FF_OPTIM_BISEC_SAVEZRONE Derivative Bisection

back to Fan's Intro Math for Econ, Matlab Examples, or Dynamic Asset Repositories

This is the example vignette for function: **ff_optim_bisec_savezrone** from the **MEconTools Package.** This functions solves for optimal savings/borrowing level given an anonymous function that provides the derivative of a intertemporal savings problem. The function is solves over a grid of state-space elements that are embeded in the anonymous function. By default, it iterates over 15 iterations with bisection.

The vectorized and looped bisection savings problem rely on this function to solve for optimal savings choices:

- States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Loop***: **ff_vfi_az_bisec_loop**, high precision even with small grid
- States Grid + Continuous Exact Savings as Share of Cash-on-Hand ***Vectorized***: **ff_vfi_az_bisec_vec**, precision and speed

Test FF_OPTIM_BISEC_SAVEZRONE Defaults

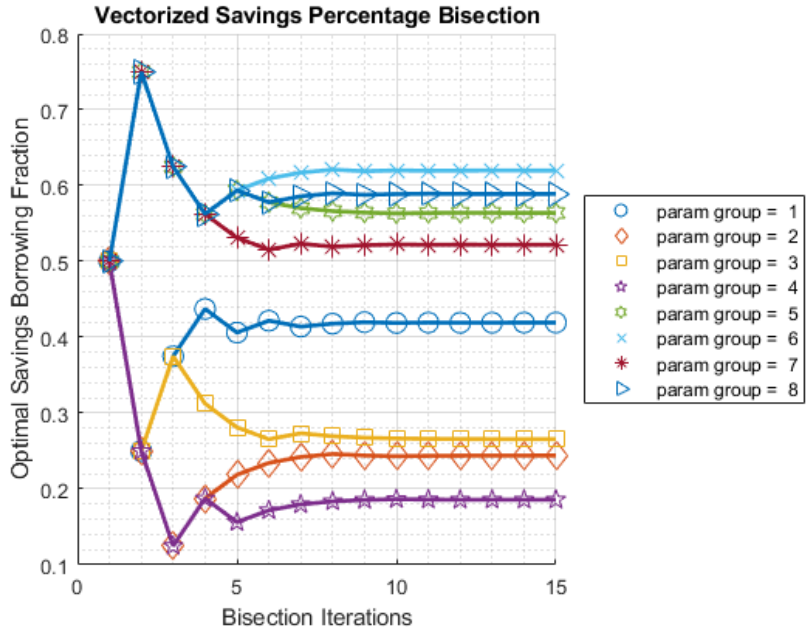
Call the function with defaults, this solves concurrently for many state-space points' optimization problems:

```
ff_optim_bisec_savezrone();
```

```
Elapsed time is 0.089585 seconds.
BISECT END: iteration=16, norm(ar_mid_fx)=0.00030653
```

	vartype	paramgroup2	paramgroup3	paramgroup4	paramgroup5	paramgroup6
a	"init"	1e-05	1e-05	1e-05	1e-05	1e-05
b	"init"	0.99999	0.99999	0.99999	0.99999	0.99999
f_a	"init"	33802	40925	67047	15411	63263
f_b	"init"	-46789	-1.2672e+05	-1.8532e+05	-67518	-48900
it1_fp	"fatx"	-0.25973	-1.7159	-2.3655	-1.0421	0.28726
it1_p	"x"	0.5	0.5	0.5	0.5	0.5
it2_fp	"fatx"	0.72822	-0.052631	0.21087	-0.28379	-1.1125
it2_p	"x"	0.25	0.25	0.25	0.25	0.74999
it3_fp	"fatx"	0.15277	1.8256	-1.1773	0.46124	-0.29179
it3_p	"x"	0.375	0.12501	0.375	0.12501	0.625
it4_fp	"fatx"	-0.059183	0.62299	-0.55013	-0.0090579	0.0069602
it4_p	"x"	0.4375	0.18751	0.3125	0.18751	0.5625
it5_fp	"fatx"	0.044028	0.2488	-0.19454	0.1861	-0.13821
it5_p	"x"	0.40625	0.21876	0.28125	0.15626	0.59375
it6_fp	"fatx"	-0.0080863	0.090981	0.00054305	0.081339	-0.064832
it6_p	"x"	0.42188	0.23438	0.26563	0.17188	0.57812
it7_fp	"fatx"	0.017822	0.017593	-0.098707	0.034591	-0.028768
it7_p	"x"	0.41406	0.24219	0.27344	0.17969	0.57031
it8_fp	"fatx"	0.0048335	-0.017893	-0.049532	0.012405	-0.010865
it8_p	"x"	0.41797	0.2461	0.26954	0.1836	0.5664
it9_fp	"fatx"	-0.0016347	-0.00024633	-0.02461	0.0015865	-0.0019434
it9_p	"x"	0.41992	0.24415	0.26758	0.18555	0.56445
it10_fp	"fatx"	0.0015973	0.0086488	-0.012063	-0.0037571	0.0025106
it10_p	"x"	0.41895	0.24317	0.26661	0.18653	0.56348
it11_fp	"fatx"	-1.9235e-05	0.0041952	-0.0057672	-0.0010907	0.00028416
it11_p	"x"	0.41944	0.24366	0.26612	0.18604	0.56396
it12_fp	"fatx"	0.00078889	0.0019729	-0.0026139	0.00024655	-0.0008295
it12_p	"x"	0.41919	0.2439	0.26587	0.1858	0.56421
it13_fp	"fatx"	0.00038479	0.00086292	-0.0010359	-0.00042242	-0.00027263
it13_p	"x"	0.41931	0.24402	0.26575	0.18592	0.56409
it14_fp	"fatx"	0.00018277	0.0003082	-0.00024654	-8.8022e-05	5.7721e-06

it14_p	"x"	0.41937	0.24408	0.26569	0.18586	0.56402
it15_fp	"fatx"	8.1766e-05	3.0909e-05	0.00014822	7.9241e-05	-0.00013343
it15_p	"x"	0.4194	0.24412	0.26566	0.18583	0.56406
it15_level	"level"	0.56205	-0.070025	0.044431	-0.039424	1.0402
exactSoluSaveborrFrac	"exact"	0.41943	0.24412	0.26567	0.18584	0.56403
exactSoluSaveborrLevel	"exact"	0.56211	-0.070022	0.044438	-0.039403	1.0402
exactSoluSaveborrFracGap	"exact"	2.4705e-05	3.402e-06	1.1458e-05	1.4456e-05	2.9252e-05
exactSoluSaveborrLevelGap	"exact"	5.28e-05	2.6845e-06	6.1825e-06	2.1411e-05	5.9818e-05




```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coef
ar_opti_foc_obj	1	1	2	8	1	8	0.00050535	6.3168e-05	9.4141e-05	1.4
ar_opti_save_frac	2	2	2	8	1	8	3.41	0.42626	0.17279	0.40

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7	c8
r1	8.1766e-05	3.0909e-05	0.00014822	7.9241e-05	-0.00013343	0.00015981	1.8966e-05	0.0001

xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7	c8
r1	0.4194	0.24412	0.26566	0.18583	0.56406	0.6199	0.522	0.58908

Test FF_OPTIM_BISEC_SAVEZRONE One Individual

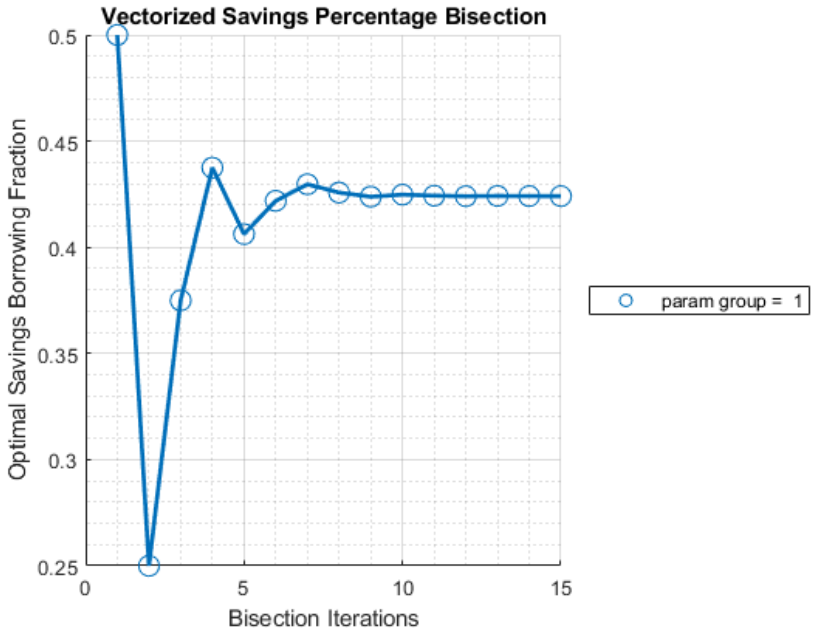
Bisection for savings choice at one state:

```
% Generate the state-space and function
[fl_z1, fl_z2, fl_r, fl_beta] = deal(0.4730, 0.6252, 0.0839, 0.7365);
% ffi_intertemporal_max is a function in ff_optim_bisec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, fl_z1, fl_z2, fl_r, fl_beta);
% Call Function
```

```
bl_verbose = true;
ff_optim_bisec_savezrone(fc_deri_wth_uniroot, bl_verbose);
```

```
BISECT END: iteration=16, norm(ar_mid_fx)=0.00016724
```

	vartype	paramgroup2
a	"init"	1e-05
b	"init"	0.99999
f_a	"init"	70155
f_b	"init"	-95255
it1_fp	"fatx"	-0.502
it1_p	"x"	0.5
it2_fp	"fatx"	1.5361
it2_p	"x"	0.25
it3_fp	"fatx"	0.34671
it3_p	"x"	0.375
it4_fp	"fatx"	-0.089881
it4_p	"x"	0.4375
it5_fp	"fatx"	0.12259
it5_p	"x"	0.40625
it6_fp	"fatx"	0.015276
it6_p	"x"	0.42188
it7_fp	"fatx"	-0.037529
it7_p	"x"	0.42969
it8_fp	"fatx"	-0.011188
it8_p	"x"	0.42578
it9_fp	"fatx"	0.0020277
it9_p	"x"	0.42383
it10_fp	"fatx"	-0.0045843
it10_p	"x"	0.42481
it11_fp	"fatx"	-0.0012793
it11_p	"x"	0.42432
it12_fp	"fatx"	0.00037392
it12_p	"x"	0.42407
it13_fp	"fatx"	-0.00045276
it13_p	"x"	0.4242
it14_fp	"fatx"	-3.9436e-05
it14_p	"x"	0.42413
it15_fp	"fatx"	0.00016724
it15_p	"x"	0.4241
it15_level	"level"	-0.13158




```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	value
ar_opti_foc_obj	1	1	0.00016724
ar_opti_save_frac	2	2	0.4241

Test FF_OPTIM_BISEC_SAVEZRONE Six Individual States

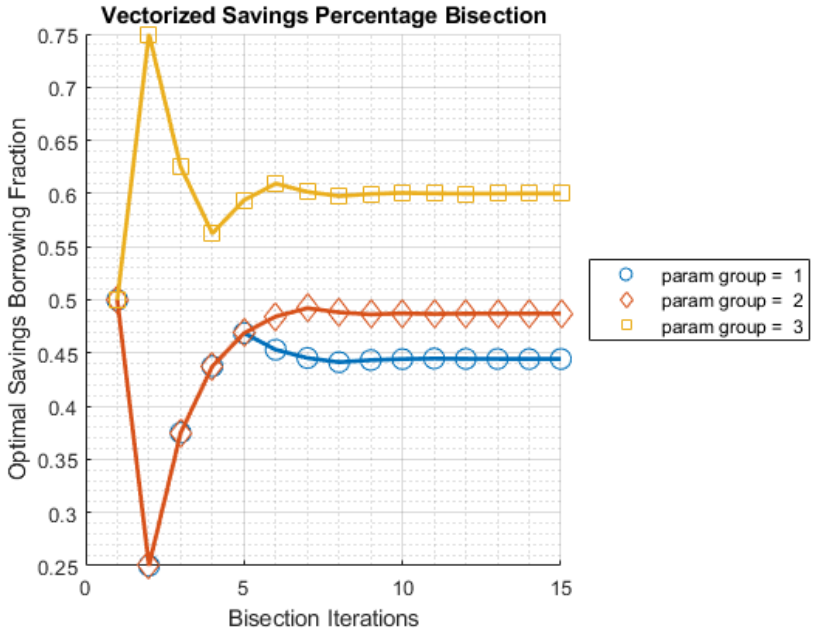
Solve the two period intertemporal optimization problem with only 6 individual states:

```
% Generate the state-space and function
ar_z1 = [1,2,3]';
ar_z2 = [3,2,1]';
ar_r = [1.05, 1.50, 1.30]';
ar_beta = [0.80, 0.95, 1.50]';
mt_fc_inputs = [ar_z1, ar_z2, ar_r, ar_beta];
% ffi_intertemporal_max is a function in ff_optim_bisec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = true;
ff_optim_bisec_savezrone(fc_deri_wth_uniroot, bl_verbose);
```

```
BISECT END: iteration=16, norm(ar_mid_fx)=8.9847e-05
```

	vartype	paramgroup2	paramgroup3	paramgroup4
a	"init"	1e-05	1e-05	1e-05
b	"init"	0.99999	0.99999	0.99999
f_a	"init"	32475	33928	43671
f_b	"init"	-40594	-35714	-29113
it1_fp	"fatx"	-0.16238	-0.035714	0.29114
it1_p	"x"	0.5	0.5	0.5
it2_fp	"fatx"	0.75773	0.88092	-0.58225
it2_p	"x"	0.25	0.25	0.74999

it3_fp	"fatx"	0.21649	0.33333	-0.077629
it3_p	"x"	0.375	0.375	0.625
it4_fp	"fatx"	0.020615	0.14059	0.11091
it4_p	"x"	0.4375	0.4375	0.5625
it5_fp	"fatx"	-0.07132	0.051539	0.018865
it5_p	"x"	0.46875	0.46875	0.59375
it6_fp	"fatx"	-0.025599	0.0078193	-0.028659
it6_p	"x"	0.45313	0.48438	0.60937
it7_fp	"fatx"	-0.0025711	-0.013955	-0.0047386
it7_p	"x"	0.44531	0.49219	0.60156
it8_fp	"fatx"	0.0090001	-0.0030715	0.0071001
it8_p	"x"	0.44141	0.48828	0.59765
it9_fp	"fatx"	0.0032093	0.0023727	0.0011903
it9_p	"x"	0.44336	0.48633	0.59961
it10_fp	"fatx"	0.00031783	-0.00034971	-0.0017717
it10_p	"x"	0.44434	0.4873	0.60058
it11_fp	"fatx"	-0.0011269	0.0010114	-0.00029011
it11_p	"x"	0.44483	0.48682	0.6001
it12_fp	"fatx"	-0.00040464	0.00033083	0.00045024
it12_p	"x"	0.44458	0.48706	0.59985
it13_fp	"fatx"	-4.3425e-05	-9.4396e-06	8.0103e-05
it13_p	"x"	0.44446	0.48718	0.59997
it14_fp	"fatx"	0.0001372	0.0001607	-0.000105
it14_p	"x"	0.4444	0.48712	0.60003
it15_fp	"fatx"	4.6884e-05	7.5628e-05	-1.2444e-05
it15_p	"x"	0.44443	0.48715	0.6
it15_level	"level"	-0.3686	0.56403	1.6261



---

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coef
ar_opti_foc_obj	1	1	2	3	1	3	0.00011007	3.6689e-05	4.4913e-05	1.2
ar_opti_save_frac	2	2	2	3	1	3	1.5316	0.51053	0.080379	0.15

xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxxx

	c1	c2	c3
r1	4.6884e-05	7.5628e-05	-1.2444e-05

```
xxx TABLE:ar_opti_save_frac xxxxxxxxxxxxxxxxx
            c1         c2       c3
          _______    _______    ___

    r1    0.44443    0.48715    0.6
```

Test FF_OPTIM_BISEC_SAVEZRONE Speed

Test Speed doing 6.25 million bisections for a savings problem:

```
% Generate the state-space and function
rng(123);
it_draws = 6250000; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
% ffi_intertemporal_max is a function in ff_optim_bisec_savezrone for testing
fc_deri_wth_uniroot = @(x) ffi_intertemporal_max(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
[ar_opti_save_frac, ar_opti_save_level] = ff_optim_bisec_savezrone(fc_deri_wth_uniroot, bl_verb
```

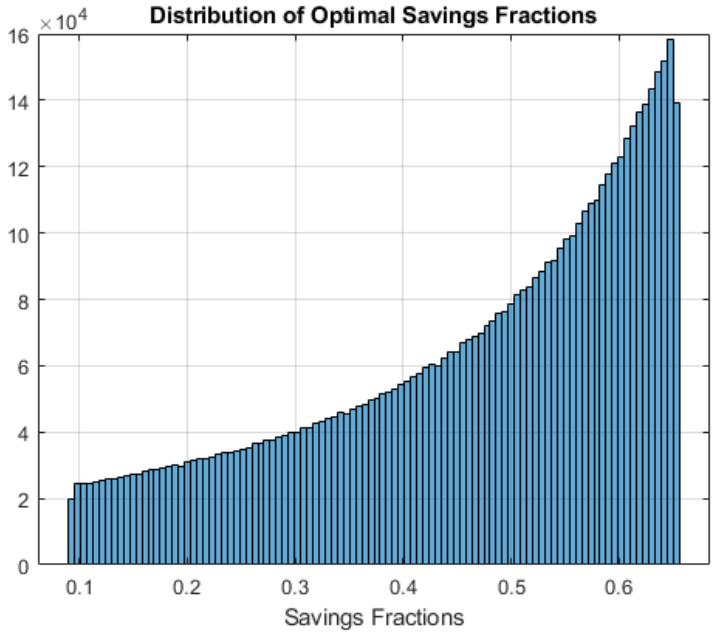
```
Elapsed time is 3.908350 seconds.
```

```
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
mp_container_map('ar_opti_save_frac') = ar_opti_save_frac;
mp_container_map('ar_opti_save_level') = ar_opti_save_level;
mp_container_map('ar_opti_save_frac_notnan') = ar_opti_save_frac(~isnan(ar_opti_save_frac));
ff_container_map_display(mp_container_map);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                              i    idx    ndim     numel       rowN      colN       sum         mean        std
                              _    ___    ____    _______    _______    ____    _________    _______    _______

    ar_opti_save_frac         1     1      2      6.25e+06    6.25e+06    1       2.884e+06    0.46144    0.15306
    ar_opti_save_frac_notnan  2     2      2      6.25e+06    6.25e+06    1       2.884e+06    0.46144    0.15306
    ar_opti_save_level        3     3      2      6.25e+06    6.25e+06    1      2.9482e+06    0.47172    0.66667
```

```
figure();
histogram(ar_opti_save_frac(~isnan(ar_opti_save_frac)),100);
title('Distribution of Optimal Savings Fractions');
xlabel('Savings Fractions');
grid on;
```



Define Two Period Intertemporal FOC Log Utility No Shock

See Household's Utility Maximization Problem and Two-Period Borrowing and Savings Problem given Endowments.

```
function [ar_deri_zero, ar_saveborr_level] = ffi_intertemporal_max(ar_saveborr_frac, z1, z2, r,
    ar_saveborr_level = ar_saveborr_frac.*(z1+z2./(1+r)) - z2./(1+r);
    ar_deri_zero = 1./(ar_saveborr_level-z1) + (beta.*(r+1))./(z2 + ar_saveborr_level.*(r+1));
end
```