

Matlab Toolbox Heterogeneous Agents Dynamic Programming

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Contents

Preface	5
1 Savings Dynamic Programming	7
1.1 FF_VFI_AZ_LOOP Dynamic Programming Asset Problem with Shocks Loop	7
1.2 FF_VFI_AZ_VEC Dynamic Programming Asset Problem with Shocks Vectorized	17
2 Summarize Policy and Value	29
2.1 FF_SUMM_ND_ARRAY Examples	29
3 Distributional Analysis	35
3.1 FF_SIMU_STATS Examples	35
3.2 FF_DISC_RAND_VAR_STATS Examples	40
3.3 FF_DISC_RAND_VAR_MASS2OUTCOMES Examples	45
3.4 FF_DISC_RAND_VAR_MASS2COVCOR Examples	48
4 Graphs	53
4.1 FF_GRAPH_GRID Examples: X, Y and Color Line Plots	53
5 Data Structures	61
5.1 FF_SAVEBORR_GRID Example for Generating Asset Grid	61
6 Common Functions	69
6.1 FFY_TAUCHEN AR1 Shock Discretization Example	69
6.2 FFY_ROUWENHORST AR1 Shock Discretization Example	74
7 Support Tools	83
7.1 FF_CONTAINER_MAP_DISPLAY Examples	83
A Index and Code Links	89
A.1 Savings Dynamic Programming links	89
A.2 Summarize Policy and Value links	89
A.3 Distributional Analysis links	89
A.4 Graphs links	90
A.5 Data Structures links	90
A.6 Common Functions links	90
A.7 Support Tools links	90

Preface

This is a work-in-progress Matlab package consisting of functions that facilitate Dynamic Programming and Related Tasks. Materials gathered from various [projects](#) in which Matlab code is used. Files are the [MEconTools](#) repository. Matlab files are linked below by section with livescript files. Tested with [Matlab](#) 2019a ([The MathWorks Inc, 2019](#)).

This bookdown file is a collection of mlx based vignettes for functions that are available from [MEconTools](#). Each Vignette file contains various examples for invoking each function. The goal of this repository is to make it easier to find/re-use codes produced for various projects.

From other repositories: For dynamic borrowing and savings problems, see [Dynamic Asset Repository](#); For code examples, see also [R Example Code](#), [Matlab Example Code](#), and [Stata Example Code](#); For intro stat with R, see [Intro Statistics for Undergraduates](#), and intro Math with Matlab, see [Intro Mathematics for Economists](#). See [here](#) for all of [Fan](#)'s public repositories.

The site is built using [Bookdown](#) ([Xie, 2020](#)).

Please contact [FanWangEcon](#) for issues or problems.

Chapter 1

Savings Dynamic Programming

1.1 FF_VFI_AZ_LOOP Dynamic Programming Asset Problem with Shocks Loop

Go back to [fan's MEconTools Toolbox \(bookdown\)](#), [Matlab Code Examples Repository \(bookdown\)](#), or [Math for Econ with Matlab Repository \(bookdown\)](#).

This is the example vignette for function: `ff_vfi_az_loop` from the [MEconTools Package](#). This function solves the dynamica programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon. This is the looped code, it is extremely slow for larger state-space problems.

1.1.1 Test FF_VFI_AZ_LOOP Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the `mp_params`.

```
%mp_params
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('fl_ccra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_loop(mp_params);
```

Elapsed time is 0.446290 seconds.

xx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari	min
	-	---	----	-----	-----	-----	-----	-----	-----	-----	---
ap	1	1	2	350	50	7	8427.6	24.079	14.27	0.59263	0

xxx TABLE:ap xxxxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7
	-----	-----	-----	-----	-----	-----	-----
r1	0	0	0	0	0	0	2.0408
r2	0	0	0	1.0204	1.0204	1.0204	3.0612
r3	1.0204	1.0204	1.0204	2.0408	2.0408	2.0408	4.0816
r4	2.0408	2.0408	2.0408	2.0408	3.0612	3.0612	5.102
r5	3.0612	3.0612	3.0612	3.0612	4.0816	4.0816	6.1224

r46	43.878	43.878	43.878	43.878	43.878	44.898	45.918
r47	44.898	44.898	44.898	44.898	44.898	45.918	46.939
r48	45.918	45.918	45.918	45.918	45.918	46.939	47.959
r49	46.939	46.939	46.939	46.939	46.939	47.959	48.98
r50	47.959	47.959	47.959	47.959	47.959	48.98	50

1.1.2 Test FF_VFI_AZ_LOOP Control Outputs

Run the function first without any outputs;

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = false;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
```

Run the function and show policy function for savings choice. For `ls_ffcmd`, `ls_ffsna`, `ls_ffgrh`, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'ap'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'ap'};
ff_vfi_az_loop(mp_params, mp_support);
```

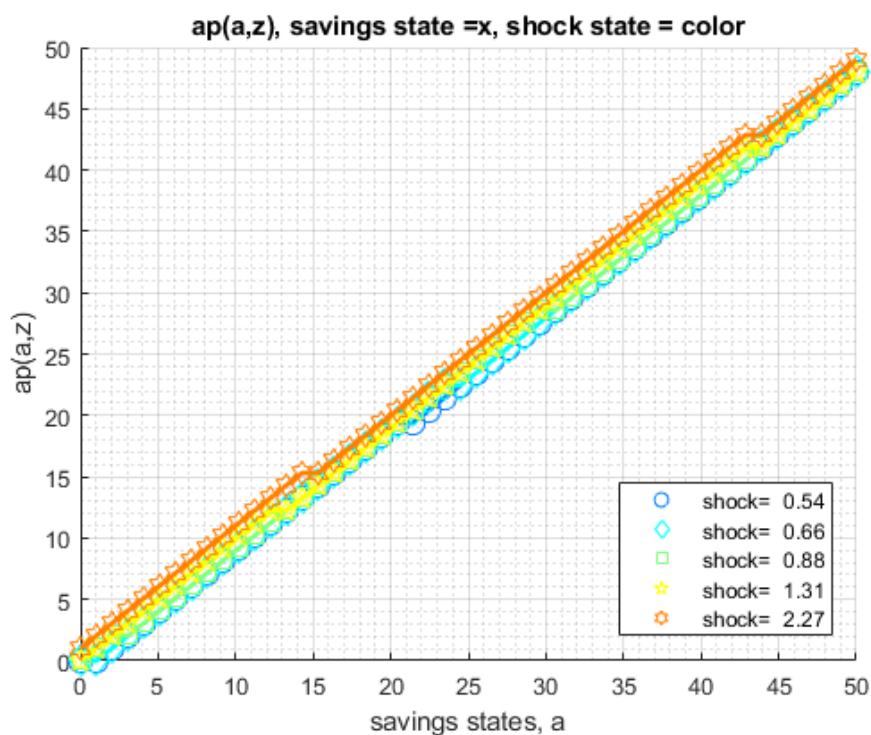
Elapsed time is 0.252449 seconds.

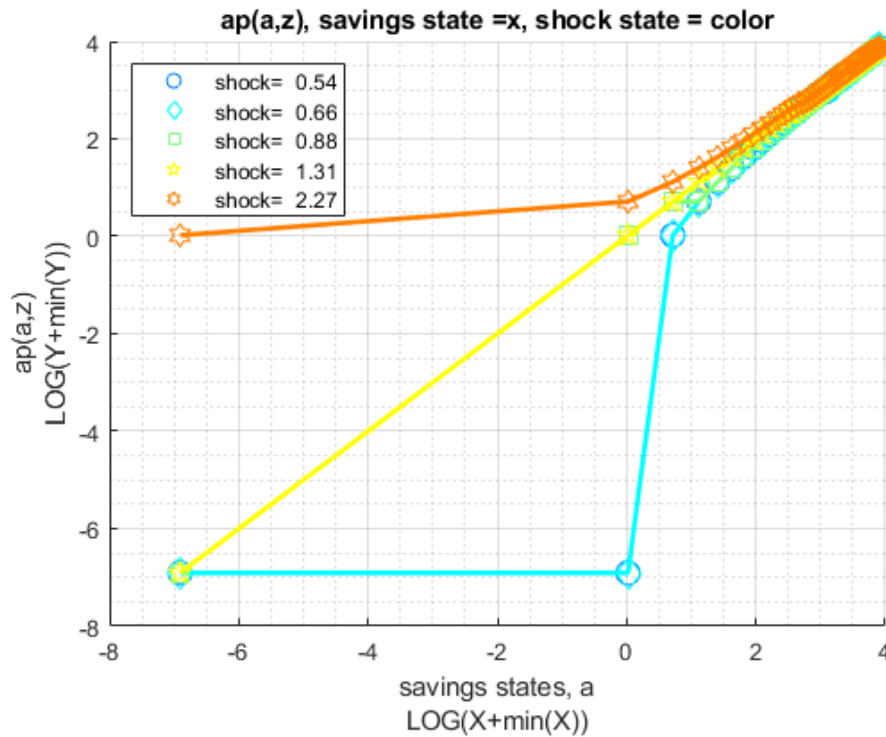
```
xxx ff_vfi_az_vec, outcome=ap xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

group	a	mean_z_0_54195	mean_z_0_66401	mean_z_0_88162	mean_z_1_3095	mean_z
-----	-----	-----	-----	-----	-----	-----
1	0	0	0	0	0	1.0
2	1.0204	0	0	1.0204	1.0204	2.0
3	2.0408	1.0204	1.0204	2.0408	2.0408	3.0
4	3.0612	2.0408	2.0408	2.0408	3.0612	4.0
5	4.0816	3.0612	3.0612	3.0612	4.0816	5.
6	5.102	4.0816	4.0816	4.0816	5.102	6.1
7	6.1224	5.102	5.102	5.102	6.1224	7.1
8	7.1429	6.1224	6.1224	6.1224	7.1429	8.1
9	8.1633	7.1429	7.1429	7.1429	8.1633	9.1
10	9.1837	8.1633	8.1633	8.1633	9.1837	10.
11	10.204	9.1837	9.1837	9.1837	10.204	11.
12	11.224	10.204	10.204	10.204	11.224	12.
13	12.245	11.224	11.224	11.224	12.245	13.
14	13.265	12.245	12.245	12.245	12.245	14.
15	14.286	13.265	13.265	13.265	13.265	15.
16	15.306	14.286	14.286	14.286	14.286	15.
17	16.327	15.306	15.306	15.306	15.306	16.
18	17.347	16.327	16.327	16.327	16.327	17.
19	18.367	17.347	17.347	17.347	17.347	18.
20	19.388	18.367	18.367	18.367	18.367	19.
21	20.408	19.388	19.388	19.388	19.388	20.
22	21.429	19.388	20.408	20.408	20.408	21.

1.1. FF_VFI_AZ_LOOP DYNAMIC PROGRAMMING ASSET PROBLEM WITH SHOCKS LOOP9

23	22.449	20.408	21.429	21.429	21.429	22.
24	23.469	21.429	22.449	22.449	22.449	23.
25	24.49	22.449	22.449	23.469	23.469	24.
26	25.51	23.469	23.469	24.49	24.49	25.
27	26.531	24.49	24.49	25.51	25.51	26.
28	27.551	25.51	25.51	26.531	26.531	27.
29	28.571	26.531	26.531	27.551	27.551	28.
30	29.592	27.551	27.551	28.571	28.571	29.
31	30.612	28.571	28.571	28.571	29.592	30.
32	31.633	29.592	29.592	29.592	30.612	31.
33	32.653	30.612	30.612	30.612	31.633	32.
34	33.673	31.633	31.633	31.633	32.653	33.
35	34.694	32.653	32.653	32.653	33.673	34.
36	35.714	33.673	33.673	33.673	34.694	35.
37	36.735	34.694	34.694	34.694	35.714	36.
38	37.755	35.714	35.714	35.714	36.735	37.
39	38.776	36.735	36.735	36.735	37.755	38.
40	39.796	37.755	37.755	37.755	38.776	39.
41	40.816	38.776	38.776	38.776	39.796	40.
42	41.837	39.796	39.796	39.796	40.816	41.
43	42.857	40.816	40.816	40.816	41.837	42.
44	43.878	41.837	41.837	41.837	41.837	42.
45	44.898	42.857	42.857	42.857	42.857	43.
46	45.918	43.878	43.878	43.878	43.878	44.
47	46.939	44.898	44.898	44.898	44.898	45.
48	47.959	45.918	45.918	45.918	45.918	46.
49	48.98	46.939	46.939	46.939	46.939	47.
50	50	47.959	47.959	47.959	47.959	48.





Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_loop(mp_params, mp_support);
```

Elapsed time is 1.625022 seconds.

xx

CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)

xx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefva
	-	---	----	-----	-----	----	-----	-----	-----	-----
ap	1	1	2	900	100	9	21825	24.25	14.089	0.58
savefraccoh	2	2	2	900	100	9	411.21	0.4569	0.2651	0.5802

xxx TABLE:ap xxxxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7	c8	
	-----	-----	-----	-----	-----	-----	-----	-----	-----
r1	0	0	0	0	0	0	0.50505	1.5152	3
r2	0	0	0	0	0.50505	0.50505	1.0101	1.5152	3
r3	0.50505	0.50505	0.50505	0.50505	0.50505	1.0101	1.5152	2.0202	4
r4	1.0101	1.0101	1.0101	1.0101	1.0101	1.5152	2.0202	2.5253	4
r5	1.5152	1.5152	1.5152	1.5152	1.5152	2.0202	2.5253	3.0303	5
r96	45.455	45.455	45.455	45.96	45.96	45.96	46.465	47.475	4
r97	45.96	45.96	45.96	46.465	46.465	46.465	46.97	47.98	4
r98	46.465	46.465	46.465	46.465	46.97	46.97	47.475	48.485	
r99	46.97	46.97	46.97	46.97	47.475	47.475	47.98	48.99	

1.1. FF_VFI_AZ_LOOP DYNAMIC PROGRAMMING ASSET PROBLEM WITH SHOCKS LOOP11

r100	47.475	47.475	47.475	47.475	47.98	47.98	48.485	49.495
xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxxxxxxx								
	c1	c2	c3	c4	c5	c6	c7	
	-----	-----	-----	-----	-----	-----	-----	
r1	0	0	0	0	0	0	0.0094749	
r2	0	0	0	0	0.009643	0.0095804	0.01895	
r3	0.0097386	0.0097261	0.0097083	0.0096824	0.009643	0.019161	0.028425	
r4	0.019477	0.019452	0.019417	0.019365	0.019286	0.028741	0.0379	
r5	0.029216	0.029178	0.029125	0.029047	0.028929	0.038321	0.047374	
r96	0.87647	0.87535	0.87375	0.8811	0.87751	0.87181	0.87169	
r97	0.88621	0.88507	0.88346	0.89078	0.88716	0.88139	0.88116	
r98	0.89595	0.8948	0.89317	0.89078	0.8968	0.89097	0.89064	
r99	0.90569	0.90452	0.90287	0.90046	0.90644	0.90055	0.90011	
r100	0.91543	0.91425	0.91258	0.91014	0.91609	0.91013	0.90959	

1.1.3 Test FF_VFI_AZ_LOOP Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0;
ff_vfi_az_loop(mp_params, mp_support);
```

Elapsed time is 0.068015 seconds.

xx

CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)

xx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefv
	-	---	----	-----	-----	-----	-----	-----	-----	-----
savefraccoh	1	1	2	250	50	5	48.774	0.1951	0.23298	1.19

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5
	-----	-----	-----	-----	-----
r1	0	0	0	0	0.0058555
r2	0	0	0	0	0.0058555
r3	0	0	0	0	0.0058555
r4	0	0	0	0	0.0058555
r5	0	0	0	0	0.0058555
r46	0.62112	0.61921	0.61584	0.60931	0.59509

```

r47    0.66655    0.6645    0.66088    0.65388    0.63861
r48    0.71414    0.71195    0.70807    0.70057    0.68421
r49    0.76395    0.7616    0.75745    0.74943    0.73193
r50    0.81602    0.81351    0.80908    0.80051    0.78182

% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_loop(mp_params, mp_support);

Elapsed time is 0.291535 seconds.
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean      std      coefv
          -      ---      ----      -----      ----      ----      -
savefraccoh    1      1      2      250      50      5      59.526    0.2381    0.27148    1.14

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxxxxxxxx
          c1      c2      c3      c4      c5
          -----
r1         0         0    0.00051196    0.005772    0.021238
r2         0         0    0.00051196    0.005772    0.021238
r3         0         0    0.00051196    0.005772    0.021238
r4         0         0    0.00099992    0.005772    0.021238
r5         0         0    0.00099992    0.0079177    0.021238
r46    0.73495    0.73278    0.72894    0.7215    0.70527
r47    0.78505    0.78273    0.77862    0.77068    0.75334
r48    0.83737    0.83489    0.83052    0.82204    0.80355
r49    0.89196    0.88933    0.88466    0.87564    0.85594
r50    0.94888    0.94608    0.94111    0.93151    0.91056

```

1.1.4 Test FF_VFI_AZ_LOOP Changing Risk Aversion

Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

```

mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';

```

Solve the model with different risk aversion levels, higher preferences for risk:

```

% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_loop(mp_params, mp_support);

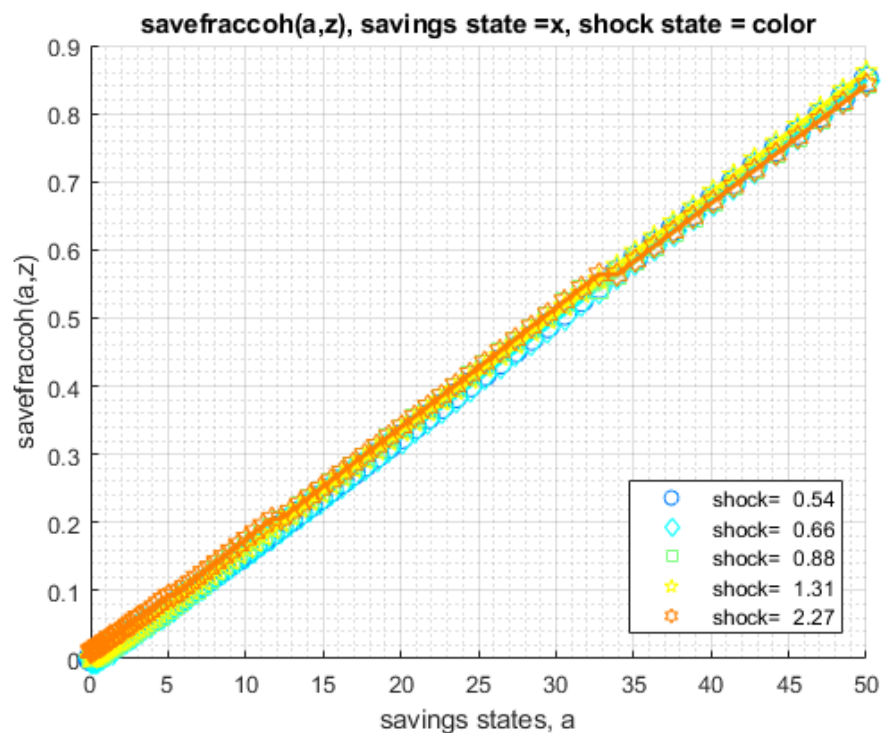
```

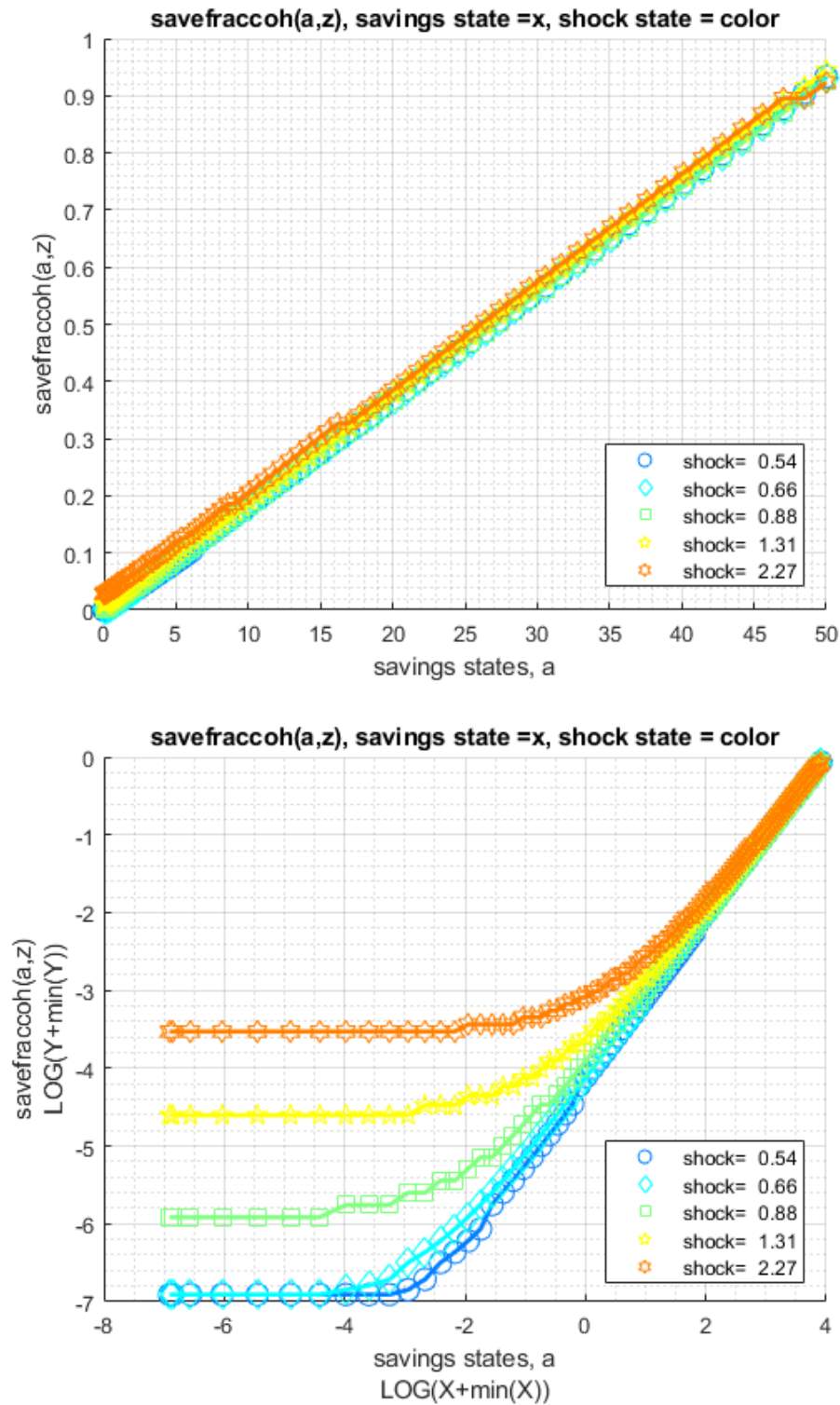
Elapsed time is 0.546316 seconds.

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

```





1.1.5 Test FF_VFI_AZ_LOOP with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```

```

mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 5;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';

```

Lower standard deviation of shock:

```

% Lower Risk Aversion
mp_params('fl_shk_std') = 0.05;
ff_vfi_az_loop(mp_params, mp_support);

```

Elapsed time is 0.933647 seconds.

xx

CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)

xx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefv
	-	---	----	-----	-----	----	-----	-----	-----	-----
savefraccoh	1	1	2	500	100	5	112.7	0.22539	0.26207	1.16

xxx TABLE:savefraccoh xxx

	c1	c2	c3	c4	c5
	-----	-----	-----	-----	-----
r1	0	0	0	0	0.00049994
r2	0	0	0	0	0.00049994
r3	0	0	0	0	0.00049994
r4	0	0	0	0	0.00049994
r5	0	0	0	0	0.00049994
r96	0.79191	0.79066	0.81492	0.81313	0.81102
r97	0.81774	0.81644	0.8412	0.83936	0.83718
r98	0.84411	0.84277	0.86805	0.86615	0.86389
r99	0.87105	0.86967	0.89546	0.8935	0.89117
r100	0.89855	0.89713	0.92344	0.92142	0.91902

Higher shock standard deviation: low shock high asset save more, high shock more asset save less, high shock low asset save more:

```

% Higher Risk Aversion
mp_params('fl_shk_std') = 0.25;
ff_vfi_az_loop(mp_params, mp_support);

```

Elapsed time is 0.894904 seconds.

xx

CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)

xx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefv
	-	---	----	-----	-----	----	-----	-----	-----	-----
savefraccoh	1	1	2	500	100	5	115.6	0.23119	0.25857	1.11

xxx TABLE:savefraccoh xxx

	c1	c2	c3	c4	c5
	-----	-----	-----	-----	-----
r1	0	0	0.00021288	0.0066707	0.033639
r2	0	0	0.00021288	0.0066707	0.033639

1.2.2 Test FF_VFI_AZ_VEC Control Outputs

Run the function first without any outputs;

```
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_timer') = false;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
```

Run the function and show policy function for savings choice. For `ls_ffcmd`, `ls_ffsna`, `ls_ffgrh`, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'ap'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'ap'};
ff_vfi_az_vec(mp_params, mp_support);
```

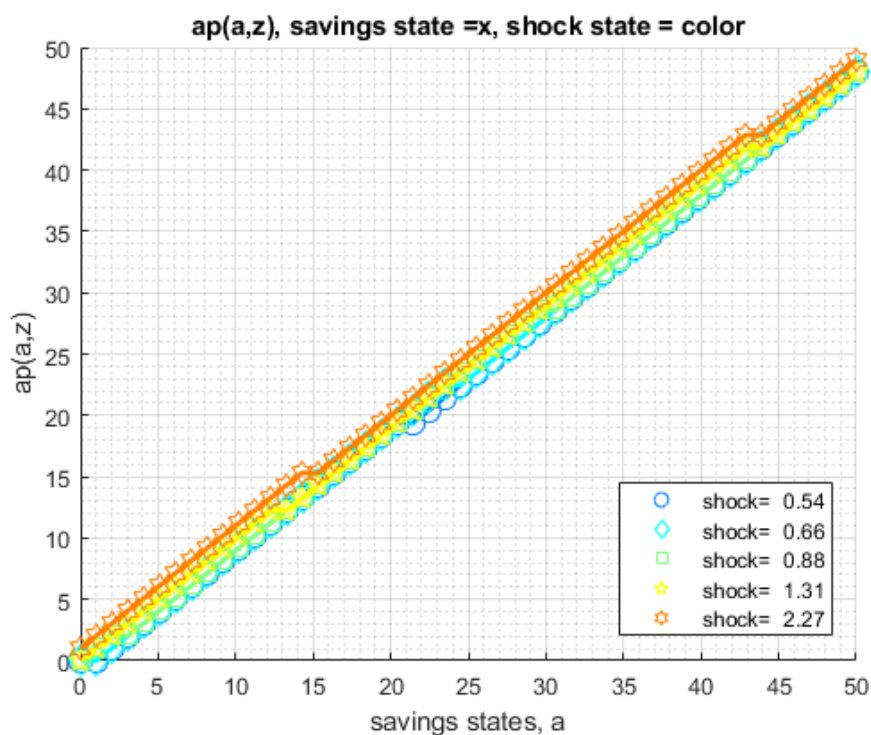
Elapsed time is 0.016286 seconds.

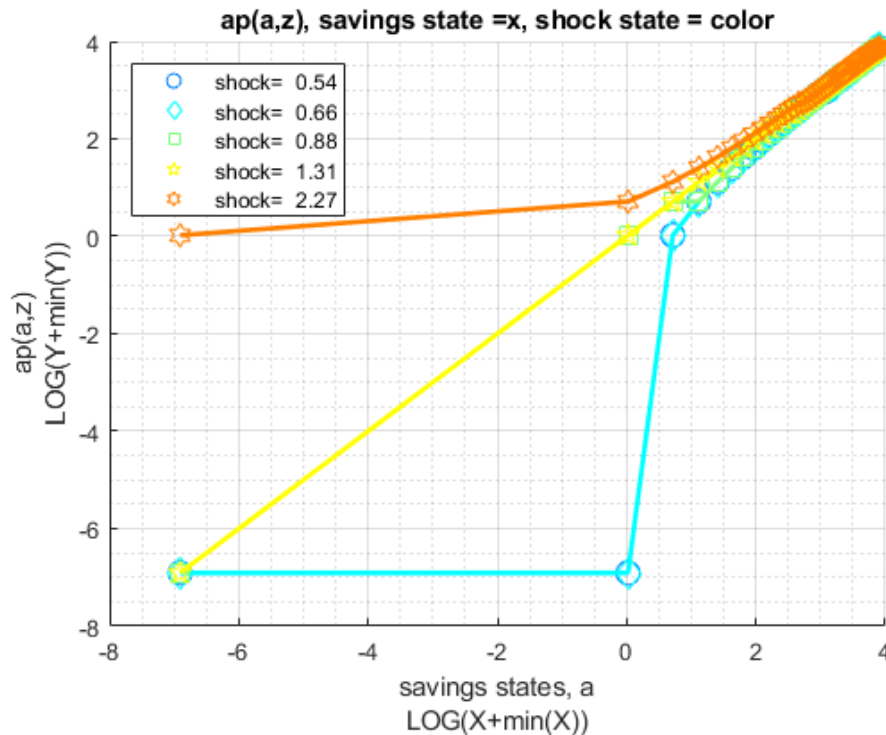
xxx ff_vfi_az_vec, outcome=ap xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

group	a	mean_z_0_54195	mean_z_0_66401	mean_z_0_88162	mean_z_1_3095	mean_z
-----	-----	-----	-----	-----	-----	-----
1	0	0	0	0	0	1.0
2	1.0204	0	0	1.0204	1.0204	2.0
3	2.0408	1.0204	1.0204	2.0408	2.0408	3.0
4	3.0612	2.0408	2.0408	2.0408	3.0612	4.0
5	4.0816	3.0612	3.0612	3.0612	4.0816	5.
6	5.102	4.0816	4.0816	4.0816	5.102	6.1
7	6.1224	5.102	5.102	5.102	6.1224	7.1
8	7.1429	6.1224	6.1224	6.1224	7.1429	8.1
9	8.1633	7.1429	7.1429	7.1429	8.1633	9.1
10	9.1837	8.1633	8.1633	8.1633	9.1837	10.
11	10.204	9.1837	9.1837	9.1837	10.204	11.
12	11.224	10.204	10.204	10.204	11.224	12.
13	12.245	11.224	11.224	11.224	12.245	13.
14	13.265	12.245	12.245	12.245	13.265	14.
15	14.286	13.265	13.265	13.265	14.286	15.
16	15.306	14.286	14.286	14.286	15.306	16.
17	16.327	15.306	15.306	15.306	16.327	17.
18	17.347	16.327	16.327	16.327	17.347	18.
19	18.367	17.347	17.347	17.347	18.367	19.
20	19.388	18.367	18.367	18.367	19.388	20.
21	20.408	19.388	19.388	19.388	20.408	21.
22	21.429	20.408	20.408	20.408	21.429	22.
23	22.449	21.429	21.429	21.429	22.449	23.
24	23.469	22.449	22.449	22.449	23.469	24.
25	24.49	23.469	23.469	24.49	24.49	25.
26	25.51	24.49	24.49	25.51	25.51	26.
27	26.531	25.51	25.51	26.531	26.531	27.
28	27.551	26.531	26.531			

1.2. FF_VFI_AZ_VEC DYNAMIC PROGRAMMING ASSET PROBLEM WITH SHOCKS VECTORIZED19

29	28.571	26.531	26.531	27.551	27.551	28.
30	29.592	27.551	27.551	28.571	28.571	29.
31	30.612	28.571	28.571	28.571	29.592	30.
32	31.633	29.592	29.592	29.592	30.612	31.
33	32.653	30.612	30.612	30.612	31.633	32.
34	33.673	31.633	31.633	31.633	32.653	33.
35	34.694	32.653	32.653	32.653	33.673	34.
36	35.714	33.673	33.673	33.673	34.694	35.
37	36.735	34.694	34.694	34.694	35.714	36.
38	37.755	35.714	35.714	35.714	36.735	37.
39	38.776	36.735	36.735	36.735	37.755	38.
40	39.796	37.755	37.755	37.755	38.776	39.
41	40.816	38.776	38.776	38.776	39.796	40.
42	41.837	39.796	39.796	39.796	40.816	41.
43	42.857	40.816	40.816	40.816	41.837	42.
44	43.878	41.837	41.837	41.837	41.837	42.
45	44.898	42.857	42.857	42.857	42.857	43.
46	45.918	43.878	43.878	43.878	43.878	44.
47	46.939	44.898	44.898	44.898	44.898	45.
48	47.959	45.918	45.918	45.918	45.918	46.
49	48.98	46.939	46.939	46.939	46.939	47.
50	50	47.959	47.959	47.959	47.959	48.





Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_vec(mp_params, mp_support);
```

Elapsed time is 0.120582 seconds.

xx

CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)

xx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coef
ap	1	1	2	900	100	9	21825	24.25	14.089	0.
savefraccoh	2	2	2	900	100	9	752.38	0.83597	0.13497	0.16

xxx TABLE:ap xxxxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7	c8	
r1	0	0	0	0	0	0	0.50505	1.5152	3
r2	0	0	0	0	0.50505	0.50505	1.0101	1.5152	3
r3	0.50505	0.50505	0.50505	0.50505	0.50505	1.0101	1.5152	2.0202	4
r4	1.0101	1.0101	1.0101	1.0101	1.0101	1.5152	2.0202	2.5253	4
r5	1.5152	1.5152	1.5152	1.5152	1.5152	2.0202	2.5253	3.0303	5
r96	45.455	45.455	45.455	45.96	45.96	45.96	46.465	47.475	4
r97	45.96	45.96	45.96	46.465	46.465	46.465	46.97	47.98	4
r98	46.465	46.465	46.465	46.465	46.97	46.97	47.475	48.485	
r99	46.97	46.97	46.97	46.97	47.475	47.475	47.98	48.99	


```

r747    0.80218    0.80112    0.7996    0.79739    0.79402    0.79208    0.78643    0.78008
r748    0.80245    0.80139    0.79988    0.79767    0.79432    0.7924    0.78677    0.78046
r749    0.80272    0.80167    0.80016    0.79796    0.79462    0.79271    0.78711    0.78085
r750    0.80299    0.80194    0.80044    0.79825    0.79492    0.79303    0.78745    0.78124

% Higher Savings Incentives
mp_params('fl_beta') = 0.95;
mp_params('fl_r') = 0.04;
ff_vfi_az_vec(mp_params, mp_support);

Elapsed time is 2.419509 seconds.
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
              i      idx      ndim      numel      rowN      colN      sum      mean      std      coef
              -      ---      ----      -----      ----      ----      -      -      -      -
savefraccoh    1      1      2      6750      750      9      4491.9    0.66547    0.28771    0.43

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxxxxxxxx
              c1      c2      c3      c4      c5      c6      c7      c8
              -----
r1              0      0      0      0      0.031818    0.14726    0.31047    0.48484
r2              0      0      0      0      0.031818    0.14726    0.31047    0.48484
r3              0      0      0      0      0.031818    0.14726    0.31047    0.48484
r4              0      0      0      0      0.031818    0.14726    0.31047    0.48484
r5              0      0      0      0      0.031818    0.14726    0.31047    0.48484
r746    0.92742    0.93    0.9283    0.92581    0.92578    0.92349    0.92443    0.91686
r747    0.9275    0.93007    0.92838    0.9259    0.92588    0.92361    0.92457    0.91706
r748    0.92757    0.93014    0.92846    0.92599    0.92598    0.92373    0.92472    0.91359
r749    0.92764    0.93022    0.92854    0.92608    0.92608    0.92384    0.92115    0.91014
r750    0.92772    0.93029    0.92862    0.92617    0.92618    0.92396    0.9213    0.90671

```

1.2.4 Test FF_VFI_AZ_VEC Changing Risk Aversion

Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

```

mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {'savefraccoh'};
mp_params = containers.Map('KeyType','char', 'ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 9;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';

```

Solve the model with different risk aversion levels, higher preferences for risk:

```

% Lower Risk Aversion
mp_params('fl_crra') = 0.5;
ff_vfi_az_vec(mp_params, mp_support);

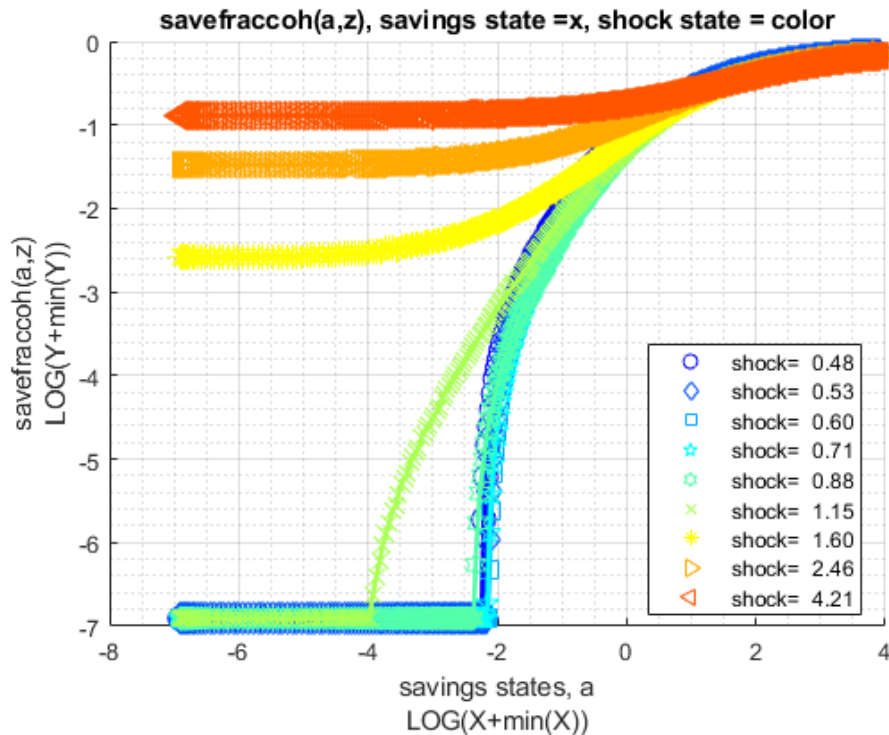
```

Elapsed time is 1.950318 seconds.

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

```

When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_vec(mp_params, mp_support);
```

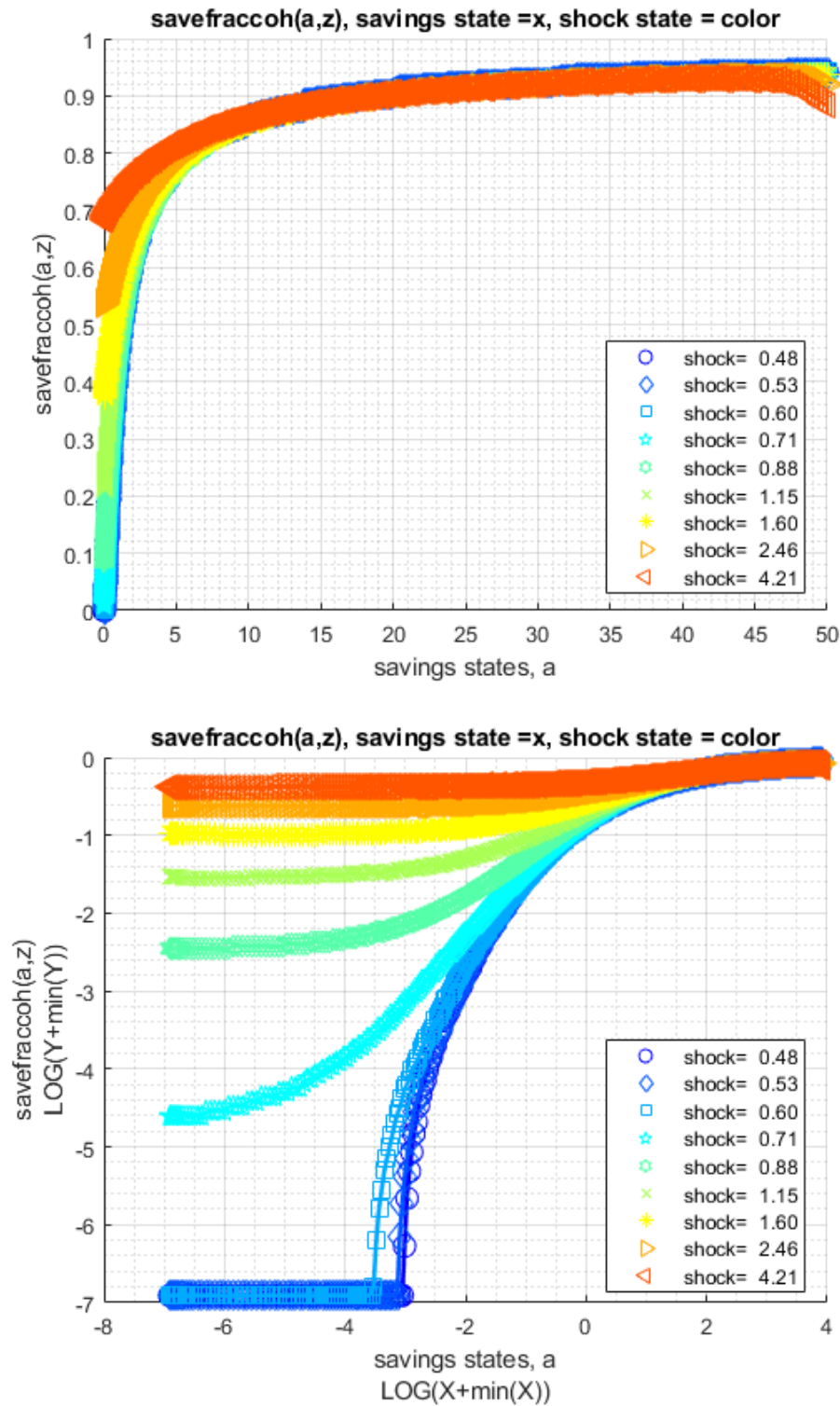
Elapsed time is 2.480325 seconds.

CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefv
savefraccoh	1	1	2	6750	750	9	4639.3	0.6873	0.28204	0.410

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7	c8
r1	0	0	0	0.008995	0.085095	0.21314	0.37277	0.53628
r2	0	0	0	0.008995	0.085095	0.21314	0.37277	0.53628
r3	0	0	0	0.008995	0.085095	0.21314	0.37277	0.53628
r4	0	0	0	0.008995	0.085095	0.21314	0.37277	0.53628
r5	0	0	0	0.0089949	0.085094	0.21314	0.37277	0.53628
r746	0.94083	0.9396	0.94168	0.93912	0.93904	0.94041	0.93743	0.92949
r747	0.94091	0.93969	0.94176	0.93921	0.93914	0.93674	0.93758	0.92969
r748	0.94098	0.93977	0.94184	0.93931	0.93924	0.93686	0.93772	0.92618
r749	0.94106	0.93985	0.94192	0.9394	0.93934	0.93699	0.93787	0.92269
r750	0.94113	0.93993	0.942	0.93949	0.93944	0.93711	0.93801	0.91921



1.2.5 Test FF_VFI_AZ_VEC with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
```


1.2. *FF_VFI_AZ_VEC DYNAMIC PROGRAMMING ASSET PROBLEM WITH SHOCKS VECTORIZED*27

r3	0	0	0	0	0.011319	0.12886	0.32464	0.53487
r4	0	0	0	0	0.011319	0.12886	0.32464	0.53487
r5	0	0	0	0	0.011319	0.12886	0.32464	0.53487
r746	0.91612	0.91885	0.9173	0.91484	0.91448	0.91454	0.91098	0.90731
r747	0.91622	0.91896	0.91741	0.91496	0.9146	0.91469	0.91117	0.90394
r748	0.91633	0.91906	0.91751	0.91507	0.91473	0.91483	0.91136	0.90422
r749	0.91643	0.91916	0.91762	0.91519	0.91486	0.91498	0.91154	0.90449
r750	0.91653	0.91926	0.91773	0.91531	0.91498	0.91512	0.91173	0.90115

Chapter 2

Summarize Policy and Value

2.1 FF_SUMM_ND_ARRAY Examples

Go back to [fan's MEconTools Toolbox \(bookdown\)](#), [Matlab Code Examples Repository \(bookdown\)](#), or [Math for Econ with Matlab Repository \(bookdown\)](#).

This is the example vignette for function: `ff_summ_nd_array` from the [MEconTools Package](#). This function summarizes policy and value functions over states.

2.1.1 Test FF_SUMM_ND_ARRAY Defaults

Call the function with defaults.

```
ff_summ_nd_array();
```

```
xxx  Summ over (a,z), condi age as cols, kids/marriage as rows  xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
      group      marry      kids      mean_age_18      mean_age_19      mean_age_20      mean_age_21
      ----      -
      1          0          1          0.52456          0.51689          0.48412          0.54526
      2          1          1          0.49355          0.52906          0.5583           0.47342
      3          0          2          0.49085          0.51315          0.45158          0.43201
      4          1          2          0.58096          0.50596          0.47985          0.58791
      5          0          3          0.57811          0.6068           0.55221          0.50677
      6          1          3          0.53023          0.49258          0.48728          0.43352
      7          0          4          0.50339          0.48449          0.53618          0.45993
      8          1          4          0.44418          0.5223           0.55657          0.48583
```

2.1.2 Test FF_SUMM_ND_ARRAY with Random 2 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random 2D dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(5,4);
bl_print_table = true;
ar_st_stats = ["mean"];
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'a', linspace(0,1,size(mn_polval,1))});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'z', linspace(-1,1,size(mn_polval,2))});
disp(mn_polval);
```

0.6965	0.4231	0.3432	0.7380
0.2861	0.9808	0.7290	0.1825
0.2269	0.6848	0.4386	0.1755
0.5513	0.4809	0.0597	0.5316
0.7195	0.3921	0.3980	0.5318

Second, show the entire matrix (no labels):

```
it_aggd = 0;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);
```

```
xxx Random 2D dimensional Array Testing Summarizing xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
group   vardim2   mean_vardim1_1   mean_vardim1_2   mean_vardim1_3   mean_vardim1_4   mean
-----
1       1         0.69647         0.28614         0.22685         0.55131         0
2       2         0.42311         0.98076         0.68483         0.48093         0
3       3         0.34318         0.72905         0.43857         0.059678        0
4       4         0.738          0.18249         0.17545         0.53155         0
```

Third, rotate row and column, and now with labels:

```
it_aggd = 0;
bl_row = 1;
ar_permute = [2,1];
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);
```

```
xxx Random 2D dimensional Array Testing Summarizing xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
group   a   mean_z__1   mean_z__0_33333   mean_z_0_33333   mean_z_1
-----
1       0   0.69647   0.42311         0.34318         0.738
2       0.25 0.28614   0.98076         0.72905         0.18249
3       0.5  0.22685   0.68483         0.43857         0.17545
4       0.75 0.55131   0.48093         0.059678        0.53155
5       1   0.71947   0.39212         0.39804         0.53183
```

Fourth, dimension one as columns, average over dim 2:

```
it_aggd = 1;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);
```

```
xxx Random 2D dimensional Array Testing Summarizing xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
group   x   mean_z__1   mean_z__0_33333   mean_z_0_33333   mean_z_1
-----
1       1   0.49605   0.59235         0.3937          0.43186
```

Fifth, dimension one as rows, average over dim 2:

```
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc);
```

```
xxx Random 2D dimensional Array Testing Summarizing xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
group   z   sum   mean   std   coefvari   min   max
```

-----	-----	-----	-----	-----	-----	-----	-----
1	-1	2.4802	0.49605	0.22895	2.1666	0.22685	0.71947
2	-0.33333	2.9617	0.59235	0.24524	2.4154	0.39212	0.98076
3	0.33333	1.9685	0.3937	0.23907	1.6468	0.059678	0.72905
4	1	2.1593	0.43186	0.24575	1.7573	0.17545	0.738

Sixth, dimension two as rows, average over dim 1:

```
ar_permute = [2,1];
it_aggd = 1;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, ...
    cl_mp_datasetdesc, ar_permute);
```

xxx	Random 2D dimensional Array Testing Summarizing					xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx		
group	a	sum	mean	std	coefvari	min	max	
-----	-----	-----	-----	-----	-----	-----	-----	
1	0	2.2007	0.55019	0.19636	2.8019	0.34318	0.738	
2	0.25	2.1784	0.54461	0.37514	1.4518	0.18249	0.98076	
3	0.5	1.5257	0.38143	0.23212	1.6432	0.17545	0.68483	
4	0.75	1.6235	0.40587	0.23269	1.7443	0.059678	0.55131	
5	1	2.0415	0.51036	0.15361	3.3226	0.39212	0.71947	

2.1.3 Test FF_SUMM_ND_ARRAY with Random 6 Dimensional Matrix

Summarize over 6 dimensional array, iteratively change how many dimensions to group over.

First, generate matrix:

```
st_title = "Random ND dimensional Array Testing Summarizing";
rng(123)
mn_polval = rand(8,7,6,5,4,3);
bl_print_table = true;
ar_st_stats = ["mean"];
```

Second, summarize over the first four dimensions, row group others:

```
it_aggd = 4;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);
```

xxx	Random	ND dimensional	Array	Testing	Summarizing	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx			
	group	vardim5	vardim6	sum	mean	std	coefvari	min	max
	-----	-----	-----	-----	-----	-----	-----	-----	-----
	1	1	1	836.78	0.49808	0.29255	1.7026	8.1888e-05	0.99964
	2	2	1	842.15	0.50128	0.28968	1.7305	6.7838e-05	0.99936
	3	3	1	831.45	0.49491	0.28851	1.7154	0.00091373	0.99989
	4	4	1	843.9	0.50232	0.28154	1.7842	0.00012471	0.99731
	5	1	2	838.99	0.4994	0.2911	1.7156	0.00029749	0.99938
	6	2	2	830.81	0.49453	0.28634	1.7271	0.00027113	0.9992
	7	3	2	832.59	0.49559	0.28682	1.7279	0.00035994	0.99936
	8	4	2	820.42	0.48835	0.29032	1.6821	0.00096259	0.99896
	9	1	3	870.56	0.51819	0.29111	1.7801	0.0010616	0.99951
	10	2	3	854.68	0.50874	0.28458	1.7877	0.001884	0.99965
	11	3	3	838.29	0.49898	0.2891	1.726	0.0019192	0.99945
	12	4	3	842.83	0.50169	0.2877	1.7438	0.00016871	0.99963

Third, summarize over the first four dimensions, column group 5th, and row group others:

```

it_aggd = 4;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ["sum"], it_aggd, bl_row);

```

```

xxx Random ND dimensional Array Testing Summarizing xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
  group   vardim6   sum_vardim5_1   sum_vardim5_2   sum_vardim5_3   sum_vardim5_4
  -----
    1      1      836.78      842.15      831.45      843.9
    2      2      838.99      830.81      832.59      820.42
    3      3      870.56      854.68      838.29      842.83

```

Fourth, summarize over the first five dimensions, column group 6th, no row groups:

```

it_aggd = 5;
bl_row = 1;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ["mean", "std"], it_aggd, bl_row);

```

```

xxx Random ND dimensional Array Testing Summarizing xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
  group   x   mean_vardim6_1   mean_vardim6_2   mean_vardim6_3   std_vardim6_1   std_vardim6
  -----
    1      1      0.49915      0.49447      0.5069      0.28805      0.28862

```

Fifth, summarize over all six dimensions, summary statistics over the entire dataframe:

```

it_aggd = 6;
bl_row = 0;
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row);

```

```

xxx Random ND dimensional Array Testing Summarizing xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
  group   x   sum   mean   std   coefvari   min   max
  -----
    1      1  10083  0.50017  0.28831  1.7349  6.7838e-05  0.99989

```

2.1.4 Test FF_SUMM_ND_ARRAY with Random 7 Dimensional Matrix with All Parameters

Given a random seven dimensional matrix, average over the 2nd, 4th and 5th dimensionals. Show as row groups the 3, 6 and 7th dimensions, and row groups the 1st dimension. Show Coefficient of Variation only.

```

st_title = "avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim.";
rng(123)
mn_polval = rand(3,10,2,10,10,2,3);
ar_permute = [2,4,5,1,3,6,7];
bl_print_table = true;
ar_st_stats = ["coefvari"];
it_aggd = 3; % mean over 3 dims
bl_row = 1; % one var for row group
cl_mp_datasetdesc = {};
cl_mp_datasetdesc{1} = containers.Map({'name', 'labval'}, ...
    {'age', [18, 19, 20]});
cl_mp_datasetdesc{2} = containers.Map({'name', 'labval'}, ...
    {'savings', linspace(0,1,10)});
cl_mp_datasetdesc{3} = containers.Map({'name', 'labval'}, ...
    {'borrsave', [-1,+1]});
cl_mp_datasetdesc{4} = containers.Map({'name', 'labval'}, ...
    {'shocka', linspace(-5,5,10)});
cl_mp_datasetdesc{5} = containers.Map({'name', 'labval'}, ...

```



```

    {'shockb', linspace(-5,5,10)});
cl_mp_datasetdesc{6} = containers.Map({'name', 'labval'}, ...
    {'marry', [0,1]});
cl_mp_datasetdesc{7} = containers.Map({'name', 'labval'}, ...
    {'region', [1,2,3]});
% call function
ff_summ_nd_array(st_title, mn_polval, bl_print_table, ar_st_stats, it_aggd, bl_row, cl_mp_datasetdes

xxx  avg VALUE 2+4+5th dims. groups 3+6+7th dims, and row groups the 1st dim. xxxxxxxxxxxxxxxxxxxxxxxx
      group    borrsave    marry    region    cv_age_18    cv_age_19    cv_age_20
      -----
      1         -1         0         1         1.7607         1.7534         1.7065
      2          1         0         1         1.6566         1.7501         1.7042
      3         -1         1         1         1.6608         1.7658         1.7291
      4          1         1         1         1.756          1.7479         1.7606
      5         -1         0         2         1.7314         1.7506         1.786
      6          1         0         2         1.7347         1.728          1.738
      7         -1         1         2         1.7811         1.755          1.7568
      8          1         1         2         1.7445         1.7398         1.7746
      9         -1         0         3         1.7025         1.7286          1.69
     10          1         0         3          1.74         1.7549         1.7356
     11         -1         1         3         1.7147         1.7287         1.7341
     12          1         1         3         1.7919         1.7313         1.7452

```


Chapter 3

Distributional Analysis

3.1 FF_SIMU_STATS Examples

Go back to [fan's MEconTools Toolbox \(bookdown\)](#), [Matlab Code Examples Repository \(bookdown\)](#), or [Math for Econ with Matlab Repository \(bookdown\)](#).

This is the example vignette for function: `ff_simu_stats` from the **MEconTools Package**. This is a gate-way function that computes mean, percentiles, covariance etc between several variables.

3.1.1 Test FF_SIMU_STATS Defaults

Call the function with defaults.

```
ff_simu_stats();
```

```
xxx tb_outcomes: all stats xxx
OriginalVariableNames    cl_mt_pol_a    cl_mt_pol_c
-----
{'mean'                  }    -0.11081      8.8423
{'sd'                    }      4.1239      6.5845
{'coefofvar'             }    -37.215      0.74466
{'min'                   }      -7          -6.3772
{'max'                   }       9          21.786
{'pYis0'                 }    0.064259      0
{'pYls0'                 }    0.54867      0.027329
{'pYgr0'                 }    0.38707      0.97267
{'pYisMINY'              }    0.051764      0.015232
{'pYisMAXY'              }    0.027329      0.046484
{'p1'                    }      -7          -6.3772
{'p10'                   }      -6          0.27238
{'p25'                   }      -3          5.2138
{'p50'                   }      -1          6.5321
{'p75'                   }       3          13.799
{'p90'                   }       5          16.887
{'p99'                   }       9          21.786
{'fl_cov_cl_mt_pol_a'}    17.007      -22.084
{'fl_cor_cl_mt_pol_a'}     1          -0.81327
{'fl_cov_cl_mt_pol_c'}   -22.084      43.356
{'fl_cor_cl_mt_pol_c'}   -0.81327      1
{'fracByP1'              }    3.2699     -0.010985
{'fracByP10'             }    5.9889     -0.013362
{'fracByP25'             }   14.165      0.041007
{'fracByP50'             }   16.208      0.1893
```

{'fracByP75'}	}	12.702	0.59539
{'fracByP90'}	}	6.6611	0.8307
{'fracByP99'}	}	1	1

3.1.2 Test FF_SIMU_STATS Four States-Points Matrix

Over some (a,z) states that is 3 by 3, c matrix, generate all stats

```
% Set Parameters
mt_x_of_s = [1, 2, 3.0;...
            3, 1, 1.5;...
            4, 3, 2.0];
mt_y_of_s = [2, -10, 9.0;...
            5, 1.1, 3.0;...
            1, 3, -1.5];
mt_z_of_s = [1.1, 2, 3.3;...
            2.3, 1, 1.5;...
            4, 2.5, 2.0];
mp_cl_mt_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s, zeros(1)};
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
% Mass
rng(123);
mt_f_of_s = rand(size(mt_x_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s, mp_cl_mt_xyz_of_s);
```

```
xxx tb_outcomes: all stats xxx
OriginalVariableNames    cl_mt_x_of_s    cl_mt_y_of_s    cl_mt_z_of_s
-----
{'mean'                  }          2.0763          1.9323          2.0668
{'sd'                    }          0.9071          5.2239          0.9042
{'coefofvar'             }          0.43688         2.7034          0.43749
{'min'                   }          1             -10             1
{'max'                   }          4              9              4
{'pYis0'                 }          0              0              0
{'pYls0'                 }          0             0.20441         0
{'pYgr0'                 }          1             0.79559         1
{'pYisMINY'              }          0.28039         0.10917         0.14247
{'pYisMAXY'              }          0.044922        0.19422         0.044922
{'p1'                    }          1             -10             1
{'p10'                   }          1             -10             1
{'p25'                   }          1             1.1             1.1
{'p50'                   }          2              2              2
{'p75'                   }          3              5              2.5
{'p90'                   }          3              9              3.3
{'p99'                   }          4              9              4
{'fl_cov_cl_mt_x_of_s'}    0.82282         1.589           0.78646
{'fl_cor_cl_mt_x_of_s'}    1              0.33534         0.95887
{'fl_cov_cl_mt_y_of_s'}    1.589          27.289          1.8353
{'fl_cor_cl_mt_y_of_s'}    0.33534         1              0.38856
{'fl_cov_cl_mt_z_of_s'}    0.78646         1.8353          0.81758
{'fl_cor_cl_mt_z_of_s'}    0.95887         0.38856         1
{'fracByP1'              }          0.13504        -0.56498        0.068934
{'fracByP10'             }          0.13504        -0.56498        0.068934
```

{'fracByP25'}	}	0.13504	-0.53456	0.14234
{'fracByP50'}	}	0.42991	-0.39181	0.43856
{'fracByP75'}	}	0.91346	0.095425	0.60296
{'fracByP90'}	}	0.91346	1	0.91306
{'fracByP99'}	}	1	1	1

3.1.3 Test FF_SIMU_STATS Four States-Points Matrix Single Column Inputs

Same as before, but now inputs are single column, should have identical results:

```
% Array Inputs
```

```
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
```

```
mp_cl_mt_xyz_of_s('cl_mt_x_of_s') = {mt_x_of_s(:), zeros(1)};
```

```
mp_cl_mt_xyz_of_s('cl_mt_y_of_s') = {mt_y_of_s(:), zeros(1)};
```

```
mp_cl_mt_xyz_of_s('cl_mt_z_of_s') = {mt_z_of_s(:), zeros(1)};
```

```
mp_cl_mt_xyz_of_s('ar_st_y_name') = ["cl_mt_x_of_s", "cl_mt_y_of_s", "cl_mt_z_of_s"];
```

```
% Call Function
```

```
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_mt_xyz_of_s);
```

```
xxx tb_outcomes: all stats xxx
```

OriginalVariableNames	cl_mt_x_of_s	cl_mt_y_of_s	cl_mt_z_of_s
-----	-----	-----	-----
{'mean'}	2.0763	1.9323	2.0668
{'sd'}	0.9071	5.2239	0.9042
{'coefofvar'}	0.43688	2.7034	0.43749
{'min'}	1	-10	1
{'max'}	4	9	4
{'pYis0'}	0	0	0
{'pYls0'}	0	0.20441	0
{'pYgr0'}	1	0.79559	1
{'pYisMINY'}	0.28039	0.10917	0.14247
{'pYisMAXY'}	0.044922	0.19422	0.044922
{'p1'}	1	-10	1
{'p10'}	1	-10	1
{'p25'}	1	1.1	1.1
{'p50'}	2	2	2
{'p75'}	3	5	2.5
{'p90'}	3	9	3.3
{'p99'}	4	9	4
{'fl_cov_cl_mt_x_of_s'}	0.82282	1.589	0.78646
{'fl_cor_cl_mt_x_of_s'}	1	0.33534	0.95887
{'fl_cov_cl_mt_y_of_s'}	1.589	27.289	1.8353
{'fl_cor_cl_mt_y_of_s'}	0.33534	1	0.38856
{'fl_cov_cl_mt_z_of_s'}	0.78646	1.8353	0.81758
{'fl_cor_cl_mt_z_of_s'}	0.95887	0.38856	1
{'fracByP1'}	0.13504	-0.56498	0.068934
{'fracByP10'}	0.13504	-0.56498	0.068934
{'fracByP25'}	0.13504	-0.53456	0.14234
{'fracByP50'}	0.42991	-0.39181	0.43856
{'fracByP75'}	0.91346	0.095425	0.60296
{'fracByP90'}	0.91346	1	0.91306
{'fracByP99'}	1	1	1

3.1.4 Test FF_SIMU_STATS Print Many Details

The Same As before, but now control which percentiles and other details to display.

```

% Array Inputs
mp_cl_ar_xyz_of_s = containers.Map('KeyType','char', 'ValueType','any');
mp_cl_ar_xyz_of_s('cl_ar_x_of_s') = {mt_x_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('cl_ar_z_of_s') = {mt_z_of_s(:), zeros(1)};
mp_cl_ar_xyz_of_s('ar_st_y_name') = ["cl_ar_x_of_s", "cl_ar_z_of_s"];

% controls
mp_support = containers.Map('KeyType','char', 'ValueType','any');
mp_support('bl_display_detail') = false;
mp_support('bl_display_final') = true;
mp_support('bl_display_drvm2outcomes') = false;
mp_support('ar_fl_percentiles') = [25 50 75];
mp_support('bl_display_drvstats') = true;
mp_support('bl_display_drvm2covcor') = false;

% Call Function
mp_cl_mt_xyz_of_s_out = ff_simu_stats(mt_f_of_s(:), mp_cl_ar_xyz_of_s, mp_support);

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: cl_ar_x_of_s
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
-----

fl_choice_mean
    2.0763

fl_choice_sd
    0.9071

fl_choice_coefofvar
    0.4369

fl_choice_prob_zero
    0

fl_choice_prob_below_zero
    0

fl_choice_prob_above_zero
    1

fl_choice_prob_max
    0.0449

tb_disc_cumu
    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass    CDF    cumsumFrac
    -----
           1                0.28039                28.039        0.13504
          1.5                0.13561                41.6         0.23301
           2                0.20441                62.041        0.42991
           3                0.33466                95.508        0.91346
           4                0.044922                100          1

    cl_ar_x_of_sDiscreteVal    cl_ar_x_of_sDiscreteValProbMass    CDF    cumsumFrac
    -----
           1                0.28039                28.039        0.13504

```

1.5	0.13561	41.6	0.23301
2	0.20441	62.041	0.42991
3	0.33466	95.508	0.91346
4	0.044922	100	1

tb_prob_drv percentiles	cl_ar_x_of_sDiscreteValPercentileValues	fracOfSumHeldBelowThisPercentile
-----	-----	-----
25	1	0.13504
50	2	0.42991
75	3	0.91346

 xxx
 Summary Statistics for: cl_ar_z_of_s
 xxx

fl_choice_mean
 2.0668

fl_choice_sd
 0.9042

fl_choice_coefofvar
 0.4375

fl_choice_prob_zero
 0

fl_choice_prob_below_zero
 0

fl_choice_prob_above_zero
 1

fl_choice_prob_max
 0.0449

tb_disc_cumu cl_ar_z_of_sDiscreteVal	cl_ar_z_of_sDiscreteValProbMass	CDF	cumsumFrac
-----	-----	-----	-----
1	0.14247	14.247	0.068934
1.1	0.13792	28.039	0.14234
1.5	0.13561	41.6	0.24076
2	0.20441	62.041	0.43856
2.3	0.056663	67.708	0.50162
2.5	0.083786	76.086	0.60296
3.3	0.19422	95.508	0.91306
4	0.044922	100	1

cl_ar_z_of_sDiscreteVal	cl_ar_z_of_sDiscreteValProbMass	CDF	cumsumFrac
-----	-----	-----	-----
1	0.14247	14.247	0.068934
1.1	0.13792	28.039	0.14234
1.5	0.13561	41.6	0.24076


```
fl_choice_mean
-1.0000
```

```
fl_choice_sd
2.5100
```

```
fl_choice_coefofvar
-2.5100
```

```
fl_choice_prob_zero
0.1416
```

```
fl_choice_prob_below_zero
0.5888
```

```
fl_choice_prob_above_zero
0.2696
```

```
fl_choice_prob_max
2.0589e-16
```

```
tb_disc_cumu
  binomDiscreteVal  binomDiscreteValProbMass  CDF  cumsumFrac
  -----
      -10          2.2539e-05      0.0022539  0.00022539
      -9          0.00028979      0.031233   0.0028335
      -8          0.0018008       0.21132   0.01724
      -7          0.0072034       0.93166   0.067664
      -6          0.020838        3.0155    0.19269
      -5          0.04644         7.6595    0.42489
      -4          0.082928       15.952    0.75661
      -3          0.12185        28.138    1.1222
      -2          0.15014        43.152    1.4224
      -1          0.15729        58.881    1.5797
```

```
  binomDiscreteVal  binomDiscreteValProbMass  CDF  cumsumFrac
  -----
      11          6.0392e-06      100      1
      12          1.0588e-06      100      1
      13          1.5784e-07      100      1
      14          1.973e-08       100      1
      15          2.0293e-09      100      1
      16          1.6725e-10      100      1
      17          1.0619e-11      100      1
      18          4.8762e-13      100      1
      19          1.4412e-14      100      1
      20          2.0589e-16      100      1
```

```
tb_prob_drv
  percentiles  binomDiscreteValPercentileValues  fracOfSumHeldBelowThisPercentile
  -----
      0.1          -8          0.01724
      1           -6          0.19269
      5           -5          0.42489
     10           -4          0.75661
```

15	-4	0.75661
20	-3	1.1222
25	-3	1.1222
35	-2	1.4224
50	-1	1.5797
65	0	1.5797
75	1	1.4694
80	1	1.4694
85	2	1.3197
90	2	1.3197
95	3	1.1865
99	5	1.0412
99.9	7	1.0052

3.2.2 Test FF_DISC_RAND_VAR_STATS 0 and 1 Random Variable

The simplest discrete random variable has two values, zero or one. The probability of zero is 30 percent, and 70 percent is the probability of one.

```
% Parameters
% 1. specify the random variable
st_var_name = 'bernoulli';
ar_choice_unique_sorted = [0, 1];
ar_choice_prob = [0.3, 0.7];
% 2. percentiles of interest
ar_fl_percentiles = [0.1 5 25 50 75 95 99.9];
% 3. print results
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: bernoulli
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
-----
```

```
fl_choice_mean
    0.7000
```

```
fl_choice_sd
    0.4583
```

```
fl_choice_coefofvar
    0.6547
```

```
fl_choice_prob_zero
    0.3000
```

```
fl_choice_prob_below_zero
    0
```

```
fl_choice_prob_above_zero
    0.7000
```

```
fl_choice_prob_max
    0.7000
```

```

tb_disc_cumu
  bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
  -----
          0                0.3                30          0
          1                0.7               100          1

  bernoulliDiscreteVal    bernoulliDiscreteValProbMass    CDF    cumsumFrac
  -----
          0                0.3                30          0
          1                0.7               100          1

tb_prob_drv
  percentiles    bernoulliDiscreteValPercentileValues    fracOfSumHeldBelowThisPercentile
  -----
    0.1                0                                0
     5                0                                0
    25                0                                0
    50                1                                1
    75                1                                1
    95                1                                1
   99.9                1                                1

```

3.2.3 Test FF_DISC_RAND_VAR_STATS with Poisson

[Poisson random variable](#), with mean equals to ten, summarize over unsymmetric percentiles. Note that the poisson random variable has no upper bound.

```

% Parameters
% 1. specify the random variable
st_var_name = 'poisson';
mu = 10;
ar_choice_unique_sorted = 0:1:50;
ar_choice_prob = poisspdf(ar_choice_unique_sorted, mu);
% 2. percentiles of interest, unsymmetric
ar_fl_percentiles = [0.1 5 10 25 50 90 95 99 99.9 99.99 99.999 99.9999];
% 3. print results
bl_display_drvstats = true;
% Call Function
[ds_stats_map] = ff_disc_rand_var_stats(st_var_name, ...
    ar_choice_unique_sorted, ar_choice_prob, ...
    ar_fl_percentiles, bl_display_drvstats);

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Summary Statistics for: poisson
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
-----

fl_choice_mean
    10

fl_choice_sd
    3.1623

fl_choice_coefofvar
    0.3162

```

```
fl_choice_prob_zero
4.5400e-05
```

```
fl_choice_prob_below_zero
0
```

```
fl_choice_prob_above_zero
1.0000
```

```
fl_choice_prob_max
1.4927e-19
```

```
tb_disc_cumu
```

poissonDiscreteVal	poissonDiscreteValProbMass	CDF	cumsumFrac
-----	-----	-----	-----
0	4.54e-05	0.00454	0
1	0.000454	0.04994	4.54e-05
2	0.00227	0.27694	0.0004994
3	0.0075667	1.0336	0.0027694
4	0.018917	2.9253	0.010336
5	0.037833	6.7086	0.029253
6	0.063055	13.014	0.067086
7	0.090079	22.022	0.13014
8	0.1126	33.282	0.22022
9	0.12511	45.793	0.33282

poissonDiscreteVal	poissonDiscreteValProbMass	CDF	cumsumFrac
-----	-----	---	-----
41	1.3571e-13	100	1
42	3.2313e-14	100	1
43	7.5146e-15	100	1
44	1.7079e-15	100	1
45	3.7953e-16	100	1
46	8.2506e-17	100	1
47	1.7554e-17	100	1
48	3.6572e-18	100	1
49	7.4636e-19	100	1
50	1.4927e-19	100	1

```
tb_prob_drv
```

percentiles	poissonDiscreteValPercentileValues	fracOfSumHeldBelowThisPercentile
-----	-----	-----
0.1	2	0.0004994
5	5	0.029253
10	6	0.067086
25	8	0.22022
50	10	0.45793
90	14	0.86446
95	15	0.91654
99	18	0.98572
99.9	21	0.99841
99.99	24	0.99988
99.999	26	0.99998
100	28	1

```
% Print out full Stored Matrix
% Note that the outputs are single row arrays.
ff_container_map_display(ds_stats_map, 100, 100)

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

          i      idx      ndim      numel      rowN      colN      mean      std      coe
          -      ---      ----      -----      ----      ----      -
ar_choice_perc_fracheld      1      1      2      12      1      12      0.62833      0.435      0.6
ar_choice_percentiles      2      2      2      12      1      12      14.75      8.7399      0.5
ar_fl_percentiles      3      3      2      12      1      12      64.499      42.887      0.6

xxx TABLE:ar_choice_perc_fracheld xxxxxxxxxxxxxxxxxxxxxxxx
          c1      c2      c3      c4      c5      c6      c7      c8
          -----
r1      0.0004994      0.029253      0.067086      0.22022      0.45793      0.86446      0.91654      0.98572

xxx TABLE:ar_choice_percentiles xxxxxxxxxxxxxxxxxxxxxxxx
          c1      c2      c3      c4      c5      c6      c7      c8      c9      c10      c11      c12
          --      --      --      --      --      --      --      --      --      ---      ---      ---
r1      2      5      6      8      10      14      15      18      21      24      26      28

xxx TABLE:ar_fl_percentiles xxxxxxxxxxxxxxxxxxxxxxxx
          c1      c2      c3      c4      c5      c6      c7      c8      c9      c10      c11      c12
          ---      --      --      --      --      --      --      --      ---      ---      ---      ---
r1      0.1      5      10      25      50      90      95      99      99.9      99.99      99.999      100

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: ds_stats_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

          i      idx      value
          --      ---      -----
fl_choice_coefofvar      1      4      0.31623
fl_choice_max      2      5      50
fl_choice_mean      3      6      10
fl_choice_min      4      7      0
fl_choice_prob_above_zero      5      8      0.99995
fl_choice_prob_below_zero      6      9      0
fl_choice_prob_max      7      10      1.4927e-19
fl_choice_prob_min      8      11      4.54e-05
fl_choice_prob_zero      9      12      4.54e-05
fl_choice_sd      10      13      3.1623
```

3.3 FF_DISC_RAND_VAR_MASS2OUTCOMES Examples

Go back to [fan's MEconTools Toolbox \(bookdown\)](#), [Matlab Code Examples Repository \(bookdown\)](#), or [Math for Econ with Matlab Repository \(bookdown\)](#).

This is the example vignette for function: [ff_disc_rand_var_mass2outcomes](#) from the [MEconTools Package](#). This function generates sorted discrete random variable from state-space joint distri-

bution.

3.3.1 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2outcomes();
```

```
INPUT f(a,z): mt_dist_bystates
  0.0289  0.0465  0.0228  0.0036  0.0001
  0.0241  0.0930  0.0857  0.0241  0.0015
  0.0080  0.0744  0.1285  0.0643  0.0074
  0.0013  0.0297  0.0964  0.0857  0.0186
  0.0001  0.0059  0.0361  0.0571  0.0232
  0.0000  0.0005  0.0054  0.0152  0.0116
```

```
INPUT y(a,z): mt_choice_bystates
  -5  -4  -5  -4  -4
  -3  -2  -3  -2  -3
  -1  -1  -1   0   0
   1   1   2   3   1
   4   3   3   4   3
   5   6   5   6   6
```

```
OUTPUT f(y): ar_choice_prob_byY
  0.0518
  0.0502
  0.1113
  0.1171
  0.2109
  0.0717
  0.0497
  0.0964
  0.1510
  0.0572
  0.0054
  0.0273
```

```
OUTPUT f(y,z): mt_choice_prob_byYZ
  0.0289  0  0.0228  0  0
    0  0.0465  0  0.0036  0.0001
  0.0241  0  0.0857  0  0.0015
    0  0.0930  0  0.0241  0
  0.0080  0.0744  0.1285  0  0
    0  0  0  0.0643  0.0074
  0.0013  0.0297  0  0  0.0186
    0  0  0.0964  0  0
    0  0.0059  0.0361  0.0857  0.0232
  0.0001  0  0  0.0571  0
  0.0000  0  0.0054  0  0
    0  0.0005  0  0.0152  0.0116
```

```
OUTPUT f(y,a): mt_choice_prob_byYA
  0.0518  0  0  0  0  0
  0.0502  0  0  0  0  0
    0  0.1113  0  0  0  0
    0  0.1171  0  0  0  0
    0  0  0.2109  0  0  0
    0  0  0.0717  0  0  0
```

0	0	0	0.0497	0	0
0	0	0	0.0964	0	0
0	0	0	0.0857	0.0653	0
0	0	0	0	0.0572	0
0	0	0	0	0	0.0054
0	0	0	0	0	0.0273

OUTPUT f(y) and y in table: tb_choice_drv_cur_byY

binomtestOutcomes	probMassFunction
-------------------	------------------

-5	0.051764
-4	0.050217
-3	0.11126
-2	0.11706
-1	0.21092
0	0.071696
1	0.049682
2	0.096388
3	0.15102
4	0.057231
5	0.0054256
6	0.027329

3.3.2 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Four States-Points

Over some (a,z) states that is 2 by 2, matrix or vectorized inputs identical results.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2;3,1];
% stationary mass over assets adn shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);

0.4039    1.0000
0.2971    2.0000
0.2990    3.0000
```

Same as before, but now inputs are single column:

```
% Call Function
[ar_f_of_y, ar_y_unique_sorted] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s(:), mt_f_of_s);
disp([ar_f_of_y ar_y_unique_sorted]);

0.4039    1.0000
0.2971    2.0000
0.2990    3.0000
```

3.3.3 Test FF_DISC_RAND_VAR_MASS2OUTCOMES Conditional Mass Outputs

Same inputs as before, but now, also output additional conditional statistics, $f(y, a)$, where a is the row state variable for $f(a, z)$. For conditional statistics, must provide matrix based inputs.

```
% Set Parameters
st_y_name = 'consumption';
% consumption matrix: c(a,z)
mt_c_of_s = [1,2,0.5;
             3,1,2.0];
% stationary mass over assets and shocks: f(a,z)
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
% Call Function
[ar_f_of_y, ar_y_unique_sorted, mt_f_of_y_srow, mt_f_of_y_scol] = ...
    ff_disc_rand_var_mass2outcomes(st_y_name, mt_c_of_s, mt_f_of_s);
% print
disp([ar_f_of_y ar_y_unique_sorted]);

    0.2695    0.5000
    0.3765    1.0000
    0.2649    2.0000
    0.0891    3.0000

disp(mt_f_of_y_srow);

    0.2695         0
    0.1215    0.2550
    0.1217    0.1432
         0    0.0891

disp(mt_f_of_y_scol);

         0         0    0.2695
    0.1215    0.2550         0
         0    0.1217    0.1432
    0.0891         0         0
```

3.4 FF_DISC_RAND_VAR_MASS2COVCOR Examples

Go back to [fan's MEconTools Toolbox \(bookdown\)](#), [Matlab Code Examples Repository \(bookdown\)](#), or [Math for Econ with Matlab Repository \(bookdown\)](#).

This is the example vignette for function: `ff_disc_rand_var_mass2covcor` from the [MEconTools Package](#). This function calculates covariance and correlation based for two discrete random variables.

3.4.1 Test FF_DISC_RAND_VAR_MASS2COVCOR Defaults

Call the function with defaults.

```
ff_disc_rand_var_mass2covcor();

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: covvar_input_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

         i      idx      ndim      numel      rowN      colN      mean      std      coefvari      --
         -      ---      ----      -----      ----      ----      -
-----
```


mt_f_of_s	1	5	2	30	6	5	0.033333	0.035743	1.0723	3.0723
mt_x_of_s	2	6	2	30	6	5	0.83333	5.3051	6.3661	11.5014
mt_y_of_s	3	7	2	30	6	5	8.3259	7.1913	0.86373	16.3771

```
xxx TABLE:mt_f_of_s xxxxxxxxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5
	-----	-----	-----	-----	-----
r1	0.028917	0.046484	0.022848	0.0036146	0.000119
r2	0.024097	0.092967	0.085679	0.024097	0.0014875
r3	0.0080324	0.074374	0.12852	0.064259	0.0074374
r4	0.0013387	0.02975	0.096388	0.085679	0.018593
r5	0.00011156	0.0059499	0.036146	0.057119	0.023242
r6	3.7187e-06	0.00047599	0.0054218	0.015232	0.011621

```
xxx TABLE:mt_x_of_s xxxxxxxxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5
	--	--	--	--	--
r1	-7	-6	-7	-6	-6
r2	-5	-3	-5	-3	-4
r3	-2	-1	-1	0	-1
r4	2	2	3	4	2
r5	6	5	5	6	5
r6	8	9	7	9	9

[illegible]

	c1	c2	c3	c4	c5
	-----	-----	-----	-----	-----
r1	13.231	21.786	18.136	19.35	13.901
r2	9.946	16.887	9.6914	15.71	8.6906
r3	16.255	6.2166	13.799	5.2138	11.641
r4	12.628	2.7525	6.5321	0.27238	13.357
r5	5.8844	4.0352	6.05	0.14102	0.50318
r6	3.5617	-0.72091	5.1855	-6.3772	-4.4805

XX

CONTAINER NAME: covvar_input_map Scalars

XX

	i	idx	value
	-	---	-----
fl_x_mean	1	1	-0.11081
fl_x_sd	2	2	4.1239
fl_y_mean	3	3	8.8423
fl_y_sd	4	4	6.5845

XX

CONTAINER NAME: covvar output map ND Array (Matrix etc)

[illegible]

	i	idx	ndim	numel	rowN	colN	mean	std
	-	---	----	-----	-----	-----	-----	-----
mt_cov_component_weighted	1	1	2	30	6	5	-0.73612	1.0404

mt_x_devi_from_mean	2	2	2	30	6	5	0.94415	5.3051
mt_x_y_multiply	3	3	2	30	6	5	-31.321	36.564
mt_y_devi_from_mean	4	4	2	30	6	5	-0.51644	7.1913

xxx TABLE:mt_cov_component_weighted xxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5
	-----	-----	-----	-----	-----
r1	-0.87434	-3.5432	-1.4628	-0.22368	-0.0035451
r2	-0.13003	-2.1607	-0.35565	-0.47814	0.00087767
r3	-0.11248	0.17365	-0.56642	-0.025838	-0.018507
r4	0.010697	-0.38241	-0.69273	-3.0184	0.17717
r5	-0.0020165	-0.14618	-0.51584	-3.0371	-0.99056
r6	-0.00015927	-0.041473	-0.14098	-2.1121	-1.4106

xxx TABLE:mt_x_devi_from_mean xxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5
	-----	-----	-----	-----	-----
r1	-6.8892	-5.8892	-6.8892	-5.8892	-5.8892
r2	-4.8892	-2.8892	-4.8892	-2.8892	-3.8892
r3	-1.8892	-0.88919	-0.88919	0.11081	-0.88919
r4	2.1108	2.1108	3.1108	4.1108	2.1108
r5	6.1108	5.1108	5.1108	6.1108	5.1108
r6	8.1108	9.1108	7.1108	9.1108	9.1108

xxx TABLE:mt_x_y_multiply xxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5
	-----	-----	-----	-----	-----
r1	-30.237	-76.225	-64.023	-61.882	-29.792
r2	-5.396	-23.242	-4.151	-19.842	0.59004
r3	-14.003	2.3348	-4.4073	-0.40209	-2.4884
r4	7.9905	-12.854	-7.1868	-35.23	9.5287
r5	-18.075	-24.568	-14.271	-53.172	-42.62
r6	-42.83	-87.129	-26.003	-138.66	-121.38

xxx TABLE:mt_y_devi_from_mean xxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5
	-----	-----	-----	-----	-----
r1	4.389	12.943	9.2933	10.508	5.0587
r2	1.1037	8.0444	0.84902	6.8677	-0.15171
r3	7.4123	-2.6258	4.9566	-3.6286	2.7985
r4	3.7855	-6.0898	-2.3103	-8.57	4.5142
r5	-2.9579	-4.8071	-2.7924	-8.7013	-8.3392
r6	-5.2806	-9.5633	-3.6568	-15.22	-13.323

fl_cov
-22.0835

fl_cor
-0.8133

3.4.2 Test FF_DISC_RAND_VAR_MASS2COVCOR Four States-Points

Over some (a,z) states that is 2 by 2, c matrix, and y matrix, find correlation. Positively related.

% Set Parameters

```

mt_c_of_s = [1,2;3,1];
mt_y_of_s = [2,10;5,1.1];
rng(123);
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=1.4446,cor=0.65723

```

Same as before, but now inputs are single column:

```

% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s(:), mt_y_of_s(:), mt_f_of_s(:), bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=1.4446,cor=0.65723

```

3.4.3 Test FF_DISC_RAND_VAR_MASS2COVCOR Two Random Vectors

Generate two random vectors, with random or even mass, correlation should be zero:

```

% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...
    mt_c_of_s, mt_y_of_s, mt_f_of_s, bl_display_drvm2covcor);
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);

cov=-57.6533,cor=-0.062023

```

3.4.4 Test FF_DISC_RAND_VAR_MASS2COVCOR Provide Mean and SD

Same as above, but now provide means and sd for x and y directly. The results are the same as when mean and sd are calculated inside the function.

```

% Set Parameters
rng(4567);
mt_c_of_s = rand([20,1])*100;
mt_y_of_s = rand([20,1])*100;
mt_f_of_s = rand(size(mt_c_of_s));
mt_f_of_s = mt_f_of_s/sum(mt_f_of_s, 'all');
fl_c_mean = sum(mt_f_of_s.*mt_c_of_s);
fl_c_sd = sqrt(sum(mt_f_of_s.*(mt_c_of_s-fl_c_mean).^2));
fl_y_mean = sum(mt_f_of_s.*mt_y_of_s);
fl_y_sd = sqrt(sum(mt_f_of_s.*(mt_y_of_s-fl_y_mean).^2));
bl_display_drvm2covcor = false;
% Call Function
[fl_cov_xy, fl_cor_xy] = ff_disc_rand_var_mass2covcor(...

```

```
mt_c_of_s, mt_y_of_s, mt_f_of_s, ...  
fl_c_mean, fl_c_sd, ...  
fl_y_mean, fl_y_sd, bl_display_drvm2covcor);  
display(['cov=' num2str(fl_cov_xy) ',cor=', num2str(fl_cor_xy)]);  
  
cov=-57.6533,cor=-0.062023
```

Chapter 4

Graphs

4.1 FF_GRAPH_GRID Examples: X, Y and Color Line Plots

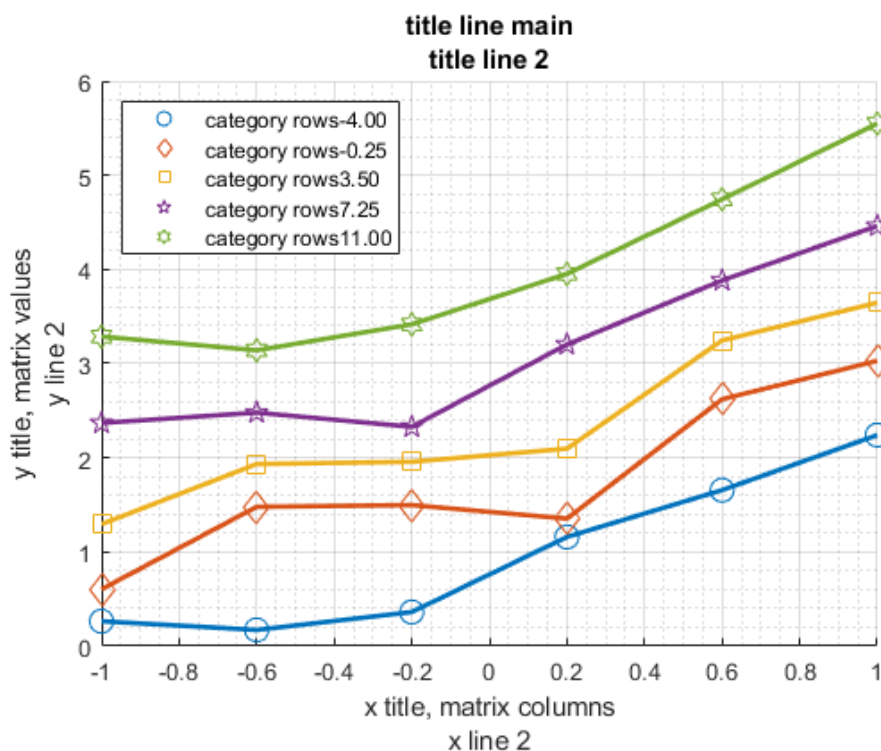
Go back to [fan's MEconTools Toolbox \(bookdown\)](#), [Matlab Code Examples Repository \(bookdown\)](#), or [Math for Econ with Matlab Repository \(bookdown\)](#).

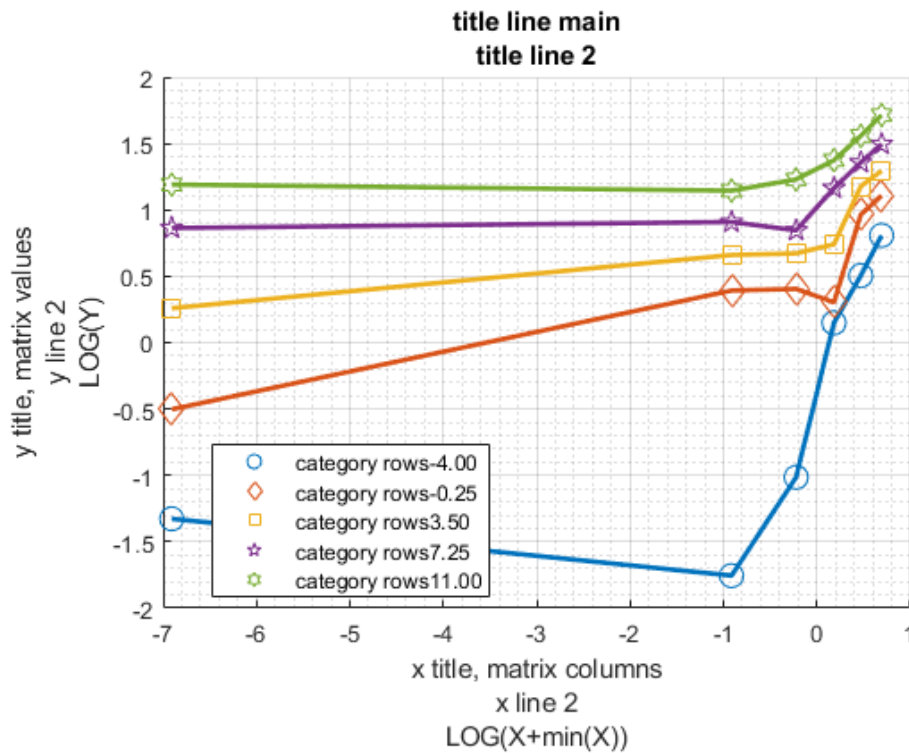
This is the example vignette for function: `ff_graph_grid` from the **MEconTools Package**. This function can graph out value and policy functions given one state vector (x-axis), conditional on other states (line groups). Can handle a few lines (scatter + lines), or many groups (jet spectrum).

4.1.1 Test FF_GRAPH_GRID Defaults

Call the function with defaults.

```
ff_graph_grid();
```

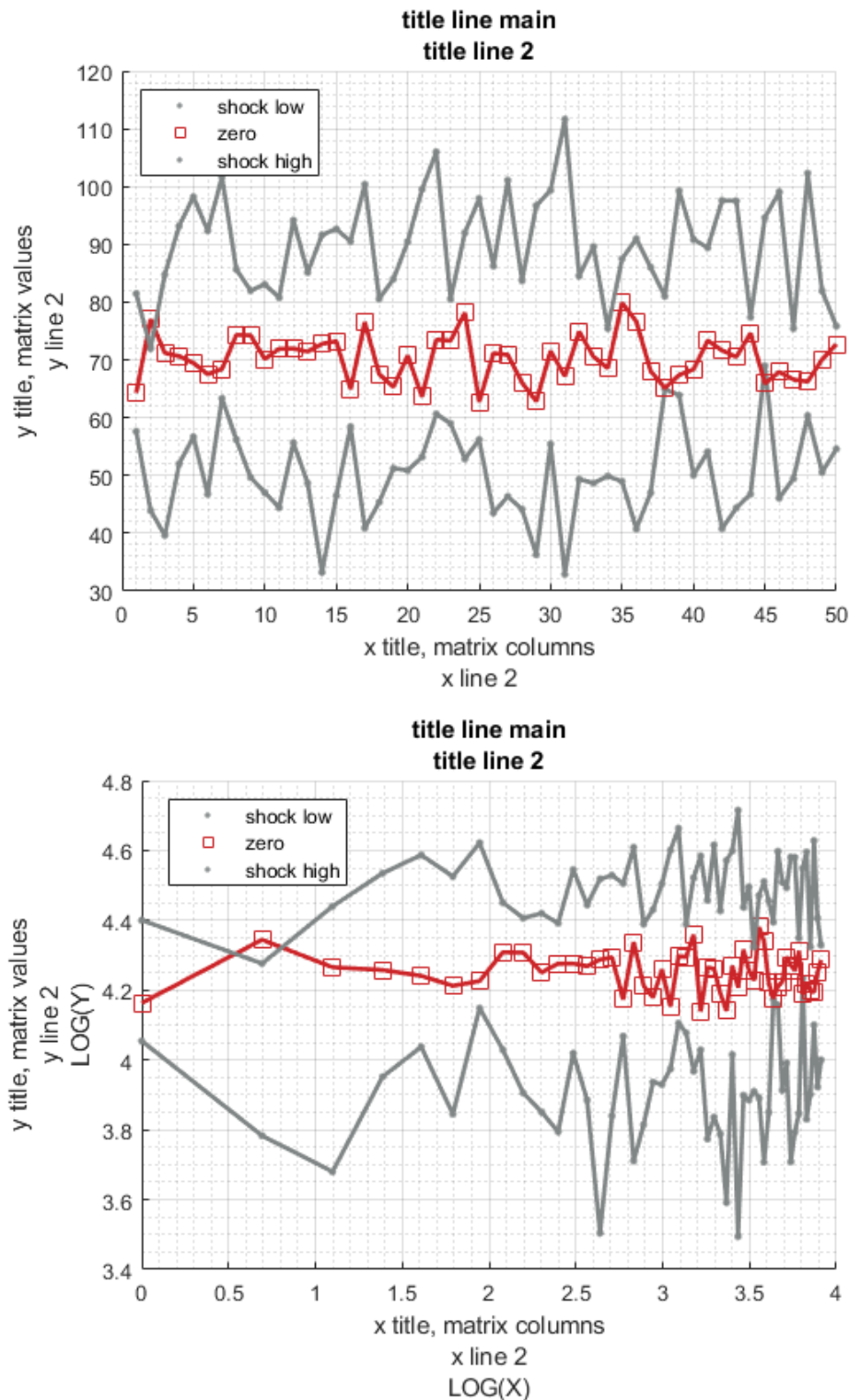




4.1.2 Test FF_GRAPH_GRID Random Matrix Pick Markers and Colors

Call the function with defaults.

```
rng(123);
mt_value = [normrnd(50,10,[1, 50]); ...
            normrnd(70,5,[1, 50]);...
            normrnd(90,10,[1, 50])];
ar_row_grid = ["shock low", "zero", "shock high"];
ar_col_grid = 1:50;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_scatter_shapes') = { '.', 's', '.' };
mp_support_graph('cl_colors') = {'gray', 'red', 'gray'};
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



4.1.3 Test FF_GRAPH_GRID Two Random Normal Lines and Labels

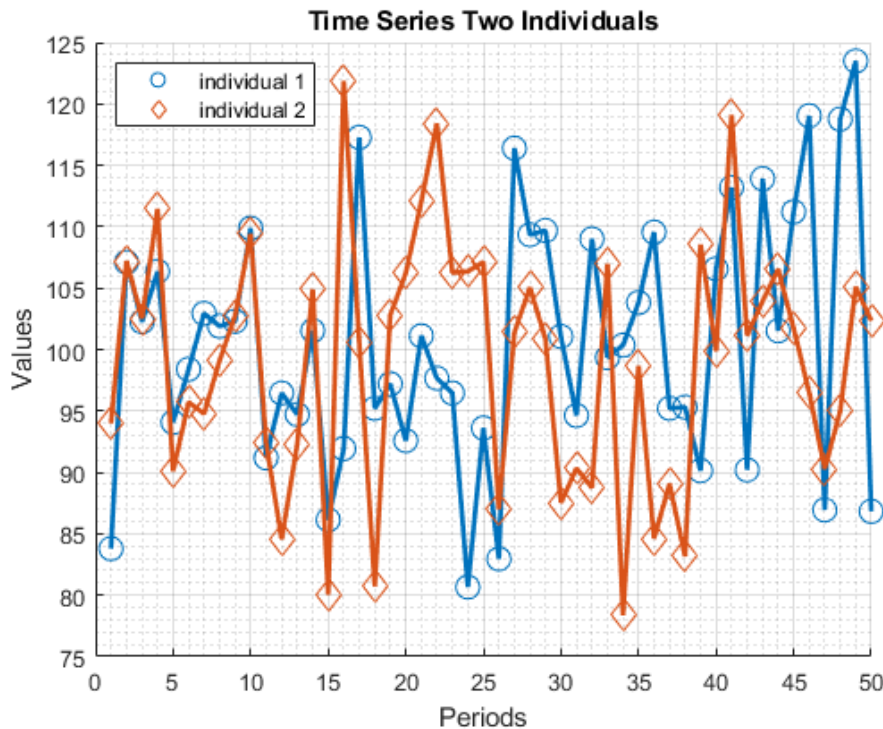
There are two autoregressive time series, plot out the time two time series.

```
% Generate the two time series
rng(456);
mt_value = normrnd(100,10,[2, 50]);
ar_row_grid = ["individual 1", "individual 2"];
ar_col_grid = 1:50;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
```

```

mp_support_graph('cl_st_graph_title') = {'Time Series Two Individuals'};
mp_support_graph('cl_st_ytitle') = {'Values'};
mp_support_graph('cl_st_xtitle') = {'Periods'};
mp_support_graph('bl_graph_logy') = false; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);

```



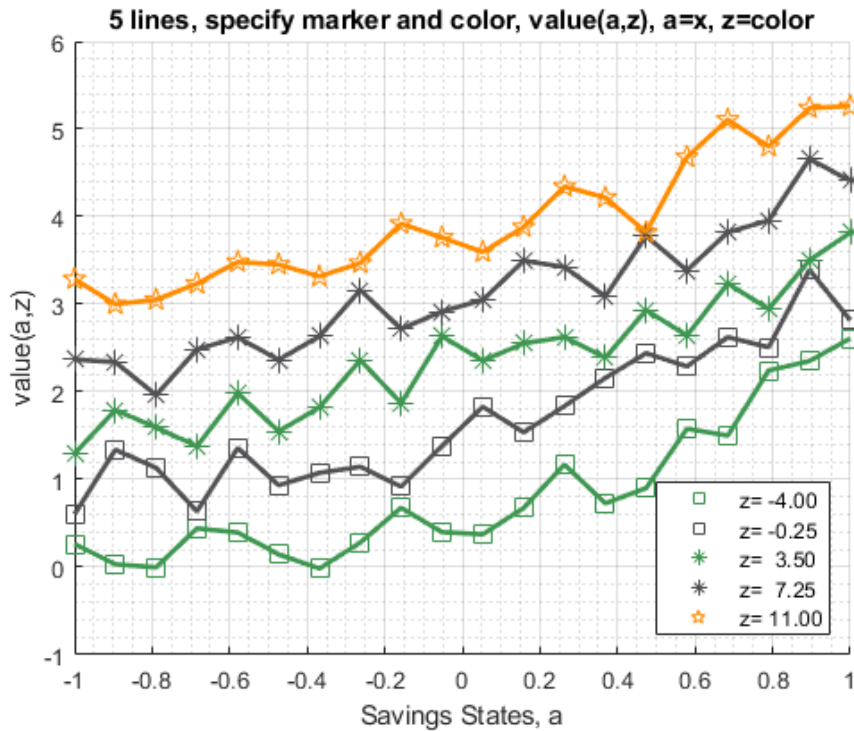
4.1.4 Test FF_GRAPH_GRID 6 Lines Pick Marker and Colors

Plot many lines, with auto legend.

```

% Generate some Data
rng(456);
ar_row_grid = linspace(-4, 11, 5);
ar_col_grid = linspace(-1, 1, 20);
rng(123);
mt_value = 0.2*ar_row_grid + exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'5 lines, specify marker and color, value(a,z), a=x, z=colo
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'southeast';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
mp_support_graph('cl_scatter_shapes') = {'s', 's', '*', '*', 'p'};
mp_support_graph('cl_colors') = {'green', 'black', 'green', 'black', 'orange'};
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);

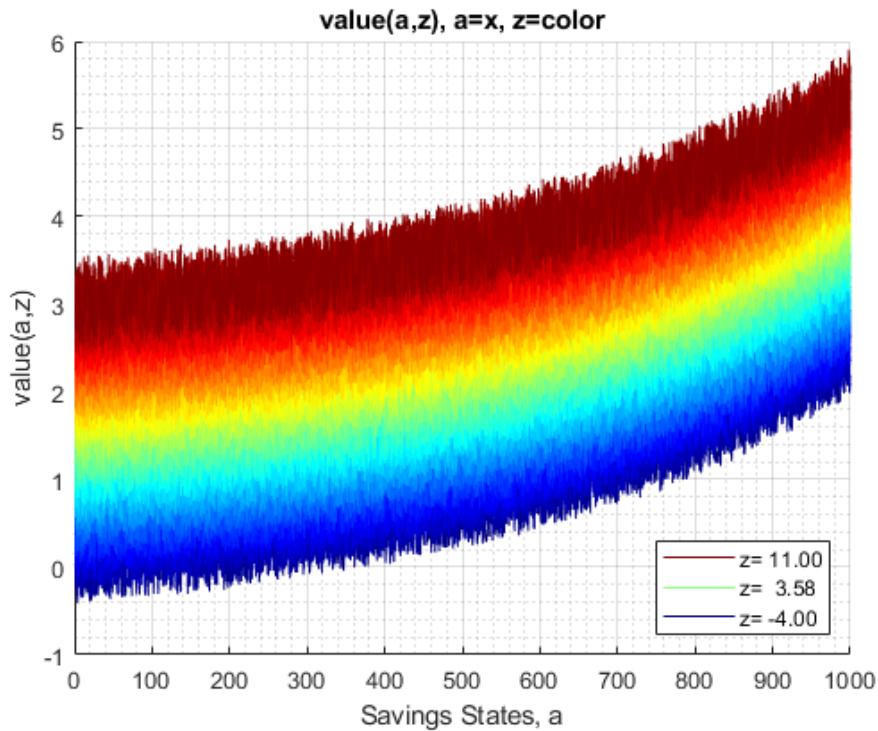
```

4.1.5 Test FF_GRAPH_GRID Many Lines

Plot many lines, with auto legend.

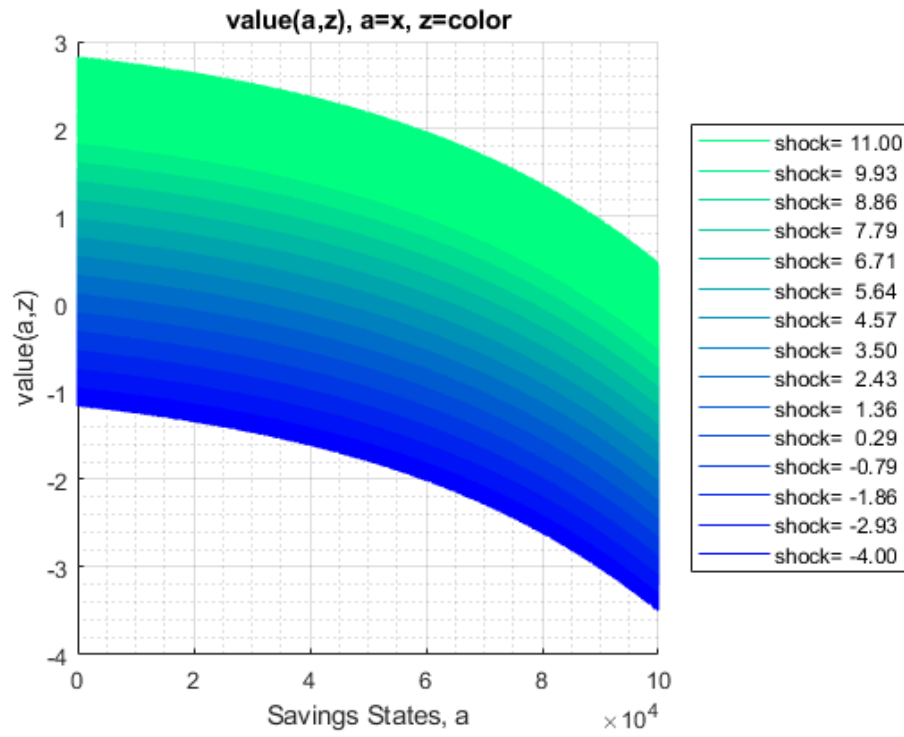
```
% Generate some Data
rng(456);
ar_row_grid = linspace(-4, 11, 100);
ar_col_grid = linspace(-1, 1, 1000);
rng(123);
mt_value = 0.2*ar_row_grid + exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('cl_legend_loc') = 'southeast';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'z=';
mp_support_graph('it_legend_select') = 3; % how many shock legends to show
mp_support_graph('st_rounding') = '6.2f'; % format shock legend
mp_support_graph('cl_colors') = 'jet'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



4.1.6 Test FF_GRAPH_GRID Many Lines Legend Exogenous

Plot many lines, exogenously set legend

```
% Generate the two time series
rng(456);
ar_row_grid = linspace(-4, 11, 15);
ar_col_grid = linspace(-1, 1, 100000);
rng(123);
mt_value = 0.2*ar_row_grid - exp(ar_col_grid) + rand([length(ar_row_grid), length(ar_col_grid)]);
% setting shock vector name exogenously here
ar_row_grid = string(num2str(ar_row_grid, "shock=%6.2f"));
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'value(a,z), a=x, z=color'};
mp_support_graph('cl_st_ytitle') = {'value(a,z)'};
mp_support_graph('cl_st_xtitle') = {'Savings States, a'};
mp_support_graph('st_legend_loc') = 'eastoutside';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('it_legend_select') = 15;
mp_support_graph('cl_colors') = 'winter'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



Chapter 5

Data Structures

5.1 FF_SAVEBORR_GRID Example for Generating Asset Grid

Go back to [fan's MEconTools Toolbox \(bookdown\)](#), [Matlab Code Examples Repository \(bookdown\)](#), or [Math for Econ with Matlab Repository \(bookdown\)](#).

This is the example vignette for function: `ff_saveborr_grid` from the **MEconTools Package**. This function generates variously spaced savings/borrowing states/choices grid.

5.1.1 Test FF_SAVEBORR_GRID Defaults

Call the function with defaults.

```
ff_saveborr_grid();
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean      std      coefv
          -      ---      ----      -----      ----      ----      ----      ----      ----      ----
ar_fl_saveborr    1      1      2      25      25      1      216.7      8.668      13.363      1.54

xxx TABLE:ar_fl_saveborr xxxxxxxxxxxxxxxxxxxxxxx
      c1
      -----
r1          0
r2      0.029558
r3      0.067855
r4      0.11748
r5      0.18177
r6      0.26507
r7      0.37301
r8      0.51286
r9      0.69407
r10     0.92885
r11     1.2331
r12     1.6272
r13     2.1379
r14     2.7996
```

```

r15      3.657
r16      4.7679
r17      6.2072
r18      8.0722
r19     10.489
r20     13.62
r21     17.676
r22     22.932
r23     29.743
r24     38.567
r25      50

```

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

```

	i	idx	value
	-	---	-----
grid_evenlog_threshold	1	2	1
grid_log10space_x1	2	3	0.3
grid_log10space_x2	3	4	3
grid_powerspace_power	4	5	3

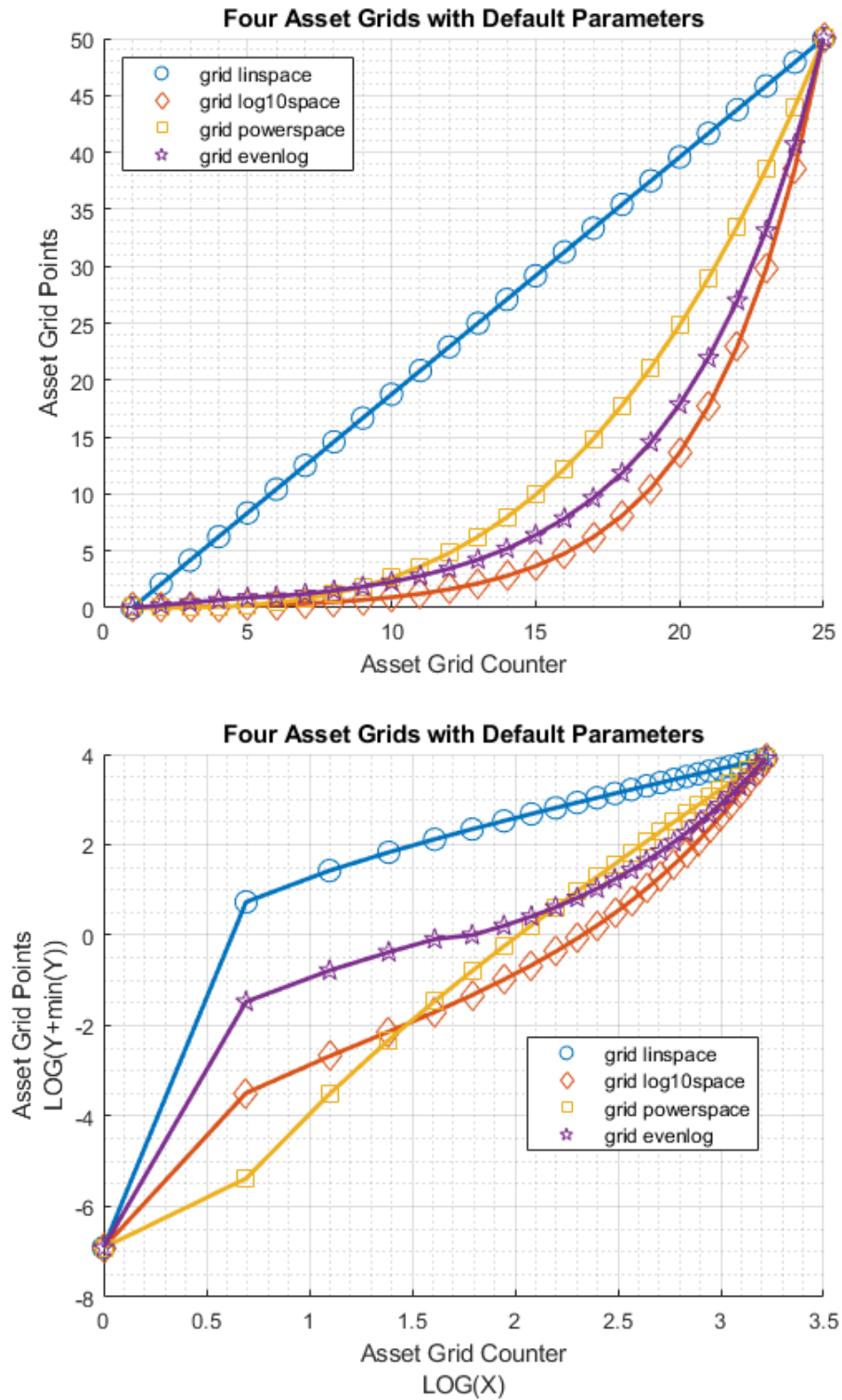
5.1.2 Test FF_SAVEBORR_GRID Default Linear Grid, Log Grid, Power Grid, Threshold Grid

Call the function with defaults.

```

% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
% Four types of grid points
st_grid_type = 'grid_linspace';
[ar_fl_saveborr_linspace] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_log10space';
[ar_fl_saveborr_log10space] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_powerspace';
[ar_fl_saveborr_powerspace] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
st_grid_type = 'grid_evenlog';
[ar_fl_saveborr_evenlog] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type);
% draw four types of lines jointly
mt_value = [ar_fl_saveborr_linspace'; ar_fl_saveborr_log10space'; ...
            ar_fl_saveborr_powerspace'; ar_fl_saveborr_evenlog'];
ar_row_grid = ["grid linspace", "grid log10space", "grid powerspace", "grid evenlog"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Four Asset Grids with Default Parameters'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);

```



5.1.3 Test FF_SAVEBORR_GRID Log Grid Changing Parameters

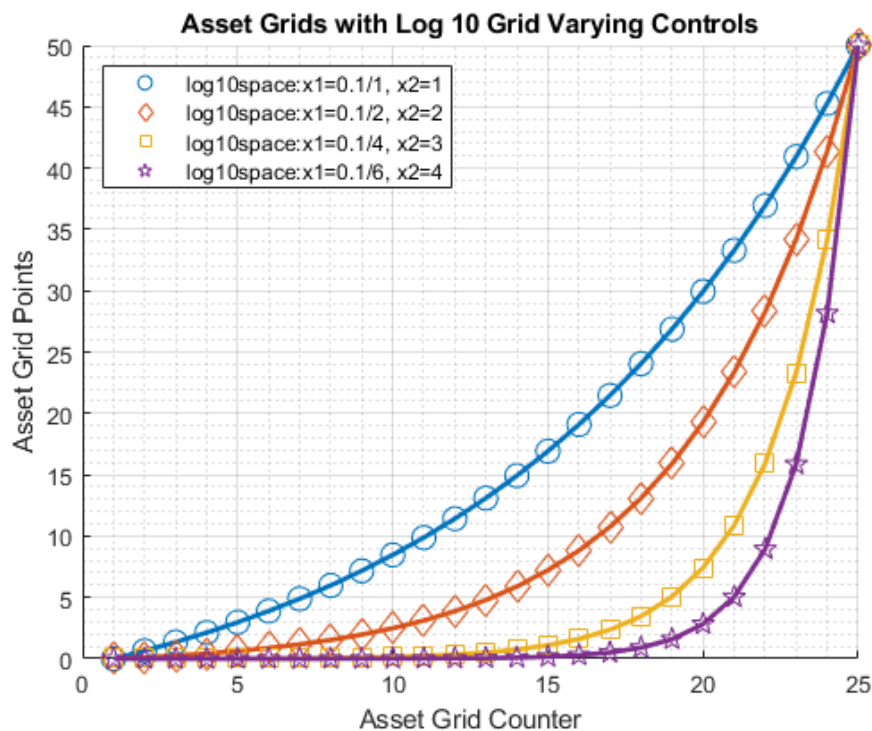
Log grid, same min and max, change log X1 and X2 points

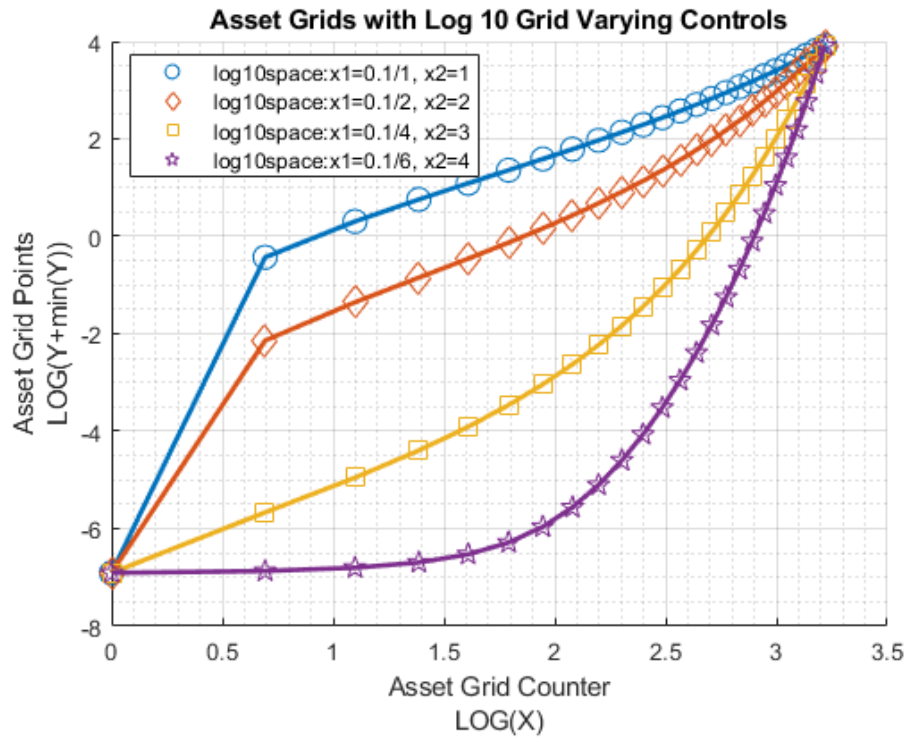
```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_log10space';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_log10space_x1') = 0.1;
```

```

mp_grid_control('grid_log10space_x2') = 1;
[ar_fl_log10space_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/2;
mp_grid_control('grid_log10space_x2') = 1*2;
[ar_fl_log10space_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/4;
mp_grid_control('grid_log10space_x2') = 1*4;
[ar_fl_log10space_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
mp_grid_control('grid_log10space_x1') = 0.1/6;
mp_grid_control('grid_log10space_x2') = 1*6;
[ar_fl_log10space_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_contr
% draw four types of lines jointly
mt_value = [ar_fl_log10space_a'; ar_fl_log10space_b'; ...
            ar_fl_log10space_c'; ar_fl_log10space_d'];
ar_row_grid = [...
    "log10space:x1=0.1/1, x2=1", ...
    "log10space:x1=0.1/2, x2=2", ...
    "log10space:x1=0.1/4, x2=3", ...
    "log10space:x1=0.1/6, x2=4"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Log 10 Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);

```

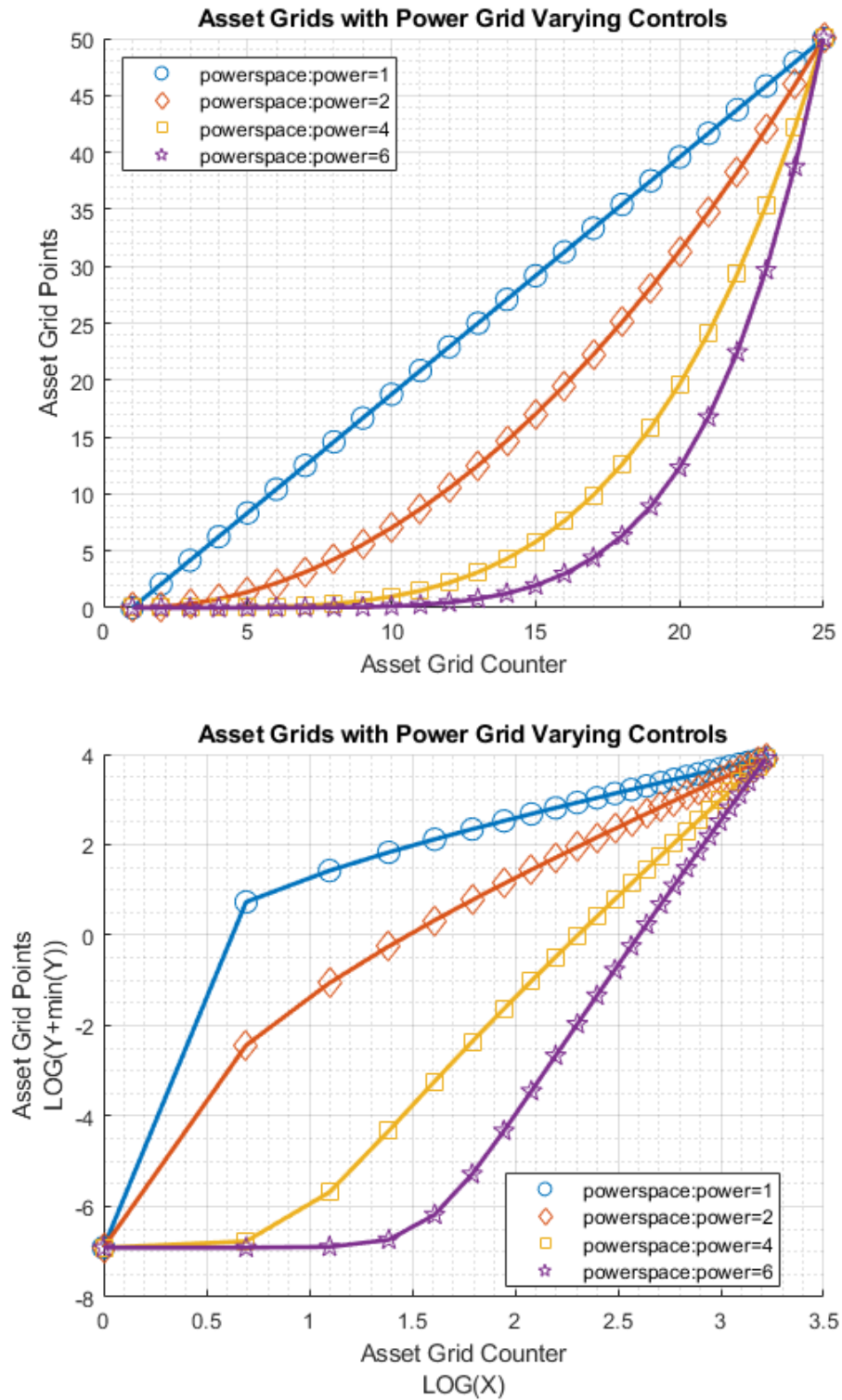




5.1.4 Test FF_SAVEBORR_GRID Power Grid Changing Parameters

Log grid, same min and max, change log X1 and X2 points

```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_powerspace';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char', 'ValueType','any');
mp_grid_control('grid_powerspace_power') = 1;
[ar_fl_powerspace_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control);
mp_grid_control('grid_powerspace_power') = 2;
[ar_fl_powerspace_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control);
mp_grid_control('grid_powerspace_power') = 4;
[ar_fl_powerspace_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control);
mp_grid_control('grid_powerspace_power') = 6;
[ar_fl_powerspace_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control);
% draw four types of lines jointly
mt_value = [ar_fl_powerspace_a'; ar_fl_powerspace_b'; ...
            ar_fl_powerspace_c'; ar_fl_powerspace_d'];
ar_row_grid = [...
    "powerspace:power=1", ...
    "powerspace:power=2", ...
    "powerspace:power=4", ...
    "powerspace:power=6"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Power Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



5.1.5 Test FF_SAVEBORR_GRID Threshold Grid Changing Parameters

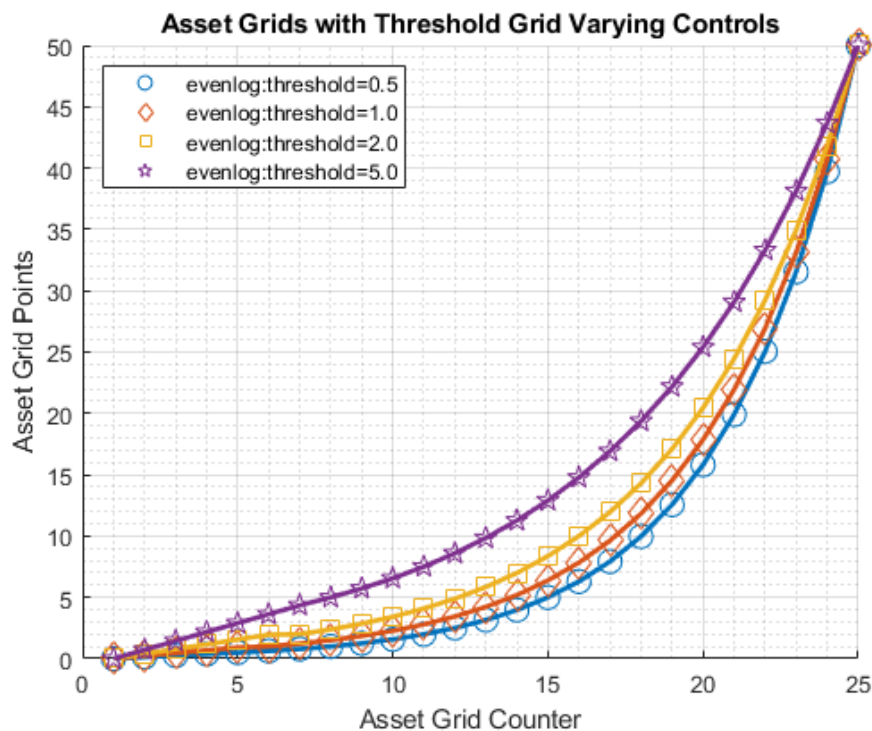
Threshold Grid, Changing Threshold Levels. Initial segments below threshold are linspace, then logspace.

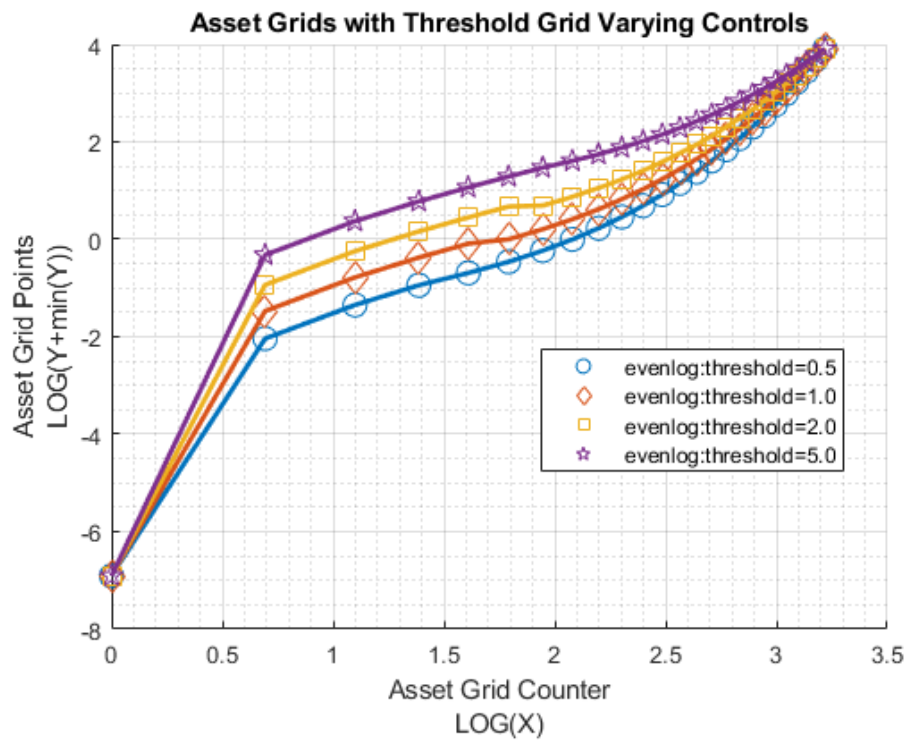
```
% Same min and max and grid points
[fl_a_min, fl_a_max, it_a_points] = deal(0,50,25);
st_grid_type = 'grid_evenlog';
% Four types of grid points
mp_grid_control = containers.Map('KeyType','char','ValueType','any');
mp_grid_control('grid_evenlog_threshold') = 0.50;
```

```

[ar_fl_evenlog_a] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 1.00;
[ar_fl_evenlog_b] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 2;
[ar_fl_evenlog_c] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
mp_grid_control('grid_evenlog_threshold') = 5;
[ar_fl_evenlog_d] = ff_saveborr_grid(fl_a_min, fl_a_max, it_a_points, st_grid_type, mp_grid_control)
% draw four types of lines jointly
mt_value = [ar_fl_evenlog_a'; ar_fl_evenlog_b'; ...
    ar_fl_evenlog_c'; ar_fl_evenlog_d'];
ar_row_grid = [...
    "evenlog:threshold=0.5", ...
    "evenlog:threshold=1.0", ...
    "evenlog:threshold=2.0", ...
    "evenlog:threshold=5.0"];
ar_col_grid = 1:it_a_points;
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Asset Grids with Threshold Grid Varying Controls'};
mp_support_graph('cl_st_ytitle') = {'Asset Grid Points'};
mp_support_graph('cl_st_xtitle') = {'Asset Grid Counter'};
mp_support_graph('bl_graph_logy') = true; % do not log
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);

```





Chapter 6

Common Functions

6.1 FFY_TAUCHEN AR1 Shock Discretization Example

Go back to fan's [MEconTools](#) Toolbox ([bookdown](#)), [Matlab Code Examples](#) Repository ([bookdown](#)), or [Math for Econ with Matlab](#) Repository ([bookdown](#)).

This is the example vignette for function: [ffiy_tauschen](#) from the [MEconTools Package](#). : See also the [ffiy_rouwenhorst](#) function from the [MEconTools Package](#). This function discretize a mean zero AR1 process, uses Tauchen (1986). See [AR 1 Example](#) for some details on how the AR1 process works. And See [Kopecky and Suen \(2010\)](#).

6.1.1 Test FFY_TAUCHEN Defaults

Call the function with defaults. Default sd bounds arer plus and minus 4. This is used in the following examples, unless otherwise specified as the 5th parameter.

```
ffiy_tauschen();
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
              i      idx      ndim      numel      rowN      colN      sum      mean      std      coef
              -      ---      ----      -
ar_disc_ar1      1      1      2      5      5      1      0      0      0.79057
mt_disc_ar1_trans 2      6      2      25      5      5      5      0.2      0.27623      1.3

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
c1
----
r1      -1
r2      -0.5
r3      0
r4      0.5
r5      1

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxxxxxx
c1      c2      c3      c4      c5
-----
r1      0.22663      0.73331      0.040048      1.0689e-05      7.3923e-12
r2      0.012224      0.58648      0.39831      0.0029797      7.605e-08
```

r3	8.8417e-05	0.10556	0.7887	0.10556	8.8417e-05
r4	7.605e-08	0.0029797	0.39831	0.58648	0.012224
r5	7.3923e-12	1.0689e-05	0.040048	0.73331	0.22663

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      value
          -      ---      -----
fl_ar1_persistence  1      2      0.6
fl_ar1_step         2      3      0.5
fl_shk_std          3      4      0.2
it_std_bound        4      5      4

```

6.1.2 Test FFY_TAUCHEN Specify Parameters

With a grid of 10 points, the sd bounds on Tauchen and Rouwenhorst are identical. With the not extremely persistent shock process here, the Tauchen and Rouwenhorst Results are very similar.

```

[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose, it_std_bound] = ...
    deal(0.60, 0.10, 10, true, 3);
ffy_tauschen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose, it_std_bound);

```

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean
          -      ---      ----      -----      ----      ----      -----      -----
ar_disc_ar1         1      1      2         10         10         1      -7.2164e-16      -7.2164e-17
mt_disc_ar1_trans   2      6      2        100         10         10         10         0.1

```

```

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
c1

```

```

-----
r1      -0.375
r2      -0.29167
r3      -0.20833
r4      -0.125
r5      -0.041667
r6      0.041667
r7      0.125
r8      0.20833
r9      0.29167
r10     0.375

```

```

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxxxxxx
c1      c2      c3      c4      c5      c6      c7

```

```

-----
r1      0.13933      0.26196      0.31887      0.20154      0.066066      0.011201      0.0009785
r2      0.056673      0.16995      0.30658      0.28713      0.1396      0.035167      0.004575
r3      0.01861      0.087039      0.23281      0.32308      0.23281      0.087039      0.01684
r4      0.0048925      0.035167      0.1396      0.28713      0.30658      0.16995      0.04884
r5      0.0010235      0.011201      0.066066      0.20154      0.31887      0.26196      0.1116

```

r6	0.00016962	0.0028101	0.02466	0.11169	0.26196	0.31887	0.2015
r7	2.2197e-05	0.00055483	0.0072547	0.048841	0.16995	0.30658	0.2871
r8	2.2881e-06	8.6129e-05	0.0016806	0.016841	0.087039	0.23281	0.3230
r9	1.8543e-07	1.0503e-05	0.00030628	0.0045756	0.035167	0.1396	0.2871
r10	1.1798e-08	1.0053e-06	4.3874e-05	0.00097859	0.011201	0.066066	0.2015

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

```

	i	idx	value
	-	---	-----
fl_ar1_persistence	1	2	0.6
fl_ar1_step	2	3	0.083333
fl_shk_std	3	4	0.1
it_std_bound	4	5	3

6.1.3 Test FFY_TAUCHEN High Persistence, Low SD

```

[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.01, 7, true);
ffynettauchen(ffl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

```

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

```

	i	idx	ndim	numel	rowN	colN	sum	mean
	-	---	----	-----	----	----	-----	-----
ar_disc_ar1	1	1	2	7	7	1	-5.5511e-17	-7.9302e-18
mt_disc_ar1_trans	2	6	2	49	7	7	7	0.14286

```

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
c1

```

```

-----
r1      -0.28355
r2      -0.18903
r3      -0.094517
r4      -2.7756e-17
r5       0.094517
r6       0.18903
r7       0.28355

```

```

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxx
c1      c2      c3

```

	c1	c2	c3	c4	c5	c6	
	-----	-----	-----	-----	-----	-----	---
r1	1	4.4497e-06	0	0	0	0	
r2	4.4412e-07	1	2.8552e-06	0	0	0	
r3	1.632e-46	7.1638e-07	1	1.8164e-06	0	0	
r4	9.6185e-124	6.3021e-46	1.1456e-06	1	1.1456e-06	0	
r5	6.3206e-239	8.9712e-123	2.4121e-45	1.8164e-06	1	7.1638e-07	
r6	0	1.426e-237	8.2932e-122	9.1503e-45	2.8552e-06	1	4.
r7	0	0	3.1885e-236	7.5984e-121	3.4405e-44	4.4497e-06	

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      value
          -      ---      -
fl_ar1_persistence  1      2      0.99
fl_ar1_step         2      3      0.094517
fl_shk_std          3      4      0.01
it_std_bound        4      5      4

```

6.1.4 Test FFY_TAUCHEN Low Persistence, Low SD

```

[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffynet(fly_ar1_persistence, fly_shk_std, it_disc_points, bl_verbose);

```

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean      std
          -      ---      ----      -
ar_disc_ar1      1      1      2      7      7      1      0      0      0.028805
mt_disc_ar1_trans 2      6      2      49      7      7      7      0.14286      0.17448

```

```

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
      c1

```

```

-----
r1    -0.040002
r2    -0.026668
r3    -0.013334
r4      0
r5     0.013334
r6     0.026668
r7     0.040002

```

```

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxx

```

	c1	c2	c3	c4	c5	c6	c7
	-----	-----	-----	-----	-----	-----	-----
r1	0.00049475	0.024497	0.24044	0.4947	0.21921	0.020299	0.00037109
r2	0.00047179	0.023751	0.23685	0.49488	0.2227	0.020954	0.00038948
r3	0.00044982	0.023024	0.23329	0.495	0.22621	0.021626	0.0004087
r4	0.0004288	0.022316	0.22974	0.49504	0.22974	0.022316	0.0004288
r5	0.0004087	0.021626	0.22621	0.495	0.23329	0.023024	0.00044982
r6	0.00038948	0.020954	0.2227	0.49488	0.23685	0.023751	0.00047179
r7	0.00037109	0.020299	0.21921	0.4947	0.24044	0.024497	0.00049475

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      value
          -      ---      -

```



```

fl_ar1_persistence  1    2    0.01
fl_ar1_step         2    3    0.013334
fl_shk_std          3    4    0.01
it_std_bound        4    5    4

```

6.1.5 Test FFY_TAUCHEN High Persistence, High SD

```

[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
  deal(0.99, 0.99, 7, true);
ffynettauchen(ffy_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);

```

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

```

	i	idx	ndim	numel	rowN	colN	sum	mean
	-	---	----	-----	----	----	-----	-----
ar_disc_ar1	1	1	2	7	7	1	-3.5527e-15	-5.0753e-16
mt_disc_ar1_trans	2	6	2	49	7	7	7	0.14286

```

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
c1

```

```

-----
r1  -28.072
r2  -18.714
r3  -9.3572
r4   0
r5   9.3572
r6   18.714
r7   28.072

```

```

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxxxxxx

```

	c1	c2	c3	c4	c5	c6	
	-----	-----	-----	-----	-----	-----	---
r1	1	4.4497e-06	0	0	0	0	
r2	4.4412e-07	1	2.8552e-06	0	0	0	
r3	1.632e-46	7.1638e-07	1	1.8164e-06	0	0	
r4	9.6185e-124	6.3021e-46	1.1456e-06	1	1.1456e-06	0	
r5	6.3206e-239	8.9712e-123	2.4121e-45	1.8164e-06	1	7.1638e-07	
r6	0	1.426e-237	8.2932e-122	9.1503e-45	2.8552e-06	1	4.
r7	0	0	3.1885e-236	7.5984e-121	3.4405e-44	4.4497e-06	

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

```

	i	idx	value
	-	---	-----
fl_ar1_persistence	1	2	0.99
fl_ar1_step	2	3	9.3572
fl_shk_std	3	4	0.99
it_std_bound	4	5	4

6.1.6 Test FFY_TAUCHEN Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffynet_tauchen(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

          i      idx      ndim      numel      rowN      colN      sum      mean      std
          -      ---      ----      -
ar_disc_ar1      1      1      2      7      7      1      0      0      0.028805
mt_disc_ar1_trans 2      6      2      49      7      7      7      0.14286  0.17448

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
      c1
      -----
r1    -0.040002
r2    -0.026668
r3    -0.013334
r4           0
r5     0.013334
r6     0.026668
r7     0.040002

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4      c5      c6      c7
      -----
r1    0.00049475  0.024497  0.24044  0.4947  0.21921  0.020299  0.00037109
r2    0.00047179  0.023751  0.23685  0.49488  0.2227  0.020954  0.00038948
r3    0.00044982  0.023024  0.23329  0.495  0.22621  0.021626  0.0004087
r4    0.0004288  0.022316  0.22974  0.49504  0.22974  0.022316  0.0004288
r5    0.0004087  0.021626  0.22621  0.495  0.23329  0.023024  0.00044982
r6    0.00038948  0.020954  0.2227  0.49488  0.23685  0.023751  0.00047179
r7    0.00037109  0.020299  0.21921  0.4947  0.24044  0.024497  0.00049475

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

          i      idx      value
          -      ---      -
fl_ar1_persistence 1      2      0.01
fl_ar1_step        2      3      0.013334
fl_shk_std         3      4      0.01
it_std_bound       4      5      4
```

6.2 FFY_ROUWENHORST AR1 Shock Discretization Example

Go back to [fan's MEconTools Toolbox \(bookdown\)](#), [Matlab Code Examples Repository \(bookdown\)](#), or [Math for Econ with Matlab Repository \(bookdown\)](#).

This is the example vignette for function: `ff_y_rouwenhorst` from the **MEconTools Package**. See also `ff_y_tauschen` function from the **MEconTools Package**. This function discretize a mean zero AR1 process, uses Rouwenhorst (1995). See [AR 1 Example](#) for some details on how the AR1 process works. And See [Kopecky and Suen \(2010\)](#).

6.2.1 Test FFY_ROUWENHORST Defaults

Call the function with defaults.

```
ff_y_rouwenhorst();
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
              i      idx      ndim      numel      rowN      colN      sum      mean      std      coef
              -      ---      ----      -
ar_disc_ar1      1      1      2      5      5      1      0      0      0.39528
mt_disc_ar1_trans 2      11     2      25     5      5      5      0.2    0.18246    0.91

xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
      c1
      ----
r1      -0.5
r2      -0.25
r3       0
r4      0.25
r5      0.5

xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4      c5
      -----
r1      0.4096    0.4096    0.1536    0.0256    0.0016
r2      0.1024    0.4864    0.3264    0.0784    0.0064
r3      0.0256    0.2176    0.5136    0.2176    0.0256
r4      0.0064    0.0784    0.3264    0.4864    0.1024
r5      0.0016    0.0256    0.1536    0.4096    0.4096

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
              i      idx      value
              -      ---      ----
fl_ar1_beg      1      2      -0.5
fl_ar1_end      2      3      0.5
fl_ar1_persistence 3      4      0.6
fl_ar1_step      4      5      0.25
fl_p0           5      6      0.8
fl_q0           6      7      0.8
fl_shk_std      7      8      0.2
fl_sig_ar1      8      9      0.25
it_std_bound    9     10      0
```

6.2.2 Test FFY_ROUWENHORST Specify Parameters

With a grid of 10 points, the Rouwenhorst bounds on standard deviations are equal to Tauchen bounds of 3. With the not extremely persistent shock process here, the Tauchen and Rouwenhorst Results are very similar.

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.60, 0.10, 10, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean
          -      ---      ----      -
ar_disc_ar1      1       1       2       10       10       1      5.5511e-17  5.5511e-18  0
mt_disc_ar1_trans 2      11       2      100       10      10       10       0.1      0.
```

```
xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
      c1
```

```
-----
r1      -0.375
r2      -0.29167
r3      -0.20833
r4      -0.125
r5      -0.041667
r6       0.041667
r7       0.125
r8       0.20833
r9       0.29167
r10      0.375
```

```
xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7
	-----	-----	-----	-----	-----	-----	-----
r1	0.13422	0.30199	0.30199	0.17616	0.06606	0.016515	0.0027525
r2	0.033554	0.20133	0.32716	0.26424	0.12662	0.038535	0.0075694
r3	0.0083886	0.081789	0.26267	0.32755	0.21401	0.082747	0.019741
r4	0.0020972	0.028312	0.14038	0.30946	0.30369	0.15877	0.047989
r5	0.00052429	0.009044	0.061145	0.20246	0.33477	0.25969	0.10585
r6	0.00013107	0.0027525	0.023642	0.10585	0.25969	0.33477	0.20246
r7	3.2768e-05	0.00081101	0.0084603	0.047989	0.15877	0.30369	0.30946
r8	8.192e-06	0.00023347	0.0028677	0.019741	0.082747	0.21401	0.32755
r9	2.048e-06	6.6048e-05	0.00093389	0.0075694	0.038535	0.12662	0.26424
r10	5.12e-07	1.8432e-05	0.00029491	0.0027525	0.016515	0.06606	0.17616

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	value
	-	---	-----
fl_ar1_beg	1	2	-0.375
fl_ar1_end	2	3	0.375

fl_ar1_persistence	3	4	0.6
fl_ar1_step	4	5	0.083333
fl_p0	5	6	0.8
fl_q0	6	7	0.8
fl_shk_std	7	8	0.1
fl_sig_ar1	8	9	0.125
it_std_bound	9	10	0

6.2.3 Test FFY_ROUWENHORST High Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.01, 7, true);
ffy_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean      std      c
          -      ---      ----      -----      ----      ----      ---      -
ar_disc_ar1      1      1      2      7      7      1      0      0      0.12503
mt_disc_ar1_trans 2      11     2     49      7      7      7     0.14286    0.34148
```

```
xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
      c1
```

```
-----
r1    -0.17364
r2    -0.11576
r3    -0.05788
r4      0
r5     0.05788
r6     0.11576
r7     0.17364
```

```
xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4      c5      c6      c7
      -----
r1      0.97037    0.029257  0.00036756  2.4627e-06  9.2815e-09  1.8656e-11  1.5625
r2      0.0048762    0.9705    0.024382  0.00024504  1.2314e-06  3.0938e-09  3.1094
r3      2.4504e-05    0.009753    0.97057  0.019506  0.00014703  4.9254e-07  6.1877
r4      1.2313e-07    7.3513e-05    0.01463    0.97059    0.01463    7.3513e-05  1.2313
r5      6.1877e-10    4.9254e-07    0.00014703  0.019506    0.97057    0.009753  2.4504
r6      3.1094e-12    3.0938e-09    1.2314e-06  0.00024504  0.024382    0.9705    0.004
r7      1.5625e-14    1.8656e-11    9.2815e-09  2.4627e-06  0.00036756  0.029257    0.9
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      value
          -      ---      -
fl_ar1_beg      1      2     -0.17364
fl_ar1_end      2      3      0.17364
fl_ar1_persistence 3      4      0.99
```

fl_ar1_step	4	5	0.05788
fl_p0	5	6	0.995
fl_q0	6	7	0.995
fl_shk_std	7	8	0.01
fl_sig_ar1	8	9	0.070888
it_std_bound	9	10	0

6.2.4 Test FFY_ROUWENHORST Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffyr_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean      std
          -      ---      ----      -
ar_disc_ar1      1       1       2         7         7         1         0         0      0.017639
mt_disc_ar1_trans 2      11       2        49         7         7         7      0.14286    0.10985
```

```
xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
      c1
```

```
-----
r1      -0.024496
r2      -0.016331
r3      -0.0081654
r4         0
r5      0.0081654
r6      0.016331
r7      0.024496
```

```
xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7
	-----	-----	-----	-----	-----	-----	-----
r1	0.016586	0.097547	0.23904	0.31241	0.22966	0.090047	0.014711
r2	0.016258	0.096266	0.23749	0.31247	0.23124	0.091266	0.015008
r3	0.015936	0.094997	0.23594	0.31251	0.23281	0.092497	0.015311
r4	0.01562	0.093741	0.23438	0.31252	0.23438	0.093741	0.01562
r5	0.015311	0.092497	0.23281	0.31251	0.23594	0.094997	0.015936
r6	0.015008	0.091266	0.23124	0.31247	0.23749	0.096266	0.016258
r7	0.014711	0.090047	0.22966	0.31241	0.23904	0.097547	0.016586

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	value
	-	---	-----
fl_ar1_beg	1	2	-0.024496
fl_ar1_end	2	3	0.024496
fl_ar1_persistence	3	4	0.01
fl_ar1_step	4	5	0.0081654

fl_p0	5	6	0.505
fl_q0	6	7	0.505
fl_shk_std	7	8	0.01
fl_sig_ar1	8	9	0.010001
it_std_bound	9	10	0

6.2.5 Test FFY_ROUWENHORST High Persistence, High SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.99, 0.99, 7, true);
ffyr_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean
          -      ---      ----      -----      ----      ----      -
ar_disc_ar1      1      1      2      7      7      1      3.5527e-15      5.0753e-16
mt_disc_ar1_trans 2      11      2      49      7      7      7      0.14286
xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
    c1
    -----
r1      -17.19
r2      -11.46
r3      -5.7301
r4       0
r5      5.7301
r6      11.46
r7      17.19
xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxx
    c1      c2      c3      c4      c5      c6      c7
    -----
r1      0.97037      0.029257      0.00036756      2.4627e-06      9.2815e-09      1.8656e-11      1.5625
r2      0.0048762      0.9705      0.024382      0.00024504      1.2314e-06      3.0938e-09      3.1094
r3      2.4504e-05      0.009753      0.97057      0.019506      0.00014703      4.9254e-07      6.1877
r4      1.2313e-07      7.3513e-05      0.01463      0.97059      0.01463      7.3513e-05      1.2313
r5      6.1877e-10      4.9254e-07      0.00014703      0.019506      0.97057      0.009753      2.4504
r6      3.1094e-12      3.0938e-09      1.2314e-06      0.00024504      0.024382      0.9705      0.004
r7      1.5625e-14      1.8656e-11      9.2815e-09      2.4627e-06      0.00036756      0.029257      0.9
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      value
          -      ---      -----
fl_ar1_beg      1      2      -17.19
fl_ar1_end      2      3      17.19
fl_ar1_persistence 3      4      0.99
fl_ar1_step      4      5      5.7301
fl_p0      5      6      0.995
```

fl_q0	6	7	0.995
fl_shk_std	7	8	0.99
fl_sig_ar1	8	9	7.0179
it_std_bound	9	10	0

6.2.6 Test FFY_ROUWENHORST Low Persistence, Low SD

```
[fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose] = ...
    deal(0.01, 0.01, 7, true);
ffyr_rouwenhorst(fl_ar1_persistence, fl_shk_std, it_disc_points, bl_verbose);
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean      std
          -      ---      ----      -
ar_disc_ar1      1      1      2      7      7      1      0      0      0.017639
mt_disc_ar1_trans 2     11      2     49      7      7      7     0.14286    0.10985
```

```
xxx TABLE:ar_disc_ar1 xxxxxxxxxxxxxxxxxxxxxxxx
c1
```

```
-----
r1      -0.024496
r2      -0.016331
r3      -0.0081654
r4       0
r5      0.0081654
r6      0.016331
r7      0.024496
```

```
xxx TABLE:mt_disc_ar1_trans xxxxxxxxxxxxxxxxxxxxxxxx
```

	c1	c2	c3	c4	c5	c6	c7
	-----	-----	-----	-----	-----	-----	-----
r1	0.016586	0.097547	0.23904	0.31241	0.22966	0.090047	0.014711
r2	0.016258	0.096266	0.23749	0.31247	0.23124	0.091266	0.015008
r3	0.015936	0.094997	0.23594	0.31251	0.23281	0.092497	0.015311
r4	0.01562	0.093741	0.23438	0.31252	0.23438	0.093741	0.01562
r5	0.015311	0.092497	0.23281	0.31251	0.23594	0.094997	0.015936
r6	0.015008	0.091266	0.23124	0.31247	0.23749	0.096266	0.016258
r7	0.014711	0.090047	0.22966	0.31241	0.23904	0.097547	0.016586

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	value
	-	---	-----
fl_ar1_beg	1	2	-0.024496
fl_ar1_end	2	3	0.024496
fl_ar1_persistence	3	4	0.01
fl_ar1_step	4	5	0.0081654
fl_p0	5	6	0.505
fl_q0	6	7	0.505

fl_shk_std	7	8	0.01
fl_sig_ar1	8	9	0.010001
it_std_bound	9	10	0

Chapter 7

Support Tools

7.1 FF_CONTAINER_MAP_DISPLAY Examples

Go back to [fan's MEconTools Toolbox \(bookdown\)](#), [Matlab Code Examples Repository \(bookdown\)](#), or [Math for Econ with Matlab Repository \(bookdown\)](#).

This is the example vignette for function: [ff_container_map_display](#) from the [MEconTools Package](#). This function summarizes statistics of matrixes stored in a container map, as well as scalar, string, function and other values stored in container maps.

7.1.1 Test FF_CONTAINER_MAP_DISPLAY Defaults

Call the function with defaults.

```
ff_container_map_display();
```

```
-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
          i      idx      ndim      numel      rowN      colN      sum      mean      std
          --      ---      ----      -----      ----      ----      -
mat_1          1       7       2          12         3         4      6.5142     0.54285    0.2232
mat_2          2       8       2        2650        50        53     1313.3     0.49559    0.29232
mat_2_boolean  3       9       2        2650        50        53       1361     0.51358    0.49991
mat_3          4      10       2         4         2         2       1.8111     0.45277    0.45111
tensor_1       5      15       3         16         2         8       7.3043     0.45652    0.27787
tensor_2       6      16       3         75         3        25      40.195     0.53593    0.29044
tensor_3       7      17       2         4         1         4       1.6926     0.42315    0.37389
tesseract_1    8      18       4         72         3        24      34.321     0.47669    0.26374
tesseract_2    9      19       4         20         2        10       8.4191     0.42096    0.28981
tesseract_bl_3 10     20       4         10         1        10         3         0.3       0.48305

xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxxxx
          c1          c2          c3          c4
          -----
r1      0.69647      0.55131      0.98076      0.39212
r2      0.28614      0.71947      0.68483      0.34318
r3      0.22685      0.42311      0.48093      0.72905

xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxxxx
          c1          c2          c3          c4          c50          c51          c52          c53
```

```

-----
r1      0.43857      0.6249      0.17108      0.56564      0.072152      0.67855      0.61667      0.540
r2      0.059678     0.67469     0.82911     0.084904     0.63289     0.27236     0.32528     0.249
r3      0.39804      0.84234     0.33867     0.58267     0.046367     0.44513     0.075047     0.78
r4       0.738      0.083195     0.55237     0.81484     0.50561     0.11117     0.59532     0.356
r5      0.18249      0.76368     0.57855     0.33707     0.10653     0.028681     0.7435     0.918
r46     0.6813      0.55326     0.88786     0.69983     0.83758     0.16382     0.74191     0.0656
r47     0.87546     0.85445     0.69631     0.66117     0.97069     0.79092     0.42466     0.787
r48     0.51042     0.38484     0.44033     0.049097     0.017768     0.33302     0.24401     0.979
r49     0.66931     0.31679     0.43821     0.7923      0.12979     0.75311     0.79466     0.0790
r50     0.58594     0.35426     0.7651      0.51872     0.86415     0.58281     0.84795     0.45

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4      c50      c51      c52      c53
-----
r1     true     false     false     true     true     false     true     true
r2     true     false     true      true     false     false     true     true
r3     false     true     false     true     false     true     false     true
r4     false     true     false     false     false     true     true     true
r5     true      true      true     false     true     false     false     true
r46    false     true      true     false     true     true     true     true
r47    true      true      true     true      true     true     false     false
r48    true      false     false     false     true     true     false     true
r49    true      true      false     true      true     true     false     false
r50    false     false     false     false     false     false     false     false

xxx TABLE:mat_3 xxxxxxxxxxxxxxxxxxxx
      c1      c2
-----
r1     0.00012471    0.13253
r2     0.88615      0.79226

xxx TABLE:tensor_1 xxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4      c5      c6      c7      c8
-----
r1     0.019363     0.34271     0.52167     0.53703     0.75756     0.68839     0.8345     0.26597
r2     0.018091     0.33355     0.11738     0.77857     0.81933     0.28644     0.6157     0.368

xxx TABLE:tensor_2 xxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4      c22      c23      c24      c25
-----
r1     0.51866     0.40495     0.48278     0.99731     0.46584     0.62976     0.035924     0.10505
r2     0.028692     0.37408     0.24149     0.35201     0.66054     0.87243     0.0024293     0.81088
r3     0.87339     0.19457     0.83212     0.15315     0.77859     0.96663     0.2501     0.8056

xxx TABLE:tensor_3 xxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4
-----
r1     0.1219     0.5119     0.91553     0.14329

xxx TABLE:tesseract_1 xxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4      c21      c22      c23      c24

```

```

-----
r1    0.64531    0.59299    0.32115    0.67653    0.90328    0.56911    0.52562    0.12014
r2    0.74558    0.5007    0.46142    0.21384    0.35564    0.13732    0.155    0.23786
r3    0.91137    0.46403    0.18118    0.049919    0.46246    0.46842    0.75348    0.64547

xxx TABLE:tesseract_2 xxxxxxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4      c7      c8      c9      c10
-----
r1    0.28898    0.48211    0.44359    0.97146    0.61782    0.65121    0.80715    0.11605
r2    0.094493    0.34941    0.17595    0.14192    0.16754    0.57097    0.043114    0.70518

xxx TABLE:tesseract_bl_3 xxxxxxxxxxxxxxxxxxxxxxxx
      c1      c2      c3      c4      c7      c8      c9      c10
-----
r1    false    false    true    true    false    true    false    false

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
      i      idx      value
      -      ---      -
boolean_1    1      1      1
empty        2      2      NaN
mat_4        3      11     0.74898
string_float_1 4      13     1021.1
string_int_1  5      14     1021

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
String
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
      i      idx      string
      ---      ----      -
list_string_1 "1"    "5"    "col1;col2;col3;col4"
list_string_2 "2"    "6"    "row1;row2;row3;row4"
string_1      "3"    "12"   "Table Name"

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Functions
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
      i      idx      functionString
      ---      ---      -
func1    "1"    "3"    "@(x)1+2+x"
func2    "2"    "4"    "@(x,y)x*1+sqrt(y)"

```

7.1.2 Test FF_CONTAINER_MAP_DISPLAY summarize Matrix Only

Three large matrixes, show summaries

```
% Create Container
```

```
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
```

```

rng(123);
mp_container_map('mat_1') = rand(100,100);
mp_container_map('mat_2') = rand(100,100)*2 + 1;
mp_container_map('mat_2_boolean') = (rand(100,100) > 0.5);
% Will only print
ff_container_map_display(mp_container_map);

```

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	co
	-	---	----	-----	----	----	-----	-----	-----	--
mat_1	1	1	2	10000	100	100	4982.3	0.49823	0.28829	0.
mat_2	2	2	2	10000	100	100	20029	2.0029	0.57632	0.
mat_2_boolean	3	3	2	10000	100	100	4995	0.4995	0.50002	1

7.1.3 Test FF_CONTAINER_MAP_DISPLAY Show Matrix Subset

A container map with three small matrixes, print only only 2 rows and 3 columns.

```

% Create Container
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
rng(789);
mp_container_map('mat_1') = rand(3,4);
mp_container_map('mat_2') = rand(50,53);
mp_container_map('mat_2_boolean') = (rand(50,53) > 0.5);
% Will only print
ff_container_map_display(mp_container_map, 2, 3);

```

```

-----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	co
	-	---	----	-----	----	----	-----	-----	-----	--
mat_1	1	1	2	12	3	4	4.9876	0.41564	0.33586	0.
mat_2	2	2	2	2650	50	53	1324.3	0.49973	0.28834	0.
mat_2_boolean	3	3	2	2650	50	53	1350	0.50943	0.50001	0.

```

xxx TABLE:mat_1 xxxxxxxxxxxxxxxxxxxxxxx

```

	c1	c2	c3	c4
	-----	-----	-----	-----
r1	0.32333	0.62442	0.01062	0.53815
r3	0.79378	0.75889	0.11104	0.55157

```

xxx TABLE:mat_2 xxxxxxxxxxxxxxxxxxxxxxx

```

	c1	c2	c52	c53
	-----	-----	-----	-----
r1	0.72837	0.20976	0.74583	0.22321
r50	0.52812	0.545	0.49521	0.29826

```

xxx TABLE:mat_2_boolean xxxxxxxxxxxxxxxxxxxxxxx

```

	c1	c2	c52	c53
	-----	-----	-----	-----

r1	false	true	true	true
r50	true	false	false	true

Appendix A

Index and Code Links

A.1 Savings Dynamic Programming links

1. [Looped Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**](#)
 - Slow looped solution.
 - Given preferences, some AR(1) shock process, solve the infinite horizon household savings dynamic programming problem. The state-space and choice-space share the same asset grid.
 - **MEconTools**: `ff_vfi_az_loop()`
2. [Vectorized Infinite Horizon Dynamic Savings Problem: **mlx** | **m** | **pdf** | **html**](#)
 - Faster vectorized solution.
 - Given preferences, some AR(1) shock process, solve the infinite horizon household savings dynamic programming problem. The state-space and choice-space share the same asset grid.
 - **MEconTools**: `ff_vfi_az_vec()`

A.2 Summarize Policy and Value links

1. [Summarize ND Array Policy and Value Functions: **mlx** | **m** | **pdf** | **html**](#)
 - Given an NDarray matrix with N1, N2, ..., ND dimensions. Generate average and standard deviation for the 3rd dimension, grouping by the other dimensions.
 - For example, show the 5th dimension as the column groups, and the other variables generate combinations shown as rows.
 - The resulting summary statistics table contains mean and standard deviation among other statistics over the policy or value contained in the ND array.
 - **MEconTools**: `ff_summ_nd_array()`

A.3 Distributional Analysis links

1. [Gateway Joint Probability Mass Statistics: **mlx** | **m** | **pdf** | **html**](#)
 - Given probability mass function $f(s)$, and information $y(s)$, $x(s)$, $z(s)$ at each element of the state-space, compute statistics for each variable, y , x , z , which are all discrete random variables.
 - Compute their correlation and covariance.
 - **MEconTools**: `ff_simu_stats()`
2. [Discrete Random Variable Distributional Statistics: **mlx** | **m** | **pdf** | **html**](#)
 - Model simulation generates discrete random variables, calculate mean, standard deviation, min, max, percentiles, and proportion of outcomes held by x percentiles, etc.
 - **MEconTools**: `ff_disc_rand_var_stats()`
3. [Generate Discrete Random Variable: **mlx** | **m** | **pdf** | **html**](#)
 - Given mass at state space points, and y , c , a , z and other outcomes or other information at each corresponding state space points, generate discrete random variable, with unique sorted values, and mass for each unique sorted values.

- Generate additional joint distributions: if initial distribution is over $f(a,z)$, generate joint distribution of $f(y,a)$ or $f(y,z)$.
- **MEconTools**: `ff_disc_rand_var_mass2outcomes()`
- 4. **Discrete Random Variable Correlation and Covariance**: [mlx](#) | [m](#) | [pdf](#) | [html](#)
 - Given probability mass function $f(s)$, $X(s)$, and $Y(s)$, compute the covariance and correlation between X and Y .
 - **MEconTools**: `ff_disc_rand_var_mass2covcor()`

A.4 Graphs links

1. **Multiple Line Graph Function**: [mlx](#) | [m](#) | [pdf](#) | [html](#)
 - Grid based Graph, x-axis one param, color another param, over outcomes.
 - **MEconTools**: `ff_graph_grid()`

A.5 Data Structures links

1. **Log and Power Spaced Asset and Choice Grids**: [mlx](#) | [m](#) | [pdf](#) | [html](#)
 - Generate linear, log-space, power-space, or threshold-cut asset or choice grids.
 - **MEconTools**: `ff_saveborr_grid()`

A.6 Common Functions links

1. **Discretize AR1 Normal Shock Tauchen (1986)**: [mlx](#) | [m](#) | [pdf](#) | [html](#)
 - Mean zero AR(1) shock discretize following Tauchen (1986).
 - **MEconTools**: `ffy_tauchen()`
2. **Discretize AR1 Normal Shock Rouwenhorst (1995)**: [mlx](#) | [m](#) | [pdf](#) | [html](#)
 - Mean zero AR(1) shock discretize following Rouwenhorst (1995).
 - **MEconTools**: `ffy_rouwenhorst()`

A.7 Support Tools links

1. **Organizes and Prints Container Map Key and Values**: [mlx](#) | [m](#) | [pdf](#) | [html](#)
 - Summarizes the contents of a map container by data types. Includes, scalar, array, matrix, string, functions, tensors (3-tuples), tesseracts (4-tuples).
 - **MEconTools**: `ff_container_map_display()`

Bibliography

The MathWorks Inc (2019). *MATLAB*. Matlab package version 2019b.

Xie, Y. (2020). *bookdown: Authoring Books and Technical Documents with R Markdown*. R package version 0.18.