FF_OPTIM_MLSEC_SAVEZRONE Derivative Multisection

back to Fan's Intro Math for Econ, Matlab Examples, or Dynamic Asset Repositories

This is the example vignette for function: **ff_optim_mlsec_savezrone** from the **MEconTools Package.** This functions solves for optimal savings/borrowing level given an anonymous function that provides the derivative of a intertemporal savings problem. This is a vectorized function solved with multi-section (multiple points bisection concurrently).

The vectorized and looped bisection savings problem rely on this function to solve for optimal savings choices:

- States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Loop***: **ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Vectorized***: **ff_vfi_az_bisec_vec**, precision and speed

Test FF_OPTIM_MLSEC_SAVEZRONE One Individual

Bisection for savings choice at one state:

```
% Generate the state-space and function
[fl_z1, fl_z2, fl_r, fl_beta] = deal(0.4730, 0.6252, 0.0839, 0.7365);
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, fl_z1, fl_z2, fl_r, fl_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
% optimally borrowing given the parameters here
mp_mlsec_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mlsec_ctrlinfo('it_mzoom_jnt_pnts') = 10;
mp_mlsec_ctrlinfo('it_mzoom_max_iter') = 4;
[fl_opti_save_frac, fl_opti_save_level] = ...
    ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, bl_timer, mp_mlsec_ctrlinfo)
```

```
Elapsed time is 0.002805 seconds.
fl_opti_save_frac = 0.4241
fl_opti_save_level = -0.1316
```

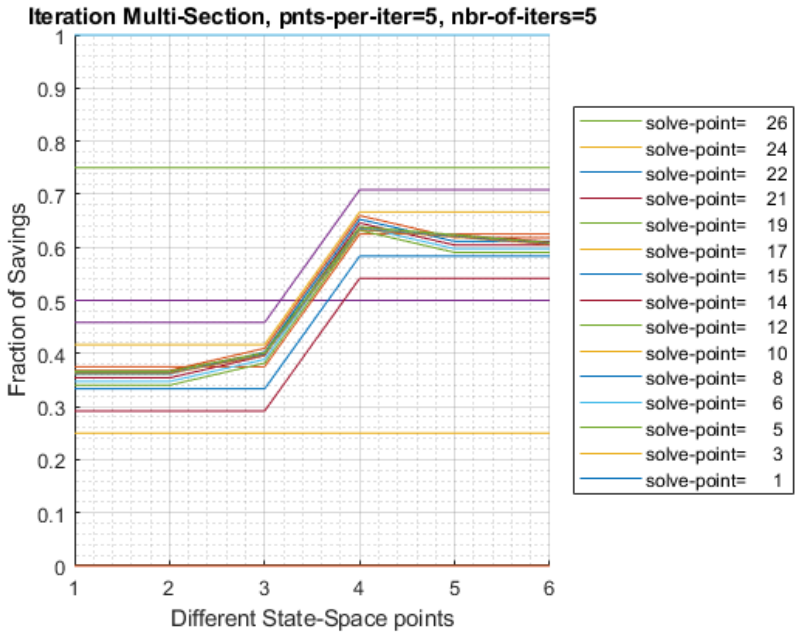
Test FF_OPTIM_MLSEC_SAVEZRONE 5 Individuals 5 Iterations 5 Points Per Iteration

5 grid points per iteration, and 5 iterations.

```
% Generate the state-space and function
rng(123);
it_draws = 6; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
```

```
bl_verbose = true;
bl_timer = true;
mp_mlsec_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mlsec_ctrlinfo('it_mlsect_jnt_pnts') = 5;
mp_mlsec_ctrlinfo('it_mlsect_max_iter') = 5;
ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, bl_timer, mp_mlsec_ctrlinfo);
```

iter	cl_row_names_a	Var1	Var2	Var3	Var4	Var5	Var6
0	"point=1"	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05
1	"point=1"	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05
1	"point=2"	0.25001	0.25001	0.25001	0.25001	0.25001	0.25001
1	"point=3"	0.5	0.5	0.5	0.5	0.5	0.5
1	"point=4"	0.75	0.75	0.75	0.75	0.75	0.75
1	"point=5"	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
2	"point=1"	0.29167	0.29167	0.29167	0.54167	0.54167	0.54167
2	"point=2"	0.33334	0.33334	0.33334	0.58333	0.58333	0.58333
2	"point=3"	0.375	0.375	0.375	0.625	0.625	0.625
2	"point=4"	0.41667	0.41667	0.41667	0.66666	0.66666	0.66666
2	"point=5"	0.45833	0.45833	0.45833	0.70833	0.70833	0.70833
3	"point=1"	0.34028	0.34028	0.38195	0.63194	0.59028	0.59028
3	"point=2"	0.34723	0.34723	0.38889	0.63889	0.59722	0.59722
3	"point=3"	0.35417	0.35417	0.39584	0.64583	0.60416	0.60416
3	"point=4"	0.36111	0.36111	0.40278	0.65277	0.61111	0.61111
3	"point=5"	0.36806	0.36806	0.40972	0.65972	0.61805	0.61805
4	"point=1"	0.36227	0.36227	0.39699	0.6331	0.61921	0.60532
4	"point=2"	0.36343	0.36343	0.39815	0.63426	0.62037	0.60648
4	"point=3"	0.36459	0.36459	0.39931	0.63541	0.62153	0.60764
4	"point=4"	0.36574	0.36574	0.40046	0.63657	0.62268	0.60879
4	"point=5"	0.3669	0.3669	0.40162	0.63773	0.62384	0.60995
5	"point=1"	0.36594	0.36594	0.40066	0.63792	0.62288	0.60783
5	"point=2"	0.36613	0.36613	0.40085	0.63811	0.62307	0.60802
5	"point=3"	0.36632	0.36632	0.40104	0.63831	0.62326	0.60822
5	"point=4"	0.36652	0.36652	0.40124	0.6385	0.62345	0.60841
5	"point=5"	0.36671	0.36671	0.40143	0.63869	0.62365	0.6086




```
Elapsed time is 0.406584 seconds.
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xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	co
ar_opti_foc_obj	1	1	2	6	6	1	-0.00037648	-6.2746e-05	0.00042601	-6
ar_opti_save_frac	2	2	2	6	6	1	3.0037	0.50061	0.13506	0.

```
xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxx
```

	c1
r1	7.0837e-05
r2	-0.0002782
r3	0.00017713
r4	0.00055875
r5	-0.00023392
r6	-0.00067107

```
xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxx
```

	c1
r1	0.36642
r2	0.36661
r3	0.40153
r4	0.63821
r5	0.62297
r6	0.60793

Test FF_OPTIM_MLSEC_SAVEZRONE 8 Individuals 3 Iterations 10 Points Per Iteration

10 grid points per iteration, and 3 iterations.

```
% Generate the state-space and function
rng(123);
it_draws = 8; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
bl_timer = true;
mp_mlsec_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mlsec_ctrlinfo('it_mlsect_jnt_pnts') = 10;
mp_mlsec_ctrlinfo('it_mlsect_max_iter') = 3;
ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, bl_timer, mp_mlsec_ctrlinfo);
```

iter	cl_row_names_a	Var1	Var2	Var3	Var4	Var5	Var6	Var7	Var8
0	"point=1"	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05
1	"point=1"	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05
1	"point=2"	0.11112	0.11112	0.11112	0.11112	0.11112	0.11112	0.11112	0.11112
1	"point=3"	0.22223	0.22223	0.22223	0.22223	0.22223	0.22223	0.22223	0.22223
1	"point=4"	0.33334	0.33334	0.33334	0.33334	0.33334	0.33334	0.33334	0.33334
1	"point=5"	0.44445	0.44445	0.44445	0.44445	0.44445	0.44445	0.44445	0.44445
1	"point=6"	0.55555	0.55555	0.55555	0.55555	0.55555	0.55555	0.55555	0.55555
1	"point=7"	0.66666	0.66666	0.66666	0.66666	0.66666	0.66666	0.66666	0.66666

1	"point=8"	0.77777	0.77777	0.77777	0.77777	0.77777	0.77777	0.77777	0.77777
1	"point=9"	0.88888	0.88888	0.88888	0.88888	0.88888	0.88888	0.88888	0.88888
1	"point=10"	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
2	"point=1"	0.34344	0.23233	0.23233	0.23233	0.56566	0.56566	0.45455	0.56566
2	"point=2"	0.35354	0.24243	0.24243	0.24243	0.57576	0.57576	0.46465	0.57576
2	"point=3"	0.36364	0.25253	0.25253	0.25253	0.58586	0.58586	0.47475	0.58586
2	"point=4"	0.37374	0.26263	0.26263	0.26263	0.59596	0.59596	0.48485	0.59596
2	"point=5"	0.38384	0.27273	0.27273	0.27273	0.60606	0.60606	0.49495	0.60606
2	"point=6"	0.39394	0.28283	0.28283	0.28283	0.61616	0.61616	0.50505	0.61616
2	"point=7"	0.40404	0.29293	0.29293	0.29293	0.62626	0.62626	0.51515	0.62626
2	"point=8"	0.41414	0.30303	0.30303	0.30303	0.63636	0.63636	0.52525	0.63636
2	"point=9"	0.42424	0.31314	0.31314	0.31314	0.64646	0.64646	0.53535	0.64646
2	"point=10"	0.43434	0.32324	0.32324	0.32324	0.65656	0.65656	0.54545	0.65656
3	"point=1"	0.42516	0.27365	0.29385	0.23325	0.55647	0.60698	0.51607	0.57667
3	"point=2"	0.42608	0.27457	0.29477	0.23417	0.55739	0.60789	0.51699	0.57759
3	"point=3"	0.427	0.27549	0.29569	0.23508	0.55831	0.60881	0.51791	0.57851
3	"point=4"	0.42792	0.2764	0.29661	0.236	0.55923	0.60973	0.51882	0.57943
3	"point=5"	0.42884	0.27732	0.29752	0.23692	0.56015	0.61065	0.51974	0.58035
3	"point=6"	0.42975	0.27824	0.29844	0.23784	0.56106	0.61157	0.52066	0.58127
3	"point=7"	0.43067	0.27916	0.29936	0.23876	0.56198	0.61249	0.52158	0.58218
3	"point=8"	0.43159	0.28008	0.30028	0.23967	0.5629	0.6134	0.5225	0.5831
3	"point=9"	0.43251	0.281	0.3012	0.24059	0.56382	0.61432	0.52342	0.58402
3	"point=10"	0.43343	0.28191	0.30212	0.24151	0.56474	0.61524	0.52433	0.58494


```
Elapsed time is 0.633499 seconds.
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xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefva
ar_opti_foc_obj	1	1	2	8	8	1	0.0033175	0.00041468	0.0029592	7.136
ar_opti_save_frac	2	2	2	8	8	1	3.5124	0.43905	0.15005	0.3417

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx

	c1
r1	0.00087102
r2	0.0033354
r3	-0.0044871
r4	0.001317

```
    r5     -0.0017862
    r6      0.0050249
    r7    -0.00058496
    r8    -0.00037273

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxx
            c1
          _______

    r1    0.42838
    r2    0.28054
    r3     0.2989
    r4    0.23371
    r5    0.55877
    r6    0.61019
    r7     0.5202
    r8    0.58172
```

Test FF_OPTIM_MLSEC_SAVEZRONE Speed

Test Speed doing 6.25 million multisections for a savings problem:

```
% Generate the state-space and function
rng(123);
it_draws = 6250000; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
[ar_opti_save_frac, ar_opti_save_level] = ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verb
```

```
Elapsed time is 13.672896 seconds.
```

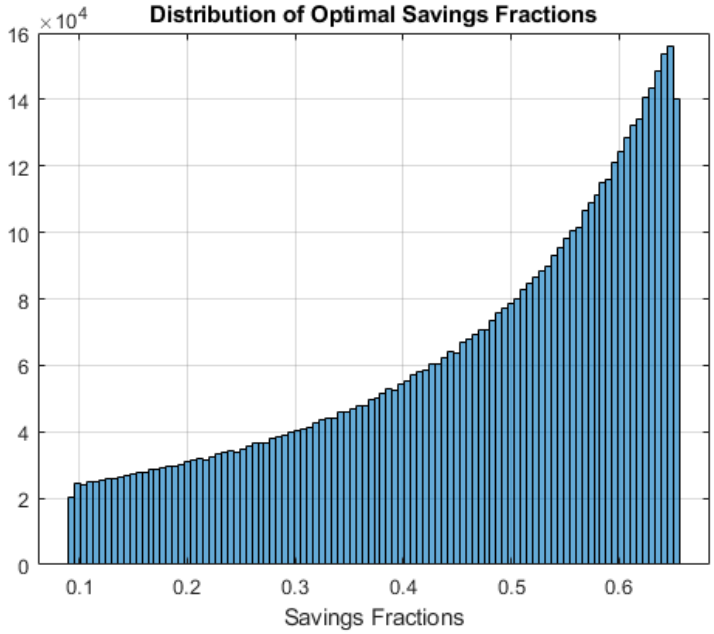
```
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
mp_container_map('ar_opti_save_frac') = ar_opti_save_frac;
mp_container_map('ar_opti_save_level') = ar_opti_save_level;
mp_container_map('ar_opti_save_frac_notnan') = ar_opti_save_frac(~isnan(ar_opti_save_frac));
ff_container_map_display(mp_container_map);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                              i    idx    ndim     numel       rowN       colN       sum         mean        std
                              _    ___    ____    _______    _______    ____    _________    _______    _______

    ar_opti_save_frac         1     1      2      6.25e+06    6.25e+06     1       2.884e+06    0.46144    0.15306
    ar_opti_save_frac_notnan  2     2      2      6.25e+06    6.25e+06     1       2.884e+06    0.46144    0.15306
    ar_opti_save_level        3     3      2      6.25e+06    6.25e+06     1      2.9482e+06    0.47172    0.66667
```

```
figure();
histogram(ar_opti_save_frac(~isnan(ar_opti_save_frac)),100);
title('Distribution of Optimal Savings Fractions');
xlabel('Savings Fractions');
```

```
grid on;
```



Define Two Period Intertemporal FOC Log Utility No Shock

See Household's Utility Maximization Problem and Two-Period Borrowing and Savings Problem given Endowments.

```
function [ar_deri_zero, ar_saveborr_level] = ffi_intertemporal_max(ar_saveborr_frac, z1, z2, r,
    ar_saveborr_level = ar_saveborr_frac.*(z1+z2./(1+r)) - z2./(1+r);
    ar_deri_zero = 1./(ar_saveborr_level-z1) + (beta.*(r+1))./(z2 + ar_saveborr_level.*(r+1));
end
```