Matlab Toolbox for Heterogeneous Agents Dynamic Programming

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Contents

Preface

This is a work-in-progress Matlab package consisting of functions that facilitate Dynamic Programming and Related Tasks. Materials gathered from various projects in which Matlab code is used. Files are the MEconTools repository. Matlab files are linked below by section with livescript files. Tested with Matlab 2019a (The MathWorks Inc, 2019).

This bookdown file is a collection of mlx based vignettes for functions that are available from MEconTools. Each Vignette file contains various examples for invoking each function. The goal of this repository is to make it easier to find/re-use codes produced for various projects.

From other repositories: For dynamic borrowing and savings problems, see Dynamic Asset Repository; For code examples, see also R Example Code, Matlab Example Code, and Stata Example Code; For intro stat with R, see Intro Statistics for Undergraduates, and intro Math with Matlab, see Intro Mathematics for Economists. See here for all of Fan's public repositories.

The site is built using Bookdown (Xie, 2020).

Please contact FanWangEcon for issues or problems.

Chapter 1

Summarize Policy and Value

1.1 FF_SUMM_ND_ARRAY Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_summ_nd_array** from the **MEconTools Package**. This function summarizes policy and value functions over states.

1.1.1 Test FF_SUMM_ND_ARRAY Defaults

Call the function with defaults.

```
ff_summ_nd_array();
```

```
xxx  Summ over (a,z), condi age as cols, kids/marriage as rows  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group    marry    kids    mn_age_18    mn_age_19    mn_age_20    mn_age_21    cv_age_18    cv_ag
    -----    -----    ----    ---------    ---------    ---------    ---------    ---------    -----

      1        0        1       0.59079      0.53324      0.55055      0.48708       1.9219       1.81
      2        1        1       0.49876       0.5033      0.48682      0.45402       1.7356       1.59
      3        0        2       0.50857       0.4829      0.49712      0.52998       1.7159       1.67
      4        1        2       0.45619      0.51721      0.50414      0.56312       1.6098       1.77
      5        0        3       0.52992      0.56536      0.41866      0.50231       1.8123       1.89
      6        1        3       0.53958      0.54057      0.52793       0.4703       1.8546       1.89
      7        0        4       0.46439      0.49755      0.52478      0.55786       1.5849       1.81
      8        1        4        0.4126      0.48144      0.47836      0.48858       1.4588       1.60
```

1.1.2 Test FF_SUMM_ND_ARRAY with Random 2 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random 2D dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(5,4);
bl_print_table = true;
ar_st_stats = ["mean"];
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'a', linspace(0,1,size(mn_polval,1))});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'z', linspace(-1,1,size(mn_polval,2))});
disp(mn_polval);
```

```
    0.6965    0.4231    0.3432    0.7380
    0.2861    0.9808    0.7290    0.1825
    0.2269    0.6848    0.4386    0.1755
    0.5513    0.4809    0.0597    0.5316
    0.7195    0.3921    0.3980    0.5318
```

Second, show the entire matrix (no labels):

```
it_aggd = 0;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim2    mn_vardim1_1    mn_vardim1_2    mn_vardim1_3    mn_vardim1_4    mn_vardim1_5
    -----    -------    ------------    ------------    ------------    ------------    ------------

      1         1          0.69647         0.28614         0.22685          0.55131         0.71947
      2         2          0.42311         0.98076         0.68483          0.48093         0.39212
      3         3          0.34318         0.72905         0.43857         0.059678         0.39804
      4         4            0.738         0.18249         0.17545          0.53155         0.53183
```

Third, rotate row and column, and now with labels:

```
it_aggd = 0;
bl_row = 1;
ar_permute = [2,1];
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group     a      mn_z__1    mn_z__0_33333    mn_z_0_33333    mn_z_1
    -----    ----    -------    -------------    ------------    -------

      1         0    0.69647       0.42311          0.34318        0.738
      2      0.25    0.28614       0.98076          0.72905      0.18249
      3       0.5    0.22685       0.68483          0.43857      0.17545
      4      0.75    0.55131       0.48093         0.059678      0.53155
      5         1    0.71947       0.39212          0.39804      0.53183
```

Fourth, dimension one as columns, average over dim 2:

```
it_aggd = 1;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x    mn_z__1    mn_z__0_33333    mn_z_0_33333    mn_z_1
    -----    -    -------    -------------    ------------    -------

      1      1    0.49605       0.59235          0.3937       0.43186
```

Fifth, dimension one as rows, average over dim 2:

```
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group       z         mean        std      coefvari      min         max
```

1	-1	0.49605	0.22895	2.1666	0.22685	0.71947
2	-0.33333	0.59235	0.24524	2.4154	0.39212	0.98076
3	0.33333	0.3937	0.23907	1.6468	0.059678	0.72905
4	1	0.43186	0.24575	1.7573	0.17545	0.738

Sixth, dimension two as rows, average over dim 1:

```
ar_permute = [2,1];
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);
```

```
xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxx
    group     a       mean        std      coefvari      min         max
    -----    ----    -------    -------    --------    --------    -------

      1        0     0.55019    0.19636     2.8019      0.34318      0.738
      2      0.25    0.54461    0.37514     1.4518      0.18249    0.98076
      3       0.5    0.38143    0.23212     1.6432      0.17545    0.68483
      4      0.75    0.40587    0.23269     1.7443     0.059678    0.55131
      5        1     0.51036    0.15361     3.3226      0.39212    0.71947
```

1.1.3 Test FF_SUMM_ND_ARRAY with Random 6 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random ND dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(8,7,6,5,4,3);
bl_print_table = true;
ar_st_stats = ["mean"];
```

Second, summarize over the first four dimensions, row group others:

```
it_aggd = 4;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);
```

```
xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim5    vardim6     mean        std      coefvari       min          max
    -----    -------    -------    -------    -------    --------    ----------    -------

      1        1          1       0.49808    0.29255     1.7026     8.1888e-05    0.99964
      2        2          1       0.50128    0.28968     1.7305     6.7838e-05    0.99936
      3        3          1       0.49491    0.28851     1.7154     0.00091373    0.99989
      4        4          1       0.50232    0.28154     1.7842     0.00012471    0.99731
      5        1          2        0.4994     0.2911     1.7156     0.00029749    0.99938
      6        2          2       0.49453    0.28634     1.7271     0.00027113     0.9992
      7        3          2       0.49559    0.28682     1.7279     0.00035994    0.99936
      8        4          2       0.48835    0.29032     1.6821     0.00096259    0.99896
      9        1          3       0.51819    0.29111     1.7801      0.0010616    0.99951
     10        2          3       0.50874    0.28458     1.7877       0.001884    0.99965
     11        3          3       0.49898     0.2891      1.726      0.0019192    0.99945
     12        4          3       0.50169     0.2877     1.7438     0.00016871    0.99963
```

Third, summarize over the first four dimensions, column group 5th, and row group others:

```
it_aggd = 4;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim6    mn_vardim5_1    mn_vardim5_2    mn_vardim5_3    mn_vardim5_4
    -----    -------    ------------    ------------    ------------    ------------

      1         1         0.49808         0.50128         0.49491         0.50232
      2         2          0.4994         0.49453         0.49559         0.48835
      3         3         0.51819         0.50874         0.49898         0.50169
```

Fourth, summarize over the first five dimensions, column group 6th, no row groups:

```
it_aggd = 5;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x    mn_vardim6_1    mn_vardim6_2    mn_vardim6_3
    -----    -    ------------    ------------    ------------

      1      1      0.49915         0.49447          0.5069
```

Fifth, summarize over all six dimensions, summary statistics over the entire dataframe:

```
it_aggd = 6;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x     mean        std      coefvari       min          max
    -----    -    -------    -------    --------    ----------    -------

      1      1    0.50017    0.28831     1.7349     6.7838e-05    0.99989
```

1.1.4 Test FF_SUMM_ND_ARRAY with Random 7 Dimensional Matrix with All Parameters

Given a random seven dimensional matrix, average over the 2nd, 4th and 5th dimensionals. Show as row groups the 3, 6 and 7th dimensions, and row groups the 1st dimension.

```
st_title = "avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim.";
rng(123)
mn_polval = rand(3,10,2,10,10,2,3);
ar_permute = [2,4,5,1,3,6,7];
bl_print_table = true;
ar_st_stats = ["mean", "coefvari"];
it_aggd = 3; % mean over 3 dims
bl_row = 1; % one var for row group
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'age', [18, 19, 20]});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'savings', linspace(0,1,10)});
cl_mp_datasetdesc{3} = containers.Map({'name', 'labval'}, ...
    {'borrsave', [-1,+1]});
cl_mp_datasetdesc{4} = containers.Map({'name', 'labval'}, ...
    {'shocka', linspace(-5,5,10)});
cl_mp_datasetdesc{5} = containers.Map({'name', 'labval'}, ...
    {'shockb', linspace(-5,5,10)});
```

```
cl_mp_datasetdesc{6} = containers.Map({'name', 'labval'}, ...
    {'marry', [0,1]});
cl_mp_datasetdesc{7} = containers.Map({'name', 'labval'}, ...
    {'region', [1,2,3]});
% call function
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, cl_mp_datasetdes

xxx  avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim.  xxxxxxxxxxxxxxxxxxxx
    group    borrsave    marry    region    mn_age_18    mn_age_19    mn_age_20    cv_age_18    cv_a
    -----    --------    -----    ------    ---------    ---------    ---------    ---------    ----

      1         -1         0        1        0.50503      0.50389      0.49788      1.7607       1.7
      2          1         0        1         0.4829      0.50795      0.49205      1.6566       1.7
      3         -1         1        1        0.48123      0.50734      0.50109      1.6608       1.7
      4          1         1        1        0.49987      0.49852      0.49519       1.756       1.7
      5         -1         0        2        0.49859      0.50866      0.51752      1.7314       1.7
      6          1         0        2        0.50451      0.49802      0.50439      1.7347        1.
      7         -1         1        2        0.50967      0.49651      0.50556      1.7811        1.
      8          1         1        2        0.50209      0.49224      0.50252      1.7445       1.7
      9         -1         0        3        0.48885      0.49229      0.49692      1.7025       1.7
     10          1         0        3        0.49534      0.50183      0.50266        1.74       1.7
     11         -1         1        3        0.50312      0.50535      0.48959      1.7147       1.7
     12          1         1        3        0.51204      0.49998      0.50738      1.7919       1.7
```

Chapter 2

Distributional Analysis

2.1 FF_SIMU_STATS Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_simu_stats** from the **MEconTools Package.** This is a gate-way function that computes mean, percentiles, covariance etc between several variables.

2.1.1 Test FF_SIMU_STATS Defaults

Call the function with defaults.

```
ff_simu_stats();
```

```
xxx tb_outcomes: all stats xxx
    OriginalVariableNames     cl_mt_pol_a    cl_mt_pol_c
    ---------------------     -----------    -----------

    {'mean'              }     -0.11081         8.8423
    {'sd'                }       4.1239         6.5845
    {'coefofvar'         }      -37.215        0.74466
    {'min'               }           -7        -6.3772
    {'max'               }            9         21.786
    {'pYis0'             }     0.064259              0
    {'pYls0'             }      0.54867       0.027329
    {'pYgr0'             }      0.38707        0.97267
    {'pYisMINY'          }     0.051764       0.015232
    {'pYisMAXY'          }     0.027329       0.046484
    {'p1'                }           -7        -6.3772
    {'p10'               }           -6        0.27238
    {'p25'               }           -3         5.2138
    {'p50'               }           -1         6.5321
    {'p75'               }            3         13.799
    {'p90'               }            5         16.887
    {'p99'               }            9         21.786
    {'fl_cov_cl_mt_pol_a'}       17.007        -22.084
    {'fl_cor_cl_mt_pol_a'}            1       -0.81327
    {'fl_cov_cl_mt_pol_c'}      -22.084         43.356
    {'fl_cor_cl_mt_pol_c'}     -0.81327              1
    {'fracByP1'          }       3.2699      -0.010985
    {'fracByP10'         }       5.9889      -0.013362
    {'fracByP25'         }       14.165       0.041007
    {'fracByP50'         }       16.208         0.1893
```

```
    {'fracByP75'         }       12.702         0.59539
    {'fracByP90'         }       6.6611          0.8307
    {'fracByP99'         }            1               1
```

2.1.2 Test FF_SIMU_STATS Four States-Points Matrix

Over some (a,z) states that is 3 by 3, c matrix, generate all stats

```
% Set Parameters
mt_x_of_s = [1, 2,  3.0;...
             3, 1,  1.5;...
             4, 3,  2.0];
mt_y_of_s = [2, -10, 9.0;...
             5, 1.1,3.0;...
             1, 3,  -1.5];
mt_z_of_s = [1.1, 2,3.3;...
             2.3, 1,1.5;...
             4, 2.5,2.0];
mp_cl_mt_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
% Mass
rng(123);
mt_f_of_s = rand(size(mt_x_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s, mp_cl_mt_xyz_of_s);

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_mt_x_of_s    cl_mt_y_of_s    cl_mt_z_of_s
    ______________________    ____________    ____________    ____________

    {'mean'               }        2.0763          1.9323          2.0668
    {'sd'                 }        0.9071          5.2239          0.9042
    {'coefofvar'          }       0.43688          2.7034         0.43749
    {'min'                }             1             -10               1
    {'max'                }             4               9               4
    {'pYis0'              }             0               0               0
    {'pYls0'              }             0         0.20441               0
    {'pYgr0'              }             1         0.79559               1
    {'pYisMINY'           }       0.28039         0.10917         0.14247
    {'pYisMAXY'           }      0.044922         0.19422        0.044922
    {'p1'                 }             1             -10               1
    {'p10'                }             1             -10               1
    {'p25'                }             1             1.1             1.1
    {'p50'                }             2               2               2
    {'p75'                }             3               5             2.5
    {'p90'                }             3               9             3.3
    {'p99'                }             4               9               4
    {'fl_cov_cl_mt_x_of_s'}       0.82282           1.589         0.78646
    {'fl_cor_cl_mt_x_of_s'}             1         0.33534         0.95887
    {'fl_cov_cl_mt_y_of_s'}         1.589          27.289          1.8353
    {'fl_cor_cl_mt_y_of_s'}       0.33534               1         0.38856
    {'fl_cov_cl_mt_z_of_s'}       0.78646          1.8353         0.81758
    {'fl_cor_cl_mt_z_of_s'}       0.95887         0.38856               1
    {'fracByP1'           }       0.13504        -0.56498        0.068934
    {'fracByP10'          }       0.13504        -0.56498        0.068934
```

```
    {'fracByP25'          }       0.13504        -0.53456         0.14234
    {'fracByP50'          }       0.42991        -0.39181         0.43856
    {'fracByP75'          }       0.91346        0.095425         0.60296
    {'fracByP90'          }       0.91346               1         0.91306
    {'fracByP99'          }             1               1               1
```

2.1.3 Test FF_SIMU_STATS Four States-Points Matrix Single Column Inputs

Same as before, but now inputs are single column, should have identical results:

```
% Array Inputs
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_mt_xyz_of_s);

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_mt_x_of_s    cl_mt_y_of_s    cl_mt_z_of_s
    ______________________    ____________    ____________    ____________

    {'mean'               }        2.0763          1.9323          2.0668
    {'sd'                 }        0.9071          5.2239          0.9042
    {'coefofvar'          }       0.43688          2.7034         0.43749
    {'min'                }             1             -10               1
    {'max'                }             4               9               4
    {'pYis0'              }             0               0               0
    {'pYls0'              }             0         0.20441               0
    {'pYgr0'              }             1         0.79559               1
    {'pYisMINY'           }       0.28039         0.10917         0.14247
    {'pYisMAXY'           }      0.044922         0.19422        0.044922
    {'p1'                 }             1             -10               1
    {'p10'                }             1             -10               1
    {'p25'                }             1             1.1             1.1
    {'p50'                }             2               2               2
    {'p75'                }             3               5             2.5
    {'p90'                }             3               9             3.3
    {'p99'                }             4               9               4
    {'fl_cov_cl_mt_x_of_s'}       0.82282           1.589         0.78646
    {'fl_cor_cl_mt_x_of_s'}             1         0.33534         0.95887
    {'fl_cov_cl_mt_y_of_s'}         1.589          27.289          1.8353
    {'fl_cor_cl_mt_y_of_s'}       0.33534               1         0.38856
    {'fl_cov_cl_mt_z_of_s'}       0.78646          1.8353         0.81758
    {'fl_cor_cl_mt_z_of_s'}       0.95887         0.38856               1
    {'fracByP1'           }       0.13504        -0.56498        0.068934
    {'fracByP10'          }       0.13504        -0.56498        0.068934
    {'fracByP25'          }       0.13504        -0.53456         0.14234
    {'fracByP50'          }       0.42991        -0.39181         0.43856
    {'fracByP75'          }       0.91346        0.095425         0.60296
    {'fracByP90'          }       0.91346               1         0.91306
    {'fracByP99'          }             1               1               1
```

2.1.4 Test FF_SIMU_STATS Print Many Details

The Same As before, but now control which percentiles and other details to display.

```
% Array Inputs
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_ar_xyz_of_s('cl_ar_x_of_s') = {mt_x_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('cl_ar_z_of_s') = {mt_z_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('ar_st_y_name') = ["cl_ar_x_of_s", "cl_ar_z_of_s"];

% controls
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_display_detail') = false;
mp_support('bl_display_final') = true;
mp_support('bl_display_drvm2outcomes') = false;
mp_support('ar_fl_percentiles') = [25 50 75];
mp_support('bl_display_drvstats') = true;
mp_support('bl_display_drvm2covcor') = false;

% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_ar_xyz_of_s, mp_support);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: cl_ar_x_of_s
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    2.0763

fl_choice_sd
    0.9071

fl_choice_coefofvar
    0.4369

fl_choice_prob_zero
     0

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
     1

fl_choice_prob_max
    0.0449

tb_disc_cumu
    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

                1                          0.28039                28.039     0.13504
              1.5                          0.13561                 41.6      0.23301
                2                          0.20441                62.041     0.42991
                3                          0.33466                95.508     0.91346
                4                         0.044922                   100           1

    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

                1                          0.28039                28.039     0.13504
```

```
            1.5                        0.13561                41.6     0.23301
              2                        0.20441              62.041     0.42991
              3                        0.33466              95.508     0.91346
              4                       0.044922                 100           1

tb_prob_drv
    percentiles    cl_ar_x_of_sDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ---------------------------------------    --------------------------------

        25                          1                                  0.13504
        50                          2                                  0.42991
        75                          3                                  0.91346

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: cl_ar_z_of_s
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    2.0668

fl_choice_sd
    0.9042

fl_choice_coefofvar
    0.4375

fl_choice_prob_zero
     0

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
     1

fl_choice_prob_max
    0.0449

tb_disc_cumu
    cl_ar_z_of_sDiscreteVal    cl_ar_z_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

              1                          0.14247                14.247     0.068934
            1.1                          0.13792                28.039      0.14234
            1.5                          0.13561                  41.6      0.24076
              2                          0.20441                62.041      0.43856
            2.3                          0.056663               67.708      0.50162
            2.5                          0.083786               76.086      0.60296
            3.3                           0.19422               95.508      0.91306
              4                         0.044922                  100           1

    cl_ar_z_of_sDiscreteVal    cl_ar_z_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

              1                          0.14247                14.247     0.068934
            1.1                          0.13792                28.039      0.14234
            1.5                          0.13561                  41.6      0.24076
```

```
             2                    0.20441          62.041       0.43856
           2.3                   0.056663          67.708       0.50162
           2.5                   0.083786          76.086       0.60296
           3.3                    0.19422          95.508       0.91306
             4                   0.044922             100             1

tb_prob_drv
    percentiles    cl_ar_z_of_sDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ---------------------------------------    --------------------------------

        25                          1.1                                0.14234
        50                            2                                0.43856
        75                          2.5                                0.60296

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_ar_x_of_s    cl_ar_z_of_s
    ----------------------    ------------    ------------

    {'mean'               }        2.0763          2.0668
    {'sd'                 }        0.9071          0.9042
    {'coefofvar'          }       0.43688         0.43749
    {'min'                }             1               1
    {'max'                }             4               4
    {'pYis0'              }             0               0
    {'pYls0'              }             0               0
    {'pYgr0'              }             1               1
    {'pYisMINY'           }       0.28039         0.14247
    {'pYisMAXY'           }      0.044922        0.044922
    {'p25'                }             1             1.1
    {'p50'                }             2               2
    {'p75'                }             3             2.5
    {'fl_cov_cl_ar_x_of_s'}       0.82282         0.78646
    {'fl_cor_cl_ar_x_of_s'}             1         0.95887
    {'fl_cov_cl_ar_z_of_s'}       0.78646         0.81758
    {'fl_cor_cl_ar_z_of_s'}       0.95887               1
    {'fracByP25'          }       0.13504         0.14234
    {'fracByP50'          }       0.42991         0.43856
    {'fracByP75'          }       0.91346         0.60296
```

2.2 FF_DISC_RAND_VAR_STATS Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_stats** from the **MEconTools Package.** This function summarizes statistics of matrixes stored in a container map, as well as scalar, string, function and other values stored in container maps.

2.2.1 Test FF_DISC_RAND_VAR_STATS Defaults

Call the function with defaults.

```
ff_disc_rand_var_stats();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: binom
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
```

```
fl_choice_mean
   -1.0000

fl_choice_sd
    2.5100

fl_choice_coefofvar
   -2.5100

fl_choice_prob_zero
    0.1416

fl_choice_prob_below_zero
    0.5888

fl_choice_prob_above_zero
    0.2696

fl_choice_prob_max
   2.0589e-16

tb_disc_cumu
    binomDiscreteVal    binomDiscreteValProbMass       CDF       cumsumFrac
    ----------------    ------------------------    ---------    ----------

          -10                 2.2539e-05           0.0022539    0.00022539
           -9                 0.00028979            0.031233     0.0028335
           -8                  0.0018008              0.21132       0.01724
           -7                  0.0072034              0.93166      0.067664
           -6                   0.020838               3.0155       0.19269
           -5                    0.04644               7.6595       0.42489
           -4                   0.082928               15.952       0.75661
           -3                    0.12185               28.138        1.1222
           -2                    0.15014               43.152        1.4224
           -1                    0.15729               58.881        1.5797

    binomDiscreteVal    binomDiscreteValProbMass    CDF    cumsumFrac
    ----------------    ------------------------    ---    ----------

           11                 6.0392e-06           100         1
           12                 1.0588e-06           100         1
           13                 1.5784e-07           100         1
           14                  1.973e-08           100         1
           15                 2.0293e-09           100         1
           16                 1.6725e-10           100         1
           17                 1.0619e-11           100         1
           18                 4.8762e-13           100         1
           19                 1.4412e-14           100         1
           20                 2.0589e-16           100         1

tb_prob_drv
    percentiles    binomDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    --------------------------------    --------------------------------

        0.1                       -8                                0.01724
          1                       -6                                0.19269
          5                       -5                                0.42489
         10                       -4                                0.75661
```

15	-4	0.75661
20	-3	1.1222
25	-3	1.1222
35	-2	1.4224
50	-1	1.5797
65	0	1.5797
75	1	1.4694
80	1	1.4694
85	2	1.3197
90	2	1.3197
95	3	1.1865
99	5	1.0412
99.9	7	1.0052

2.2.2 Test FF_DISC_RAND_VAR_STATS 0 and 1 Random Variable

The simplest discrete random variable has two values, zero or one. The probability of zero is 30 percent, and 70 percent is the probability of one.

```
% Parameters
% 1. specify the random variable
st_var_name = 'bernoulli';
ar_choice_unique_sorted = [0, 1];
ar_choice_prob = [0.3, 0.7];
% 2. percentiles of interest
ar_fl_percentiles = [0.1 5 25 50 75 95 99.9];
% 3. print resutls
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: bernoulli
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    0.7000

fl_choice_sd
    0.4583

fl_choice_coefofvar
    0.6547

fl_choice_prob_zero
    0.3000

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
    0.7000

fl_choice_prob_max
    0.7000
```

```
tb_disc_cumu
    bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
    --------------------    ----------------------------    ---    ----------

              0                           0.3                30        0
              1                           0.7               100        1

    bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
    --------------------    ----------------------------    ---    ----------

              0                           0.3                30        0
              1                           0.7               100        1

tb_prob_drv
    percentiles    bernoulliDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ------------------------------------    --------------------------------

        0.1                         0                                  0
          5                         0                                  0
         25                         0                                  0
         50                         1                                  1
         75                         1                                  1
         95                         1                                  1
       99.9                         1                                  1
```

2.2.3 Test FF_DISC_RAND_VAR_STATS with Poisson

Poisson random variable, with mean equals to ten, summarize over umsymmetric percentiles. Note that the poisson random variable has no upper bound.

```
% Parameters
% 1. specify the random variable
st_var_name = 'poisson';
mu = 10;
ar_choice_unique_sorted = 0:1:50;
ar_choice_prob = poisspdf(ar_choice_unique_sorted, mu);
% 2. percentiles of interest, unsymmetric
ar_fl_percentiles = [0.1 5 10 25 50 90 95 99 99.9 99.99 99.999 99.9999];
% 3. print resutls
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: poisson
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    10

fl_choice_sd
    3.1623

fl_choice_coefofvar
    0.3162
```

```
fl_choice_prob_zero
   4.5400e-05

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
    1.0000

fl_choice_prob_max
   1.4927e-19

tb_disc_cumu
    poissonDiscreteVal    poissonDiscreteValProbMass      CDF      cumsumFrac
    ------------------    --------------------------    -------    ----------

            0                      4.54e-05             0.00454           0
            1                      0.000454             0.04994     4.54e-05
            2                       0.00227             0.27694    0.0004994
            3                     0.0075667              1.0336    0.0027694
            4                      0.018917              2.9253     0.010336
            5                      0.037833              6.7086     0.029253
            6                      0.063055              13.014     0.067086
            7                      0.090079              22.022      0.13014
            8                        0.1126              33.282      0.22022
            9                       0.12511              45.793      0.33282

    poissonDiscreteVal    poissonDiscreteValProbMass    CDF    cumsumFrac
    ------------------    --------------------------    ---    ----------

            41                    1.3571e-13            100        1
            42                    3.2313e-14            100        1
            43                    7.5146e-15            100        1
            44                    1.7079e-15            100        1
            45                    3.7953e-16            100        1
            46                    8.2506e-17            100        1
            47                    1.7554e-17            100        1
            48                    3.6572e-18            100        1
            49                    7.4636e-19            100        1
            50                    1.4927e-19            100        1

tb_prob_drv
    percentiles    poissonDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ----------------------------------    --------------------------------

         0.1                       2                                0.0004994
           5                       5                                 0.029253
          10                       6                                 0.067086
          25                       8                                  0.22022
          50                      10                                  0.45793
          90                      14                                  0.86446
          95                      15                                  0.91654
          99                      18                                  0.98572
        99.9                      21                                  0.99841
       99.99                      24                                  0.99988
      99.999                      26                                  0.99998
         100                      28                                        1
```

```
% Print out full Stored Matrix
% Note that the outputs are single row arrays.
ff_container_map_display(ds_stats_map, 100, 100)

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                              i    idx    ndim    numel    rowN    colN     mean       std      coe
                              -    ---    ----    -----    ----    ----    -------    ------    ---

    ar_choice_perc_fracheld   1     1      2       12       1       12     0.62833     0.435    0.6
    ar_choice_percentiles     2     2      2       12       1       12       14.75    8.7399    0.5
    ar_fl_percentiles         3     3      2       12       1       12      64.499    42.887    0.6

xxx TABLE:ar_choice_perc_fracheld xxxxxxxxxxxxxxxxx
             c1           c2           c3          c4          c5          c6          c7          c8
          ---------    --------    --------    -------    -------    -------    -------    -------

    r1    0.0004994    0.029253    0.067086    0.22022    0.45793    0.86446    0.91654    0.98572

xxx TABLE:ar_choice_percentiles xxxxxxxxxxxxxxxxx
          c1    c2    c3    c4    c5    c6    c7    c8    c9    c10    c11    c12
          --    --    --    --    --    --    --    --    --    ---    ---    ---

    r1    2     5     6     8     10    14    15    18    21    24     26     28

xxx TABLE:ar_fl_percentiles xxxxxxxxxxxxxxxxx
          c1     c2    c3    c4    c5    c6    c7    c8     c9      c10      c11      c12
          ---    --    --    --    --    --    --    --    ----    -----    ------    ---

    r1    0.1    5     10    25    50    90    95    99    99.9    99.99    99.999    100

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i     idx      value
                                --    ---    ----------

    fl_choice_coefofvar          1     4        0.31623
    fl_choice_max                2     5             50
    fl_choice_mean               3     6             10
    fl_choice_min                4     7              0
    fl_choice_prob_above_zero    5     8        0.99995
    fl_choice_prob_below_zero    6     9              0
    fl_choice_prob_max           7    10     1.4927e-19
    fl_choice_prob_min           8    11       4.54e-05
    fl_choice_prob_zero          9    12       4.54e-05
    fl_choice_sd                10    13         3.1623
```

2.3 FF_DISC_RAND_VAR_MASS2OUTCOMES Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_mass2outcomes** from the **MEconTools Package.** This function generates sorted discrete random variable from state-space joint distri-

bution.

2.3.1 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2outcomes();

INPUT f(a,z): mt_dist_bystates
    0.0289    0.0465    0.0228    0.0036    0.0001
    0.0241    0.0930    0.0857    0.0241    0.0015
    0.0080    0.0744    0.1285    0.0643    0.0074
    0.0013    0.0297    0.0964    0.0857    0.0186
    0.0001    0.0059    0.0361    0.0571    0.0232
    0.0000    0.0005    0.0054    0.0152    0.0116

INPUT y(a,z): mt_choice_bystates
    -5    -4    -5    -4    -4
    -3    -2    -3    -2    -3
    -1    -1    -1     0     0
     1     1     2     3     1
     4     3     3     4     3
     5     6     5     6     6

OUTPUT f(y): ar_choice_prob_byY
    0.0518
    0.0502
    0.1113
    0.1171
    0.2109
    0.0717
    0.0497
    0.0964
    0.1510
    0.0572
    0.0054
    0.0273

OUTPUT f(y,z): mt_choice_prob_byYZ
    0.0289         0    0.0228         0         0
         0    0.0465         0    0.0036    0.0001
    0.0241         0    0.0857         0    0.0015
         0    0.0930         0    0.0241         0
    0.0080    0.0744    0.1285         0         0
         0         0         0    0.0643    0.0074
    0.0013    0.0297         0         0    0.0186
         0         0    0.0964         0         0
         0    0.0059    0.0361    0.0857    0.0232
    0.0001         0         0    0.0571         0
    0.0000         0    0.0054         0         0
         0    0.0005         0    0.0152    0.0116

OUTPUT f(y,a): mt_choice_prob_byYA
    0.0518         0         0         0         0         0
    0.0502         0         0         0         0         0
         0    0.1113         0         0         0         0
         0    0.1171         0         0         0         0
         0         0    0.2109         0         0         0
         0         0    0.0717         0         0         0
```

```
         0         0         0    0.0497         0         0
         0         0         0    0.0964         0         0
         0         0         0    0.0857    0.0653         0
         0         0         0         0    0.0572         0
         0         0         0         0         0    0.0054
         0         0         0         0         0    0.0273

OUTPUT f(y) and y in table: tb_choice_drv_cur_byY
    binomtestOutcomes    probMassFunction
    -----------------    ----------------

           -5                0.051764
           -4                0.050217
           -3                 0.11126
           -2                 0.11706
           -1                 0.21092
            0                0.071696
            1                0.049682
            2                0.096388
            3                 0.15102
            4                0.057231
            5               0.0054256
            6                0.027329
```

2.3.2 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Four States-Points

Over some (a,z) states that is 2 by 2, matrix or vectorized inputs identical results.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2;3,1];
% stationary mass over assets adn shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);

    0.4039    1.0000
    0.2971    2.0000
    0.2990    3.0000
```

Same as before, but now inputs are single column:

```
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s(:), mt_f_of_s);
disp([ar_f_of_y ar_y_unique_sorted]);

    0.4039    1.0000
    0.2971    2.0000
    0.2990    3.0000
```

2.3.3 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Conditional Mass Outputs

Same inputs as before, but now, also output additional conditional statistis, f(y, a), where a is the row state variable for f(a,z). For conditional statistics, must provide matrix based inputs.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2,0.5;
             3,1,2.0];
% stationary mass over assets adn shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted, mt_f_of_y_srow, mt_f_of_y_scol] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);

    0.2695    0.5000
    0.3765    1.0000
    0.2649    2.0000
    0.0891    3.0000

disp(mt_f_of_y_srow);

    0.2695         0
    0.1215    0.2550
    0.1217    0.1432
         0    0.0891

disp(mt_f_of_y_scol);

         0         0    0.2695
    0.1215    0.2550         0
         0    0.1217    0.1432
    0.0891         0         0
```

2.4 FF_DISC_RAND_VAR_MASS2COVCOR Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_mass2covcor** from the **MEconTools Package.** This function calculates covariance and correlation based for two discrete random variables.

2.4.1 Test FF_DISC_RAND_VAR_MASS2COVCOR Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2covcor();

-----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_input_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    ndim    numel    rowN    colN      mean        std       coefvari
                 -    ---    ----    -----    ----    ----    --------    --------    --------    --
```

mt_f_of_s	1	5	2	30	6	5	0.033333	0.035743	1.0723	3.
mt_x_of_s	2	6	2	30	6	5	0.83333	5.3051	6.3661	
mt_y_of_s	3	7	2	30	6	5	8.3259	7.1913	0.86373	

xxx TABLE:mt_f_of_s xxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5
r1	0.028917	0.046484	0.022848	0.0036146	0.000119
r2	0.024097	0.092967	0.085679	0.024097	0.0014875
r3	0.0080324	0.074374	0.12852	0.064259	0.0074374
r4	0.0013387	0.02975	0.096388	0.085679	0.018593
r5	0.00011156	0.0059499	0.036146	0.057119	0.023242
r6	3.7187e-06	0.00047599	0.0054218	0.015232	0.011621

xxx TABLE:mt_x_of_s xxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5
r1	-7	-6	-7	-6	-6
r2	-5	-3	-5	-3	-4
r3	-2	-1	-1	0	-1
r4	2	2	3	4	2
r5	6	5	5	6	5
r6	8	9	7	9	9

xxx TABLE:mt_y_of_s xxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5
r1	13.231	21.786	18.136	19.35	13.901
r2	9.946	16.887	9.6914	15.71	8.6906
r3	16.255	6.2166	13.799	5.2138	11.641
r4	12.628	2.7525	6.5321	0.27238	13.357
r5	5.8844	4.0352	6.05	0.14102	0.50318
r6	3.5617	-0.72091	5.1855	-6.3772	-4.4805

---

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_input_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

	i	idx	value
fl_x_mean	1	1	-0.11081
fl_x_sd	2	2	4.1239
fl_y_mean	3	3	8.8423
fl_y_sd	4	4	6.5845

---

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_output_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

	i	idx	ndim	numel	rowN	colN	mean	std
mt_cov_component_weighted	1	1	2	30	6	5	-0.73612	1.0404

```
    mt_x_devi_from_mean          2     2      2       30       6       5       0.94415    5.3051
    mt_x_y_multiply              3     3      2       30       6       5       -31.321    36.564
    mt_y_devi_from_mean          4     4      2       30       6       5      -0.51644    7.1913

xxx TABLE:mt_cov_component_weighted xxxxxxxxxxxxxxxxx
              c1            c2           c3          c4           c5
          -----------    ---------    --------    ---------    ----------

    r1       -0.87434      -3.5432     -1.4628     -0.22368    -0.0035451
    r2       -0.13003      -2.1607    -0.35565     -0.47814    0.00087767
    r3       -0.11248      0.17365    -0.56642    -0.025838     -0.018507
    r4       0.010697     -0.38241    -0.69273      -3.0184       0.17717
    r5     -0.0020165     -0.14618    -0.51584      -3.0371      -0.99056
    r6    -0.00015927    -0.041473    -0.14098      -2.1121       -1.4106

xxx TABLE:mt_x_devi_from_mean xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          -------    --------    --------    -------    --------

    r1    -6.8892     -5.8892     -6.8892    -5.8892     -5.8892
    r2    -4.8892     -2.8892     -4.8892    -2.8892     -3.8892
    r3    -1.8892    -0.88919    -0.88919    0.11081    -0.88919
    r4     2.1108      2.1108      3.1108     4.1108      2.1108
    r5     6.1108      5.1108      5.1108     6.1108      5.1108
    r6     8.1108      9.1108      7.1108     9.1108      9.1108

xxx TABLE:mt_x_y_multiply xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          -------    -------    -------    --------    -------

    r1    -30.237    -76.225    -64.023     -61.882    -29.792
    r2     -5.396    -23.242     -4.151     -19.842    0.59004
    r3    -14.003     2.3348    -4.4073    -0.40209    -2.4884
    r4     7.9905    -12.854    -7.1868      -35.23     9.5287
    r5    -18.075    -24.568    -14.271     -53.172     -42.62
    r6     -42.83    -87.129    -26.003     -138.66    -121.38

xxx TABLE:mt_y_devi_from_mean xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          -------    -------    -------    -------    --------

    r1      4.389      12.943     9.2933      10.508      5.0587
    r2     1.1037      8.0444    0.84902      6.8677    -0.15171
    r3     7.4123    -2.6258      4.9566    -3.6286      2.7985
    r4     3.7855    -6.0898    -2.3103      -8.57      4.5142
    r5    -2.9579    -4.8071    -2.7924    -8.7013     -8.3392
    r6    -5.2806    -9.5633    -3.6568     -15.22     -13.323

fl_cov
  -22.0835

fl_cor
   -0.8133
```

2.4.2 Test FF_DISC_RAND_VAR_MASS2COVCOR Four States-Points

Over some (a,z) states that is 2 by 2, c matrix, and y matrix, find correlation. Positively related.

```
% Set Parameters
```

```
mt_c_of_s = [1,2;3,1];
mt_y_of_s = [2,10;5,1.1];
rng(123);
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=1.4446,cor=0.65723
```

Same as before, but now inputs are single column:

```
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s(:), mt_y_of_s(:), mt_f_of_s(:), bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=1.4446,cor=0.65723
```

2.4.3 Test FF_DISC_RAND_VAR_MASS2COVCOR Two Random Vectors

Generate two random vectors, with random or even mass, correlation should be zero:

```
% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=-57.6533,cor=-0.062023
```

2.4.4 Test FF_DISC_RAND_VAR_MASS2COVCOR Provide Mean and SD

Same as above, but now provide means and sd for x andy directly. The results are the same as when mean and sd are calculated inside the function.

```
% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
fl_c_mean = sum(mt_f_of_s.*mt_c_of_s);
fl_c_sd = sqrt(sum(mt_f_of_s.*(mt_c_of_s-fl_c_mean).^2));
fl_y_mean = sum(mt_f_of_s.*mt_y_of_s);
fl_y_sd = sqrt(sum(mt_f_of_s.*(mt_y_of_s-fl_y_mean).^2));
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
```

```
    mt_c_of_s, mt_y_of_s, mt_f_of_s, ...
    fl_c_mean, fl_c_sd, ...
    fl_y_mean, fl_y_sd, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=-57.6533,cor=-0.062023
```

Chapter 3

Graphs

3.1 FF_GRAPH_GRID Examples: X, Y and Color Line Plots

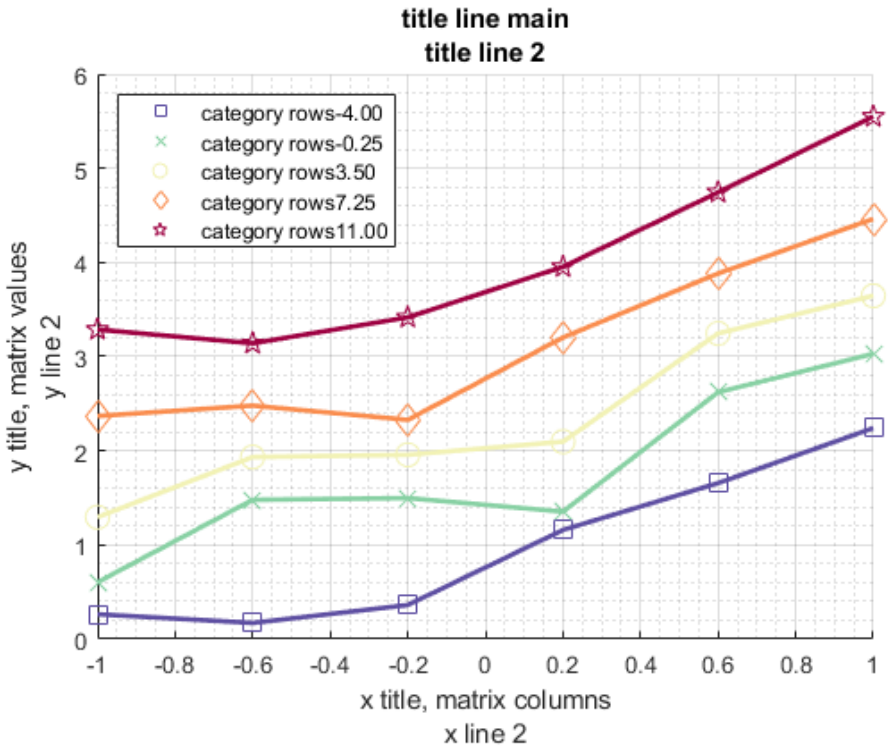
Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

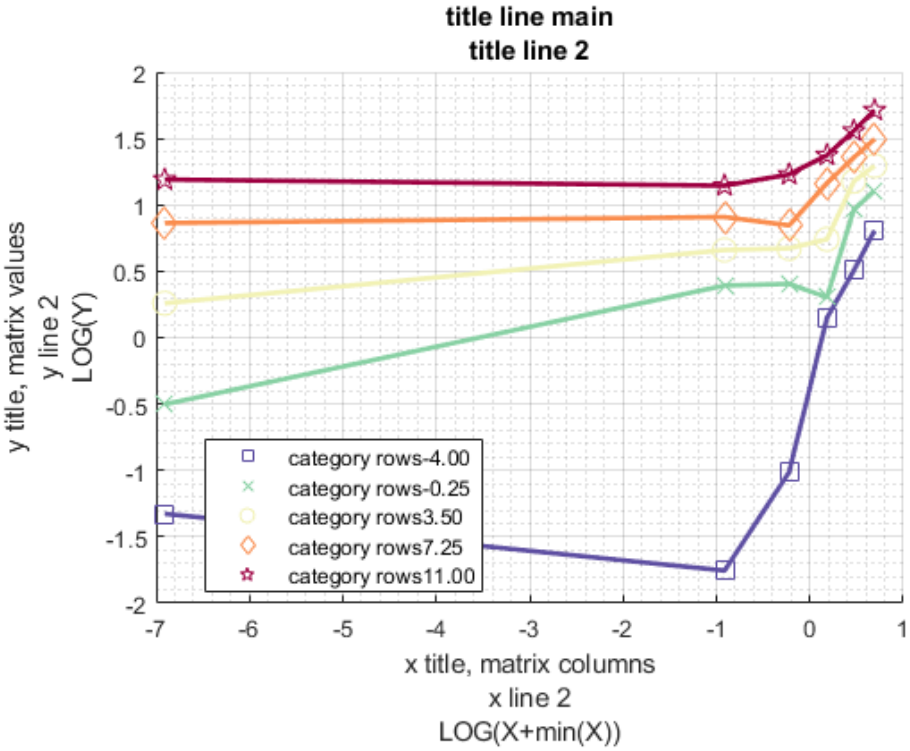
This is the example vignette for function: **ff_graph_grid** from the **MEconTools Package.** This function can graph out value and policy functions given one state vector (x-axis), conditional on other states (line groups). Can handle a few lines (scatter + lines), or many groups (jet spectrum).

3.1.1 Test FF_GRAPH_GRID Defaults

Call the function with defaults.

```
ff_graph_grid();
```

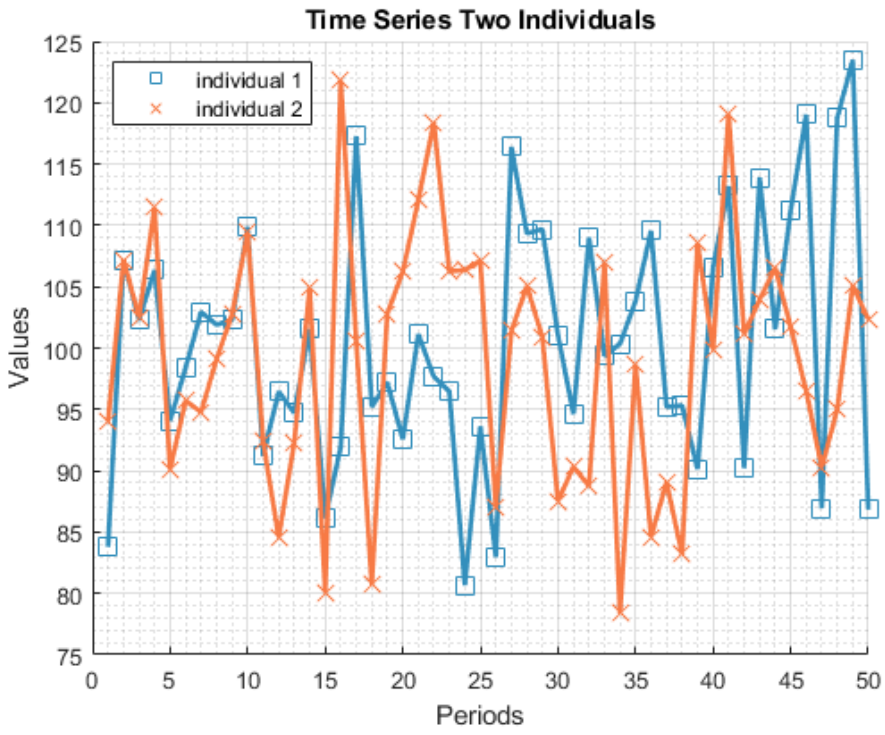




3.1.2 Test FF_GRAPH_GRID Two Random Normal Lines and Labels

There are two autoregressive time series, plot out the time two time series.

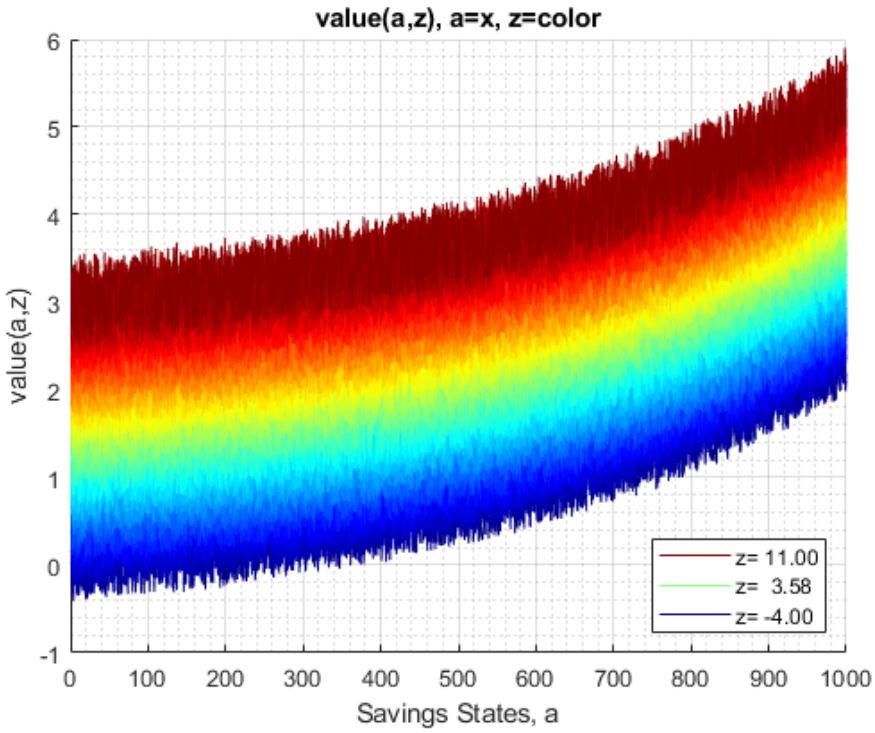
```
% Generate the two time series
rng(456);
mt_value = normrnd(100,10,[2, 50]);
ar_row_grid = ["individual 1", "individual 2"];
ar_col_grid = 1:50;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Time Series Two Individuals'};
mp_support_graph('cl_st_ytitle') = {'Values'};
mp_support_graph('cl_st_xtitle') = {'Periods'};
mp_support_graph('bl_graph_logy') = false; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



3.1.3 Test FF_GRAPH_GRID Many Lines

Plot many lines, with auto legend.

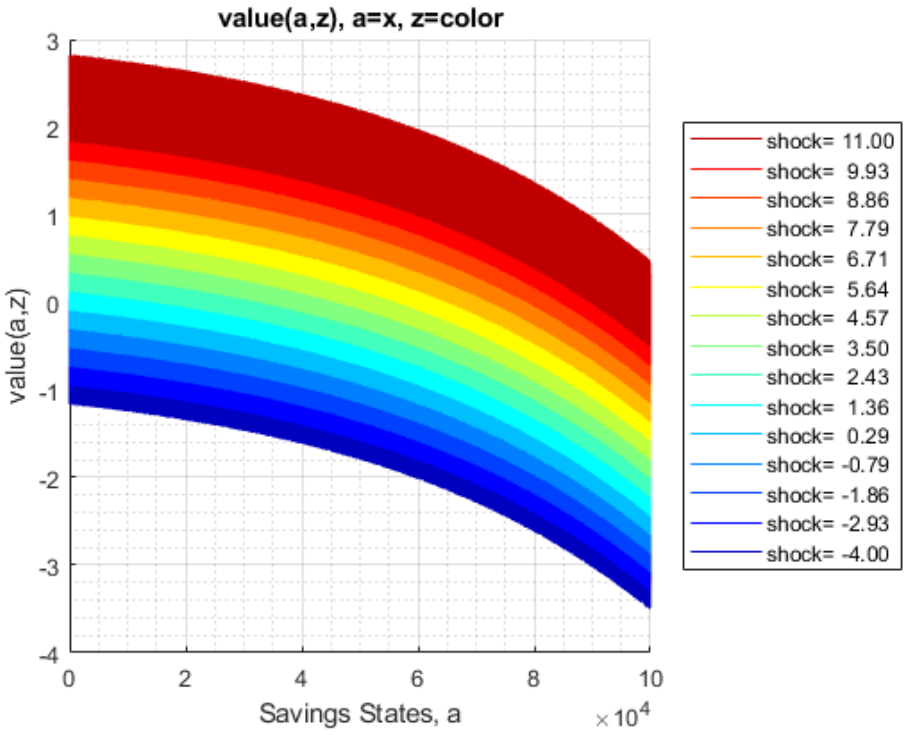
```
% Generate some Data
rng(456);
ar_row_grid = linspace(-4, 11, 100);
ar_col_grid = linspace(-1, 1, 1000);
rng(123);
mt_value = 0.2*ar_row_grid' + exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'southeast';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



3.1.4 Test FF_GRAPH_GRID Many Lines Legend Exogenous

Plot many lines, exogenously set legend

```
% Generate the two time series
rng(456);
ar_row_grid = linspace(-4, 11, 15);
ar_col_grid = linspace(-1, 1, 100000);
rng(123);
mt_value = 0.2*ar_row_grid' - exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% setting shock vector name exogenously here
ar_row_grid = string(num2str(ar_row_grid', "shock=%6.2f"));
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'eastoutside';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('it_legend_select') = 15;
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```


value(a,z), a=x, z=color
value(a,z)
Savings States, a
×10^4
shock= 11.00
shock= 9.93
shock= 8.86
shock= 7.79
shock= 6.71
shock= 5.64
shock= 4.57
shock= 3.50
shock= 2.43
shock= 1.36
shock= 0.29
shock= -0.79
shock= -1.86
shock= -2.93
shock= -4.00

Chapter 4

Support Tools

4.1 FF_CONTAINER_MAP_DISPLAY Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_container_map_display** from the **MEconTools Package.** This function summarizes statistics of matrixes stored in a container map, as well as scalar, string, function and other values stored in container maps.

4.1.1 Test FF_CONTAINER_MAP_DISPLAY Defaults

Call the function with defaults.

```
ff_container_map_display();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	mean	std	coefvari
mat_1	1	7	2	12	3	4	0.54285	0.2232	0.41115
mat_2	2	8	2	2650	50	53	0.49559	0.29232	0.58985
mat_2_boolean	3	9	2	2650	50	53	0.51358	0.49991	0.97337
mat_3	4	10	2	4	2	2	0.45277	0.45111	0.99635
tensor_1	5	15	3	16	2	8	0.45652	0.27787	0.60867
tensor_2	6	16	3	75	3	25	0.53593	0.29044	0.54194
tensor_3	7	17	2	4	1	4	0.42315	0.37389	0.88359
tesseract_1	8	18	4	72	3	24	0.47669	0.26374	0.55327
tesseract_2	9	19	4	20	2	10	0.42096	0.28981	0.68846
tesseract_bl_3	10	20	4	10	1	10	0.3	0.48305	1.6102

```
xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4
r1	0.69647	0.55131	0.98076	0.39212
r2	0.28614	0.71947	0.68483	0.34318
r3	0.22685	0.42311	0.48093	0.72905

```
xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c50	c51	c52	c53

```
          --------    --------    -------    --------    --------    --------    --------    ------

    r1     0.43857      0.6249    0.17108     0.56564    0.072152     0.67855     0.61667     0.540
    r2    0.059678     0.67469    0.82911    0.084904     0.63289     0.27236     0.32528     0.249
    r3     0.39804     0.84234    0.33867     0.58267    0.046367     0.44513    0.075047      0.78
    r4       0.738    0.083195    0.55237     0.81484     0.50561     0.11117     0.59532     0.356
    r5     0.18249     0.76368    0.57855     0.33707     0.10653    0.028681      0.7435     0.918
    r46     0.6813     0.55326    0.88786     0.69983     0.83758     0.16382     0.74191    0.0656
    r47    0.87546     0.85445    0.69631     0.66117     0.97069     0.79092     0.42466     0.787
    r48    0.51042     0.38484    0.44033    0.049097    0.017768     0.33302     0.24401     0.979
    r49    0.66931     0.31679    0.43821      0.7923     0.12979     0.75311     0.79466    0.0790
    r50    0.58594     0.35426     0.7651     0.51872     0.86415     0.58281     0.84795      0.45

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxx
          c1       c2       c3       c4       c50      c51      c52      c53
          -----    -----    -----    -----    -----    -----    -----    -----

    r1    true     false    false    true     true     false    true     true
    r2    true     false    true     true     false    false    true     true
    r3    false    true     false    true     false    true     false    true
    r4    false    true     false    false    false    true     true     true
    r5    true     true     true     false    true     false    false    true
    r46   false    true     true     false    true     true     true     true
    r47   true     true     true     true     true     true     false    false
    r48   true     false    false    false    true     true     false    true
    r49   true     true     false    true     true     true     false    false
    r50   false    false    false    false    false    false    false    false

xxx TABLE:mat_3 xxxxxxxxxxxxxxxxx
             c1          c2
          ----------    -------

    r1    0.00012471    0.13253
    r2       0.88615    0.79226

xxx TABLE:tensor_1 xxxxxxxxxxxxxxxxx
            c1         c2         c3         c4         c5         c6         c7         c8
          --------    -------    -------    -------    -------    -------    ------    -------

    r1    0.019363    0.34271    0.52167    0.53703    0.75756    0.68839    0.8345    0.26597
    r2    0.018091    0.33355    0.11738    0.77857    0.81933    0.28644    0.6157      0.368

xxx TABLE:tensor_2 xxxxxxxxxxxxxxxxx
            c1         c2         c3         c4         c22        c23         c24         c25
          --------    -------    -------    -------    -------    -------    ---------    -------

    r1     0.51866    0.40495    0.48278    0.99731    0.46584    0.62976     0.035924    0.10505
    r2    0.028692    0.37408    0.24149    0.35201    0.66054    0.87243    0.0024293    0.81088
    r3     0.87339    0.19457    0.83212    0.15315    0.77859    0.96663       0.2501     0.8056

xxx TABLE:tensor_3 xxxxxxxxxxxxxxxxx
            c1         c2         c3          c4
          ------    ------    -------    -------

    r1    0.1219    0.5119    0.91553    0.14329

xxx TABLE:tesseract_1 xxxxxxxxxxxxxxxxx
            c1         c2         c3          c4         c21        c22        c23        c24
```

```
          -------    -------    -------    --------    -------    -------    -------    -------

    r1    0.64531    0.59299    0.32115     0.67653    0.90328    0.56911    0.52562    0.12014
    r2    0.74558     0.5007    0.46142     0.21384    0.35564    0.13732      0.155    0.23786
    r3    0.91137    0.46403    0.18118    0.049919    0.46246    0.46842    0.75348    0.64547

xxx TABLE:tesseract_2 xxxxxxxxxxxxxxxxx
             c1         c2         c3         c4         c7         c8          c9         c10
          --------    -------    -------    -------    -------    -------    --------    -------

    r1     0.28898    0.48211    0.44359    0.97146    0.61782    0.65121     0.80715    0.11605
    r2    0.094493    0.34941    0.17595    0.14192    0.16754    0.57097    0.043114    0.70518

xxx TABLE:tesseract_bl_3 xxxxxxxxxxxxxxxxx
           c1       c2       c3       c4       c7       c8       c9       c10
          -----    -----    -----    -----    -----    -----    -----    -----

    r1    false    false    true     true     false    true     false    false

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i    idx     value
                      -    ---    -------

    boolean_1         1     1           1
    empty             2     2         NaN
    mat_4             3    11     0.74898
    string_float_1    4    13      1021.1
    string_int_1      5    14        1021

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 String
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i     idx            string
                     ---    ----    ---------------------

    list_string_1    "1"    "5"     "col1;col2;col3;col4"
    list_string_2    "2"    "6"     "row1;row2;row3;row4"
    string_1         "3"    "12"    "Table Name"

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 Functions
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
              i     idx      functionString
             ---    ---    -------------------

    func1    "1"    "3"    "@(x)1+2+x"
    func2    "2"    "4"    "@(x,y)x*1+sqrt(y)"
```

4.1.2 Test FF_CONTAINER_MAP_DISPLAY summarize Matrix Only

Three large matrixes, show summaries

```
% Create Container
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
```

```
rng(123);
mp_container_map('mat_1') = rand(100,100);
mp_container_map('mat_2') = rand(100,100)*2 + 1;
mp_container_map('mat_2_boolean') = (rand(100,100) > 0.5);
% Will only print
ff_container_map_display(mp_container_map);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                     i    idx    ndim    numel    rowN    colN     mean        std      coefvari
                     -    ---    ----    -----    ----    ----    -------    -------    --------

    mat_1            1     1      2      10000    100     100     0.49823    0.28829    0.57863
    mat_2            2     2      2      10000    100     100      2.0029    0.57632    0.28774
    mat_2_boolean    3     3      2      10000    100     100      0.4995    0.50002     1.0011
```

4.1.3 Test FF_CONTAINER_MAP_DISPLAY Show Matrix Subset

A container map with three small matrixes, print only only 2 rows and 3 columns.

```
% Create Container
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
rng(789);
mp_container_map('mat_1') = rand(3,4);
mp_container_map('mat_2') = rand(50,53);
mp_container_map('mat_2_boolean') = (rand(50,53) > 0.5);
% Will only print
ff_container_map_display(mp_container_map, 2, 3);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                     i    idx    ndim    numel    rowN    colN     mean        std      coefvari
                     -    ---    ----    -----    ----    ----    -------    -------    --------

    mat_1            1     1      2        12       3       4     0.41564    0.33586    0.80805
    mat_2            2     2      2      2650      50      53     0.49973    0.28834    0.57699
    mat_2_boolean    3     3      2      2650      50      53     0.50943    0.50001    0.98149

xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4
          -------    -------    -------    -------

    r1    0.32333    0.62442    0.01062    0.53815
    r3    0.79378    0.75889    0.11104    0.55157

xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxx
             c1         c2         c52        c53
           -------    -------    -------    -------

    r1     0.72837    0.20976    0.74583    0.22321
    r50    0.52812      0.545    0.49521    0.29826

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxx
            c1        c2        c52       c53
           -----     -----     -----     -----
```

```
r1     false    true     true     true
r50    true     false    false    true
```

Appendix A

Index and Code Links

A.1 Summarize Policy and Value links

1. Summarize ND Array Policy and Value Functions: **mlx** | **m** | **pdf** | **html**
   - Given an NDarray matrix with N1, N2, ..., ND dimensions. Generate average and standard deviation for the 3rd dimension, grouping by the other dimensions.
   - For example, show the 5th dimension as the column groups, and the other variables generate combinations shown as rows.
   - The resulting summary statistics table contains mean and standard deviation among other statistics over the policy or value contained in the ND array.
   - **MEconTools**: *ff_summ_nd_array()*

A.2 Distributional Analysis links

1. Gateway Joint Probability Mass Statistics: **mlx** | **m** | **pdf** | **html**
   - Given probability mass function f(s), and information y(s), x(s), z(s) at each element of the state-space, compute statistics for each variable, y, x, z, which are all discrete random variables.
   - Compute their correlation and covariance.
   - **MEconTools**: *ff_simu_stats()*
2. Discrete Random Variable Distributional Statistics: **mlx** | **m** | **pdf** | **html**
   - Model simulation generates discrete random variables, calculate mean, standard deviation, min, max, percentiles, and proportion of outcomes held by x percentiles, etc.
   - **MEconTools**: *ff_disc_rand_var_stats()*
3. Generate Discrete Random Variable: **mlx** | **m** | **pdf** | **html**
   - Given mass at state space points, and y, c, a, z and other outcomes or other information at each corresponding state space points, generate discrete random variable, with unique sorted values, and mass for each unique sorted values.
   - Generate additional joint distributions: if initial distribution is over f(a,z), generate joint distribution of f(y,a) or f(y,z).
   - **MEconTools**: *ff_disc_rand_var_mass2outcomes()*
4. Discrete Random Variable Correlation and Covariance: **mlx** | **m** | **pdf** | **html**
   - Given probability mass function f(s), X(s), and Y(s), compute the covariance and correlation betwen X and Y.
   - **MEconTools**: *ff_disc_rand_var_mass2covcor()*

A.3 Graphs links

1. Multiple Line Graph Function: **mlx** | **m** | **pdf** | **html**
   - Grid based Graph, x-axis one param, color another param, over outcomes.
   - **MEconTools**: *ff_graph_grid()*

A.4 Support Tools links

1. Organizes and Prints Container Map Key and Values: **mlx** | **m** | **pdf** | **html**
   - Summarizes the contents of a map container by data types. Includes, scalar, array, matrix, string, functions, tensors (3-tuples), tesseracts (4-tuples).
   - **MEconTools**: *ff_container_map_display()*

Bibliography

The MathWorks Inc (2019). *MATLAB*. Matlab package version 2019b.

Xie, Y. (2020). *bookdown: Authoring Books and Technical Documents with R Markdown*. R package version 0.18.