# Matlab Toolbox Heterogeneous Agents Dynamic Programming

Fan Wang

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# Contents

# Preface

This is a work-in-progress Matlab package consisting of functions that facilitate Dynamic Programming and Related Tasks. Materials gathered from various projects in which Matlab code is used. Some of the solutions/algorithms are research outputs developed for specific research papers, other algorithms and methods are commonly-used. Files are the MEconTools repository. Matlab files are linked below by section with livescript files. Tested with Matlab 2019a (The MathWorks Inc, 2019).

> Download and install the Matlab toolbox: MEconTools.mltbx

This bookdown file is a collection of mlx based vignettes for functions that are available from MEconTools. Each Vignette file contains various examples for invoking each function. The goal of this repository is to make it easier to find/re-use codes produced for various projects.

From other repositories: For dynamic borrowing and savings problems, see Dynamic Asset Repository; For code examples, see also R Example Code, Matlab Example Code, and Stata Example Code; For intro stat with R, see Intro Statistics for Undergraduates, and intro Math with Matlab, see Intro Mathematics for Economists. See here for all of Fan's public repositories.

The site is built using Bookdown (Xie, 2020).

Please contact FanWangEcon for issues or problems.

# Chapter 1

# Savings Dynamic Programming

## 1.1 FF_VFI_AZ_LOOP Dynamic Savings Problem Loop Common Grid

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_vfi_az_loop** from the **MEconTools Package.** This function solves the dynamic programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon. This is the looped code, it is slow for larger state-space problems. The code uses common grid, with the same state space and choice space grids.

**Links to Four Code:**

Four Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid : **ff_vfi_az_loop**, slow should use for testing new models
- Common Choice and States Grid : **ff_vfi_az_vec**, fast good for many purposes
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand :**ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand : **ff_vfi_az_bisec_vec**, precision and speed

The four sample codes are written for the standard dynamic savings problem with AR(1) shock that is one of the core problems introduced in first sessions of graduate Economics courses. The code can be easily adapted to accomand multiple assets, savings and borrowing, discrete and continuous choice, etc. A large proportion of dynamic economic models are based on the underlying structure of solving a model with endogenous states and exogenous shocks, and that is what the (a,z) model does. In general, one should write looped code first to make sure the economics is correct, then vectorized code can be adopted to increase speed.

### 1.1.1 Test FF_VFI_AZ_LOOP Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_loop(mp_params);
```

```
Elapsed time is 1.291175 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum      mean      std     coefvari    min
          -    ---    ----    -----    ----    ----    -----    ------    -----    --------    ---

    ap    1     1       2      700      100      7     16864    24.091    14.08    0.58446      0

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
             c1         c2         c3         c4         c5         c6         c7
           -------    -------    -------    -------    -------    -------    ------

    r1          0          0          0          0          0    0.50505    2.0202
    r2          0          0          0    0.50505    0.50505     1.0101    2.5253
    r3    0.50505    0.50505    0.50505    0.50505     1.0101     1.5152    3.0303
    r4     1.0101     1.0101     1.0101     1.0101     1.5152     2.0202    3.5354
    r5     1.5152     1.5152     1.5152     1.5152     2.0202     2.5253    4.0404
    r96    45.455     45.455      45.96      45.96      45.96      46.97    48.485
    r97     45.96      45.96      45.96     46.465     46.465     47.475     48.99
    r98    46.465     46.465     46.465      46.97      46.97      47.98     48.99
    r99     46.97      46.97      46.97     47.475     47.475     48.485    49.495
    r100   47.475     47.475     47.475      47.98      47.98      48.99        50
```

### 1.1.2 Test FF_VFI_AZ_LOOP Speed Tests

Call the function with different a and z grid size, print out speed:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
```

A grid 50, shock grid 5:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_loop(mp_params, mp_support);
```

```
Elapsed time is 0.223217 seconds.
```

A grid 100, shock grid 7:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
ff_vfi_az_loop(mp_params, mp_support);
```

```
Elapsed time is 1.284511 seconds.
```

A grid 200, shock grid 9:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 200;
mp_params('it_z_n') = 9;
ff_vfi_az_loop(mp_params, mp_support);
```

```
Elapsed time is 6.325330 seconds.
```

### 1.1.3 Test FF_VFI_AZ_LOOP Control Outputs

Run the function first without any outputs;

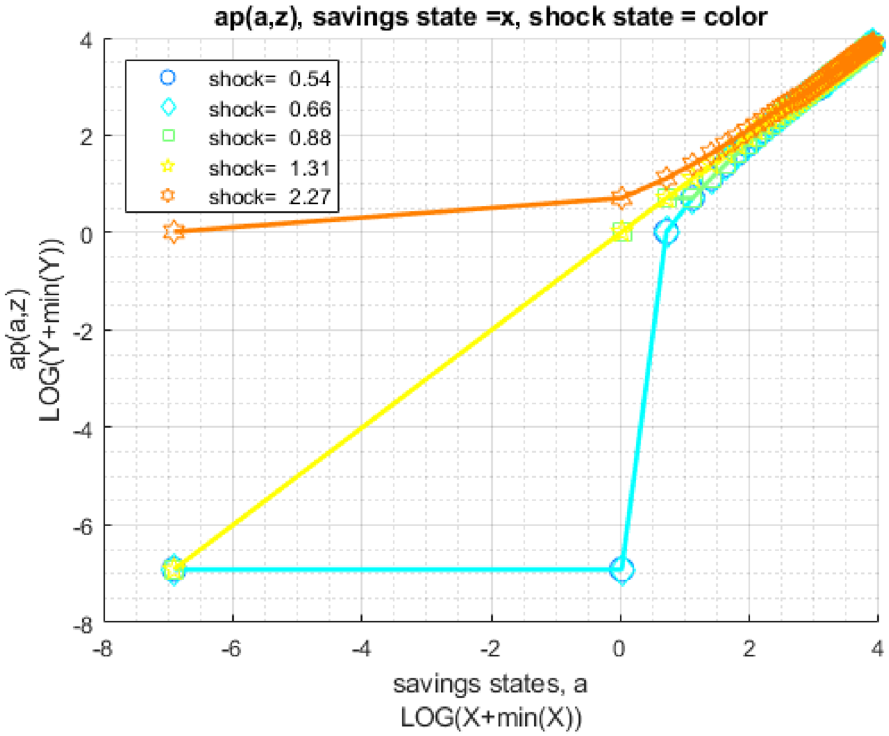
```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = false;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
```

Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'ap'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'ap'};
ff_vfi_az_loop(mp_params, mp_support);
```

```
Elapsed time is 0.313830 seconds.
xxx  ff_vfi_az_vec, outcome=ap  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group      a       mean_z_0_54195    mean_z_0_66401    mean_z_0_88162    mean_z_1_3095    mean_z
    -----    ------    --------------    --------------    --------------    -------------    ------

      1          0           0                 0                 0                0           1.0
      2     1.0204           0                 0            1.0204           1.0204           2.0
      3     2.0408      1.0204            1.0204            2.0408           2.0408           3.0
      4     3.0612      2.0408            2.0408            2.0408           3.0612           4.0
      5     4.0816      3.0612            3.0612            3.0612           4.0816            5.
      6      5.102      4.0816            4.0816            4.0816            5.102           6.1
      7     6.1224       5.102             5.102             5.102           6.1224           7.1
      8     7.1429      6.1224            6.1224            6.1224           7.1429           8.1
      9     8.1633      7.1429            7.1429            7.1429           8.1633           9.1
     10     9.1837      8.1633            8.1633            8.1633           9.1837           10.
     11     10.204      9.1837            9.1837            9.1837           10.204           11.
     12     11.224      10.204            10.204            10.204           11.224           12.
     13     12.245      11.224            11.224            11.224           12.245           13.
     14     13.265      12.245            12.245            12.245           12.245           14.
     15     14.286      13.265            13.265            13.265           13.265           15.
     16     15.306      14.286            14.286            14.286           14.286           15.
     17     16.327      15.306            15.306            15.306           15.306           16.
     18     17.347      16.327            16.327            16.327           16.327           17.
     19     18.367      17.347            17.347            17.347           17.347           18.
     20     19.388      18.367            18.367            18.367           18.367           19.
     21     20.408      19.388            19.388            19.388           19.388           20.
     22     21.429      19.388            20.408            20.408           20.408           21.
     23     22.449      20.408            21.429            21.429           21.429           22.
     24     23.469      21.429            22.449            22.449           22.449           23.
     25      24.49      22.449            22.449            23.469           23.469            24
     26      25.51      23.469            23.469             24.49            24.49            25
     27     26.531       24.49             24.49             25.51            25.51           26.
     28     27.551       25.51             25.51            26.531           26.531           27.
```

| | | | | | | |
|---|---|---|---|---|---|---|
| 29 | 28.571 | 26.531 | 26.531 | 27.551 | 27.551 | 28. |
| 30 | 29.592 | 27.551 | 27.551 | 28.571 | 28.571 | 29. |
| 31 | 30.612 | 28.571 | 28.571 | 28.571 | 29.592 | 30. |
| 32 | 31.633 | 29.592 | 29.592 | 29.592 | 30.612 | 31. |
| 33 | 32.653 | 30.612 | 30.612 | 30.612 | 31.633 | 32. |
| 34 | 33.673 | 31.633 | 31.633 | 31.633 | 32.653 | 33. |
| 35 | 34.694 | 32.653 | 32.653 | 32.653 | 33.673 | 34. |
| 36 | 35.714 | 33.673 | 33.673 | 33.673 | 34.694 | 35. |
| 37 | 36.735 | 34.694 | 34.694 | 34.694 | 35.714 | 36. |
| 38 | 37.755 | 35.714 | 35.714 | 35.714 | 36.735 | 37. |
| 39 | 38.776 | 36.735 | 36.735 | 36.735 | 37.755 | 38. |
| 40 | 39.796 | 37.755 | 37.755 | 37.755 | 38.776 | 39. |
| 41 | 40.816 | 38.776 | 38.776 | 38.776 | 39.796 | 40. |
| 42 | 41.837 | 39.796 | 39.796 | 39.796 | 40.816 | 41. |
| 43 | 42.857 | 40.816 | 40.816 | 40.816 | 41.837 | 42. |
| 44 | 43.878 | 41.837 | 41.837 | 41.837 | 41.837 | 42. |
| 45 | 44.898 | 42.857 | 42.857 | 42.857 | 42.857 | 43. |
| 46 | 45.918 | 43.878 | 43.878 | 43.878 | 43.878 | 44. |
| 47 | 46.939 | 44.898 | 44.898 | 44.898 | 44.898 | 45. |
| 48 | 47.959 | 45.918 | 45.918 | 45.918 | 45.918 | 46. |
| 49 | 48.98 | 46.939 | 46.939 | 46.939 | 46.939 | 47. |
| 50 | 50 | 47.959 | 47.959 | 47.959 | 47.959 | 48 |




Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_loop(mp_params, mp_support);
```

```
Elapsed time is 1.867278 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                   _    ___    ____    _____    ____    ____    ______    _______    _______    ____

    ap             1     1      2       900     100      9      21825      24.25     14.089      0.
    savefraccoh    2     2      2       900     100      9     752.38    0.83597    0.13497    0.16

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
                c1          c2          c3          c4          c5          c6          c7          c8
              _______    _______    _______    _______    _______    _______    _______    ______    _

    r1              0          0          0          0          0          0    0.50505    1.5152    3
    r2              0          0          0          0    0.50505    0.50505     1.0101    1.5152    3
    r3        0.50505    0.50505    0.50505    0.50505    0.50505     1.0101     1.5152    2.0202    4
    r4         1.0101     1.0101     1.0101     1.0101     1.0101     1.5152     2.0202    2.5253    4
    r5         1.5152     1.5152     1.5152     1.5152     1.5152     2.0202     2.5253    3.0303    5
    r96        45.455     45.455     45.455      45.96      45.96      45.96     46.465    47.475    4
    r97         45.96      45.96      45.96     46.465     46.465     46.465      46.97     47.98    4
    r98        46.465     46.465     46.465     46.465      46.97      46.97     47.475    48.485
    r99         46.97      46.97      46.97      46.97     47.475     47.475      47.98     48.99
```

```
    r100     47.475     47.475     47.475     47.475      47.98      47.98     48.485    49.495

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
                c1          c2          c3          c4          c5          c6          c7          c8
            -------     -------     -------     -------     -------     -------     -------     -------

    r1            0           0           0           0           0           0     0.24587     0.48182
    r2            0           0           0           0      0.3075     0.25444     0.39276     0.41371
    r3      0.30679     0.29486     0.27938     0.25939      0.2338     0.40362     0.49043      0.4833
    r4       0.4668     0.45285     0.43438     0.40981     0.37721     0.50166     0.56006     0.53755
    r5      0.56502     0.55132     0.53293     0.50802     0.47415     0.57101     0.61221     0.58103
    r96     0.91292      0.9117     0.90997     0.91752     0.91364     0.90746     0.90692     0.90732
    r97     0.91357     0.91236     0.91064     0.91812     0.91427     0.90815     0.90761     0.90799
    r98      0.9142       0.913      0.9113     0.90882     0.91489     0.90882     0.90828     0.90865
    r99     0.91482     0.91363     0.91195     0.90949     0.91549     0.90949     0.90894     0.90929
    r100    0.91543     0.91425     0.91258     0.91014     0.91609     0.91013     0.90959     0.90992
```

### 1.1.4 Test FF_VFI_AZ_LOOP Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.113265 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean       std      coefv
                   -    ---    ----    -----    ----    ----    ------    -------    ------    -----

    savefraccoh    1     1      2       250      50      5      118.68    0.47472    0.2843    0.598

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
               c1          c2          c3          c4          c5
           -------     -------     -------     -------     -------

    r1           0           0           0           0     0.10642
    r2           0           0           0           0      0.1064
    r3           0           0           0           0     0.10629
    r4           0           0           0           0       0.106
    r5           0           0           0           0     0.10543
    r46    0.79096     0.78787     0.78241     0.77191     0.74922
```

```
    r47    0.79553    0.79262    0.78747    0.77755    0.75606
    r48     0.7999    0.79715    0.79229     0.7829    0.76254
    r49    0.80407    0.80147    0.79687    0.78799    0.76868
    r50    0.80805    0.80559    0.80125    0.79284     0.7745
```

```
% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_loop(mp_params, mp_support);
```

```
Elapsed time is 0.327279 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2       250      50      5      160.99    0.64394    0.29947    0.46

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2          c3         c4         c5
           -------    -------    --------    -------    -------

    r1           0          0    0.024103    0.18484    0.40057
    r2           0          0    0.024094     0.1848    0.40051
    r3           0          0    0.024028    0.18446    0.40008
    r4           0          0    0.046583    0.18354    0.39894
    r5           0          0    0.045925    0.24935    0.39672
    r46    0.94526    0.94167     0.93533    0.92312    0.89672
    r47    0.94628    0.94291     0.93696    0.92548    0.90059
    r48    0.94722    0.94405     0.93846    0.92766    0.90418
    r49    0.94808    0.94511     0.93984    0.92966    0.90749
    r50    0.94888    0.94608     0.94111    0.93151    0.91056
```

### 1.1.5 Test FF_VFI_AZ_LOOP Changing Risk Aversion

Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

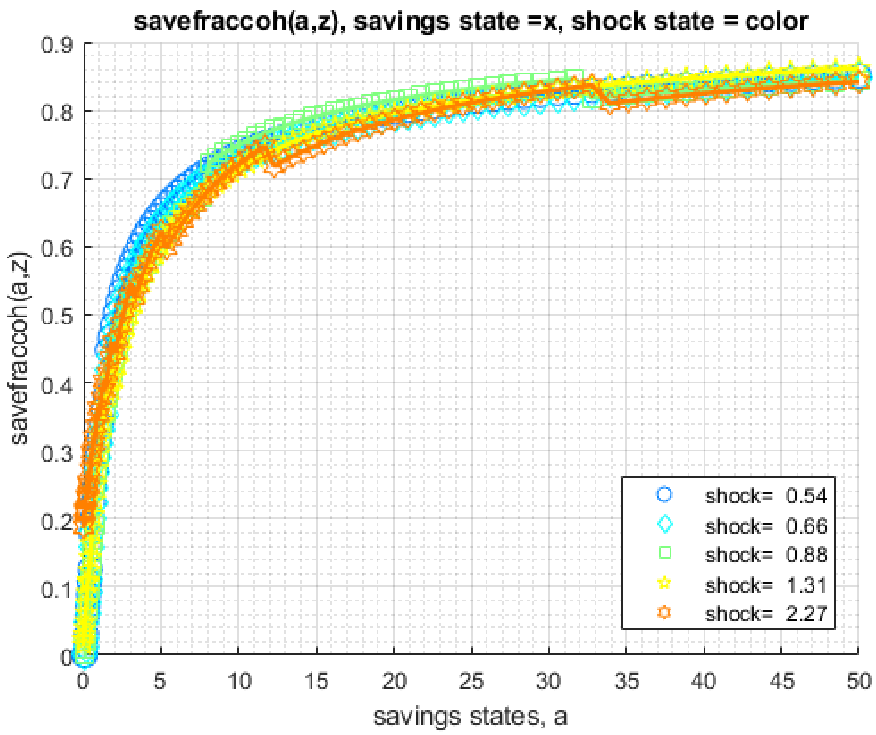
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

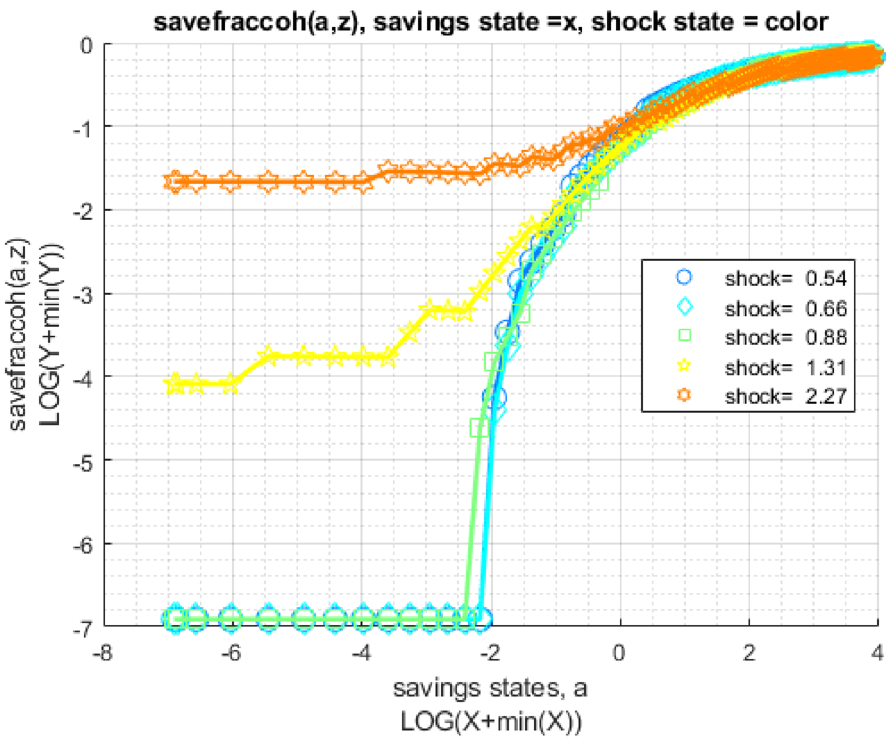
Solve the model with different risk aversion levels, higher preferences for risk:

```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_loop(mp_params, mp_support);
```

```
Elapsed time is 0.581794 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

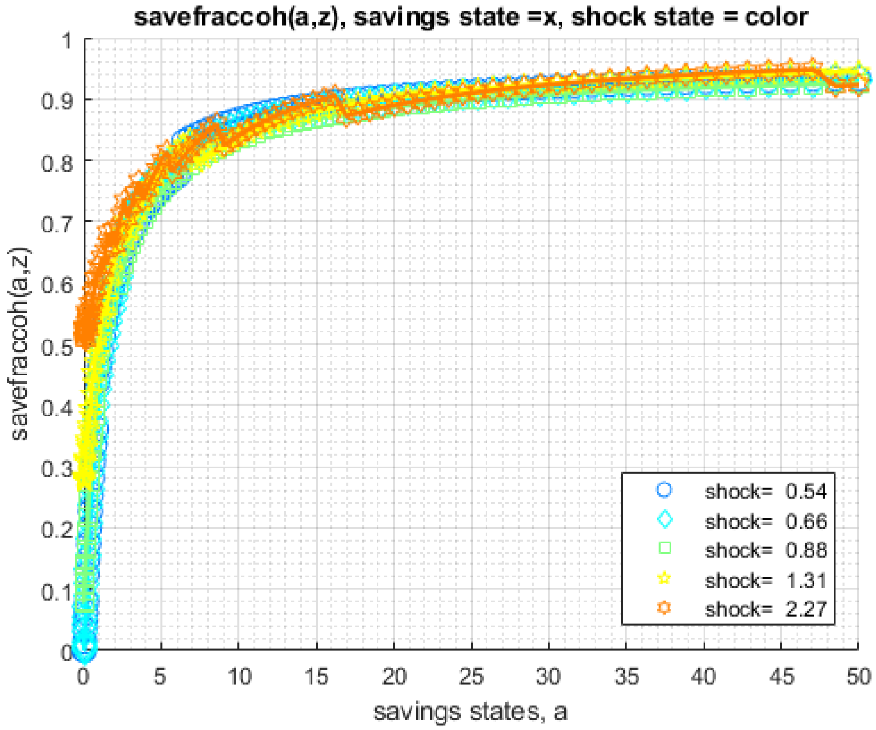
```
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i     idx    ndim     numel     rowN     colN     sum       mean       std       coef
                   -     ---    ----     -----     ----     ----    ------    -------    -------    ----

    savefraccoh    1      1       2       500      100       5      268.82    0.53764    0.29852    0.55

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2         c3          c4          c5
            -------    -------    -------    --------    -------

    r1            0          0          0    0.015741    0.18847
    r2            0          0          0     0.01574    0.18847
    r3            0          0          0    0.015737    0.18844
    r4            0          0          0    0.015728    0.18838
    r5            0          0          0    0.022367    0.18825
    r96     0.84455    0.84169    0.83664     0.85445    0.83255
    r97     0.84611    0.84333    0.83842     0.85626    0.83496
    r98     0.84763    0.84493    0.84016     0.85803    0.83729
    r99     0.84911    0.84648    0.84185     0.85974    0.83956
    r100    0.85055      0.848    0.84349     0.86141    0.84176
```

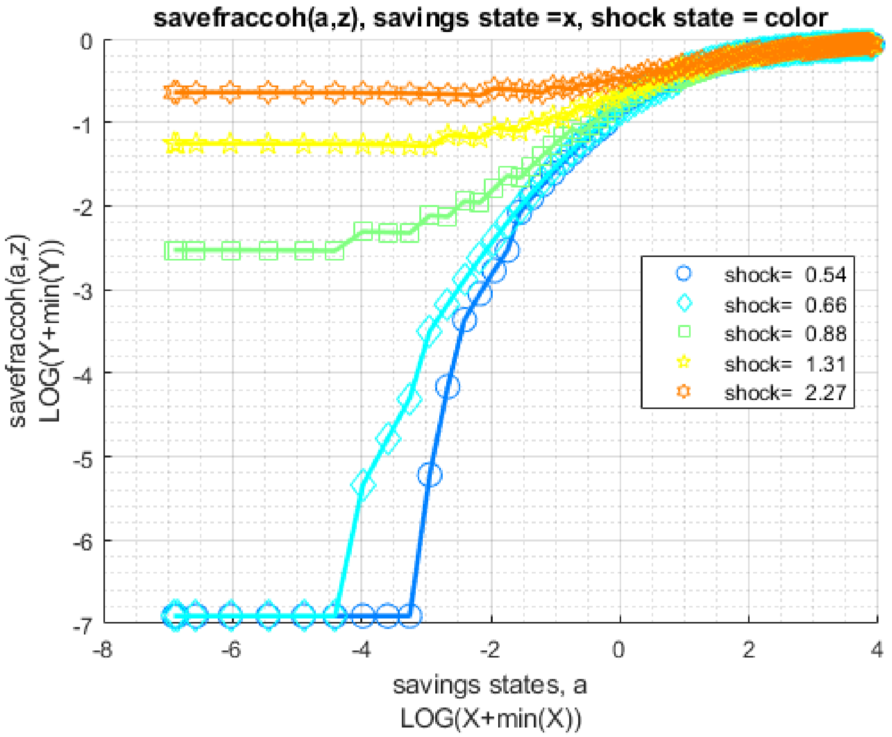





When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.937495 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                   _    ___    ____    _____    ____    ____    ______    _______    _______    ____

    savefraccoh    1     1      2       500      100      5     335.64    0.67129    0.28688    0.42

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2          c3          c4         c5
            _______    _______    ________    _______    _______

    r1            0          0    0.078907    0.28472    0.52731
    r2            0          0    0.078904    0.28471     0.5273
    r3            0          0    0.078878    0.28465    0.52723
    r4            0          0    0.078808    0.28448    0.52705
    r5            0          0    0.078672    0.28415    0.52669
    r96     0.93086    0.92771     0.92215    0.94079    0.94593
    r97     0.93161    0.92855     0.92315    0.94183    0.94739
    r98     0.93233    0.92936     0.92411    0.94283     0.9488
    r99     0.93303    0.93015     0.92505    0.94379    0.92164
    r100    0.93371    0.93091     0.92595    0.94471    0.92317
```




### 1.1.6 Test FF_VFI_AZ_LOOP with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.957457 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum      mean       std       coefva
                   -    ---    ----    -----    ----    ----    -----    ------    -------    ------

    savefraccoh    1     1       2      500      100      5     294.1    0.5882    0.32083    0.5454

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2         c3          c4         c5
            -------    -------    -------    --------    -------

    r1            0          0          0    0.034556    0.11424
    r2            0          0          0    0.034555    0.11424
    r3            0          0          0    0.034546    0.11422
    r4            0          0          0    0.034523    0.11416
    r5            0          0          0    0.034478    0.11404
    r96     0.89673    0.89421    0.91986     0.91499    0.90808
    r97     0.89789    0.89545    0.92093      0.9162    0.90948
    r98     0.89903    0.89665    0.92196     0.91737    0.91084
    r99     0.90013    0.89782    0.92295      0.9185    0.91215
    r100    0.90119    0.89896    0.92392     0.91959    0.91342
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.923630 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1       2      500      100      5     350.37    0.70073    0.26741    0.38

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2          c3          c4         c5
            -------    -------    --------    -------    -------

    r1            0          0    0.030722    0.36969    0.77072
    r2            0          0     0.03072    0.36967    0.77071
```

```
r3           0          0      0.0307    0.36958    0.77068
r4           0          0    0.030646    0.36933     0.7706
r5           0          0    0.030543    0.36885    0.77044
r96    0.90975    0.90819      0.9038    0.91513    0.88687
r97    0.91053    0.90902     0.90476    0.91633    0.89076
r98    0.91129    0.90982     0.90569     0.9175    0.86794
r99    0.91204    0.91061      0.9066    0.91862    0.84583
r100   0.91276    0.91138     0.90748    0.91971    0.82439
```

## 1.2 FF_VFI_AZ_VEC Dynamic Savings Problem Vectorized Common Grid

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_vfi_az_vec** from the **MEconTools Package.** This function solves (vectorized) the dynamic programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon.

The code uses common grid, with the same state space and choice space grids. **ff_vfi_az_bisec_vec** from the **MEconTools Package** solves the same problem but using continuous exact percentage asset choices, which is more precise than the solution here, and perhaps a little bit slower.

This is the vectorized code, its speed is much faster than the looped code. The function is designed to have small memory footprint and requires low computing resources, yet is fast.

**Links to Four Code:**

Four Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid : **ff_vfi_az_loop**, slow should use for testing new models
- Common Choice and States Grid : **ff_vfi_az_vec**, fast good for many purposes
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand :**ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand : **ff_vfi_az_bisec_vec**, precision and speed

### 1.2.1 Test FF_VFI_AZ_VEC Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
ff_vfi_az_vec(mp_params);
```

```
Elapsed time is 0.128918 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN    sum      mean      std     coefvari    min
          -    ---    ----    -----    ----    ----    -----    ------    -----   --------    ---

    ap    1     1      2       700     100      7      16864    24.091    14.08    0.58446     0
```

```
xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c6         c7
            -------    -------    -------    -------    -------    -------    ------

    r1            0          0          0          0          0    0.50505    2.0202
    r2            0          0          0    0.50505    0.50505     1.0101    2.5253
    r3      0.50505    0.50505    0.50505    0.50505     1.0101     1.5152    3.0303
    r4       1.0101     1.0101     1.0101     1.0101     1.5152     2.0202    3.5354
    r5       1.5152     1.5152     1.5152     1.5152     2.0202     2.5253    4.0404
    r96      45.455     45.455      45.96      45.96      45.96      46.97    48.485
    r97       45.96      45.96      45.96     46.465     46.465     47.475     48.99
    r98      46.465     46.465     46.465      46.97      46.97      47.98     48.99
    r99       46.97      46.97      46.97     47.475     47.475     48.485    49.495
    r100     47.475     47.475     47.475      47.98      47.98      48.99        50
```

### 1.2.2 Test FF_VFI_AZ_BISEC_VEC Speed Tests

Call the function with different a and z grid size, print out speed:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
```

A grid 200, shock grid 9:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 200;
mp_params('it_z_n') = 9;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.220867 seconds.
```

A grid 750, shock grid 15:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 15;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 3.573648 seconds.
```

A grid 600, shock grid 45:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 600;
mp_params('it_z_n') = 45;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 8.398580 seconds.
```

### 1.2.3 Test FF_VFI_AZ_VEC Control Outputs

Run the function first without any outputs;

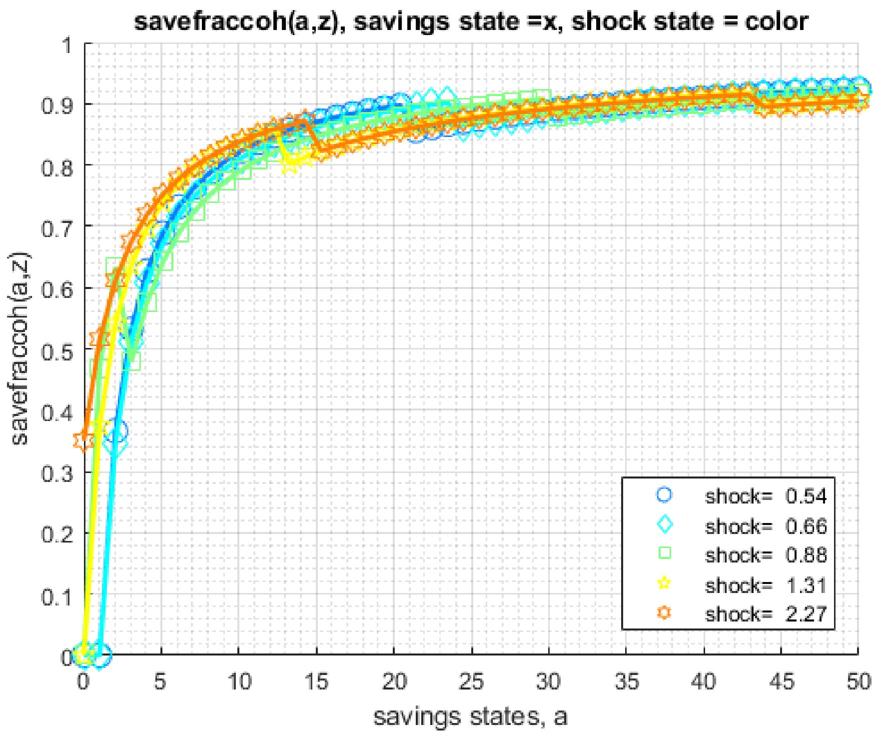
```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = false;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
```

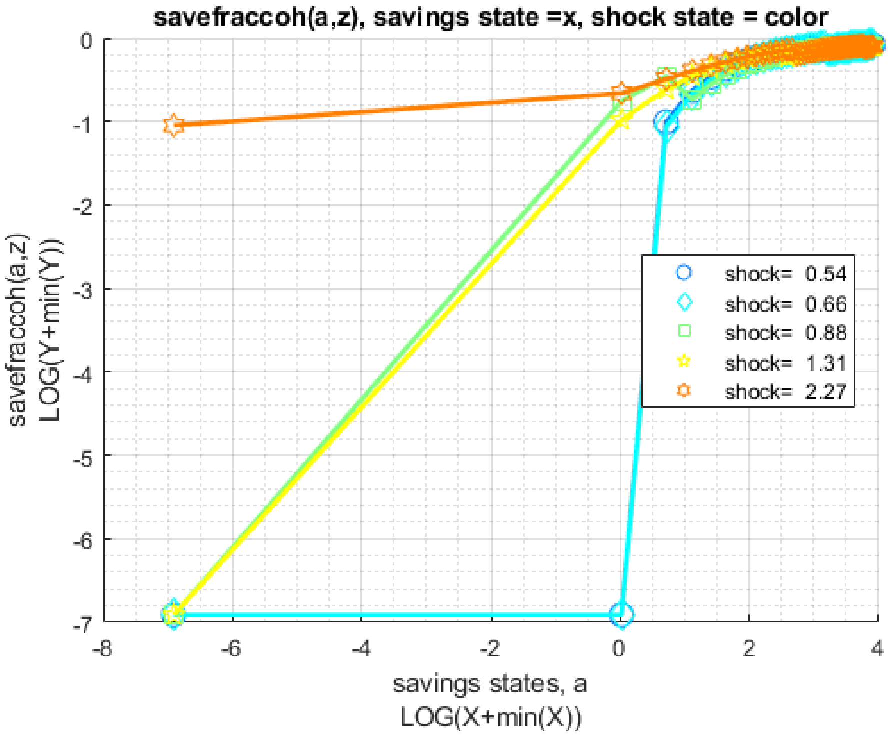
Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: ‘v’, ‘ap’, ‘c’, ‘y’, ‘coh’, ‘savefraccoh’. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
ff_vfi_az_vec(mp_params, mp_support);
```

```
Elapsed time is 0.014484 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group      a       mean_z_0_54195    mean_z_0_66401    mean_z_0_88162    mean_z_1_3095    mean_z
    -----    ------    --------------    --------------    --------------    -------------    ------

      1          0            0                 0                 0                0            0.
      2     1.0204            0                 0           0.46928          0.37487          0.5
      3     2.0408      0.36632           0.34687           0.63373          0.54163          0.6
      4     3.0612      0.53265           0.51178           0.47837          0.63592          0.6
      5     4.0816      0.62764           0.60816           0.57627          0.69655          0.7
      6      5.102      0.68908           0.67137           0.64196          0.73882          0.7
      7     6.1224      0.73208           0.71603           0.68909          0.76996          0.7
      8     7.1429      0.76386           0.74926           0.72456          0.79387          0.7
      9     8.1633       0.7883           0.77494           0.75221          0.81279          0.8
     10     9.1837      0.80769           0.79539           0.77438          0.82815          0.8
     11     10.204      0.82343           0.81206           0.79254          0.84086          0.
     12     11.224      0.83648           0.82591            0.8077          0.85155          0.8
     13     12.245      0.84747           0.83759           0.82053          0.86067          0.8
     14     13.265      0.85685           0.84758           0.83155          0.80173          0.8
     15     14.286      0.86495           0.85622            0.8411          0.81288          0.8
     16     15.306      0.87201           0.86377           0.84947          0.82268          0.8
     17     16.327      0.87823           0.87043           0.85685          0.83137          0.8
     18     17.347      0.88374           0.87633           0.86342          0.83912          0.8
     19     18.367      0.88866           0.88161            0.8693          0.84608          0.8
     20     19.388      0.89309           0.88635            0.8746          0.85237          0.8
     21     20.408      0.89708           0.89064           0.87939          0.85807          0.8
     22     21.429      0.85567           0.89454           0.88375          0.86327          0.8
     23     22.449      0.86096           0.89809           0.88773          0.86803          0.8
     24     23.469      0.86581           0.90135           0.89138          0.87241          0.8
     25      24.49      0.87026           0.86502           0.89474          0.87644          0.8
     26      25.51      0.87436            0.8693           0.89784          0.88017          0.8
     27     26.531      0.87816           0.87327           0.90071          0.88362          0.8
     28     27.551      0.88168           0.87695           0.90338          0.88684          0.8
     29     28.571      0.88496           0.88037           0.90586          0.88984          0.8
     30     29.592      0.88802           0.88357           0.90818          0.89264          0.8
     31     30.612      0.89087           0.88655           0.87896          0.89527          0.8
     32     31.633      0.89355           0.88935           0.88197          0.89773          0.8
     33     32.653      0.89606           0.89198            0.8848          0.90005          0.8
     34     33.673      0.89843           0.89446           0.88747          0.90223          0.8
     35     34.694      0.90065           0.89679           0.88998          0.90429          0.9
     36     35.714      0.90275           0.89899           0.89235          0.90624          0.9
     37     36.735      0.90474           0.90107            0.8946          0.90809          0.9
     38     37.755      0.90662           0.90304           0.89673          0.90984          0.9
     39     38.776      0.90841           0.90491           0.89874           0.9115          0.9
```

| | | | | | | |
|---|---|---|---|---|---|---|
| 40 | 39.796 | 0.9101 | 0.90669 | 0.90066 | 0.91308 | 0.9 |
| 41 | 40.816 | 0.91171 | 0.90838 | 0.90249 | 0.91458 | 0.9 |
| 42 | 41.837 | 0.91325 | 0.90998 | 0.90422 | 0.91601 | 0.9 |
| 43 | 42.857 | 0.91471 | 0.91152 | 0.90588 | 0.91738 | 0.9 |
| 44 | 43.878 | 0.9161 | 0.91298 | 0.90746 | 0.89681 | 0.8 |
| 45 | 44.898 | 0.91743 | 0.91438 | 0.90897 | 0.89854 | 0.8 |
| 46 | 45.918 | 0.91871 | 0.91571 | 0.91042 | 0.90019 | 0.8 |
| 47 | 46.939 | 0.91993 | 0.91699 | 0.91181 | 0.90178 | 0.8 |
| 48 | 47.959 | 0.9211 | 0.91822 | 0.91313 | 0.9033 | 0.9 |
| 49 | 48.98 | 0.92222 | 0.91939 | 0.91441 | 0.90475 | 0.9 |
| 50 | 50 | 0.92329 | 0.92052 | 0.91563 | 0.90615 | 0.9 |






Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_vec(mp_params, mp_support);
```

```
Elapsed time is 0.127807 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    ap             1     1      2       900      100      9     21825      24.25     14.089      0.
    savefraccoh    2     2      2       900      100      9     752.38    0.83597    0.13497    0.16

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
                c1         c2         c3         c4         c5         c6         c7         c8
             -------    -------    -------    -------    -------    -------    -------    ------    -

    r1             0          0          0          0          0          0    0.50505    1.5152    3
    r2             0          0          0          0    0.50505    0.50505     1.0101    1.5152    3
    r3       0.50505    0.50505    0.50505    0.50505    0.50505     1.0101     1.5152    2.0202    4
    r4        1.0101     1.0101     1.0101     1.0101     1.0101     1.5152     2.0202    2.5253    4
    r5        1.5152     1.5152     1.5152     1.5152     1.5152     2.0202     2.5253    3.0303    5
    r96       45.455     45.455     45.455      45.96      45.96      45.96     46.465    47.475    4
    r97        45.96      45.96      45.96     46.465     46.465     46.465      46.97     47.98    4
    r98       46.465     46.465     46.465     46.465      46.97      46.97     47.475    48.485
    r99        46.97      46.97      46.97      46.97     47.475     47.475      47.98     48.99
```

```
    r100     47.475     47.475     47.475     47.475      47.98      47.98     48.485    49.495

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c6         c7         c8
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0          0    0.24587    0.48182
    r2            0          0          0          0     0.3075    0.25444    0.39276    0.41371
    r3      0.30679    0.29486    0.27938    0.25939     0.2338    0.40362    0.49043     0.4833
    r4       0.4668    0.45285    0.43438    0.40981    0.37721    0.50166    0.56006    0.53755
    r5      0.56502    0.55132    0.53293    0.50802    0.47415    0.57101    0.61221    0.58103
    r96     0.91292     0.9117    0.90997    0.91752    0.91364    0.90746    0.90692    0.90732
    r97     0.91357    0.91236    0.91064    0.91812    0.91427    0.90815    0.90761    0.90799
    r98      0.9142      0.913     0.9113    0.90882    0.91489    0.90882    0.90828    0.90865
    r99     0.91482    0.91363    0.91195    0.90949    0.91549    0.90949    0.90894    0.90929
    r100    0.91543    0.91425    0.91258    0.91014    0.91609    0.91013    0.90959    0.90992
```

### 1.2.4 Test FF_VFI_AZ_VEC Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.771613 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2      6750     750      9      3318.8    0.49167    0.27768    0.56

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c6          c7          c8
            -------    -------    -------    -------    -------    -------    --------    -------

    r1            0          0          0          0          0          0    0.023475    0.13289
    r2            0          0          0          0          0          0    0.023475    0.13289
    r3            0          0          0          0          0          0    0.023475    0.13289
    r4            0          0          0          0          0          0    0.023475    0.13289
    r5            0          0          0          0          0          0    0.023475    0.13289
    r746     0.8044    0.80333    0.80182    0.79961    0.79626    0.79093      0.7887     0.7824
```

```
    r747    0.80465    0.80359    0.80209    0.79989    0.79655    0.79124     0.78903    0.78277
    r748    0.80491    0.80385    0.80235    0.80016    0.79683    0.79154     0.78936    0.78315
    r749    0.80517    0.80411    0.80262    0.80043    0.79712    0.79185     0.78969    0.78352
    r750    0.80542    0.80437    0.80288    0.80071     0.7974    0.79215     0.79002    0.78389
```

```
% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_vec(mp_params, mp_support);
```

```
Elapsed time is 2.484993 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2      6750     750      9      4491.9    0.66547    0.28771    0.43

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4          c5         c6         c7         c8
            -------    -------    -------    -------    --------    -------    -------    -------

    r1            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r2            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r3            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r4            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r5            0          0          0          0    0.031818    0.14726    0.31047    0.48484
    r746    0.92742       0.93     0.9283    0.92581     0.92578    0.92349    0.92443    0.91686
    r747     0.9275    0.93007    0.92838     0.9259     0.92588    0.92361    0.92457    0.91706
    r748    0.92757    0.93014    0.92846    0.92599     0.92598    0.92373    0.92472    0.91359
    r749    0.92764    0.93022    0.92854    0.92608     0.92608    0.92384    0.92115    0.91014
    r750    0.92772    0.93029    0.92862    0.92617     0.92618    0.92396     0.9213    0.90671
```

### 1.2.5 Test FF_VFI_AZ_VEC Changing Risk Aversion

Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

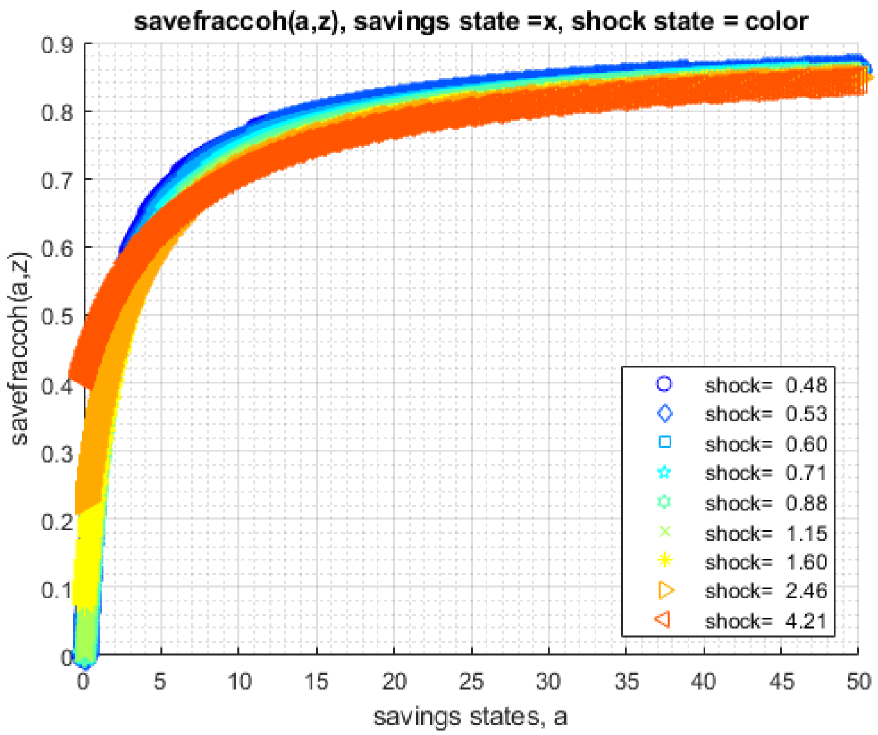
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

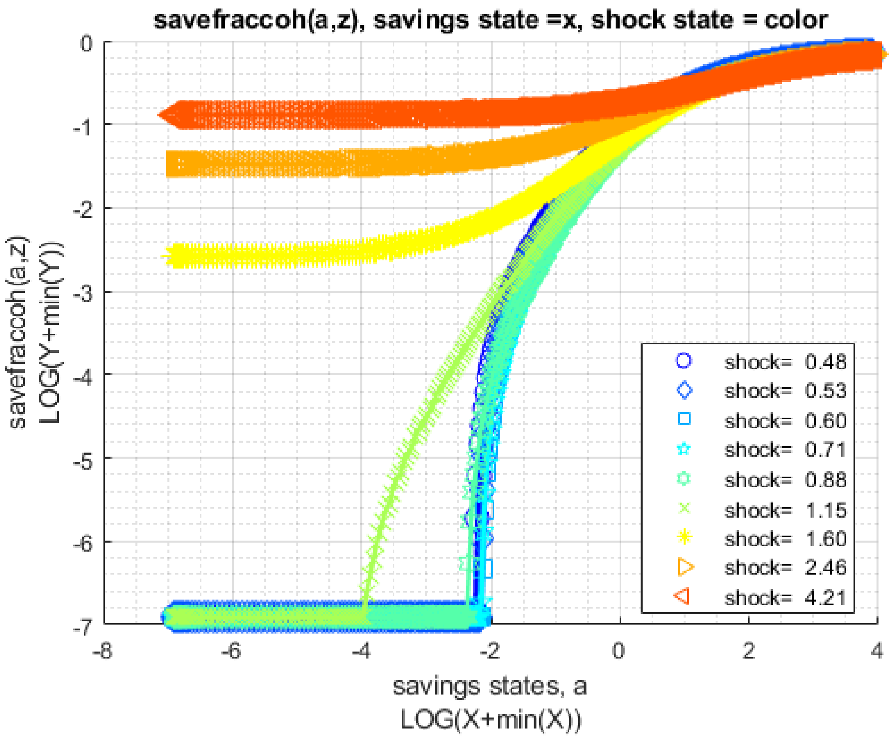
Solve the model with different risk aversion levels, higher preferences for risk:

```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_vec(mp_params, mp_support);
```

```
Elapsed time is 1.991475 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

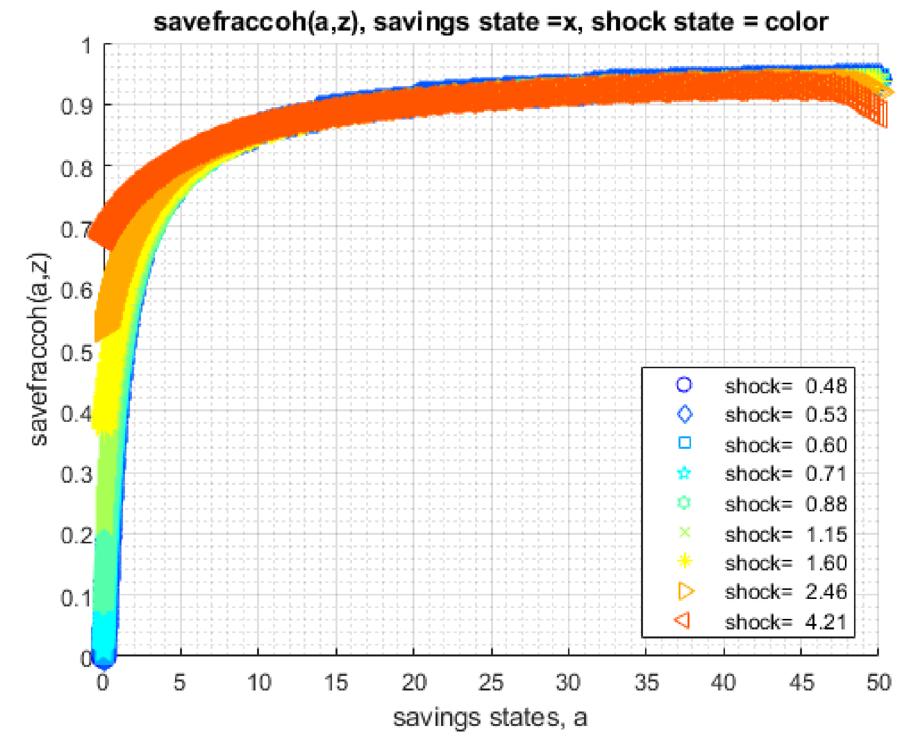
```
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean       std      coefv
                   -    ---    ----    -----    ----    ----    ------    -------    ------    -----

    savefraccoh    1     1      2      6750     750      9      3735.9    0.55347    0.2897    0.523

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
               c1         c2         c3         c4         c5         c6          c7          c8
             -------    -------    -------    -------    -------    -------    --------    -------

    r1             0          0          0          0          0          0    0.075021    0.22812
    r2             0          0          0          0          0          0    0.075021    0.22812
    r3             0          0          0          0          0          0    0.075021    0.22812
    r4             0          0          0          0          0          0    0.075021    0.22812
    r5             0          0          0          0          0          0    0.075021    0.22812
    r746     0.85928    0.85816    0.85657    0.85425    0.85428     0.8522     0.84972    0.84635
    r747     0.85946    0.85834    0.85676    0.85444    0.85449    0.85242     0.84997    0.84665
    r748     0.85963    0.85852    0.85694    0.85464    0.85469    0.85264     0.85021    0.84694
    r749     0.85981     0.8587    0.85713    0.85483    0.85489    0.85286     0.85046    0.84723
    r750     0.85998    0.85888    0.85731    0.85502    0.85509    0.85307      0.8507    0.84752
```

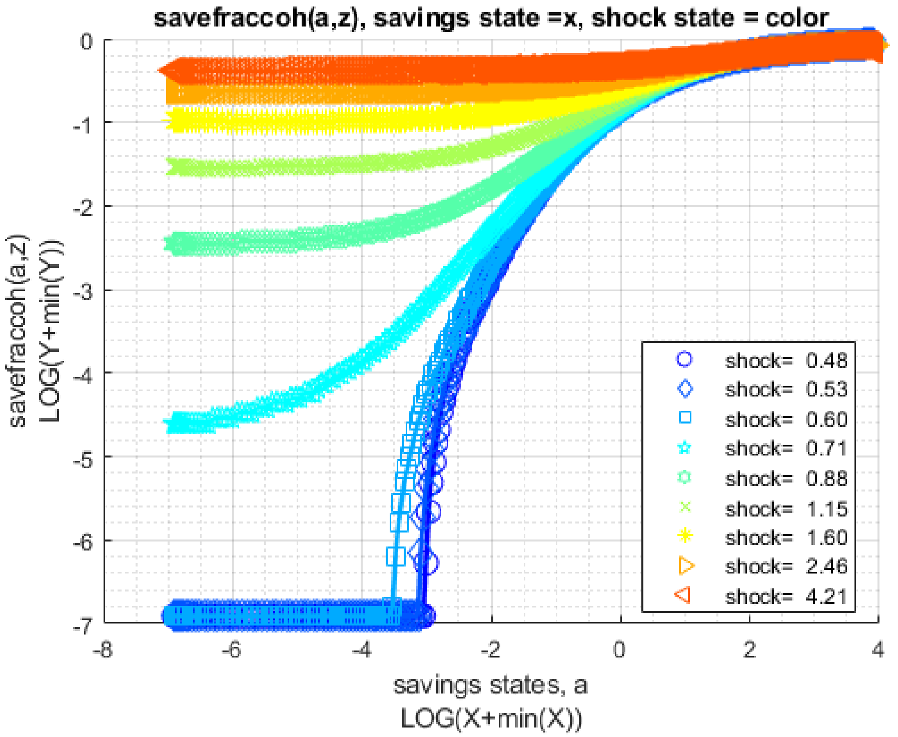





When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 2.026442 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim     numel     rowN     colN      sum        mean        std        coefv
                   _    ___    ____     _____     ____     ____     ______     ______     _______     -----

    savefraccoh    1     1       2      6750      750        9      4639.3     0.6873     0.28204     0.410

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
                 c1          c2          c3           c4            c5          c6          c7          c8
              _______     _______     _______     _________     _________     _______     _______     _______

    r1              0           0           0      0.008995      0.085095     0.21314     0.37277     0.53628
    r2              0           0           0      0.008995      0.085095     0.21314     0.37277     0.53628
    r3              0           0           0      0.008995      0.085095     0.21314     0.37277     0.53628
    r4              0           0           0      0.008995      0.085095     0.21314     0.37277     0.53628
    r5              0           0           0     0.0089949      0.085094     0.21314     0.37277     0.53628
    r746      0.94083      0.9396     0.94168       0.93912       0.93904     0.94041     0.93743     0.92949
    r747      0.94091     0.93969     0.94176       0.93921       0.93914     0.93674     0.93758     0.92969
    r748      0.94098     0.93977     0.94184       0.93931       0.93924     0.93686     0.93772     0.92618
    r749      0.94106     0.93985     0.94192        0.9394       0.93934     0.93699     0.93787     0.92269
    r750      0.94113     0.93993       0.942       0.93949       0.93944     0.93711     0.93801     0.91921
```




### 1.2.6 Test FF_VFI_AZ_VEC with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_vec(mp_params, mp_support);
```

```
Elapsed time is 2.065989 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN    sum      mean        std       coefva
                   -    ---    ----    -----    ----    ----    ----    -------    -------    ------

    savefraccoh    1     1      2      6750     750      9      4026    0.59644    0.31533    0.5286

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5          c6          c7          c8
            -------    -------    -------    -------    -------    --------    --------    -------

    r1            0          0          0          0          0    0.012569    0.062884    0.13754
    r2            0          0          0          0          0    0.012569    0.062884    0.13754
    r3            0          0          0          0          0    0.012569    0.062884    0.13754
    r4            0          0          0          0          0    0.012569    0.062884    0.13754
    r5            0          0          0          0          0    0.012569    0.062884    0.13754
    r746    0.91375    0.91251    0.91101    0.91289    0.91057     0.91138     0.91136    0.91021
    r747    0.91387    0.91264    0.91114    0.91302    0.91072     0.91153     0.91152    0.91039
    r748    0.91399    0.91277    0.91127    0.91315    0.91086     0.91168     0.91168    0.91056
    r749    0.91411    0.91289     0.9114    0.91329      0.911     0.91183     0.91183    0.91073
    r750    0.91423    0.91302    0.91153    0.91342    0.91114     0.91197     0.91199     0.9109
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_vec(mp_params, mp_support);
```

```
Elapsed time is 2.184888 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2      6750     750      9      4687.4    0.69442    0.27109    0.39

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4          c5          c6         c7         c8
            -------    -------    -------    -------    --------    -------    -------    -------

    r1            0          0          0          0    0.030619    0.24561    0.55369    0.80189
    r2            0          0          0          0    0.030619    0.24561    0.55369    0.80189
```

```
r3            0          0          0          0    0.030619     0.2456    0.55369    0.80189
r4            0          0          0          0    0.030619     0.2456    0.55369    0.80189
r5            0          0          0          0    0.030618     0.2456    0.55369    0.80189
r746    0.93365    0.93335     0.9328    0.93173     0.92941    0.92713    0.92079     0.8402
r747    0.93371    0.93341    0.93286     0.9318     0.92949    0.92723    0.92095    0.83734
r748    0.93378    0.93348    0.93293    0.93187     0.92957    0.92733    0.92111    0.83449
r749    0.93384    0.93354      0.933    0.93194     0.92965    0.92743    0.92127    0.83166
r750     0.9339     0.9336    0.93306    0.93201     0.92973    0.92753    0.92143    0.82883
```

## 1.3 FF_VFI_AZ_BISEC_LOOP Dynamic Savings Problem Loop Continuous Choice

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function:**ff_vfi_az_bisec_loop**from the **MEconTools Package.** This function solves the dynamic programming problem for a (a,z) model. Households can save (face some borrowing constraint), and face AR(1) shock z. The problem is solved over the infinite horizon. This is the looped code, it is slow for larger state-space problems. The code uses continuous choices, solved with bisection. The state-space is on a grid, but choice grids are in terms of percentage of resources to save and solved exactly.

**Links to Four Code:**

Four Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid : **ff_vfi_az_loop**, slow should use for testing new models
- Common Choice and States Grid : **ff_vfi_az_vec**, fast good for many purposes
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand :**ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand : **ff_vfi_az_bisec_vec**, precision and speed

### 1.3.1 Test FF_VFI_AZ_BISEC_LOOP Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_bisec_loop(mp_params);

Elapsed time is 31.520504 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum      mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    -----    ------    ------    --------    ---

    ap    1     1       2      700      100      7     16866    24.094    14.071    0.58399      0

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
```

|  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
|  | ------- | ------- | ------- | ------- | ------- | ------- | ------ |
| r1 | 0 | 0 | 0 | 0 | 0.13188 | 0.66203 | 1.9859 |
| r2 | 0.25914 | 0.26426 | 0.29511 | 0.39221 | 0.57697 | 1.1208 | 2.4569 |
| r3 | 0.65371 | 0.66543 | 0.70966 | 0.82502 | 1.029 | 1.582 | 2.9298 |
| r4 | 1.0748 | 1.0921 | 1.1447 | 1.2698 | 1.5151 | 2.0481 | 3.4046 |
| r5 | 1.5152 | 1.5319 | 1.5903 | 1.721 | 2.0011 | 2.5252 | 3.8802 |
| r96 | 45.561 | 45.615 | 45.712 | 45.887 | 46.192 | 46.835 | 48.252 |
| r97 | 46.049 | 46.104 | 46.201 | 46.377 | 46.681 | 47.325 | 48.743 |
| r98 | 46.54 | 46.593 | 46.69 | 46.866 | 47.171 | 47.815 | 49.235 |
| r99 | 47.029 | 47.082 | 47.179 | 47.356 | 47.661 | 48.304 | 49.734 |
| r100 | 47.518 | 47.572 | 47.67 | 47.845 | 48.15 | 48.793 | 50.252 |

### 1.3.2 Test FF_VFI_AZ_BISEC_LOOP Speed Tests

Call the function with different a and z grid size, print out speed:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
```

A grid 50, shock grid 5:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_bisec_loop(mp_params, mp_support);
```

```
Elapsed time is 9.142315 seconds.
```

A grid 100, shock grid 7:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
ff_vfi_az_bisec_loop(mp_params, mp_support);
```

```
Elapsed time is 26.910198 seconds.
```

A grid 200, shock grid 9:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 200;
mp_params('it_z_n') = 9;
ff_vfi_az_bisec_loop(mp_params, mp_support);
```

```
Elapsed time is 74.127079 seconds.
```

### 1.3.3 Test FF_VFI_AZ_BISEC_LOOP Control Outputs

Run the function first without any outputs;

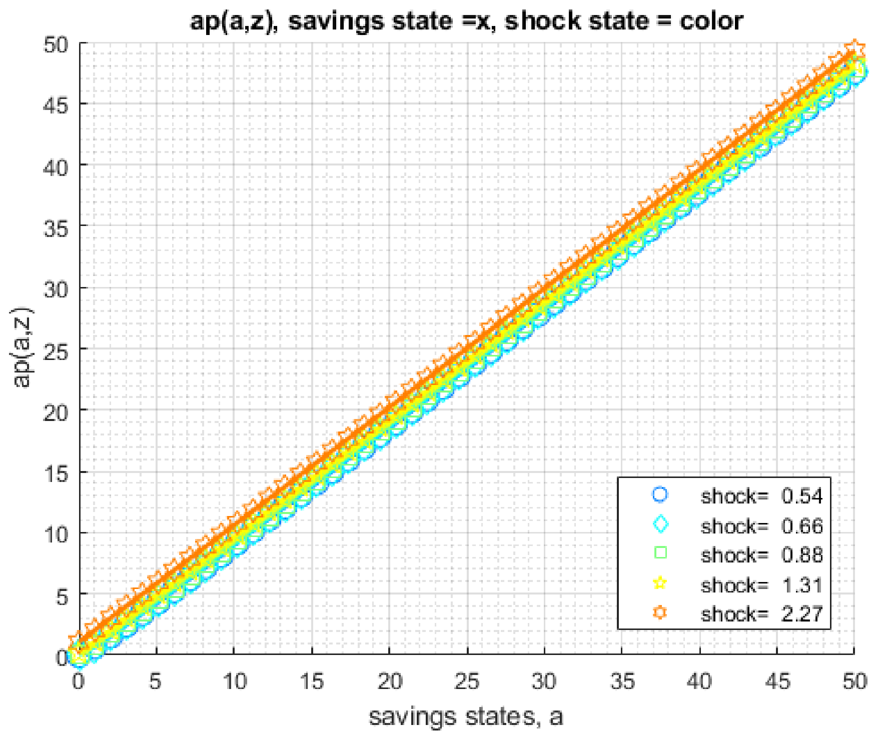
```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = false;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
```

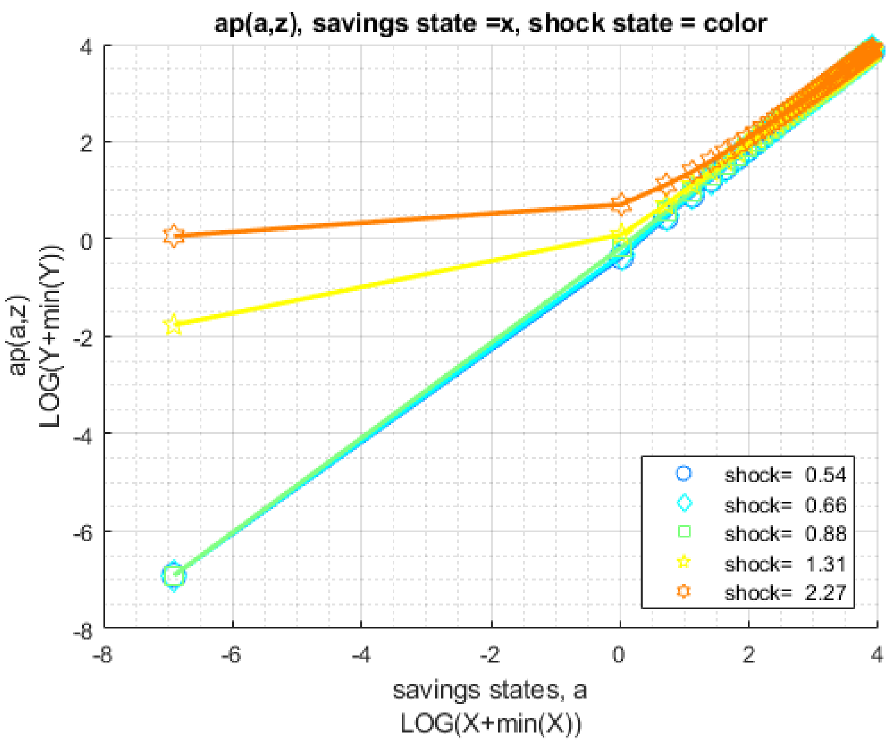
Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'ap'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'ap'};
ff_vfi_az_bisec_loop(mp_params, mp_support);
```

```
Elapsed time is 9.221215 seconds.
xxx  ff_vfi_az_vec, outcome=ap  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group      a       mean_z_0_54195    mean_z_0_66401    mean_z_0_88162    mean_z_1_3095    mean_z
    -----    ------    --------------    --------------    --------------    -------------    ------

      1           0              0                 0                 0          0.17037        1.0
      2      1.0204        0.69295           0.72405           0.84423            1.099        2.0
      3      2.0408         1.5665            1.6143            1.7589           2.0408        3.0
      4      3.0612         2.4757             2.534            2.6903           3.0612        4.0
      5      4.0816         3.4036            3.4686            3.6319           4.0274        4.9
      6       5.102         4.3431            4.4131            4.5822           4.9848        5.9
      7      6.1224         5.2908            5.3648            5.5384           5.9466        6.9
      8      7.1429         6.2445            6.3216            6.4989           6.9116        7.8
      9      8.1633         7.2029            7.2827             7.463           7.8793        8.8
     10      9.1837         8.1651            8.2469            8.4301           8.8495        9.8
     11      10.204         9.1837            9.2139            9.3992           9.8212        10.
     12      11.224         10.175            10.204             10.37           10.795        11.
     13      12.245         11.141            11.225            11.343           11.769        12.
     14      13.265         12.109            12.198            12.316           12.745        13.
     15      14.286         13.079            13.169            13.291           13.721         14
     16      15.306         14.051            14.142            14.286           14.699          1
     17      16.327         15.026            15.117            15.306           15.677         16
     18      17.347         16.001            16.093            16.289           16.655         17
     19      18.367         16.978            17.069            17.265           17.634        18.
     20      19.388         17.955            18.048            18.243           18.614        19.
     21      20.408         18.934            19.026            19.223           19.595        20.
     22      21.429         19.913            20.006            20.203           20.577        21.
     23      22.449         20.894            20.986            21.184           21.559        22.
     24      23.469         21.875            21.968            22.166           22.542        23.
     25       24.49         22.856             22.95            23.148           23.525        24.
     26       25.51         23.838            23.932            24.131           24.509        25.
     27      26.531         24.821            24.915            25.114            25.51        26.
     28      27.551         25.804            25.899            26.098           26.531        27.
     29      28.571         26.788            26.883            27.082           27.524        28.
     30      29.592         27.772            27.867            28.067           28.507        29.
     31      30.612         28.757            28.852            29.052           29.492        30.
     32      31.633         29.742            29.837            30.037           30.477        31.
     33      32.653         30.727            30.822            31.023           31.463        32.
     34      33.673         31.712            31.808            32.009           32.449         33
     35      34.694         32.698            32.795            32.995           33.435        34.
     36      35.714         33.685            33.781            33.982           34.422        35.
     37      36.735         34.694            34.768            34.969            35.41        36.
     38      37.755         35.714            35.755            35.955           36.397        37.
     39      38.776         36.703            36.741            36.942           37.385        38.
     40      39.796         37.689            37.755             37.93           38.372        39.
     41      40.816         38.676            38.774            38.918           39.361        40.
     42      41.837         39.663            39.761            39.906           40.349        41.
```

| | | | | | | |
|---|---|---|---|---|---|---|
| 43 | 42.857 | 40.65 | 40.749 | 40.894 | 41.337 | 42. |
| 44 | 43.878 | 41.637 | 41.736 | 41.881 | 42.326 | 43. |
| 45 | 44.898 | 42.624 | 42.724 | 42.87 | 43.314 | 44. |
| 46 | 45.918 | 43.612 | 43.711 | 43.877 | 44.303 | 45. |
| 47 | 46.939 | 44.6 | 44.7 | 44.898 | 45.293 | 46. |
| 48 | 47.959 | 45.589 | 45.688 | 45.893 | 46.281 | 47. |
| 49 | 48.98 | 46.577 | 46.676 | 46.881 | 47.27 | 4 |
| 50 | 50 | 47.566 | 47.664 | 47.87 | 48.259 | 49. |




Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_bisec_loop(mp_params, mp_support);
```

```
Elapsed time is 35.765637 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean       std      coefv
                   -    ---    ----    -----    ----    ----    ------    -------    ------    -----

    ap             1     1      2       900      100      9      21835     24.261    14.095    0.580
    savefraccoh    2     2      2       900      100      9     754.27    0.83808    0.1259    0.150

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5          c6          c7         c8
            -------    -------    -------    -------    -------    --------    -------    ------

    r1            0          0          0          0          0    0.082559    0.50504    1.2988
    r2      0.26067    0.25936    0.26888    0.30308    0.39296     0.52492    0.96211    1.7672
    r3      0.65383    0.65589    0.67297    0.71974    0.82473      1.0101     1.4185    2.2377
    r4       1.0734     1.0789     1.1015     1.1556     1.2679      1.4909     1.8821    2.7095
    r5       1.5151     1.5159     1.5427     1.6019       1.72      1.9489      2.349    3.1825
    r96      45.547      45.58     45.636      45.73     45.888      46.134     46.603     47.52
    r97      46.036     46.069     46.126      46.22     46.377      46.622     47.092    48.009
    r98      46.525     46.559     46.615      46.71     46.867      47.112     47.583    48.501
    r99      47.014     47.049     47.104     47.198     47.357      47.601     48.072    48.992
    r100     47.503     47.537     47.593     47.687     47.845      48.091     48.561    49.495

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5          c6          c7         c8
            -------    -------    -------    -------    -------    --------    -------    -------

    r1            0          0          0          0          0    0.056268    0.24587    0.41301
    r2      0.23098      0.217    0.20843    0.21203    0.23925     0.26445     0.3741    0.48253
    r3      0.39717    0.38292    0.37227    0.36965    0.38179     0.40361    0.45915    0.53532
    r4      0.49605    0.48369    0.47368    0.46883    0.47347     0.49364    0.52177    0.57677
    r5      0.56502    0.55159    0.54262    0.53709    0.53825     0.55086    0.56947    0.61021
    r96     0.91477    0.91422    0.91361    0.91294    0.91221      0.9109    0.90961    0.90818
    r97     0.91508    0.91453    0.91395    0.91328    0.91254     0.91123    0.90998    0.90855
    r98     0.91538    0.91486    0.91425    0.91361    0.91288     0.91157    0.91035    0.90894
    r99     0.91569    0.91517    0.91456    0.91392    0.91322      0.9119    0.91068    0.90934
    r100    0.91596    0.91544    0.91486    0.91422    0.91352     0.91224    0.91102    0.90992
```

### 1.3.4 Test FF_VFI_AZ_BISEC_LOOP Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_bisec_loop(mp_params, mp_support);
```

```
Elapsed time is 2.672239 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2       250      50      5      119.77    0.47907    0.28808    0.60

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
             c1         c2         c3         c4         c5
           -------    -------    -------    -------    -------

    r1           0          0          0          0    0.10641
    r2           0          0          0          0    0.10641
    r3           0          0          0          0    0.10629
    r4           0          0          0          0    0.10601
    r5           0          0          0          0    0.10793
    r46    0.79096    0.78788    0.78242    0.77293    0.76768
    r47    0.79554    0.79261    0.78749    0.77754    0.77168
    r48    0.79991    0.79716    0.79228    0.78291     0.7754
    r49    0.80406    0.80146    0.79688      0.788    0.77891
    r50    0.80699    0.80396     0.8003    0.79283    0.78218
```

```
% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_bisec_loop(mp_params, mp_support);
```

```
Elapsed time is 11.445094 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2       250      50      5      162.74    0.65097    0.29744    0.45

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
             c1         c2          c3         c4         c5
           -------    -------    --------    -------    -------

    r1           0          0    0.029138    0.21236    0.45384
    r2           0          0    0.029535    0.21258     0.4539
```

```
    r3          0          0     0.03219    0.21401    0.45448
    r4          0          0    0.039301    0.21795    0.45607
    r5          0          0    0.045923    0.22542    0.45909
    r46    0.9221    0.92124     0.92029    0.91929    0.91587
    r47   0.92408    0.92329     0.92237    0.92142    0.91816
    r48   0.92591    0.92518     0.92432    0.92344    0.92057
    r49   0.92762    0.92692     0.92612    0.92536    0.92347
    r50   0.92924     0.9286     0.92792    0.92737    0.92783
```

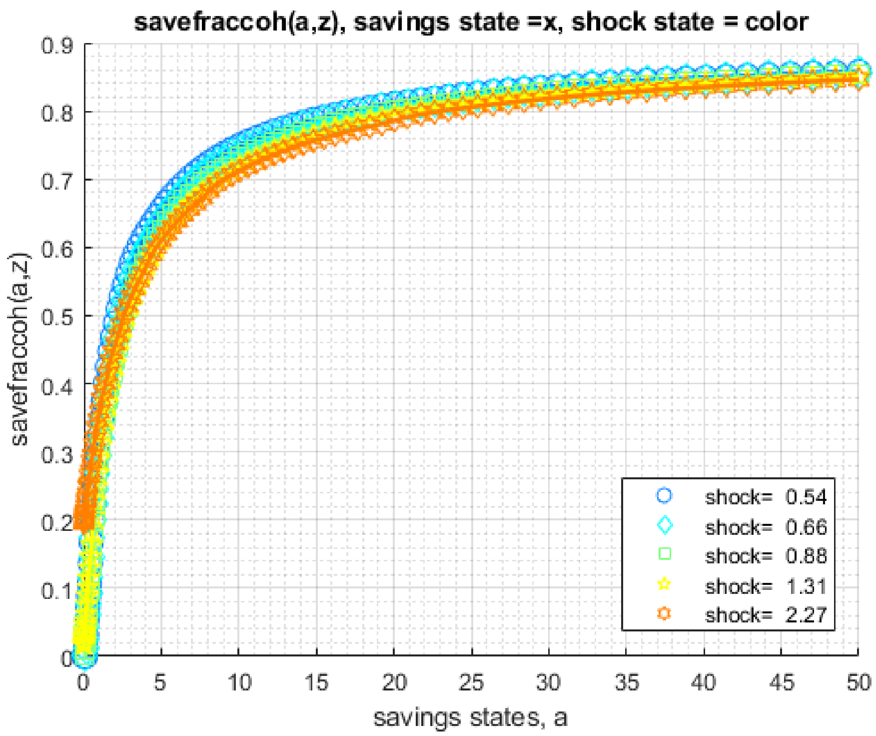
### 1.3.5 Test FF_VFI_AZ_BISEC_LOOP Changing Risk Aversion

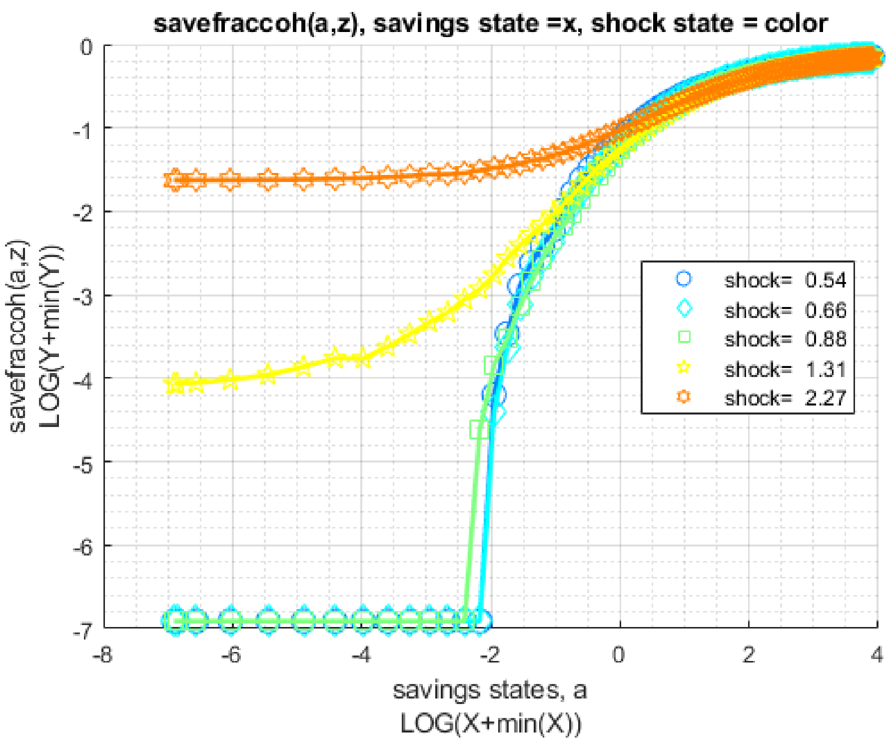
Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 20.016228 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                  i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                  -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh   1     1      2       500      100      5     270.24    0.54048    0.30018     0.5

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
             c1         c2         c3          c4         c5
           -------    -------    -------    --------    -------

    r1           0          0          0    0.016108    0.19552
    r2           0          0          0    0.016138    0.19555
    r3           0          0          0    0.016352    0.19564
    r4           0          0          0    0.016962    0.19591
    r5           0          0          0    0.018091    0.19646
    r96    0.85334    0.85127    0.84879     0.84602    0.84037
    r97    0.85456    0.85255    0.85017     0.84748     0.8419
    r98    0.85578    0.85383    0.85151     0.84889    0.84339
    r99    0.85694    0.85502    0.85279     0.85023    0.84483
    r100   0.85804    0.85621    0.85404     0.85154    0.84623
```



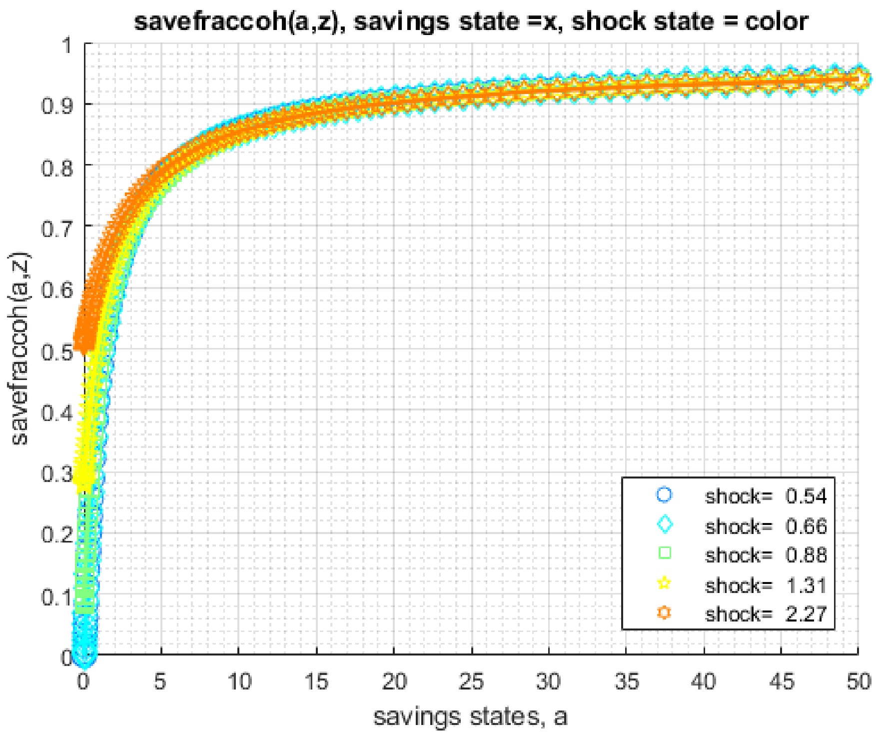



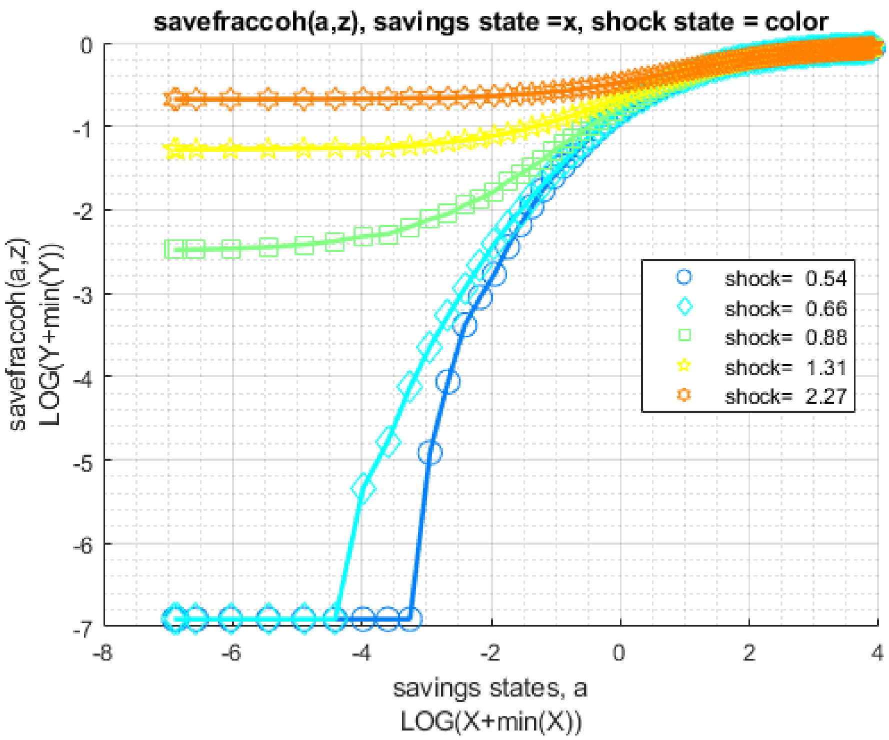

When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_bisec_loop(mp_params, mp_support);
```

```
Elapsed time is 19.070686 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
```

```
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean       std       coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2       500      100      5     337.39    0.67477    0.28798    0.42

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1          c2           c3          c4          c5
            -------     -------     --------     -------     -------

    r1            0           0     0.082635      0.2781      0.5078
    r2            0           0     0.082665     0.27813      0.5078
    r3            0           0      0.08297     0.27828     0.50786
    r4            0           0     0.083794     0.27871     0.50801
    r5            0           0     0.085381     0.27956     0.50835
    r96     0.93751     0.93699      0.93641     0.93586     0.93482
    r97     0.93839      0.9379      0.93732      0.9368     0.93586
    r98     0.93925     0.93876      0.93824     0.93775     0.93702
    r99      0.9401     0.93961      0.93909     0.93867     0.93839
    r100    0.94089     0.94044      0.93998     0.93961     0.94013
```






### 1.3.6 Test FF_VFI_AZ_BISEC_LOOP with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 19.744605 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                   _    ___    ____    _____    ____    ____    ______    _______    _______    ----

    savefraccoh    1     1      2       500     100      5     295.24    0.59048    0.32213    0.54

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          _______     _______     _______     ________     _______
```

```
    r1          0          0          0    0.030451    0.12142
    r2          0          0          0    0.030481    0.12145
    r3          0          0          0    0.030725    0.12164
    r4          0          0          0    0.031366    0.12209
    r5          0          0          0    0.032648    0.12304
    r96    0.90824    0.90775    0.90726     0.90675    0.90629
    r97    0.90943    0.90894    0.90845     0.90797    0.90751
    r98    0.91056     0.9101    0.90961     0.90916     0.9087
    r99    0.91166     0.9112    0.91074     0.91029    0.90983
    r100    0.9127    0.91227    0.91184     0.91138    0.91096
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 17.556461 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2       500     100      5      354.06    0.70811    0.27055    0.38

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2          c3          c4         c5
            -------    -------    ---------    -------    -------

    r1          0          0     0.030725    0.36968    0.77064
    r2          0          0     0.030725    0.36968    0.77064
    r3          0          0     0.030695    0.36959    0.77064
    r4          0          0     0.030634    0.36934    0.77061
    r5          0          0     0.030542    0.36885    0.77043
    r96    0.92429    0.92289      0.92091    0.91688    0.92026
    r97    0.92496    0.92362      0.92173    0.91795    0.92231
    r98    0.92564    0.92432      0.92252    0.91898    0.92429
    r99    0.92628    0.92503      0.92332    0.91999    0.92625
    r100   0.92689     0.9257      0.92408    0.92103    0.92811
```

## 1.4 FF_VFI_AZ_BISEC_VEC Dynamic Savings Problem Vectorized Continuous Exact

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_vfi_az_bisec_vec** from the **MEconTools Package.** This function solves the dynamic programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon. This is a vectorized code, it is much faster for larger state-space problems then looped code.

The code uses continuous choices, solved with bi(multi)section. The state-space is on a grid, but choice grids are in terms of percentage of resources available, which is individual specific, to save and solved exactly up to ((1/(2)^16)*100=0.001525878) percentage of cash on hand. THe **ff_vfi_az_vec** from the **MEconTools Package** solves the same problem using vectorized common grid code where the choice set and state space share the same grid.

This is the vectorized code, its speed is much faster than the looped code. The function is designed to have small memory footprint and requires low computing resources, yet is fast.

**Links to Four Code:**

Four Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid : **ff_vfi_az_loop**, slow should use for testing new models
- Common Choice and States Grid : **ff_vfi_az_vec**, fast good for many purposes
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand :**ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand : **ff_vfi_az_bisec_vec**, precision and speed

### 1.4.1 Test FF_VFI_AZ_BISEC_VEC Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_bisec_vec(mp_params);

Elapsed time is 0.668798 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN    sum      mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    -----    ------    ------    --------    ---

    ap    1     1      2       700     100      7     16866    24.094    14.071    0.58399      0

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c6        c7
            -------    -------    -------    -------    -------    -------    ------

    r1            0          0          0          0    0.13188    0.66203    1.9859
    r2      0.25914    0.26426    0.29511    0.39221    0.57697     1.1208    2.4569
    r3      0.65371    0.66543    0.70966    0.82502      1.029      1.582    2.9298
    r4       1.0748     1.0921     1.1447     1.2698     1.5151     2.0481    3.4046
    r5       1.5152     1.5319     1.5903      1.721     2.0011     2.5252    3.8802
    r96      45.561     45.615     45.712     45.887     46.192     46.835    48.252
    r97      46.049     46.104     46.201     46.377     46.681     47.325    48.743
    r98       46.54     46.593      46.69     46.866     47.171     47.815    49.235
    r99      47.029     47.082     47.179     47.356     47.661     48.304    49.734
    r100     47.518     47.572      47.67     47.845      48.15     48.793    50.252
```

### 1.4.2 Test FF_VFI_AZ_BISEC_VEC Speed Tests

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
```

```
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
```

A grid 50, shock grid 5:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 0.336083 seconds.
```

A grid 750, shock grid 15:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 15;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 23.612756 seconds.
```

A grid 600, shock grid 45:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 600;
mp_params('it_z_n') = 45;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 46.281741 seconds.
```

### 1.4.3 Test FF_VFI_AZ_BISEC_VEC Control Outputs

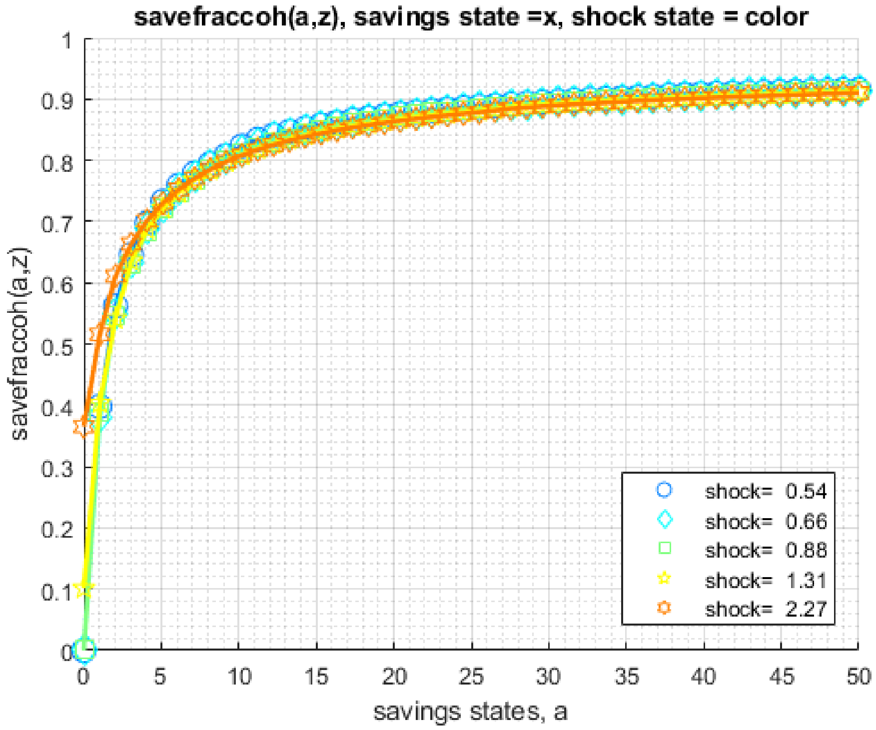
Run the function first without any outputs;

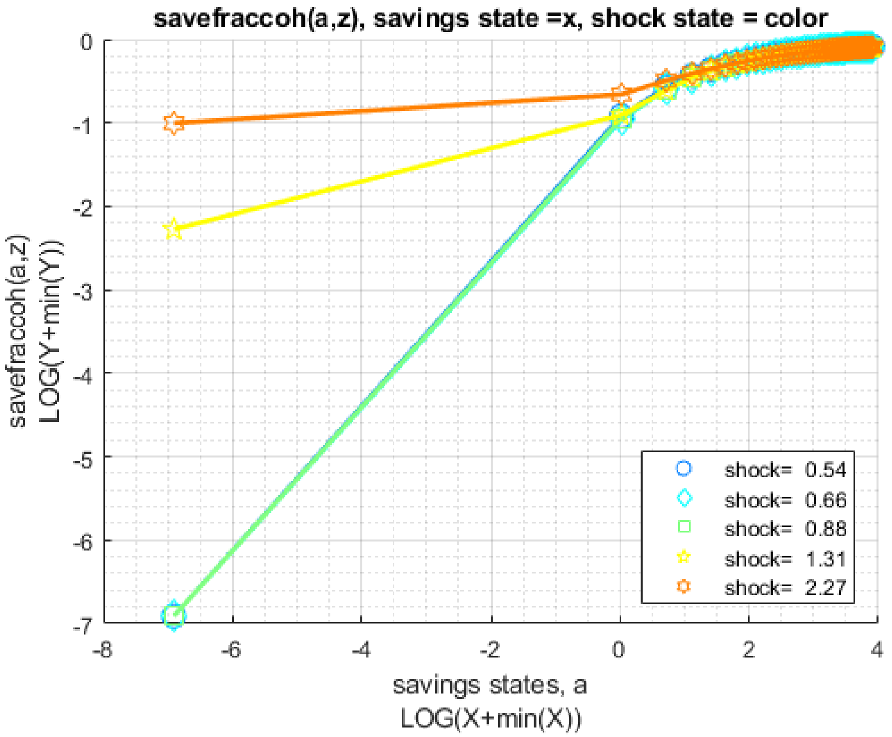
```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = false;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
```

Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: ‘v’, ‘ap’, ‘c’, ‘y’, ‘coh’, ‘savefraccoh’. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 0.470134 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group      a       mean_z_0_54195    mean_z_0_66401    mean_z_0_88162    mean_z_1_3095    mean_z
    -----    ------    --------------    --------------    --------------    -------------    ------

      1          0            0                 0                 0             0.10165         0.3
```

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 1.0204 | 0.39833 | 0.38191 | 0.38826 | 0.40373 | 0.5 |
| 3 | 2.0408 | 0.56236 | 0.54875 | 0.54619 | 0.54161 | 0.6 |
| 4 | 3.0612 | 0.64616 | 0.63545 | 0.6306 | 0.63591 | 0.6 |
| 5 | 4.0816 | 0.69783 | 0.6891 | 0.6837 | 0.6873 | 0.7 |
| 6 | 5.102 | 0.73323 | 0.7259 | 0.72068 | 0.72184 | 0. |
| 7 | 6.1224 | 0.75917 | 0.75291 | 0.74803 | 0.74784 | 0.7 |
| 8 | 7.1429 | 0.77909 | 0.77363 | 0.76911 | 0.76817 | 0.7 |
| 9 | 8.1633 | 0.79493 | 0.79011 | 0.78593 | 0.78452 | 0.7 |
| 10 | 9.1837 | 0.80787 | 0.80354 | 0.79969 | 0.79801 | 0.7 |
| 11 | 10.204 | 0.82343 | 0.81474 | 0.81114 | 0.8093 | 0.8 |
| 12 | 11.224 | 0.83412 | 0.82591 | 0.82084 | 0.81895 | 0.8 |
| 13 | 12.245 | 0.84116 | 0.83759 | 0.82917 | 0.82725 | 0.8 |
| 14 | 13.265 | 0.84733 | 0.84434 | 0.83637 | 0.83448 | 0.8 |
| 15 | 14.286 | 0.85282 | 0.84998 | 0.84272 | 0.84083 | 0.8 |
| 16 | 15.306 | 0.8577 | 0.85508 | 0.84947 | 0.84647 | 0.8 |
| 17 | 16.327 | 0.86213 | 0.85966 | 0.85685 | 0.85151 | 0.8 |
| 18 | 17.347 | 0.86613 | 0.86378 | 0.86143 | 0.856 | 0.8 |
| 19 | 18.367 | 0.86976 | 0.8675 | 0.86521 | 0.86008 | 0.8 |
| 20 | 19.388 | 0.87305 | 0.87092 | 0.86869 | 0.86384 | 0.8 |
| 21 | 20.408 | 0.87608 | 0.87403 | 0.8719 | 0.86726 | 0.8 |
| 22 | 21.429 | 0.87885 | 0.8769 | 0.87486 | 0.8704 | 0. |
| 23 | 22.449 | 0.88145 | 0.87955 | 0.8776 | 0.87333 | 0.8 |
| 24 | 23.469 | 0.88383 | 0.88203 | 0.88013 | 0.87601 | 0.8 |
| 25 | 24.49 | 0.88602 | 0.88432 | 0.88248 | 0.87852 | 0.8 |
| 26 | 25.51 | 0.8881 | 0.88645 | 0.88468 | 0.88087 | 0.8 |
| 27 | 26.531 | 0.89002 | 0.88844 | 0.88673 | 0.88361 | 0.8 |
| 28 | 27.551 | 0.89185 | 0.89033 | 0.88865 | 0.88685 | 0.8 |
| 29 | 28.571 | 0.89353 | 0.89207 | 0.89045 | 0.88895 | 0.8 |
| 30 | 29.592 | 0.89515 | 0.89371 | 0.89216 | 0.89063 | 0.8 |
| 31 | 30.612 | 0.89664 | 0.89527 | 0.89375 | 0.89225 | 0.8 |
| 32 | 31.633 | 0.89808 | 0.89674 | 0.89524 | 0.89378 | 0.8 |
| 33 | 32.653 | 0.89942 | 0.89811 | 0.89667 | 0.89521 | 0.8 |
| 34 | 33.673 | 0.90067 | 0.89942 | 0.89802 | 0.89658 | 0.8 |
| 35 | 34.694 | 0.90189 | 0.90067 | 0.8993 | 0.8979 | 0.8 |
| 36 | 35.714 | 0.90305 | 0.90186 | 0.90052 | 0.89915 | 0.8 |
| 37 | 36.735 | 0.90473 | 0.90299 | 0.90168 | 0.90034 | 0.8 |
| 38 | 37.755 | 0.90662 | 0.90406 | 0.90278 | 0.90147 | 0.8 |
| 39 | 38.776 | 0.90763 | 0.90507 | 0.90382 | 0.90256 | 0.9 |
| 40 | 39.796 | 0.90852 | 0.90668 | 0.90482 | 0.90357 | 0. |
| 41 | 40.816 | 0.90937 | 0.90833 | 0.9058 | 0.90458 | 0.9 |
| 42 | 41.837 | 0.91019 | 0.90919 | 0.90671 | 0.90552 | 0.9 |
| 43 | 42.857 | 0.91099 | 0.91001 | 0.9076 | 0.90641 | 0.9 |
| 44 | 43.878 | 0.91172 | 0.91077 | 0.90842 | 0.90729 | 0.9 |
| 45 | 44.898 | 0.91245 | 0.91154 | 0.90925 | 0.90812 | 0. |
| 46 | 45.918 | 0.91315 | 0.91224 | 0.91041 | 0.90891 | 0.9 |
| 47 | 46.939 | 0.91383 | 0.91294 | 0.91181 | 0.90971 | 0.9 |
| 48 | 47.959 | 0.9145 | 0.91361 | 0.91264 | 0.91044 | 0.9 |
| 49 | 48.98 | 0.91511 | 0.91425 | 0.91328 | 0.91114 | 0.9 |
| 50 | 50 | 0.91572 | 0.91486 | 0.91392 | 0.91181 | 0.9 |




Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

```
Elapsed time is 0.716025 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean       std      coefv
                   -    ---    ----    -----    ----    ----    ------    -------    ------    -----

    ap             1     1      2       900     100      9       21835     24.261    14.095    0.580
    savefraccoh    2     2      2       900     100      9      754.27    0.83808    0.1259    0.150

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5          c6          c7         c8
            -------    -------    -------    -------    -------    --------    -------    ------

    r1            0          0          0          0          0    0.082559    0.50504    1.2988
    r2      0.26067    0.25936    0.26888    0.30308    0.39296     0.52492    0.96211    1.7672
    r3      0.65383    0.65589    0.67297    0.71974    0.82473      1.0101     1.4185    2.2377
    r4       1.0734     1.0789     1.1015     1.1556     1.2679      1.4909     1.8821    2.7095
    r5       1.5151     1.5159     1.5427     1.6019       1.72      1.9489      2.349    3.1825
    r96      45.547      45.58     45.636      45.73     45.888      46.134     46.603     47.52
    r97      46.036     46.069     46.126      46.22     46.377      46.622     47.092    48.009
    r98      46.525     46.559     46.615      46.71     46.867      47.112     47.583    48.501
    r99      47.014     47.049     47.104     47.198     47.357      47.601     48.072    48.992
    r100     47.503     47.537     47.593     47.687     47.845      48.091     48.561    49.495

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5          c6          c7         c8
            -------    -------    -------    -------    -------    --------    -------    -------

    r1            0          0          0          0          0    0.056268    0.24587    0.41301
    r2      0.23098      0.217    0.20843    0.21203    0.23925     0.26445     0.3741    0.48253
    r3      0.39717    0.38292    0.37227    0.36965    0.38179     0.40361    0.45915    0.53532
    r4      0.49605    0.48369    0.47368    0.46883    0.47347     0.49364    0.52177    0.57677
    r5      0.56502    0.55159    0.54262    0.53709    0.53825     0.55086    0.56947    0.61021
    r96     0.91477    0.91422    0.91361    0.91294    0.91221      0.9109    0.90961    0.90818
    r97     0.91508    0.91453    0.91395    0.91328    0.91254     0.91123    0.90998    0.90855
    r98     0.91538    0.91486    0.91425    0.91361    0.91288     0.91157    0.91035    0.90894
    r99     0.91569    0.91517    0.91456    0.91392    0.91322      0.9119    0.91068    0.90934
    r100    0.91596    0.91544    0.91486    0.91422    0.91352     0.91224    0.91102    0.90992
```

### 1.4.4 Test FF_VFI_AZ_BISEC_VEC Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
```

```
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

```
Elapsed time is 4.541023 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

| | i | idx | ndim | numel | rowN | colN | sum | mean | std | coef |
|---|---|---|---|---|---|---|---|---|---|---|
| | - | --- | ---- | ----- | ---- | ---- | ------ | ------- | ------- | ---- |
| savefraccoh | 1 | 1 | 2 | 6750 | 750 | 9 | 3318.4 | 0.49162 | 0.27766 | 0.56 |

```
xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
```

| | c1 | c2 | c3 | c4 | c5 | c6 | c7 | c8 |
|---|---|---|---|---|---|---|---|---|
| | ------- | ------- | ------- | ------- | ------- | ------- | -------- | ------- |
| r1 | 0 | 0 | 0 | 0 | 0 | 0 | 0.023584 | 0.1329 |
| r2 | 0 | 0 | 0 | 0 | 0 | 0 | 0.023584 | 0.1329 |
| r3 | 0 | 0 | 0 | 0 | 0 | 0 | 0.023584 | 0.1329 |
| r4 | 0 | 0 | 0 | 0 | 0 | 0 | 0.023584 | 0.1329 |
| r5 | 0 | 0 | 0 | 0 | 0 | 0 | 0.023584 | 0.1329 |
| r746 | 0.80439 | 0.80299 | 0.80094 | 0.79856 | 0.79588 | 0.79222 | 0.78825 | 0.78263 |
| r747 | 0.80467 | 0.80314 | 0.8011 | 0.79875 | 0.79606 | 0.7924 | 0.78846 | 0.78285 |
| r748 | 0.80491 | 0.80329 | 0.80125 | 0.7989 | 0.79621 | 0.79255 | 0.78864 | 0.78315 |
| r749 | 0.80515 | 0.80341 | 0.80137 | 0.79905 | 0.7964 | 0.79273 | 0.78883 | 0.78352 |
| r750 | 0.80534 | 0.80357 | 0.80152 | 0.7992 | 0.79655 | 0.79292 | 0.78904 | 0.78388 |

```
% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

```
Elapsed time is 17.994960 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

| | i | idx | ndim | numel | rowN | colN | sum | mean | std | coef |
|---|---|---|---|---|---|---|---|---|---|---|
| | - | --- | ---- | ----- | ---- | ---- | ------ | ------- | ------- | ---- |
| savefraccoh | 1 | 1 | 2 | 6750 | 750 | 9 | 4493.5 | 0.66571 | 0.28784 | 0.43 |

```
xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
```

| | c1 | c2 | c3 | c4 | c5 | c6 | c7 | c8 |
|---|---|---|---|---|---|---|---|---|
| | ------- | ------- | ------- | ------- | -------- | ------- | ------- | ------- |
| r1 | 0 | 0 | 0 | 0 | 0.032007 | 0.15008 | 0.31087 | 0.48467 |
| r2 | 0 | 0 | 0 | 0 | 0.032007 | 0.15008 | 0.31087 | 0.48467 |
| r3 | 0 | 0 | 0 | 0 | 0.032007 | 0.15008 | 0.31087 | 0.48467 |
| r4 | 0 | 0 | 0 | 0 | 0.032007 | 0.15008 | 0.31087 | 0.48467 |
| r5 | 0 | 0 | 0 | 0 | 0.032007 | 0.15008 | 0.31087 | 0.48467 |
| r746 | 0.9289 | 0.9285 | 0.92805 | 0.92734 | 0.92664 | 0.92594 | 0.92515 | 0.92545 |
| r747 | 0.92902 | 0.9286 | 0.92814 | 0.92747 | 0.92673 | 0.92606 | 0.9253 | 0.92573 |
| r748 | 0.92911 | 0.92869 | 0.92826 | 0.92756 | 0.92686 | 0.92618 | 0.92545 | 0.926 |
| r749 | 0.92921 | 0.92881 | 0.92835 | 0.92768 | 0.92698 | 0.92631 | 0.92564 | 0.92631 |
| r750 | 0.9293 | 0.9289 | 0.92844 | 0.92777 | 0.92707 | 0.92643 | 0.92591 | 0.92658 |

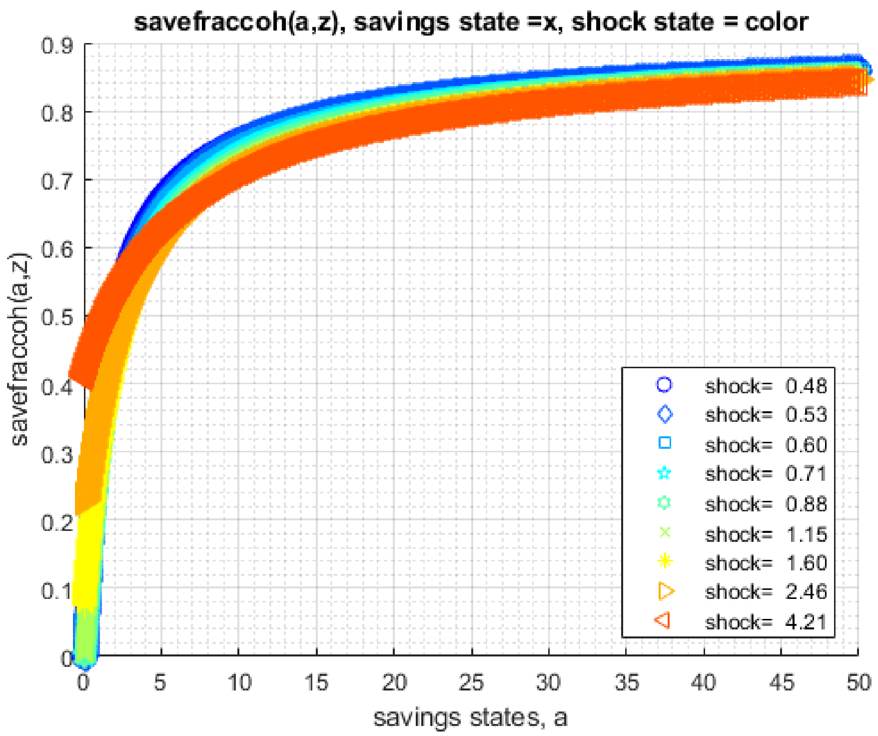
### 1.4.5 Test FF_VFI_AZ_BISEC_VEC Changing Risk Aversion

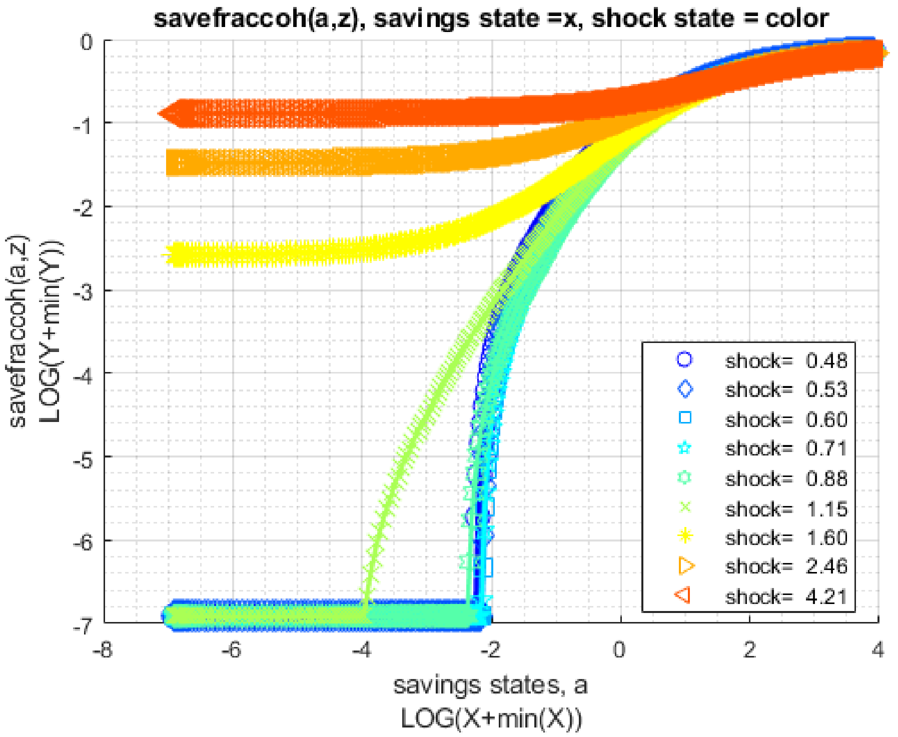
Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 13.815578 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                   -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh    1     1      2      6750     750      9      3735.7    0.55343    0.28972     0.5

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
             c1         c2         c3         c4         c5         c6          c7         c8
           -------    -------    -------    -------    -------    -------    --------    -------

    r1           0          0          0          0          0          0    0.074609    0.22661
    r2           0          0          0          0          0          0    0.074609    0.22661
    r3           0          0          0          0          0          0    0.074609    0.22661
    r4           0          0          0          0          0          0    0.074609    0.22661
    r5           0          0          0          0          0          0    0.074609    0.22664
    r746   0.85941    0.85828    0.85703     0.8556    0.85398    0.85178     0.84907    0.84583
    r747   0.85957    0.85844    0.85719    0.85575    0.85413    0.85194     0.84925    0.84602
    r748   0.85969    0.85859    0.85734     0.8559    0.85429    0.85212     0.84943     0.8462
    r749   0.85984    0.85871    0.85749    0.85606    0.85447    0.85227     0.84962    0.84638
    r750   0.85999    0.85889    0.85761    0.85621    0.85462    0.85246     0.84977    0.84657
```




When risk aversion increases, at every state-space point, the household wants to save more.

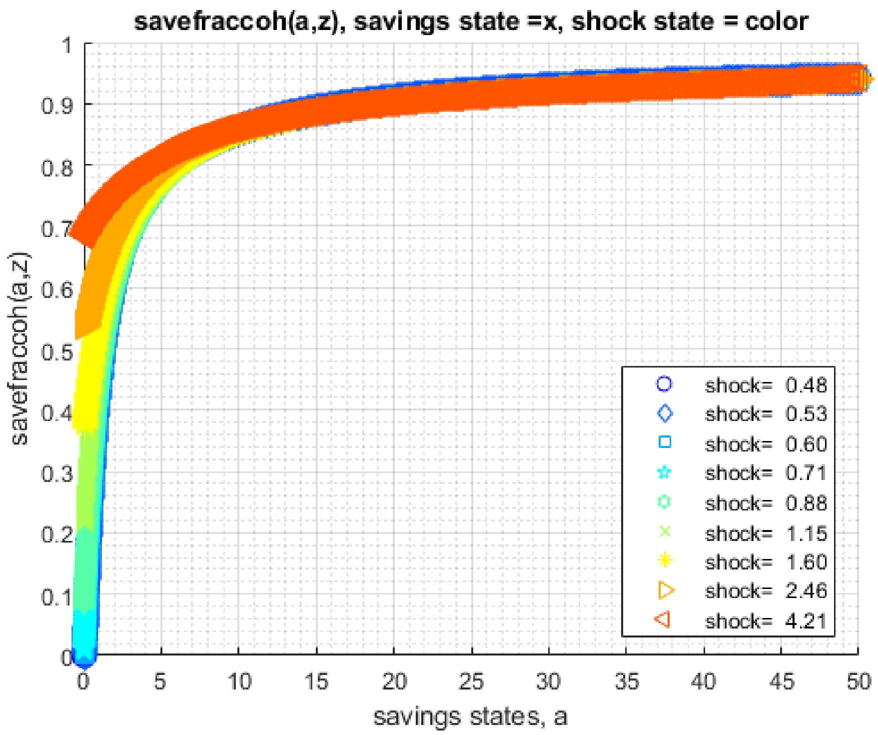
```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 13.688997 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
```

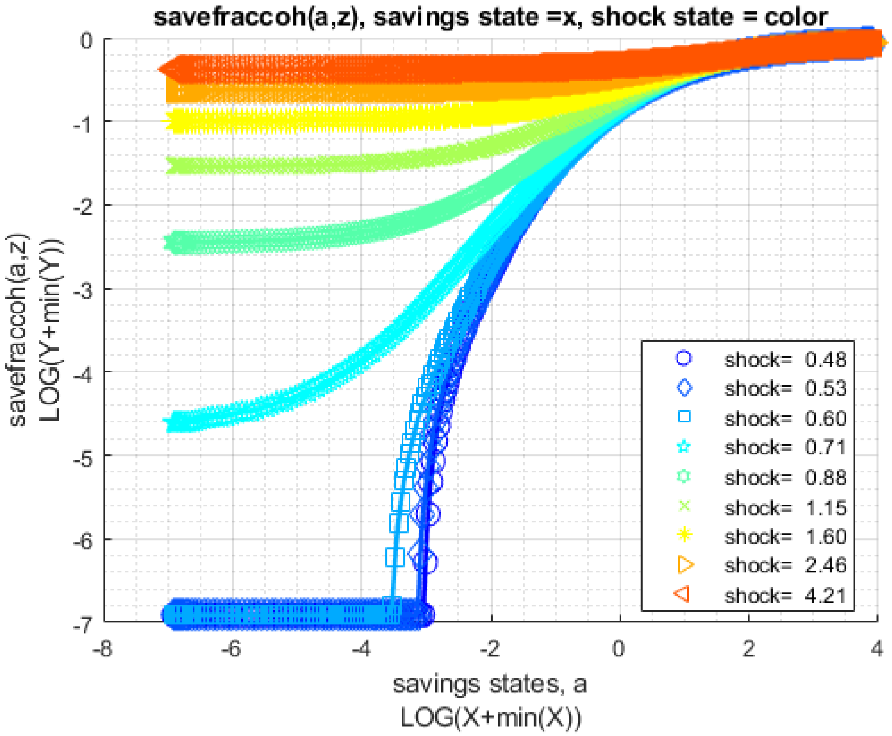
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

| | i | idx | ndim | numel | rowN | colN | sum | mean | std | coefvar |
|---|---|---|---|---|---|---|---|---|---|---|
| savefraccoh | 1 | 1 | 2 | 6750 | 750 | 9 | 4640 | 0.68741 | 0.2821 | 0.41039 |

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx

| | c1 | c2 | c3 | c4 | c5 | c6 | c7 | c8 |
|---|---|---|---|---|---|---|---|---|
| r1 | 0 | 0 | 0 | 0.0089972 | 0.085107 | 0.21343 | 0.37139 | 0.53578 |
| r2 | 0 | 0 | 0 | 0.0089972 | 0.085107 | 0.21343 | 0.37139 | 0.53578 |
| r3 | 0 | 0 | 0 | 0.0089972 | 0.085107 | 0.21343 | 0.37139 | 0.53578 |
| r4 | 0 | 0 | 0 | 0.0089972 | 0.085107 | 0.21343 | 0.37139 | 0.53578 |
| r5 | 0 | 0 | 0 | 0.0089972 | 0.085107 | 0.21343 | 0.37139 | 0.53578 |
| r746 | 0.94105 | 0.94074 | 0.94041 | 0.93992 | 0.93943 | 0.93897 | 0.93848 | 0.93885 |
| r747 | 0.94114 | 0.94083 | 0.94053 | 0.94004 | 0.93955 | 0.93909 | 0.93864 | 0.93909 |
| r748 | 0.94126 | 0.94095 | 0.94062 | 0.94016 | 0.93964 | 0.93922 | 0.93879 | 0.93931 |
| r749 | 0.94135 | 0.94105 | 0.94074 | 0.94025 | 0.93976 | 0.93934 | 0.93894 | 0.93955 |
| r750 | 0.94144 | 0.94117 | 0.94083 | 0.94038 | 0.93989 | 0.93946 | 0.93915 | 0.93976 |






### 1.4.6 Test FF_VFI_AZ_BISEC_VEC with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 14.533433 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum       mean       std      coefv
                   -    ---    ----    -----    ----    ----    ------    -------    ------    -----

    savefraccoh    1     1      2      6750     750      9      4025.4    0.59636    0.3153     0.52

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
            c1          c2          c3          c4          c5           c6           c7          c8
          -------     -------     -------     -------     -------     --------     --------     -------
```

```
    r1           0          0          0          0          0   0.012568   0.063073    0.13604
    r2           0          0          0          0          0   0.012568   0.063073    0.13604
    r3           0          0          0          0          0   0.012598   0.063073    0.13604
    r4           0          0          0          0          0   0.012598   0.063073    0.13604
    r5           0          0          0          0          0   0.012598   0.063073    0.13604
    r746   0.91276    0.91248    0.91196    0.91163    0.91111    0.91077    0.91025    0.90977
    r747   0.91291     0.9126    0.91209    0.91178    0.91126    0.91093    0.91041    0.90992
    r748   0.91303    0.91276    0.91224    0.91193    0.91138    0.91108    0.91056    0.91004
    r749   0.91318    0.91288    0.91236    0.91206    0.91154     0.9112    0.91068    0.91019
    r750   0.91331      0.913    0.91251    0.91221    0.91169    0.91135    0.91083    0.91035
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 14.829381 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                   i    idx    ndim    numel    rowN    colN     sum      mean        std      coef
                   -    ---    ----    -----    ----    ----    ------   -------    -------    ----

    savefraccoh    1     1      2      6750     750      9     5105.2    0.75633    0.26373     0.3

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
             c1         c2         c3         c4          c5         c6         c7         c8
          -------    -------    -------    -------    --------    -------    -------    -------

    r1           0          0          0          0    0.031641     0.2456    0.55455    0.80573
    r2           0          0          0          0    0.031641     0.2456    0.55455    0.80573
    r3           0          0          0          0    0.031641     0.2456    0.55455    0.80573
    r4           0          0          0          0    0.031671     0.2456    0.55455    0.80573
    r5           0          0          0          0    0.031671     0.2456    0.55455    0.80573
    r746   0.93336    0.93287    0.93226    0.93149      0.9303     0.9289    0.92725    0.93293
    r747   0.93342    0.93293    0.93232    0.93159      0.9304    0.92899    0.92737    0.93317
    r748   0.93348    0.93299    0.93241    0.93165     0.93046    0.92905     0.9275    0.93339
    r749   0.93354    0.93305    0.93247    0.93171     0.93055    0.92914    0.92762    0.93363
    r750    0.9336    0.93311    0.93253    0.93177     0.93061    0.92924    0.92771    0.93384
```

# Chapter 2

# Summarize Policy and Value

## 2.1 FF_SUMM_ND_ARRAY Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_summ_nd_array** from the **MEconTools Package.** This function summarizes policy and value functions over states.

### 2.1.1 Test FF_SUMM_ND_ARRAY Defaults

Call the function with defaults.

```
ff_summ_nd_array();
```

```
xxx  Summ over (a,z), condi age as cols, kids/marriage as rows  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    marry    kids    mean_age_18    mean_age_19    mean_age_20    mean_age_21
    _____    _____    ____    ___________    ___________    ___________    ___________

      1        0       1        0.52456        0.51689        0.48412        0.54526
      2        1       1        0.49355        0.52906         0.5583        0.47342
      3        0       2        0.49085        0.51315        0.45158        0.43201
      4        1       2        0.58096        0.50596        0.47985        0.58791
      5        0       3        0.57811         0.6068        0.55221        0.50677
      6        1       3        0.53023        0.49258        0.48728        0.43352
      7        0       4        0.50339        0.48449        0.53618        0.45993
      8        1       4        0.44418         0.5223        0.55657        0.48583
```

### 2.1.2 Test FF_SUMM_ND_ARRAY with Random 2 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random 2D dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(5,4);
bl_print_table = true;
ar_st_stats = ["mean"];
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'a', linspace(0,1,size(mn_polval,1))});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'z', linspace(-1,1,size(mn_polval,2))});
disp(mn_polval);
```

```
    0.6965    0.4231    0.3432    0.7380
    0.2861    0.9808    0.7290    0.1825
    0.2269    0.6848    0.4386    0.1755
    0.5513    0.4809    0.0597    0.5316
    0.7195    0.3921    0.3980    0.5318
```

Second, show the entire matrix (no labels):

```
it_aggd = 0;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim2    mean_vardim1_1    mean_vardim1_2    mean_vardim1_3    mean_vardim1_4    mean
    -----    -------    --------------    --------------    --------------    --------------    ----

      1         1          0.69647           0.28614           0.22685           0.55131         0
      2         2          0.42311           0.98076           0.68483           0.48093         0
      3         3          0.34318           0.72905           0.43857          0.059678         0
      4         4            0.738           0.18249           0.17545           0.53155         0
```

Third, rotate row and column, and now with labels:

```
it_aggd = 0;
bl_row = 1;
ar_permute = [2,1];
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group     a      mean_z__1    mean_z__0_33333    mean_z_0_33333    mean_z_1
    -----    ----    ---------    ---------------    --------------    --------

      1        0      0.69647        0.42311            0.34318          0.738
      2      0.25     0.28614        0.98076            0.72905        0.18249
      3       0.5     0.22685        0.68483            0.43857        0.17545
      4      0.75     0.55131        0.48093           0.059678        0.53155
      5        1      0.71947        0.39212            0.39804        0.53183
```

Fourth, dimension one as columns, average over dim 2:

```
it_aggd = 1;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x    mean_z__1    mean_z__0_33333    mean_z_0_33333    mean_z_1
    -----    -    ---------    ---------------    --------------    --------

      1      1     0.49605         0.59235            0.3937         0.43186
```

Fifth, dimension one as rows, average over dim 2:

```
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group      z        sum       mean        std      coefvari      min        max
```

```
    -----    --------    ------    -------    -------    --------    --------    -------

      1            -1    2.4802    0.49605    0.22895     2.1666      0.22685    0.71947
      2      -0.33333    2.9617    0.59235    0.24524     2.4154      0.39212    0.98076
      3       0.33333    1.9685     0.3937    0.23907     1.6468     0.059678    0.72905
      4             1    2.1593    0.43186    0.24575     1.7573      0.17545      0.738
```

Sixth, dimension two as rows, average over dim 1:

```
ar_permute = [2,1];
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);
```

```
xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group     a       sum       mean       std      coefvari      min         max
    -----    ----    ------    -------    -------    --------    --------    -------

      1         0    2.2007    0.55019    0.19636     2.8019      0.34318      0.738
      2      0.25    2.1784    0.54461    0.37514     1.4518      0.18249    0.98076
      3       0.5    1.5257    0.38143    0.23212     1.6432      0.17545    0.68483
      4      0.75    1.6235    0.40587    0.23269     1.7443     0.059678    0.55131
      5         1    2.0415    0.51036    0.15361     3.3226      0.39212    0.71947
```

### 2.1.3 Test FF_SUMM_ND_ARRAY with Random 6 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random ND dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(8,7,6,5,4,3);
bl_print_table = true;
ar_st_stats = ["mean"];
```

Second, summarize over the first four dimensions, row group others:

```
it_aggd = 4;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);
```

```
xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim5    vardim6     sum       mean        std      coefvari       min           max
    -----    -------    -------    ------    -------    -------    --------    ----------    -------

      1         1          1       836.78    0.49808    0.29255     1.7026     8.1888e-05    0.99964
      2         2          1       842.15    0.50128    0.28968     1.7305     6.7838e-05    0.99936
      3         3          1       831.45    0.49491    0.28851     1.7154     0.00091373    0.99989
      4         4          1        843.9    0.50232    0.28154     1.7842     0.00012471    0.99731
      5         1          2       838.99     0.4994     0.2911     1.7156     0.00029749    0.99938
      6         2          2       830.81    0.49453    0.28634     1.7271     0.00027113     0.9992
      7         3          2       832.59    0.49559    0.28682     1.7279     0.00035994    0.99936
      8         4          2       820.42    0.48835    0.29032     1.6821     0.00096259    0.99896
      9         1          3       870.56    0.51819    0.29111     1.7801      0.0010616    0.99951
     10         2          3       854.68    0.50874    0.28458     1.7877       0.001884    0.99965
     11         3          3       838.29    0.49898     0.2891      1.726      0.0019192    0.99945
     12         4          3       842.83    0.50169     0.2877     1.7438     0.00016871    0.99963
```

Third, summarize over the first four dimensions, column group 5th, and row group others:

```
it_aggd = 4;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ["sum"], it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim6    sum_vardim5_1    sum_vardim5_2    sum_vardim5_3    sum_vardim5_4
    _____    _______    _____________    _____________    _____________    _____________

      1         1          836.78           842.15           831.45            843.9
      2         2          838.99           830.81           832.59           820.42
      3         3          870.56           854.68           838.29           842.83
```

Fourth, summarize over the first five dimensions, column group 6th, no row groups:

```
it_aggd = 5;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ["mean", "std"], it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x    mean_vardim6_1    mean_vardim6_2    mean_vardim6_3    std_vardim6_1    std_vardim6
    _____    _    ______________    ______________    ______________    _____________    ___________

      1      1       0.49915           0.49447            0.5069           0.28805          0.28862
```

Fifth, summarize over all six dimensions, summary statistics over the entire dataframe:

```
it_aggd = 6;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x     sum      mean        std      coefvari       min           max
    _____    _    _____    _______    _______    ________    __________    _______

      1      1    10083    0.50017    0.28831     1.7349     6.7838e-05    0.99989
```

### 2.1.4 Test FF_SUMM_ND_ARRAY with Random 7 Dimensional Matrix with All Parameters

Given a random seven dimensional matrix, average over the 2nd, 4th and 5th dimensionals. Show as row groups the 3, 6 and 7th dimensions, and row groups the 1st dimension. Show Coefficient of Variation only.

```
st_title = "avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim.";
rng(123)
mn_polval = rand(3,10,2,10,10,2,3);
ar_permute = [2,4,5,1,3,6,7];
bl_print_table = true;
ar_st_stats = ["coefvari"];
it_aggd = 3; % mean over 3 dims
bl_row = 1; % one var for row group
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'age', [18, 19, 20]});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'savings', linspace(0,1,10)});
cl_mp_datasetdesc{3} = containers.Map({'name', 'labval'}, ...
    {'borrsave', [-1,+1]});
cl_mp_datasetdesc{4} = containers.Map({'name', 'labval'}, ...
    {'shocka', linspace(-5,5,10)});
cl_mp_datasetdesc{5} = containers.Map({'name', 'labval'}, ...
```

```
    {'shockb', linspace(-5,5,10)});
cl_mp_datasetdesc{6} = containers.Map({'name', 'labval'}, ...
    {'marry', [0,1]});
cl_mp_datasetdesc{7} = containers.Map({'name', 'labval'}, ...
    {'region', [1,2,3]});
% call function
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, cl_mp_datasetdes

xxx  avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim.  xxxxxxxxxxxxxxxxxxxxx
    group    borrsave    marry    region    cv_age_18    cv_age_19    cv_age_20
    _____    ________    _____    ______    _________    _________    _________

      1         -1         0        1        1.7607       1.7534       1.7065
      2          1         0        1        1.6566       1.7501       1.7042
      3         -1         1        1        1.6608       1.7658       1.7291
      4          1         1        1         1.756       1.7479       1.7606
      5         -1         0        2        1.7314       1.7506        1.786
      6          1         0        2        1.7347        1.728        1.738
      7         -1         1        2        1.7811        1.755       1.7568
      8          1         1        2        1.7445       1.7398       1.7746
      9         -1         0        3        1.7025       1.7286         1.69
     10          1         0        3          1.74       1.7549       1.7356
     11         -1         1        3        1.7147       1.7287       1.7341
     12          1         1        3        1.7919       1.7313       1.7452
```

# Chapter 3

# Distributional Analysis

## 3.1 FF_SIMU_STATS Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_simu_stats** from the **MEconTools Package.** This is a gate-way function that computes mean, percentiles, covariance etc between several variables.

### 3.1.1 Test FF_SIMU_STATS Defaults

Call the function with defaults.

```
ff_simu_stats();
```

```
xxx tb_outcomes: all stats xxx
    OriginalVariableNames     cl_mt_pol_a    cl_mt_pol_c
    ______________________    ___________    ___________

    {'mean'              }     -0.11081         8.8423
    {'sd'                }       4.1239         6.5845
    {'coefofvar'         }      -37.215        0.74466
    {'min'               }           -7        -6.3772
    {'max'               }            9         21.786
    {'pYis0'             }     0.064259              0
    {'pYls0'             }      0.54867       0.027329
    {'pYgr0'             }      0.38707        0.97267
    {'pYisMINY'          }     0.051764       0.015232
    {'pYisMAXY'          }     0.027329       0.046484
    {'p1'                }           -7        -6.3772
    {'p10'               }           -6        0.27238
    {'p25'               }           -3         5.2138
    {'p50'               }           -1         6.5321
    {'p75'               }            3         13.799
    {'p90'               }            5         16.887
    {'p99'               }            9         21.786
    {'fl_cov_cl_mt_pol_a'}       17.007        -22.084
    {'fl_cor_cl_mt_pol_a'}            1       -0.81327
    {'fl_cov_cl_mt_pol_c'}      -22.084         43.356
    {'fl_cor_cl_mt_pol_c'}     -0.81327              1
    {'fracByP1'          }       3.2699      -0.010985
    {'fracByP10'         }       5.9889      -0.013362
    {'fracByP25'         }       14.165       0.041007
    {'fracByP50'         }       16.208         0.1893
```

```
    {'fracByP75'        }       12.702         0.59539
    {'fracByP90'        }       6.6611          0.8307
    {'fracByP99'        }            1               1
```

### 3.1.2 Test FF_SIMU_STATS Four States-Points Matrix

Over some (a,z) states that is 3 by 3, c matrix, generate all stats

```
% Set Parameters
mt_x_of_s = [1, 2,  3.0;...
             3, 1,  1.5;...
             4, 3,  2.0];
mt_y_of_s = [2, -10, 9.0;...
             5, 1.1,3.0;...
             1, 3,  -1.5];
mt_z_of_s = [1.1, 2,3.3;...
             2.3, 1,1.5;...
             4, 2.5,2.0];
mp_cl_mt_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
% Mass
rng(123);
mt_f_of_s = rand(size(mt_x_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s, mp_cl_mt_xyz_of_s);
```

```
xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_mt_x_of_s    cl_mt_y_of_s    cl_mt_z_of_s
    _______________________    ____________    ____________    ____________

    {'mean'               }       2.0763          1.9323          2.0668
    {'sd'                 }       0.9071          5.2239          0.9042
    {'coefofvar'          }      0.43688          2.7034         0.43749
    {'min'                }            1             -10               1
    {'max'                }            4               9               4
    {'pYis0'              }            0               0               0
    {'pYls0'              }            0         0.20441               0
    {'pYgr0'              }            1         0.79559               1
    {'pYisMINY'           }      0.28039         0.10917         0.14247
    {'pYisMAXY'           }     0.044922         0.19422        0.044922
    {'p1'                 }            1             -10               1
    {'p10'                }            1             -10               1
    {'p25'                }            1             1.1             1.1
    {'p50'                }            2               2               2
    {'p75'                }            3               5             2.5
    {'p90'                }            3               9             3.3
    {'p99'                }            4               9               4
    {'fl_cov_cl_mt_x_of_s'}      0.82282           1.589         0.78646
    {'fl_cor_cl_mt_x_of_s'}            1         0.33534         0.95887
    {'fl_cov_cl_mt_y_of_s'}        1.589          27.289          1.8353
    {'fl_cor_cl_mt_y_of_s'}      0.33534               1         0.38856
    {'fl_cov_cl_mt_z_of_s'}      0.78646          1.8353         0.81758
    {'fl_cor_cl_mt_z_of_s'}      0.95887         0.38856               1
    {'fracByP1'           }      0.13504        -0.56498        0.068934
    {'fracByP10'          }      0.13504        -0.56498        0.068934
```

```
    {'fracByP25'            }       0.13504         -0.53456          0.14234
    {'fracByP50'            }       0.42991         -0.39181          0.43856
    {'fracByP75'            }       0.91346         0.095425          0.60296
    {'fracByP90'            }       0.91346                1          0.91306
    {'fracByP99'            }             1                1                1
```

### 3.1.3 Test FF_SIMU_STATS Four States-Points Matrix Single Column Inputs

Same as before, but now inputs are single column, should have identical results:

```
% Array Inputs
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_mt_xyz_of_s);
```

```
xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_mt_x_of_s    cl_mt_y_of_s    cl_mt_z_of_s
    _______________________    ____________    ____________    ____________

    {'mean'                 }        2.0763          1.9323          2.0668
    {'sd'                   }        0.9071          5.2239          0.9042
    {'coefofvar'            }       0.43688          2.7034         0.43749
    {'min'                  }             1             -10               1
    {'max'                  }             4               9               4
    {'pYis0'                }             0               0               0
    {'pYls0'                }             0         0.20441               0
    {'pYgr0'                }             1         0.79559               1
    {'pYisMINY'             }       0.28039         0.10917         0.14247
    {'pYisMAXY'             }      0.044922         0.19422        0.044922
    {'p1'                   }             1             -10               1
    {'p10'                  }             1             -10               1
    {'p25'                  }             1             1.1             1.1
    {'p50'                  }             2               2               2
    {'p75'                  }             3               5             2.5
    {'p90'                  }             3               9             3.3
    {'p99'                  }             4               9               4
    {'fl_cov_cl_mt_x_of_s'}       0.82282           1.589         0.78646
    {'fl_cor_cl_mt_x_of_s'}             1         0.33534         0.95887
    {'fl_cov_cl_mt_y_of_s'}         1.589          27.289          1.8353
    {'fl_cor_cl_mt_y_of_s'}       0.33534               1         0.38856
    {'fl_cov_cl_mt_z_of_s'}       0.78646          1.8353         0.81758
    {'fl_cor_cl_mt_z_of_s'}       0.95887         0.38856               1
    {'fracByP1'             }       0.13504        -0.56498        0.068934
    {'fracByP10'            }       0.13504        -0.56498        0.068934
    {'fracByP25'            }       0.13504        -0.53456         0.14234
    {'fracByP50'            }       0.42991        -0.39181         0.43856
    {'fracByP75'            }       0.91346        0.095425         0.60296
    {'fracByP90'            }       0.91346               1         0.91306
    {'fracByP99'            }             1               1               1
```

### 3.1.4 Test FF_SIMU_STATS Print Many Details

The Same As before, but now control which percentiles and other details to display.

```
% Array Inputs
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_ar_xyz_of_s('cl_ar_x_of_s') = {mt_x_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('cl_ar_z_of_s') = {mt_z_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('ar_st_y_name') = ["cl_ar_x_of_s", "cl_ar_z_of_s"];

% controls
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_display_detail') = false;
mp_support('bl_display_final') = true;
mp_support('bl_display_drvm2outcomes') = false;
mp_support('ar_fl_percentiles') = [25 50 75];
mp_support('bl_display_drvstats') = true;
mp_support('bl_display_drvm2covcor') = false;

% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_ar_xyz_of_s, mp_support);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: cl_ar_x_of_s
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    2.0763

fl_choice_sd
    0.9071

fl_choice_coefofvar
    0.4369

fl_choice_prob_zero
     0

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
     1

fl_choice_prob_max
    0.0449

tb_disc_cumu
    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass     CDF      cumsumFrac
    _______________________    _______________________________    ______    __________

               1                           0.28039                28.039     0.13504
             1.5                           0.13561                  41.6     0.23301
               2                           0.20441                62.041     0.42991
               3                           0.33466                95.508     0.91346
               4                          0.044922                   100           1

    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass     CDF      cumsumFrac
    _______________________    _______________________________    ______    __________

               1                           0.28039                28.039     0.13504
```

```
           1.5                        0.13561                41.6     0.23301
             2                        0.20441              62.041     0.42991
             3                        0.33466              95.508     0.91346
             4                       0.044922                 100           1

tb_prob_drv
    percentiles    cl_ar_x_of_sDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ---------------------------------------    --------------------------------

        25                            1                                    0.13504
        50                            2                                    0.42991
        75                            3                                    0.91346

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: cl_ar_z_of_s
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    2.0668

fl_choice_sd
    0.9042

fl_choice_coefofvar
    0.4375

fl_choice_prob_zero
     0

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
     1

fl_choice_prob_max
    0.0449

tb_disc_cumu
    cl_ar_z_of_sDiscreteVal    cl_ar_z_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

             1                        0.14247              14.247     0.068934
           1.1                        0.13792              28.039      0.14234
           1.5                        0.13561                41.6      0.24076
             2                        0.20441              62.041      0.43856
           2.3                       0.056663              67.708      0.50162
           2.5                       0.083786              76.086      0.60296
           3.3                        0.19422              95.508      0.91306
             4                       0.044922                 100            1

    cl_ar_z_of_sDiscreteVal    cl_ar_z_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

             1                        0.14247              14.247     0.068934
           1.1                        0.13792              28.039      0.14234
           1.5                        0.13561                41.6      0.24076
```

```
               2                    0.20441          62.041        0.43856
             2.3                   0.056663          67.708        0.50162
             2.5                   0.083786          76.086        0.60296
             3.3                    0.19422          95.508        0.91306
               4                   0.044922             100              1

tb_prob_drv
    percentiles    cl_ar_z_of_sDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ---------------------------------------    --------------------------------

        25                           1.1                                  0.14234
        50                             2                                  0.43856
        75                           2.5                                  0.60296

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_ar_x_of_s    cl_ar_z_of_s
    _______________________    ____________    ____________

    {'mean'               }         2.0763          2.0668
    {'sd'                 }         0.9071          0.9042
    {'coefofvar'          }        0.43688         0.43749
    {'min'                }              1               1
    {'max'                }              4               4
    {'pYis0'              }              0               0
    {'pYls0'              }              0               0
    {'pYgr0'              }              1               1
    {'pYisMINY'           }        0.28039         0.14247
    {'pYisMAXY'           }       0.044922        0.044922
    {'p25'                }              1             1.1
    {'p50'                }              2               2
    {'p75'                }              3             2.5
    {'fl_cov_cl_ar_x_of_s'}        0.82282         0.78646
    {'fl_cor_cl_ar_x_of_s'}              1         0.95887
    {'fl_cov_cl_ar_z_of_s'}        0.78646         0.81758
    {'fl_cor_cl_ar_z_of_s'}        0.95887               1
    {'fracByP25'          }        0.13504         0.14234
    {'fracByP50'          }        0.42991         0.43856
    {'fracByP75'          }        0.91346         0.60296
```

## 3.2 FF_DISC_RAND_VAR_STATS Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_stats** from the **MEconTools Package.** This function summarizes statistics of matrixes stored in a container map, as well as scalar, string, function and other values stored in container maps.

### 3.2.1 Test FF_DISC_RAND_VAR_STATS Defaults

Call the function with defaults.

```
ff_disc_rand_var_stats();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: binom
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
```

```
fl_choice_mean
   -1.0000

fl_choice_sd
    2.5100

fl_choice_coefofvar
   -2.5100

fl_choice_prob_zero
    0.1416

fl_choice_prob_below_zero
    0.5888

fl_choice_prob_above_zero
    0.2696

fl_choice_prob_max
   2.0589e-16

tb_disc_cumu
    binomDiscreteVal    binomDiscreteValProbMass       CDF       cumsumFrac
    ________________    ________________________    _________    __________

          -10                  2.2539e-05           0.0022539    0.00022539
           -9                  0.00028979            0.031233     0.0028335
           -8                   0.0018008             0.21132       0.01724
           -7                   0.0072034             0.93166      0.067664
           -6                    0.020838              3.0155       0.19269
           -5                     0.04644              7.6595       0.42489
           -4                    0.082928              15.952       0.75661
           -3                     0.12185              28.138        1.1222
           -2                     0.15014              43.152        1.4224
           -1                     0.15729              58.881        1.5797

    binomDiscreteVal    binomDiscreteValProbMass    CDF    cumsumFrac
    ________________    ________________________    ___    __________

           11                  6.0392e-06           100        1
           12                  1.0588e-06           100        1
           13                  1.5784e-07           100        1
           14                   1.973e-08           100        1
           15                  2.0293e-09           100        1
           16                  1.6725e-10           100        1
           17                  1.0619e-11           100        1
           18                  4.8762e-13           100        1
           19                  1.4412e-14           100        1
           20                  2.0589e-16           100        1

tb_prob_drv
    percentiles    binomDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    ___________    ________________________________    ________________________________

        0.1                       -8                                0.01724
          1                       -6                                0.19269
          5                       -5                                0.42489
         10                       -4                                0.75661
```

```
         15                    -4                        0.75661
         20                    -3                         1.1222
         25                    -3                         1.1222
         35                    -2                         1.4224
         50                    -1                         1.5797
         65                     0                         1.5797
         75                     1                         1.4694
         80                     1                         1.4694
         85                     2                         1.3197
         90                     2                         1.3197
         95                     3                         1.1865
         99                     5                         1.0412
       99.9                     7                         1.0052
```

### 3.2.2 Test FF_DISC_RAND_VAR_STATS 0 and 1 Random Variable

The simplest discrete random variable has two values, zero or one. The probability of zero is 30 percent, and 70 percent is the probability of one.

```
% Parameters
% 1. specify the random variable
st_var_name = 'bernoulli';
ar_choice_unique_sorted = [0, 1];
ar_choice_prob = [0.3, 0.7];
% 2. percentiles of interest
ar_fl_percentiles = [0.1 5 25 50 75 95 99.9];
% 3. print resutls
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: bernoulli
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    0.7000

fl_choice_sd
    0.4583

fl_choice_coefofvar
    0.6547

fl_choice_prob_zero
    0.3000

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
    0.7000

fl_choice_prob_max
    0.7000
```

```
tb_disc_cumu
    bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
    ____________________    ____________________________    ___    __________

             0                          0.3                  30        0
             1                          0.7                 100        1

    bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
    ____________________    ____________________________    ___    __________

             0                          0.3                  30        0
             1                          0.7                 100        1

tb_prob_drv
    percentiles    bernoulliDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    ___________    ____________________________________    ________________________________

        0.1                         0                                     0
          5                         0                                     0
         25                         0                                     0
         50                         1                                     1
         75                         1                                     1
         95                         1                                     1
       99.9                         1                                     1
```

### 3.2.3 Test FF_DISC_RAND_VAR_STATS with Poisson

Poisson random variable, with mean equals to ten, summarize over umsymmetric percentiles. Note that the poisson random variable has no upper bound.

```
% Parameters
% 1. specify the random variable
st_var_name = 'poisson';
mu = 10;
ar_choice_unique_sorted = 0:1:50;
ar_choice_prob = poisspdf(ar_choice_unique_sorted, mu);
% 2. percentiles of interest, unsymmetric
ar_fl_percentiles = [0.1 5 10 25 50 90 95 99 99.9 99.99 99.999 99.9999];
% 3. print resutls
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: poisson
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    10

fl_choice_sd
    3.1623

fl_choice_coefofvar
    0.3162
```

```
fl_choice_prob_zero
   4.5400e-05

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
    1.0000

fl_choice_prob_max
   1.4927e-19

tb_disc_cumu
    poissonDiscreteVal    poissonDiscreteValProbMass      CDF      cumsumFrac
    ------------------    --------------------------    -------    ----------

            0                      4.54e-05             0.00454            0
            1                      0.000454             0.04994     4.54e-05
            2                       0.00227             0.27694    0.0004994
            3                     0.0075667              1.0336    0.0027694
            4                      0.018917              2.9253     0.010336
            5                      0.037833              6.7086     0.029253
            6                      0.063055              13.014     0.067086
            7                      0.090079              22.022      0.13014
            8                        0.1126              33.282      0.22022
            9                       0.12511              45.793      0.33282

    poissonDiscreteVal    poissonDiscreteValProbMass    CDF    cumsumFrac
    ------------------    --------------------------    ---    ----------

            41                    1.3571e-13            100        1
            42                    3.2313e-14            100        1
            43                    7.5146e-15            100        1
            44                    1.7079e-15            100        1
            45                    3.7953e-16            100        1
            46                    8.2506e-17            100        1
            47                    1.7554e-17            100        1
            48                    3.6572e-18            100        1
            49                    7.4636e-19            100        1
            50                    1.4927e-19            100        1

tb_prob_drv
    percentiles    poissonDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ----------------------------------    --------------------------------

        0.1                        2                                0.0004994
          5                        5                                 0.029253
         10                        6                                 0.067086
         25                        8                                  0.22022
         50                       10                                  0.45793
         90                       14                                  0.86446
         95                       15                                  0.91654
         99                       18                                  0.98572
       99.9                       21                                  0.99841
      99.99                       24                                  0.99988
     99.999                       26                                  0.99998
        100                       28                                        1
```

```
% Print out full Stored Matrix
% Note that the outputs are single row arrays.
ff_container_map_display(ds_stats_map, 100, 100)

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                              i    idx    ndim    numel    rowN    colN     mean      std     coe
                              -    ---    ----    -----    ----    ----    -------   ------   ---

    ar_choice_perc_fracheld   1     1      2       12       1       12     0.62833    0.435   0.6
    ar_choice_percentiles     2     2      2       12       1       12       14.75   8.7399   0.5
    ar_fl_percentiles         3     3      2       12       1       12      64.499   42.887   0.6

xxx TABLE:ar_choice_perc_fracheld xxxxxxxxxxxxxxxxxx
              c1           c2          c3          c4         c5         c6         c7         c8
          _________    ________    ________    _______    _______    _______    _______    _______

    r1    0.0004994    0.029253    0.067086    0.22022    0.45793    0.86446    0.91654    0.98572

xxx TABLE:ar_choice_percentiles xxxxxxxxxxxxxxxxxx
          c1    c2    c3    c4    c5    c6    c7    c8    c9    c10    c11    c12
          __    __    __    __    __    __    __    __    __    ___    ___    ___

    r1    2     5     6     8     10    14    15    18    21    24     26     28

xxx TABLE:ar_fl_percentiles xxxxxxxxxxxxxxxxxx
          c1     c2    c3    c4    c5    c6    c7    c8     c9      c10      c11      c12
          ___    __    __    __    __    __    __    __    ____    _____    ______    ___

    r1    0.1    5     10    25    50    90    95    99    99.9    99.99    99.999    100

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i     idx       value
                                __    ___    __________

    fl_choice_coefofvar         1      4        0.31623
    fl_choice_max               2      5             50
    fl_choice_mean              3      6             10
    fl_choice_min               4      7              0
    fl_choice_prob_above_zero   5      8        0.99995
    fl_choice_prob_below_zero   6      9              0
    fl_choice_prob_max          7     10     1.4927e-19
    fl_choice_prob_min          8     11       4.54e-05
    fl_choice_prob_zero         9     12       4.54e-05
    fl_choice_sd               10     13         3.1623
```

## 3.3 FF_DISC_RAND_VAR_MASS2OUTCOMES Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_mass2outcomes** from the **MEconTools Package.** This function generates sorted discrete random variable from state-space joint distribution.

### 3.3.1 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2outcomes();

INPUT f(a,z): mt_dist_bystates
    0.0289    0.0465    0.0228    0.0036    0.0001
    0.0241    0.0930    0.0857    0.0241    0.0015
    0.0080    0.0744    0.1285    0.0643    0.0074
    0.0013    0.0297    0.0964    0.0857    0.0186
    0.0001    0.0059    0.0361    0.0571    0.0232
    0.0000    0.0005    0.0054    0.0152    0.0116

INPUT y(a,z): mt_choice_bystates
    -5    -4    -5    -4    -4
    -3    -2    -3    -2    -3
    -1    -1    -1     0     0
     1     1     2     3     1
     4     3     3     4     3
     5     6     5     6     6

OUTPUT f(y): ar_choice_prob_byY
    0.0518
    0.0502
    0.1113
    0.1171
    0.2109
    0.0717
    0.0497
    0.0964
    0.1510
    0.0572
    0.0054
    0.0273

OUTPUT f(y,z): mt_choice_prob_byYZ
    0.0289         0    0.0228         0         0
         0    0.0465         0    0.0036    0.0001
    0.0241         0    0.0857         0    0.0015
         0    0.0930         0    0.0241         0
    0.0080    0.0744    0.1285         0         0
         0         0         0    0.0643    0.0074
    0.0013    0.0297         0         0    0.0186
         0         0    0.0964         0         0
         0    0.0059    0.0361    0.0857    0.0232
    0.0001         0         0    0.0571         0
    0.0000         0    0.0054         0         0
         0    0.0005         0    0.0152    0.0116

OUTPUT f(y,a): mt_choice_prob_byYA
    0.0518         0         0         0         0         0
    0.0502         0         0         0         0         0
         0    0.1113         0         0         0         0
         0    0.1171         0         0         0         0
         0         0    0.2109         0         0         0
         0         0    0.0717         0         0         0
```

```
         0         0         0    0.0497         0         0
         0         0         0    0.0964         0         0
         0         0         0    0.0857    0.0653         0
         0         0         0         0    0.0572         0
         0         0         0         0         0    0.0054
         0         0         0         0         0    0.0273

OUTPUT f(y) and y in table: tb_choice_drv_cur_byY
    binomtestOutcomes    probMassFunction
    _________________    ________________

           -5                0.051764
           -4                0.050217
           -3                 0.11126
           -2                 0.11706
           -1                 0.21092
            0                0.071696
            1                0.049682
            2                0.096388
            3                 0.15102
            4                0.057231
            5               0.0054256
            6                0.027329
```

### 3.3.2 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Four States-Points

Over some (a,z) states that is 2 by 2, matrix or vectorized inputs identical results.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2;3,1];
% stationary mass over assets adn shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);
```

```
    0.4039    1.0000
    0.2971    2.0000
    0.2990    3.0000
```

Same as before, but now inputs are single column:

```
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s(:), mt_f_of_s);
disp([ar_f_of_y ar_y_unique_sorted]);
```

```
    0.4039    1.0000
    0.2971    2.0000
    0.2990    3.0000
```

### 3.3.3 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Conditional Mass Outputs

Same inputs as before, but now, also output additional conditional statistis, f(y, a), where a is the row state variable for f(a,z). For conditional statistics, must provide matrix based inputs.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2,0.5;
             3,1,2.0];
% stationary mass over assets adn shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted, mt_f_of_y_srow, mt_f_of_y_scol] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);
```

```
    0.2695    0.5000
    0.3765    1.0000
    0.2649    2.0000
    0.0891    3.0000
```

```
disp(mt_f_of_y_srow);
```

```
    0.2695         0
    0.1215    0.2550
    0.1217    0.1432
         0    0.0891
```

```
disp(mt_f_of_y_scol);
```

```
         0         0    0.2695
    0.1215    0.2550         0
         0    0.1217    0.1432
    0.0891         0         0
```

## 3.4 FF_DISC_RAND_VAR_MASS2COVCOR Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_mass2covcor** from the **MEconTools Package.** This function calculates covariance and correlation based for two discrete random variables.

### 3.4.1 Test FF_DISC_RAND_VAR_MASS2COVCOR Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2covcor();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_input_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    ndim    numel    rowN    colN      mean        std       coefvari
                 -    ---    ----    -----    ----    ----    --------    --------    --------    --
```

```
   mt_f_of_s    1     5      2       30       6       5      0.033333    0.035743     1.0723     3.
   mt_x_of_s    2     6      2       30       6       5       0.83333      5.3051     6.3661
   mt_y_of_s    3     7      2       30       6       5        8.3259      7.1913    0.86373

xxx TABLE:mt_f_of_s xxxxxxxxxxxxxxxxxx
              c1            c2            c3            c4            c5
          ----------    ----------    ---------     ---------     ---------

    r1      0.028917      0.046484      0.022848     0.0036146      0.000119
    r2      0.024097      0.092967      0.085679      0.024097     0.0014875
    r3     0.0080324      0.074374       0.12852      0.064259     0.0074374
    r4     0.0013387       0.02975      0.096388      0.085679      0.018593
    r5    0.00011156     0.0059499      0.036146      0.057119      0.023242
    r6    3.7187e-06    0.00047599     0.0054218      0.015232      0.011621

xxx TABLE:mt_x_of_s xxxxxxxxxxxxxxxxxx
          c1    c2    c3    c4    c5
          --    --    --    --    --

    r1    -7    -6    -7    -6    -6
    r2    -5    -3    -5    -3    -4
    r3    -2    -1    -1     0    -1
    r4     2     2     3     4     2
    r5     6     5     5     6     5
    r6     8     9     7     9     9

xxx TABLE:mt_y_of_s xxxxxxxxxxxxxxxxxx
            c1          c2          c3         c4          c5
          ------    --------    ------    -------    -------

    r1    13.231      21.786    18.136      19.35     13.901
    r2     9.946      16.887    9.6914      15.71     8.6906
    r3    16.255      6.2166    13.799     5.2138     11.641
    r4    12.628      2.7525    6.5321    0.27238     13.357
    r5    5.8844      4.0352      6.05    0.14102    0.50318
    r6    3.5617    -0.72091    5.1855    -6.3772    -4.4805

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_input_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                i    idx     value
                -    ---    --------

    fl_x_mean   1     1     -0.11081
    fl_x_sd     2     2       4.1239
    fl_y_mean   3     3       8.8423
    fl_y_sd     4     4       6.5845

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_output_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i    idx    ndim    numel    rowN    colN      mean       std
                                -    ---    ----    -----    ----    ----    --------    ------

    mt_cov_component_weighted   1     1      2       30       6       5      -0.73612    1.0404
```

```
    mt_x_devi_from_mean          2     2      2       30       6       5        0.94415     5.3051
    mt_x_y_multiply              3     3      2       30       6       5        -31.321     36.564
    mt_y_devi_from_mean          4     4      2       30       6       5       -0.51644     7.1913

xxx TABLE:mt_cov_component_weighted xxxxxxxxxxxxxxxxxx
              c1            c2           c3          c4            c5
          -----------    ---------    --------    ---------    ----------

    r1      -0.87434      -3.5432     -1.4628     -0.22368    -0.0035451
    r2      -0.13003      -2.1607    -0.35565     -0.47814    0.00087767
    r3      -0.11248      0.17365    -0.56642    -0.025838     -0.018507
    r4      0.010697     -0.38241    -0.69273      -3.0184       0.17717
    r5    -0.0020165     -0.14618    -0.51584      -3.0371      -0.99056
    r6   -0.00015927    -0.041473    -0.14098      -2.1121       -1.4106

xxx TABLE:mt_x_devi_from_mean xxxxxxxxxxxxxxxxxx
            c1          c2          c3          c4          c5
          -------    --------    --------    -------    --------

    r1    -6.8892     -5.8892     -6.8892    -5.8892     -5.8892
    r2    -4.8892     -2.8892     -4.8892    -2.8892     -3.8892
    r3    -1.8892    -0.88919    -0.88919    0.11081    -0.88919
    r4     2.1108      2.1108      3.1108     4.1108      2.1108
    r5     6.1108      5.1108      5.1108     6.1108      5.1108
    r6     8.1108      9.1108      7.1108     9.1108      9.1108

xxx TABLE:mt_x_y_multiply xxxxxxxxxxxxxxxxxx
            c1          c2         c3           c4          c5
          -------    -------    -------    --------    -------

    r1    -30.237    -76.225    -64.023     -61.882    -29.792
    r2     -5.396    -23.242     -4.151     -19.842    0.59004
    r3    -14.003     2.3348    -4.4073    -0.40209    -2.4884
    r4     7.9905    -12.854    -7.1868      -35.23     9.5287
    r5    -18.075    -24.568    -14.271     -53.172     -42.62
    r6     -42.83    -87.129    -26.003     -138.66    -121.38

xxx TABLE:mt_y_devi_from_mean xxxxxxxxxxxxxxxxxx
            c1          c2          c3          c4          c5
          -------    -------    -------    -------    --------

    r1      4.389     12.943     9.2933     10.508      5.0587
    r2     1.1037     8.0444    0.84902     6.8677    -0.15171
    r3     7.4123    -2.6258     4.9566    -3.6286      2.7985
    r4     3.7855    -6.0898    -2.3103      -8.57      4.5142
    r5    -2.9579    -4.8071    -2.7924    -8.7013     -8.3392
    r6    -5.2806    -9.5633    -3.6568     -15.22     -13.323

fl_cov
  -22.0835

fl_cor
   -0.8133
```

### 3.4.2 Test FF_DISC_RAND_VAR_MASS2COVCOR Four States-Points

Over some (a,z) states that is 2 by 2, c matrix, and y matrix, find correlation. Positively related.

```
% Set Parameters
```

```
mt_c_of_s = [1,2;3,1];
mt_y_of_s = [2,10;5,1.1];
rng(123);
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);
```

```
cov=1.4446,cor=0.65723
```

Same as before, but now inputs are single column:

```
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s(:), mt_y_of_s(:), mt_f_of_s(:), bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);
```

```
cov=1.4446,cor=0.65723
```

### 3.4.3 Test FF_DISC_RAND_VAR_MASS2COVCOR Two Random Vectors

Generate two random vectors, with random or even mass, correlation should be zero:

```
% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);
```

```
cov=-57.6533,cor=-0.062023
```

### 3.4.4 Test FF_DISC_RAND_VAR_MASS2COVCOR Provide Mean and SD

Same as above, but now provide means and sd for x andy directly. The results are the same as when mean and sd are calculated inside the function.

```
% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
fl_c_mean = sum(mt_f_of_s.*mt_c_of_s);
fl_c_sd = sqrt(sum(mt_f_of_s.*(mt_c_of_s-fl_c_mean).^2));
fl_y_mean = sum(mt_f_of_s.*mt_y_of_s);
fl_y_sd = sqrt(sum(mt_f_of_s.*(mt_y_of_s-fl_y_mean).^2));
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
```

```
    mt_c_of_s, mt_y_of_s, mt_f_of_s, ...
    fl_c_mean, fl_c_sd, ...
    fl_y_mean, fl_y_sd, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);
```

```
cov=-57.6533,cor=-0.062023
```

# Chapter 4

# Optimizers

## 4.1 FF_OPTIM_BISEC_SAVEZRONE Derivative Bisection

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_optim_bisec_savezrone** from the **MEconTools Package.** This functions solves for optimal savings/borrowing level given an anonymous function that provides the derivative of a intertemporal savings problem. The function is solves over a grid of state-space elements that are embeded in the anonymous function. By default, it iterates over 15 iterations with bisection.

The vectorized and looped bisection savings problem rely on this function to solve for optimal savings choices:

- States Grid + Continuous Exact Savings as Share of Cash-on-Hand :**ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand : **ff_vfi_az_bisec_vec**, precision and speed

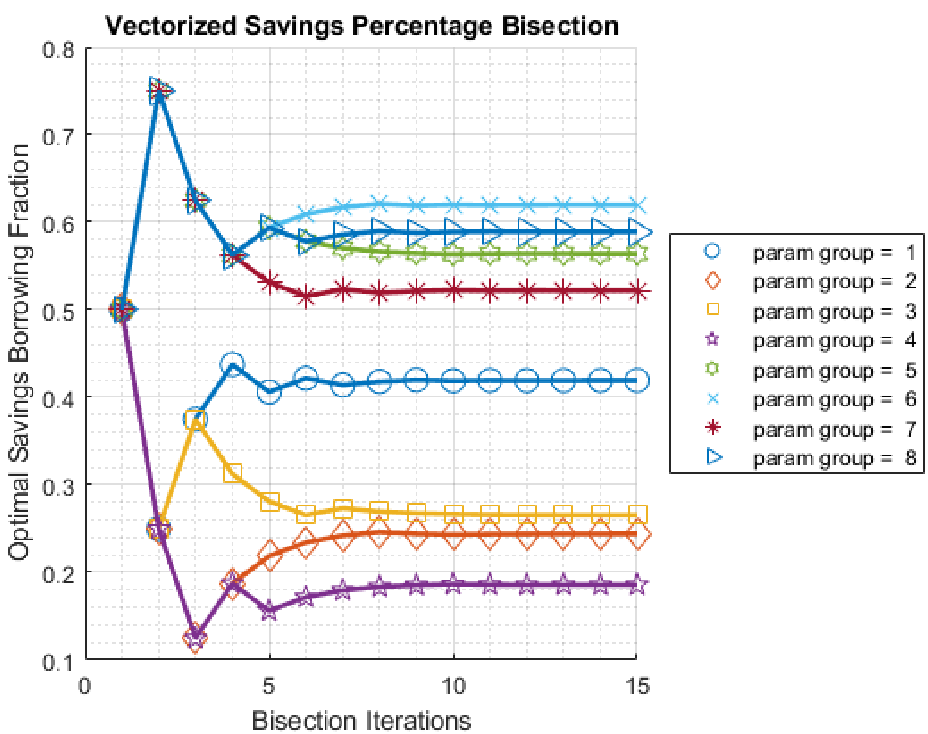
### 4.1.1 Test FF_OPTIM_BISEC_SAVEZRONE Defaults

Call the function with defaults, this solves concurrently for many state-space points' optimization problems:

```
ff_optim_bisec_savezrone();
```

```
Elapsed time is 0.089585 seconds.
BISECT END: iteration=16, norm(ar_mid_fx)=0.00030653
                                  vartype     paramgroup2    paramgroup3    paramgroup4    paramgroup5
                                  -------     -----------    -----------    -----------    -----------

    a                             "init"            1e-05          1e-05          1e-05          1e-05
    b                             "init"          0.99999        0.99999        0.99999        0.99999
    f_a                           "init"            33802          40925          67047          15411
    f_b                           "init"           -46789    -1.2672e+05    -1.8532e+05         -67518
    it1_fp                        "fatx"         -0.25973        -1.7159        -2.3655        -1.0421
    it1_p                         "x"                 0.5            0.5            0.5            0.5
    it2_fp                        "fatx"          0.72822      -0.052631        0.21087       -0.28379
    it2_p                         "x"                0.25           0.25           0.25           0.25
    it3_fp                        "fatx"          0.15277         1.8256        -1.1773        0.46124
    it3_p                         "x"               0.375        0.12501          0.375        0.12501
    it4_fp                        "fatx"        -0.059183        0.62299       -0.55013     -0.0090579
    it4_p                         "x"              0.4375        0.18751         0.3125        0.18751
```

| | | | | | |
|---|---|---|---|---|---|
| it5_fp | "fatx" | 0.044028 | 0.2488 | -0.19454 | 0.1861 |
| it5_p | "x" | 0.40625 | 0.21876 | 0.28125 | 0.15626 |
| it6_fp | "fatx" | -0.0080863 | 0.090981 | 0.00054305 | 0.081339 |
| it6_p | "x" | 0.42188 | 0.23438 | 0.26563 | 0.17188 |
| it7_fp | "fatx" | 0.017822 | 0.017593 | -0.098707 | 0.034591 |
| it7_p | "x" | 0.41406 | 0.24219 | 0.27344 | 0.17969 |
| it8_fp | "fatx" | 0.0048335 | -0.017893 | -0.049532 | 0.012405 |
| it8_p | "x" | 0.41797 | 0.2461 | 0.26954 | 0.1836 |
| it9_fp | "fatx" | -0.0016347 | -0.00024633 | -0.02461 | 0.0015865 |
| it9_p | "x" | 0.41992 | 0.24415 | 0.26758 | 0.18555 |
| it10_fp | "fatx" | 0.0015973 | 0.0086488 | -0.012063 | -0.0037571 |
| it10_p | "x" | 0.41895 | 0.24317 | 0.26661 | 0.18653 |
| it11_fp | "fatx" | -1.9235e-05 | 0.0041952 | -0.0057672 | -0.0010907 |
| it11_p | "x" | 0.41944 | 0.24366 | 0.26612 | 0.18604 |
| it12_fp | "fatx" | 0.00078889 | 0.0019729 | -0.0026139 | 0.00024655 |
| it12_p | "x" | 0.41919 | 0.2439 | 0.26587 | 0.1858 |
| it13_fp | "fatx" | 0.00038479 | 0.00086292 | -0.0010359 | -0.00042242 |
| it13_p | "x" | 0.41931 | 0.24402 | 0.26575 | 0.18592 |
| it14_fp | "fatx" | 0.00018277 | 0.0003082 | -0.00024654 | -8.8022e-05 |
| it14_p | "x" | 0.41937 | 0.24408 | 0.26569 | 0.18586 |
| it15_fp | "fatx" | 8.1766e-05 | 3.0909e-05 | 0.00014822 | 7.9241e-05 |
| it15_p | "x" | 0.4194 | 0.24412 | 0.26566 | 0.18583 |
| it15_level | "level" | 0.56205 | -0.070025 | 0.044431 | -0.039424 |
| exactSoluSaveborrFrac | "exact" | 0.41943 | 0.24412 | 0.26567 | 0.18584 |
| exactSoluSaveborrLevel | "exact" | 0.56211 | -0.070022 | 0.044438 | -0.039403 |
| exactSoluSaveborrFracGap | "exact" | 2.4705e-05 | 3.402e-06 | 1.1458e-05 | 1.4456e-05 |
| exactSoluSaveborrLevelGap | "exact" | 5.28e-05 | 2.6845e-06 | 6.1825e-06 | 2.1411e-05 |




```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim     numel    rowN    colN       sum          mean
                         -    ---    ----     -----    ----    ----    ----------    ----------    --
```

```
    ar_opti_foc_obj      1     1     2       8       1       8      0.00050535    6.3168e-05    9.
    ar_opti_save_frac    2     2     2       8       1       8            3.41       0.42626

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx
              c1            c2            c3            c4            c5             c6            c
          ----------    ----------    ----------    ----------    -----------    ----------    -----

    r1    8.1766e-05    3.0909e-05    0.00014822    7.9241e-05    -0.00013343    0.00015981    1.896

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxxx
            c1        c2         c3         c4         c5         c6       c7         c8
          ------    -------    -------    -------    -------    ------    -----    -------

    r1    0.4194    0.24412    0.26566    0.18583    0.56406    0.6199    0.522    0.58908
```

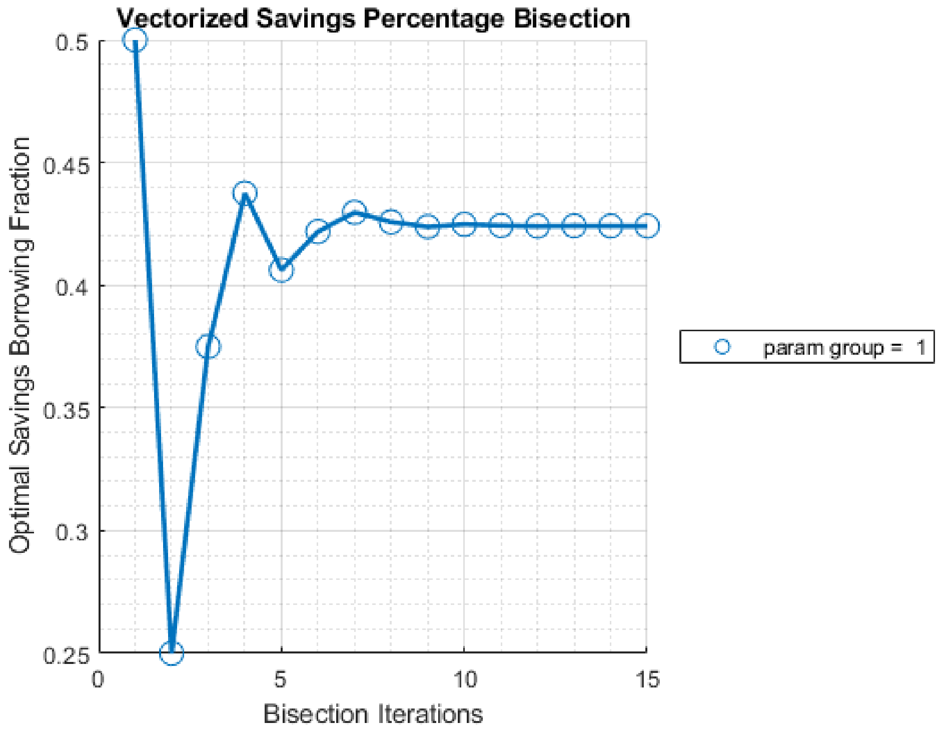
### 4.1.2 Test FF_OPTIM_BISEC_SAVEZRONE One Individual

Bisection for savings choice at one state:

```
% Generate the state-space and function
[fl_z1, fl_z2, fl_r, fl_beta] = deal(0.4730, 0.6252, 0.0839, 0.7365);
% ffi_intertemporal_max is a function in ff_optim_bisec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, fl_z1, fl_z2, fl_r, fl_beta);
% Call Function
bl_verbose = true;
ff_optim_bisec_savezrone(fc_deri_wth_uniroot, bl_verbose);
```

```
BISECT END: iteration=16, norm(ar_mid_fx)=0.00016724
                  vartype    paramgroup2
                  -------    -----------

    a             "init"           1e-05
    b             "init"         0.99999
    f_a           "init"           70155
    f_b           "init"          -95255
    it1_fp        "fatx"          -0.502
    it1_p         "x"                0.5
    it2_fp        "fatx"          1.5361
    it2_p         "x"               0.25
    it3_fp        "fatx"         0.34671
    it3_p         "x"              0.375
    it4_fp        "fatx"       -0.089881
    it4_p         "x"             0.4375
    it5_fp        "fatx"         0.12259
    it5_p         "x"            0.40625
    it6_fp        "fatx"        0.015276
    it6_p         "x"            0.42188
    it7_fp        "fatx"       -0.037529
    it7_p         "x"            0.42969
    it8_fp        "fatx"       -0.011188
    it8_p         "x"            0.42578
    it9_fp        "fatx"       0.0020277
    it9_p         "x"            0.42383
    it10_fp       "fatx"      -0.0045843
    it10_p        "x"            0.42481
    it11_fp       "fatx"      -0.0012793
    it11_p        "x"            0.42432
    it12_fp       "fatx"      0.00037392
```

```
    it12_p        "x"           0.42407
    it13_fp       "fatx"     -0.00045276
    it13_p        "x"            0.4242
    it14_fp       "fatx"     -3.9436e-05
    it14_p        "x"           0.42413
    it15_fp       "fatx"      0.00016724
    it15_p        "x"            0.4241
    it15_level    "level"      -0.13158
```



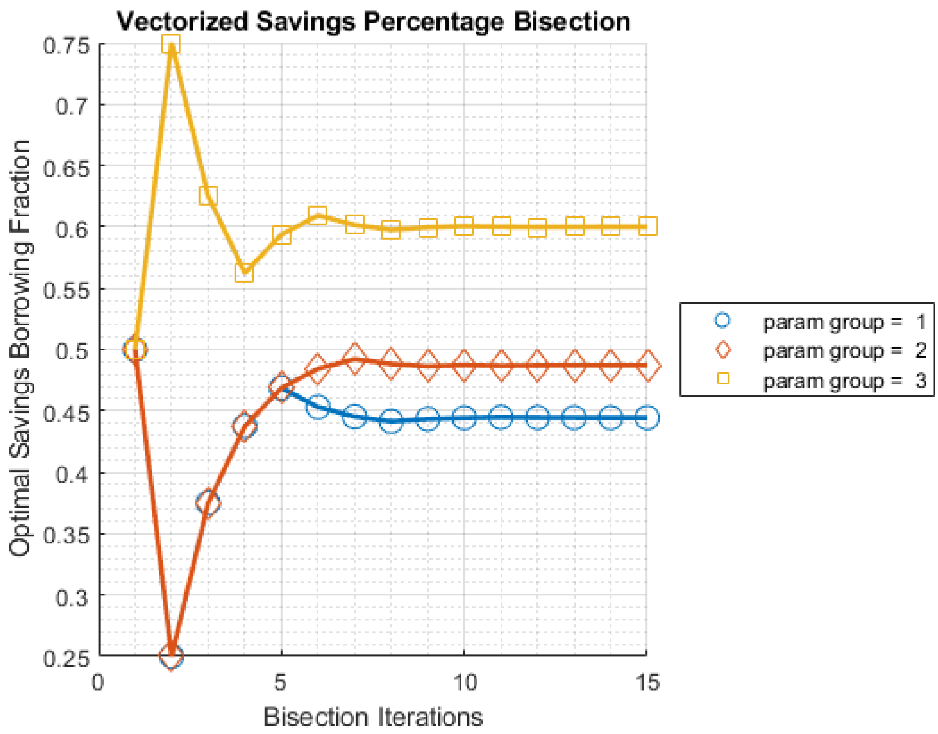

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx      value
                         -    ---    ----------

    ar_opti_foc_obj      1     1     0.00016724
    ar_opti_save_frac    2     2         0.4241
```

### 4.1.3 Test FF_OPTIM_BISEC_SAVEZRONE Six Individual States

Solve the two period intertemporal optimization problem with only 6 individual states:

```
% Generate the state-space and function
ar_z1 = [1,2,3]';
ar_z2 = [3,2,1]';
ar_r = [1.05, 1.50, 1.30]';
ar_beta = [0.80, 0.95, 1.50]';
mt_fc_inputs = [ar_z1, ar_z2, ar_r, ar_beta];
% ffi_intertemporal_max is a function in ff_optim_bisec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
ff_optim_bisec_savezrone(fc_deri_wth_uniroot, bl_verbose);
```

```
BISECT END: iteration=16, norm(ar_mid_fx)=8.9847e-05
                  vartype    paramgroup2    paramgroup3    paramgroup4
                  -------    -----------    -----------    -----------

    a             "init"          1e-05          1e-05          1e-05
    b             "init"        0.99999        0.99999        0.99999
    f_a           "init"          32475          33928          43671
    f_b           "init"         -40594         -35714         -29113
    it1_fp        "fatx"       -0.16238      -0.035714        0.29114
    it1_p         "x"               0.5            0.5            0.5
    it2_fp        "fatx"        0.75773        0.88092       -0.58225
    it2_p         "x"              0.25           0.25        0.74999
    it3_fp        "fatx"        0.21649        0.33333      -0.077629
    it3_p         "x"             0.375          0.375          0.625
    it4_fp        "fatx"       0.020615        0.14059        0.11091
    it4_p         "x"            0.4375         0.4375         0.5625
    it5_fp        "fatx"       -0.07132       0.051539       0.018865
    it5_p         "x"           0.46875        0.46875        0.59375
    it6_fp        "fatx"      -0.025599      0.0078193      -0.028659
    it6_p         "x"           0.45313        0.48438        0.60937
    it7_fp        "fatx"     -0.0025711      -0.013955     -0.0047386
    it7_p         "x"           0.44531        0.49219        0.60156
    it8_fp        "fatx"      0.0090001     -0.0030715      0.0071001
    it8_p         "x"           0.44141        0.48828        0.59765
    it9_fp        "fatx"      0.0032093      0.0023727      0.0011903
    it9_p         "x"           0.44336        0.48633        0.59961
    it10_fp       "fatx"     0.00031783    -0.00034971     -0.0017717
    it10_p        "x"           0.44434         0.4873        0.60058
    it11_fp       "fatx"     -0.0011269      0.0010114    -0.00029011
    it11_p        "x"           0.44483        0.48682         0.6001
    it12_fp       "fatx"    -0.00040464     0.00033083     0.00045024
    it12_p        "x"           0.44458        0.48706        0.59985
    it13_fp       "fatx"    -4.3425e-05    -9.4396e-06     8.0103e-05
    it13_p        "x"           0.44446        0.48718        0.59997
    it14_fp       "fatx"      0.0001372      0.0001607      -0.000105
    it14_p        "x"            0.4444        0.48712        0.60003
    it15_fp       "fatx"     4.6884e-05     7.5628e-05    -1.2444e-05
    it15_p        "x"           0.44443        0.48715            0.6
    it15_level    "level"       -0.3686        0.56403         1.6261
```




```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN       sum            mean        

                         -    ---    ----    -----    ----    ----    ----------     ----------    --

    ar_opti_foc_obj      1     1      2        3       1       3      0.00011007     3.6689e-05    4.
    ar_opti_save_frac    2     2      2        3       1       3          1.5316        0.51053

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx
              c1            c2             c3
          ----------    ----------    -----------

    r1    4.6884e-05    7.5628e-05    -1.2444e-05

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxxx
            c1         c2        c3
          -------    -------    ---

    r1    0.44443    0.48715    0.6
```

### 4.1.4 Test FF_OPTIM_BISEC_SAVEZRONE Speed

Test Speed doing 6.25 million bisections for a savings problem:

```
% Generate the state-space and function
rng(123);
it_draws = 6250000; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
% ffi_intertemporal_max is a function in ff_optim_bisec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
```

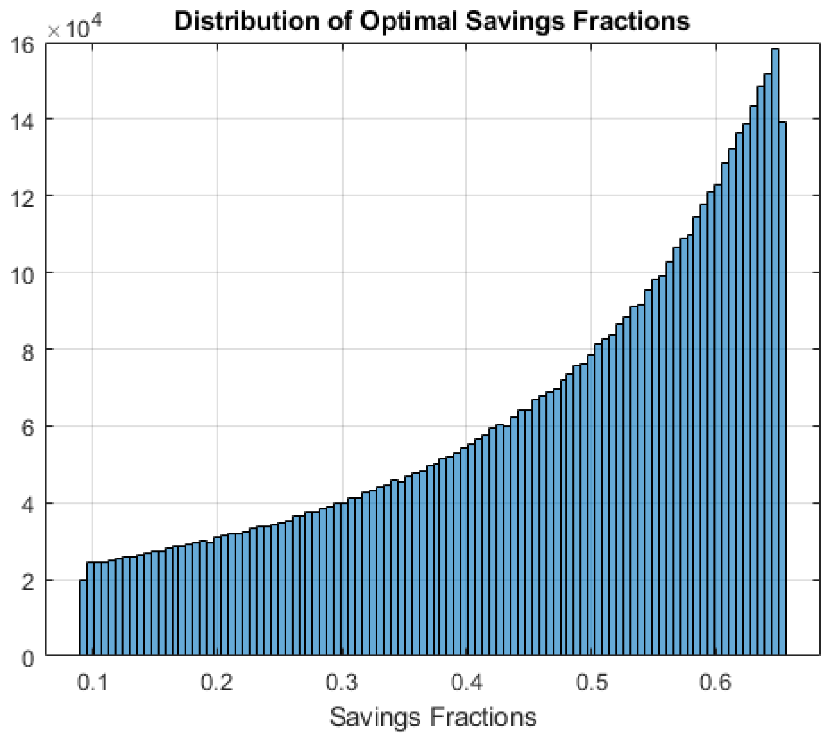
```
% Call Function
bl_verbose = false;
bl_timer = true;
[ar_opti_save_frac, ar_opti_save_level] = ff_optim_bisec_savezrone(fc_deri_wth_uniroot, bl_verbose, 
```

```
Elapsed time is 3.908350 seconds.
```

```
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
mp_container_map('ar_opti_save_frac') = ar_opti_save_frac;
mp_container_map('ar_opti_save_level') = ar_opti_save_level;
mp_container_map('ar_opti_save_frac_notnan') = ar_opti_save_frac(~isnan(ar_opti_save_frac));
ff_container_map_display(mp_container_map);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i    idx    ndim     numel        rowN      colN       sum         m
                                -    ---    ----    --------    --------    ----    ----------    --

    ar_opti_save_frac           1     1      2      6.25e+06    6.25e+06     1       2.884e+06    0.
    ar_opti_save_frac_notnan    2     2      2      6.25e+06    6.25e+06     1       2.884e+06    0.
    ar_opti_save_level          3     3      2      6.25e+06    6.25e+06     1      2.9482e+06    0.
```

```
figure();
histogram(ar_opti_save_frac(~isnan(ar_opti_save_frac)),100);
title('Distribution of Optimal Savings Fractions');
xlabel('Savings Fractions');
grid on;
```




### 4.1.5 Define Two Period Intertemporal FOC Log Utility No Shock

See Household's Utility Maximization Problem and Two-Period Borrowing and Savings Problem given Endowments.

```
function [ar_deri_zero, ar_saveborr_level] = ffi_intertemporal_max(ar_saveborr_frac, z1, z2, r, beta
    ar_saveborr_level = ar_saveborr_frac.*(z1+z2./(1+r)) - z2./(1+r);
    ar_deri_zero = 1./(ar_saveborr_level-z1) + (beta.*(r+1))./(z2 + ar_saveborr_level.*(r+1));
end
```

## 4.2 FF_OPTIM_MLSEC_SAVEZRONE Derivative Multisection

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_optim_mlsec_savezrone** from the **MEconTools Package.** This functions solves for optimal savings/borrowing level given an anonymous function that provides the derivative of a intertemporal savings problem. This is a vectorized function solved with multi-section (multiple points bisection concurrently).

The vectorized and looped bisection savings problem rely on this function to solve for optimal savings choices:

- States Grid + Continuous Exact Savings as Share of Cash-on-Hand :**ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand : **ff_vfi_az_bisec_vec**, precision and speed

### 4.2.1 Test FF_OPTIM_MLSEC_SAVEZRONE One Individual

Bisection for savings choice at one state:

```
% Generate the state-space and function
[fl_z1, fl_z2, fl_r, fl_beta] = deal(0.4730, 0.6252, 0.0839, 0.7365);
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, fl_z1, fl_z2, fl_r, fl_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
% optimally borrowing given the parameters here
mp_mlsec_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mlsec_ctrlinfo('it_mzoom_jnt_pnts') = 10;
mp_mlsec_ctrlinfo('it_mzoom_max_iter') = 4;
[fl_opti_save_frac, fl_opti_save_level] = ...
    ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, bl_timer, mp_mlsec_ctrlinfo)
```

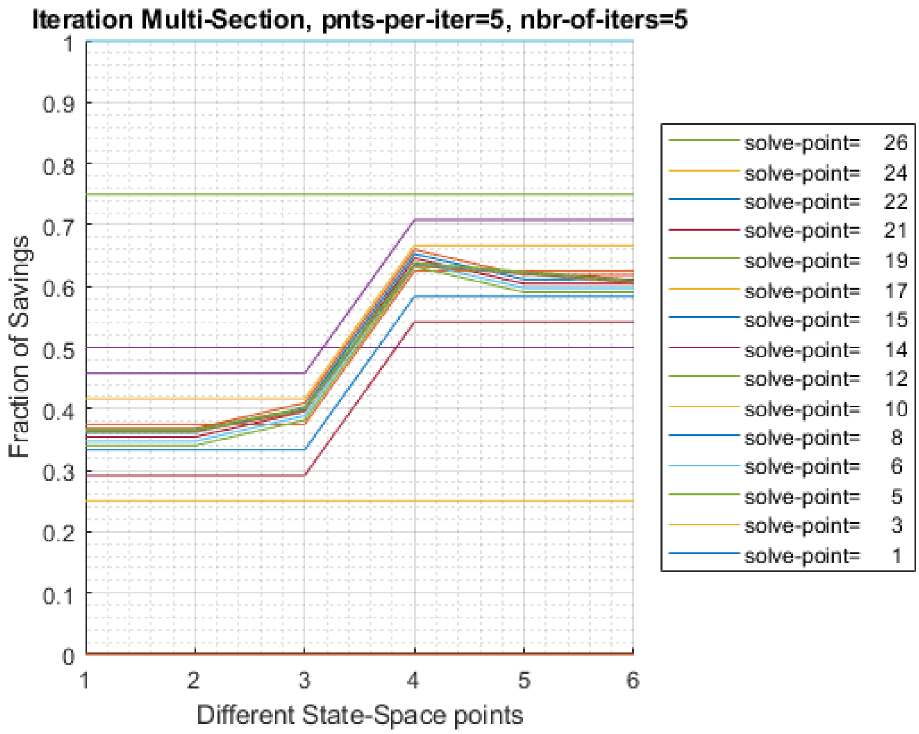
```
Elapsed time is 0.002805 seconds.
fl_opti_save_frac = 0.4241
fl_opti_save_level = -0.1316
```

### 4.2.2 Test FF_OPTIM_MLSEC_SAVEZRONE 5 Individuals 5 Iterations 5 Points Per Iteration

5 grid points per iteration, and 5 iterations.

```
% Generate the state-space and function
rng(123);
it_draws = 6; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
```

```
% Call Function
bl_verbose = true;
bl_timer = true;
mp_mlsec_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mlsec_ctrlinfo('it_mlsect_jnt_pnts') = 5;
mp_mlsec_ctrlinfo('it_mlsect_max_iter') = 5;
ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, bl_timer, mp_mlsec_ctrlinfo);

    iter    cl_row_names_a     Var1       Var2       Var3       Var4       Var5       Var6
    ----    --------------    -------    -------    -------    -------    -------    -------

     0        "point=1"         1e-05      1e-05      1e-05      1e-05      1e-05      1e-05
     1        "point=1"         1e-05      1e-05      1e-05      1e-05      1e-05      1e-05
     1        "point=2"       0.25001    0.25001    0.25001    0.25001    0.25001    0.25001
     1        "point=3"           0.5        0.5        0.5        0.5        0.5        0.5
     1        "point=4"          0.75       0.75       0.75       0.75       0.75       0.75
     1        "point=5"       0.99999    0.99999    0.99999    0.99999    0.99999    0.99999
     2        "point=1"       0.29167    0.29167    0.29167    0.54167    0.54167    0.54167
     2        "point=2"       0.33334    0.33334    0.33334    0.58333    0.58333    0.58333
     2        "point=3"         0.375      0.375      0.375      0.625      0.625      0.625
     2        "point=4"       0.41667    0.41667    0.41667    0.66666    0.66666    0.66666
     2        "point=5"       0.45833    0.45833    0.45833    0.70833    0.70833    0.70833
     3        "point=1"       0.34028    0.34028    0.38195    0.63194    0.59028    0.59028
     3        "point=2"       0.34723    0.34723    0.38889    0.63889    0.59722    0.59722
     3        "point=3"       0.35417    0.35417    0.39584    0.64583    0.60416    0.60416
     3        "point=4"       0.36111    0.36111    0.40278    0.65277    0.61111    0.61111
     3        "point=5"       0.36806    0.36806    0.40972    0.65972    0.61805    0.61805
     4        "point=1"       0.36227    0.36227    0.39699     0.6331    0.61921    0.60532
     4        "point=2"       0.36343    0.36343    0.39815    0.63426    0.62037    0.60648
     4        "point=3"       0.36459    0.36459    0.39931    0.63541    0.62153    0.60764
     4        "point=4"       0.36574    0.36574    0.40046    0.63657    0.62268    0.60879
     4        "point=5"        0.3669     0.3669    0.40162    0.63773    0.62384    0.60995
     5        "point=1"       0.36594    0.36594    0.40066    0.63792    0.62288    0.60783
     5        "point=2"       0.36613    0.36613    0.40085    0.63811    0.62307    0.60802
     5        "point=3"       0.36632    0.36632    0.40104    0.63831    0.62326    0.60822
     5        "point=4"       0.36652    0.36652    0.40124     0.6385    0.62345    0.60841
     5        "point=5"       0.36671    0.36671    0.40143    0.63869    0.62365     0.6086
```




```
Elapsed time is 0.406584 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN        sum            mean
                         -    ---    ----    -----    ----    ----    -----------    -----------

    ar_opti_foc_obj      1     1      2        6        6       1     -0.00037648    -6.2746e-05
    ar_opti_save_frac    2     2      2        6        6       1          3.0037        0.50061

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx
              c1
          -----------

    r1     7.0837e-05
    r2     -0.0002782
    r3     0.00017713
    r4     0.00055875
    r5    -0.00023392
    r6    -0.00067107

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxxx
            c1
          -------

    r1    0.36642
    r2    0.36661
    r3    0.40153
    r4    0.63821
    r5    0.62297
    r6    0.60793
```

### 4.2.3 Test FF_OPTIM_MLSEC_SAVEZRONE 8 Individuals 3 Iterations 10 Points Per Iteration

10 grid points per iteration, and 3 iterations.

```
% Generate the state-space and function
rng(123);
it_draws = 8; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
bl_timer = true;
mp_mlsec_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mlsec_ctrlinfo('it_mlsect_jnt_pnts') = 10;
mp_mlsec_ctrlinfo('it_mlsect_max_iter') = 3;
ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, bl_timer, mp_mlsec_ctrlinfo);
```

| iter | cl_row_names_a | Var1 | Var2 | Var3 | Var4 | Var5 | Var6 | Var |
|---|---|---|---|---|---|---|---|---|
| 0 | "point=1" | 1e-05 | 1e-05 | 1e-05 | 1e-05 | 1e-05 | 1e-05 | 1e |
| 1 | "point=1" | 1e-05 | 1e-05 | 1e-05 | 1e-05 | 1e-05 | 1e-05 | 1e |
| 1 | "point=2" | 0.11112 | 0.11112 | 0.11112 | 0.11112 | 0.11112 | 0.11112 | 0.11 |
| 1 | "point=3" | 0.22223 | 0.22223 | 0.22223 | 0.22223 | 0.22223 | 0.22223 | 0.22 |
| 1 | "point=4" | 0.33334 | 0.33334 | 0.33334 | 0.33334 | 0.33334 | 0.33334 | 0.33 |
| 1 | "point=5" | 0.44445 | 0.44445 | 0.44445 | 0.44445 | 0.44445 | 0.44445 | 0.44 |
| 1 | "point=6" | 0.55555 | 0.55555 | 0.55555 | 0.55555 | 0.55555 | 0.55555 | 0.55 |
| 1 | "point=7" | 0.66666 | 0.66666 | 0.66666 | 0.66666 | 0.66666 | 0.66666 | 0.66 |
| 1 | "point=8" | 0.77777 | 0.77777 | 0.77777 | 0.77777 | 0.77777 | 0.77777 | 0.77 |
| 1 | "point=9" | 0.88888 | 0.88888 | 0.88888 | 0.88888 | 0.88888 | 0.88888 | 0.88 |
| 1 | "point=10" | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99 |
| 2 | "point=1" | 0.34344 | 0.23233 | 0.23233 | 0.23233 | 0.56566 | 0.56566 | 0.45 |
| 2 | "point=2" | 0.35354 | 0.24243 | 0.24243 | 0.24243 | 0.57576 | 0.57576 | 0.46 |
| 2 | "point=3" | 0.36364 | 0.25253 | 0.25253 | 0.25253 | 0.58586 | 0.58586 | 0.47 |
| 2 | "point=4" | 0.37374 | 0.26263 | 0.26263 | 0.26263 | 0.59596 | 0.59596 | 0.48 |
| 2 | "point=5" | 0.38384 | 0.27273 | 0.27273 | 0.27273 | 0.60606 | 0.60606 | 0.49 |
| 2 | "point=6" | 0.39394 | 0.28283 | 0.28283 | 0.28283 | 0.61616 | 0.61616 | 0.50 |
| 2 | "point=7" | 0.40404 | 0.29293 | 0.29293 | 0.29293 | 0.62626 | 0.62626 | 0.51 |
| 2 | "point=8" | 0.41414 | 0.30303 | 0.30303 | 0.30303 | 0.63636 | 0.63636 | 0.52 |
| 2 | "point=9" | 0.42424 | 0.31314 | 0.31314 | 0.31314 | 0.64646 | 0.64646 | 0.53 |
| 2 | "point=10" | 0.43434 | 0.32324 | 0.32324 | 0.32324 | 0.65656 | 0.65656 | 0.54 |
| 3 | "point=1" | 0.42516 | 0.27365 | 0.29385 | 0.23325 | 0.55647 | 0.60698 | 0.51 |
| 3 | "point=2" | 0.42608 | 0.27457 | 0.29477 | 0.23417 | 0.55739 | 0.60789 | 0.51 |
| 3 | "point=3" | 0.427 | 0.27549 | 0.29569 | 0.23508 | 0.55831 | 0.60881 | 0.51 |
| 3 | "point=4" | 0.42792 | 0.2764 | 0.29661 | 0.236 | 0.55923 | 0.60973 | 0.51 |
| 3 | "point=5" | 0.42884 | 0.27732 | 0.29752 | 0.23692 | 0.56015 | 0.61065 | 0.51 |
| 3 | "point=6" | 0.42975 | 0.27824 | 0.29844 | 0.23784 | 0.56106 | 0.61157 | 0.52 |
| 3 | "point=7" | 0.43067 | 0.27916 | 0.29936 | 0.23876 | 0.56198 | 0.61249 | 0.52 |
| 3 | "point=8" | 0.43159 | 0.28008 | 0.30028 | 0.23967 | 0.5629 | 0.6134 | 0.5 |
| 3 | "point=9" | 0.43251 | 0.281 | 0.3012 | 0.24059 | 0.56382 | 0.61432 | 0.52 |
| 3 | "point=10" | 0.43343 | 0.28191 | 0.30212 | 0.24151 | 0.56474 | 0.61524 | 0.52 |

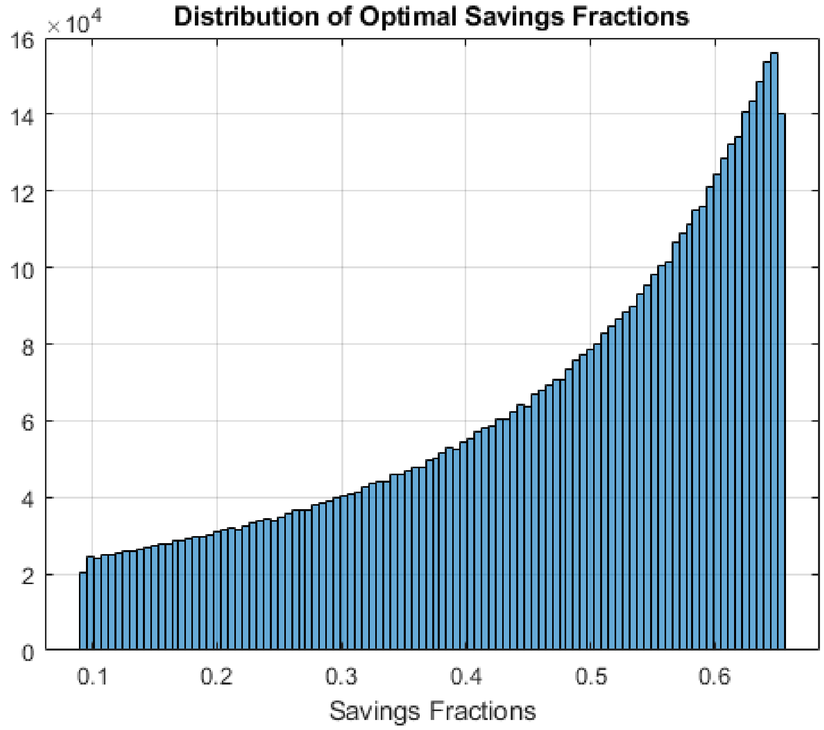

```
Elapsed time is 0.633499 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN      sum          mean         ---
                         -    ---    ----    -----    ----    ----    ---------    ----------    ---

    ar_opti_foc_obj      1     1      2        8        8       1     0.0033175    0.00041468    0.0
    ar_opti_save_frac    2     2      2        8        8       1        3.5124       0.43905      0

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx
              c1
          -----------

    r1     0.00087102
    r2      0.0033354
    r3     -0.0044871
    r4       0.001317
    r5     -0.0017862
    r6      0.0050249
    r7    -0.00058496
    r8    -0.00037273

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxxx
            c1
          -------

    r1    0.42838
    r2    0.28054
    r3     0.2989
    r4    0.23371
    r5    0.55877
    r6    0.61019
```

```
    r7     0.5202
    r8    0.58172
```

### 4.2.4 Test FF_OPTIM_MLSEC_SAVEZRONE Speed

Test Speed doing 6.25 million multisections for a savings problem:

```
% Generate the state-space and function
rng(123);
it_draws = 6250000; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
[ar_opti_save_frac, ar_opti_save_level] = ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, 

Elapsed time is 13.672896 seconds.

mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
mp_container_map('ar_opti_save_frac') = ar_opti_save_frac;
mp_container_map('ar_opti_save_level') = ar_opti_save_level;
mp_container_map('ar_opti_save_frac_notnan') = ar_opti_save_frac(~isnan(ar_opti_save_frac));
ff_container_map_display(mp_container_map);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i     idx    ndim     numel        rowN       colN       sum         m
                                -     ---    ----    --------    --------     ----    ----------    --

    ar_opti_save_frac           1      1       2     6.25e+06    6.25e+06      1       2.884e+06    0.
    ar_opti_save_frac_notnan    2      2       2     6.25e+06    6.25e+06      1       2.884e+06    0.
    ar_opti_save_level          3      3       2     6.25e+06    6.25e+06      1      2.9482e+06    0.

figure();
histogram(ar_opti_save_frac(~isnan(ar_opti_save_frac)),100);
title('Distribution of Optimal Savings Fractions');
xlabel('Savings Fractions');
grid on;
```




### 4.2.5 Define Two Period Intertemporal FOC Log Utility No Shock

See Household's Utility Maximization Problem and Two-Period Borrowing and Savings Problem given Endowments.

```
function [ar_deri_zero, ar_saveborr_level] = ffi_intertemporal_max(ar_saveborr_frac, z1, z2, r, beta
    ar_saveborr_level = ar_saveborr_frac.*(z1+z2./(1+r)) - z2./(1+r);
    ar_deri_zero = 1./(ar_saveborr_level-z1) + (beta.*(r+1))./(z2 + ar_saveborr_level.*(r+1));
end
```

## 4.3 FF_OPTIM_MZOOM_SAVEZRONE Derivative Multisection

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_optim_mzoom_savezrone** from the **MEconTools Package.** This functions solves for optimal savings/borrowing level given an anonymous function that provides the utility (not derivative) of a intertemporal savings problem. This is a vectorized function solves for multiple state-space elements at the same time. The function allows for controls of iteration counts, the number of evaluations per iteration, and how much to "zoom-in" for each iteration around the last iteration's maximum/optimal choice.

Note that if first order conditions are available this method should not be used, but **ff_optim_mlsec_savezrone** should be used. **ff_optim_mlsec_savezrone** relies on bisection. In the first example below more values are needed to achieve the same precision than under **ff_optim_mlsec_savezrone.** However, increasing might not expensive given vectorization, should increase time cost linearly in generally. MZOOM is much more robust than bisection based methods. And by increasing the number of points evaluated per iteration, in limited number of iterations, the approximately exact optimal savings choice can be found.

The vectorized zooming savings problem rely on this function to solve for optimal savings choices:

- States Grid + Approximate Continuous Exact Savings (zoom) as Share of Cash-on-Hand : **ff_vfi_az_zoom_vec**, precision and speed

### 4.3.1 Test FF_OPTIM_MZOOM_SAVEZRONE One Individual

Bisection for savings choice at one state:

```
% Generate the state-space and function
[fl_z1, fl_z2, fl_r, fl_beta] = deal(0.4730, 0.6252, 0.0839, 0.7365);
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_util = @(x) ffi_intertemporal_util(x, fl_z1, fl_z2, fl_r, fl_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
% optimally borrowing given the parameters here
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 15;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 10;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0.25;
[fl_opti_save_frac, fl_opti_save_level] = ...
    ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo)

Elapsed time is 0.004395 seconds.
fl_opti_save_frac = 0.4241
fl_opti_save_level = -0.1316
```

### 4.3.2 Test FF_OPTIM_MZOOM_SAVEZRONE 4 Individuals 3 Iterations 50 Points Per Iteration

5 grid points per iteration, and 5 iterations.

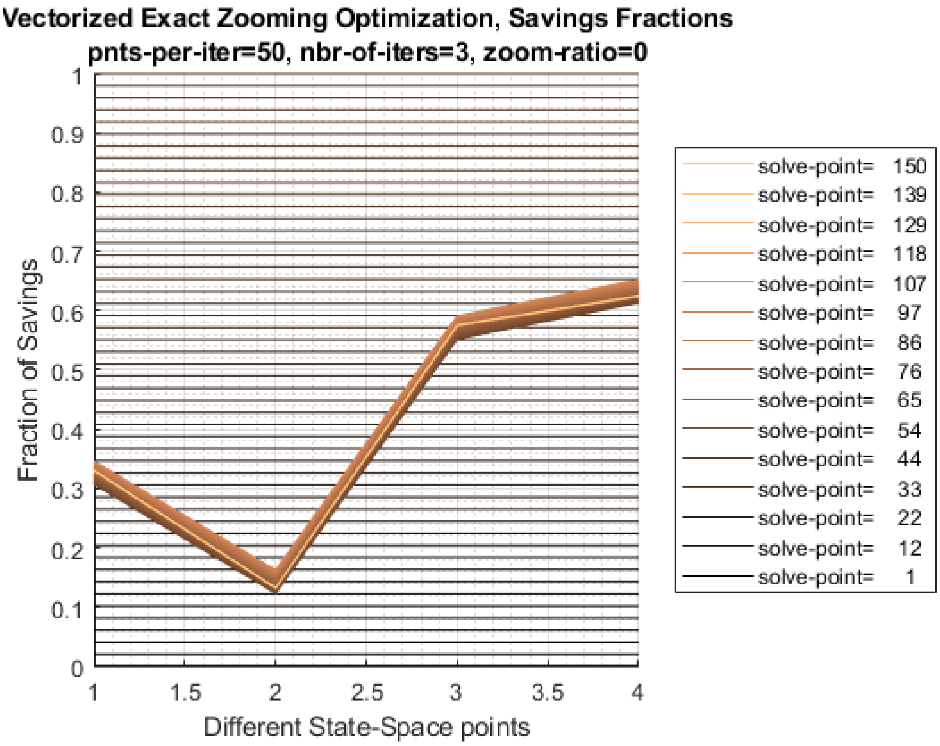
```
% Generate the state-space and function
rng(123);
it_draws = 4; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_util = @(x) ffi_intertemporal_util(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
bl_timer = true;
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 50;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 3;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0;
ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo);
```

| iter | cl_row_names_a | Var1 | Var2 | Var3 | Var4 |
|---|---|---|---|---|---|
| 1 | "point=1" | 1e-05 | 1e-05 | 1e-05 | 1e-05 |
| 1 | "point=2" | 0.020418 | 0.020418 | 0.020418 | 0.020418 |
| 1 | "point=3" | 0.040826 | 0.040826 | 0.040826 | 0.040826 |
| 1 | "point=4" | 0.061233 | 0.061233 | 0.061233 | 0.061233 |
| 1 | "point=5" | 0.081641 | 0.081641 | 0.081641 | 0.081641 |
| 1 | "point=6" | 0.10205 | 0.10205 | 0.10205 | 0.10205 |
| 1 | "point=7" | 0.12246 | 0.12246 | 0.12246 | 0.12246 |
| 1 | "point=8" | 0.14286 | 0.14286 | 0.14286 | 0.14286 |
| 1 | "point=9" | 0.16327 | 0.16327 | 0.16327 | 0.16327 |
| 1 | "point=10" | 0.18368 | 0.18368 | 0.18368 | 0.18368 |
| 1 | "point=11" | 0.20409 | 0.20409 | 0.20409 | 0.20409 |
| 1 | "point=12" | 0.2245 | 0.2245 | 0.2245 | 0.2245 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | "point=13" | 0.2449 | 0.2449 | 0.2449 | 0.2449 |
| 1 | "point=14" | 0.26531 | 0.26531 | 0.26531 | 0.26531 |
| 1 | "point=15" | 0.28572 | 0.28572 | 0.28572 | 0.28572 |
| 1 | "point=16" | 0.30613 | 0.30613 | 0.30613 | 0.30613 |
| 1 | "point=17" | 0.32653 | 0.32653 | 0.32653 | 0.32653 |
| 1 | "point=18" | 0.34694 | 0.34694 | 0.34694 | 0.34694 |
| 1 | "point=19" | 0.36735 | 0.36735 | 0.36735 | 0.36735 |
| 1 | "point=20" | 0.38776 | 0.38776 | 0.38776 | 0.38776 |
| 1 | "point=21" | 0.40817 | 0.40817 | 0.40817 | 0.40817 |
| 1 | "point=22" | 0.42857 | 0.42857 | 0.42857 | 0.42857 |
| 1 | "point=23" | 0.44898 | 0.44898 | 0.44898 | 0.44898 |
| 1 | "point=24" | 0.46939 | 0.46939 | 0.46939 | 0.46939 |
| 1 | "point=25" | 0.4898 | 0.4898 | 0.4898 | 0.4898 |
| 1 | "point=26" | 0.5102 | 0.5102 | 0.5102 | 0.5102 |
| 1 | "point=27" | 0.53061 | 0.53061 | 0.53061 | 0.53061 |
| 1 | "point=28" | 0.55102 | 0.55102 | 0.55102 | 0.55102 |
| 1 | "point=29" | 0.57143 | 0.57143 | 0.57143 | 0.57143 |
| 1 | "point=30" | 0.59183 | 0.59183 | 0.59183 | 0.59183 |
| 1 | "point=31" | 0.61224 | 0.61224 | 0.61224 | 0.61224 |
| 1 | "point=32" | 0.63265 | 0.63265 | 0.63265 | 0.63265 |
| 1 | "point=33" | 0.65306 | 0.65306 | 0.65306 | 0.65306 |
| 1 | "point=34" | 0.67347 | 0.67347 | 0.67347 | 0.67347 |
| 1 | "point=35" | 0.69387 | 0.69387 | 0.69387 | 0.69387 |
| 1 | "point=36" | 0.71428 | 0.71428 | 0.71428 | 0.71428 |
| 1 | "point=37" | 0.73469 | 0.73469 | 0.73469 | 0.73469 |
| 1 | "point=38" | 0.7551 | 0.7551 | 0.7551 | 0.7551 |
| 1 | "point=39" | 0.7755 | 0.7755 | 0.7755 | 0.7755 |
| 1 | "point=40" | 0.79591 | 0.79591 | 0.79591 | 0.79591 |
| 1 | "point=41" | 0.81632 | 0.81632 | 0.81632 | 0.81632 |
| 1 | "point=42" | 0.83673 | 0.83673 | 0.83673 | 0.83673 |
| 1 | "point=43" | 0.85714 | 0.85714 | 0.85714 | 0.85714 |
| 1 | "point=44" | 0.87754 | 0.87754 | 0.87754 | 0.87754 |
| 1 | "point=45" | 0.89795 | 0.89795 | 0.89795 | 0.89795 |
| 1 | "point=46" | 0.91836 | 0.91836 | 0.91836 | 0.91836 |
| 1 | "point=47" | 0.93877 | 0.93877 | 0.93877 | 0.93877 |
| 1 | "point=48" | 0.95917 | 0.95917 | 0.95917 | 0.95917 |
| 1 | "point=49" | 0.97958 | 0.97958 | 0.97958 | 0.97958 |
| 1 | "point=50" | 0.99999 | 0.99999 | 0.99999 | 0.99999 |
| 2 | "point=1" | 0.30693 | 0.12326 | 0.55182 | 0.61304 |
| 2 | "point=2" | 0.30773 | 0.12406 | 0.55262 | 0.61384 |
| 2 | "point=3" | 0.30853 | 0.12486 | 0.55342 | 0.61464 |
| 2 | "point=4" | 0.30933 | 0.12566 | 0.55422 | 0.61544 |
| 2 | "point=5" | 0.31013 | 0.12646 | 0.55502 | 0.61624 |
| 2 | "point=6" | 0.31093 | 0.12726 | 0.55582 | 0.61704 |
| 2 | "point=7" | 0.31173 | 0.12806 | 0.55662 | 0.61784 |
| 2 | "point=8" | 0.31253 | 0.12886 | 0.55742 | 0.61865 |
| 2 | "point=9" | 0.31333 | 0.12966 | 0.55822 | 0.61945 |
| 2 | "point=10" | 0.31413 | 0.13046 | 0.55902 | 0.62025 |
| 2 | "point=11" | 0.31493 | 0.13126 | 0.55982 | 0.62105 |
| 2 | "point=12" | 0.31573 | 0.13206 | 0.56062 | 0.62185 |
| 2 | "point=13" | 0.31653 | 0.13286 | 0.56142 | 0.62265 |
| 2 | "point=14" | 0.31733 | 0.13366 | 0.56222 | 0.62345 |
| 2 | "point=15" | 0.31813 | 0.13446 | 0.56302 | 0.62425 |
| 2 | "point=16" | 0.31893 | 0.13526 | 0.56382 | 0.62505 |
| 2 | "point=17" | 0.31973 | 0.13606 | 0.56462 | 0.62585 |
| 2 | "point=18" | 0.32053 | 0.13686 | 0.56542 | 0.62665 |
| 2 | "point=19" | 0.32133 | 0.13766 | 0.56623 | 0.62745 |
| 2 | "point=20" | 0.32213 | 0.13846 | 0.56703 | 0.62825 |

| | | | | | |
|---|---|---|---|---|---|
| 2 | "point=21" | 0.32293 | 0.13926 | 0.56783 | 0.62905 |
| 2 | "point=22" | 0.32373 | 0.14006 | 0.56863 | 0.62985 |
| 2 | "point=23" | 0.32453 | 0.14086 | 0.56943 | 0.63065 |
| 2 | "point=24" | 0.32533 | 0.14166 | 0.57023 | 0.63145 |
| 2 | "point=25" | 0.32613 | 0.14246 | 0.57103 | 0.63225 |
| 2 | "point=26" | 0.32693 | 0.14326 | 0.57183 | 0.63305 |
| 2 | "point=27" | 0.32773 | 0.14406 | 0.57263 | 0.63385 |
| 2 | "point=28" | 0.32853 | 0.14487 | 0.57343 | 0.63465 |
| 2 | "point=29" | 0.32934 | 0.14567 | 0.57423 | 0.63545 |
| 2 | "point=30" | 0.33014 | 0.14647 | 0.57503 | 0.63625 |
| 2 | "point=31" | 0.33094 | 0.14727 | 0.57583 | 0.63705 |
| 2 | "point=32" | 0.33174 | 0.14807 | 0.57663 | 0.63785 |
| 2 | "point=33" | 0.33254 | 0.14887 | 0.57743 | 0.63865 |
| 2 | "point=34" | 0.33334 | 0.14967 | 0.57823 | 0.63945 |
| 2 | "point=35" | 0.33414 | 0.15047 | 0.57903 | 0.64025 |
| 2 | "point=36" | 0.33494 | 0.15127 | 0.57983 | 0.64105 |
| 2 | "point=37" | 0.33574 | 0.15207 | 0.58063 | 0.64185 |
| 2 | "point=38" | 0.33654 | 0.15287 | 0.58143 | 0.64265 |
| 2 | "point=39" | 0.33734 | 0.15367 | 0.58223 | 0.64345 |
| 2 | "point=40" | 0.33814 | 0.15447 | 0.58303 | 0.64425 |
| 2 | "point=41" | 0.33894 | 0.15527 | 0.58383 | 0.64506 |
| 2 | "point=42" | 0.33974 | 0.15607 | 0.58463 | 0.64586 |
| 2 | "point=43" | 0.34054 | 0.15687 | 0.58543 | 0.64666 |
| 2 | "point=44" | 0.34134 | 0.15767 | 0.58623 | 0.64746 |
| 2 | "point=45" | 0.34214 | 0.15847 | 0.58703 | 0.64826 |
| 2 | "point=46" | 0.34294 | 0.15927 | 0.58783 | 0.64906 |
| 2 | "point=47" | 0.34374 | 0.16007 | 0.58863 | 0.64986 |
| 2 | "point=48" | 0.34454 | 0.16087 | 0.58943 | 0.65066 |
| 2 | "point=49" | 0.34534 | 0.16167 | 0.59023 | 0.65146 |
| 2 | "point=50" | 0.34614 | 0.16247 | 0.59103 | 0.65226 |
| 3 | "point=1" | 0.32937 | 0.13129 | 0.57426 | 0.62348 |
| 3 | "point=2" | 0.3294 | 0.13132 | 0.57429 | 0.62351 |
| 3 | "point=3" | 0.32943 | 0.13135 | 0.57432 | 0.62354 |
| 3 | "point=4" | 0.32946 | 0.13139 | 0.57435 | 0.62357 |
| 3 | "point=5" | 0.32949 | 0.13142 | 0.57439 | 0.6236 |
| 3 | "point=6" | 0.32952 | 0.13145 | 0.57442 | 0.62364 |
| 3 | "point=7" | 0.32955 | 0.13148 | 0.57445 | 0.62367 |
| 3 | "point=8" | 0.32959 | 0.13151 | 0.57448 | 0.6237 |
| 3 | "point=9" | 0.32962 | 0.13154 | 0.57451 | 0.62373 |
| 3 | "point=10" | 0.32965 | 0.13157 | 0.57454 | 0.62376 |
| 3 | "point=11" | 0.32968 | 0.13161 | 0.57457 | 0.62379 |
| 3 | "point=12" | 0.32971 | 0.13164 | 0.5746 | 0.62382 |
| 3 | "point=13" | 0.32974 | 0.13167 | 0.57464 | 0.62385 |
| 3 | "point=14" | 0.32977 | 0.1317 | 0.57467 | 0.62389 |
| 3 | "point=15" | 0.32981 | 0.13173 | 0.5747 | 0.62392 |
| 3 | "point=16" | 0.32984 | 0.13176 | 0.57473 | 0.62395 |
| 3 | "point=17" | 0.32987 | 0.13179 | 0.57476 | 0.62398 |
| 3 | "point=18" | 0.3299 | 0.13182 | 0.57479 | 0.62401 |
| 3 | "point=19" | 0.32993 | 0.13186 | 0.57482 | 0.62404 |
| 3 | "point=20" | 0.32996 | 0.13189 | 0.57486 | 0.62407 |
| 3 | "point=21" | 0.32999 | 0.13192 | 0.57489 | 0.62411 |
| 3 | "point=22" | 0.33003 | 0.13195 | 0.57492 | 0.62414 |
| 3 | "point=23" | 0.33006 | 0.13198 | 0.57495 | 0.62417 |
| 3 | "point=24" | 0.33009 | 0.13201 | 0.57498 | 0.6242 |
| 3 | "point=25" | 0.33012 | 0.13204 | 0.57501 | 0.62423 |
| 3 | "point=26" | 0.33015 | 0.13208 | 0.57504 | 0.62426 |
| 3 | "point=27" | 0.33018 | 0.13211 | 0.57508 | 0.62429 |
| 3 | "point=28" | 0.33021 | 0.13214 | 0.57511 | 0.62433 |

| | | | | | |
|---|---|---|---|---|---|
| 3 | "point=29" | 0.33025 | 0.13217 | 0.57514 | 0.62436 |
| 3 | "point=30" | 0.33028 | 0.1322 | 0.57517 | 0.62439 |
| 3 | "point=31" | 0.33031 | 0.13223 | 0.5752 | 0.62442 |
| 3 | "point=32" | 0.33034 | 0.13226 | 0.57523 | 0.62445 |
| 3 | "point=33" | 0.33037 | 0.1323 | 0.57526 | 0.62448 |
| 3 | "point=34" | 0.3304 | 0.13233 | 0.5753 | 0.62451 |
| 3 | "point=35" | 0.33043 | 0.13236 | 0.57533 | 0.62455 |
| 3 | "point=36" | 0.33046 | 0.13239 | 0.57536 | 0.62458 |
| 3 | "point=37" | 0.3305 | 0.13242 | 0.57539 | 0.62461 |
| 3 | "point=38" | 0.33053 | 0.13245 | 0.57542 | 0.62464 |
| 3 | "point=39" | 0.33056 | 0.13248 | 0.57545 | 0.62467 |
| 3 | "point=40" | 0.33059 | 0.13252 | 0.57548 | 0.6247 |
| 3 | "point=41" | 0.33062 | 0.13255 | 0.57551 | 0.62473 |
| 3 | "point=42" | 0.33065 | 0.13258 | 0.57555 | 0.62477 |
| 3 | "point=43" | 0.33068 | 0.13261 | 0.57558 | 0.6248 |
| 3 | "point=44" | 0.33072 | 0.13264 | 0.57561 | 0.62483 |
| 3 | "point=45" | 0.33075 | 0.13267 | 0.57564 | 0.62486 |
| 3 | "point=46" | 0.33078 | 0.1327 | 0.57567 | 0.62489 |
| 3 | "point=47" | 0.33081 | 0.13273 | 0.5757 | 0.62492 |
| 3 | "point=48" | 0.33084 | 0.13277 | 0.57573 | 0.62495 |
| 3 | "point=49" | 0.33087 | 0.1328 | 0.57577 | 0.62498 |
| 3 | "point=50" | 0.3309 | 0.13283 | 0.5758 | 0.62502 |




| iter | cl_row_names_a | Var1 | Var2 | Var3 | Var4 |
|---|---|---|---|---|---|
| 1 | "point=1" | -3.6912 | -1.9565 | -12.83 | -14.789 |
| 1 | "point=2" | 0.058694 | -0.80561 | -2.4984 | -2.1254 |
| 1 | "point=3" | 0.38043 | -0.72015 | -1.5784 | -0.99337 |
| 1 | "point=4" | 0.55947 | -0.67935 | -1.0493 | -0.34024 |
| 1 | "point=5" | 0.67979 | -0.65711 | -0.68055 | 0.11647 |
| 1 | "point=6" | 0.7677 | -0.64529 | -0.39997 | 0.46531 |
| 1 | "point=7" | 0.8349 | -0.64026 | -0.17534 | 0.74571 |
| 1 | "point=8" | 0.88763 | -0.6401 | 0.010483 | 0.9787 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | "point=9" | 0.92959 | -0.64367 | 0.16774 | 1.1768 |
| 1 | "point=10" | 0.96316 | -0.65026 | 0.30302 | 1.3481 |
| 1 | "point=11" | 0.98996 | -0.65938 | 0.4208 | 1.4981 |
| 1 | "point=12" | 1.0111 | -0.67071 | 0.52427 | 1.6308 |
| 1 | "point=13" | 1.0275 | -0.684 | 0.61578 | 1.7489 |
| 1 | "point=14" | 1.0397 | -0.6991 | 0.69709 | 1.8547 |
| 1 | "point=15" | 1.0482 | -0.71588 | 0.76958 | 1.9499 |
| 1 | "point=16" | 1.0533 | -0.73426 | 0.83429 | 2.0357 |
| 1 | "point=17" | 1.0554 | -0.75419 | 0.8921 | 2.1132 |
| 1 | "point=18" | 1.0546 | -0.77564 | 0.94367 | 2.1833 |
| 1 | "point=19" | 1.0512 | -0.79861 | 0.98955 | 2.2467 |
| 1 | "point=20" | 1.0451 | -0.82309 | 1.0302 | 2.3039 |
| 1 | "point=21" | 1.0366 | -0.8491 | 1.066 | 2.3554 |
| 1 | "point=22" | 1.0256 | -0.87669 | 1.0971 | 2.4015 |
| 1 | "point=23" | 1.0123 | -0.90591 | 1.124 | 2.4425 |
| 1 | "point=24" | 0.99654 | -0.93682 | 1.1466 | 2.4788 |
| 1 | "point=25" | 0.97838 | -0.9695 | 1.1652 | 2.5104 |
| 1 | "point=26" | 0.95775 | -1.004 | 1.1798 | 2.5375 |
| 1 | "point=27" | 0.93459 | -1.0406 | 1.1905 | 2.5602 |
| 1 | "point=28" | 0.90881 | -1.0792 | 1.1973 | 2.5785 |
| 1 | "point=29" | 0.88029 | -1.1202 | 1.2002 | 2.5925 |
| 1 | "point=30" | 0.84886 | -1.1635 | 1.1991 | 2.6022 |
| 1 | "point=31" | 0.81434 | -1.2096 | 1.1938 | 2.6073 |
| 1 | "point=32" | 0.77649 | -1.2587 | 1.1843 | 2.6078 |
| 1 | "point=33" | 0.73504 | -1.3109 | 1.1703 | 2.6035 |
| 1 | "point=34" | 0.68964 | -1.3668 | 1.1514 | 2.594 |
| 1 | "point=35" | 0.63987 | -1.4268 | 1.1274 | 2.5792 |
| 1 | "point=36" | 0.58522 | -1.4913 | 1.0978 | 2.5584 |
| 1 | "point=37" | 0.52505 | -1.5611 | 1.062 | 2.5312 |
| 1 | "point=38" | 0.45857 | -1.6369 | 1.0192 | 2.4968 |
| 1 | "point=39" | 0.38475 | -1.7198 | 0.96837 | 2.4541 |
| 1 | "point=40" | 0.3023 | -1.8111 | 0.90834 | 2.4021 |
| 1 | "point=41" | 0.20947 | -1.9126 | 0.83737 | 2.3388 |
| 1 | "point=42" | 0.10391 | -2.0266 | 0.75313 | 2.2622 |
| 1 | "point=43" | -0.017693 | -2.1564 | 0.65234 | 2.1687 |
| 1 | "point=44" | -0.16019 | -2.3069 | 0.53016 | 2.0538 |
| 1 | "point=45" | -0.33112 | -2.4857 | 0.37908 | 1.9097 |
| 1 | "point=46" | -0.54312 | -2.7054 | 0.18649 | 1.724 |
| 1 | "point=47" | -0.81989 | -2.9896 | -0.071303 | 1.4729 |
| 1 | "point=48" | -1.2146 | -3.3917 | -0.44748 | 1.1033 |
| 1 | "point=49" | -1.8971 | -4.0814 | -1.1118 | 0.44547 |
| 1 | "point=50" | -9.5085 | -11.7 | -8.7054 | -7.1418 |
| 2 | "point=1" | 1.0535 | -0.64017 | 1.1975 | 2.6074 |
| 2 | "point=2" | 1.0536 | -0.64009 | 1.1977 | 2.6075 |
| 2 | "point=3" | 1.0537 | -0.64001 | 1.1979 | 2.6076 |
| 2 | "point=4" | 1.0539 | -0.63995 | 1.198 | 2.6077 |
| 2 | "point=5" | 1.054 | -0.63989 | 1.1982 | 2.6077 |
| 2 | "point=6" | 1.0541 | -0.63983 | 1.1984 | 2.6078 |
| 2 | "point=7" | 1.0542 | -0.63979 | 1.1985 | 2.6079 |
| 2 | "point=8" | 1.0543 | -0.63975 | 1.1986 | 2.6079 |
| 2 | "point=9" | 1.0544 | -0.63971 | 1.1988 | 2.608 |
| 2 | "point=10" | 1.0545 | -0.63969 | 1.1989 | 2.608 |
| 2 | "point=11" | 1.0546 | -0.63967 | 1.199 | 2.6081 |
| 2 | "point=12" | 1.0547 | -0.63966 | 1.1992 | 2.6081 |
| 2 | "point=13" | 1.0548 | -0.63965 | 1.1993 | 2.6081 |
| 2 | "point=14" | 1.0548 | -0.63965 | 1.1994 | 2.6081 |
| 2 | "point=15" | 1.0549 | -0.63966 | 1.1995 | 2.6081 |
| 2 | "point=16" | 1.055 | -0.63967 | 1.1996 | 2.6081 |

| | | | | | |
|---|---|---|---|---|---|
| 2 | "point=17" | 1.0551 | -0.63969 | 1.1997 | 2.6081 |
| 2 | "point=18" | 1.0551 | -0.63971 | 1.1998 | 2.6081 |
| 2 | "point=19" | 1.0552 | -0.63975 | 1.1998 | 2.6081 |
| 2 | "point=20" | 1.0552 | -0.63978 | 1.1999 | 2.6081 |
| 2 | "point=21" | 1.0553 | -0.63983 | 1.2 | 2.608 |
| 2 | "point=22" | 1.0553 | -0.63988 | 1.2 | 2.608 |
| 2 | "point=23" | 1.0553 | -0.63993 | 1.2001 | 2.6079 |
| 2 | "point=24" | 1.0554 | -0.63999 | 1.2001 | 2.6079 |
| 2 | "point=25" | 1.0554 | -0.64006 | 1.2002 | 2.6078 |
| 2 | "point=26" | 1.0554 | -0.64013 | 1.2002 | 2.6077 |
| 2 | "point=27" | 1.0555 | -0.64021 | 1.2002 | 2.6077 |
| 2 | "point=28" | 1.0555 | -0.64029 | 1.2003 | 2.6076 |
| 2 | "point=29" | 1.0555 | -0.64038 | 1.2003 | 2.6075 |
| 2 | "point=30" | 1.0555 | -0.64048 | 1.2003 | 2.6074 |
| 2 | "point=31" | 1.0555 | -0.64058 | 1.2003 | 2.6073 |
| 2 | "point=32" | 1.0555 | -0.64069 | 1.2003 | 2.6071 |
| 2 | "point=33" | 1.0555 | -0.6408 | 1.2003 | 2.607 |
| 2 | "point=34" | 1.0555 | -0.64091 | 1.2003 | 2.6069 |
| 2 | "point=35" | 1.0555 | -0.64104 | 1.2002 | 2.6067 |
| 2 | "point=36" | 1.0554 | -0.64116 | 1.2002 | 2.6066 |
| 2 | "point=37" | 1.0554 | -0.64129 | 1.2002 | 2.6064 |
| 2 | "point=38" | 1.0554 | -0.64143 | 1.2001 | 2.6063 |
| 2 | "point=39" | 1.0554 | -0.64157 | 1.2001 | 2.6061 |
| 2 | "point=40" | 1.0553 | -0.64172 | 1.2001 | 2.6059 |
| 2 | "point=41" | 1.0553 | -0.64188 | 1.2 | 2.6057 |
| 2 | "point=42" | 1.0552 | -0.64203 | 1.1999 | 2.6056 |
| 2 | "point=43" | 1.0552 | -0.6422 | 1.1999 | 2.6053 |
| 2 | "point=44" | 1.0551 | -0.64236 | 1.1998 | 2.6051 |
| 2 | "point=45" | 1.0551 | -0.64254 | 1.1997 | 2.6049 |
| 2 | "point=46" | 1.055 | -0.64271 | 1.1996 | 2.6047 |
| 2 | "point=47" | 1.0549 | -0.64289 | 1.1995 | 2.6045 |
| 2 | "point=48" | 1.0549 | -0.64308 | 1.1994 | 2.6042 |
| 2 | "point=49" | 1.0548 | -0.64327 | 1.1993 | 2.604 |
| 2 | "point=50" | 1.0547 | -0.64347 | 1.1992 | 2.6037 |
| 3 | "point=1" | 1.0555 | -0.63967 | 1.2003 | 2.6081 |
| 3 | "point=2" | 1.0555 | -0.63967 | 1.2003 | 2.6081 |
| 3 | "point=3" | 1.0555 | -0.63967 | 1.2003 | 2.6081 |
| 3 | "point=4" | 1.0555 | -0.63967 | 1.2003 | 2.6081 |
| 3 | "point=5" | 1.0555 | -0.63967 | 1.2003 | 2.6081 |
| 3 | "point=6" | 1.0555 | -0.63967 | 1.2003 | 2.6081 |
| 3 | "point=7" | 1.0555 | -0.63967 | 1.2003 | 2.6081 |
| 3 | "point=8" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=9" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=10" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=11" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=12" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=13" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=14" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=15" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=16" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=17" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=18" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=19" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=20" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=21" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=22" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=23" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |
| 3 | "point=24" | 1.0555 | -0.63966 | 1.2003 | 2.6081 |

```
3        "point=25"      1.0555   -0.63966     1.2003      2.6081
3        "point=26"      1.0555   -0.63966     1.2003      2.6081
3        "point=27"      1.0555   -0.63966     1.2003      2.6081
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3        "point=34"      1.0555   -0.63965     1.2003      2.6081
3        "point=35"      1.0555   -0.63965     1.2003      2.6081
3        "point=36"      1.0555   -0.63965     1.2003      2.6081
3        "point=37"      1.0555   -0.63965     1.2003      2.6081
3        "point=38"      1.0555   -0.63965     1.2003      2.6081
3        "point=39"      1.0555   -0.63965     1.2003      2.6081
3        "point=40"      1.0555   -0.63965     1.2003      2.6081
3        "point=41"      1.0555   -0.63965     1.2003      2.6081
3        "point=42"      1.0555   -0.63965     1.2003      2.6081
3        "point=43"      1.0555   -0.63965     1.2003      2.6081
3        "point=44"      1.0555   -0.63965     1.2003      2.6081
3        "point=45"      1.0555   -0.63965     1.2003      2.6081
3        "point=46"      1.0555   -0.63965     1.2003      2.6081
3        "point=47"      1.0555   -0.63965     1.2003      2.6081
3        "point=48"      1.0555   -0.63965     1.2003      2.6081
3        "point=49"      1.0555   -0.63965     1.2003      2.6081
3        "point=50"      1.0555   -0.63965     1.2003      2.6081
```
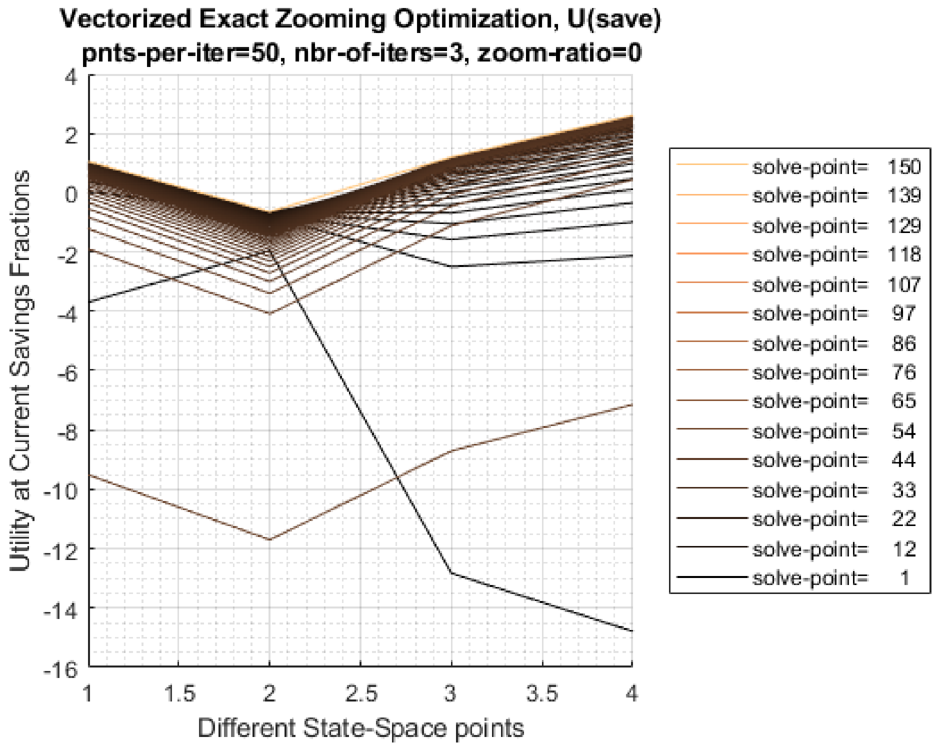




```
Elapsed time is 1.292487 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx   ndim    numel    rowN    colN     sum      mean      std
                         -    ---   ----    -----    ----    ----    ------    ------    ------
```

```
    ar_opti_foc_obj      1     1      2        4       1       4      4.2243    1.0561    1.3298
    ar_opti_save_frac    2     2      2        4       4       1       1.664     0.416    0.2284

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx
            c1         c2         c3        c4
          ------    --------    ------    ------

    r1    1.0555    -0.63965    1.2003    2.6081

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxxx
            c1
          -------

    r1    0.33086
    r2    0.13278
    r3    0.57575
    r4    0.62461
```

### 4.3.3 Test FF_OPTIM_MZOOM_SAVEZRONE 8 Individuals 3 Iterations 10 Points Per Iteration, 0.25 zoom in ratio

10 grid points per iteration, and 3 iterations.

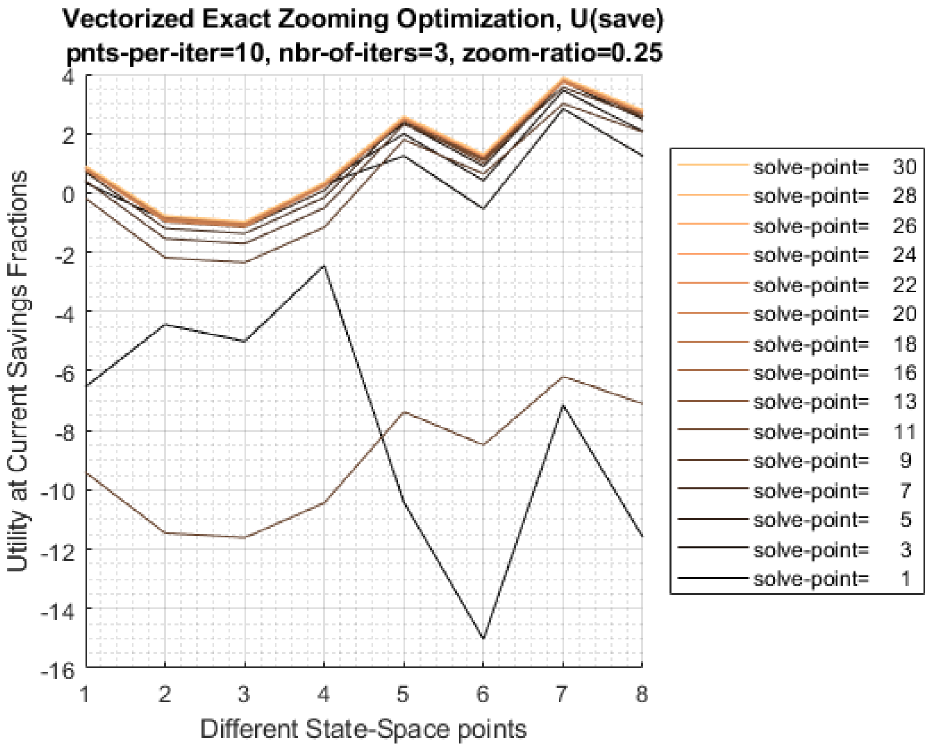
```
% Generate the state-space and function
rng(123);
it_draws = 8; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_util = @(x) ffi_intertemporal_util(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
bl_timer = true;
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 10;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 3;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0.25;
ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo);
```

```
    iter    cl_row_names_a     Var1       Var2       Var3       Var4       Var5       Var6       Var
    ----    --------------    -------    -------    -------    -------    -------    -------    ----

     1        "point=1"         1e-05      1e-05      1e-05      1e-05      1e-05      1e-05      1e
     1        "point=2"       0.11112    0.11112    0.11112    0.11112    0.11112    0.11112    0.11
     1        "point=3"       0.22223    0.22223    0.22223    0.22223    0.22223    0.22223    0.22
     1        "point=4"       0.33334    0.33334    0.33334    0.33334    0.33334    0.33334    0.33
     1        "point=5"       0.44445    0.44445    0.44445    0.44445    0.44445    0.44445    0.44
     1        "point=6"       0.55555    0.55555    0.55555    0.55555    0.55555    0.55555    0.55
     1        "point=7"       0.66666    0.66666    0.66666    0.66666    0.66666    0.66666    0.66
     1        "point=8"       0.77777    0.77777    0.77777    0.77777    0.77777    0.77777    0.77
     1        "point=9"       0.88888    0.88888    0.88888    0.88888    0.88888    0.88888    0.88
     1        "point=10"      0.99999    0.99999    0.99999    0.99999    0.99999    0.99999    0.99
     2        "point=1"       0.28788    0.20455    0.20455    0.12122    0.37121    0.37121    0.37
     2        "point=2"       0.32576    0.24243    0.24243     0.1591    0.40909    0.40909    0.40
     2        "point=3"       0.36364    0.28031    0.28031    0.19698    0.44697    0.44697    0.44
     2        "point=4"       0.40152    0.31819    0.31819    0.23485    0.48485    0.48485    0.48
     2        "point=5"        0.4394    0.35606    0.35606    0.27273    0.52273    0.52273    0.52
```

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | "point=6" | 0.47727 | 0.39394 | 0.39394 | 0.31061 | 0.5606 | 0.5606 | 0.5 |
| 2 | "point=7" | 0.51515 | 0.43182 | 0.43182 | 0.34849 | 0.59848 | 0.59848 | 0.59 |
| 2 | "point=8" | 0.55303 | 0.4697 | 0.4697 | 0.38637 | 0.63636 | 0.63636 | 0.63 |
| 2 | "point=9" | 0.59091 | 0.50758 | 0.50758 | 0.42424 | 0.67424 | 0.67424 | 0.67 |
| 2 | "point=10" | 0.62879 | 0.54545 | 0.54545 | 0.46212 | 0.71212 | 0.71212 | 0.71 |
| 3 | "point=1" | 0.34987 | 0.20972 | 0.20972 | 0.15479 | 0.46161 | 0.49001 | 0.4 |
| 3 | "point=2" | 0.3645 | 0.22435 | 0.22435 | 0.16943 | 0.47624 | 0.50465 | 0.44 |
| 3 | "point=3" | 0.37913 | 0.23899 | 0.23899 | 0.18406 | 0.49087 | 0.51928 | 0.46 |
| 3 | "point=4" | 0.39377 | 0.25362 | 0.25362 | 0.1987 | 0.50551 | 0.53392 | 0.4 |
| 3 | "point=5" | 0.4084 | 0.26826 | 0.26826 | 0.21333 | 0.52014 | 0.54855 | 0.49 |
| 3 | "point=6" | 0.42304 | 0.28289 | 0.28289 | 0.22797 | 0.53478 | 0.56319 | 0.50 |
| 3 | "point=7" | 0.43767 | 0.29752 | 0.29752 | 0.2426 | 0.54941 | 0.57782 | 0.52 |
| 3 | "point=8" | 0.45231 | 0.31216 | 0.31216 | 0.25724 | 0.56405 | 0.59246 | 0.53 |
| 3 | "point=9" | 0.46694 | 0.32679 | 0.32679 | 0.27187 | 0.57868 | 0.60709 | 0.55 |
| 3 | "point=10" | 0.48158 | 0.34143 | 0.34143 | 0.28651 | 0.59332 | 0.62173 | 0.56 |


| iter | cl_row_names_a | Var1 | Var2 | Var3 | Var4 | Var5 | Var6 |
|---|---|---|---|---|---|---|---|
| 1 | "point=1" | -6.5286 | -4.4312 | -4.9951 | -2.4407 | -10.415 | -15.025 |
| 1 | "point=2" | 0.34227 | -0.90966 | -1.148 | 0.28691 | 1.2451 | -0.53687 |
| 1 | "point=3" | 0.7287 | -0.77242 | -0.98657 | 0.36508 | 1.9879 | 0.4163 |
| 1 | "point=4" | 0.87872 | -0.76818 | -0.96816 | 0.33477 | 2.3463 | 0.89785 |
| 1 | "point=5" | 0.91222 | -0.83811 | -1.028 | 0.24031 | 2.5277 | 1.1666 |
| 1 | "point=6" | 0.85648 | -0.97408 | -1.1562 | 0.085331 | 2.5867 | 1.2933 |
| 1 | "point=7" | 0.70558 | -1.1905 | -1.3663 | -0.14666 | 2.5296 | 1.2915 |
| 1 | "point=8" | 0.41577 | -1.5358 | -1.7061 | -0.50502 | 2.319 | 1.1277 |
| 1 | "point=9" | -0.17716 | -2.1767 | -2.3424 | -1.1573 | 1.7947 | 0.64395 |
| 1 | "point=10" | -9.4046 | -11.446 | -11.608 | -10.437 | -7.3721 | -8.4872 |
| 2 | "point=1" | 0.8347 | -0.78233 | -0.99938 | 0.30205 | 2.4239 | 1.0081 |
| 2 | "point=2" | 0.87277 | -0.76475 | -0.97586 | 0.34105 | 2.4846 | 1.0983 |
| 2 | "point=3" | 0.89748 | -0.75933 | -0.96536 | 0.36018 | 2.5303 | 1.1709 |
| 2 | "point=4" | 0.91044 | -0.76388 | -0.96549 | 0.36559 | 2.5622 | 1.2275 |
| 2 | "point=5" | 0.91269 | -0.7771 | -0.97477 | 0.36049 | 2.581 | 1.269 |

```
2     "point=6"    0.90477   -0.79823  -0.99237   0.34672   2.5867    1.296
2     "point=7"    0.88684    -0.8269   -1.0178   0.32535   2.5793   1.3084
2     "point=8"    0.85872   -0.86304    -1.051   0.29697   2.5578   1.3055
2     "point=9"    0.81987   -0.90685   -1.0921   0.26182   2.5209   1.2862
2     "point=10"   0.76932   -0.95877   -1.1415   0.21989   2.4664   1.2483
3     "point=1"    0.88992    -0.7791  -0.99528   0.33777   2.5443    1.234
3     "point=2"     0.8979   -0.77144  -0.98526    0.3479   2.5562    1.251
3     "point=3"    0.90413    -0.7658  -0.97741   0.35543   2.5661   1.2659
3     "point=4"    0.90869     -0.762  -0.97154    0.3607   2.5741   1.2785
3     "point=5"    0.91163   -0.75989  -0.96746   0.36397   2.5801    1.289
3     "point=6"    0.91299   -0.75934  -0.96506   0.36546   2.5842   1.2974
3     "point=7"    0.91281   -0.76025  -0.96421   0.36532   2.5864   1.3035
3     "point=8"    0.91112   -0.76255  -0.96482    0.3637   2.5866   1.3074
3     "point=9"    0.90792   -0.76615  -0.96683    0.3607   2.5849   1.3091
3     "point=10"   0.90324   -0.77102  -0.97016   0.35641   2.5811   1.3085
```




```
Elapsed time is 1.047404 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN     sum       mean        std
                         _    ___    ____    _____    ____    ____    ______    _______    _______

    ar_opti_foc_obj      1     1      2        8        1       8     10.125     1.2656      1.731
    ar_opti_save_frac    2     2      2        8        8       1     3.3843    0.42304    0.15074

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx
            c1          c2          c3          c4         c5         c6         c7         c8
          _______    ________    ________    _______    ______    ______    ______    ______

    r1    0.91163    -0.75989    -0.96506    0.36397    2.5864    1.3074    3.8892    2.7911

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxxx
```

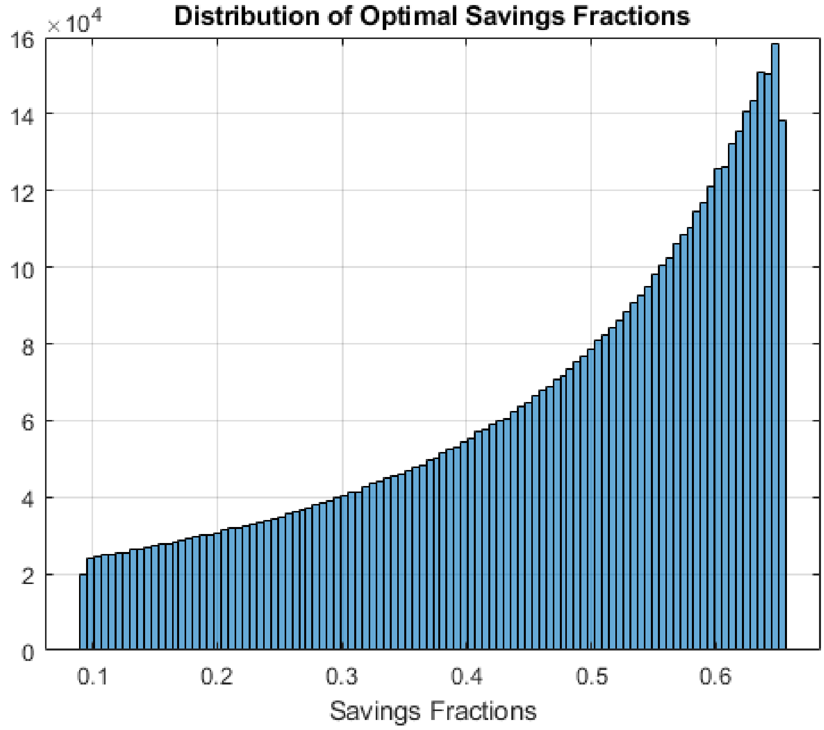
```
           c1
         -------

    r1     0.4084
    r2    0.26826
    r3    0.28289
    r4    0.21333
    r5    0.54941
    r6    0.59246
    r7    0.50637
    r8    0.56319
```

### 4.3.4 Test FF_OPTIM_MZOOM_SAVEZRONE Speed

Test Speed doing 6.25 million state-spcae points for a savings problem:

```
% Generate the state-space and function
rng(123);
it_draws = 6250000; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_util = @(x) ffi_intertemporal_util(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
% set parameters
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 20;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 10;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0.25;
[ar_opti_save_frac, ar_opti_save_level] = ...
    ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo);

Elapsed time is 54.241104 seconds.

mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
mp_container_map('ar_opti_save_frac') = ar_opti_save_frac;
mp_container_map('ar_opti_save_level') = ar_opti_save_level;
mp_container_map('ar_opti_save_frac_notnan') = ar_opti_save_frac(~isnan(ar_opti_save_frac));
ff_container_map_display(mp_container_map);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i    idx    ndim     numel        rowN      colN       sum         m
                                -    ---    ----    --------    --------    ----    ----------    --

    ar_opti_save_frac           1     1      2      6.25e+06    6.25e+06     1      2.8839e+06    0.
    ar_opti_save_frac_notnan    2     2      2      6.25e+06    6.25e+06     1      2.8839e+06    0.
    ar_opti_save_level          3     3      2      6.25e+06    6.25e+06     1      2.9481e+06    0.

figure();
histogram(ar_opti_save_frac(~isnan(ar_opti_save_frac)),100);
title('Distribution of Optimal Savings Fractions');
xlabel('Savings Fractions');
```

```
grid on;
```




### 4.3.5 Define Two Period Intertemporal Log Utility No Shock Utility Function

See Household's Utility Maximization Problem and Two-Period Borrowing and Savings Problem given Endowments.

```
function [ar_util, ar_saveborr_level] = ...
    ffi_intertemporal_util(ar_saveborr_frac, z1, z2, r, beta)

ar_saveborr_level = ar_saveborr_frac.*(z1+z2./(1+r)) - z2./(1+r);
ar_util = log(z1 - ar_saveborr_level) + beta.*log(ar_saveborr_level.*(1+r) + z2);

end
```

# Chapter 5

# Graphs

## 5.1 FF_GRAPH_GRID Examples: X, Y and Color Line Plots

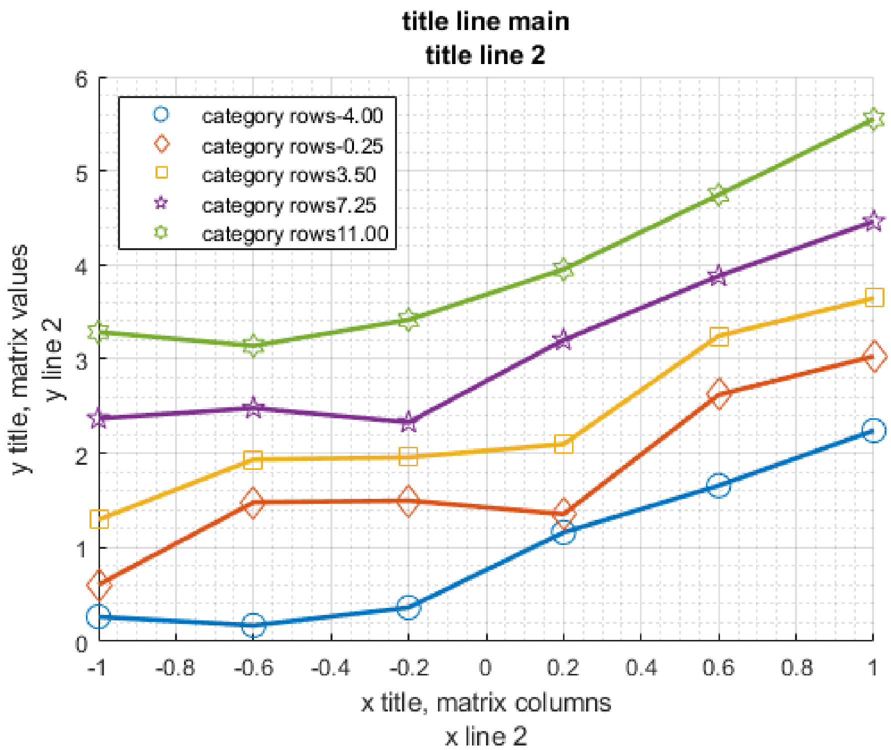
Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

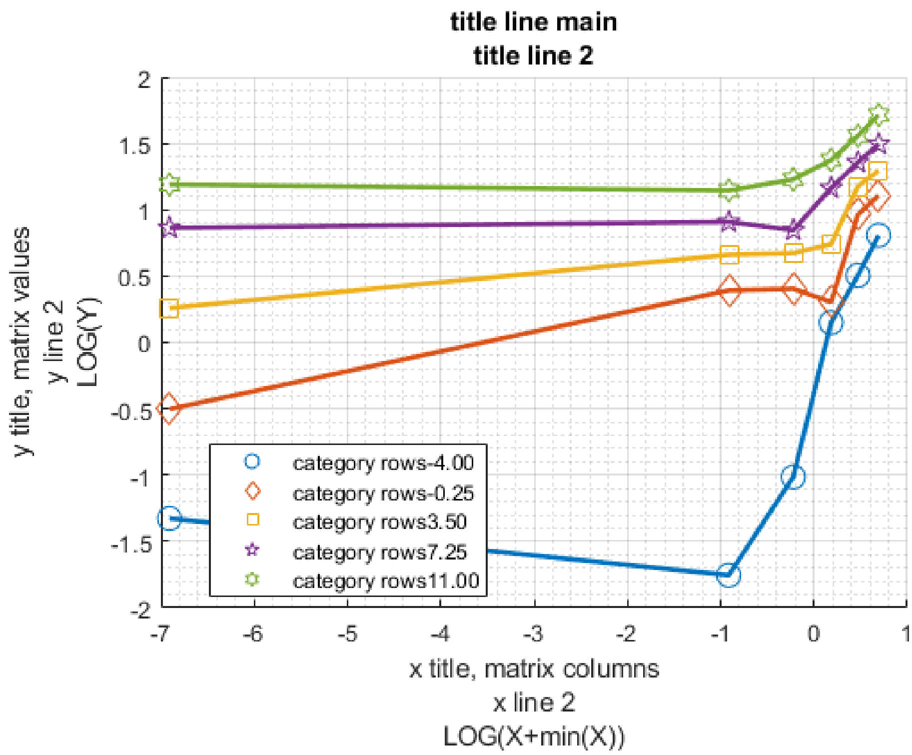
This is the example vignette for function: **ff_graph_grid** from the **MEconTools Package.** This function can graph out value and policy functions given one state vector (x-axis), conditional on other states (line groups). Can handle a few lines (scatter + lines), or many groups (jet spectrum).

### 5.1.1 Test FF_GRAPH_GRID Defaults

Call the function with defaults.

```
ff_graph_grid();
```

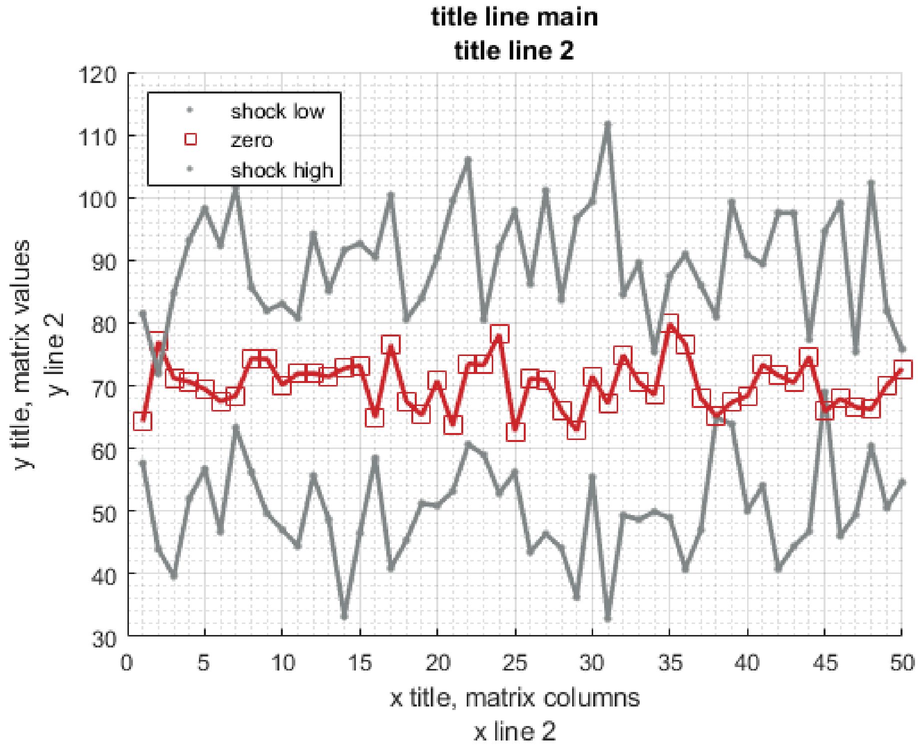


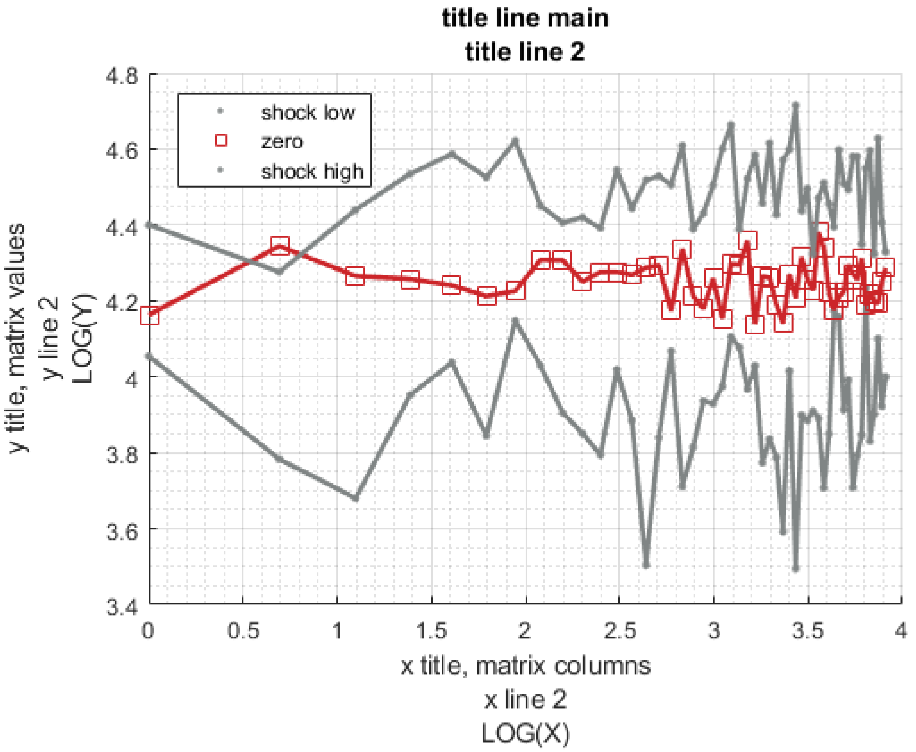



### 5.1.2 Test FF_GRAPH_GRID Random Matrix Pick Markers and Colors

Call the function with defaults.

```
rng(123);
mt_value = [normrnd(50,10,[1, 50]); ...
            normrnd(70,5,[1, 50]);...
            normrnd(90,10,[1, 50])];
ar_row_grid = ["shock low", "zero", "shock high"];
ar_col_grid = 1:50;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_scatter_shapes') = { '.', 's' ,'.' };
mp_support_graph('cl_colors') = {'gray', 'red', 'gray'};
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



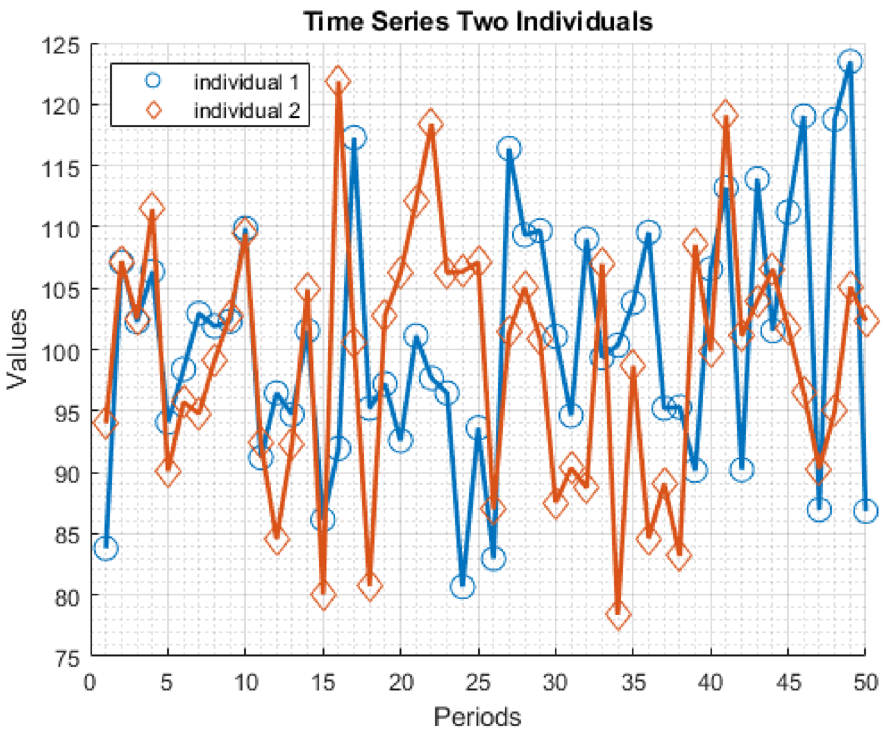




### 5.1.3 Test FF_GRAPH_GRID Two Random Normal Lines and Labels

There are two autoregressive time series, plot out the time two time series.

```
% Generate the two time series
rng(456);
mt_value = normrnd(100,10,[2, 50]);
ar_row_grid = ["individual 1", "individual 2"];
ar_col_grid = 1:50;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
```

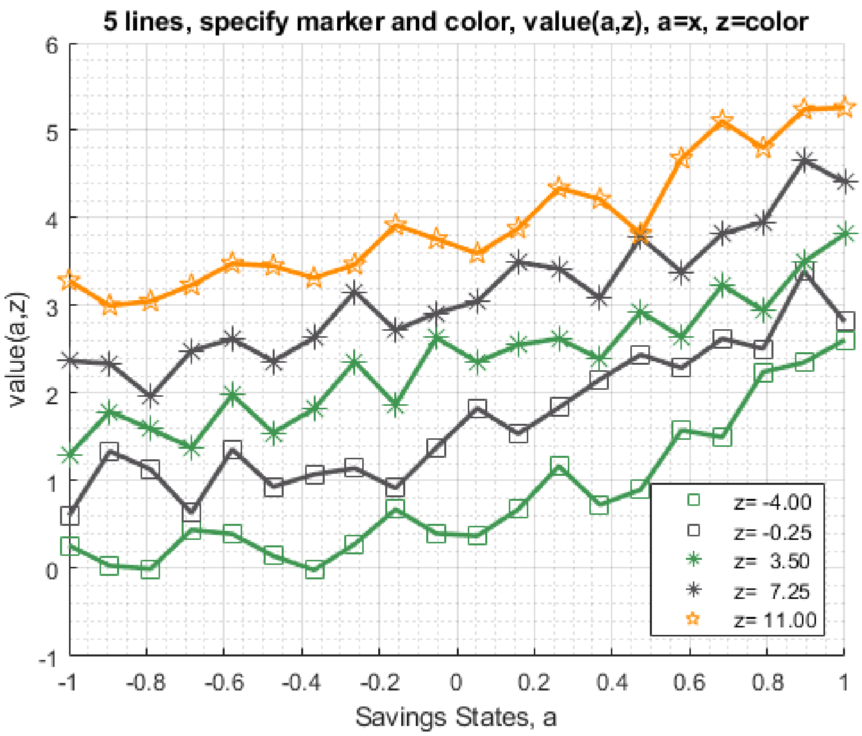
```
mp_support_graph('cl_st_graph_title') = {'Time Series Two Individuals'};
mp_support_graph('cl_st_ytitle') = {'Values'};
mp_support_graph('cl_st_xtitle') = {'Periods'};
mp_support_graph('bl_graph_logy') = false; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```




### 5.1.4 Test FF_GRAPH_GRID 6 Lines Pick Marker and Colors

Plot many lines, with auto legend.

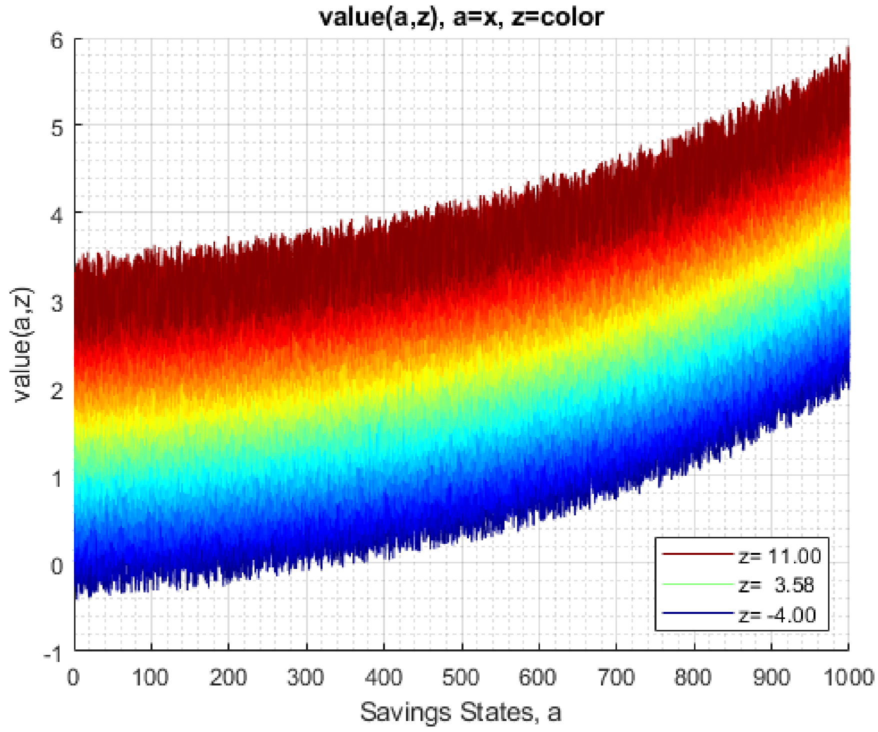
```
% Generate some Data
rng(456);
ar_row_grid = linspace(-4, 11, 5);
ar_col_grid = linspace(-1, 1, 20);
rng(123);
mt_value = 0.2*ar_row_grid' + exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'5 lines, specify marker and color, value(a,z), a=x, z=colo
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'southeast';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
mp_support_graph('cl_scatter_shapes') = {'s', 's', '*', '*', 'p'};
mp_support_graph('cl_colors') = {'green', 'black', 'green', 'black', 'orange'};
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```




### 5.1.5 Test FF_GRAPH_GRID Many Lines

Plot many lines, with auto legend.

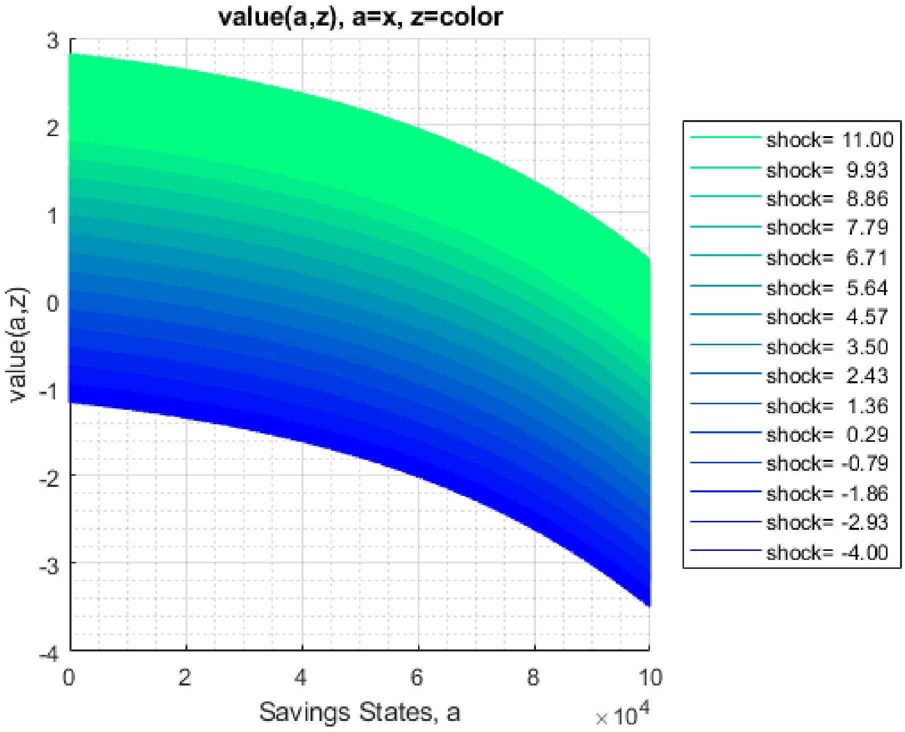
```
% Generate some Data
rng(456);
ar_row_grid = linspace(-4, 11, 100);
ar_col_grid = linspace(-1, 1, 1000);
rng(123);
mt_value = 0.2*ar_row_grid' + exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'southeast';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
mp_support_graph('cl_colors') = 'jet'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```




### 5.1.6 Test FF_GRAPH_GRID Many Lines Legend Exogenous

Plot many lines, exogenously set legend

```
% Generate the two time series
rng(456);
ar_row_grid = linspace(-4, 11, 15);
ar_col_grid = linspace(-1, 1, 100000);
rng(123);
mt_value = 0.2*ar_row_grid' - exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% setting shock vector name exogenously here
ar_row_grid = string(num2str(ar_row_grid', "shock=%6.2f"));
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'eastoutside';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('it_legend_select') = 15;
mp_support_graph('cl_colors') = 'winter'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



value(a,z), a=x, z=color
value(a,z)
Savings States, a
×10^4
shock= 11.00
shock= 9.93
shock= 8.86
shock= 7.79
shock= 6.71
shock= 5.64
shock= 4.57
shock= 3.50
shock= 2.43
shock= 1.36
shock= 0.29
shock= -0.79
shock= -1.86
shock= -2.93
shock= -4.00

# Chapter 6

# Support Tools

## 6.1 FF_CONTAINER_MAP_DISPLAY Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_container_map_display** from the **MEconTools Package.** This function summarizes statistics of matrixes stored in a container map, as well as scalar, string, function and other values stored in container maps.

### 6.1.1 Test FF_CONTAINER_MAP_DISPLAY Defaults

Call the function with defaults.

```
ff_container_map_display();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i     idx    ndim    numel    rowN    colN     sum       mean        std
                      --    ---    ----    -----    ----    ----    ------    -------    -------

    mat_1              1     7      2        12      3       4      6.5142    0.54285     0.2232
    mat_2              2     8      2      2650     50      53      1313.3    0.49559    0.29232
    mat_2_boolean      3     9      2      2650     50      53        1361    0.51358    0.49991
    mat_3              4    10      2         4      2       2      1.8111    0.45277    0.45111
    tensor_1           5    15      3        16      2       8      7.3043    0.45652    0.27787
    tensor_2           6    16      3        75      3      25      40.195    0.53593    0.29044
    tensor_3           7    17      2         4      1       4      1.6926    0.42315    0.37389
    tesseract_1        8    18      4        72      3      24      34.321    0.47669    0.26374
    tesseract_2        9    19      4        20      2      10      8.4191    0.42096    0.28981
    tesseract_bl_3    10    20      4        10      1      10           3        0.3    0.48305

xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4
          -------    -------    -------    -------

    r1    0.69647    0.55131    0.98076    0.39212
    r2    0.28614    0.71947    0.68483    0.34318
    r3    0.22685    0.42311    0.48093    0.72905

xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxx
              c1          c2         c3          c4         c50         c51         c52         c53
```

```
         --------   --------   -------   --------   --------   --------   --------   ------
    r1    0.43857     0.6249   0.17108    0.56564   0.072152    0.67855    0.61667    0.540
    r2   0.059678    0.67469   0.82911   0.084904    0.63289    0.27236    0.32528    0.249
    r3    0.39804    0.84234   0.33867    0.58267   0.046367    0.44513   0.075047     0.78
    r4      0.738   0.083195   0.55237    0.81484    0.50561    0.11117    0.59532    0.356
    r5    0.18249    0.76368   0.57855    0.33707    0.10653   0.028681     0.7435    0.918
    r46    0.6813    0.55326   0.88786    0.69983    0.83758    0.16382    0.74191   0.0656
    r47   0.87546    0.85445   0.69631    0.66117    0.97069    0.79092    0.42466    0.787
    r48   0.51042    0.38484   0.44033   0.049097   0.017768    0.33302    0.24401    0.979
    r49   0.66931    0.31679   0.43821     0.7923    0.12979    0.75311    0.79466   0.0790
    r50   0.58594    0.35426    0.7651    0.51872    0.86415    0.58281    0.84795     0.45

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxxx
           c1       c2       c3       c4       c50      c51      c52      c53

         -----    -----    -----    -----    -----    -----    -----    -----

    r1   true     false    false    true     true     false    true     true
    r2   true     false    true     true     false    false    true     true
    r3   false    true     false    true     false    true     false    true
    r4   false    true     false    false    false    true     true     true
    r5   true     true     true     false    true     false    false    true
    r46  false    true     true     false    true     true     true     true
    r47  true     true     true     true     true     true     false    false
    r48  true     false    false    false    true     true     false    true
    r49  true     true     false    true     true     true     false    false
    r50  false    false    false    false    false    false    false    false

xxx TABLE:mat_3 xxxxxxxxxxxxxxxxxx
             c1          c2

        ----------    -------

    r1  0.00012471    0.13253
    r2     0.88615    0.79226

xxx TABLE:tensor_1 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4         c5         c6         c7        c8

        --------    -------    -------    -------    -------    -------    ------    -------

    r1  0.019363    0.34271    0.52167    0.53703    0.75756    0.68839    0.8345    0.26597
    r2  0.018091    0.33355    0.11738    0.77857    0.81933    0.28644    0.6157      0.368

xxx TABLE:tensor_2 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4         c22        c23         c24         c25

        --------    -------    -------    -------    -------    -------    ---------    -------

    r1   0.51866    0.40495    0.48278    0.99731    0.46584    0.62976     0.035924    0.10505
    r2  0.028692    0.37408    0.24149    0.35201    0.66054    0.87243    0.0024293    0.81088
    r3   0.87339    0.19457    0.83212    0.15315    0.77859    0.96663       0.2501     0.8056

xxx TABLE:tensor_3 xxxxxxxxxxxxxxxxxx
           c1        c2        c3         c4

        ------    ------    -------    -------

    r1  0.1219    0.5119    0.91553    0.14329

xxx TABLE:tesseract_1 xxxxxxxxxxxxxxxxxx
           c1         c2         c3          c4          c21         c22         c23         c24
```

```
          -------    -------    -------    --------    -------    -------    -------    -------

    r1    0.64531    0.59299    0.32115     0.67653    0.90328    0.56911    0.52562    0.12014
    r2    0.74558     0.5007    0.46142     0.21384    0.35564    0.13732      0.155    0.23786
    r3    0.91137    0.46403    0.18118    0.049919    0.46246    0.46842    0.75348    0.64547

xxx TABLE:tesseract_2 xxxxxxxxxxxxxxxxxx
             c1          c2         c3         c4         c7         c8          c9          c10
          --------    -------    -------    -------    -------    -------    --------    -------

    r1     0.28898    0.48211    0.44359    0.97146    0.61782    0.65121     0.80715    0.11605
    r2    0.094493    0.34941    0.17595    0.14192    0.16754    0.57097    0.043114    0.70518

xxx TABLE:tesseract_bl_3 xxxxxxxxxxxxxxxxxx
           c1       c2       c3       c4       c7       c8       c9       c10
          -----    -----    -----    -----    -----    -----    -----    -----

    r1    false    false    true     true     false    true     false    false

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i    idx     value
                      -    ---    -------

    boolean_1         1     1           1
    empty             2     2         NaN
    mat_4             3    11     0.74898
    string_float_1    4    13      1021.1
    string_int_1      5    14        1021

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 String
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i     idx           string
                     ---    ----    ---------------------

    list_string_1    "1"    "5"     "col1;col2;col3;col4"
    list_string_2    "2"    "6"     "row1;row2;row3;row4"
    string_1         "3"    "12"    "Table Name"

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 Functions
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
              i      idx      functionString
             ---     ---    -------------------

    func1    "1"     "3"    "@(x)1+2+x"
    func2    "2"     "4"    "@(x,y)x*1+sqrt(y)"
```

### 6.1.2 Test FF_CONTAINER_MAP_DISPLAY summarize Matrix Only

Three large matrixes, show summaries

```
% Create Container
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
```

```
rng(123);
mp_container_map('mat_1') = rand(100,100);
mp_container_map('mat_2') = rand(100,100)*2 + 1;
mp_container_map('mat_2_boolean') = (rand(100,100) > 0.5);
% Will only print
ff_container_map_display(mp_container_map);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                     i    idx    ndim    numel    rowN    colN     sum       mean        std      co
                     -    ---    ----    -----    ----    ----    ------    -------    -------    --

    mat_1            1     1      2      10000    100     100     4982.3    0.49823    0.28829    0.
    mat_2            2     2      2      10000    100     100      20029     2.0029    0.57632    0.
    mat_2_boolean    3     3      2      10000    100     100       4995     0.4995    0.50002     1
```

### 6.1.3 Test FF_CONTAINER_MAP_DISPLAY Show Matrix Subset

A container map with three small matrixes, print only only 2 rows and 3 columns.

```
% Create Container
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
rng(789);
mp_container_map('mat_1') = rand(3,4);
mp_container_map('mat_2') = rand(50,53);
mp_container_map('mat_2_boolean') = (rand(50,53) > 0.5);
% Will only print
ff_container_map_display(mp_container_map, 2, 3);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                     i    idx    ndim    numel    rowN    colN     sum       mean        std      co
                     -    ---    ----    -----    ----    ----    ------    -------    -------    --

    mat_1            1     1      2        12       3       4     4.9876    0.41564    0.33586    0.
    mat_2            2     2      2      2650      50      53     1324.3    0.49973    0.28834    0.
    mat_2_boolean    3     3      2      2650      50      53       1350    0.50943    0.50001    0.

xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxx
           c1         c2         c3         c4
         -------    -------    -------    -------

    r1    0.32333    0.62442    0.01062    0.53815
    r3    0.79378    0.75889    0.11104    0.55157

xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxx
            c1         c2         c52        c53
          -------    -------    -------    -------

    r1     0.72837    0.20976    0.74583    0.22321
    r50    0.52812      0.545    0.49521    0.29826

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxxx
           c1       c2       c52      c53
          -----    -----    -----    -----
```

```
r1     false    true     true     true
r50    true     false    false    true
```

# Chapter 7

# Data Structures

## 7.1 FF_SAVEBORR_GRID Example for Generating Asset Grid

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_saveborr_grid** from the **MEconTools Package.** This function generates variously spaced savings/borrowing states/choices grid.

### 7.1.1 Test FF_SAVEBORR_GRID Defaults

Call the function with defaults.

```
ff_saveborr_grid();
```

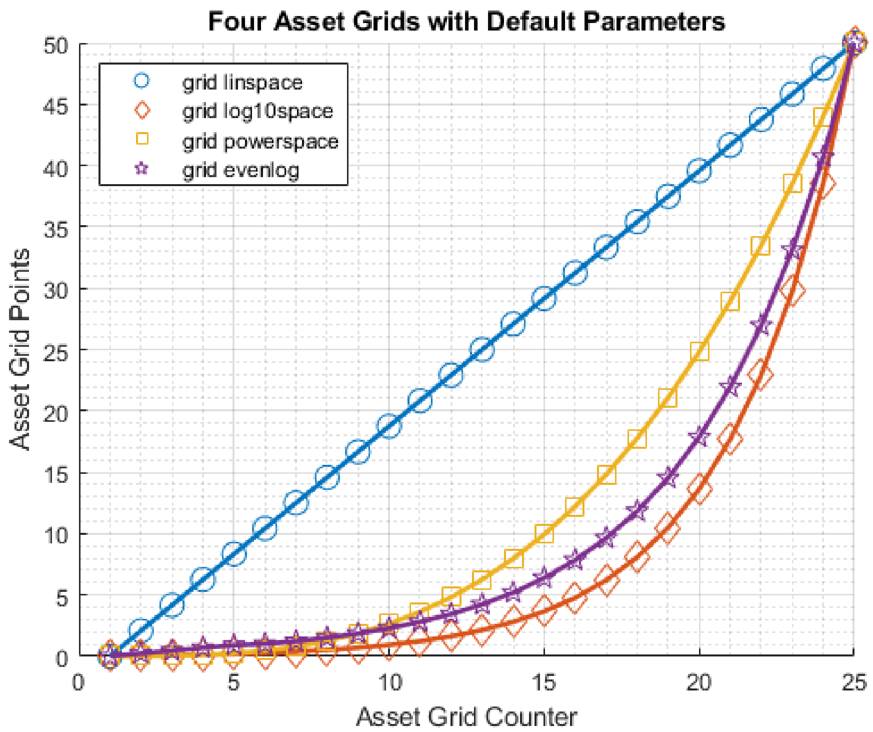
```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i    idx    ndim    numel    rowN    colN    sum      mean     std      coefv
                      -    ---    ----    -----    ----    ----    -----    -----    ------   -----

    ar_fl_saveborr    1     1      2       25       25      1      216.7    8.668    13.363    1.54

xxx TABLE:ar_fl_saveborr xxxxxxxxxxxxxxxxxx
              c1
           --------

    r1            0
    r2     0.029558
    r3     0.067855
    r4      0.11748
    r5      0.18177
    r6      0.26507
    r7      0.37301
    r8      0.51286
    r9      0.69407
    r10     0.92885
    r11      1.2331
    r12      1.6272
    r13      2.1379
    r14      2.7996
```

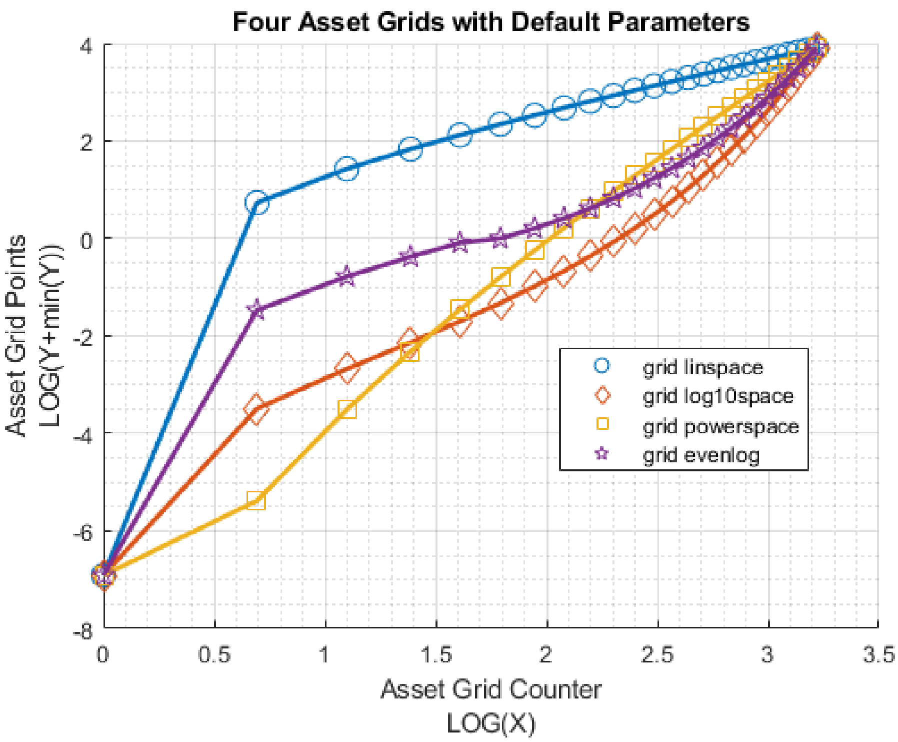
```
    r15       3.657
    r16      4.7679
    r17      6.2072
    r18      8.0722
    r19      10.489
    r20       13.62
    r21      17.676
    r22      22.932
    r23      29.743
    r24      38.567
    r25          50

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                              i    idx    value
                              -    ---    -----

    grid_evenlog_threshold    1     2        1
    grid_log10space_x1        2     3      0.3
    grid_log10space_x2        3     4        3
    grid_powerspace_power     4     5        3
```

### 7.1.2 Test FF_SAVEBORR_GRID Default Linear Grid, Log Grid, Power Grid, Threshold Grid

Call the function with defaults.

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
% Four types of grid points
st_grid_type = 'grid_linspace';
[ar_fl_saveborr_linspace] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_log10space';
[ar_fl_saveborr_log10space] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_powerspace';
[ar_fl_saveborr_powerspace] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_evenlog';
[ar_fl_saveborr_evenlog] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
% draw four types of lines jointly
mt_value = [ar_fl_saveborr_linspace'; ar_fl_saveborr_log10space'; ...
    ar_fl_saveborr_powerspace'; ar_fl_saveborr_evenlog'];
ar_row_grid = ["grid linspace", "grid log10space", "grid powerspace", "grid evenlog"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Four Asset Grids with Default Parameters'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



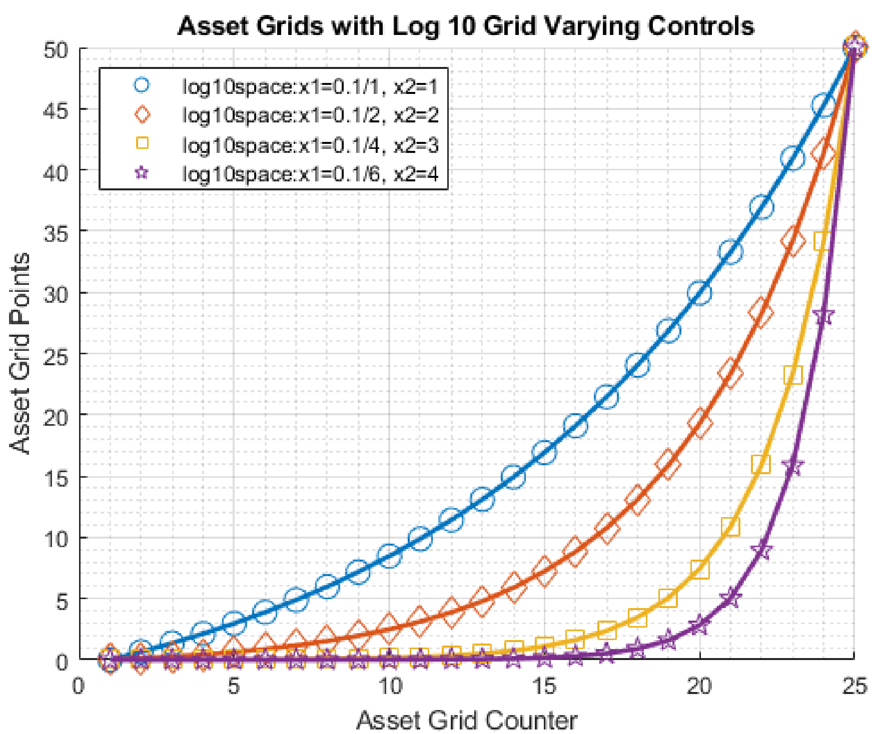



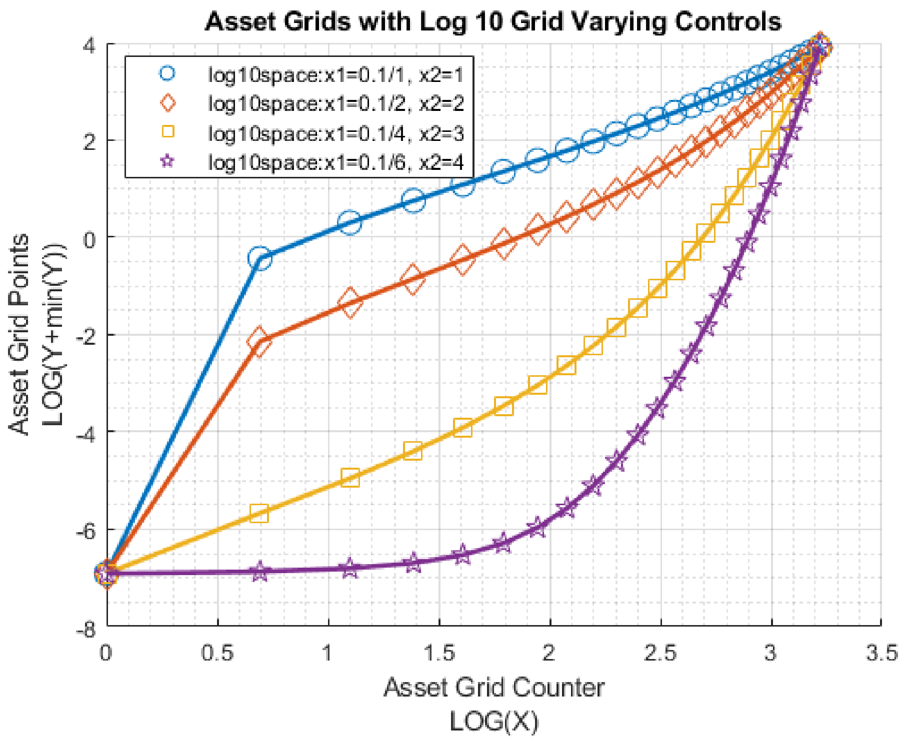

### 7.1.3 Test FF_SAVEBORR_GRID Log Grid Changing Parameters

Log grid, same min and max, change log X1 and X2 points

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_log10space';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_log10space_x1') = 0.1;
```

```
mp_grid_control('grid_log10space_x2') = 1;
[ar_fl_log10space_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/2;
mp_grid_control('grid_log10space_x2') = 1*2;
[ar_fl_log10space_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/4;
mp_grid_control('grid_log10space_x2') = 1*4;
[ar_fl_log10space_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/6;
mp_grid_control('grid_log10space_x2') = 1*6;
[ar_fl_log10space_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
% draw four types of lines jointly
mt_value = [ar_fl_log10space_a'; ar_fl_log10space_b'; ...
    ar_fl_log10space_c'; ar_fl_log10space_d'];
ar_row_grid = [...
    "log10space:x1=0.1/1, x2=1", ...
    "log10space:x1=0.1/2, x2=2", ...
    "log10space:x1=0.1/4, x2=3", ...
    "log10space:x1=0.1/6, x2=4"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Log 10 Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```

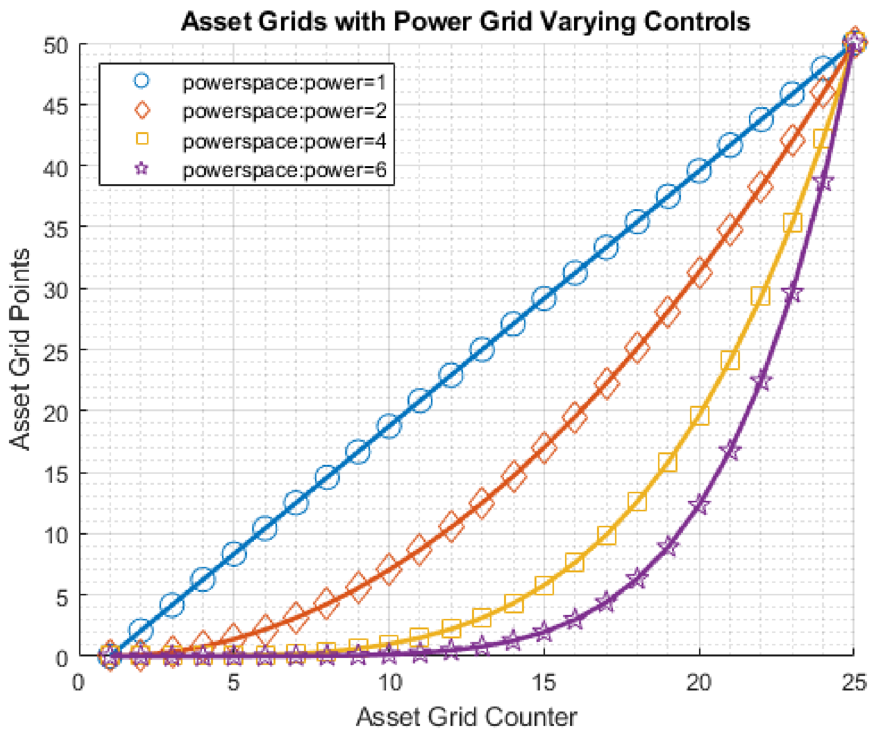


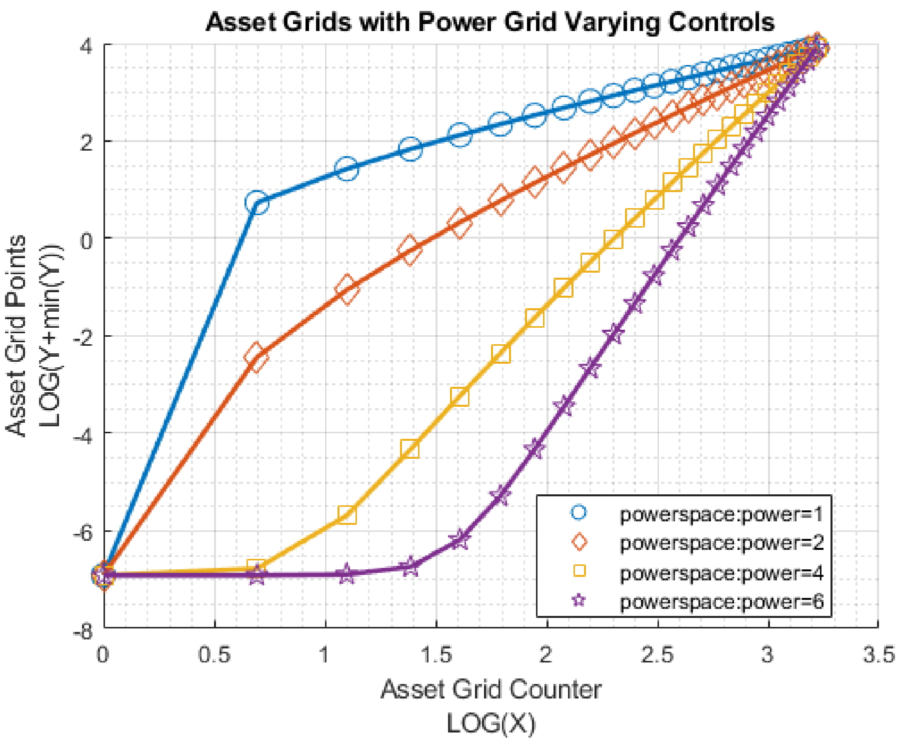



### 7.1.4 Test FF_SAVEBORR_GRID Power Grid Changing Parameters

Log grid, same min and max, change log X1 and X2 points

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_powerspace';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_powerspace_power') = 1;
[ar_fl_powerspace_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_powerspace_power') = 2;
[ar_fl_powerspace_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_powerspace_power') = 4;
[ar_fl_powerspace_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_powerspace_power') = 6;
[ar_fl_powerspace_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
% draw four types of lines jointly
mt_value = [ar_fl_powerspace_a'; ar_fl_powerspace_b'; ...
    ar_fl_powerspace_c'; ar_fl_powerspace_d'];
ar_row_grid = [...
    "powerspace:power=1", ...
    "powerspace:power=2", ...
    "powerspace:power=4", ...
    "powerspace:power=6"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Power Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



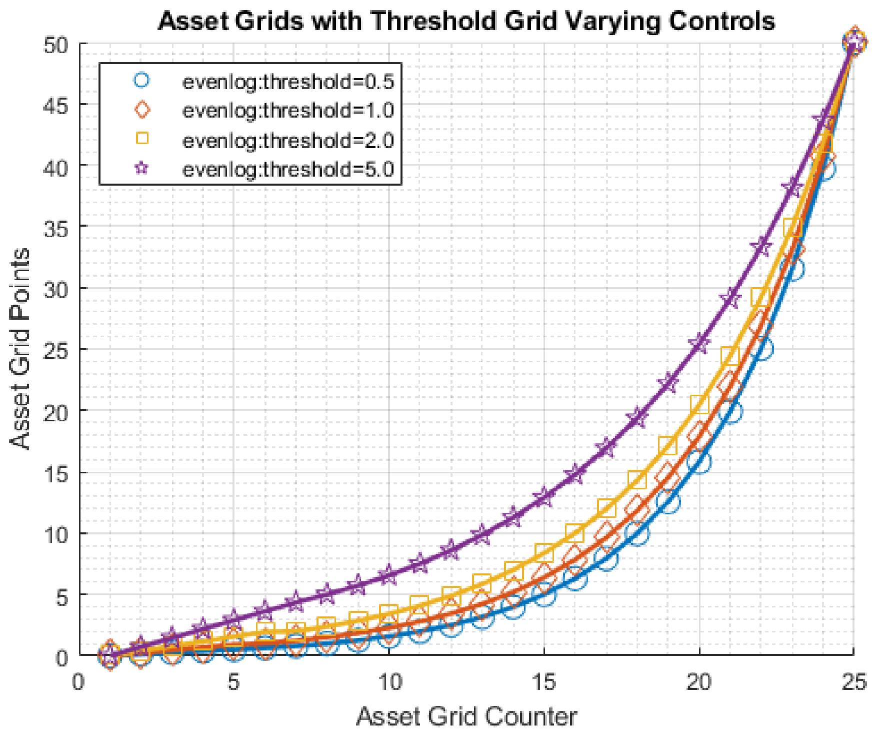



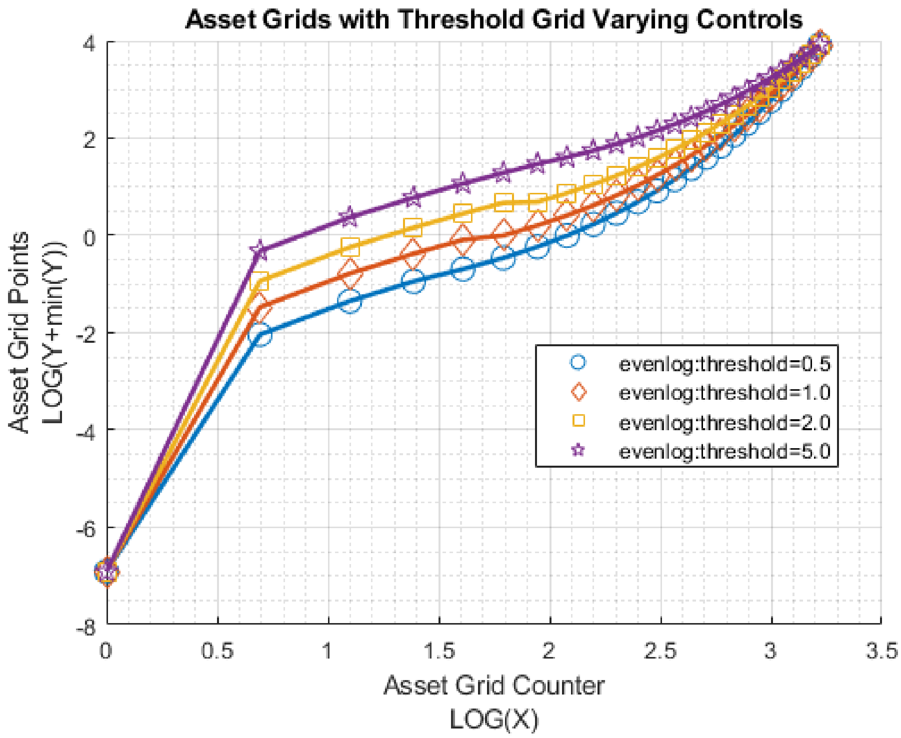

### 7.1.5 Test FF_SAVEBORR_GRID Threshold Grid Changing Parameters

Threshold Grid, Changing Threshold Levels. Initial segments below threshold are linspace, then logspace.

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_evenlog';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_evenlog_threshold') = 0.50;
```

```
[ar_fl_evenlog_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 1.00;
[ar_fl_evenlog_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 2;
[ar_fl_evenlog_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 5;
[ar_fl_evenlog_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
% draw four types of lines jointly
mt_value = [ar_fl_evenlog_a'; ar_fl_evenlog_b'; ...
    ar_fl_evenlog_c'; ar_fl_evenlog_d'];
ar_row_grid = [...
    "evenlog:threshold=0.5", ...
    "evenlog:threshold=1.0", ...
    "evenlog:threshold=2.0", ...
    "evenlog:threshold=5.0"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Threshold Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```




Asset Grids with Threshold Grid Varying Controls
Asset Grid Points
LOG(Y+min(Y))
Asset Grid Counter
LOG(X)
evenlog:threshold=0.5
evenlog:threshold=1.0
evenlog:threshold=2.0
evenlog:threshold=5.0
4
2
0
-2
-4
-6
-8
0
0.5
1
1.5
2
2.5
3
3.5

# Chapter 8

# Common Functions

## 8.1 FFY_TAUCHEN AR1 Shock Discretization Example

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ffy_tauchen** from the **MEconTools Package.** : See also the **ffy_rouwenhorst** function from the **MEconTools Package.** This function discretize a mean zero AR1 process, uses Tauchen (1986). See AR 1 Example for some details on how the AR1 process works. And See Kopecky and Suen (2010).

### 8.1.1 Test FFY_TAUCHEN Defaults

Call the function with defaults. Default sd bounds arer plus and minus 4. This is used in the following examples, unless otherwise specified as the 5th parameter.

```
ffy_tauchen();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum    mean      std       coef
                         -    ---    ----    -----    ----    ----    ---    ----    -------    ----

    ar_disc_ar1          1     1      2        5        5       1       0       0    0.79057
    mt_disc_ar1_trans    2     6      2       25        5       5       5     0.2    0.27623     1.3

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
           c1
          ----

    r1      -1
    r2    -0.5
    r3       0
    r4     0.5
    r5       1

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1            c2           c3            c4             c5
          ----------    ----------    --------    ----------    ----------

    r1       0.22663       0.73331    0.040048    1.0689e-05    7.3923e-12
    r2      0.012224       0.58648     0.39831     0.0029797     7.605e-08
```

```
    r3    8.8417e-05       0.10556      0.7887       0.10556    8.8417e-05
    r4     7.605e-08     0.0029797     0.39831       0.58648      0.012224
    r5    7.3923e-12    1.0689e-05    0.040048       0.73331       0.22663

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    -----

    fl_ar1_persistence    1     2      0.6
    fl_ar1_step           2     3      0.5
    fl_shk_std            3     4      0.2
    it_std_bound          4     5        4
```

### 8.1.2 Test FFY_TAUCHEN Specify Parameters

With a grid of 10 points, the sd bounds on Tauchen and Rouwenhorst are identical. With the not extremely persistent shock process here, the Tauchen and Rouwenhorst Results are very similar.

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose, it_std_bound] = ...
    deal(0.60, 0.10, 10, true, 3);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose, it_std_bound);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN        sum            mean
                         -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1          1     1      2        10      10       1     -7.2164e-16    -7.2164e-17
    mt_disc_ar1_trans    2     6      2       100      10      10              10            0.1

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
           ---------

    r1       -0.375
    r2     -0.29167
    r3     -0.20833
    r4       -0.125
    r5    -0.041667
    r6     0.041667
    r7        0.125
    r8      0.20833
    r9      0.29167
    r10       0.375

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
               c1            c2            c3            c4           c5           c6           c7
           ----------    ----------    ----------    ----------    --------    --------    ---------

    r1        0.13933       0.26196       0.31887       0.20154    0.066066    0.011201    0.0009785
    r2       0.056673       0.16995       0.30658       0.28713      0.1396    0.035167     0.004575
    r3        0.01861      0.087039       0.23281       0.32308     0.23281    0.087039      0.01684
    r4      0.0048925      0.035167        0.1396       0.28713     0.30658     0.16995      0.04884
    r5      0.0010235      0.011201      0.066066       0.20154     0.31887     0.26196       0.1116
```

```
    r6     0.00016962     0.0028101       0.02466       0.11169      0.26196      0.31887       0.2015
    r7     2.2197e-05    0.00055483     0.0072547      0.048841      0.16995      0.30658       0.2871
    r8     2.2881e-06    8.6129e-05     0.0016806      0.016841     0.087039      0.23281       0.3230
    r9     1.8543e-07    1.0503e-05    0.00030628     0.0045756     0.035167       0.1396       0.2871
    r10    1.1798e-08    1.0053e-06    4.3874e-05    0.00097859     0.011201     0.066066       0.2015

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2          0.6
    fl_ar1_step           2     3     0.083333
    fl_shk_std            3     4          0.1
    it_std_bound          4     5            3
```

### 8.1.3 Test FFY_TAUCHEN High Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.01, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN        sum            mean
                         -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1          1     1      2        7       7       1      -5.5511e-17    -7.9302e-18
    mt_disc_ar1_trans    2     6      2       49       7       7                7        0.14286

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
          -----------

    r1       -0.28355
    r2       -0.18903
    r3      -0.094517
    r4    -2.7756e-17
    r5       0.094517
    r6        0.18903
    r7        0.28355

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1             c2             c3             c4             c5            c6
          -----------    -----------    -----------    -----------    ----------    ----------    --

    r1              1     4.4497e-06              0              0             0             0
    r2     4.4412e-07              1     2.8552e-06              0             0             0
    r3      1.632e-46     7.1638e-07              1     1.8164e-06             0             0
    r4    9.6185e-124     6.3021e-46     1.1456e-06              1    1.1456e-06             0
    r5    6.3206e-239    8.9712e-123     2.4121e-45     1.8164e-06             1    7.1638e-07
    r6              0     1.426e-237    8.2932e-122     9.1503e-45    2.8552e-06             1    4.
    r7              0              0    3.1885e-236    7.5984e-121    3.4405e-44    4.4497e-06
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2         0.99
    fl_ar1_step           2     3     0.094517
    fl_shk_std            3     4         0.01
    it_std_bound          4     5            4
```

### 8.1.4 Test FFY_TAUCHEN Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std
                         -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1          1     1      2        7       7       1       0          0     0.028805
    mt_disc_ar1_trans    2     6      2       49       7       7       7    0.14286      0.17448

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
          ---------

    r1    -0.040002
    r2    -0.026668
    r3    -0.013334
    r4            0
    r5     0.013334
    r6     0.026668
    r7     0.040002

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1            c2          c3          c4         c5          c6            c7
          ----------    --------    -------    -------    -------    --------    ----------

    r1    0.00049475    0.024497    0.24044     0.4947    0.21921    0.020299    0.00037109
    r2    0.00047179    0.023751    0.23685    0.49488     0.2227    0.020954    0.00038948
    r3    0.00044982    0.023024    0.23329      0.495    0.22621    0.021626     0.0004087
    r4     0.0004288    0.022316    0.22974    0.49504    0.22974    0.022316     0.0004288
    r5     0.0004087    0.021626    0.22621      0.495    0.23329    0.023024    0.00044982
    r6    0.00038948    0.020954     0.2227    0.49488    0.23685    0.023751    0.00047179
    r7    0.00037109    0.020299    0.21921     0.4947    0.24044    0.024497    0.00049475

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------
```

```
    fl_ar1_persistence    1     2         0.01
    fl_ar1_step           2     3     0.013334
    fl_shk_std            3     4         0.01
    it_std_bound          4     5            4
```

### 8.1.5 Test FFY_TAUCHEN High Persistence, High SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.99, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN        sum            mean
                         -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1          1     1      2        7       7       1      -3.5527e-15    -5.0753e-16
    mt_disc_ar1_trans    2     6      2       49       7       7                7        0.14286

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
             c1
          -------

    r1    -28.072
    r2    -18.714
    r3    -9.3572
    r4          0
    r5     9.3572
    r6     18.714
    r7     28.072

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
               c1             c2             c3             c4             c5            c6
          -----------    -----------    -----------    -----------    ----------    ----------    --

    r1              1    4.4497e-06              0              0             0             0
    r2     4.4412e-07             1     2.8552e-06              0             0             0
    r3      1.632e-46    7.1638e-07              1     1.8164e-06             0             0
    r4    9.6185e-124     6.3021e-46    1.1456e-06              1    1.1456e-06             0
    r5    6.3206e-239    8.9712e-123     2.4121e-45    1.8164e-06             1    7.1638e-07
    r6              0     1.426e-237    8.2932e-122     9.1503e-45    2.8552e-06             1    4.
    r7              0             0     3.1885e-236    7.5984e-121     3.4405e-44    4.4497e-06

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    ------

    fl_ar1_persistence    1     2       0.99
    fl_ar1_step           2     3     9.3572
    fl_shk_std            3     4       0.99
    it_std_bound          4     5          4
```

### 8.1.6 Test FFY_TAUCHEN Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                        i    idx    ndim    numel    rowN    colN    sum      mean          std
                        -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1         1     1      2        7       7       1       0           0    0.028805
    mt_disc_ar1_trans   2     6      2       49       7       7       7     0.14286     0.17448

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
             c1
          ---------

    r1    -0.040002
    r2    -0.026668
    r3    -0.013334
    r4            0
    r5     0.013334
    r6     0.026668
    r7     0.040002

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1            c2          c3         c4         c5          c6            c7
          ----------    --------    -------    -------    -------    --------    ----------

    r1    0.00049475    0.024497    0.24044     0.4947    0.21921    0.020299    0.00037109
    r2    0.00047179    0.023751    0.23685    0.49488     0.2227    0.020954    0.00038948
    r3    0.00044982    0.023024    0.23329      0.495    0.22621    0.021626     0.0004087
    r4     0.0004288    0.022316    0.22974    0.49504    0.22974    0.022316     0.0004288
    r5     0.0004087    0.021626    0.22621      0.495    0.23329    0.023024    0.00044982
    r6    0.00038948    0.020954     0.2227    0.49488    0.23685    0.023751    0.00047179
    r7    0.00037109    0.020299    0.21921     0.4947    0.24044    0.024497    0.00049475

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx     value
                         -    ---    --------

    fl_ar1_persistence   1     2         0.01
    fl_ar1_step          2     3     0.013334
    fl_shk_std           3     4         0.01
    it_std_bound         4     5            4
```

## 8.2 FFY_ROUWENHORST AR1 Shock Discretization Example

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ffy_rouwenhorst** from the **MEconTools Package.** See also **ffy_tauchen** function from the **MEconTools Package.** This function discretize a mean zero AR1 process, uses Rouwenhorst (1995). See AR 1 Example for some details on how the AR1 process works. And See Kopecky and Suen (2010).

### 8.2.1 Test FFY_ROUWENHORST Defaults

Call the function with defaults.

```
ffy_rouwenhorst();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum    mean      std      coef
                         -    ---    ----    -----    ----    ----    ---    ----    -------    ----

    ar_disc_ar1          1     1      2        5       5       1       0       0     0.39528
    mt_disc_ar1_trans    2    11      2       25       5       5       5     0.2     0.18246    0.91

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
             -----

    r1        -0.5
    r2       -0.25
    r3           0
    r4        0.25
    r5         0.5

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
               c1         c2         c3         c4         c5
             ------     ------     ------     ------     ------

    r1       0.4096     0.4096     0.1536     0.0256     0.0016
    r2       0.1024     0.4864     0.3264     0.0784     0.0064
    r3       0.0256     0.2176     0.5136     0.2176     0.0256
    r4       0.0064     0.0784     0.3264     0.4864     0.1024
    r5       0.0016     0.0256     0.1536     0.4096     0.4096

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    -----

    fl_ar1_beg            1     2     -0.5
    fl_ar1_end            2     3      0.5
    fl_ar1_persistence    3     4      0.6
    fl_ar1_step           4     5     0.25
    fl_p0                 5     6      0.8
    fl_q0                 6     7      0.8
    fl_shk_std            7     8      0.2
    fl_sig_ar1            8     9     0.25
    it_std_bound          9    10        0
```

### 8.2.2 Test FFY_ROUWENHORST Specify Parameters

With a grid of 10 points, the Rwouenhorst bounds on standard deviations are equall to Tauchen bounds of 3. With the not extremely persistent shock process here, the Tauchen and Rouwenhorst Results are very similar.

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.60, 0.10, 10, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN       sum           mean
                         -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1          1     1      2        10      10       1     5.5511e-17    5.5511e-18     0
    mt_disc_ar1_trans    2    11      2       100      10      10             10           0.1    0.

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
           ---------

    r1        -0.375
    r2      -0.29167
    r3      -0.20833
    r4        -0.125
    r5     -0.041667
    r6      0.041667
    r7         0.125
    r8       0.20833
    r9       0.29167
    r10        0.375

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
               c1            c2            c3            c4           c5          c6           c7
           ----------    ----------    ----------    ---------    --------    --------    ---------

    r1        0.13422       0.30199       0.30199      0.17616     0.06606    0.016515    0.0027525
    r2       0.033554       0.20133       0.32716      0.26424     0.12662    0.038535    0.0075694
    r3      0.0083886      0.081789       0.26267      0.32755     0.21401    0.082747     0.019741
    r4      0.0020972      0.028312       0.14038      0.30946     0.30369     0.15877     0.047989
    r5     0.00052429      0.009044      0.061145      0.20246     0.33477     0.25969      0.10585
    r6     0.00013107     0.0027525      0.023642      0.10585     0.25969     0.33477      0.20246
    r7     3.2768e-05    0.00081101     0.0084603     0.047989     0.15877     0.30369      0.30946
    r8      8.192e-06    0.00023347     0.0028677     0.019741    0.082747     0.21401      0.32755
    r9      2.048e-06    6.6048e-05    0.00093389    0.0075694    0.038535     0.12662      0.26424
    r10      5.12e-07    1.8432e-05    0.00029491    0.0027525    0.016515     0.06606      0.17616

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_beg            1     2       -0.375
    fl_ar1_end            2     3        0.375
```

```
    fl_ar1_persistence    3     4          0.6
    fl_ar1_step           4     5     0.083333
    fl_p0                 5     6          0.8
    fl_q0                 6     7          0.8
    fl_shk_std            7     8          0.1
    fl_sig_ar1            8     9        0.125
    it_std_bound          9    10            0
```

### 8.2.3 Test FFY_ROUWENHORST High Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.01, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std      c
                         -    ---    ----    -----    ----    ----    ---    -------    -------    -

    ar_disc_ar1          1     1      2        7        7       1      0          0     0.12503
    mt_disc_ar1_trans    2    11      2       49        7       7      7    0.14286     0.34148

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
             c1
          --------

    r1    -0.17364
    r2    -0.11576
    r3    -0.05788
    r4           0
    r5     0.05788
    r6     0.11576
    r7     0.17364

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1            c2            c3            c4            c5            c6            c7
          ----------    ----------    ----------    ----------    ----------    ----------    ------

    r1       0.97037      0.029257    0.00036756    2.4627e-06    9.2815e-09    1.8656e-11    1.5625
    r2     0.0048762        0.9705      0.024382    0.00024504    1.2314e-06    3.0938e-09    3.1094
    r3    2.4504e-05      0.009753       0.97057      0.019506    0.00014703    4.9254e-07    6.1877
    r4    1.2313e-07    7.3513e-05       0.01463       0.97059       0.01463    7.3513e-05    1.2313
    r5    6.1877e-10    4.9254e-07    0.00014703      0.019506       0.97057      0.009753    2.4504
    r6    3.1094e-12    3.0938e-09    1.2314e-06    0.00024504      0.024382        0.9705     0.004
    r7    1.5625e-14    1.8656e-11    9.2815e-09    2.4627e-06    0.00036756      0.029257       0.9

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_beg            1     2     -0.17364
    fl_ar1_end            2     3      0.17364
    fl_ar1_persistence    3     4         0.99
```

```
    fl_ar1_step          4    5     0.05788
    fl_p0                5    6       0.995
    fl_q0                6    7       0.995
    fl_shk_std           7    8        0.01
    fl_sig_ar1           8    9    0.070888
    it_std_bound         9   10           0
```

### 8.2.4 Test FFY_ROUWENHORST Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std
                         -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1          1     1      2        7       7       1       0           0    0.017639
    mt_disc_ar1_trans    2    11      2       49       7       7       7     0.14286     0.10985

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
          ----------

    r1     -0.024496
    r2     -0.016331
    r3    -0.0081654
    r4             0
    r5     0.0081654
    r6      0.016331
    r7      0.024496

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
             c1          c2          c3         c4         c5          c6          c7
          --------    --------    -------    -------    -------    --------    --------

    r1    0.016586    0.097547    0.23904    0.31241    0.22966    0.090047    0.014711
    r2    0.016258    0.096266    0.23749    0.31247    0.23124    0.091266    0.015008
    r3    0.015936    0.094997    0.23594    0.31251    0.23281    0.092497    0.015311
    r4     0.01562    0.093741    0.23438    0.31252    0.23438    0.093741     0.01562
    r5    0.015311    0.092497    0.23281    0.31251    0.23594    0.094997    0.015936
    r6    0.015008    0.091266    0.23124    0.31247    0.23749    0.096266    0.016258
    r7    0.014711    0.090047    0.22966    0.31241    0.23904    0.097547    0.016586

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx      value
                          -    ---    ---------

    fl_ar1_beg            1     2     -0.024496
    fl_ar1_end            2     3      0.024496
    fl_ar1_persistence    3     4          0.01
    fl_ar1_step           4     5     0.0081654
```

```
    fl_p0                5     6        0.505
    fl_q0                6     7        0.505
    fl_shk_std           7     8         0.01
    fl_sig_ar1           8     9     0.010001
    it_std_bound         9    10            0
```

### 8.2.5 Test FFY_ROUWENHORST High Persistence, High SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.99, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN       sum           mean       --
                         -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1          1     1      2        7       7       1      3.5527e-15    5.0753e-16    1
    mt_disc_ar1_trans    2    11      2       49       7       7               7       0.14286    0.

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
            c1
          -------

    r1     -17.19
    r2     -11.46
    r3    -5.7301
    r4          0
    r5     5.7301
    r6      11.46
    r7      17.19

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1            c2            c3            c4            c5            c6            c7
          ----------    ----------    ----------    ----------    ----------    ----------    ------

    r1       0.97037      0.029257    0.00036756    2.4627e-06    9.2815e-09    1.8656e-11    1.5625
    r2     0.0048762        0.9705      0.024382    0.00024504    1.2314e-06    3.0938e-09    3.1094
    r3    2.4504e-05      0.009753       0.97057      0.019506    0.00014703    4.9254e-07    6.1877
    r4    1.2313e-07    7.3513e-05       0.01463       0.97059       0.01463    7.3513e-05    1.2313
    r5    6.1877e-10    4.9254e-07    0.00014703      0.019506       0.97057      0.009753    2.4504
    r6    3.1094e-12    3.0938e-09    1.2314e-06    0.00024504      0.024382        0.9705     0.004
    r7    1.5625e-14    1.8656e-11    9.2815e-09    2.4627e-06    0.00036756      0.029257       0.9

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    ------

    fl_ar1_beg            1     2    -17.19
    fl_ar1_end            2     3     17.19
    fl_ar1_persistence    3     4      0.99
    fl_ar1_step           4     5    5.7301
    fl_p0                 5     6     0.995
```

```
    fl_q0                6     7      0.995
    fl_shk_std           7     8       0.99
    fl_sig_ar1           8     9     7.0179
    it_std_bound         9    10          0
```

### 8.2.6 Test FFY_ROUWENHORST Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std
                         -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1          1     1      2        7       7       1       0           0    0.017639
    mt_disc_ar1_trans    2    11      2       49       7       7       7     0.14286     0.10985

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
          ----------

    r1     -0.024496
    r2     -0.016331
    r3    -0.0081654
    r4             0
    r5     0.0081654
    r6      0.016331
    r7      0.024496

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
             c1          c2          c3         c4         c5          c6          c7
          --------    --------    -------    -------    -------    --------    --------

    r1    0.016586    0.097547    0.23904    0.31241    0.22966    0.090047    0.014711
    r2    0.016258    0.096266    0.23749    0.31247    0.23124    0.091266    0.015008
    r3    0.015936    0.094997    0.23594    0.31251    0.23281    0.092497    0.015311
    r4     0.01562    0.093741    0.23438    0.31252    0.23438    0.093741     0.01562
    r5    0.015311    0.092497    0.23281    0.31251    0.23594    0.094997    0.015936
    r6    0.015008    0.091266    0.23124    0.31247    0.23749    0.096266    0.016258
    r7    0.014711    0.090047    0.22966    0.31241    0.23904    0.097547    0.016586

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx      value
                          -    ---    ---------

    fl_ar1_beg            1     2     -0.024496
    fl_ar1_end            2     3      0.024496
    fl_ar1_persistence    3     4          0.01
    fl_ar1_step           4     5     0.0081654
    fl_p0                 5     6         0.505
    fl_q0                 6     7         0.505
```

```
fl_shk_std           7    8         0.01
fl_sig_ar1           8    9     0.010001
it_std_bound         9   10            0
```

# Appendix A

# Index and Code Links

## A.1 Savings Dynamic Programming links

1. Looped Grid Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Common grid looped solution
   - Slow, but easy to modify, useful for developing new models
   - Given preferences, some AR(1) shock process, solve the infinite horizon household savings dynamic programming problem. The state-space and choice-space share the same asset grid.
   - **MEconTools**: *ff_vfi_az_loop()*
2. Vectorized Grid Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Common grid vectorized solution
   - Fast, sufficiently approximate value(a,z), but c(a,z) not precise
   - Given preferences, some AR(1) shock process, solve the infinite horizon household savings dynamic programming problem. The state-space and choice-space share the same asset grid.
   - **MEconTools**: *ff_vfi_az_vec()*
3. Looped Exact Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Infinite horizon constrained dynamic savings problem with persistent shock.
   - The state-space is on a grid, the choice space are continuous percentages of cash-on-hand.
   - Looped exact savings-percentage algorithm, slow but high precision at low grid size.
   - Solves for EV(ap,z) given shock state and for a savings choice. Bisection based on FOC with analytical du(c(ap))/dap and spline slopes dEV(ap,z)/dap.
   - **MEconTools**: *ff_vfi_az_bisec_loop() + ff_optim_bisec_savezrone()*
4. Vectorized Exact Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Exact choice vectorized solution
   - Fast, approximates choices with higher precision (given value grid, accurate up to 0.001525878 percent of individual-specific cash-on-hand)
   - Given preferences, some AR(1) shock process, solve the infinite horizon household savings dynamic programming problem. The state-space is on a grid. The choice space are continuous percentages of cash-on-hand.
   - **MEconTools**: *ff_vfi_az_bisec_vec()*

## A.2 Summarize Policy and Value links

1. Summarize ND Array Policy and Value Functions: **mlx** | **m** | **pdf** | **html**
   - Given an NDarray matrix with N1, N2, ..., ND dimensions. Generate average and standard deviation for the 3rd dimension, grouping by the other dimensions.
   - For example, show the 5th dimension as the column groups, and the other variables generate combinations shown as rows.
   - The resulting summary statistics table contains mean and standard deviation among other statistics over the policy or value contained in the ND array.
   - **MEconTools**: *ff_summ_nd_array()*

## A.3 Distributional Analysis links

1. Gateway Joint Probability Mass Statistics: **mlx** | **m** | **pdf** | **html**
   - Given probability mass function f(s), and information y(s), x(s), z(s) at each element of the state-space, compute statistics for each variable, y, x, z, which are all discrete random variables.
   - Compute their correlation and covariance.
   - **MEconTools**: *ff_simu_stats()*
2. Discrete Random Variable Distributional Statistics: **mlx** | **m** | **pdf** | **html**
   - Model simulation generates discrete random variables, calculate mean, standard deviation, min, max, percentiles, and proportion of outcomes held by x percentiles, etc.
   - **MEconTools**: *ff_disc_rand_var_stats()*
3. Generate Discrete Random Variable: **mlx** | **m** | **pdf** | **html**
   - Given mass at state space points, and y, c, a, z and other outcomes or other information at each corresponding state space points, generate discrete random variable, with unique sorted values, and mass for each unique sorted values.
   - Generate additional joint distributions: if initial distribution is over f(a,z), generate joint distribution of f(y,a) or f(y,z).
   - **MEconTools**: *ff_disc_rand_var_mass2outcomes()*
4. Discrete Random Variable Correlation and Covariance: **mlx** | **m** | **pdf** | **html**
   - Given probability mass function f(s), X(s), and Y(s), compute the covariance and correlation betwen X and Y.
   - **MEconTools**: *ff_disc_rand_var_mass2covcor()*

## A.4 Optimizers links

1. Bisection Exact Optimal Savings Share Multiple States: **mlx** | **m** | **pdf** | **html**
   - Given a First Order Condition function handle that takes the fraction of resources (cash-on-hand) saved as the input, solve for the optimal savings fraction via bisection. Solve this concurrently for many elements of the state-space. The function handle contains the FOC with parameters and state-space elements embeded.
   - **MEconTools**: *ff_optim_bisec_savezrone()*
2. Multisection Exact Optimal Savings Share Multiple States: **mlx** | **m** | **pdf** | **html**
   - Given a First Order Condition function handle that takes the fraction of resources (cash-on-hand) saved as the input, solve for the optimal savings fraction via multisection where there are multiple evaluations per iteration of the FOC. Solve this concurrently for many elements of the state-space. The function handle contains the FOC with parameters and state-space elements embeded.
   - **MEconTools**: *ff_optim_mlsec_savezrone()*
3. Vectorized Zooming Exact Optimal Savings Share Multiple States: **mlx** | **m** | **pdf** | **html**
   - Given a Utility (not FOC) function handle that takes the fraction of resources (cash-on-hand) saved as the input, solve for the optimal savings fraction via iterative zooming where there are multiple evaluations per iteration of the utility function. Solve this concurrently for many elements of the state-space. The function handle contains the utilty function with parameters and state-space elements embeded.
   - **MEconTools**: *ff_optim_mzoom_savezrone()*

## A.5 Graphs links

1. Multiple Line Graph Function: **mlx** | **m** | **pdf** | **html**
   - Grid based Graph, x-axis one param, color another param, over outcomes.
   - **MEconTools**: *ff_graph_grid()*

## A.6 Support Tools links

1. Organizes and Prints Container Map Key and Values: **mlx** | **m** | **pdf** | **html**

- Summarizes the contents of a map container by data types. Includes, scalar, array, matrix, string, functions, tensors (3-tuples), tesseracts (4-tuples).
- **MEconTools**: *ff_container_map_display()*

## A.7 Data Structures links

1. Log and Power Spaced Asset and Choice Grids: **mlx** | **m** | **pdf** | **html**
   - Generate linear, log-space, power-space, or threshold-cut asset or choice grids.
   - **MEconTools**: *ff_saveborr_grid()*

## A.8 Common Functions links

1. Discretize AR1 Normal Shock Tauchen (1986): **mlx** | **m** | **pdf** | **html**
   - Mean zero AR(1) shock discretize following Tauchen (1986).
   - **MEconTools**: *ffy_tauchen()*
2. Discretize AR1 Normal Shock Rouwenhorst (1995): **mlx** | **m** | **pdf** | **html**
   - Mean zero AR(1) shock discretize following Rouwenhorst (1995).
   - **MEconTools**: *ffy_rouwenhorst()*

# Bibliography


The MathWorks Inc (2019). *MATLAB*. Matlab package version 2019b.

Xie, Y. (2020). *bookdown: Authoring Books and Technical Documents with R Markdown*. R package version 0.18.