### FF OPTIM MZOOM SAVEZRONE Derivative Multisection

#### back to Fan's Intro Math for Econ, Matlab Examples, or Dynamic Asset Repositories

This is the example vignette for function: **ff\_optim\_mzoom\_savezrone** from the **MEconTools Package.** This functions solves for optimal savings/borrowing level given an anonymous function that provides the utility (not derivative) of a intertemporal savings problem. This is a vectorized function solves for multiple state-space elements at the same time. The function allows for controls of iteration counts, the number of evaluations per iteration, and how much to "zoom-in" for each iteration around the last iteration's maximum/optimal choice.

Note that if first order conditions are available this method should not be used, but <code>ff\_optim\_mlsec\_savezrone</code> should be used. <code>ff\_optim\_mlsec\_savezrone</code> relies on bisection. In the first example below more <code>it\_mzoom\_int\_pnts</code> values are needed to achieve the same precision than under <code>ff\_optim\_mlsec\_savezrone</code>. However, increasing <code>it\_mzoom\_int\_pnts</code> might not expensive given vectorization, should increase time cost linearly in generally. MZOOM is much more robust than bisection based methods. And by increasing the number of points evaluated per iteration, in limited number of iterations, the approximately exact optimal savings choice can be found.

The vectorized zooming savings problem rely on this function to solve for optimal savings choices:

States Grid + Approximate Continuous Exact Savings (zoom) as Share of Cash-on-Hand <u>Vectorized</u>:
 ff\_vfi\_az\_zoom\_vec, precision and speed

#### Test FF\_OPTIM\_MZOOM\_SAVEZRONE One Individual

Bisection for savings choice at one state:

```
% Generate the state-space and function
[fl_z1, fl_z2, fl_r, fl_beta] = deal(0.4730, 0.6252, 0.0839, 0.7365);
% ffi_intertemporal_max is a function in ff_optim_mlsec_savezrone for testing
fc_util = @(x) ffi_intertemporal_util(x, fl_z1, fl_z2, fl_r, fl_beta);
% Call Function
bl_verbose = false;
bl_timer = true;
% optimally borrowing given the parameters here
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 15;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 10;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0.25;
[fl_opti_save_frac, fl_opti_save_level] = ...
ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo)
```

```
Elapsed time is 0.011586 seconds.
fl_opti_save_frac = 0.4241
fl_opti_save_level = -0.1316
```

## Test FF\_OPTIM\_MZOOM\_SAVEZRONE 4 Individuals 3 Iterations 50 Points Per Iteration

5 grid points per iteration, and 5 iterations.

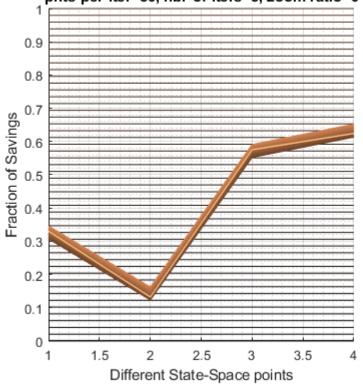
```
% Generate the state-space and function
rng(123);
it_draws = 4; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar beta = [rand(round(it draws/2),1)*0.9+0.1; rand(round(it draws/2),1)*0.9+1];
fc_util = \omega(x) ffi_intertemporal_util(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl verbose = true;
bl_timer = true;
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 50;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 3;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0;
[fl_opti_save_frac, fl_opti_save_level] = ...
    ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo);
```

iter	cl_row_names_a	Var1	Var2	Var3	Var4
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1	"point=2"	0.020418	0.020418	0.020418	0.020418
1	"point=3"	0.040826	0.040826	0.040826	0.040826
1	"point=4"	0.061233	0.061233	0.061233	0.061233
1	"point=5"	0.081641	0.081641	0.081641	0.081641
1	"point=6"	0.10205	0.10205	0.10205	0.10205
1	"point=7"	0.12246	0.12246	0.12246	0.12246
1	"point=8"	0.14286	0.14286	0.14286	0.14286
1	"point=9"	0.16327	0.16327	0.16327	0.16327
1	"point=10"	0.18368	0.18368	0.18368	0.18368
1	"point=11"	0.20409	0.20409	0.20409	0.20409
1	"point=12"	0.2245	0.2245	0.2245	0.2245
1	"point=13"	0.2449	0.2449	0.2449	0.2449
1	"point=14"	0.26531	0.26531	0.26531	0.26531
1	"point=15"	0.28572	0.28572	0.28572	0.28572
1	"point=16"	0.30613	0.30613	0.30613	0.30613
1	"point=17"	0.32653	0.32653	0.32653	0.32653
1	"point=18"	0.34694	0.34694	0.34694	0.34694
1	"point=19"	0.36735	0.36735	0.36735	0.36735
1	"point=20"	0.38776	0.38776	0.38776	0.38776
1	"point=21"	0.40817	0.40817	0.40817	0.40817
1	"point=22"	0.42857	0.42857	0.42857	0.42857
1	"point=23"	0.44898	0.44898	0.44898	0.44898
1	"point=24"	0.46939	0.46939	0.46939	0.46939
1	"point=25"	0.4898	0.4898	0.4898	0.4898
1	"point=26"	0.5102	0.5102	0.5102	0.5102
1	"point=27"	0.53061	0.53061	0.53061	0.53061
1	"point=28"	0.55102	0.55102	0.55102	0.55102
1	"point=29"	0.57143	0.57143	0.57143	0.57143
1	"point=30"	0.59183	0.59183	0.59183	0.59183
1	"point=31"	0.61224	0.61224	0.61224	0.61224
1	"point=32"	0.63265	0.63265	0.63265	0.63265
1	"point=33"	0.65306	0.65306	0.65306	0.65306
1	"point=34"	0.67347	0.67347	0.67347	0.67347
1	"point=35"	0.69387	0.69387	0.69387	0.69387
1	"point=36"	0.71428	0.71428	0.71428	0.71428
1	"point=37"	0.73469	0.73469	0.73469	0.73469
1	"point=38"	0.7551	0.7551	0.7551	0.7551
1	"point=39"	0.7755	0.7755	0.7755	0.7755
1	"point=40"	0.79591	0.79591	0.79591	0.79591

1	"point=41"	0.81632	0.81632	0.81632	0.81632
1	"point=42"	0.83673	0.83673	0.83673	0.83673
1	"point=43"	0.85714	0.85714	0.85714	0.85714
	•				
1	"point=44"	0.87754	0.87754	0.87754	0.87754
1	"point=45"	0.89795	0.89795	0.89795	0.89795
1	"point=46"	0.91836	0.91836	0.91836	0.91836
1	"point=47"	0.93877	0.93877	0.93877	0.93877
1	"point=48"	0.95917	0.95917	0.95917	0.95917
1	"point=49"	0.97958	0.97958	0.97958	0.97958
1	"point=50"	0.99999	0.99999	0.99999	0.99999
2	"point=1"	0.30693	0.12326	0.55182	0.61304
2	"point=2"	0.30773	0.12406	0.55262	0.61384
2	"point=3"	0.30853	0.12486	0.55342	0.61464
	•				
2	"point=4"	0.30933	0.12566	0.55422	0.61544
2	"point=5"	0.31013	0.12646	0.55502	0.61624
2	"point=6"	0.31093	0.12726	0.55582	0.61704
	•				
2	"point=7"	0.31173	0.12806	0.55662	0.61784
2	"point=8"	0.31253	0.12886	0.55742	0.61865
2	"point=9"	0.31333	0.12966	0.55822	0.61945
2	•			0.55902	
	"point=10"	0.31413	0.13046		0.62025
2	"point=11"	0.31493	0.13126	0.55982	0.62105
2	"point=12"	0.31573	0.13206	0.56062	0.62185
2	"point=13"	0.31653	0.13286	0.56142	0.62265
	•				
2	"point=14"	0.31733	0.13366	0.56222	0.62345
2	"point=15"	0.31813	0.13446	0.56302	0.62425
2	"point=16"	0.31893	0.13526	0.56382	0.62505
2	"point=17"	0.31973	0.13606	0.56462	0.62585
2	"point=18"	0.32053	0.13686	0.56542	0.62665
2	"point=19"	0.32133	0.13766	0.56623	0.62745
2	"point=20"	0.32213	0.13846	0.56703	0.62825
2	"point=21"	0.32293	0.13926	0.56783	0.62905
2	"point=22"	0.32373	0.14006	0.56863	0.62985
2	"point=23"	0.32453	0.14086	0.56943	0.63065
	•				
2	"point=24"	0.32533	0.14166	0.57023	0.63145
2	"point=25"	0.32613	0.14246	0.57103	0.63225
2	"point=26"	0.32693	0.14326	0.57183	0.63305
2	"point=27"	0.32773	0.14406	0.57263	0.63385
2	"point=28"	0.32853	0.14487	0.57343	0.63465
2	"point=29"	0.32934	0.14567	0.57423	0.63545
2	"point=30"	0.33014	0.14647	0.57503	0.63625
	-				
2	"point=31"	0.33094	0.14727	0.57583	0.63705
2	"point=32"	0.33174	0.14807	0.57663	0.63785
2	"point=33"	0.33254	0.14887	0.57743	0.63865
2	"point=34"	0.33334	0.14967	0.57823	0.63945
2	"point=35"	0.33414	0.15047	0.57903	0.64025
2	"point=36"	0.33494	0.15127	0.57983	0.64105
2	"point=37"	0.33574	0.15207	0.58063	0.64185
2	"point=38"	0.33654	0.15287	0.58143	0.64265
2	"point=39"	0.33734	0.15367	0.58223	0.64345
2	"point=40"	0.33814	0.15447	0.58303	0.64425
2	"point=41"	0.33894	0.15527	0.58383	0.64506
2	"point=42"	0.33974	0.15607	0.58463	0.64586
2	"point=43"	0.34054	0.15687	0.58543	0.64666
2	"point=44"				0.64746
	•	0.34134	0.15767	0.58623	
2	"point=45"	0.34214	0.15847	0.58703	0.64826
2	"point=46"	0.34294	0.15927	0.58783	0.64906
2	"point=47"	0.34374	0.16007	0.58863	0.64986
2	"point=48"	0.34454	0.16087	0.58943	0.65066
2	"point=49"	0.34534	0.16167	0.59023	0.65146
2	"point=50"	0.34614	0.16247	0.59103	0.65226
3					
	"point=1"	0.32937	0.13129	0.57426	0.62348
3	"point=2"	0.3294	0.13132	0.57429	0.62351
3	"point=3"	0.32943	0.13135	0.57432	0.62354
3	"point=4"	0.32946	0.13139	0.57435	0.62357
	•				
3	"point=5"	0.32949	0.13142	0.57439	0.6236

3	"point=6"	0.32952	0.13145	0.57442	0.62364
3	"point=7"	0.32955	0.13148	0.57445	0.62367
3	"point=8"	0.32959	0.13151	0.57448	0.6237
3	"point=9"	0.32962	0.13154	0.57451	0.62373
3	"point=10"	0.32965	0.13157	0.57454	0.62376
3	"point=11"	0.32968	0.13161	0.57457	0.62379
3	"point=12"	0.32971	0.13164	0.5746	0.62382
3	"point=13"	0.32974	0.13167	0.57464	0.62385
3	"point=14"	0.32977	0.1317	0.57467	0.62389
3	"point=15"	0.32981	0.13173	0.5747	0.62392
3	"point=16"	0.32984	0.13176	0.57473	0.62395
3	"point=17"	0.32987	0.13179	0.57476	0.62398
3	"point=18"	0.3299	0.13182	0.57479	0.62401
3	"point=19"	0.32993	0.13186	0.57482	0.62404
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3	"point=22"	0.33003	0.13195	0.57492	0.62414
3	"point=23"	0.33006	0.13198	0.57495	0.62417
3	"point=24"	0.33009	0.13201	0.57498	0.6242
3	"point=25"	0.33012	0.13204	0.57501	0.62423
3	"point=26"	0.33015	0.13208	0.57504	0.62426
3	"point=27"	0.33018	0.13211	0.57508	0.62429
3	"point=28"	0.33021	0.13214	0.57511	0.62433
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3	"point=30"	0.33028	0.1322	0.57517	0.62439
3	"point=31"	0.33031	0.13223	0.5752	0.62442
3	"point=32"	0.33034	0.13226	0.57523	0.62445
3	"point=33"	0.33037	0.1323	0.57526	0.62448
3	"point=34"	0.3304	0.13233	0.5753	0.62451
3	"point=35"	0.33043	0.13236	0.57533	0.62455
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3	"point=37"	0.3305	0.13242	0.57539	0.62461
3	"point=38"	0.33053	0.13245	0.57542	0.62464
3	"point=39"	0.33056	0.13248	0.57545	0.62467
3	"point=40"	0.33059	0.13252	0.57548	0.6247
3	"point=41"	0.33062	0.13255	0.57551	0.62473
3	"point=42"	0.33065	0.13258	0.57555	0.62477
3	"point=43"	0.33068	0.13261	0.57558	0.6248
3	"point=44"	0.33072	0.13264	0.57561	0.62483
3	"point=45"	0.33075	0.13267	0.57564	0.62486
3	"point=46"	0.33078	0.1327	0.57567	0.62489
3	"point=47"	0.33081	0.13273	0.5757	0.62492
3	"point=48"	0.33084	0.13277	0.57573	0.62495
3	"point=49"	0.33087	0.1328	0.57577	0.62498
3	"point=50"	0.3309	0.13283	0.5758	0.62502
	•				

# Vectorized Exact Zooming Optimization, Savings Fractions pnts-per-iter=50, nbr-of-iters=3, zoom-ratio=0



	150
solve-point=	
solve-point=	139
solve-point=	129
solve-point=	118
solve-point=	107
solve-point=	97
solve-point=	86
solve-point=	76
solve-point=	65
solve-point=	54
solve-point=	44
solve-point=	33
solve-point=	22
solve-point=	12
solve-point=	1

Var4

iter	cl_row_names_a	Var1	Var2	Var3							
1	"point=1"	-3.6912	-1.9565	-12.83							
1	"point=2"	0.058694	-0.80561	-2.4984							
1	"point=3"	0.38043	-0.72015	-1.5784							
1	"point=4"	0.55947	-0.67935	-1.0493							

1

"point=32"

1	"point=1"	-3.6912	-1.9565	-12.83	-14.789
1	"point=2"	0.058694	-0.80561	-2.4984	-2.1254
1	"point=3"	0.38043	-0.72015	-1.5784	-0.99337
1	"point=4"	0.55947	-0.67935	-1.0493	-0.34024
1	"point=5"	0.67979	-0.65711	-0.68055	0.11647
1	"point=6"	0.7677	-0.64529	-0.39997	0.46531
1	"point=7"	0.8349	-0.64026	-0.17534	0.74571
1	"point=8"	0.88763	-0.6401	0.010483	0.9787
1	"point=9"	0.92959	-0.64367	0.16774	1.1768
1	"point=10"	0.96316	-0.65026	0.30302	1.3481
1	"point=11"	0.98996	-0.65938	0.4208	1.4981
1	"point=12"	1.0111	-0.67071	0.52427	1.6308
1	"point=13"	1.0275	-0.684	0.61578	1.7489
1	"point=14"	1.0397	-0.6991	0.69709	1.8547
1	"point=15"	1.0482	-0.71588	0.76958	1.9499
1	"point=16"	1.0533	-0.73426	0.83429	2.0357
1	"point=17"	1.0554	-0.75419	0.8921	2.1132
1	"point=18"	1.0546	-0.77564	0.94367	2.1833
1	"point=19"	1.0512	-0.79861	0.98955	2.2467
1	"point=20"	1.0451	-0.82309	1.0302	2.3039
1	"point=21"	1.0366	-0.8491	1.066	2.3554
1	"point=22"	1.0256	-0.87669	1.0971	2.4015
1	"point=23"	1.0123	-0.90591	1.124	2.4425
1	"point=24"	0.99654	-0.93682	1.1466	2.4788
1	"point=25"	0.97838	-0.9695	1.1652	2.5104
1	"point=26"	0.95775	-1.004	1.1798	2.5375
1	"point=27"	0.93459	-1.0406	1.1905	2.5602
1	"point=28"	0.90881	-1.0792	1.1973	2.5785
1	"point=29"	0.88029	-1.1202	1.2002	2.5925
1	"point=30"	0.84886	-1.1635	1.1991	2.6022
1	"point=31"	0.81434	-1.2096	1.1938	2.6073

0.77649

1.1843

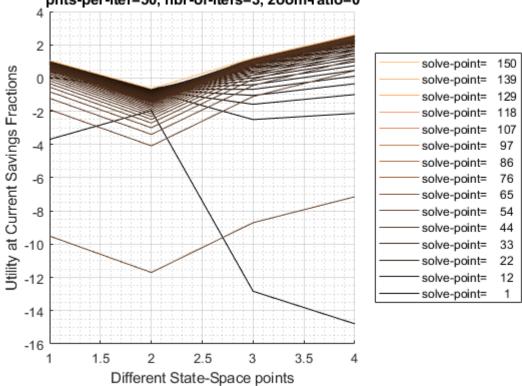
2.6078

-1.2587

1	"point=33"	0.73504	-1.3109	1.1703	2.6035
1	"point=34"	0.68964	-1.3668	1.1514	2.594
1	"point=35"	0.63987	-1.4268	1.1274	2.5792
1	"point=36"	0.58522	-1.4913	1.0978	2.5584
1	"point=37"	0.52505	-1.5611	1.062	2.5312
1	"point=38"	0.45857	-1.6369	1.0192	2.4968
1	"point=39"	0.38475	-1.7198	0.96837	2.4541
1	"point=40"	0.3023	-1.8111	0.90834	2.4021
1	"point=41"	0.20947	-1.9126	0.83737	2.3388
1	"point=42"	0.10391	-2.0266	0.75313	2.2622
			-2.1564		
1	"point=43"	-0.017693		0.65234	2.1687
1	"point=44"	-0.16019	-2.3069	0.53016	2.0538
1	"point=45"	-0.33112	-2.4857	0.37908	1.9097
1	"point=46"	-0.54312	-2.7054	0.18649	1.724
1	"point=47"	-0.81989	-2.9896	-0.071303	1.4729
1	"point=48"	-1.2146	-3.3917	-0.44748	1.1033
1	"point=49"	-1.8971	-4.0814	-1.1118	0.44547
1	"point=50"	-9.5085	-11.7		-7.1418
				-8.7054	
2	"point=1"	1.0535	-0.64017	1.1975	2.6074
2	"point=2"	1.0536	-0.64009	1.1977	2.6075
2	"point=3"	1.0537	-0.64001	1.1979	2.6076
2	"point=4"	1.0539	-0.63995	1.198	2.6077
2	"point=5"	1.054	-0.63989	1.1982	2.6077
2	"point=6"	1.0541	-0.63983	1.1984	2.6078
	•				
2	"point=7"	1.0542	-0.63979	1.1985	2.6079
2	"point=8"	1.0543	-0.63975	1.1986	2.6079
2	"point=9"	1.0544	-0.63971	1.1988	2.608
2	"point=10"	1.0545	-0.63969	1.1989	2.608
2	"point=11"	1.0546	-0.63967	1.199	2.6081
2	"point=12"	1.0547	-0.63966	1.1992	2.6081
2	"point=13"	1.0548	-0.63965	1.1993	2.6081
2	"point=14"	1.0548	-0.63965	1.1994	2.6081
2	"point=15"	1.0549	-0.63966	1.1995	2.6081
2	"point=16"	1.055	-0.63967	1.1996	2.6081
2	"point=17"	1.0551	-0.63969	1.1997	2.6081
2	"point=18"	1.0551	-0.63971	1.1998	2.6081
2	"point=19"	1.0552	-0.63975	1.1998	2.6081
2	"point=20"	1.0552	-0.63978	1.1999	2.6081
2	"point=21"	1.0553	-0.63983	1.2	2.608
2	"point=22"	1.0553	-0.63988	1.2	2.608
2	"point=23"	1.0553	-0.63993	1.2001	2.6079
2	"point=24"	1.0554	-0.63999	1.2001	2.6079
2	"point=25"	1.0554	-0.64006	1.2002	2.6078
2	"point=26"	1.0554	-0.64013	1.2002	2.6077
2	"point=27"	1.0555	-0.64021	1.2002	2.6077
2	"point=28"				
	•	1.0555	-0.64029	1.2003	2.6076
2	"point=29"	1.0555	-0.64038	1.2003	2.6075
2	"point=30"	1.0555	-0.64048	1.2003	2.6074
2	"point=31"	1.0555	-0.64058	1.2003	2.6073
2	"point=32"	1.0555	-0.64069	1.2003	2.6071
2	"point=33"	1.0555	-0.6408	1.2003	2.607
2	"point=34"	1.0555	-0.64091	1.2003	2.6069
2	"point=35"	1.0555	-0.64104	1.2002	2.6067
2	"point=36"	1.0554	-0.64116	1.2002	2.6066
2	"point=37"	1.0554	-0.64129	1.2002	2.6064
2	"point=38"	1.0554	-0.64143	1.2001	2.6063
2	"point=39"	1.0554	-0.64157	1.2001	2.6061
2	"point=40"	1.0553	-0.64172	1.2001	2.6059
2	"point=41"	1.0553	-0.64188	1.2	2.6057
2	"point=42"	1.0552	-0.64203	1.1999	2.6056
2	"point=43"	1.0552	-0.6422	1.1999	2.6053
2	"point=44"	1.0551	-0.64236	1.1998	2.6051
2	"point=45"	1.0551	-0.64254	1.1997	2.6049
2	"point=46"	1.055	-0.64271	1.1996	2.6047
2	"point=47"	1.0549	-0.64289	1.1995	2.6045
_	F-0	,	2.0.205		

2	"point=48"	1.0549	-0.64308	1.1994	2.6042
2	"point=49"	1.0548	-0.64327	1.1993	2.604
2	"point=50"	1.0547	-0.64347	1.1992	2.6037
3	"point=1"	1.0555	-0.63967	1.2003	2.6081
3	"point=2"	1.0555	-0.63967	1.2003	2.6081
3	"point=3"	1.0555	-0.63967	1.2003	2.6081
3	"point=4"	1.0555	-0.63967	1.2003	2.6081
3	"point=5"	1.0555	-0.63967	1.2003	2.6081
3	"point=6"	1.0555	-0.63967	1.2003	2.6081
3	"point=7"	1.0555	-0.63967	1.2003	2.6081
3	"point=8"	1.0555	-0.63966	1.2003	2.6081
3	"point=9"	1.0555	-0.63966	1.2003	2.6081
3	"point=10"	1.0555	-0.63966	1.2003	2.6081
3	"point=11"	1.0555	-0.63966	1.2003	2.6081
3	"point=12"	1.0555	-0.63966	1.2003	2.6081
3	"point=13"	1.0555	-0.63966	1.2003	2.6081
3	"point=14"	1.0555	-0.63966	1.2003	2.6081
3	"point=15"	1.0555	-0.63966	1.2003	2.6081
3	"point=16"	1.0555	-0.63966	1.2003	2.6081
3	"point=17"	1.0555	-0.63966	1.2003	2.6081
3	"point=18"	1.0555	-0.63966	1.2003	2.6081
3	"point=19"	1.0555	-0.63966	1.2003	2.6081
3	"point=20"	1.0555	-0.63966	1.2003	2.6081
3	"point=21"	1.0555	-0.63966	1.2003	2.6081
3	"point=22"	1.0555	-0.63966	1.2003	2.6081
3	"point=23"	1.0555	-0.63966	1.2003	2.6081
3	"point=24"	1.0555	-0.63966	1.2003	2.6081
3	"point=25"	1.0555	-0.63966	1.2003	2.6081
3	"point=26"	1.0555	-0.63966	1.2003	2.6081
3	"point=27"	1.0555	-0.63966	1.2003	2.6081
3	"point=28"	1.0555	-0.63966	1.2003	2.6081
3	"point=29"	1.0555	-0.63966	1.2003	2.6081
3	"point=30"	1.0555	-0.63966	1.2003	2.6081
3	"point=31"	1.0555	-0.63965	1.2003	2.6081
3	"point=32"	1.0555	-0.63965	1.2003	2.6081
3	"point=33"	1.0555	-0.63965	1.2003	2.6081
3	"point=34"	1.0555	-0.63965	1.2003	2.6081
3	"point=35"	1.0555	-0.63965	1.2003	2.6081
3	"point=36"	1.0555	-0.63965	1.2003	2.6081
3	"point=37"	1.0555	-0.63965	1.2003	2.6081
3	"point=38"	1.0555	-0.63965	1.2003	2.6081
3	"point=39"	1.0555	-0.63965	1.2003	2.6081
3	"point=40"	1.0555	-0.63965	1.2003	2.6081
3	"point=41"	1.0555	-0.63965	1.2003	2.6081
3	"point=42"	1.0555	-0.63965	1.2003	2.6081
3	"point=43"	1.0555	-0.63965	1.2003	2.6081
3	"point=44"	1.0555	-0.63965	1.2003	2.6081
3	"point=45"	1.0555	-0.63965	1.2003	2.6081
3	"point=46"	1.0555	-0.63965	1.2003	2.6081
3	"point=47"	1.0555	-0.63965	1.2003	2.6081
3	"point=48"	1.0555	-0.63965	1.2003	2.6081
3	"point=49"	1.0555	-0.63965	1.2003	2.6081
3	"point=50"	1.0555	-0.63965	1.2003	2.6081
J	horuc-20	1.0000	-0.03503	1.2003	2.0001

#### Vectorized Exact Zooming Optimization, U(save) pnts-per-iter=50, nbr-of-iters=3, zoom-ratio=0



Elapsed time is 1.304303 seconds.

CONTAINER NAME: mp\_container\_map ND Array (Matrix etc)

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

	i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari
	_									
ar_opti_foc_obj	1	1	2	4	1	4	4.2243	1.0561	1.3298	1.2592
ar_opti_save_frac	2	2	2	4	4	1	1.664	0.416	0.2284	0.54904
ar_opti_save_level	3	3	2	4	1	4	0.80247	0.20062	0.37807	1.8845

xxx TABLE:ar\_opti\_foc\_obj xxxxxxxxxxxxxxxxx

**c1** c2 с3 с4 -0.63965 2.6081 1.0555 1.2003

xxx TABLE:ar\_opti\_save\_frac xxxxxxxxxxxxxxxxxxxx

r1 0.33086 r2 0.13278

r1

r3 0.57575

r4 0.62461

xxx TABLE:ar\_opti\_save\_level xxxxxxxxxxxxxxxxxx

**c1** c2 **c**3 c4 r1 0.37401 -0.070015 -0.15125 0.64972

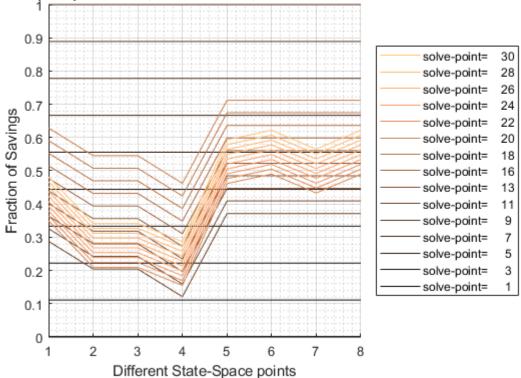
# Test FF\_OPTIM\_MZOOM\_SAVEZRONE 8 Individuals 3 Iterations 10 Points Per Iteration, 0.25 zoom in ratio

10 grid points per iteration, and 3 iterations.

```
% Generate the state-space and function
rng(123);
it_draws = 8; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
fc_util = @(x) ffi_intertemporal_util(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl verbose = true;
bl timer = true;
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 10;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 3;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0.25;
[fl_opti_save_frac, fl_opti_save_level] = ...
    ff_optim_mzoom_savezrone(fc_util, bl_verbose, bl_timer, mp_mzoom_ctrlinfo);
```

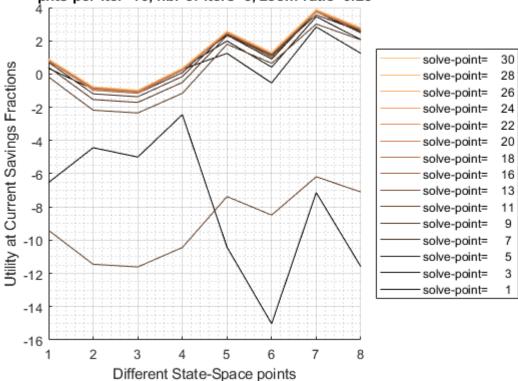
iter	cl_row_names_a	Var1	Var2	Var3	Var4	Var5	Var6	Var7	Var8
1	"point=1"	1e-05							
1	"point=2"	0.11112	0.11112	0.11112	0.11112	0.11112	0.11112	0.11112	0.11112
1	"point=3"	0.22223	0.22223	0.22223	0.22223	0.22223	0.22223	0.22223	0.22223
1	"point=4"	0.33334	0.33334	0.33334	0.33334	0.33334	0.33334	0.33334	0.33334
1	"point=5"	0.44445	0.44445	0.44445	0.44445	0.44445	0.44445	0.44445	0.44445
1	"point=6"	0.55555	0.55555	0.55555	0.55555	0.55555	0.55555	0.55555	0.55555
1	"point=7"	0.66666	0.66666	0.66666	0.66666	0.66666	0.66666	0.66666	0.66666
1	"point=8"	0.77777	0.77777	0.77777	0.77777	0.77777	0.77777	0.77777	0.77777
1	"point=9"	0.88888	0.88888	0.88888	0.88888	0.88888	0.88888	0.88888	0.88888
1	"point=10"	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
2	"point=1"	0.28788	0.20455	0.20455	0.12122	0.37121	0.37121	0.37121	0.37121
2	"point=2"	0.32576	0.24243	0.24243	0.1591	0.40909	0.40909	0.40909	0.40909
2	"point=3"	0.36364	0.28031	0.28031	0.19698	0.44697	0.44697	0.44697	0.44697
2	"point=4"	0.40152	0.31819	0.31819	0.23485	0.48485	0.48485	0.48485	0.48485
2	"point=5"	0.4394	0.35606	0.35606	0.27273	0.52273	0.52273	0.52273	0.52273
2	"point=6"	0.47727	0.39394	0.39394	0.31061	0.5606	0.5606	0.5606	0.5606
2	"point=7"	0.51515	0.43182	0.43182	0.34849	0.59848	0.59848	0.59848	0.59848
2	"point=8"	0.55303	0.4697	0.4697	0.38637	0.63636	0.63636	0.63636	0.63636
2	"point=9"	0.59091	0.50758	0.50758	0.42424	0.67424	0.67424	0.67424	0.67424
2	"point=10"	0.62879	0.54545	0.54545	0.46212	0.71212	0.71212	0.71212	0.71212
3	"point=1"	0.34987	0.20972	0.20972	0.15479	0.46161	0.49001	0.4332	0.49001
3	"point=2"	0.3645	0.22435	0.22435	0.16943	0.47624	0.50465	0.44783	0.50465
3	"point=3"	0.37913	0.23899	0.23899	0.18406	0.49087	0.51928	0.46247	0.51928
3	"point=4"	0.39377	0.25362	0.25362	0.1987	0.50551	0.53392	0.4771	0.53392
3	"point=5"	0.4084	0.26826	0.26826	0.21333	0.52014	0.54855	0.49174	0.54855
3	"point=6"	0.42304	0.28289	0.28289	0.22797	0.53478	0.56319	0.50637	0.56319
3	"point=7"	0.43767	0.29752	0.29752	0.2426	0.54941	0.57782	0.52101	0.57782
3	"point=8"	0.45231	0.31216	0.31216	0.25724	0.56405	0.59246	0.53564	0.59246
3	"point=9"	0.46694	0.32679	0.32679	0.27187	0.57868	0.60709	0.55027	0.60709
3	"point=10"	0.48158	0.34143	0.34143	0.28651	0.59332	0.62173	0.56491	0.62173

# Vectorized Exact Zooming Optimization, Savings Fractions pnts-per-iter=10, nbr-of-iters=3, zoom-ratio=0.25



	Dillerent S	tate-Space	pontis						
iter	cl_row_names_a	Var1	Var2	Var3	Var4	Var5	Var6	Var7	Var
1	"point=1"	-6.5286	-4.4312	-4.9951	-2.4407	-10.415	-15.025	-7.1352	-11.
1	"point=2"	0.34227	-0.90966	-1.148	0.28691	1.2451	-0.53687	2.835	1.
1	"point=3"	0.7287	-0.77242	-0.98657	0.36508	1.9879	0.4163	3.452	2.0
1	"point=4"	0.87872	-0.76818	-0.96816	0.33477	2.3463	0.89785	3.737	2.4
1	"point=5"	0.91222	-0.83811	-1.028	0.24031	2.5277	1.1666	3.8662	2.7
1	"point=6"	0.85648	-0.97408	-1.1562	0.085331	2.5867	1.2933	3.8847	2.7
1	"point=7"	0.70558	-1.1905	-1.3663	-0.14666	2.5296	1.2915	3.7944	2.7
1	"point=8"	0.41577	-1.5358	-1.7061	-0.50502	2.319	1.1277	3.5559	2.5
1	"point=9"	-0.17716	-2.1767	-2.3424	-1.1573	1.7947	0.64395	3.0074	2.6
1	"point=10"	-9.4046	-11.446	-11.608	-10.437	-7.3721	-8.4872	-6.1808	-7.6
2	"point=1"	0.8347	-0.78233	-0.99938	0.30205	2.4239	1.0081	3.795	2.5
2	"point=2"	0.87277	-0.76475	-0.97586	0.34105	2.4846	1.0983	3.8381	2.6
2	"point=3"	0.89748	-0.75933	-0.96536	0.36018	2.5303	1.1709	3.8677	2.7
2	"point=4"	0.91044	-0.76388	-0.96549	0.36559	2.5622	1.2275	3.8849	2.7
2	"point=5"	0.91269	-0.7771	-0.97477	0.36049	2.581	1.269	3.89	2.
2	"point=6"	0.90477	-0.79823	-0.99237	0.34672	2.5867	1.296	3.883	2.7
2	"point=7"	0.88684	-0.8269	-1.0178	0.32535	2.5793	1.3084	3.8637	2.7
2	"point=8"	0.85872	-0.86304	-1.051	0.29697	2.5578	1.3055	3.831	2.7
2	"point=9"	0.81987	-0.90685	-1.0921	0.26182	2.5209	1.2862	3.7837	2.
2	"point=10"	0.76932	-0.95877	-1.1415	0.21989	2.4664	1.2483	3.7192	2.7
3	"point=1"	0.88992	-0.7791	-0.99528	0.33777	2.5443	1.234	3.8584	2.7
3	"point=2"	0.8979	-0.77144	-0.98526	0.3479	2.5562	1.251	3.8683	2.7
3	"point=3"	0.90413	-0.7658	-0.97741	0.35543	2.5661	1.2659	3.8762	2.
3	"point=4"	0.90869	-0.762	-0.97154	0.3607	2.5741	1.2785	3.8824	2.7
3	"point=5"	0.91163	-0.75989	-0.96746	0.36397	2.5801	1.289	3.8867	2.7
3	"point=6"	0.91299	-0.75934	-0.96506	0.36546	2.5842	1.2974	3.8892	2.7
3	"point=7"	0.91281	-0.76025	-0.96421	0.36532	2.5864	1.3035	3.89	2.7
3	"point=8"	0.91112	-0.76255	-0.96482	0.3637	2.5866	1.3074	3.889	2.7
3	"point=9"	0.90792	-0.76615	-0.96683	0.3607	2.5849	1.3091	3.8861	2.7
3	"point=10"	0.90324	-0.77102	-0.97016	0.35641	2.5811	1.3085	3.8815	2.7

# Vectorized Exact Zooming Optimization, U(save) pnts-per-iter=10, nbr-of-iters=3, zoom-ratio=0.25



Elapsed time is 0.997040 seconds.

xxxxxxxxxxxxxxxxxxxxxxxx	i i	idx	ndim	numel	rowN	colN	sum	mean	std	coefvari
	-									
ar_opti_foc_obj	1	1	2	8	1	8	10.125	1.2656	1.731	1.3677
ar_opti_save_frac	2	2	2	8	8	1	3.3843	0.42304	0.15074	0.35632
ar_opti_save_level	3	3	2	8	1	8	4.965	0.62062	0.71498	1.152
xxx TABLE:ar_opti_foc_obj xxxxxxxxxxxxxxxx										
c1	c2		c3	c4	c5	c	6 (	.7 c8		

r1	0.4084
r2	0.26826
r3	0.28289
r4	0.21333
r5	0.54941
r6	0.59246
r7	0.50637
r8	0.56319

xxx TABLE:ar\_opti\_save\_level xxxxxxxxxxxxxxxxxx

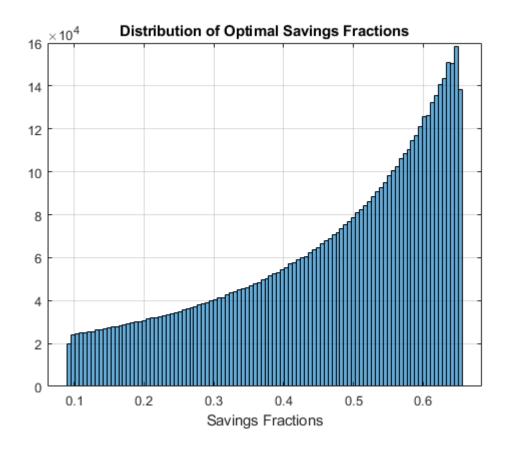
c1 c2 c3 c4 c5 c6 c7 c8

r1 0.53854 -0.050976 0.053729 0.0013119 1.0103 0.45895 2.098 0.85508

### Test FF\_OPTIM\_MZOOM\_SAVEZRONE Speed

Test Speed doing 6.25 million state-spcae points for a savings problem:

```
% Generate the state-space and function
rng(123);
it draws = 6250000; % must be even number
ar_z1 = exp(rand([it_draws,1])*3-1.5);
ar_z2 = exp(rand([it_draws,1])*3-1.5);
ar_r = (rand(it_draws,1)*10.0);
ar_beta = [rand(round(it_draws/2),1)*0.9+0.1; rand(round(it_draws/2),1)*0.9+1];
% ffi intertemporal max is a function in ff optim mlsec savezrone for testing
fc_util = @(x) ffi_intertemporal_util(x, ar_z1, ar_z2, ar_r, ar_beta);
% Call Function
bl verbose = false;
bl_timer = true;
% set parameters
mp_mzoom_ctrlinfo = containers.Map('KeyType','char', 'ValueType','any');
mp_mzoom_ctrlinfo('it_mzoom_jnt_pnts') = 20;
mp_mzoom_ctrlinfo('it_mzoom_max_iter') = 10;
mp_mzoom_ctrlinfo('it_mzoom_zm_ratio') = 0.25;
[ar_opti_save_frac, ar_opti_save_level] = ...
    ff optim mzoom savezrone(fc util, bl verbose, bl timer, mp mzoom ctrlinfo);
Elapsed time is 64.837799 seconds.
mp_container_map = containers.Map('KeyType','char', 'ValueType','any');
mp_container_map('ar_opti_save_frac') = ar_opti_save_frac;
mp_container_map('ar_opti_save_level') = ar_opti_save_level;
mp_container_map('ar_opti_save_frac_notnan') = ar_opti_save_frac(~isnan(ar_opti_save_frac));
ff_container_map_display(mp_container_map);
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
i
                              idx
                                    ndim
                                           numel
                                                      rowN
                                                               colN
                                                                        sum
                                                                                  mean
                                                                                            std
   ar opti save frac
                          1
                               1
                                     2
                                           6.25e+06
                                                     6.25e+06
                                                               1
                                                                     2.8839e+06
                                                                                 0.46142
                                                                                          0.15305
   ar_opti_save_frac_notnan
                          2
                               2
                                     2
                                           6.25e+06
                                                     6.25e+06
                                                               1
                                                                     2.8839e+06
                                                                                 0.46142
                                                                                          0.15305
   ar_opti_save_level
                                           6.25e+06
                                                     6.25e+06
                                                                     2.9481e+06
                                                                                 0.47169
                                                                                          0.66665
figure();
histogram(ar_opti_save_frac(~isnan(ar_opti_save_frac)),100);
title('Distribution of Optimal Savings Fractions');
xlabel('Savings Fractions');
grid on;
```



### Define Two Period Intertemporal Log Utility No Shock Utility Function

See Household's Utility Maximization Problem and Two-Period Borrowing and Savings Problem given Endowments.

```
function [ar_util, ar_saveborr_level] = ...
    ffi_intertemporal_util(ar_saveborr_frac, z1, z2, r, beta)

ar_saveborr_level = ar_saveborr_frac.*(z1+z2./(1+r)) - z2./(1+r);
ar_util = log(z1 - ar_saveborr_level) + beta.*log(ar_saveborr_level.*(1+r) + z2);
end
```