscreteValProbMass       CDF       cumsumFrac
    ----------------    ------------------------    ---------    ----------

          -10                 2.2539e-05           0.0022539    0.00022539
           -9                 0.00028979            0.031233     0.0028335
           -8                  0.0018008             0.21132       0.01724
           -7                  0.0072034             0.93166      0.067664
           -6                   0.020838              3.0155       0.19269
           -5                    0.04644              7.6595       0.42489
           -4                   0.082928              15.952       0.75661
           -3                    0.12185              28.138        1.1222
           -2                    0.15014              43.152        1.4224
           -1                    0.15729              58.881        1.5797

    binomDiscreteVal    binomDiscreteValProbMass    CDF    cumsumFrac
    ----------------    ------------------------    ---    ----------

           11                 6.0392e-06           100         1
           12                 1.0588e-06           100         1
           13                 1.5784e-07           100         1
           14                  1.973e-08           100         1
           15                 2.0293e-09           100         1
           16                 1.6725e-10           100         1
           17                 1.0619e-11           100         1
           18                 4.8762e-13           100         1
           19                 1.4412e-14           100         1
           20                 2.0589e-16           100         1

tb_prob_drv
    percentiles    binomDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    --------------------------------    --------------------------------

        0.1                       -8                                0.01724
          1                       -6                                0.19269
          5                       -5                                0.42489
         10                       -4                                0.75661
```

15	-4	0.75661
20	-3	1.1222
25	-3	1.1222
35	-2	1.4224
50	-1	1.5797
65	0	1.5797
75	1	1.4694
80	1	1.4694
85	2	1.3197
90	2	1.3197
95	3	1.1865
99	5	1.0412
99.9	7	1.0052

3.2.2 Test FF_DISC_RAND_VAR_STATS 0 and 1 Random Variable

The simplest discrete random variable has two values, zero or one. The probability of zero is 30 percent, and 70 percent is the probability of one.

```
% Parameters
% 1. specify the random variable
st_var_name = 'bernoulli';
ar_choice_unique_sorted = [0, 1];
ar_choice_prob = [0.3, 0.7];
% 2. percentiles of interest
ar_fl_percentiles = [0.1 5 25 50 75 95 99.9];
% 3. print resutls
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: bernoulli
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    0.7000

fl_choice_sd
    0.4583

fl_choice_coefofvar
    0.6547

fl_choice_prob_zero
    0.3000

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
    0.7000

fl_choice_prob_max
    0.7000
```

```
tb_disc_cumu
    bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
    --------------------    ----------------------------    ---    ----------

              0                           0.3                30        0
              1                           0.7               100        1

    bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
    --------------------    ----------------------------    ---    ----------

              0                           0.3                30        0
              1                           0.7               100        1

tb_prob_drv
    percentiles    bernoulliDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ------------------------------------    --------------------------------

        0.1                         0                                  0
          5                         0                                  0
         25                         0                                  0
         50                         1                                  1
         75                         1                                  1
         95                         1                                  1
       99.9                         1                                  1
```

3.2.3 Test FF_DISC_RAND_VAR_STATS with Poisson

Poisson random variable, with mean equals to ten, summarize over umsymmetric percentiles. Note that the poisson random variable has no upper bound.

```
% Parameters
% 1. specify the random variable
st_var_name = 'poisson';
mu = 10;
ar_choice_unique_sorted = 0:1:50;
ar_choice_prob = poisspdf(ar_choice_unique_sorted, mu);
% 2. percentiles of interest, unsymmetric
ar_fl_percentiles = [0.1 5 10 25 50 90 95 99 99.9 99.99 99.999 99.9999];
% 3. print resutls
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: poisson
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    10

fl_choice_sd
    3.1623

fl_choice_coefofvar
    0.3162
```

```
fl_choice_prob_zero
   4.5400e-05

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
    1.0000

fl_choice_prob_max
   1.4927e-19

tb_disc_cumu
    poissonDiscreteVal    poissonDiscreteValProbMass      CDF      cumsumFrac
    ------------------    --------------------------    -------    ----------

            0                     4.54e-05             0.00454           0
            1                     0.000454             0.04994     4.54e-05
            2                      0.00227             0.27694    0.0004994
            3                    0.0075667              1.0336    0.0027694
            4                     0.018917              2.9253     0.010336
            5                     0.037833              6.7086     0.029253
            6                     0.063055              13.014     0.067086
            7                     0.090079              22.022      0.13014
            8                       0.1126              33.282      0.22022
            9                      0.12511              45.793      0.33282

    poissonDiscreteVal    poissonDiscreteValProbMass    CDF    cumsumFrac
    ------------------    --------------------------    ---    ----------

            41                    1.3571e-13            100        1
            42                    3.2313e-14            100        1
            43                    7.5146e-15            100        1
            44                    1.7079e-15            100        1
            45                    3.7953e-16            100        1
            46                    8.2506e-17            100        1
            47                    1.7554e-17            100        1
            48                    3.6572e-18            100        1
            49                    7.4636e-19            100        1
            50                    1.4927e-19            100        1

tb_prob_drv
    percentiles    poissonDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ----------------------------------    --------------------------------

         0.1                       2                                0.0004994
           5                       5                                 0.029253
          10                       6                                 0.067086
          25                       8                                  0.22022
          50                      10                                  0.45793
          90                      14                                  0.86446
          95                      15                                  0.91654
          99                      18                                  0.98572
        99.9                      21                                  0.99841
       99.99                      24                                  0.99988
      99.999                      26                                  0.99998
         100                      28                                        1
```

```
% Print out full Stored Matrix
% Note that the outputs are single row arrays.
ff_container_map_display(ds_stats_map, 100, 100)

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                              i    idx    ndim    numel    rowN    colN     mean       std      coe
                              -    ---    ----    -----    ----    ----    -------    ------    ---

    ar_choice_perc_fracheld   1     1      2       12       1       12     0.62833     0.435    0.6
    ar_choice_percentiles     2     2      2       12       1       12       14.75    8.7399    0.5
    ar_fl_percentiles         3     3      2       12       1       12      64.499    42.887    0.6

xxx TABLE:ar_choice_perc_fracheld xxxxxxxxxxxxxxxxx
             c1           c2           c3          c4          c5          c6          c7          c8
          ---------    --------    --------    -------    -------    -------    -------    -------

    r1    0.0004994    0.029253    0.067086    0.22022    0.45793    0.86446    0.91654    0.98572

xxx TABLE:ar_choice_percentiles xxxxxxxxxxxxxxxxx
          c1    c2    c3    c4    c5    c6    c7    c8    c9    c10    c11    c12
          --    --    --    --    --    --    --    --    --    ---    ---    ---

    r1    2     5     6     8     10    14    15    18    21    24     26     28

xxx TABLE:ar_fl_percentiles xxxxxxxxxxxxxxxxx
          c1     c2    c3    c4    c5    c6    c7    c8     c9      c10      c11      c12
          ---    --    --    --    --    --    --    --    ----    -----    ------    ---

    r1    0.1    5     10    25    50    90    95    99    99.9    99.99    99.999    100

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i     idx      value
                                --    ---    ----------

    fl_choice_coefofvar          1     4        0.31623
    fl_choice_max                2     5             50
    fl_choice_mean               3     6             10
    fl_choice_min                4     7              0
    fl_choice_prob_above_zero    5     8        0.99995
    fl_choice_prob_below_zero    6     9              0
    fl_choice_prob_max           7    10     1.4927e-19
    fl_choice_prob_min           8    11       4.54e-05
    fl_choice_prob_zero          9    12       4.54e-05
    fl_choice_sd                10    13         3.1623
```

3.3 FF_DISC_RAND_VAR_MASS2OUTCOMES Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_mass2outcomes** from the **MEconTools Package.** This function generates sorted discrete random variable from state-space joint distri-

bution.

3.3.1 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2outcomes();

INPUT f(a,z): mt_dist_bystates
    0.0289    0.0465    0.0228    0.0036    0.0001
    0.0241    0.0930    0.0857    0.0241    0.0015
    0.0080    0.0744    0.1285    0.0643    0.0074
    0.0013    0.0297    0.0964    0.0857    0.0186
    0.0001    0.0059    0.0361    0.0571    0.0232
    0.0000    0.0005    0.0054    0.0152    0.0116

INPUT y(a,z): mt_choice_bystates
    -5    -4    -5    -4    -4
    -3    -2    -3    -2    -3
    -1    -1    -1     0     0
     1     1     2     3     1
     4     3     3     4     3
     5     6     5     6     6

OUTPUT f(y): ar_choice_prob_byY
    0.0518
    0.0502
    0.1113
    0.1171
    0.2109
    0.0717
    0.0497
    0.0964
    0.1510
    0.0572
    0.0054
    0.0273

OUTPUT f(y,z): mt_choice_prob_byYZ
    0.0289         0    0.0228         0         0
         0    0.0465         0    0.0036    0.0001
    0.0241         0    0.0857         0    0.0015
         0    0.0930         0    0.0241         0
    0.0080    0.0744    0.1285         0         0
         0         0         0    0.0643    0.0074
    0.0013    0.0297         0         0    0.0186
         0         0    0.0964         0         0
         0    0.0059    0.0361    0.0857    0.0232
    0.0001         0         0    0.0571         0
    0.0000         0    0.0054         0         0
         0    0.0005         0    0.0152    0.0116

OUTPUT f(y,a): mt_choice_prob_byYA
    0.0518         0         0         0         0         0
    0.0502         0         0         0         0         0
         0    0.1113         0         0         0         0
         0    0.1171         0         0         0         0
         0         0    0.2109         0         0         0
         0         0    0.0717         0         0         0
```

```
         0         0         0    0.0497         0         0
         0         0         0    0.0964         0         0
         0         0         0    0.0857    0.0653         0
         0         0         0         0    0.0572         0
         0         0         0         0         0    0.0054
         0         0         0         0         0    0.0273

OUTPUT f(y) and y in table: tb_choice_drv_cur_byY
    binomtestOutcomes    probMassFunction
    -----------------    ----------------

           -5                0.051764
           -4                0.050217
           -3                 0.11126
           -2                 0.11706
           -1                 0.21092
            0                0.071696
            1                0.049682
            2                0.096388
            3                 0.15102
            4                0.057231
            5               0.0054256
            6                0.027329
```

3.3.2 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Four States-Points

Over some (a,z) states that is 2 by 2, matrix or vectorized inputs identical results.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2;3,1];
% stationary mass over assets adn shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);

    0.4039    1.0000
    0.2971    2.0000
    0.2990    3.0000
```

Same as before, but now inputs are single column:

```
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s(:), mt_f_of_s);
disp([ar_f_of_y ar_y_unique_sorted]);

    0.4039    1.0000
    0.2971    2.0000
    0.2990    3.0000
```

3.3.3 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Conditional Mass Outputs

Same inputs as before, but now, also output additional conditional statistis, f(y, a), where a is the row state variable for f(a,z). For conditional statistics, must provide matrix based inputs.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2,0.5;
             3,1,2.0];
% stationary mass over assets adn shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted, mt_f_of_y_srow, mt_f_of_y_scol] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);

    0.2695    0.5000
    0.3765    1.0000
    0.2649    2.0000
    0.0891    3.0000

disp(mt_f_of_y_srow);

    0.2695         0
    0.1215    0.2550
    0.1217    0.1432
         0    0.0891

disp(mt_f_of_y_scol);

         0         0    0.2695
    0.1215    0.2550         0
         0    0.1217    0.1432
    0.0891         0         0
```

3.4 FF_DISC_RAND_VAR_MASS2COVCOR Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_mass2covcor** from the **MEconTools Package.** This function calculates covariance and correlation based for two discrete random variables.

3.4.1 Test FF_DISC_RAND_VAR_MASS2COVCOR Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2covcor();

-----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_input_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    ndim    numel    rowN    colN      mean        std       coefvari
                 -    ---    ----    -----    ----    ----    --------    --------    --------    --
```

```
    mt_f_of_s    1     5      2       30       6       5      0.033333    0.035743     1.0723     3.
    mt_x_of_s    2     6      2       30       6       5       0.83333      5.3051     6.3661
    mt_y_of_s    3     7      2       30       6       5        8.3259      7.1913    0.86373

xxx TABLE:mt_f_of_s xxxxxxxxxxxxxxxxx
              c1            c2           c3          c4          c5
          ----------    ----------    ---------    ---------    ---------

    r1      0.028917      0.046484     0.022848    0.0036146     0.000119
    r2      0.024097      0.092967     0.085679     0.024097    0.0014875
    r3     0.0080324      0.074374      0.12852     0.064259    0.0074374
    r4     0.0013387       0.02975     0.096388     0.085679     0.018593
    r5    0.00011156     0.0059499     0.036146     0.057119     0.023242
    r6    3.7187e-06    0.00047599    0.0054218     0.015232     0.011621

xxx TABLE:mt_x_of_s xxxxxxxxxxxxxxxxx
          c1    c2    c3    c4    c5
          --    --    --    --    --

    r1    -7    -6    -7    -6    -6
    r2    -5    -3    -5    -3    -4
    r3    -2    -1    -1     0    -1
    r4     2     2     3     4     2
    r5     6     5     5     6     5
    r6     8     9     7     9     9

xxx TABLE:mt_y_of_s xxxxxxxxxxxxxxxxx
            c1          c2          c3         c4          c5
          ------    --------    ------    -------    -------

    r1    13.231      21.786    18.136      19.35     13.901
    r2     9.946      16.887    9.6914      15.71     8.6906
    r3    16.255      6.2166    13.799     5.2138     11.641
    r4    12.628      2.7525    6.5321    0.27238     13.357
    r5    5.8844      4.0352      6.05    0.14102    0.50318
    r6    3.5617    -0.72091    5.1855    -6.3772    -4.4805

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_input_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                i    idx     value
                -    ---    --------

    fl_x_mean   1     1     -0.11081
    fl_x_sd     2     2       4.1239
    fl_y_mean   3     3       8.8423
    fl_y_sd     4     4       6.5845

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_output_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i    idx    ndim    numel    rowN    colN      mean       std
                                -    ---    ----    -----    ----    ----    --------    ------

    mt_cov_component_weighted   1     1      2       30       6       5      -0.73612    1.0404
```

```
    mt_x_devi_from_mean          2     2      2       30       6       5       0.94415    5.3051
    mt_x_y_multiply              3     3      2       30       6       5       -31.321    36.564
    mt_y_devi_from_mean          4     4      2       30       6       5      -0.51644    7.1913

xxx TABLE:mt_cov_component_weighted xxxxxxxxxxxxxxxxx
              c1            c2           c3          c4           c5
          -----------    ---------    --------    ---------    ----------

    r1       -0.87434      -3.5432     -1.4628     -0.22368    -0.0035451
    r2       -0.13003      -2.1607    -0.35565     -0.47814    0.00087767
    r3       -0.11248      0.17365    -0.56642    -0.025838     -0.018507
    r4       0.010697     -0.38241    -0.69273      -3.0184       0.17717
    r5     -0.0020165     -0.14618    -0.51584      -3.0371      -0.99056
    r6    -0.00015927    -0.041473    -0.14098      -2.1121       -1.4106

xxx TABLE:mt_x_devi_from_mean xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          -------    --------    --------    -------    --------

    r1    -6.8892     -5.8892     -6.8892    -5.8892     -5.8892
    r2    -4.8892     -2.8892     -4.8892    -2.8892     -3.8892
    r3    -1.8892    -0.88919    -0.88919    0.11081    -0.88919
    r4     2.1108      2.1108      3.1108     4.1108      2.1108
    r5     6.1108      5.1108      5.1108     6.1108      5.1108
    r6     8.1108      9.1108      7.1108     9.1108      9.1108

xxx TABLE:mt_x_y_multiply xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          -------    -------    -------    --------    -------

    r1    -30.237    -76.225    -64.023     -61.882    -29.792
    r2     -5.396    -23.242     -4.151     -19.842    0.59004
    r3    -14.003     2.3348    -4.4073    -0.40209    -2.4884
    r4     7.9905    -12.854    -7.1868      -35.23     9.5287
    r5    -18.075    -24.568    -14.271     -53.172     -42.62
    r6     -42.83    -87.129    -26.003     -138.66    -121.38

xxx TABLE:mt_y_devi_from_mean xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          -------    -------    -------    -------    --------

    r1      4.389      12.943     9.2933      10.508      5.0587
    r2     1.1037      8.0444    0.84902      6.8677    -0.15171
    r3     7.4123    -2.6258      4.9566    -3.6286      2.7985
    r4     3.7855    -6.0898    -2.3103      -8.57      4.5142
    r5    -2.9579    -4.8071    -2.7924    -8.7013     -8.3392
    r6    -5.2806    -9.5633    -3.6568     -15.22     -13.323

fl_cov
  -22.0835

fl_cor
   -0.8133
```

3.4.2 Test FF_DISC_RAND_VAR_MASS2COVCOR Four States-Points

Over some (a,z) states that is 2 by 2, c matrix, and y matrix, find correlation. Positively related.

```
% Set Parameters
```

```
mt_c_of_s = [1,2;3,1];
mt_y_of_s = [2,10;5,1.1];
rng(123);
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=1.4446,cor=0.65723
```

Same as before, but now inputs are single column:

```
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s(:), mt_y_of_s(:), mt_f_of_s(:), bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=1.4446,cor=0.65723
```

3.4.3 Test FF_DISC_RAND_VAR_MASS2COVCOR Two Random Vectors

Generate two random vectors, with random or even mass, correlation should be zero:

```
% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=-57.6533,cor=-0.062023
```

3.4.4 Test FF_DISC_RAND_VAR_MASS2COVCOR Provide Mean and SD

Same as above, but now provide means and sd for x andy directly. The results are the same as when mean and sd are calculated inside the function.

```
% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
fl_c_mean = sum(mt_f_of_s.*mt_c_of_s);
fl_c_sd = sqrt(sum(mt_f_of_s.*(mt_c_of_s-fl_c_mean).^2));
fl_y_mean = sum(mt_f_of_s.*mt_y_of_s);
fl_y_sd = sqrt(sum(mt_f_of_s.*(mt_y_of_s-fl_y_mean).^2));
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
```

```
    mt_c_of_s, mt_y_of_s, mt_f_of_s, ...
    fl_c_mean, fl_c_sd, ...
    fl_y_mean, fl_y_sd, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=-57.6533,cor=-0.062023
```

Chapter 4

Graphs

4.1 FF_GRAPH_GRID Examples: X, Y and Color Line Plots

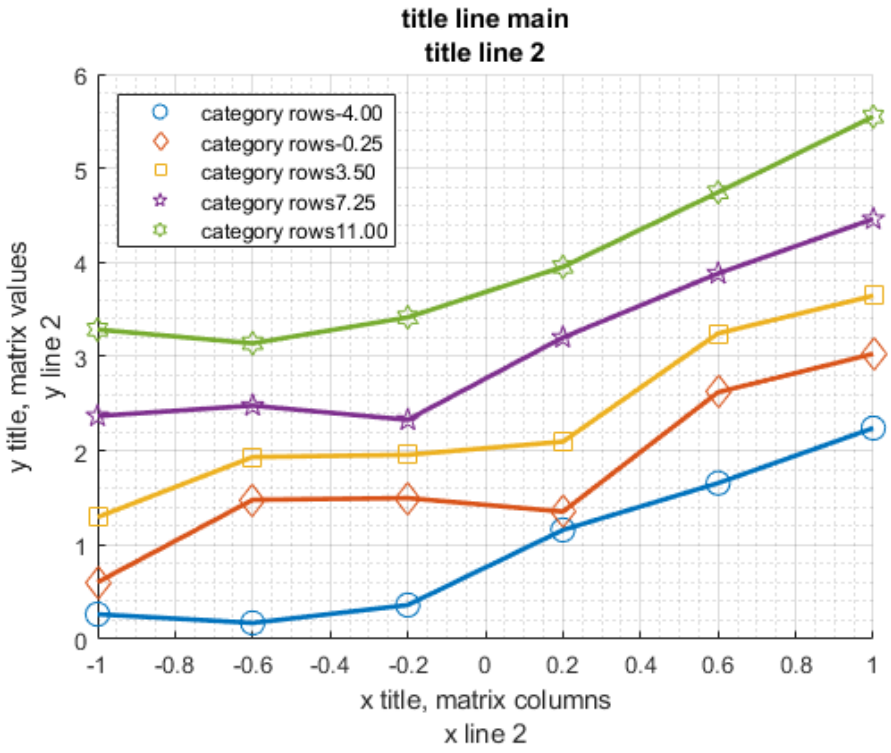
Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

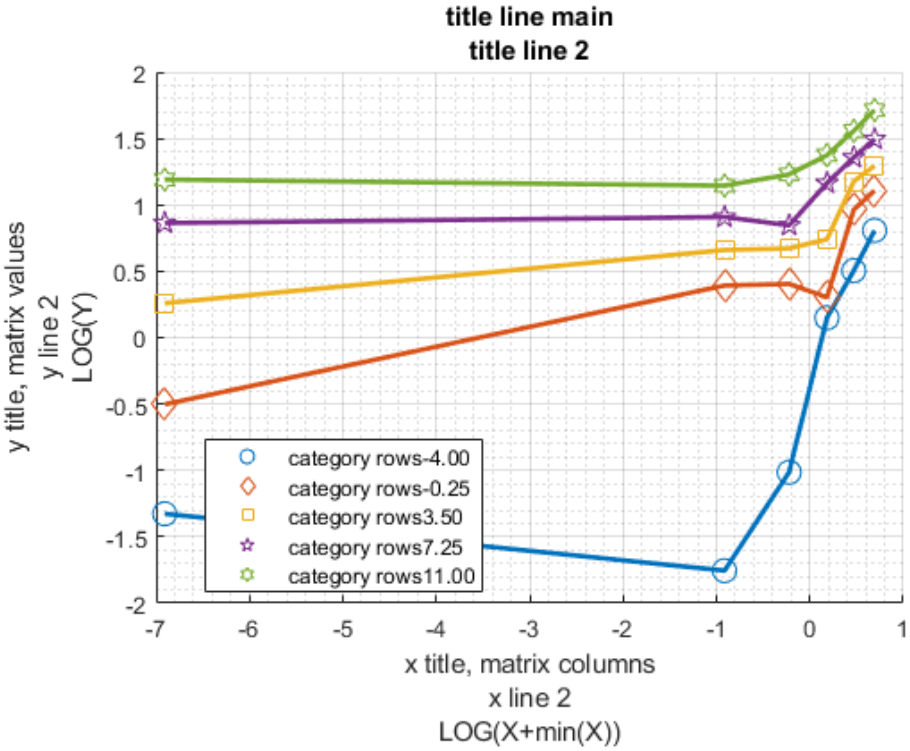
This is the example vignette for function: **ff_graph_grid** from the **MEconTools Package.** This function can graph out value and policy functions given one state vector (x-axis), conditional on other states (line groups). Can handle a few lines (scatter + lines), or many groups (jet spectrum).

4.1.1 Test FF_GRAPH_GRID Defaults

Call the function with defaults.

```
ff_graph_grid();
```

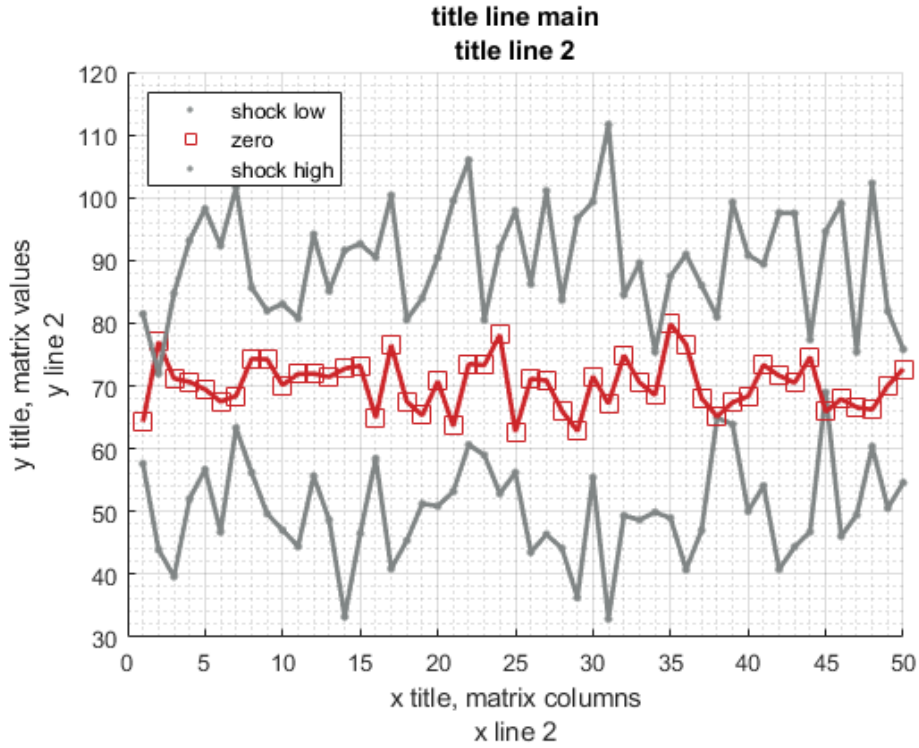


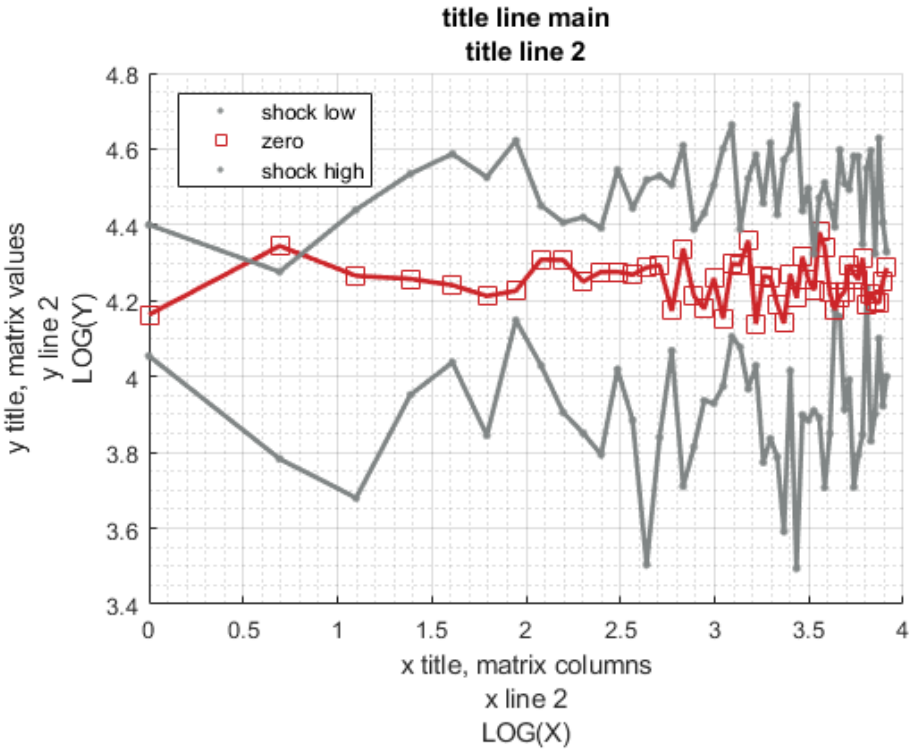


4.1.2 Test FF_GRAPH_GRID Random Matrix Pick Markers and Colors

Call the function with defaults.

```
rng(123);
mt_value = [normrnd(50,10,[1, 50]); ...
            normrnd(70,5,[1, 50]);...
            normrnd(90,10,[1, 50])];
ar_row_grid = ["shock low", "zero", "shock high"];
ar_col_grid = 1:50;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_scatter_shapes') = { '.', 's' ,'.' };
mp_support_graph('cl_colors') = {'gray', 'red', 'gray'};
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



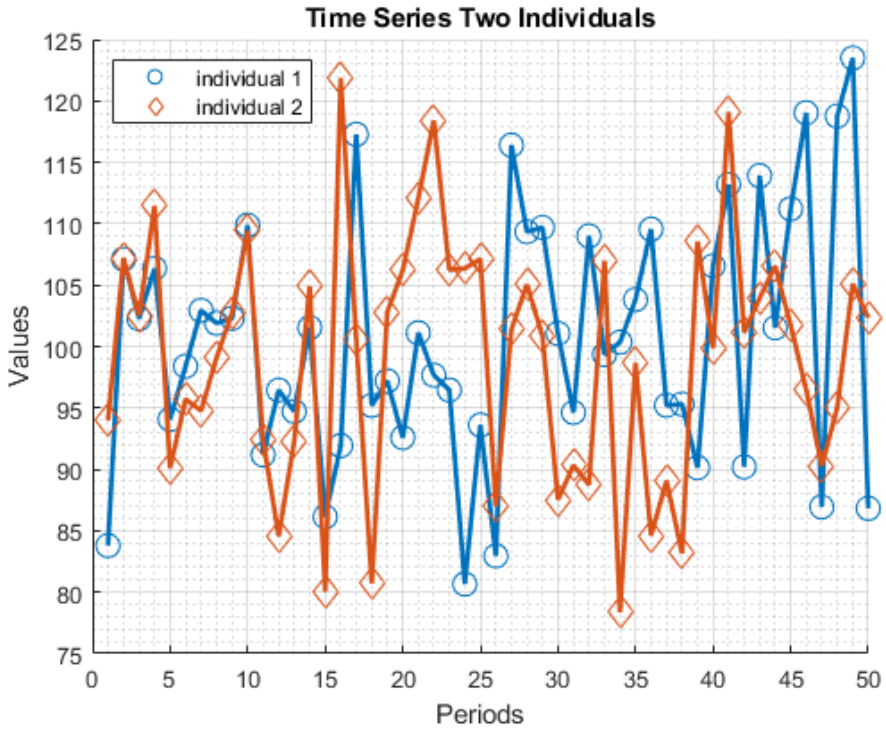


4.1.3 Test FF_GRAPH_GRID Two Random Normal Lines and Labels

There are two autoregressive time series, plot out the time two time series.

```
% Generate the two time series
rng(456);
mt_value = normrnd(100,10,[2, 50]);
ar_row_grid = ["individual 1", "individual 2"];
ar_col_grid = 1:50;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
```

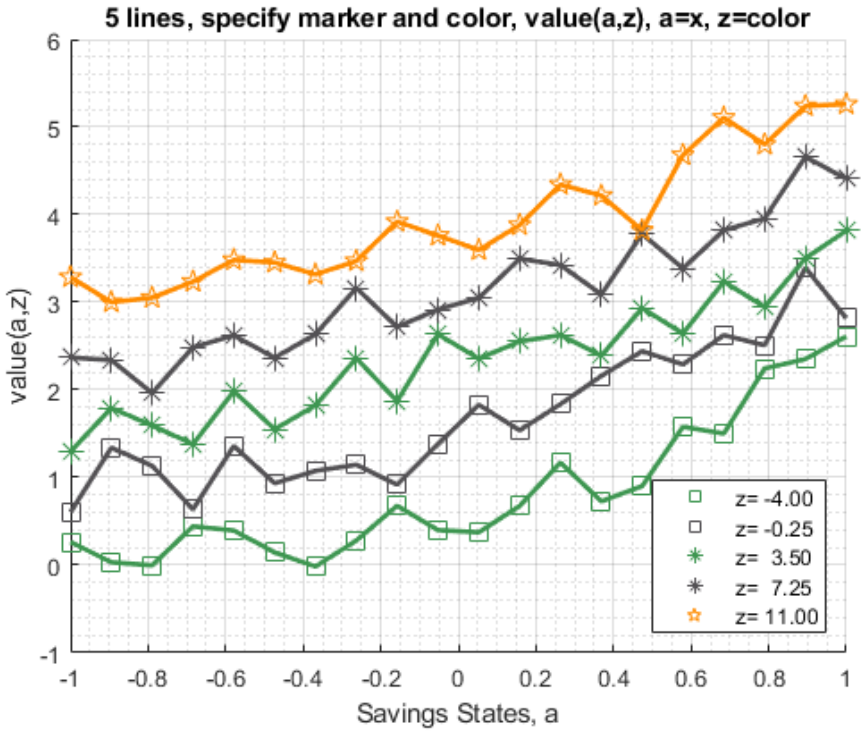
```
mp_support_graph('cl_st_graph_title') = {'Time Series Two Individuals'};
mp_support_graph('cl_st_ytitle') = {'Values'};
mp_support_graph('cl_st_xtitle') = {'Periods'};
mp_support_graph('bl_graph_logy') = false; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



4.1.4 Test FF_GRAPH_GRID 6 Lines Pick Marker and Colors

Plot many lines, with auto legend.

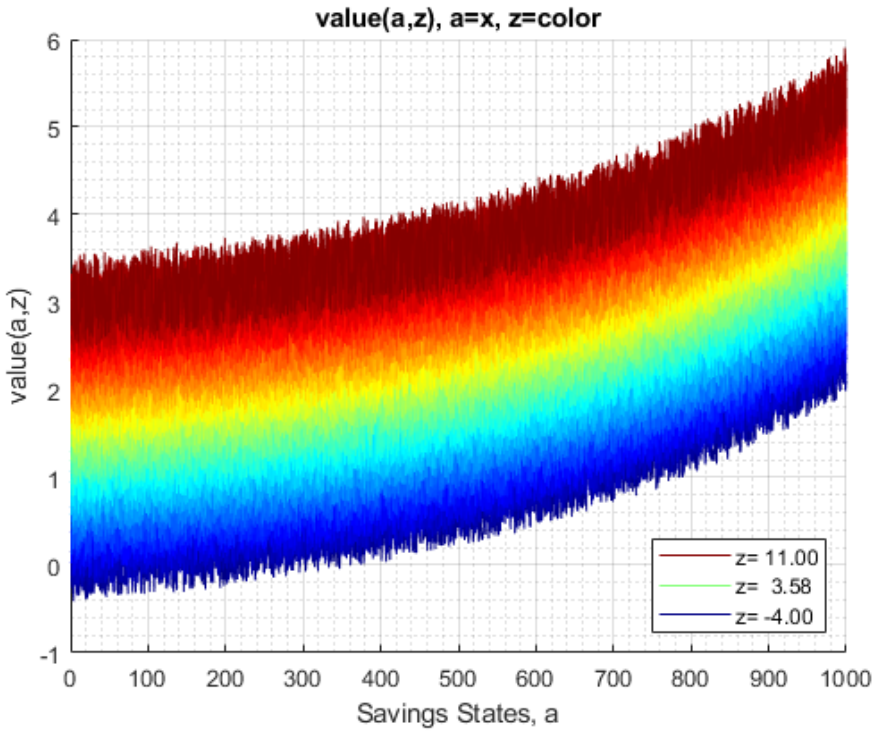
```
% Generate some Data
rng(456);
ar_row_grid = linspace(-4, 11, 5);
ar_col_grid = linspace(-1, 1, 20);
rng(123);
mt_value = 0.2*ar_row_grid' + exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'5 lines, specify marker and color, value(a,z), a=x, z=colo
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'southeast';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
mp_support_graph('cl_scatter_shapes') = {'s', 's', '*', '*', 'p'};
mp_support_graph('cl_colors') = {'green', 'black', 'green', 'black', 'orange'};
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



4.1.5 Test FF_GRAPH_GRID Many Lines

Plot many lines, with auto legend.

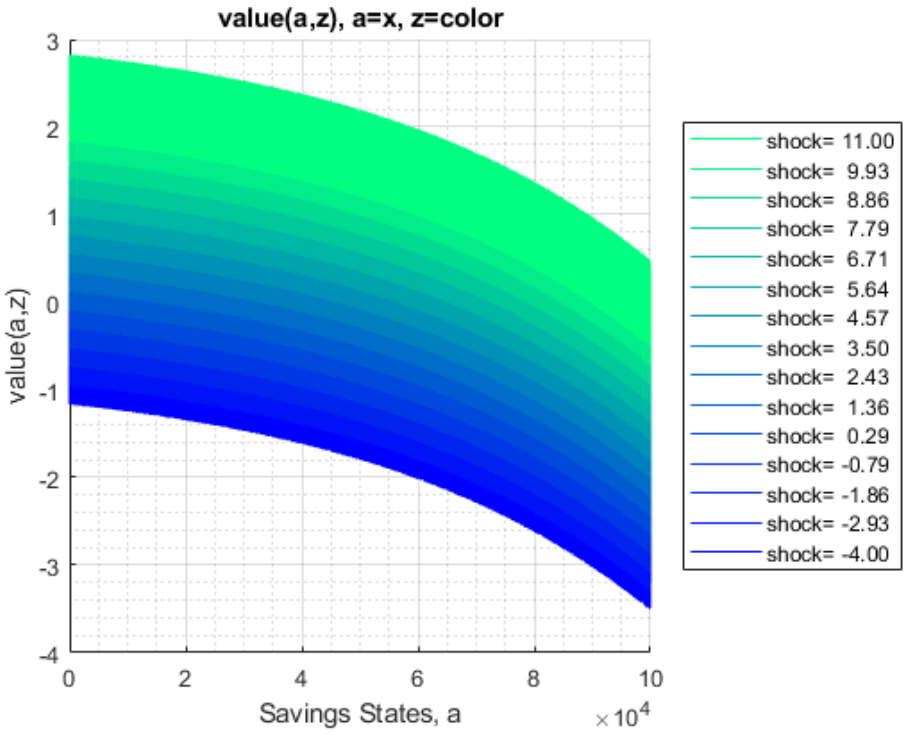
```
% Generate some Data
rng(456);
ar_row_grid = linspace(-4, 11, 100);
ar_col_grid = linspace(-1, 1, 1000);
rng(123);
mt_value = 0.2*ar_row_grid' + exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'southeast';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
mp_support_graph('cl_colors') = 'jet'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



4.1.6 Test FF_GRAPH_GRID Many Lines Legend Exogenous

Plot many lines, exogenously set legend

```
% Generate the two time series
rng(456);
ar_row_grid = linspace(-4, 11, 15);
ar_col_grid = linspace(-1, 1, 100000);
rng(123);
mt_value = 0.2*ar_row_grid' - exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% setting shock vector name exogenously here
ar_row_grid = string(num2str(ar_row_grid', "shock=%6.2f"));
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'eastoutside';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('it_legend_select') = 15;
mp_support_graph('cl_colors') = 'winter'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```


value(a,z), a=x, z=color
value(a,z)
Savings States, a
×10^4
shock= 11.00
shock= 9.93
shock= 8.86
shock= 7.79
shock= 6.71
shock= 5.64
shock= 4.57
shock= 3.50
shock= 2.43
shock= 1.36
shock= 0.29
shock= -0.79
shock= -1.86
shock= -2.93
shock= -4.00

Chapter 5

Data Structures

5.1 FF_SAVEBORR_GRID Example for Generating Asset Grid

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_saveborr_grid** from the **MEconTools Package.** This function generates variously spaced savings/borrowing states/choices grid.

5.1.1 Test FF_SAVEBORR_GRID Defaults

Call the function with defaults.

```
ff_saveborr_grid();
```

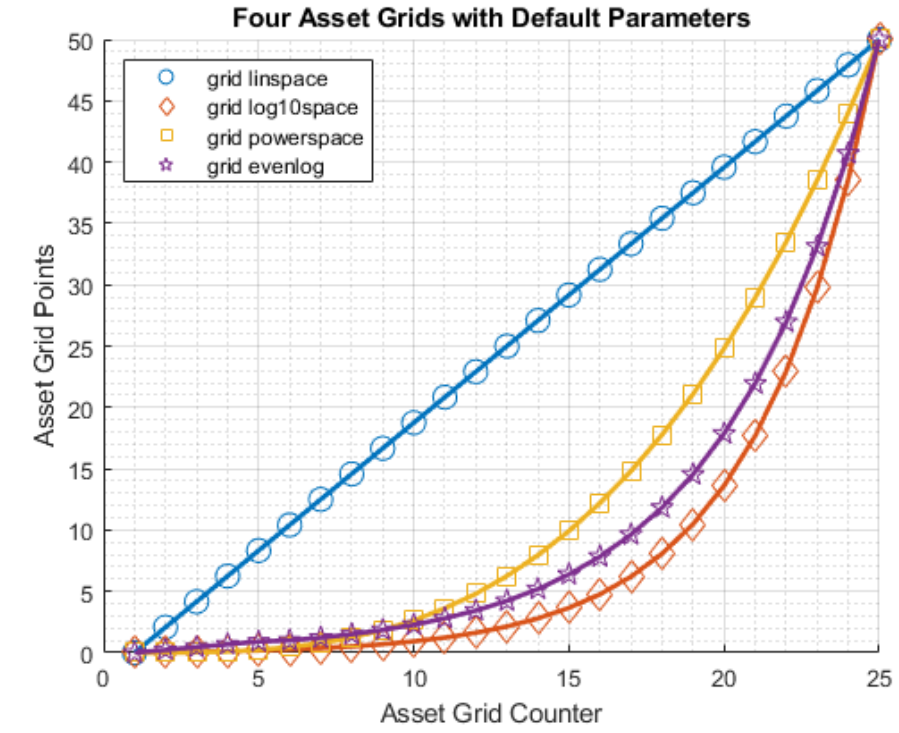
```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i    idx    ndim    numel    rowN    colN     sum     mean      std      coefv
                      -    ---    ----    -----    ----    ----    -----    -----    ------    -----

    ar_fl_saveborr    1     1      2       25       25      1      216.7    8.668    13.363     1.54

xxx TABLE:ar_fl_saveborr xxxxxxxxxxxxxxxxxx
              c1
           --------

    r1           0
    r2     0.029558
    r3     0.067855
    r4      0.11748
    r5      0.18177
    r6      0.26507
    r7      0.37301
    r8      0.51286
    r9      0.69407
    r10     0.92885
    r11      1.2331
    r12      1.6272
    r13      2.1379
    r14      2.7996
```

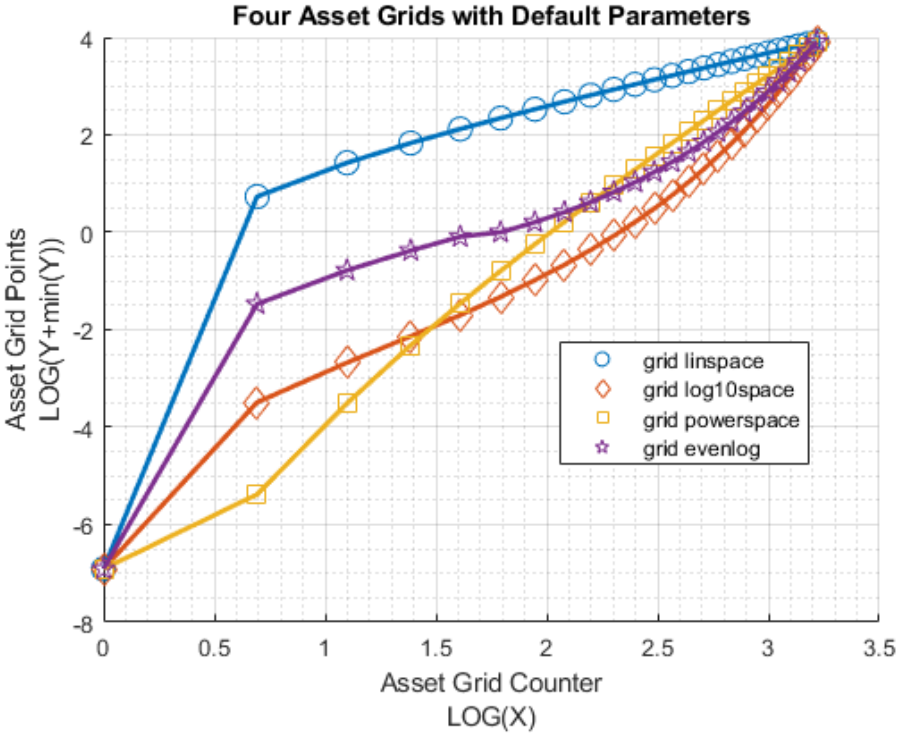
```
    r15       3.657
    r16      4.7679
    r17      6.2072
    r18      8.0722
    r19      10.489
    r20       13.62
    r21      17.676
    r22      22.932
    r23      29.743
    r24      38.567
    r25          50

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                               i    idx    value
                               -    ---    -----

    grid_evenlog_threshold     1     2        1
    grid_log10space_x1         2     3      0.3
    grid_log10space_x2         3     4        3
    grid_powerspace_power      4     5        3
```

5.1.2 Test FF_SAVEBORR_GRID Default Linear Grid, Log Grid, Power Grid, Threshold Grid

Call the function with defaults.

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
% Four types of grid points
st_grid_type = 'grid_linspace';
[ar_fl_saveborr_linspace] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_log10space';
[ar_fl_saveborr_log10space] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_powerspace';
[ar_fl_saveborr_powerspace] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_evenlog';
[ar_fl_saveborr_evenlog] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
% draw four types of lines jointly
mt_value = [ar_fl_saveborr_linspace'; ar_fl_saveborr_log10space'; ...
    ar_fl_saveborr_powerspace'; ar_fl_saveborr_evenlog'];
ar_row_grid = ["grid linspace", "grid log10space", "grid powerspace", "grid evenlog"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Four Asset Grids with Default Parameters'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



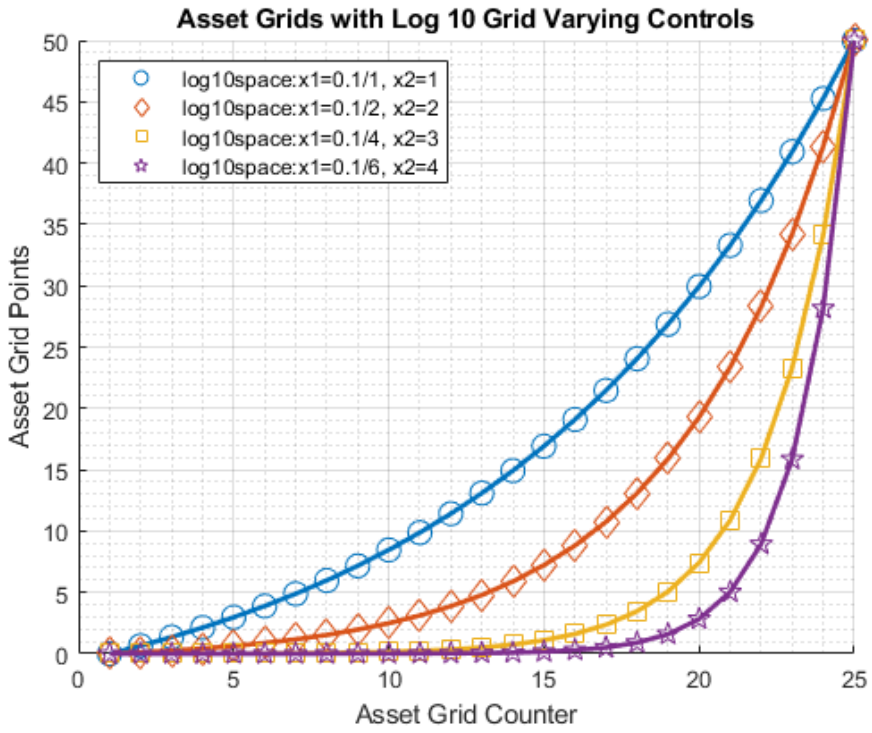


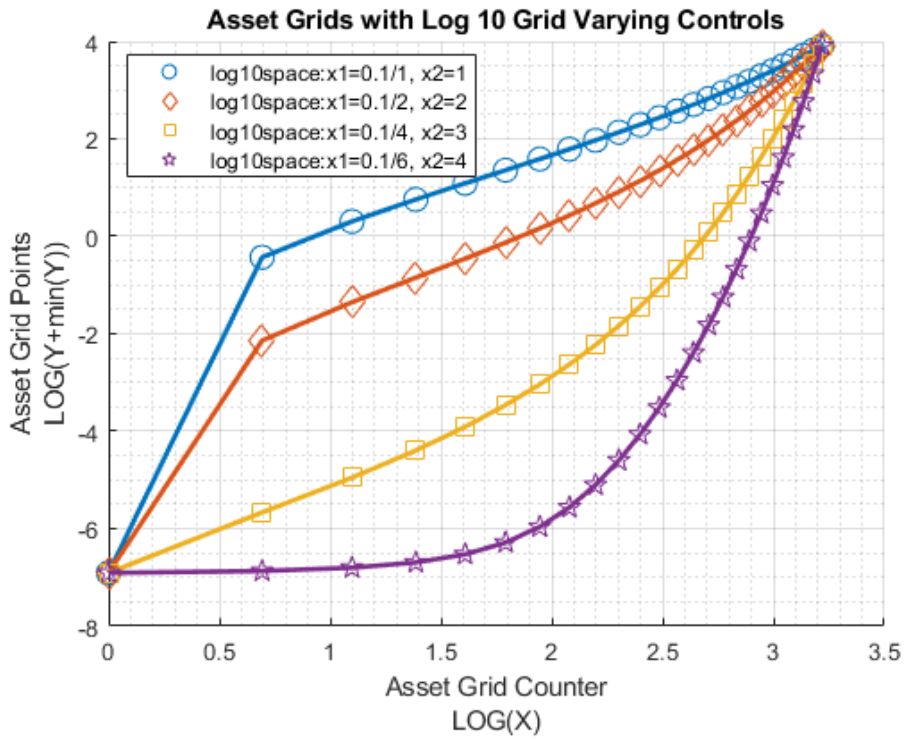
5.1.3 Test FF_SAVEBORR_GRID Log Grid Changing Parameters

Log grid, same min and max, change log X1 and X2 points

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_log10space';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_log10space_x1') = 0.1;
```

```
mp_grid_control('grid_log10space_x2') = 1;
[ar_fl_log10space_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/2;
mp_grid_control('grid_log10space_x2') = 1*2;
[ar_fl_log10space_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/4;
mp_grid_control('grid_log10space_x2') = 1*4;
[ar_fl_log10space_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/6;
mp_grid_control('grid_log10space_x2') = 1*6;
[ar_fl_log10space_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
% draw four types of lines jointly
mt_value = [ar_fl_log10space_a'; ar_fl_log10space_b'; ...
    ar_fl_log10space_c'; ar_fl_log10space_d'];
ar_row_grid = [...
    "log10space:x1=0.1/1, x2=1", ...
    "log10space:x1=0.1/2, x2=2", ...
    "log10space:x1=0.1/4, x2=3", ...
    "log10space:x1=0.1/6, x2=4"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Log 10 Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```

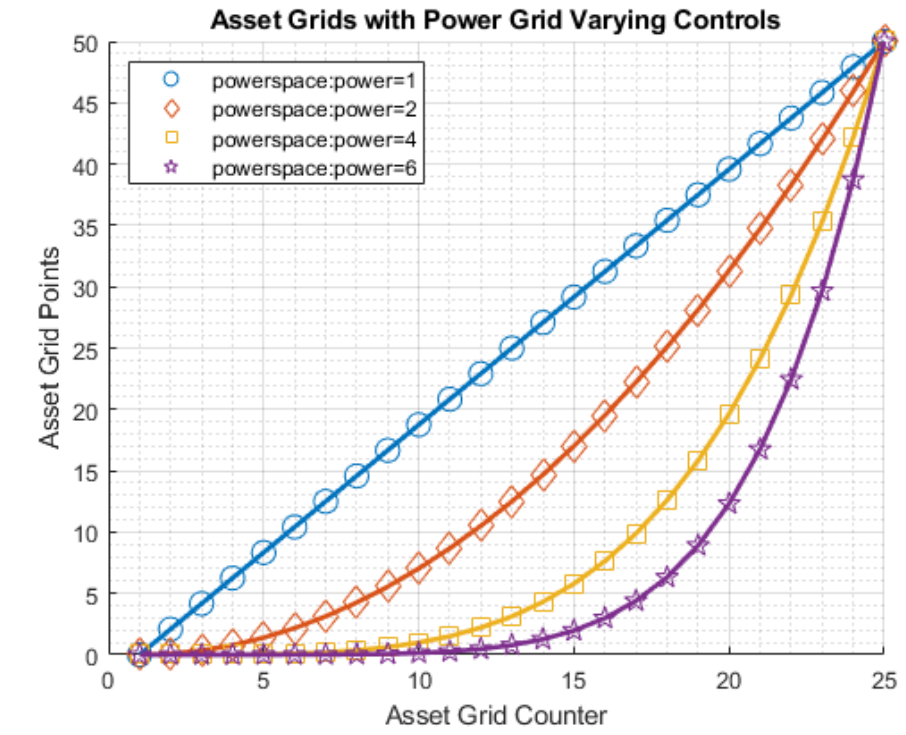


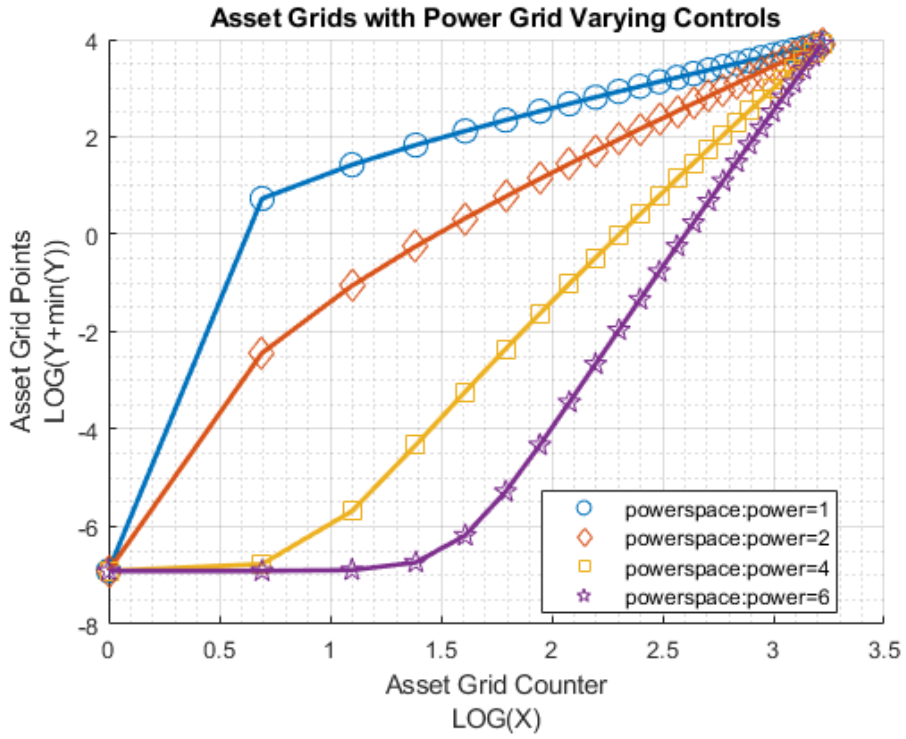


5.1.4 Test FF_SAVEBORR_GRID Power Grid Changing Parameters

Log grid, same min and max, change log X1 and X2 points

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_powerspace';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_powerspace_power') = 1;
[ar_fl_powerspace_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_powerspace_power') = 2;
[ar_fl_powerspace_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_powerspace_power') = 4;
[ar_fl_powerspace_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_powerspace_power') = 6;
[ar_fl_powerspace_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
% draw four types of lines jointly
mt_value = [ar_fl_powerspace_a'; ar_fl_powerspace_b'; ...
    ar_fl_powerspace_c'; ar_fl_powerspace_d'];
ar_row_grid = [...
    "powerspace:power=1", ...
    "powerspace:power=2", ...
    "powerspace:power=4", ...
    "powerspace:power=6"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Power Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



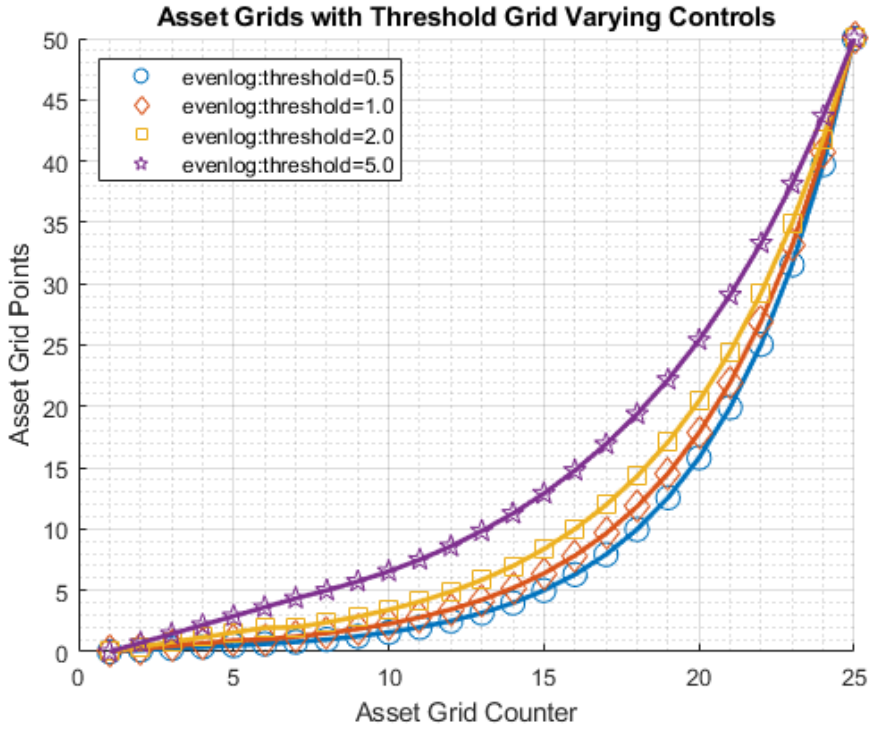


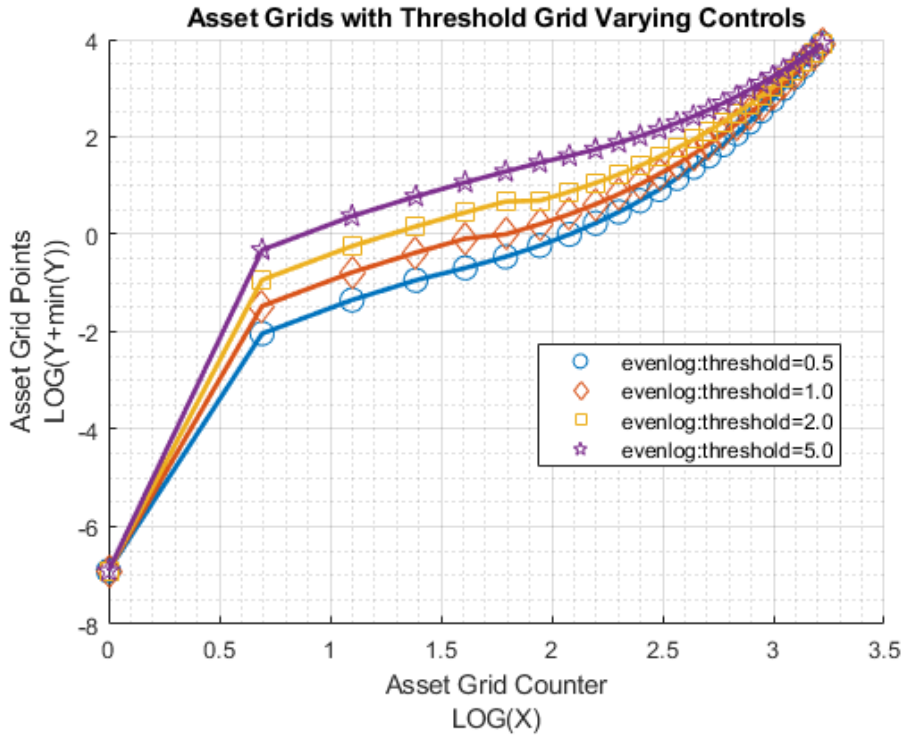
5.1.5 Test FF_SAVEBORR_GRID Threshold Grid Changing Parameters

Threshold Grid, Changing Threshold Levels. Initial segments below threshold are linspace, then logspace.

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_evenlog';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_evenlog_threshold') = 0.50;
```

```
[ar_fl_evenlog_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 1.00;
[ar_fl_evenlog_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 2;
[ar_fl_evenlog_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 5;
[ar_fl_evenlog_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
% draw four types of lines jointly
mt_value = [ar_fl_evenlog_a'; ar_fl_evenlog_b'; ...
    ar_fl_evenlog_c'; ar_fl_evenlog_d'];
ar_row_grid = [...
    "evenlog:threshold=0.5", ...
    "evenlog:threshold=1.0", ...
    "evenlog:threshold=2.0", ...
    "evenlog:threshold=5.0"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Threshold Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```




Asset Grids with Threshold Grid Varying Controls
Asset Grid Points
LOG(Y+min(Y))
Asset Grid Counter
LOG(X)
evenlog:threshold=0.5
evenlog:threshold=1.0
evenlog:threshold=2.0
evenlog:threshold=5.0
4
2
0
-2
-4
-6
-8
0
0.5
1
1.5
2
2.5
3
3.5

Chapter 6

Common Functions

6.1 FFY_TAUCHEN AR1 Shock Discretization Example

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ffy_tauchen** from the **MEconTools Package.** : See also the **ffy_rouwenhorst** function from the **MEconTools Package.** This function discretize a mean zero AR1 process, uses Tauchen (1986). See AR 1 Example for some details on how the AR1 process works. And See Kopecky and Suen (2010).

6.1.1 Test FFY_TAUCHEN Defaults

Call the function with defaults. Default sd bounds arer plus and minus 4. This is used in the following examples, unless otherwise specified as the 5th parameter.

```
ffy_tauchen();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum    mean      std      coef
                         -    ---    ----    -----    ----    ----    ---    ----    -------    ----

    ar_disc_ar1          1     1      2        5       5       1       0       0     0.79057
    mt_disc_ar1_trans    2     6      2       25       5       5       5     0.2     0.27623     1.3

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
           c1
          ----

    r1      -1
    r2    -0.5
    r3       0
    r4     0.5
    r5       1

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1             c2             c3             c4             c5
          ----------     ----------     --------     ----------     ----------

    r1       0.22663        0.73331     0.040048     1.0689e-05     7.3923e-12
    r2      0.012224        0.58648      0.39831      0.0029797      7.605e-08
```

```
    r3    8.8417e-05       0.10556      0.7887       0.10556    8.8417e-05
    r4     7.605e-08     0.0029797     0.39831       0.58648     0.012224
    r5    7.3923e-12    1.0689e-05    0.040048       0.73331       0.22663

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    -----

    fl_ar1_persistence    1     2      0.6
    fl_ar1_step           2     3      0.5
    fl_shk_std            3     4      0.2
    it_std_bound          4     5        4
```

6.1.2 Test FFY_TAUCHEN Specify Parameters

With a grid of 10 points, the sd bounds on Tauchen and Rouwenhorst are identical. With the not extremely persistent shock process here, the Tauchen and Rouwenhorst Results are very similar.

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose, it_std_bound] = ...
    deal(0.60, 0.10, 10, true, 3);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose, it_std_bound);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN        sum           mean
                          -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1           1     1      2        10      10       1     -7.2164e-16    -7.2164e-17
    mt_disc_ar1_trans     2     6      2       100      10      10              10            0.1

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
           ---------

    r1        -0.375
    r2      -0.29167
    r3      -0.20833
    r4        -0.125
    r5     -0.041667
    r6      0.041667
    r7         0.125
    r8       0.20833
    r9       0.29167
    r10        0.375

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1            c2            c3            c4           c5           c6           c7
           ----------    ----------    ----------    ----------    --------    --------    ---------

    r1        0.13933       0.26196       0.31887       0.20154    0.066066    0.011201    0.0009785
    r2       0.056673       0.16995       0.30658       0.28713      0.1396    0.035167     0.004575
    r3        0.01861      0.087039       0.23281       0.32308     0.23281    0.087039      0.01684
    r4      0.0048925      0.035167        0.1396       0.28713     0.30658     0.16995      0.04884
    r5      0.0010235      0.011201      0.066066       0.20154     0.31887     0.26196       0.1116
```

```
    r6     0.00016962     0.0028101       0.02466       0.11169     0.26196     0.31887       0.2015
    r7     2.2197e-05    0.00055483     0.0072547      0.048841     0.16995     0.30658       0.2871
    r8     2.2881e-06    8.6129e-05     0.0016806      0.016841    0.087039     0.23281       0.3230
    r9     1.8543e-07    1.0503e-05    0.00030628     0.0045756    0.035167      0.1396       0.2871
    r10    1.1798e-08    1.0053e-06    4.3874e-05    0.00097859    0.011201    0.066066       0.2015

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2         0.6
    fl_ar1_step           2     3    0.083333
    fl_shk_std            3     4         0.1
    it_std_bound          4     5           3
```

6.1.3 Test FFY_TAUCHEN High Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.01, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN        sum           mean
                          -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1           1     1      2        7       7       1      -5.5511e-17    -7.9302e-18
    mt_disc_ar1_trans     2     6      2       49       7       7                7        0.14286

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
          -----------

    r1       -0.28355
    r2       -0.18903
    r3      -0.094517
    r4     -2.7756e-17
    r5       0.094517
    r6        0.18903
    r7        0.28355

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1             c2             c3             c4             c5             c6          --
          -----------    -----------    -----------    -----------    ----------    ----------    --

    r1              1     4.4497e-06              0              0             0             0
    r2     4.4412e-07              1     2.8552e-06              0             0             0
    r3      1.632e-46     7.1638e-07              1     1.8164e-06             0             0
    r4    9.6185e-124     6.3021e-46     1.1456e-06              1    1.1456e-06             0
    r5    6.3206e-239    8.9712e-123     2.4121e-45     1.8164e-06             1    7.1638e-07
    r6              0     1.426e-237    8.2932e-122     9.1503e-45    2.8552e-06             1    4.
    r7              0              0    3.1885e-236    7.5984e-121    3.4405e-44    4.4497e-06
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2         0.99
    fl_ar1_step           2     3     0.094517
    fl_shk_std            3     4         0.01
    it_std_bound          4     5            4
```

6.1.4 Test FFY_TAUCHEN Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN    sum     mean        std
                          -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1           1     1      2        7       7       1       0           0    0.028805
    mt_disc_ar1_trans     2     6      2       49       7       7       7     0.14286     0.17448

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
             c1
          ---------

    r1    -0.040002
    r2    -0.026668
    r3    -0.013334
    r4            0
    r5     0.013334
    r6     0.026668
    r7     0.040002

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1           c2          c3          c4          c5          c6           c7
          ----------    --------    -------    -------    -------    --------    ----------

    r1    0.00049475    0.024497    0.24044     0.4947    0.21921    0.020299    0.00037109
    r2    0.00047179    0.023751    0.23685    0.49488     0.2227    0.020954    0.00038948
    r3    0.00044982    0.023024    0.23329      0.495    0.22621    0.021626     0.0004087
    r4     0.0004288    0.022316    0.22974    0.49504    0.22974    0.022316     0.0004288
    r5     0.0004087    0.021626    0.22621      0.495    0.23329    0.023024    0.00044982
    r6    0.00038948    0.020954     0.2227    0.49488    0.23685    0.023751    0.00047179
    r7    0.00037109    0.020299    0.21921     0.4947    0.24044    0.024497    0.00049475

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------
```

```
    fl_ar1_persistence    1     2         0.01
    fl_ar1_step           2     3     0.013334
    fl_shk_std            3     4         0.01
    it_std_bound          4     5            4
```

6.1.5 Test FFY_TAUCHEN High Persistence, High SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.99, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN        sum           mean
                          -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1           1     1      2        7       7       1      -3.5527e-15    -5.0753e-16
    mt_disc_ar1_trans     2     6      2       49       7       7                7        0.14286

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
            c1
          -------

    r1    -28.072
    r2    -18.714
    r3    -9.3572
    r4          0
    r5     9.3572
    r6     18.714
    r7     28.072

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1              c2              c3              c4              c5              c6
          -----------    -----------    -----------    -----------    ----------    ----------    --

    r1              1     4.4497e-06              0              0             0             0
    r2     4.4412e-07              1     2.8552e-06              0             0             0
    r3      1.632e-46     7.1638e-07              1     1.8164e-06             0             0
    r4    9.6185e-124     6.3021e-46     1.1456e-06              1    1.1456e-06             0
    r5    6.3206e-239    8.9712e-123     2.4121e-45     1.8164e-06             1    7.1638e-07
    r6              0     1.426e-237    8.2932e-122     9.1503e-45    2.8552e-06             1    4.
    r7              0              0    3.1885e-236    7.5984e-121    3.4405e-44    4.4497e-06

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    ------

    fl_ar1_persistence    1     2       0.99
    fl_ar1_step           2     3     9.3572
    fl_shk_std            3     4       0.99
    it_std_bound          4     5          4
```

6.1.6 Test FFY_TAUCHEN Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN    sum     mean        std
                          -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1           1     1      2        7       7       1       0          0    0.028805
    mt_disc_ar1_trans     2     6      2       49       7       7       7    0.14286     0.17448

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
             c1
          ---------

    r1    -0.040002
    r2    -0.026668
    r3    -0.013334
    r4            0
    r5     0.013334
    r6     0.026668
    r7     0.040002

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1           c2          c3          c4          c5          c6           c7
          ----------    --------    -------    -------    -------    --------    ----------

    r1    0.00049475    0.024497    0.24044     0.4947    0.21921    0.020299    0.00037109
    r2    0.00047179    0.023751    0.23685    0.49488     0.2227    0.020954    0.00038948
    r3    0.00044982    0.023024    0.23329      0.495    0.22621    0.021626     0.0004087
    r4     0.0004288    0.022316    0.22974    0.49504    0.22974    0.022316     0.0004288
    r5     0.0004087    0.021626    0.22621      0.495    0.23329    0.023024    0.00044982
    r6    0.00038948    0.020954     0.2227    0.49488    0.23685    0.023751    0.00047179
    r7    0.00037109    0.020299    0.21921     0.4947    0.24044    0.024497    0.00049475

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2        0.01
    fl_ar1_step           2     3    0.013334
    fl_shk_std            3     4        0.01
    it_std_bound          4     5           4
```

6.2 FFY_ROUWENHORST AR1 Shock Discretization Example

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ffy_rouwenhorst** from the **MEconTools Package**. See also **ffy_tauchen** function from the **MEconTools Package**. This function discretize a mean zero AR1 process, uses Rouwenhorst (1995). See AR 1 Example for some details on how the AR1 process works. And See Kopecky and Suen (2010).

6.2.1 Test FFY_ROUWENHORST Defaults

Call the function with defaults.

```
ffy_rouwenhorst();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN    sum    mean      std      coef
                          -    ---    ----    -----    ----    ----    ---    ----    -------    ----

    ar_disc_ar1           1     1      2        5       5       1       0       0     0.39528
    mt_disc_ar1_trans     2    11      2       25       5       5       5     0.2     0.18246    0.91

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
           c1
          -----

    r1     -0.5
    r2    -0.25
    r3        0
    r4     0.25
    r5      0.5

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
            c1        c2        c3        c4        c5
          ------    ------    ------    ------    ------

    r1    0.4096    0.4096    0.1536    0.0256    0.0016
    r2    0.1024    0.4864    0.3264    0.0784    0.0064
    r3    0.0256    0.2176    0.5136    0.2176    0.0256
    r4    0.0064    0.0784    0.3264    0.4864    0.1024
    r5    0.0016    0.0256    0.1536    0.4096    0.4096

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    -----

    fl_ar1_beg            1     2     -0.5
    fl_ar1_end            2     3      0.5
    fl_ar1_persistence    3     4      0.6
    fl_ar1_step           4     5     0.25
    fl_p0                 5     6      0.8
    fl_q0                 6     7      0.8
    fl_shk_std            7     8      0.2
    fl_sig_ar1            8     9     0.25
    it_std_bound          9    10        0
```

6.2.2 Test FFY_ROUWENHORST Specify Parameters

With a grid of 10 points, the Rwouenhorst bounds on standard deviations are equall to Tauchen bounds of 3. With the not extremely persistent shock process here, the Tauchen and Rouwenhorst Results are very similar.

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.60, 0.10, 10, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN       sum           mean          --
                         -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1          1     1      2        10      10       1     5.5511e-17    5.5511e-18     0
    mt_disc_ar1_trans    2    11      2       100      10      10             10           0.1    0.

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
           ---------

    r1        -0.375
    r2       -0.29167
    r3       -0.20833
    r4         -0.125
    r5      -0.041667
    r6       0.041667
    r7          0.125
    r8        0.20833
    r9        0.29167
    r10         0.375

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1             c2             c3             c4             c5             c6             c7
           ----------    ----------    ----------    ---------    --------    --------    ---------

    r1        0.13422       0.30199       0.30199      0.17616     0.06606    0.016515    0.0027525
    r2       0.033554       0.20133       0.32716      0.26424     0.12662    0.038535    0.0075694
    r3      0.0083886      0.081789       0.26267      0.32755     0.21401    0.082747     0.019741
    r4      0.0020972      0.028312       0.14038      0.30946     0.30369     0.15877     0.047989
    r5     0.00052429      0.009044      0.061145      0.20246     0.33477     0.25969      0.10585
    r6     0.00013107     0.0027525      0.023642      0.10585     0.25969     0.33477      0.20246
    r7     3.2768e-05    0.00081101     0.0084603     0.047989     0.15877     0.30369      0.30946
    r8      8.192e-06    0.00023347     0.0028677     0.019741    0.082747     0.21401      0.32755
    r9      2.048e-06    6.6048e-05    0.00093389    0.0075694    0.038535     0.12662      0.26424
    r10      5.12e-07    1.8432e-05    0.00029491    0.0027525    0.016515     0.06606      0.17616

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_beg            1     2       -0.375
    fl_ar1_end            2     3        0.375
```

```
    fl_ar1_persistence    3     4          0.6
    fl_ar1_step           4     5     0.083333
    fl_p0                 5     6          0.8
    fl_q0                 6     7          0.8
    fl_shk_std            7     8          0.1
    fl_sig_ar1            8     9        0.125
    it_std_bound          9    10            0
```

6.2.3 Test FFY_ROUWENHORST High Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.01, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std      c
                         -    ---    ----    -----    ----    ----    ---    -------    -------    -

    ar_disc_ar1          1     1      2        7       7       1       0           0    0.12503
    mt_disc_ar1_trans    2    11      2       49       7       7       7     0.14286    0.34148

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
             c1
          --------

    r1    -0.17364
    r2    -0.11576
    r3    -0.05788
    r4           0
    r5     0.05788
    r6     0.11576
    r7     0.17364

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1            c2            c3            c4            c5            c6            c7
          ----------    ----------    ----------    ----------    ----------    ----------    ------

    r1       0.97037      0.029257    0.00036756    2.4627e-06    9.2815e-09    1.8656e-11    1.5625
    r2     0.0048762        0.9705      0.024382    0.00024504    1.2314e-06    3.0938e-09    3.1094
    r3    2.4504e-05      0.009753       0.97057      0.019506    0.00014703    4.9254e-07    6.1877
    r4    1.2313e-07    7.3513e-05       0.01463       0.97059       0.01463    7.3513e-05    1.2313
    r5    6.1877e-10    4.9254e-07    0.00014703      0.019506       0.97057      0.009753    2.4504
    r6    3.1094e-12    3.0938e-09    1.2314e-06    0.00024504      0.024382        0.9705     0.004
    r7    1.5625e-14    1.8656e-11    9.2815e-09    2.4627e-06    0.00036756      0.029257       0.9

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx     value
                         -    ---    --------

    fl_ar1_beg           1     2     -0.17364
    fl_ar1_end           2     3      0.17364
    fl_ar1_persistence   3     4         0.99
```

```
    fl_ar1_step           4     5      0.05788
    fl_p0                 5     6       0.995
    fl_q0                 6     7       0.995
    fl_shk_std            7     8        0.01
    fl_sig_ar1            8     9    0.070888
    it_std_bound          9    10           0
```

6.2.4 Test FFY_ROUWENHORST Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std
                         -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1          1     1      2        7       7       1       0           0    0.017639
    mt_disc_ar1_trans    2    11      2       49       7       7       7     0.14286     0.10985

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
          ----------

    r1     -0.024496
    r2     -0.016331
    r3    -0.0081654
    r4             0
    r5     0.0081654
    r6      0.016331
    r7      0.024496

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5          c6          c7
          --------    --------    -------    -------    -------    --------    --------

    r1    0.016586    0.097547    0.23904    0.31241    0.22966    0.090047    0.014711
    r2    0.016258    0.096266    0.23749    0.31247    0.23124    0.091266    0.015008
    r3    0.015936    0.094997    0.23594    0.31251    0.23281    0.092497    0.015311
    r4     0.01562    0.093741    0.23438    0.31252    0.23438    0.093741     0.01562
    r5    0.015311    0.092497    0.23281    0.31251    0.23594    0.094997    0.015936
    r6    0.015008    0.091266    0.23124    0.31247    0.23749    0.096266    0.016258
    r7    0.014711    0.090047    0.22966    0.31241    0.23904    0.097547    0.016586

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx      value
                         -    ---    ---------

    fl_ar1_beg           1     2     -0.024496
    fl_ar1_end           2     3      0.024496
    fl_ar1_persistence   3     4          0.01
    fl_ar1_step          4     5     0.0081654
```

```
    fl_p0                5     6         0.505
    fl_q0                6     7         0.505
    fl_shk_std           7     8          0.01
    fl_sig_ar1           8     9      0.010001
    it_std_bound         9    10             0
```

6.2.5 Test FFY_ROUWENHORST High Persistence, High SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.99, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN       sum           mean          --

    ar_disc_ar1          1     1      2        7       7       1      3.5527e-15    5.0753e-16     1
    mt_disc_ar1_trans    2    11      2       49       7       7               7       0.14286    0.

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
            c1
          -------

    r1     -17.19
    r2     -11.46
    r3    -5.7301
    r4          0
    r5     5.7301
    r6      11.46
    r7      17.19

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1            c2            c3            c4            c5            c6            c7
          ----------    ----------    ----------    ----------    ----------    ----------    ------

    r1       0.97037      0.029257    0.00036756    2.4627e-06    9.2815e-09    1.8656e-11    1.5625
    r2     0.0048762        0.9705      0.024382    0.00024504    1.2314e-06    3.0938e-09    3.1094
    r3    2.4504e-05      0.009753       0.97057      0.019506    0.00014703    4.9254e-07    6.1877
    r4    1.2313e-07    7.3513e-05       0.01463       0.97059       0.01463    7.3513e-05    1.2313
    r5    6.1877e-10    4.9254e-07    0.00014703      0.019506       0.97057      0.009753    2.4504
    r6    3.1094e-12    3.0938e-09    1.2314e-06    0.00024504      0.024382        0.9705     0.004
    r7    1.5625e-14    1.8656e-11    9.2815e-09    2.4627e-06    0.00036756      0.029257       0.9

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    value
                         -    ---    ------

    fl_ar1_beg           1     2     -17.19
    fl_ar1_end           2     3      17.19
    fl_ar1_persistence   3     4       0.99
    fl_ar1_step          4     5     5.7301
    fl_p0                5     6      0.995
```

```
    fl_q0                6     7      0.995
    fl_shk_std           7     8       0.99
    fl_sig_ar1           8     9     7.0179
    it_std_bound         9    10          0
```

6.2.6 Test FFY_ROUWENHORST Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std
                         -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1          1     1      2        7       7       1       0           0    0.017639
    mt_disc_ar1_trans    2    11      2       49       7       7       7     0.14286     0.10985

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
          ----------

    r1     -0.024496
    r2     -0.016331
    r3    -0.0081654
    r4             0
    r5     0.0081654
    r6      0.016331
    r7      0.024496

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
          --------    --------    -------    -------    -------    --------    --------

    r1    0.016586    0.097547    0.23904    0.31241    0.22966    0.090047    0.014711
    r2    0.016258    0.096266    0.23749    0.31247    0.23124    0.091266    0.015008
    r3    0.015936    0.094997    0.23594    0.31251    0.23281    0.092497    0.015311
    r4     0.01562    0.093741    0.23438    0.31252    0.23438    0.093741     0.01562
    r5    0.015311    0.092497    0.23281    0.31251    0.23594    0.094997    0.015936
    r6    0.015008    0.091266    0.23124    0.31247    0.23749    0.096266    0.016258
    r7    0.014711    0.090047    0.22966    0.31241    0.23904    0.097547    0.016586

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx      value
                         -    ---    ---------

    fl_ar1_beg           1     2     -0.024496
    fl_ar1_end           2     3      0.024496
    fl_ar1_persistence   3     4          0.01
    fl_ar1_step          4     5     0.0081654
    fl_p0                5     6         0.505
    fl_q0                6     7         0.505
```

```
fl_shk_std            7     8          0.01
fl_sig_ar1            8     9      0.010001
it_std_bound          9    10             0
```

Chapter 7

Support Tools

7.1 FF_CONTAINER_MAP_DISPLAY Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_container_map_display** from the **MEconTools Package.** This function summarizes statistics of matrixes stored in a container map, as well as scalar, string, function and other values stored in container maps.

7.1.1 Test FF_CONTAINER_MAP_DISPLAY Defaults

Call the function with defaults.

```
ff_container_map_display();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                        i     idx     ndim     numel     rowN     colN      sum       mean        std
                        --    ---     ----     -----     ----     ----     ------     -------     -------

    mat_1               1      7       2        12        3        4      6.5142     0.54285      0.2232
    mat_2               2      8       2      2650       50       53      1313.3     0.49559     0.29232
    mat_2_boolean       3      9       2      2650       50       53        1361     0.51358     0.49991
    mat_3               4     10       2         4        2        2      1.8111     0.45277     0.45111
    tensor_1            5     15       3        16        2        8      7.3043     0.45652     0.27787
    tensor_2            6     16       3        75        3       25      40.195     0.53593     0.29044
    tensor_3            7     17       2         4        1        4      1.6926     0.42315     0.37389
    tesseract_1         8     18       4        72        3       24      34.321     0.47669     0.26374
    tesseract_2         9     19       4        20        2       10      8.4191     0.42096     0.28981
    tesseract_bl_3     10     20       4        10        1       10           3         0.3     0.48305

xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxx
             c1         c2         c3         c4
           -------    -------    -------    -------

    r1     0.69647    0.55131    0.98076    0.39212
    r2     0.28614    0.71947    0.68483    0.34318
    r3     0.22685    0.42311    0.48093    0.72905

xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxx
              c1          c2         c3          c4         c50         c51         c52         c53
```

```
          --------    --------    -------    --------    --------    --------    --------    ------

    r1     0.43857      0.6249    0.17108     0.56564    0.072152     0.67855     0.61667     0.540
    r2    0.059678     0.67469    0.82911    0.084904     0.63289     0.27236     0.32528     0.249
    r3     0.39804     0.84234    0.33867     0.58267    0.046367     0.44513    0.075047      0.78
    r4       0.738    0.083195    0.55237     0.81484     0.50561     0.11117     0.59532     0.356
    r5     0.18249     0.76368    0.57855     0.33707     0.10653    0.028681      0.7435     0.918
    r46     0.6813     0.55326    0.88786     0.69983     0.83758     0.16382     0.74191    0.0656
    r47    0.87546     0.85445    0.69631     0.66117     0.97069     0.79092     0.42466     0.787
    r48    0.51042     0.38484    0.44033    0.049097    0.017768     0.33302     0.24401     0.979
    r49    0.66931     0.31679    0.43821      0.7923     0.12979     0.75311     0.79466    0.0790
    r50    0.58594     0.35426     0.7651     0.51872     0.86415     0.58281     0.84795      0.45

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxx
          c1       c2       c3       c4       c50      c51      c52      c53
          -----    -----    -----    -----    -----    -----    -----    -----

    r1    true     false    false    true     true     false    true     true
    r2    true     false    true     true     false    false    true     true
    r3    false    true     false    true     false    true     false    true
    r4    false    true     false    false    false    true     true     true
    r5    true     true     true     false    true     false    false    true
    r46   false    true     true     false    true     true     true     true
    r47   true     true     true     true     true     true     false    false
    r48   true     false    false    false    true     true     false    true
    r49   true     true     false    true     true     true     false    false
    r50   false    false    false    false    false    false    false    false

xxx TABLE:mat_3 xxxxxxxxxxxxxxxxx
             c1          c2
          ----------    -------

    r1    0.00012471    0.13253
    r2       0.88615    0.79226

xxx TABLE:tensor_1 xxxxxxxxxxxxxxxxx
            c1         c2         c3         c4         c5         c6         c7         c8
          --------    -------    -------    -------    -------    -------    ------    -------

    r1    0.019363    0.34271    0.52167    0.53703    0.75756    0.68839    0.8345    0.26597
    r2    0.018091    0.33355    0.11738    0.77857    0.81933    0.28644    0.6157      0.368

xxx TABLE:tensor_2 xxxxxxxxxxxxxxxxx
            c1         c2         c3         c4         c22        c23         c24         c25
          --------    -------    -------    -------    -------    -------    ---------    -------

    r1     0.51866    0.40495    0.48278    0.99731    0.46584    0.62976     0.035924    0.10505
    r2    0.028692    0.37408    0.24149    0.35201    0.66054    0.87243    0.0024293    0.81088
    r3     0.87339    0.19457    0.83212    0.15315    0.77859    0.96663       0.2501     0.8056

xxx TABLE:tensor_3 xxxxxxxxxxxxxxxxx
            c1         c2         c3          c4
          ------    ------    -------    -------

    r1    0.1219    0.5119    0.91553    0.14329

xxx TABLE:tesseract_1 xxxxxxxxxxxxxxxxx
            c1         c2         c3          c4         c21        c22        c23        c24
```

```
          -------    -------    -------    --------    -------    -------    -------    -------

    r1    0.64531    0.59299    0.32115     0.67653    0.90328    0.56911    0.52562    0.12014
    r2    0.74558     0.5007    0.46142     0.21384    0.35564    0.13732      0.155    0.23786
    r3    0.91137    0.46403    0.18118    0.049919    0.46246    0.46842    0.75348    0.64547

xxx TABLE:tesseract_2 xxxxxxxxxxxxxxxxx
             c1         c2         c3         c4         c7         c8         c9         c10
          --------    -------    -------    -------    -------    -------    --------    -------

    r1     0.28898    0.48211    0.44359    0.97146    0.61782    0.65121     0.80715    0.11605
    r2    0.094493    0.34941    0.17595    0.14192    0.16754    0.57097    0.043114    0.70518

xxx TABLE:tesseract_bl_3 xxxxxxxxxxxxxxxxx
           c1       c2       c3       c4       c7       c8       c9       c10
          -----    -----    -----    -----    -----    -----    -----    -----

    r1    false    false    true     true     false    true     false    false

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i    idx     value
                      -    ---    -------

    boolean_1         1     1           1
    empty             2     2         NaN
    mat_4             3    11     0.74898
    string_float_1    4    13      1021.1
    string_int_1      5    14        1021

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 String
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i     idx            string
                     ---    ----    ---------------------

    list_string_1    "1"    "5"     "col1;col2;col3;col4"
    list_string_2    "2"    "6"     "row1;row2;row3;row4"
    string_1         "3"    "12"    "Table Name"

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 Functions
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
              i     idx      functionString
             ---    ---    -------------------

    func1    "1"    "3"    "@(x)1+2+x"
    func2    "2"    "4"    "@(x,y)x*1+sqrt(y)"
```

7.1.2 Test FF_CONTAINER_MAP_DISPLAY summarize Matrix Only

Three large matrixes, show summaries

```
% Create Container
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
```

```
rng(123);
mp_container_map('mat_1') = rand(100,100);
mp_container_map('mat_2') = rand(100,100)*2 + 1;
mp_container_map('mat_2_boolean') = (rand(100,100) > 0.5);
% Will only print
ff_container_map_display(mp_container_map);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                     i    idx    ndim    numel    rowN    colN     sum       mean        std      co
                     -    ---    ----    -----    ----    ----    ------    -------    -------    --

    mat_1            1     1      2      10000    100     100     4982.3    0.49823    0.28829    0.
    mat_2            2     2      2      10000    100     100      20029     2.0029    0.57632    0.
    mat_2_boolean    3     3      2      10000    100     100       4995     0.4995    0.50002     1
```

7.1.3 Test FF_CONTAINER_MAP_DISPLAY Show Matrix Subset

A container map with three small matrixes, print only only 2 rows and 3 columns.

```
% Create Container
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
rng(789);
mp_container_map('mat_1') = rand(3,4);
mp_container_map('mat_2') = rand(50,53);
mp_container_map('mat_2_boolean') = (rand(50,53) > 0.5);
% Will only print
ff_container_map_display(mp_container_map, 2, 3);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                     i    idx    ndim    numel    rowN    colN     sum       mean        std      co
                     -    ---    ----    -----    ----    ----    ------    -------    -------    --

    mat_1            1     1      2        12       3       4     4.9876    0.41564    0.33586    0.
    mat_2            2     2      2      2650      50      53     1324.3    0.49973    0.28834    0.
    mat_2_boolean    3     3      2      2650      50      53       1350    0.50943    0.50001    0.

xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4
          -------    -------    -------    -------

    r1    0.32333    0.62442    0.01062    0.53815
    r3    0.79378    0.75889    0.11104    0.55157

xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxx
             c1         c2         c52        c53
           -------    -------    -------    -------

    r1     0.72837    0.20976    0.74583    0.22321
    r50    0.52812      0.545    0.49521    0.29826

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxx
             c1        c2        c52       c53
           -----     -----     -----     -----
```

```
r1     false    true     true     true
r50    true     false    false    true
```

Appendix A

Index and Code Links

A.1 Savings Dynamic Programming links

1. Looped Solution for Infinite Horizon Optimal Savings Dynamic Programming Problem: **mlx** | **m** | **pdf** | **html**
   - Slow looped solution.
   - Given preferences, some AR(1) shock process, solve the infinite horizon household savings dynamic programming problem. The state-space and choice-space share the same asset grid.
   - **MEconTools**: *ff_vfi_az_loop()*
2. Vectorized Solution for Infinite Horizon Optimal Savings Dynamic Programming Problem: **mlx** | **m** | **pdf** | **html**
   - Faster vectorized solution.
   - Given preferences, some AR(1) shock process, solve the infinite horizon household savings dynamic programming problem. The state-space and choice-space share the same asset grid.
   - **MEconTools**: *ff_vfi_az_vec()*

A.2 Summarize Policy and Value links

1. Summarize ND Array Policy and Value Functions: **mlx** | **m** | **pdf** | **html**
   - Given an NDarray matrix with N1, N2, ..., ND dimensions. Generate average and standard deviation for the 3rd dimension, grouping by the other dimensions.
   - For example, show the 5th dimension as the column groups, and the other variables generate combinations shown as rows.
   - The resulting summary statistics table contains mean and standard deviation among other statistics over the policy or value contained in the ND array.
   - **MEconTools**: *ff_summ_nd_array()*

A.3 Distributional Analysis links

1. Gateway Joint Probability Mass Statistics: **mlx** | **m** | **pdf** | **html**
   - Given probability mass function f(s), and information y(s), x(s), z(s) at each element of the state-space, compute statistics for each variable, y, x, z, which are all discrete random variables.
   - Compute their correlation and covariance.
   - **MEconTools**: *ff_simu_stats()*
2. Discrete Random Variable Distributional Statistics: **mlx** | **m** | **pdf** | **html**
   - Model simulation generates discrete random variables, calculate mean, standard deviation, min, max, percentiles, and proportion of outcomes held by x percentiles, etc.
   - **MEconTools**: *ff_disc_rand_var_stats()*
3. Generate Discrete Random Variable: **mlx** | **m** | **pdf** | **html**

- Given mass at state space points, and y, c, a, z and other outcomes or other information at each corresponding state space points, generate discrete random variable, with unique sorted values, and mass for each unique sorted values.
- Generate additional joint distributions: if initial distribution is over f(a,z), generate joint distribution of f(y,a) or f(y,z).
- **MEconTools**: *ff_disc_rand_var_mass2outcomes()*

4. Discrete Random Variable Correlation and Covariance: **mlx** | **m** | **pdf** | **html**
   - Given probability mass function f(s), X(s), and Y(s), compute the covariance and correlation betwen X and Y.
   - **MEconTools**: *ff_disc_rand_var_mass2covcor()*

A.4 Graphs links

1. Multiple Line Graph Function: **mlx** | **m** | **pdf** | **html**
   - Grid based Graph, x-axis one param, color another param, over outcomes.
   - **MEconTools**: *ff_graph_grid()*

A.5 Data Structures links

1. Log and Power Spaced Asset and Choice Grids: **mlx** | **m** | **pdf** | **html**
   - Generate linear, log-space, power-space, or threshold-cut asset or choice grids.
   - **MEconTools**: *ff_saveborr_grid()*

A.6 Common Functions links

1. Discretize AR1 Normal Shock Tauchen (1986): **mlx** | **m** | **pdf** | **html**
   - Mean zero AR(1) shock discretize following Tauchen (1986).
   - **MEconTools**: *ffy_tauchen()*
2. Discretize AR1 Normal Shock Rouwenhorst (1995): **mlx** | **m** | **pdf** | **html**
   - Mean zero AR(1) shock discretize following Rouwenhorst (1995).
   - **MEconTools**: *ffy_rouwenhorst()*

A.7 Support Tools links

1. Organizes and Prints Container Map Key and Values: **mlx** | **m** | **pdf** | **html**
   - Summarizes the contents of a map container by data types. Includes, scalar, array, matrix, string, functions, tensors (3-tuples), tesseracts (4-tuples).
   - **MEconTools**: *ff_container_map_display()*

Bibliography

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Xie, Y. (2020). *bookdown: Authoring Books and Technical Documents with R Markdown*. R package version 0.18.