

FF_VFI_AZ_BISEC_VEC Savings Vectorized Exact (FOC) Examples

[back to Fan's Intro Math for Econ](#), [Matlab Examples](#), or [Dynamic Asset Repositories](#)

This is the example vignette for function: `ff_vfi_az_bisec_vec` from the [MEconTools Package](#). This function solves the dynamic programming problem for a (a,z) model. Households can save a, and face AR(1) shock z. The problem is solved over the infinite horizon.

This is the vectorized code, its speed is much faster than the looped code. The function is designed to have small memory footprint and requires low computing resources, yet is fast.

The code uses **continuous choices**, solved with bi(multi)section. The state-space is on a grid, but choice grids are in terms of percentage of resources available, which is individual specific, to save and solved exactly up to $((1/(2)^{16}) * 100 = 0.001525878)$ percentage of cash on hand. The `ff_vfi_az_vec` from the [MEconTools Package](#) solves the same problem using vectorized common grid code where the choice set and state space share the same grid. The common grid function is faster, but less precise for the same number of asset grid points.

Links to Other Code:

Core Savings/Borrowing Dynamic Programming Solution Functions that are functions in the [MEconTools Package](#).

- Common Choice and States Grid **Loop**: `ff_vfi_az_loop`
- Common Choice and States Grid **Vectorized**: `ff_vfi_az_vec`
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC, **Loop**: `ff_vfi_az_bisec_loop`
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, rely on FOC **Vectorized**: `ff_vfi_az_bisec_vec`
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, **Loop**: `ff_vfi_az_mzoom_loop`
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand, VALUE comparison, **Vectorized**: `ff_vfi_az_mzoom_vec`

Test FF_VFI_AZ_BISEC_VEC Defaults

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the `mp_params`.

```
%mp_params
mp_params = containers.Map('KeyType','char','ValueType','any');
mp_params('fl_crra') = 1.5;
mp_params('fl_beta') = 0.94;
% call function
ff_vfi_az_bisec_vec(mp_params);
```

Elapsed time is 1.762201 seconds.

```

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari	min	max
	—	—	—	—	—	—	—	—	—	—	—	—
ap	1	1	2	700	100	7	9863.4	14.091	14.388	1.0211	0	50.117

```

xxx TABLE:ap XXXXXXXXXXXXXXXXXXXXXXX

```

	c1	c2	c3	c4	c5	c6	c7
	—	—	—	—	—	—	—
r1	0	0	0	0.053491	0.25574	0.60604	1.1157
r2	0	0	0	0.053998	0.25571	0.6066	1.1163
r3	0	0	0	0.056449	0.25576	0.60907	1.1187
r4	0	0	0	0.061799	0.26016	0.6109	1.1239
r5	0	0	0	0.066463	0.26897	0.61141	1.1327
r96	43.388	43.52	43.701	43.925	44.222	44.68	45.228
r97	44.566	44.695	44.878	45.103	45.398	45.856	46.403
r98	45.761	45.892	46.072	46.298	46.592	47.05	47.597
r99	46.973	47.107	47.286	47.514	47.806	48.263	48.815
r100	48.206	48.338	48.519	48.746	49.037	49.497	50.117

Test FF_VFI_AZ_BISEC_VEC Speed Tests

Call the function with defaults. By default, shows the asset policy function summary. Model parameters can be changed by the mp_params.

```

mp_support = containers.Map('KeyType','char','ValueType','any');
mp_support('bl_timer') = true;
mp_support('ls_ffcmd') = {};
% A grid 50, shock grid 5:
mp_params = containers.Map('KeyType','char','ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
ff_vfi_az_bisec_vec(mp_params, mp_support);

```

Elapsed time is 0.792541 seconds.

```

% A grid 750, shock grid 15:
mp_params = containers.Map('KeyType','char','ValueType','any');
mp_params('it_a_n') = 750;
mp_params('it_z_n') = 15;
ff_vfi_az_bisec_vec(mp_params, mp_support);

```

Elapsed time is 43.095190 seconds.

```

% A grid 600, shock grid 45:
mp_params = containers.Map('KeyType','char','ValueType','any');
mp_params('it_a_n') = 600;
mp_params('it_z_n') = 45;
ff_vfi_az_bisec_vec(mp_params, mp_support);

```

Elapsed time is 80.139775 seconds.

Test FF_VFI_AZ_BISEC_VEC Control Outputs

Run the function first without any outputs;

```
mp_params = containers.Map('KeyType','char','ValueType','any');
mp_params('it_a_n') = 50;
mp_params('it_z_n') = 5;
mp_support = containers.Map('KeyType','char','ValueType','any');
mp_support('bl_timer') = true;
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {};
ff_vfi_az_vec(mp_params, mp_support);
```

Elapsed time is 0.029901 seconds.

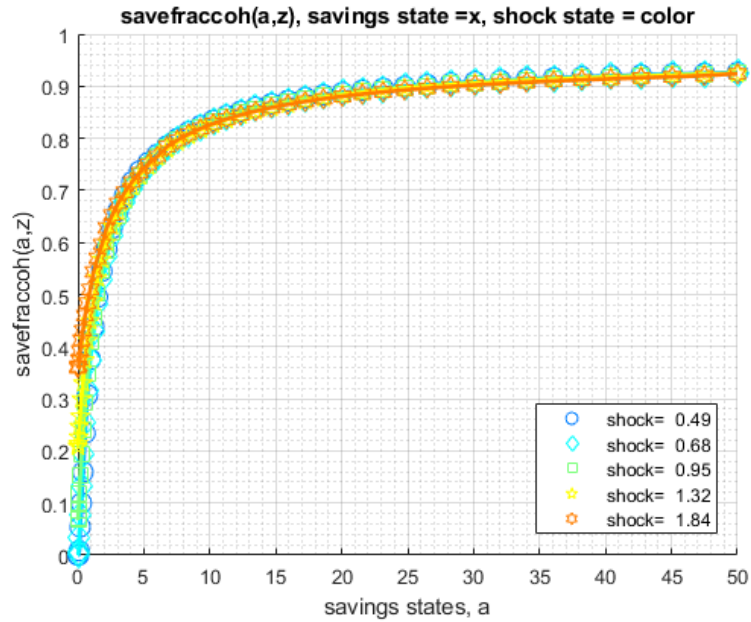
Run the function and show policy function for savings choice. For ls_ffcmd, ls_ffsna, ls_ffgrh, can include these: 'v', 'ap', 'c', 'y', 'coh', 'savefraccoh'. These are value, aprime savings choice, consumption, income, cash on hand, and savings fraction as cash-on-hand.

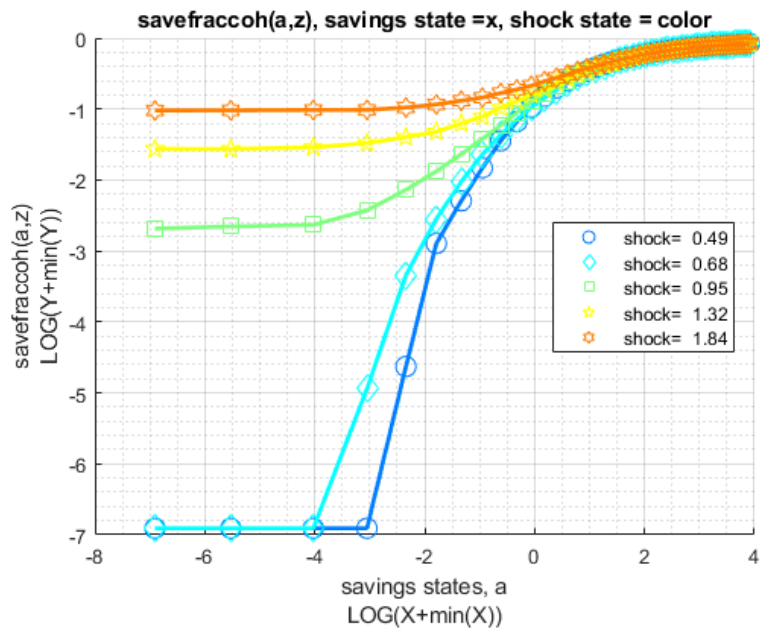
```
mp_support = containers.Map('KeyType','char','ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
% ls_ffcmd: summary print which outcomes
mp_support('ls_ffcmd') = {};
% ls_ffsna: detail print which outcomes
mp_support('ls_ffsna') = {'savefraccoh'};
% ls_ffgrh: graphical print which outcomes
mp_support('ls_ffgrh') = {'savefraccoh'};
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

Elapsed time is 0.494900 seconds.

group	a	mean_z_0_4858	mean_z_0_67798	mean_z_0_9462	mean_z_1_3205	mean_z_1_8429
1	0	0	0	0.067239	0.20859	0.35953
2	0.002975	0	0	0.069375	0.20829	0.36032
3	0.016829	0	0	0.070901	0.2139	0.36215
4	0.046375	0	0.0061439	0.087319	0.2266	0.36264
5	0.095198	0.0087684	0.034403	0.1168	0.2468	0.37473
6	0.1663	0.054361	0.077248	0.1522	0.26639	0.39151
7	0.26234	0.099892	0.13132	0.19388	0.29929	0.41281
8	0.38568	0.15958	0.19309	0.24112	0.33017	0.43088
9	0.53852	0.23417	0.25553	0.29215	0.37436	0.45969
10	0.72291	0.3071	0.31656	0.34812	0.41153	0.48386
11	0.94076	0.37595	0.37503	0.40842	0.44925	0.50992
12	1.1939	0.43881	0.42941	0.45755	0.48697	0.54367
13	1.484	0.49509	0.48129	0.50381	0.53262	0.56979
14	1.8128	0.54489	0.53018	0.54642	0.56778	0.59634
15	2.1817	0.58871	0.57382	0.58548	0.60055	0.6282
16	2.5924	0.62716	0.61258	0.62076	0.63101	0.65249
17	3.0463	0.66079	0.64682	0.65243	0.65884	0.6752
18	3.5449	0.69027	0.67709	0.68069	0.68423	0.69638
19	4.0894	0.71621	0.70376	0.70596	0.70724	0.71591
20	4.6813	0.73703	0.72732	0.72848	0.72799	0.73385
21	5.3218	0.75326	0.74813	0.7485	0.74673	0.75021
22	6.0121	0.76913	0.76657	0.76632	0.76364	0.76535
23	6.7536	0.78536	0.78286	0.78231	0.77889	0.7842
24	7.5474	0.79983	0.79745	0.79653	0.79269	0.79678
25	8.3948	0.81271	0.81039	0.80929	0.80514	0.80831

26	9.2967	0.82418	0.82198	0.82076	0.81637	0.81875
27	10.254	0.8345	0.83242	0.83114	0.82656	0.82833
28	11.269	0.84377	0.84176	0.84042	0.83584	0.83706
29	12.342	0.85214	0.85024	0.84884	0.8442	0.84499
30	13.473	0.85964	0.85781	0.85647	0.85183	0.85232
31	14.665	0.86648	0.86471	0.86337	0.85879	0.85897
32	15.918	0.87264	0.87099	0.86965	0.86507	0.86507
33	17.233	0.87826	0.87667	0.87533	0.87161	0.87063
34	18.611	0.88338	0.88186	0.88052	0.87771	0.87582
35	20.053	0.88802	0.88656	0.88528	0.88326	0.88052
36	21.56	0.8923	0.89089	0.88967	0.88833	0.88485
37	23.133	0.89614	0.89486	0.89364	0.8926	0.88888
38	24.773	0.89974	0.89852	0.8973	0.89626	0.8926
39	26.481	0.90304	0.90182	0.90072	0.89968	0.89608
40	28.258	0.90603	0.90493	0.90383	0.90279	0.89925
41	30.104	0.90884	0.90774	0.9067	0.90572	0.90218
42	32.021	0.9114	0.91036	0.90932	0.90841	0.90493
43	34.01	0.91378	0.9128	0.91183	0.91091	0.90749
44	36.07	0.91598	0.91506	0.91408	0.91317	0.90987
45	38.204	0.91805	0.91714	0.91622	0.91537	0.91207
46	40.412	0.91994	0.91909	0.91817	0.91732	0.91415
47	42.695	0.92171	0.92086	0.92001	0.91921	0.9161
48	45.053	0.92336	0.92257	0.92171	0.92092	0.91799
49	47.488	0.92489	0.92409	0.92336	0.92257	0.92025
50	50	0.92629	0.92562	0.92489	0.92428	0.92403





Run the function and show summaries for savings and fraction of coh saved:

```
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 9;
mp_support('ls_ffcmd') = {'ap', 'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_support('bl_vfi_store_all') = true; % store c(a,z), y(a,z)
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

Elapsed time is 1.164186 seconds.

 xxx
 CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
 xxx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari	min
ap	1	1	2	900	100	9	12926	14.362	14.544	1.0127	0
savefraccoh	2	2	2	900	100	9	621.24	0.69027	0.26896	0.38965	0

xxx TABLE:ap xxxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7	c8	c9
r1	0	0	0	0	0.087442	0.27778	0.58243	1.0038	1.5724
r2	0	0	0	0	0.087962	0.27828	0.58297	1.0044	1.5731
r3	0	0	0	0	0.090477	0.28074	0.58547	1.0069	1.5755
r4	0	0	0	0.00055771	0.09279	0.28605	0.5907	1.0122	1.5808
r5	0	0	0	0.0059496	0.09602	0.29477	0.59952	1.0209	1.5895
r96	43.845	43.923	44.022	44.198	44.428	44.722	45.103	45.546	46.186
r97	45.031	45.101	45.208	45.384	45.613	45.91	46.293	46.735	47.382
r98	46.237	46.297	46.411	46.59	46.818	47.115	47.501	47.948	48.605
r99	47.46	47.512	47.635	47.812	48.041	48.34	48.726	49.191	49.869
r100	48.703	48.746	48.878	49.055	49.283	49.586	49.978	50.495	51.171

xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7	c8	c9
--	----	----	----	----	----	----	----	----	----

r1	0	0	0	0	0.066018	0.16569	0.27445	0.37369	0.46243
r2	0	0	0	0	0.066384	0.16593	0.27463	0.37381	0.46256
r3	0	0	0	0	0.068154	0.16715	0.27549	0.37442	0.46292
r4	0	0	0	0.00052879	0.069619	0.16978	0.27726	0.37564	0.46378
r5	0	0	0	0.0055946	0.071572	0.17405	0.28025	0.37766	0.46512
r96	0.92458	0.92354	0.92226	0.92171	0.92116	0.92055	0.91994	0.91842	0.91811
r97	0.92531	0.92416	0.92306	0.92251	0.92196	0.92141	0.92086	0.91933	0.91915
r98	0.92605	0.9247	0.92379	0.9233	0.92275	0.9222	0.92171	0.92031	0.92031
r99	0.92672	0.92525	0.92452	0.92403	0.92348	0.923	0.92251	0.92147	0.92184
r100	0.92739	0.9258	0.92525	0.92477	0.92422	0.92379	0.92342	0.92336	0.92367

Test FF_VFI_AZ_BISEC_VEC Change Interest Rate and Discount

Show only save fraction of cash on hand:

```
mp_support = containers.Map('KeyType','char','ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char','ValueType','any');
mp_params('it_a_n') = 100;
mp_params('it_z_n') = 7;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with several different interest rates and discount factor:

```
% Lower Savings Incentives
mp_params('fl_beta') = 0.80;
mp_params('fl_r') = 0.01;
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

Elapsed time is 0.271658 seconds.

```
-----
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari	min	max
	—	—	—	—	—	—	—	—	—	—	—	—
savefraccoh	1	1	2	700	100	7	357.85	0.51122	0.27528	0.53848	0	0

```
xxx TABLE:savefraccoh XXXXXXXXXXXXXXXXXXXX
```

	c1	c2	c3	c4	c5	c6	c7
	—	—	—	—	—	—	—
r1	0	0	0	0	0	0.00022362	0.041544
r2	0	0	0	0	0	0.00022362	0.041544
r3	0	0	0	0	0	0.0011391	0.041544
r4	0	0	0	0	0	0.0016884	0.041483
r5	0	0	0	0	0	0.0034584	0.04136
r96	0.79586	0.79275	0.78945	0.78591	0.78225	0.77853	0.77059
r97	0.79684	0.79379	0.79055	0.78713	0.78359	0.77993	0.77212
r98	0.79782	0.79482	0.79171	0.78835	0.78488	0.78127	0.77365
r99	0.79873	0.79586	0.79275	0.78951	0.7861	0.78262	0.77548
r100	0.79965	0.79684	0.79385	0.79061	0.78732	0.7839	0.7781

% Higher Savings Incentives

```
mp_params('fl_beta') = 0.95;  
mp_params('fl_r') = 0.04;  
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

Elapsed time is 0.971218 seconds.

XX

CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)

XX

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari	min
savefraccoh	1	1	2	700	100	7	481.37	0.68768	0.27118	0.39435	0

xxx TABLE:savefraccoh XXXXXXXXXXXXXXXXXXXXXXX

	c1	c2	c3	c4	c5	c6	c7
r1	0	0	0	0.065774	0.18076	0.30655	0.41654
r2	0	0	0	0.066201	0.18101	0.30674	0.4166
r3	0	0	0	0.06791	0.18223	0.30747	0.41709
r4	0	0	0	0.069619	0.18467	0.30759	0.41812
r5	0	0	0	0.071694	0.18876	0.30838	0.41983
r96	0.92428	0.92245	0.92178	0.92116	0.92049	0.91872	0.91824
r97	0.92501	0.92324	0.92257	0.92196	0.92129	0.91958	0.91921
r98	0.92574	0.92397	0.92336	0.92275	0.92208	0.92049	0.92025
r99	0.92647	0.9247	0.92409	0.92348	0.92287	0.92147	0.92159
r100	0.92702	0.92544	0.92483	0.92422	0.92373	0.92336	0.92348

Test FF_VFI_AZ_BISEC_VEC Changing Risk Aversion

Here, again, show fraction of coh saved in summary tabular form, but also show it graphically.

```
mp_support = containers.Map('KeyType','char','ValueType','any');  
mp_support('bl_print_params') = false;  
mp_support('bl_print_iterinfo') = false;  
mp_support('ls_ffcmd') = {'savefraccoh'};  
mp_support('ls_ffsna') = {};  
mp_support('ls_ffgrh') = {'savefraccoh'};  
mp_params = containers.Map('KeyType','char','ValueType','any');  
mp_params('it_a_n') = 100;  
mp_params('it_z_n') = 7;  
mp_params('fl_a_max') = 50;  
mp_params('st_grid_type') = 'grid_powerspace';
```

Solve the model with different risk aversion levels, higher preferences for risk:

% Lower Risk Aversion

```
mp_params('fl_crra') = 0.5;  
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

Elapsed time is 0.873752 seconds.

XX

CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)

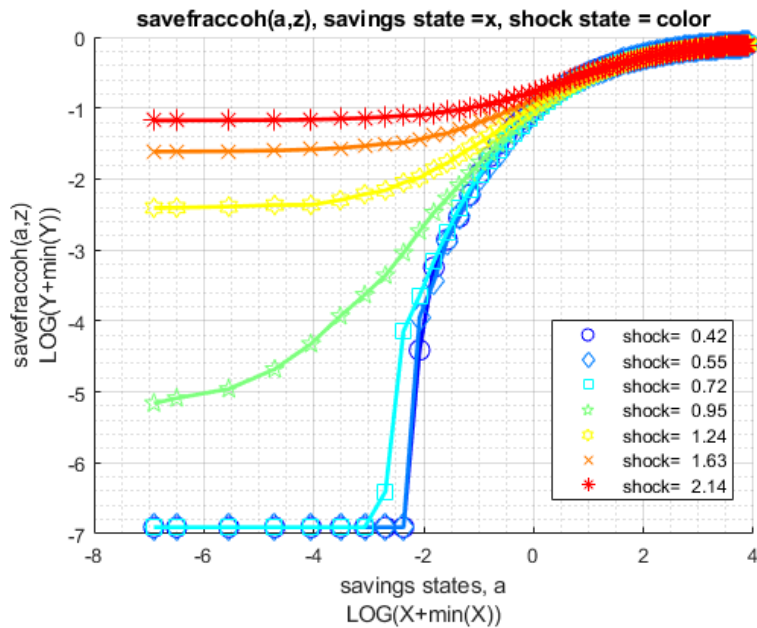
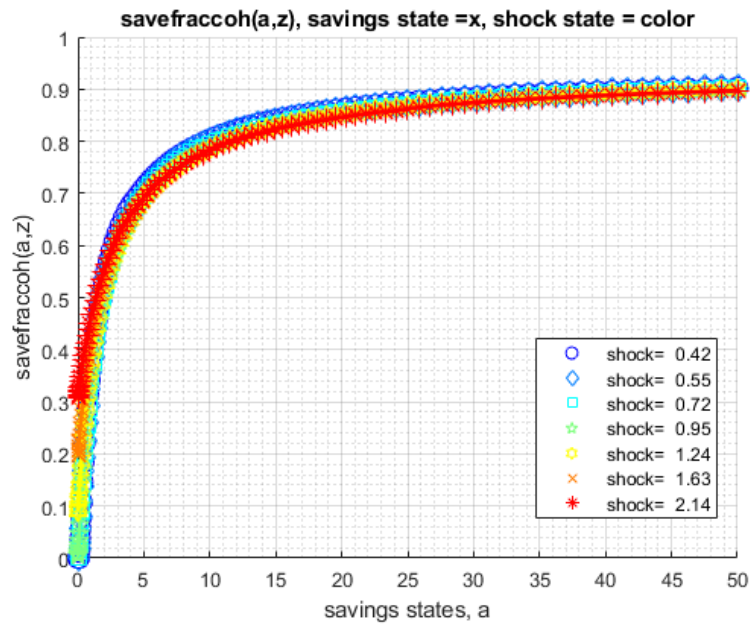
XX

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari	min
--	---	-----	------	-------	------	------	-----	------	-----	----------	-----

savefraccoh	1	1	2	700	100	7	452.13	0.6459	0.28031	0.43398	0	0.
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xxx TABLE:savefraccoh xxxxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7
r1	0	0	0	0.0047401	0.089089	0.19822	0.30783
r2	0	0	0	0.0051674	0.089394	0.1984	0.30796
r3	0	0	0	0.0060218	0.090676	0.19926	0.30851
r4	0	0	0	0.0082801	0.092812	0.20115	0.30973
r5	0	0	0	0.012247	0.092995	0.2042	0.31174
r96	0.90047	0.89925	0.89828	0.8973	0.89632	0.89376	0.89297
r97	0.90127	0.90017	0.89919	0.89828	0.8973	0.8948	0.89394
r98	0.90206	0.90102	0.90011	0.89919	0.89828	0.89577	0.89498
r99	0.90279	0.90188	0.90102	0.90011	0.89919	0.89681	0.8959
r100	0.90359	0.90273	0.90188	0.90096	0.90011	0.89803	0.89687



When risk aversion increases, at every state-space point, the household wants to save more.

```
% Higher Risk Aversion
mp_params('fl_crra') = 5;
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

Elapsed time is 0.970314 seconds.

XX

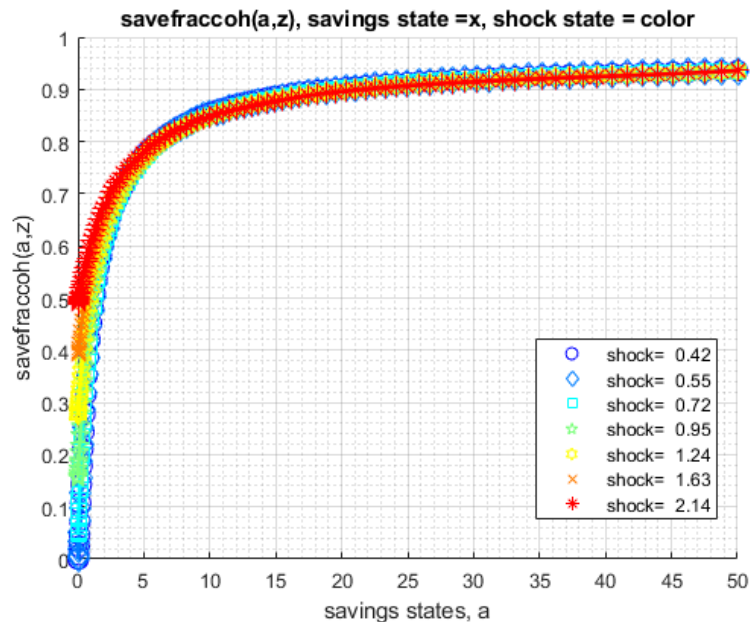
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)

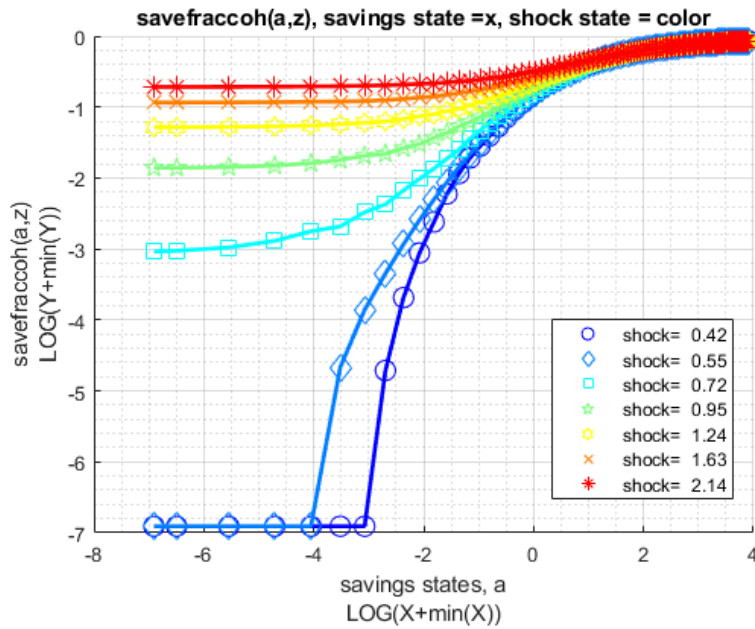
XX

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari	min	max
savefraccoh	1	1	2	700	100	7	502.71	0.71816	0.25437	0.3542	0	0

xxx TABLE:savefraccoh XXXXXXXXXXXXXXXXXXXXXXX

	c1	c2	c3	c4	c5	c6	c7
r1	0	0	0.047037	0.15537	0.27573	0.3909	0.48782
r2	0	0	0.047525	0.15531	0.27591	0.39102	0.48795
r3	0	0	0.049844	0.1569	0.27695	0.3917	0.48837
r4	0	0	0.054788	0.16025	0.27915	0.3931	0.48929
r5	0	0	0.062905	0.16569	0.28275	0.39542	0.49075
r96	0.93307	0.93258	0.93203	0.93154	0.9302	0.92995	0.92971
r97	0.93374	0.93325	0.93276	0.93227	0.93111	0.93105	0.93117
r98	0.93441	0.93398	0.93349	0.93307	0.93209	0.93227	0.9327
r99	0.93508	0.93465	0.93423	0.93392	0.93331	0.93368	0.93435
r100	0.93575	0.93539	0.93508	0.9349	0.93496	0.93526	0.93587





Test FF_VFI_AZ_BISEC_VEC with Higher Uncertainty

Increase the standard deviation of the Shock.

```
mp_support = containers.Map('KeyType','char','ValueType','any');
mp_support('bl_print_params') = false;
mp_support('bl_print_iterinfo') = false;
mp_support('ls_ffcmd') = {'savefraccoh'};
mp_support('ls_ffsna') = {};
mp_support('ls_ffgrh') = {};
mp_params = containers.Map('KeyType','char','ValueType','any');
mp_params('it_a_n') = 150;
mp_params('it_z_n') = 15;
mp_params('fl_a_max') = 50;
mp_params('st_grid_type') = 'grid_powerspace';
% graph color spectrum
mp_params('cl_colors') = 'copper';
```

Lower standard deviation of shock:

```
% Lower Risk Aversion
mp_params('fl_shk_std') = 0.10;
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

Elapsed time is 2.595920 seconds.

```
-----
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari	min	max
savefraccoh	1	1	2	2250	150	15	1507.5	0.67001	0.28668	0.42788	0	0

```
xxx TABLE:savefraccoh XXXXXXXXXXXXXXXXXXXX
```

	c1	c2	c3	c4	c5	c11	c12	c13	c14	c15
r1	0	0	0	0	0	0.13847	0.18485	0.23026	0.27378	0.31717
r2	0	0	0	0	0	0.13853	0.18491	0.23032	0.27384	0.31723
r3	0	0	0	0	0	0.13895	0.18528	0.23063	0.27408	0.31757
r4	0	0	0	0	0	0.13987	0.18607	0.2313	0.27469	0.31819
r5	0	0	0	0	0	0.14011	0.18735	0.2324	0.27567	0.31881
r146	0.92373	0.92354	0.9233	0.92312	0.92287	0.92086	0.92068	0.92049	0.91952	0.91919
r147	0.92422	0.92403	0.92385	0.92361	0.92342	0.92141	0.92123	0.92098	0.92007	0.91974
r148	0.9247	0.92452	0.92434	0.92409	0.92391	0.9219	0.92171	0.92153	0.92062	0.92029
r149	0.92519	0.92501	0.92483	0.92458	0.9244	0.92245	0.92226	0.92208	0.92116	0.92083
r150	0.92568	0.9255	0.92531	0.92507	0.92489	0.92293	0.92275	0.92257	0.92245	0.92233

```
% Higher Risk Aversion
mp_params('fl_shk_std') = 0.40;
ff_vfi_az_bisec_vec(mp_params, mp_support);
```

```

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CONTAINER NAME: mp_ffcmd ND Array (Matrix etc)
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

```

[illegible]