Matlab Toolbox Heterogeneous Agents Dynamic Programming

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Contents

Preface

This is a work-in-progress Matlab package consisting of functions that facilitate Dynamic Programming and Related Tasks. Materials gathered from various projects in which Matlab code is used. Some of the solutions/algorithms are research outputs developed for specific research papers, other algorithms and methods are commonly-used. Files are the MEconTools repository. Matlab files are linked below by section with livescript files. Tested with Matlab 2019a (The MathWorks Inc, 2019).

Download and install the Matlab toolbox: MEconTools.mltbx

This bookdown file is a collection of mlx based vignettes for functions that are available from MEconTools. Each Vignette file contains various examples for invoking each function. The goal of this repository is to make it easier to find/re-use codes produced for various projects.

From other repositories: For dynamic borrowing and savings problems, see Dynamic Asset Repository; For code examples, see also R Example Code, Matlab Example Code, and Stata Example Code; For intro stat with R, see Intro Statistics for Undergraduates, and intro Math with Matlab, see Intro Mathematics for Economists. See here for all of Fan's public repositories.

The site is built using Bookdown (Xie, 2020).

Please contact FanWangEcon for issues or problems.

Chapter 1

Savings Dynamic Programming

1.1 FF_VFI_AZ_LOOP Savings Loop Grid Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_vfi_az_loop** from the **MEconTools Package.** This function solves the dynamic programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon.

This is the **looped** code, it is slow for larger state-space problems. The code uses **common grid**, with the same state space and choice space grids.

Links to Other Code:

Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid ***Loop***: **ff_vfi_az_loop**
- Common Choice and States Grid ***Vectorized***: **ff_vfi_az_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC, ***Loop***:**ff_vfi_az_bisec_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC ***Vectorized***: **ff_vfi_az_bisec_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Loop***:**ff_vfi_az_mzoom_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Vectorized***: **ff_vfi_az_mzoom_vec**

The sample codes are written for the standard dynamic savings problem. The code can be adapted for multiple assets, savings and borrowing, discrete and continuous choice, etc. A large proportion of dynamic economic models are based on the underlying structure of solving a model with endogenous states and exogenous shocks, and that is what the (a,z) model does. In general, one can write looped code first to make sure the economics is correct, then vectorized code can be adopted to increase speed.

1.1.1 Test FF_VFI_AZ_LOOP Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
```

```
ff_vfi_az_loop(mp_params);

Elapsed time is 2.378952 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum       mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    ------    ------    ------    --------    ---

    ap    1     1      2       700     100      7      9855.1    14.079    14.408     1.0234      0

xxx TABLE:ap xxxxxxxxxxxxxxxxx
              c1        c2        c3         c4         c5         c6         c7
            ------    ------    ------    --------    -------    -------    ------

    r1           0         0         0    0.045213    0.25576    0.61095    1.0362
    r2           0         0         0    0.045213    0.25576    0.61095    1.0362
    r3           0         0         0    0.045213    0.25576    0.61095    1.0362
    r4           0         0         0     0.06647    0.25576    0.61095    1.0362
    r5           0         0         0     0.06647    0.25576    0.61095     1.164
    r96     43.924    43.924    43.924      43.924     43.924     45.102    45.102
    r97     45.102    45.102    45.102      45.102     45.102     46.298    46.298
    r98     46.298    46.298    46.298      46.298     46.298     47.513    47.513
    r99     47.513    47.513    47.513      47.513     47.513     48.747    48.747
    r100    48.747    48.747    48.747      48.747     48.747         50        50
```

1.1.2 Test FF_VFI_AZ_BISEC_VEC Speed Tests

Call the function with different a and z grid size, print out speed:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
% A grid 50, shock grid 5:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.715890 seconds.

% A grid 750, shock grid 15:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 15;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 300.576571 seconds.

% A grid 600, shock grid 45:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 600;
mp_params('it_z_n') = 45;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 910.111661 seconds.
```

1.1.3 Test FF_VFI_AZ_LOOP Control Outputs

Run the function first without any outputs, but only the timer.

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {};
ff_vfi_az_loop(mp_params, mp_support);
```

```
Elapsed time is 0.400105 seconds.
```

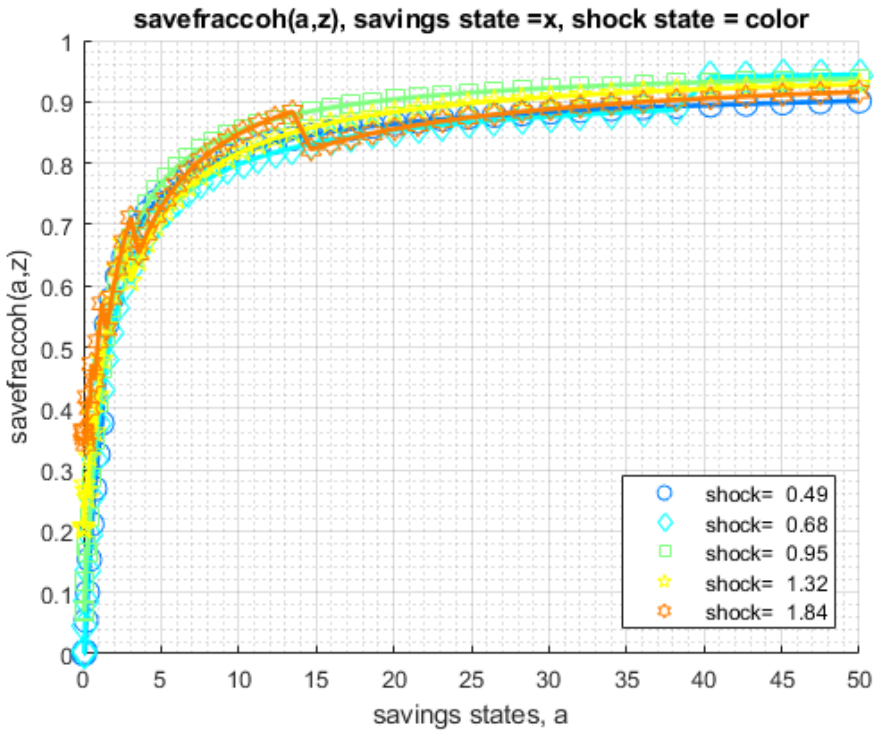
Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

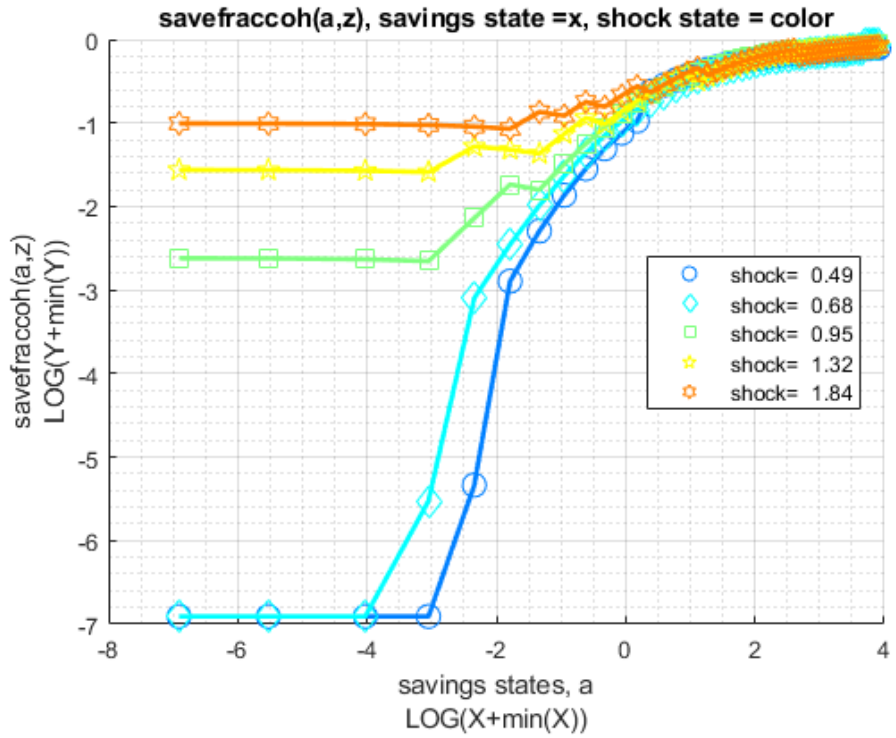
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
ff_vfi_az_loop(mp_params, mp_support);
```

```
Elapsed time is 0.410866 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxx
```

group	a	mean_z_0_4858	mean_z_0_67798	mean_z_0_9462	mean_z_1_3205	mean_z
1	0	0	0	0.071865	0.20862	0.3
2	0.002975	0	0	0.071698	0.20827	0.3
3	0.016829	0	0	0.070928	0.20666	0.3
4	0.046375	0	0.0029827	0.069341	0.20331	0.3
5	0.095198	0.0038183	0.044243	0.11681	0.27649	0.3
6	0.1663	0.054362	0.084837	0.17517	0.26637	0.3
7	0.26234	0.099899	0.13609	0.16422	0.25383	0.4
8	0.38568	0.15381	0.19428	0.22348	0.32132	0.4
9	0.53852	0.21153	0.25554	0.28573	0.39055	0.4
10	0.72291	0.26934	0.31659	0.34814	0.36175	0.4
11	0.94076	0.3247	0.37504	0.40848	0.42229	0.5
12	1.1939	0.37617	0.42941	0.46521	0.4802	0.5
13	1.484	0.53695	0.47898	0.51743	0.5344	0.
14	1.8128	0.57847	0.52356	0.56473	0.58429	0.5
15	2.1817	0.61468	0.56329	0.6071	0.62958	0.6
16	2.5924	0.6462	0.5985	0.64475	0.67028	0.6
17	3.0463	0.67365	0.62963	0.67804	0.60721	0.7
18	3.5449	0.69762	0.65713	0.70737	0.6404	0.6
19	4.0894	0.71859	0.68142	0.73318	0.67021	0.6
20	4.6813	0.73701	0.70293	0.75587	0.6969	0.7
21	5.3218	0.75325	0.722	0.77584	0.72078	0.7
22	6.0121	0.76763	0.73895	0.79344	0.74211	0.7
23	6.7536	0.7804	0.75407	0.80897	0.76119	0.7
24	7.5474	0.7918	0.76759	0.8227	0.77824	0.8

25	8.3948	0.80201	0.77972	0.83486	0.79351	0.8
26	9.2967	0.81119	0.79063	0.84567	0.80719	0.8
27	10.254	0.81947	0.80049	0.8553	0.81948	0.8
28	11.269	0.82697	0.80941	0.86389	0.83053	0.8
29	12.342	0.83379	0.81752	0.87159	0.84048	0.8
30	13.473	0.84001	0.8249	0.87849	0.84946	0.8
31	14.665	0.84569	0.83165	0.8847	0.85759	0.8
32	15.918	0.8509	0.83782	0.8903	0.86495	0.8
33	17.233	0.8557	0.8435	0.89536	0.87163	0.8
34	18.611	0.86012	0.84872	0.89995	0.8777	0.8
35	20.053	0.86421	0.85354	0.90411	0.88324	0.8
36	21.56	0.86799	0.858	0.9079	0.8883	0.8
37	23.133	0.87151	0.86214	0.91136	0.89292	0.8
38	24.773	0.87479	0.86598	0.91452	0.89716	0.8
39	26.481	0.87784	0.86955	0.91741	0.90105	0.8
40	28.258	0.8807	0.87289	0.92007	0.90463	0.8
41	30.104	0.88337	0.87601	0.92251	0.90793	0.8
42	32.021	0.88588	0.87893	0.92475	0.91097	0.
43	34.01	0.88824	0.88166	0.92683	0.91378	0.8
44	36.07	0.89046	0.88423	0.92874	0.91638	0.8
45	38.204	0.89256	0.88665	0.93052	0.91879	0.9
46	40.412	0.89453	0.9403	0.93216	0.92102	0.9
47	42.695	0.8964	0.94141	0.93368	0.9231	0.9
48	45.053	0.89817	0.94245	0.9351	0.92504	0.9
49	47.488	0.89985	0.94341	0.93642	0.92684	0.
50	50	0.90144	0.9443	0.93765	0.92853	0.9





Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 3.281815 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    ap              1     1      2       900     100      9       12904     14.338     14.524      1.
    savefraccoh     2     2      2       900     100      9      619.51    0.68834    0.26953    0.39

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1        c2        c3          c4           c5         c6         c7         c8
            ------    ------    ------    ----------    --------    -------    -------    ------

    r1           0         0         0             0    0.092813    0.25576    0.61095    1.0362
    r2           0         0         0             0    0.092813    0.25576    0.61095    1.0362
    r3           0         0         0             0    0.092813    0.25576    0.61095    1.0362
    r4           0         0         0    0.00051272    0.092813    0.25576    0.61095    1.0362
    r5           0         0         0     0.0029004    0.092813    0.25576    0.61095    1.0362
    r96     43.924    43.924    43.924        43.924       43.924     45.102     45.102     45.102
    r97     45.102    45.102    45.102        45.102       45.102     46.298     46.298     46.298
    r98     46.298    46.298    46.298        46.298       46.298     47.513     47.513     47.513
    r99     47.513    47.513    47.513        47.513       47.513     48.747     48.747     48.747
```

```
    r100    48.747    48.747    48.747        48.747      48.747         50         50         50

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3           c4           c5         c6         c7         c8
            -------    -------    -------    ----------    --------    -------    -------    -------

    r1            0          0          0             0    0.070073    0.15255    0.28789    0.38573
    r2            0          0          0             0    0.070045     0.1525    0.28781    0.38565
    r3            0          0          0             0    0.069914    0.15228    0.28748     0.3853
    r4            0          0          0    0.00048613    0.069636     0.1518    0.28676    0.38454
    r5            0          0          0     0.0027273    0.069182    0.15101    0.28559    0.38329
    r96     0.92625    0.92358    0.92022         0.916     0.91072    0.92836    0.91992    0.90945
    r97     0.92676    0.92416    0.92088       0.91677     0.91162    0.92918    0.92095    0.91073
    r98     0.92727    0.92473    0.92153       0.91752     0.91249    0.92998    0.92194    0.91196
    r99     0.92776    0.92528    0.92216       0.91824     0.91333    0.93076    0.92291    0.91315
    r100    0.92823    0.92581    0.92277       0.91895     0.91416    0.93151    0.92384    0.91431
```

1.1.4 Test FF_VFI_AZ_LOOP Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.825240 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                  i    idx    ndim    numel    rowN    colN     sum       mean      std       coefva
                  -    ---    ----    -----    ----    ----    ------    ------    ------    ------

    savefraccoh   1     1      2       700     100      7      357.49    0.5107    0.2755    0.5394

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5           c6           c7
            -------    -------    -------    -------    -------    ----------    --------

    r1            0          0          0          0          0     0.0002246    0.041573
    r2            0          0          0          0          0    0.00022455    0.041566
    r3            0          0          0          0          0     0.0012689    0.041533
    r4            0          0          0          0          0      0.001266    0.041462
    r5            0          0          0          0          0     0.0034759    0.041345
    r96     0.78455    0.78145    0.79995    0.79456     0.7876       0.77865     0.76719
```

```
    r97     0.78669    0.78366    0.77972    0.79679    0.78998       0.78122     0.77001
    r98     0.78878    0.78582    0.78197    0.79897    0.79231       0.78374     0.77276
    r99     0.79084    0.78794    0.78417    0.77927    0.79459        0.7862     0.77545
    r100    0.79285    0.79001    0.78633    0.78154    0.79682        0.7886     0.77808

% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 2.386791 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       700     100      7      479.94    0.68563    0.27152    0.39

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2           c3           c4          c5          c6          c7
            -------    -------    ----------    --------    -------    -------    -------

    r1            0          0             0     0.07007    0.17967    0.30874    0.43404
    r2            0          0             0    0.070042    0.17961    0.30866    0.43396
    r3            0          0             0    0.069911    0.17935    0.30833     0.4336
    r4            0          0             0    0.069633    0.17881    0.30762    0.43284
    r5            0          0    0.00049972    0.069179    0.17792    0.30645    0.43158
    r96     0.92489    0.92134       0.91672     0.91072    0.92717    0.91691    0.92776
    r97     0.92544    0.92198       0.91747     0.91162    0.92802    0.91801    0.92895
    r98     0.92598     0.9226        0.9182     0.91249    0.92885    0.91908     0.9301
    r99      0.9265     0.9232       0.91891     0.91333    0.92965    0.92011    0.93121
    r100      0.927    0.92379        0.9196     0.91416    0.93042     0.9211    0.90914
```

1.1.5 Test FF_VFI_AZ_LOOP Changing Risk Aversion

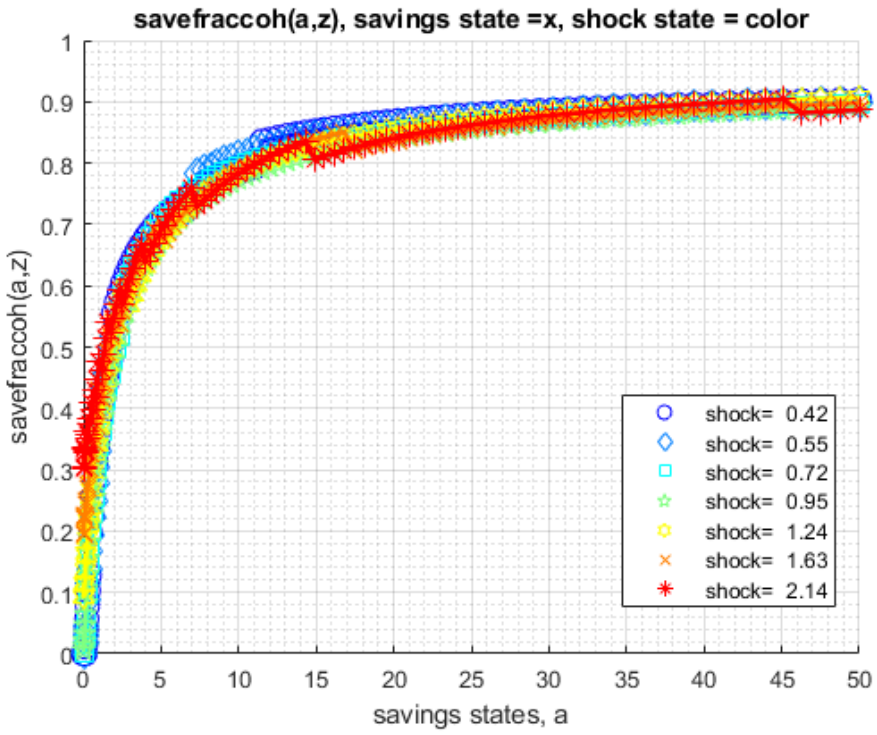
Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

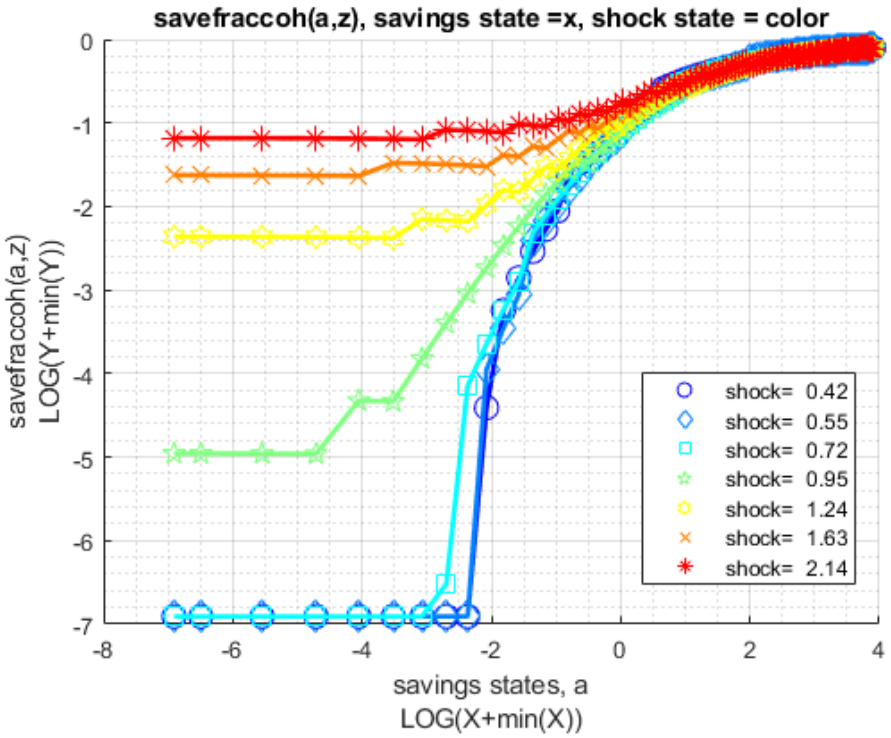
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 1.327261 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

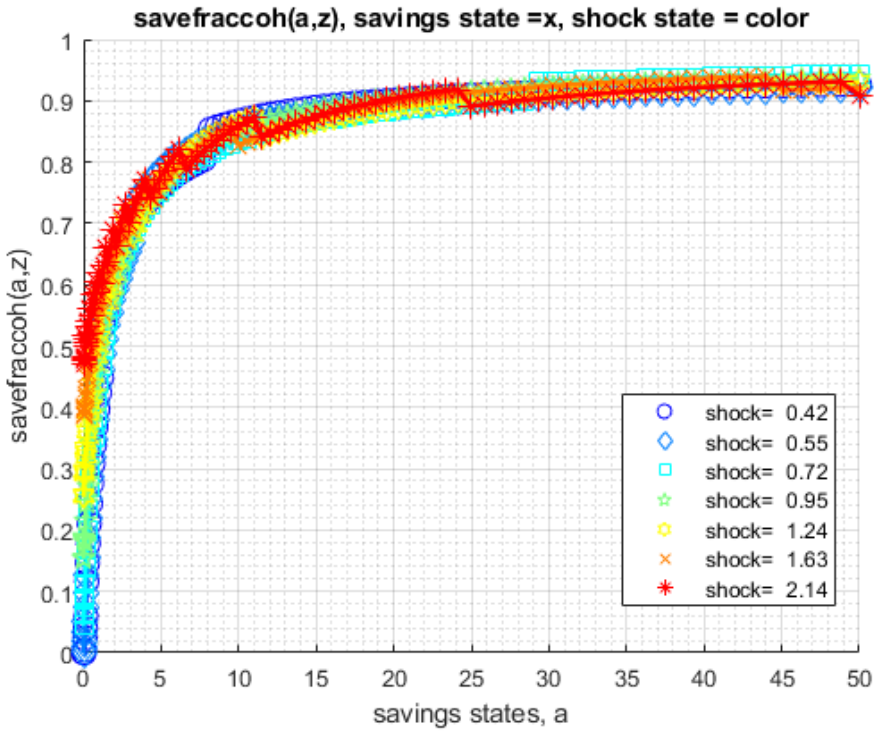
```
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum       mean       std       coefv
                    -     ---     ----     -----     ----     ----     ------     -------     ------     -----

    savefraccoh     1      1       2        700      100       7       450.35     0.64336     0.2803     0.435

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
            -------     -------     -------     ---------     --------     -------     -------

    r1            0           0           0     0.0060341     0.093241     0.19572     0.30604
    r2            0           0           0     0.0060316     0.093213     0.19567     0.30599
    r3            0           0           0     0.0060204      0.09308     0.19546     0.30574
    r4            0           0           0     0.0059964     0.092798     0.19501      0.3052
    r5            0           0           0      0.012229     0.092335     0.19427     0.30431
    r96     0.90049     0.89703     0.89253       0.88669     0.90296     0.89297     0.90379
    r97     0.90128     0.89791     0.89351       0.88781     0.90404     0.89429     0.88181
    r98     0.90205     0.89876     0.89447       0.88891      0.9051     0.89557     0.88337
    r99      0.9028     0.89959     0.89541       0.88998     0.90612     0.89681     0.88489
    r100    0.90354      0.9004     0.89632       0.89101     0.90711     0.89802     0.88636
```

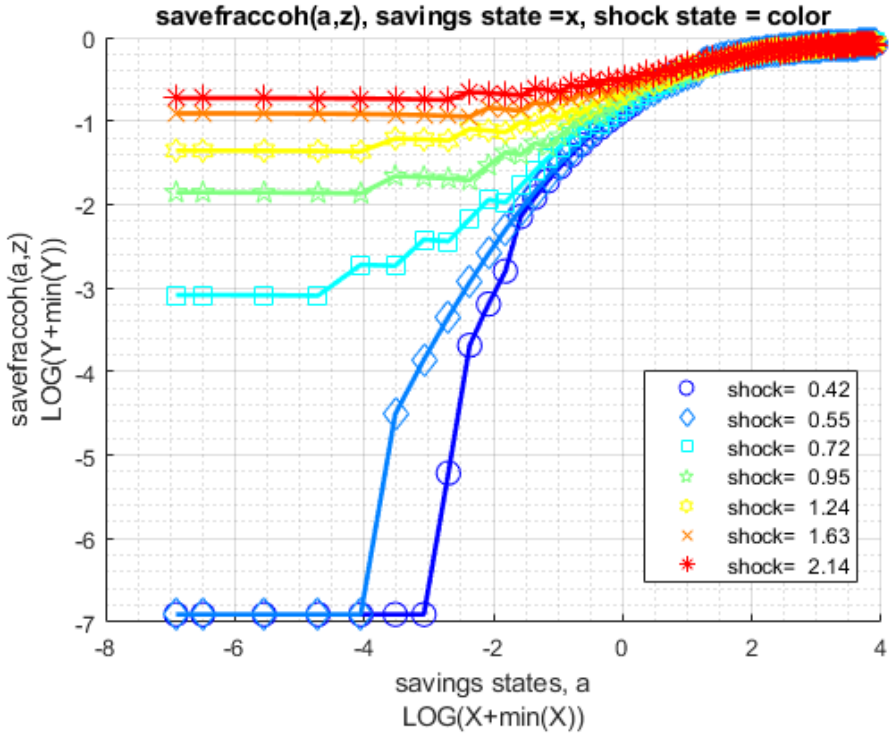




When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 2.680109 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1      1       2      700     100       7     500.59    0.71513    0.25488    0.35

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2          c3         c4         c5         c6         c7
            -------    -------    --------    -------    -------    -------    -------

    r1            0          0    0.044811    0.15534    0.25694    0.40177    0.48276
    r2            0          0    0.044787    0.15528    0.25686    0.40168    0.48268
    r3            0          0    0.044678    0.15499     0.2565    0.40124    0.48228
    r4            0          0    0.044445    0.15437    0.25572    0.40032    0.48143
    r5            0          0    0.064784    0.15337    0.25445    0.39879    0.48003
    r96     0.92489    0.92134     0.94129    0.93513    0.92717    0.91691    0.92776
    r97     0.92544    0.92198      0.9418     0.9358    0.92802    0.91801    0.92895
    r98     0.92598     0.9226      0.9423    0.93644    0.92885    0.91908     0.9301
    r99      0.9265     0.9232     0.94278    0.93706    0.92965    0.92011    0.93121
    r100      0.927    0.92379     0.94324    0.93765    0.93042     0.9211    0.90914
```





1.1.6 Test FF_VFI_AZ_LOOP with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 150;
mp_params('it_z_n') = 15;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
% graph color spectrum
mp_params('cl_colors') = 'copper';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 13.492999 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1506.3    0.66947    0.28673     0.4

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c11         c12         c13
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0    0.14061     0.1891    0.24154
    r2            0          0          0          0          0     0.1406    0.18908    0.24152
    r3            0          0          0          0          0    0.14053      0.189    0.24142
    r4            0          0          0          0          0    0.14038    0.18881     0.2412
    r5            0          0          0          0          0    0.14013    0.18851    0.24085
    r146    0.93087    0.92957    0.92815    0.92661    0.92492    0.92712    0.92403    0.92069
    r147    0.93121    0.92994    0.92854    0.92702    0.92537    0.92768    0.92465    0.92135
    r148    0.93156     0.9303    0.92893    0.92743    0.92581    0.92823    0.92525    0.92201
    r149    0.93189    0.93065     0.9293    0.92783    0.92623    0.92878    0.92584    0.92264
    r150    0.93222      0.931    0.92967    0.92823    0.92665     0.9293    0.92641    0.92327
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 18.680264 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1678.8    0.74614    0.22779    0.30

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c11         c12         c13
            -------    -------    -------    -------    -------    -------    -------    -------
```

r1	0	0	0	0	0	0.53612	0.59853	0.67884
r2	0	0	0	0	0	0.53609	0.5985	0.67882
r3	0	0	0	0	0	0.53594	0.59839	0.67873
r4	0	0	0	0	0	0.53563	0.59814	0.67853
r5	0	0	0	0	0	0.53511	0.59774	0.67821
r146	0.92696	0.9262	0.92513	0.92359	0.92142	0.91653	0.9078	0.88992
r147	0.92721	0.92647	0.92541	0.9239	0.92176	0.91741	0.90895	0.89144
r148	0.92746	0.92673	0.92569	0.92421	0.9221	0.91827	0.91007	0.87813
r149	0.9277	0.92698	0.92596	0.9245	0.92243	0.9191	0.89605	0.86507
r150	0.92794	0.92724	0.92623	0.9248	0.92276	0.90467	0.88233	0.85227

1.2 FF_VFI_AZ_VEC Savings Vectorized Grid Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_vfi_az_vec** from the **MEconTools Package.** This function solves the dynamic programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon.

This is the **vectorized** code, its speed is much faster than the looped code. The function is designed to have small memory footprint and requires low computing resources, yet is fast.

The code uses **common grid**, with the same state space and choice space grids. **ff_vfi_az_bisec_vec** from the **MEconTools Package** solves the same problem but using continuous exact percentage asset choices, which is more precise than the solution here, and perhaps a little bit slower and relies on First Order Conditions. The **ff_vfi_az_mzoom_vec** also solves the same class of problems with continuous exact percentage asset choices, and does not rely on First Order Conditions, but is slower than **ff_vfi_az_bisec_vec.**

Links to Other Code:

Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid ***Loop***: **ff_vfi_az_loop**
- Common Choice and States Grid ***Vectorized***: **ff_vfi_az_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC, ***Loop***:**ff_vfi_az_bisec_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC ***Vectorized***: **ff_vfi_az_bisec_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Loop***:**ff_vfi_az_mzoom_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Vectorized***: **ff_vfi_az_mzoom_vec**

1.2.1 Test FF_VFI_AZ_VEC Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
ff_vfi_az_vec(mp_params);

Elapsed time is 0.136223 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

```
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum       mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    ------    ------    ------    --------    ---

    ap    1     1      2       700     100      7      9855.1    14.079    14.408     1.0234      0

xxx TABLE:ap xxxxxxxxxxxxxxxxx
              c1        c2        c3         c4         c5         c6         c7
            ------    ------    ------    --------    -------    -------    ------

    r1           0         0         0    0.045213    0.25576    0.61095    1.0362
    r2           0         0         0    0.045213    0.25576    0.61095    1.0362
    r3           0         0         0    0.045213    0.25576    0.61095    1.0362
    r4           0         0         0     0.06647    0.25576    0.61095    1.0362
    r5           0         0         0     0.06647    0.25576    0.61095     1.164
    r96     43.924    43.924    43.924      43.924     43.924     45.102    45.102
    r97     45.102    45.102    45.102      45.102     45.102     46.298    46.298
    r98     46.298    46.298    46.298      46.298     46.298     47.513    47.513
    r99     47.513    47.513    47.513      47.513     47.513     48.747    48.747
    r100    48.747    48.747    48.747      48.747     48.747         50        50
```

1.2.2 Test FF_VFI_AZ_BISEC_VEC Speed Tests

Call the function with different a and z grid size, print out speed:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
% A grid 50, shock grid 5:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.025309 seconds.

% A grid 750, shock grid 15:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 15;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 4.855482 seconds.

% A grid 600, shock grid 45:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 600;
mp_params('it_z_n') = 45;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 12.201130 seconds.
```

1.2.3 Test FF_VFI_AZ_VEC Control Outputs

Run the function first without any outputs, but only the timer.

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
```

```
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {};
ff_vfi_az_vec(mp_params, mp_support);
```

```
Elapsed time is 0.022504 seconds.
```

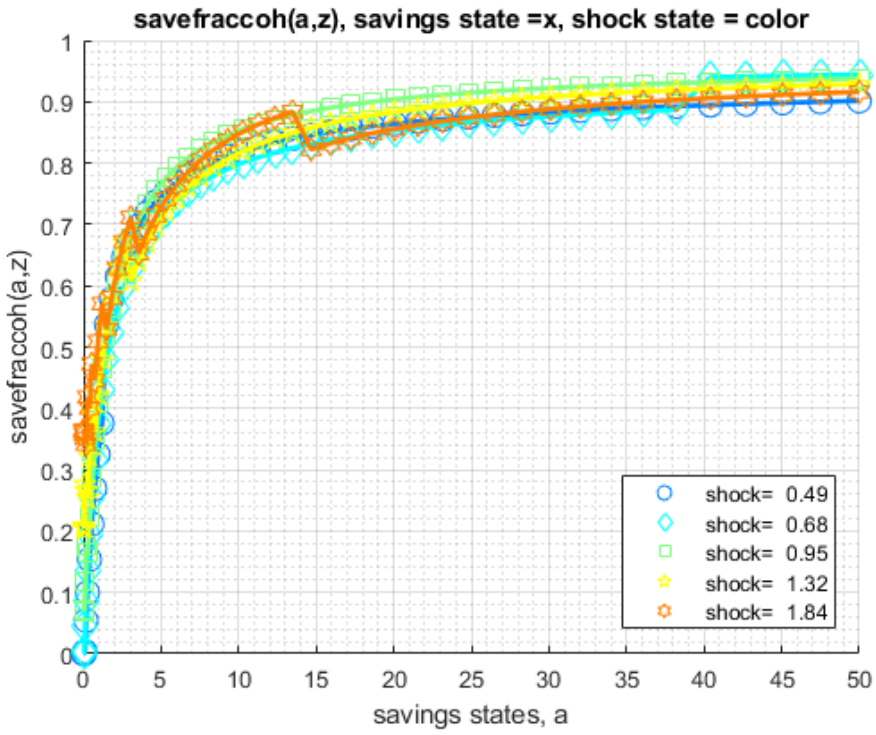
Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

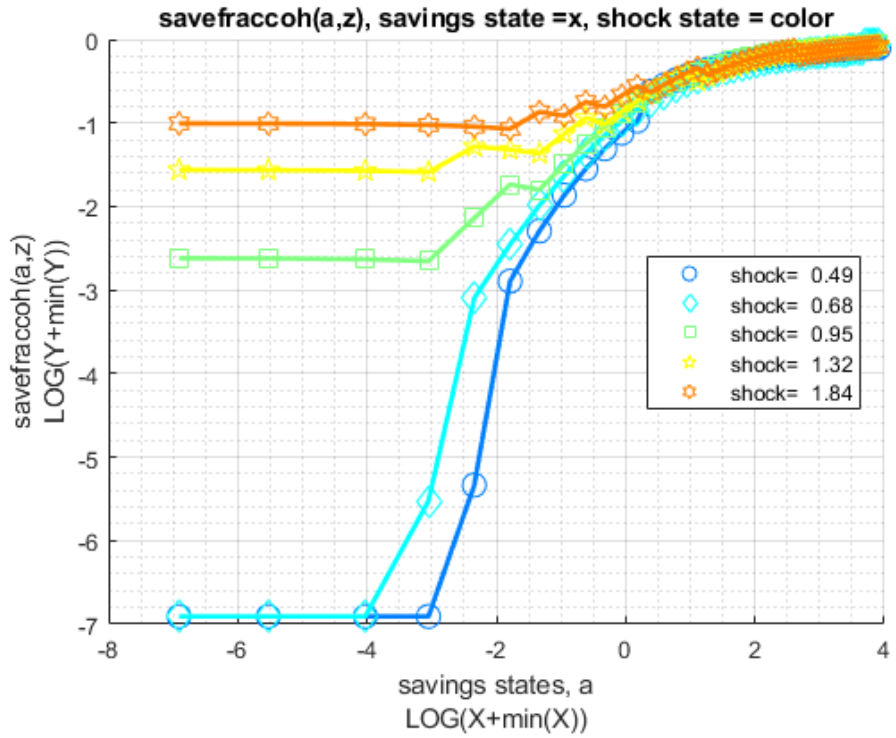
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
ff_vfi_az_vec(mp_params, mp_support);
```

```
Elapsed time is 0.041571 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxx
```

group	a	mean_z_0_4858	mean_z_0_67798	mean_z_0_9462	mean_z_1_3205	mean_z
1	0	0	0	0.071865	0.20862	0.3
2	0.002975	0	0	0.071698	0.20827	0.3
3	0.016829	0	0	0.070928	0.20666	0.3
4	0.046375	0	0.0029827	0.069341	0.20331	0.3
5	0.095198	0.0038183	0.044243	0.11681	0.27649	0.3
6	0.1663	0.054362	0.084837	0.17517	0.26637	0.3
7	0.26234	0.099899	0.13609	0.16422	0.25383	0.4
8	0.38568	0.15381	0.19428	0.22348	0.32132	0.4
9	0.53852	0.21153	0.25554	0.28573	0.39055	0.4
10	0.72291	0.26934	0.31659	0.34814	0.36175	0.4
11	0.94076	0.3247	0.37504	0.40848	0.42229	0.5
12	1.1939	0.37617	0.42941	0.46521	0.4802	0.5
13	1.484	0.53695	0.47898	0.51743	0.5344	0.
14	1.8128	0.57847	0.52356	0.56473	0.58429	0.5
15	2.1817	0.61468	0.56329	0.6071	0.62958	0.6
16	2.5924	0.6462	0.5985	0.64475	0.67028	0.6
17	3.0463	0.67365	0.62963	0.67804	0.60721	0.7
18	3.5449	0.69762	0.65713	0.70737	0.6404	0.6
19	4.0894	0.71859	0.68142	0.73318	0.67021	0.6
20	4.6813	0.73701	0.70293	0.75587	0.6969	0.7
21	5.3218	0.75325	0.722	0.77584	0.72078	0.7
22	6.0121	0.76763	0.73895	0.79344	0.74211	0.7
23	6.7536	0.7804	0.75407	0.80897	0.76119	0.7
24	7.5474	0.7918	0.76759	0.8227	0.77824	0.8
25	8.3948	0.80201	0.77972	0.83486	0.79351	0.8
26	9.2967	0.81119	0.79063	0.84567	0.80719	0.8
27	10.254	0.81947	0.80049	0.8553	0.81948	0.8
28	11.269	0.82697	0.80941	0.86389	0.83053	0.8
29	12.342	0.83379	0.81752	0.87159	0.84048	0.8

30	13.473	0.84001	0.8249	0.87849	0.84946	0.8
31	14.665	0.84569	0.83165	0.8847	0.85759	0.8
32	15.918	0.8509	0.83782	0.8903	0.86495	0.8
33	17.233	0.8557	0.8435	0.89536	0.87163	0.8
34	18.611	0.86012	0.84872	0.89995	0.8777	0.8
35	20.053	0.86421	0.85354	0.90411	0.88324	0.8
36	21.56	0.86799	0.858	0.9079	0.8883	0.8
37	23.133	0.87151	0.86214	0.91136	0.89292	0.8
38	24.773	0.87479	0.86598	0.91452	0.89716	0.8
39	26.481	0.87784	0.86955	0.91741	0.90105	0.8
40	28.258	0.8807	0.87289	0.92007	0.90463	0.8
41	30.104	0.88337	0.87601	0.92251	0.90793	0.8
42	32.021	0.88588	0.87893	0.92475	0.91097	0.
43	34.01	0.88824	0.88166	0.92683	0.91378	0.8
44	36.07	0.89046	0.88423	0.92874	0.91638	0.8
45	38.204	0.89256	0.88665	0.93052	0.91879	0.9
46	40.412	0.89453	0.9403	0.93216	0.92102	0.9
47	42.695	0.8964	0.94141	0.93368	0.9231	0.9
48	45.053	0.89817	0.94245	0.9351	0.92504	0.9
49	47.488	0.89985	0.94341	0.93642	0.92684	0.
50	50	0.90144	0.9443	0.93765	0.92853	0.9





Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.230510 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                  i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                  -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    ap            1     1      2       900     100      9       12904     14.338     14.524      1.
    savefraccoh   2     2      2       900     100      9      619.51    0.68834    0.26953    0.39

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1        c2        c3          c4           c5         c6         c7         c8
            ------    ------    ------    ----------    --------    -------    -------    ------

    r1           0         0         0             0    0.092813    0.25576    0.61095    1.0362
    r2           0         0         0             0    0.092813    0.25576    0.61095    1.0362
    r3           0         0         0             0    0.092813    0.25576    0.61095    1.0362
    r4           0         0         0    0.00051272    0.092813    0.25576    0.61095    1.0362
    r5           0         0         0     0.0029004    0.092813    0.25576    0.61095    1.0362
    r96     43.924    43.924    43.924        43.924       43.924     45.102     45.102     45.102
    r97     45.102    45.102    45.102        45.102       45.102     46.298     46.298     46.298
    r98     46.298    46.298    46.298        46.298       46.298     47.513     47.513     47.513
    r99     47.513    47.513    47.513        47.513       47.513     48.747     48.747     48.747
```

```
    r100    48.747    48.747    48.747        48.747      48.747         50         50         50

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3           c4           c5         c6         c7         c8
            -------    -------    -------    ----------    --------    -------    -------    -------

    r1            0          0          0             0    0.070073    0.15255    0.28789    0.38573
    r2            0          0          0             0    0.070045     0.1525    0.28781    0.38565
    r3            0          0          0             0    0.069914    0.15228    0.28748     0.3853
    r4            0          0          0    0.00048613    0.069636     0.1518    0.28676    0.38454
    r5            0          0          0     0.0027273    0.069182    0.15101    0.28559    0.38329
    r96     0.92625    0.92358    0.92022         0.916     0.91072    0.92836    0.91992    0.90945
    r97     0.92676    0.92416    0.92088       0.91677     0.91162    0.92918    0.92095    0.91073
    r98     0.92727    0.92473    0.92153       0.91752     0.91249    0.92998    0.92194    0.91196
    r99     0.92776    0.92528    0.92216       0.91824     0.91333    0.93076    0.92291    0.91315
    r100    0.92823    0.92581    0.92277       0.91895     0.91416    0.93151    0.92384    0.91431
```

1.2.4 Test FF_VFI_AZ_VEC Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.058079 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                  i    idx    ndim    numel    rowN    colN     sum       mean      std       coefva
                  -    ---    ----    -----    ----    ----    ------    ------    ------    ------

    savefraccoh   1     1      2       700     100      7      357.49    0.5107    0.2755    0.5394

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5           c6           c7
            -------    -------    -------    -------    -------    ----------    --------

    r1            0          0          0          0          0     0.0002246    0.041573
    r2            0          0          0          0          0    0.00022455    0.041566
    r3            0          0          0          0          0     0.0012689    0.041533
    r4            0          0          0          0          0      0.001266    0.041462
    r5            0          0          0          0          0     0.0034759    0.041345
    r96     0.78455    0.78145    0.79995    0.79456     0.7876       0.77865     0.76719
```

```
    r97     0.78669    0.78366    0.77972    0.79679    0.78998       0.78122     0.77001
    r98     0.78878    0.78582    0.78197    0.79897    0.79231       0.78374     0.77276
    r99     0.79084    0.78794    0.78417    0.77927    0.79459        0.7862     0.77545
    r100    0.79285    0.79001    0.78633    0.78154    0.79682        0.7886     0.77808

% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.177867 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       700     100      7      479.94    0.68563    0.27152    0.39

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2           c3           c4          c5          c6          c7
            -------    -------    ----------    --------    -------    -------    -------

    r1            0          0             0     0.07007    0.17967    0.30874    0.43404
    r2            0          0             0    0.070042    0.17961    0.30866    0.43396
    r3            0          0             0    0.069911    0.17935    0.30833     0.4336
    r4            0          0             0    0.069633    0.17881    0.30762    0.43284
    r5            0          0    0.00049972    0.069179    0.17792    0.30645    0.43158
    r96     0.92489    0.92134       0.91672     0.91072    0.92717    0.91691    0.92776
    r97     0.92544    0.92198       0.91747     0.91162    0.92802    0.91801    0.92895
    r98     0.92598     0.9226        0.9182     0.91249    0.92885    0.91908     0.9301
    r99      0.9265     0.9232       0.91891     0.91333    0.92965    0.92011    0.93121
    r100      0.927    0.92379        0.9196     0.91416    0.93042     0.9211    0.90914
```

1.2.5 Test FF_VFI_AZ_VEC Changing Risk Aversion

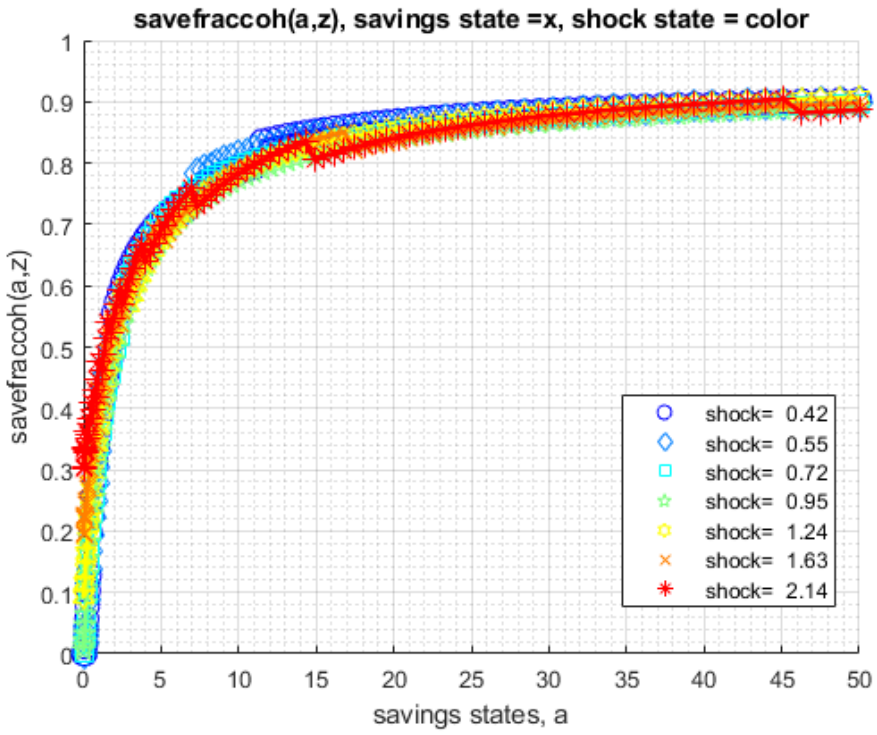
Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

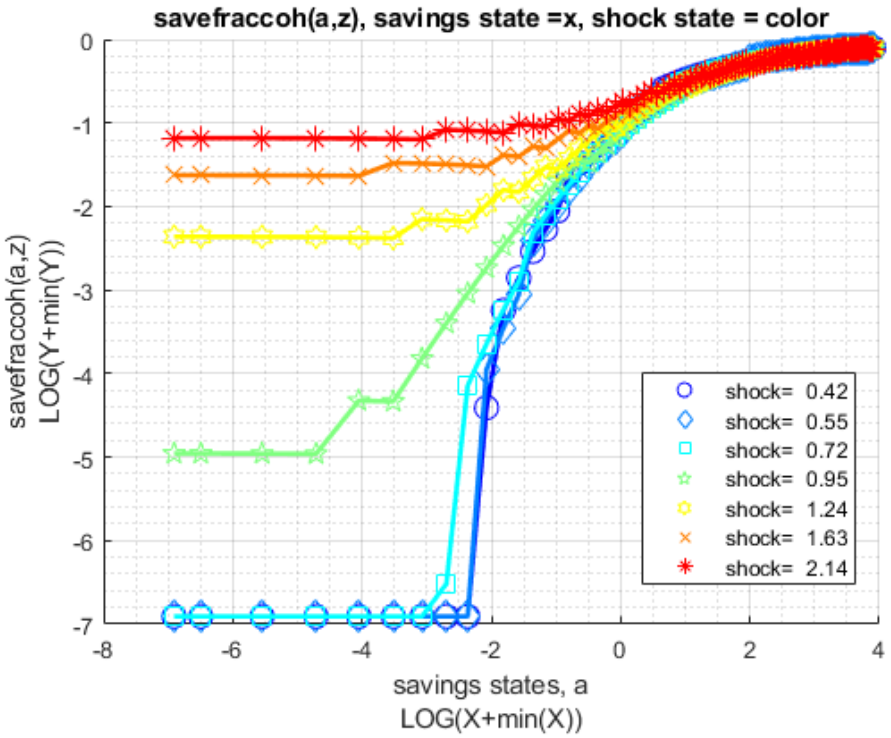
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.181638 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

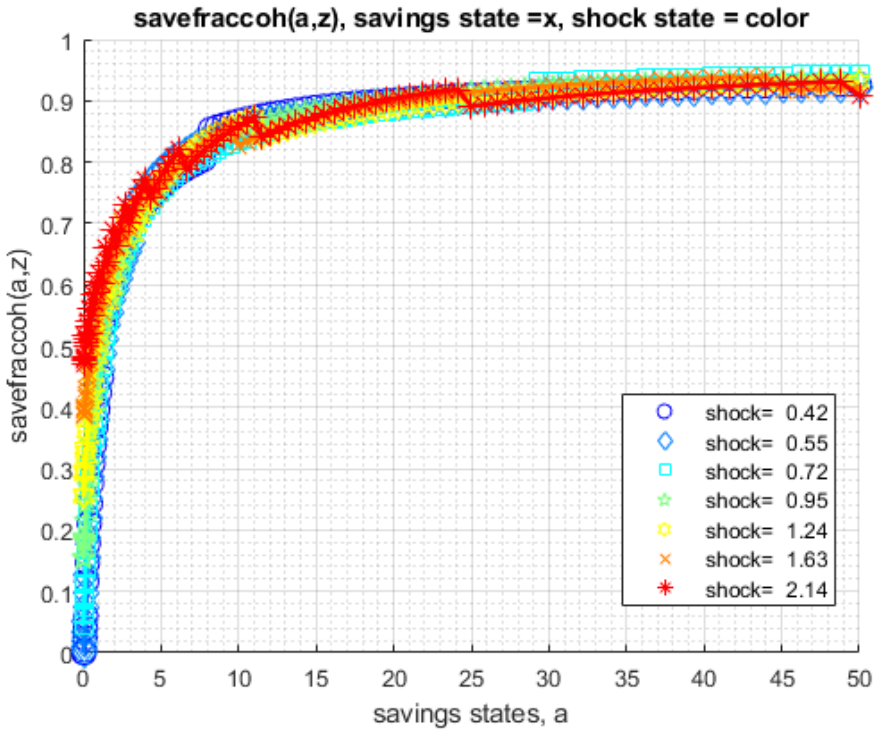
```
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum       mean       std       coefv
                    -     ---     ----     -----     ----     ----     ------     -------     ------     -----

    savefraccoh     1      1       2        700      100       7       450.35     0.64336     0.2803     0.435

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
            -------     -------     -------     ---------     --------     -------     -------

    r1            0           0           0     0.0060341     0.093241     0.19572     0.30604
    r2            0           0           0     0.0060316     0.093213     0.19567     0.30599
    r3            0           0           0     0.0060204      0.09308     0.19546     0.30574
    r4            0           0           0     0.0059964     0.092798     0.19501      0.3052
    r5            0           0           0      0.012229     0.092335     0.19427     0.30431
    r96     0.90049     0.89703     0.89253       0.88669     0.90296     0.89297     0.90379
    r97     0.90128     0.89791     0.89351       0.88781     0.90404     0.89429     0.88181
    r98     0.90205     0.89876     0.89447       0.88891      0.9051     0.89557     0.88337
    r99      0.9028     0.89959     0.89541       0.88998     0.90612     0.89681     0.88489
    r100    0.90354      0.9004     0.89632       0.89101     0.90711     0.89802     0.88636
```

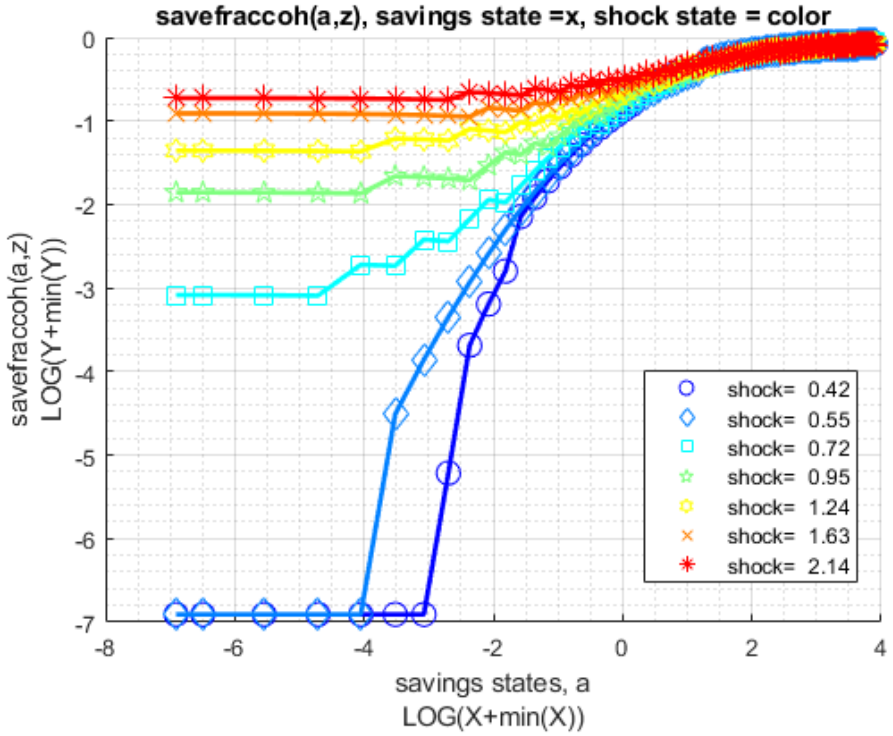




When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.152901 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1      1       2      700     100       7     500.59    0.71513    0.25488    0.35

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
            -------    -------    --------    -------    -------    -------    -------

    r1            0          0    0.044811    0.15534    0.25694    0.40177    0.48276
    r2            0          0    0.044787    0.15528    0.25686    0.40168    0.48268
    r3            0          0    0.044678    0.15499     0.2565    0.40124    0.48228
    r4            0          0    0.044445    0.15437    0.25572    0.40032    0.48143
    r5            0          0    0.064784    0.15337    0.25445    0.39879    0.48003
    r96     0.92489    0.92134     0.94129    0.93513    0.92717    0.91691    0.92776
    r97     0.92544    0.92198      0.9418     0.9358    0.92802    0.91801    0.92895
    r98     0.92598     0.9226      0.9423    0.93644    0.92885    0.91908     0.9301
    r99      0.9265     0.9232     0.94278    0.93706    0.92965    0.92011    0.93121
    r100      0.927    0.92379     0.94324    0.93765    0.93042     0.9211    0.90914
```





1.2.6 Test FF_VFI_AZ_VEC with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 150;
mp_params('it_z_n') = 15;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
% graph color spectrum
mp_params('cl_colors') = 'copper';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.544499 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1506.3    0.66947    0.28673     0.4

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c11         c12         c13
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0    0.14061     0.1891    0.24154
    r2            0          0          0          0          0     0.1406    0.18908    0.24152
    r3            0          0          0          0          0    0.14053      0.189    0.24142
    r4            0          0          0          0          0    0.14038    0.18881     0.2412
    r5            0          0          0          0          0    0.14013    0.18851    0.24085
    r146    0.93087    0.92957    0.92815    0.92661    0.92492    0.92712    0.92403    0.92069
    r147    0.93121    0.92994    0.92854    0.92702    0.92537    0.92768    0.92465    0.92135
    r148    0.93156     0.9303    0.92893    0.92743    0.92581    0.92823    0.92525    0.92201
    r149    0.93189    0.93065     0.9293    0.92783    0.92623    0.92878    0.92584    0.92264
    r150    0.93222      0.931    0.92967    0.92823    0.92665     0.9293    0.92641    0.92327
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 0.515060 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1678.8    0.74614    0.22779    0.30

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c11         c12         c13
            -------    -------    -------    -------    -------    -------    -------    -------
```

r1	0	0	0	0	0	0.53612	0.59853	0.67884
r2	0	0	0	0	0	0.53609	0.5985	0.67882
r3	0	0	0	0	0	0.53594	0.59839	0.67873
r4	0	0	0	0	0	0.53563	0.59814	0.67853
r5	0	0	0	0	0	0.53511	0.59774	0.67821
r146	0.92696	0.9262	0.92513	0.92359	0.92142	0.91653	0.9078	0.88992
r147	0.92721	0.92647	0.92541	0.9239	0.92176	0.91741	0.90895	0.89144
r148	0.92746	0.92673	0.92569	0.92421	0.9221	0.91827	0.91007	0.87813
r149	0.9277	0.92698	0.92596	0.9245	0.92243	0.9191	0.89605	0.86507
r150	0.92794	0.92724	0.92623	0.9248	0.92276	0.90467	0.88233	0.85227

1.3 FF_VFI_AZ_BISEC_LOOP Savings Loop Exact (FOC) Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function:**ff_vfi_az_bisec_loop**from the **MEconTools Package.** This function solves the dynamic programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon.

This is the **looped** code, it is slow for larger state-space problems.

The code uses **continuous** choices, solved with bisection. The state-space is on a grid, but choice grids are in terms of **percentage of resources** to save and solved exactly.

Links to Other Code:

Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid ***Loop***: **ff_vfi_az_loop**
- Common Choice and States Grid ***Vectorized***: **ff_vfi_az_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC, ***Loop***:**ff_vfi_az_bisec_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC ***Vectorized***: **ff_vfi_az_bisec_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Loop***:**ff_vfi_az_mzoom_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Vectorized***: **ff_vfi_az_mzoom_vec**

1.3.1 Test FF_VFI_AZ_BISEC_LOOP Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_bisec_loop(mp_params);

Elapsed time is 33.158577 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

```
          i    idx    ndim    numel    rowN    colN     sum       mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    ------    ------    ------    --------    ---

    ap    1     1      2       700     100      7      9863.4    14.091    14.388     1.0211      0

xxx TABLE:ap xxxxxxxxxxxxxxxxx
              c1        c2        c3         c4         c5         c6         c7
            ------    ------    ------    --------    -------    -------    ------

    r1           0         0         0    0.053491    0.25574    0.60604    1.1157
    r2           0         0         0    0.053998    0.25571     0.6066    1.1163
    r3           0         0         0    0.056449    0.25576    0.60907    1.1187
    r4           0         0         0    0.061799    0.26016     0.6109    1.1239
    r5           0         0         0    0.066463    0.26897    0.61141    1.1327
    r96     43.388     43.52    43.701      43.925     44.222      44.68    45.228
    r97     44.566    44.695    44.878      45.103     45.398     45.856    46.403
    r98     45.761    45.892    46.072      46.298     46.592      47.05    47.597
    r99     46.973    47.107    47.286      47.514     47.806     48.263    48.815
    r100    48.206    48.338    48.519      48.746     49.037     49.497    50.117
```

1.3.2 Test FF_VFI_AZ_BISEC_LOOP Speed Tests

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
% A grid 50, shock grid 5:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 14.819629 seconds.

% A grid 750, shock grid 15:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 15;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 783.169420 seconds.

%A grid 600, shock grid 45:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 600;
mp_params('it_z_n') = 45;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 1955.142516 seconds.
```

1.3.3 Test FF_VFI_AZ_BISEC_LOOP Control Outputs

Run the function first without any outputs;

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
```

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {};
ff_vfi_az_vec(mp_params, mp_support);
```

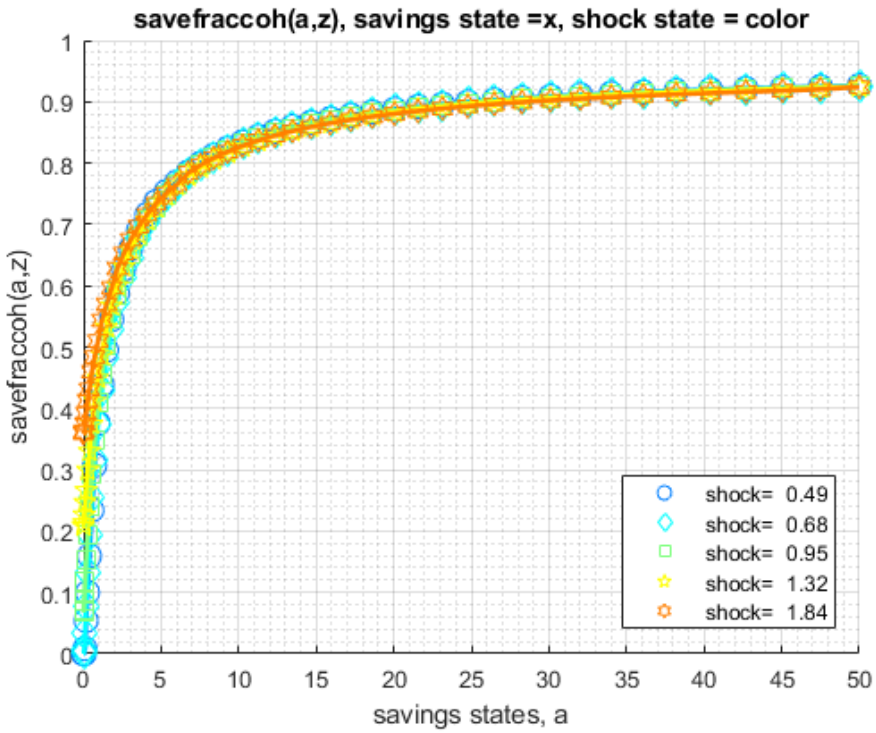
```
Elapsed time is 0.122166 seconds.
```

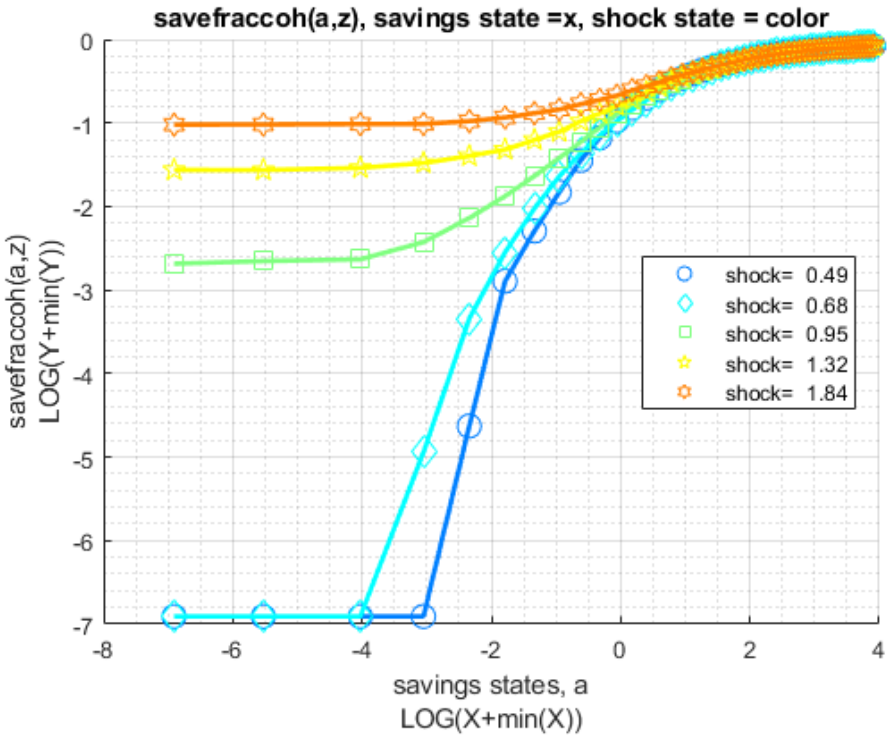
Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
ff_vfi_az_bisec_loop(mp_params, mp_support);
```

```
Elapsed time is 20.812511 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group       a        mean_z_0_4858    mean_z_0_67798    mean_z_0_9462    mean_z_1_3205    mean_z
    -----    --------    -------------    --------------    -------------    -------------    ------

      1            0              0                0          0.067239          0.20859          0.3
      2     0.002975              0                0          0.069375          0.20829          0.3
      3     0.016829              0                0          0.070901           0.2139          0.3
      4     0.046375              0        0.0061439          0.087319           0.2266          0.3
      5     0.095198      0.0087684         0.034403            0.1168           0.2468          0.3
      6       0.1663       0.054361         0.077248            0.1522          0.26639          0.3
      7      0.26234       0.099892          0.13132           0.19388          0.29929          0.4
      8      0.38568        0.15958          0.19309           0.24112          0.33017          0.4
      9      0.53852        0.23417          0.25553           0.29215          0.37436          0.4
     10      0.72291         0.3071          0.31656           0.34812          0.41153          0.4
     11      0.94076        0.37595          0.37503           0.40842          0.44925          0.5
     12       1.1939        0.43881          0.42941           0.45755          0.48697          0.5
     13        1.484        0.49509          0.48129           0.50381          0.53262          0.5
     14       1.8128        0.54489          0.53018           0.54642          0.56778          0.5
     15       2.1817        0.58871          0.57382           0.58548          0.60055          0.
     16       2.5924        0.62716          0.61258           0.62076          0.63101          0.6
     17       3.0463        0.66079          0.64682           0.65243          0.65884          0.
     18       3.5449        0.69027          0.67709           0.68069          0.68423          0.6
     19       4.0894        0.71621          0.70376           0.70596          0.70724          0.7
     20       4.6813        0.73703          0.72732           0.72848          0.72799          0.7
     21       5.3218        0.75326          0.74813            0.7485          0.74673          0.7
     22       6.0121        0.76913          0.76657           0.76632          0.76364          0.7
     23       6.7536        0.78536          0.78286           0.78231          0.77889          0.
     24       7.5474        0.79983          0.79745           0.79653          0.79269          0.7
     25       8.3948        0.81271          0.81039           0.80929          0.80514          0.8
     26       9.2967        0.82418          0.82198           0.82076          0.81637          0.8
     27       10.254         0.8345          0.83242           0.83114          0.82656          0.8
     28       11.269        0.84377          0.84176           0.84042          0.83584          0.8
     29       12.342        0.85214          0.85024           0.84884           0.8442          0.8
     30       13.473        0.85964          0.85781           0.85647          0.85183          0.8
```

31	14.665	0.86648	0.86471	0.86337	0.85879	0.8
32	15.918	0.87264	0.87099	0.86965	0.86507	0.8
33	17.233	0.87826	0.87667	0.87533	0.87161	0.8
34	18.611	0.88338	0.88186	0.88052	0.87771	0.8
35	20.053	0.88802	0.88656	0.88528	0.88326	0.8
36	21.56	0.8923	0.89089	0.88967	0.88833	0.8
37	23.133	0.89614	0.89486	0.89364	0.8926	0.8
38	24.773	0.89974	0.89852	0.8973	0.89626	0.
39	26.481	0.90304	0.90182	0.90072	0.89968	0.8
40	28.258	0.90603	0.90493	0.90383	0.90279	0.8
41	30.104	0.90884	0.90774	0.9067	0.90572	0.9
42	32.021	0.9114	0.91036	0.90932	0.90841	0.9
43	34.01	0.91378	0.9128	0.91183	0.91091	0.9
44	36.07	0.91598	0.91506	0.91408	0.91317	0.9
45	38.204	0.91805	0.91714	0.91622	0.91537	0.9
46	40.412	0.91994	0.91909	0.91817	0.91732	0.9
47	42.695	0.92171	0.92086	0.92001	0.91921	0.
48	45.053	0.92336	0.92257	0.92171	0.92092	0.9
49	47.488	0.92489	0.92409	0.92336	0.92257	0.9
50	50	0.92629	0.92562	0.92489	0.92428	0.9





Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 57.010652 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    ap              1     1      2       900     100      9       12926     14.362     14.544     1.0
    savefraccoh     2     2      2       900     100      9      621.24    0.69027    0.26896    0.38

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1        c2        c3          c4          c5         c6         c7         c8
            ------    ------    ------    ----------    --------    -------    -------    ------

    r1           0         0         0             0    0.087442    0.27778    0.58243    1.0038
    r2           0         0         0             0    0.087962    0.27828    0.58297    1.0044
    r3           0         0         0             0    0.090477    0.28074    0.58547    1.0069
    r4           0         0         0    0.00055771     0.09279    0.28605     0.5907    1.0122
    r5           0         0         0     0.0059496     0.09602    0.29477    0.59952    1.0209
    r96     43.845    43.923    44.022        44.198       44.428     44.722     45.103     45.546
    r97     45.031    45.101    45.208        45.384       45.613      45.91     46.293     46.735
    r98     46.237    46.297    46.411         46.59       46.818     47.115     47.501     47.948
    r99      47.46    47.512    47.635        47.812       48.041      48.34     48.726     49.191
```

```
    r100    48.703    48.746    48.878        49.055      49.283     49.586     49.978    50.495

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3           c4           c5         c6         c7         c8
            -------    -------    -------    ----------    --------    -------    -------    -------

    r1            0          0          0             0    0.066018    0.16569    0.27445    0.37369
    r2            0          0          0             0    0.066384    0.16593    0.27463    0.37381
    r3            0          0          0             0    0.068154    0.16715    0.27549    0.37442
    r4            0          0          0    0.00052879    0.069619    0.16978    0.27726    0.37564
    r5            0          0          0     0.0055946    0.071572    0.17405    0.28025    0.37766
    r96     0.92458    0.92354    0.92226       0.92171     0.92116    0.92055    0.91994    0.91842
    r97     0.92531    0.92416    0.92306       0.92251     0.92196    0.92141    0.92086    0.91933
    r98     0.92605     0.9247    0.92379        0.9233     0.92275     0.9222    0.92171    0.92031
    r99     0.92672    0.92525    0.92452       0.92403     0.92348      0.923    0.92251    0.92147
    r100    0.92739     0.9258    0.92525       0.92477     0.92422    0.92379    0.92342    0.92336
```

1.3.4 Test FF_VFI_AZ_BISEC_LOOP Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 10.824225 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                  i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                  -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh   1     1      2       700     100      7      357.85    0.51122    0.27528    0.53

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5           c6           c7
            -------    -------    -------    -------    -------    ----------    --------

    r1            0          0          0          0          0    0.00022362    0.041544
    r2            0          0          0          0          0    0.00022362    0.041544
    r3            0          0          0          0          0     0.0011391    0.041544
    r4            0          0          0          0          0     0.0016884    0.041483
    r5            0          0          0          0          0     0.0034584     0.04136
    r96     0.79586    0.79275    0.78945    0.78591    0.78225       0.77853     0.77059
```

r97	0.79684	0.79379	0.79055	0.78713	0.78359	0.77993	0.77212
r98	0.79782	0.79482	0.79171	0.78835	0.78488	0.78127	0.77365
r99	0.79873	0.79586	0.79275	0.78951	0.7861	0.78262	0.77548
r100	0.79965	0.79684	0.79385	0.79061	0.78732	0.7839	0.7781

```
% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 53.369195 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coef
	-	---	----	-----	----	----	------	-------	-------	----
savefraccoh	1	1	2	700	100	7	481.37	0.68768	0.27118	0.39

```
xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7
	-------	-------	-------	--------	-------	-------	-------
r1	0	0	0	0.065774	0.18076	0.30655	0.41654
r2	0	0	0	0.066201	0.18101	0.30674	0.4166
r3	0	0	0	0.06791	0.18223	0.30747	0.41709
r4	0	0	0	0.069619	0.18467	0.30759	0.41812
r5	0	0	0	0.071694	0.18876	0.30838	0.41983
r96	0.92428	0.92245	0.92178	0.92116	0.92049	0.91872	0.91824
r97	0.92501	0.92324	0.92257	0.92196	0.92129	0.91958	0.91921
r98	0.92574	0.92397	0.92336	0.92275	0.92208	0.92049	0.92025
r99	0.92647	0.9247	0.92409	0.92348	0.92287	0.92147	0.92159
r100	0.92702	0.92544	0.92483	0.92422	0.92373	0.92336	0.92348

1.3.5 Test FF_VFI_AZ_BISEC_LOOP Changing Risk Aversion

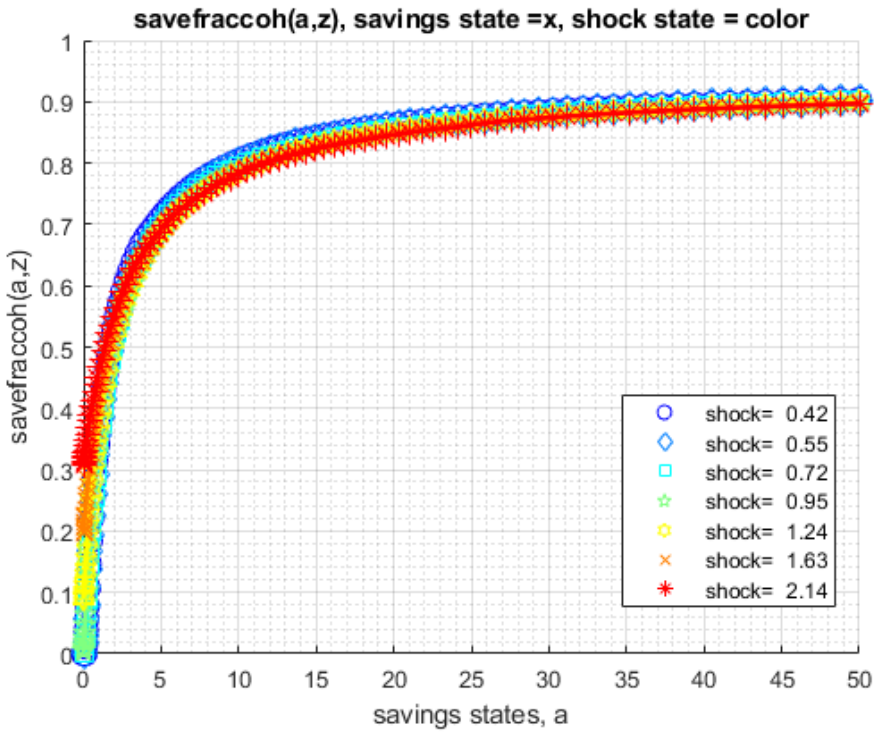
Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

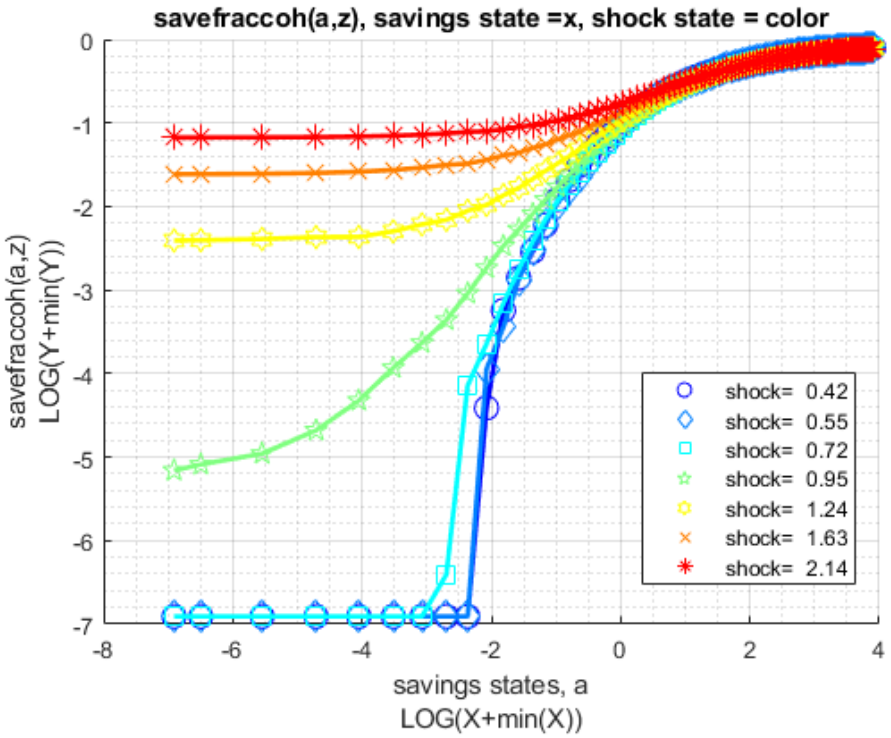
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 47.635241 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

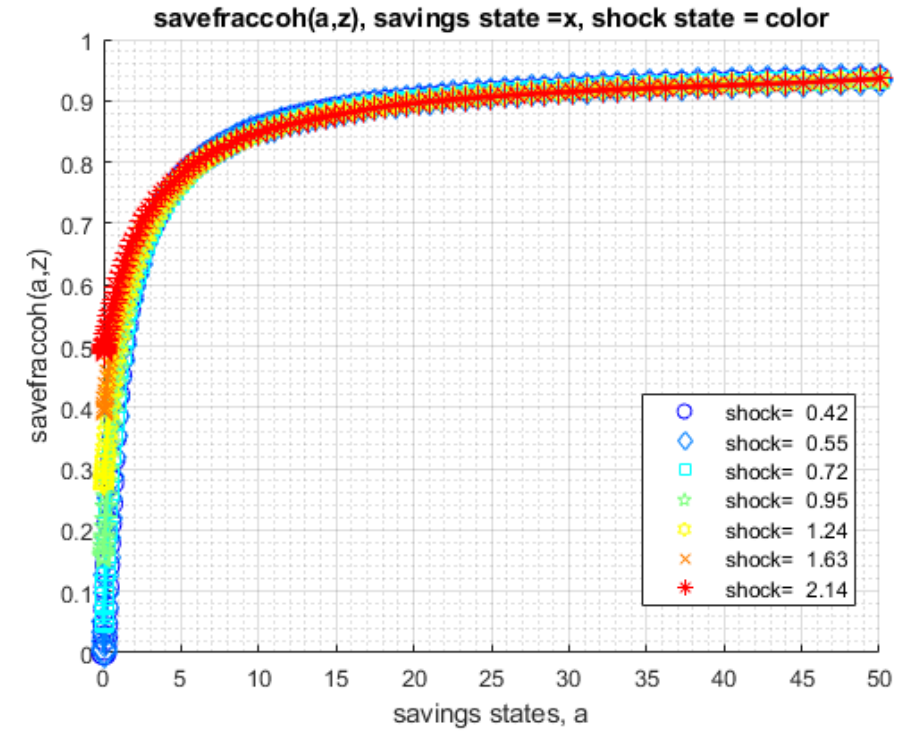
```
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum       mean       std       coefv
                    -     ---     ----     -----     ----     ----     ------     ------     -------     -----

    savefraccoh     1      1       2        700      100       7       452.13     0.6459     0.28031     0.433

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
            -------     -------     -------     ---------     --------     -------     -------

    r1            0           0           0     0.0047401     0.089089     0.19822     0.30783
    r2            0           0           0     0.0051674     0.089394      0.1984     0.30796
    r3            0           0           0     0.0060218     0.090676     0.19926     0.30851
    r4            0           0           0     0.0082801     0.092812     0.20115     0.30973
    r5            0           0           0      0.012247     0.092995      0.2042     0.31174
    r96     0.90047     0.89925     0.89828        0.8973     0.89632     0.89376     0.89297
    r97     0.90127     0.90017     0.89919     0.89828        0.8973      0.8948     0.89394
    r98     0.90206     0.90102     0.90011     0.89919     0.89828     0.89577     0.89498
    r99     0.90279     0.90188     0.90102     0.90011     0.89919     0.89681      0.8959
    r100    0.90359     0.90273     0.90188     0.90096     0.90011     0.89803     0.89687
```

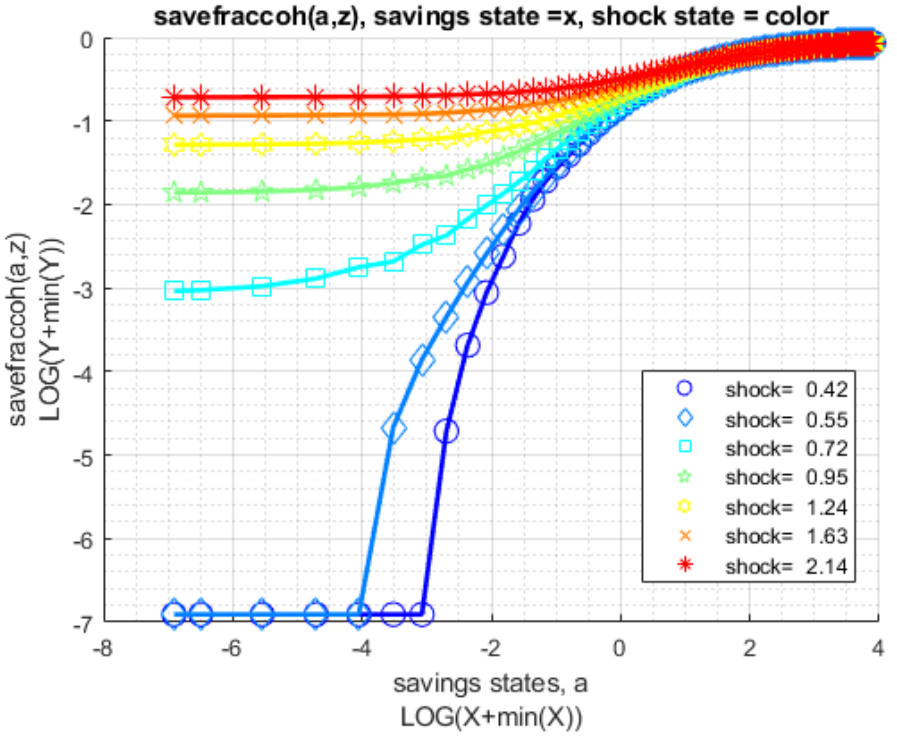




When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 46.937845 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1      1       2      700     100       7     502.71    0.71816    0.25437     0.3

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2          c3         c4         c5         c6         c7
            -------    -------    --------    -------    -------    -------    -------

    r1            0          0    0.047037    0.15537    0.27573     0.3909    0.48782
    r2            0          0    0.047525    0.15531    0.27591    0.39102    0.48795
    r3            0          0    0.049844     0.1569    0.27695     0.3917    0.48837
    r4            0          0    0.054788    0.16025    0.27915     0.3931    0.48929
    r5            0          0    0.062905    0.16569    0.28275    0.39542    0.49075
    r96     0.93307    0.93258     0.93203    0.93154     0.9302    0.92995    0.92971
    r97     0.93374    0.93325     0.93276    0.93227    0.93111    0.93105    0.93117
    r98     0.93441    0.93398     0.93349    0.93307    0.93209    0.93227     0.9327
    r99     0.93508    0.93465     0.93423    0.93392    0.93331    0.93368    0.93435
    r100    0.93575    0.93539     0.93508     0.9349    0.93496    0.93526    0.93587
```





1.3.6 Test FF_VFI_AZ_BISEC_LOOP with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 150;
mp_params('it_z_n') = 15;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
% graph color spectrum
mp_params('cl_colors') = 'copper';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 150.979328 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1507.5    0.67001    0.28668    0.42

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c11         c12         c13
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0    0.13847    0.18485    0.23026
    r2            0          0          0          0          0    0.13853    0.18491    0.23032
    r3            0          0          0          0          0    0.13895    0.18528    0.23063
    r4            0          0          0          0          0    0.13987    0.18607     0.2313
    r5            0          0          0          0          0    0.14011    0.18735     0.2324
    r146    0.92373    0.92354     0.9233    0.92312    0.92287    0.92086    0.92068    0.92049
    r147    0.92422    0.92403    0.92385    0.92361    0.92342    0.92141    0.92123    0.92098
    r148     0.9247    0.92452    0.92434    0.92409    0.92391     0.9219    0.92171    0.92153
    r149    0.92519    0.92501    0.92483    0.92458     0.9244    0.92245    0.92226    0.92208
    r150    0.92568     0.9255    0.92531    0.92507    0.92489    0.92293    0.92275    0.92257
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_bisec_loop(mp_params, mp_support);

Elapsed time is 136.803951 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1685.6    0.74914    0.22909     0.3

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c11         c12         c13
            -------    -------    -------    -------    -------    -------    -------    -------
```

r1	0	0	0	0	0	0.5264	0.61264	0.68271
r2	0	0	0	0	0	0.52646	0.61264	0.68271
r3	0	0	0	0	0	0.52658	0.6127	0.68271
r4	0	0	0	0	0	0.52682	0.61288	0.68283
r5	0	0	0	0	0	0.52731	0.61313	0.68295
r146	0.92983	0.92928	0.92873	0.92806	0.92739	0.92269	0.92354	0.9258
r147	0.9302	0.92971	0.9291	0.92849	0.92788	0.92361	0.92477	0.9269
r148	0.93056	0.93008	0.92953	0.92892	0.92831	0.92458	0.92593	0.928
r149	0.93093	0.93044	0.92995	0.92934	0.92873	0.9258	0.92702	0.9291
r150	0.9313	0.93087	0.93032	0.92977	0.92916	0.92696	0.92818	0.93014

1.4 FF_VFI_AZ_BISEC_VEC Savings Vectorized Exact (FOC) Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_vfi_az_bisec_vec** from the **MEconTools Package.** This function solves the dynamic programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon.

This is the vectorized code, its speed is much faster than the looped code. The function is designed to have small memory footprint and requires low computing resources, yet is fast.

The code uses **continuous choices**, solved with bi(multi)section. The state-space is on a grid, but choice grids are in terms of percentage of resources available, which is individual specific, to save and solved exactly up to ((1/(2)^16)*100=0.001525878) percentage of cash on hand. The **ff_vfi_az_vec** from the **MEconTools Package** solves the same problem using vectorized common grid code where the choice set and state space share the same grid. The common grid function is faster, but less precise for the same number of asset grid points.

Links to Other Code:

Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid ***Loop***: **ff_vfi_az_loop**
- Common Choice and States Grid ***Vectorized***: **ff_vfi_az_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC, ***Loop***:**ff_vfi_az_bisec_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC ***Vectorized***: **ff_vfi_az_bisec_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Loop***:**ff_vfi_az_mzoom_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Vectorized***: **ff_vfi_az_mzoom_vec**

1.4.1 Test FF_VFI_AZ_BISEC_VEC Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_bisec_vec(mp_params);
```

```
Elapsed time is 1.762201 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum       mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    ------    ------    ------    --------    ---

    ap    1     1      2       700     100      7      9863.4    14.091    14.388     1.0211      0

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1        c2        c3         c4         c5         c6         c7
            ------    ------    ------    --------    -------    -------    ------

    r1           0         0         0    0.053491    0.25574    0.60604    1.1157
    r2           0         0         0    0.053998    0.25571     0.6066    1.1163
    r3           0         0         0    0.056449    0.25576    0.60907    1.1187
    r4           0         0         0    0.061799    0.26016     0.6109    1.1239
    r5           0         0         0    0.066463    0.26897    0.61141    1.1327
    r96     43.388     43.52    43.701      43.925     44.222      44.68    45.228
    r97     44.566    44.695    44.878      45.103     45.398     45.856    46.403
    r98     45.761    45.892    46.072      46.298     46.592      47.05    47.597
    r99     46.973    47.107    47.286      47.514     47.806     48.263    48.815
    r100    48.206    48.338    48.519      48.746     49.037     49.497    50.117
```

1.4.2 Test FF_VFI_AZ_BISEC_VEC Speed Tests

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
% A grid 50, shock grid 5:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 0.792541 seconds.

% A grid 750, shock grid 15:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 15;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 43.095190 seconds.

% A grid 600, shock grid 45:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 600;
mp_params('it_z_n') = 45;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 80.139775 seconds.
```

1.4.3 Test FF_VFI_AZ_BISEC_VEC Control Outputs

Run the function first without any outputs;

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {};
ff_vfi_az_vec(mp_params, mp_support);
```

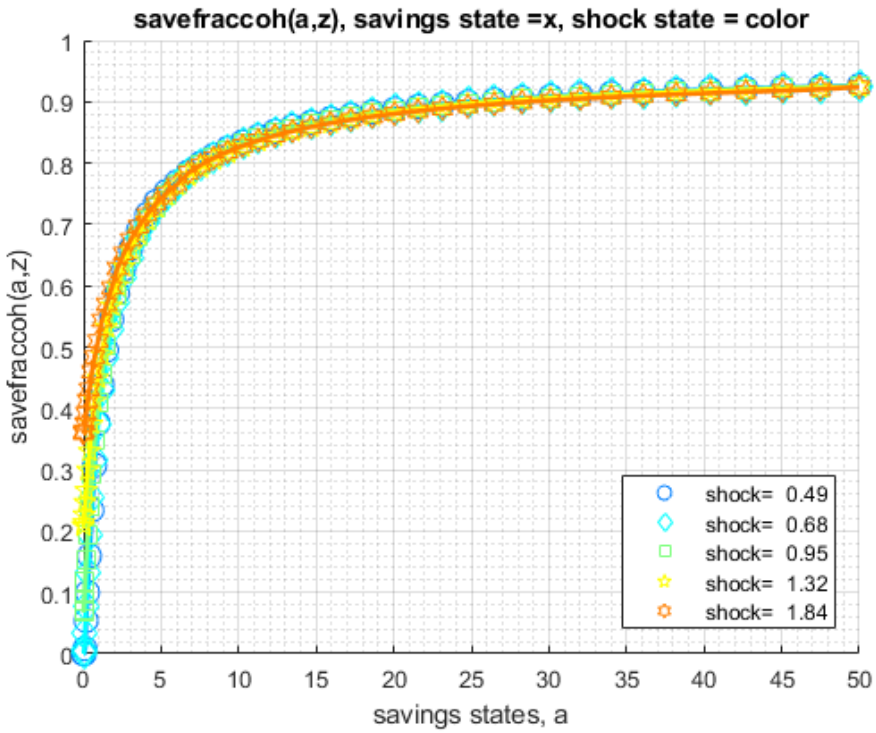
```
Elapsed time is 0.029901 seconds.
```

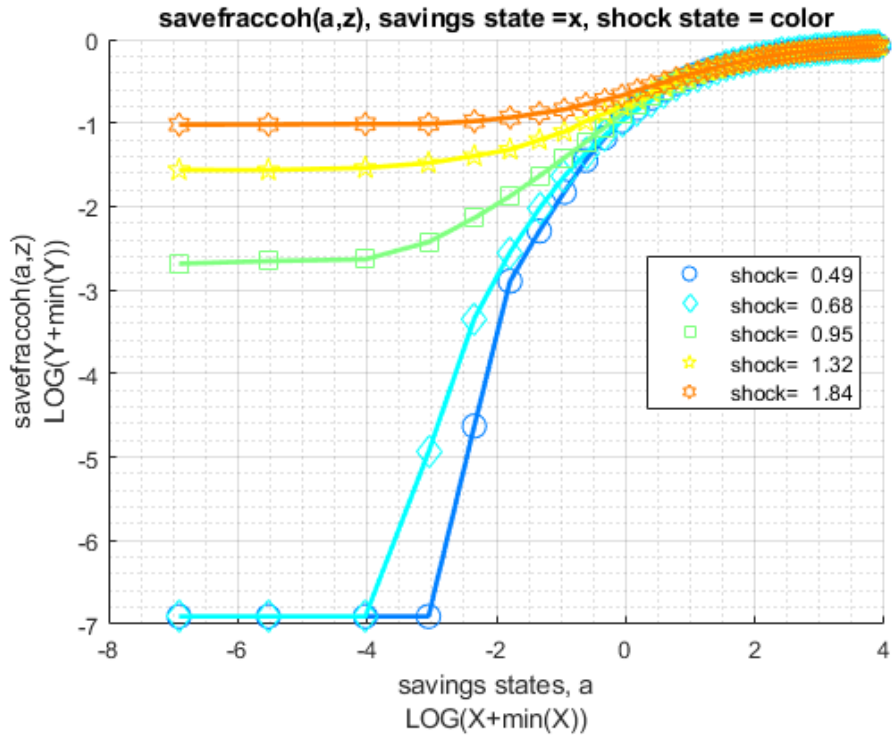
Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

```
Elapsed time is 0.494900 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group       a        mean_z_0_4858    mean_z_0_67798    mean_z_0_9462    mean_z_1_3205    mean_z
    -----    -------    -------------    --------------    -------------    -------------    ------

      1            0              0                0          0.067239          0.20859          0.3
      2     0.002975              0                0          0.069375          0.20829          0.3
      3     0.016829              0                0          0.070901           0.2139          0.3
      4     0.046375              0        0.0061439          0.087319           0.2266          0.3
      5     0.095198      0.0087684         0.034403            0.1168           0.2468          0.3
      6       0.1663       0.054361         0.077248            0.1522          0.26639          0.3
      7      0.26234       0.099892          0.13132           0.19388          0.29929          0.4
      8      0.38568        0.15958          0.19309           0.24112          0.33017          0.4
      9      0.53852        0.23417          0.25553           0.29215          0.37436          0.4
     10      0.72291         0.3071          0.31656           0.34812          0.41153          0.4
     11      0.94076        0.37595          0.37503           0.40842          0.44925          0.5
     12       1.1939        0.43881          0.42941           0.45755          0.48697          0.5
     13        1.484        0.49509          0.48129           0.50381          0.53262          0.5
     14       1.8128        0.54489          0.53018           0.54642          0.56778          0.5
     15       2.1817        0.58871          0.57382           0.58548          0.60055          0.
     16       2.5924        0.62716          0.61258           0.62076          0.63101          0.6
     17       3.0463        0.66079          0.64682           0.65243          0.65884          0.
     18       3.5449        0.69027          0.67709           0.68069          0.68423          0.6
     19       4.0894        0.71621          0.70376           0.70596          0.70724          0.7
     20       4.6813        0.73703          0.72732           0.72848          0.72799          0.7
     21       5.3218        0.75326          0.74813            0.7485          0.74673          0.7
     22       6.0121        0.76913          0.76657           0.76632          0.76364          0.7
     23       6.7536        0.78536          0.78286           0.78231          0.77889          0.
     24       7.5474        0.79983          0.79745           0.79653          0.79269          0.7
```

25	8.3948	0.81271	0.81039	0.80929	0.80514	0.8
26	9.2967	0.82418	0.82198	0.82076	0.81637	0.8
27	10.254	0.8345	0.83242	0.83114	0.82656	0.8
28	11.269	0.84377	0.84176	0.84042	0.83584	0.8
29	12.342	0.85214	0.85024	0.84884	0.8442	0.8
30	13.473	0.85964	0.85781	0.85647	0.85183	0.8
31	14.665	0.86648	0.86471	0.86337	0.85879	0.8
32	15.918	0.87264	0.87099	0.86965	0.86507	0.8
33	17.233	0.87826	0.87667	0.87533	0.87161	0.8
34	18.611	0.88338	0.88186	0.88052	0.87771	0.8
35	20.053	0.88802	0.88656	0.88528	0.88326	0.8
36	21.56	0.8923	0.89089	0.88967	0.88833	0.8
37	23.133	0.89614	0.89486	0.89364	0.8926	0.8
38	24.773	0.89974	0.89852	0.8973	0.89626	0.
39	26.481	0.90304	0.90182	0.90072	0.89968	0.8
40	28.258	0.90603	0.90493	0.90383	0.90279	0.8
41	30.104	0.90884	0.90774	0.9067	0.90572	0.9
42	32.021	0.9114	0.91036	0.90932	0.90841	0.9
43	34.01	0.91378	0.9128	0.91183	0.91091	0.9
44	36.07	0.91598	0.91506	0.91408	0.91317	0.9
45	38.204	0.91805	0.91714	0.91622	0.91537	0.9
46	40.412	0.91994	0.91909	0.91817	0.91732	0.9
47	42.695	0.92171	0.92086	0.92001	0.91921	0.
48	45.053	0.92336	0.92257	0.92171	0.92092	0.9
49	47.488	0.92489	0.92409	0.92336	0.92257	0.9
50	50	0.92629	0.92562	0.92489	0.92428	0.9





Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 1.164186 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    ap              1      1       2      900     100       9      12926     14.362     14.544      1.0
    savefraccoh     2      2       2      900     100       9     621.24    0.69027    0.26896     0.38

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1         c2         c3          c4           c5         c6         c7         c8
            ------     ------     ------     ----------    --------    -------    -------    ------

    r1           0          0          0              0     0.087442    0.27778    0.58243    1.0038
    r2           0          0          0              0     0.087962    0.27828    0.58297    1.0044
    r3           0          0          0              0     0.090477    0.28074    0.58547    1.0069
    r4           0          0          0    0.00055771      0.09279    0.28605     0.5907    1.0122
    r5           0          0          0     0.0059496      0.09602    0.29477    0.59952    1.0209
    r96     43.845     43.923     44.022         44.198        44.428     44.722     45.103     45.546
    r97     45.031     45.101     45.208         45.384        45.613      45.91     46.293     46.735
    r98     46.237     46.297     46.411          46.59        46.818     47.115     47.501     47.948
    r99      47.46     47.512     47.635         47.812        48.041      48.34     48.726     49.191
```

```
    r100    48.703    48.746    48.878        49.055      49.283     49.586     49.978    50.495

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3           c4           c5         c6         c7         c8
            -------    -------    -------    ----------    --------    -------    -------    -------

    r1            0          0          0             0    0.066018    0.16569    0.27445    0.37369
    r2            0          0          0             0    0.066384    0.16593    0.27463    0.37381
    r3            0          0          0             0    0.068154    0.16715    0.27549    0.37442
    r4            0          0          0    0.00052879    0.069619    0.16978    0.27726    0.37564
    r5            0          0          0     0.0055946    0.071572    0.17405    0.28025    0.37766
    r96     0.92458    0.92354    0.92226       0.92171     0.92116    0.92055    0.91994    0.91842
    r97     0.92531    0.92416    0.92306       0.92251     0.92196    0.92141    0.92086    0.91933
    r98     0.92605     0.9247    0.92379        0.9233     0.92275     0.9222    0.92171    0.92031
    r99     0.92672    0.92525    0.92452       0.92403     0.92348      0.923    0.92251    0.92147
    r100    0.92739     0.9258    0.92525       0.92477     0.92422    0.92379    0.92342    0.92336
```

1.4.4 Test FF_VFI_AZ_BISEC_VEC Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 0.271658 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                  i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                  -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh   1     1      2       700     100      7      357.85    0.51122    0.27528    0.53

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5           c6           c7
            -------    -------    -------    -------    -------    ----------    --------

    r1            0          0          0          0          0    0.00022362    0.041544
    r2            0          0          0          0          0    0.00022362    0.041544
    r3            0          0          0          0          0     0.0011391    0.041544
    r4            0          0          0          0          0     0.0016884    0.041483
    r5            0          0          0          0          0     0.0034584     0.04136
    r96     0.79586    0.79275    0.78945    0.78591    0.78225       0.77853     0.77059
```

```
    r97     0.79684    0.79379    0.79055    0.78713    0.78359       0.77993     0.77212
    r98     0.79782    0.79482    0.79171    0.78835    0.78488       0.78127     0.77365
    r99     0.79873    0.79586    0.79275    0.78951     0.7861       0.78262     0.77548
    r100    0.79965    0.79684    0.79385    0.79061    0.78732        0.7839      0.7781
```

```
% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 0.971218 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       700     100      7      481.37    0.68768    0.27118    0.39

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
            -------    -------    -------    --------    -------    -------    -------

    r1            0          0          0    0.065774    0.18076    0.30655    0.41654
    r2            0          0          0    0.066201    0.18101    0.30674     0.4166
    r3            0          0          0     0.06791    0.18223    0.30747    0.41709
    r4            0          0          0    0.069619    0.18467    0.30759    0.41812
    r5            0          0          0    0.071694    0.18876    0.30838    0.41983
    r96     0.92428    0.92245    0.92178     0.92116    0.92049    0.91872    0.91824
    r97     0.92501    0.92324    0.92257     0.92196    0.92129    0.91958    0.91921
    r98     0.92574    0.92397    0.92336     0.92275    0.92208    0.92049    0.92025
    r99     0.92647     0.9247    0.92409     0.92348    0.92287    0.92147    0.92159
    r100    0.92702    0.92544    0.92483     0.92422    0.92373    0.92336    0.92348
```

1.4.5 Test FF_VFI_AZ_BISEC_VEC Changing Risk Aversion

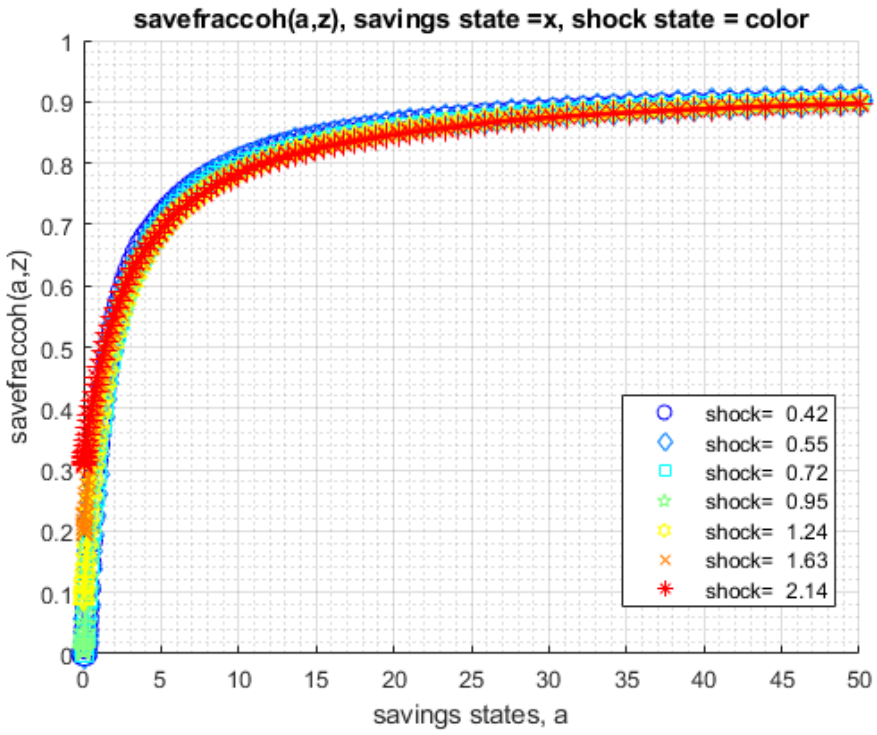
Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

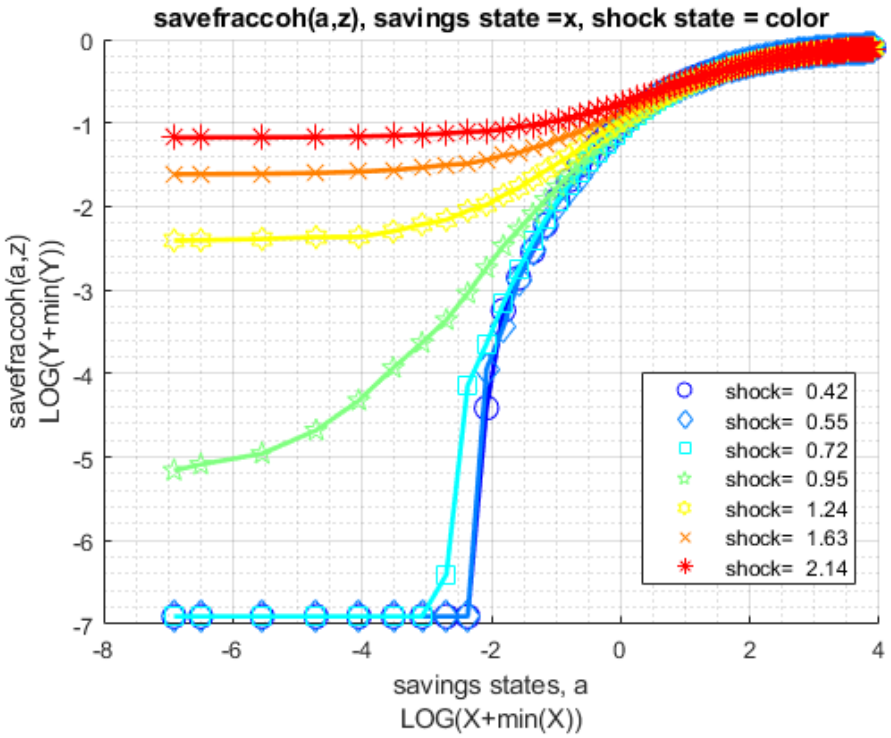
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 0.873752 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

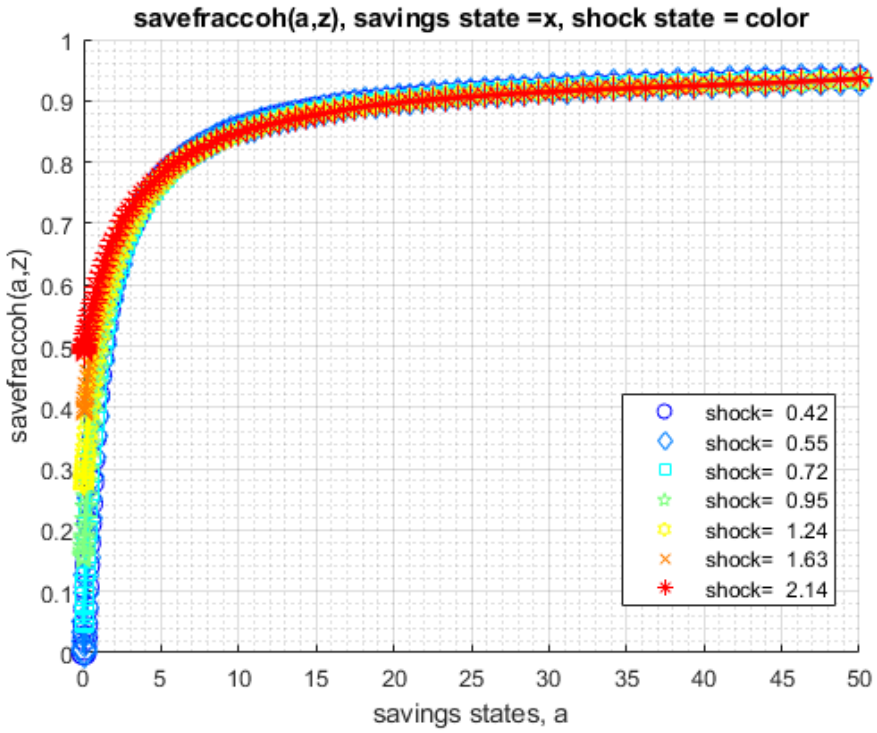
```
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum       mean       std       coefv
                    -     ---     ----     -----     ----     ----      ------    ------     -------     -----

    savefraccoh     1      1       2        700      100       7       452.13     0.6459     0.28031     0.433

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
            -------     -------     -------     ---------     --------     -------     -------

    r1            0           0           0     0.0047401     0.089089     0.19822     0.30783
    r2            0           0           0     0.0051674     0.089394      0.1984     0.30796
    r3            0           0           0     0.0060218     0.090676     0.19926     0.30851
    r4            0           0           0     0.0082801     0.092812     0.20115     0.30973
    r5            0           0           0      0.012247     0.092995      0.2042     0.31174
    r96     0.90047     0.89925     0.89828        0.8973     0.89632     0.89376     0.89297
    r97     0.90127     0.90017     0.89919     0.89828        0.8973      0.8948     0.89394
    r98     0.90206     0.90102     0.90011     0.89919     0.89828     0.89577     0.89498
    r99     0.90279     0.90188     0.90102     0.90011     0.89919     0.89681      0.8959
    r100    0.90359     0.90273     0.90188     0.90096     0.90011     0.89803     0.89687
```

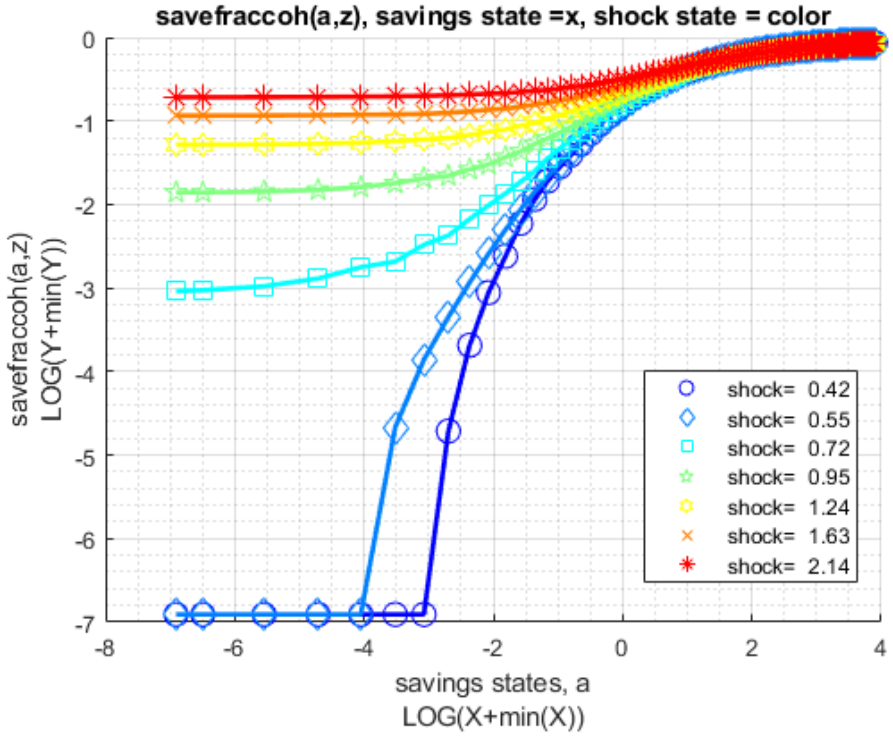




When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 0.970314 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std       coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1      1       2      700     100       7     502.71    0.71816    0.25437     0.3

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2          c3         c4         c5         c6         c7
            -------    -------    --------    -------    -------    -------    -------

    r1            0          0    0.047037    0.15537    0.27573     0.3909    0.48782
    r2            0          0    0.047525    0.15531    0.27591    0.39102    0.48795
    r3            0          0    0.049844     0.1569    0.27695     0.3917    0.48837
    r4            0          0    0.054788    0.16025    0.27915     0.3931    0.48929
    r5            0          0    0.062905    0.16569    0.28275    0.39542    0.49075
    r96     0.93307    0.93258     0.93203    0.93154     0.9302    0.92995    0.92971
    r97     0.93374    0.93325     0.93276    0.93227    0.93111    0.93105    0.93117
    r98     0.93441    0.93398     0.93349    0.93307    0.93209    0.93227     0.9327
    r99     0.93508    0.93465     0.93423    0.93392    0.93331    0.93368    0.93435
    r100    0.93575    0.93539     0.93508     0.9349    0.93496    0.93526    0.93587
```





1.4.6 Test FF_VFI_AZ_BISEC_VEC with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 150;
mp_params('it_z_n') = 15;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
% graph color spectrum
mp_params('cl_colors') = 'copper';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 2.595920 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1507.5    0.67001    0.28668    0.42

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c11         c12         c13
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0    0.13847    0.18485    0.23026
    r2            0          0          0          0          0    0.13853    0.18491    0.23032
    r3            0          0          0          0          0    0.13895    0.18528    0.23063
    r4            0          0          0          0          0    0.13987    0.18607     0.2313
    r5            0          0          0          0          0    0.14011    0.18735     0.2324
    r146    0.92373    0.92354     0.9233    0.92312    0.92287    0.92086    0.92068    0.92049
    r147    0.92422    0.92403    0.92385    0.92361    0.92342    0.92141    0.92123    0.92098
    r148     0.9247    0.92452    0.92434    0.92409    0.92391     0.9219    0.92171    0.92153
    r149    0.92519    0.92501    0.92483    0.92458     0.9244    0.92245    0.92226    0.92208
    r150    0.92568     0.9255    0.92531    0.92507    0.92489    0.92293    0.92275    0.92257
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_bisec_vec(mp_params, mp_support);

Elapsed time is 2.805227 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1685.6    0.74914    0.22909     0.3

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c11         c12         c13
            -------    -------    -------    -------    -------    -------    -------    -------
```

r1	0	0	0	0	0	0.5264	0.61264	0.68271
r2	0	0	0	0	0	0.52646	0.61264	0.68271
r3	0	0	0	0	0	0.52658	0.6127	0.68271
r4	0	0	0	0	0	0.52682	0.61288	0.68283
r5	0	0	0	0	0	0.52731	0.61313	0.68295
r146	0.92983	0.92928	0.92873	0.92806	0.92739	0.92269	0.92354	0.9258
r147	0.9302	0.92971	0.9291	0.92849	0.92788	0.92361	0.92477	0.9269
r148	0.93056	0.93008	0.92953	0.92892	0.92831	0.92458	0.92593	0.928
r149	0.93093	0.93044	0.92995	0.92934	0.92873	0.9258	0.92702	0.9291
r150	0.9313	0.93087	0.93032	0.92977	0.92916	0.92696	0.92818	0.93014

1.5 FF_VFI_AZ_MZOOM_LOOP Savings Loop Exact (VALUE) Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function:**ff_vfi_az_mzoom_loop** from the **MEconTools Package.** This function solves the dynamic programming problem for a (a,z) model. The state-space is on a grid, but choice grids are in terms of **percentage of resources** to save and solved exactly.

This is a **looped** code for **continuous** choices, solved with the **mzoom** algorithm. In contrast to the **bisection** based solution, this is slower, but this does not rely on first order conditions.

Links to Other Code:

Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid ***Loop***: **ff_vfi_az_loop**
- Common Choice and States Grid ***Vectorized***: **ff_vfi_az_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC, ***Loop***:**ff_vfi_az_bisec_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC ***Vectorized***: **ff_vfi_az_bisec_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Loop***:**ff_vfi_az_mzoom_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Vectorized***: **ff_vfi_az_mzoom_vec**

1.5.1 Test FF_VFI_AZ_MZOOM_LOOP Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_mzoom_loop(mp_params);

Elapsed time is 83.956071 seconds.
-----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum       mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    ------    ------    ------    --------    ---
```

```
    ap    1     1      2       700     100      7      9861.5    14.088    14.386     1.0212      0

xxx TABLE:ap xxxxxxxxxxxxxxxxx
              c1        c2        c3         c4         c5         c6         c7
            ------    ------    ------    --------    -------    -------    ------

    r1           0         0         0     0.05343    0.25568    0.60598    1.1155
    r2           0         0         0    0.053451    0.25571    0.60652    1.1161
    r3           0         0         0    0.056468    0.25574    0.60897    1.1174
    r4           0         0         0    0.061232    0.25995    0.61042    1.1238
    r5           0         0         0    0.065929     0.2689    0.61091    1.1323
    r96     43.387    43.517      43.7      43.922     44.221     44.657    45.225
    r97     44.562    44.694    44.876      45.095     45.392     45.847    46.394
    r98     45.758     45.89    46.071      46.287     46.583     47.037    47.596
    r99     46.972    47.103    47.285        47.5     47.794     48.247    48.812
    r100    48.183    48.337    48.518      48.732     49.025     49.478    50.115
```

1.5.2 Test FF_VFI_AZ_MZOOM_LOOP Speed Tests

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
```

A grid 50, shock grid 5:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 26.554641 seconds.
```

A grid 750, shock grid 15:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 15;
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 2148.508425 seconds.
```

A grid 600, shock grid 45:

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 600;
mp_params('it_z_n') = 45;
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 8507.097739 seconds.
```

1.5.3 Test FF_VFI_AZ_MZOOM_LOOP Control Outputs

Run the function first without any outputs, but only the timer.

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
```

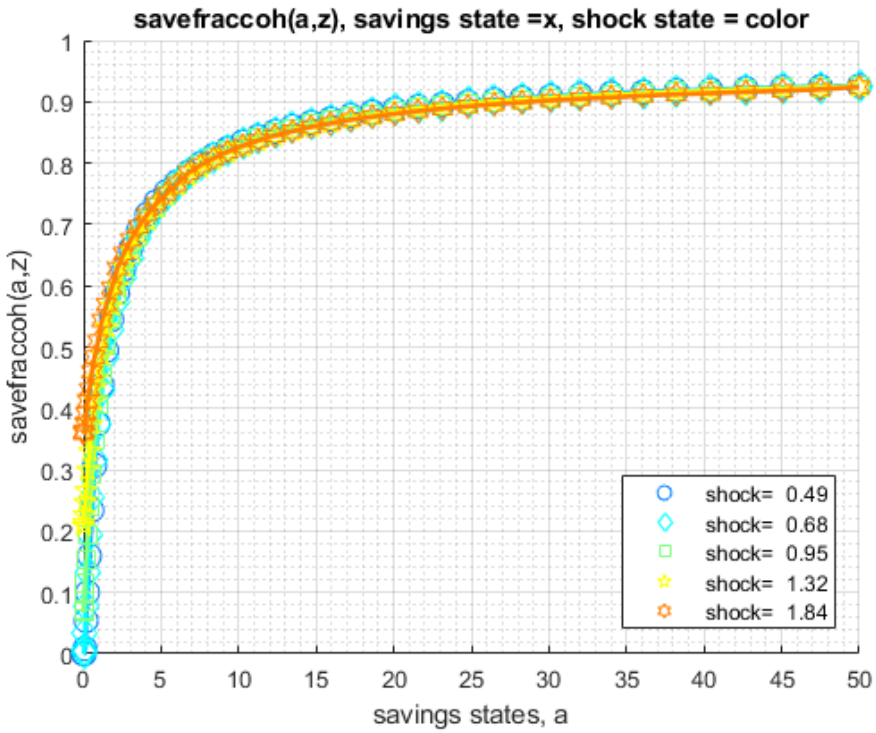
```
mp_support('bl_timer') = true;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {};
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 24.011245 seconds.
```

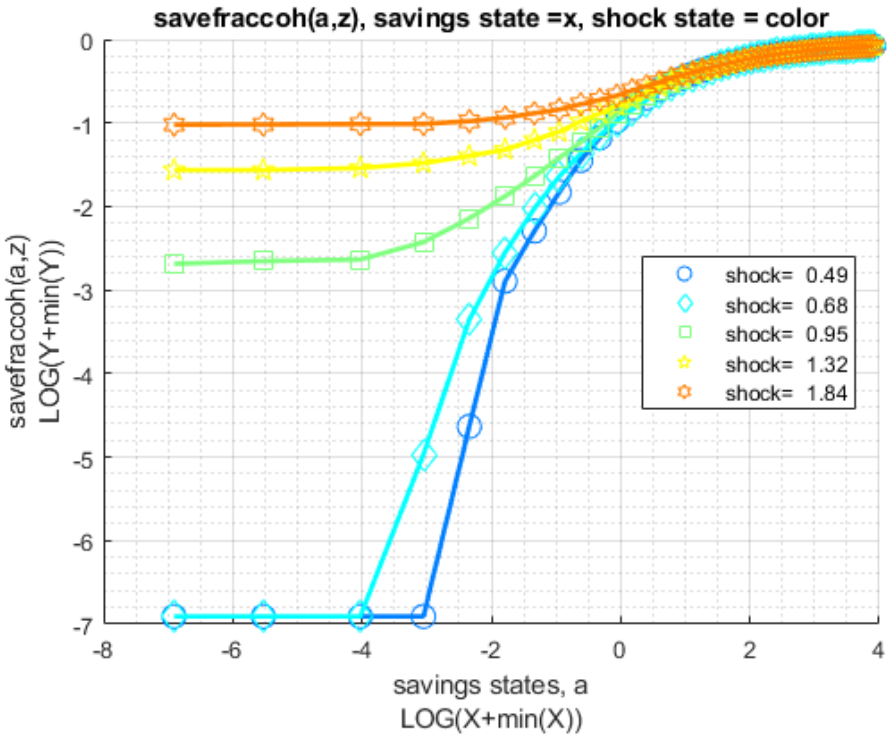
Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 23.773078 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxxx
```

group	a	mean_z_0_4858	mean_z_0_67798	mean_z_0_9462	mean_z_1_3205	mean_z
1	0	0	0	0.067148	0.2084	0.3
2	0.002975	0	0	0.069345	0.20826	0.3
3	0.016829	0	0	0.070749	0.2136	0.3
4	0.046375	0	0.0059631	0.08732	0.22641	0.3
5	0.095198	0.008725	0.033935	0.11637	0.24674	0.
6	0.1663	0.054327	0.077152	0.15198	0.26635	0.3
7	0.26234	0.099882	0.13131	0.1936	0.29922	0.4
8	0.38568	0.15954	0.1928	0.24107	0.33005	0.4
9	0.53852	0.23411	0.25482	0.29164	0.37407	0.
10	0.72291	0.30704	0.31604	0.34806	0.41148	0.4
11	0.94076	0.37567	0.37487	0.40768	0.44925	0.5
12	1.1939	0.43849	0.42939	0.4573	0.48691	0.5
13	1.484	0.49491	0.48129	0.50332	0.53253	0.5
14	1.8128	0.54486	0.53013	0.54642	0.56773	0.5
15	2.1817	0.58868	0.57335	0.58545	0.60016	0.6
16	2.5924	0.6271	0.61254	0.62056	0.63057	0.6
17	3.0463	0.66058	0.6468	0.65237	0.65884	0.6
18	3.5449	0.69019	0.67699	0.68069	0.68379	0.6
19	4.0894	0.71615	0.70375	0.7058	0.70719	0.
20	4.6813	0.73661	0.72701	0.72843	0.72781	0.7
21	5.3218	0.75302	0.7481	0.74821	0.74661	0.7
22	6.0121	0.76912	0.76622	0.76622	0.76342	0.7
23	6.7536	0.78503	0.78285	0.78223	0.77885	0.7
24	7.5474	0.79943	0.79703	0.79623	0.79223	0.7
25	8.3948	0.81264	0.81024	0.8093	0.80504	0.8
26	9.2967	0.82384	0.82198	0.82064	0.81634	0.8
27	10.254	0.83447	0.83225	0.83065	0.82653	0.8
28	11.269	0.84345	0.84174	0.84025	0.83545	0.8
29	12.342	0.85185	0.85017	0.84865	0.84417	0.8
30	13.473	0.85962	0.85746	0.85642	0.85178	0.8
31	14.665	0.86626	0.86466	0.86306	0.85873	0.8

32	15.918	0.87226	0.87066	0.86959	0.86504	0.8
33	17.233	0.87786	0.87626	0.87529	0.87146	0.8
34	18.611	0.88332	0.88182	0.88026	0.87766	0.8
35	20.053	0.888	0.88656	0.88507	0.88267	0.8
36	21.56	0.89187	0.89087	0.88947	0.88825	0.8
37	23.133	0.89587	0.89484	0.89347	0.89256	0.8
38	24.773	0.8997	0.89827	0.89727	0.89587	0.8
39	26.481	0.903	0.90147	0.90066	0.89964	0.8
40	28.258	0.90601	0.90467	0.90376	0.90278	0.8
41	30.104	0.90881	0.9077	0.90628	0.90547	0.9
42	32.021	0.91137	0.91035	0.90908	0.90838	0.9
43	34.01	0.91377	0.91275	0.91148	0.91068	0.9
44	36.07	0.91595	0.91468	0.91388	0.91308	0.9
45	38.204	0.91788	0.91708	0.91617	0.91531	0.9
46	40.412	0.91948	0.91868	0.91788	0.91708	0.9
47	42.695	0.92168	0.92085	0.91998	0.91915	0.9
48	45.053	0.92331	0.92251	0.92171	0.92091	0.9
49	47.488	0.92485	0.92408	0.92331	0.92254	0.
50	50	0.92588	0.92555	0.92485	0.92423	0.9



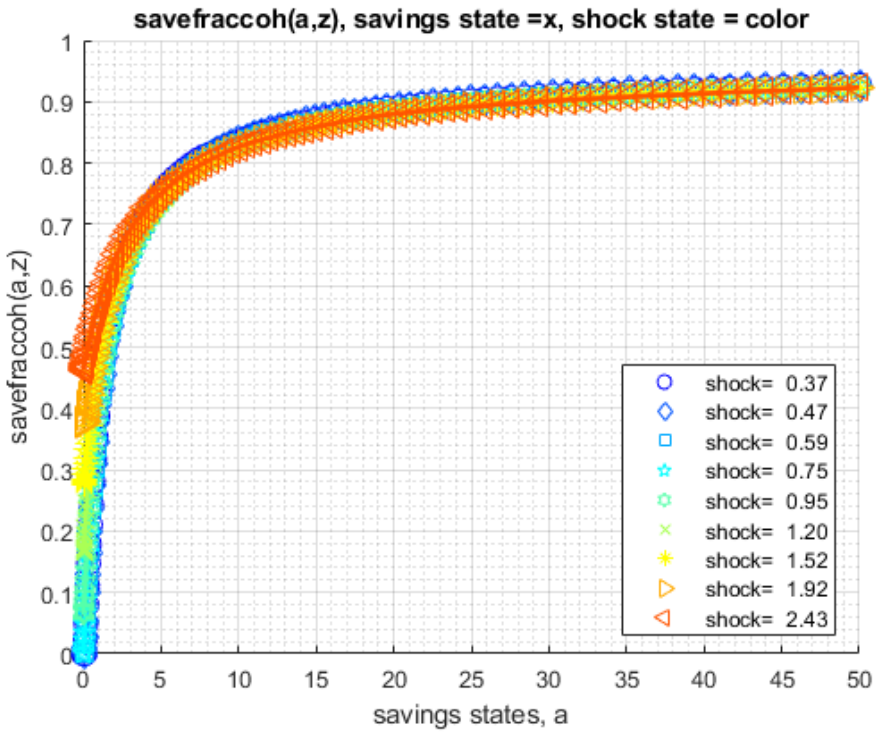


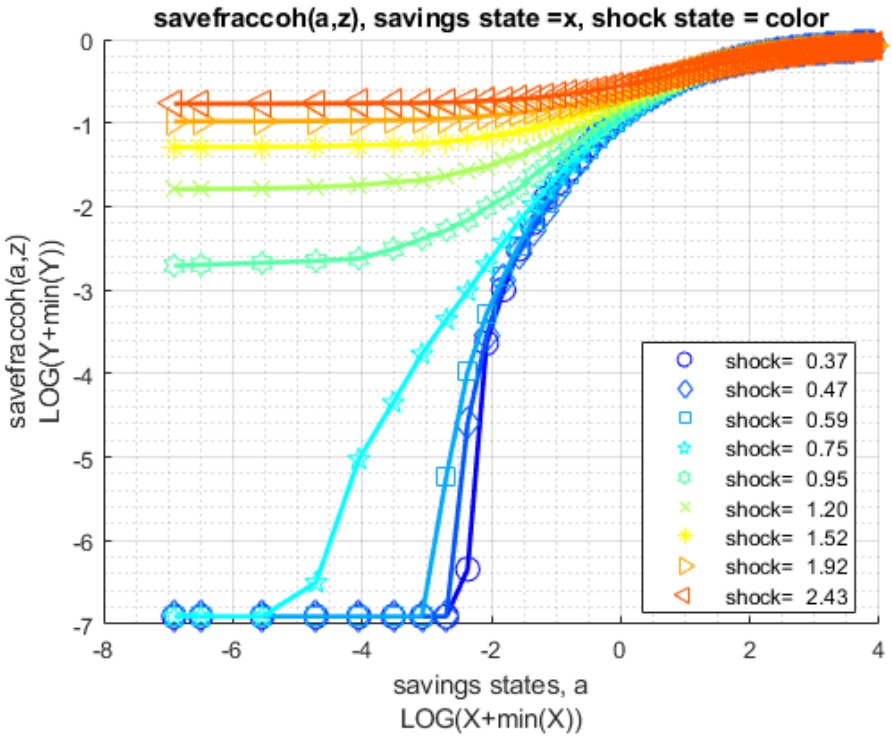
Run the function and show summaries for savings and fraction of coh saved:

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
% mp_params('fl_crra') = 1.5;
% mp_params('fl_beta') = 0.94;
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
% call function
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 111.419370 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxxxx
    group        a         mean_z_0_36853    mean_z_0_46648    mean_z_0_59047    mean_z_0_74742    m
    _____    __________    ______________    ______________    ______________    ______________    _

      1              0                0                 0                 0                 0
      2     0.00051272                0                 0                 0                 0
      3      0.0029004                0                 0                 0                 0
      4      0.0079925                0                 0                 0        0.00050216
      5       0.016407                0                 0                 0          0.005563
      6       0.028662                0                 0                 0          0.011926
      7       0.045213                0                 0                 0          0.022095
      8        0.06647                0                 0         0.0043625          0.033935
      9       0.092813       0.00076108        0.0091251          0.017748          0.047979
```

10	0.12459	0.02539	0.027791	0.036336	0.066347
11	0.16214	0.049062	0.054743	0.057497	0.087289
12	0.20576	0.080353	0.076351	0.084213	0.11115
13	0.25576	0.11036	0.10076	0.11357	0.13677
14	0.31242	0.14798	0.12866	0.14076	0.16483
15	0.37601	0.17839	0.16439	0.16895	0.194
16	0.4468	0.2098	0.20032	0.1988	0.22401
17	0.52503	0.24246	0.23721	0.23371	0.25482
18	0.61095	0.28123	0.27422	0.26803	0.28577
19	0.7048	0.31861	0.30964	0.30224	0.31644
20	0.8068	0.35352	0.34406	0.33561	0.34646
21	0.91719	0.38727	0.37774	0.36766	0.37639
22	1.0362	0.42001	0.40688	0.39888	0.40495
23	1.164	0.4501	0.43289	0.42881	0.43266
24	1.3008	0.47851	0.45746	0.45719	0.45922
25	1.4468	0.50572	0.48514	0.48371	0.48451
26	1.6023	0.53093	0.51118	0.50952	0.50892
27	1.7673	0.55214	0.53571	0.53333	0.53173
28	1.9422	0.57052	0.55854	0.55614	0.55374
29	2.127	0.58782	0.58031	0.57735	0.57415
30	2.3221	0.60768	0.60016	0.59758	0.59375
31	2.5275	0.62577	0.61947	0.61496	0.61226
32	2.7434	0.64351	0.63697	0.63101	0.62956
33	2.97	0.65976	0.65338	0.64537	0.64591
34	3.2075	0.67458	0.66898	0.66058	0.66124
35	3.456	0.68919	0.68379	0.67538	0.67538
36	3.7158	0.7022	0.69739	0.68939	0.68928
37	3.9869	0.7146	0.7098	0.7022	0.70205
38	4.2696	0.72668	0.7218	0.7146	0.7138
39	4.564	0.73741	0.73341	0.7262	0.7254
40	4.8702	0.74798	0.74381	0.73711	0.73581
41	5.1884	0.75768	0.75382	0.74727	0.74581
42	5.5188	0.76679	0.7618	0.75684	0.75542
43	5.8615	0.77502	0.76862	0.76542	0.76422
44	6.2166	0.78303	0.77658	0.77422	0.77262
45	6.5844	0.79063	0.78452	0.78223	0.78063
46	6.9649	0.79783	0.79196	0.78983	0.78823
47	7.3583	0.80499	0.79863	0.79695	0.79543
48	7.7647	0.81024	0.80566	0.80343	0.80231
49	8.1844	0.81504	0.81184	0.81003	0.80862
50	8.6173	0.81984	0.81744	0.81584	0.81424
51	9.0637	0.82544	0.82351	0.82144	0.82031
52	9.5237	0.83065	0.82881	0.82664	0.82544
53	9.9975	0.83545	0.83385	0.83217	0.83065
54	10.485	0.84025	0.83863	0.83697	0.83545
55	10.987	0.84494	0.84315	0.84155	0.84023
56	11.502	0.84919	0.84705	0.84585	0.84425
57	12.032	0.85319	0.85156	0.85002	0.84785
58	12.577	0.85666	0.85506	0.85396	0.85174
59	13.136	0.86064	0.85906	0.85746	0.85506
60	13.709	0.86386	0.86226	0.86122	0.85826
61	14.298	0.86706	0.86596	0.86461	0.86138
62	14.901	0.87052	0.86906	0.86746	0.86464
63	15.519	0.87306	0.87215	0.87066	0.86746
64	16.152	0.87626	0.87466	0.87378	0.87066
65	16.801	0.87866	0.87779	0.87626	0.8736
66	17.465	0.88163	0.88026	0.87923	0.87626
67	18.144	0.88409	0.88267	0.88179	0.87866

68	18.839	0.88646	0.88507	0.88422	0.88107
69	19.55	0.88867	0.88747	0.88653	0.88347
70	20.277	0.89087	0.88947	0.88867	0.88587
71	21.02	0.89267	0.89187	0.89087	0.88787
72	21.778	0.89493	0.89347	0.89267	0.89027
73	22.553	0.89667	0.89582	0.89487	0.89187
74	23.345	0.89827	0.89747	0.89667	0.89422
75	24.152	0.90034	0.89907	0.89827	0.89587
76	24.977	0.90204	0.90111	0.89987	0.89747
77	25.818	0.90361	0.90274	0.90147	0.89907
78	26.675	0.90515	0.90387	0.90307	0.90067
79	27.55	0.90628	0.90547	0.90467	0.90227
80	28.441	0.90788	0.90708	0.90547	0.90387
81	29.35	0.90908	0.9086	0.90708	0.90547
82	30.276	0.91068	0.90988	0.90825	0.90697
83	31.219	0.91195	0.91121	0.90908	0.90828
84	32.179	0.91308	0.91228	0.91035	0.90958
85	33.157	0.91388	0.91361	0.91148	0.91068
86	34.153	0.91543	0.91468	0.91228	0.91198
87	35.166	0.91628	0.91548	0.9138	0.91308
88	36.198	0.91708	0.91688	0.91468	0.91388
89	37.247	0.91851	0.91786	0.91548	0.91527
90	38.314	0.91946	0.91868	0.91691	0.91628
91	39.399	0.92028	0.91948	0.91788	0.91708
92	40.503	0.92108	0.92028	0.91868	0.91788
93	41.625	0.92188	0.92108	0.91948	0.91868
94	42.765	0.92268	0.92188	0.92028	0.92001
95	43.924	0.92348	0.92268	0.92108	0.92085
96	45.102	0.92428	0.92348	0.92188	0.92168
97	46.298	0.92508	0.92414	0.92268	0.92248
98	47.513	0.92588	0.92469	0.92348	0.92325
99	48.747	0.92668	0.92508	0.92428	0.92398
100	50	0.92737	0.9258	0.92508	0.92428





1.5.4 Test FF_VFI_AZ_MZOOM_LOOP Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 294.329574 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum        mean        std       coefv
                    -     ---     ----     -----     ----     ----     ------     ------     -------     -----

    savefraccoh     1      1       2       6750      750       9       3468.2     0.5138     0.27192     0.529

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7	c8
r1	0	0	0	0	0	0	0	0.02073
r2	0	0	0	0	0	0	0	0.02073
r3	0	0	0	0	0	0	0	0.02073
r4	0	0	0	0	0	0	0	0.02073
r5	0	0	0	0	0	0	0	0.02073
r746	0.8008	0.79843	0.7959	0.79303	0.78983	0.78663	0.78303	0.77903
r747	0.80092	0.79855	0.79603	0.79303	0.79058	0.78713	0.78362	0.77953
r748	0.80102	0.79863	0.79615	0.7935	0.79063	0.78729	0.78378	0.77972
r749	0.80103	0.79863	0.79623	0.79369	0.79063	0.78743	0.78383	0.77983
r750	0.80103	0.79904	0.79623	0.79378	0.79063	0.78743	0.78383	0.77983

```
% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 1309.412430 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefv
savefraccoh	1	1	2	6750	750	9	4667.7	0.6915	0.26685	0.38

```
xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7	c8
r1	0	0	0	0	0.0647	0.16668	0.27352	0.37327
r2	0	0	0	0	0.0647	0.16668	0.27352	0.37327
r3	0	0	0	0	0.064731	0.16668	0.27352	0.37327
r4	0	0	0	0	0.064731	0.16668	0.27355	0.37327
r5	0	0	0	0	0.064747	0.16671	0.27355	0.37327
r746	0.92657	0.92588	0.92508	0.92428	0.92348	0.92268	0.92235	0.92188
r747	0.92664	0.92588	0.92508	0.92428	0.92402	0.92318	0.92248	0.92188
r748	0.92668	0.92588	0.92508	0.92478	0.92411	0.92328	0.9226	0.92188
r749	0.92668	0.92588	0.92555	0.92488	0.9242	0.9234	0.92268	0.92254
r750	0.92668	0.92588	0.92565	0.92497	0.92427	0.92348	0.92268	0.92268

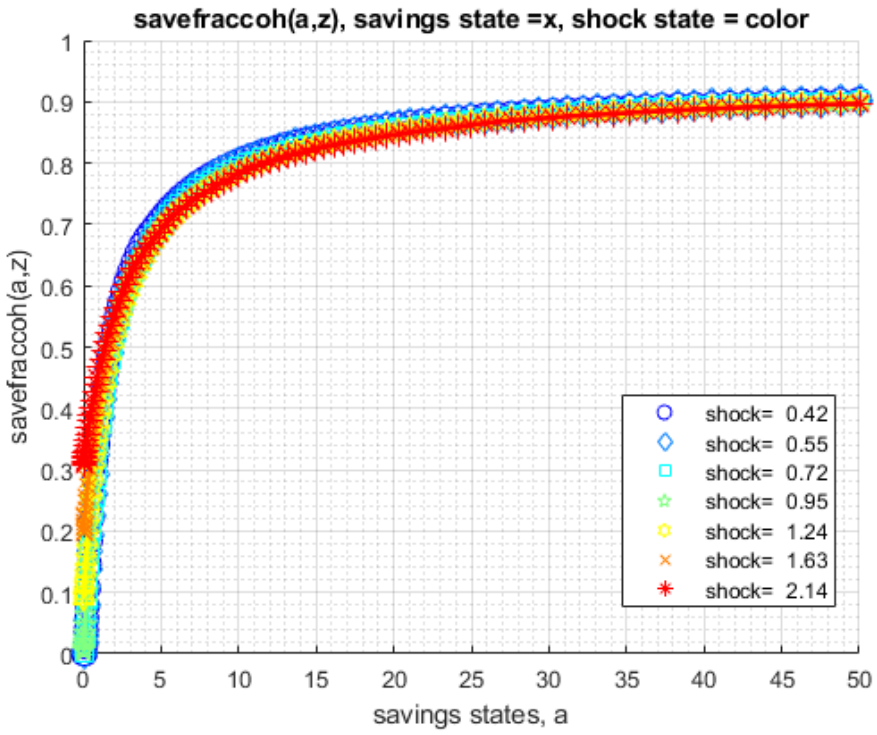
1.5.5 Test FF_VFI_AZ_MZOOM_LOOP Changing Risk Aversion

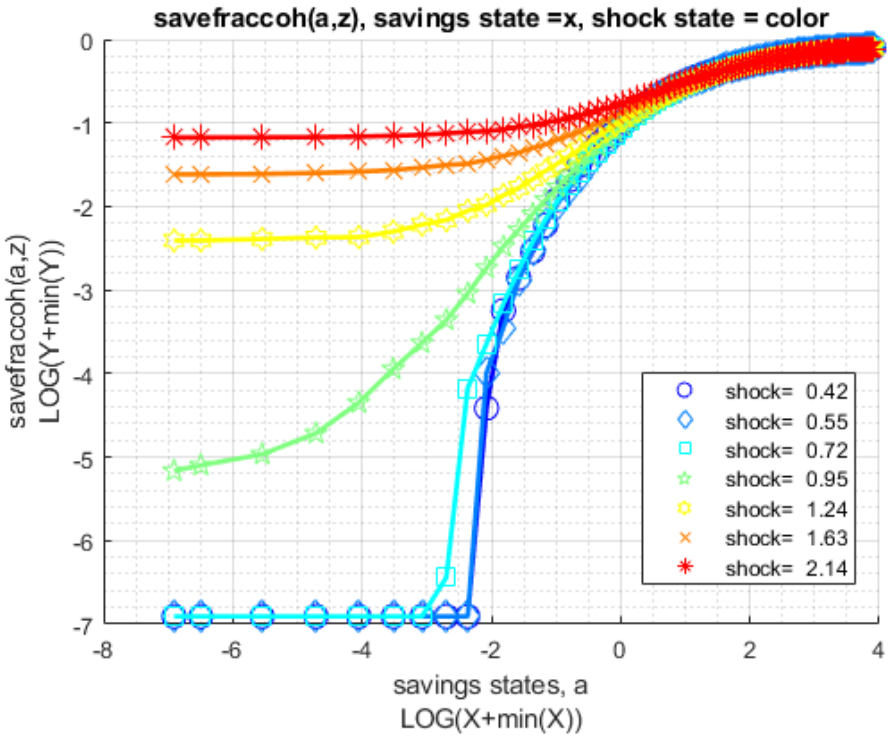
Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

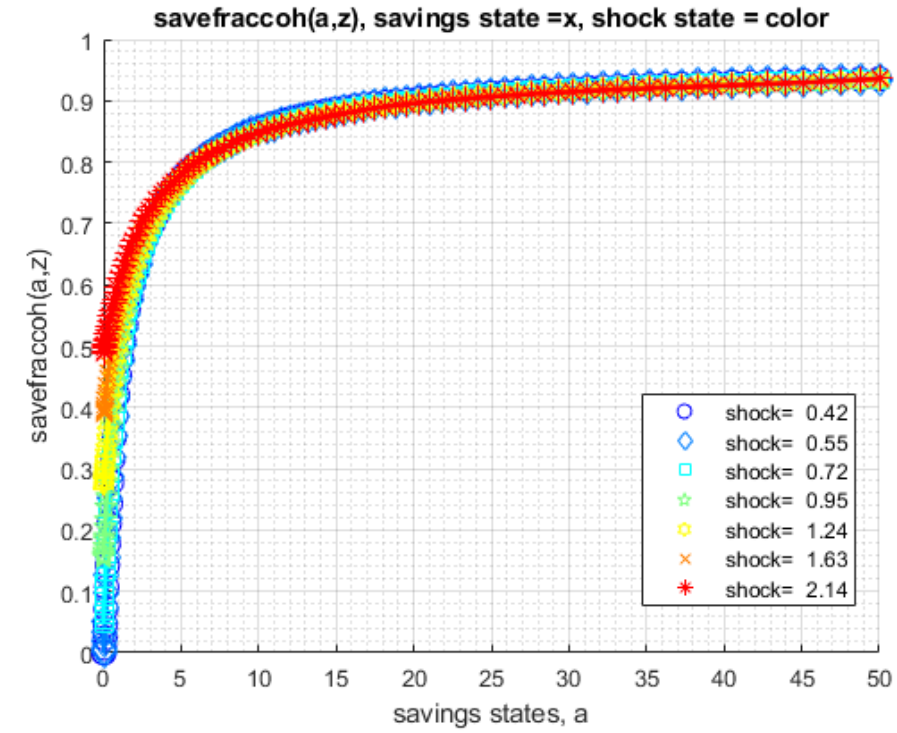
```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 84.461743 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum        mean         std        coef
                    -     ---     ----     -----     ----     ----     ------     -------     -------     ----

    savefraccoh     1      1       2        700      100       7       452.03     0.64575     0.28029     0.43

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4           c5          c6          c7
            -------     -------     -------     ---------     --------     -------     -------

    r1            0           0           0     0.0047077     0.089109       0.198     0.30781
    r2            0           0           0     0.0051079     0.089156       0.198     0.30793
    r3            0           0           0     0.0059631     0.090679      0.1988     0.30848
    r4            0           0           0     0.0079639     0.092358     0.20109     0.30964
    r5            0           0           0      0.011926     0.092758     0.20413     0.31171
    r96     0.90047     0.89907     0.89826       0.89727      0.89587     0.89347     0.89267
    r97     0.90127     0.89987     0.89907       0.89822      0.89727     0.89477     0.89394
    r98     0.90204     0.90067     0.89987       0.89907      0.89822     0.89573     0.89493
    r99     0.90278     0.90147     0.90067       0.89987      0.89907     0.89667     0.89587
    r100    0.90354     0.90227     0.90147       0.90067      0.89987     0.89801     0.89667
```

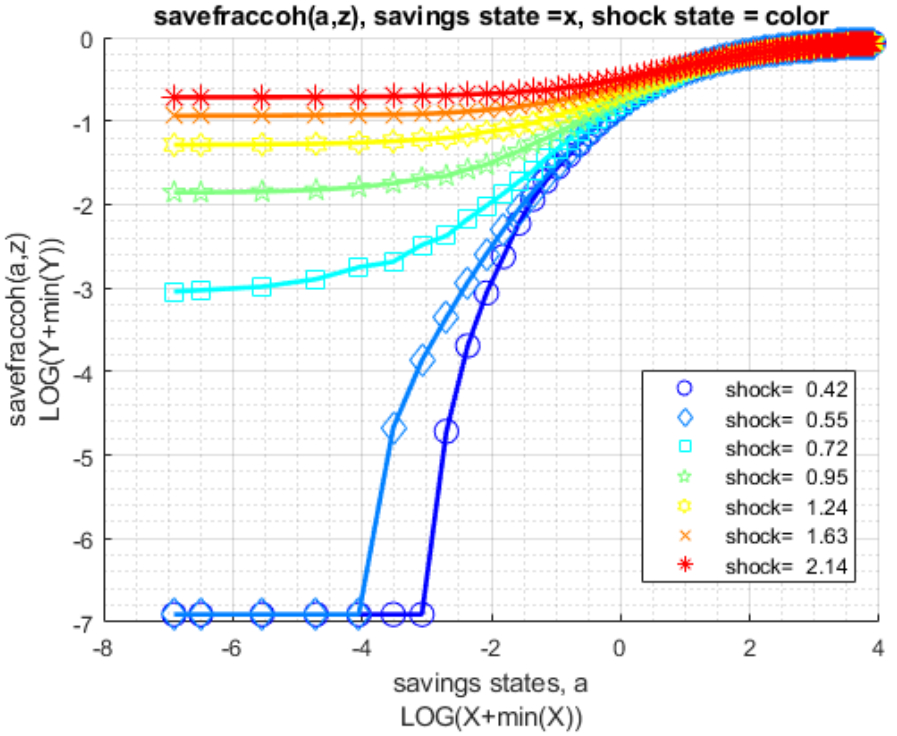




When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 88.697274 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum     mean       std       coefvar
                    -    ---    ----    -----    ----    ----    -----    -----    -------    -------

    savefraccoh     1      1       2      700     100       7     502.6    0.718    0.25437    0.35427

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1         c2          c3         c4         c5         c6         c7
            -------    -------    --------    -------    -------    -------    -------

    r1            0          0     0.04674    0.15532    0.27563    0.39047    0.48771
    r2            0          0    0.047493    0.15525    0.27563    0.39101    0.48771
    r3            0          0    0.049541    0.15685    0.27693    0.39127    0.48834
    r4            0          0    0.054343    0.16018    0.27883    0.39287    0.48923
    r5            0          0    0.062848    0.16566    0.28272    0.39528    0.49071
    r96     0.93269    0.93251     0.93189    0.93108    0.93014    0.92988    0.92968
    r97     0.93349    0.93322     0.93269    0.93189    0.93107    0.93104    0.93108
    r98     0.93429    0.93349     0.93347    0.93269    0.93189    0.93189    0.93269
    r99     0.93507    0.93429     0.93424    0.93349    0.93331    0.93349    0.93429
    r100    0.93575    0.93509     0.93507    0.93488    0.93491    0.93509    0.93587
```





1.5.6 Test FF_VFI_AZ_MZOOM_LOOP with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 150;
mp_params('it_z_n') = 15;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 304.022067 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1507.2    0.66985    0.28667    0.42

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c11        c12        c13
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0    0.13838    0.18479    0.23021
    r2            0          0          0          0          0    0.13838    0.18479    0.23027
    r3            0          0          0          0          0    0.13894    0.18526    0.23041
    r4            0          0          0          0          0    0.13987    0.18606    0.23121
    r5            0          0          0          0          0    0.13998    0.18719    0.23201
    r146    0.92348    0.92348    0.92328    0.92268    0.92268    0.92085    0.92028    0.92028
    r147     0.9242    0.92398    0.92348    0.92348    0.92337    0.92108    0.92108    0.92097
    r148    0.92428    0.92428    0.92428    0.92408    0.92348    0.92188    0.92171    0.92108
    r149    0.92508    0.92497    0.92478    0.92428    0.92428    0.92241    0.92188    0.92188
    r150    0.92565    0.92508    0.92508    0.92507    0.92485    0.92268    0.92268    0.92254
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_mzoom_loop(mp_params, mp_support);

Elapsed time is 304.175092 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1685.2    0.74898    0.22908    0.30

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c11        c12        c13
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0    0.52613    0.61256    0.68259
    r2            0          0          0          0          0    0.52613    0.61256    0.68259
```

r3	0	0	0	0	0	0.52613	0.61256	0.68259
r4	0	0	0	0	0	0.52682	0.61256	0.68259
r5	0	0	0	0	0	0.52693	0.61309	0.68259
r146	0.92948	0.92925	0.92828	0.92805	0.92737	0.92263	0.92348	0.92577
r147	0.93017	0.92948	0.92868	0.92828	0.92748	0.92348	0.92428	0.92668
r148	0.93028	0.93005	0.92948	0.92891	0.92827	0.92428	0.92587	0.92799
r149	0.93091	0.93028	0.92948	0.92931	0.92828	0.92574	0.92668	0.92904
r150	0.93108	0.93082	0.93027	0.92948	0.92868	0.92668	0.92814	0.93008

1.6 FF_VFI_AZ_MZOOM_VEC Savings Vectorized Exact (VALUE) Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function:**ff_vfi_az_mzoom_vec** from the **MEconTools Package.** This function solves the dynamic programming problem for a (a,z) model. The state-space is on a grid, but choice grids are in terms of percentage of resources to save and solved exactly.

This is a **vectorized** code for **continuous** choices, solved with the **mzoom** algorithm. In contrast to the **bisection** based solution, this is slower, but this does not rely on first order conditions.

Links to Other Code:

Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the **MEconTools Package.** :

- Common Choice and States Grid ***Loop***: **ff_vfi_az_loop**
- Common Choice and States Grid ***Vectorized***: **ff_vfi_az_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC, ***Loop***:**ff_vfi_az_bisec_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC ***Vectorized***: **ff_vfi_az_bisec_vec**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Loop***:**ff_vfi_az_mzoom_loop**
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, ***Vectorized***: **ff_vfi_az_mzoom_vec**

1.6.1 Test FF_VFI_AZ_MZOOM_VEC Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_mzoom_vec(mp_params);

Elapsed time is 6.126702 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum       mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    ------    ------    ------    --------    ---

    ap    1     1      2       700     100      7      9861.5    14.088    14.386     1.0212      0
```

```
xxx TABLE:ap xxxxxxxxxxxxxxxxx
              c1        c2        c3         c4         c5         c6         c7
            ------    ------    ------    --------    -------    -------    ------

    r1           0         0         0     0.05343    0.25568    0.60598    1.1155
    r2           0         0         0    0.053451    0.25571    0.60652    1.1161
    r3           0         0         0    0.056468    0.25574    0.60897    1.1174
    r4           0         0         0    0.061232    0.25995    0.61042    1.1238
    r5           0         0         0    0.065929     0.2689    0.61091    1.1323
    r96     43.387    43.517      43.7      43.922     44.221     44.657    45.225
    r97     44.562    44.694    44.876      45.095     45.392     45.847    46.394
    r98     45.758     45.89    46.071      46.287     46.583     47.037    47.596
    r99     46.972    47.103    47.285        47.5     47.794     48.247    48.812
    r100    48.183    48.337    48.518      48.732     49.025     49.478    50.115
```

1.6.2 Test FF_VFI_AZ_MZOOM_VEC Speed Tests

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
% A grid 50, shock grid 5:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 1.996365 seconds.

% A grid 750, shock grid 15:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 15;
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 337.171768 seconds.

% A grid 600, shock grid 45:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 600;
mp_params('it_z_n') = 45;
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 1758.273287 seconds.
```

1.6.3 Test FF_VFI_AZ_MZOOM_VEC Control Outputs

Run the function first without any outputs, but only the timer.

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
```

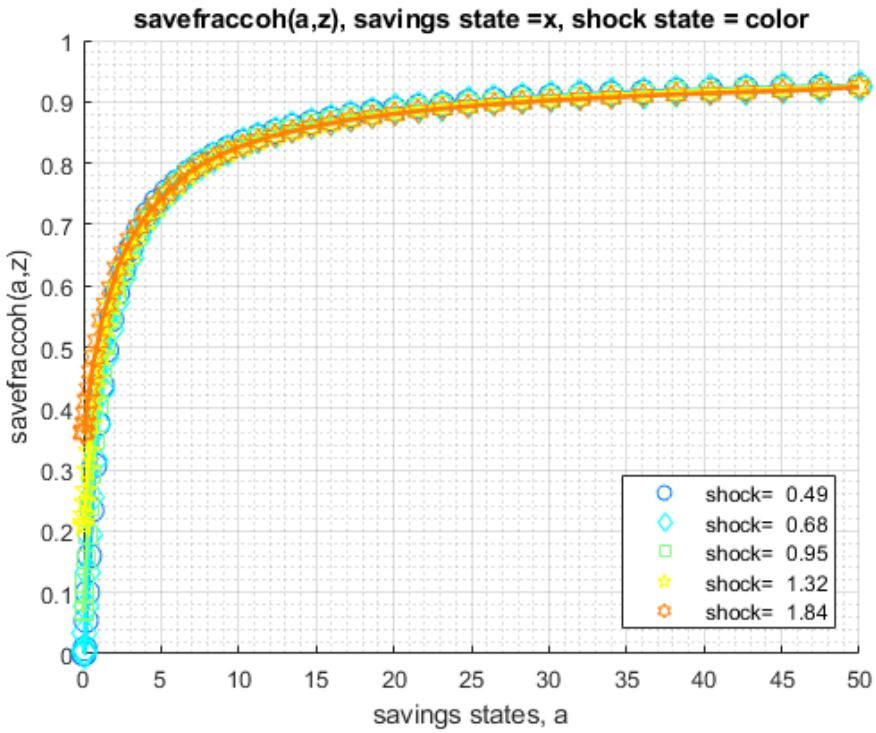
```
mp_support('ls_ffcmd') = {};
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 1.091918 seconds.
```

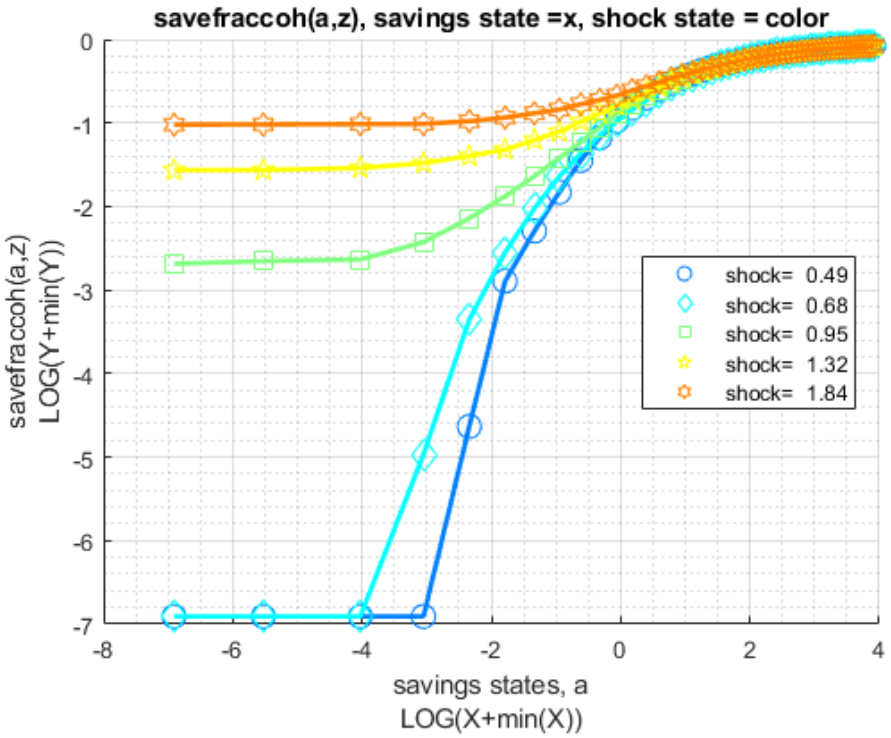
Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 1.090424 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

group	a	mean_z_0_4858	mean_z_0_67798	mean_z_0_9462	mean_z_1_3205	mean_z
1	0	0	0	0.067148	0.2084	0.3
2	0.002975	0	0	0.069345	0.20826	0.3
3	0.016829	0	0	0.070749	0.2136	0.3
4	0.046375	0	0.0059631	0.08732	0.22641	0.3
5	0.095198	0.008725	0.033935	0.11637	0.24674	0.
6	0.1663	0.054327	0.077152	0.15198	0.26635	0.3
7	0.26234	0.099882	0.13131	0.1936	0.29922	0.4
8	0.38568	0.15954	0.1928	0.24107	0.33005	0.4
9	0.53852	0.23411	0.25482	0.29164	0.37407	0.
10	0.72291	0.30704	0.31604	0.34806	0.41148	0.4
11	0.94076	0.37567	0.37487	0.40768	0.44925	0.5
12	1.1939	0.43849	0.42939	0.4573	0.48691	0.5
13	1.484	0.49491	0.48129	0.50332	0.53253	0.5
14	1.8128	0.54486	0.53013	0.54642	0.56773	0.5
15	2.1817	0.58868	0.57335	0.58545	0.60016	0.6
16	2.5924	0.6271	0.61254	0.62056	0.63057	0.6
17	3.0463	0.66058	0.6468	0.65237	0.65884	0.6
18	3.5449	0.69019	0.67699	0.68069	0.68379	0.6
19	4.0894	0.71615	0.70375	0.7058	0.70719	0.
20	4.6813	0.73661	0.72701	0.72843	0.72781	0.7
21	5.3218	0.75302	0.7481	0.74821	0.74661	0.7
22	6.0121	0.76912	0.76622	0.76622	0.76342	0.7
23	6.7536	0.78503	0.78285	0.78223	0.77885	0.7
24	7.5474	0.79943	0.79703	0.79623	0.79223	0.7
25	8.3948	0.81264	0.81024	0.8093	0.80504	0.8
26	9.2967	0.82384	0.82198	0.82064	0.81634	0.8
27	10.254	0.83447	0.83225	0.83065	0.82653	0.8
28	11.269	0.84345	0.84174	0.84025	0.83545	0.8
29	12.342	0.85185	0.85017	0.84865	0.84417	0.8
30	13.473	0.85962	0.85746	0.85642	0.85178	0.8
31	14.665	0.86626	0.86466	0.86306	0.85873	0.8
32	15.918	0.87226	0.87066	0.86959	0.86504	0.8
33	17.233	0.87786	0.87626	0.87529	0.87146	0.8
34	18.611	0.88332	0.88182	0.88026	0.87766	0.8

35	20.053	0.888	0.88656	0.88507	0.88267	0.8
36	21.56	0.89187	0.89087	0.88947	0.88825	0.8
37	23.133	0.89587	0.89484	0.89347	0.89256	0.8
38	24.773	0.8997	0.89827	0.89727	0.89587	0.8
39	26.481	0.903	0.90147	0.90066	0.89964	0.8
40	28.258	0.90601	0.90467	0.90376	0.90278	0.8
41	30.104	0.90881	0.9077	0.90628	0.90547	0.9
42	32.021	0.91137	0.91035	0.90908	0.90838	0.9
43	34.01	0.91377	0.91275	0.91148	0.91068	0.9
44	36.07	0.91595	0.91468	0.91388	0.91308	0.9
45	38.204	0.91788	0.91708	0.91617	0.91531	0.9
46	40.412	0.91948	0.91868	0.91788	0.91708	0.9
47	42.695	0.92168	0.92085	0.91998	0.91915	0.9
48	45.053	0.92331	0.92251	0.92171	0.92091	0.9
49	47.488	0.92485	0.92408	0.92331	0.92254	0.
50	50	0.92588	0.92555	0.92485	0.92423	0.9



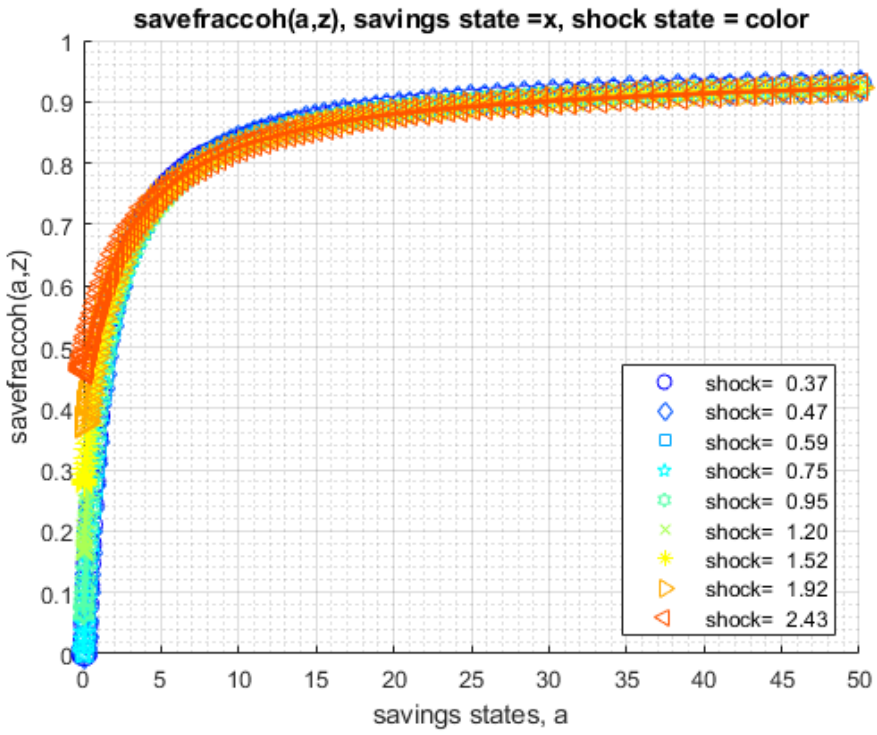


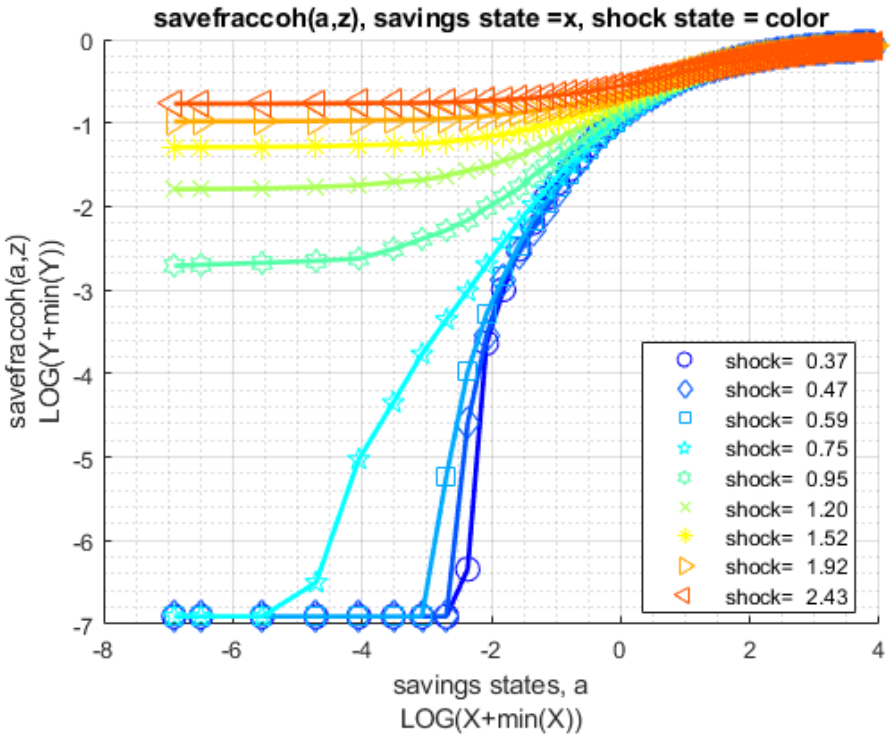
Run the function and show summaries for savings and fraction of coh saved:

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
% mp_params('fl_crra') = 1.5;
% mp_params('fl_beta') = 0.94;
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
% call function
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 5.173849 seconds.
xxx  ff_vfi_az_vec, outcome=savefraccoh  xxxxxxxxxxxxxxxxxxxxxxxxxxxx
    group        a         mean_z_0_36853    mean_z_0_46648    mean_z_0_59047    mean_z_0_74742    m
    -----    ----------    --------------    --------------    --------------    --------------    -

      1              0                 0                 0                 0                 0
      2     0.00051272                 0                 0                 0                 0
      3      0.0029004                 0                 0                 0                 0
      4      0.0079925                 0                 0                 0        0.00050216
      5       0.016407                 0                 0                 0          0.005563
      6       0.028662                 0                 0                 0          0.011926
      7       0.045213                 0                 0                 0          0.022095
      8        0.06647                 0                 0         0.0043625          0.033935
      9       0.092813       0.00076108         0.0091251          0.017748          0.047979
```

10	0.12459	0.02539	0.027791	0.036336	0.066347
11	0.16214	0.049062	0.054743	0.057497	0.087289
12	0.20576	0.080353	0.076351	0.084213	0.11115
13	0.25576	0.11036	0.10076	0.11357	0.13677
14	0.31242	0.14798	0.12866	0.14076	0.16483
15	0.37601	0.17839	0.16439	0.16895	0.194
16	0.4468	0.2098	0.20032	0.1988	0.22401
17	0.52503	0.24246	0.23721	0.23371	0.25482
18	0.61095	0.28123	0.27422	0.26803	0.28577
19	0.7048	0.31861	0.30964	0.30224	0.31644
20	0.8068	0.35352	0.34406	0.33561	0.34646
21	0.91719	0.38727	0.37774	0.36766	0.37639
22	1.0362	0.42001	0.40688	0.39888	0.40495
23	1.164	0.4501	0.43289	0.42881	0.43266
24	1.3008	0.47851	0.45746	0.45719	0.45922
25	1.4468	0.50572	0.48514	0.48371	0.48451
26	1.6023	0.53093	0.51118	0.50952	0.50892
27	1.7673	0.55214	0.53571	0.53333	0.53173
28	1.9422	0.57052	0.55854	0.55614	0.55374
29	2.127	0.58782	0.58031	0.57735	0.57415
30	2.3221	0.60768	0.60016	0.59758	0.59375
31	2.5275	0.62577	0.61947	0.61496	0.61226
32	2.7434	0.64351	0.63697	0.63101	0.62956
33	2.97	0.65976	0.65338	0.64537	0.64591
34	3.2075	0.67458	0.66898	0.66058	0.66124
35	3.456	0.68919	0.68379	0.67538	0.67538
36	3.7158	0.7022	0.69739	0.68939	0.68928
37	3.9869	0.7146	0.7098	0.7022	0.70205
38	4.2696	0.72668	0.7218	0.7146	0.7138
39	4.564	0.73741	0.73341	0.7262	0.7254
40	4.8702	0.74798	0.74381	0.73711	0.73581
41	5.1884	0.75768	0.75382	0.74727	0.74581
42	5.5188	0.76679	0.7618	0.75684	0.75542
43	5.8615	0.77502	0.76862	0.76542	0.76422
44	6.2166	0.78303	0.77658	0.77422	0.77262
45	6.5844	0.79063	0.78452	0.78223	0.78063
46	6.9649	0.79783	0.79196	0.78983	0.78823
47	7.3583	0.80499	0.79863	0.79695	0.79543
48	7.7647	0.81024	0.80566	0.80343	0.80231
49	8.1844	0.81504	0.81184	0.81003	0.80862
50	8.6173	0.81984	0.81744	0.81584	0.81424
51	9.0637	0.82544	0.82351	0.82144	0.82031
52	9.5237	0.83065	0.82881	0.82664	0.82544
53	9.9975	0.83545	0.83385	0.83217	0.83065
54	10.485	0.84025	0.83863	0.83697	0.83545
55	10.987	0.84494	0.84315	0.84155	0.84023
56	11.502	0.84919	0.84705	0.84585	0.84425
57	12.032	0.85319	0.85156	0.85002	0.84785
58	12.577	0.85666	0.85506	0.85396	0.85174
59	13.136	0.86064	0.85906	0.85746	0.85506
60	13.709	0.86386	0.86226	0.86122	0.85826
61	14.298	0.86706	0.86596	0.86461	0.86138
62	14.901	0.87052	0.86906	0.86746	0.86464
63	15.519	0.87306	0.87215	0.87066	0.86746
64	16.152	0.87626	0.87466	0.87378	0.87066
65	16.801	0.87866	0.87779	0.87626	0.8736
66	17.465	0.88163	0.88026	0.87923	0.87626
67	18.144	0.88409	0.88267	0.88179	0.87866

68	18.839	0.88646	0.88507	0.88422	0.88107
69	19.55	0.88867	0.88747	0.88653	0.88347
70	20.277	0.89087	0.88947	0.88867	0.88587
71	21.02	0.89267	0.89187	0.89087	0.88787
72	21.778	0.89493	0.89347	0.89267	0.89027
73	22.553	0.89667	0.89582	0.89487	0.89187
74	23.345	0.89827	0.89747	0.89667	0.89422
75	24.152	0.90034	0.89907	0.89827	0.89587
76	24.977	0.90204	0.90111	0.89987	0.89747
77	25.818	0.90361	0.90274	0.90147	0.89907
78	26.675	0.90515	0.90387	0.90307	0.90067
79	27.55	0.90628	0.90547	0.90467	0.90227
80	28.441	0.90788	0.90708	0.90547	0.90387
81	29.35	0.90908	0.9086	0.90708	0.90547
82	30.276	0.91068	0.90988	0.90825	0.90697
83	31.219	0.91195	0.91121	0.90908	0.90828
84	32.179	0.91308	0.91228	0.91035	0.90958
85	33.157	0.91388	0.91361	0.91148	0.91068
86	34.153	0.91543	0.91468	0.91228	0.91198
87	35.166	0.91628	0.91548	0.9138	0.91308
88	36.198	0.91708	0.91688	0.91468	0.91388
89	37.247	0.91851	0.91786	0.91548	0.91527
90	38.314	0.91946	0.91868	0.91691	0.91628
91	39.399	0.92028	0.91948	0.91788	0.91708
92	40.503	0.92108	0.92028	0.91868	0.91788
93	41.625	0.92188	0.92108	0.91948	0.91868
94	42.765	0.92268	0.92188	0.92028	0.92001
95	43.924	0.92348	0.92268	0.92108	0.92085
96	45.102	0.92428	0.92348	0.92188	0.92168
97	46.298	0.92508	0.92414	0.92268	0.92248
98	47.513	0.92588	0.92469	0.92348	0.92325
99	48.747	0.92668	0.92508	0.92428	0.92398
100	50	0.92737	0.9258	0.92508	0.92428





1.6.4 Test FF_VFI_AZ_MZOOM_VEC Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 37.005214 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum        mean        std       coefv
                    -     ---     ----     -----     ----     ----     ------     ------     -------     -----

    savefraccoh     1      1       2       6750      750       9       3468.2     0.5138     0.27192     0.529

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7	c8
r1	0	0	0	0	0	0	0	0.02073
r2	0	0	0	0	0	0	0	0.02073
r3	0	0	0	0	0	0	0	0.02073
r4	0	0	0	0	0	0	0	0.02073
r5	0	0	0	0	0	0	0	0.02073
r746	0.8008	0.79843	0.7959	0.79303	0.78983	0.78663	0.78303	0.77903
r747	0.80092	0.79855	0.79603	0.79303	0.79058	0.78713	0.78362	0.77953
r748	0.80102	0.79863	0.79615	0.7935	0.79063	0.78729	0.78378	0.77972
r749	0.80103	0.79863	0.79623	0.79369	0.79063	0.78743	0.78383	0.77983
r750	0.80103	0.79904	0.79623	0.79378	0.79063	0.78743	0.78383	0.77983

```
% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 159.606266 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefv
savefraccoh	1	1	2	6750	750	9	4667.7	0.6915	0.26685	0.38

```
xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7	c8
r1	0	0	0	0	0.0647	0.16668	0.27352	0.37327
r2	0	0	0	0	0.0647	0.16668	0.27352	0.37327
r3	0	0	0	0	0.064731	0.16668	0.27352	0.37327
r4	0	0	0	0	0.064731	0.16668	0.27355	0.37327
r5	0	0	0	0	0.064747	0.16671	0.27355	0.37327
r746	0.92657	0.92588	0.92508	0.92428	0.92348	0.92268	0.92235	0.92188
r747	0.92664	0.92588	0.92508	0.92428	0.92402	0.92318	0.92248	0.92188
r748	0.92668	0.92588	0.92508	0.92478	0.92411	0.92328	0.9226	0.92188
r749	0.92668	0.92588	0.92555	0.92488	0.9242	0.9234	0.92268	0.92254
r750	0.92668	0.92588	0.92565	0.92497	0.92427	0.92348	0.92268	0.92268

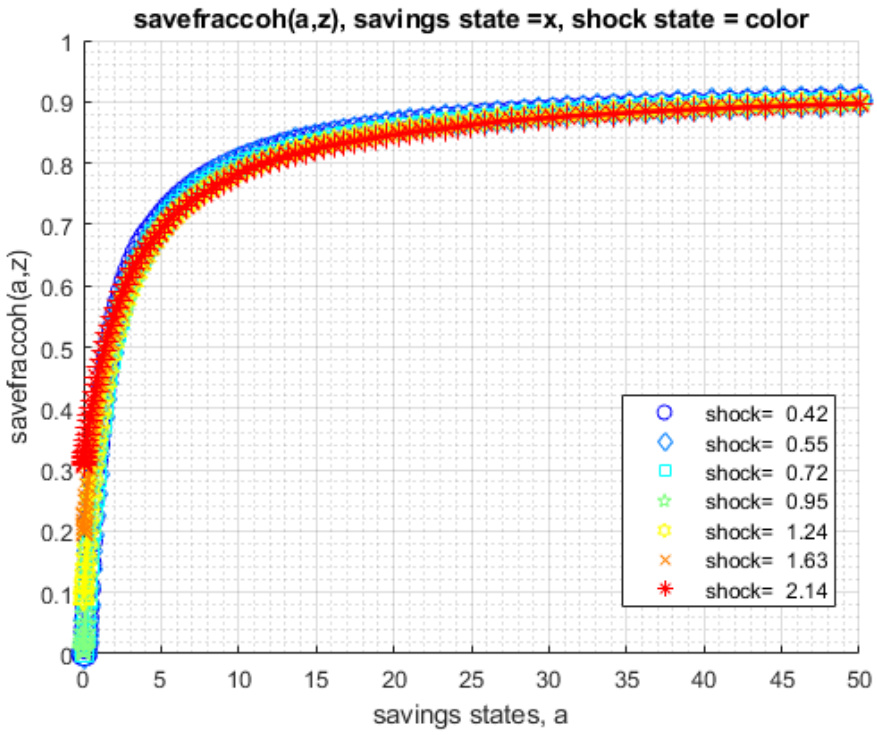
1.6.5 Test FF_VFI_AZ_MZOOM_VEC Changing Risk Aversion

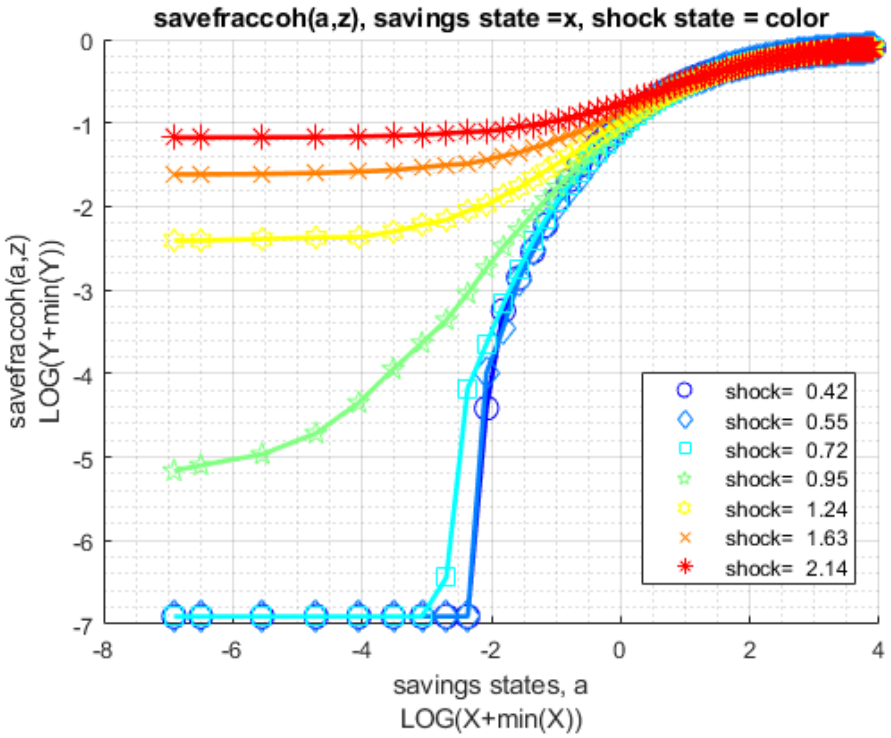
Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

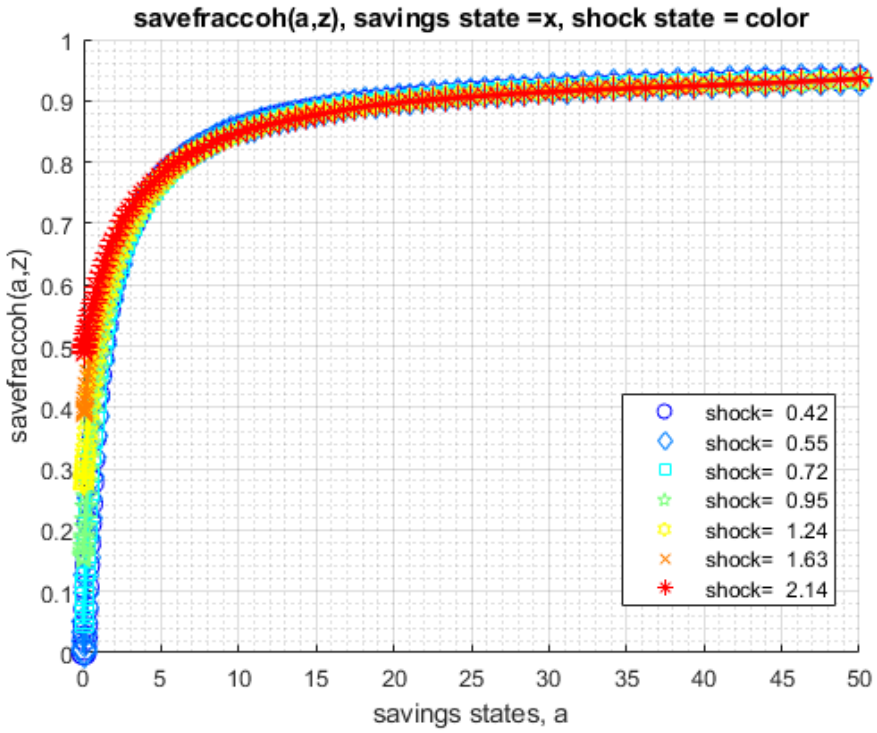
```
% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 3.409484 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx     ndim     numel     rowN     colN      sum        mean         std        coef
                    -     ---     ----     -----     ----     ----     ------     -------     -------     ----

    savefraccoh     1      1       2        700      100       7       452.03     0.64575     0.28029     0.43

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1          c2          c3          c4           c5          c6          c7
            -------     -------     -------     ---------     --------     -------     -------

    r1            0           0           0     0.0047077     0.089109       0.198     0.30781
    r2            0           0           0     0.0051079     0.089156       0.198     0.30793
    r3            0           0           0     0.0059631     0.090679      0.1988     0.30848
    r4            0           0           0     0.0079639     0.092358     0.20109     0.30964
    r5            0           0           0      0.011926     0.092758     0.20413     0.31171
    r96     0.90047     0.89907     0.89826       0.89727      0.89587     0.89347     0.89267
    r97     0.90127     0.89987     0.89907       0.89822      0.89727     0.89477     0.89394
    r98     0.90204     0.90067     0.89987       0.89907      0.89822     0.89573     0.89493
    r99     0.90278     0.90147     0.90067       0.89987      0.89907     0.89667     0.89587
    r100    0.90354     0.90227     0.90147       0.90067      0.89987     0.89801     0.89667
```

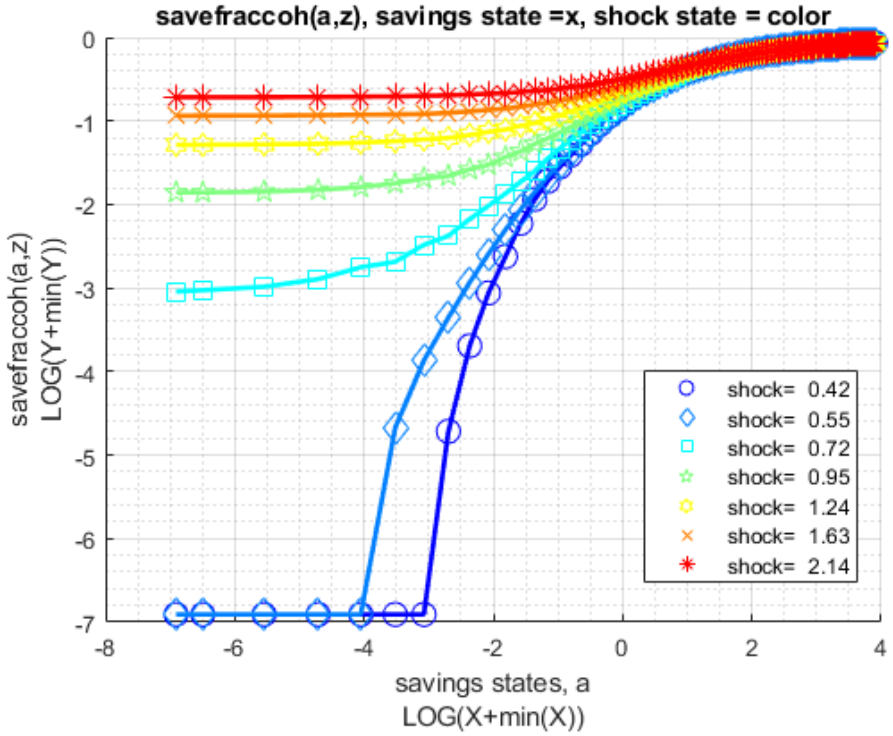




When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 4.012888 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum     mean       std       coefvar
                    -    ---    ----    -----    ----    ----    -----    -----    -------    -------

    savefraccoh     1      1       2      700     100       7     502.6    0.718    0.25437    0.35427

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxx
              c1          c2          c3          c4          c5          c6          c7
            -------     -------     --------    -------     -------     -------     -------

    r1            0           0     0.04674     0.15532     0.27563     0.39047     0.48771
    r2            0           0    0.047493     0.15525     0.27563     0.39101     0.48771
    r3            0           0    0.049541     0.15685     0.27693     0.39127     0.48834
    r4            0           0    0.054343     0.16018     0.27883     0.39287     0.48923
    r5            0           0    0.062848     0.16566     0.28272     0.39528     0.49071
    r96     0.93269     0.93251     0.93189     0.93108     0.93014     0.92988     0.92968
    r97     0.93349     0.93322     0.93269     0.93189     0.93107     0.93104     0.93108
    r98     0.93429     0.93349     0.93347     0.93269     0.93189     0.93189     0.93269
    r99     0.93507     0.93429     0.93424     0.93349     0.93331     0.93349     0.93429
    r100    0.93575     0.93509     0.93507     0.93488     0.93491     0.93509     0.93587
```





1.6.6 Test FF_VFI_AZ_MZOOM_VEC with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 150;
mp_params('it_z_n') = 15;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 16.599473 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1507.2    0.66985    0.28667    0.42

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c11        c12        c13
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0    0.13838    0.18479    0.23021
    r2            0          0          0          0          0    0.13838    0.18479    0.23027
    r3            0          0          0          0          0    0.13894    0.18526    0.23041
    r4            0          0          0          0          0    0.13987    0.18606    0.23121
    r5            0          0          0          0          0    0.13998    0.18719    0.23201
    r146    0.92348    0.92348    0.92328    0.92268    0.92268    0.92085    0.92028    0.92028
    r147     0.9242    0.92398    0.92348    0.92348    0.92337    0.92108    0.92108    0.92097
    r148    0.92428    0.92428    0.92428    0.92408    0.92348    0.92188    0.92171    0.92108
    r149    0.92508    0.92497    0.92478    0.92428    0.92428    0.92241    0.92188    0.92188
    r150    0.92565    0.92508    0.92508    0.92507    0.92485    0.92268    0.92268    0.92254
```

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_mzoom_vec(mp_params, mp_support);

Elapsed time is 16.323916 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coef
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ----

    savefraccoh     1     1      2       2250     150      15     1685.2    0.74898    0.22908    0.30

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c11        c12        c13
            -------    -------    -------    -------    -------    -------    -------    -------

    r1            0          0          0          0          0    0.52613    0.61256    0.68259
    r2            0          0          0          0          0    0.52613    0.61256    0.68259
```

r3	0	0	0	0	0	0.52613	0.61256	0.68259
r4	0	0	0	0	0	0.52682	0.61256	0.68259
r5	0	0	0	0	0	0.52693	0.61309	0.68259
r146	0.92948	0.92925	0.92828	0.92805	0.92737	0.92263	0.92348	0.92577
r147	0.93017	0.92948	0.92868	0.92828	0.92748	0.92348	0.92428	0.92668
r148	0.93028	0.93005	0.92948	0.92891	0.92827	0.92428	0.92587	0.92799
r149	0.93091	0.93028	0.92948	0.92931	0.92828	0.92574	0.92668	0.92904
r150	0.93108	0.93082	0.93027	0.92948	0.92868	0.92668	0.92814	0.93008

Chapter 2

Stationary Distribution

2.1 FF_DS_AZ_LOOP Dynamic Savings Loop Discrete Distribution

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_ds_az_loop** from the **MEconTools Package.** F(a,z) discrete probability mass function given policy function solution with discretized savings choices.

- Distribution for Common Choice and States Grid ***Loop***: ff_ds_az_cts_loop
- Distribution for States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Loop***: **ff_ds_az_cts_loop**
- Distribution for States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Vectorized***: **ff_ds_az_cts_vec**

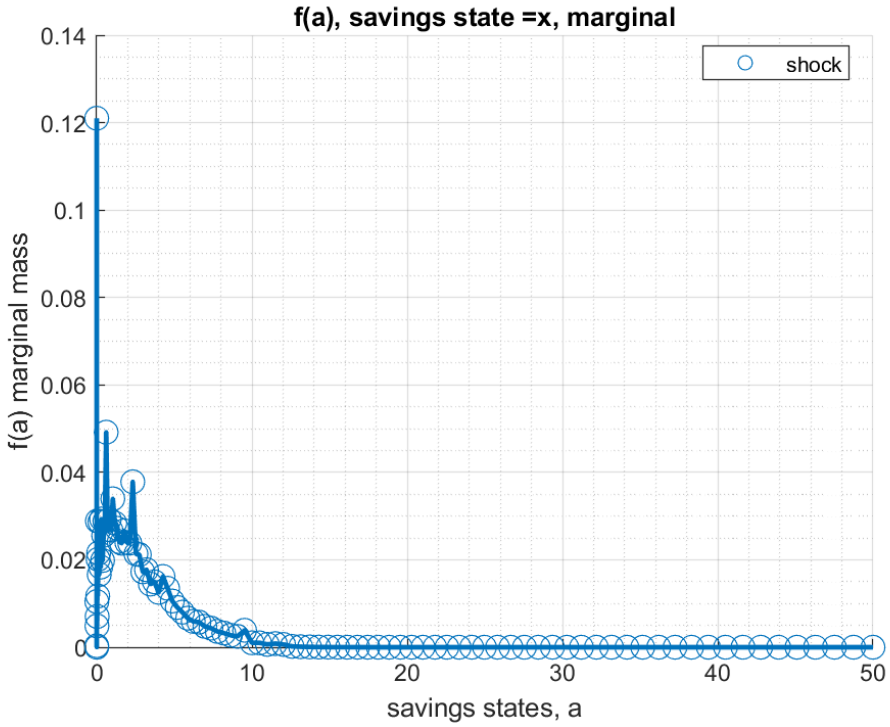
2.1.1 Test FF_DS_AZ_LOOP Defaults

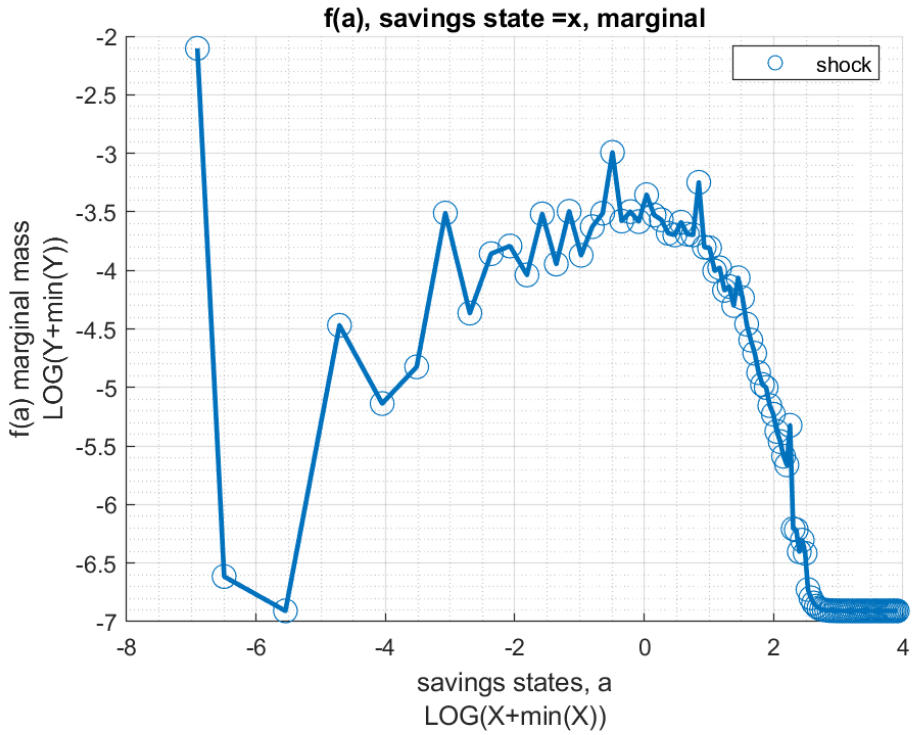
Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_ds_az_loop(mp_params);

Elapsed time is 0.191238 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum       mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    ------    ------    ------    --------    ---

    ap    1     1      2       700     100      7      9855.1    14.079    14.408     1.0234      0

xxx TABLE:ap xxxxxxxxxxxxxxxxx
              c1        c2        c3         c4         c5         c6         c7
            ------    ------    ------    --------    -------    -------    ------

    r1           0         0         0    0.045213    0.25576    0.61095    1.0362
```

```
    r2          0         0         0    0.045213    0.25576    0.61095    1.0362
    r3          0         0         0    0.045213    0.25576    0.61095    1.0362
    r4          0         0         0     0.06647    0.25576    0.61095    1.0362
    r5          0         0         0     0.06647    0.25576    0.61095     1.164
    r96    43.924    43.924    43.924      43.924     43.924     45.102    45.102
    r97    45.102    45.102    45.102      45.102     45.102     46.298    46.298
    r98    46.298    46.298    46.298      46.298     46.298     47.513    47.513
    r99    47.513    47.513    47.513      47.513     47.513     48.747    48.747
    r100   48.747    48.747    48.747      48.747     48.747         50        50

FF_DS_AZ_LOOP finished. Distribution took = 0.14487
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ddcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN    sum      mean          std       coefvari
          -    ---    ----    -----    ----    ----    ---    ---------    ---------    --------

    fa    1      1       2      100     100       1      1         0.01     0.016114     1.6114
    faz   2      2       2      700     100       7      1       0.0014     0.0035847    2.5093
    fz    3      3       2        7       7       1      1      0.14286      0.11742    0.82196

xxx TABLE:fa xxxxxxxxxxxxxxxxx
               c1
          ----------

    r1          0.121
    r2      0.00034068
    r3               0
    r4        0.010458
    r5       0.0048751
    r96     1.1148e-21
    r97      3.227e-22
    r98     7.9165e-23
    r99     1.4982e-23
    r100    1.7037e-24

xxx TABLE:faz xxxxxxxxxxxxxxxxx
               c1            c2            c3            c4            c5            c6
          ----------    ----------    ----------    ----------    ----------    ----------    ----

    r1       0.0084023       0.03778      0.052693      0.018985     0.0029243    0.00020787    5.63
    r2      0.00018105     0.0001207    3.3528e-05    4.9671e-06    4.1392e-07    1.8397e-08    3.40
    r3               0             0             0             0             0             0
    r4      0.00016518      0.002081      0.005593     0.0022334    0.00035834    2.6032e-05     7.1
    r5      0.00021881    0.00067299     0.0026761     0.0011123    0.00018127    1.3278e-05    3.66
    r96     1.7183e-25    2.8942e-24    2.2565e-23    1.0675e-22    3.1764e-22    4.9586e-22    1.68
    r97     3.2228e-26     6.111e-25    5.3384e-24    2.7969e-23    9.0055e-23    1.4769e-22    5.10
    r98     4.5065e-27    1.0023e-25    1.0174e-24    6.0677e-24      2.15e-23    3.7371e-23    1.31
    r99     3.8775e-28    1.0954e-26      1.38e-25    9.8022e-25    3.9213e-24    7.3193e-24    2.61
    r100    1.1692e-29    5.3148e-28    9.7109e-27    8.9563e-26    4.2252e-25    8.6574e-25    3.15

xxx TABLE:fz xxxxxxxxxxxxxxxxx
             c1
          --------

    r1    0.015625
    r2     0.09375
```

```
r3      0.23438
r4       0.3125
r5      0.23438
r6      0.09375
r7     0.015625
```





2.1.2 Test FF_DS_AZ_LOOP Speed Tests

Call the function with different a and z grid size, print out speed:

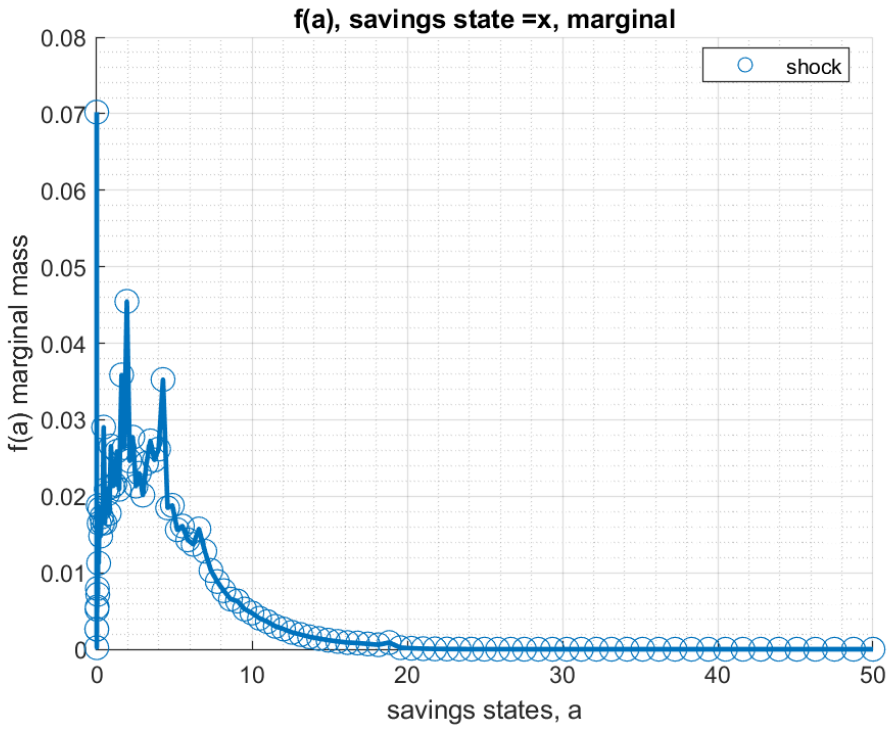
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
```

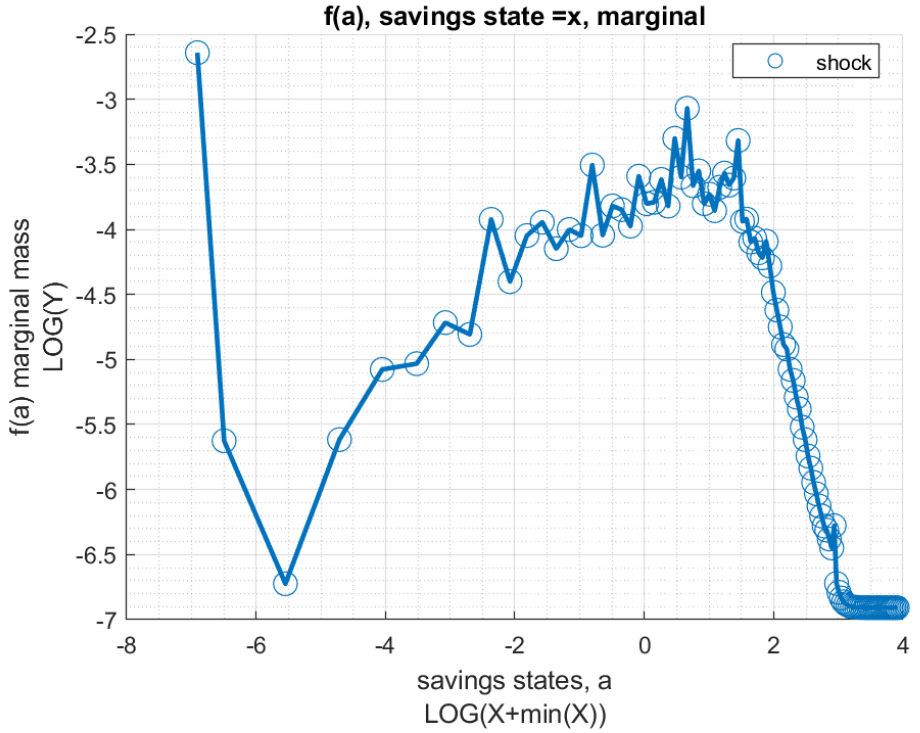
```
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {};
mp_support('bl_show_stats_table') = false;
% A grid 50, shock grid 5:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_ds_az_loop(mp_params, mp_support);

Elapsed time is 0.021787 seconds.
FF_DS_AZ_LOOP finished. Distribution took = 0.046636

% A grid 100, shock grid 7:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
ff_ds_az_loop(mp_params, mp_support);

Elapsed time is 0.218465 seconds.
FF_DS_AZ_LOOP finished. Distribution took = 0.13608

% A grid 200, shock grid 9:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 200;
mp_params('it_z_n') = 9;
ff_ds_az_loop(mp_params, mp_support);

Elapsed time is 0.489370 seconds.
FF_DS_AZ_LOOP finished. Distribution took = 0.35393
```

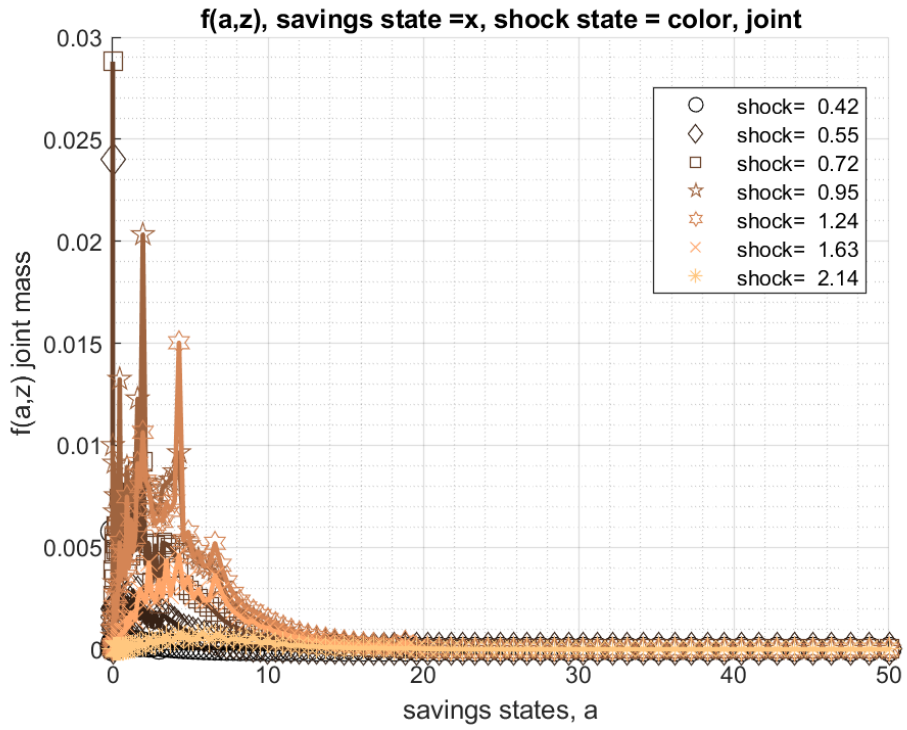
2.1.3 Test FF_DS_AZ_LOOP A grid 100 Shock grid 7

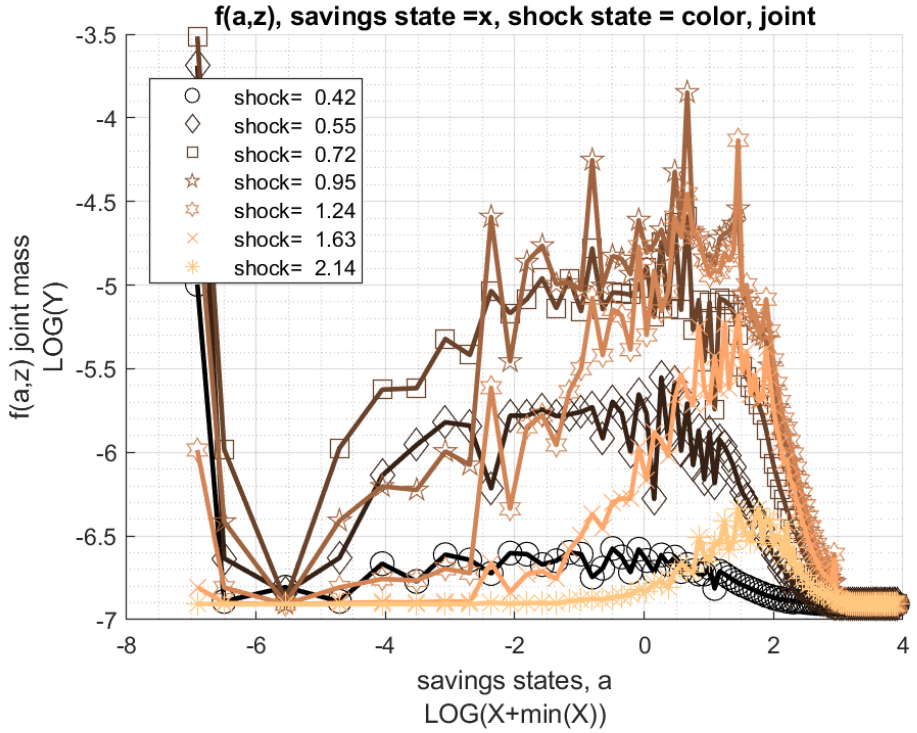
Call the function with different a and z grid size, print out speed:

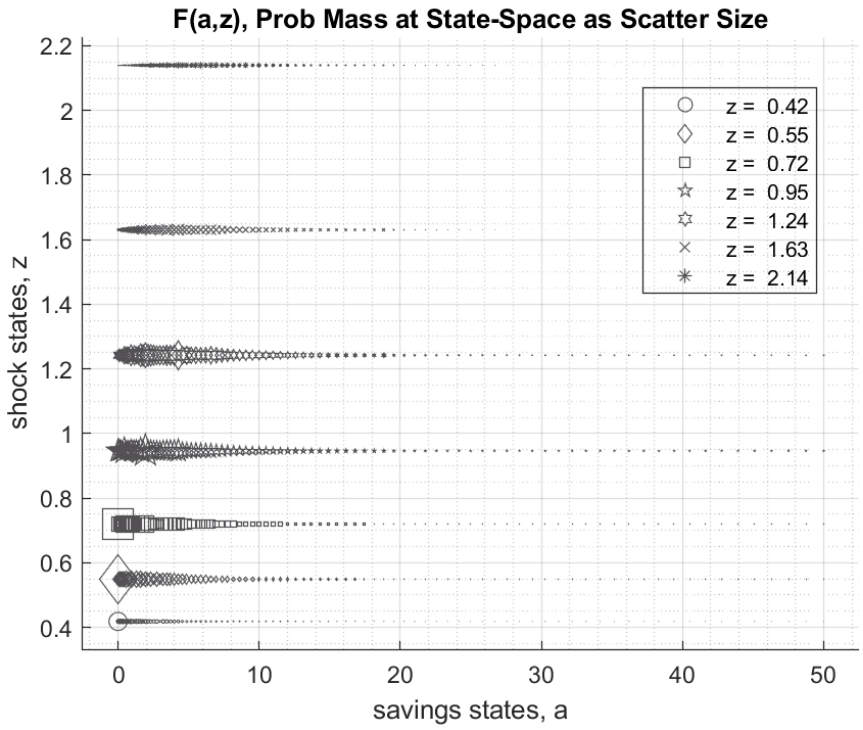
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {'faz','fa'};
mp_support('bl_show_stats_table') = true;
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
ff_ds_az_loop(mp_params, mp_support);

Elapsed time is 0.217312 seconds.
FF_DS_AZ_LOOP finished. Distribution took = 0.1105
```


f(a), savings state =x, marginal
shock
f(a) marginal mass
savings states, a
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
10
20
30
40
50


f(a), savings state =x, marginal
shock
f(a) marginal mass
LOG(Y)
savings states, a
LOG(X+min(X))
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4


f(a,z), savings state =x, shock state = color, joint
f(a,z) joint mass
savings states, a
shock= 0.42
shock= 0.55
shock= 0.72
shock= 0.95
shock= 1.24
shock= 1.63
shock= 2.14
0.03
0.025
0.02
0.015
0.01
0.005
0
0
10
20
30
40
50


f(a,z), savings state =x, shock state = color, joint
f(a,z) joint mass
LOG(Y)
savings states, a
LOG(X+min(X))
shock= 0.42
shock= 0.55
shock= 0.72
shock= 0.95
shock= 1.24
shock= 1.63
shock= 2.14
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4



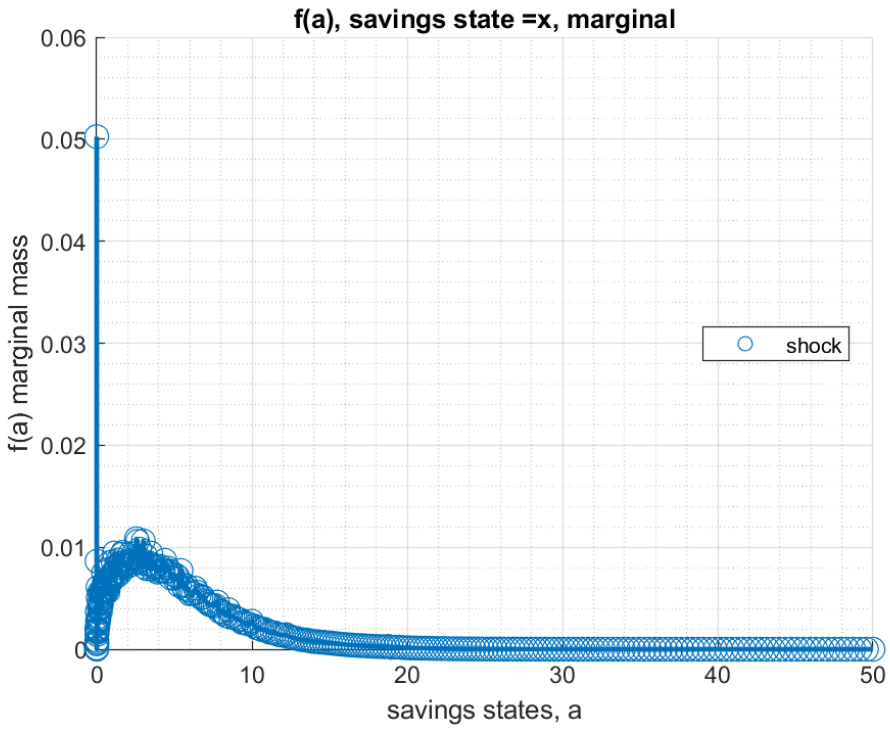
xxx tb_outcomes: all stats xxx

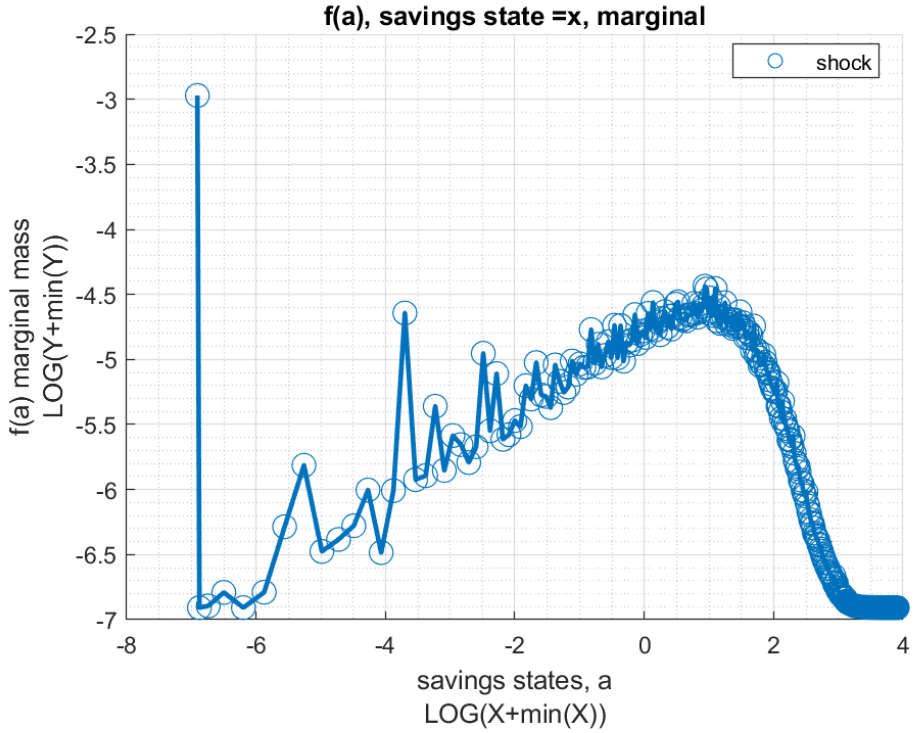
OriginalVariableNames	ap	v	c	y	coh
{'mean' }	2.7094	6.6576	1.5089	1.5084	4.2183
{'unweighted_sum' }	1439.4	7299.4	1545.9	1473.6	11549
{'sd' }	2.8976	2.0599	0.35843	0.52611	3.2096
{'coefofvar' }	1.0694	0.3094	0.23755	0.34879	0.76088
{'gini' }	0.53346	0.17414	0.13326	0.19097	0.39103
{'min' }	0	1.6927	0.58543	0.58543	0.58543
{'max' }	50	19.139	4.9969	4.9969	54.997
{'pYis0' }	0.070216	0	0	0	0
{'pYls0' }	0	0	0	0	0
{'pYgr0' }	0.92978	1	1	1	1
{'pYisMINY' }	0.070216	0.0057675	0.0057675	0.0057675	0.0057675
{'pYisMAXY' }	2.1143e-10	3.7149e-11	3.7149e-11	3.7149e-11	3.7149e-11
{'p0_01' }	0	1.6927	0.58543	0.58543	0.58543
{'p0_1' }	0	1.6927	0.58543	0.58543	0.58543
{'p1' }	0	2.7674	0.76855	0.61362	0.76855
{'p5' }	0	3.273	0.91608	0.77504	1.009
{'p10' }	0.06647	4.0961	1.0308	0.92803	1.1055
{'p20' }	0.37601	4.8781	1.2371	1.0319	1.555
{'p25' }	0.52503	5.2636	1.2781	1.0731	1.8354
{'p30' }	0.7048	5.4822	1.3424	1.1472	2.0866
{'p40' }	1.3008	6.0574	1.3953	1.3424	2.6774
{'p50' }	1.9422	6.542	1.4931	1.4023	3.3444
{'p60' }	2.5275	7.1265	1.6174	1.4954	4.1208
{'p70' }	3.456	7.657	1.6502	1.7803	5.1554
{'p75' }	3.9869	8.0469	1.733	1.824	5.7555
{'p80' }	4.564	8.4125	1.8179	1.8875	6.1793
{'p90' }	6.5844	9.3821	1.9734	2.3349	8.568
{'p95' }	8.1844	10.225	2.1388	2.4776	10.358
{'p99' }	13.136	11.834	2.3359	3.1677	15.511
{'p99_9' }	18.839	13.486	2.7733	3.4782	21.332

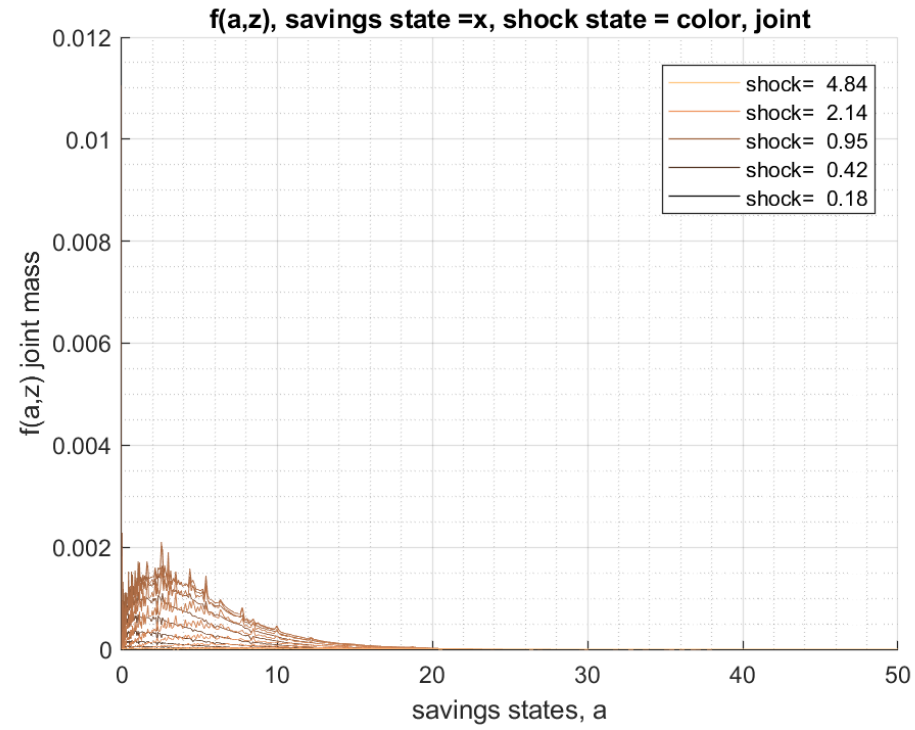
{'p99_99' }	21.778	14.354	3.0939	3.7505	24.78
{'fl_cov_ap' }	8.396	5.2587	0.88866	0.93721	9.2847
{'fl_cor_ap' }	1	0.88106	0.85565	0.61478	0.99833
{'fl_cov_v' }	5.2587	4.243	0.71989	0.93806	5.9786
{'fl_cor_v' }	0.88106	1	0.97505	0.86559	0.90428
{'fl_cov_c' }	0.88866	0.71989	0.12847	0.15253	1.0171
{'fl_cor_c' }	0.85565	0.97505	1	0.80886	0.88413
{'fl_cov_y' }	0.93721	0.93806	0.15253	0.2768	1.0897
{'fl_cor_y' }	0.61478	0.86559	0.80886	1	0.64534
{'fl_cov_coh' }	9.2847	5.9786	1.0171	1.0897	10.302
{'fl_cor_coh' }	0.99833	0.90428	0.88413	0.64534	1
{'fl_cov_savefraccoh'}	0.58458	0.453	0.079518	0.080824	0.6641
{'fl_cor_savefraccoh'}	0.7919	0.86321	0.8708	0.603	0.81215
{'fracByP0_01' }	0	0.0014664	0.0022377	0.0022385	0.00080043
{'fracByP0_1' }	0	0.0014664	0.0022377	0.0022385	0.00080043
{'fracByP1' }	0	0.0029302	0.01567	0.00403	0.0055106
{'fracByP5' }	0	0.021763	0.026172	0.02466	0.015702
{'fracByP10' }	0.0004071	0.050764	0.058937	0.05144	0.022123
{'fracByP20' }	0.0096198	0.1171	0.13549	0.11855	0.05416
{'fracByP25' }	0.017608	0.15851	0.17677	0.15694	0.074837
{'fracByP30' }	0.02761	0.19906	0.21973	0.19018	0.09783
{'fracByP40' }	0.071719	0.28454	0.3135	0.28477	0.15542
{'fracByP50' }	0.15388	0.38017	0.40577	0.38385	0.23227
{'fracByP60' }	0.21684	0.48325	0.51534	0.46249	0.31381
{'fracByP70' }	0.32573	0.59393	0.62048	0.57438	0.42716
{'fracByP75' }	0.39815	0.65416	0.68002	0.63899	0.4882
{'fracByP80' }	0.48482	0.72413	0.732	0.69931	0.55881
{'fracByP90' }	0.6819	0.84902	0.85906	0.8281	0.73338
{'fracByP95' }	0.79123	0.91664	0.92592	0.90812	0.83969
{'fracByP99' }	0.9433	0.98136	0.98418	0.97889	0.95655
{'fracByP99_9' }	0.99595	0.99805	0.99819	0.99776	0.99501
{'fracByP99_99' }	0.99934	0.99982	0.99985	0.9998	0.99938

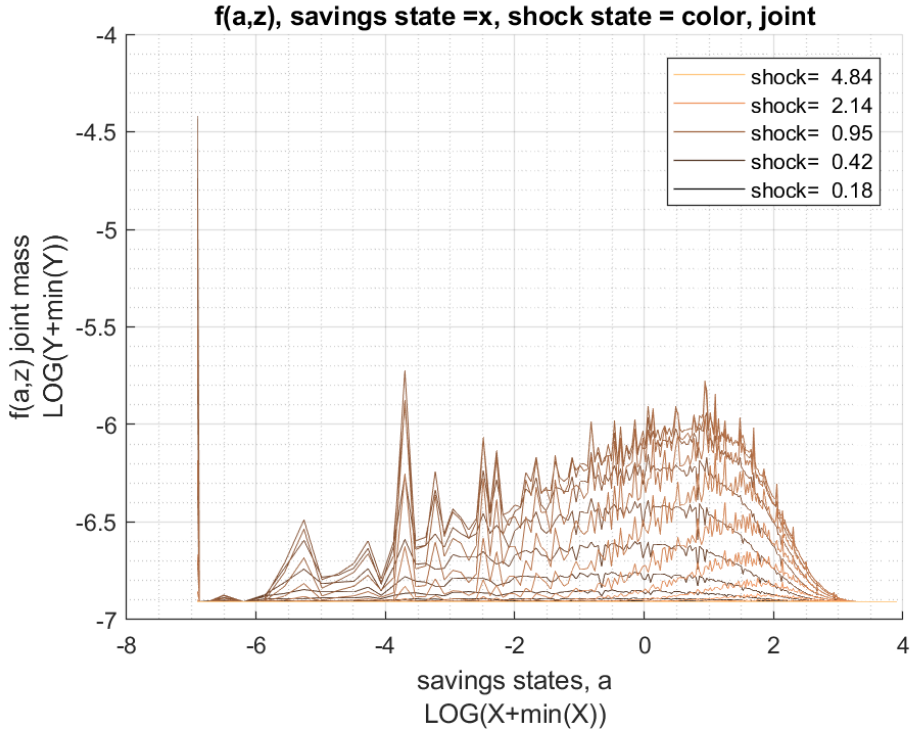
2.1.4 Test FF_DS_AZ_LOOP A grid 300 Shock Grid 25

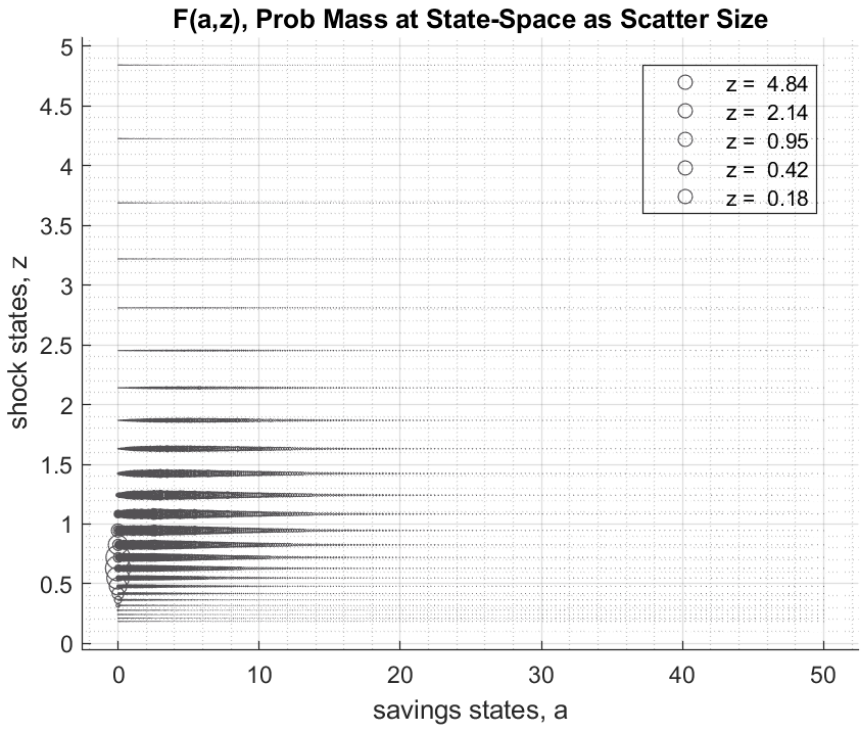
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {'faz','fa'};
mp_support('bl_show_stats_table') = true;
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 300;
mp_params('it_z_n') = 25;
ff_ds_az_loop(mp_params, mp_support);

Elapsed time is 1.356902 seconds.
FF_DS_AZ_LOOP finished. Distribution took = 1.3706
```


f(a), savings state =x, marginal
f(a) marginal mass
savings states, a
shock
0.06
0.05
0.04
0.03
0.02
0.01
0
0
10
20
30
40
50


f(a), savings state =x, marginal
f(a) marginal mass
LOG(Y+min(Y))
savings states, a
LOG(X+min(X))
shock
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4


f(a,z), savings state =x, shock state = color, joint
shock= 4.84
shock= 2.14
shock= 0.95
shock= 0.42
shock= 0.18
f(a,z) joint mass
0.012
0.01
0.008
0.006
0.004
0.002
0
0
10
20
30
40
50
savings states, a


f(a,z), savings state =x, shock state = color, joint
shock= 4.84
shock= 2.14
shock= 0.95
shock= 0.42
shock= 0.18
f(a,z) joint mass
LOG(Y+min(Y))
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4
savings states, a
LOG(X+min(X))



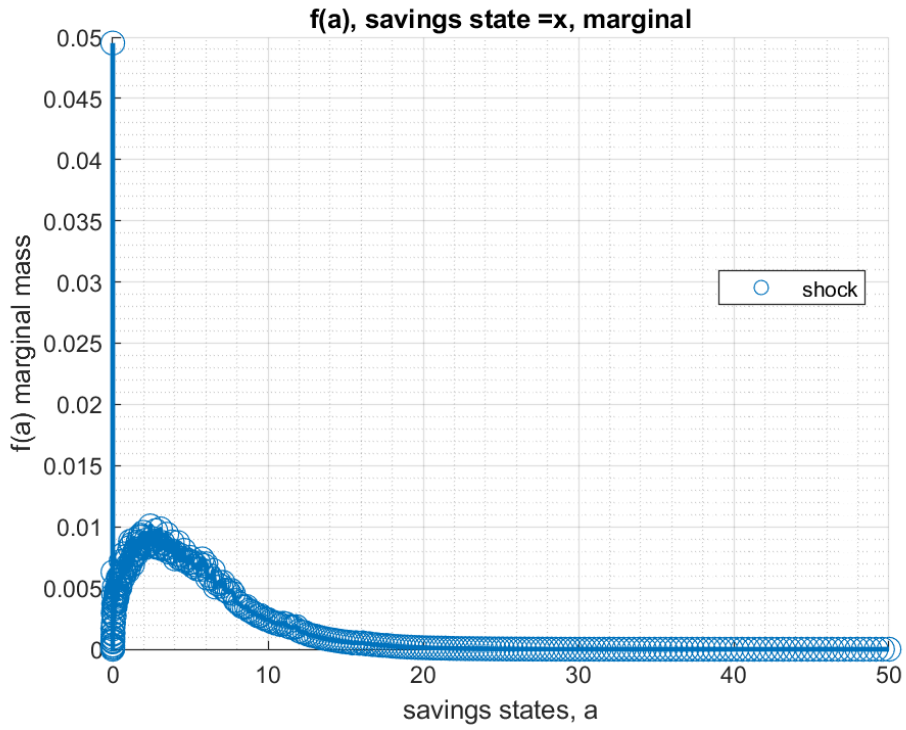
xxx tb_outcomes: all stats xxx

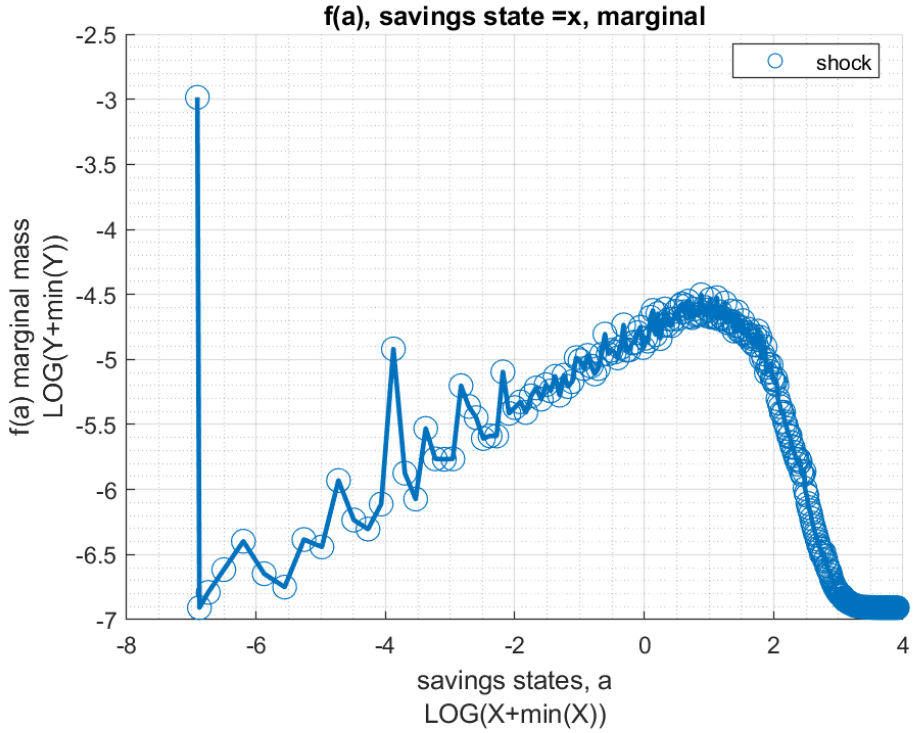
OriginalVariableNames	ap	v	c	y	coh
{'mean' }	3.1835	6.9106	1.5286	1.5274	4.7121
{'unweighted_sum' }	4296.5	79518	16864	19751	1.2716e+05
{'sd' }	3.2831	2.152	0.35175	0.53521	3.5973
{'coefofvar' }	1.0313	0.31141	0.2301	0.35041	0.76341
{'gini' }	0.52466	0.17565	0.12887	0.19155	0.39536
{'min' }	0	-2.7621	0.25871	0.25871	0.25871
{'max' }	50	20.027	8.7798	8.7798	58.78
{'pYis0' }	0.050267	0	0	0	0
{'pYls0' }	0	7.4299e-05	0	0	0
{'pYgr0' }	0.94973	0.99993	1	1	1
{'pYisMINY' }	0.050267	3.1587e-08	3.1587e-08	3.1587e-08	3.1587e-08
{'pYisMAXY' }	2.3964e-09	9.6288e-14	9.6288e-14	9.6288e-14	9.6288e-14
{'p0_01' }	0	0.33524	0.44588	0.42089	0.44588
{'p0_1' }	0	1.0281	0.51088	0.51088	0.51088
{'p1' }	0	2.3294	0.67069	0.67069	0.67069
{'p5' }	0	3.531	0.9348	0.80006	1.0088
{'p10' }	0.10107	4.1808	1.0877	0.90775	1.2209
{'p20' }	0.48982	5.0629	1.248	1.0638	1.7564
{'p25' }	0.7256	5.3749	1.3048	1.157	2.0452
{'p30' }	0.97897	5.7085	1.3561	1.192	2.3425
{'p40' }	1.5756	6.2702	1.4389	1.3331	2.9951
{'p50' }	2.2184	6.8025	1.5235	1.4352	3.7422
{'p60' }	2.9972	7.3608	1.6237	1.5724	4.6044
{'p70' }	4.012	7.977	1.7017	1.7487	5.6899
{'p75' }	4.5871	8.3254	1.7349	1.8191	6.3522
{'p80' }	5.3173	8.7116	1.8227	1.9222	7.1504
{'p90' }	7.5009	9.7584	1.9829	2.2334	9.526
{'p95' }	9.6743	10.633	2.1133	2.5088	11.809
{'p99' }	14.854	12.286	2.3901	3.1545	17.176
{'p99_9' }	21.166	14.023	2.7913	3.9726	23.779

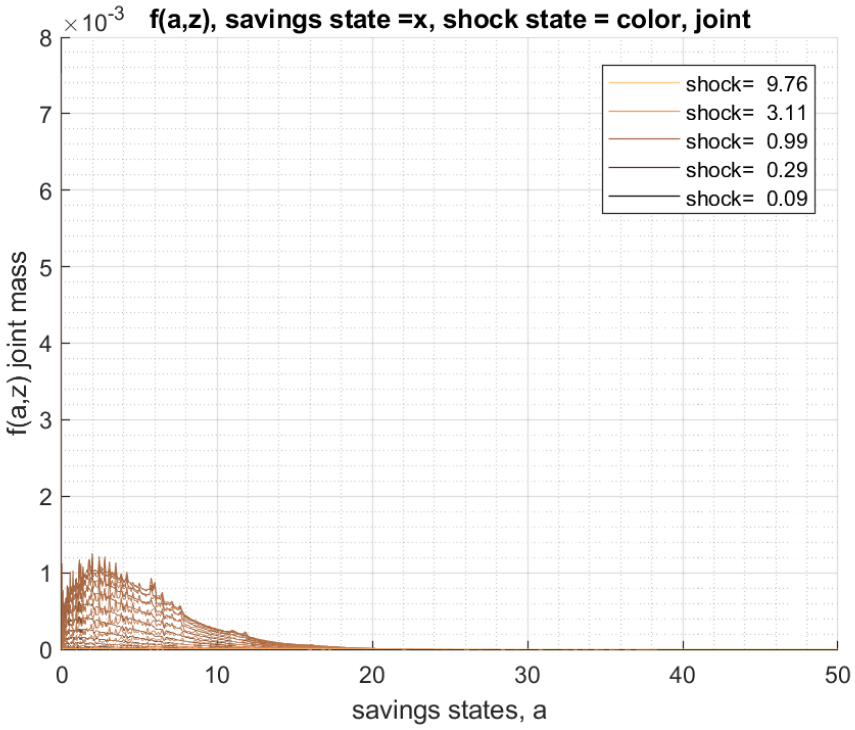
{'p99_99'	}	26.803	15.357	3.0931	4.7968	29.914
{'fl_cov_ap'	}	10.779	6.2944	1.019	1.0643	11.798
{'fl_cor_ap'	}	1	0.89089	0.88234	0.60566	0.99894
{'fl_cov_v'	}	6.2944	4.6311	0.7528	0.97564	7.0472
{'fl_cor_v'	}	0.89089	1	0.9945	0.84708	0.91033
{'fl_cov_c'	}	1.019	0.7528	0.12373	0.15568	1.1427
{'fl_cor_c'	}	0.88234	0.9945	1	0.82696	0.90306
{'fl_cov_y'	}	1.0643	0.97564	0.15568	0.28645	1.2199
{'fl_cor_y'	}	0.60566	0.84708	0.82696	1	0.63363
{'fl_cov_coh'	}	11.798	7.0472	1.1427	1.2199	12.941
{'fl_cor_coh'	}	0.99894	0.91033	0.90306	0.63363	1
{'fl_cov_savefraccoh'	}	0.64446	0.46366	0.077608	0.077311	0.72207
{'fl_cor_savefraccoh'	}	0.78015	0.85631	0.8769	0.57411	0.79776
{'fracByP0_01'	}	0	7.366e-06	9.1288e-05	2.5324e-05	2.9613e-05
{'fracByP0_1'	}	0	0.00015226	0.00040756	0.00048297	0.00013202
{'fracByP1'	}	0	0.0031657	0.0040997	0.0058265	0.0013172
{'fracByP5'	}	0	0.020854	0.026015	0.023308	0.010613
{'fracByP10'	}	0.0007829	0.049187	0.059665	0.051833	0.020313
{'fracByP20'	}	0.010458	0.1169	0.13673	0.11782	0.052147
{'fracByP25'	}	0.020375	0.15489	0.17838	0.15407	0.072616
{'fracByP30'	}	0.033945	0.19501	0.22212	0.1924	0.09561
{'fracByP40'	}	0.076084	0.28102	0.3131	0.2752	0.15182
{'fracByP50'	}	0.13323	0.3766	0.41016	0.36618	0.22332
{'fracByP60'	}	0.21876	0.4783	0.51311	0.46472	0.31143
{'fracByP70'	}	0.32789	0.58936	0.62182	0.57246	0.4201
{'fracByP75'	}	0.39329	0.64823	0.67676	0.63063	0.48449
{'fracByP80'	}	0.47094	0.70976	0.73532	0.69204	0.55555
{'fracByP90'	}	0.66575	0.84269	0.85851	0.82742	0.72907
{'fracByP95'	}	0.8001	0.91584	0.92543	0.90488	0.84038
{'fracByP99'	}	0.94734	0.98115	0.98337	0.97713	0.95746
{'fracByP99_9'	}	0.99324	0.99789	0.99809	0.99717	0.99445
{'fracByP99_99'	}	0.99909	0.99977	0.99979	0.99967	0.99931

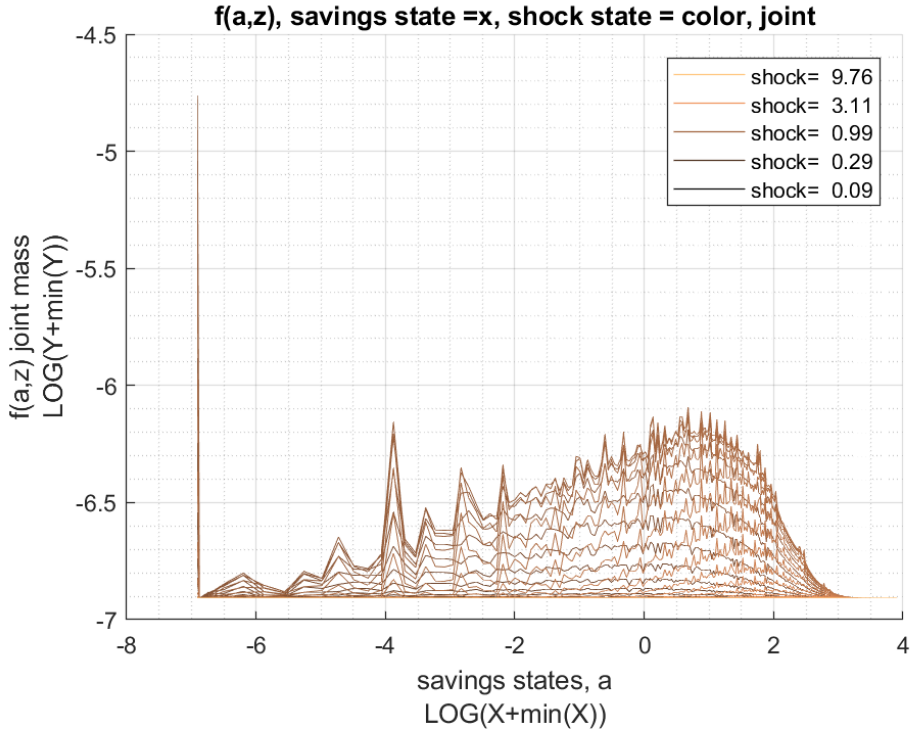
2.1.5 Test FF_DS_AZ_LOOP A grid 300 Shock Grid 50

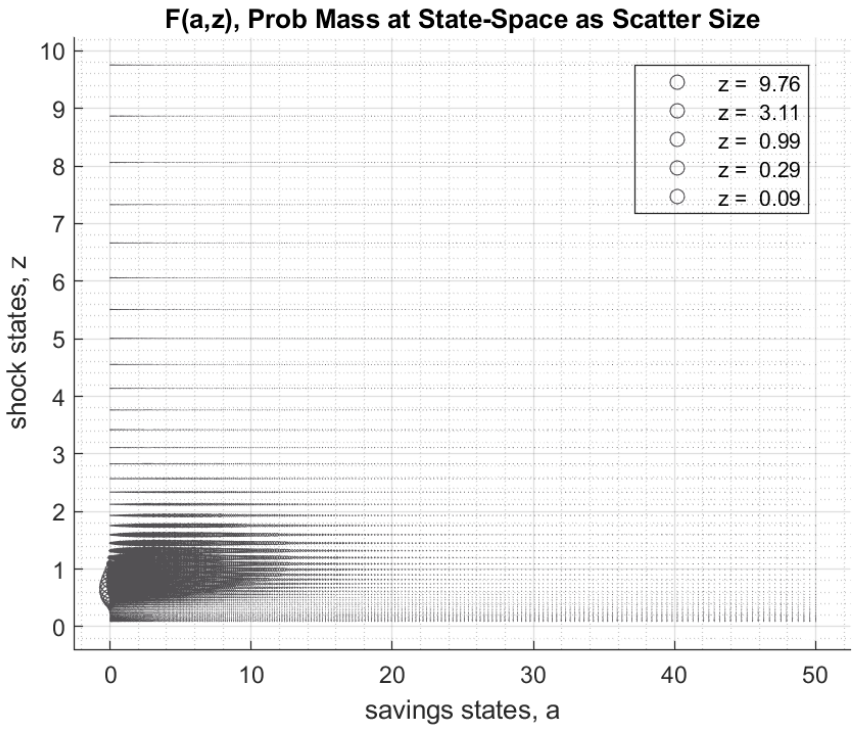
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {'faz','fa'};
mp_support('bl_show_stats_table') = true;
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 300;
mp_params('it_z_n') = 50;
ff_ds_az_loop(mp_params, mp_support);

Elapsed time is 3.256673 seconds.
FF_DS_AZ_LOOP finished. Distribution took = 3.3311
```


f(a), savings state =x, marginal
f(a) marginal mass
savings states, a
shock
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
10
20
30
40
50


f(a), savings state =x, marginal
f(a) marginal mass
LOG(Y+min(Y))
savings states, a
LOG(X+min(X))
shock
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4


f(a,z), savings state =x, shock state = color, joint
×10^-3
f(a,z) joint mass
savings states, a
shock= 9.76
shock= 3.11
shock= 0.99
shock= 0.29
shock= 0.09


f(a,z), savings state =x, shock state = color, joint
f(a,z) joint mass
LOG(Y+min(Y))
savings states, a
LOG(X+min(X))
shock= 9.76
shock= 3.11
shock= 0.99
shock= 0.29
shock= 0.09



xxx tb_outcomes: all stats xxx

OriginalVariableNames	ap	v	c	y	coh
{'mean' }	3.26	6.9484	1.5319	1.5305	4.7919
{'unweighted_sum' }	4296.5	1.6217e+05	35821	53309	2.6813e+05
{'sd' }	3.3166	2.1606	0.35167	0.5364	3.6315
{'coefofvar' }	1.0174	0.31094	0.22956	0.35048	0.75783
{'gini' }	0.52112	0.17551	0.12829	0.19134	0.39468
{'min' }	0	-7.6871	0.12843	0.12843	0.12843
{'max' }	50	20.751	15.657	15.657	65.657
{'pYis0' }	0.049546	0	0	0	0
{'pYls0' }	0	0.00011924	0	0	0
{'pYgr0' }	0.95045	0.99988	1	1	1
{'pYisMINY' }	0.049546	1.1021e-15	1.1021e-15	1.1021e-15	1.1021e-15
{'pYisMAXY' }	5.1436e-09	3.0978e-19	3.0978e-19	3.0978e-19	3.0978e-19
{'p0_01' }	0	-0.20486	0.40271	0.40271	0.40271
{'p0_1' }	0	1.2135	0.53589	0.488	0.53589
{'p1' }	0	2.3687	0.71312	0.64833	0.71312
{'p5' }	0.00050419	3.5428	0.94895	0.8071	0.96945
{'p10' }	0.11149	4.2401	1.0944	0.93681	1.2484
{'p20' }	0.51629	5.0791	1.255	1.072	1.7729
{'p25' }	0.75904	5.4237	1.3033	1.1504	2.067
{'p30' }	1.0189	5.7339	1.3518	1.2006	2.3841
{'p40' }	1.6286	6.2919	1.446	1.3198	3.0593
{'p50' }	2.2834	6.8389	1.5355	1.4423	3.8053
{'p60' }	3.0751	7.4137	1.613	1.5765	4.7113
{'p70' }	4.1046	8.0318	1.7011	1.7318	5.8286
{'p75' }	4.7891	8.3723	1.7435	1.8266	6.5055
{'p80' }	5.5379	8.765	1.8035	1.9295	7.3201
{'p90' }	7.6355	9.7879	1.9921	2.2457	9.6214
{'p95' }	9.8311	10.68	2.1096	2.5308	11.976
{'p99' }	14.653	12.305	2.407	3.1554	17.087
{'p99_9' }	21.166	14.067	2.7771	4.0255	23.953

{'p99_99' }	27.382	15.467	3.1325	4.887	30.554
{'fl_cov_ap' }	11	6.3988	1.032	1.0771	12.032
{'fl_cor_ap' }	1	0.89298	0.88481	0.60546	0.99898
{'fl_cov_v' }	6.3988	4.668	0.75538	0.97839	7.1542
{'fl_cor_v' }	0.89298	1	0.99418	0.84423	0.91183
{'fl_cov_c' }	1.032	0.75538	0.12367	0.15613	1.1557
{'fl_cor_c' }	0.88481	0.99418	1	0.82768	0.90493
{'fl_cov_y' }	1.0771	0.97839	0.15613	0.28772	1.2333
{'fl_cor_y' }	0.60546	0.84423	0.82768	1	0.63312
{'fl_cov_coh' }	12.032	7.1542	1.1557	1.2333	13.188
{'fl_cor_coh' }	0.99898	0.91183	0.90493	0.63312	1
{'fl_cov_savefraccoh'}	0.65387	0.46619	0.077331	0.076912	0.7312
{'fl_cor_savefraccoh'}	0.78182	0.85567	0.87203	0.56861	0.79848
{'fracByP0_01' }	0	-7.082e-06	2.6291e-05	3.0744e-05	8.4044e-06
{'fracByP0_1' }	0	8.1705e-05	0.00058298	0.00029929	0.00018591
{'fracByP1' }	0	0.0025872	0.0055744	0.0043199	0.0017463
{'fracByP5' }	5.9482e-08	0.02063	0.028475	0.023256	0.0085179
{'fracByP10' }	0.00083251	0.049013	0.059787	0.051875	0.020182
{'fracByP20' }	0.01069	0.11692	0.13707	0.11785	0.051473
{'fracByP25' }	0.021006	0.15459	0.17869	0.15432	0.071586
{'fracByP30' }	0.034297	0.19493	0.22235	0.19226	0.095063
{'fracByP40' }	0.076942	0.2811	0.31433	0.27537	0.15173
{'fracByP50' }	0.13547	0.37553	0.41049	0.36597	0.22294
{'fracByP60' }	0.21688	0.47822	0.51321	0.46464	0.31179
{'fracByP70' }	0.32617	0.58918	0.6213	0.57279	0.42106
{'fracByP75' }	0.40001	0.64825	0.67795	0.6311	0.48455
{'fracByP80' }	0.47816	0.71036	0.73507	0.69272	0.55654
{'fracByP90' }	0.67319	0.84299	0.85862	0.82739	0.73089
{'fracByP95' }	0.80347	0.91616	0.92515	0.90483	0.84244
{'fracByP99' }	0.94675	0.98117	0.98325	0.97691	0.95831
{'fracByP99_9' }	0.99284	0.99789	0.9981	0.99713	0.99445
{'fracByP99_99' }	0.99909	0.99977	0.99979	0.99966	0.9993

2.2 FF_DS_AZ_CTS_LOOP Dynamic Savings Loop Continuous Distribution

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_ds_az_cts_loop** from the **MEconTools Package.** F(a,z) discrete probability mass function given policy function solution with continuous savings choices.

- Distribution for Common Choice and States Grid ***Loop***: ff_ds_az_cts_loop
- Distribution for States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Loop***: **ff_ds_az_cts_loop**
- Distribution for States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Vectorized***: **ff_ds_az_cts_vec**

2.2.1 Test FF_DS_AZ_CTS_LOOP Defaults

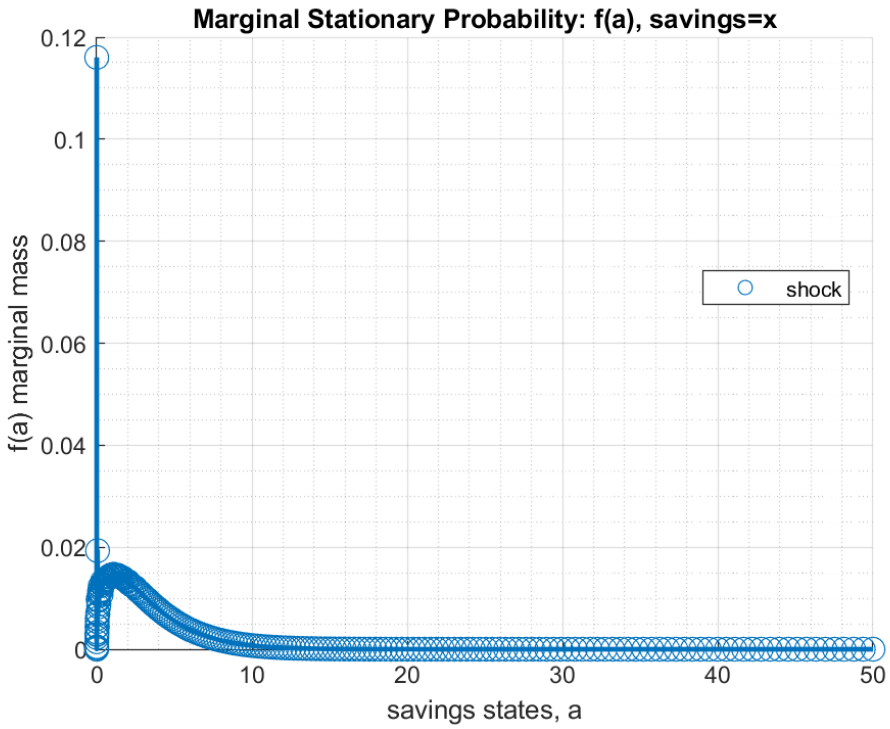
Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

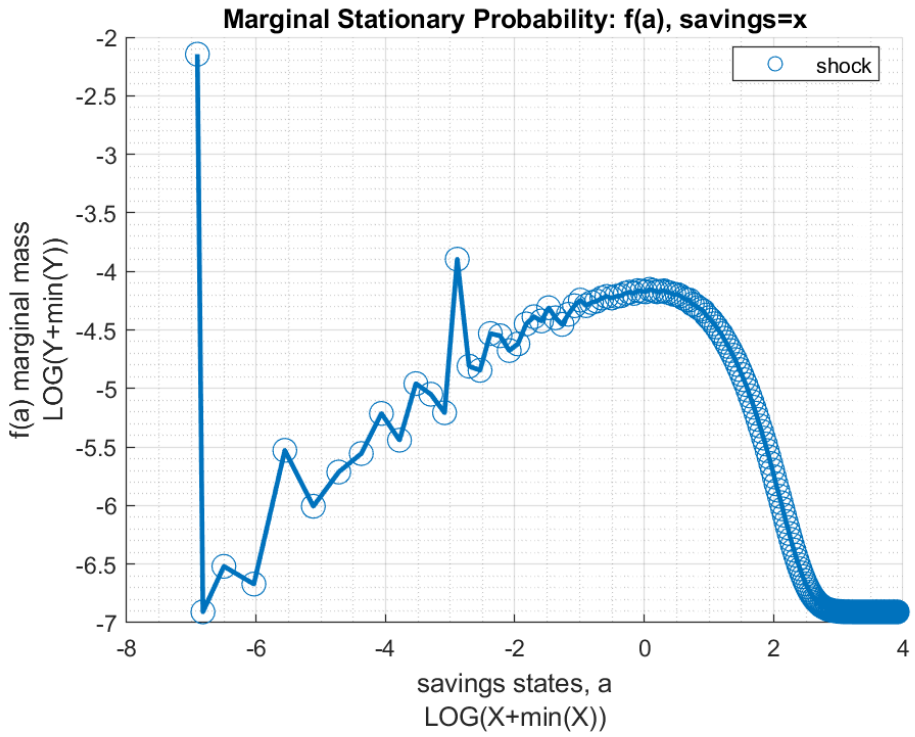
```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
```

```
ff_ds_az_cts_loop(mp_params);

Elapsed time is 1.912182 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN     sum      mean      std      coefvari    min
          -    ---    ----    -----    ----    ----    -----    ------    ------    --------    ---

    ap    1     1      2       3000     200      15     42703    14.234    14.307     1.0051      0

xxx TABLE:ap xxxxxxxxxxxxxxxxx
              c1        c2        c3        c4        c5        c11        c12       c13       c14
            ------    ------    ------    ------    ------    -------    -------    ------    ------

    r1           0         0         0         0         0    0.58655    0.89911    1.2884    1.7803
    r2           0         0         0         0         0    0.58671    0.89914    1.2885    1.7804
    r3           0         0         0         0         0     0.5871    0.89961    1.2888    1.7808
    r4           0         0         0         0         0    0.58803    0.90058    1.2898    1.7817
    r5           0         0         0         0         0    0.58953    0.90208    1.2914    1.7831
    r196    45.655    45.699    45.725    45.798    45.889     47.025     47.404    47.828    48.358
    r197    46.257    46.303    46.326    46.401    46.492     47.626     48.005    48.432    48.965
    r198    46.863     46.91    46.931    47.007    47.097     48.232     48.611    49.041     49.59
    r199    47.472    47.521    47.542    47.617    47.711     48.843     49.222    49.658    50.235
    r200    48.088    48.134    48.157    48.232    48.326     49.459     49.841    50.311    50.885

FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.69766
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ddcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i    idx    ndim    numel    rowN    colN    sum       mean          std        coefvari
          -    ---    ----    -----    ----    ----    ---    ----------    ---------    --------

    fa    1     1      2        200     200       1      1          0.005    0.0096174     1.9235
    faz   2     2      2       3000     200      15      1     0.00033333    0.0011636     3.4908
    fz    3     3      2         15      15       1      1       0.066667     0.076895     1.1534

xxx TABLE:fa xxxxxxxxxxxxxxxxx
              c1
            ----------

    r1         0.11604
    r2               0
    r3       0.0004751
    r4      0.00026799
    r5       0.0029727
    r196    3.5618e-14
    r197    2.1735e-14
    r198     1.329e-14
    r199    8.3938e-15
    r200    8.2751e-15

xxx TABLE:faz xxxxxxxxxxxxxxxxx
              c1            c2            c3            c4            c5            c11           c
            ----------    ----------    ----------    ----------    ----------    ----------    ----
```

r1	4.1559e-05	0.00053618	0.0031141	0.010616	0.023097	9.8338e-05	8.18
r2	0	0	0	0	0	0	
r3	2.0452e-10	1.1226e-08	2.5837e-07	3.2065e-06	2.2865e-05	1.2294e-06	1.06
r4	8.6656e-10	2.8074e-08	3.684e-07	2.7287e-06	1.4098e-05	6.831e-07	5.94
r5	9.2776e-08	2.9148e-06	3.479e-05	0.00019689	0.00056423	2.3628e-06	1.93
r196	1.6685e-22	7.5909e-21	1.5483e-19	1.8762e-18	1.5117e-17	7.3723e-15	8.18
r197	4.6363e-23	2.3916e-21	5.523e-20	7.5562e-19	6.8327e-18	4.5113e-15	5.00
r198	8.2487e-24	4.9336e-22	1.3328e-20	2.1488e-19	2.2991e-18	2.8157e-15	3.08
r199	6.6913e-25	5.3279e-23	1.9003e-21	4.0019e-20	5.5219e-19	1.9017e-15	2.02
r200	2.8381e-26	2.725e-24	1.1911e-22	3.1319e-21	5.5136e-20	1.4819e-15	2.26

```
xxx TABLE:fz xxxxxxxxxxxxxxxxx
              c1

          ----------

    r1     6.1035e-05
    r2     0.00085449
    r3      0.0055542
    r4        0.022217
    r5        0.061096
    r11       0.061096
    r12       0.022217
    r13     0.0055542
    r14    0.00085449
    r15    6.1035e-05
```





xxx tb_outcomes: all stats xxx

OriginalVariableNames	ap	v	c	y	coh
{'mean' }	1.675	5.0913	1.4673	1.467	3.1423
{'unweighted_sum' }	42703	26797	7295.8	6979.8	49998
{'sd' }	2.0062	1.7215	0.36267	0.51485	2.3189
{'coefofvar' }	1.1977	0.33813	0.24717	0.35095	0.73794
{'gini' }	0.59404	0.19113	0.13962	0.19161	0.37632
{'min' }	0	-1.2641	0.38052	0.38052	0.38052
{'max' }	51.591	16.787	5.0209	6.6099	56.61
{'pYis0' }	0.11606	0	0	0	0
{'pYls0' }	0	0.00066766	0	0	0
{'pYgr0' }	0.88394	0.99933	1	1	1
{'pYisMINY' }	0.11606	4.1559e-05	4.1559e-05	4.1559e-05	4.1559e-05
{'pYisMAXY' }	3.1409e-16	3.1409e-16	5.148e-16	3.1409e-16	3.1409e-16
{'p0_01' }	0	-0.34507	0.45473	0.45473	0.45473
{'p0_1' }	0	0.52204	0.54342	0.54342	0.54342
{'p1' }	0	1.3412	0.6494	0.6494	0.6494
{'p5' }	0	2.1813	0.85431	0.77605	0.88697
{'p10' }	0	2.8514	0.96477	0.92741	1.002
{'p20' }	0.10665	3.5986	1.1516	1.0358	1.3244
{'p25' }	0.21483	3.8501	1.2354	1.1105	1.4524
{'p30' }	0.32994	4.2218	1.284	1.129	1.6395
{'p40' }	0.60561	4.5759	1.3788	1.3244	1.999
{'p50' }	0.9866	5.0443	1.4671	1.363	2.4484
{'p60' }	1.4331	5.4957	1.5615	1.5828	2.9924
{'p70' }	2.0261	5.9595	1.6562	1.6429	3.671
{'p75' }	2.4055	6.2377	1.7089	1.7094	4.0981
{'p80' }	2.8929	6.5441	1.7669	1.9106	4.6329
{'p90' }	4.3431	7.3623	1.9254	2.123	6.2699
{'p95' }	5.7881	8.0262	2.0625	2.4019	7.7831
{'p99' }	8.9453	9.2776	2.3421	2.9539	11.327
{'p99_9' }	13.367	10.599	2.6636	3.7357	15.962

{'p99_99' }	17.333	11.639	2.9483	4.3328	20.294
{'fl_cov_ap' }	4.0248	2.8944	0.61038	0.64355	4.6352
{'fl_cor_ap' }	1	0.83807	0.83891	0.62307	0.99637
{'fl_cov_v' }	2.8944	2.9636	0.62238	0.79332	3.5168
{'fl_cor_v' }	0.83807	1	0.99685	0.89507	0.88097
{'fl_cov_c' }	0.61038	0.62238	0.13153	0.16405	0.74192
{'fl_cor_c' }	0.83891	0.99685	1	0.87859	0.8822
{'fl_cov_y' }	0.64355	0.79332	0.16405	0.26507	0.80761
{'fl_cor_y' }	0.62307	0.89507	0.87859	1	0.67647
{'fl_cov_coh' }	4.6352	3.5168	0.74192	0.80761	5.3771
{'fl_cor_coh' }	0.99637	0.88097	0.8822	0.67647	1
{'fl_cov_savefraccoh'}	0.41772	0.36874	0.079746	0.079867	0.49746
{'fl_cor_savefraccoh'}	0.83512	0.85912	0.88192	0.6222	0.86045
{'fracByP0_01' }	0	-4.8153e-05	0.00017799	0.00018159	8.3115e-05
{'fracByP0_1' }	0	0.00027167	0.0013548	0.0014279	0.00063242
{'fracByP1' }	0	0.0032852	0.0063125	0.0069982	0.0029338
{'fracByP5' }	0	0.016969	0.025021	0.024262	0.011819
{'fracByP10' }	0	0.044207	0.05664	0.064855	0.026579
{'fracByP20' }	0.0026834	0.1115	0.13073	0.11733	0.067668
{'fracByP25' }	0.0076113	0.14492	0.17311	0.15549	0.086
{'fracByP30' }	0.015302	0.19105	0.21762	0.19333	0.11182
{'fracByP40' }	0.043894	0.27218	0.30467	0.27748	0.16912
{'fracByP50' }	0.089861	0.36738	0.40369	0.36807	0.23805
{'fracByP60' }	0.16112	0.46928	0.50828	0.46652	0.3263
{'fracByP70' }	0.26525	0.58046	0.61519	0.57507	0.4298
{'fracByP75' }	0.33325	0.64122	0.67431	0.63025	0.49166
{'fracByP80' }	0.41265	0.70474	0.73277	0.69273	0.56293
{'fracByP90' }	0.62139	0.84051	0.85792	0.82668	0.73375
{'fracByP95' }	0.77085	0.91406	0.9245	0.90615	0.84324
{'fracByP99' }	0.93558	0.98098	0.98317	0.97729	0.95807
{'fracByP99_9' }	0.99103	0.99787	0.99814	0.9972	0.99438
{'fracByP99_99' }	0.99886	0.99977	0.99979	0.99969	0.99931

2.2.2 Test FF__DS__AZ__CTS__LOOP Speed Tests

Call the function with different a and z grid size, print out speed:

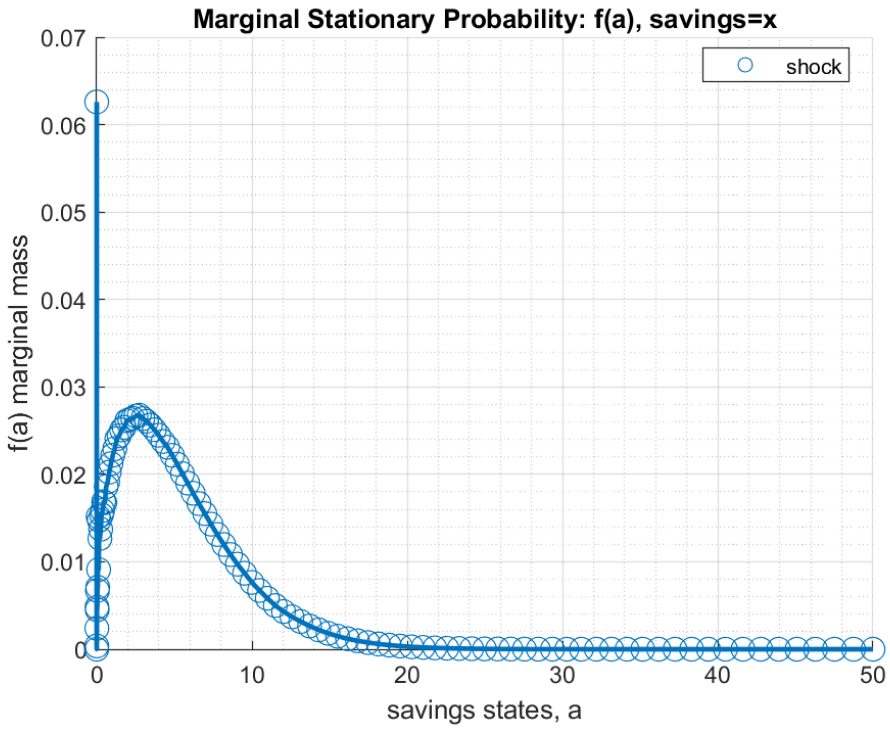
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {};
mp_support('bl_show_stats_table') = false;
% A grid 50, shock grid 5:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_ds_az_cts_loop(mp_params, mp_support);

Elapsed time is 0.466529 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.065434

% A grid 100, shock grid 7:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
ff_ds_az_cts_loop(mp_params, mp_support);

Elapsed time is 0.930211 seconds.
```

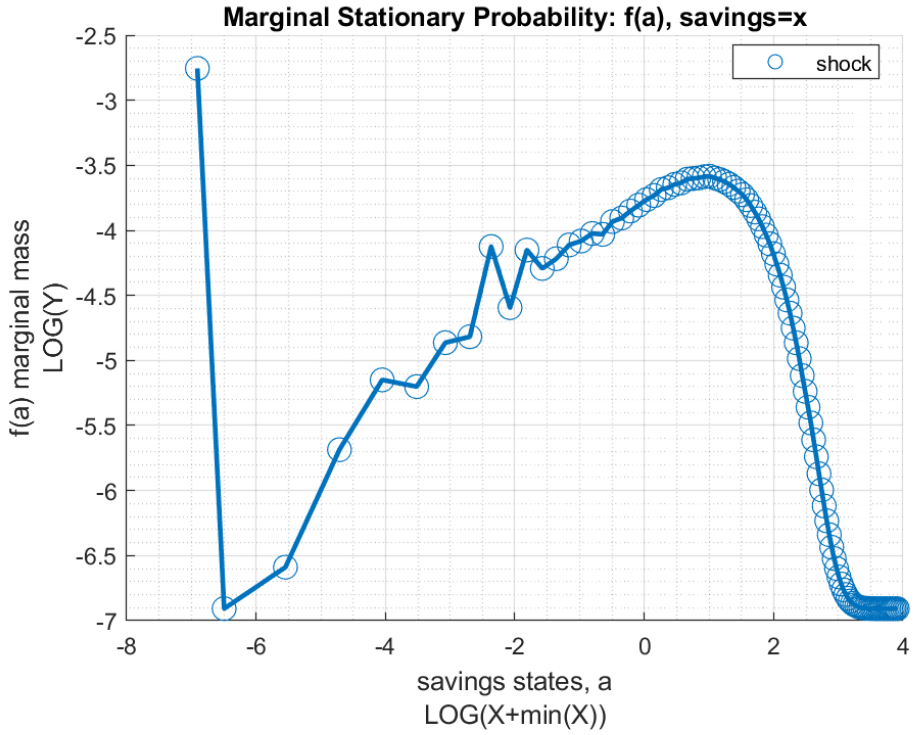
```
FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.20136

% A grid 200, shock grid 9:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 200;
mp_params('it_z_n') = 9;
ff_ds_az_cts_loop(mp_params, mp_support);

Elapsed time is 1.614469 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.52925
```

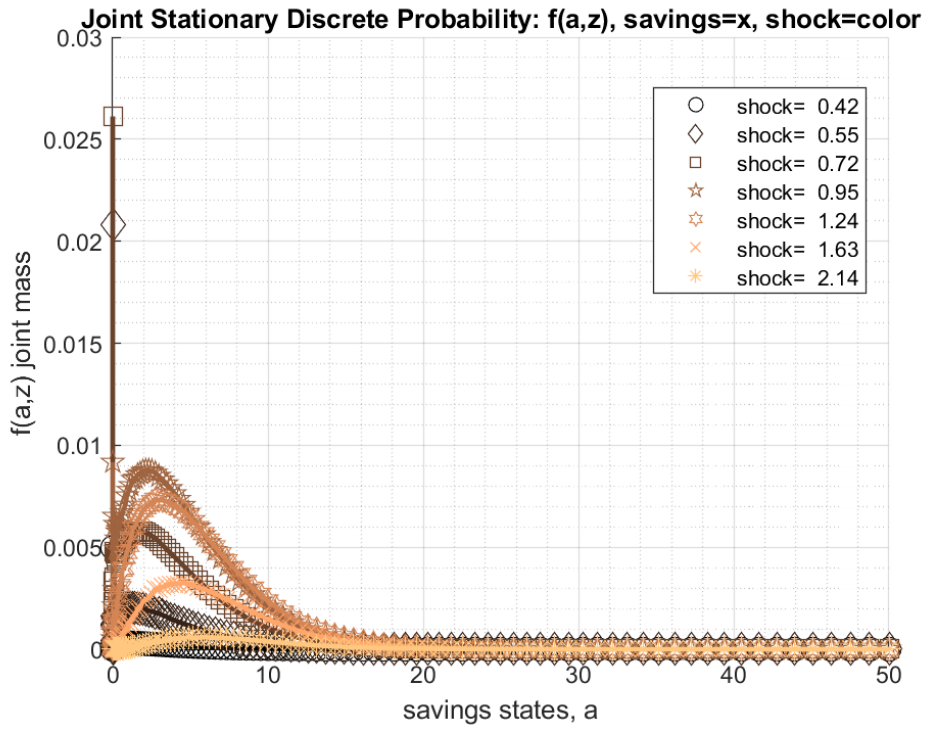
2.2.3 Test FF_DS_AZ_CTS_LOOP A grid 100 Shock grid 7

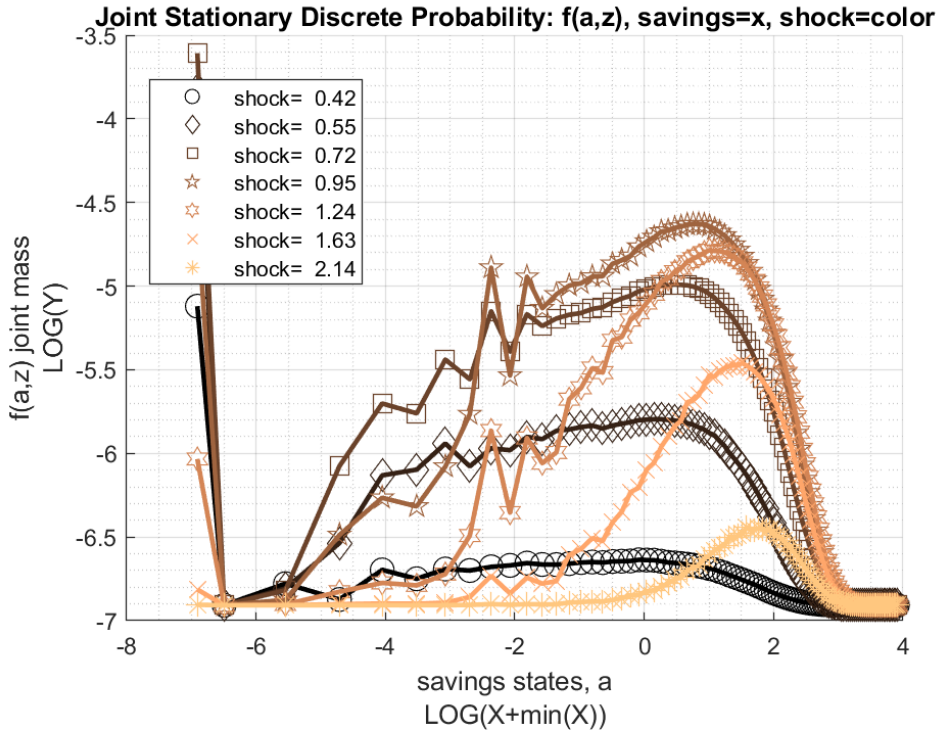
Call the function with different a and z grid size, print out speed:

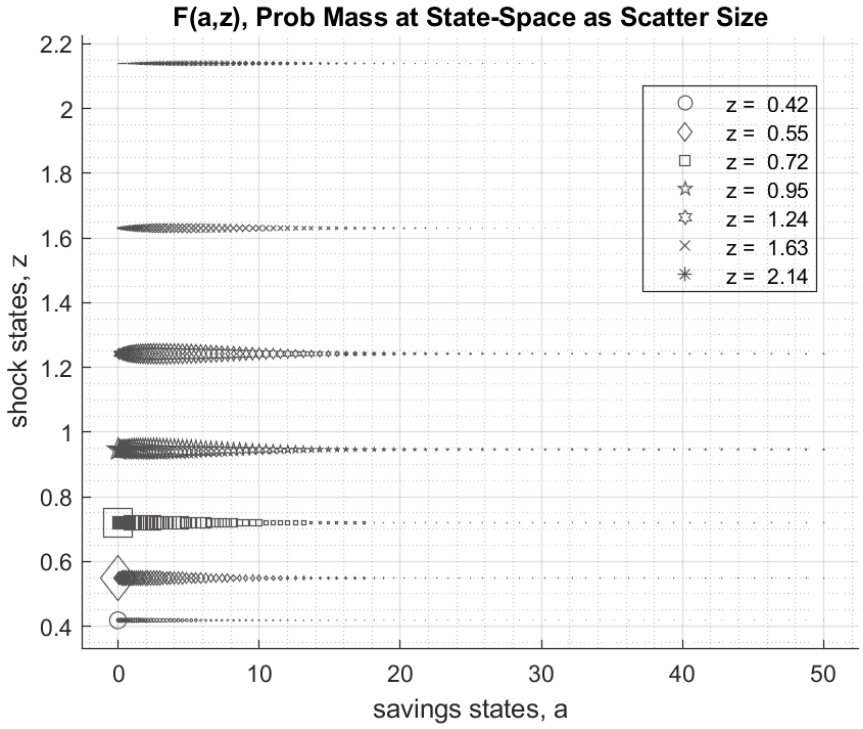
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {'faz','fa'};
mp_support('bl_show_stats_table') = true;
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
ff_ds_az_cts_loop(mp_params, mp_support);

Elapsed time is 0.899597 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.25939
```




Marginal Stationary Probability: f(a), savings=x
shock
f(a) marginal mass
LOG(Y)
savings states, a
LOG(X+min(X))
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4


Joint Stationary Discrete Probability: f(a,z), savings=x, shock=color
shock= 0.42
shock= 0.55
shock= 0.72
shock= 0.95
shock= 1.24
shock= 1.63
shock= 2.14
f(a,z) joint mass
savings states, a
0.03
0.025
0.02
0.015
0.01
0.005
0
0
10
20
30
40
50





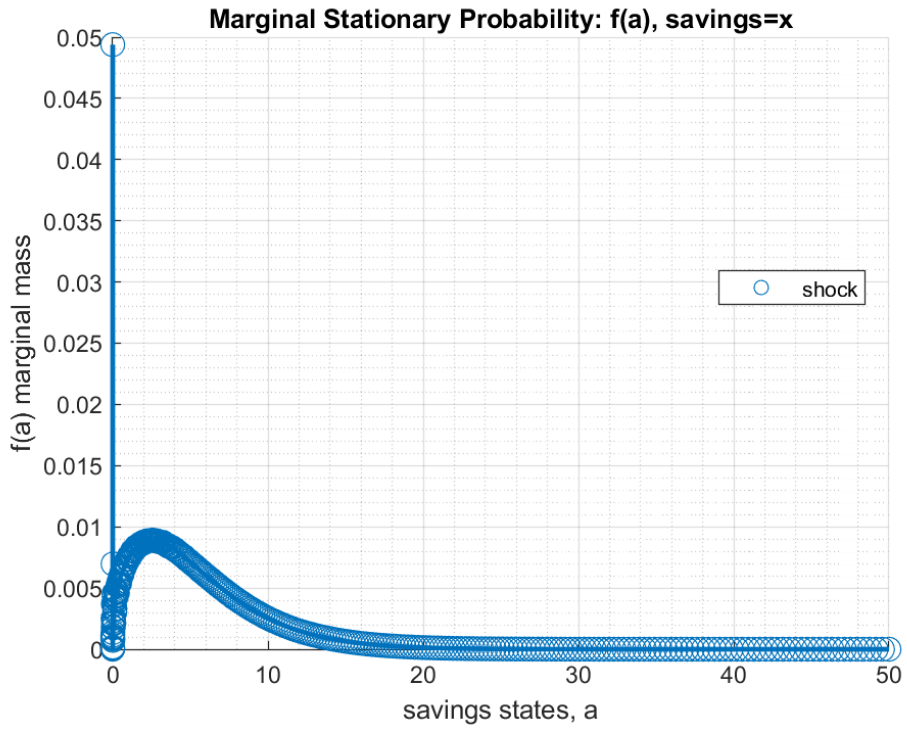
xxx tb_outcomes: all stats xxx

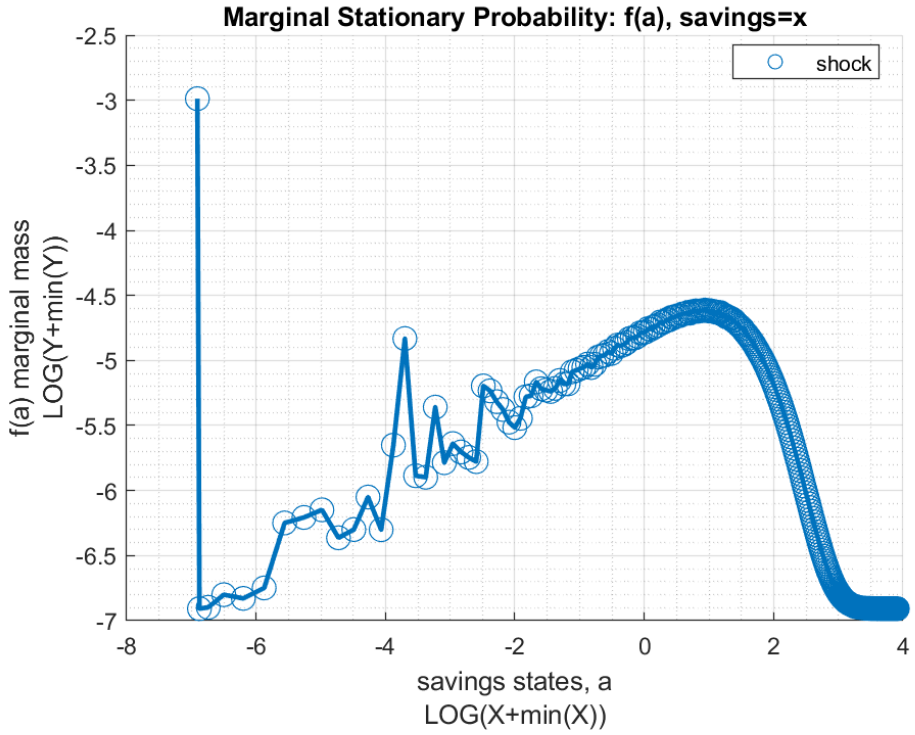
OriginalVariableNames	ap	v	c	y	coh
{'mean' }	3.2216	6.9329	1.5295	1.5289	4.7511
{'unweighted_sum' }	10019	7323.6	1530.6	1473.6	11549
{'sd' }	3.2562	2.1508	0.34914	0.5307	3.5687
{'coefofvar' }	1.0107	0.31024	0.22827	0.34711	0.75113
{'gini' }	0.52352	0.17526	0.12797	0.19065	0.3936
{'min' }	0	1.7008	0.58543	0.58543	0.58543

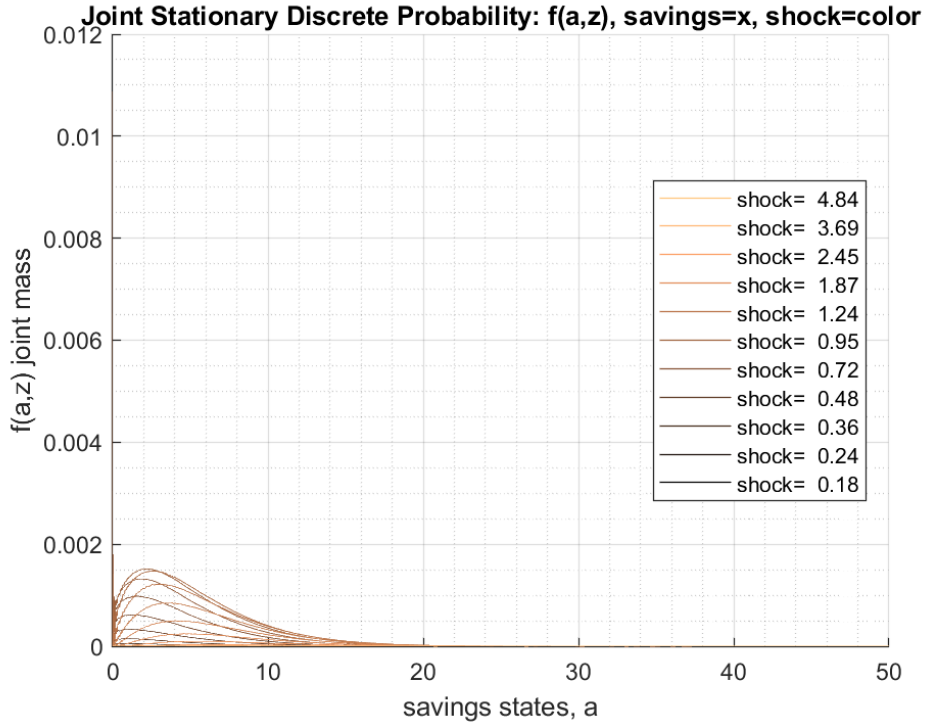
{'max' }	50.789	19.213	4.21	4.9969	54.997
{'pYis0' }	0.062608	0	0	0	0
{'pYls0' }	0	0	0	0	0
{'pYgr0' }	0.93739	1	1	1	1
{'pYisMINY' }	0.062608	0.0049772	0.0049772	0.0049772	0.0049772
{'pYisMAXY' }	2.9501e-11	2.9501e-11	3.1223e-11	2.9501e-11	2.9501e-11
{'p0_01' }	0	1.7008	0.58543	0.58543	0.58543
{'p0_1' }	0	1.7008	0.58543	0.58543	0.58543
{'p1' }	0	2.9492	0.76855	0.62688	0.76855
{'p5' }	0	3.4945	0.97884	0.78105	1.009
{'p10' }	0.092835	4.1716	1.0603	0.97609	1.223
{'p20' }	0.47609	5.1938	1.2588	1.0456	1.7419
{'p25' }	0.7311	5.3812	1.3008	1.094	2.0576
{'p30' }	0.97803	5.6276	1.351	1.188	2.3618
{'p40' }	1.5512	6.3139	1.4528	1.349	3.0158
{'p50' }	2.233	6.8328	1.5245	1.4175	3.7588
{'p60' }	3.0801	7.416	1.6192	1.5453	4.6604
{'p70' }	4.105	8.0461	1.7025	1.7909	5.7649
{'p75' }	4.6992	8.4292	1.7544	1.84	6.4292
{'p80' }	5.4329	8.7432	1.8159	1.9097	7.3478
{'p90' }	7.7004	9.7559	1.9663	2.3407	9.5263
{'p95' }	9.7011	10.662	2.1066	2.5036	11.722
{'p99' }	14.279	12.148	2.3613	3.1795	16.608
{'p99_9' }	19.899	13.734	2.6792	3.5223	22.615
{'p99_99' }	25.265	14.885	2.9563	3.7789	28.175
{'fl_cov_ap' }	10.603	6.2617	1.0053	1.0453	11.608
{'fl_cor_ap' }	1	0.89408	0.8843	0.60489	0.99896
{'fl_cov_v' }	6.2617	4.626	0.74802	0.96794	7.0097
{'fl_cor_v' }	0.89408	1	0.99613	0.848	0.91325
{'fl_cov_c' }	1.0053	0.74802	0.1219	0.15425	1.1272
{'fl_cor_c' }	0.8843	0.99613	1	0.83252	0.9047
{'fl_cov_y' }	1.0453	0.96794	0.15425	0.28164	1.1995
{'fl_cor_y' }	0.60489	0.848	0.83252	1	0.63337
{'fl_cov_coh' }	11.608	7.0097	1.1272	1.1995	12.735
{'fl_cor_coh' }	0.99896	0.91325	0.9047	0.63337	1
{'fl_cov_savefraccoh'}	0.65544	0.47179	0.078595	0.078136	0.73404
{'fl_cor_savefraccoh'}	0.78925	0.86007	0.88265	0.57729	0.8065
{'fracByP0_01' }	0	0.001221	0.0019051	0.0019058	0.00061329
{'fracByP0_1' }	0	0.001221	0.0019051	0.0019058	0.00061329
{'fracByP1' }	0	0.011511	0.013437	0.0039104	0.0042425
{'fracByP5' }	0	0.021279	0.026546	0.024488	0.012268
{'fracByP10' }	0.0006892	0.05109	0.059758	0.051739	0.020676
{'fracByP20' }	0.0099846	0.12278	0.1366	0.12131	0.052438
{'fracByP25' }	0.019425	0.15429	0.17945	0.15485	0.072434
{'fracByP30' }	0.032212	0.19399	0.22206	0.19029	0.094665
{'fracByP40' }	0.0737	0.28144	0.31482	0.27941	0.15063
{'fracByP50' }	0.1321	0.3768	0.41124	0.37234	0.22365
{'fracByP60' }	0.21336	0.48025	0.51513	0.4642	0.31463
{'fracByP70' }	0.3254	0.59015	0.62157	0.57794	0.42288
{'fracByP75' }	0.39769	0.65462	0.67967	0.6363	0.48537
{'fracByP80' }	0.47503	0.71232	0.73844	0.70062	0.56134
{'fracByP90' }	0.67403	0.84445	0.86104	0.82867	0.73331
{'fracByP95' }	0.80886	0.92029	0.92647	0.90776	0.84668
{'fracByP99' }	0.95057	0.98162	0.98401	0.97831	0.96163
{'fracByP99_9' }	0.99336	0.99797	0.99826	0.99778	0.99494
{'fracByP99_99' }	0.99924	0.99979	0.99981	0.99977	0.9994

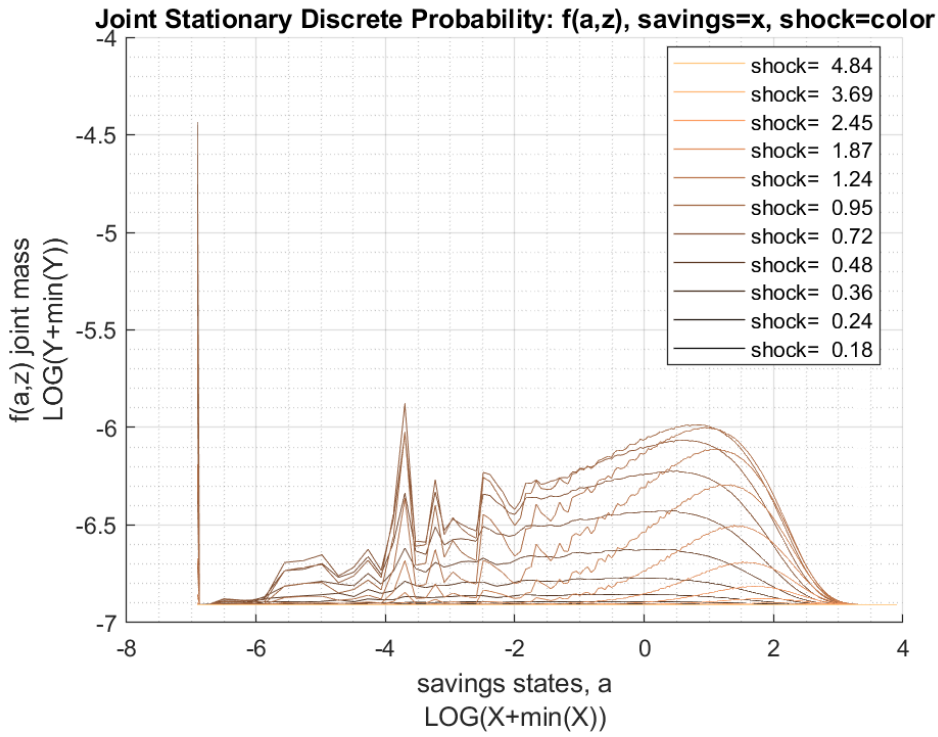
2.2.4 Test FF_DS_AZ_CTS_LOOP A grid 300 Shock grid 25

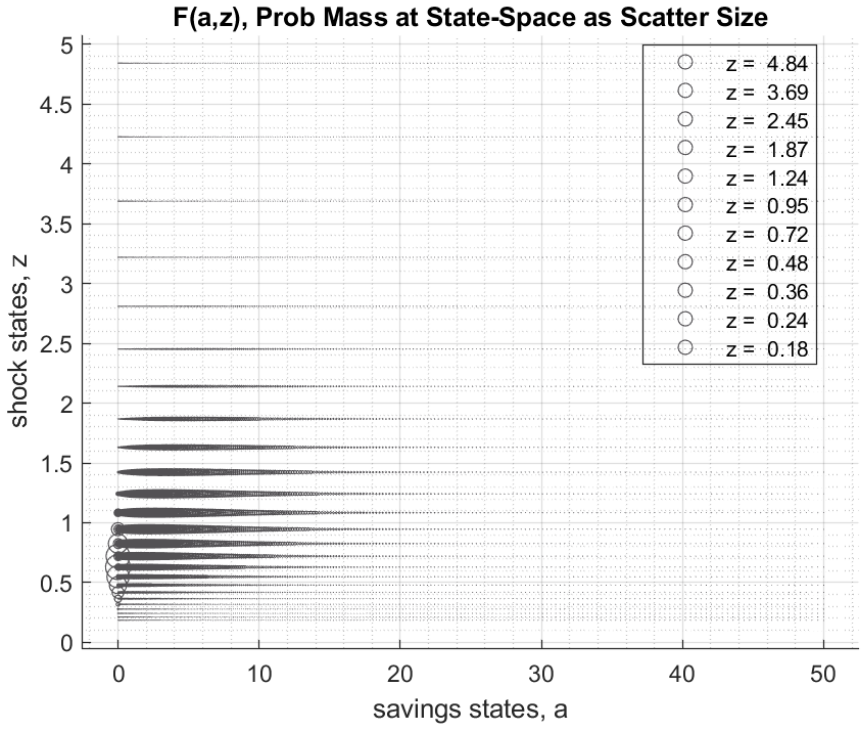
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {'faz','fa'};
mp_support('bl_show_stats_table') = true;
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 300;
mp_params('it_z_n') = 25;
ff_ds_az_cts_loop(mp_params, mp_support);

Elapsed time is 7.769713 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 2.2408
```




Marginal Stationary Probability: f(a), savings=x
shock
f(a) marginal mass
LOG(Y+min(Y))
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4
savings states, a
LOG(X+min(X))


Joint Stationary Discrete Probability: f(a,z), savings=x, shock=color
f(a,z) joint mass
0.012
0.01
0.008
0.006
0.004
0.002
0
shock= 4.84
shock= 3.69
shock= 2.45
shock= 1.87
shock= 1.24
shock= 0.95
shock= 0.72
shock= 0.48
shock= 0.36
shock= 0.24
shock= 0.18
0
10
20
30
40
50
savings states, a



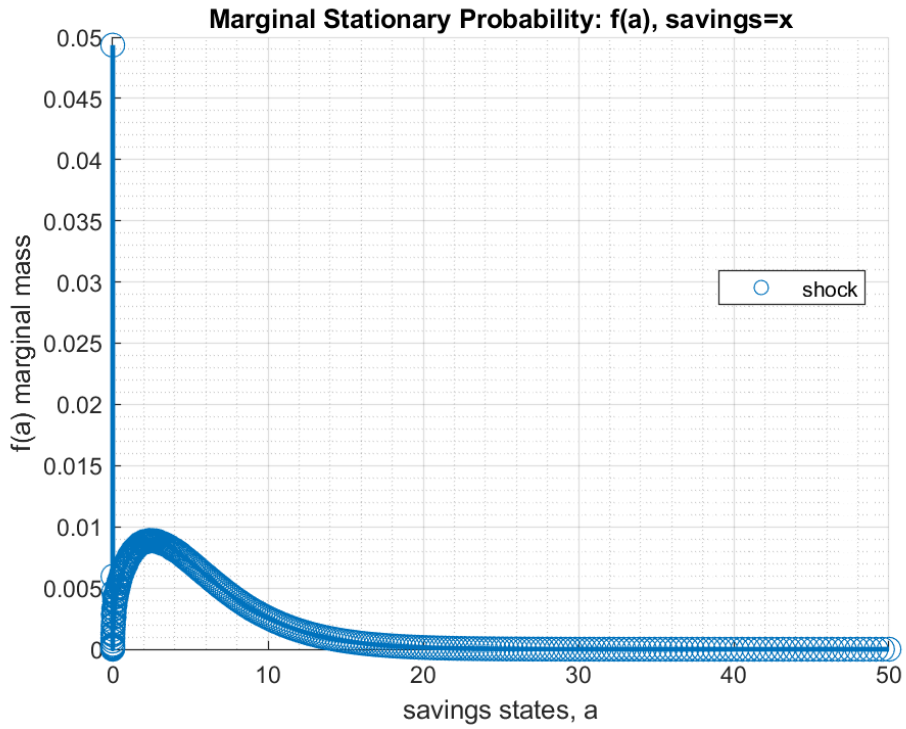


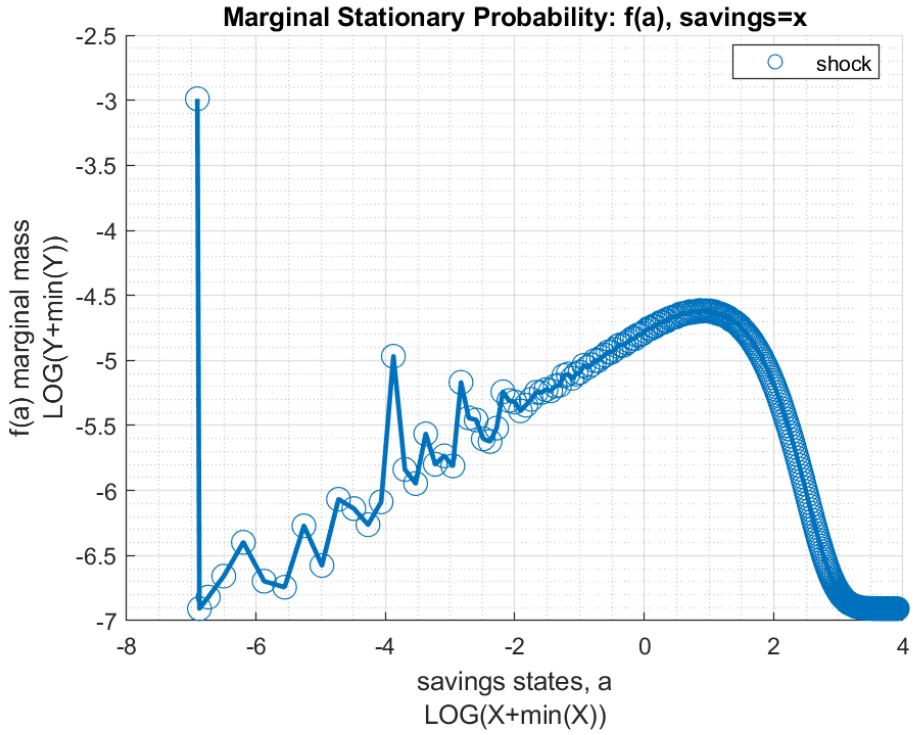

```
xxx tb_outcomes: all stats xxx
    OriginalVariableNames         ap           v           c           y           coh
    ---------------------     ----------    ----------    ----------    ----------    ----------

    {'mean'               }        3.2612        6.9497        1.5318        1.5305         4.793
    {'unweighted_sum'     }     1.1043e+05         79555         16733         19751     1.2716e+05
    {'sd'                 }        3.3352        2.1663       0.35078       0.5359        3.6495
    {'coefofvar'          }        1.0227       0.31171         0.229       0.35014       0.76143
    {'gini'               }       0.52534       0.17597       0.12824       0.19145       0.39608
    {'min'                }             0            -2.7616       0.25871       0.25871       0.25871
```

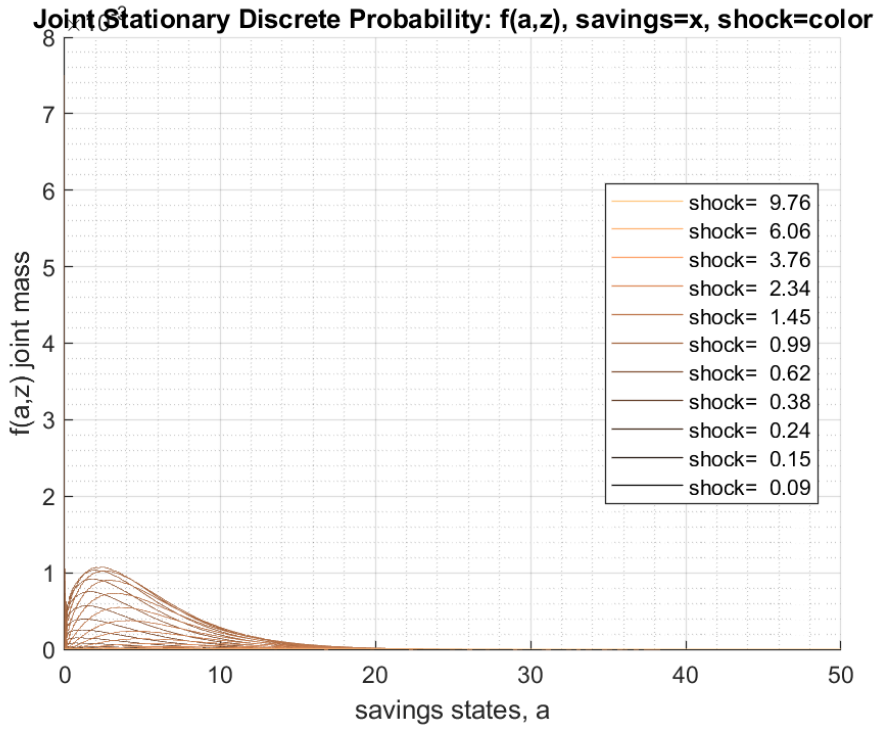
{'max' }	54.451	20.418	4.3301	8.7798	58.78
{'pYis0' }	0.04941	0	0	0	0
{'pYls0' }	0	7.3281e-05	0	0	0
{'pYgr0' }	0.95059	0.99993	1	1	1
{'pYisMINY' }	0.04941	3.1163e-08	3.1163e-08	3.1163e-08	3.1163e-08
{'pYisMAXY' }	2.8477e-13	2.8477e-13	1.121e-13	2.8477e-13	2.8477e-13
{'p0_01' }	0	0.33584	0.44588	0.42374	0.44588
{'p0_1' }	0	1.0287	0.51088	0.51088	0.51088
{'p1' }	0	2.33	0.67226	0.67069	0.67505
{'p5' }	0.0027154	3.5353	0.94151	0.8016	1.0088
{'p10' }	0.11496	4.1978	1.0921	0.9095	1.2356
{'p20' }	0.51133	5.096	1.2504	1.0657	1.779
{'p25' }	0.75298	5.4004	1.3077	1.1577	2.0685
{'p30' }	1.004	5.7312	1.3565	1.1951	2.3792
{'p40' }	1.5834	6.298	1.4458	1.3352	3.0372
{'p50' }	2.2686	6.8433	1.5287	1.441	3.7996
{'p60' }	3.0898	7.4098	1.6132	1.5764	4.6904
{'p70' }	4.0971	8.0297	1.7037	1.7526	5.7899
{'p75' }	4.7228	8.3787	1.7552	1.8223	6.462
{'p80' }	5.4827	8.7742	1.8144	1.9267	7.2769
{'p90' }	7.7718	9.8224	1.9746	2.2406	9.6945
{'p95' }	9.9683	10.704	2.1148	2.5163	12.048
{'p99' }	14.759	12.325	2.3956	3.157	17.176
{'p99_9' }	21.215	14.066	2.7525	3.9803	23.946
{'p99_99' }	27.205	15.415	3.0759	4.7968	30.277
{'fl_cov_ap' }	11.123	6.4528	1.0361	1.0808	12.16
{'fl_cor_ap' }	1	0.89313	0.88563	0.60472	0.999
{'fl_cov_v' }	6.4528	4.6928	0.75717	0.98035	7.21
{'fl_cor_v' }	0.89313	1	0.99643	0.84447	0.91198
{'fl_cov_c' }	1.0361	0.75717	0.12304	0.15594	1.1592
{'fl_cor_c' }	0.88563	0.99643	1	0.82954	0.90548
{'fl_cov_y' }	1.0808	0.98035	0.15594	0.28718	1.2368
{'fl_cor_y' }	0.60472	0.84447	0.82954	1	0.63237
{'fl_cov_coh' }	12.16	7.21	1.1592	1.2368	13.319
{'fl_cor_coh' }	0.999	0.91198	0.90548	0.63237	1
{'fl_cov_savefraccoh'}	0.65691	0.46786	0.07767	0.077234	0.73458
{'fl_cor_savefraccoh'}	0.78162	0.85705	0.87868	0.57192	0.79876
{'fracByP0_01' }	0	7.2341e-06	8.9677e-05	2.5415e-05	2.8657e-05
{'fracByP0_1' }	0	0.00014925	0.00040034	0.00047536	0.00012777
{'fracByP1' }	0	0.0031002	0.004056	0.0057421	0.0012982
{'fracByP5' }	4.4271e-07	0.020663	0.026101	0.023318	0.010275
{'fracByP10' }	0.00081444	0.049128	0.059669	0.051817	0.020124
{'fracByP20' }	0.010142	0.11647	0.13733	0.1174	0.051401
{'fracByP25' }	0.0197	0.15487	0.17845	0.15395	0.07176
{'fracByP30' }	0.033115	0.19474	0.22243	0.19298	0.095014
{'fracByP40' }	0.07268	0.28138	0.31442	0.27544	0.15079
{'fracByP50' }	0.13241	0.3756	0.41097	0.36527	0.22198
{'fracByP60' }	0.21444	0.47892	0.51282	0.46572	0.31091
{'fracByP70' }	0.323	0.58868	0.62139	0.57261	0.41949
{'fracByP75' }	0.39061	0.6478	0.67743	0.63129	0.48319
{'fracByP80' }	0.46952	0.70943	0.73587	0.6919	0.55532
{'fracByP90' }	0.66831	0.84297	0.85906	0.82754	0.72955
{'fracByP95' }	0.80219	0.91616	0.92541	0.90507	0.84194
{'fracByP99' }	0.94613	0.98125	0.98339	0.97711	0.95822
{'fracByP99_9' }	0.9927	0.9979	0.99812	0.99719	0.99443
{'fracByP99_99' }	0.99909	0.99977	0.99979	0.99967	0.99932

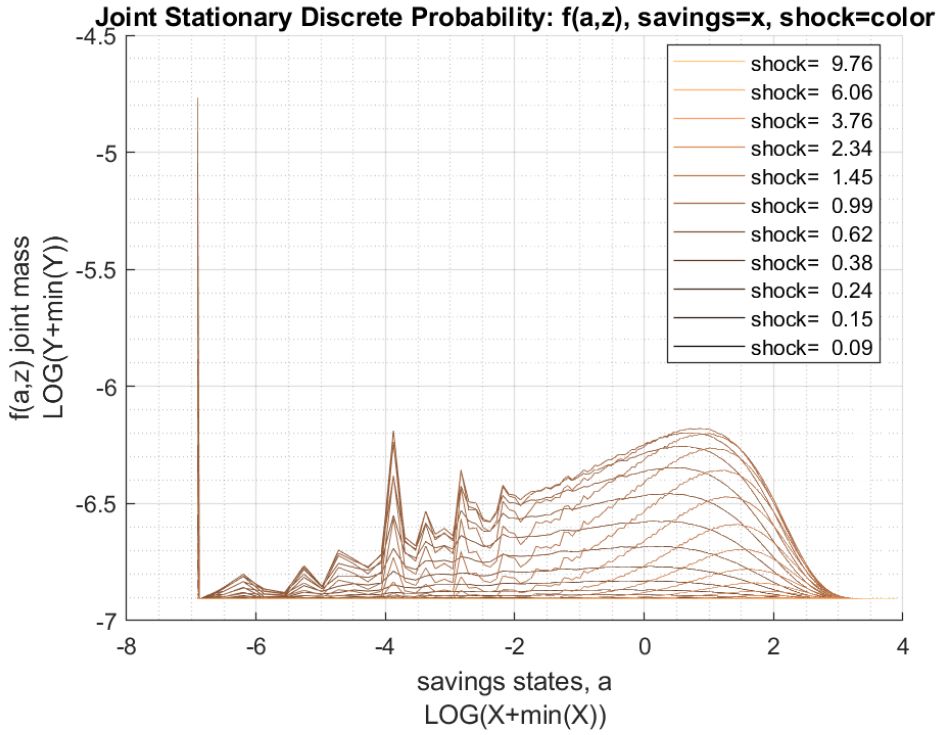
2.2.5 Test FF_DS_AZ_CTS_LOOP A grid 300 Shock grid 50

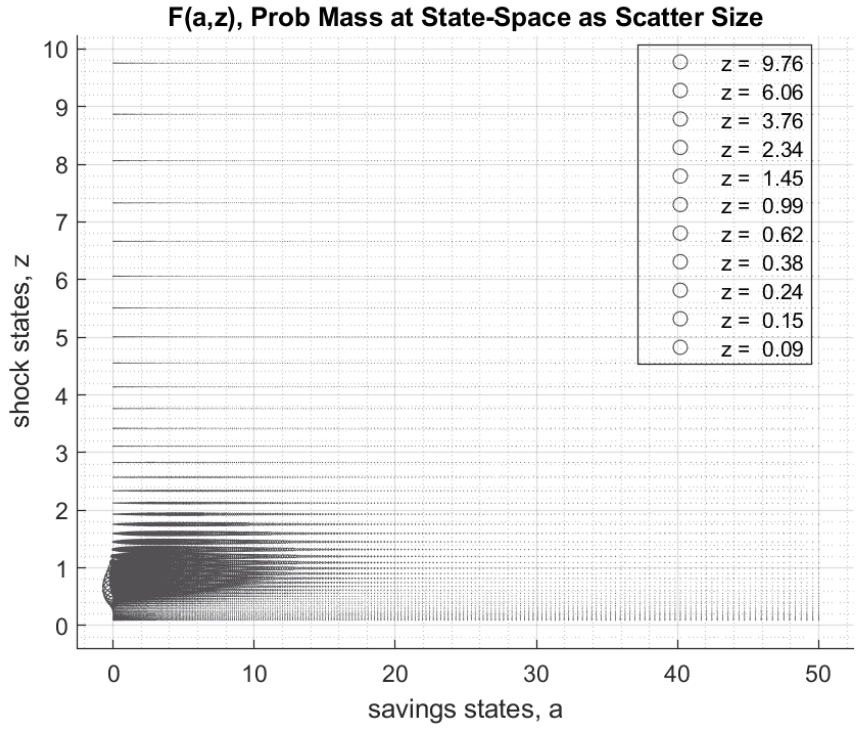
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {'faz','fa'};
mp_support('bl_show_stats_table') = true;
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 300;
mp_params('it_z_n') = 50;
ff_ds_az_cts_loop(mp_params, mp_support);

Elapsed time is 13.966894 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 5.0619
```




Marginal Stationary Probability: f(a), savings=x
shock
f(a) marginal mass
LOG(Y+min(Y))
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4
savings states, a
LOG(X+min(X))


Joint Stationary Discrete Probability: f(a,z), savings=x, shock=color
f(a,z) joint mass
8
7
6
5
4
3
2
1
0
0
10
20
30
40
50
savings states, a
shock= 9.76
shock= 6.06
shock= 3.76
shock= 2.34
shock= 1.45
shock= 0.99
shock= 0.62
shock= 0.38
shock= 0.24
shock= 0.15
shock= 0.09






```
xxx tb_outcomes: all stats xxx
    OriginalVariableNames         ap             v             c             y             coh
    _____________________    __________    ___________    __________    __________    __________

    {'mean'              }        3.2794          6.957        1.5328        1.5312        4.8122
    {'unweighted_sum'    }     2.3346e+05     1.6237e+05         34668         53309     2.6813e+05
    {'sd'                }        3.3623         2.1722       0.35142       0.53693        3.6772
    {'coefofvar'         }        1.0253        0.31224       0.22927       0.35065       0.76415
    {'gini'              }       0.52595        0.17618       0.12829       0.19144        0.3969
    {'min'               }             0        -7.6866       0.12843       0.12843       0.12843
```

{'max' }	61.275	22.164	4.3849	15.657	65.657
{'pYis0' }	0.049376	0	0	0	0
{'pYls0' }	0	0.00011917	0	0	0
{'pYgr0' }	0.95062	0.99988	1	1	1
{'pYisMINY' }	0.049376	1.1048e-15	1.1048e-15	1.1048e-15	1.1048e-15
{'pYisMAXY' }	1.584e-18	1.584e-18	5.0847e-19	1.584e-18	1.584e-18
{'p0_01' }	0	-0.20427	0.40271	0.40271	0.40271
{'p0_1' }	0	1.2141	0.53589	0.48816	0.53589
{'p1' }	0	2.3693	0.71312	0.64833	0.71312
{'p5' }	0.001023	3.5435	0.94895	0.80724	0.96945
{'p10' }	0.11645	4.2417	1.0917	0.93681	1.2501
{'p20' }	0.50875	5.08	1.2515	1.072	1.7735
{'p25' }	0.75899	5.4247	1.3061	1.1504	2.0649
{'p30' }	1.0156	5.7325	1.3564	1.2011	2.3741
{'p40' }	1.6036	6.2932	1.4459	1.3198	3.0387
{'p50' }	2.2768	6.8406	1.5297	1.4423	3.8053
{'p60' }	3.0945	7.4051	1.6122	1.5771	4.7002
{'p70' }	4.113	8.0338	1.7042	1.7334	5.8225
{'p75' }	4.7604	8.3794	1.7554	1.8278	6.4985
{'p80' }	5.5142	8.7771	1.8143	1.9295	7.3239
{'p90' }	7.8048	9.8378	1.9756	2.2476	9.7629
{'p95' }	10.007	10.714	2.1161	2.5336	12.107
{'p99' }	14.9	12.348	2.407	3.1578	17.285
{'p99_9' }	21.501	14.13	2.7694	4.0322	24.216
{'p99_99' }	27.735	15.514	3.1037	4.8946	30.851
{'fl_cov_ap' }	11.305	6.5234	1.0466	1.0907	12.352
{'fl_cor_ap' }	1	0.89316	0.88579	0.60415	0.99902
{'fl_cov_v' }	6.5234	4.7186	0.76066	0.98362	7.2841
{'fl_cor_v' }	0.89316	1	0.99645	0.84334	0.9119
{'fl_cov_c' }	1.0466	0.76066	0.1235	0.15645	1.1701
{'fl_cor_c' }	0.88579	0.99645	1	0.82914	0.9055
{'fl_cov_y' }	1.0907	0.98362	0.15645	0.2883	1.2471
{'fl_cor_y' }	0.60415	0.84334	0.82914	1	0.63165
{'fl_cov_coh' }	12.352	7.2841	1.1701	1.2471	13.522
{'fl_cor_coh' }	0.99902	0.9119	0.9055	0.63165	1
{'fl_cov_savefraccoh'}	0.66084	0.46879	0.077707	0.0772	0.73855
{'fl_cor_savefraccoh'}	0.78009	0.85658	0.87766	0.57067	0.79716
{'fracByP0_01' }	0	-7.0657e-06	2.6272e-05	3.0716e-05	8.3673e-06
{'fracByP0_1' }	0	8.1733e-05	0.00058172	0.0003	0.00018482
{'fracByP1' }	0	0.0025825	0.0055755	0.0043105	0.0017358
{'fracByP5' }	1.3446e-07	0.020553	0.028388	0.023343	0.0084443
{'fracByP10' }	0.00082822	0.048923	0.059616	0.051792	0.020041
{'fracByP20' }	0.010119	0.11678	0.1368	0.1176	0.051426
{'fracByP25' }	0.019764	0.15445	0.17846	0.15402	0.071298
{'fracByP30' }	0.033198	0.19437	0.22195	0.19279	0.094487
{'fracByP40' }	0.072799	0.28088	0.31405	0.27516	0.15079
{'fracByP50' }	0.13186	0.37535	0.41129	0.36559	0.22202
{'fracByP60' }	0.21318	0.47748	0.51316	0.46495	0.30966
{'fracByP70' }	0.32222	0.58845	0.62103	0.57307	0.41837
{'fracByP75' }	0.39045	0.64744	0.67785	0.63075	0.48233
{'fracByP80' }	0.46786	0.7092	0.73555	0.69205	0.55399
{'fracByP90' }	0.66756	0.84275	0.8587	0.82726	0.72947
{'fracByP95' }	0.80166	0.91607	0.92521	0.90478	0.84112
{'fracByP99' }	0.94602	0.98111	0.98335	0.97699	0.95791
{'fracByP99_9' }	0.99264	0.99789	0.9981	0.99714	0.99438
{'fracByP99_99' }	0.99908	0.99977	0.99979	0.99966	0.9993

2.3 FF_DS_AZ_CTS_VEC Dynamic Savings Vectorized Continuous Distribution

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_ds_az_cts_vec** from the **MEconTools Package.** F(a,z) discrete probability mass function given policy function solution with continuous savings choices, vectorized.

- Distribution for Common Choice and States Grid ***Loop***: ff_ds_az_cts_loop
- Distribution for States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Loop***: **ff_ds_az_cts_loop**
- Distribution for States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Vectorized***: **ff_ds_az_cts_vec**

2.3.1 Test FF_DS_AZ_CTS_VEC Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_ds_az_cts_vec(mp_params);

Elapsed time is 2.185467 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i     idx     ndim     numel     rowN     colN      sum      mean      std      coefvari     min
          -     ---     ----     -----     ----     ----     -----     ------    ------    --------     ---

    ap    1      1       2       3000      200       15      42703     14.234    14.307     1.0051       0

xxx TABLE:ap xxxxxxxxxxxxxxxxxx
              c1         c2         c3         c4         c5         c11        c12        c13        c14
            ------     ------     ------     ------     ------     -------    -------    ------     ------

    r1           0          0          0          0          0     0.58655    0.89911    1.2884     1.7803
    r2           0          0          0          0          0     0.58671    0.89914    1.2885     1.7804
    r3           0          0          0          0          0      0.5871    0.89961    1.2888     1.7808
    r4           0          0          0          0          0     0.58803    0.90058    1.2898     1.7817
    r5           0          0          0          0          0     0.58953    0.90208    1.2914     1.7831
    r196    45.655     45.699     45.725     45.798     45.889     47.025     47.404     47.828     48.358
    r197    46.257     46.303     46.326     46.401     46.492     47.626     48.005     48.432     48.965
    r198    46.863      46.91     46.931     47.007     47.097     48.232     48.611     49.041      49.59
    r199    47.472     47.521     47.542     47.617     47.711     48.843     49.222     49.658     50.235
    r200    48.088     48.134     48.157     48.232     48.326     49.459     49.841     50.311     50.885

FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.13145
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ddcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari
fa	1	1	2	200	200	1	1	0.005	0.0096174	1.9235
faz	2	2	2	3000	200	15	1	0.00033333	0.0011636	3.4908
fz	3	3	2	15	15	1	1	0.066667	0.076895	1.1534

xxx TABLE:fa xxxxxxxxxxxxxxxxx

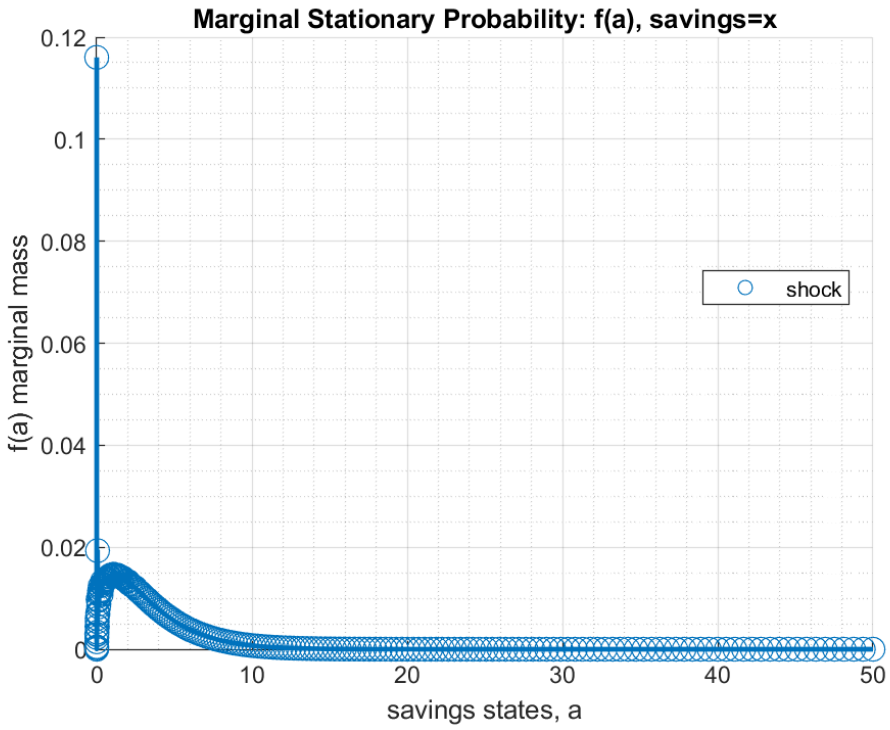
	c1
r1	0.11604
r2	0
r3	0.0004751
r4	0.00026799
r5	0.0029727
r196	3.5618e-14
r197	2.1735e-14
r198	1.329e-14
r199	8.3938e-15
r200	8.2751e-15

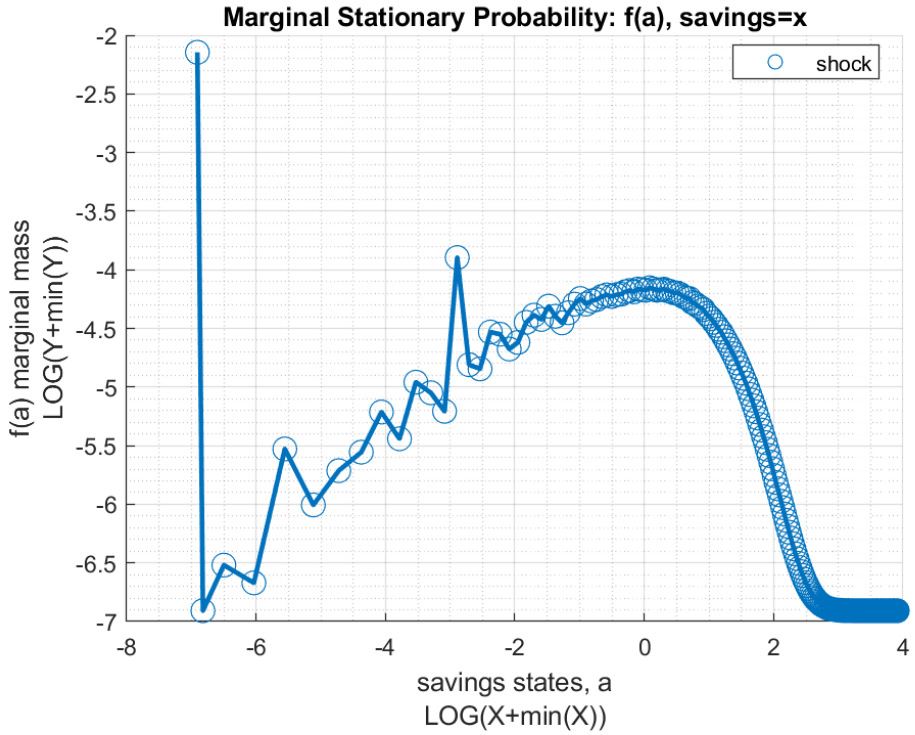
xxx TABLE:faz xxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c11	c
r1	4.1559e-05	0.00053618	0.0031141	0.010616	0.023097	9.8338e-05	8.18
r2	0	0	0	0	0	0	
r3	2.0452e-10	1.1226e-08	2.5837e-07	3.2065e-06	2.2865e-05	1.2294e-06	1.06
r4	8.6656e-10	2.8074e-08	3.684e-07	2.7287e-06	1.4098e-05	6.831e-07	5.94
r5	9.2776e-08	2.9148e-06	3.479e-05	0.00019689	0.00056423	2.3628e-06	1.93
r196	1.6685e-22	7.5909e-21	1.5483e-19	1.8762e-18	1.5117e-17	7.3723e-15	8.18
r197	4.6363e-23	2.3916e-21	5.523e-20	7.5562e-19	6.8327e-18	4.5113e-15	5.00
r198	8.2487e-24	4.9336e-22	1.3328e-20	2.1488e-19	2.2991e-18	2.8157e-15	3.08
r199	6.6913e-25	5.3279e-23	1.9003e-21	4.0019e-20	5.5219e-19	1.9017e-15	2.02
r200	2.8381e-26	2.725e-24	1.1911e-22	3.1319e-21	5.5136e-20	1.4819e-15	2.26

xxx TABLE:fz xxxxxxxxxxxxxxxxx

	c1
r1	6.1035e-05
r2	0.00085449
r3	0.0055542
r4	0.022217
r5	0.061096
r11	0.061096
r12	0.022217
r13	0.0055542
r14	0.00085449
r15	6.1035e-05





xxx tb_outcomes: all stats xxx

OriginalVariableNames	ap	v	c	y	coh
{'mean' }	1.675	5.0913	1.4673	1.467	3.1423
{'unweighted_sum' }	42703	26797	7295.8	6979.8	49998
{'sd' }	2.0062	1.7215	0.36267	0.51485	2.3189
{'coefofvar' }	1.1977	0.33813	0.24717	0.35095	0.73794
{'gini' }	0.59404	0.19113	0.13962	0.19161	0.37632
{'min' }	0	-1.2641	0.38052	0.38052	0.38052

{'max' }	51.591	16.787	5.0209	6.6099	56.61
{'pYis0' }	0.11606	0	0	0	0
{'pYls0' }	0	0.00066766	0	0	0
{'pYgr0' }	0.88394	0.99933	1	1	1
{'pYisMINY' }	0.11606	4.1559e-05	4.1559e-05	4.1559e-05	4.1559e-05
{'pYisMAXY' }	3.1409e-16	3.1409e-16	5.148e-16	3.1409e-16	3.1409e-16
{'p0_01' }	0	-0.34507	0.45473	0.45473	0.45473
{'p0_1' }	0	0.52204	0.54342	0.54342	0.54342
{'p1' }	0	1.3412	0.6494	0.6494	0.6494
{'p5' }	0	2.1813	0.85431	0.77605	0.88697
{'p10' }	0	2.8514	0.96477	0.92741	1.002
{'p20' }	0.10665	3.5986	1.1516	1.0358	1.3244
{'p25' }	0.21483	3.8501	1.2354	1.1105	1.4524
{'p30' }	0.32994	4.2218	1.284	1.129	1.6395
{'p40' }	0.60561	4.5759	1.3788	1.3244	1.999
{'p50' }	0.9866	5.0443	1.4671	1.363	2.4484
{'p60' }	1.4331	5.4957	1.5615	1.5828	2.9924
{'p70' }	2.0261	5.9595	1.6562	1.6429	3.671
{'p75' }	2.4055	6.2377	1.7089	1.7094	4.0981
{'p80' }	2.8929	6.5441	1.7669	1.9106	4.6329
{'p90' }	4.3431	7.3623	1.9254	2.123	6.2699
{'p95' }	5.7881	8.0262	2.0625	2.4019	7.7831
{'p99' }	8.9453	9.2776	2.3421	2.9539	11.327
{'p99_9' }	13.367	10.599	2.6636	3.7357	15.962
{'p99_99' }	17.333	11.639	2.9483	4.3328	20.294
{'fl_cov_ap' }	4.0248	2.8944	0.61038	0.64355	4.6352
{'fl_cor_ap' }	1	0.83807	0.83891	0.62307	0.99637
{'fl_cov_v' }	2.8944	2.9636	0.62238	0.79332	3.5168
{'fl_cor_v' }	0.83807	1	0.99685	0.89507	0.88097
{'fl_cov_c' }	0.61038	0.62238	0.13153	0.16405	0.74192
{'fl_cor_c' }	0.83891	0.99685	1	0.87859	0.8822
{'fl_cov_y' }	0.64355	0.79332	0.16405	0.26507	0.80761
{'fl_cor_y' }	0.62307	0.89507	0.87859	1	0.67647
{'fl_cov_coh' }	4.6352	3.5168	0.74192	0.80761	5.3771
{'fl_cor_coh' }	0.99637	0.88097	0.8822	0.67647	1
{'fl_cov_savefraccoh'}	0.41772	0.36874	0.079746	0.079867	0.49746
{'fl_cor_savefraccoh'}	0.83512	0.85912	0.88192	0.6222	0.86045
{'fracByP0_01' }	0	-4.8153e-05	0.00017799	0.00018159	8.3115e-05
{'fracByP0_1' }	0	0.00027167	0.0013548	0.0014279	0.00063242
{'fracByP1' }	0	0.0032852	0.0063125	0.0069982	0.0029338
{'fracByP5' }	0	0.016969	0.025021	0.024262	0.011819
{'fracByP10' }	0	0.044207	0.05664	0.064855	0.026579
{'fracByP20' }	0.0026834	0.1115	0.13073	0.11733	0.067668
{'fracByP25' }	0.0076113	0.14492	0.17311	0.15549	0.086
{'fracByP30' }	0.015302	0.19105	0.21762	0.19333	0.11182
{'fracByP40' }	0.043894	0.27218	0.30467	0.27748	0.16912
{'fracByP50' }	0.089861	0.36738	0.40369	0.36807	0.23805
{'fracByP60' }	0.16112	0.46928	0.50828	0.46652	0.3263
{'fracByP70' }	0.26525	0.58046	0.61519	0.57507	0.4298
{'fracByP75' }	0.33325	0.64122	0.67431	0.63025	0.49166
{'fracByP80' }	0.41265	0.70474	0.73277	0.69273	0.56293
{'fracByP90' }	0.62139	0.84051	0.85792	0.82668	0.73375
{'fracByP95' }	0.77085	0.91406	0.9245	0.90615	0.84324
{'fracByP99' }	0.93558	0.98098	0.98317	0.97729	0.95807
{'fracByP99_9' }	0.99103	0.99787	0.99814	0.9972	0.99438
{'fracByP99_99' }	0.99886	0.99977	0.99979	0.99969	0.99931

2.3.2 Test FF_DS_AZ_CTS_VEC Speed Tests

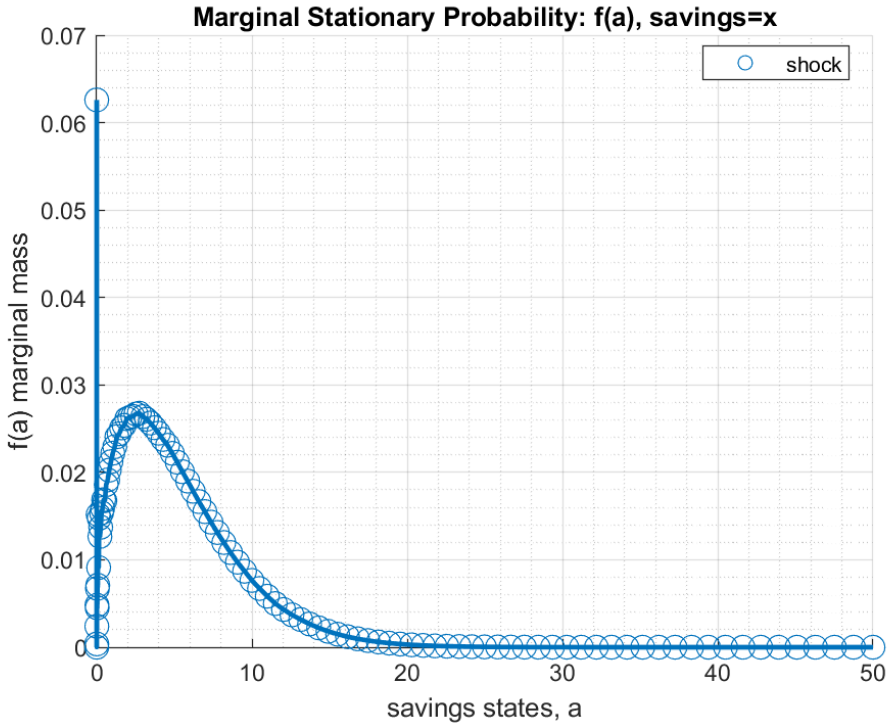
Call the function with different a and z grid size, print out speed:

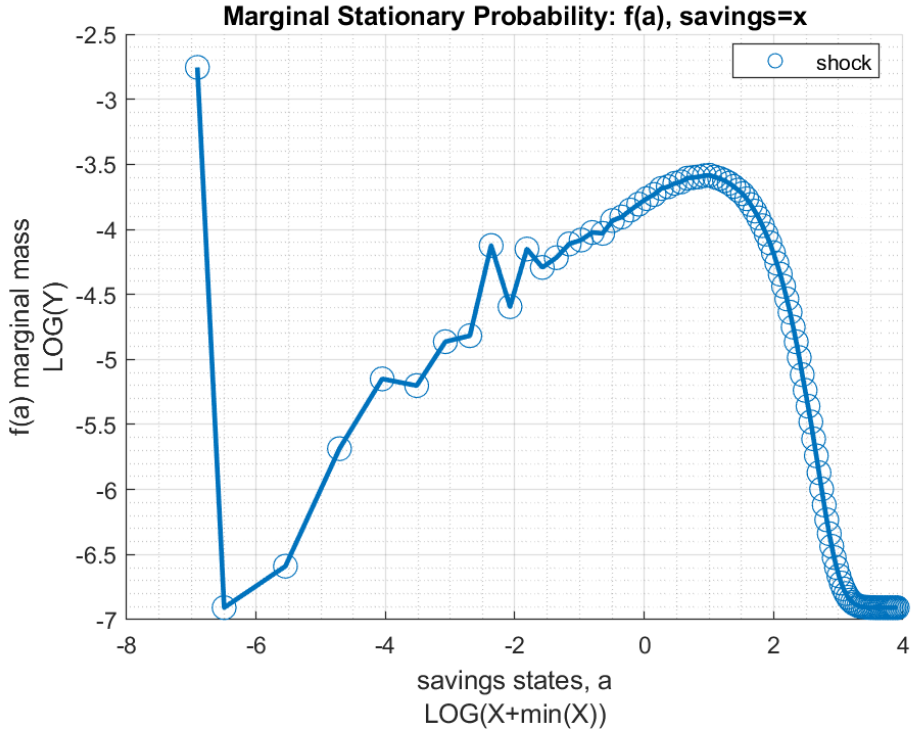
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {};
mp_support('bl_show_stats_table') = false;
% A grid 50, shock grid 5:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_ds_az_cts_vec(mp_params, mp_support);

Elapsed time is 0.459956 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.015748

% A grid 100, shock grid 7:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
ff_ds_az_cts_vec(mp_params, mp_support);

Elapsed time is 0.938024 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.046035

% A grid 200, shock grid 9:
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 200;
mp_params('it_z_n') = 9;
ff_ds_az_cts_vec(mp_params, mp_support);

Elapsed time is 1.696573 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.12795
```

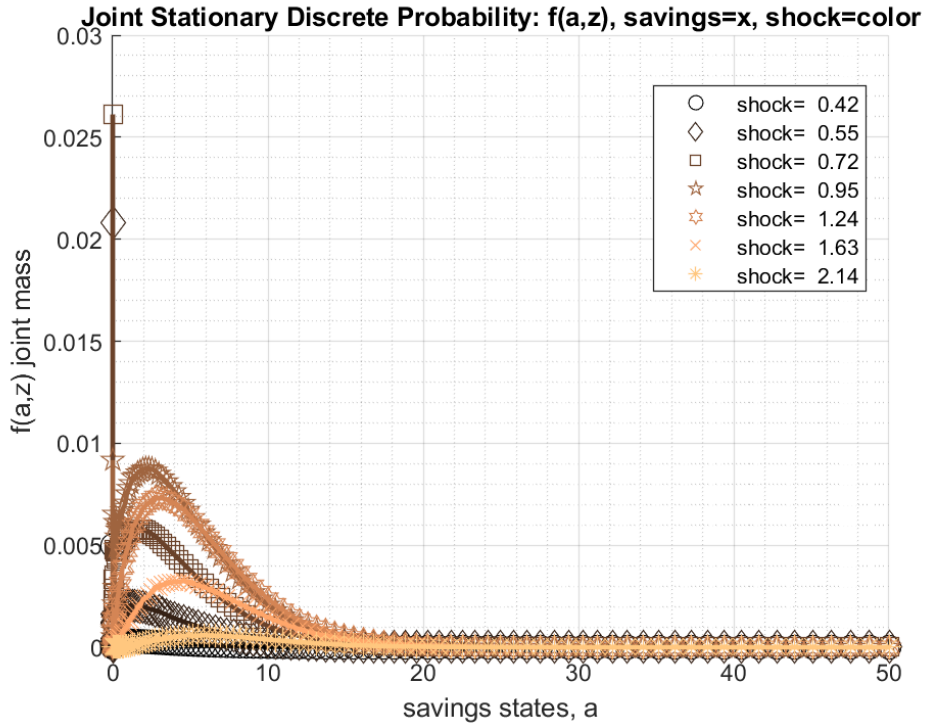
2.3.3 Test FF_DS_AZ_CTS_VEC A grid 100 Shock grid 7

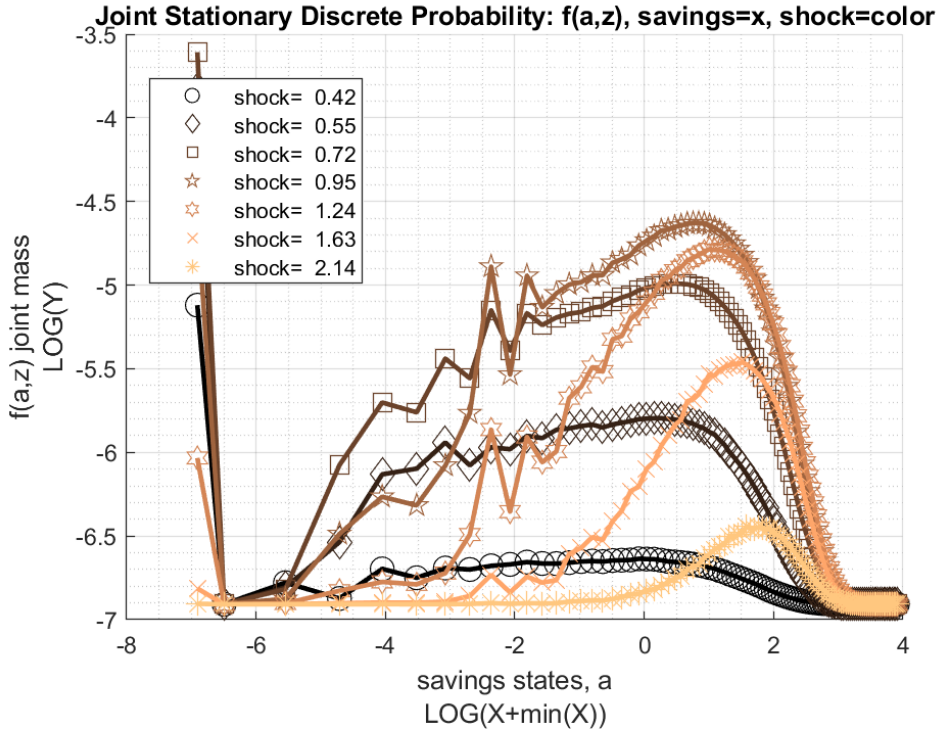
Call the function with different a and z grid size, print out speed:

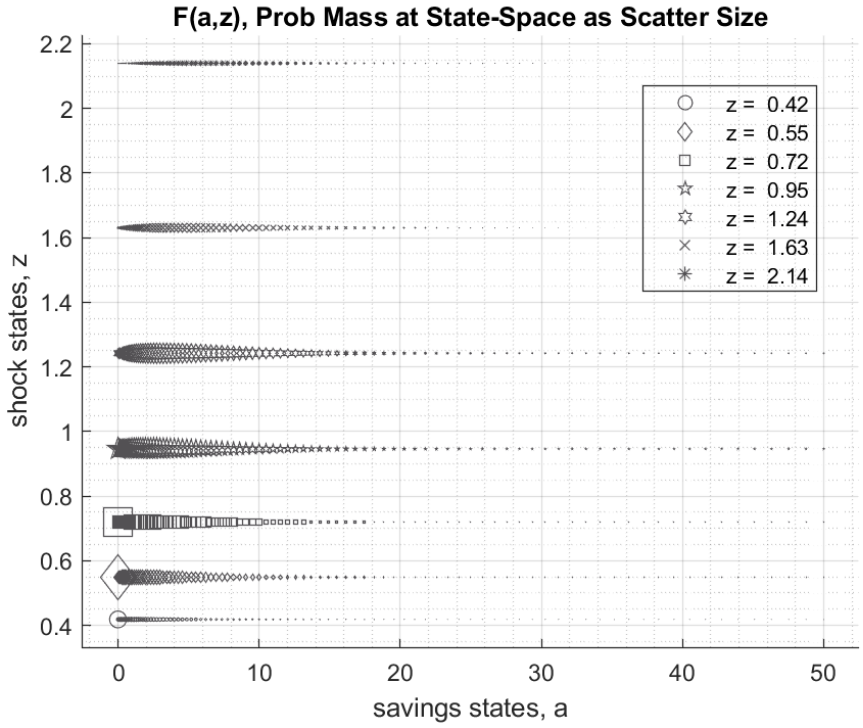
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {'faz','fa'};
mp_support('bl_show_stats_table') = true;
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
ff_ds_az_cts_vec(mp_params, mp_support);

Elapsed time is 0.931254 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.069571
```


Marginal Stationary Probability: f(a), savings=x
shock
f(a) marginal mass
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
10
20
30
40
50
savings states, a


Marginal Stationary Probability: f(a), savings=x
shock
f(a) marginal mass
LOG(Y)
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4
savings states, a
LOG(X+min(X))


Joint Stationary Discrete Probability: f(a,z), savings=x, shock=color
f(a,z) joint mass
savings states, a
shock= 0.42
shock= 0.55
shock= 0.72
shock= 0.95
shock= 1.24
shock= 1.63
shock= 2.14
0.03
0.025
0.02
0.015
0.01
0.005
0
0
10
20
30
40
50


Joint Stationary Discrete Probability: f(a,z), savings=x, shock=color
f(a,z) joint mass
LOG(Y)
savings states, a
LOG(X+min(X))
shock= 0.42
shock= 0.55
shock= 0.72
shock= 0.95
shock= 1.24
shock= 1.63
shock= 2.14
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4



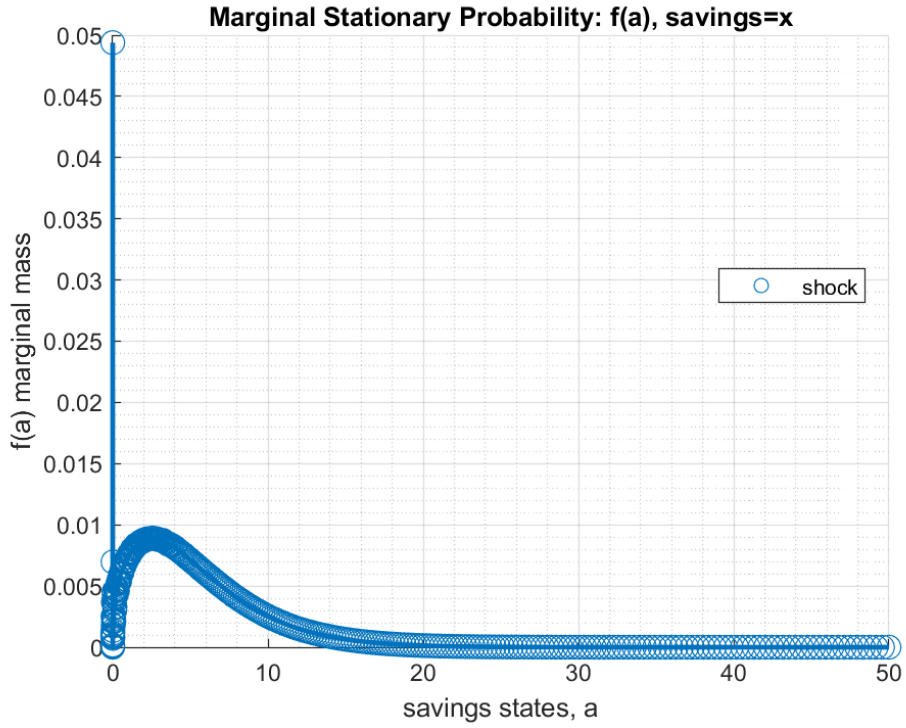
xxx tb_outcomes: all stats xxx

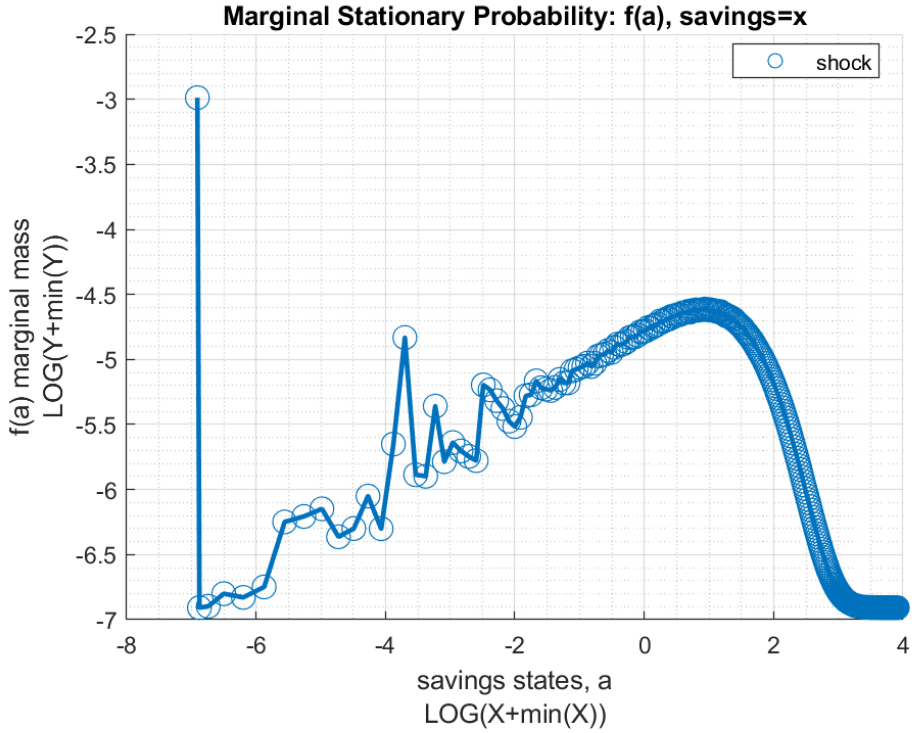
OriginalVariableNames	ap	v	c	y	coh
{'mean' }	3.2216	6.9329	1.5295	1.5289	4.7511
{'unweighted_sum' }	10019	7323.6	1530.6	1473.6	11549
{'sd' }	3.2562	2.1508	0.34914	0.5307	3.5687
{'coefofvar' }	1.0107	0.31024	0.22827	0.34711	0.75113
{'gini' }	0.52352	0.17526	0.12797	0.19065	0.3936
{'min' }	0	1.7008	0.58543	0.58543	0.58543
{'max' }	50.789	19.213	4.21	4.9969	54.997
{'pYis0' }	0.062608	0	0	0	0
{'pYls0' }	0	0	0	0	0
{'pYgr0' }	0.93739	1	1	1	1
{'pYisMINY' }	0.062608	0.0049772	0.0049772	0.0049772	0.0049772
{'pYisMAXY' }	2.9501e-11	2.9501e-11	3.1223e-11	2.9501e-11	2.9501e-11
{'p0_01' }	0	1.7008	0.58543	0.58543	0.58543
{'p0_1' }	0	1.7008	0.58543	0.58543	0.58543
{'p1' }	0	2.9492	0.76855	0.62688	0.76855
{'p5' }	0	3.4945	0.97884	0.78105	1.009
{'p10' }	0.092835	4.1716	1.0603	0.97609	1.223
{'p20' }	0.47609	5.1938	1.2588	1.0456	1.7419
{'p25' }	0.7311	5.3812	1.3008	1.094	2.0576
{'p30' }	0.97803	5.6276	1.351	1.188	2.3618
{'p40' }	1.5512	6.3139	1.4528	1.349	3.0158
{'p50' }	2.233	6.8328	1.5245	1.4175	3.7588
{'p60' }	3.0801	7.416	1.6192	1.5453	4.6604
{'p70' }	4.105	8.0461	1.7025	1.7909	5.7649
{'p75' }	4.6992	8.4292	1.7544	1.84	6.4292
{'p80' }	5.4329	8.7432	1.8159	1.9097	7.3478
{'p90' }	7.7004	9.7559	1.9663	2.3407	9.5263
{'p95' }	9.7011	10.662	2.1066	2.5036	11.722
{'p99' }	14.279	12.148	2.3613	3.1795	16.608
{'p99_9' }	19.899	13.734	2.6792	3.5223	22.615

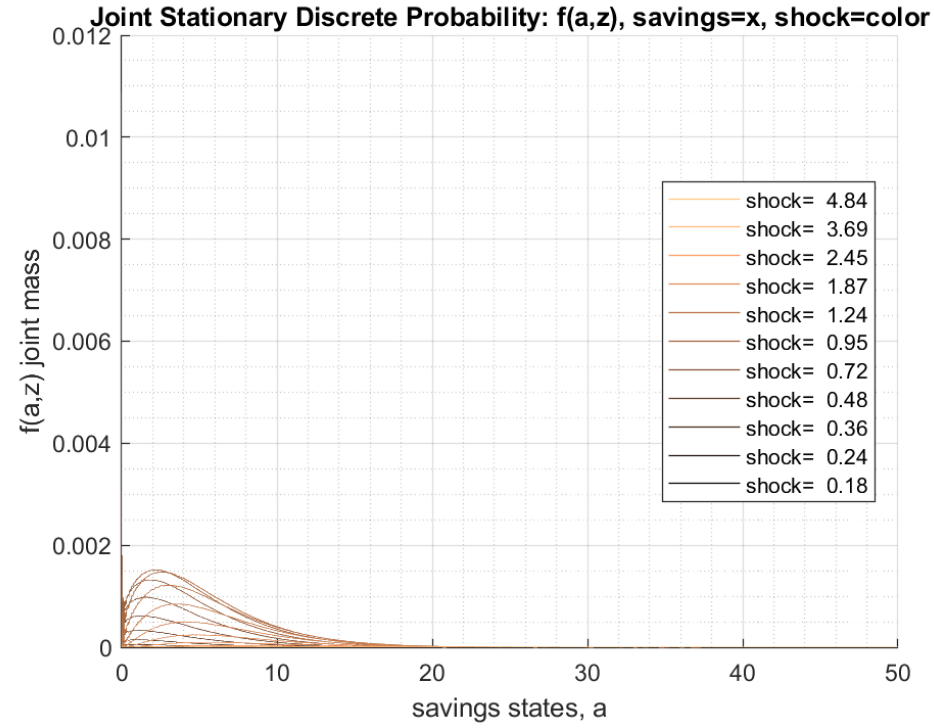
{'p99_99'	}	25.265	14.885	2.9563	3.7789	28.175
{'fl_cov_ap'	}	10.603	6.2617	1.0053	1.0453	11.608
{'fl_cor_ap'	}	1	0.89408	0.8843	0.60489	0.99896
{'fl_cov_v'	}	6.2617	4.626	0.74802	0.96794	7.0097
{'fl_cor_v'	}	0.89408	1	0.99613	0.848	0.91325
{'fl_cov_c'	}	1.0053	0.74802	0.1219	0.15425	1.1272
{'fl_cor_c'	}	0.8843	0.99613	1	0.83252	0.9047
{'fl_cov_y'	}	1.0453	0.96794	0.15425	0.28164	1.1995
{'fl_cor_y'	}	0.60489	0.848	0.83252	1	0.63337
{'fl_cov_coh'	}	11.608	7.0097	1.1272	1.1995	12.735
{'fl_cor_coh'	}	0.99896	0.91325	0.9047	0.63337	1
{'fl_cov_savefraccoh'	}	0.65544	0.47179	0.078595	0.078136	0.73404
{'fl_cor_savefraccoh'	}	0.78925	0.86007	0.88265	0.57729	0.8065
{'fracByP0_01'	}	0	0.001221	0.0019051	0.0019058	0.00061329
{'fracByP0_1'	}	0	0.001221	0.0019051	0.0019058	0.00061329
{'fracByP1'	}	0	0.011511	0.013437	0.0039104	0.0042425
{'fracByP5'	}	0	0.021279	0.026546	0.024488	0.012268
{'fracByP10'	}	0.0006892	0.05109	0.059758	0.051739	0.020676
{'fracByP20'	}	0.0099846	0.12278	0.1366	0.12131	0.052438
{'fracByP25'	}	0.019425	0.15429	0.17945	0.15485	0.072434
{'fracByP30'	}	0.032212	0.19399	0.22206	0.19029	0.094665
{'fracByP40'	}	0.0737	0.28144	0.31482	0.27941	0.15063
{'fracByP50'	}	0.1321	0.3768	0.41124	0.37234	0.22365
{'fracByP60'	}	0.21336	0.48025	0.51513	0.4642	0.31463
{'fracByP70'	}	0.3254	0.59015	0.62157	0.57794	0.42288
{'fracByP75'	}	0.39769	0.65462	0.67967	0.6363	0.48537
{'fracByP80'	}	0.47503	0.71232	0.73844	0.70062	0.56134
{'fracByP90'	}	0.67403	0.84445	0.86104	0.82867	0.73331
{'fracByP95'	}	0.80886	0.92029	0.92647	0.90776	0.84668
{'fracByP99'	}	0.95057	0.98162	0.98401	0.97831	0.96163
{'fracByP99_9'	}	0.99336	0.99797	0.99826	0.99778	0.99494
{'fracByP99_99'	}	0.99924	0.99979	0.99981	0.99977	0.9994

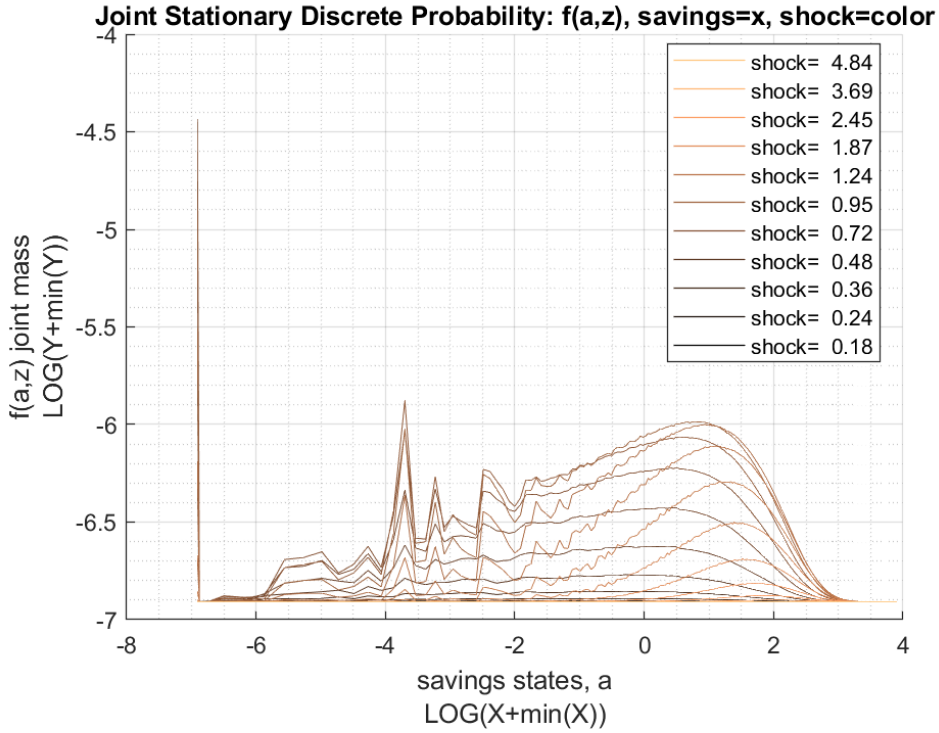
2.3.4 Test FF_DS_AZ_CTS_VEC A grid 300 Shock grid 25

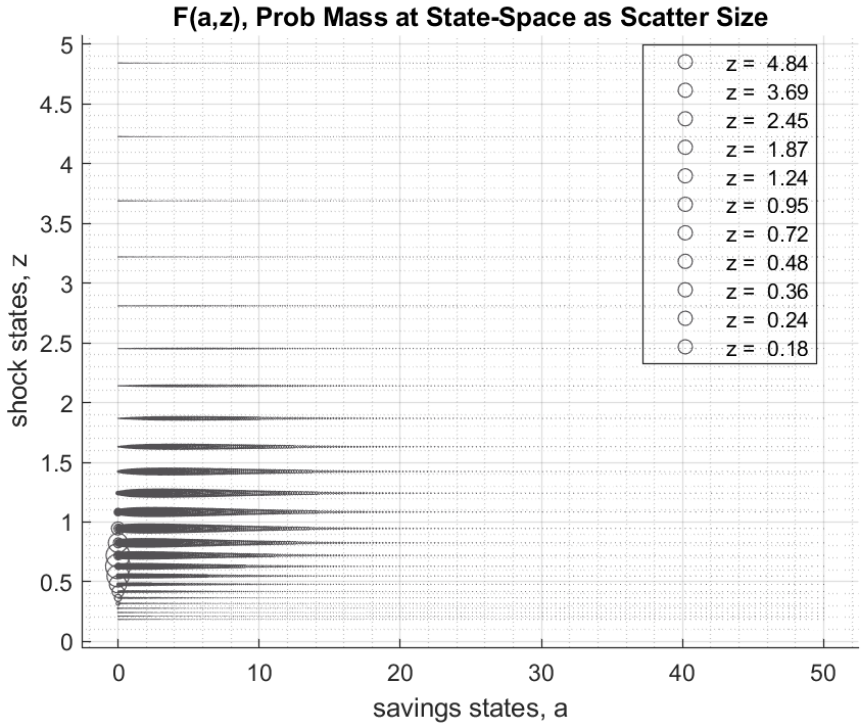
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {'faz','fa'};
mp_support('bl_show_stats_table') = true;
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 300;
mp_params('it_z_n') = 25;
ff_ds_az_cts_vec(mp_params, mp_support);

Elapsed time is 7.884421 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 0.34095
```


Marginal Stationary Probability: f(a), savings=x
f(a) marginal mass
savings states, a
shock
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
10
20
30
40
50


Marginal Stationary Probability: f(a), savings=x
f(a) marginal mass
LOG(Y+min(Y))
savings states, a
LOG(X+min(X))
shock
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4


Joint Stationary Discrete Probability: f(a,z), savings=x, shock=color
f(a,z) joint mass
savings states, a
shock= 4.84
shock= 3.69
shock= 2.45
shock= 1.87
shock= 1.24
shock= 0.95
shock= 0.72
shock= 0.48
shock= 0.36
shock= 0.24
shock= 0.18


Joint Stationary Discrete Probability: f(a,z), savings=x, shock=color
f(a,z) joint mass
LOG(Y+min(Y))
savings states, a
LOG(X+min(X))
shock= 4.84
shock= 3.69
shock= 2.45
shock= 1.87
shock= 1.24
shock= 0.95
shock= 0.72
shock= 0.48
shock= 0.36
shock= 0.24
shock= 0.18



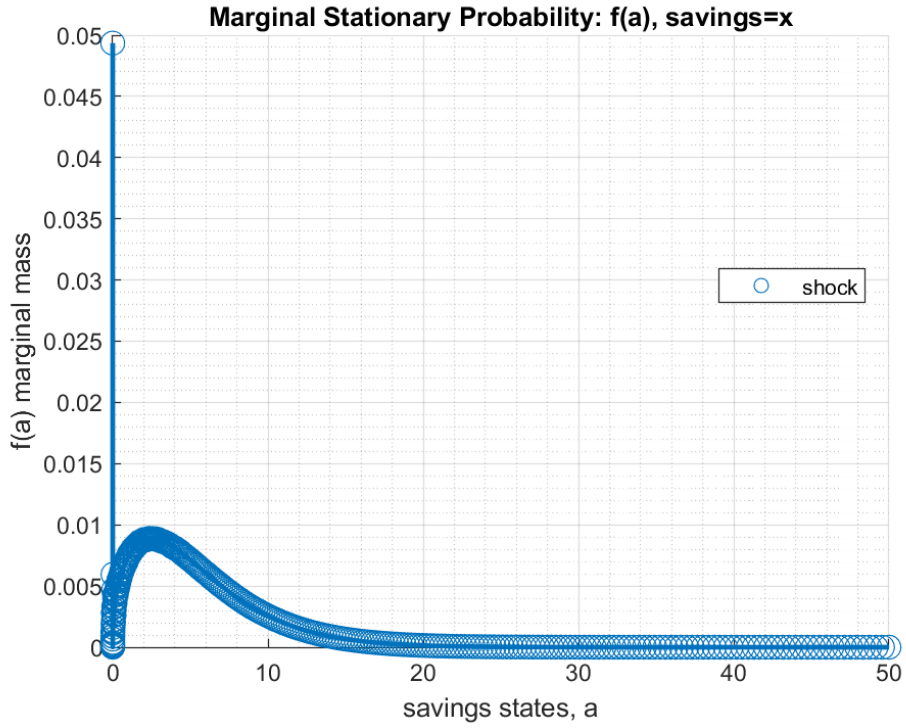
xxx tb_outcomes: all stats xxx

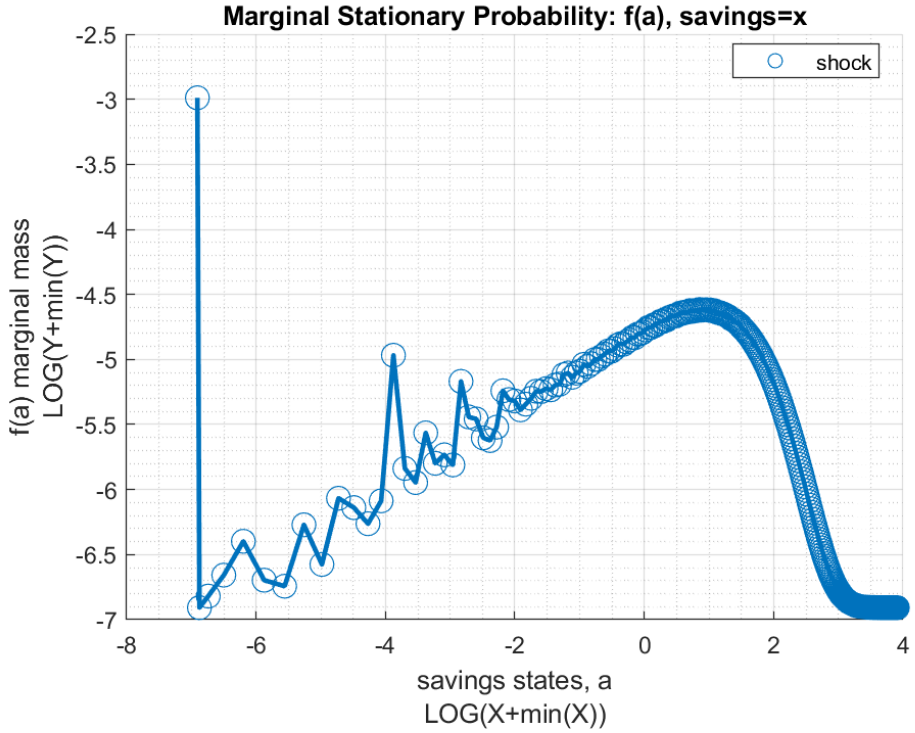
OriginalVariableNames	ap	v	c	y	coh
{'mean' }	3.2612	6.9497	1.5318	1.5305	4.793
{'unweighted_sum' }	1.1043e+05	79555	16733	19751	1.2716e+05
{'sd' }	3.3352	2.1663	0.35078	0.5359	3.6495
{'coefofvar' }	1.0227	0.31171	0.229	0.35014	0.76143
{'gini' }	0.52534	0.17597	0.12824	0.19145	0.39608
{'min' }	0	-2.7616	0.25871	0.25871	0.25871
{'max' }	54.451	20.418	4.3301	8.7798	58.78
{'pYis0' }	0.04941	0	0	0	0
{'pYls0' }	0	7.3281e-05	0	0	0
{'pYgr0' }	0.95059	0.99993	1	1	1
{'pYisMINY' }	0.04941	3.1163e-08	3.1163e-08	3.1163e-08	3.1163e-08
{'pYisMAXY' }	2.8477e-13	2.8477e-13	1.121e-13	2.8477e-13	2.8477e-13
{'p0_01' }	0	0.33584	0.44588	0.42374	0.44588
{'p0_1' }	0	1.0287	0.51088	0.51088	0.51088
{'p1' }	0	2.33	0.67226	0.67069	0.67505
{'p5' }	0.0027154	3.5353	0.94151	0.8016	1.0088
{'p10' }	0.11496	4.1978	1.0921	0.9095	1.2356
{'p20' }	0.51133	5.096	1.2504	1.0657	1.779
{'p25' }	0.75298	5.4004	1.3077	1.1577	2.0685
{'p30' }	1.004	5.7312	1.3565	1.1951	2.3792
{'p40' }	1.5834	6.298	1.4458	1.3352	3.0372
{'p50' }	2.2686	6.8433	1.5287	1.441	3.7996
{'p60' }	3.0898	7.4098	1.6132	1.5764	4.6904
{'p70' }	4.0971	8.0297	1.7037	1.7526	5.7899
{'p75' }	4.7228	8.3787	1.7552	1.8223	6.462
{'p80' }	5.4827	8.7742	1.8144	1.9267	7.2769
{'p90' }	7.7718	9.8224	1.9746	2.2406	9.6945
{'p95' }	9.9683	10.704	2.1148	2.5163	12.048
{'p99' }	14.759	12.325	2.3956	3.157	17.176
{'p99_9' }	21.215	14.066	2.7525	3.9803	23.946

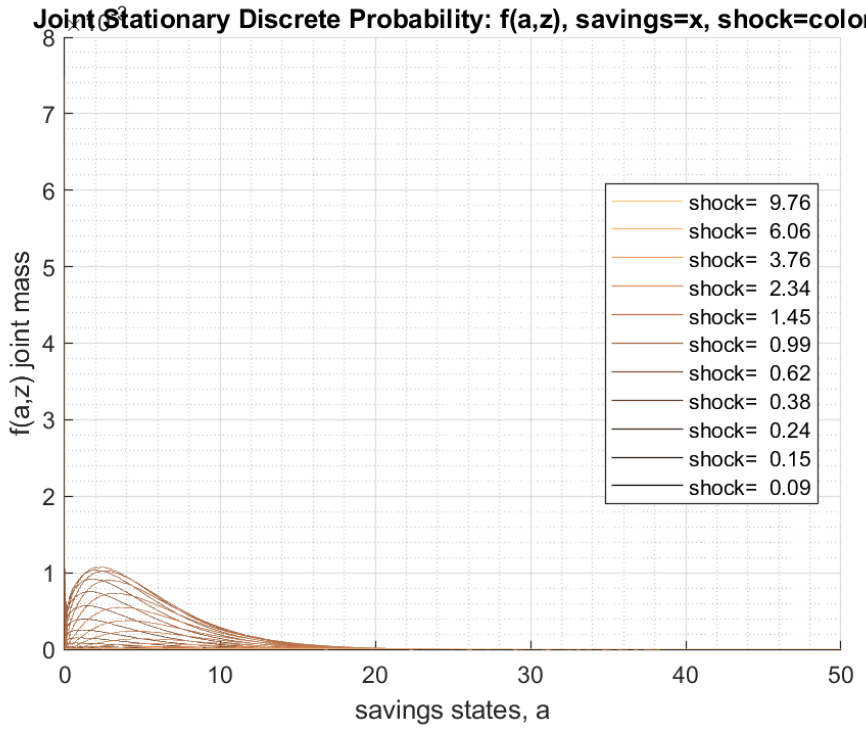
{'p99_99' }	27.205	15.415	3.0759	4.7968	30.277
{'fl_cov_ap' }	11.123	6.4528	1.0361	1.0808	12.16
{'fl_cor_ap' }	1	0.89313	0.88563	0.60472	0.999
{'fl_cov_v' }	6.4528	4.6928	0.75717	0.98035	7.21
{'fl_cor_v' }	0.89313	1	0.99643	0.84447	0.91198
{'fl_cov_c' }	1.0361	0.75717	0.12304	0.15594	1.1592
{'fl_cor_c' }	0.88563	0.99643	1	0.82954	0.90548
{'fl_cov_y' }	1.0808	0.98035	0.15594	0.28718	1.2368
{'fl_cor_y' }	0.60472	0.84447	0.82954	1	0.63237
{'fl_cov_coh' }	12.16	7.21	1.1592	1.2368	13.319
{'fl_cor_coh' }	0.999	0.91198	0.90548	0.63237	1
{'fl_cov_savefraccoh'}	0.65691	0.46786	0.07767	0.077234	0.73458
{'fl_cor_savefraccoh'}	0.78162	0.85705	0.87868	0.57192	0.79876
{'fracByP0_01' }	0	7.2341e-06	8.9677e-05	2.5415e-05	2.8657e-05
{'fracByP0_1' }	0	0.00014925	0.00040034	0.00047536	0.00012777
{'fracByP1' }	0	0.0031002	0.004056	0.0057421	0.0012982
{'fracByP5' }	4.4271e-07	0.020663	0.026101	0.023318	0.010275
{'fracByP10' }	0.00081444	0.049128	0.059669	0.051817	0.020124
{'fracByP20' }	0.010142	0.11647	0.13733	0.1174	0.051401
{'fracByP25' }	0.0197	0.15487	0.17845	0.15395	0.07176
{'fracByP30' }	0.033115	0.19474	0.22243	0.19298	0.095014
{'fracByP40' }	0.07268	0.28138	0.31442	0.27544	0.15079
{'fracByP50' }	0.13241	0.3756	0.41097	0.36527	0.22198
{'fracByP60' }	0.21444	0.47892	0.51282	0.46572	0.31091
{'fracByP70' }	0.323	0.58868	0.62139	0.57261	0.41949
{'fracByP75' }	0.39061	0.6478	0.67743	0.63129	0.48319
{'fracByP80' }	0.46952	0.70943	0.73587	0.6919	0.55532
{'fracByP90' }	0.66831	0.84297	0.85906	0.82754	0.72955
{'fracByP95' }	0.80219	0.91616	0.92541	0.90507	0.84194
{'fracByP99' }	0.94613	0.98125	0.98339	0.97711	0.95822
{'fracByP99_9' }	0.9927	0.9979	0.99812	0.99719	0.99443
{'fracByP99_99' }	0.99909	0.99977	0.99979	0.99967	0.99932

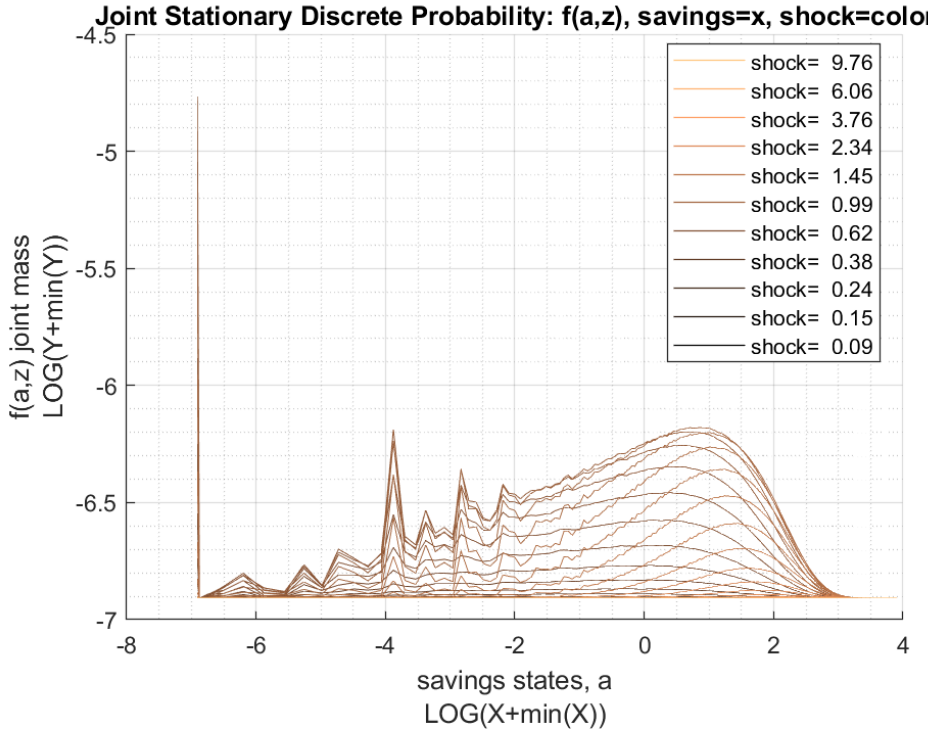
2.3.5 Test FF_DS_AZ_CTS_VEC A grid 300 Shock grid 50

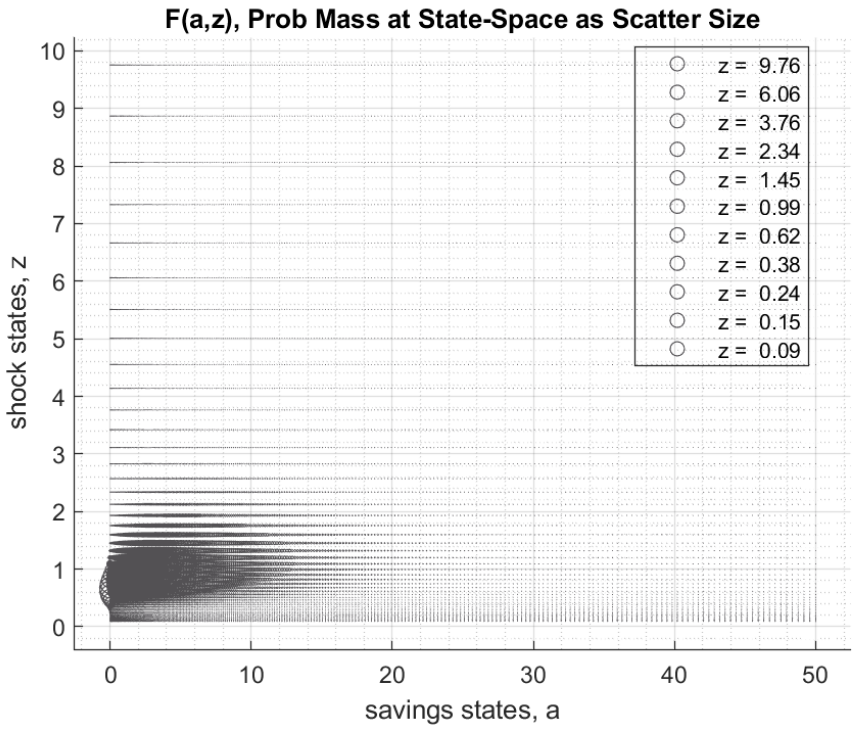
```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
mp_support('ls_ddcmd') = {};
mp_support('ls_ddgrh') = {'faz','fa'};
mp_support('bl_show_stats_table') = true;
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 300;
mp_params('it_z_n') = 50;
ff_ds_az_cts_vec(mp_params, mp_support);

Elapsed time is 14.233149 seconds.
FF_DS_AZ_CTS_LOOP finished. Distribution took = 1.2257
```


Marginal Stationary Probability: f(a), savings=x
f(a) marginal mass
savings states, a
shock
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
10
20
30
40
50


Marginal Stationary Probability: f(a), savings=x
f(a) marginal mass
LOG(Y+min(Y))
savings states, a
LOG(X+min(X))
shock
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-8
-6
-4
-2
0
2
4


Joint Stationary Discrete Probability: f(a,z), savings=x, shock=color
×10^-3
f(a,z) joint mass
savings states, a
shock= 9.76
shock= 6.06
shock= 3.76
shock= 2.34
shock= 1.45
shock= 0.99
shock= 0.62
shock= 0.38
shock= 0.24
shock= 0.15
shock= 0.09


Joint Stationary Discrete Probability: f(a,z), savings=x, shock=color
f(a,z) joint mass
LOG(Y+min(Y))
savings states, a
LOG(X+min(X))
shock= 9.76
shock= 6.06
shock= 3.76
shock= 2.34
shock= 1.45
shock= 0.99
shock= 0.62
shock= 0.38
shock= 0.24
shock= 0.15
shock= 0.09



xxx tb_outcomes: all stats xxx

OriginalVariableNames	ap	v	c	y	coh
{'mean' }	3.2794	6.957	1.5328	1.5312	4.8122
{'unweighted_sum' }	2.3346e+05	1.6237e+05	34668	53309	2.6813e+05
{'sd' }	3.3623	2.1722	0.35142	0.53693	3.6772
{'coefofvar' }	1.0253	0.31224	0.22927	0.35065	0.76415
{'gini' }	0.52595	0.17618	0.12829	0.19144	0.3969
{'min' }	0	-7.6866	0.12843	0.12843	0.12843
{'max' }	61.275	22.164	4.3849	15.657	65.657
{'pYis0' }	0.049376	0	0	0	0
{'pYls0' }	0	0.00011917	0	0	0
{'pYgr0' }	0.95062	0.99988	1	1	1
{'pYisMINY' }	0.049376	1.1048e-15	1.1048e-15	1.1048e-15	1.1048e-15
{'pYisMAXY' }	1.584e-18	1.584e-18	5.0847e-19	1.584e-18	1.584e-18
{'p0_01' }	0	-0.20427	0.40271	0.40271	0.40271
{'p0_1' }	0	1.2141	0.53589	0.48816	0.53589
{'p1' }	0	2.3693	0.71312	0.64833	0.71312
{'p5' }	0.001023	3.5435	0.94895	0.80724	0.96945
{'p10' }	0.11645	4.2417	1.0917	0.93681	1.2501
{'p20' }	0.50875	5.08	1.2515	1.072	1.7735
{'p25' }	0.75899	5.4247	1.3061	1.1504	2.0649
{'p30' }	1.0156	5.7325	1.3564	1.2011	2.3741
{'p40' }	1.6036	6.2932	1.4459	1.3198	3.0387
{'p50' }	2.2768	6.8406	1.5297	1.4423	3.8053
{'p60' }	3.0945	7.4051	1.6122	1.5771	4.7002
{'p70' }	4.113	8.0338	1.7042	1.7334	5.8225
{'p75' }	4.7604	8.3794	1.7554	1.8278	6.4985
{'p80' }	5.5142	8.7771	1.8143	1.9295	7.3239
{'p90' }	7.8048	9.8378	1.9756	2.2476	9.7629
{'p95' }	10.007	10.714	2.1161	2.5336	12.107
{'p99' }	14.9	12.348	2.407	3.1578	17.285
{'p99_9' }	21.501	14.13	2.7694	4.0322	24.216

{'p99_99' }	27.735	15.514	3.1037	4.8946	30.851
{'fl_cov_ap' }	11.305	6.5234	1.0466	1.0907	12.352
{'fl_cor_ap' }	1	0.89316	0.88579	0.60415	0.99902
{'fl_cov_v' }	6.5234	4.7186	0.76066	0.98362	7.2841
{'fl_cor_v' }	0.89316	1	0.99645	0.84334	0.9119
{'fl_cov_c' }	1.0466	0.76066	0.1235	0.15645	1.1701
{'fl_cor_c' }	0.88579	0.99645	1	0.82914	0.9055
{'fl_cov_y' }	1.0907	0.98362	0.15645	0.2883	1.2471
{'fl_cor_y' }	0.60415	0.84334	0.82914	1	0.63165
{'fl_cov_coh' }	12.352	7.2841	1.1701	1.2471	13.522
{'fl_cor_coh' }	0.99902	0.9119	0.9055	0.63165	1
{'fl_cov_savefraccoh'}	0.66084	0.46879	0.077707	0.0772	0.73855
{'fl_cor_savefraccoh'}	0.78009	0.85658	0.87766	0.57067	0.79716
{'fracByP0_01' }	0	-7.0657e-06	2.6272e-05	3.0716e-05	8.3673e-06
{'fracByP0_1' }	0	8.1733e-05	0.00058172	0.0003	0.00018482
{'fracByP1' }	0	0.0025825	0.0055755	0.0043105	0.0017358
{'fracByP5' }	1.3446e-07	0.020553	0.028388	0.023343	0.0084443
{'fracByP10' }	0.00082822	0.048923	0.059616	0.051792	0.020041
{'fracByP20' }	0.010119	0.11678	0.1368	0.1176	0.051426
{'fracByP25' }	0.019764	0.15445	0.17846	0.15402	0.071298
{'fracByP30' }	0.033198	0.19437	0.22195	0.19279	0.094487
{'fracByP40' }	0.072799	0.28088	0.31405	0.27516	0.15079
{'fracByP50' }	0.13186	0.37535	0.41129	0.36559	0.22202
{'fracByP60' }	0.21318	0.47748	0.51316	0.46495	0.30966
{'fracByP70' }	0.32222	0.58845	0.62103	0.57307	0.41837
{'fracByP75' }	0.39045	0.64744	0.67785	0.63075	0.48233
{'fracByP80' }	0.46786	0.7092	0.73555	0.69205	0.55399
{'fracByP90' }	0.66756	0.84275	0.8587	0.82726	0.72947
{'fracByP95' }	0.80166	0.91607	0.92521	0.90478	0.84112
{'fracByP99' }	0.94602	0.98111	0.98335	0.97699	0.95791
{'fracByP99_9' }	0.99264	0.99789	0.9981	0.99714	0.99438
{'fracByP99_99' }	0.99908	0.99977	0.99979	0.99966	0.9993

Chapter 3

Summarize Policy and Value

3.1 FF_SUMM_ND_ARRAY Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_summ_nd_array** from the **MEconTools Package**. This function summarizes policy and value functions over states.

3.1.1 Test FF_SUMM_ND_ARRAY Defaults

Call the function with defaults.

```
ff_summ_nd_array();
```

```
xxx  Summ over (a,z), condi age as cols, kids/marriage as rows  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    marry    kids    mean_age_18    mean_age_19    mean_age_20    mean_age_21
    _____    _____    ____    ___________    ___________    ___________    ___________

      1        0        1        0.53448        0.44448         0.5053        0.52914
      2        1        1         0.4564        0.44512        0.44998        0.51775
      3        0        2        0.52415        0.49903        0.48403        0.44429
      4        1        2        0.49235        0.43684        0.44717        0.45226
      5        0        3         0.4668        0.52676        0.49386        0.51855
      6        1        3        0.47097        0.60345        0.58319        0.46238
      7        0        4        0.55484        0.53601        0.53069        0.49323
      8        1        4         0.5283        0.44091        0.53317        0.51062
```

3.1.2 Test FF_SUMM_ND_ARRAY with Random 2 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random 2D dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(5,4);
bl_print_table = true;
ar_st_stats = ["mean"];
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'a', linspace(0,1,size(mn_polval,1))});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'z', linspace(-1,1,size(mn_polval,2))});
disp(mn_polval);
```

```
    0.6965    0.4231    0.3432    0.7380
    0.2861    0.9808    0.7290    0.1825
    0.2269    0.6848    0.4386    0.1755
    0.5513    0.4809    0.0597    0.5316
    0.7195    0.3921    0.3980    0.5318
```

Second, show the entire matrix (no labels):

```
it_aggd = 0;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim2    mean_vardim1_1    mean_vardim1_2    mean_vardim1_3    mean_vardim1_4    mean
    -----    -------    --------------    --------------    --------------    --------------    ----

      1         1          0.69647           0.28614           0.22685            0.55131          0
      2         2          0.42311           0.98076           0.68483            0.48093          0
      3         3          0.34318           0.72905           0.43857           0.059678          0
      4         4            0.738           0.18249           0.17545            0.53155          0
```

Third, rotate row and column, and now with labels:

```
it_aggd = 0;
bl_row = 1;
ar_permute = [2,1];
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group     a      mean_z__1    mean_z__0_33333    mean_z_0_33333    mean_z_1
    -----    ----    ---------    ---------------    --------------    --------

      1         0     0.69647         0.42311            0.34318         0.738
      2      0.25     0.28614         0.98076            0.72905       0.18249
      3       0.5     0.22685         0.68483            0.43857       0.17545
      4      0.75     0.55131         0.48093           0.059678       0.53155
      5         1     0.71947         0.39212            0.39804       0.53183
```

Fourth, dimension one as columns, average over dim 2:

```
it_aggd = 1;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x    mean_z__1    mean_z__0_33333    mean_z_0_33333    mean_z_1
    -----    -    ---------    ---------------    --------------    --------

      1      1     0.49605         0.59235            0.3937        0.43186
```

Fifth, dimension one as rows, average over dim 2:

```
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);

xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group       z         sum       mean        std      coefvari      min         max
```

1	-1	2.4802	0.49605	0.22895	2.1666	0.22685	0.71947
2	-0.33333	2.9617	0.59235	0.24524	2.4154	0.39212	0.98076
3	0.33333	1.9685	0.3937	0.23907	1.6468	0.059678	0.72905
4	1	2.1593	0.43186	0.24575	1.7573	0.17545	0.738

Sixth, dimension two as rows, average over dim 1:

```
ar_permute = [2,1];
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);
```

```
xxx  Random 2D dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxx
    group     a       sum       mean        std      coefvari      min         max
    -----    ----    ------    -------    -------    --------    --------    -------

      1        0     2.2007    0.55019    0.19636     2.8019      0.34318      0.738
      2      0.25    2.1784    0.54461    0.37514     1.4518      0.18249    0.98076
      3       0.5    1.5257    0.38143    0.23212     1.6432      0.17545    0.68483
      4      0.75    1.6235    0.40587    0.23269     1.7443     0.059678    0.55131
      5        1     2.0415    0.51036    0.15361     3.3226      0.39212    0.71947
```

3.1.3 Test FF_SUMM_ND_ARRAY with Random 6 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random ND dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(8,7,6,5,4,3);
bl_print_table = true;
ar_st_stats = ["mean"];
```

Second, summarize over the first four dimensions, row group others:

```
it_aggd = 4;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);
```

```
xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim5    vardim6     sum       mean        std      coefvari       min          max
    -----    -------    -------    ------    -------    -------    --------    ----------    -------

      1         1          1       836.78    0.49808    0.29255     1.7026     8.1888e-05    0.99964
      2         2          1       842.15    0.50128    0.28968     1.7305     6.7838e-05    0.99936
      3         3          1       831.45    0.49491    0.28851     1.7154     0.00091373    0.99989
      4         4          1        843.9    0.50232    0.28154     1.7842     0.00012471    0.99731
      5         1          2       838.99     0.4994     0.2911     1.7156     0.00029749    0.99938
      6         2          2       830.81    0.49453    0.28634     1.7271     0.00027113     0.9992
      7         3          2       832.59    0.49559    0.28682     1.7279     0.00035994    0.99936
      8         4          2       820.42    0.48835    0.29032     1.6821     0.00096259    0.99896
      9         1          3       870.56    0.51819    0.29111     1.7801      0.0010616    0.99951
     10         2          3       854.68    0.50874    0.28458     1.7877       0.001884    0.99965
     11         3          3       838.29    0.49898     0.2891      1.726       0.0019192    0.99945
     12         4          3       842.83    0.50169     0.2877     1.7438     0.00016871    0.99963
```

Third, summarize over the first four dimensions, column group 5th, and row group others:

```
it_aggd = 4;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ["sum"], it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    vardim6    sum_vardim5_1    sum_vardim5_2    sum_vardim5_3    sum_vardim5_4
    -----    -------    -------------    -------------    -------------    -------------

      1          1          836.78           842.15           831.45            843.9
      2          2          838.99           830.81           832.59           820.42
      3          3          870.56           854.68           838.29           842.83
```

Fourth, summarize over the first five dimensions, column group 6th, no row groups:

```
it_aggd = 5;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ["mean", "std"], it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x    mean_vardim6_1    mean_vardim6_2    mean_vardim6_3    std_vardim6_1    std_vardim6
    -----    -    --------------    --------------    --------------    -------------    -----------

      1      1       0.49915           0.49447            0.5069           0.28805          0.28862
```

Fifth, summarize over all six dimensions, summary statistics over the entire dataframe:

```
it_aggd = 6;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

xxx  Random ND dimensional Array Testing Summarizing  xxxxxxxxxxxxxxxxxxxxxxxxxxx
    group    x     sum      mean        std      coefvari       min          max
    -----    -    -----    -------    -------    --------    ----------    -------

      1      1    10083    0.50017    0.28831     1.7349     6.7838e-05    0.99989
```

3.1.4 Test FF_SUMM_ND_ARRAY with Random 7 Dimensional Matrix with All Parameters

Given a random seven dimensional matrix, average over the 2nd, 4th and 5th dimensionals. Show as row groups the 3, 6 and 7th dimensions, and row groups the 1st dimension. Show Coefficient of Variation only.

```
st_title = "avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim.";
rng(123)
mn_polval = rand(3,10,2,10,10,2,3);
ar_permute = [2,4,5,1,3,6,7];
bl_print_table = true;
ar_st_stats = ["coefvari"];
it_aggd = 3; % mean over 3 dims
bl_row = 1; % one var for row group
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'age', [18, 19, 20]});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'savings', linspace(0,1,10)});
cl_mp_datasetdesc{3} = containers.Map({'name', 'labval'}, ...
    {'borrsave', [-1,+1]});
cl_mp_datasetdesc{4} = containers.Map({'name', 'labval'}, ...
    {'shocka', linspace(-5,5,10)});
cl_mp_datasetdesc{5} = containers.Map({'name', 'labval'}, ...
```

```
    {'shockb', linspace(-5,5,10)});
cl_mp_datasetdesc{6} = containers.Map({'name', 'labval'}, ...
    {'marry', [0,1]});
cl_mp_datasetdesc{7} = containers.Map({'name', 'labval'}, ...
    {'region', [1,2,3]});
% call function
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, cl_mp_datasetdes

xxx  avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim.  xxxxxxxxxxxxxxxxxxxx
    group    borrsave    marry    region    cv_age_18    cv_age_19    cv_age_20
    -----    --------    -----    ------    ---------    ---------    ---------

      1         -1         0        1        1.7607       1.7534       1.7065
      2          1         0        1        1.6566       1.7501       1.7042
      3         -1         1        1        1.6608       1.7658       1.7291
      4          1         1        1         1.756       1.7479       1.7606
      5         -1         0        2        1.7314       1.7506        1.786
      6          1         0        2        1.7347        1.728        1.738
      7         -1         1        2        1.7811        1.755       1.7568
      8          1         1        2        1.7445       1.7398       1.7746
      9         -1         0        3        1.7025       1.7286         1.69
     10          1         0        3          1.74       1.7549       1.7356
     11         -1         1        3        1.7147       1.7287       1.7341
     12          1         1        3        1.7919       1.7313       1.7452
```

Chapter 4

Distributional Analysis

4.1 FF_SIMU_STATS Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_simu_stats** from the **MEconTools Package.** This is a gate-way function that computes mean, percentiles, covariance etc between several variables.

4.1.1 Test FF_SIMU_STATS Defaults

Call the function with defaults.

```
ff_simu_stats();
```

```
xxx tb_outcomes: all stats xxx
    OriginalVariableNames     cl_mt_pol_a    cl_mt_pol_c
    ----------------------    -----------    -----------

    {'mean'              }     -0.11081         8.8423
    {'sd'                }       4.1239         6.5845
    {'coefofvar'         }      -37.215        0.74466
    {'min'               }           -7        -6.3772
    {'max'               }            9         21.786
    {'pYis0'             }     0.064259              0
    {'pYls0'             }      0.54867       0.027329
    {'pYgr0'             }      0.38707        0.97267
    {'pYisMINY'          }     0.051764       0.015232
    {'pYisMAXY'          }     0.027329       0.046484
    {'p1'                }           -7        -6.3772
    {'p10'               }           -6        0.27238
    {'p25'               }           -3         5.2138
    {'p50'               }           -1         6.5321
    {'p75'               }            3         13.799
    {'p90'               }            5         16.887
    {'p99'               }            9         21.786
    {'fl_cov_cl_mt_pol_a'}       17.007        -22.084
    {'fl_cor_cl_mt_pol_a'}            1       -0.81327
    {'fl_cov_cl_mt_pol_c'}      -22.084         43.356
    {'fl_cor_cl_mt_pol_c'}     -0.81327              1
    {'fracByP1'          }       3.2699      -0.010985
    {'fracByP10'         }       5.9889      -0.013362
    {'fracByP25'         }       14.165       0.041007
    {'fracByP50'         }       16.208         0.1893
```

```
    {'fracByP75'         }       12.702         0.59539
    {'fracByP90'         }       6.6611          0.8307
    {'fracByP99'         }            1               1
```

4.1.2 Test FF_SIMU_STATS Four States-Points Matrix

Over some (a,z) states that is 3 by 3, c matrix, generate all stats

```
% Set Parameters
mt_x_of_s = [1, 2,  3.0;...
             3, 1,  1.5;...
             4, 3,  2.0];
mt_y_of_s = [2, -10, 9.0;...
             5, 1.1,3.0;...
             1, 3,  -1.5];
mt_z_of_s = [1.1, 2,3.3;...
             2.3, 1,1.5;...
             4, 2.5,2.0];
mp_cl_mt_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
% Mass
rng(123);
mt_f_of_s = rand(size(mt_x_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s, mp_cl_mt_xyz_of_s);

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_mt_x_of_s    cl_mt_y_of_s    cl_mt_z_of_s
    ______________________    ____________    ____________    ____________

    {'mean'               }        2.0763          1.9323          2.0668
    {'sd'                 }        0.9071          5.2239          0.9042
    {'coefofvar'          }       0.43688          2.7034         0.43749
    {'min'                }             1             -10               1
    {'max'                }             4               9               4
    {'pYis0'              }             0               0               0
    {'pYls0'              }             0         0.20441               0
    {'pYgr0'              }             1         0.79559               1
    {'pYisMINY'           }       0.28039         0.10917         0.14247
    {'pYisMAXY'           }      0.044922         0.19422        0.044922
    {'p1'                 }             1             -10               1
    {'p10'                }             1             -10               1
    {'p25'                }             1             1.1             1.1
    {'p50'                }             2               2               2
    {'p75'                }             3               5             2.5
    {'p90'                }             3               9             3.3
    {'p99'                }             4               9               4
    {'fl_cov_cl_mt_x_of_s'}       0.82282           1.589         0.78646
    {'fl_cor_cl_mt_x_of_s'}             1         0.33534         0.95887
    {'fl_cov_cl_mt_y_of_s'}         1.589          27.289          1.8353
    {'fl_cor_cl_mt_y_of_s'}       0.33534               1         0.38856
    {'fl_cov_cl_mt_z_of_s'}       0.78646          1.8353         0.81758
    {'fl_cor_cl_mt_z_of_s'}       0.95887         0.38856               1
    {'fracByP1'           }       0.13504        -0.56498        0.068934
    {'fracByP10'          }       0.13504        -0.56498        0.068934
```

```
    {'fracByP25'          }       0.13504        -0.53456         0.14234
    {'fracByP50'          }       0.42991        -0.39181         0.43856
    {'fracByP75'          }       0.91346        0.095425         0.60296
    {'fracByP90'          }       0.91346               1         0.91306
    {'fracByP99'          }             1               1               1
```

4.1.3 Test FF_SIMU_STATS Four States-Points Matrix Single Column Inputs

Same as before, but now inputs are single column, should have identical results:

```
% Array Inputs
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s(:), zeros(1)};
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_mt_xyz_of_s);

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_mt_x_of_s    cl_mt_y_of_s    cl_mt_z_of_s
    ______________________    ____________    ____________    ____________

    {'mean'               }        2.0763          1.9323          2.0668
    {'sd'                 }        0.9071          5.2239          0.9042
    {'coefofvar'          }       0.43688          2.7034         0.43749
    {'min'                }             1             -10               1
    {'max'                }             4               9               4
    {'pYis0'              }             0               0               0
    {'pYls0'              }             0         0.20441               0
    {'pYgr0'              }             1         0.79559               1
    {'pYisMINY'           }       0.28039         0.10917         0.14247
    {'pYisMAXY'           }      0.044922         0.19422        0.044922
    {'p1'                 }             1             -10               1
    {'p10'                }             1             -10               1
    {'p25'                }             1             1.1             1.1
    {'p50'                }             2               2               2
    {'p75'                }             3               5             2.5
    {'p90'                }             3               9             3.3
    {'p99'                }             4               9               4
    {'fl_cov_cl_mt_x_of_s'}       0.82282           1.589         0.78646
    {'fl_cor_cl_mt_x_of_s'}             1         0.33534         0.95887
    {'fl_cov_cl_mt_y_of_s'}         1.589          27.289          1.8353
    {'fl_cor_cl_mt_y_of_s'}       0.33534               1         0.38856
    {'fl_cov_cl_mt_z_of_s'}       0.78646          1.8353         0.81758
    {'fl_cor_cl_mt_z_of_s'}       0.95887         0.38856               1
    {'fracByP1'           }       0.13504        -0.56498        0.068934
    {'fracByP10'          }       0.13504        -0.56498        0.068934
    {'fracByP25'          }       0.13504        -0.53456         0.14234
    {'fracByP50'          }       0.42991        -0.39181         0.43856
    {'fracByP75'          }       0.91346        0.095425         0.60296
    {'fracByP90'          }       0.91346               1         0.91306
    {'fracByP99'          }             1               1               1
```

4.1.4 Test FF_SIMU_STATS Print Many Details

The Same As before, but now control which percentiles and other details to display.

```
% Array Inputs
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_ar_xyz_of_s('cl_ar_x_of_s') = {mt_x_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('cl_ar_z_of_s') = {mt_z_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('ar_st_y_name') = ["cl_ar_x_of_s", "cl_ar_z_of_s"];

% controls
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_display_detail') = false;
mp_support('bl_display_final') = true;
mp_support('bl_display_drvm2outcomes') = false;
mp_support('ar_fl_percentiles') = [25 50 75];
mp_support('bl_display_drvstats') = true;
mp_support('bl_display_drvm2covcor') = false;

% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_ar_xyz_of_s, mp_support);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: cl_ar_x_of_s
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    2.0763

fl_choice_sd
    0.9071

fl_choice_coefofvar
    0.4369

fl_choice_prob_zero
     0

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
     1

fl_choice_prob_max
    0.0449

tb_disc_cumu
    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

                1                          0.28039                28.039     0.13504
              1.5                          0.13561                 41.6      0.23301
                2                          0.20441                62.041     0.42991
                3                          0.33466                95.508     0.91346
                4                         0.044922                   100           1

    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

                1                          0.28039                28.039     0.13504
```

```
            1.5                        0.13561                41.6     0.23301
              2                        0.20441              62.041     0.42991
              3                        0.33466              95.508     0.91346
              4                       0.044922                 100           1

tb_prob_drv
    percentiles    cl_ar_x_of_sDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ---------------------------------------    --------------------------------

        25                           1                                  0.13504
        50                           2                                  0.42991
        75                           3                                  0.91346

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: cl_ar_z_of_s
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    2.0668

fl_choice_sd
    0.9042

fl_choice_coefofvar
    0.4375

fl_choice_prob_zero
     0

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
     1

fl_choice_prob_max
    0.0449

tb_disc_cumu
    cl_ar_z_of_sDiscreteVal    cl_ar_z_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

              1                          0.14247                 14.247     0.068934
            1.1                          0.13792                 28.039      0.14234
            1.5                          0.13561                   41.6      0.24076
              2                          0.20441                 62.041      0.43856
            2.3                          0.056663                67.708      0.50162
            2.5                          0.083786                76.086      0.60296
            3.3                           0.19422                95.508      0.91306
              4                         0.044922                   100           1

    cl_ar_z_of_sDiscreteVal    cl_ar_z_of_sDiscreteValProbMass     CDF      cumsumFrac
    -----------------------    -------------------------------    ------    ----------

              1                          0.14247                 14.247     0.068934
            1.1                          0.13792                 28.039      0.14234
            1.5                          0.13561                   41.6      0.24076
```

```
              2                      0.20441              62.041      0.43856
            2.3                     0.056663              67.708      0.50162
            2.5                     0.083786              76.086      0.60296
            3.3                      0.19422              95.508      0.91306
              4                     0.044922                 100            1

tb_prob_drv
    percentiles    cl_ar_z_of_sDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ---------------------------------------    --------------------------------

        25                          1.1                                0.14234
        50                            2                                0.43856
        75                          2.5                                0.60296

xxx tb_outcomes: all stats xxx
     OriginalVariableNames     cl_ar_x_of_s    cl_ar_z_of_s
    ----------------------    ------------    ------------

    {'mean'               }        2.0763          2.0668
    {'sd'                 }        0.9071          0.9042
    {'coefofvar'          }       0.43688         0.43749
    {'min'                }             1               1
    {'max'                }             4               4
    {'pYis0'              }             0               0
    {'pYls0'              }             0               0
    {'pYgr0'              }             1               1
    {'pYisMINY'           }       0.28039         0.14247
    {'pYisMAXY'           }      0.044922        0.044922
    {'p25'                }             1             1.1
    {'p50'                }             2               2
    {'p75'                }             3             2.5
    {'fl_cov_cl_ar_x_of_s'}       0.82282         0.78646
    {'fl_cor_cl_ar_x_of_s'}             1         0.95887
    {'fl_cov_cl_ar_z_of_s'}       0.78646         0.81758
    {'fl_cor_cl_ar_z_of_s'}       0.95887               1
    {'fracByP25'          }       0.13504         0.14234
    {'fracByP50'          }       0.42991         0.43856
    {'fracByP75'          }       0.91346         0.60296
```

4.2 FF_DISC_RAND_VAR_STATS Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_stats** from the **MEconTools Package**. This function summarizes statistics of matrixes stored in a container map, as well as scalar, string, function and other values stored in container maps.

4.2.1 Test FF_DISC_RAND_VAR_STATS Defaults

Call the function with defaults.

```
ff_disc_rand_var_stats();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: binom
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
```

```
fl_choice_mean
   -1.0000

fl_choice_sd
    2.5100

fl_choice_coefofvar
   -2.5100

fl_choice_prob_zero
    0.1416

fl_choice_prob_below_zero
    0.5888

fl_choice_prob_above_zero
    0.2696

fl_choice_prob_max
   2.0589e-16

tb_disc_cumu
    binomDiscreteVal    binomDiscreteValProbMass       CDF       cumsumFrac
    ----------------    ------------------------    ---------    ----------

          -10                 2.2539e-05           0.0022539    0.00022539
           -9                 0.00028979            0.031233     0.0028335
           -8                  0.0018008             0.21132       0.01724
           -7                  0.0072034             0.93166      0.067664
           -6                   0.020838              3.0155       0.19269
           -5                    0.04644              7.6595       0.42489
           -4                   0.082928              15.952       0.75661
           -3                    0.12185              28.138        1.1222
           -2                    0.15014              43.152        1.4224
           -1                    0.15729              58.881        1.5797

    binomDiscreteVal    binomDiscreteValProbMass    CDF    cumsumFrac
    ----------------    ------------------------    ---    ----------

           11                 6.0392e-06           100         1
           12                 1.0588e-06           100         1
           13                 1.5784e-07           100         1
           14                  1.973e-08           100         1
           15                 2.0293e-09           100         1
           16                 1.6725e-10           100         1
           17                 1.0619e-11           100         1
           18                 4.8762e-13           100         1
           19                 1.4412e-14           100         1
           20                 2.0589e-16           100         1

tb_prob_drv
    percentiles    binomDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    --------------------------------    --------------------------------

        0.1                       -8                                0.01724
          1                       -6                                0.19269
          5                       -5                                0.42489
         10                       -4                                0.75661
```

15	-4	0.75661
20	-3	1.1222
25	-3	1.1222
35	-2	1.4224
50	-1	1.5797
65	0	1.5797
75	1	1.4694
80	1	1.4694
85	2	1.3197
90	2	1.3197
95	3	1.1865
99	5	1.0412
99.9	7	1.0052

4.2.2 Test FF_DISC_RAND_VAR_STATS 0 and 1 Random Variable

The simplest discrete random variable has two values, zero or one. The probability of zero is 30 percent, and 70 percent is the probability of one.

```
% Parameters
% 1. specify the random variable
st_var_name = 'bernoulli';
ar_choice_unique_sorted = [0, 1];
ar_choice_prob = [0.3, 0.7];
% 2. percentiles of interest
ar_fl_percentiles = [0.1 5 25 50 75 95 99.9];
% 3. print resutls
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: bernoulli
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    0.7000

fl_choice_sd
    0.4583

fl_choice_coefofvar
    0.6547

fl_choice_prob_zero
    0.3000

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
    0.7000

fl_choice_prob_max
    0.7000
```

```
tb_disc_cumu
    bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
    --------------------    ----------------------------    ---    ----------

              0                           0.3                30        0
              1                           0.7               100        1

    bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
    --------------------    ----------------------------    ---    ----------

              0                           0.3                30        0
              1                           0.7               100        1

tb_prob_drv
    percentiles    bernoulliDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ------------------------------------    --------------------------------

        0.1                         0                                  0
          5                         0                                  0
         25                         0                                  0
         50                         1                                  1
         75                         1                                  1
         95                         1                                  1
       99.9                         1                                  1
```

4.2.3 Test FF_DISC_RAND_VAR_STATS with Poisson

Poisson random variable, with mean equals to ten, summarize over umsymmetric percentiles. Note that the poisson random variable has no upper bound.

```
% Parameters
% 1. specify the random variable
st_var_name = 'poisson';
mu = 10;
ar_choice_unique_sorted = 0:1:50;
ar_choice_prob = poisspdf(ar_choice_unique_sorted, mu);
% 2. percentiles of interest, unsymmetric
ar_fl_percentiles = [0.1 5 10 25 50 90 95 99 99.9 99.99 99.999 99.9999];
% 3. print resutls
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: poisson
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
----------------------------------------
fl_choice_mean
    10

fl_choice_sd
    3.1623

fl_choice_coefofvar
    0.3162
```

```
fl_choice_prob_zero
   4.5400e-05

fl_choice_prob_below_zero
     0

fl_choice_prob_above_zero
    1.0000

fl_choice_prob_max
   1.4927e-19

tb_disc_cumu
    poissonDiscreteVal    poissonDiscreteValProbMass      CDF      cumsumFrac
    ------------------    --------------------------    -------    ----------

            0                      4.54e-05             0.00454           0
            1                      0.000454             0.04994     4.54e-05
            2                       0.00227             0.27694    0.0004994
            3                     0.0075667              1.0336    0.0027694
            4                      0.018917              2.9253     0.010336
            5                      0.037833              6.7086     0.029253
            6                      0.063055              13.014     0.067086
            7                      0.090079              22.022      0.13014
            8                        0.1126              33.282      0.22022
            9                       0.12511              45.793      0.33282

    poissonDiscreteVal    poissonDiscreteValProbMass    CDF    cumsumFrac
    ------------------    --------------------------    ---    ----------

            41                    1.3571e-13            100        1
            42                    3.2313e-14            100        1
            43                    7.5146e-15            100        1
            44                    1.7079e-15            100        1
            45                    3.7953e-16            100        1
            46                    8.2506e-17            100        1
            47                    1.7554e-17            100        1
            48                    3.6572e-18            100        1
            49                    7.4636e-19            100        1
            50                    1.4927e-19            100        1

tb_prob_drv
    percentiles    poissonDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
    -----------    ----------------------------------    --------------------------------

         0.1                       2                                0.0004994
           5                       5                                 0.029253
          10                       6                                 0.067086
          25                       8                                  0.22022
          50                      10                                  0.45793
          90                      14                                  0.86446
          95                      15                                  0.91654
          99                      18                                  0.98572
        99.9                      21                                  0.99841
       99.99                      24                                  0.99988
      99.999                      26                                  0.99998
         100                      28                                        1
```

```
% Print out full Stored Matrix
% Note that the outputs are single row arrays.
ff_container_map_display(ds_stats_map, 100, 100)

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                               i    idx    ndim    numel    rowN    colN     sum       mean       st
                               -    ---    ----    -----    ----    ----    ------    -------    ---

    ar_choice_perc_fracheld    1     1      2       12       1       12       7.54    0.62833     0.
    ar_choice_percentiles      2     2      2       12       1       12        177      14.75    8.7
    ar_fl_percentiles          3     3      2       12       1       12     773.99     64.499    42.

xxx TABLE:ar_choice_perc_fracheld xxxxxxxxxxxxxxxxx
             c1           c2           c3          c4          c5          c6          c7          c8
          ---------    --------    --------    -------    -------    -------    -------    -------

    r1    0.0004994    0.029253    0.067086    0.22022    0.45793    0.86446    0.91654    0.98572

xxx TABLE:ar_choice_percentiles xxxxxxxxxxxxxxxxx
          c1    c2    c3    c4    c5    c6    c7    c8    c9    c10    c11    c12
          --    --    --    --    --    --    --    --    --    ---    ---    ---

    r1    2     5     6     8     10    14    15    18    21    24     26     28

xxx TABLE:ar_fl_percentiles xxxxxxxxxxxxxxxxx
          c1     c2    c3    c4    c5    c6    c7    c8     c9       c10       c11       c12
          ---    --    --    --    --    --    --    --    ----    -----    ------    ---

    r1    0.1    5     10    25    50    90    95    99    99.9    99.99    99.999    100

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                 i     idx      value
                                 --    ---    ----------

    fl_choice_coefofvar           1     4        0.31623
    fl_choice_max                 2     5             50
    fl_choice_mean                3     6             10
    fl_choice_min                 4     7              0
    fl_choice_prob_above_zero     5     8        0.99995
    fl_choice_prob_below_zero     6     9              0
    fl_choice_prob_max            7    10     1.4927e-19
    fl_choice_prob_min            8    11       4.54e-05
    fl_choice_prob_zero           9    12       4.54e-05
    fl_choice_sd                 10    13         3.1623
```

4.3 FF_DISC_RAND_VAR_MASS2OUTCOMES Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_mass2outcomes** from the **MEconTools Package.** This function generates sorted discrete random variable from state-space joint distri-

bution.

4.3.1 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2outcomes();

INPUT f(a,z): mt_dist_bystates
    0.0289    0.0465    0.0228    0.0036    0.0001
    0.0241    0.0930    0.0857    0.0241    0.0015
    0.0080    0.0744    0.1285    0.0643    0.0074
    0.0013    0.0297    0.0964    0.0857    0.0186
    0.0001    0.0059    0.0361    0.0571    0.0232
    0.0000    0.0005    0.0054    0.0152    0.0116

INPUT y(a,z): mt_choice_bystates
    -5    -4    -5    -4    -4
    -3    -2    -3    -2    -3
    -1    -1    -1     0     0
     1     1     2     3     1
     4     3     3     4     3
     5     6     5     6     6

OUTPUT f(y): ar_choice_prob_byY
    0.0518
    0.0502
    0.1113
    0.1171
    0.2109
    0.0717
    0.0497
    0.0964
    0.1510
    0.0572
    0.0054
    0.0273

OUTPUT f(y,z): mt_choice_prob_byYZ
    0.0289         0    0.0228         0         0
         0    0.0465         0    0.0036    0.0001
    0.0241         0    0.0857         0    0.0015
         0    0.0930         0    0.0241         0
    0.0080    0.0744    0.1285         0         0
         0         0         0    0.0643    0.0074
    0.0013    0.0297         0         0    0.0186
         0         0    0.0964         0         0
         0    0.0059    0.0361    0.0857    0.0232
    0.0001         0         0    0.0571         0
    0.0000         0    0.0054         0         0
         0    0.0005         0    0.0152    0.0116

OUTPUT f(y,a): mt_choice_prob_byYA
    0.0518         0         0         0         0         0
    0.0502         0         0         0         0         0
         0    0.1113         0         0         0         0
         0    0.1171         0         0         0         0
         0         0    0.2109         0         0         0
         0         0    0.0717         0         0         0
```

```
         0         0         0    0.0497         0         0
         0         0         0    0.0964         0         0
         0         0         0    0.0857    0.0653         0
         0         0         0         0    0.0572         0
         0         0         0         0         0    0.0054
         0         0         0         0         0    0.0273

OUTPUT f(y) and y in table: tb_choice_drv_cur_byY
    binomtestOutcomes    probMassFunction
    -----------------    ----------------

           -5                0.051764
           -4                0.050217
           -3                 0.11126
           -2                 0.11706
           -1                 0.21092
            0                0.071696
            1                0.049682
            2                0.096388
            3                 0.15102
            4                0.057231
            5               0.0054256
            6                0.027329
```

4.3.2 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Four States-Points

Over some (a,z) states that is 2 by 2, matrix or vectorized inputs identical results.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2;3,1];
% stationary mass over assets adn shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);

    0.4039    1.0000
    0.2971    2.0000
    0.2990    3.0000
```

Same as before, but now inputs are single column:

```
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s(:), mt_f_of_s);
disp([ar_f_of_y ar_y_unique_sorted]);

    0.4039    1.0000
    0.2971    2.0000
    0.2990    3.0000
```

4.3.3 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Conditional Mass Outputs

Same inputs as before, but now, also output additional conditional statistis, f(y, a), where a is the row state variable for f(a,z). For conditional statistics, must provide matrix based inputs.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2,0.5;
             3,1,2.0];
% stationary mass over assets adn shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted, mt_f_of_y_srow, mt_f_of_y_scol] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);

    0.2695    0.5000
    0.3765    1.0000
    0.2649    2.0000
    0.0891    3.0000

disp(mt_f_of_y_srow);

    0.2695         0
    0.1215    0.2550
    0.1217    0.1432
         0    0.0891

disp(mt_f_of_y_scol);

         0         0    0.2695
    0.1215    0.2550         0
         0    0.1217    0.1432
    0.0891         0         0
```

4.4 FF_DISC_RAND_VAR_MASS2COVCOR Examples

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_disc_rand_var_mass2covcor** from the **MEconTools Package.** This function calculates covariance and correlation based for two discrete random variables.

4.4.1 Test FF_DISC_RAND_VAR_MASS2COVCOR Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2covcor();

-----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_input_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    ndim    numel    rowN    colN     sum        mean        std       coef
                 -    ---    ----    -----    ----    ----    ------    --------    --------    ----
```

```
    mt_f_of_s    1     5      2       30       6       5          1    0.033333    0.035743     1.0
    mt_x_of_s    2     6      2       30       6       5         25     0.83333      5.3051     6.3
    mt_y_of_s    3     7      2       30       6       5     249.78      8.3259      7.1913    0.86

xxx TABLE:mt_f_of_s xxxxxxxxxxxxxxxxx
              c1            c2           c3          c4          c5
          ----------    ----------    ---------    ---------    ---------

    r1      0.028917      0.046484     0.022848    0.0036146     0.000119
    r2      0.024097      0.092967     0.085679     0.024097    0.0014875
    r3     0.0080324      0.074374      0.12852     0.064259    0.0074374
    r4     0.0013387       0.02975     0.096388     0.085679     0.018593
    r5    0.00011156     0.0059499     0.036146     0.057119     0.023242
    r6    3.7187e-06    0.00047599    0.0054218     0.015232     0.011621

xxx TABLE:mt_x_of_s xxxxxxxxxxxxxxxxx
          c1    c2    c3    c4    c5
          --    --    --    --    --

    r1    -7    -6    -7    -6    -6
    r2    -5    -3    -5    -3    -4
    r3    -2    -1    -1     0    -1
    r4     2     2     3     4     2
    r5     6     5     5     6     5
    r6     8     9     7     9     9

xxx TABLE:mt_y_of_s xxxxxxxxxxxxxxxxx
            c1          c2          c3         c4          c5
          ------    --------    ------    -------    -------

    r1    13.231      21.786    18.136      19.35     13.901
    r2     9.946      16.887    9.6914      15.71     8.6906
    r3    16.255      6.2166    13.799     5.2138     11.641
    r4    12.628      2.7525    6.5321    0.27238     13.357
    r5    5.8844      4.0352      6.05    0.14102    0.50318
    r6    3.5617    -0.72091    5.1855    -6.3772    -4.4805

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_input_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                i    idx     value
                -    ---    --------

    fl_x_mean   1     1     -0.11081
    fl_x_sd     2     2       4.1239
    fl_y_mean   3     3       8.8423
    fl_y_sd     4     4       6.5845

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_output_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i    idx    ndim    numel    rowN    colN      sum        mean
                                -    ---    ----    -----    ----    ----    -------    --------

    mt_cov_component_weighted   1     1      2       30       6       5      -22.084    -0.73612
```

```
    mt_x_devi_from_mean          2     2      2       30       6       5       28.324     0.94415
    mt_x_y_multiply              3     3      2       30       6       5      -939.63     -31.321
    mt_y_devi_from_mean          4     4      2       30       6       5      -15.493    -0.51644

xxx TABLE:mt_cov_component_weighted xxxxxxxxxxxxxxxxx
              c1            c2           c3          c4           c5
          -----------    ---------    --------    ---------    ----------

    r1       -0.87434      -3.5432     -1.4628     -0.22368    -0.0035451
    r2       -0.13003      -2.1607    -0.35565     -0.47814    0.00087767
    r3       -0.11248      0.17365    -0.56642    -0.025838     -0.018507
    r4       0.010697     -0.38241    -0.69273      -3.0184       0.17717
    r5     -0.0020165     -0.14618    -0.51584      -3.0371      -0.99056
    r6    -0.00015927    -0.041473    -0.14098      -2.1121       -1.4106

xxx TABLE:mt_x_devi_from_mean xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          -------    --------    --------    -------    --------

    r1    -6.8892     -5.8892     -6.8892    -5.8892     -5.8892
    r2    -4.8892     -2.8892     -4.8892    -2.8892     -3.8892
    r3    -1.8892    -0.88919    -0.88919    0.11081    -0.88919
    r4     2.1108      2.1108      3.1108     4.1108      2.1108
    r5     6.1108      5.1108      5.1108     6.1108      5.1108
    r6     8.1108      9.1108      7.1108     9.1108      9.1108

xxx TABLE:mt_x_y_multiply xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          -------    -------    -------    --------    -------

    r1    -30.237    -76.225    -64.023     -61.882    -29.792
    r2     -5.396    -23.242     -4.151     -19.842    0.59004
    r3    -14.003     2.3348    -4.4073    -0.40209    -2.4884
    r4     7.9905    -12.854    -7.1868      -35.23     9.5287
    r5    -18.075    -24.568    -14.271     -53.172     -42.62
    r6     -42.83    -87.129    -26.003     -138.66    -121.38

xxx TABLE:mt_y_devi_from_mean xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5
          -------    -------    -------    -------    --------

    r1      4.389      12.943     9.2933      10.508      5.0587
    r2     1.1037      8.0444    0.84902      6.8677    -0.15171
    r3     7.4123    -2.6258      4.9566    -3.6286      2.7985
    r4     3.7855    -6.0898    -2.3103      -8.57      4.5142
    r5    -2.9579    -4.8071    -2.7924    -8.7013     -8.3392
    r6    -5.2806    -9.5633    -3.6568     -15.22     -13.323

fl_cov
  -22.0835

fl_cor
   -0.8133
```

4.4.2 Test FF_DISC_RAND_VAR_MASS2COVCOR Four States-Points

Over some (a,z) states that is 2 by 2, c matrix, and y matrix, find correlation. Positively related.

```
% Set Parameters
```

```
mt_c_of_s = [1,2;3,1];
mt_y_of_s = [2,10;5,1.1];
rng(123);
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=1.4446,cor=0.65723
```

Same as before, but now inputs are single column:

```
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s(:), mt_y_of_s(:), mt_f_of_s(:), bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=1.4446,cor=0.65723
```

4.4.3 Test FF_DISC_RAND_VAR_MASS2COVCOR Two Random Vectors

Generate two random vectors, with random or even mass, correlation should be zero:

```
% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=-57.6533,cor=-0.062023
```

4.4.4 Test FF_DISC_RAND_VAR_MASS2COVCOR Provide Mean and SD

Same as above, but now provide means and sd for x andy directly. The results are the same as when mean and sd are calculated inside the function.

```
% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
fl_c_mean = sum(mt_f_of_s.*mt_c_of_s);
fl_c_sd = sqrt(sum(mt_f_of_s.*(mt_c_of_s-fl_c_mean).^2));
fl_y_mean = sum(mt_f_of_s.*mt_y_of_s);
fl_y_sd = sqrt(sum(mt_f_of_s.*(mt_y_of_s-fl_y_mean).^2));
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
```

```
    mt_c_of_s, mt_y_of_s, mt_f_of_s, ...
    fl_c_mean, fl_c_sd, ...
    fl_y_mean, fl_y_sd, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=-57.6533,cor=-0.062023
```

Chapter 5

Optimizers

5.1 faFF_OPTIM_BISEC_SAVEZRONE Derivative Bisection

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_optim_bisec_savezrone** from the **MEconTools Package.** This functions solves for optimal savings/borrowing level given an anonymous function that provides the derivative of a intertemporal savings problem. The function is solves over a grid of state-space elements that are embeded in the anonymous function. By default, it iterates over 15 iterations with bisection.

The vectorized and looped bisection savings problem rely on this function to solve for optimal savings choices:

- States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Loop***:**ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Vectorized***: **ff_vfi_az_bisec_vec**, precision and speed

5.1.1 Test FF_OPTIM_BISEC_SAVEZRONE Defaults

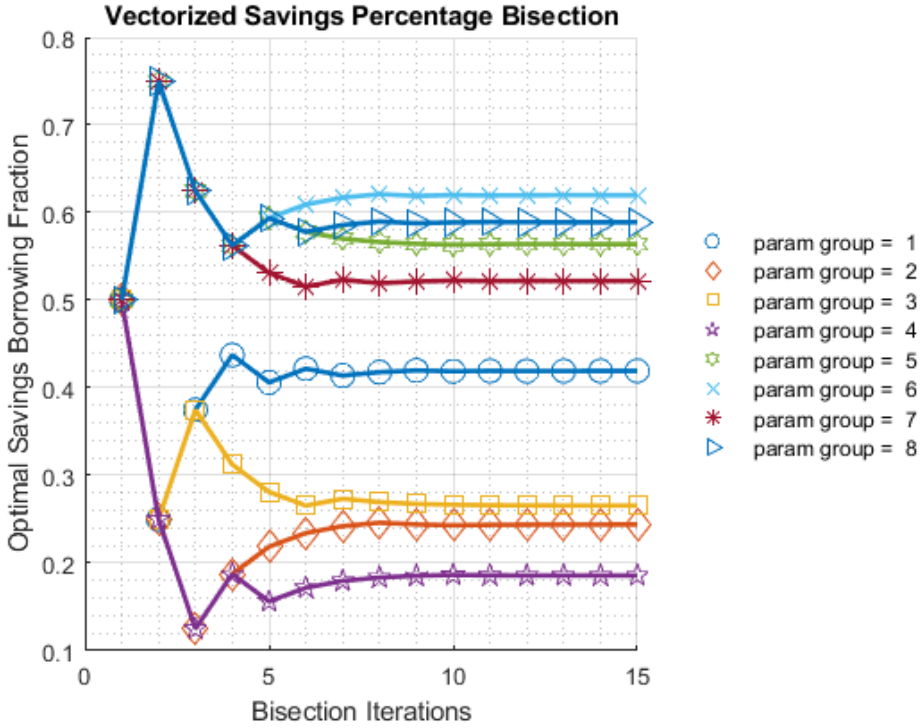
Call the function with defaults, this solves concurrently for many state-space points' optimization problems:

```
ff_optim_bisec_savezrone();
```

```
Elapsed time is 0.105423 seconds.
BISECT END: iteration=16, norm(ar_mid_fx)=0.00030653
```

	vartype	paramgroup2	paramgroup3	paramgroup4	paramgroup5
	-------	----------	----------	----------	----------
a	"init"	1e-05	1e-05	1e-05	1e-05
b	"init"	0.99999	0.99999	0.99999	0.99999
f_a	"init"	33802	40925	67047	15411
f_b	"init"	-46789	-1.2672e+05	-1.8532e+05	-67518
it1_fp	"fatx"	-0.25973	-1.7159	-2.3655	-1.0421
it1_p	"x"	0.5	0.5	0.5	0.5
it2_fp	"fatx"	0.72822	-0.052631	0.21087	-0.28379
it2_p	"x"	0.25	0.25	0.25	0.25
it3_fp	"fatx"	0.15277	1.8256	-1.1773	0.46124
it3_p	"x"	0.375	0.12501	0.375	0.12501
it4_fp	"fatx"	-0.059183	0.62299	-0.55013	-0.0090579

it4_p	"x"	0.4375	0.18751	0.3125	0.18751
it5_fp	"fatx"	0.044028	0.2488	-0.19454	0.1861
it5_p	"x"	0.40625	0.21876	0.28125	0.15626
it6_fp	"fatx"	-0.0080863	0.090981	0.00054305	0.081339
it6_p	"x"	0.42188	0.23438	0.26563	0.17188
it7_fp	"fatx"	0.017822	0.017593	-0.098707	0.034591
it7_p	"x"	0.41406	0.24219	0.27344	0.17969
it8_fp	"fatx"	0.0048335	-0.017893	-0.049532	0.012405
it8_p	"x"	0.41797	0.2461	0.26954	0.1836
it9_fp	"fatx"	-0.0016347	-0.00024633	-0.02461	0.0015865
it9_p	"x"	0.41992	0.24415	0.26758	0.18555
it10_fp	"fatx"	0.0015973	0.0086488	-0.012063	-0.0037571
it10_p	"x"	0.41895	0.24317	0.26661	0.18653
it11_fp	"fatx"	-1.9235e-05	0.0041952	-0.0057672	-0.0010907
it11_p	"x"	0.41944	0.24366	0.26612	0.18604
it12_fp	"fatx"	0.00078889	0.0019729	-0.0026139	0.00024655
it12_p	"x"	0.41919	0.2439	0.26587	0.1858
it13_fp	"fatx"	0.00038479	0.00086292	-0.0010359	-0.00042242
it13_p	"x"	0.41931	0.24402	0.26575	0.18592
it14_fp	"fatx"	0.00018277	0.0003082	-0.00024654	-8.8022e-05
it14_p	"x"	0.41937	0.24408	0.26569	0.18586
it15_fp	"fatx"	8.1766e-05	3.0909e-05	0.00014822	7.9241e-05
it15_p	"x"	0.4194	0.24412	0.26566	0.18583
it15_level	"level"	0.56205	-0.070025	0.044431	-0.039424
exactSoluSaveborrFrac	"exact"	0.41943	0.24412	0.26567	0.18584
exactSoluSaveborrLevel	"exact"	0.56211	-0.070022	0.044438	-0.039403
exactSoluSaveborrFracGap	"exact"	2.4705e-05	3.402e-06	1.1458e-05	1.4456e-05
exactSoluSaveborrLevelGap	"exact"	5.28e-05	2.6845e-06	6.1825e-06	2.1411e-05




```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i     idx     ndim     numel     rowN     colN        sum           mean
```

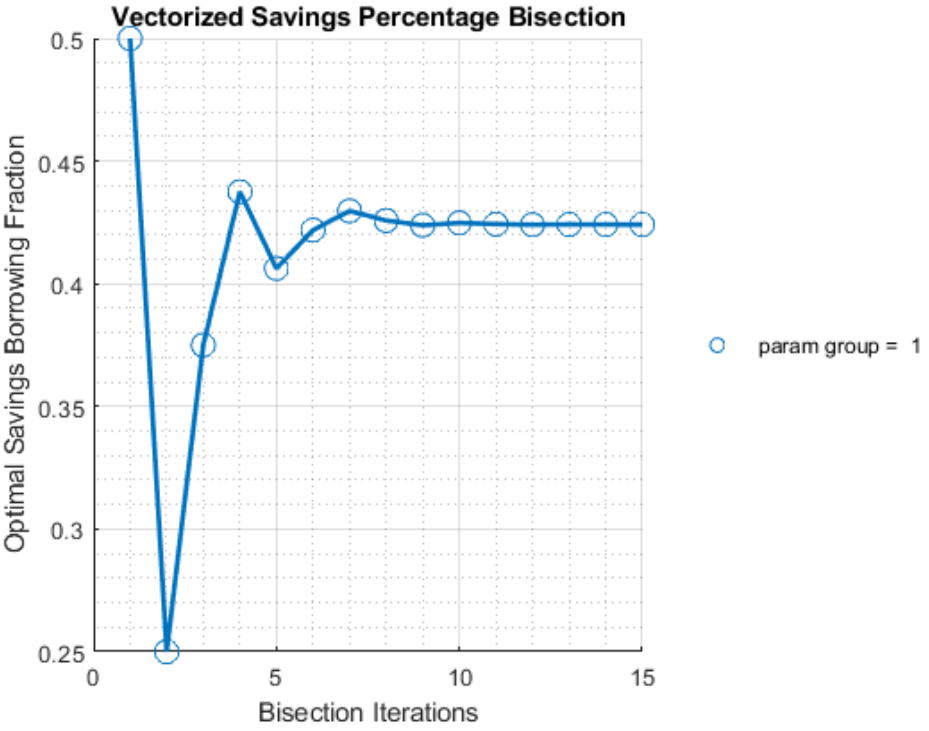
```
                        -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_opti_foc_obj     1     1      2        8       1       8      0.00050535    6.3168e-05    9.
    ar_opti_save_frac   2     2      2        8       1       8            3.41       0.42626

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxx
              c1             c2             c3             c4             c5             c6             c
          ----------     ----------     ----------     ----------     -----------    ----------     -----

    r1    8.1766e-05     3.0909e-05     0.00014822     7.9241e-05     -0.00013343    0.00015981     1.896

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxx
            c1         c2          c3          c4          c5          c6         c7         c8
          ------     -------     -------     -------     -------     ------     -----     -------

    r1    0.4194     0.24412     0.26566     0.18583     0.56406     0.6199     0.522     0.58908
```

5.1.2 Test FF_OPTIM_BISEC_SAVEZRONE One Individual

Bisection for savings choice at one state:

```
% Generate the state-space and function
[fl_z1, fl_z2, fl_r, fl_beta] = deal(0.4730, 0.6252, 0.0839, 0.7365);
% ffi_intertemporal_max is a function in ff_optim_bisec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, fl_z1, fl_z2, fl_r, fl_beta);
% Call Function
bl_verbose = true;
ff_optim_bisec_savezrone(fc_deri_wth_uniroot, bl_verbose);

BISECT END: iteration=16, norm(ar_mid_fx)=0.00016724
                  vartype    paramgroup2
                  -------    -----------

    a             "init"           1e-05
    b             "init"         0.99999
    f_a           "init"           70155
    f_b           "init"          -95255
    it1_fp        "fatx"          -0.502
    it1_p         "x"                0.5
    it2_fp        "fatx"          1.5361
    it2_p         "x"              0.25
    it3_fp        "fatx"         0.34671
    it3_p         "x"              0.375
    it4_fp        "fatx"       -0.089881
    it4_p         "x"              0.4375
    it5_fp        "fatx"         0.12259
    it5_p         "x"              0.40625
    it6_fp        "fatx"        0.015276
    it6_p         "x"              0.42188
    it7_fp        "fatx"       -0.037529
    it7_p         "x"              0.42969
    it8_fp        "fatx"       -0.011188
    it8_p         "x"              0.42578
    it9_fp        "fatx"       0.0020277
    it9_p         "x"              0.42383
    it10_fp       "fatx"      -0.0045843
    it10_p        "x"              0.42481
    it11_fp       "fatx"      -0.0012793
    it11_p        "x"              0.42432
```

```
    it12_fp       "fatx"      0.00037392
    it12_p        "x"            0.42407
    it13_fp       "fatx"     -0.00045276
    it13_p        "x"              0.4242
    it14_fp       "fatx"     -3.9436e-05
    it14_p        "x"            0.42413
    it15_fp       "fatx"      0.00016724
    it15_p        "x"              0.4241
    it15_level    "level"       -0.13158
```



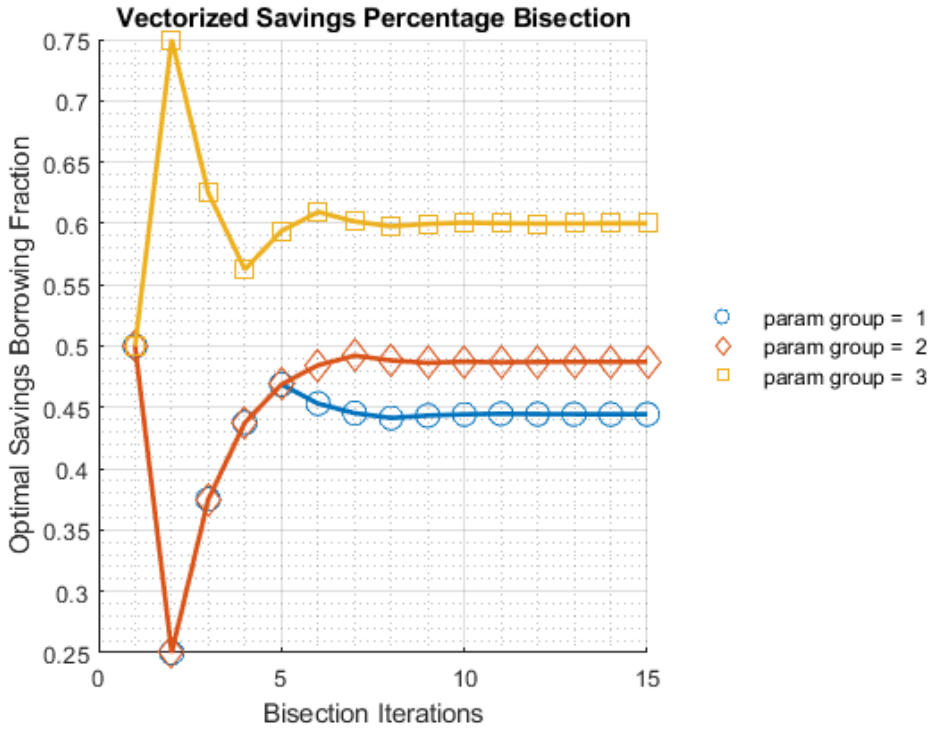

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx      value
                          -    ---    ----------

    ar_opti_foc_obj       1     1     0.00016724
    ar_opti_save_frac     2     2        0.4241
```

5.1.3 Test FF_OPTIM_BISEC_SAVEZRONE Six Individual States

Solve the two period intertemporal optimization problem with only 6 individual states:

```
% Generate the state-space and function
ar_z1 = [1,2,3]';
ar_z2 = [3,2,1]';
ar_r = [1.05, 1.50, 1.30]';
ar_beta = [0.80, 0.95, 1.50]';
mt_fc_inputs = [ar_z1, ar_z2, ar_r, ar_beta];
% ffi_intertemporal_max is a function in ff_optim_bisec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
ff_optim_bisec_savezrone(fc_deri_wth_uniroot, bl_verbose);
```

```
BISECT END: iteration=16, norm(ar_mid_fx)=8.9847e-05
                vartype    paramgroup2    paramgroup3    paramgroup4
                -------    -----------    -----------    -----------

    a           "init"           1e-05          1e-05          1e-05
    b           "init"         0.99999        0.99999        0.99999
    f_a         "init"           32475          33928          43671
    f_b         "init"          -40594         -35714         -29113
    it1_fp      "fatx"        -0.16238      -0.035714        0.29114
    it1_p       "x"                0.5            0.5            0.5
    it2_fp      "fatx"         0.75773        0.88092       -0.58225
    it2_p       "x"               0.25           0.25        0.74999
    it3_fp      "fatx"         0.21649        0.33333       -0.077629
    it3_p       "x"              0.375          0.375          0.625
    it4_fp      "fatx"        0.020615        0.14059        0.11091
    it4_p       "x"              0.4375         0.4375         0.5625
    it5_fp      "fatx"        -0.07132       0.051539       0.018865
    it5_p       "x"              0.46875        0.46875        0.59375
    it6_fp      "fatx"       -0.025599      0.0078193      -0.028659
    it6_p       "x"              0.45313        0.48438        0.60937
    it7_fp      "fatx"      -0.0025711      -0.013955     -0.0047386
    it7_p       "x"              0.44531        0.49219        0.60156
    it8_fp      "fatx"       0.0090001     -0.0030715      0.0071001
    it8_p       "x"              0.44141        0.48828        0.59765
    it9_fp      "fatx"       0.0032093      0.0023727      0.0011903
    it9_p       "x"              0.44336        0.48633        0.59961
    it10_fp     "fatx"      0.00031783    -0.00034971     -0.0017717
    it10_p      "x"              0.44434         0.4873        0.60058
    it11_fp     "fatx"      -0.0011269      0.0010114    -0.00029011
    it11_p      "x"              0.44483        0.48682         0.6001
    it12_fp     "fatx"     -0.00040464     0.00033083     0.00045024
    it12_p      "x"              0.44458        0.48706        0.59985
    it13_fp     "fatx"     -4.3425e-05    -9.4396e-06     8.0103e-05
    it13_p      "x"              0.44446        0.48718        0.59997
    it14_fp     "fatx"       0.0001372      0.0001607       -0.000105
    it14_p      "x"              0.4444        0.48712        0.60003
    it15_fp     "fatx"      4.6884e-05     7.5628e-05    -1.2444e-05
    it15_p      "x"              0.44443        0.48715            0.6
    it15_level  "level"          -0.3686        0.56403         1.6261
```



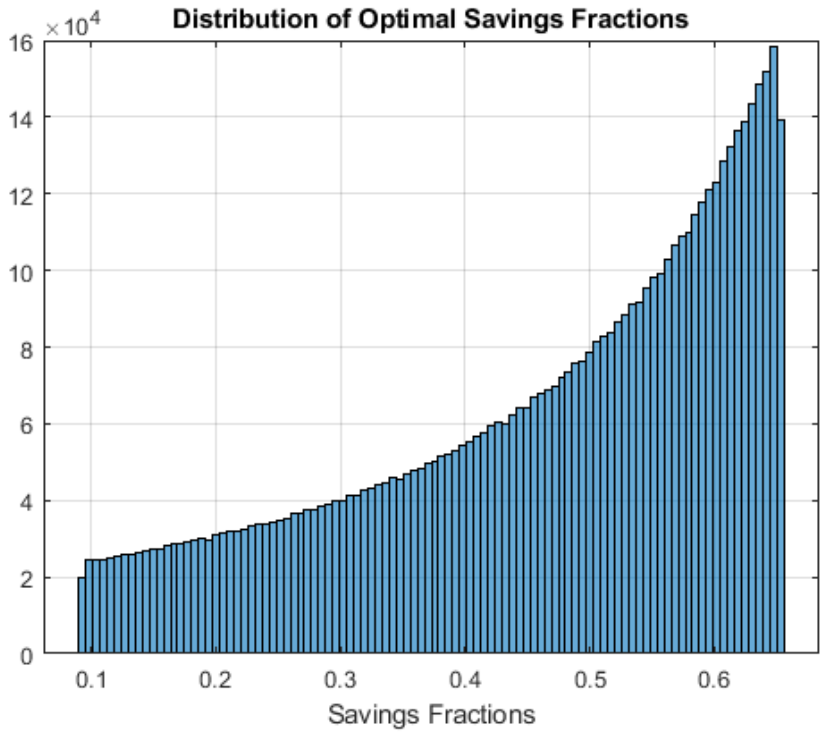

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN       sum           mean          --
                          -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_opti_foc_obj       1     1      2        3       1       3      0.00011007    3.6689e-05    4.
    ar_opti_save_frac     2     2      2        3       1       3          1.5316        0.51053

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx
              c1             c2             c3
          ----------    ----------    -----------

    r1    4.6884e-05    7.5628e-05    -1.2444e-05

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxxx
            c1          c2        c3
          -------    -------    ---

    r1    0.44443    0.48715    0.6
```

5.1.4 Test FF_OPTIM_BISEC_SAVEZRONE Speed

Test Speed doing 6.25 million bisections for a savings problem:

```
% Generate the state-space and function
rng(123);
it_draws = 6250000; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
% ffi_intertemporal_max is a function in ff_optim_bisec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
```

```
% Call Function
bl_verbose = false;
bl_timer = true;
[ar_opti_save_frac, ar_opti_save_level] = ff_optim_bisec_savezrone(fc_deri_wth_uniroot, bl_verbose,

Elapsed time is 2.570982 seconds.

mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
mp_container_map('ar_opti_save_frac') = ar_opti_save_frac;
mp_container_map('ar_opti_save_level') = ar_opti_save_level;
mp_container_map('ar_opti_save_frac_notnan') = ar_opti_save_frac(~isnan(ar_opti_save_frac));
ff_container_map_display(mp_container_map);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i     idx     ndim      numel         rowN       colN        sum          m
                                -     ---     ----     --------     --------     ----     ----------     --

    ar_opti_save_frac           1      1       2       6.25e+06     6.25e+06      1        2.884e+06     0.
    ar_opti_save_frac_notnan    2      2       2       6.25e+06     6.25e+06      1        2.884e+06     0.
    ar_opti_save_level          3      3       2       6.25e+06     6.25e+06      1       2.9482e+06     0.

figure();
histogram(ar_opti_save_frac(~isnan(ar_opti_save_frac)),100);
title('Distribution of Optimal Savings Fractions');
xlabel('Savings Fractions');
grid on;
```



5.1.5 Define Two Period Intertemporal FOC Log Utility No Shock

See Household's Utility Maximization Problem and Two-Period Borrowing and Savings Problem given Endowments.

```
function [ar_deri_zero, ar_saveborr_level] = ffi_intertemporal_max(ar_saveborr_frac, z1, z2, r, beta
    ar_saveborr_level = ar_saveborr_frac.*(z1+z2./(1+r)) - z2./(1+r);
    ar_deri_zero = 1./(ar_saveborr_level-z1) + (beta.*(r+1))./(z2 + ar_saveborr_level.*(r+1));
end
```

5.2 FF_OPTIM_MLSEC_SAVEZRONE Derivative Multisection

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_optim_mlsec_savezrone** from the **MEconTools Package.** This functions solves for optimal savings/borrowing level given an anonymous function that provides the derivative of a intertemporal savings problem. This is a vectorized function solved with multi-section (multiple points bisection concurrently).

The vectorized and looped bisection savings problem rely on this function to solve for optimal savings choices:

- States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Loop***:**ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Vectorized***: **ff_vfi_az_bisec_vec**, precision and speed

5.2.1 Test FF_OPTIM_MLSEC_SAVEZRONE One Individual

Bisection for savings choice at one state:

```
% Generate the state-space and function
[fl_z1, fl_z2, fl_r, fl_beta] = deal(0.4730, 0.6252, 0.0839, 0.7365);
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, fl_z1, fl_z2, fl_r, fl_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
% optimally borrowing given the parameters here
mp_mlsec_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mlsec_ctrlinfo('it_mzoom_jnt_pnts') = 10;
mp_mlsec_ctrlinfo('it_mzoom_max_iter') = 4;
[fl_opti_save_frac, fl_opti_save_level] = ...
    ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, bl_timer, mp_mlsec_ctrlinfo)
```

```
Elapsed time is 0.011265 seconds.
fl_opti_save_frac = 0.4241
fl_opti_save_level = -0.1316
```

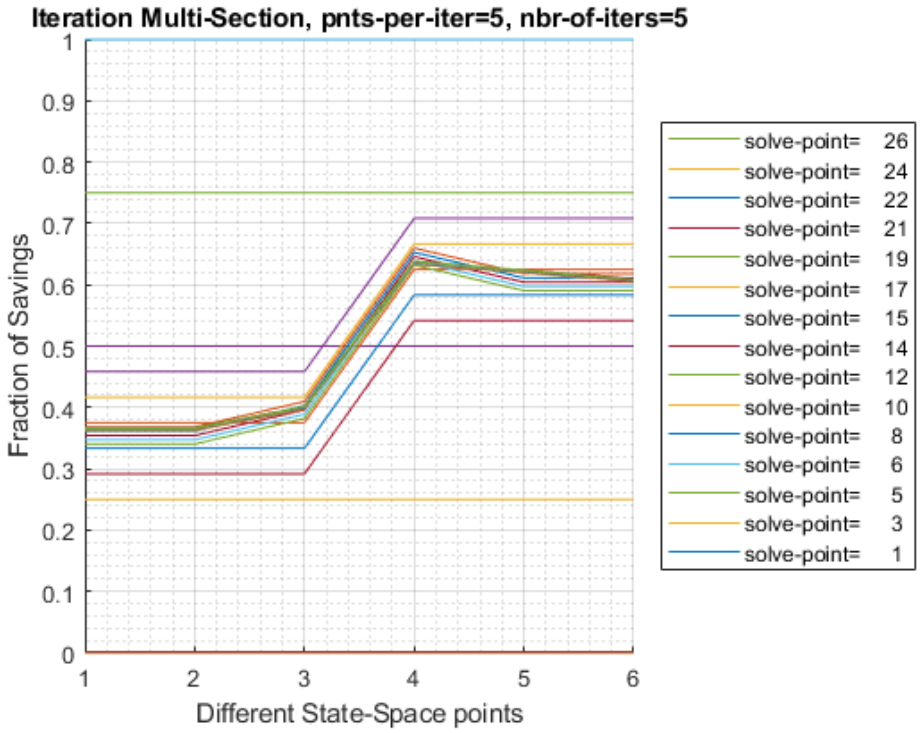
5.2.2 Test FF_OPTIM_MLSEC_SAVEZRONE 5 Individuals 5 Iterations 5 Points Per Iteration

5 grid points per iteration, and 5 iterations.

```
% Generate the state-space and function
rng(123);
it_draws = 6; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
```

```
% Call Function
bl_verbose = true;
bl_timer = true;
mp_mlsec_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mlsec_ctrlinfo('it_mlsect_jnt_pnts') = 5;
mp_mlsec_ctrlinfo('it_mlsect_max_iter') = 5;
ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, bl_timer, mp_mlsec_ctrlinfo);
```

iter	cl_row_names_a	Var1	Var2	Var3	Var4	Var5	Var6
0	"point=1"	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05
1	"point=1"	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05
1	"point=2"	0.25001	0.25001	0.25001	0.25001	0.25001	0.25001
1	"point=3"	0.5	0.5	0.5	0.5	0.5	0.5
1	"point=4"	0.75	0.75	0.75	0.75	0.75	0.75
1	"point=5"	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
2	"point=1"	0.29167	0.29167	0.29167	0.54167	0.54167	0.54167
2	"point=2"	0.33334	0.33334	0.33334	0.58333	0.58333	0.58333
2	"point=3"	0.375	0.375	0.375	0.625	0.625	0.625
2	"point=4"	0.41667	0.41667	0.41667	0.66666	0.66666	0.66666
2	"point=5"	0.45833	0.45833	0.45833	0.70833	0.70833	0.70833
3	"point=1"	0.34028	0.34028	0.38195	0.63194	0.59028	0.59028
3	"point=2"	0.34723	0.34723	0.38889	0.63889	0.59722	0.59722
3	"point=3"	0.35417	0.35417	0.39584	0.64583	0.60416	0.60416
3	"point=4"	0.36111	0.36111	0.40278	0.65277	0.61111	0.61111
3	"point=5"	0.36806	0.36806	0.40972	0.65972	0.61805	0.61805
4	"point=1"	0.36227	0.36227	0.39699	0.6331	0.61921	0.60532
4	"point=2"	0.36343	0.36343	0.39815	0.63426	0.62037	0.60648
4	"point=3"	0.36459	0.36459	0.39931	0.63541	0.62153	0.60764
4	"point=4"	0.36574	0.36574	0.40046	0.63657	0.62268	0.60879
4	"point=5"	0.3669	0.3669	0.40162	0.63773	0.62384	0.60995
5	"point=1"	0.36594	0.36594	0.40066	0.63792	0.62288	0.60783
5	"point=2"	0.36613	0.36613	0.40085	0.63811	0.62307	0.60802
5	"point=3"	0.36632	0.36632	0.40104	0.63831	0.62326	0.60822
5	"point=4"	0.36652	0.36652	0.40124	0.6385	0.62345	0.60841
5	"point=5"	0.36671	0.36671	0.40143	0.63869	0.62365	0.6086



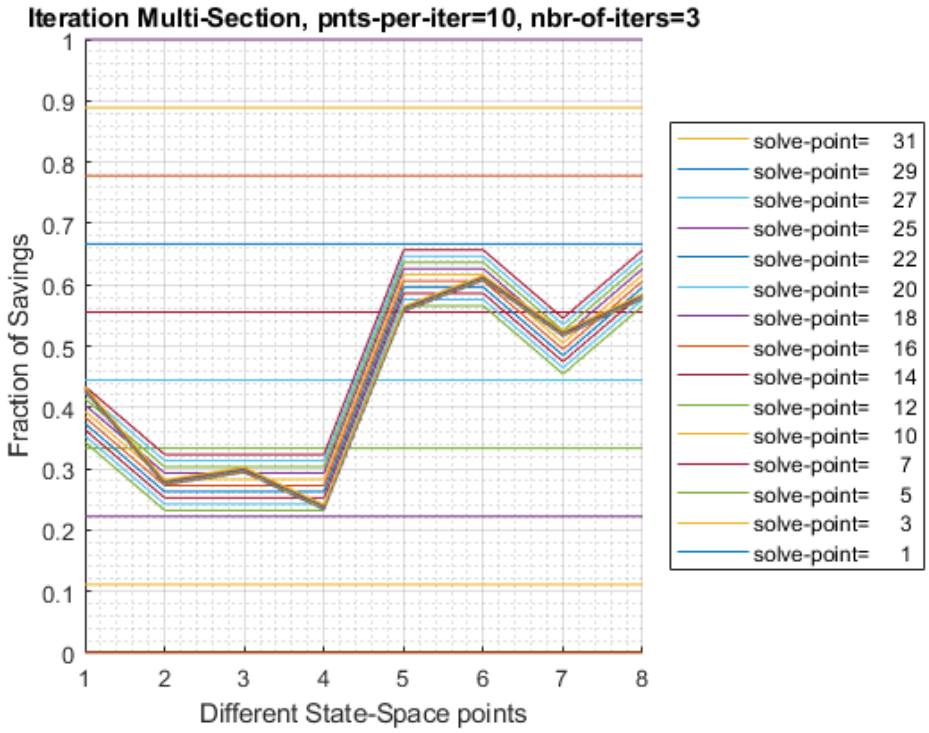

```
Elapsed time is 0.495996 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN        sum           mean
                          -    ---    ----    -----    ----    ----    -----------    -----------

    ar_opti_foc_obj       1     1      2        6       6       1      -0.00037648    -6.2746e-05
    ar_opti_save_frac     2     2      2        6       6       1           3.0037        0.50061

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx
              c1
           -----------

    r1      7.0837e-05
    r2      -0.0002782
    r3      0.00017713
    r4      0.00055875
    r5     -0.00023392
    r6     -0.00067107

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxx
             c1
           -------

    r1     0.36642
    r2     0.36661
    r3     0.40153
    r4     0.63821
    r5     0.62297
    r6     0.60793
```

5.2.3 Test FF_OPTIM_MLSEC_SAVEZRONE 8 Individuals 3 Iterations 10 Points Per Iteration

10 grid points per iteration, and 3 iterations.

```
% Generate the state-space and function
rng(123);
it_draws = 8; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
bl_timer = true;
mp_mlsec_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mlsec_ctrlinfo('it_mlsect_jnt_pnts') = 10;
mp_mlsec_ctrlinfo('it_mlsect_max_iter') = 3;
ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose, bl_timer, mp_mlsec_ctrlinfo);
```

iter	cl_row_names_a	Var1	Var2	Var3	Var4	Var5	Var6	Var
0	"point=1"	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05	1e
1	"point=1"	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05	1e
1	"point=2"	0.11112	0.11112	0.11112	0.11112	0.11112	0.11112	0.11
1	"point=3"	0.22223	0.22223	0.22223	0.22223	0.22223	0.22223	0.22
1	"point=4"	0.33334	0.33334	0.33334	0.33334	0.33334	0.33334	0.33
1	"point=5"	0.44445	0.44445	0.44445	0.44445	0.44445	0.44445	0.44
1	"point=6"	0.55555	0.55555	0.55555	0.55555	0.55555	0.55555	0.55
1	"point=7"	0.66666	0.66666	0.66666	0.66666	0.66666	0.66666	0.66
1	"point=8"	0.77777	0.77777	0.77777	0.77777	0.77777	0.77777	0.77
1	"point=9"	0.88888	0.88888	0.88888	0.88888	0.88888	0.88888	0.88
1	"point=10"	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99
2	"point=1"	0.34344	0.23233	0.23233	0.23233	0.56566	0.56566	0.45
2	"point=2"	0.35354	0.24243	0.24243	0.24243	0.57576	0.57576	0.46
2	"point=3"	0.36364	0.25253	0.25253	0.25253	0.58586	0.58586	0.47
2	"point=4"	0.37374	0.26263	0.26263	0.26263	0.59596	0.59596	0.48
2	"point=5"	0.38384	0.27273	0.27273	0.27273	0.60606	0.60606	0.49
2	"point=6"	0.39394	0.28283	0.28283	0.28283	0.61616	0.61616	0.50
2	"point=7"	0.40404	0.29293	0.29293	0.29293	0.62626	0.62626	0.51
2	"point=8"	0.41414	0.30303	0.30303	0.30303	0.63636	0.63636	0.52
2	"point=9"	0.42424	0.31314	0.31314	0.31314	0.64646	0.64646	0.53
2	"point=10"	0.43434	0.32324	0.32324	0.32324	0.65656	0.65656	0.54
3	"point=1"	0.42516	0.27365	0.29385	0.23325	0.55647	0.60698	0.51
3	"point=2"	0.42608	0.27457	0.29477	0.23417	0.55739	0.60789	0.51
3	"point=3"	0.427	0.27549	0.29569	0.23508	0.55831	0.60881	0.51
3	"point=4"	0.42792	0.2764	0.29661	0.236	0.55923	0.60973	0.51
3	"point=5"	0.42884	0.27732	0.29752	0.23692	0.56015	0.61065	0.51
3	"point=6"	0.42975	0.27824	0.29844	0.23784	0.56106	0.61157	0.52
3	"point=7"	0.43067	0.27916	0.29936	0.23876	0.56198	0.61249	0.52
3	"point=8"	0.43159	0.28008	0.30028	0.23967	0.5629	0.6134	0.5
3	"point=9"	0.43251	0.281	0.3012	0.24059	0.56382	0.61432	0.52
3	"point=10"	0.43343	0.28191	0.30212	0.24151	0.56474	0.61524	0.52



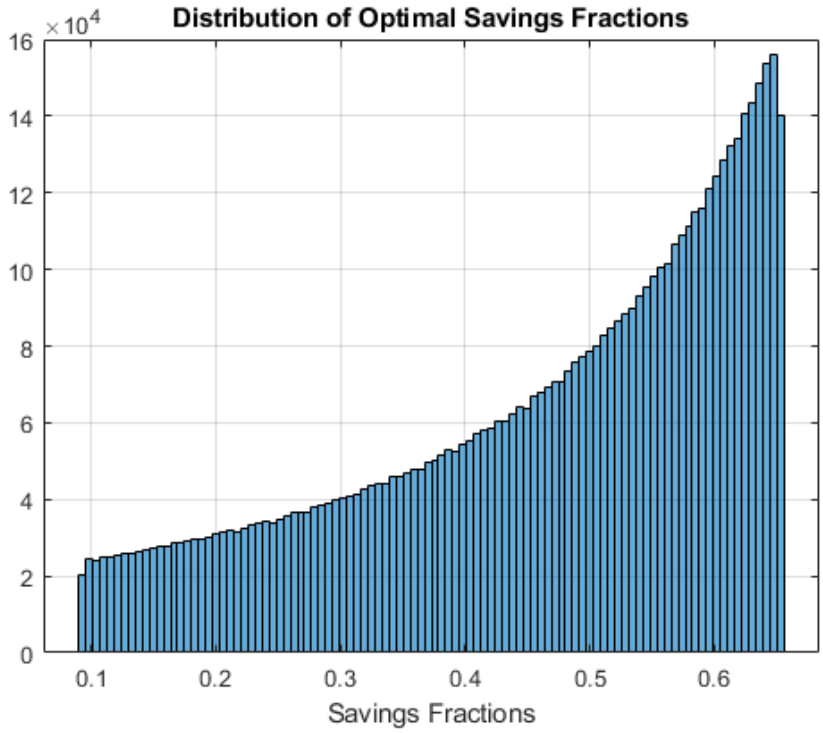

```
Elapsed time is 0.486844 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN       sum          mean          ---
                          -    ---    ----    -----    ----    ----    ---------    ----------    ---

    ar_opti_foc_obj       1     1      2        8       8       1      0.0033175    0.00041468    0.0
    ar_opti_save_frac     2     2      2        8       8       1         3.5124       0.43905      0

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx
              c1
           -----------

    r1      0.00087102
    r2       0.0033354
    r3      -0.0044871
    r4        0.001317
    r5      -0.0017862
    r6       0.0050249
    r7     -0.00058496
    r8     -0.00037273

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxxx
              c1
           -------

    r1     0.42838
    r2     0.28054
    r3      0.2989
    r4     0.23371
    r5     0.55877
    r6     0.61019
```

```
    r7     0.5202
    r8    0.58172
```

5.2.4 Test FF_OPTIM_MLSEC_SAVEZRONE Speed

Test Speed doing 6.25 million multisections for a savings problem:

```
% Generate the state-space and function
rng(123);
it_draws = 6250000; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
[ar_opti_save_frac, ar_opti_save_level] = ff_optim_mlsec_savezrone(fc_deri_wth_uniroot, bl_verbose,

Elapsed time is 16.390434 seconds.

mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
mp_container_map('ar_opti_save_frac') = ar_opti_save_frac;
mp_container_map('ar_opti_save_level') = ar_opti_save_level;
mp_container_map('ar_opti_save_frac_notnan') = ar_opti_save_frac(~isnan(ar_opti_save_frac));
ff_container_map_display(mp_container_map);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                              i    idx    ndim     numel        rowN      colN       sum          m
                              -    ---    ----    --------    --------    ----    ----------    --

    ar_opti_save_frac         1     1      2      6.25e+06    6.25e+06     1       2.884e+06    0.
    ar_opti_save_frac_notnan  2     2      2      6.25e+06    6.25e+06     1       2.884e+06    0.
    ar_opti_save_level        3     3      2      6.25e+06    6.25e+06     1      2.9482e+06    0.

figure();
histogram(ar_opti_save_frac(~isnan(ar_opti_save_frac)),100);
title('Distribution of Optimal Savings Fractions');
xlabel('Savings Fractions');
grid on;
```



5.2.5 Define Two Period Intertemporal FOC Log Utility No Shock

See Household's Utility Maximization Problem and Two-Period Borrowing and Savings Problem given Endowments.

```
function [ar_deri_zero, ar_saveborr_level] = ffi_intertemporal_max(ar_saveborr_frac, z1, z2, r, beta
    ar_saveborr_level = ar_saveborr_frac.*(z1+z2./(1+r)) - z2./(1+r);
    ar_deri_zero = 1./(ar_saveborr_level-z1) + (beta.*(r+1))./(z2 + ar_saveborr_level.*(r+1));
end
```

5.3 FF_OPTIM_MZOOM_SAVEZRONE Derivative Multi-section

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_optim_mzoom_savezrone** from the **MEconTools Package.** This functions solves for optimal savings/borrowing level given an anonymous function that provides the utility (not derivative) of a intertemporal savings problem. This is a vectorized function solves for multiple state-space elements at the same time. The function allows for controls of iteration counts, the number of evaluations per iteration, and how much to "zoom-in" for each iteration around the last iteration's maximum/optimal choice.

Note that if first order conditions are available this method should not be used, but **ff_optim_mlsec_savezrone** should be used. **ff_optim_mlsec_savezrone** relies on bisection. In the first example below more *it_mzoom_jnt_pnts* values are needed to achieve the same precision than under **ff_optim_mlsec_savezrone.** However, increasing *it_mzoom_jnt_pnts* might not expensive given vectorization, should increase time cost linearly in generally. MZOOM is much more robust than bisection based methods. And by increasing the number of points evaluated per iteration, in limited number of iterations, the approximately exact optimal savings choice can be found.

The vectorized zooming savings problem rely on this function to solve for optimal savings choices:

- States Grid + Approximate Continuous Exact Savings (zoom) as Share of Cash-on-Hand ***Vectorized***: **ff_vfi_az_zoom_vec**, precision and speed

5.3.1 Test FF_OPTIM_MZOOM_SAVEZRONE One Individual

Bisection for savings choice at one state:

```
% Generate the state-space and function
[fl_z1, fl_z2, fl_r, fl_beta] = deal(0.4730, 0.6252, 0.0839, 0.7365);
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_util = @(x) ffi_intertemporal_util(x, fl_z1, fl_z2, fl_r, fl_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
% optimally borrowing given the parameters here
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 15;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 10;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0.25;
[fl_opti_save_frac, fl_opti_save_level] = ...
    ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo)

Elapsed time is 0.011586 seconds.
fl_opti_save_frac = 0.4241
fl_opti_save_level = -0.1316
```

5.3.2 Test FF_OPTIM_MZOOM_SAVEZRONE 4 Individuals 3 Iterations 50 Points Per Iteration

5 grid points per iteration, and 5 iterations.

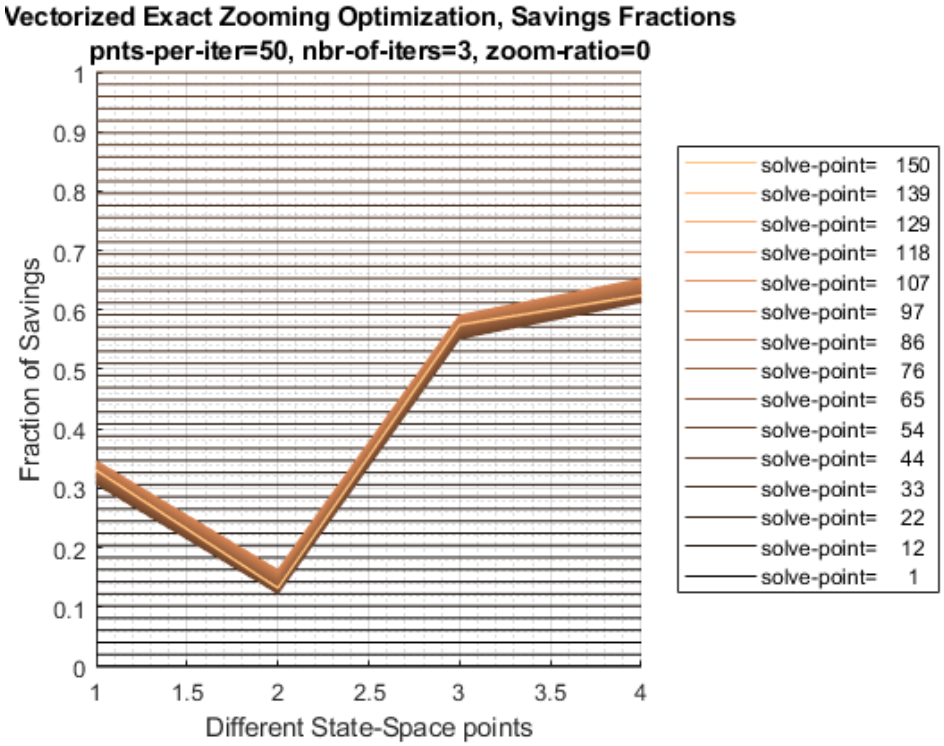
```
% Generate the state-space and function
rng(123);
it_draws = 4; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_util = @(x) ffi_intertemporal_util(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
bl_timer = true;
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 50;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 3;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0;
[fl_opti_save_frac, fl_opti_save_level] = ...
    ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo);
```

iter	cl_row_names_a	Var1	Var2	Var3	Var4
1	"point=1"	1e-05	1e-05	1e-05	1e-05
1	"point=2"	0.020418	0.020418	0.020418	0.020418
1	"point=3"	0.040826	0.040826	0.040826	0.040826
1	"point=4"	0.061233	0.061233	0.061233	0.061233
1	"point=5"	0.081641	0.081641	0.081641	0.081641
1	"point=6"	0.10205	0.10205	0.10205	0.10205
1	"point=7"	0.12246	0.12246	0.12246	0.12246
1	"point=8"	0.14286	0.14286	0.14286	0.14286
1	"point=9"	0.16327	0.16327	0.16327	0.16327
1	"point=10"	0.18368	0.18368	0.18368	0.18368
1	"point=11"	0.20409	0.20409	0.20409	0.20409

1	"point=12"	0.2245	0.2245	0.2245	0.2245
1	"point=13"	0.2449	0.2449	0.2449	0.2449
1	"point=14"	0.26531	0.26531	0.26531	0.26531
1	"point=15"	0.28572	0.28572	0.28572	0.28572
1	"point=16"	0.30613	0.30613	0.30613	0.30613
1	"point=17"	0.32653	0.32653	0.32653	0.32653
1	"point=18"	0.34694	0.34694	0.34694	0.34694
1	"point=19"	0.36735	0.36735	0.36735	0.36735
1	"point=20"	0.38776	0.38776	0.38776	0.38776
1	"point=21"	0.40817	0.40817	0.40817	0.40817
1	"point=22"	0.42857	0.42857	0.42857	0.42857
1	"point=23"	0.44898	0.44898	0.44898	0.44898
1	"point=24"	0.46939	0.46939	0.46939	0.46939
1	"point=25"	0.4898	0.4898	0.4898	0.4898
1	"point=26"	0.5102	0.5102	0.5102	0.5102
1	"point=27"	0.53061	0.53061	0.53061	0.53061
1	"point=28"	0.55102	0.55102	0.55102	0.55102
1	"point=29"	0.57143	0.57143	0.57143	0.57143
1	"point=30"	0.59183	0.59183	0.59183	0.59183
1	"point=31"	0.61224	0.61224	0.61224	0.61224
1	"point=32"	0.63265	0.63265	0.63265	0.63265
1	"point=33"	0.65306	0.65306	0.65306	0.65306
1	"point=34"	0.67347	0.67347	0.67347	0.67347
1	"point=35"	0.69387	0.69387	0.69387	0.69387
1	"point=36"	0.71428	0.71428	0.71428	0.71428
1	"point=37"	0.73469	0.73469	0.73469	0.73469
1	"point=38"	0.7551	0.7551	0.7551	0.7551
1	"point=39"	0.7755	0.7755	0.7755	0.7755
1	"point=40"	0.79591	0.79591	0.79591	0.79591
1	"point=41"	0.81632	0.81632	0.81632	0.81632
1	"point=42"	0.83673	0.83673	0.83673	0.83673
1	"point=43"	0.85714	0.85714	0.85714	0.85714
1	"point=44"	0.87754	0.87754	0.87754	0.87754
1	"point=45"	0.89795	0.89795	0.89795	0.89795
1	"point=46"	0.91836	0.91836	0.91836	0.91836
1	"point=47"	0.93877	0.93877	0.93877	0.93877
1	"point=48"	0.95917	0.95917	0.95917	0.95917
1	"point=49"	0.97958	0.97958	0.97958	0.97958
1	"point=50"	0.99999	0.99999	0.99999	0.99999
2	"point=1"	0.30693	0.12326	0.55182	0.61304
2	"point=2"	0.30773	0.12406	0.55262	0.61384
2	"point=3"	0.30853	0.12486	0.55342	0.61464
2	"point=4"	0.30933	0.12566	0.55422	0.61544
2	"point=5"	0.31013	0.12646	0.55502	0.61624
2	"point=6"	0.31093	0.12726	0.55582	0.61704
2	"point=7"	0.31173	0.12806	0.55662	0.61784
2	"point=8"	0.31253	0.12886	0.55742	0.61865
2	"point=9"	0.31333	0.12966	0.55822	0.61945
2	"point=10"	0.31413	0.13046	0.55902	0.62025
2	"point=11"	0.31493	0.13126	0.55982	0.62105
2	"point=12"	0.31573	0.13206	0.56062	0.62185
2	"point=13"	0.31653	0.13286	0.56142	0.62265
2	"point=14"	0.31733	0.13366	0.56222	0.62345
2	"point=15"	0.31813	0.13446	0.56302	0.62425
2	"point=16"	0.31893	0.13526	0.56382	0.62505
2	"point=17"	0.31973	0.13606	0.56462	0.62585
2	"point=18"	0.32053	0.13686	0.56542	0.62665
2	"point=19"	0.32133	0.13766	0.56623	0.62745

```
2       "point=20"      0.32213     0.13846     0.56703     0.62825
2       "point=21"      0.32293     0.13926     0.56783     0.62905
2       "point=22"      0.32373     0.14006     0.56863     0.62985
2       "point=23"      0.32453     0.14086     0.56943     0.63065
2       "point=24"      0.32533     0.14166     0.57023     0.63145
2       "point=25"      0.32613     0.14246     0.57103     0.63225
2       "point=26"      0.32693     0.14326     0.57183     0.63305
2       "point=27"      0.32773     0.14406     0.57263     0.63385
2       "point=28"      0.32853     0.14487     0.57343     0.63465
2       "point=29"      0.32934     0.14567     0.57423     0.63545
2       "point=30"      0.33014     0.14647     0.57503     0.63625
2       "point=31"      0.33094     0.14727     0.57583     0.63705
2       "point=32"      0.33174     0.14807     0.57663     0.63785
2       "point=33"      0.33254     0.14887     0.57743     0.63865
2       "point=34"      0.33334     0.14967     0.57823     0.63945
2       "point=35"      0.33414     0.15047     0.57903     0.64025
2       "point=36"      0.33494     0.15127     0.57983     0.64105
2       "point=37"      0.33574     0.15207     0.58063     0.64185
2       "point=38"      0.33654     0.15287     0.58143     0.64265
2       "point=39"      0.33734     0.15367     0.58223     0.64345
2       "point=40"      0.33814     0.15447     0.58303     0.64425
2       "point=41"      0.33894     0.15527     0.58383     0.64506
2       "point=42"      0.33974     0.15607     0.58463     0.64586
2       "point=43"      0.34054     0.15687     0.58543     0.64666
2       "point=44"      0.34134     0.15767     0.58623     0.64746
2       "point=45"      0.34214     0.15847     0.58703     0.64826
2       "point=46"      0.34294     0.15927     0.58783     0.64906
2       "point=47"      0.34374     0.16007     0.58863     0.64986
2       "point=48"      0.34454     0.16087     0.58943     0.65066
2       "point=49"      0.34534     0.16167     0.59023     0.65146
2       "point=50"      0.34614     0.16247     0.59103     0.65226
3       "point=1"       0.32937     0.13129     0.57426     0.62348
3       "point=2"        0.3294     0.13132     0.57429     0.62351
3       "point=3"       0.32943     0.13135     0.57432     0.62354
3       "point=4"       0.32946     0.13139     0.57435     0.62357
3       "point=5"       0.32949     0.13142     0.57439      0.6236
3       "point=6"       0.32952     0.13145     0.57442     0.62364
3       "point=7"       0.32955     0.13148     0.57445     0.62367
3       "point=8"       0.32959     0.13151     0.57448      0.6237
3       "point=9"       0.32962     0.13154     0.57451     0.62373
3       "point=10"      0.32965     0.13157     0.57454     0.62376
3       "point=11"      0.32968     0.13161     0.57457     0.62379
3       "point=12"      0.32971     0.13164      0.5746     0.62382
3       "point=13"      0.32974     0.13167     0.57464     0.62385
3       "point=14"      0.32977      0.1317     0.57467     0.62389
3       "point=15"      0.32981     0.13173      0.5747     0.62392
3       "point=16"      0.32984     0.13176     0.57473     0.62395
3       "point=17"      0.32987     0.13179     0.57476     0.62398
3       "point=18"       0.3299     0.13182     0.57479     0.62401
3       "point=19"      0.32993     0.13186     0.57482     0.62404
3       "point=20"      0.32996     0.13189     0.57486     0.62407
3       "point=21"      0.32999     0.13192     0.57489     0.62411
3       "point=22"      0.33003     0.13195     0.57492     0.62414
3       "point=23"      0.33006     0.13198     0.57495     0.62417
3       "point=24"      0.33009     0.13201     0.57498      0.6242
3       "point=25"      0.33012     0.13204     0.57501     0.62423
3       "point=26"      0.33015     0.13208     0.57504     0.62426
3       "point=27"      0.33018     0.13211     0.57508     0.62429
```

3	"point=28"	0.33021	0.13214	0.57511	0.62433
3	"point=29"	0.33025	0.13217	0.57514	0.62436
3	"point=30"	0.33028	0.1322	0.57517	0.62439
3	"point=31"	0.33031	0.13223	0.5752	0.62442
3	"point=32"	0.33034	0.13226	0.57523	0.62445
3	"point=33"	0.33037	0.1323	0.57526	0.62448
3	"point=34"	0.3304	0.13233	0.5753	0.62451
3	"point=35"	0.33043	0.13236	0.57533	0.62455
3	"point=36"	0.33046	0.13239	0.57536	0.62458
3	"point=37"	0.3305	0.13242	0.57539	0.62461
3	"point=38"	0.33053	0.13245	0.57542	0.62464
3	"point=39"	0.33056	0.13248	0.57545	0.62467
3	"point=40"	0.33059	0.13252	0.57548	0.6247
3	"point=41"	0.33062	0.13255	0.57551	0.62473
3	"point=42"	0.33065	0.13258	0.57555	0.62477
3	"point=43"	0.33068	0.13261	0.57558	0.6248
3	"point=44"	0.33072	0.13264	0.57561	0.62483
3	"point=45"	0.33075	0.13267	0.57564	0.62486
3	"point=46"	0.33078	0.1327	0.57567	0.62489
3	"point=47"	0.33081	0.13273	0.5757	0.62492
3	"point=48"	0.33084	0.13277	0.57573	0.62495
3	"point=49"	0.33087	0.1328	0.57577	0.62498
3	"point=50"	0.3309	0.13283	0.5758	0.62502

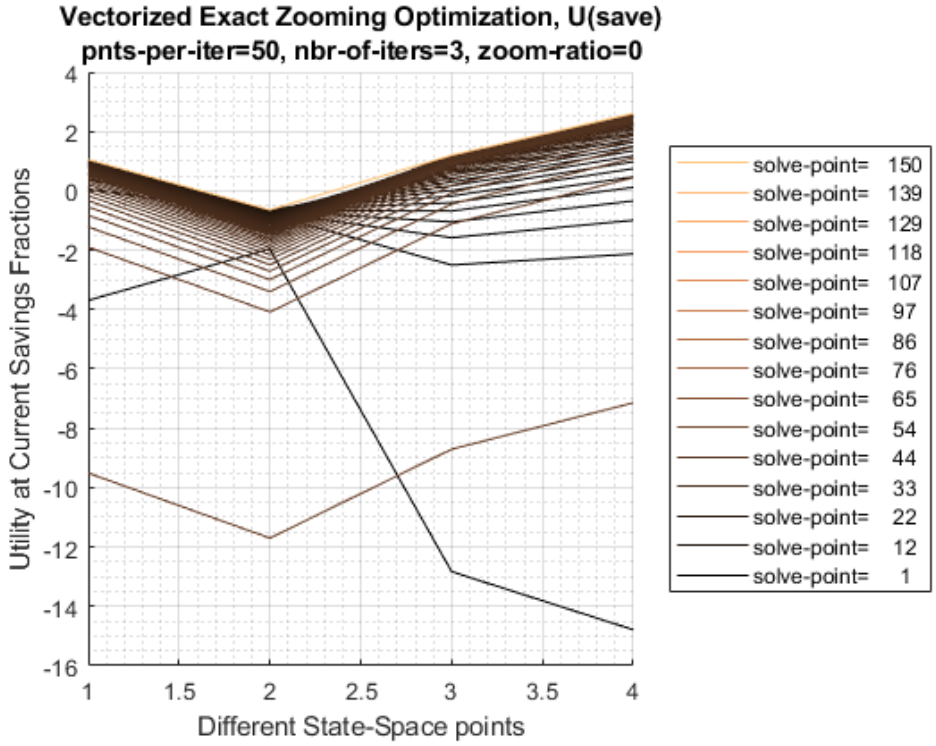


iter	cl_row_names_a	Var1	Var2	Var3	Var4
1	"point=1"	-3.6912	-1.9565	-12.83	-14.789
1	"point=2"	0.058694	-0.80561	-2.4984	-2.1254
1	"point=3"	0.38043	-0.72015	-1.5784	-0.99337
1	"point=4"	0.55947	-0.67935	-1.0493	-0.34024
1	"point=5"	0.67979	-0.65711	-0.68055	0.11647
1	"point=6"	0.7677	-0.64529	-0.39997	0.46531
1	"point=7"	0.8349	-0.64026	-0.17534	0.74571

1	"point=8"	0.88763	-0.6401	0.010483	0.9787
1	"point=9"	0.92959	-0.64367	0.16774	1.1768
1	"point=10"	0.96316	-0.65026	0.30302	1.3481
1	"point=11"	0.98996	-0.65938	0.4208	1.4981
1	"point=12"	1.0111	-0.67071	0.52427	1.6308
1	"point=13"	1.0275	-0.684	0.61578	1.7489
1	"point=14"	1.0397	-0.6991	0.69709	1.8547
1	"point=15"	1.0482	-0.71588	0.76958	1.9499
1	"point=16"	1.0533	-0.73426	0.83429	2.0357
1	"point=17"	1.0554	-0.75419	0.8921	2.1132
1	"point=18"	1.0546	-0.77564	0.94367	2.1833
1	"point=19"	1.0512	-0.79861	0.98955	2.2467
1	"point=20"	1.0451	-0.82309	1.0302	2.3039
1	"point=21"	1.0366	-0.8491	1.066	2.3554
1	"point=22"	1.0256	-0.87669	1.0971	2.4015
1	"point=23"	1.0123	-0.90591	1.124	2.4425
1	"point=24"	0.99654	-0.93682	1.1466	2.4788
1	"point=25"	0.97838	-0.9695	1.1652	2.5104
1	"point=26"	0.95775	-1.004	1.1798	2.5375
1	"point=27"	0.93459	-1.0406	1.1905	2.5602
1	"point=28"	0.90881	-1.0792	1.1973	2.5785
1	"point=29"	0.88029	-1.1202	1.2002	2.5925
1	"point=30"	0.84886	-1.1635	1.1991	2.6022
1	"point=31"	0.81434	-1.2096	1.1938	2.6073
1	"point=32"	0.77649	-1.2587	1.1843	2.6078
1	"point=33"	0.73504	-1.3109	1.1703	2.6035
1	"point=34"	0.68964	-1.3668	1.1514	2.594
1	"point=35"	0.63987	-1.4268	1.1274	2.5792
1	"point=36"	0.58522	-1.4913	1.0978	2.5584
1	"point=37"	0.52505	-1.5611	1.062	2.5312
1	"point=38"	0.45857	-1.6369	1.0192	2.4968
1	"point=39"	0.38475	-1.7198	0.96837	2.4541
1	"point=40"	0.3023	-1.8111	0.90834	2.4021
1	"point=41"	0.20947	-1.9126	0.83737	2.3388
1	"point=42"	0.10391	-2.0266	0.75313	2.2622
1	"point=43"	-0.017693	-2.1564	0.65234	2.1687
1	"point=44"	-0.16019	-2.3069	0.53016	2.0538
1	"point=45"	-0.33112	-2.4857	0.37908	1.9097
1	"point=46"	-0.54312	-2.7054	0.18649	1.724
1	"point=47"	-0.81989	-2.9896	-0.071303	1.4729
1	"point=48"	-1.2146	-3.3917	-0.44748	1.1033
1	"point=49"	-1.8971	-4.0814	-1.1118	0.44547
1	"point=50"	-9.5085	-11.7	-8.7054	-7.1418
2	"point=1"	1.0535	-0.64017	1.1975	2.6074
2	"point=2"	1.0536	-0.64009	1.1977	2.6075
2	"point=3"	1.0537	-0.64001	1.1979	2.6076
2	"point=4"	1.0539	-0.63995	1.198	2.6077
2	"point=5"	1.054	-0.63989	1.1982	2.6077
2	"point=6"	1.0541	-0.63983	1.1984	2.6078
2	"point=7"	1.0542	-0.63979	1.1985	2.6079
2	"point=8"	1.0543	-0.63975	1.1986	2.6079
2	"point=9"	1.0544	-0.63971	1.1988	2.608
2	"point=10"	1.0545	-0.63969	1.1989	2.608
2	"point=11"	1.0546	-0.63967	1.199	2.6081
2	"point=12"	1.0547	-0.63966	1.1992	2.6081
2	"point=13"	1.0548	-0.63965	1.1993	2.6081
2	"point=14"	1.0548	-0.63965	1.1994	2.6081
2	"point=15"	1.0549	-0.63966	1.1995	2.6081

```
2       "point=16"        1.055    -0.63967      1.1996      2.6081
2       "point=17"       1.0551    -0.63969      1.1997      2.6081
2       "point=18"       1.0551    -0.63971      1.1998      2.6081
2       "point=19"       1.0552    -0.63975      1.1998      2.6081
2       "point=20"       1.0552    -0.63978      1.1999      2.6081
2       "point=21"       1.0553    -0.63983         1.2       2.608
2       "point=22"       1.0553    -0.63988         1.2       2.608
2       "point=23"       1.0553    -0.63993      1.2001      2.6079
2       "point=24"       1.0554    -0.63999      1.2001      2.6079
2       "point=25"       1.0554    -0.64006      1.2002      2.6078
2       "point=26"       1.0554    -0.64013      1.2002      2.6077
2       "point=27"       1.0555    -0.64021      1.2002      2.6077
2       "point=28"       1.0555    -0.64029      1.2003      2.6076
2       "point=29"       1.0555    -0.64038      1.2003      2.6075
2       "point=30"       1.0555    -0.64048      1.2003      2.6074
2       "point=31"       1.0555    -0.64058      1.2003      2.6073
2       "point=32"       1.0555    -0.64069      1.2003      2.6071
2       "point=33"       1.0555     -0.6408      1.2003       2.607
2       "point=34"       1.0555    -0.64091      1.2003      2.6069
2       "point=35"       1.0555    -0.64104      1.2002      2.6067
2       "point=36"       1.0554    -0.64116      1.2002      2.6066
2       "point=37"       1.0554    -0.64129      1.2002      2.6064
2       "point=38"       1.0554    -0.64143      1.2001      2.6063
2       "point=39"       1.0554    -0.64157      1.2001      2.6061
2       "point=40"       1.0553    -0.64172      1.2001      2.6059
2       "point=41"       1.0553    -0.64188         1.2      2.6057
2       "point=42"       1.0552    -0.64203      1.1999      2.6056
2       "point=43"       1.0552     -0.6422      1.1999      2.6053
2       "point=44"       1.0551    -0.64236      1.1998      2.6051
2       "point=45"       1.0551    -0.64254      1.1997      2.6049
2       "point=46"        1.055    -0.64271      1.1996      2.6047
2       "point=47"       1.0549    -0.64289      1.1995      2.6045
2       "point=48"       1.0549    -0.64308      1.1994      2.6042
2       "point=49"       1.0548    -0.64327      1.1993       2.604
2       "point=50"       1.0547    -0.64347      1.1992      2.6037
3       "point=1"        1.0555    -0.63967      1.2003      2.6081
3       "point=2"        1.0555    -0.63967      1.2003      2.6081
3       "point=3"        1.0555    -0.63967      1.2003      2.6081
3       "point=4"        1.0555    -0.63967      1.2003      2.6081
3       "point=5"        1.0555    -0.63967      1.2003      2.6081
3       "point=6"        1.0555    -0.63967      1.2003      2.6081
3       "point=7"        1.0555    -0.63967      1.2003      2.6081
3       "point=8"        1.0555    -0.63966      1.2003      2.6081
3       "point=9"        1.0555    -0.63966      1.2003      2.6081
3       "point=10"       1.0555    -0.63966      1.2003      2.6081
3       "point=11"       1.0555    -0.63966      1.2003      2.6081
3       "point=12"       1.0555    -0.63966      1.2003      2.6081
3       "point=13"       1.0555    -0.63966      1.2003      2.6081
3       "point=14"       1.0555    -0.63966      1.2003      2.6081
3       "point=15"       1.0555    -0.63966      1.2003      2.6081
3       "point=16"       1.0555    -0.63966      1.2003      2.6081
3       "point=17"       1.0555    -0.63966      1.2003      2.6081
3       "point=18"       1.0555    -0.63966      1.2003      2.6081
3       "point=19"       1.0555    -0.63966      1.2003      2.6081
3       "point=20"       1.0555    -0.63966      1.2003      2.6081
3       "point=21"       1.0555    -0.63966      1.2003      2.6081
3       "point=22"       1.0555    -0.63966      1.2003      2.6081
3       "point=23"       1.0555    -0.63966      1.2003      2.6081
```

```
3        "point=24"        1.0555    -0.63966       1.2003       2.6081
3        "point=25"        1.0555    -0.63966       1.2003       2.6081
3        "point=26"        1.0555    -0.63966       1.2003       2.6081
3        "point=27"        1.0555    -0.63966       1.2003       2.6081
3        "point=28"        1.0555    -0.63966       1.2003       2.6081
3        "point=29"        1.0555    -0.63966       1.2003       2.6081
3        "point=30"        1.0555    -0.63966       1.2003       2.6081
3        "point=31"        1.0555    -0.63965       1.2003       2.6081
3        "point=32"        1.0555    -0.63965       1.2003       2.6081
3        "point=33"        1.0555    -0.63965       1.2003       2.6081
3        "point=34"        1.0555    -0.63965       1.2003       2.6081
3        "point=35"        1.0555    -0.63965       1.2003       2.6081
3        "point=36"        1.0555    -0.63965       1.2003       2.6081
3        "point=37"        1.0555    -0.63965       1.2003       2.6081
3        "point=38"        1.0555    -0.63965       1.2003       2.6081
3        "point=39"        1.0555    -0.63965       1.2003       2.6081
3        "point=40"        1.0555    -0.63965       1.2003       2.6081
3        "point=41"        1.0555    -0.63965       1.2003       2.6081
3        "point=42"        1.0555    -0.63965       1.2003       2.6081
3        "point=43"        1.0555    -0.63965       1.2003       2.6081
3        "point=44"        1.0555    -0.63965       1.2003       2.6081
3        "point=45"        1.0555    -0.63965       1.2003       2.6081
3        "point=46"        1.0555    -0.63965       1.2003       2.6081
3        "point=47"        1.0555    -0.63965       1.2003       2.6081
3        "point=48"        1.0555    -0.63965       1.2003       2.6081
3        "point=49"        1.0555    -0.63965       1.2003       2.6081
3        "point=50"        1.0555    -0.63965       1.2003       2.6081
```




```
Elapsed time is 1.304303 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i     idx     ndim     numel     rowN     colN       sum        mean         std
```

```
                          -    ---    ----    -----    ----    ----    -------    -------    -------

    ar_opti_foc_obj       1     1      2        4       1       4       4.2243     1.0561     1.3298
    ar_opti_save_frac     2     2      2        4       4       1        1.664      0.416     0.2284
    ar_opti_save_level    3     3      2        4       1       4      0.80247    0.20062    0.37807

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxx
            c1          c2          c3         c4
          ------    --------    ------    ------

    r1    1.0555    -0.63965    1.2003    2.6081

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxx
            c1
          -------

    r1    0.33086
    r2    0.13278
    r3    0.57575
    r4    0.62461

xxx TABLE:ar_opti_save_level xxxxxxxxxxxxxxxxx
            c1           c2           c3          c4
          -------    ---------    --------    -------

    r1    0.37401    -0.070015    -0.15125    0.64972
```

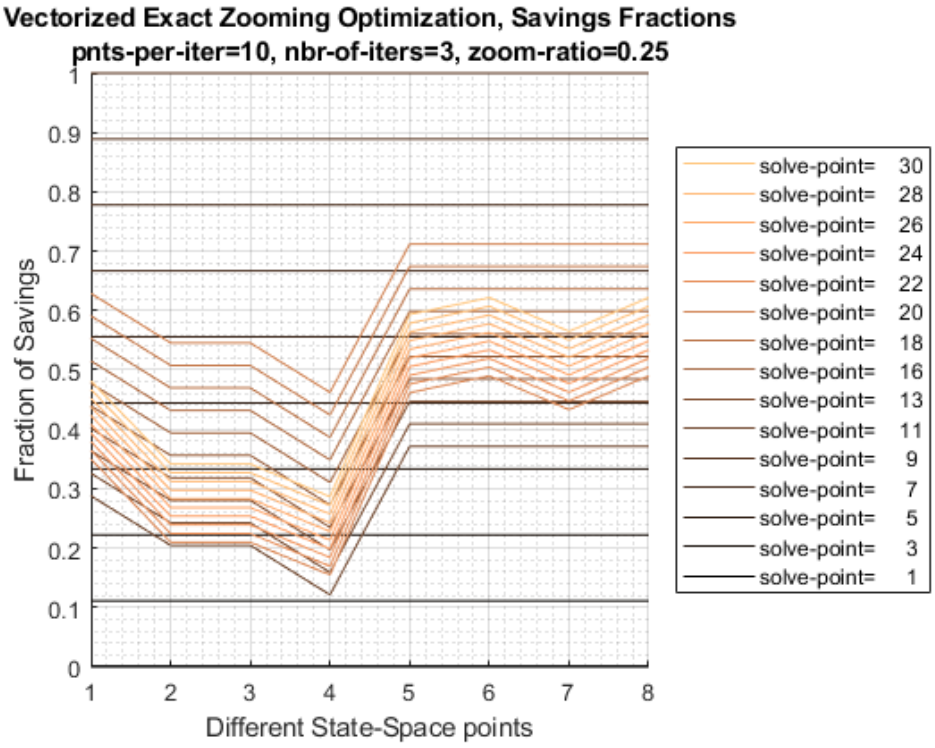
5.3.3 Test FF_OPTIM_MZOOM_SAVEZRONE 8 Individuals 3 Iterations 10 Points Per Iteration, 0.25 zoom in ratio

10 grid points per iteration, and 3 iterations.

```
% Generate the state-space and function
rng(123);
it_draws = 8; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_util = @(x) ffi_intertemporal_util(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
bl_timer = true;
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 10;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 3;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0.25;
[fl_opti_save_frac, fl_opti_save_level] = ...
    ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo);
```

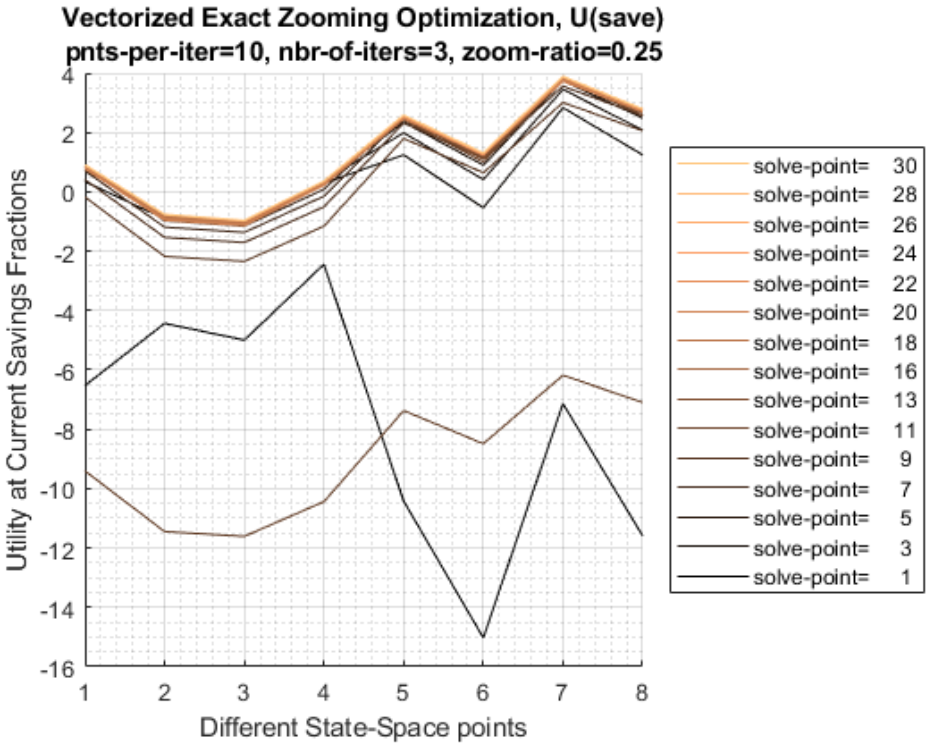
iter	cl_row_names_a	Var1	Var2	Var3	Var4	Var5	Var6	Var
1	"point=1"	1e-05	1e-05	1e-05	1e-05	1e-05	1e-05	1e
1	"point=2"	0.11112	0.11112	0.11112	0.11112	0.11112	0.11112	0.11
1	"point=3"	0.22223	0.22223	0.22223	0.22223	0.22223	0.22223	0.22
1	"point=4"	0.33334	0.33334	0.33334	0.33334	0.33334	0.33334	0.33
1	"point=5"	0.44445	0.44445	0.44445	0.44445	0.44445	0.44445	0.44
1	"point=6"	0.55555	0.55555	0.55555	0.55555	0.55555	0.55555	0.55

1	"point=7"	0.66666	0.66666	0.66666	0.66666	0.66666	0.66666	0.66
1	"point=8"	0.77777	0.77777	0.77777	0.77777	0.77777	0.77777	0.77
1	"point=9"	0.88888	0.88888	0.88888	0.88888	0.88888	0.88888	0.88
1	"point=10"	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99
2	"point=1"	0.28788	0.20455	0.20455	0.12122	0.37121	0.37121	0.37
2	"point=2"	0.32576	0.24243	0.24243	0.1591	0.40909	0.40909	0.40
2	"point=3"	0.36364	0.28031	0.28031	0.19698	0.44697	0.44697	0.44
2	"point=4"	0.40152	0.31819	0.31819	0.23485	0.48485	0.48485	0.48
2	"point=5"	0.4394	0.35606	0.35606	0.27273	0.52273	0.52273	0.52
2	"point=6"	0.47727	0.39394	0.39394	0.31061	0.5606	0.5606	0.5
2	"point=7"	0.51515	0.43182	0.43182	0.34849	0.59848	0.59848	0.59
2	"point=8"	0.55303	0.4697	0.4697	0.38637	0.63636	0.63636	0.63
2	"point=9"	0.59091	0.50758	0.50758	0.42424	0.67424	0.67424	0.67
2	"point=10"	0.62879	0.54545	0.54545	0.46212	0.71212	0.71212	0.71
3	"point=1"	0.34987	0.20972	0.20972	0.15479	0.46161	0.49001	0.4
3	"point=2"	0.3645	0.22435	0.22435	0.16943	0.47624	0.50465	0.44
3	"point=3"	0.37913	0.23899	0.23899	0.18406	0.49087	0.51928	0.46
3	"point=4"	0.39377	0.25362	0.25362	0.1987	0.50551	0.53392	0.4
3	"point=5"	0.4084	0.26826	0.26826	0.21333	0.52014	0.54855	0.49
3	"point=6"	0.42304	0.28289	0.28289	0.22797	0.53478	0.56319	0.50
3	"point=7"	0.43767	0.29752	0.29752	0.2426	0.54941	0.57782	0.52
3	"point=8"	0.45231	0.31216	0.31216	0.25724	0.56405	0.59246	0.53
3	"point=9"	0.46694	0.32679	0.32679	0.27187	0.57868	0.60709	0.55
3	"point=10"	0.48158	0.34143	0.34143	0.28651	0.59332	0.62173	0.56



iter	cl_row_names_a	Var1	Var2	Var3	Var4	Var5	Var6
1	"point=1"	-6.5286	-4.4312	-4.9951	-2.4407	-10.415	-15.025
1	"point=2"	0.34227	-0.90966	-1.148	0.28691	1.2451	-0.53687
1	"point=3"	0.7287	-0.77242	-0.98657	0.36508	1.9879	0.4163
1	"point=4"	0.87872	-0.76818	-0.96816	0.33477	2.3463	0.89785
1	"point=5"	0.91222	-0.83811	-1.028	0.24031	2.5277	1.1666
1	"point=6"	0.85648	-0.97408	-1.1562	0.085331	2.5867	1.2933

1	"point=7"	0.70558	-1.1905	-1.3663	-0.14666	2.5296	1.2915
1	"point=8"	0.41577	-1.5358	-1.7061	-0.50502	2.319	1.1277
1	"point=9"	-0.17716	-2.1767	-2.3424	-1.1573	1.7947	0.64395
1	"point=10"	-9.4046	-11.446	-11.608	-10.437	-7.3721	-8.4872
2	"point=1"	0.8347	-0.78233	-0.99938	0.30205	2.4239	1.0081
2	"point=2"	0.87277	-0.76475	-0.97586	0.34105	2.4846	1.0983
2	"point=3"	0.89748	-0.75933	-0.96536	0.36018	2.5303	1.1709
2	"point=4"	0.91044	-0.76388	-0.96549	0.36559	2.5622	1.2275
2	"point=5"	0.91269	-0.7771	-0.97477	0.36049	2.581	1.269
2	"point=6"	0.90477	-0.79823	-0.99237	0.34672	2.5867	1.296
2	"point=7"	0.88684	-0.8269	-1.0178	0.32535	2.5793	1.3084
2	"point=8"	0.85872	-0.86304	-1.051	0.29697	2.5578	1.3055
2	"point=9"	0.81987	-0.90685	-1.0921	0.26182	2.5209	1.2862
2	"point=10"	0.76932	-0.95877	-1.1415	0.21989	2.4664	1.2483
3	"point=1"	0.88992	-0.7791	-0.99528	0.33777	2.5443	1.234
3	"point=2"	0.8979	-0.77144	-0.98526	0.3479	2.5562	1.251
3	"point=3"	0.90413	-0.7658	-0.97741	0.35543	2.5661	1.2659
3	"point=4"	0.90869	-0.762	-0.97154	0.3607	2.5741	1.2785
3	"point=5"	0.91163	-0.75989	-0.96746	0.36397	2.5801	1.289
3	"point=6"	0.91299	-0.75934	-0.96506	0.36546	2.5842	1.2974
3	"point=7"	0.91281	-0.76025	-0.96421	0.36532	2.5864	1.3035
3	"point=8"	0.91112	-0.76255	-0.96482	0.3637	2.5866	1.3074
3	"point=9"	0.90792	-0.76615	-0.96683	0.3607	2.5849	1.3091
3	"point=10"	0.90324	-0.77102	-0.97016	0.35641	2.5811	1.3085




```
Elapsed time is 0.997040 seconds.
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std
	-	---	----	-----	----	----	------	-------	-------
ar_opti_foc_obj	1	1	2	8	1	8	10.125	1.2656	1.731

```
    ar_opti_save_frac     2     2      2        8       8       1      3.3843    0.42304    0.15074
    ar_opti_save_level    3     3      2        8       1       8       4.965    0.62062    0.71498

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxx
            c1          c2          c3         c4         c5         c6         c7         c8
          -------    --------    --------    -------    ------    ------    ------    ------

    r1    0.91163    -0.75989    -0.96506    0.36397    2.5864    1.3074    3.8892    2.7911

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxx
            c1
          -------

    r1     0.4084
    r2    0.26826
    r3    0.28289
    r4    0.21333
    r5    0.54941
    r6    0.59246
    r7    0.50637
    r8    0.56319

xxx TABLE:ar_opti_save_level xxxxxxxxxxxxxxxxx
            c1          c2           c3          c4          c5         c6         c7         c8
          -------    ---------    --------    ---------    ------    -------    -----    -------

    r1    0.53854    -0.050976    0.053729    0.0013119    1.0103    0.45895    2.098    0.85508
```

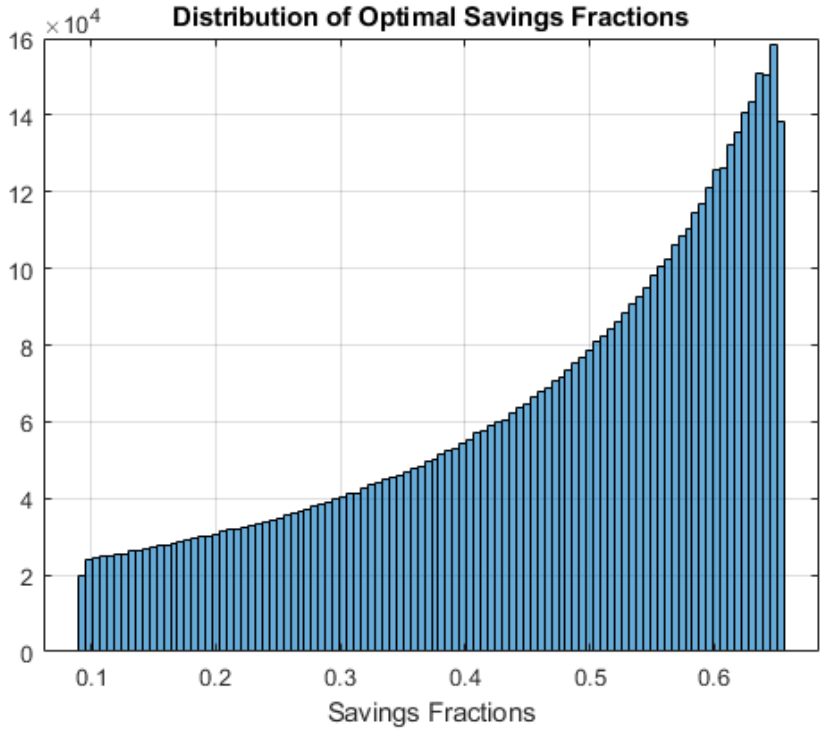
5.3.4 Test FF_OPTIM_MZOOM_SAVEZRONE Speed

Test Speed doing 6.25 million state-spcae points for a savings problem:

```
% Generate the state-space and function
rng(123);
it_draws = 6250000; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_util = @(x) ffi_intertemporal_util(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
% set parameters
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 20;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 10;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0.25;
[ar_opti_save_frac, ar_opti_save_level] = ...
    ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo);

Elapsed time is 64.837799 seconds.

mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
mp_container_map('ar_opti_save_frac') = ar_opti_save_frac;
mp_container_map('ar_opti_save_level') = ar_opti_save_level;
mp_container_map('ar_opti_save_frac_notnan') = ar_opti_save_frac(~isnan(ar_opti_save_frac));
ff_container_map_display(mp_container_map);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                i     idx     ndim      numel         rowN       colN        sum          m
                                -     ---     ----     --------     --------     ----     ----------     --

    ar_opti_save_frac           1      1       2       6.25e+06     6.25e+06      1       2.8839e+06     0.
    ar_opti_save_frac_notnan    2      2       2       6.25e+06     6.25e+06      1       2.8839e+06     0.
    ar_opti_save_level          3      3       2       6.25e+06     6.25e+06      1       2.9481e+06     0.
```

```
figure();
histogram(ar_opti_save_frac(~isnan(ar_opti_save_frac)),100);
title('Distribution of Optimal Savings Fractions');
xlabel('Savings Fractions');
grid on;
```



5.3.5 Define Two Period Intertemporal Log Utility No Shock Utility Function

See Household's Utility Maximization Problem and Two-Period Borrowing and Savings Problem given Endowments.

```
function [ar_util, ar_saveborr_level] = ...
    ffi_intertemporal_util(ar_saveborr_frac, z1, z2, r, beta)

ar_saveborr_level = ar_saveborr_frac.*(z1+z2./(1+r)) - z2./(1+r);
ar_util = log(z1 - ar_saveborr_level) + beta.*log(ar_saveborr_level.*(1+r) + z2);

end
```

Chapter 6

Graphs

6.1 FF_GRAPH_GRID Examples: X, Y and Color Line Plots

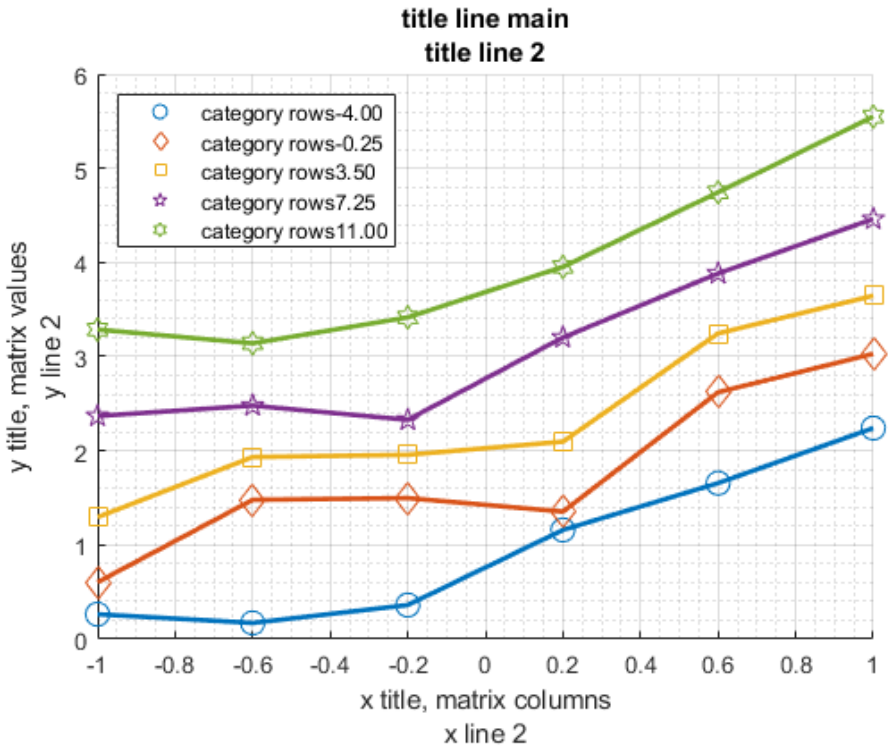
Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

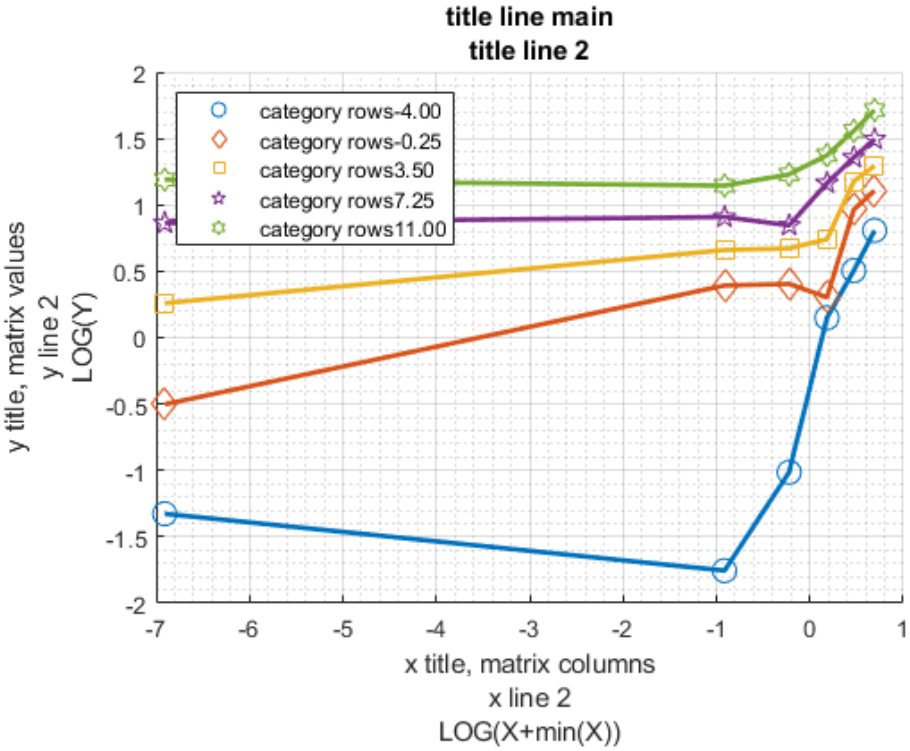
This is the example vignette for function: **ff_graph_grid** from the **MEconTools Package.** This function can graph out value and policy functions given one state vector (x-axis), conditional on other states (line groups). Can handle a few lines (scatter + lines), or many groups (jet spectrum). Can handle policy and value function graphs, or distributional plots.

6.1.1 Test FF_GRAPH_GRID Defaults

Call the function with defaults.

```
ff_graph_grid();
```



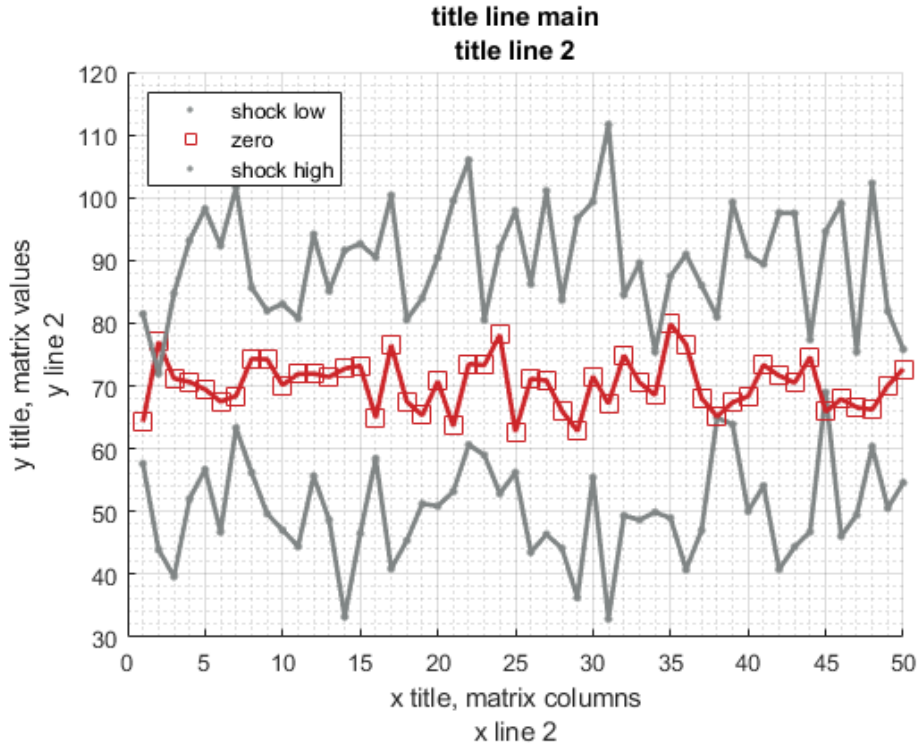


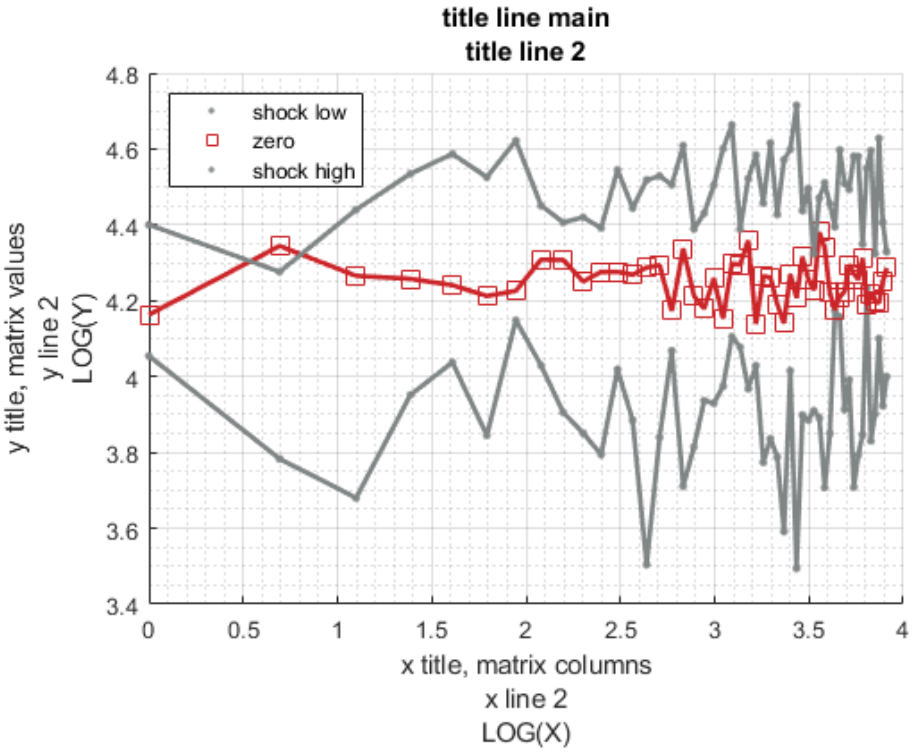
6.1.2 Test FF_GRAPH_GRID Random Matrix Pick Markers and Colors

Call the function with defaults. Graph a matrix, each row of the matrix is a separate line, each column a point along the x-axis, value of the matrix are plotting on the y-axis.

- ar_row_grid: the values associated with each row, what will show up in the legend
- ar_col_grid: the values associated with each column
- mt_support_graph: various controls, color, etc...

```
rng(123);
mt_value = [normrnd(50,10,[1, 50]); ...
            normrnd(70,5,[1, 50]);...
            normrnd(90,10,[1, 50])];
ar_row_grid = ["shock low", "zero", "shock high"];
ar_col_grid = 1:50;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_scatter_shapes') = { '.', 's' ,'.' };
mp_support_graph('cl_colors') = {'gray', 'red', 'gray'};
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



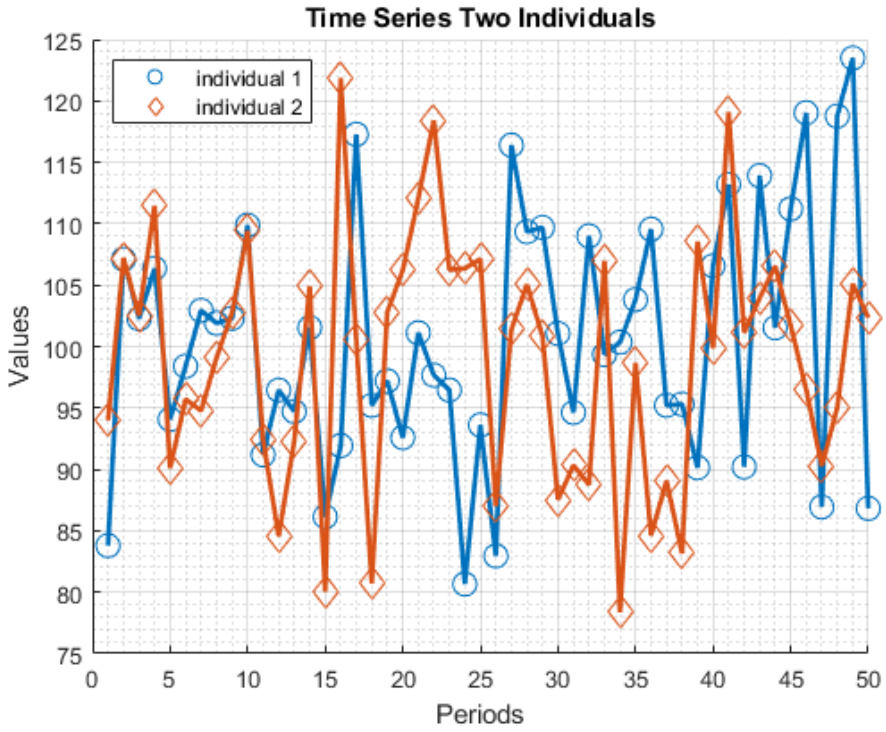


6.1.3 Test FF_GRAPH_GRID Two Random Normal Lines and Labels

There are two autoregressive time series, plot out the time two time series.

```
% Generate the two time series
rng(456);
mt_value = normrnd(100,10,[2, 50]);
ar_row_grid = ["individual 1", "individual 2"];
ar_col_grid = 1:50;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
```

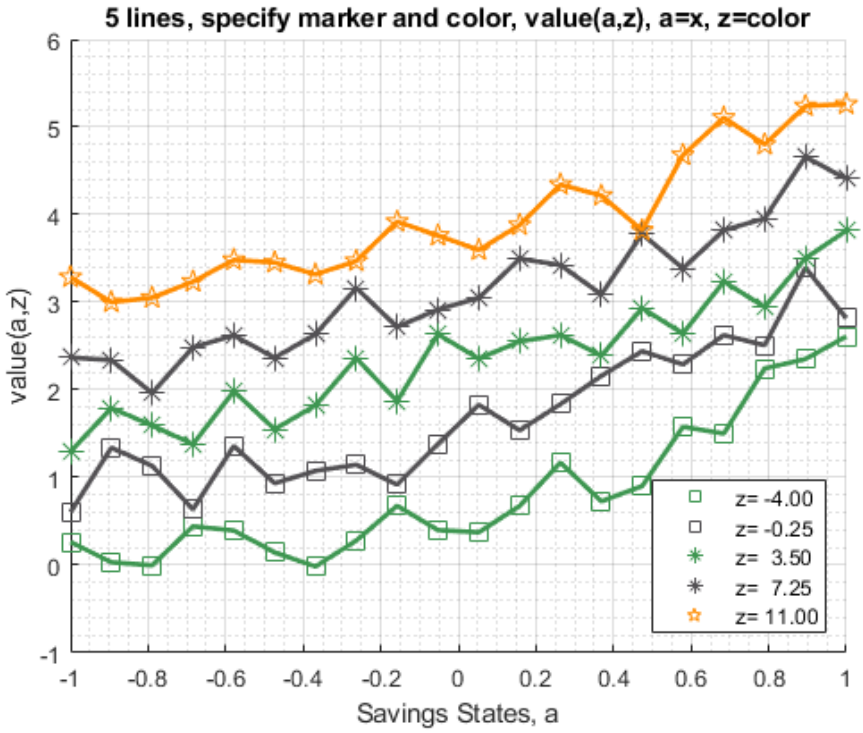
```
mp_support_graph('cl_st_graph_title') = {'Time Series Two Individuals'};
mp_support_graph('cl_st_ytitle') = {'Values'};
mp_support_graph('cl_st_xtitle') = {'Periods'};
mp_support_graph('bl_graph_logy') = false; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



6.1.4 Test FF_GRAPH_GRID 6 Lines Pick Marker and Colors

Plot many lines, with auto legend.

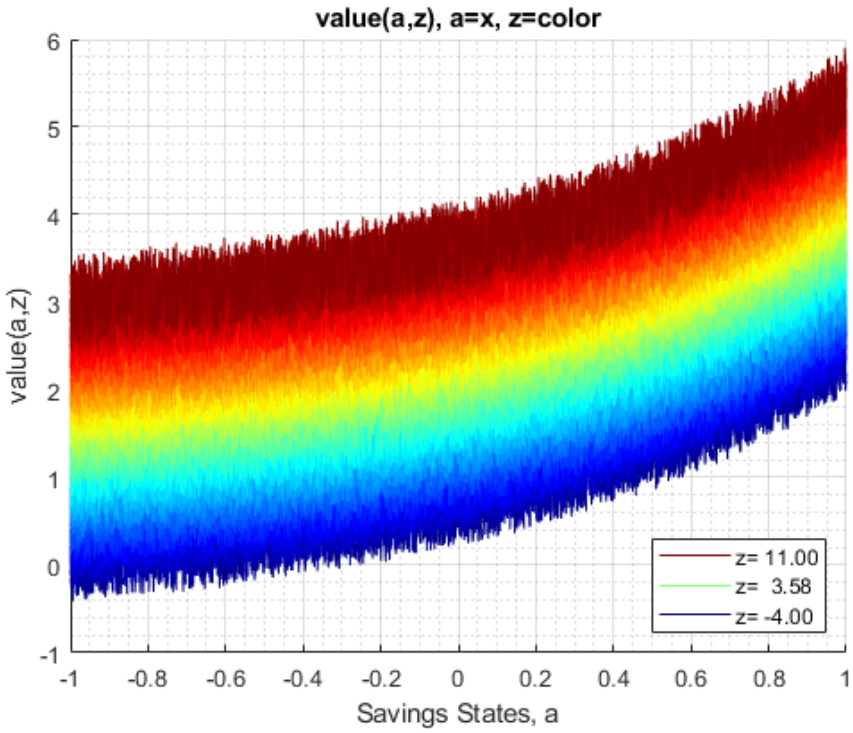
```
% Generate some Data
rng(456);
ar_row_grid = linspace(-4, 11, 5);
ar_col_grid = linspace(-1, 1, 20);
rng(123);
mt_value = 0.2*ar_row_grid' + exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'5 lines, specify marker and color, value(a,z), a=x, z=colo
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'southeast';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
mp_support_graph('cl_scatter_shapes') = {'s', 's', '*', '*', 'p'};
mp_support_graph('cl_colors') = {'green', 'black', 'green', 'black', 'orange'};
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



6.1.5 Test FF_GRAPH_GRID Many Lines

Plot many lines, with auto legend.

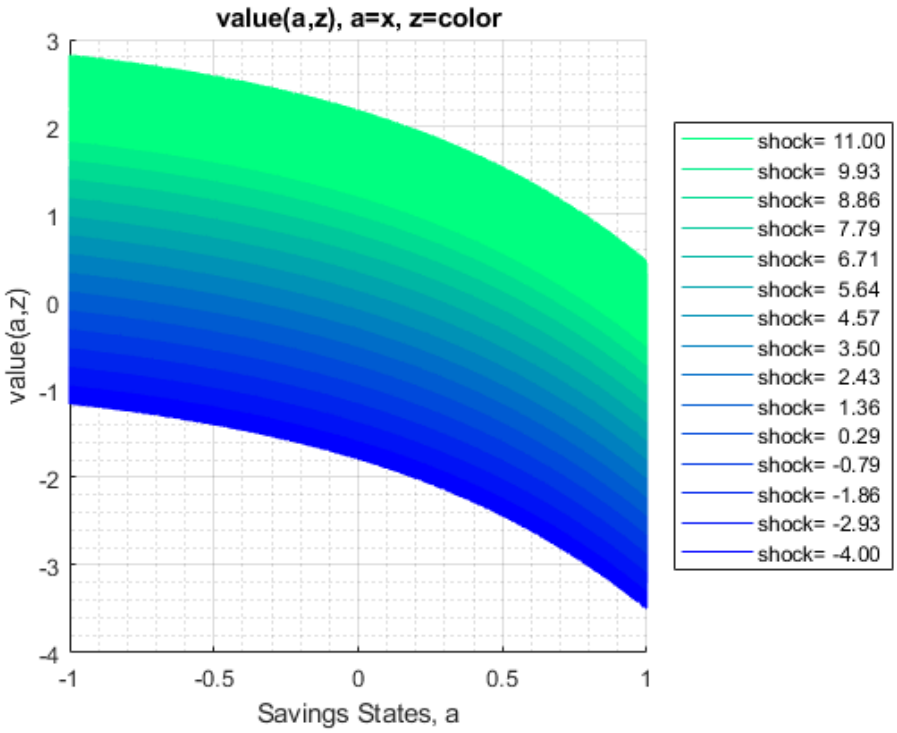
```
% Generate some Data
rng(456);
ar_row_grid = linspace(-4, 11, 100);
ar_col_grid = linspace(-1, 1, 1000);
rng(123);
mt_value = 0.2*ar_row_grid' + exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'southeast';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
mp_support_graph('cl_colors') = 'jet'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



6.1.6 Test FF_GRAPH_GRID Many Lines Legend Exogenous

Plot many lines, exogenously set legend

```
% Generate the two time series
rng(456);
ar_row_grid = linspace(-4, 11, 15);
ar_col_grid = linspace(-1, 1, 100000);
rng(123);
mt_value = 0.2*ar_row_grid' - exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% setting shock vector name exogenously here
ar_row_grid = string(num2str(ar_row_grid', "shock=%6.2f"));
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'eastoutside';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('it_legend_select') = 15;
mp_support_graph('cl_colors') = 'winter'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



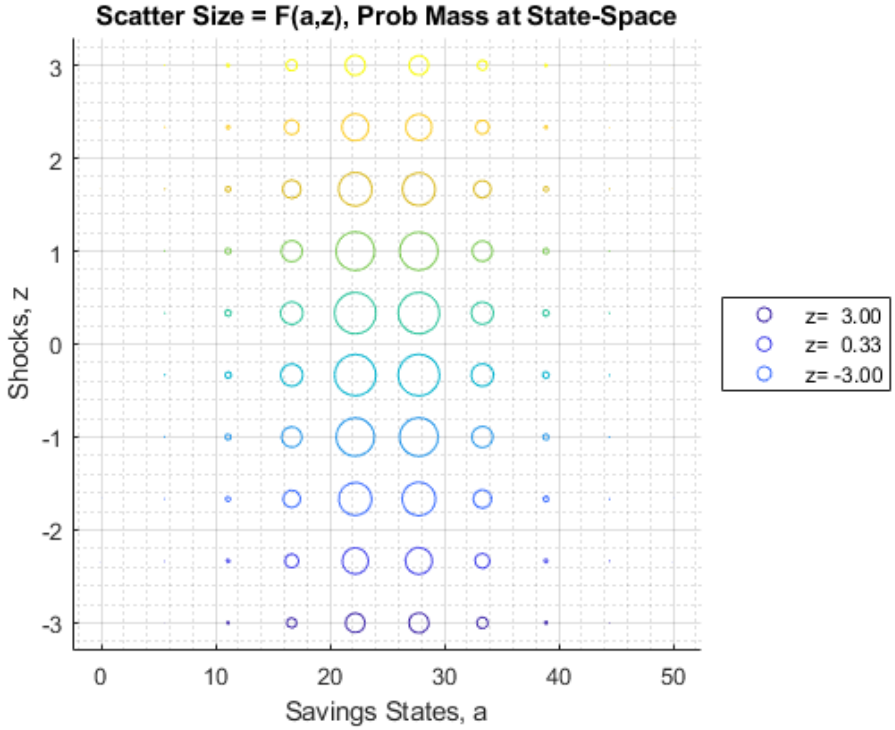
6.1.7 Test FF_GRAPH_GRID Joint Probability Mass Output as Scatter Size

Along two dimensions of the state-space, we might want to visualize the probability mass distribution P(a,z) . We will show A and Z as the X and Y dimensions, and use Scatter size for mass at each point.

In the default mode, each ar_row_grid can be a string array, providing labels for each data matrix row, shown with different colors. Here, the ar_row_grid must be numeric.

```
% Joint Normal Mass
rng(456);
mu = [0 25];
sigma = [3 -0.3; -0.3 25];
ar_z = linspace(-3, 3, 10);
ar_a = linspace(0, 50, 10);
[mt_a, mt_z] = meshgrid(ar_a, ar_z);
mt_x = [mt_z(:) mt_a(:)];
ar_prob = mvnpdf(mt_x, mu, sigma);
mt_prob = reshape(ar_prob,length(ar_a),length(ar_z));
mt_prob = mt_prob/sum(mt_prob, 'all');
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Scatter Size = F(a,z), Prob Mass at State-Space'};
mp_support_graph('cl_st_ytitle') = {'Shocks, z'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'eastoutside';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
mp_support_graph('cl_colors') = 'parula'; % any predefined matlab colormap
mp_support_graph('it_dist_csize_multiple') = 5000;

% Call function
```

```
ar_row_grid = ar_z;
ar_col_grid = ar_a;
st_figtype = 'dist';
ff_graph_grid(mt_prob, (ar_z), ar_col_grid, mp_support_graph, st_figtype);
```



Chapter 7

Support Tools

7.1 FF_CONTAINER_MAP_DISPLAY Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_container_map_display** from the **MEconTools Package.** This function summarizes statistics of matrixes stored in a container map, as well as scalar, string, function and other values stored in container maps.

7.1.1 Test FF_CONTAINER_MAP_DISPLAY Defaults

Call the function with defaults.

```
ff_container_map_display();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std
mat_1	1	7	2	12	3	4	6.5142	0.54285	0.2232
mat_2	2	8	2	2650	50	53	1313.3	0.49559	0.29232
mat_2_boolean	3	9	2	2650	50	53	1361	0.51358	0.49991
mat_3	4	10	2	4	2	2	1.8111	0.45277	0.45111
tensor_1	5	15	3	16	2	8	7.3043	0.45652	0.27787
tensor_2	6	16	3	75	3	25	40.195	0.53593	0.29044
tensor_3	7	17	2	4	1	4	1.6926	0.42315	0.37389
tesseract_1	8	18	4	72	3	24	34.321	0.47669	0.26374
tesseract_2	9	19	4	20	2	10	8.4191	0.42096	0.28981
tesseract_bl_3	10	20	4	10	1	10	3	0.3	0.48305

```
xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4
r1	0.69647	0.55131	0.98076	0.39212
r2	0.28614	0.71947	0.68483	0.34318
r3	0.22685	0.42311	0.48093	0.72905

```
xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c50	c51	c52	c53

```
          --------    --------    -------    --------    --------    --------    --------    ------

    r1     0.43857      0.6249    0.17108     0.56564    0.072152     0.67855     0.61667     0.540
    r2    0.059678     0.67469    0.82911    0.084904     0.63289     0.27236     0.32528     0.249
    r3     0.39804     0.84234    0.33867     0.58267    0.046367     0.44513    0.075047      0.78
    r4       0.738    0.083195    0.55237     0.81484     0.50561     0.11117     0.59532     0.356
    r5     0.18249     0.76368    0.57855     0.33707     0.10653    0.028681      0.7435     0.918
    r46     0.6813     0.55326    0.88786     0.69983     0.83758     0.16382     0.74191    0.0656
    r47    0.87546     0.85445    0.69631     0.66117     0.97069     0.79092     0.42466     0.787
    r48    0.51042     0.38484    0.44033    0.049097    0.017768     0.33302     0.24401     0.979
    r49    0.66931     0.31679    0.43821      0.7923     0.12979     0.75311     0.79466    0.0790
    r50    0.58594     0.35426     0.7651     0.51872     0.86415     0.58281     0.84795      0.45

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxx
          c1       c2       c3       c4       c50      c51      c52      c53
          -----    -----    -----    -----    -----    -----    -----    -----

    r1    true     false    false    true     true     false    true     true
    r2    true     false    true     true     false    false    true     true
    r3    false    true     false    true     false    true     false    true
    r4    false    true     false    false    false    true     true     true
    r5    true     true     true     false    true     false    false    true
    r46   false    true     true     false    true     true     true     true
    r47   true     true     true     true     true     true     false    false
    r48   true     false    false    false    true     true     false    true
    r49   true     true     false    true     true     true     false    false
    r50   false    false    false    false    false    false    false    false

xxx TABLE:mat_3 xxxxxxxxxxxxxxxxx
             c1          c2
          ----------    -------

    r1    0.00012471    0.13253
    r2       0.88615    0.79226

xxx TABLE:tensor_1 xxxxxxxxxxxxxxxxx
            c1         c2         c3         c4         c5         c6         c7         c8
          --------    -------    -------    -------    -------    -------    ------    -------

    r1    0.019363    0.34271    0.52167    0.53703    0.75756    0.68839    0.8345    0.26597
    r2    0.018091    0.33355    0.11738    0.77857    0.81933    0.28644    0.6157      0.368

xxx TABLE:tensor_2 xxxxxxxxxxxxxxxxx
            c1         c2         c3         c4         c22        c23         c24         c25
          --------    -------    -------    -------    -------    -------    ---------    -------

    r1     0.51866    0.40495    0.48278    0.99731    0.46584    0.62976     0.035924    0.10505
    r2    0.028692    0.37408    0.24149    0.35201    0.66054    0.87243    0.0024293    0.81088
    r3     0.87339    0.19457    0.83212    0.15315    0.77859    0.96663       0.2501     0.8056

xxx TABLE:tensor_3 xxxxxxxxxxxxxxxxx
            c1         c2         c3          c4
          ------    ------    -------    -------

    r1    0.1219    0.5119    0.91553    0.14329

xxx TABLE:tesseract_1 xxxxxxxxxxxxxxxxx
            c1         c2         c3          c4         c21        c22        c23        c24
```

```
          -------    -------    -------    --------    -------    -------    -------    -------

    r1    0.64531    0.59299    0.32115     0.67653    0.90328    0.56911    0.52562    0.12014
    r2    0.74558     0.5007    0.46142     0.21384    0.35564    0.13732      0.155    0.23786
    r3    0.91137    0.46403    0.18118    0.049919    0.46246    0.46842    0.75348    0.64547

xxx TABLE:tesseract_2 xxxxxxxxxxxxxxxxx
             c1         c2         c3         c4         c7         c8          c9         c10
          --------    -------    -------    -------    -------    -------    --------    -------

    r1     0.28898    0.48211    0.44359    0.97146    0.61782    0.65121     0.80715    0.11605
    r2    0.094493    0.34941    0.17595    0.14192    0.16754    0.57097    0.043114    0.70518

xxx TABLE:tesseract_bl_3 xxxxxxxxxxxxxxxxx
           c1       c2       c3       c4       c7       c8       c9       c10
          -----    -----    -----    -----    -----    -----    -----    -----

    r1    false    false    true     true     false    true     false    false

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i    idx     value
                      -    ---    -------

    boolean_1         1     1           1
    empty             2     2         NaN
    mat_4             3    11     0.74898
    string_float_1    4    13      1021.1
    string_int_1      5    14        1021

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 String
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i     idx            string
                     ---    ----    ---------------------

    list_string_1    "1"    "5"     "col1;col2;col3;col4"
    list_string_2    "2"    "6"     "row1;row2;row3;row4"
    string_1         "3"    "12"    "Table Name"

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
 Functions
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
              i     idx      functionString
             ---    ---    -------------------

    func1    "1"    "3"    "@(x)1+2+x"
    func2    "2"    "4"    "@(x,y)x*1+sqrt(y)"
```

7.1.2 Test FF_CONTAINER_MAP_DISPLAY summarize Matrix Only

Three large matrixes, show summaries

```
% Create Container
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
```

```
rng(123);
mp_container_map('mat_1') = rand(100,100);
mp_container_map('mat_2') = rand(100,100)*2 + 1;
mp_container_map('mat_2_boolean') = (rand(100,100) > 0.5);
% Will only print
ff_container_map_display(mp_container_map);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                     i    idx    ndim    numel    rowN    colN     sum       mean        std      co
                     -    ---    ----    -----    ----    ----    ------    -------    -------    --

    mat_1            1     1      2      10000    100     100     4982.3    0.49823    0.28829    0.
    mat_2            2     2      2      10000    100     100      20029     2.0029    0.57632    0.
    mat_2_boolean    3     3      2      10000    100     100       4995     0.4995    0.50002     1
```

7.1.3 Test FF_CONTAINER_MAP_DISPLAY Show Matrix Subset

A container map with three small matrixes, print only only 2 rows and 3 columns.

```
% Create Container
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
rng(789);
mp_container_map('mat_1') = rand(3,4);
mp_container_map('mat_2') = rand(50,53);
mp_container_map('mat_2_boolean') = (rand(50,53) > 0.5);
% Will only print
ff_container_map_display(mp_container_map, 2, 3);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                     i    idx    ndim    numel    rowN    colN     sum       mean        std      co
                     -    ---    ----    -----    ----    ----    ------    -------    -------    --

    mat_1            1     1      2        12       3       4     4.9876    0.41564    0.33586    0.
    mat_2            2     2      2      2650      50      53     1324.3    0.49973    0.28834    0.
    mat_2_boolean    3     3      2      2650      50      53       1350    0.50943    0.50001    0.

xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4
          -------    -------    -------    -------

    r1    0.32333    0.62442    0.01062    0.53815
    r3    0.79378    0.75889    0.11104    0.55157

xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxx
             c1         c2         c52        c53
           -------    -------    -------    -------

    r1     0.72837    0.20976    0.74583    0.22321
    r50    0.52812      0.545    0.49521    0.29826

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxxx
             c1        c2        c52       c53
           -------    -------    -------    -------
```

```
r1     false    true     true     true
r50    true     false    false    true
```

Chapter 8

Data Structures

8.1 FF_SAVEBORR_GRID Example for Generating Asset Grid

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_saveborr_grid** from the **MEconTools Package.** This function generates variously spaced savings/borrowing states/choices grid.

8.1.1 Test FF_SAVEBORR_GRID Defaults

Call the function with defaults.

```
ff_saveborr_grid();
```

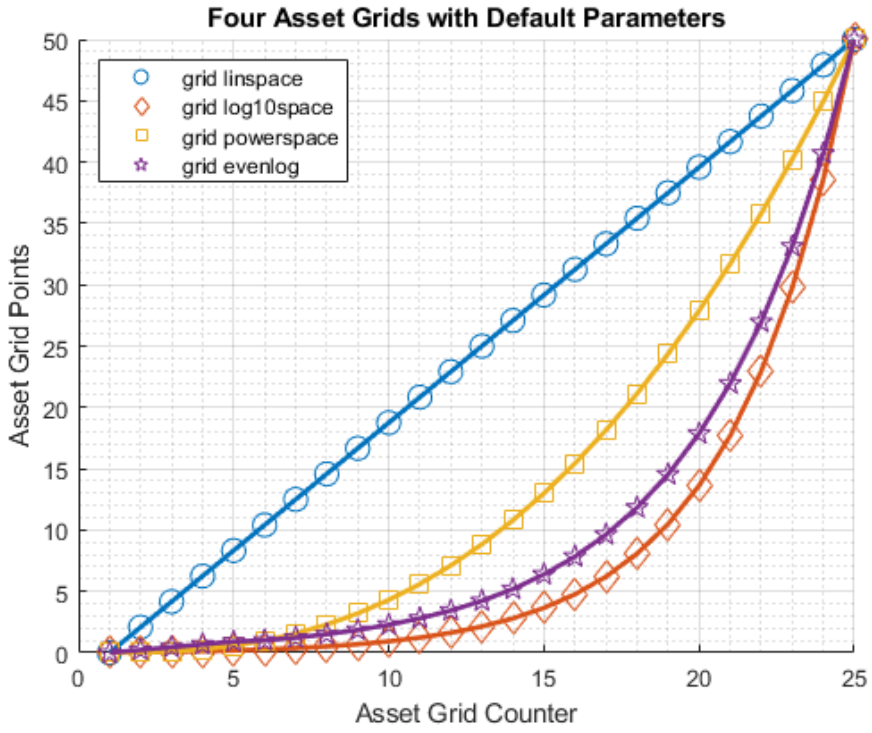
```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i    idx    ndim    numel    rowN    colN     sum       mean      std      coe
                      -    ---    ----    -----    ----    ----    ------    ------    ------    ---

    ar_fl_saveborr    1     1      2       25       25      1      385.93    15.437    15.324    0.9

xxx TABLE:ar_fl_saveborr xxxxxxxxxxxxxxxxxx
             c1
           ------

    r1          1
    r2     1.0174
    r3     1.0982
    r4     1.2707
    r5     1.5557
    r6     1.9707
    r7     2.5312
    r8     3.2512
    r9     4.1434
    r10    5.2196
    r11    6.4912
    r12    7.9687
    r13    9.6621
    r14    11.581
```

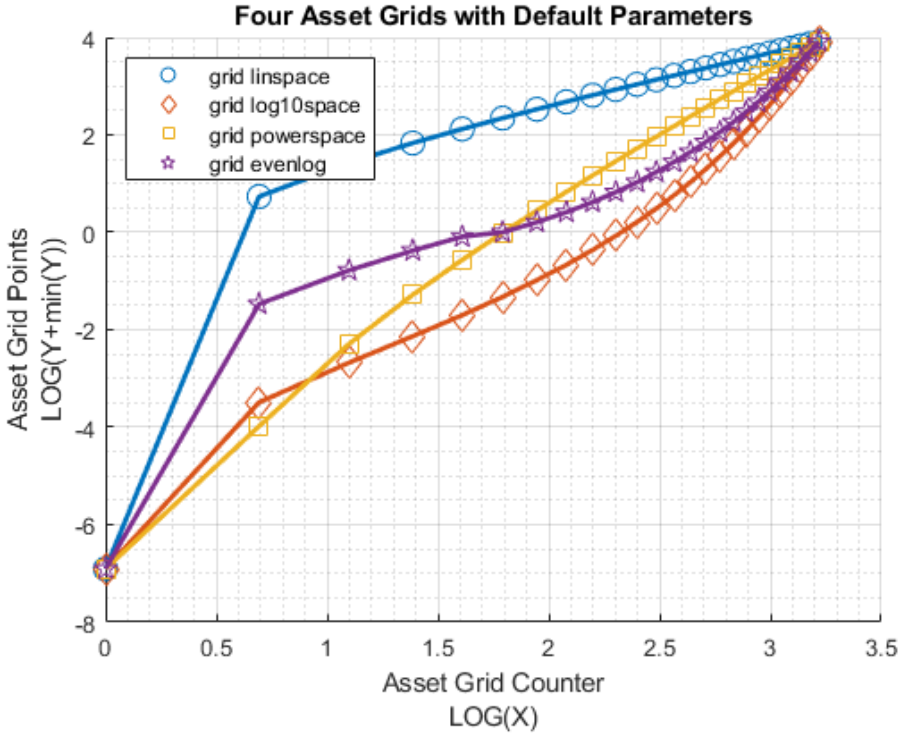
```
    r15    13.735
    r16    16.132
    r17    18.781
    r18    21.691
    r19     24.87
    r20    28.324
    r21    32.063
    r22    36.093
    r23    40.421
    r24    45.054
    r25        50

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                               i    idx    value
                               -    ---    -----

    grid_evenlog_threshold     1     2       1
    grid_log10space_x1         2     3     0.3
    grid_log10space_x2         3     4       3
    grid_powerspace_power      4     5     2.5
```

8.1.2 Test FF_SAVEBORR_GRID Default Linear Grid, Log Grid, Power Grid, Threshold Grid

Call the function with defaults.

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
% Four types of grid points
st_grid_type = 'grid_linspace';
[ar_fl_saveborr_linspace] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_log10space';
[ar_fl_saveborr_log10space] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_powerspace';
[ar_fl_saveborr_powerspace] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_evenlog';
[ar_fl_saveborr_evenlog] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
% draw four types of lines jointly
mt_value = [ar_fl_saveborr_linspace'; ar_fl_saveborr_log10space'; ...
    ar_fl_saveborr_powerspace'; ar_fl_saveborr_evenlog'];
ar_row_grid = ["grid linspace", "grid log10space", "grid powerspace", "grid evenlog"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Four Asset Grids with Default Parameters'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



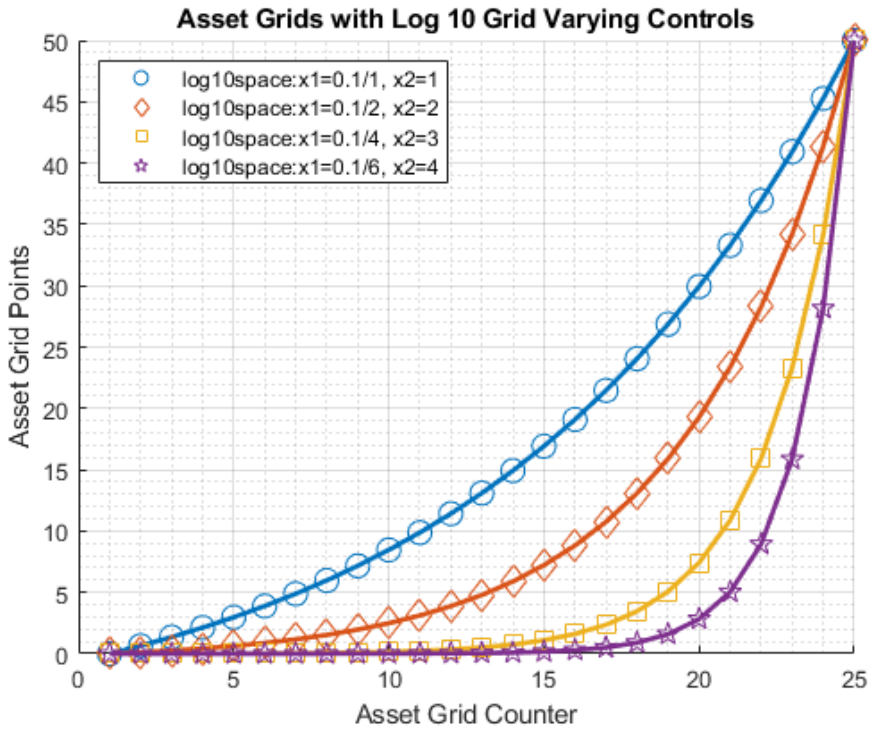


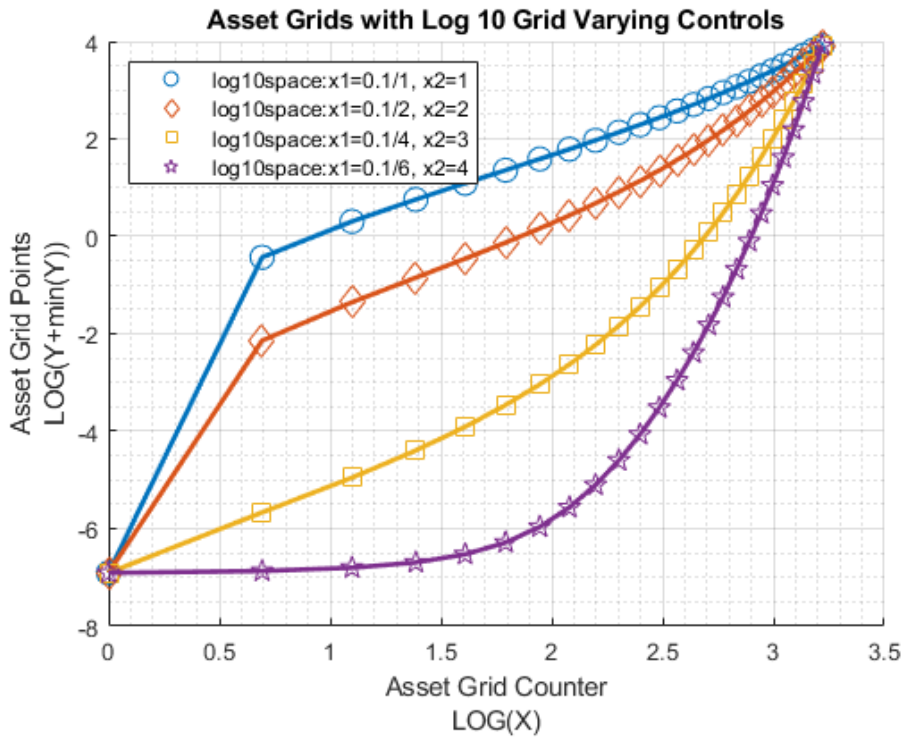
8.1.3 Test FF_SAVEBORR_GRID Log Grid Changing Parameters

Log grid, same min and max, change log X1 and X2 points

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_log10space';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_log10space_x1') = 0.1;
```

```
mp_grid_control('grid_log10space_x2') = 1;
[ar_fl_log10space_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/2;
mp_grid_control('grid_log10space_x2') = 1*2;
[ar_fl_log10space_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/4;
mp_grid_control('grid_log10space_x2') = 1*4;
[ar_fl_log10space_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/6;
mp_grid_control('grid_log10space_x2') = 1*6;
[ar_fl_log10space_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
% draw four types of lines jointly
mt_value = [ar_fl_log10space_a'; ar_fl_log10space_b'; ...
    ar_fl_log10space_c'; ar_fl_log10space_d'];
ar_row_grid = [...
    "log10space:x1=0.1/1, x2=1", ...
    "log10space:x1=0.1/2, x2=2", ...
    "log10space:x1=0.1/4, x2=3", ...
    "log10space:x1=0.1/6, x2=4"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Log 10 Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```

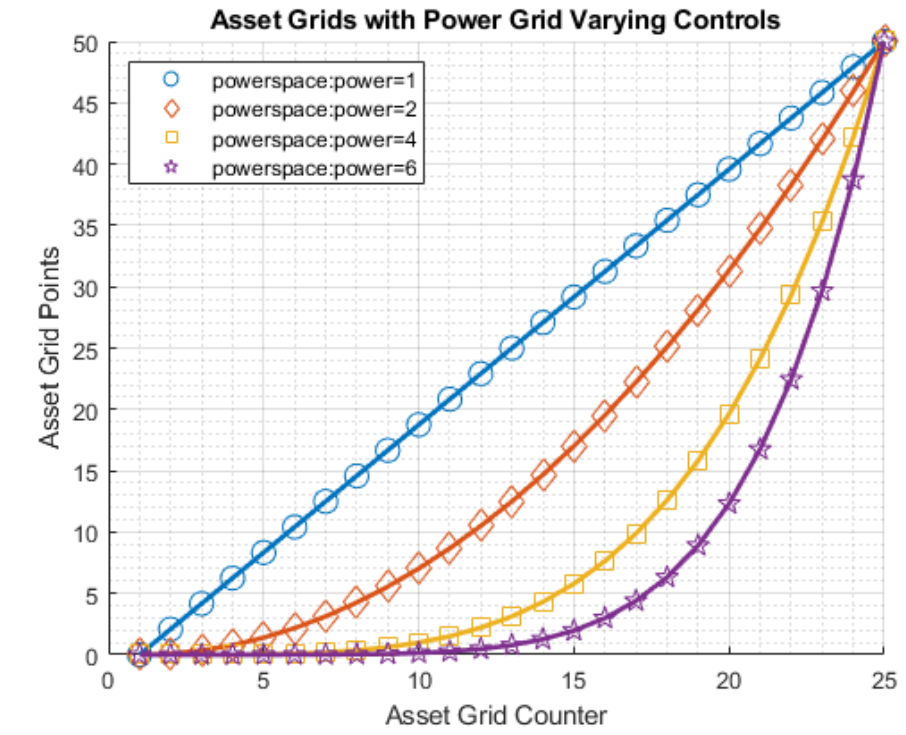


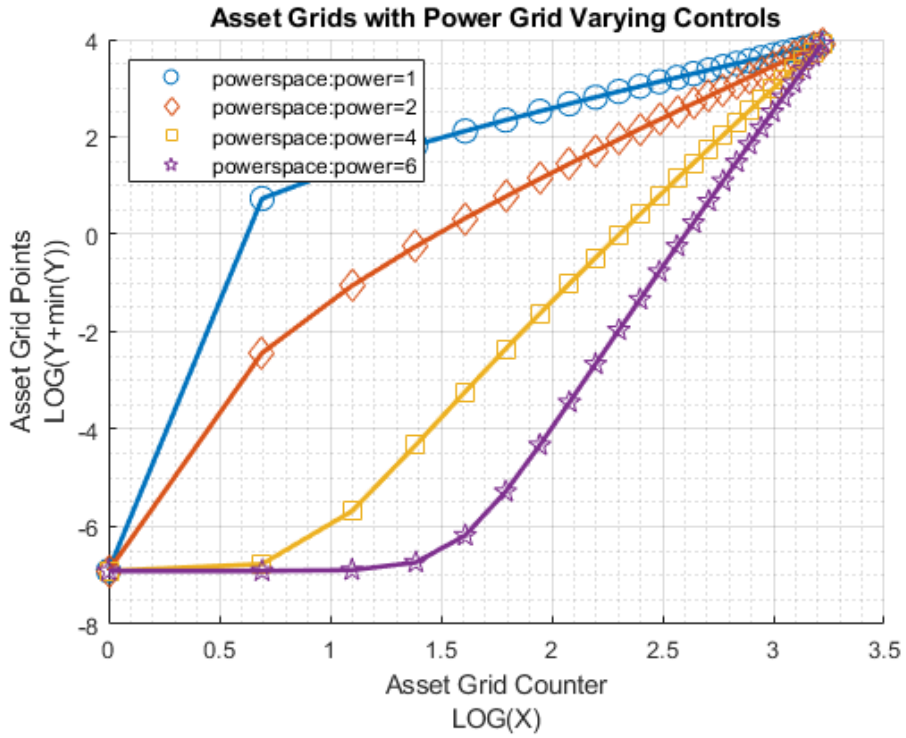


8.1.4 Test FF_SAVEBORR_GRID Power Grid Changing Parameters

Log grid, same min and max, change log X1 and X2 points

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_powerspace';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_powerspace_power') = 1;
[ar_fl_powerspace_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_powerspace_power') = 2;
[ar_fl_powerspace_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_powerspace_power') = 4;
[ar_fl_powerspace_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_powerspace_power') = 6;
[ar_fl_powerspace_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
% draw four types of lines jointly
mt_value = [ar_fl_powerspace_a'; ar_fl_powerspace_b'; ...
    ar_fl_powerspace_c'; ar_fl_powerspace_d'];
ar_row_grid = [...
    "powerspace:power=1", ...
    "powerspace:power=2", ...
    "powerspace:power=4", ...
    "powerspace:power=6"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Power Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



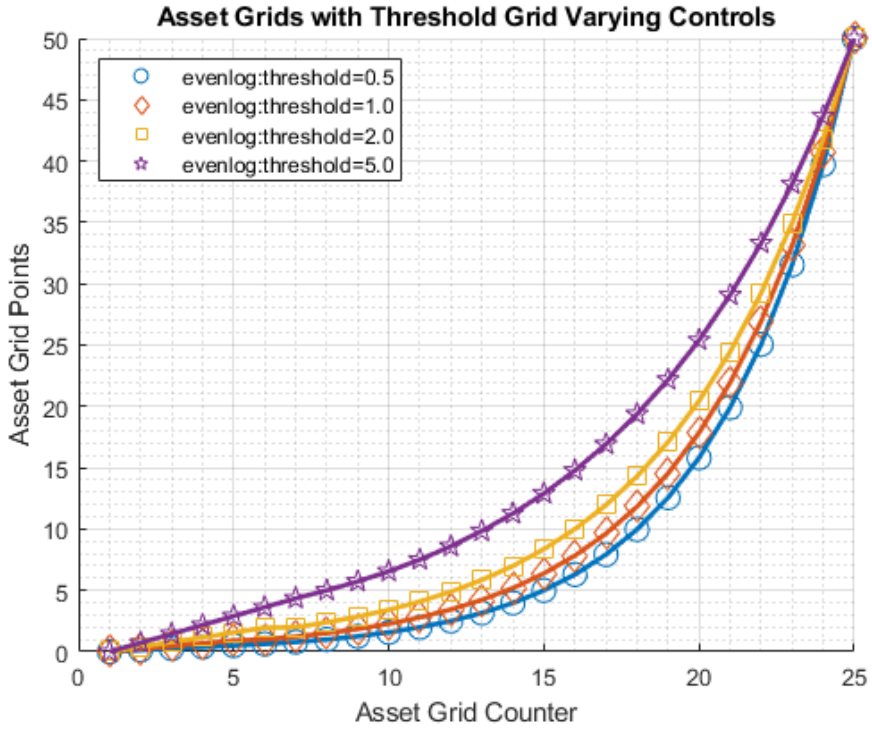


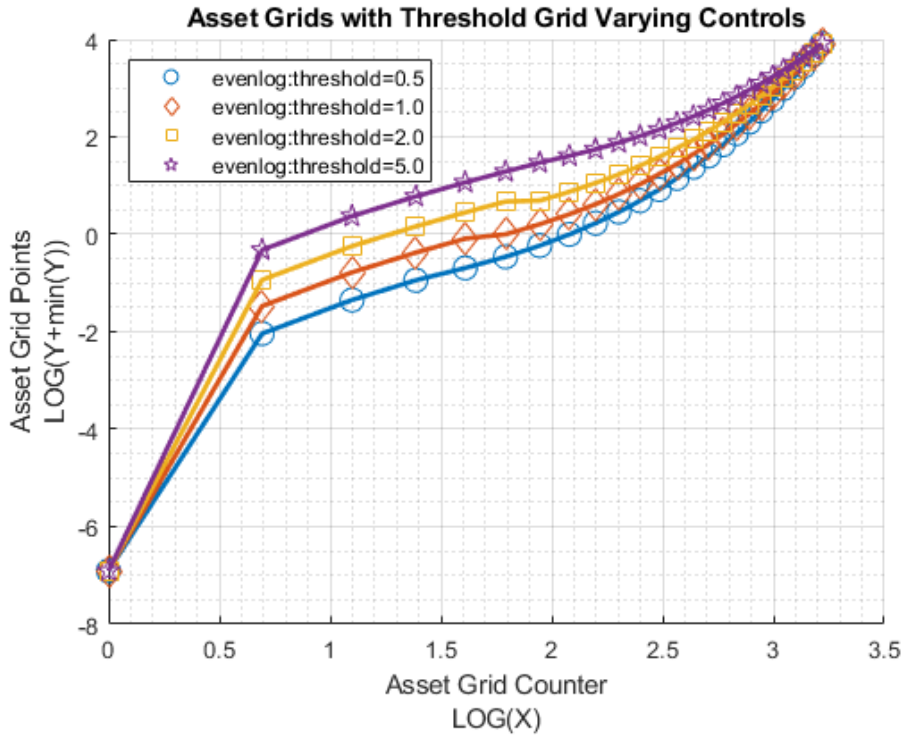
8.1.5 Test FF_SAVEBORR_GRID Threshold Grid Changing Parameters

Threshold Grid, Changing Threshold Levels. Initial segments below threshold are linspace, then logspace.

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_evenlog';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_evenlog_threshold') = 0.50;
```

```
[ar_fl_evenlog_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 1.00;
[ar_fl_evenlog_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 2;
[ar_fl_evenlog_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 5;
[ar_fl_evenlog_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
% draw four types of lines jointly
mt_value = [ar_fl_evenlog_a'; ar_fl_evenlog_b'; ...
    ar_fl_evenlog_c'; ar_fl_evenlog_d'];
ar_row_grid = [...
    "evenlog:threshold=0.5", ...
    "evenlog:threshold=1.0", ...
    "evenlog:threshold=2.0", ...
    "evenlog:threshold=5.0"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Threshold Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```




Asset Grids with Threshold Grid Varying Controls
evenlog:threshold=0.5
evenlog:threshold=1.0
evenlog:threshold=2.0
evenlog:threshold=5.0
Asset Grid Points
LOG(Y+min(Y))
Asset Grid Counter
LOG(X)
4
2
0
-2
-4
-6
-8
0
0.5
1
1.5
2
2.5
3
3.5

Chapter 9

Common Functions

9.1 FFY_TAUCHEN AR1 Shock Discretization Example

> Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ffy_tauchen** from the **MEconTools Package.** : See also the **ffy_rouwenhorst** function from the **MEconTools Package.** This function discretize a mean zero AR1 process, uses Tauchen (1986). See AR 1 Example for some details on how the AR1 process works. And See Kopecky and Suen (2010).

9.1.1 Test FFY_TAUCHEN Defaults

Call the function with defaults. Default sd bounds arer plus and minus 4. This is used in the following examples, unless otherwise specified as the 5th parameter.

```
ffy_tauchen();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum    mean      std      coef
                         -    ---    ----    -----    ----    ----    ---    ----    -------    ----

    ar_disc_ar1          1     1      2        5       5       1       0       0     0.79057
    mt_disc_ar1_trans    2     6      2       25       5       5       5     0.2     0.27623     1.3

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
           c1
          ----

    r1      -1
    r2    -0.5
    r3       0
    r4     0.5
    r5       1

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1             c2             c3             c4             c5
          ----------    ----------    --------    ----------    ----------

    r1       0.22663       0.73331    0.040048    1.0689e-05    7.3923e-12
    r2      0.012224       0.58648     0.39831     0.0029797     7.605e-08
```

```
    r3    8.8417e-05       0.10556      0.7887       0.10556    8.8417e-05
    r4     7.605e-08     0.0029797     0.39831       0.58648      0.012224
    r5    7.3923e-12    1.0689e-05    0.040048       0.73331       0.22663

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    -----

    fl_ar1_persistence    1     2      0.6
    fl_ar1_step           2     3      0.5
    fl_shk_std            3     4      0.2
    it_std_bound          4     5        4
```

9.1.2 Test FFY_TAUCHEN Specify Parameters

With a grid of 10 points, the sd bounds on Tauchen and Rouwenhorst are identical. With the not extremely persistent shock process here, the Tauchen and Rouwenhorst Results are very similar.

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose, it_std_bound] = ...
    deal(0.60, 0.10, 10, true, 3);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose, it_std_bound);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN        sum           mean
                          -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1           1     1      2        10      10       1     -7.2164e-16    -7.2164e-17
    mt_disc_ar1_trans     2     6      2       100      10      10              10            0.1

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
           ---------

    r1        -0.375
    r2      -0.29167
    r3      -0.20833
    r4        -0.125
    r5     -0.041667
    r6      0.041667
    r7         0.125
    r8       0.20833
    r9       0.29167
    r10        0.375

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1            c2            c3            c4           c5           c6           c7
           ----------    ----------    ----------    ----------    --------    --------    ---------

    r1        0.13933       0.26196       0.31887       0.20154    0.066066    0.011201    0.0009785
    r2       0.056673       0.16995       0.30658       0.28713      0.1396    0.035167     0.004575
    r3        0.01861      0.087039       0.23281       0.32308     0.23281    0.087039      0.01684
    r4      0.0048925      0.035167        0.1396       0.28713     0.30658     0.16995      0.04884
    r5      0.0010235      0.011201      0.066066       0.20154     0.31887     0.26196       0.1116
```

```
    r6     0.00016962     0.0028101       0.02466       0.11169     0.26196     0.31887       0.2015
    r7     2.2197e-05    0.00055483     0.0072547      0.048841     0.16995     0.30658       0.2871
    r8     2.2881e-06    8.6129e-05     0.0016806      0.016841    0.087039     0.23281       0.3230
    r9     1.8543e-07    1.0503e-05    0.00030628     0.0045756    0.035167      0.1396       0.2871
    r10    1.1798e-08    1.0053e-06    4.3874e-05    0.00097859    0.011201    0.066066       0.2015

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2         0.6
    fl_ar1_step           2     3    0.083333
    fl_shk_std            3     4         0.1
    it_std_bound          4     5           3
```

9.1.3 Test FFY_TAUCHEN High Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.01, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN    sum     mean        std      c
                          -    ---    ----    -----    ----    ----    ---    -------    -------    -

    ar_disc_ar1           1     1      2        7       7       1       0          0    0.15314
    mt_disc_ar1_trans     2     6      2       49       7       7       7    0.14286    0.35338

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
             c1
          ---------

    r1     -0.21266
    r2     -0.14178
    r3    -0.070888
    r4            0
    r5     0.070888
    r6      0.14178
    r7      0.21266

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1             c2             c3             c4             c5             c6
          -----------    -----------    -----------    ----------    ----------    ----------    ---

    r1        0.99957     0.00043152              0             0             0             0
    r2     0.00011382        0.99955      0.0003337             0             0             0
    r3     4.8683e-27        0.00015        0.99959    0.00025684             0             0
    r4     1.4175e-70     1.0439e-26     0.00019675       0.99961    0.00019675             0
    r5    1.9884e-135      4.986e-70     2.2273e-26    0.00025684       0.99959       0.00015
    r6    1.2359e-221     1.149e-134     1.7451e-69    4.7287e-26     0.0003337       0.99955    0.0
    r7              0    1.1738e-220    6.6059e-134     6.077e-69    9.9893e-26    0.00043152
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2         0.99
    fl_ar1_step           2     3     0.070888
    fl_shk_std            3     4         0.01
    it_std_bound          4     5            3
```

9.1.4 Test FFY_TAUCHEN Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN       sum           mean          --
                          -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1           1     1      2        7       7       1      3.4694e-18    4.9564e-19    0.
    mt_disc_ar1_trans     2     6      2       49       7       7               7       0.14286     0

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
          ---------

    r1    -0.030002
    r2    -0.020001
    r3    -0.010001
    r4            0
    r5     0.010001
    r6     0.020001
    r7     0.030002

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1           c2          c3          c4          c5          c6          c7
          ---------    --------    -------    -------    -------    --------    ---------

    r1    0.0067533    0.064018     0.2484    0.38278    0.23505    0.057298    0.0057011
    r2    0.0065668     0.06286    0.24618    0.38287    0.23728     0.05838    0.0058656
    r3    0.0063849    0.061717    0.24396    0.38292     0.2395    0.059478    0.0060344
    r4    0.0062075     0.06059    0.24173    0.38294    0.24173     0.06059    0.0062075
    r5    0.0060344    0.059478     0.2395    0.38292    0.24396    0.061717    0.0063849
    r6    0.0058656     0.05838    0.23728    0.38287    0.24618     0.06286    0.0065668
    r7    0.0057011    0.057298    0.23505    0.38278     0.2484    0.064018    0.0067533

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------
```

```
    fl_ar1_persistence    1     2         0.01
    fl_ar1_step           2     3     0.010001
    fl_shk_std            3     4         0.01
    it_std_bound          4     5            3
```

9.1.5 Test FFY_TAUCHEN High Persistence, High SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.99, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN        sum           mean
                          -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1           1     1      2        7       7       1      -7.1054e-15    -1.0151e-15
    mt_disc_ar1_trans     2     6      2       49       7       7                7        0.14286

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
          -----------

    r1        -21.054
    r2        -14.036
    r3        -7.0179
    r4    -1.7764e-15
    r5         7.0179
    r6         14.036
    r7         21.054

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1              c2              c3              c4              c5              c6
          -----------    -----------    -----------    ----------    ----------    ----------    ---

    r1        0.99957     0.00043152              0             0             0             0
    r2     0.00011382        0.99955      0.0003337             0             0             0
    r3     4.8683e-27        0.00015        0.99959    0.00025684             0             0
    r4     1.4175e-70     1.0439e-26     0.00019675       0.99961    0.00019675             0
    r5    1.9884e-135      4.986e-70     2.2273e-26    0.00025684       0.99959       0.00015
    r6    1.2359e-221     1.149e-134     1.7451e-69    4.7287e-26     0.0003337       0.99955    0.0
    r7              0    1.1738e-220    6.6059e-134     6.077e-69    9.9893e-26    0.00043152

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    ------

    fl_ar1_persistence    1     2       0.99
    fl_ar1_step           2     3     7.0179
    fl_shk_std            3     4       0.99
    it_std_bound          4     5          3
```

9.1.6 Test FFY_TAUCHEN Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN       sum           mean
                          -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1           1     1      2        7       7       1      3.4694e-18    4.9564e-19    0.
    mt_disc_ar1_trans     2     6      2       49       7       7               7       0.14286     0

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
             c1
          ---------

    r1    -0.030002
    r2    -0.020001
    r3    -0.010001
    r4            0
    r5     0.010001
    r6     0.020001
    r7     0.030002

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
             c1           c2          c3          c4          c5          c6          c7
          ---------    --------    -------    -------    -------    --------    ---------

    r1    0.0067533    0.064018     0.2484    0.38278    0.23505    0.057298    0.0057011
    r2    0.0065668     0.06286    0.24618    0.38287    0.23728     0.05838    0.0058656
    r3    0.0063849    0.061717    0.24396    0.38292     0.2395    0.059478    0.0060344
    r4    0.0062075     0.06059    0.24173    0.38294    0.24173     0.06059    0.0062075
    r5    0.0060344    0.059478     0.2395    0.38292    0.24396    0.061717    0.0063849
    r6    0.0058656     0.05838    0.23728    0.38287    0.24618     0.06286    0.0065668
    r7    0.0057011    0.057298    0.23505    0.38278     0.2484    0.064018    0.0067533

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2        0.01
    fl_ar1_step           2     3    0.010001
    fl_shk_std            3     4        0.01
    it_std_bound          4     5           3
```

9.2 FFY_ROUWENHORST AR1 Shock Discretization Example

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ffy_rouwenhorst** from the **MEconTools Package.** See also **ffy_tauchen** function from the **MEconTools Package.** This function discretize a mean zero AR1 process, uses Rouwenhorst (1995). See AR 1 Example for some details on how the AR1 process works. And See Kopecky and Suen (2010).

9.2.1 Test FFY_ROUWENHORST Defaults

Call the function with defaults.

```
ffy_rouwenhorst();
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN    sum    mean      std      coef
                          -    ---    ----    -----    ----    ----    ---    ----    -------    ----

    ar_disc_ar1           1     1      2        5       5       1       0       0     0.39528
    mt_disc_ar1_trans     2    11      2       25       5       5       5     0.2     0.18246    0.91

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
           c1
          -----

    r1     -0.5
    r2    -0.25
    r3        0
    r4     0.25
    r5      0.5

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
            c1        c2        c3        c4        c5
          ------    ------    ------    ------    ------

    r1    0.4096    0.4096    0.1536    0.0256    0.0016
    r2    0.1024    0.4864    0.3264    0.0784    0.0064
    r3    0.0256    0.2176    0.5136    0.2176    0.0256
    r4    0.0064    0.0784    0.3264    0.4864    0.1024
    r5    0.0016    0.0256    0.1536    0.4096    0.4096

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    -----

    fl_ar1_beg            1     2     -0.5
    fl_ar1_end            2     3      0.5
    fl_ar1_persistence    3     4      0.6
    fl_ar1_step           4     5     0.25
    fl_p0                 5     6      0.8
    fl_q0                 6     7      0.8
    fl_shk_std            7     8      0.2
    fl_sig_ar1            8     9     0.25
    it_std_bound          9    10        0
```

9.2.2 Test FFY_ROUWENHORST Specify Parameters

With a grid of 10 points, the Rwouenhorst bounds on standard deviations are equall to Tauchen bounds of 3. With the not extremely persistent shock process here, the Tauchen and Rouwenhorst Results are very similar.

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.60, 0.10, 10, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN       sum           mean          --
                         -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1          1     1      2        10      10       1     5.5511e-17    5.5511e-18     0
    mt_disc_ar1_trans    2    11      2       100      10      10             10           0.1    0.

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
           ---------

    r1        -0.375
    r2       -0.29167
    r3       -0.20833
    r4         -0.125
    r5       -0.041667
    r6        0.041667
    r7           0.125
    r8         0.20833
    r9         0.29167
    r10          0.375

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
               c1             c2             c3             c4             c5             c6             c7
           ----------    ----------    ----------    ---------    --------    --------    ---------

    r1        0.13422       0.30199       0.30199      0.17616     0.06606    0.016515    0.0027525
    r2       0.033554       0.20133       0.32716      0.26424     0.12662    0.038535    0.0075694
    r3      0.0083886      0.081789       0.26267      0.32755     0.21401    0.082747     0.019741
    r4      0.0020972      0.028312       0.14038      0.30946     0.30369     0.15877     0.047989
    r5     0.00052429       0.009044      0.061145      0.20246     0.33477     0.25969      0.10585
    r6     0.00013107     0.0027525      0.023642      0.10585     0.25969     0.33477      0.20246
    r7     3.2768e-05    0.00081101     0.0084603     0.047989     0.15877     0.30369      0.30946
    r8      8.192e-06    0.00023347     0.0028677     0.019741    0.082747     0.21401      0.32755
    r9      2.048e-06    6.6048e-05    0.00093389    0.0075694    0.038535     0.12662      0.26424
    r10      5.12e-07    1.8432e-05    0.00029491    0.0027525    0.016515     0.06606      0.17616

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_beg            1     2       -0.375
    fl_ar1_end            2     3        0.375
```

```
    fl_ar1_persistence    3     4          0.6
    fl_ar1_step           4     5     0.083333
    fl_p0                 5     6          0.8
    fl_q0                 6     7          0.8
    fl_shk_std            7     8          0.1
    fl_sig_ar1            8     9        0.125
    it_std_bound          9    10            0
```

9.2.3 Test FFY_ROUWENHORST High Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.90, 0.02, 7, true);
[ar_z, mt_z_trans] = ffy_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN       sum           mean          --
                         -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1          1     1      2        7       7       1      2.7756e-17    3.9651e-18    0.
    mt_disc_ar1_trans    2     6      2       49       7       7               7       0.14286     0

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
          ----------

    r1      -0.13765
    r2     -0.091766
    r3     -0.045883
    r4    1.3878e-17
    r5      0.045883
    r6      0.091766
    r7       0.13765

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1            c2            c3            c4            c5            c6            c7
          ----------    ----------    ----------    ----------    ----------    ----------    ------

    r1       0.67682       0.32022     0.0029525    2.2423e-07     1.058e-13             0
    r2      0.054147        0.7002       0.24422     0.0014299    6.5815e-08    1.8541e-14
    r3    0.00012097      0.084213       0.73627       0.17874    0.00065947    1.8356e-08    3.1086
    r4    4.8643e-09    0.00028953       0.12539       0.74865       0.12539    0.00028953    4.8643
    r5    3.0921e-15    1.8356e-08    0.00065947       0.17874       0.73627      0.084213    0.0001
    r6    2.9554e-23    1.8558e-14    6.5815e-08     0.0014299       0.24422        0.7002      0.05
    r7    4.1477e-33    2.8319e-22    1.0576e-13    2.2423e-07     0.0029525       0.32022       0.6

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx     value
                         -    ---    --------

    fl_ar1_persistence    1     2          0.9
    fl_ar1_step           2     3     0.045883
    fl_shk_std            3     4         0.02
```

```
    it_std_bound          4     5           3

ar_z_stationary = mt_z_trans^1000;
ar_z_stationary = ar_z_stationary(1,:);
fl_labor_agg = ar_z_stationary*exp(ar_z);
ar_z = exp(ar_z')/fl_labor_agg;
```

9.2.4 Test FFY_ROUWENHORST Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std
                         -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1          1     1      2        7       7       1       0           0    0.017639
    mt_disc_ar1_trans    2    11      2       49       7       7       7     0.14286     0.10985

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
          ----------

    r1     -0.024496
    r2     -0.016331
    r3    -0.0081654
    r4             0
    r5     0.0081654
    r6      0.016331
    r7      0.024496

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5          c6          c7
          --------    --------    -------    -------    -------    --------    --------

    r1    0.016586    0.097547    0.23904    0.31241    0.22966    0.090047    0.014711
    r2    0.016258    0.096266    0.23749    0.31247    0.23124    0.091266    0.015008
    r3    0.015936    0.094997    0.23594    0.31251    0.23281    0.092497    0.015311
    r4     0.01562    0.093741    0.23438    0.31252    0.23438    0.093741     0.01562
    r5    0.015311    0.092497    0.23281    0.31251    0.23594    0.094997    0.015936
    r6    0.015008    0.091266    0.23124    0.31247    0.23749    0.096266    0.016258
    r7    0.014711    0.090047    0.22966    0.31241    0.23904    0.097547    0.016586

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx      value
                         -    ---    ---------

    fl_ar1_beg           1     2     -0.024496
    fl_ar1_end           2     3      0.024496
    fl_ar1_persistence   3     4          0.01
    fl_ar1_step          4     5     0.0081654
```

```
    fl_p0                5     6         0.505
    fl_q0                6     7         0.505
    fl_shk_std           7     8          0.01
    fl_sig_ar1           8     9      0.010001
    it_std_bound         9    10             0
```

9.2.5 Test FFY_ROUWENHORST High Persistence, High SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.99, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN       sum           mean          --

    ar_disc_ar1          1     1      2        7       7       1      3.5527e-15    5.0753e-16     1
    mt_disc_ar1_trans    2    11      2       49       7       7               7       0.14286    0.

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
            c1
          -------

    r1     -17.19
    r2     -11.46
    r3    -5.7301
    r4          0
    r5     5.7301
    r6      11.46
    r7      17.19

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1            c2            c3            c4            c5            c6            c7
          ----------    ----------    ----------    ----------    ----------    ----------    ------

    r1       0.97037      0.029257    0.00036756    2.4627e-06    9.2815e-09    1.8656e-11    1.5625
    r2     0.0048762        0.9705      0.024382    0.00024504    1.2314e-06    3.0938e-09    3.1094
    r3    2.4504e-05      0.009753       0.97057      0.019506    0.00014703    4.9254e-07    6.1877
    r4    1.2313e-07    7.3513e-05       0.01463       0.97059       0.01463    7.3513e-05    1.2313
    r5    6.1877e-10    4.9254e-07    0.00014703      0.019506       0.97057      0.009753    2.4504
    r6    3.1094e-12    3.0938e-09    1.2314e-06    0.00024504      0.024382        0.9705     0.004
    r7    1.5625e-14    1.8656e-11    9.2815e-09    2.4627e-06    0.00036756      0.029257       0.9

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    value
                         -    ---    ------

    fl_ar1_beg           1     2     -17.19
    fl_ar1_end           2     3      17.19
    fl_ar1_persistence   3     4       0.99
    fl_ar1_step          4     5     5.7301
    fl_p0                5     6      0.995
```

```
    fl_q0                6     7      0.995
    fl_shk_std           7     8       0.99
    fl_sig_ar1           8     9     7.0179
    it_std_bound         9    10          0
```

9.2.6 Test FFY_ROUWENHORST Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std
                         -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1          1     1      2        7       7       1       0           0    0.017639
    mt_disc_ar1_trans    2    11      2       49       7       7       7     0.14286     0.10985

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
          ----------

    r1     -0.024496
    r2     -0.016331
    r3    -0.0081654
    r4             0
    r5     0.0081654
    r6      0.016331
    r7      0.024496

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5          c6          c7
          --------    --------    -------    -------    -------    --------    --------

    r1    0.016586    0.097547    0.23904    0.31241    0.22966    0.090047    0.014711
    r2    0.016258    0.096266    0.23749    0.31247    0.23124    0.091266    0.015008
    r3    0.015936    0.094997    0.23594    0.31251    0.23281    0.092497    0.015311
    r4     0.01562    0.093741    0.23438    0.31252    0.23438    0.093741     0.01562
    r5    0.015311    0.092497    0.23281    0.31251    0.23594    0.094997    0.015936
    r6    0.015008    0.091266    0.23124    0.31247    0.23749    0.096266    0.016258
    r7    0.014711    0.090047    0.22966    0.31241    0.23904    0.097547    0.016586

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx      value
                         -    ---    ---------

    fl_ar1_beg           1     2     -0.024496
    fl_ar1_end           2     3      0.024496
    fl_ar1_persistence   3     4          0.01
    fl_ar1_step          4     5     0.0081654
    fl_p0                5     6         0.505
    fl_q0                6     7         0.505
```

```
fl_shk_std            7     8          0.01
fl_sig_ar1            8     9      0.010001
it_std_bound          9    10             0
```

Chapter 10

System

10.1 FF_FIND_FILES Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_find_files** from the **MEconTools Package.** This function finds files in subfolders of folders.

10.1.1 Test FF_FIND_FILES Defaults

Call the function with defaults.

```
[cl_st_file_names, cl_st_folder_names] = ff_find_files();
disp(cl_st_file_names);
```

```
    'ff_az_vf_vecsv.m'    'ff_ipwkz_vf_vecsv.m'
```

```
disp(cl_st_folder_names);
```

```
    'c:\Users\fan\CodeDynaAsset\m_az\solve'    'c:\Users\fan\CodeDynaAsset\m_ipwkz\solve'
```

10.1.2 Test FF_FIND_FILES search for files in subfolders

Search for .mlx files in two subfolders:

```
st_proj_folder = 'C:\Users\fan\MEconTools\MEconTools\doc\';
cl_st_subfolder = {'generate','vfi'};
st_file_search_name = '*.mlx';
bl_verbose = true;
[cl_st_file_names, cl_st_folder_names] = ff_find_files(...
    st_proj_folder, cl_st_subfolder, st_file_search_name, bl_verbose);
```

```
C:\Users\fan\MEconTools\MEconTools\doc\generate\*.mlx
fx_saveborr_grid.mlx
C:\Users\fan\MEconTools\MEconTools\doc\generate
C:\Users\fan\MEconTools\MEconTools\doc\vfi\*.mlx
fx_vfi_az_bisec_loop.mlx
C:\Users\fan\MEconTools\MEconTools\doc\vfi
fx_vfi_az_bisec_vec.mlx
C:\Users\fan\MEconTools\MEconTools\doc\vfi
fx_vfi_az_loop.mlx
C:\Users\fan\MEconTools\MEconTools\doc\vfi
fx_vfi_az_mzoom_loop.mlx
```

```
C:\Users\fan\MEconTools\MEconTools\doc\vfi
fx_vfi_az_mzoom_vec.mlx
C:\Users\fan\MEconTools\MEconTools\doc\vfi
fx_vfi_az_vec.mlx
C:\Users\fan\MEconTools\MEconTools\doc\vfi
```

10.2 FF_MLX2HTMLPDF_RUNANDEXPORT Examples

Go back to fan's MEconTools Toolbox (bookdown), Matlab Code Examples Repository (bookdown), or Math for Econ with Matlab Repository (bookdown).

This is the example vignette for function: **ff_mlx2htmlpdf_runandexport** from the **MEconTools Package.** This file runs MLX files and converts it to HTML files.

10.2.1 Test FF_MLX2HTMLPDF_RUNANDEXPORT search for MLX files and Convert to HTML

Finds MLX files, re-run, and save to HTML in possibly another folder.

```
st_proj_folder = 'C:\Users\fan\MEconTools\MEconTools\doc\';
cl_st_subfolder = {'generate','graph'};
st_out_folder = 'C:\Users\fan\MEconTools\MEconTools\doc\sys\_test';
st_mlx_search_name = '*.mlx';
st_pub_format = 'html';
bl_run_mlx = true;
bl_run_mlx_only = false;
bl_verbose = true;
ff_mlx2htmlpdf_runandexport(...
    st_proj_folder, cl_st_subfolder, ...
    st_mlx_search_name, st_out_folder, st_pub_format, ...
    bl_run_mlx, bl_run_mlx_only, ...
    bl_verbose);

execute:fx_saveborr_grid.mlx
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                      i    idx    ndim    numel    rowN    colN     sum       mean      std       coe
                      -    ---    ----    -----    ----    ----    ------    ------    ------    ---

    ar_fl_saveborr    1     1      2       25       25      1      385.93    15.437    15.324    0.9

xxx TABLE:ar_fl_saveborr xxxxxxxxxxxxxxxxx
             c1
           ------

    r1          1
    r2     1.0174
    r3     1.0982
    r4     1.2707
    r5     1.5557
    r6     1.9707
    r7     2.5312
    r8     3.2512
    r9     4.1434
    r10    5.2196
    r11    6.4912
    r12    7.9687
```

```
    r13    9.6621
    r14    11.581
    r15    13.735
    r16    16.132
    r17    18.781
    r18    21.691
    r19     24.87
    r20    28.324
    r21    32.063
    r22    36.093
    r23    40.421
    r24    45.054
    r25        50

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                              i    idx    value
                              -    ---    -----

    grid_evenlog_threshold    1     2       1
    grid_log10space_x1        2     3     0.3
    grid_log10space_x2        3     4       3
    grid_powerspace_power     4     5     2.5
```

10.2.2 Test FF_MLX2HTMLPDF_RUNANDEXPORT re-run MLX

Finds MLX files, re-run, do NOT save HTML.

```
st_proj_folder = 'C:\Users\fan\MEconTools\MEconTools\doc\';
cl_st_subfolder = {'external'};
st_mlx_search_name = '*.mlx';
st_out_folder = '';
st_pub_format = '';
bl_run_mlx = true;
bl_run_mlx_only = true;
bl_verbose = true;
ff_mlx2htmlpdf_runandexport(...
    st_proj_folder, cl_st_subfolder, ...
    st_mlx_search_name, st_out_folder, st_pub_format, ...
    bl_run_mlx, bl_run_mlx_only, ...
    bl_verbose);

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum    mean      std      coef
                         -    ---    ----    -----    ----    ----    ---    ----    -------    ----

    ar_disc_ar1          1     1      2        5       5       1       0       0     0.39528
    mt_disc_ar1_trans    2    11      2       25       5       5       5     0.2     0.18246    0.91

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
           c1
          -----

    r1     -0.5
```

```
    r2    -0.25
    r3        0
    r4     0.25
    r5      0.5

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
             c1         c2         c3         c4         c5
           ------     ------     ------     ------     ------

    r1     0.4096     0.4096     0.1536     0.0256     0.0016
    r2     0.1024     0.4864     0.3264     0.0784     0.0064
    r3     0.0256     0.2176     0.5136     0.2176     0.0256
    r4     0.0064     0.0784     0.3264     0.4864     0.1024
    r5     0.0016     0.0256     0.1536     0.4096     0.4096

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                           i    idx    value
                           -    ---    -----

    fl_ar1_beg             1     2     -0.5
    fl_ar1_end             2     3      0.5
    fl_ar1_persistence     3     4      0.6
    fl_ar1_step            4     5     0.25
    fl_p0                  5     6      0.8
    fl_q0                  6     7      0.8
    fl_shk_std             7     8      0.2
    fl_sig_ar1             8     9     0.25
    it_std_bound           9    10        0

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                           i    idx    ndim    numel    rowN    colN       sum           mean          --
                           -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1            1     1      2        10      10       1     5.5511e-17    5.5511e-18     0
    mt_disc_ar1_trans      2    11      2       100      10      10             10           0.1    0.

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
             c1
           ---------

    r1       -0.375
    r2     -0.29167
    r3     -0.20833
    r4       -0.125
    r5    -0.041667
    r6     0.041667
    r7        0.125
    r8      0.20833
    r9      0.29167
    r10       0.375

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7
r1	0.13422	0.30199	0.30199	0.17616	0.06606	0.016515	0.0027525
r2	0.033554	0.20133	0.32716	0.26424	0.12662	0.038535	0.0075694
r3	0.0083886	0.081789	0.26267	0.32755	0.21401	0.082747	0.019741
r4	0.0020972	0.028312	0.14038	0.30946	0.30369	0.15877	0.047989
r5	0.00052429	0.009044	0.061145	0.20246	0.33477	0.25969	0.10585
r6	0.00013107	0.0027525	0.023642	0.10585	0.25969	0.33477	0.20246
r7	3.2768e-05	0.00081101	0.0084603	0.047989	0.15877	0.30369	0.30946
r8	8.192e-06	0.00023347	0.0028677	0.019741	0.082747	0.21401	0.32755
r9	2.048e-06	6.6048e-05	0.00093389	0.0075694	0.038535	0.12662	0.26424
r10	5.12e-07	1.8432e-05	0.00029491	0.0027525	0.016515	0.06606	0.17616

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	value
fl_ar1_beg	1	2	-0.375
fl_ar1_end	2	3	0.375
fl_ar1_persistence	3	4	0.6
fl_ar1_step	4	5	0.083333
fl_p0	5	6	0.8
fl_q0	6	7	0.8
fl_shk_std	7	8	0.1
fl_sig_ar1	8	9	0.125
it_std_bound	9	10	0

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	
ar_disc_ar1	1	1	2	7	7	1	2.7756e-17	3.9651e-18	0.
mt_disc_ar1_trans	2	6	2	49	7	7	7	0.14286	0

```
xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
```

	c1
r1	-0.13765
r2	-0.091766
r3	-0.045883
r4	1.3878e-17
r5	0.045883
r6	0.091766
r7	0.13765

```
xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7
r1	0.67682	0.32022	0.0029525	2.2423e-07	1.058e-13	0	

```
    r2      0.054147        0.7002       0.24422     0.0014299    6.5815e-08    1.8541e-14
    r3    0.00012097      0.084213       0.73627       0.17874    0.00065947    1.8356e-08    3.1086
    r4    4.8643e-09    0.00028953       0.12539       0.74865       0.12539    0.00028953    4.8643
    r5    3.0921e-15    1.8356e-08    0.00065947       0.17874       0.73627      0.084213    0.0001
    r6    2.9554e-23    1.8558e-14    6.5815e-08     0.0014299       0.24422        0.7002      0.05
    r7    4.1477e-33    2.8319e-22    1.0576e-13    2.2423e-07     0.0029525       0.32022       0.6

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2          0.9
    fl_ar1_step           2     3     0.045883
    fl_shk_std            3     4         0.02
    it_std_bound          4     5            3

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std
                         -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1          1     1      2        7       7       1       0           0    0.017639
    mt_disc_ar1_trans    2    11      2       49       7       7       7     0.14286     0.10985

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
          ----------

    r1     -0.024496
    r2     -0.016331
    r3    -0.0081654
    r4             0
    r5     0.0081654
    r6      0.016331
    r7      0.024496

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5          c6          c7
          --------    --------    -------    -------    -------    --------    --------

    r1    0.016586    0.097547    0.23904    0.31241    0.22966    0.090047    0.014711
    r2    0.016258    0.096266    0.23749    0.31247    0.23124    0.091266    0.015008
    r3    0.015936    0.094997    0.23594    0.31251    0.23281    0.092497    0.015311
    r4     0.01562    0.093741    0.23438    0.31252    0.23438    0.093741     0.01562
    r5    0.015311    0.092497    0.23281    0.31251    0.23594    0.094997    0.015936
    r6    0.015008    0.091266    0.23124    0.31247    0.23749    0.096266    0.016258
    r7    0.014711    0.090047    0.22966    0.31241    0.23904    0.097547    0.016586

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx      value
```

```
                          -    ---    ---------

    fl_ar1_beg            1     2     -0.024496
    fl_ar1_end            2     3      0.024496
    fl_ar1_persistence    3     4          0.01
    fl_ar1_step           4     5     0.0081654
    fl_p0                 5     6         0.505
    fl_q0                 6     7         0.505
    fl_shk_std            7     8          0.01
    fl_sig_ar1            8     9      0.010001
    it_std_bound          9    10             0

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN       sum           mean          --
                          -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1           1     1      2        7       7       1      3.5527e-15    5.0753e-16     1
    mt_disc_ar1_trans     2    11      2       49       7       7               7       0.14286    0.

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
             c1
           -------

    r1      -17.19
    r2      -11.46
    r3     -5.7301
    r4           0
    r5      5.7301
    r6       11.46
    r7       17.19

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1             c2             c3             c4             c5             c6             c7
           ----------    ----------    ----------    ----------    ----------    ----------    ------

    r1        0.97037      0.029257    0.00036756    2.4627e-06    9.2815e-09    1.8656e-11    1.5625
    r2      0.0048762        0.9705      0.024382    0.00024504    1.2314e-06    3.0938e-09    3.1094
    r3     2.4504e-05      0.009753       0.97057      0.019506    0.00014703    4.9254e-07    6.1877
    r4     1.2313e-07    7.3513e-05       0.01463       0.97059       0.01463    7.3513e-05    1.2313
    r5     6.1877e-10    4.9254e-07    0.00014703      0.019506       0.97057      0.009753    2.4504
    r6     3.1094e-12    3.0938e-09    1.2314e-06    0.00024504      0.024382        0.9705     0.004
    r7     1.5625e-14    1.8656e-11    9.2815e-09    2.4627e-06    0.00036756      0.029257       0.9

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    ------

    fl_ar1_beg            1     2     -17.19
    fl_ar1_end            2     3      17.19
    fl_ar1_persistence    3     4       0.99
    fl_ar1_step           4     5     5.7301
    fl_p0                 5     6      0.995
```

```
    fl_q0                6     7      0.995
    fl_shk_std           7     8       0.99
    fl_sig_ar1           8     9     7.0179
    it_std_bound         9    10          0

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean        std
                         -    ---    ----    -----    ----    ----    ---    -------    --------

    ar_disc_ar1          1     1      2        7       7       1       0          0    0.017639
    mt_disc_ar1_trans    2    11      2       49       7       7       7    0.14286     0.10985

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
             c1
          ----------

    r1     -0.024496
    r2     -0.016331
    r3    -0.0081654
    r4             0
    r5     0.0081654
    r6      0.016331
    r7      0.024496

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5          c6          c7
          --------    --------    -------    -------    -------    --------    --------

    r1    0.016586    0.097547    0.23904    0.31241    0.22966    0.090047    0.014711
    r2    0.016258    0.096266    0.23749    0.31247    0.23124    0.091266    0.015008
    r3    0.015936    0.094997    0.23594    0.31251    0.23281    0.092497    0.015311
    r4     0.01562    0.093741    0.23438    0.31252    0.23438    0.093741     0.01562
    r5    0.015311    0.092497    0.23281    0.31251    0.23594    0.094997    0.015936
    r6    0.015008    0.091266    0.23124    0.31247    0.23749    0.096266    0.016258
    r7    0.014711    0.090047    0.22966    0.31241    0.23904    0.097547    0.016586

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx      value
                         -    ---    ---------

    fl_ar1_beg           1     2     -0.024496
    fl_ar1_end           2     3      0.024496
    fl_ar1_persistence   3     4          0.01
    fl_ar1_step          4     5     0.0081654
    fl_p0                5     6         0.505
    fl_q0                6     7         0.505
    fl_shk_std           7     8          0.01
    fl_sig_ar1           8     9      0.010001
    it_std_bound         9    10             0

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

```
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum    mean      std      coef
                         -    ---    ----    -----    ----    ----    ---    ----    -------    ----

    ar_disc_ar1          1     1      2        5       5       1       0       0     0.79057
    mt_disc_ar1_trans    2     6      2       25       5       5       5     0.2     0.27623     1.3

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
           c1
          ----

    r1      -1
    r2    -0.5
    r3       0
    r4     0.5
    r5       1

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1            c2           c3           c4           c5
          ----------    ----------    --------    ----------    ----------

    r1       0.22663       0.73331    0.040048    1.0689e-05    7.3923e-12
    r2      0.012224       0.58648     0.39831     0.0029797     7.605e-08
    r3    8.8417e-05       0.10556      0.7887       0.10556    8.8417e-05
    r4     7.605e-08     0.0029797     0.39831       0.58648      0.012224
    r5    7.3923e-12    1.0689e-05    0.040048       0.73331       0.22663

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    -----

    fl_ar1_persistence    1     2      0.6
    fl_ar1_step           2     3      0.5
    fl_shk_std            3     4      0.2
    it_std_bound          4     5        4

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN        sum           mean
                         -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1          1     1      2        10      10       1     -7.2164e-16    -7.2164e-17
    mt_disc_ar1_trans    2     6      2       100      10      10               10            0.1

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
           ---------

    r1       -0.375
    r2     -0.29167
    r3     -0.20833
    r4       -0.125
```

```
    r5     -0.041667
    r6      0.041667
    r7         0.125
    r8       0.20833
    r9       0.29167
    r10        0.375

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1            c2            c3            c4           c5           c6           c7
           ----------    ----------    ----------    ----------    --------    --------    ---------

    r1        0.13933       0.26196       0.31887       0.20154    0.066066    0.011201    0.0009785
    r2       0.056673       0.16995       0.30658       0.28713      0.1396    0.035167     0.004575
    r3        0.01861      0.087039       0.23281       0.32308     0.23281    0.087039      0.01684
    r4      0.0048925      0.035167        0.1396       0.28713     0.30658     0.16995      0.04884
    r5      0.0010235      0.011201      0.066066       0.20154     0.31887     0.26196       0.1116
    r6     0.00016962     0.0028101       0.02466       0.11169     0.26196     0.31887       0.2015
    r7     2.2197e-05    0.00055483     0.0072547      0.048841     0.16995     0.30658       0.2871
    r8     2.2881e-06    8.6129e-05     0.0016806      0.016841    0.087039     0.23281       0.3230
    r9     1.8543e-07    1.0503e-05    0.00030628     0.0045756    0.035167      0.1396       0.2871
    r10    1.1798e-08    1.0053e-06    4.3874e-05    0.00097859    0.011201    0.066066       0.2015

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2          0.6
    fl_ar1_step           2     3     0.083333
    fl_shk_std            3     4          0.1
    it_std_bound          4     5            3

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                         i    idx    ndim    numel    rowN    colN    sum     mean         std       c
                         -    ---    ----    -----    ----    ----    ---    -------    -------    -

    ar_disc_ar1          1     1      2        7       7       1       0           0    0.15314
    mt_disc_ar1_trans    2     6      2       49       7       7       7     0.14286    0.35338

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
             c1
          ---------

    r1     -0.21266
    r2     -0.14178
    r3    -0.070888
    r4            0
    r5     0.070888
    r6      0.14178
    r7      0.21266

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1             c2             c3             c4            c5            c6
```

r1	0.99957	0.00043152	0	0	0	0	
r2	0.00011382	0.99955	0.0003337	0	0	0	
r3	4.8683e-27	0.00015	0.99959	0.00025684	0	0	
r4	1.4175e-70	1.0439e-26	0.00019675	0.99961	0.00019675	0	
r5	1.9884e-135	4.986e-70	2.2273e-26	0.00025684	0.99959	0.00015	
r6	1.2359e-221	1.149e-134	1.7451e-69	4.7287e-26	0.0003337	0.99955	0.0
r7	0	1.1738e-220	6.6059e-134	6.077e-69	9.9893e-26	0.00043152	

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	value
fl_ar1_persistence	1	2	0.99
fl_ar1_step	2	3	0.070888
fl_shk_std	3	4	0.01
it_std_bound	4	5	3

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	
ar_disc_ar1	1	1	2	7	7	1	3.4694e-18	4.9564e-19	0.
mt_disc_ar1_trans	2	6	2	49	7	7	7	0.14286	0

```
xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
```

	c1
r1	-0.030002
r2	-0.020001
r3	-0.010001
r4	0
r5	0.010001
r6	0.020001
r7	0.030002

```
xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7
r1	0.0067533	0.064018	0.2484	0.38278	0.23505	0.057298	0.0057011
r2	0.0065668	0.06286	0.24618	0.38287	0.23728	0.05838	0.0058656
r3	0.0063849	0.061717	0.24396	0.38292	0.2395	0.059478	0.0060344
r4	0.0062075	0.06059	0.24173	0.38294	0.24173	0.06059	0.0062075
r5	0.0060344	0.059478	0.2395	0.38292	0.24396	0.061717	0.0063849
r6	0.0058656	0.05838	0.23728	0.38287	0.24618	0.06286	0.0065668
r7	0.0057011	0.057298	0.23505	0.38278	0.2484	0.064018	0.0067533

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

```
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2         0.01
    fl_ar1_step           2     3     0.010001
    fl_shk_std            3     4         0.01
    it_std_bound          4     5            3

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN        sum           mean
                          -    ---    ----    -----    ----    ----    -----------    -----------

    ar_disc_ar1           1     1      2        7       7       1      -7.1054e-15    -1.0151e-15
    mt_disc_ar1_trans     2     6      2       49       7       7                7        0.14286

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxx
              c1
          -----------

    r1        -21.054
    r2        -14.036
    r3        -7.0179
    r4    -1.7764e-15
    r5         7.0179
    r6         14.036
    r7         21.054

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxx
              c1              c2              c3              c4              c5              c6
          -----------    -----------    -----------    ----------    ----------    ----------    ---

    r1        0.99957     0.00043152              0             0             0             0
    r2     0.00011382        0.99955      0.0003337             0             0             0
    r3     4.8683e-27        0.00015        0.99959    0.00025684             0             0
    r4     1.4175e-70     1.0439e-26     0.00019675       0.99961    0.00019675             0
    r5    1.9884e-135      4.986e-70     2.2273e-26    0.00025684       0.99959       0.00015
    r6    1.2359e-221     1.149e-134     1.7451e-69    4.7287e-26     0.0003337       0.99955    0.0
    r7              0    1.1738e-220    6.6059e-134     6.077e-69    9.9893e-26    0.00043152

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    value
                          -    ---    ------

    fl_ar1_persistence    1     2       0.99
    fl_ar1_step           2     3     7.0179
    fl_shk_std            3     4       0.99
    it_std_bound          4     5          3

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

```
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx    ndim    numel    rowN    colN       sum           mean
                          -    ---    ----    -----    ----    ----    ----------    ----------    --

    ar_disc_ar1           1     1      2        7       7       1      3.4694e-18    4.9564e-19    0.
    mt_disc_ar1_trans     2     6      2       49       7       7               7       0.14286     0

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxx
              c1
          ---------

    r1    -0.030002
    r2    -0.020001
    r3    -0.010001
    r4            0
    r5     0.010001
    r6     0.020001
    r7     0.030002

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxx
              c1           c2          c3          c4          c5          c6          c7
          ---------    --------    -------    -------    -------    --------    ---------

    r1    0.0067533    0.064018     0.2484    0.38278    0.23505    0.057298    0.0057011
    r2    0.0065668     0.06286    0.24618    0.38287    0.23728     0.05838    0.0058656
    r3    0.0063849    0.061717    0.24396    0.38292     0.2395    0.059478    0.0060344
    r4    0.0062075     0.06059    0.24173    0.38294    0.24173     0.06059    0.0062075
    r5    0.0060344    0.059478     0.2395    0.38292    0.24396    0.061717    0.0063849
    r6    0.0058656     0.05838    0.23728    0.38287    0.24618     0.06286    0.0065668
    r7    0.0057011    0.057298    0.23505    0.38278     0.2484    0.064018    0.0067533

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                          i    idx     value
                          -    ---    --------

    fl_ar1_persistence    1     2         0.01
    fl_ar1_step           2     3     0.010001
    fl_shk_std            3     4         0.01
    it_std_bound          4     5            3
```

Appendix A

Index and Code Links

A.1 Savings Dynamic Programming links

1. Looped Grid Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Infinite horizon constrained dynamic savings problem with persistent shock.
   - The state-space and choice-space share the same asset grid.
   - Looped algorithm, slow but easy to modify, useful for developing new models.
   - **MEconTools**: *ff_vfi_az_loop()*
2. Vectorized Grid Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Vectorized version of ff_vfi_az_loop(), fast and sufficiently approximate value(a,z), but choices not precise.
   - Broadcast and vectorized evaluation and maximization.
   - Solve u(c) once, and retrieve with cell arrays.
   - **MEconTools**: *ff_vfi_az_vec()*
3. Looped Exact FOC Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Infinite horizon constrained dynamic savings problem with persistent shock.
   - The state-space is on a grid, the choice space are continuous percentages of cash-on-hand.
   - Looped exact savings-percentage algorithm, slow but high precision at low grid size.
   - Solves for EV(ap,z) given shock state and for a savings choice. Bisection based on FOC with analytical du(c(ap))/dap and spline slopes dEV(ap,z)/dap.
   - **MEconTools**: *ff_vfi_az_bisec_loop()* + *ff_optim_bisec_savezrone()*
4. Vectorized Exact FOC Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Vectorized version of ff_vfi_az_bisec_loop() exact savings-percentage algorithm, high precision and high speed.
   - **MEconTools**: *ff_vfi_az_bisec_vec()* + *ff_optim_bisec_savezrone()*
5. Looped Exact Value Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Looped infinite horizon constrained dynamic savings problem with persistent shock.
   - The state-space is on a grid, the choice space are continuous percentages of cash-on-hand.
   - Evaluate value at choice grid iteratively by zooming-in to construct finer savings percentages.
   - **MEconTools**: *ff_vfi_az_mzoom_loop()* + *ff_optim_mzoom_savezrone()*
6. Vectorized Exact Value Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Vectorized version of ff_vfi_az_mzoom_loop() exact savings-percentage algorithm.
   - **MEconTools**: *ff_vfi_az_mzoom_vec()* + *ff_optim_mzoom_savezrone()*

A.2 Stationary Distribution links

1. Looped Grid Stationary Distribution Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Stationary distribution for infinite horizon constrained dynamic savings problem with persistent shock.
   - The state-space and choice-space share the same asset grid.
   - Looped algorithm.
   - **MEconTools**: *ff_ds_az_loop()*

2. Looped Exact Stationary Distribution Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - Stationary distribution for infinite horizon constrained dynamic savings problem with persistent shock.
   - The state-space is on a grid, the choice space are continuous percentages of cash-on-hand.
   - Looped algorithm.
   - **MEconTools**: *ff_ds_az_cts_loop()*
3. Vectorized Exact Stationary Distribution Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**
   - This is the vectorized version of ff_ds_az_cts_loop().
   - **MEconTools**: *ff_ds_az_cts_vec(8)*

A.3 Summarize Policy and Value links

1. Summarize ND Array Policy and Value Functions: **mlx** | **m** | **pdf** | **html**
   - Given an NDarray matrix with N1, N2, ..., ND dimensions. Generate average and standard deviation for the 3rd dimension, grouping by the other dimensions.
   - For example, show the 5th dimension as the column groups, and the other variables generate combinations shown as rows.
   - The resulting summary statistics table contains mean and standard deviation among other statistics over the policy or value contained in the ND array.
   - **MEconTools**: *ff_summ_nd_array()*

A.4 Distributional Analysis links

1. Gateway Joint Probability Mass Statistics: **mlx** | **m** | **pdf** | **html**
   - Given model policy functions and stationary distribution, compute distributional statistics.
   - Given discrete probability mass function f(s), and information y(s), x(s), z(s) at each element of the state-space, compute statistics for each variable, y, x, z, which are all discrete random variables.
   - Compute correlation and covariance betwen input discrete random variables.
   - **MEconTools**: *ff_simu_stats()*
2. Discrete Random Variable Distributional Statistics: **mlx** | **m** | **pdf** | **html**
   - Model simulation generates discrete random variables, calculate mean, standard deviation, min, max, percentiles, and proportion of outcomes held by x percentiles, etc.
   - **MEconTools**: *ff_disc_rand_var_stats()*
3. Generate Discrete Random Variable: **mlx** | **m** | **pdf** | **html**
   - Given mass at state space points, and y, c, a, z and other outcomes or other information at each corresponding state space points, generate discrete random variable, with unique sorted values, and mass for each unique sorted values.
   - Generate additional joint distributions: if initial distribution is over f(a,z), generate joint distribution of f(y,a) or f(y,z).
   - **MEconTools**: *ff_disc_rand_var_mass2outcomes()*
4. Discrete Random Variable Correlation and Covariance: **mlx** | **m** | **pdf** | **html**
   - Given probability mass function f(s), X(s), and Y(s), compute the covariance and correlation betwen X and Y.
   - **MEconTools**: *ff_disc_rand_var_mass2covcor()*

A.5 Optimizers links

1. Bisection Exact Optimal Savings Share Multiple States: **mlx** | **m** | **pdf** | **html**
   - Given a First Order Condition function handle that takes the fraction of resources (cash-on-hand) saved as the input, solve for the optimal savings fraction via bisection. Solve this concurrently for many elements of the state-space. The function handle contains the FOC with parameters and state-space elements embeded.
   - **MEconTools**: *ff_optim_bisec_savezrone()*
2. Multisection Exact Optimal Savings Share Multiple States: **mlx** | **m** | **pdf** | **html**
   - Given a First Order Condition function handle that takes the fraction of resources (cash-on-hand) saved as the input, solve for the optimal savings fraction via multisection where there

are multiple evaluations per iteration of the FOC. Solve this concurrently for many elements of the state-space. The function handle contains the FOC with parameters and state-space elements embeded.
    - **MEconTools**: *ff_optim_mlsec_savezrone()*
3. Vectorized Zooming Exact Optimal Savings Share Multiple States: **mlx** | **m** | **pdf** | **html**
    - Given a Utility (not FOC) function handle that takes the fraction of resources (cash-on-hand) saved as the input, solve for the optimal savings fraction via iterative zooming where there are multiple evaluations per iteration of the utility function. Solve this concurrently for many elements of the state-space. The function handle contains the utilty function with parameters and state-space elements embeded.
    - **MEconTools**: *ff_optim_mzoom_savezrone()*

A.6 Graphs links

1. Multiple Line Graph Function: **mlx** | **m** | **pdf** | **html**
    - Policy and Value Function graphs, x-axis one state, color another state, y-axis are value and policies.
    - Distributional graphs, x-axis one state, y-axis and color another state, size show distributional mass.
    - **MEconTools**: *ff_graph_grid()*

A.7 Support Tools links

1. Organizes and Prints Container Map Key and Values: **mlx** | **m** | **pdf** | **html**
    - Summarizes the contents of a map container by data types. Includes, scalar, array, matrix, string, functions, tensors (3-tuples), tesseracts (4-tuples).
    - **MEconTools**: *ff_container_map_display()*

A.8 Data Structures links

1. Log and Power Spaced Asset and Choice Grids: **mlx** | **m** | **pdf** | **html**
    - Generate linear, log-space, power-space, or threshold-cut asset or choice grids.
    - **MEconTools**: *ff_saveborr_grid()*

A.9 Common Functions links

1. Discretize AR1 Normal Shock Tauchen (1986): **mlx** | **m** | **pdf** | **html**
    - Mean zero AR(1) shock discretize following Tauchen (1986).
    - **MEconTools**: *ffy_tauchen()*
2. Discretize AR1 Normal Shock Rouwenhorst (1995): **mlx** | **m** | **pdf** | **html**
    - Mean zero AR(1) shock discretize following Rouwenhorst (1995).
    - **MEconTools**: *ffy_rouwenhorst()*

A.10 System links

1. Search and Find File Names: **mlx** | **m** | **pdf** | **html**
    - Search and find file names.
    - **MEconTools**: *ff_find_files()*
2. Execute and Export Livescript Files: **mlx** | **m** | **pdf** | **html**
    - Find livescript (mlx) files, execute MLX files, and export MLX files to HTML in a different folder.
    - **MEconTools**: *ff_mlx2htmlpdf_runandexport()*

Bibliography

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